3.1: PROBLEM DEFINITION

Apply the grid method to cases a, b, c and d.

a.)

Situation:

Pressure values need to be converted.

Find:

Calculate the gage pressure (kPa) corresponding to 8 in. H_2O (vacuum).

Solution:

$$p = \frac{8 \text{ in. H20}}{} \Bigg| \frac{\text{Pa}}{0.00402 \text{ in. H20}} \Bigg| \frac{\text{kPa}}{1000 \text{ Pa}}$$

$$p = 1.99 \text{ kPa-vacuum} = \boxed{-1.99 \text{ kPa-gage}}$$

b.)

Situation:

Pressure values need to be converted.

Find:

Calculate the gage pressure (psig) corresponding to 120 kPa-abs.

Properties:

$p_{\text{atm}} = 14.70\text{psi}$.

Solution:

$$p_{\text{abs}} = \left(\frac{120 \text{ kPa}}{1}\right)\left(\frac{14.70 \text{ psi}}{101.3 \text{ kPa}}\right) = 17.4 \text{ psia}$$

$$p_{\text{gage}} = p_{\text{abs}} - p_{\text{atm}} = (17.4 \text{ psia}) - (14.70 \text{ psia}) = 2.71 \text{ psi}$$

$$\boxed{p_{\text{gage}} = 2.71 \text{ psig}}$$

c.)

Situation:

Pressure values need to be converted.

Find:

Calculate the absolute pressure (psia) corresponding to a pressure of 0.5 bar (gage).

Properties:

$p_{atm} = 14.70\text{psi}$.

Solution:

$$p_{\text{gage}} = \left(\frac{0.5 \text{ bar}}{1}\right)\left(\frac{14.70 \text{ psi}}{1.013 \text{ bar}}\right) = 7.25 \text{ psig}$$

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}} = (7.25 \text{ psig}) + (14.70 \text{ psia}) = 21.9 \text{ psia}$$

$$\boxed{p_{\text{abs}} = 21.9 \text{ psia}}$$

d.)

Situation:

Pressure values need to be converted.

Find:

Calculate the pressure (kPa-abs) corresponding to a blood pressure of 120 mm-Hg.

Properties:

Solution:

$$p_{\text{gage}} = \left(\frac{120 \text{ mm-Hg}}{1}\right)\left(\frac{101.3 \text{ kPa}}{760 \text{ mm-Hg}}\right) = 17.00 \text{ kPa-gage}$$

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}} = (101.3 \text{ kPa}) + (17.00 \text{ kPa-gage}) = 118 \text{ kPagage}$$

$$\boxed{p_{\text{abs}} = 118 \text{ kPa gage}}$$

3.2: PROBLEM DEFINITION

Apply the grid method to:

a.)

Situation:

A sphere contains an ideal gas.

Find:

Calculate the density of helium at a gage pressure of 20 in. H_2O.

Properties:

From Table A.2: $R_{\text{helium}} = 2077\,\text{J/kg}\cdot\text{K}$.

Solution:

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}} = 101.3\,\text{kPa} + \left(\frac{20\text{ in. H2O}}{1}\right)\left(\frac{248.8\,\text{Pa}}{1.0\text{ in. H2O}}\right) = 106.3\,\text{kPa}$$

Ideal gas law:

$$\rho = \frac{p}{RT} = \left(\frac{106.3\,\text{kPa}}{1}\right)\left(\frac{\text{kg K}}{2077\,\text{J}}\right)\left(\frac{1}{293.2\,\text{K}}\right)\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)\left(\frac{\text{J}}{\text{N m}}\right)\left(\frac{\text{N}}{\text{Pa m}^2}\right)$$

$$\boxed{\rho = 0.175\ \text{kg/m}^3}$$

b.)

Situation:

A sphere contains an ideal gas.

Find:

Calculate the density of argon at a vacuum pressure of 3 psi.

Properties:

From Table A.2: $R_{\text{methane}} = 518\,\text{J/kg}\cdot\text{K}$.

Solution:

$$p_{\text{abs}} = p_{\text{atm}} - p_{\text{vacuum}} = 101.3\,\text{kPa} - \left(\frac{3\text{ psi}}{1}\right)\left(\frac{101.3\,\text{kPa}}{14.696\text{ psi}}\right) = 80.62\,\text{kPa}$$

Ideal gas law:

$$\rho = \frac{p}{RT} = \left(\frac{80.62\,\text{kPa}}{1}\right)\left(\frac{\text{kg K}}{518\,\text{J}}\right)\left(\frac{1}{293.2\,\text{K}}\right)\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)\left(\frac{\text{J}}{\text{N m}}\right)\left(\frac{\text{N}}{\text{Pa m}^2}\right)$$

$$\boxed{\rho = 0.531\ \text{kg/m}^3}$$

3.3: PROBLEM DEFINITION

Using Section 3.1 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words and equations in ways that enhance the effectiveness of your communication.

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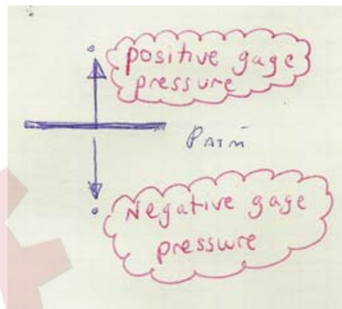
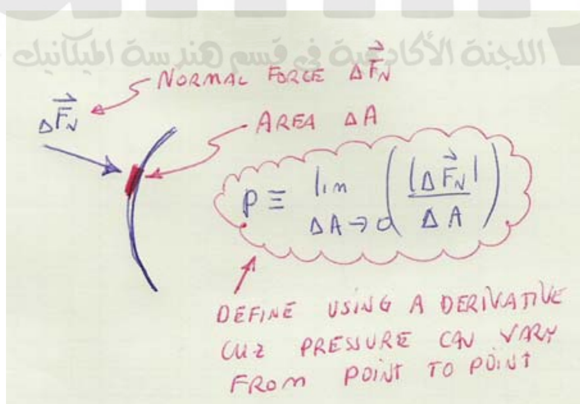
a. What are five important facts that engineers need to know about pressure?

- Pressure is often expressed using "gage pressure," where gage pressure is the difference between local atmospheric pressure and actual pressure.



- Primary dimensions of pressure are M/LT^2.
- Vacuum pressure = negative gage pressure. Negative vacuum pressure = gage pressure.
- Pressure is often expressed as length of a fluid column; e.g. the pressure of air in a duct is 10 inches of water column.
- pressure is defined using a derivative



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b. What are five common instances in which people use gage pressure?

- car tire pressure is expressed as gage pressure.
- blood pressure measured by a doctor is a gage pressure.
- the pressure inside a pressure cooker is expressed as a gage pressure.
- a Bourdon-tube pressure gage gives a pressure reading as a gage pressure.
- the pressure that a scuba diver feels is usually expressed as a gage pressure; e.g. a diver at a depth of 10 m will experience a pressure of 1 atm.

---

c. What are the most common units for pressure?

- Pa, psi, psf
- length of a column of water(in. H20; ft H2O)
- length of a column of mercury (mm Hg; in. Hg)
- bar

---

d. Why is pressure defined using a derivative?
Pressure is defined as a derivative because pressure can vary at every point along a surface.

---

e. How is pressure similar to shear stress? How does pressure differ from shear stress?

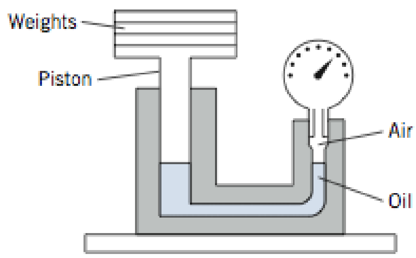
- Similarities
  - Both pressure and shear stress give a ratio of force to area.
  - Both pressure and shear stress apply at a point (they are defined using a derivative.
  - Pressure and shear stress have the same units.
  - Both pressure and shear stress are types of "stress."
- Differences: (the easy way to show differences is to make a table as shown below)

Attribute	Pressure	Shear Stress
direction of associated force	associated with force normal to area	associated with force tangent to an area
presence in a hydrostatic fluid	pressure is non-zero	shear stress is zero
typical magnitude	much larger than shear stress	much smaller than pressure
main physical cause	associated with weight of fluid & motion of fluid (non-viscous effects)	associated with motion of fluid (viscous effects)

3.4: PROBLEM DEFINITION

Situation:

A Crosby gage tester is applied to calibrate a pressure gage.
Indicated pressure on the gage is $p = 200\,\text{kPa}$.
$W = 140\,\text{N}$, $D = 0.03\,\text{m}$.



Find:

Percent error in gage reading.

PLAN

1. Calculate the pressure that the gage should be indicating (true pressure).
2. Compare this true pressure with the actual pressure.

SOLUTION

1. True pressure

$$\begin{aligned} p_{\text{true}} &= \frac{F}{A} \\ &= \frac{140\,\text{N}}{(\pi/4 \times 0.03^2)\ \text{m}^2} \\ &= 198,049\,\text{kPa} \end{aligned}$$

2. Percent error

$$\begin{aligned} \%\ \text{Error} &= \frac{(p_{\text{recorded}} - p_{\text{true}})\,100}{p_{\text{true}}} \\ &= \frac{(200\,\text{kPa} - 198\,\text{kPa})\,100}{198\,\text{kPa}} \\ &= 1.0101\% \end{aligned}$$

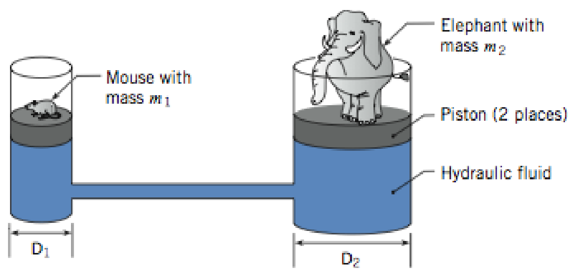
$$\boxed{\%\ \text{Error} = 1.01\%}$$

3.5: PROBLEM DEFINITION

Situation:

A hydraulic machine is used to provide a mechanical advantage.

$m_1 = 0.025\,\text{kg}$, $m_2 = 7500\,\text{kg}$.



Find:

(a) Derive an algebraic equation for the mechanical advantage.

(b) Calculate D_1 and D_2 so the mouse can support the elephant.

Assumptions:

- Neglect the mass of the pistons.
- Neglect the friction between the piston and the cylinder wall.
- The pistons are at the same elevation; thus, the pressure acting on the bottom of each piston is the same.
- A mouse can fit onto a piston of diameter $D_1 = 70$ mm.

PLAN

1. Define "mechanical advantage."
2. Derive an equation for the pressure acting on piston 1.
3. Derive an equation for the pressure acting on piston 2.
4. Derive an equation for mechanical advantage by combining steps 2 and 3.
5. Calculate D_2 by using the result of step 4.

SOLUTION

1. Mechanical advantage.

$$\left\{ \begin{array}{c} \text{Mechanical} \\ \text{advantage} \end{array} \right\} = \frac{\text{Weight "lifted" by the mouse}}{\text{Weight of the mouse}} = \frac{W_2}{W_1} \qquad (1)$$

where W_2 is the weight of the elephant, and W_1 is the weight of the mouse.

2. Equilibrium (piston 1):

$$\begin{aligned} W_1 &= p\left(\frac{\pi D_1^2}{4}\right) \\ p &= W_1\left(\frac{4}{\pi D_1^2}\right) \end{aligned} \tag{2}$$

3. Equilibrium (piston 2):

$$\begin{aligned} W_2 &= p\left(\frac{\pi D_2^2}{4}\right) \\ p &= W_2\left(\frac{4}{\pi D_2^2}\right) \end{aligned} \tag{3}$$

4. Combine Eqs. (2) and (3):

$$p = W_1\left(\frac{4}{\pi D_1^2}\right) = W_2\left(\frac{4}{\pi D_2^2}\right) \tag{5}$$

Solve Eq. (5) for mechanical advantage:

$$\boxed{\frac{W_2}{W_1} = \left(\frac{D_2}{D_1}\right)^2}$$

5. Calculate D_2.

$$\begin{aligned} \frac{W_2}{W_1} &= \left(\frac{D_2}{D_1}\right)^2 \\ \frac{(7500\,\mathrm{kg})\,(9.80\,\mathrm{m/\,s^2})}{(0.025\,\mathrm{kg})\,(9.80\,\mathrm{m/\,s^2})} &= 300000 = \left(\frac{D_2}{0.07\,\mathrm{m}}\right)^2 \\ D_2 &= 38.3\,\mathrm{m} \end{aligned}$$

The ratio of (D_2/D_1) needs to be $\sqrt{300,000}$. If $D_1 = 70\,\mathrm{mm}$, then $D_2 = 38.3\,\mathrm{m}$.

REVIEW

1. Notice. The mechanical advantage varies as the diameter ratio squared.
2. The mouse needs a mechanical advantage of 300,000:1. This results in a piston that is impractical (diameter = 38.3 m = 126 ft !).

3.6: PROBLEM DEFINITION

Situation:
To work the problem, data was recorded from a parked vehicle. Relevant information:

- Left front tire of a parked VW Passat 2003 GLX Wagon (with 4-motion).
- Bridgestone snow tires on the vehicle.
- Inflation pressure = 36 psig. This value was found by using a conventional "stick-type" tire pressure gage.
- Contact Patch: $5.88\,\text{in} \times 7.5\,\text{in}$. The 7.5 inch dimension is across the tread. These data were found by measuring with a ruler.
- Weight on the front axle = 2514 lbf. This data was recorded from a sticker on the driver side door jamb. The owners manual states that this is maximum weight (car + occupants + cargo).

Assumptions:

- The weight on the car axle without a load is 2000 lbf. Thus, the load acting on the left front tire is 1000 lbf.
- The thickness of the tire tread is 1 inch. The thickness of the tire sidewall is 1/2 inch.
- The contact path is flat and rectangular.
- Neglect any tensile force carried by the material of the tire.

Find:
Measure the size of the contact patch.
Calculate the size of the contact patch.
Compare the measurement with the calculation and discuss.

PLAN

To estimate the area of contact, apply equilibrium to the contact patch.

SOLUTION

Equilibrium in the vertical direction applied to a section of the car tire

$$p_i A_i = F_{\text{pavement}}$$

where p_i is the inflation pressure, A_i is the area of the contact patch on the inside of the tire and F_{pavement} is the normal force due to the pavement. Thus,

$$\begin{aligned} A_i &= \frac{F_{\text{pavement}}}{p_i} \\ &= \frac{1000\,\text{lbf}}{36\,\text{lbf/in}^2} \\ &= 27.8\,\text{in}^2 \end{aligned}$$

Comparison. The actual contact patch has an area $A_o = 5.88\,\text{in} \times 7.5\,\text{in} = 44.1\,\text{in}^2$. Using the assumed thickness of rubber, this would correspond to an inside contact area of $A_o = 4.88\,\text{in} \times 5.5\,\text{in} = 26.8\,\text{in}^2$.Thus, the predicted contact area $\left(27.8\,\text{in}^2\right)$ and the measured contact area $\left(26.8\,\text{in}^2\right)$ agree to within about 1 part in 25 or about 4%.

REVIEW

The comparison between predicted and measured contact area is highly dependent on the assumptions made.

Problem 3.7

Apply the grid method to calculations involving the hydrostatic equation:

$$\Delta p = \gamma \Delta z = \rho g \Delta z$$

Note: Unit cancellations are not shown in this solution.

a.)

Situation:

Pressure varies with elevation.

$\Delta z = 10\,\text{ft}$.

Find:

Pressure change (kPa).

Properties:

$\rho = 90\,\text{lb}/\,\text{ft}^3$.

Solution:

Convert density to units of kg/m^3:

$$\rho = \left(\frac{90\ \text{lbm}}{\text{ft}^3}\right)\left(\frac{35.315\,\text{ft}^3}{\text{m}^3}\right)\left(\frac{1.0\,\text{kg}}{2.2046\ \text{lbm}}\right) = 1442\ \frac{\text{kg}}{\text{m}^3}$$

Calculate the pressure change:

$$\Delta p = \rho g \Delta z = \left(\frac{1442\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{10\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.208\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\ \text{m}\cdot\ \text{s}^2}{\text{kg}}\right)$$

$$\boxed{\Delta p = 43.1\,\text{kPa}}$$

b.)

Situation:

Pressure varies with elevation.

$\Delta z = 22\,\text{m},\ S = 0.8$.

Find:

Pressure change (psf).

Properties:

$\gamma = 62.4\,\text{lbf}/\,\text{ft}^3$.

Solution:

$$\Delta p = \gamma \Delta z = S\ \gamma_{H2O}\ \Delta z = \left(\frac{(0.8\cdot 62.4)\ \text{lbf}}{\text{ft}^3}\right)\left(\frac{22\,\text{m}}{1.0}\right)\left(\frac{3.2808\,\text{ft}}{\text{m}}\right)$$

$$\boxed{\Delta p = 3600\ \text{psf}}$$

c.)

Situation:

Pressure varies with elevation.
$\Delta z = 1000\,\text{ft}$.

Find:

Pressure change (in H_2O).

Properties:

air, $\rho = 1.2\,\text{kg}/\,\text{m}^3$.

Solution:

$$\Delta p = \rho g \Delta z = \left(\frac{1.2\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{1000\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.281\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\ \text{m}\cdot\ \text{s}^2}{\text{kg}}\right)\left(\frac{\text{in.-H2O}}{248.4\,\text{Pa}}\right)$$

$$\boxed{\Delta p = 14.8\ \text{in}\ \text{H}_2\text{O}}$$

d.)

Situation:

Pressure varies with elevation.
$\Delta p = 1/6\,\text{atm}$, $S = 13$.

Find:

Elevation change (mm).

Properties:

$\gamma = 9810\,\text{N}/\,\text{m}^3$, $p_{atm} = 101.3\,\text{kPa}$.

Solution:

d. Calculate Δz (mm) corresponding to $S = 13$ and $\Delta p = 1/6\,\text{atm}$.

$$\Delta z = \frac{\Delta p}{\gamma} = \frac{\Delta p}{S\gamma_{H2O}} = \left(\frac{1/6\,\text{atm}}{1.0}\right)\left(\frac{\text{m}^3}{(13\cdot 9810)\ \text{N}}\right)\left(\frac{101.3\times 10^3\,\text{Pa}}{\text{atm}}\right)\left(\frac{1000\,\text{mm}}{1.0\,\text{m}}\right)$$

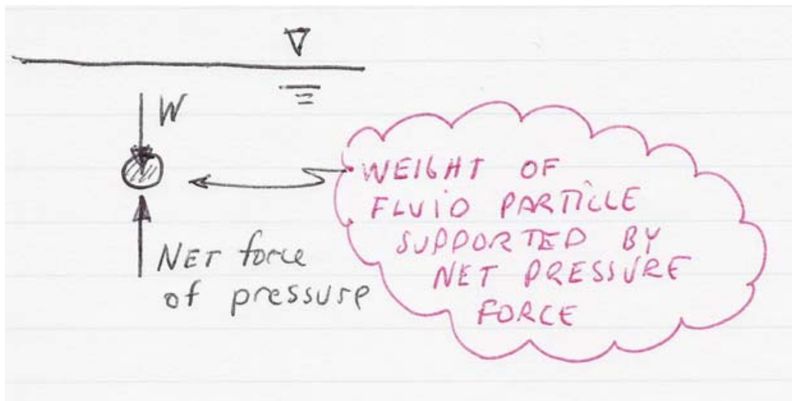
$$\boxed{\Delta z = 132\,\text{mm}}$$

Problem 3.8

Using Section 3.2 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words and equations in ways that enhance the effectiveness of your communication.

a. What does hydrostatic mean? How do engineers identify if a fluid is hydrostatic?

- Each fluid particle within the body is in force equilibrium(z-direction) with the net force due to pressure balancing the weight of the particle. Here, the z-direction is aligned with the gravity vector.



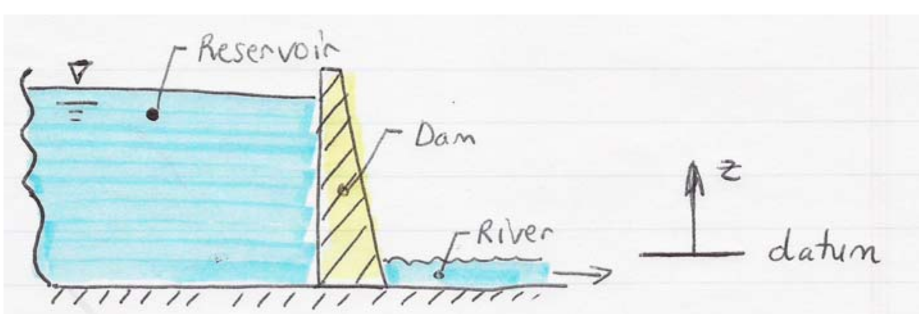
- Engineers establish hydrostatic conditions by analyzing the forces acting in the z-direction.

b. What are common forms of the hydrostatic equation? Are the forms equivalent or are they different?

- There are three common forms; these are given in Table F.2 (front of book).
- These equations are equivalent because you can start with any of the equations and derive the other two.

c. What is a datum? How do engineers establish a datum?

- A datum is a fixed reference point from which elevations are measured.



- Engineers select a datum that makes calculations easy. For example, select a datum on the free surface of a river below a dam so that all elevations are positive.

---

d. What are the main ideas of Eq. (3.5)? That is, what is the meaning of this equation?

$$p_z = p + \gamma z = \text{constant}$$

This equation means that the sum of $(p + \gamma z)$ has the same numerical value at every location within a body of fluid.

---

e. What assumptions need to be satisfied to apply the hydrostatic equation?

$$p_z = p + \gamma z = \text{constant}$$

This equation is valid when

- the density of the fluid is constant at all locations.
- equilibrium is satisfied in the z-direction (net force of pressure balances weight of the fluid particle.

Problem 3.9

Apply the grid method to each situation below. Unit cancellations are not shown in these solutions.

a.)

Situation:

Pressure varies with elevation.
$\Delta z = 8\,\text{ft}$.

Find:

Pressure change (Pa).

Properties:

air, $\rho = 1.2\,\text{kg/ m}^3$.

Solution:

$$\Delta p = \rho g \Delta z$$

$$\begin{aligned}\Delta p &= \rho g \Delta z \\ &= \left(\frac{1.2\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{8\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.281\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\text{m}\cdot\text{s}^2}{\text{kg}}\right)\end{aligned}$$

$$\boxed{\Delta p = 28.7\,\text{Pa}}$$

b.)

Situation:

Pressure increases with depth in the ocean.
Pressure reading is 2 atm gage.

Find:

Water depth (m).

Properties:

Seawater, Table A.4, $S = 1.03$, $\gamma = 10070\,\text{N/ m}^3$.

Solution:

$$\Delta z = \frac{\Delta p}{\gamma} = \left(\frac{2.0\,\text{atm}}{1.0}\right)\left(\frac{\text{m}^3}{10070\,\text{N}}\right)\left(\frac{101.3\times 10^3\,\text{Pa}}{\text{atm}}\right)\left(\frac{\text{N}}{\text{Pa}\,\text{m}^2}\right)$$

$$\boxed{\Delta z = 20.1\,\text{m}}$$

c.)

Situation:

Pressure decreases with elevation in the atmosphere.
$\Delta z = 1200\,\text{ft}$.

Find:

Pressure (mbar).

Assumptions:
Density of air is constant.

Properties:
Air, $\rho = 1.1\,\mathrm{kg/\,m^3}$.

Solution:

$$\Delta p = \rho g \Delta z = \left(\frac{1.1\,\mathrm{kg}}{\mathrm{m^3}}\right)\left(\frac{9.81\,\mathrm{m}}{\mathrm{s^2}}\right)\left(\frac{-1200\,\mathrm{ft}}{1.0}\right)\left(\frac{\mathrm{m}}{3.281\,\mathrm{ft}}\right)\left(\frac{\mathrm{Pa\cdot\ m\cdot\ s^2}}{\mathrm{kg}}\right) = -3947\,\mathrm{Pa}$$

Pressure at summit:

$$p_{\text{summit}} = p_{\text{base}} + \Delta p = 940\,\mathrm{mbar} - \left(\frac{3947\,\mathrm{Pa}}{1.0}\right)\left(\frac{10^{-2}\,\mathrm{mbar}}{\mathrm{Pa}}\right)$$

$$\boxed{p_{\text{summit}} = 901\,\mathrm{mbar\ (absolute)}}$$

d.)

Situation:
Pressure increases with depth in a lake.
$\Delta z = 350\,\mathrm{m}$.

Find:
Pressure (MPa).

Properties:
Water, $\gamma = 9810\,\mathrm{N/\,m^3}$.

Solution:

$$\begin{aligned}\Delta p &= \gamma \Delta z \\ &= \left(\frac{9810\,\mathrm{N}}{\mathrm{m^3}}\right)\left(\frac{350\,\mathrm{m}}{1.0}\right)\left(\frac{\mathrm{Pa\cdot\ m^2}}{\mathrm{N}}\right)\left(\frac{\mathrm{MPa}}{10^6\,\mathrm{Pa}}\right)\end{aligned}$$

$$\boxed{p_{\max} = 3.4\,\mathrm{MPa\ (gage)\ [about\ 34\ atmospheres]}}$$

e.)

Situation:
Pressure increase with water depth in a standpipe.
$\Delta z = 60\,\mathrm{m}$.

Find:
Pressure (kPa).

Properties:
Water, $\gamma = 9810\,\mathrm{N/\,m^3}$.

Solution:

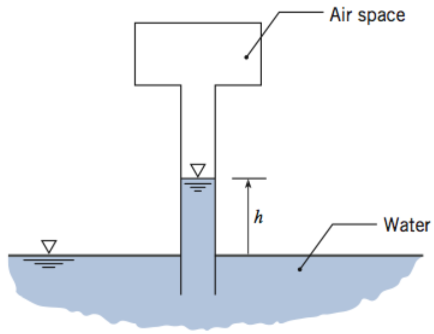
$$
\begin{aligned}
\Delta p &= \gamma \Delta z \\
&= \left(\frac{9810\,\mathrm{N}}{\mathrm{m}^3}\right)\left(\frac{60\,\mathrm{m}}{1.0}\right)\left(\frac{\mathrm{Pa}\cdot\ \mathrm{m}^2}{\mathrm{N}}\right)\left(\frac{\mathrm{kPa}}{10^3\,\mathrm{Pa}}\right)
\end{aligned}
$$

$$
\boxed{p_{\mathrm{max}} = 589\,\mathrm{kPa\ (gage)\ [nearly\ 6\ atmospheres]}}
$$

3.10: PROBLEM DEFINITION

Situation:

Air above a long tube is pressurized.
Initial state: $p_{\text{air1}} = 50\,\text{kPa-vacuum}$
Final state: $p_{\text{air2}} = 25\,\text{kPa-vacuum}$.



Find:

Will h increase or decrease?
The change in water column height (Δh) in meters.

Assumptions:

Atmospheric pressure is 100 kPa.

Properties:

Water (20 °C), Table A.5, $\gamma = 9790\,\text{N/m}^3$.

PLAN

Since pressure increases, the water column height will decrease. Use absolute pressure in the hydrostatic equation.

1. Find h (initial state) by applying the hydrostatic equation.
2. Find h (final state) by applying the hydrostatic equation.
3. Find the change in height by $\Delta h = h(\text{final state}) - h\,(\text{initial state})$.

SOLUTION

1. Initial State. Locate point 1 on the reservoir surface; point 2 on the water surface inside the tube:

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{100\,\text{kPa}}{9790\,\text{N/m}^3} + 0 &= \frac{50\,\text{kPa}}{9790\,\text{N/m}^3} + h \\ h\,(\text{initial state}) &= 5.107\,\text{m} \end{aligned}$$

2. Final State:

$$
\begin{aligned}
\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\
\frac{100\,\text{kPa}}{9790\,\text{N}/\,\text{m}^3} + 0 &= \frac{75\,\text{kPa}}{9790\,\text{N}/\,\text{m}^3} + h \\
h\,(\text{final state}) &= 2.554\,\text{m}
\end{aligned}
$$

3. Change in height:

$$
\begin{aligned}
\Delta h &= h(\text{final state}) - h\,(\text{initial state}) \\
&= 2.554\,\text{m} - 5.107\,\text{m} = -2.55\,\text{m}
\end{aligned}
$$

$$\boxed{\text{The height has decreased by 2.55 m.}}$$

REVIEW

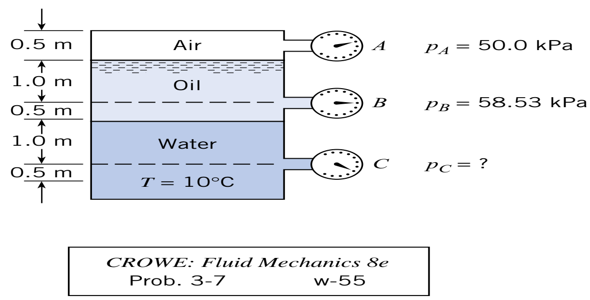
Tip! In the hydrostatic equation, use gage pressure or absolute pressure. Using vacuum pressure will give a wrong answer.

3.11: PROBLEM DEFINITION

Situation:
A closed tank contains air, oil, and water.

Find:
Specific gravity of oil.
Pressure at C (kPa-gage).

Sketch:



Properties:
Water (10 °C), Table A.5, $\gamma = 9810\,\text{N/m}^3$.

PLAN

1. Find the oil specific gravity by applying the hydrostatic equation from A to B.
2. Apply the hydrostatic equation to the water.
3. Apply the hydrostatic equation to the oil.
4. Find the pressure at C by combining results for steps 2 and 3.

SOLUTION

1. Hydrostatic equation (from oil surface to elevation B):

$$\begin{aligned} p_A + \gamma z_A &= p_B + \gamma z_B \\ 50,000\text{ N/m}^2 + \gamma_{\text{oil}}\,(1\text{ m}) &= 58,530\text{ N/m}^2 + \gamma_{\text{oil}}\,(0\text{ m}) \\ \gamma_{\text{oil}} &= 8530\text{ N/m}^3 \end{aligned}$$

Specific gravity:

$$S = \frac{\gamma_{\text{oil}}}{\gamma_{\text{water}}} = \frac{8530\text{ N/m}^3}{9810\text{ N/m}^3}$$

$$\boxed{S_{\text{oil}} = 0.87}$$

2. Hydrostatic equation (in water):

$$p_c = (p_{\text{btm of oil}}) + \gamma_{\text{water}}\,(1\,\text{m})$$

3. Hydrostatic equation (in oil):

$$p_{\text{btm of oil}} = (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m})$$

4. Combine equations:

$$\begin{aligned} p_c &= (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m}) + \gamma_{\text{water}}\,(1\,\text{m}) \\ &= \left(58,530\,\text{Pa} + 8530\,\text{N/m}^3 \times 0.5\,\text{m}\right) + 9810\,\text{N/m}^3\,(1\,\text{m}) \\ &= 72,605\ \text{N/m}^2 \end{aligned}$$

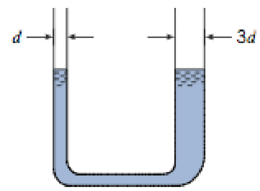
$$\boxed{p_c = 72.6\ \text{kPa-gage}}$$

3.12: PROBLEM DEFINITION

Situation:

A manometer is described in the problem statement.

$d_{\text{left}} = 1\,\text{mm}$, $d_{\text{right}} = 3\,\text{mm}$.



Find:

Water surface level in the left tube as compared to the right tube.

SOLUTION

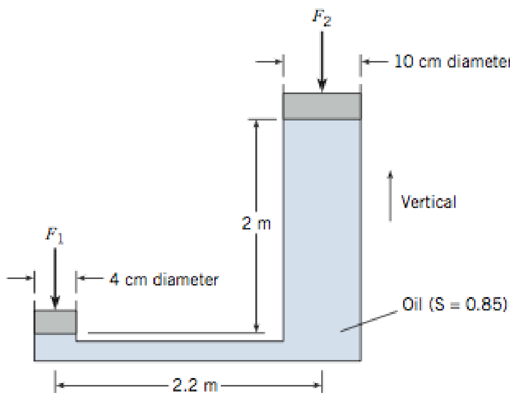
(a) The water surface level in the left tube will be higher because of greater surface tension effects for that tube.

3.13: PROBLEM DEFINITION

Situation:

A force is applied to a piston.

$F_1 = 200\,\mathrm{N}$, $d_1 = 4\,\mathrm{cm}$, $d_2 = 10\,\mathrm{cm}$.



Find:

Force resisted by piston.

Assumptions:

Neglect piston weight.

PLAN

Apply the hydrostatic equation and equilibrium.

SOLUTION

1. Equilibrium (piston 1)

$$\begin{aligned} F_1 &= p_1 A_1 \\ p_1 &= \frac{F_1}{A_1} \\ &= \frac{4 \times 200\,\mathrm{N}}{\pi \cdot (0.04\,\mathrm{m})^2\ \mathrm{m}^2} \\ &= 1.592 \times 10^5\,\mathrm{Pa} \end{aligned}$$

2. Hydrostatic equation

$$\begin{aligned} p_2 + \gamma z_2 &= p_1 + \gamma z_1 \\ p_2 &= p_1 + (S\gamma_{\mathrm{water}})(z_1 - z_2) \\ &= 1.592 \times 10^5\,\mathrm{Pa} + \left(0.85 \times 9810\,\mathrm{N/\,m^3}\right)(-2\,\mathrm{m}) \\ &= 1.425 \times 10^5\,\mathrm{Pa} \end{aligned}$$

3. Equilibrium (piston 2)

$$\begin{aligned} F_2 &= p_2 A_2 \\ &= \left(1.425 \times 10^5 \,\mathrm{N/\,m^2}\right)\left(\frac{\pi\,(0.1\,\mathrm{m})^2}{4}\right) \\ &= 1119\,\mathrm{N} \end{aligned}$$

$$\boxed{F_2 = 1120 \text{ N}}$$

3.14: PROBLEM DEFINITION

Situation:

A diver goes underwater.
$\Delta z = 50\,\text{m}$.

Find:

Gage pressure (kPa).
Ratio of pressure to normal atmospheric pressure.

Properties:

Water (20 °C), Table A.5, $\gamma = 9790\,\text{N/m}^3$.

PLAN

1. Apply the hydrostatic equation.
2. Calculate the pressure ratio (use absolute pressure values).

SOLUTION

1. Hydrostatic equation

$$\begin{aligned} p &= \gamma \Delta z = 9790\,\text{N/m}^3 \times 50\,\text{m} \\ &= 489,500\ \text{N/m}^2 \end{aligned}$$

$$\boxed{p = 490\ \text{kPa gage}}$$

2. Calculate pressure ratio

$$\frac{p_{50}}{p_{\text{atm}}} = \frac{489.5\,\text{kPa} + 101.3\,\text{kPa}}{101.3\,\text{kPa}}$$

$$\boxed{\frac{p_{50}}{p_{\text{atm}}} = 5.83}$$

3.15: PROBLEM DEFINITION

Situation:
Water and kerosene are in a tank.
$z_{\text{water}} = 1\,\text{m}$, $z_{\text{kerosene}} = 0.75\,\text{m}$.

Find:
Gage pressure at bottom of tank (kPa-gage).

Properties:
Water (20 °C), Table A.5, $\gamma_w = 9790\ \text{N/m}^3$.
Kerosene (20 °C), Table A.4, $\gamma_k = 8010\ \text{N/m}^3$.

SOLUTION

Manometer equation (add up pressure from the top of the tank to the bottom of the tank).

$$p_{\text{atm}} + \gamma_{\text{k}}\ (0.75\,\text{m}) + \gamma_{\text{w}}\ (1.0\,\text{m}) = p_{\text{btm}}$$

Solve for pressure

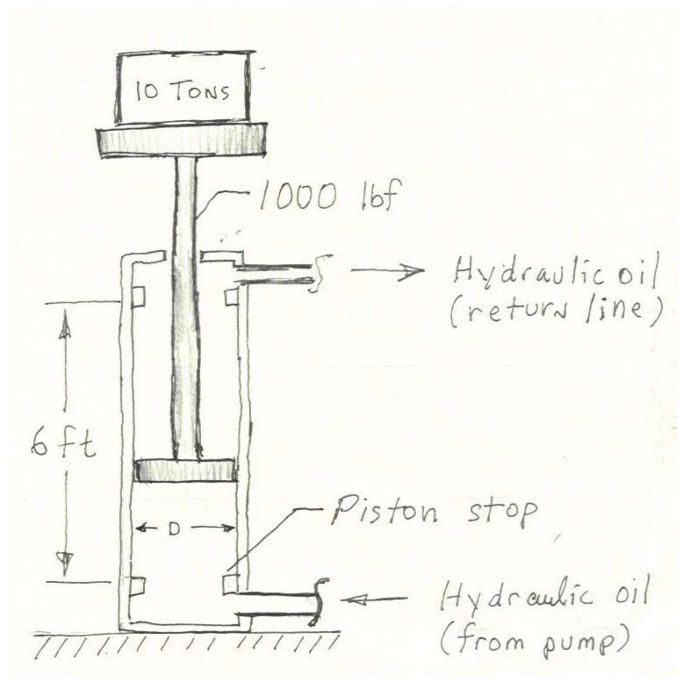
$$\begin{aligned} p_{\text{btm}} &= 0 + \gamma_{\text{k}}\ (0.75\,\text{m}) + \gamma_{\text{w}}\ (1.0\,\text{m}) \\ &= \left(8010\,\text{N/}\,\text{m}^3\right)(0.75\,\text{m}) + \left(9790\,\text{N/}\,\text{m}^3\right)(1.0\,\text{m}) \\ &= 15.8\,\text{kPa} \end{aligned}$$

$$\boxed{p_{\text{btm}} = 15.8\,\text{kPa gage}}$$

3.16: PROBLEM DEFINITION

Situation:

A hydraulic lift is being designed.
$W_{\max} = 10$ ton $= 20000\,\text{lbf}$, $W_{parts} = 1000\,\text{lbf}$.
$\Delta L = 6\,\text{ft}$, $\Delta t = 20\,\text{s}$.
Diameter range: $2 - 8\,\text{in}$.
Pressure range: $200 - 3000$ psig.
Available pumping capacity: 5, 10, 15 gpm.



Find:

Select a hydraulic pump capacity (gpm).
Select a cylinder diameter (D).

PLAN

Apply equilibrium to find the smallest bore diameter (D) that works. Then find the largest bore diameter that works by considering the lift speed requirement. Select bore and pump combinations that meet the desired specifications.

SOLUTION

Equilibrium (piston)

$$F = pA$$

where $F = 21,000\,\text{lbf}$ is the load that needs to be lifted and p is the pressure on the bottom of the piston. Maximum pressure is 3000 psig so minimum bore area is

$$\begin{aligned} A_{\text{min}} &= \frac{F}{p_{\text{max}}} \\ &= \frac{21,000\,\text{lbf}}{3000\,\text{in}^2} \\ &= 7.0\ \text{in}^2 \end{aligned}$$

Corresponding minimum bore diameter is

$$\begin{aligned} D &= \sqrt{\frac{4}{\pi}A} \\ D_{\text{min}} &= 2.98\ \text{in} \end{aligned}$$

The pump needs to provide enough flow to raise the lift in 20 seconds.

$$A\Delta L = \dot{V}\Delta t$$

where A is the bore area, ΔL is stroke (lift height), $\dot{V}$ is the volume/time of fluid provided by the pump, and Δt is the time. Thus, the maximum bore area is

$$A_{\text{max}} = \frac{\dot{V}\Delta t}{\Delta L}$$

Conversion from gallons to cubic feet $\left(\text{ft}^3\right)$: 7.48 gal=1 ft^3. Thus, the maximum bore diameter for three pumps (to meet the lift speed specification) is given in the table below.

pump (gpm)	pump (cfm)	A (ft^2)	D_{max} (in)
5	0.668	0.037	2.61
10	1.337	0.074	3.68
15	2.01	0.116	4.61

Since the minimum bore diameter is 2.98 in., the 5 gpm pump will not work. The 10 gpm pump can be used with a 3 in. bore. The 15 gpm pump can be used with a 3 or 4 in. bore.

1.) The 10 gpm pump will work with a bore diameter between 3.0 and 3.6 inches.

2.) The 15 gpm pump will work with a bore diameter between 3.0 and 4.6 inches.

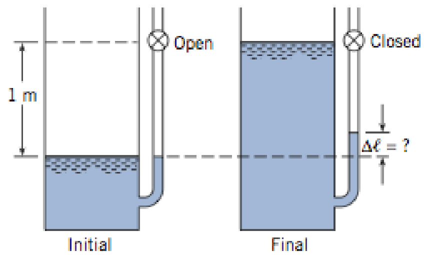
REVIEW

1. These are preliminary design values. Other issues such as pressure drop in the hydraulic lines and valves would have to be considered.

2. We recommend selecting the 15 gpm pump and a 4.5 inch bore to provide latitude to handle pressure losses, and to reduce the maximum system pressure.

3.17: PROBLEM DEFINITION

Situation:
Initial State: Water levels as shown. Valve in open.
Final State: Water is added to the tank with the valve closed.



Find:
Increase of water level $\Delta\ell$ in manometer (in meters).

Properties:
Water (20 °C), Table A.5, γ_w=9790 N/m^3.
$p_{atm} = 100\,\text{kPa}$.

Assumptions: Ideal gas.

PLAN

Apply the hydrostatic equation and the ideal gas law.

SOLUTION

Ideal gas law (mole form; apply to air in the manometer tube)

$$pV\!\!\!/ = n\Re T$$

Because the number of moles (n) and temperature (T) are constants, the ideal gas reduces to Boyle's equation.

$$p_1 V\!\!\!/_1 = p_2 V\!\!\!/_2 \tag{1}$$

State 1 (before air is compressed)

$$\begin{aligned} p_1 &= 100,000 \text{ N/m}^2 \text{ abs} \\ V\!\!\!/_1 &= 1 \text{ m} \times A_{\text{tube}} \end{aligned} \tag{a}$$

State 2 (after air is compressed)

$$\begin{aligned} p_2 &= 100,000 \text{ N/m}^2 + \gamma_w (1 \text{ m} - \Delta\ell) \\ V\!\!\!/_2 &= (1 \text{ m} - \Delta\ell) A_{\text{tube}} \end{aligned} \tag{b}$$

Substitute (a) and (b) into Eq. (1)

$$\begin{aligned}
p_1 \forall_1 &= p_2 \forall_2 \\
\left(100,000\,\mathrm{N/\,m^2}\right)(1\ \mathrm{m} \times A_{\mathrm{tube}}) &= \left(100,000\ \mathrm{N/m^2} + \gamma_{\mathrm{w}}(1\ \mathrm{m} - \Delta\ell)\right)(1\ \mathrm{m} - \Delta\ell)A_{\mathrm{tube}} \\
100,000\,\mathrm{N/\,m^2} &= \left(100,000\,\mathrm{N/\,m^2} + 9790\,\mathrm{N/\,m^3}\,(1 - \Delta\ell)\right)(1 - \Delta\ell)
\end{aligned}$$

Solving for $\Delta\ell$

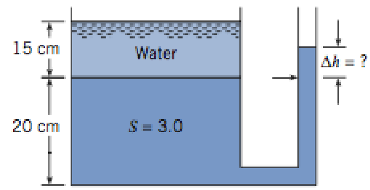
$$\boxed{\Delta\ell = 0.0824\ \mathrm{m}}$$

3.18: PROBLEM DEFINITION

Situation:

A tank is fitted with a manometer.

$S = 3$, $z_1 = 0.15\,\text{m}$.



Find:

Deflection of the manometer (cm).

Properties:

γ_{water}=9810 N/m^3.

PLAN

Apply the hydrostatic principle to the water and then to the manometer fluid.

SOLUTION

1. Hydrostatic equation (location 1 is on the free surface of the water; location 2 is the interface)

$$\begin{aligned}
\frac{p_1}{\gamma_{\text{water}}} + z_1 &= \frac{p_2}{\gamma_{\text{water}}} + z_2 \\
\frac{0\,\text{Pa}}{9810\,\text{N/m}^3} + 0.15\,\text{m} &= \frac{p_2}{9810\,\text{N/m}^3} + 0\,\text{m} \\
p_2 &= (0.15\,\text{m})\left(9810\,\text{N/m}^3\right) \\
&= 1471.5\,\text{Pa}
\end{aligned}$$

2. Hydrostatic equation (manometer fluid; let location 3 be on the free surface)

$$\begin{aligned}
\frac{p_2}{\gamma_{\text{man. fluid}}} + z_2 &= \frac{p_3}{\gamma_{\text{man. fluid}}} + z_3 \\
\frac{1471.5\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} + 0\,\text{m} &= \frac{0\,\text{Pa}}{\gamma_{\text{man. fluid}}} + \Delta h
\end{aligned}$$

3. Solve for Δh

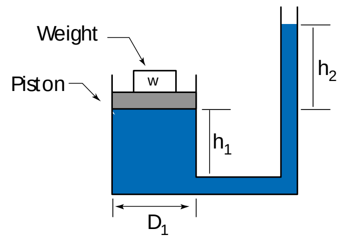
$$\begin{aligned}
\Delta h &= \frac{1471.5\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} \\
&= 0.0500\,\text{m}
\end{aligned}$$

$$\boxed{\Delta h = 5.00\ \text{cm}}$$

3.19: PROBLEM DEFINITION

Situation:

A mass sits on top of a piston situated above a reservoir of oil.



Find:

Derive an equation for h_2 in terms of the specified parameters.

Assumptions:

Neglect the mass of the piston.
Neglect friction between the piston and the cylinder wall.
The pressure at the top of the oil column is 0 kPa-gage.

PLAN

1. Relate w to pressure acting on the bottom of the piston using equilibrium.
2. Related pressure on the bottom of the piston to the oil column height using the hydrostatic equation.
3. Find h_2 by combining steps 1 and 2.

SOLUTION

1. Equilibrium (piston):

$$w = p_1 \left(\frac{\pi D_1^2}{4} \right) \tag{1}$$

2. Hydrostatic equation. (point 1 at btm of piston; point 2 at top of oil column):

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{p_1}{S\gamma_{\text{water}}} + 0 &= 0 + h_2 \\ p_1 &= S\ \gamma_{\text{water}}\ h_2 \end{aligned} \tag{2}$$

3. Combine Eqs. (1) and (2):

$$mg = S\ \gamma_{\text{water}}\ h_2 \left(\frac{\pi D_1^2}{4} \right)$$

Answer: $$h_2 = \frac{4w}{(S)(\gamma_{\text{water}})(\pi D_1^2)}$$

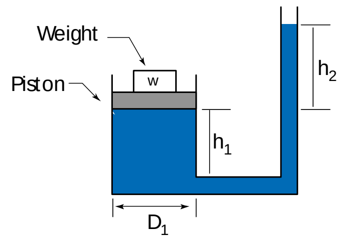
REVIEW

1. Notice. Column height h_2 increases linearly with increasing weight w.Similarly, h_2 decreases linearly with S and decreases quadratically with D_1.
2. Notice. The apparatus involved in the problem could be used to create an instrument for weighing an object.

3.20: PROBLEM DEFINITION

Situation:

A mass sits on top of a piston situated above a reservoir of oil.
$m = 10\,\text{kg}$, $S = 0.8$, $h_1 = 42\,\text{mm}$.
$D_1 = 42\,\text{mm}$, $D_2 = 5\,\text{mm}$.



Find:

Calculate h_2 (m).

Assumptions:

Neglect the mass of the piston.
Neglect friction between the piston and the cylinder wall.
The pressure at the top of the oil column is 0 kPa-gage.

PLAN

1. Relate mass m to pressure acting on the bottom of the piston using equilibrium.
2. Related pressure on the bottom of the piston to the oil column height using the hydrostatic equation.
3. Find h_2 by combining steps 1 and 2.

SOLUTION

1. Equilibrium (piston):

$$mg = p_1 \left(\frac{\pi D_1^2}{4} \right) \tag{1}$$

2. Hydrostatic equation. (point 1 at btm of piston; point 2 at top of oil column):

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{p_1}{S\gamma_{\text{water}}} + 0 &= 0 + h_2 \\ p_1 &= S\ \gamma_{\text{water}}\ h_2 \qquad (2) \end{aligned}$$

3. Combine Eqs. (1) and (2):

$$mg = S\ \gamma_{\text{water}}\ h_2 \left(\frac{\pi D_1^2}{4} \right)$$

$$h_2 = \frac{4mg}{(S)\,(\gamma_{\text{water}})\,(\pi D_1^2)} = \frac{4\,(10\,\text{kg})\,(9.81\,\text{m/}\,\text{s}^2)}{(0.8)\,(9810\,\text{N/}\,\text{m}^3)\,(\pi)\,(0.14^2\,\text{m}^2)}$$

$$\boxed{h_2 = 0.812\,\text{m}}$$

3.21: PROBLEM DEFINITION

Situation:

An odd tank contains water, air and a liquid.

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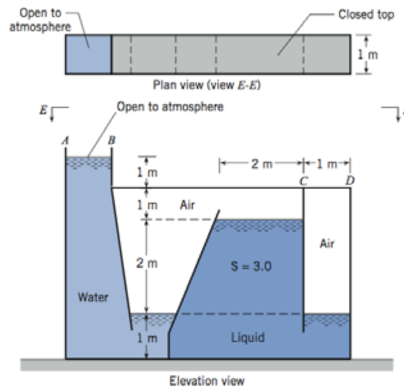


Find:

Maximum gage pressure (kPa).
Where will maximum pressure occur.
Hydrostatic force (in kN) on top of the last chamber, surface CD.

Properties:

$\gamma_{\text{water}} = 9810\ \text{N/m}^3$.

PLAN

1. To find the maximum pressure, apply the manometer equation.
2. To find the hydrostatic force, multiply pressure times area.

SOLUTION

1. Manometer eqn. (start at surface AB; neglect pressure changes in the air; end at the bottom of the liquid reservoir)

$$\begin{aligned} 0 + 4 \times \gamma_{\text{H}_2\text{O}} + 3 \times 3\gamma_{\text{H}_2\text{O}} &= p_{\max} \\ p_{\max} &= 13\,\text{m} \times 9,810\,\text{N/m}^3 \\ &= 127,530\ \text{N/m}^2 \end{aligned}$$

$$\boxed{p_{\max} = 127.5\ \text{kPa}}$$

Answer $\Rightarrow$ Maximum pressure will be at the bottom of the liquid that has a specific gravity of $S = 3$.

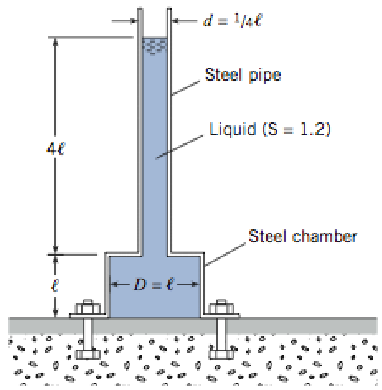
2. Hydrostatic force

$$\begin{aligned} F_{CD} &= pA \\ &= (127,530\,\text{N/m}^2 - 1\,\text{m} \times 3 \times 9810\,\text{N/m}^3) \times 1\ \text{m}^2 \\ &\boxed{F_{CD} = 98.1\ \text{kN}} \end{aligned}$$

3.22: PROBLEM DEFINITION

Situation:

A steel pipe is connected to a steel chamber.
$\ell = 2.5\,\mathrm{ft}$, $W = 600\,\mathrm{lbf}$.
$D_1 = 0.25\ell$, $z_1 = 5\ell$.
$D_2 = \ell$, $S = 1.2$.



Find:

Force exerted on chamber by bolts (lbf).

Properties:

$\gamma_{\text{water}} = 62.4\,\mathrm{lbf/\,ft^3}$.

PLAN

Apply equilibrium and the hydrostatic equation.

SOLUTION

1. Equilibrium. (system is the steel structure plus the liquid within)

$$\text{(Weight of the steel)} = \begin{array}{l}\text{(Force exerted by bolts)} + \text{(Weight of the liquid)} + \\ \text{(Pressure force acting on the bottom of the free body)}\end{array}$$

$$F_B + W_{\text{liquid}} + W_s = p_2 A_2 \tag{1}$$

2. Hydrostatic equation (location 1 is on surface; location 2 at the bottom)

$$\begin{aligned}
\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma_{\text{liquid}}} + z_2 \\
0 + 5\ell &= \frac{p_2}{1.2\gamma_{\text{water}}} + 0 \\
p_2 &= 1.2\gamma_{\text{water}} 5\ell \\
&= 1.2 \times 62.4 \times 5 \times 2.5 \\
&= 936 \text{ psfg}
\end{aligned}$$

3. Area

$$A_2 = \frac{\pi D^2}{4} = \frac{\pi \ell^2}{4} = \frac{\pi \times (2.5\,\text{ft})^2}{4} = 4.909\,\text{ft}^2$$

4. Weight of liquid

$$\begin{aligned} W_{\text{liquid}} &= \left(A_2\ell + \frac{\pi d^2}{4}4\ell\right)\gamma_{\text{liquid}} = \left(A_2\ell + \frac{\pi\ell^3}{16}\right)(1.2)\,\gamma_{\text{water}} \\ &= \left(\left(4.909\,\text{ft}^2\right)(2.5\,\text{ft}) + \frac{\pi\,(2.5\,\text{ft})^3}{16}\right)(1.2)\left(62.4\frac{\text{lbf}}{\text{ft}^3}\right) \\ &= 1148.\,7\,\text{lbf} \end{aligned}$$

5. Substitute numbers into Eq. (1)

$$\begin{aligned} F_B + (1148.\,7\,\text{lbf}) + (600\,\text{lbf}) &= \left(936\,\text{lbf}/\,\text{ft}^2\right)\left(4.909\,\text{ft}^2\right) \\ F_B &= 2846\,\text{lbf} \end{aligned}$$

$$\boxed{F_B = 2850\ \text{lbf}}$$

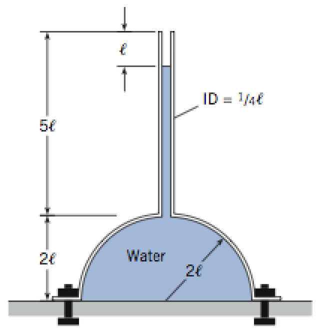
3.23: PROBLEM DEFINITION

Situation:

A metal dome with water is held down by bolts.

$W = 6\,\text{kN}$, $\ell = 80\,\text{cm}$, $h = 6\ell$.

$D_{\text{pipe}} = 0.25\ell$, $D_{\text{bottom}} = 2\ell$.



Find:

Force exerted by the bolts (kN).

Properties:

$\gamma_{\text{water}} = 9810\,\text{N/m}^3$.

PLAN

1. To derive an equation for the load on the bolts, apply equilibrium.
2. Calculate intermediate value.
2. Calculate the load on the bolts.

SOLUTION

1. Equilibrium (free body is the water plus the dome)

$$\sum F_z = 0$$

$$p_{\text{bottom}} A_{\text{bottom}} + F_{\text{bolts}} - W_{\text{H}_2\text{O}} - W_{\text{dome}} = 0$$

$$F_{\text{bolts}} = -p_{\text{bottom}} A_{\text{bottom}} + W_{\text{H}_2\text{O}} + W_{\text{dome}} \quad (1)$$

2. Intermediate calculations

$$p_{\text{bottom}} A_{\text{bottom}} = 4.8 \times 9,810\,\text{N/m}^3 \times \pi \times (1.6\,\text{m})^2 = 378.7\text{ kN}$$

$$W_{\text{H}_2\text{O}} = 9,810(3.2 \times (\pi/4) \times 0.2^2 + (2/3)\pi \times 1.6^3) = 85.1\text{ kN}$$

3. Load on bolts (apply Eq. (1))

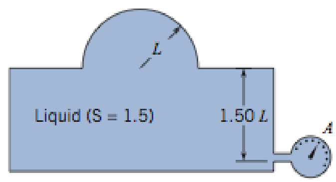
$$F_{\text{bolts}} = (-378.7 + 85.1 + 6)\text{ kN} = -288\,\text{kN}$$

$$\boxed{F_{\text{bolts}} = 288\,\text{kN (acting downward)}}$$

3.24: PROBLEM DEFINITION

Situation:

A tank under pressure with a dome on top.
$L = 2\,\text{ft}$, $W_{\text{dome}} = 1000\,\text{lbf}$.
Gage A reads 5 psig.



Find:

Vertical component of force in metal at the base of the dome (lbf).
Is the metal in tension or compression?

Properties:

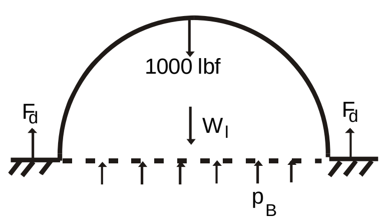
$\gamma_{\text{H}_2\text{O}} = 62.4\,\text{lbf/ft}^3$, $S = 1.5$.

PLAN

Apply equilibrium to a free body comprised of the dome plus the water within. Apply the hydrostatic principle to find the pressure at the base of the dome.

SOLUTION

Equilibrium



$$\sum F_z = 0 \quad (1)$$

$$F_d + p_B A - W_{\text{liquid}} - W_{\text{dome}} = 0 \quad (1)$$

Hydrostatic equation

$$p_B + \gamma z_B = p_A + \gamma z_A$$

$$\begin{aligned} p_B &= p_A - (\gamma_{H_2O})\, S\Delta z \\ &= (5\text{ psig})\left(144\,\text{in}^2/\text{ft}^2\right) - \left(62.4\,\text{lbf/ft}^3\right)(1.5)(3\,\text{ft}) \\ &= 439.2\text{ psfg} \end{aligned}$$

Weight of the liquid

$$
\begin{aligned}
W_{\text{liquid}} &= \left(\gamma_{H_2O}\right)(S)(V\!\!\!-) \\
&= \left(62.4\,\text{lbf}/\,\text{ft}^3\right)(1.5)\left(\frac{2}{3}\pi 2^3\,\text{ft}^3\right) \\
&= 1568\,\text{lbf}
\end{aligned}
$$

Pressure Force

$$
\begin{aligned}
F_B &= p_B A \\
&= (439.2 \text{ psfg})\left(\pi \times 2^2\,\text{ft}^2\right) \\
&= 5519\,\text{lbf}
\end{aligned}
$$

Substitute into Eq. (1).

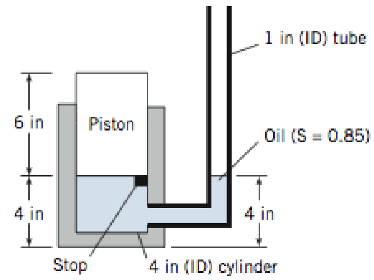
$$
\begin{aligned}
F_d &= -F_B + W_{\text{liquid}} + W_{\text{dome}} \\
&= -(5519\,\text{lbf}) + (1568\,\text{lbf}) + (1000\,\text{lbf}) \\
&= -2951\,\text{lbf}
\end{aligned}
$$

$$
\boxed{F_d = 2950\,\text{lbf} \quad \text{(metal is in tension)}}
$$

3.25: PROBLEM DEFINITION

Situation:

Oil is added to the tube so the piston rises 1 inch.
$W_{\text{piston}} = 10\,\text{lbf}$, $S = 0.85$.
$D_p = 4\,\text{in}$, $D_{\text{tube}} = 1\,\text{in}$.

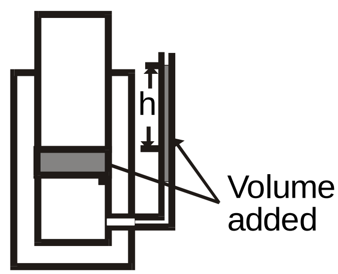


Find:

Volume of oil (in^3) that is added.

SOLUTION

Notice that the oil fills the apparatus as shown below.



Pressure acting on the bottom of the piston

$$\begin{aligned} p_p A_p &= 10 \text{ lbf} \\ p_p &= \frac{10\,\text{lbf}}{A_p} = \frac{10\,\text{lbf}}{\pi\,(4\,\text{in})^2/4} \\ &= 0.796 \text{ psig } = 114.6 \text{ psfg} \end{aligned}$$

Hydrostatic equation (apply to liquid in the tube)

$$\begin{aligned} \gamma_{\text{oil}} h &= 114.6 \text{ psfg} \\ h &= 114.6/(62.4 \times 0.85) = 2.161 \text{ ft} = 25.9 \text{ in} \end{aligned}$$

Calculate volume

$$\begin{aligned} V\!\!\!\!-_{\text{added}} &= V\!\!\!\!-_{\text{left}} + V\!\!\!\!-_{\text{right}} \\ &= \frac{\pi\,(4\,\text{in})^2\,(1\,\text{in})}{4} + \frac{\pi\,(1\,\text{in})^2\,(1\,\text{in} + 25.9\,\text{in})}{4} \end{aligned}$$

$$\boxed{V\!\!\!\!-_{\text{added}} = 33.7 \text{ in.}^3}$$

3.26: PROBLEM DEFINITION

Situation:

An air bubble rises from the bottom of a lake.

$z_{34} = 34\,\text{ft}$, $z_8 = 8\,\text{ft}$.

Find:

Ratio of the density of air within the bubble at different depths.

Assumptions:

Air is ideal gas.

Temperature is constant.

Neglect surface tension effects.

Properties:

$\gamma = 62.4\,\text{lbf/ft}^3$.

PLAN

Apply the hydrostatic equation and the ideal gas law.

SOLUTION

Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ \rho_{34} &= \frac{p_{34}}{RT};\ \rho_8 = \frac{p_8}{RT} \\ \frac{\rho_{34}}{\rho_8} &= \frac{p_{34}}{p_8} \end{aligned}$$

where p is absolute pressure (required in ideal gas law).

Hydrostatic equation

$$\begin{aligned} p_8 &= p_{\text{atm}} + \gamma\,(8\,\text{ft}) \\ &= 2120\,\text{lbf/ft}^2 + \left(62.4\ \text{lbf/ft}^3\right)(8\ \text{ft}) \\ &= 2619\ \text{lbf/ft}^2 \end{aligned}$$

$$\begin{aligned} p_{34} &= p_{\text{atm}} + \gamma\,(34\,\text{ft}) \\ &= 2120\,\text{lbf/ft}^2 + \left(62.4\ \text{lbf/ft}^3\right)(34\ \text{ft}) \\ &= 4241.6\ \text{lbf/ft}^2 \end{aligned}$$

Density ratio

$$\begin{aligned} \frac{\rho_{34}}{\rho_8} &= \frac{4241.6\ \text{lbf/ft}^2}{2619\ \text{lbf/ft}^2} \\ &= 1.620 \end{aligned}$$

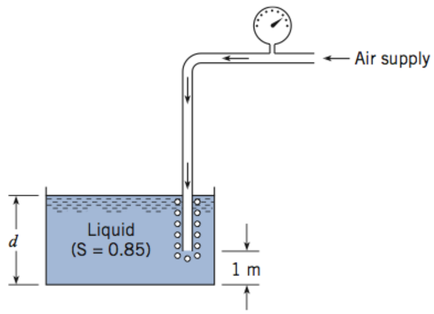
$$\boxed{\rho_{34}/\rho_8 = 1.62}$$

3.27: PROBLEM DEFINITION

Situation:

Air is injected into a tank of liquid.

Pressure reading the Bourdon tube gage is $p_{\text{gage}} = 15\,\text{kPa}$



Find:

Depth d of liquid in tank (m).

Assumptions:

Neglect the change of pressure due to the column of air in the tube.

Properties: $\gamma\,(\text{water}) = 9810\,\text{N/m}^3$, $S = 0.85$.

PLAN

1. Find the depth corresponding to $p = 15\,\text{kPa}$ using the hydrostatic equation.
2. Find d by adding 1.0 m to value from step 1.

SOLUTION

1. Hydrostatic equation

$$\begin{aligned}\Delta p &= \gamma_{\text{liquid}}\Delta z \\ \Delta z &= \frac{\Delta p}{\gamma_{\text{liquid}}} = \frac{15000\,\text{Pa}}{0.85\,(9810\,\text{N/m}^3)} = 1.80\,\text{m}\end{aligned}$$

2. Depth of tank

$$\begin{aligned}d &= \Delta z + 1\,\text{m} \\ &= 1.80\,\text{m} + 1\,\text{m} \\ &= 2.8\,\text{m}\end{aligned}$$

$$\boxed{d = 2.80\text{ m}}$$

Problem 3.28

Using the Internet and other resources, answer the following questions:

a. What are three common types of manometers? For each type, make a sketch and give a brief description.

- Sketches left as a exercise.
- Some possible types of manometer: U-tube manometer, well manometer, inclined manometer, micro-manometer.
- Note that many electronic instruments are now called manometers. These instrument are not really manometers (manometers rely on the change in level of a liquid column).

b. How would you build manometers from materials that are commonly available? Sketch your design concept.

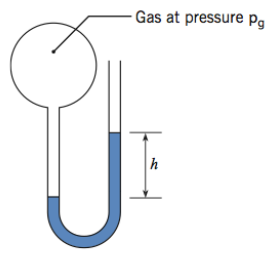
- The photo shows a design built by students at the University of Idaho. Some features to notice:

  - Use of green food coloring to enhance the visibility of the liquid.

- Attaching the tubing to a board using zip ties (fast, low cost, and effective)
- Adding a ruler so that changes in column height are easy to determine.

Problem 3.29

Apply the grid method to a U-tube manometer.



The working equation (i.e. the hydrostatic equation) is:

$$p_{\text{gas}} = \gamma_{\text{liquid}} h$$

Note: Unit cancellations are not shown in this solution.

a.)

Situation:
Water in a manometer.
$h = 1\,\text{ft}$.

Find:
Absolute pressure (psig).

Properties:
$S = 1.5$, $\gamma = 62.4\,\text{lbf}/\,\text{ft}^3$.

Solution:
First, find the gage pressure in the gas:

$$\begin{aligned} p_{\text{gas}} &= \gamma_{\text{liquid}} h = S\gamma_{\text{H2O}} h \\ &= (1.5)\left(\frac{62.4\,\text{lbf}}{\text{ft}^3}\right)\left(\frac{1\,\text{ft}}{1.0}\right)\left(\frac{1\,\text{ft}}{12\,\text{in.}}\right)^2 = 0.650 \text{ psig} \end{aligned}$$

Now, find the absolute pressure:

$$\begin{aligned} p_{\text{abs}} &= p_{\text{atm}} + p_{\text{gage}} \\ &= 14.7 \text{ psi } + 0.650 \text{ psi } = \boxed{15.4 \text{ psia}} \end{aligned}$$

b.)

Situation:
Mercury in a manometer.

Find:
Column rise (mm).

Properties:
Table A.4, $\gamma = 133000\,\text{N/m}^3$.
$p_{gas} = 1/6\,\text{atm}$, $p_{atm} = 101.3\,\text{kN}$.

Solution:
b. Find column rise in mm. The manometer uses mercury (). The gas pressure is 1/6 atm.

$$h = \frac{p_{\text{gas}}}{\gamma_{\text{liquid}}} = \left(\frac{1/6\,\text{atm}}{1.0}\right)\left(\frac{\text{m}^3}{133000\,\text{N}}\right)\left(\frac{101.3\times 10^3\,\text{N}}{1\,\text{atm}\cdot\text{m}^2}\right) = 0.1269\,\text{m}$$

$$\boxed{h = 127\,\text{mm}}$$

c.)

Situation:
Liquid in manometer.
$h = 6\,\text{in}$.

Find:
Pressure (psfg).

Properties:
$\rho = 50\,\text{lb/ft}^3$.

Solution:

$$\begin{aligned} p_{\text{gas}} &= \gamma_{\text{liquid}} h = \rho g h \\ &= \left(\frac{50\ \text{lbm}}{\text{ft}^3}\right)\left(\frac{32.2\,\text{ft}}{\text{s}^2}\right)\left(\frac{6\,\text{in}}{1.0}\right)\left(\frac{\text{lbf}\cdot\text{ft}^2}{32.2\ \text{lbm}\cdot\text{ft}}\right)\left(\frac{1.0\,\text{ft}}{12\,\text{in}}\right) \\ &= \boxed{25\ \text{psfg}} \end{aligned}$$

d.)

Situation:
Liquid in manometer.
$h = 3\,\text{m}$.

Find:
Gage pressure (bar).

Properties:
$\rho = 800\,\text{kg/m}^3$.

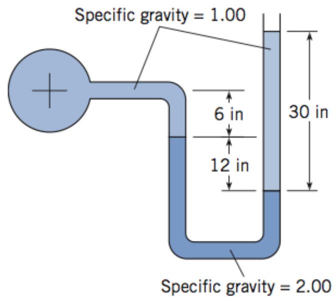
Solution:

$$\begin{aligned} p_{\text{gas}} &= \gamma_{\text{liquid}} h = \rho g h \\ &= \left(\frac{800\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{3\,\text{m}}{1.0}\right)\left(\frac{\text{Pa}\cdot\text{m}\cdot\text{s}^2}{\text{kg}}\right)\left(\frac{1\,\text{bar}}{10^5\,\text{Pa}}\right) \\ &= \boxed{0.235\,\text{bar}} \end{aligned}$$

3.30: PROBLEM DEFINITION

Situation:

A manometer is connected to a pipe.



Find:

Determine if the gage pressure at the center of the pipe is:

(a) negative

(b) positive

(c) zero

PLAN

Apply the manometer equation and justify the solution using calculations.

SOLUTION

Manometer equation. (add up pressures from the pipe center to the open end of the manometer)

$$p_{\text{pipe}} + (0.5 \text{ ft})(62.4 \text{ lbf/ft}^3) + (1 \text{ ft})(2 \times 62.4 \text{ lbf/ft}^3) - (2.5 \text{ ft})(62.4 \text{ lbf/ft}^3) = 0 \quad (1)$$

Solve Eq. (1) for the pressure in the pipe

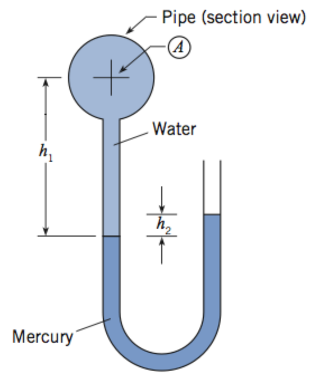
$$p_{\text{pipe}} = (-0.5 - 2 + 2.5) \text{ ft } (62.4 \text{ lbf/ft}^3) = 0$$

$$\boxed{p \,(\text{center of pipe}) = 0.0 \, \text{lbf/ ft}^2}$$

3.31: PROBLEM DEFINITION

Situation:

A manometer is connected to a pipe.
$h_1 = 16\,\text{in}$, $h_2 = 2\,\text{in}$.



Find:

Gage pressure at the center of the pipe in units of psig.

Properties:

Mercury (68 °F), Table A.4, $\gamma_{\text{Hg}} = 847\,\text{lbf/ft}^3$.
Water (70 °F), Table A.5, $\gamma_{\text{H2O}} = 62.3\,\text{lbf/ft}^3$.

PLAN

Find pressure (p_A) by applying the manometer equation from point A to the top of the mercury column.

SOLUTION

Manometer equation:

$$p_A + \left(\frac{16}{12}\,\text{ft}\right)(62.3\ \text{lbf/ft}^3) - \left(\frac{2}{12}\,\text{ft}\right)(847\ \text{lbf/ft}^3) = 0$$

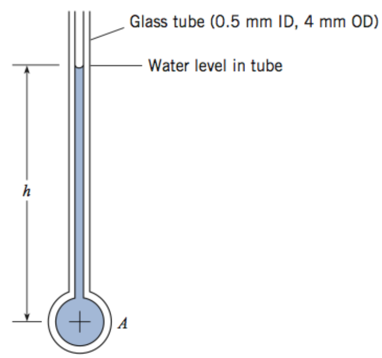
$$p_A = \left(\frac{58.1\,\text{lbf}}{\text{ft}^2}\right)\left(\frac{1.0\,\text{ft}}{12\,\text{in}}\right)^2$$

$$\boxed{p_A = 0.403\ \text{psig}}$$

3.32: PROBLEM DEFINITION

Situation:

A glass tube ($d = 0.5\,\mathrm{mm}$) is connected to a pipe containing water.
Column rise ($h = 100\,\mathrm{mm}$) is due to pressure and surface tension.



Find:

Gage pressure at the center of the pipe (Pa-gage).

Assumptions:

The contact angle is small so $\cos\theta \approx 1$ in the capillary rise equation.

Properties:

Water (20 °C), Table A-5: $\gamma = 9790\,\mathrm{N/m^3}, \sigma = 0.073\,\mathrm{N/m}$.

PLAN

1. Find the column rise due to surface tension by applying the capillary rise equation.
2. Find the column rise due to pressure by applying the hydrostatic equation.
3. Find the total column rise by combining steps 1 and 2.
4. Run calcs.

SOLUTION

1. Capillary rise equation (from chapter 2):

$$\Delta h_1 = \frac{4\sigma}{\gamma d} \tag{1}$$

2. Hydrostatic equation.

$$\Delta h_2 = \frac{p_A}{\gamma} \tag{2}$$

3. Total column rise:

$$\Delta h = \Delta h_1 + \Delta h_2 = \frac{4\sigma}{\gamma d} + \frac{p_A}{\gamma} \tag{3}$$

4. Calculations:

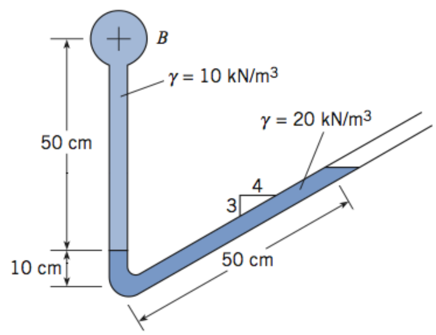
$$\Delta h_1 = \frac{4\sigma}{\gamma d} = 4\left(\frac{0.073\,\mathrm{N}}{\mathrm{m}}\right)\left(\frac{\mathrm{m}^3}{9790\,\mathrm{N}}\right)\left(\frac{1.0}{0.5\times10^{-3}\,\mathrm{m}}\right) = 0.05965\,\mathrm{m}$$

$$p_A = (\Delta h - \Delta h_1)\,\gamma = (0.1\,\text{m} - 0.05965\,\text{m})\left(9790\,\text{N/}\,\text{m}^3\right) = 395\,\text{Pa}$$

$$\boxed{p_A = 395\,\text{Pa gage}}$$

3.33: PROBLEM DEFINITION

Situation:
A tube (manometer) is connected to a pipe.



Find:
Pressure at the center of pipe B (kPa-gage).

Properties:
$\gamma_1 = 10\,\mathrm{kN/\,m^3}$, $\gamma_2 = 20\,\mathrm{kN/\,m^3}$.

PLAN

Apply the manometer equation.

SOLUTION

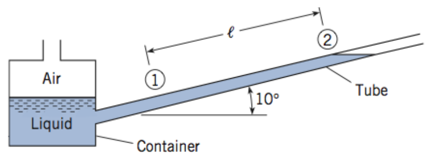
Manometer equation (add up pressures from the open end of the manometer to the center of pipe B).

$$\begin{aligned} p_B &= 0 \\ &\quad + \left(0.30\,\mathrm{m} \times 20,000\,\mathrm{N/\,m^3}\right) \\ &\quad - \left(0.1\,\mathrm{m} \times 20,000\,\mathrm{N/\,m^3}\right) \\ &\quad - \left(0.5\,\mathrm{m} \times 10,000\,\mathrm{N/\,m^3}\right) \\ &= -1000\,\mathrm{Pa} \end{aligned}$$

$$\boxed{p_B = -1.00 \ \text{kPa gage}}$$

3.34: PROBLEM DEFINITION

Situation:
State 1: air at p_{atm}, liquid in tube at elevation 1.
State 2: air is pressurized; liquid at elevation 2.
$\ell = 0.4\,\text{m}$, $D_{\text{container}} = 8D_{\text{tube}}$.



Find:
Pressure in the air within the container (Pa).

Properties:
Liquid, $\rho = 1200\,\text{kg/m}^3$.

PLAN

1. Find the decrease in liquid level in the container by applying conservation of mass.
2. Find the air pressure by applying the hydrostatic equation.

SOLUTION

1. Conservation of mass (applied to liquid)

$$\begin{aligned}
\text{Gain in mass of liq. in tube} &= \text{Loss of mass of liq. in container} \\
(\text{Volume change in tube})\,\rho_{\text{liquid}} &= (\text{Volume change in container })\,\rho_{\text{liquid}} \\
V\!\!\!-_{\text{tube}} &= V\!\!\!-_{\text{container}}
\end{aligned}$$

$$\begin{aligned}
(\pi/4)D_{\text{tube}}^2 \times \ell &= (\pi/4)D_{\text{container}}^2 \times (\Delta h)_{\text{container}} \\
(\Delta h)_{\text{container}} &= \left(\frac{D_{\text{tube}}}{D_{\text{container}}}\right)^2 \ell \\
(\Delta h)_{\text{container}} &= (1/8)^2 \times 40 \\
&= 0.625 \text{ cm}
\end{aligned}$$

2. Hydrostatic equation

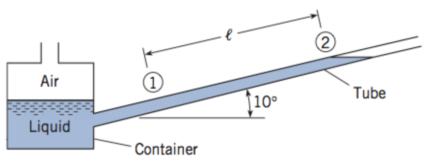
$$\begin{aligned}
p_{\text{container}} &= (\ell \sin 10^\circ + \Delta h)\rho g \\
&= [(0.4\,\text{m})\sin 10^o + 0.00625\,\text{m}]\left(1200\,\text{kg/m}^3\right)\left(9.81\,\text{m/s}^2\right)
\end{aligned}$$

$$\boxed{p_{\text{container}} = 891\,\text{Pa gage}}$$

3.35: PROBLEM DEFINITION

Situation:

State 1: air at p_{atm}, liquid in tube at elevation 1.
State 2: air is pressurized; liquid at elevation 2.
$D_{\text{container}} = 10D_{\text{tube}}$, $\ell = 3\,\text{ft}$.



Find:

Pressure in the air within the container (psfg).

Properties:

liquid, $\gamma = 50\,\text{lbf}/\,\text{ft}^3$.

PLAN

1. Find the decrease in liquid level in the container by using conservation of mass.
2. Find the pressure in the container by apply the manometer equation.

SOLUTION

1. Conservation of mass (applied to liquid)

$$\begin{aligned}
\text{Gain in mass of liq. in tube} &= \text{Loss of mass of liq. in container} \\
(\text{Volume change in tube})\,\rho_{\text{liquid}} &= (\text{Volume change in container })\,\rho_{\text{liquid}} \\
\forall_{\text{tube}} &= \forall_{\text{container}}
\end{aligned}$$

$$\begin{aligned}
(\pi/4)D_{\text{tube}}^2 \times \ell &= (\pi/4)D_{\text{container}}^2 \times (\Delta h)_{\text{container}} \\
(\Delta h)_{\text{container}} &= \left(\frac{D_{\text{tube}}}{D_{\text{container}}}\right)^2 \ell \\
(\Delta h)_{\text{container}} &= \left(\frac{1}{10}\right)^2 \times 3\,\text{ft} \\
&= 0.03\,\text{ft}
\end{aligned}$$

2. Manometer equation (point 1 = free surface of liquid in the tube; point 2 = free surface of liquid in the container)

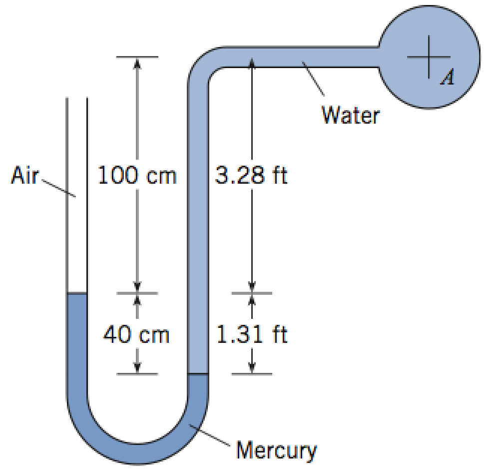
$$\begin{aligned}
p_{\text{container}} &= (\ell \sin 10^\circ + \Delta h)\gamma \\
&= (3 \sin 10^\circ + .03)\,\text{ft} \times 50\,\text{lbf}/\,\text{ft}^3 \\
&= 27.548\,\text{lbf}/\,\text{ft}^2
\end{aligned}$$

$$\boxed{p_{\text{container}} = 27.5 \text{ psfg}}$$

3.36: PROBLEM DEFINITION

Situation:

A pipe system has a manometer attached to it.



Find:

Gage pressure at center of pipe A (psi, kPa).

Properties:

Mercury, Table A.4: $\gamma = 1.33 \times 10^5\,\mathrm{N/\,m^3}$.

Water, Table A.5: $\gamma = 9810\,\mathrm{N/\,m^3}$.

PLAN

Apply the manometer equation.

SOLUTION

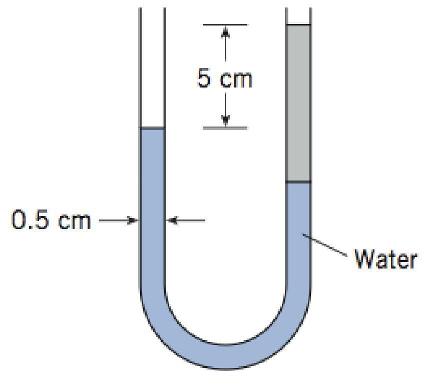
Manometer equation

$$\begin{aligned} p_A &= 1.31\,\mathrm{ft} \times 847\,\mathrm{lbf/\,ft^3} - 4.59\,\mathrm{ft} \times 62.4\,\mathrm{lbf/\,ft^3} \\ &= 823.2 \text{ psf} \\ & \boxed{p_A = 5.72 \text{ psig}} \\ p_A &= 0.4\,\mathrm{m} \times 1.33 \times 10^5\,\mathrm{N/\,m^3} - 1.4\,\mathrm{m} \times 9810\,\mathrm{N/\,m^3} \\ & \boxed{p_A = 39.5 \text{ kPa gage}} \end{aligned}$$

3.37: PROBLEM DEFINITION

Situation:

A U-tube manometer can be used to measure γ.
Initial state: A U-tube manometer contains water.
Final state: An unknown liquid ($\forall = 2\,\text{cm}^3$) is added to the right leg
$d = 0.5\,\text{cm}$, $\Delta h = 5\,\text{cm}$.



Find:

Specific weight of unknown fluid (N/m^3).

SOLUTION

1. Find the length of the column of the unknown liquid.

$$\forall = (\pi/4)(0.5\,\text{cm})^2\ell = 2\,\text{cm}^3$$

Solve for ℓ

$$\ell = 10.186\text{ cm}$$

2. Manometer equation (from water surface in left leg to liquid surface in right leg)

$$0 + (10.186\text{ cm - 5 cm})(10^{-2}\text{ m/cm})(9810\text{ N/m}^3)$$
$$-(10.186\text{ cm})(10^{-2}\text{ m/cm})\gamma_{\text{liq.}} = 0$$

Solve for $\gamma_{\text{liq.}}$

$$508.7\text{ Pa } - 0.10186\gamma_{\text{liq.}} = 0$$

$$\boxed{\gamma_{\text{liq.}} = 4995\text{ N/m}^3}$$

3.38: PROBLEM DEFINITION

Situation:

Mercury and water are poured into a tube.

$\ell_{\text{mercury}} = \ell_{\text{water}} = 375\,\text{mm}$.

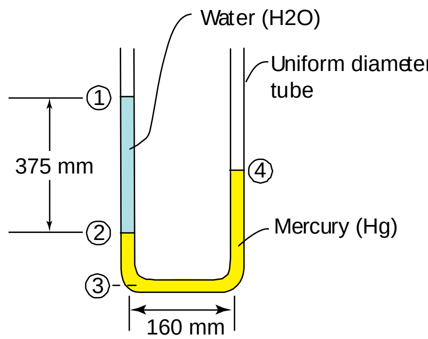
Find:

Locate the water surface (mm).

Locate the mercury surface (mm).

Find the maximum pressure in the U-tube (kPa gage).

Sketch:



Assumptions:

Uniform diameter tube.

Properties:

Mercury (20 °C), Table A.4, $\gamma_{Hg} = 133000\,\text{N/m}^3$.

Water (20 °C), Table A.5, $\gamma = 9790\,\text{N/m}^3$.

PLAN

1. Find p_2 by applying the hydrostatic equation.
2. Find $(z_4 - z_2)$ by applying the hydrostatic equation.
3. Solve $(z_2 - z_3)$ by using the fact that the mercury column has a fixed length.
4. Locate the liquid surfaces by using lengths from steps 2 and 3.
5. Solve for the maximum pressure by applying the hydrostatic equation to the mercury.

SOLUTION

1. Hydrostatic equation (apply to water column):

$$\begin{aligned} \frac{p_1}{\gamma_{H2O}} + z_1 &= \frac{p_2}{\gamma_{H2O}} + z_2 \\ 0 + z_1 &= \frac{p_2}{9710\,\text{N/m}^3} + z_2 \\ p_2 &= \left(9710\,\text{N/m}^3\right)(z_1 - z_2) = \left(9710\,\text{N/m}^3\right)(0.375\,\text{m}) = 3641\,\text{N/m}^2 \end{aligned}$$

Since the pressure across the water/mercury interface is constant, $p_{2,\text{ H2O}} = p_{2,\text{ Hg}}$.

2. Hydrostatic equation (apply to Hg column):

$$\begin{aligned}\frac{p_4}{\gamma_{\text{Hg}}} + z_4 &= \frac{p_2}{\gamma_{\text{Hg}}} + z_2 \\ 0 + z_4 &= \frac{3641\,\text{N/m}^2}{133000\,\text{N/m}^3} + z_2 \\ (z_4 - z_2) &= 27.38\,\text{m}\end{aligned}$$

3. Length constraint (length of Hg column is 375 mm):

$$(z_2 - z_3) + 160\,\text{mm} + (z_2 - z_3) + 27.38\,\text{mm} = 375\,\text{mm}$$
$$(z_2 - z_3) = 93.18\,\text{mm}$$

4. Locate surfaces:

$$\text{Water: } (z_1 - z_2) + (z_2 - z_3) = 375\,\text{mm} + 93.18\,\text{mm} = 468\,\text{mm}$$

The surface of the water is located 468 mm above the centerline of the horizontal leg

$$\text{Mercury: } (z_4 - z_2) + (z_2 - z_3) = 27.38\,\text{mm} + 93.18\,\text{mm} = 121\,\text{mm}$$

The surface of the mercury is located 121 mm above the centerline of the horizontal leg

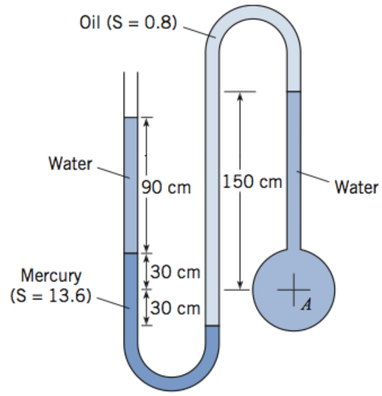
5. Hydrostatic Equation:

$$\begin{aligned}\frac{p_3}{\gamma_{\text{Hg}}} + z_3 &= \frac{p_4}{\gamma_{\text{Hg}}} + z_4 \\ p_3 &= \frac{p_4}{\gamma_{\text{Hg}}} + \gamma_{\text{Hg}}(z_4 - z_3) = 0 + \left(133000\,\text{N/m}^3\right)(0.121\,\text{m}) \\ &\boxed{p_3 = p_{\max} = 16.1\,\text{kPa gage}}\end{aligned}$$

3.39: PROBLEM DEFINITION

Situation:

A manometer tube is attached to a pipe.



Find:

Pressure at center of pipe A (kPa).

Properties:

$S_{\text{Hg}} = 13.6$, $S_{\text{oil}} = 0.8$, $\gamma_{\text{water}} = 9810\,\text{N/m}^3$.

SOLUTION

Manometer equation (apply from top of water column to point A)

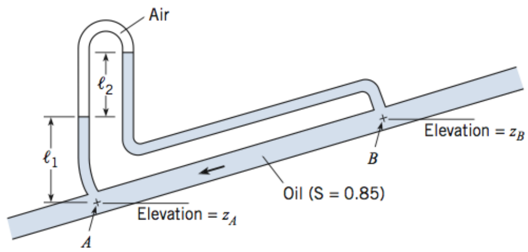
$$p_A = (0.9\,\text{m} + 0.6\,\text{m} \times 13.6 - 1.8\,\text{m} \times 0.8 + 1.5\,\text{m})9,810\,\text{N/m}^3 = 89,467 \text{ Pa}$$

$$\boxed{p_A = 89.47 \text{ kPa}}$$

3.40: PROBLEM DEFINITION

Situation:

A system is described in the problem statement.
$\ell_1 = 1\,\text{m},\ \ell_2 = 0.5\,\text{m}.$
$z_A = 10\,\text{m},\ z_B = 11\,\text{m}.$



Find:

(a) Difference in pressure between points A and B (kPa).
(b) Difference in piezometric head between points A and B (m).

Properties:

$\gamma = 9810\,\text{N/m}^3,\ S = 0.85.$

PLAN

Apply the manometer equation.

SOLUTION

Manometer equation (apply from A to B)

$$\begin{aligned} p_A - (1\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) + (0.5\,\text{m})\left(0.85 \times 9810\,\text{N/m}^3\right) &= p_B \\ p_A - p_B &= 4169\,\text{Pa} \end{aligned}$$

$$\boxed{p_A - p_B = 4.17\ \text{kPa}}$$

Piezometric head

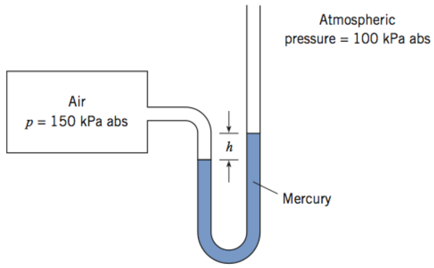
$$\begin{aligned} h_A - h_B &= \left(\frac{p_A}{\gamma} + z_A\right) - \left(\frac{p_B}{\gamma} + z_B\right) \\ &= \frac{p_A - p_B}{\gamma} + (z_A - z_B) \\ &= \frac{4169\,\text{N/m}^2}{0.85 \times 9810\,\text{N/m}^3} - 1\,\text{m} \\ &= -0.5\ \text{m} \end{aligned}$$

$$\boxed{h_A - h_B = -0.50\ \text{m}}$$

3.41: PROBLEM DEFINITION

Situation:

A manometer attached to a tank.



Find:

Manometer deflection when pressure in tank is doubled.

Properties:

$p_{atm} = 100\,\text{kPa}$, $p = 150\,\text{kPa}$.

SOLUTION

$$p - p_{atm} = \gamma h$$

For 150 kPa absolute pressure and an atmospheric pressure of 100 kPa,

$$\gamma h = 150 - 100 = 50 \text{ kPa}$$

For an absolute pressure of 300 kPa

$$\gamma h_{new} = 300 - 100 = 200 \text{ kPa}$$

Divide equations to eliminate the specific weight

$$\frac{h_{new}}{h} = \frac{200}{50} = 4.0$$

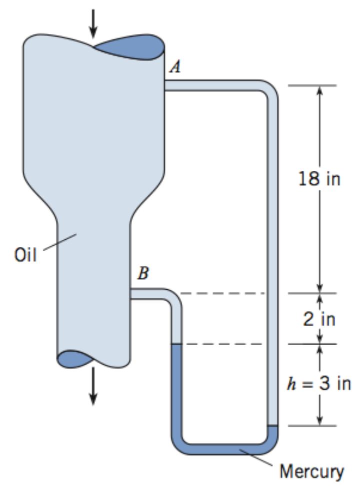
so

$$\boxed{h_{new} = 4.0h}$$

3.42: PROBLEM DEFINITION

Situation:

A manometer is tapped into a vertical conduit.



Find:

Difference in pressure between points A and B (psf).

Difference in piezometric head between points A and B (ft).

Properties:

From Table A.4, $\gamma_{Hg} = 847$ lbf/ft^3.

$$\begin{aligned}\gamma_{\text{oil}} &= (0.95)(62.4 \text{ lbf/ft}^3) \\ &= 59.28 \text{ lbf/ft}^3\end{aligned}$$

SOLUTION

Manometer equation

$$p_A + \left(\frac{18}{12}\right) \text{ ft } (\gamma_{\text{oil}}) + \left(\frac{2}{12}\right) \text{ ft. } \gamma_{\text{oil}} + \left(\frac{3}{12}\right) \text{ ft } \gamma_{\text{oil}} - \left(\frac{3}{12}\right) \text{ ft } \gamma_{\text{Hg}} - \left(\frac{2}{12}\right) \text{ ft } \gamma_{\text{oil}} = p_B$$

thus

$$\begin{aligned}p_A - p_B &= (-1.75 \text{ ft.})(59.28 \text{ lbf/ft}^3) + (0.25 \text{ ft.})(847 \text{ lbf/ft}^3) \\ & \boxed{p_A - p_B = 108 \text{ psf}}\end{aligned}$$

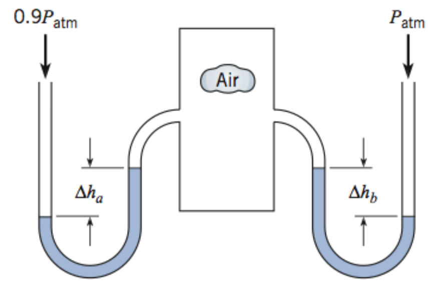
Piezometric head

$$\begin{aligned}h_A - h_B &= \frac{p_A - p_B}{\gamma_{\text{oil}}} + z_A - z_B \\ h_A - h_B &= \frac{108.01 \text{ lbf/ft}^2}{59.28 \text{ lbf/ft}^3} + (1.5 - 0) \text{ ft} \\ & \boxed{h_A - h_B = 3.32 \text{ ft}}\end{aligned}$$

3.43: PROBLEM DEFINITION

Situation:

Two manometers attached to an air tank.



Find:

Difference in deflection between manometers (m).

Properties:

$p_{\text{left}} = 0.9p_{\text{atm}}$, $p_{\text{right}} = p_{\text{atm}} = 100\,\text{kPa}$.
$\gamma_w = 9810\,\text{N/m}^3$.

SOLUTION

The pressure in the tank using manometer b is

$$p_t = p_{atm} - \gamma_w \Delta h_b$$

and using manometer a is

$$p_t = 0.9p_{atm} - \gamma_w \Delta h_a$$

Combine equations

$$p_{atm} - \gamma_w \Delta h_b = 0.9p_{atm} - \gamma_w \Delta h_a$$

or

$$0.1p_{atm} = \gamma_w(\Delta h_b - \Delta h_a)$$

Solve for the difference in deflection

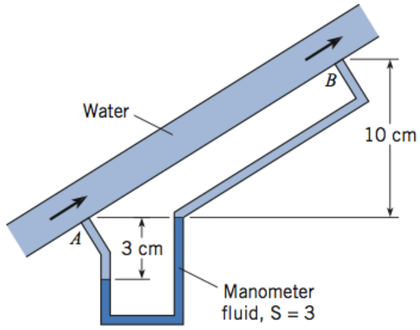
$$\begin{aligned}\Delta h_b - \Delta h_a &= \frac{0.1p_{atm}}{\gamma_w} \\ &= \frac{0.1 \times 10^5\,\text{Pa}}{9.81 \times 10^3\,\text{N/m}^3}\end{aligned}$$

$$\boxed{\Delta h_b - \Delta h_a = 1.02\text{ m}}$$

3.44: PROBLEM DEFINITION

Situation:

Manometer–measuring pressure difference in a pipe.



Find:

(a) Pressure difference $(p_A - p_B)$ in kPa.

(b) Piezometric pressure difference $(p_{zA} - p_{zB})$ in kPa.

Properties:

$S = 3.0$.

PLAN

Apply the manometer equation. Use the definition of piezometric pressure.

SOLUTION

Manometer equation (apply between points A & B)

$$p_B = p_A + 0.03\gamma_f - 0.03\gamma_m - 0.1\gamma_f$$

or

$$p_A - p_B = -0.03(\gamma_f - \gamma_m) + 0.1\gamma_f$$

Substitute in values

$$p_A - p_B = -0.03\,\text{m}(9810\,\text{N/m}^3 - 3 \times 9810\,\text{N/m}^3) + 0.1 \times 9810\,\text{N/m}^3$$

$$\boxed{p_A - p_B = 1.57\,\text{kPa}}$$

Definition of piezometric pressure

$$p_z \equiv p + \gamma z$$

Thus

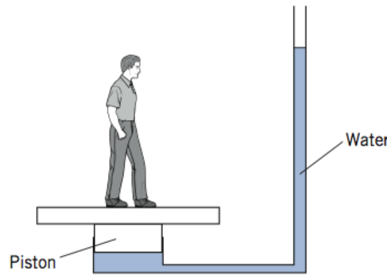
$$\begin{aligned} p_{zA} - p_{zB} &= \left(p_A + \gamma_{H2O}\, z_A\right) - \left(p_B + \gamma_{H2O}\, z_B\right) \\ &= (p_A - p_B) + \gamma_{H2O}\,(z_A - z_B) \\ &= 1.57\,\text{kPa} + \left(9.81\,\text{kN/m}^3\right)(-0.1\,\text{m}) = 0.589\text{ kPa} \end{aligned}$$

$$\boxed{p_{zA} - p_{zB} = 0.589\text{ kPa}}$$

3.45: PROBLEM DEFINITION

Situation:

A piston scale is used to measure weight.
Weight range: $60 = 250\,\text{lbf}$.
Height range: $4 - 6\,\text{ft}$ tall.



Find:

Select a piston size and standpipe diameter.

SOLUTION

First of all neglect the weight of the piston and find the piston area which will give reasonable manometer deflections. Equating the force on the piston, the piston area and the deflection of the manometer gives

$$W = \Delta h \gamma A$$

where γ is the specific weight of the water. Thus, solving for the area one has

$$A = \frac{W}{\gamma \Delta h}$$

For a four foot person weighing 60 lbf, the area for a 4 foot deflection (manometer near eye level of person) would be

$$A = \frac{60\,\text{lbf}}{62.4\,\text{lbf/}\,\text{ft}^3 \times 4\,\text{ft}^2} = 0.24\ \text{ft}^2$$

while for a 250 lbf person 6 feet tall would be

$$A = \frac{250\,\text{lbf}}{62.4\,\text{lbf/}\,\text{ft}^3 \times 6\,\text{ft}} = 0.66\ \text{ft}^2$$

It will not be possible to maintain the manometer at the eye level for each person so take a piston area of 0.5 ft^2. This would give a deflection of 1.92 ft for the 4-foot, 60 lbf person and 8 ft for the 6-foot, 250 lbf person. This is a good compromise.

The size of the standpipe does not affect the pressure. The pipe should be big enough so the person can easily see the water level and be able to read the calibration on the scale. A 1/2 inch diameter tube would probably suffice. Thus the ratio of the standpipe area to the piston area would be

$$\frac{A_{\text{pipe}}}{A_{\text{piston}}} = \frac{0.785 \times (0.5\,\text{in})^2}{0.5 \times 144\,\text{in}^2/\,\text{ft}^2} = 0.0027$$

This means that when the water level rises to 8 ft, the piston will only have moved by $0.0027 \times 8 = 0.0216$ ft or 0.26 inches.
The weight of the piston will cause an initial deflection of the manometer. If the piston weight is 5 lbf or less, the initial deflection of the manometer would be

$$\Delta h_o = \frac{W_{\text{piston}}}{\gamma A_{\text{piston}}} = 0.16 \text{ ft or } 1.92 \text{ inches}$$

This will not significantly affect the range of the manometer (between 2 and 8 feet). The system would be calibrated by putting knows weights on the scale and marking the position on the standpipe. The scale would be linear.

3.46: PROBLEM DEFINITION

Situation:

The boiling point of water decreases with elevation because p_{atm} decreases.
$z_1 = 2000\,\text{m}$, $z_2 = 4000\,\text{m}$.

Find:

Boiling point of water (°C) at z_1 and z_2.

Assumptions:

$T_{\text{sea level}} = 296$ K= 23°C.
Standard atmosphere.

Properties:

Table A.2: $R = 287\,\text{J/kg K}$.

PLAN

The pressure of boiling (p_{vapor}) corresponds to local atmospheric pressure.

1. Find the atmospheric pressure by calculating the pressure in the troposphere.
2. Find boiling temperature at 2000 m by interpolating in Table A.5.
3. Find boiling temperature at 4000 m by interpolating in Table A.5.

SOLUTION

1. Atmospheric pressure:

$$\begin{aligned} p &= p_0 \left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R} \\ &= 101.3\,\text{kPa}\left[\frac{296\,\text{K} - 5.87\,\text{K/km}(z - z_0)}{296\,\text{K}}\right]^{g/\alpha R} \end{aligned}$$

where

$$g/\alpha R = \frac{9.81\,\text{m/s}^2}{(5.87 \times 10^{-3})\ \text{K/m} \times 287\,\text{J/kg K}} = 5.823$$

So

$$p_{2000\,\text{m}} = 101.3\,\text{kPa}\left[\frac{296\,\text{K} - 5.87\,\text{K/km}(2.0\,\text{km})}{296\,\text{K}}\right]^{5.823} = 80.0\ \text{kPa}$$

$$p_{4000\,\text{m}} = 101.3\,\text{kPa}\left[\frac{296\,\text{K} - 5.87\,\text{K/km}(4.0\,\text{km})}{296\,\text{K}}\right]^{5.823} = 62.6\ \text{kPa}$$

2. Boiling temperature @ 2000 m.

$$T = 90\,^\circ\text{C} + \left(\frac{(80.0 - 70.1)\,\text{kPa}}{(101.3 - 70.1)\,\text{kPa}}\right)(10\,^\circ\text{C}) = 93.2\,^\circ\text{C}$$

$$\boxed{T_{\text{boiling, 2000 m}} \approx 93.2\,^\circ\text{C}}$$

3. Boiling temperature @ 4000 m.

$$T = 80\,^\circ\text{C} + \left(\frac{(62.6 - 47.4)\,\text{kPa}}{(70.1 - 47.4)\,\text{kPa}}\right)(10\,^\circ\text{C}) = 86.7\,^\circ\text{C}$$

$$\boxed{T_{\text{boiling, 4000 m}} \approx 86.7\,^{\circ}\text{C}}$$

3.47: PROBLEM DEFINITION

Situation:

Pressure variation from a lake to atmosphere.
$h = 10\,\text{m}$, $z_2 = 4000\,\text{m}$.

Find:

Plot pressure variation.

Assumptions:

$p_{atm} = 101.3\,\text{kPa}$.
The lake surface is at sea level.

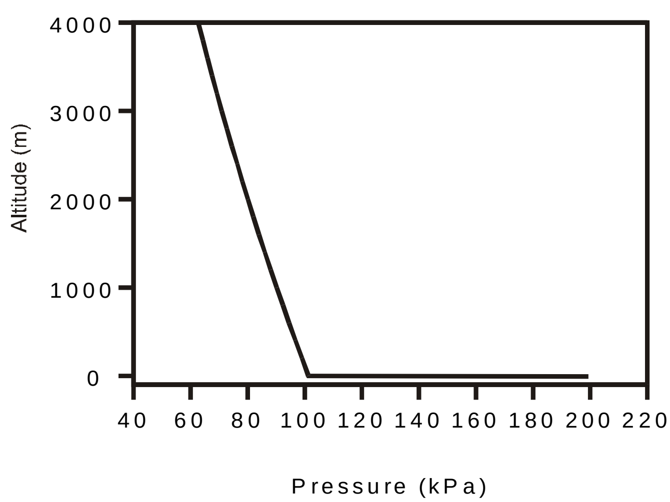
SOLUTION

Atmosphere pressure variation (troposphere)

$$p_A = 101.3\,\text{kPa}\left(1 - \frac{5.87 \times 10^{-3}\,\text{K/ m} \times z}{296\,\text{K}}\right)^{5.823}$$

Pressure in water

$$p_w = 101.3\,\text{kPa} + 9.810\,\text{N/ m}^3 \times z$$



3.48: PROBLEM DEFINITION

Situation:

A woman breathing.

$z = 18000\,\text{ft}$.

Find:

Breathing rate.

Assumptions:

Volume drawn in per breath is the same.

Air is an ideal gas.

Properties:

$T = 59\,^\circ\text{F}$, $p_{atm} = 14.7$ psia.

SOLUTION

Let $b\forall\rho$ = constant where b = breathing rate = number of breaths for each unit of time, $\forall$ = volume per breath, and ρ = mass density of air. Assume 1 is sea level and point 2 is 18,000 ft. elevation. Then

$$\begin{aligned} b_1\forall_1\rho_1 &= b_2\forall_2\rho_2 \\ b_2 &= b_1(\forall_1/\forall_2)(\rho_1/\rho_2) \\ \text{then } b_2 &= b_1(\rho_1/\rho_2) \text{ but } \rho = (p/RT) \\ \text{Thus, } b_2 &= b_1(p_1/p_2)(T_2/T_1) \\ p_2 &= p_1(T_2/T_1)^{g/\alpha R} \\ p_1/p_2 &= (T_2/T_1)^{-g/\alpha R} \\ \text{Then } b_2 &= b_1(T_2/T_1)^{1-g/\alpha R} \end{aligned}$$

Since the volume drawn in per breath is the same

$$b_2 = b_1(\rho_1/\rho_2)$$

Ideal gas law

$$b_2 = b_1(p_1/p_2)(T_2/T_1)$$
$$p_1/p_2 = (T_2/T_1)^{-g/\alpha R}$$
$$b_2 = b_1(T_2/T_1)^{1-g/\alpha R}$$

where $b_1 = 16$ breaths per minute and $T_1 = 59^\circ F = 519^\circ R$

$$\begin{aligned} T_2 &= T_1 - \alpha(z_2 - z_1) = 519 - 3.221 \times 10^{-3}(18,000 - 0) = 461.0\ ^\text{o}\text{R} \\ b_2 &= 16(461.0/519)^{1-32.2/(3.221\times10^{-3}\times1,715)} \end{aligned}$$

$$\boxed{b_2 = 28.4 \text{ breaths per minute}}$$

3.49: PROBLEM DEFINITION

Situation:

A pressure gage in an airplane.
$z_0 = 1\,\text{km}$.

Find:

Elevation (km).
Temperature ($^\circ$C).

Properties:

$a = 5.87\,^\circ\text{C/ km}$, $p_0 = 95\,\text{kPa}$.
$T_0 = 10\,^\circ\text{C}$, $p = 75\,\text{kPa}$.

SOLUTION

Atmosphere pressure variation (troposphere)

$$p = p_0\left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R}$$

$$75\,\text{kPa} = 95\,\text{kPa}\left[\frac{283\,\text{K} - 5.87\,\text{K/ km}(z-1)\,\text{km}}{283\,\text{K}}\right]^{9.81/(5.87\times10^{-3}\times287)}$$

$$\boxed{z = 2.91\ \text{km}}$$

$$T = T_0 - \alpha(z - z_0)$$

$$= 10\,^\circ\text{C} - 5.87\,^\circ\text{C/ km}(2.91 - 1)\,\text{km}$$

$$\boxed{T = -1.21^\circ\text{C}}$$

3.50: PROBLEM DEFINITION

Situation:
Denver, CO (the mile-high city) is described in the problem statement.

Find:
Pressure in both SI and traditional units.
Temperature in both SI and traditional units.
Density in both SI and traditional units.

Properties:
Air, Table A.2: $R = 287\,\text{J/kg K} = 1716\,\text{ft lbf/slug}^\circ\text{R}$.

SOLUTION

Atmosphere pressure variation (troposphere)

$$\begin{aligned}
T &= T_0 - \alpha(z - z_0) \\
&= 533\,^\circ\text{R} - 3.221 \times 10^{-3}\,^\circ\text{R/ft}(5,280 - 0)\,\text{ft} = 516^\circ\text{R} \\
&= 296\,\text{K} - 5.87 \times 10^{-3}\,\text{K/m}(1,609 - 0)\,\text{m} \\
&\boxed{T = 287 \text{ K} = 516 \text{ }^\circ\text{R}} \\
p &= p_0(T/T_0)^{g/\alpha R} \\
&= 14.7 \text{ psia}(516^\circ\text{R}/533^\circ\text{R})^{5.823} \\
&p = 12.2 \text{ psia} \\
p_a &= 101.3\,\text{kPa}(287\,\text{K}/296\,\text{K})^{9.81/(5.87\times10^{-3}\times287)} \\
&\boxed{p_a = 86.0 \text{ kPa} = 12.2 \text{ psia}}
\end{aligned}$$

Ideal gas law

$$\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{12.2 \text{ psia} \times 144\,\text{in}^2/\text{ft}^2}{1,715\,\text{ft lbf/slug}^\circ\text{R} \times 516^\circ\text{R}} \\
&= 0.00199 \text{ slugs/ft}^3 \\
\rho &= \frac{86,000\,\text{kPa}}{287\,\text{K} \times 287\,\text{J/kg K}}
\end{aligned}$$

$$\boxed{\rho = 1.04 \text{ kg/m}^3 = 0.00199 \text{ slugs/ft}^3}$$

3.51: PROBLEM DEFINITION

Situation:

A force due to pressure is acting on an airplane window.
Window is flat & elliptical.
$a = 0.3\,\mathrm{m},\ b = 0.2\,\mathrm{m}.$
$p_{\text{inside}} = 100\,\mathrm{kPa},\ z = 10\,\mathrm{km}.$

Find:

Outward force on the window (in N).

PLAN

Find the force on the window by using $F = \Delta p A$. The steps are
1. Find outside air pressure by applying Eq. (3.16) in EFM9e.
2. Find the area by using formula from Figure A.1 in EFM9e
3. Find the force F.

SOLUTION

1. Atmospheric pressure

$$\frac{g}{\alpha R} = \frac{(9.81\,\mathrm{m/s^2})}{(5.87 \times 10^{-3}\,\mathrm{K/m})\,(287\,\mathrm{J/kg\cdot K})} = 5.823$$

$$\begin{aligned} p_{\text{outside}} &= p_0 \left[\frac{T_0 - \alpha(z - z_0)}{T_0}\right]^{g/\alpha R} \\ &= 101.3\,\mathrm{kPa} \left[\frac{296\,\mathrm{K} - (5.87 \times 10^{-3}\,\mathrm{K/m})\,(10000 - 0)\,\mathrm{m}}{296\,\mathrm{K}}\right]^{5.823} \\ &= 27.97\,\mathrm{kPa} \end{aligned}$$

2. Area

$$A = \pi ab = \pi\,(0.3\,\mathrm{m})\,(0.2\,\mathrm{m}) = 0.188\,5\,\mathrm{m^2}$$

3. Force

$$F = \Delta p A = (100 - 27.97)\,\frac{\mathrm{kN}}{\mathrm{m^2}}\left(0.188\,5\,\mathrm{m^2}\right)$$

$$\boxed{F = 13.6\,\mathrm{kN}}$$

REVIEW

- While the window is small, the force is surprisingly large. This force, which is about 3100 lbf, is equal to the weight of a car!

3.52: PROBLEM DEFINITION

Situation:

Atmospheric conditions on Mars.

- Temperature at the Martian surface is $T = -63\,^{\circ}\mathrm{C} = 210\,\mathrm{K}$ The pressure at the Martian surface is $p = 7$ mbar.
- The atmosphere consists primarily of CO_2 (95.3%) with small amounts of nitrogen and argon.
- Acceleration due to gravity on the surface is 3.72 m/s^2.
- Temperature distribution: approximately constant from surface to 14 km. Temperature decreases linearly at a lapse rate of 1.5°C/km from 14 to 34 km.

Find:

Pressure at an elevation of 8 km.

Pressure at an elevation of 30 km.

Assumptions:

Assume the atmosphere is totally carbon dioxide.

Properties:

CO_2 (from Table A.2): the gas constant is R =189 J/kg·K.

PLAN

Derive equations for atmospheric pressure variation from first principles.

SOLUTION

A.) Elevation of 8 km.

Differential equation describing pressure variation in a hydrostatic fluid

$$\frac{dp}{dz} = -\rho g \tag{1}$$

Ideal gas law

$$\rho = \frac{p}{RT} \tag{2}$$

Combine Eqs. (1) and (2)

$$\frac{dp}{dz} = -\frac{p}{RT} g \tag{3}$$

Integrate Eq. (3) for constant temperature

$$\ln \frac{p}{p_o} = -\frac{(z - z_o)g}{RT} \tag{4}$$

Substitute in values

$$\begin{aligned} \ln \frac{p}{p_o} &= -\frac{(8000\,\mathrm{m})\,(3.72\,\mathrm{m/\,s^2})}{(189\,\mathrm{J/\,kg\cdot\ K})\,(210\,\mathrm{K})} \\ &= -0.7498 \end{aligned}$$

Thus

$$\begin{aligned} \frac{p}{p_o} &= \exp(-0.7498) \\ &= 0.4725 \end{aligned}$$

and

$$\begin{aligned} p &= (7\,\mathrm{mbar}) \times 0.4725 \\ &= 3.308 \text{ mbar} \end{aligned}$$

$$\boxed{p(z = 8\,\mathrm{km}) = 3.31\,\mathrm{mbar}}$$

B.) Elevation of 30 km.

Apply Eq. (4) to find the pressure at $z = 14\,\mathrm{km}$

$$\begin{aligned} \frac{p_{14\text{ km}}}{p_o} &= \exp\left[-\frac{(14000\,\mathrm{m})\,(3.72\,\mathrm{m/\,s^2})}{(189\,\mathrm{J/\,kg\cdot\ K})\,(210\,\mathrm{K})}\right] \\ &= \exp(-1.3122) \\ &= 0.2692 \\ p_{14\text{ km}} &= (7\,\mathrm{mbar})\,(0.2692) \\ &= 1.884 \text{ mbar} \end{aligned}$$

In the region of varying temperature Eq. (3) becomes

$$\frac{dp}{dz} = \frac{pg}{R[T_o + \alpha(z - z_o)]}$$

where the subscript o refers to the conditions at 14 km and α is the lapse rate above 14 km. Integrating gives

$$\frac{p}{p_o} = \left[\frac{T_o - \alpha(z - z_o)}{T_o}\right]^{g/\alpha R}$$

Calculations for $z = 30\,\mathrm{km}$.

$$\begin{aligned} \frac{p}{(1.884 \text{ mbar})} &= \left[\frac{210\,\mathrm{K} - 0.0015\,\mathrm{K/\,m}(30000 - 14000)\,\mathrm{m}}{210\,\mathrm{K}}\right]^{3.72/(0.0015\times 189)} \\ &= 0.2034 \\ p &= (1.884 \text{ mbar})\,0.2034 \\ &= 0.3832\,\mathrm{mbar} \end{aligned}$$

$$\boxed{p(z = 30\,\mathrm{km}) = 0.383\,\mathrm{mbar}}$$

3.53: PROBLEM DEFINITION

Situation:

The US standard atmosphere from 0 to 30 km is described in the problem statement.

Find:

Design a computer program that calculates the pressure and density.

SOLUTION

The following are sample values obtained using computer calculations.

altitude (km)	temperature (^{o}C)	pressure (kPa)	density (kg/m^3)
10	-35.7	27.9	0.409
15	-57.5	12.8	0.208
25	-46.1	2.75	0.042

Problem 3.54

Using Section 3.4 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words and equations in ways that enhance the effectiveness of your communication. There are many possible good answers to these questions. Here, we give some examples.

---

a. For hydrostatic conditions, what do typical pressure distributions on a panel look like? Sketch three examples that correspond to different situations.

- Arrows (which represent normal stress) are compressive.
- Arrows are normal to the panel.
- Pressure varies linearly with elevation.
- Slope of pressure with respect to elevation (dp/dz) equal the negative of specific weight $(dp/dz = -\gamma)$.

---

b. What is a center of pressure? What is a centroid of area?

- The center of pressure is an imaginary point. If pressure distribution is replaced with a statically equivalent "point force," then this resultant force acts at the "center of pressure."
- The centroid of area is the "geometric center." For a flat plate, the centroid of area is at the same location as the center of gravity for a thin uniform-density plate of that shape,

---

c. In Eq. (3.23), what does $\overline{p}$ mean? What factors influence the value of $\overline{p}$?

- P-bar ($\overline{p}$)is the pressure evaluated at the elevation of the centroid of area.
- Typically $\overline{p} = \gamma\overline{z}$. Since this equation has two variable, there are two factors that influence the value of $\overline{p}$:
  - The specific weight of the liquid.
  - The vertical distance $\overline{z}$ from liquid surface to the centroid of the panel.

---

d. What is the relationship between the pressure distribution on a panel and the resultant force?

$$\left|\overrightarrow{F}\right| = \int_{\text{panel area}} p dA$$

---

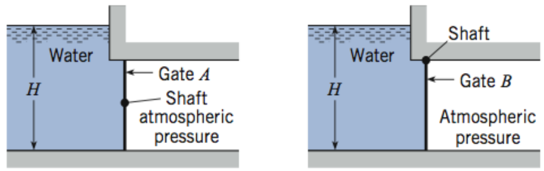
e. How far is the center of pressure from the centroid of area? What factors influence this distance?

- Distance is given by $\overline{I}/(\overline{y}A)$. Thus
  - The shape of the panel determines $\overline{I}$.
  - The depth of liquid and the angle of the panel determine $\overline{y}$.
  - The size of the panel determines A.

3.55: PROBLEM DEFINITION

Situation:

Two submerged gates are described in the problem statement.



Find:

Determine which statements are true.

(a) T_A increases with H.

(b) T_B increases with H.

(c) T_A does not change with H.

(d) T_B does not change with H.

PLAN

Apply equilibrium equations. Apply hydrostatic force equations.

SOLUTION

Let the horizontal gate dimension be given as b and the vertical dimension, h.

Torque (Gate A). Equilibrium. Sum moments about the hinge:

$$T_A = F(y_{cp} - \bar{y}) \tag{1}$$

Hydrostatic force equation (magnitude)

$$\begin{aligned} F &= \bar{p}A \\ &= \gamma\left(H - \frac{h}{2}\right) bh \end{aligned} \tag{2}$$

Hydrostatic force equation (center of pressure)

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{bh^3}{12} \frac{1}{\left(H - \frac{h}{2}\right) bh} \end{aligned} \tag{3}$$

Combine eqns. 1 to 3:

$$\begin{aligned} T_A &= F(y_{cp} - \bar{y}) \\ &= \left[\gamma\left(H - \frac{h}{2}\right) bh\right] \left[\frac{bh^3}{12} \frac{1}{\left(H - \frac{h}{2}\right) bh}\right] \\ &= \gamma \frac{bh^3}{12} \end{aligned} \tag{4}$$

Therefore, T_A does not change with H.

Torque (gate B). Equilibrium. Sum moments about the hinge:

$$T_B = F\left(\frac{h}{2} + y_{cp} - \bar{y}\right) \tag{5}$$

Combine eqns. 2, 3, and 5:

$$\begin{aligned} T_B &= F\left(\frac{h}{2} + y_{cp} - \bar{y}\right) \\ &= \left[\gamma\left(H - \frac{h}{2}\right) bh\right]\left[\frac{h}{2} + \frac{bh^3}{12}\frac{1}{\left(H - \frac{h}{2}\right) bh}\right] \\ &= \frac{\gamma h^2 b\,(3H - h)}{6} \end{aligned} \tag{6}$$

Thus, T_A is constant but T_B increases with H.

Case (b) is a correct choice.

Case (c) is a correct choice.

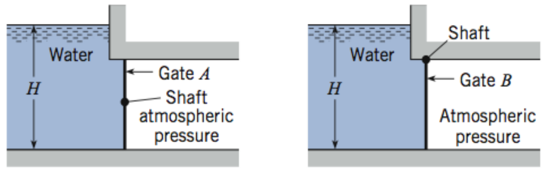
REVIEW

Case A provides an example of how to design a gate so that the torque to hold the gate closed is independent of water depth.

3.56: PROBLEM DEFINITION

Situation:

This problem involves Gate A (see sketch).



Find:

Choose the statements that are valid for Gate A.

(a) The hydrostatic force acting on the gate increases as H increases.

(b) The distance between the CP on the gate and the centroid of the gate decreases as H increases.

(c) The distance between the CP on the gate and the centroid of the gate remains constant as H increases.

(d) The torque applied to the shaft to prevent the gate from turning must be increased as H increases.

(e) The torque applied to the shaft to prevent the gate from turning remains constant as H increases.

SOLUTION

Let the horizontal gate dimension be given as b and the vertical dimension, h.

Torque (Gate A). Sum moments about the hinge:

$$T_A = F(y_{cp} - \bar{y}) \tag{1}$$

Hydrostatic force equation (magnitude)

$$\begin{aligned} F &= \bar{p}A \\ &= \gamma\left(H - \frac{h}{2}\right)bh \end{aligned} \tag{2}$$

Hydrostatic force equation (center of pressure)

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{bh^3}{12}\frac{1}{\left(H - \frac{h}{2}\right)bh} \end{aligned} \tag{3}$$

Combine eqns. 1 to 3:

$$\begin{aligned} T_A &= F(y_{cp} - \bar{y}) \\ &= \left[\gamma\left(H - \frac{h}{2}\right)bh\right]\left[\frac{bh^3}{12}\frac{1}{\left(H - \frac{h}{2}\right)bh}\right] \\ &= \gamma\frac{bh^3}{12} \end{aligned} \tag{4}$$

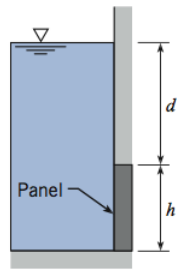
Therefore, T_A does not change with H. The correct answers are obtained by reviewing the above solution.

$$\boxed{\text{a, b, and e are valid statements.}}$$

3.57: PROBLEM DEFINITION

Situation:

Water exerts a load on square panel.
$d = 1\,\text{m}$, $h = 2\,\text{m}$



Find:

(a) Depth of the centroid (m).
(b) Resultant force on the panel (kN).
(c) Distance from the centroid to the center of pressure (m).

Properties:

Water (15 °C), Table A.5: $\gamma = 9800\,\text{N/m}^3$.

PLAN

1. Locate the centroid by inspection (center of the panel).
2. Find the pressure at the depth of the centroid using the hydrostatic equation.
2. Find the resultant force using $F = \overline{p}A$.
3. Find the distance between the centroid and the CP using $y_{cp} - \bar{y} = \overline{I}/(\bar{y}A)$

SOLUTION

1. Depth of the centroid of area:

$$\overline{z} = d + h/2 = 1\,\text{m} + (2\,\text{m})/2$$
$$\boxed{\overline{z} = 2\,\text{m}}$$

2. Hydrostatic equation:

$$\overline{p} = \gamma\overline{z} = \left(9800\,\text{N/m}^3\right)(2\,\text{m}) = 19.6\,\text{kPa}$$

3. Resultant force:

$$F = \overline{p}A = (19.6\,\text{kPa})(2\,\text{m})(2\,\text{m})$$
$$\boxed{F = 78.4\ \text{kN}}$$

4. Distance to CP:

- Find $\overline{I}$ using formula from Fig. A.1.

$$\overline{I} = \frac{bh^3}{12} = \frac{(2\,\mathrm{m})\,(2\,\mathrm{m})^3}{12} = 1.333\,\mathrm{m}^4$$

- Recognize that $\bar{y} = \overline{z} = 2\,\mathrm{m}$.

- Final calculation:

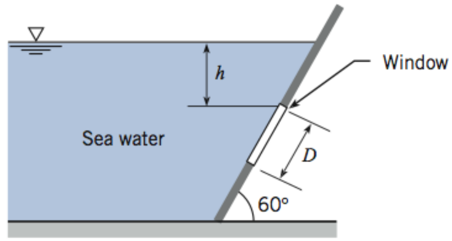
$$y_{cp} - \bar{y} = \frac{\overline{I}}{\bar{y}A} = \frac{(1.333\,\mathrm{m}^4)}{(2\,\mathrm{m})\,(2\,\mathrm{m})^2}$$

$$\boxed{y_{cp} - \bar{y} = 0.167\ \ \mathrm{m}}$$

3.58: PROBLEM DEFINITION

Situation:

Seawater exerts a load on a round viewing window.
$h = 1.2\,\text{m}$, $\theta = 60^0$, $D = 0.8\,\text{m}$



Find:

Hydrostatic force on the window (kN).
Locate the CP (center of pressure).

Properties:

Seawater: $S = 1.03$, $\gamma = 1.03 \times 9810\,\text{N/m}^3 = 10100\,\text{N/m}^3$.

PLAN

1. Find distances using trig.
2. Find the pressure at the depth of the centroid using the hydrostatic equation.
2. Find the resultant force using $F = \overline{p}A$.
3. Find the distance between the centroid and the CP using $y_{cp} - \bar{y} = \overline{I}/(\bar{y}A)$

SOLUTION

1. Distances:

- Slant height

$$\bar{y} = \frac{D}{2} + \frac{h}{\sin\theta} = \frac{0.8\,\text{m}}{2} + \frac{1.2\,\text{m}}{\sin 60^o} = 1.786\,\text{m}$$

- Depth of centroid

$$\Delta z = h + \frac{D}{2}\sin 60^o = 1.2\,\text{m} + \frac{0.8\,\text{m}}{2}\sin 60^o = 1.546\,\text{m}$$

2. Hydrostatic equation:

$$\overline{p} = \gamma\Delta z = \left(10100\,\text{N/m}^3\right)(1.546\,\text{m}) = 15.62\,\text{kPa}$$

3. Resultant force:

$$F = \overline{p}A = (15.62\,\text{kPa})\frac{\pi\,(0.8\,\text{m})^2}{4} = 7.85\ \text{kN}$$

$$\boxed{F = 7.85\ \text{kN}}$$

4. Distance to CP:

- Find $\bar{I}$ using formula from Fig. A.1.

$$\bar{I} = \frac{\pi r^4}{4} = \frac{\pi \left(0.4\,\text{m}\right)^4}{4} = 0.0201\,1\,\text{m}^4$$

- Final calculation:

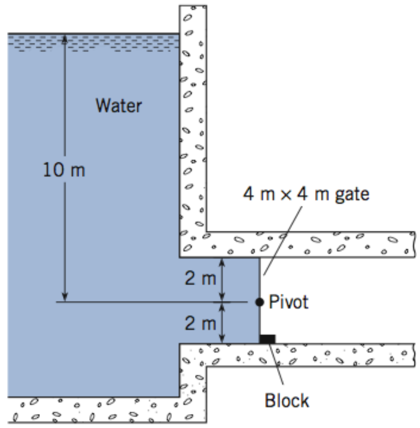
$$y_{cp} - \bar{y} = \frac{\overline{I}}{\bar{y}A} = \frac{(0.0201\,1\,\text{m}^4)}{(1.786\,\text{m})\left(\frac{\pi(0.8\,\text{m})^2}{4}\right)} = 0.0224\,\text{m}$$

$$\boxed{y_{cp} - \bar{y} = 22.4 \ \text{mm}}$$

3.59: PROBLEM DEFINITION

Situation:

Water exerts a load on a submerged gate.



Find:

Force of gate on block (kN).

SOLUTION

Hydrostatic force

$$
\begin{aligned}
F_{\text{hs}} &= \bar{p}A \\
&= \bar{y}\gamma A \\
&= (10\,\text{m}) \times \left(9810\,\text{N}/\,\text{m}^3\right) \times (4 \times 4)\ \text{m}^2 \\
&= 1.569\,6 \times 10^6\,\text{N}
\end{aligned}
$$

Center of pressure

$$
\begin{aligned}
y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\
&= \frac{bh^3/12}{\bar{y}A} \\
&= \frac{(4 \times 4^3/12)\ \text{m}^4}{(10\,\text{m})\,(4 \times 4)\ \text{m}^2} \\
&= 0.133\,33\,\text{m}
\end{aligned}
$$

Equilibrium (sum moments about the pivot)

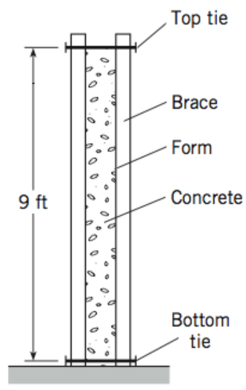
$$
\begin{aligned}
F_{\text{hs}}\,(y_{cp} - \bar{y}) - F_{\text{block}}\,(2\,\text{m}) &= 0 \\
\left(1.569\,6 \times 10^6\,\text{N}\right)(0.133\,33\,\text{m}) - F_{\text{block}}\,(2\,\text{m}) &= 0 \\
F_{\text{block}} &= 1.046 \times 10^5\,\text{N (acts to the left)}
\end{aligned}
$$

$$\boxed{F_{\text{gate}} = 105\,\text{kN (acts to the right)}}$$

3.60: PROBLEM DEFINITION

Situation:

Wet concrete is held in place with forms.
Ties are spaced on 2 feet centers.



Find:

Hydrostatic force per foot on form (lbf/ft).
Force exerted on bottom tie (lbf).

Properties:

Concrete, $\gamma = 150\,\text{lbf}/\,\text{ft}^3$.

SOLUTION

Hydrostatic force

$$
\begin{aligned}
F_{\text{hs}} &= \overline{p}A = \bar{y}\gamma A \\
&= 4.5\,\text{ft} \times 150\,\text{lbf}/\,\text{ft}^3 \times (9\,\text{ft}) \\
& \boxed{F_{\text{hs}} = 6075\,\frac{\text{lbf}}{\text{ft}}}
\end{aligned}
$$

Center of pressure

$$
\begin{aligned}
y_{cp} &= \bar{y} + \frac{\overline{I}}{\bar{y}A} \\
&= 4.5 + \frac{(1 \times 9^3)/12}{4.5 \times 9} \\
&= 6.00 \text{ ft}
\end{aligned}
$$

Equilibrium (sum moments about the top tie)

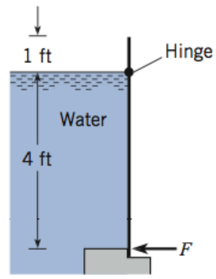
$$
\begin{aligned}
F_{\text{bottom tie}} &= \frac{F_{\text{hs}} \times y_{cp}}{h} \\
&= \frac{2\,\text{ft} \times 6075\,\text{lbf}/\,\text{ft} \times 6.00\,\text{ft}}{9\,\text{ft}} \\
& 8100\,\text{lbf}
\end{aligned}
$$

$$\boxed{F_{\text{bottom tie}} = 8100\,\text{lbf (tension)}}$$

3.61: PROBLEM DEFINITION

Situation:

A rectangular gate is hinged at the water line.
$h = 4\,\text{ft}$, $b = 10\,\text{ft}$.



Find:

Force to keep gate closed.

Properties:

From Table A.4, $\gamma_{\text{Water}} = 62.4$ lbf/ft^3.

SOLUTION

Hydrostatic Force (magnitude):

$$\begin{aligned} F_G &= \bar{p}A \\ &= \left(\gamma_{\text{H}_2\text{O}} \times \bar{y}\right)\left(40\ \text{ft}^2\right) \\ &= \left(62.4\,\text{lbf/}\,\text{ft}^3 \times 2\,\text{ft}\right)\left(40\,\text{ft}^2\right) \\ &= 4992\,\text{lbf} \end{aligned}$$

Center of pressure. Since the gate extends from the free surface of the water, F_G acts at 2/3 depth or 8/3 ft. below the water surface.

Moment Equilibrium. (sum moments about the hinge)

$$\begin{aligned} \sum M &= 0 \\ (F_G \times 8/3\,\text{ft}) - (4\,\text{ft})\,F &= 0 \end{aligned}$$

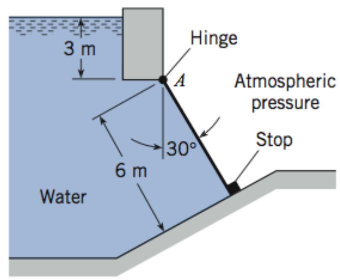
$$F = \frac{4992\,\text{lbf} \times 8/3\,\text{ft}}{4\,\text{ft}}$$

$$\boxed{F = 3330\ \text{lbf to the left}}$$

3.62: PROBLEM DEFINITION

Situation:

A submerged gate sits at an angle.
$h = 6\,\text{m}$, $b = 4\,\text{m}$, $\theta = 30\,°$.



Find:

Reaction at point A.

Assumptions:

Gate is weightless.

Properties:

Water, Table A.5: $\gamma = 9810\,\text{N/m}^3$.

PLAN

The reaction at A can be found by summing moments about the stop. The steps are
1. Find the hydrostatic force.
2. Locate the center of pressure.
3. Sum moments about the stop.

SOLUTION

1. Hydrostatic force (magnitude)

$$\begin{aligned} F &= \bar{p}A \\ &= (3\text{ m } + 3\text{ m} \times \cos 30°)(9810\text{ N/m}^3) \times 24\text{ m}^2 \\ F &= 1,318,000\text{ N} \end{aligned}$$

2. Center of pressure:

$$\begin{aligned} \bar{y} &= 3 + \frac{3}{\cos 30^{\text{o}}} \\ &= 6.464\text{ m} \\ y_{cp} - y &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{(4 \times 6^3/12)\,\text{m}^4}{6.464\text{ m} \times 24\text{ m}^2} \\ &= 0.4641\text{ m} \end{aligned}$$

3. Moment equilibrium about the stop:

$$\begin{aligned} \sum M_{\text{stop}} &= 0 \\ (6\,\text{m})\,R_A - (3\,\text{m} - 0.464\,\text{m}) \times 1,318,000\,\text{N} &= 0 \end{aligned}$$

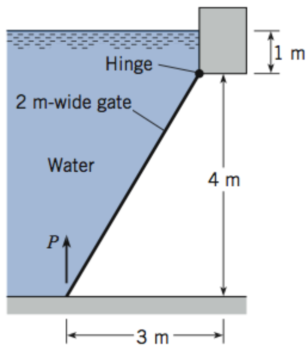
Thus

$$\boxed{R_A = 557\,\text{kN (acting normal to the gate)}}$$

3.63: PROBLEM DEFINITION

Situation:

A submerged gate holds back water.
$b = 2\,\text{m}$



Find:

Force P required to begin to open gate (kN).

Assumptions:

Gate is weightless.

Properties:

Water, Table A.5: $\gamma = 9810\,\text{N/m}^3$.

SOLUTION

The length of gate is $\sqrt{4^2 + 3^2} = 5$ m
Hydrostatic force

$$\begin{aligned} F &= \bar{p}A \\ &= (\gamma \Delta z)\, A \\ &= (9810\,\text{N/m}^3)(3\,\text{m})(2\,\text{m} \times 5\,\text{m}) \\ &= 294.3 \text{ kN} \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{((2 \times 5^3)/12)\ \text{m}^4}{(2.5\,\text{m} + 1.25\,\text{m})\,(2\,\text{m} \times 5\,\text{m})} \\ &= 0.555\,6\,\text{m} \end{aligned}$$

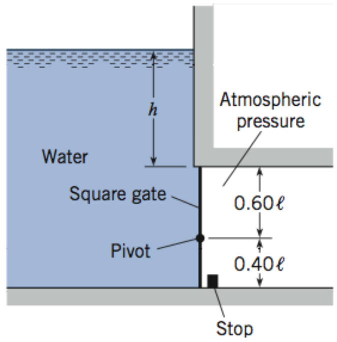
Equilibrium

$$\sum M_{\text{hinge}} = 0$$
$$294.3\,\text{kN} \times (2.5\,\text{m} + 0.555\,6\,\text{m}) - (3\,\text{m})\,P = 0$$
$$P = 299.\,75\,\text{kN}$$
$$\boxed{P = 300\,\text{kN}}$$

3.64: PROBLEM DEFINITION

Situation:

A submerged gate opens when the water level reaches a certain value.



Find:

h in terms of ℓ to open gate.

PLAN

As depth of water increase, the center of pressure will move upward. The gate will open when the center of pressure reaches the pivot.

SOLUTION

Center of pressure (when the gate opens)

$$\begin{aligned} y_{cp} - \bar{y} &= 0.60\ell - 0.5\ell \\ &= 0.10\ell \end{aligned} \tag{1}$$

Center of pressure (formula)

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{(\ell \times \ell^3)/12}{(h + \ell/2)\ell^2} \end{aligned} \tag{2}$$

Combine Eqs. (1) and (2)

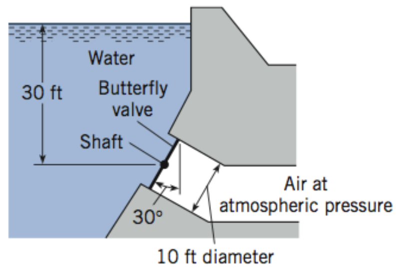
$$\begin{aligned} 0.10\ell &= \frac{(\ell \times \ell^3)/12}{(h + \ell/2)\ell^2} \\ 0.10 &= \frac{\ell}{12(h + \ell/2)} \\ h &= \frac{5}{6}\ell - \frac{1}{2}\ell \\ &= \frac{1}{3}\ell \end{aligned}$$

$$\boxed{h = \ell/3}$$

3.65: PROBLEM DEFINITION

Situation:

A butterfly valve is described in the problem statement.
$d = 10\,\text{ft}$, $\theta = 30\,°$, $\bar{y} = 30\,\text{ft}$.



Find:

Torque required to hold valve in position (ft-lbf).

SOLUTION Hydrostatic force

$$\begin{aligned} F &= \bar{p}A = \overline{y}\gamma A \\ &= (30 \text{ ft} \times 62.4 \text{ lb/ft}^3)\left(\pi \times \frac{D^2}{4}\right) \text{ ft}^2 \\ &= \left(30\,\text{ft} \times 62.4\,\text{lbf/}\,\text{ft}^3 \times \pi \times \frac{(10\,\text{ft})^2}{4}\right) \\ &= 147,027 \text{ lbf} \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} \\ &= \frac{\pi r^4/4}{\bar{y}\pi r^2} \\ &= \frac{(5\,\text{ft})^2/4}{30\,\text{ft}/0.866} \\ &= 0.1804 \text{ ft} \end{aligned}$$

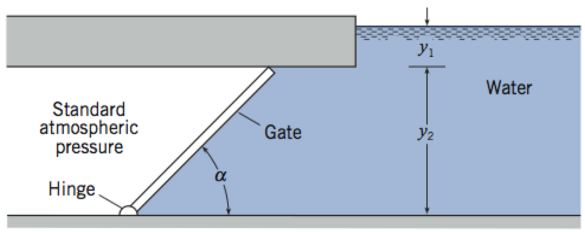
Torque

$$\begin{aligned} \text{Torque} &= 0.1804\,\text{ft} \times 147,027\,\text{lbf} \\ & \boxed{T = 26,520 \text{ ft-lbf}} \end{aligned}$$

3.66: PROBLEM DEFINITION

Situation:

A submerged gate may fall due to its weight (or be held in place by pressure).
$y_1 = 1\,\text{m}$, $y_2 = 4\,\text{m}$, $w = 1\,\text{m}$.
$W = 150\,\text{kN}$, $\alpha = 45^o$.



Find:

Will the gate fall or stay in position?

Properties:

Water (10 °C), Table A.5, $\gamma = 9810\,\text{N/m}^3$.

SOLUTION

1. Geometry

- Slant height:

$$\bar{y} = \frac{y_1 + y_2/2}{\sin\alpha} = \frac{(1 + 4/2)\,\text{m}}{\sin 45^o} = 4.243\,\text{m}$$

- Depth of centroid:

$$\Delta z = y_1 + \frac{y_2}{2} = \left(1 + \frac{4}{2}\right)\,\text{m} = 3\,\text{m}$$

- Panel surface area

$$A = \left(\frac{y_2}{\sin\alpha}\right) w = \left(\frac{4\,\text{m}}{\sin 45^o}\right)(1\,\text{m}) = 5.657\,\text{m}^2$$

2. Pressure at Centroid:

$$\bar{p} = \gamma\Delta z = \left(9810\,\text{N/m}^3\right)(3\,\text{m}) = 29.43\,\text{kPa}$$

3. Hydrostatic force:

$$F = \bar{p}A = (29.43\,\text{kPa})\left(5.657\,\text{m}^2\right) = 166.5\,\text{kN}$$

4. Distance from CP to centroid:

- Area moment of inertia from Fig. A.1:

$$\begin{aligned} \bar{I} &= \frac{wh^3}{12} \\ h &= \frac{y_2}{\sin\alpha} = \frac{4\,\text{m}}{\sin 45^o} = 5.657\,\text{m} \\ \bar{I} &= \frac{wh^3}{12} = \frac{(1\,\text{m})\,(5.657\,\text{m})^3}{12} = 15.09\,\text{m}^4 \end{aligned}$$

- Final Calculation:

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{(15.09\,\text{m}^4)}{(4.243\,\text{m})\,(5.657\,\text{m}^2)} = 0.6287\,\text{m}$$

5. Torques:

- Torque caused by hydrostatic force:

$$x_h = \frac{h}{2} - (y_{cp} - \bar{y}) = \frac{5.657\,\text{m}}{2} - 0.6287\,\text{m} = 2.200\,\text{m}$$

$$T_{\text{HS}} = Fx_h = (166.5\,\text{kN})\,(2.2\,\text{m}) = 366\,\text{kN}\cdot\,\text{m}$$

- Torque caused by the weight:

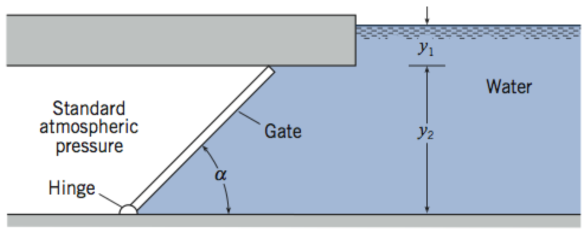
$$x_w = \frac{y_2/2}{\tan\alpha} = \frac{4\,\text{m}/2}{\tan 45^o} = 2\,\text{m}$$

$$T_{\text{W}} = Wx_w = (150\,\text{kN})\,(2\,\text{m}) = 300\,\text{kN}\cdot\,\text{m}$$

The torque caused by the hydrostatic force exceeds the torque caused by the weight: So the $\boxed{\text{gate will stay in position.}}$

3.67: PROBLEM DEFINITION

Situation:
A submerged gate may fall due to its weight.
$y_1 = 3\,\text{ft}$, $y_2 = 6\,\text{ft}$, $w = 3\,\text{ft}$.
$W = 18000\,\text{lbf}$, $\alpha = 45^o$.



Find:
Will gate fall or stay in position?

Properties:
Water (50 °F), Table A.5, $\gamma = 62.4\,\text{lbf}/\,\text{ft}^3$.

SOLUTION

1. Hydrostatic Force:

- Area:

$$A = \frac{y_2}{\sin\alpha} \times w = \frac{6\,\text{ft}}{\sin 45^o} \times 3\,\text{ft} = 25.46\,\text{ft}^2$$

- Depth of the centroid of the plate:

$$\Delta z = y_1 + \frac{y_2}{2} = 3\,\text{ft} + \frac{6\,\text{ft}}{2} = 6\,\text{ft}$$

- Final Calculation:

$$F = \bar{p}A = \gamma \Delta z A = \left(62.4\,\text{lbf}/\,\text{ft}^3\right)(6\,\text{ft})\left(25.46\,\text{ft}^2\right) = 9532\,\text{lbf}$$

2. Distance from CP to centroid:

- Area moment of inertia from Fig. A.1:

$$\begin{aligned} \bar{I} &= \frac{wh^3}{12} \\ h &= \frac{y_2}{\sin\alpha} = \frac{6\,\text{ft}}{\sin 45^o} = 8.485\,\text{ft} \\ \bar{I} &= \frac{wh^3}{12} = \frac{(3\,\text{ft})(8.485\,\text{ft})^3}{12} = 152.7\,\text{ft}^4 \end{aligned}$$

- Slant height:

$$\bar{y} = \frac{h}{2} + \frac{y_1}{\sin\alpha} = \frac{8.485\,\text{ft}}{2} + \frac{3\,\text{ft}}{\sin 45^o} = 8.485\,\text{ft}$$

- Final Calculation:

$$y_{cp} - \bar{y} = \frac{\overline{I}}{\bar{y}A} = \frac{\left(152.7\,\text{ft}^4\right)}{\left(8.485\,\text{ft}\right)\left(25.46\,\text{ft}^2\right)} = 0.7069\,\text{ft}$$

3. Torque due to weight:

   - Moment arm:

$$x_1 = \frac{y_2 \tan\alpha}{2} = \frac{\left(6\,\text{ft}\right)\left(\tan 45^o\right)}{2} = 3\,\text{ft}$$

   - Final calculation:

$$M_1 = Wx_1 = \left(18000\,\text{lbf}\right)\left(3\,\text{ft}\right) = 54000\,\text{ft lbf}$$

4. Torque due hydrostatic pressure:

   - Moment arm:

$$x_2 = h/2 - \left(y_{cp} - \bar{y}\right) = \frac{8.485\,\text{ft}}{2} - \left(0.7069\,\text{ft}\right) = 3.536\,\text{ft}$$

   - Final calculation:

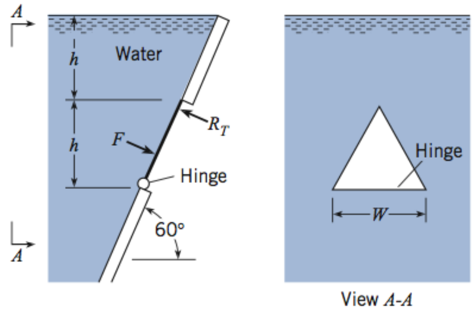
$$M_1 = Fx_2 = \left(9532\,\text{lbf}\right)\left(3.536\,\text{ft}\right) = 33705\,\text{ft lbf}$$

Since the torque due to weight exceeds the torque due to hydrostatic pressure:
$\boxed{\text{the gate will fall}}$.

3.68: PROBLEM DEFINITION

Situation:

A submerged gate is described in the problem statement.



Find:

Hydrostatic force (F) on gate.

Ratio (R_T/F) of the reaction force to the hydrostatic force.

SOLUTION

$$\begin{aligned} F &= \bar{p}A \\ &= \left(h + \frac{2h}{3}\right)\gamma\left(\frac{Wh/\sin 60^\circ}{2}\right) \end{aligned}$$

$$\boxed{F = \frac{5\gamma Wh^2}{3\sqrt{3}}}$$

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} = \frac{W(h/\sin 60^\circ)^3}{(36 \times (5h/(3\sin 60^\circ))} \times \frac{Wh}{2\sin 60^\circ} \\ &= \frac{h}{15\sqrt{3}} \\ \Sigma M &= 0 \\ R_T h/\sin 60^\circ &= F\left[\left(\frac{h}{3\sin 60^\circ}\right) - \left(\frac{h}{15\sqrt{3}}\right)\right] \end{aligned}$$

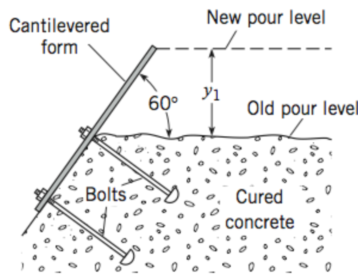
$$\boxed{\frac{R_T}{F} = \frac{3}{10}}$$

3.69: PROBLEM DEFINITION

Situation:

A concrete form is described in the problem statement.

$y_1 = 1.5\,\mathrm{m}$, $\theta = 60\,°$.



Find:

Moment at base of form per meter of length (kN·m/m).

Properties:

Concrete, $\gamma = 24\,\mathrm{kN/\,m^3}$.

Assumptions:

Assume that the form has a length of w =1 meter into the paper.

PLAN

Find the moment by muliplying the hydrostatic force by its moment arm. The plan for reaching the goal is:

1. Calculate the hydrostatic force.
2. Calculate the centroid of area using $I = bh^3/12$.
3. Calculate the center of pressure.
4. Use results from steps 1 to 4 to calculate the moment.

SOLUTION

1. Hydrostatic force

$$\begin{aligned} F &= \bar{p}A = \gamma z_c h w \\ h &= \text{height of panel} = \left(\frac{1.5\,\mathrm{m}}{\sin 60°}\right) = 1.\,732\,1\,\mathrm{m} \\ F &= \left(24000\,\mathrm{N/\,m^3}\right)\left(\frac{1.5}{2}\,\mathrm{m}\right)(1.\,732\,1\,\mathrm{m})(1\,\mathrm{m}) = 31178\,\mathrm{N} \end{aligned}$$

2. Centroid of area

$$I = \frac{bh^3}{12} = \frac{(1\,\mathrm{m})\,(1.\,732\,1\,\mathrm{m})^3}{12} = 0.4331\,\mathrm{m}^4$$

3. Center of pressure

$$
\begin{aligned}
y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} \\
\bar{y} &= (1.7321\,\mathrm{m})/2 = 0.86605\,\mathrm{m} \\
A &= hw = (1.7321\,\mathrm{m})(1\,\mathrm{m}) = 1.7321\,\mathrm{m}^2 \\
y_{cp} - \bar{y} &= \frac{(0.4331\,\mathrm{m}^4)}{(0.86605\,\mathrm{m})(1.7321\,\mathrm{m}^2)} = 0.2887\,\mathrm{m}
\end{aligned}
$$

4. Moment at base

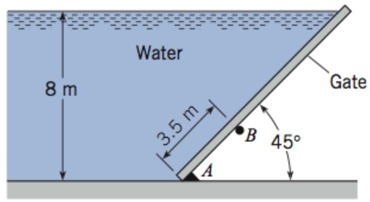
$$
\begin{aligned}
M &= (\text{force})(\text{moment arm}) \\
&= (31178\,\mathrm{N})(1.7321\,\mathrm{m}/2 - 0.2887\,\mathrm{m}) \\
&\boxed{M = 18\ \text{kN}\cdot\text{m per meter of form}}
\end{aligned}
$$

3.70: PROBLEM DEFINITION

Situation:

A submerged gate is described in the problem statement.

$\theta = 45\,°$.



Find:

Is the gate stable or unstable.

SOLUTION

$$y_{cp} = \frac{2}{3} \times \frac{8}{\cos 45°} = 7.54 \text{ m}$$

Point B is (8/cos 45°) m - 3.5 m = 7.81 m along the gate from the water surface; therefore, the gate is **unstable.**

3.71: PROBLEM DEFINITION

Situation:

Two hemispherical shells are sealed together.

$r_o = 10.5\,\text{cm}$, $r_i = 10.75\,\text{cm}$.

Find:

Force required to separate the two shells.

Assumptions:

The pressure seal is at the average radius ($r = 10.6\,\text{cm}$)

Properties:

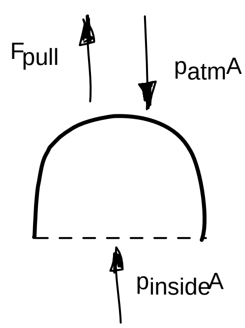
$p_{\text{atm}} = 101.3\,\text{kPa}$, $p_i = 0.25 p_{\text{atm}}$.

PLAN

1. Apply equilibrium to a freebody comprised of the top half of the shell plus the air inside.
2. Calculate the force.

SOLUTION

1. Equilibrium.



$$\begin{aligned} \sum F_y &= 0 \\ F_{\text{pull}} + p_i A - p_{\text{atm}} A &= 0 \end{aligned}$$

2. Force to separate shells.

$$\begin{aligned} F_{\text{pull}} &= (p_{\text{atm}} - p_i) A = p_{\text{atm}} (1 - 0.25) A \\ &= (1 - 0.25) \left(101000\,\text{N/ m}^2\right) \left(\pi \,(0.106\,\text{m})^2\right) \\ &= 2670\,\text{N} \end{aligned}$$

$$\boxed{F_{\text{pull}} = 2670\,\text{N}}$$

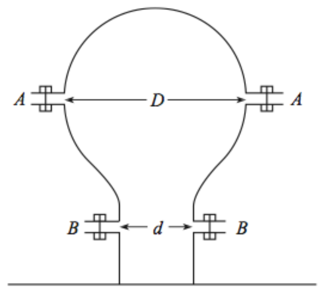
3.72: PROBLEM DEFINITION

Situation:

An air chamber is held together by bolts.

$d = 20\,\text{cm}$, $D = 40\,\text{cm}$.

A-A: 20 bolts, $D_{bolt} = 2.5\,\text{cm}$.



Find:

Number of bolts required at section B-B.

Assumptions:

Same force per bolt at B-B.

SOLUTION

Hydrostatic force

$$\begin{aligned} F \text{ per bolt at } A - A &= p(\pi/4)D^2/20 \\ p\left(\frac{\pi}{4}\right)\frac{D^2}{20} &= p\left(\frac{\pi}{4}\right)\frac{d^2}{n} \\ n &= 20 \times \left(\frac{d}{D}\right)^2 \\ &= 20 \times \left(\frac{1}{2}\right)^2 \end{aligned}$$

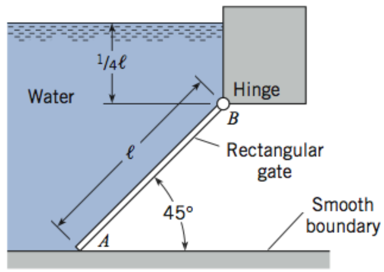
$$\boxed{n = 5 \text{ bolts}}$$

3.73: PROBLEM DEFINITION

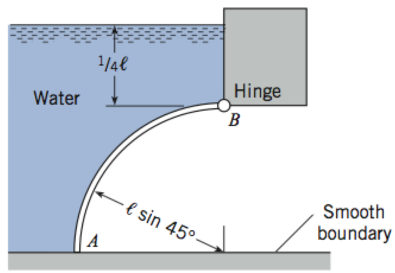
Situation:

Submerged rectangular and curved gates.

$\bar{y} = 0.25\ell$.



(a) Plane gate



Find:

Magnitude of reaction at A.

Comparison to that for a plane gate.

SOLUTION

a)

$$F_{Hydr} = \bar{p}A = (0.25\ell + 0.5\ell \times 0.707) \times \xi W\ell = 0.6036\gamma W\ell^2$$

$$y_{cp} - \bar{y} = \frac{I}{\bar{y}A} = \frac{W\ell^3/12}{((0.25\ell/0.707) + 0.5\ell) \times W\ell}$$

$$y_{cp} - \bar{y} = 0.0976\ell$$

$$\sum M_{\text{hinge}} = 0$$

$$\text{Then } -0.70R_A\ell + (0.5\ell + 0.0976\ell) \times 0.6036\gamma W\ell^2 = 0$$

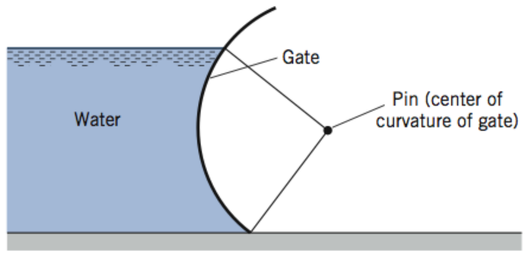
$$\boxed{R_A = 0.510\gamma W\ell^2}$$

b) The reaction here will be less because if one thinks of the applied hydrostatic force in terms of vertical and horizontal components, the horizontal component will be the same in both cases, but the vertical component will be less because there is less volume of liquid above the curved gate.

3.74: PROBLEM DEFINITION

Situation:

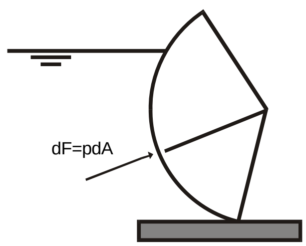
A radial gate holds back water.



Find:

Where the resultant of the pressure force acts.

SOLUTION



Consider all the differential pressure forces acting on the radial gate as shown. Because each differential pressure force acts normal to the differential area, then each differential pressure force must act through the center of curvature of the gate. Because all the differential pressure forces will be acting through the center of curvature (the pin), the resultant must also pass through this same point (the pin).

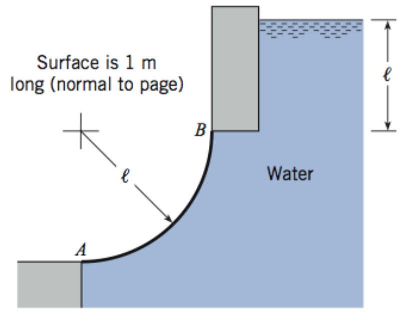
Resultant passes through the pin.

3.75: PROBLEM DEFINITION

Situation:

A curved surface is described in the problem statement.

$\ell = 1\,\text{m}$.



Find:

(a) Vertical hydrostatic force.

(b) Horizontal hydrostatic force.

(c) Resultant force.

Properties:

Water, Table A.5: $\gamma = 9810\,\text{N}/\,\text{m}^3$.

SOLUTION

$$F_V = 1\,\text{m} \times 9,810\,\text{N}/\,\text{m}^3 \times 1\,\text{m}^2 \times +(1/4)\pi \times (1\,\text{m})^2 \times 1\,\text{m} \times 9,810\,\text{N}/\,\text{m}^3$$

$$\boxed{F_V = 17,515\text{ N}}$$

$$\begin{aligned} x &= \frac{M_0}{F_V} \\ &= \frac{1 \times 1 \times 1 \times 9,810 \times 0.5 + 1 \times 9,810 \times \int_0^1 \sqrt{1-x^2}xdx}{17,515\,\text{N}} \end{aligned}$$

$$\boxed{x = 0.467\text{ m}}$$

$$\begin{aligned} F_H &= \bar{p}A \\ &= (1+0.5)9,810\,\text{N}/\,\text{m}^3 \times 1\,\text{m} \times 1\,\text{m} \end{aligned}$$

$$\boxed{F_H = 14,715\text{ N}}$$

$$\begin{aligned} y_{cp} &= \bar{y} + \bar{I}/\bar{y}A \\ &= 1.5 + \frac{(1 \times 1^3)\,\text{m}^4}{12 \times 1.5\,\text{m} \times 1\,\text{m} \times 1\,\text{m}} \end{aligned}$$

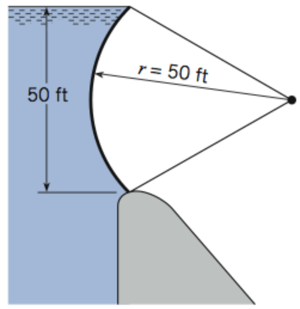
$$\boxed{y_{cp} = 1.555\text{ m}}$$

$$
\begin{aligned}
F_R &= \sqrt{(14,715\,\mathrm{N})^2 + (17,515\,\mathrm{N})^2} \\
&\boxed{F_R = 22,876\ \mathrm{N}} \\
\tan\theta &= \frac{14,715\,\mathrm{N}}{17,515\,\mathrm{N}} \\
&\boxed{\theta = 40^\circ 2'}
\end{aligned}
$$

3.76: PROBLEM DEFINITION

Situation:

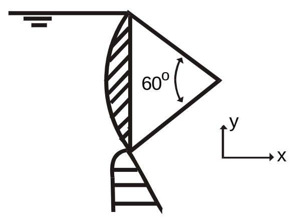
A radial gate is described in the problem statement.



Find:

Hydrostatic force acting on gate.

SOLUTION



The resultant must pass through the center of curvature of the gate. The horizontal component of hydrostatic force acting on the gate will be the hydrostatic force acting on the vertical projection of the gate or:

Hydrostatic force

$$\begin{aligned} F_H &= \bar{p}A \\ &= 25 \text{ ft} \times 62.4 \text{ lb/ft}^3 \times 40 \text{ ft} \times 50 \text{ ft} \\ F_H &= 3,120,000 \text{ lb} \end{aligned}$$

The vertical component of hydrostatic force will be the buoyant force acting on the radial gate. It will be equal in magnitude to the weight of the displaced liquid (the weight of water shown by the cross-hatched volume in the above Fig.).

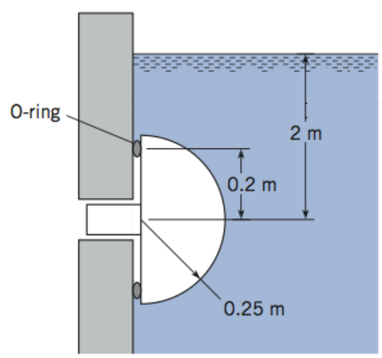
Thus,

$$
\begin{aligned}
F_V &= \gamma \forall \\
\text{where } \forall &= [(60/360)\pi \times 50^2 \text{ ft}^2 - \left(\frac{1}{2}\right) 50 \times 50 \cos 30^\circ \text{ ft}^2] \times 40 \text{ ft} \\
&= 226.5 \text{ ft}^2 \times 40 \text{ ft} \\
&= 9600 \text{ ft}^3 \\
\text{Then } F_V &= (62.4 \text{ lbf/ft}^3)(9060 \text{ ft}^3) = 565,344 \text{ lbs} \\
& \boxed{F_{\text{resultant}} = (3,120,000 \mathbf{\ i} \ + 565,344 \mathbf{\ j})\text{lbf}}
\end{aligned}
$$

acting through the center of curvature of the gate.

3.77: PROBLEM DEFINITION

Situation:
A plug sits in a hole in the side of a tank.
$z = 2\,\text{m}$, $r_{o-ring} = 0.2\,\text{m}$, $r_{plug} = 0.25\,\text{m}$.



Find:
Horizontal and vertical forces on plug.

Properties:
Water, Table A.5: $\gamma = 9810\,\text{N/m}^3$.

SOLUTION

Hydrostatic force

$$
\begin{aligned}
F_h &= \bar{p}A \\
&= \gamma z A \\
&= 9810\,\text{N/m}^3 \times 2\,\text{m} \times \pi \times (0.2\,\text{m})^2 \\
& \boxed{F_h = 2465 \text{ N}}
\end{aligned}
$$

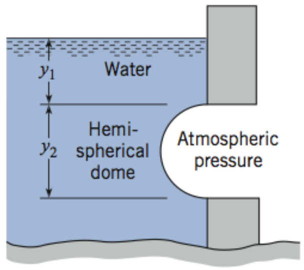
The vertical force is simply the buoyant force.

$$\begin{aligned} F_v &= \gamma \forall \\ &= 9810\,\mathrm{N/\,m^3} \times \frac{4}{6} \times \pi \times (0.25\,\mathrm{m})^3 \\ &\boxed{F_v = 321 \text{ N}} \end{aligned}$$

3.78: PROBLEM DEFINITION

Situation:

A dome below the water surface is described in the problem statement.
$y_1 = 1\,\text{m}$, $y_2 = 2\,\text{m}$.



Find:

Magnitude and direction of force to hold dome in place.

Properties:

Water (10 °C), Table A.5: $\gamma = 9810\,\text{N/m}^3$.

SOLUTION

1. Horizontal component of force.

$$\begin{aligned} F_H &= (1\,\text{m} + 1\,\text{m})9810\,\text{N/m}^3 \times \pi \times (1\,\text{m})^2 \\ &= 61,640 \text{ N } = 61.64 \text{ kN} \end{aligned}$$

2. Center of pressure.

$$\begin{aligned} (y_{cp} - \bar{y}) &= \frac{I}{\bar{y}A} \\ &= \frac{\pi \times (1\,\text{m})^4/4}{2\,\text{m} \times \pi \times (1\,\text{m})^2} \\ &= 0.125\,\text{m} \end{aligned}$$

3. Vertical component of force

$$\begin{aligned} F_V &= \left(\frac{1}{2}\right)\left(\frac{4\pi \times (1\,\text{m})^3}{3}\right) 9,810\,\text{N/m}^3 \\ &= 20,550 \text{ N} \\ F_V &= 20.6\,\text{kN} \end{aligned}$$

4. Answer

$$\boxed{F_{\text{horizontal}} = 61.6\,\text{kN (applied to the left to hold dome in place)}}$$

Line of action is 0.125 m below a horizontal line passing through the dome center

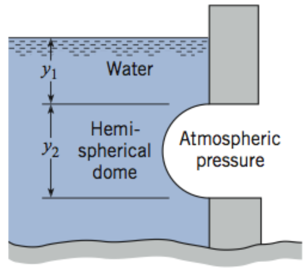
$F_{\text{vertical}} = 20.6\,\text{kN}$ (applied downward to hold dome in place)

3.79: PROBLEM DEFINITION

Situation:

A dome below the water surface.

$d = 10\,\text{ft}$



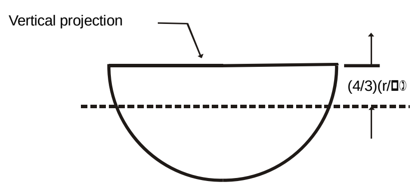
Find:

Force on the dome.

Properties:

Water, Table A.5: $\gamma = 62.4\,\text{lbf/ft}^3$.

SOLUTION

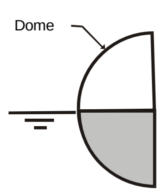


The horizontal component of the hydrostatic force acting on the dome will be the hydrostatic force acting on the vertical projection of the bottom half of the dome.

Hydrostatic force

$$
\begin{aligned}
F_H &= \bar{p}A \\
\bar{p} &= \left(\frac{4}{3}\right)\left(\frac{5}{\pi}\right)\text{ ft } (62.4\text{ lbf/ft}^3) \\
&= 132.4\text{ lbf/ft}^2 \\
F_H &= (132.4\text{ lbf/ft}^2)\left(\frac{\pi}{8}\right)(10^2)\text{ ft}^2 = 5,199\text{ lbf}
\end{aligned}
$$

The vertical component of force will be the buoyant force acting on the dome. It will be the weight of water represented by the cross-hatched region shown in the Fig. (below).



Thus,

$$
\begin{aligned}
F_V &= \gamma \forall \\
&= (62.4 \text{ lbf/ft}^3)\left(\frac{(1/6)\pi D^3}{4}\right) \text{ ft}^3 \\
F_V &= 8,168 \text{ lbf}
\end{aligned}
$$

The resultant force is then given below. This force acts through the center of curvature of the dome.

$$
\boxed{\mathbf{F}_{\text{result}} = 5,199\mathbf{i} + 8,168\mathbf{j} \text{ lbf}}
$$

Problem 3.80

Apply the grid method to each situation described below. Note: Unit cancellations are not shown in this solution.

a.)

Situation:
A basketball floating in a lake.

Find:
Find buoyant force (N).

Assumptions:
$m = 596\,\text{g}$.

Solution:
Since the basketball is floating, the buoyant force equals the weight of the ball. The mass of a basketball is between 567 and 624 grams (from encarta.msn.com on 2/12/08). Using a typical mass:

$$F_B = \text{Weight} = mg = \left(\frac{596\,\text{g}}{1.0}\right)\left(\frac{1.0\,\text{kg}}{1000\,\text{g}}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right) \approx 5.9\,\text{N}$$

$$\boxed{F_B \approx 5.9\,\text{N}}$$

b.)

Situation:
Copper sphere in kerosene.
$D = 1\,\text{mm}$.

Find:
Find buoyant force (N).

Properties:
Kerosene (20 °C), Table A.4, $\gamma = 8010\,\text{N/m}^3$.

Solution:

$$\begin{aligned} F_B &= \gamma \forall \\ F_B &= \gamma \frac{\pi D^3}{6} = \left(8010\,\text{N/m}^3\right)\frac{\pi\,(0.001\,\text{m})^3}{6} = 4.19\times10^{-6}\,\text{N} \end{aligned}$$

$$F_B = 4.19\times10^{-6}\,\text{N}$$

c.)

Situation:
Helium balloon in air.
$D = 12\,\text{in}$.

Find:
Find buoyant force (N).

Properties:
Air (20 °C), Table A.3, $\gamma = 11.8\,\mathrm{N/\,m^3}$.

Solution:

$$
\begin{aligned}
F_B &= \gamma \forall \\
F_B &= \gamma \frac{\pi D^3}{6} = \left(11.8\,\mathrm{N/\,m^3}\right)\left(\frac{\pi\,(12\,\mathrm{in.})^3}{6}\right)\left(\frac{1.0\,\mathrm{m}}{39.37\,\mathrm{in.}}\right)^3 = 0.175\,\mathrm{N}
\end{aligned}
$$

$$F_B = 0.175\,\mathrm{N}$$

Problem 3.81

Using Section 3.6 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words and equations in ways that enhance the effectiveness of your communication.

---

a. Why learn about buoyancy? That is, what are important technical problems that involve buoyant forces?

- boiling heat transfer–vapor bubbles are acted on by buoyant force.
- particles in liquids–sand carried by a river; mixing of solids in liquids.
- vessel design–ships, submarines, jet-skis, etc.

---

b. For a buoyant force, where is the center of pressure? Line of action?

- The buoyant force acts through an imaginary point called the center of pressure.
- The center of pressure is at the center-of-mass of the displaced fluid.
- For a uniform density fluid, the center of pressure is at the centroid of volume of the displaced fluid.
- The buoyant force acts parallel to an imaginary line called the "line of action." This line in parallel to the gravity vector.

---

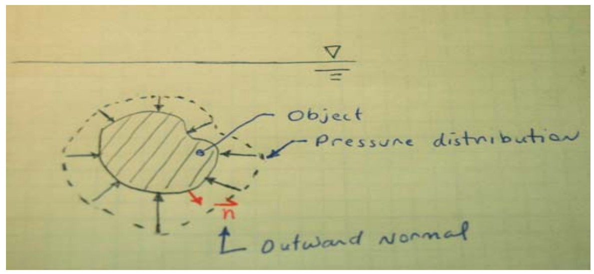
c. What is displaced volume? Why is it important?

- When the object is placed into a fluid, it move fluid away. This volume of this fluid is called the "displaced volume."
- Displaced volume is important because the buoyant forces is the product of displaced volume and specific weight.

d. What is the relationship between pressure distribution and buoyant force?

- The integral of pressure over surface area gives the buoyant force:

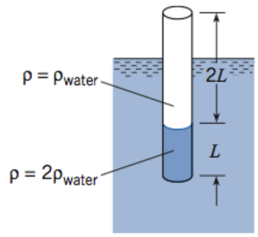
$$F = \int_A -p\hat{n}dA$$



3.82: PROBLEM DEFINITION

Situation:

A rod is floating in a liquid.



Find:

Determine if the liquid is

a. lighter than water

b. must be water

c. heavier than water.

SOLUTION

Rod weight

$$\begin{aligned} W &= (2LA\gamma_W + LA(2\gamma_W) \\ W &= 4LA\gamma_W \end{aligned} \tag{1}$$

Since part of the rod extends above the liquid,

$$F_B < \forall\gamma_{\text{Liq}} = 3LA\gamma_{\text{Liq}} \tag{2}$$

Equilibrium applied to the rod

$$W = F_B \tag{3}$$

Combine Eqs. (1), (2) and (3).

$$\begin{aligned} 4LA\gamma_W &< 3LA\gamma_{\text{Liq}} \\ \gamma_{\text{Liq}} &> \frac{4}{3}\gamma_W. \end{aligned}$$

The liquid is more dense than water so is answer $\boxed{\text{c).}}$

3.83: PROBLEM DEFINITION

Situation:

A ship is sailing from salt to fresh water.

$W = 35000$ tons $= 70 \times 10^6$ lbf.

$A = 38000\,\text{ft}^2$, $L = 800\,\text{ft}$.

Find:

Will the ship rise or settle?

Amount (ft) the ship will rise or settle.

PLAN

1. To establish whether the ship will rise or settle, apply the equilibrium equation.
2. Determine the volume displaced in both salt and fresh water.
3. Calculate the distance the ship moves.

SOLUTION

1. Equilibrium. The weight of the ship is balanced by the buoyant force

$$W = F_B = \gamma \forall$$

As the ship moves into freshwater, the specific weight of the water decreases Thus, the volume of the displaced water will increase as shown below.

$$W = F_B = (\gamma \downarrow)(\forall \uparrow)$$

Thus **the ship will settle**.

2. Volume displaced (salt water):

$$W = F_B = \gamma_s \forall_s$$

$$\forall_s = \frac{W}{\gamma_s} = \frac{70 \times 10^6\,\text{lbf}}{1.03\,(62.4\,\text{lbf/ft}^3)}$$

Volume displaced (fresh water):

$$W = F_B = \gamma_f \forall_f$$

$$\forall_f = \frac{W}{\gamma_f} = \frac{70 \times 10^6\,\text{lbf}}{62.4\,\text{lbf/ft}^3}$$

3. Distance Moved. The distance moved Δh is given by

$$\Delta \forall = A \Delta h$$

where $\Delta \forall$ is the change in displaced volume and A is the section area of the ship at the water line. Thus:

$$\left(\frac{70 \times 10^6\,\text{lbf}}{62.4\,\text{lbf/ft}^3}\right) - \left(\frac{70 \times 10^6\,\text{lbf}}{1.03\,(62.4\,\text{lbf/ft}^3)}\right) = \left(38000\,\text{ft}^2\right)\Delta h$$

Thus:

$$\boxed{\Delta h = 0.860\,\text{ft}}$$

3.84: PROBLEM DEFINITION

Situation:

A spherical buoy is anchored in salt water.

$W = 1200\,\text{N}$, $D = 1.2\,\text{m}$.

$T = 4500\,\text{N}$, $y = 20\,\text{m}$.

Find:

Weight of scrap iron (N) to be sealed in the buoy.

Properties:

Seawater, Table A.4 $\gamma_s = 10070\,\text{N/m}^3$.

PLAN

1. Find the buoyant force using the buoyant force equation.
2. Find the weight of scrap iron by applying equilibrium.

SOLUTION

1. Buoyant force equation:

$$F_B = \gamma_s \forall = \left(10070\,\text{N/m}^3\right) \frac{\pi\,(1.2\,\text{m})^3}{6} = 9111\,\text{N}$$

2. Equilibrium

$$\begin{aligned} \Sigma F_y &= 0 \\ F_B &= W_{\text{buoy}} + W_{\text{scrap}} + T \\ 9111\,\text{N} &= 1200\,\text{N} + W_{\text{scrap}} + 4500\,\text{N} \end{aligned}$$

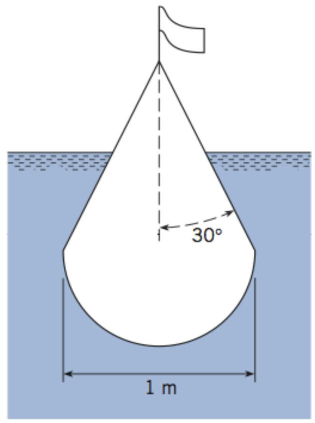
$$\boxed{W_{\text{scrap}} = 3420\,\text{N}}$$

3.85: PROBLEM DEFINITION

Situation:

A buoy has a spherical top and conical bottom.

$m = 460\,\text{kg},\ D = 1\,\text{m},\ \theta = 30\,°$.



Find:

Location of water level.

Properties:

$\rho = 1010\,\text{kg/m}^3$.

SOLUTION

The buoyant force is equal to the weight.

$$F_B = W$$

The weight of the buoy is $9.81 \times 460 = 4512$ N.
The volume of the hemisphere at the bottom of the buoy is

$$\forall = \frac{1}{2}\frac{\pi}{6}D^3 = \frac{\pi}{12}1^3 = \frac{\pi}{12}\ \text{m}^3$$

The buoyant force due to the hemisphere is

$$F_B = \frac{\pi}{12}(9.81\,\text{m/s}^2)(1010\,\text{kg/m}^3) = 2594\ \text{N}$$

Since this is less than the buoy weight, the water line must lie above the hemisphere. Let h is the distance from the top of the buoy. The volume of the cone which lies between the top of the hemisphere and the water line is

$$\begin{aligned}\forall &= \frac{\pi}{3}r_o^2 h_o - \frac{\pi}{3}r^2 h = \frac{\pi}{3}(0.5^2 \times 0.866 - h^3\tan^2 30) \\ &= 0.2267 - 0.349h^3\end{aligned}$$

The additional volume needed to support the weight is

$$\forall\!\!\!\!/ = \frac{4512\,\mathrm{N} - 2594\,\mathrm{N}}{9.81\,\mathrm{m/\,s^2} \times 1010\,\mathrm{kg/\,m^3}} = 0.1936\ \mathrm{m}^3$$

Equating the two volumes and solving for h gives

$$h^3 = \frac{0.0331}{0.349} = 0.0948\ \mathrm{m}^3$$

$$\boxed{h = 0.456\ \mathrm{m}}$$

3.86: PROBLEM DEFINITION

Situation:

In air, a rock weighs $W_{\text{air}} = 1000\,\text{N}$.

In water, a rock weighs $W_{\text{water}} = 609\,\text{kg}$.

Find:

The volume of the rock (liters).

Properties:

Water (15^oC) ,Table A.5, $\gamma = 9800\,\text{N/}\,\text{m}^3$.

PLAN

1. Apply equilibrium to the rock when it is submerged in water.
2. Solve the equation from step 1 for volume.

SOLUTION

1. Equilibrium:

$$\left\{\begin{array}{c}\text{Force to hold}\\ \text{rock stationary in water}\\ \text{(apparent weight)}\end{array}\right\} + \left\{\begin{array}{c}\text{Buoyant Force}\\ \text{on rock}\end{array}\right\} = \left\{\begin{array}{c}\text{Weight of rock}\\ \text{in air}\end{array}\right\}$$

$$\begin{aligned} W_{\text{water}} + F_B &= W_{\text{air}} \\ W_{\text{water}} + \gamma V &= W_{\text{air}} \\ 609\,\text{N} + 9810\,\text{N/}\,\text{m}^3 V &= 1000\,\text{N} \end{aligned}$$

2. Solve for volume

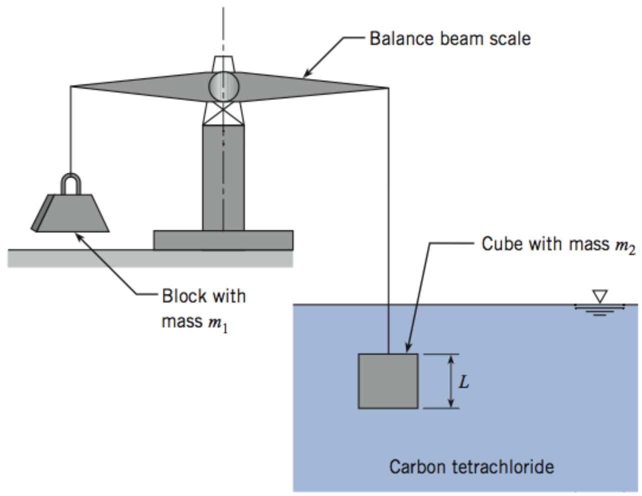
$$V = \frac{1000\,\text{N} - 609\,\text{N}}{9810\,\text{N/}\,\text{m}^3} = 0.0399\,\text{m}^3$$

$$\boxed{V = 39.9\text{ L}}$$

3.87: PROBLEM DEFINITION

Situation:

A cube is suspended in carbon tetrachloride.
$m_1 = 700\,\mathrm{g},\ L = 0.06\,\mathrm{m}$



Find:

The mass of the cube (kg).

Properties:

Carbon Tetrachloride (20^oC) ,Table A.4, $\gamma = 15600\,\mathrm{N/\,m^3}$.

PLAN

1. Find the force on the balance arm scale by finding the weight of the block.
2. Find m_2 by applying equilibrium to the cube.

SOLUTION

1. Force on balance arm:

$$\left\{\begin{array}{c}\text{Force on}\\ \text{balance arm}\end{array}\right\} = \left\{\begin{array}{c}\text{Weight}\\ \text{of block}\end{array}\right\} = mg = (0.7\,\mathrm{kg})\left(9.81\,\mathrm{m/\,s^2}\right) = 6.867\,\mathrm{N}$$

2. Equilibrium (applied to cube):

$$\begin{aligned}\left\{\begin{array}{c}\text{Force on}\\ \text{balance arm}\end{array}\right\} + \left\{\begin{array}{c}\text{Buoyant Force}\\ \text{on cube}\end{array}\right\} &= \{\text{Weight of cube}\}\\ F + \gamma\,(L_2)^3 &= m_2 g\end{aligned}$$

Solve for m_2 :

$$m_2 = \frac{F + \gamma\,(L_2)^3}{g} = \frac{(6.867\,\mathrm{N}) + (15600\,\mathrm{N/\,m^3})\,(0.06\,\mathrm{m})^3}{9.81\,\mathrm{m/\,s^2}}$$

$$\boxed{m_2 = 1.04\,\mathrm{kg}}$$

3.88: PROBLEM DEFINITION

Situation:

A block is submerged in water.

$W_{\text{water}} = 300\,\text{N}$, $W_{\text{air}} = 700\,\text{N}$.

Find:

The volume of the block (liters).

The specific weight of the material that was used to make the block (N/m^3).

Properties:

Water (15^oC) ,Table A.5, $\gamma = 9800\,\text{N/m}^3$.

PLAN

1. Find the block's volume by applying equilibrium to the block.
2. Find the specific weight by using the definition.

SOLUTION

1. Equilibrium (block submerged in water):

$$\left\{\begin{array}{c}\text{Force to hold}\\ \text{block in water}\\ \text{(apparent weight)}\end{array}\right\} + \left\{\begin{array}{c}\text{Buoyant Force}\\ \text{on block}\end{array}\right\} = \left\{\begin{array}{c}\text{Weight of block}\\ \text{in air}\end{array}\right\}$$

$$\begin{aligned} W_{\text{water}} + F_B &= W_{\text{air}} \\ W_{\text{water}} + \gamma_{\text{H2O}} V\!\!\!\!- &= W_{\text{air}} \end{aligned}$$

Solve for volume:

$$\begin{aligned} V\!\!\!\!- &= \frac{W_{\text{air}} - W_{\text{water}}}{\gamma_{\text{H2O}}} \\ &= \frac{700\,\text{N} - 300\,\text{N}}{9800\,\text{N/m}^3} = 4.08 \times 10^{-2}\,\text{m}^3 \end{aligned}$$

$$\boxed{V\!\!\!\!- = 40.8\text{ L}}$$

2. Specific weight (definition):

$$\gamma_{\text{block}} = \frac{\text{weight of block}}{\text{volume of block}} = \frac{700\,\text{N}}{4.08 \times 10^{-2}\,\text{m}^3} = 17200\,\text{N/m}^3$$

$$\boxed{\gamma_{\text{block}} = 17.2\,\text{kN/m}^3}$$

3.89: PROBLEM DEFINITION

Situation:

A cylindrical tank is filled with water.
A cylinder of wood is set afloat in the water.
$D_{\text{tank}} = 1\,\text{ft}$, $D_{\text{wood}} = 6\,\text{in.}$
$W_{\text{wood}} = 2\,\text{lbf}$, $L_{\text{wood}} = 3\,\text{in.}$

Find:

Change of water level in tank.

Properties:

Water, Table A.5: $\gamma = 62.4\,\text{lbf}/\,\text{ft}^3$.

PLAN

When the wood enters the tank, it will displace volume. This volume can be visualized as adding extra water to the tank. Thus, find this volume and use it to determine the increase in water level.

1. Find the buoyant force by applying equilibrium.
2. Find the displaced volume by applying the buoyancy equation.
3. Find the increase in water level by equating volumes

SOLUTION

1. Equilibrium

$$\text{weight of block} = \text{buoyant force on block}$$

$$F_B = W_{\text{block}} = 2\,\text{lbf}$$

2. Buoyancy equation

$$F_B = \gamma_{\text{H}_2\text{O}} \forall_D = 2\,\text{lbf}$$

$$\forall_D = \frac{2\,\text{lbf}}{62.4\,\text{lbf}/\,\text{ft}^3} = 0.03205\,\text{ft}^3$$

2. Volume

$$\forall_D = (\text{volume change}) = (\text{tank section area})\,(\text{height change})$$

$$0.03205\,\text{ft}^3 = \frac{\pi\,(1\,\text{ft})^2}{4}\Delta h$$

$$\boxed{\Delta h = 0.041\,\text{ft}}$$

3.90: PROBLEM DEFINITION

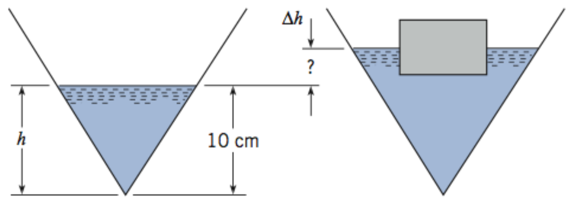
Situation:

An inverted cone contains water (state 1).

A block ($S = 0.6$) is added (state 2).

$\forall = (\pi/3)\,h^3$, $h_1 = 10\,\text{cm}$.

$\forall_{\text{block}} = 200\,\text{cm}^3$.



Find:

Change of water level.

SOLUTION

1. Equilibrium (apply to block)

$$F_B = W_{\text{block}} = (\gamma_{\text{block}})\,(\forall_{\text{block}})$$

2. Buoyancy equation

$$F_B = \left(\gamma_{H_2O}\right)(\forall_D) = (\gamma_{\text{block}})\,(\forall_{\text{block}})$$

Thus

$$(\forall_D) = \left(\frac{\gamma_{\text{block}}}{\gamma_{H_2O}}\right)(\forall_{\text{block}}) = (0.6)\left(200\,\text{cm}^3\right) = 120\,\text{cm}^3$$

3.Volume considerations.

$$\left(\begin{array}{c}\text{final}\\ \text{volume}\end{array}\right) = \left(\begin{array}{c}\text{initial water}\\ \text{volume}\end{array}\right) + \left(\begin{array}{c}\text{displaced}\\ \text{volume}\end{array}\right)$$

Calculate initial water volume

$$\forall = \frac{\pi}{3}h^3 = \frac{\pi}{3}\,(10\,\text{cm})^3 = 1047\,\text{cm}^3$$

Calculate final volume

$$\forall_{\text{final}} = \left(1047\,\text{cm}^3\right) + \left(120\,\text{cm}^3\right) = 1167\,\text{cm}^3$$

4. Increase in water level

$$\forall_{\text{final}} = \frac{\pi}{3}h_f^3$$

$$1167\,\text{cm}^3 = \frac{\pi}{3}h_f^3$$

$$h_{\mathrm{f}} = 10.368\,\mathrm{cm}$$

$$\Delta h = 10.368\,\mathrm{cm} - 10 = 0.368\,\mathrm{cm}$$

$$\boxed{\Delta h = 0.37\,\mathrm{cm}}$$

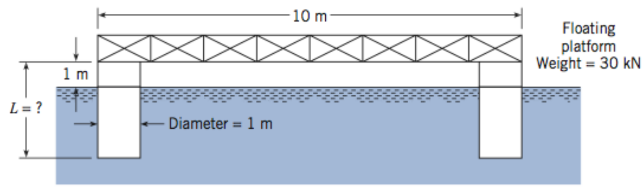
3.91: PROBLEM DEFINITION

Situation:

A platform floats in water.

$W_{\text{platform}} = 30\,\text{kN}$, $W_{\text{cylinder}} = 1\,\text{kN/ m}$.

$y = 1\,\text{m}$, $D_{\text{cylinder}} = 1\,\text{m}$.



Find:

Length of cylinder so that the platform floats 1 m above water surface.

Properties:

$\gamma_{\text{water}} = 10,000\,\text{N/ m}^3$.

SOLUTION

1.Equilibrium (vertical direction)

$$\begin{pmatrix} \text{Weight of} \\ \text{platform} \end{pmatrix} + 4 \begin{pmatrix} \text{Weight of} \\ \text{a cylinder} \end{pmatrix} = 4 \begin{pmatrix} \text{Buoyant force} \\ \text{on a cylinder} \end{pmatrix}$$

$$(30000\,\text{N}) + 4L\left(\frac{1000\,\text{N}}{\text{m}}\right) = 4\,(\gamma \text{Ṿ}_D)$$

$$(30000\,\text{N}) + 4L\left(\frac{1000\,\text{N}}{\text{m}}\right) = 4\left(\frac{10000\,\text{N}}{\text{m}^3}\right)\left(\frac{\pi\,(1\,\text{m})^2}{4}\,(L - 1\,\text{m})\right)$$

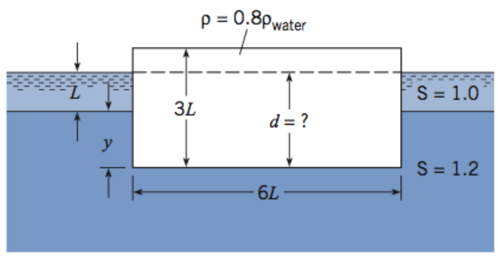
1. Solve for L

$$\boxed{L = 2.24\,\text{m}}$$

3.92: PROBLEM DEFINITION

Situation:

A block floats in two layered liquids.
$b = 6L$, $h = 3L$.



Find:

Depth block will float.

Assumptions:

The block will sink a distance y into the fluid with $S = 1.2$.

Properties:

$\rho_{block} = 0.8\rho_{water}$.

SOLUTION

1. Equilibrium.

$$\sum F_y = 0$$

$$-\begin{pmatrix}\text{Weight} \\ \text{of block}\end{pmatrix} + \begin{pmatrix}\text{Pressure force} \\ \text{on btm of block}\end{pmatrix} = 0$$

$$-\left(\forall_{\text{block}}\right)\left(\gamma_{\text{block}}\right) - p_{\text{btm}} A = 0$$

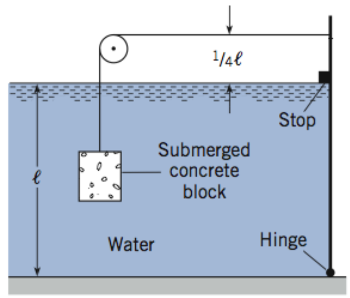
$$-(6L)^2 \times 3L \times 0.8\gamma_{\text{water}} + (L \times \gamma_{\text{water}} + y \times 1.2\gamma_W)36L^2 = 0$$

$$y = 1.167L$$
$$d = y + L$$
$$\boxed{d = 2.17L}$$

3.93: PROBLEM DEFINITION

Situation:

A submerged gate has a concrete block attached to it.
$b = 1\,\text{m}$, $\ell = 2\,\text{m}$.



Find:

Minimum volume of concrete to keep gate in closed position (m^3).

Properties:

Concrete $\gamma = 23.6\,\text{kN/}\,\text{m}^3$.

SOLUTION

Hydrostatic force on gate and CP

$$F = \bar{p}A = 1\,\text{m} \times 9,810\,\text{N/}\,\text{m}^3 \times 2\,\text{m} \times 1\,\text{m} = 19,620\text{ N}$$

$$y_{cp} - \bar{y} = \frac{I}{\bar{y}A} = \frac{1\,\text{m} \times (2\,\text{m})^3}{12 \times 1\,\text{m} \times 2\,\text{m} \times 1\,\text{m}} = 0.33\text{ m}$$

Sum moments about the hinge to find the tension in the cable

$$T = 19,620 \times \frac{1 - 0.33}{2.5} = 5,258\text{ N}$$

Equilibrium applied to concrete block

$$\begin{pmatrix}\text{Tension}\\\text{in cable}\end{pmatrix} + \begin{pmatrix}\text{Buoyant}\\\text{force}\end{pmatrix} = (\text{Weight})$$

$$T + \forall\gamma_{\text{H}_2\text{O}} = \forall\gamma_c$$

Solve for volume of block

$$\forall = \frac{T}{\gamma_c - \gamma_{\text{H}_2\text{O}}} = \frac{5258\,\text{N}}{23,600\,\text{N/}\,\text{m}^3 - 9,810\,\text{N/}\,\text{m}^3}$$

$$\boxed{\forall = 0.381\text{ m}^3}$$

3.94: PROBLEM DEFINITION

Situation:

Ice is added to a cylindrical tank holding water.
$d = 2\,\text{ft}$, $h = 4\,\text{ft}$.
$W_{ice} = 5\,\text{lb}$.

Find:

Change of water level in tank after ice is added.
Change in water level after the ice melts.
Explain all processes.

Properties:

$\gamma_{water} = 62.4\,\text{lbf/ft}^3$.

SOLUTION

Change in water level (due to addition of ice)

$$\begin{aligned} W_{\text{ice}} &= F_{\text{buoyancy}} \\ &= \Delta V_W \gamma_W \end{aligned}$$

So

$$\begin{aligned} \Delta V_W &= \frac{W_{\text{ice}}}{\gamma_W} = \frac{5\,\text{lbf}}{62.4\,\text{lbf/ft}^3} \\ &= 0.0801\ \text{ft}^3 \end{aligned}$$

Rise of water in tank (due to addition of ice)

$$\begin{aligned} \Delta h &= \frac{\Delta V_W}{A_{\text{cyl}}} \\ &= \frac{0.0801\ \text{ft}^3}{(\pi/4)(2\ \text{ft})^2} = 0.02550\,\text{ft} = 0.3060\,\text{in} \end{aligned}$$

$$\boxed{\Delta h = 0.306\,\text{in} \quad <== \text{(due to addition of ice)}}$$

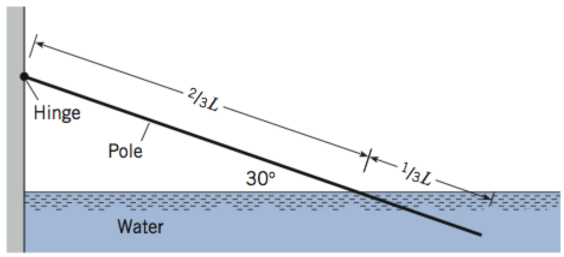
Answer $\Rightarrow$ When the ice melts, the melted water will occupy the same volume of water that the ice originally displaced; therefore, there will be no change in water surface level in the tank after the ice melts.

3.95: PROBLEM DEFINITION

Situation:

A partially submerged wood pole is attached to a wall.

$\theta = 30\,°$.



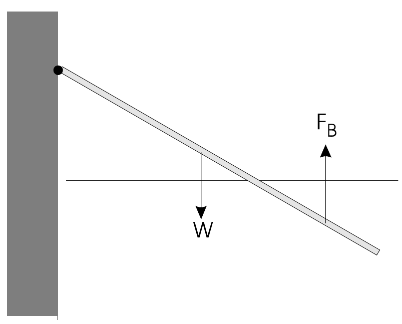
Find:

Density of wood.

Properties:

$\gamma = 9810\,\text{N/m}^3$.

SOLUTION



$$\begin{aligned} M_{\text{hinge}} &= 0 \\ -W_{\text{wood}} \times (0.5L\cos 30°) + F_{B.} \times (5/6)L\cos 30° &= 0 \\ -\gamma_{\text{wood}} \times AL \times (0.5L\cos 30°) + \left(\frac{1}{3}AL\gamma_{\text{H}_2\text{O}}\right) \times \left(\frac{5}{6}L\cos 30°\right) &= 0 \end{aligned}$$

$$\gamma_{\text{wood}} = \left(\tfrac{10}{18}\right)\gamma_{\text{H}_2\text{O}}$$

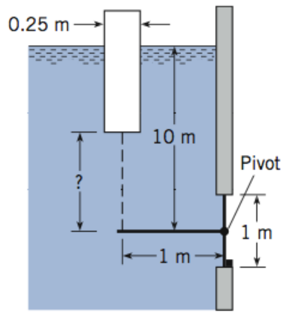
$$\boxed{\gamma_{\text{wood}} = 5,450\ \text{N/m}^3}$$

$$\boxed{\rho_{\text{wood}} = 556\ \text{kg/m}^3}$$

3.96: PROBLEM DEFINITION

Situation:

A submerged gate is described in the problem statement.
$d = 25\,\text{cm}$, $W = 200\,\text{N}$.
$y = 10\,\text{m}$, $L = 1\,\text{m}$.



Find:

Length of chain so that gate just on verge of opening.

PLAN

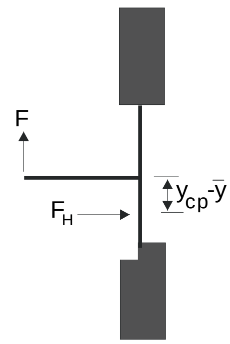
Apply hydrostatic force equations and then sum moments about the hinge.

SOLUTION

Hydrostatic force

$$
\begin{aligned}
F_H &= \bar{p}A = 10\,\text{m} \times 9,810\,\text{N/}\,\text{m}^3 \times \frac{\pi D^2}{4} \\
&= 98,100\,\text{N/}\,\text{m}^2 \times \pi \frac{\pi}{4}\,(1\,\text{m})^2 \\
&= 77,048\ \text{N} \\
y_{cp} - \bar{y} &= \frac{I}{\bar{y}A} \\
&= \frac{\pi r^4/4}{10\,\text{m} \times \pi D^2/4} \\
y_{cp} - \bar{y} &= \frac{r^2}{40} = 0.00625\ \text{m}
\end{aligned}
$$

Equilibrium



$$
\begin{aligned}
\sum M_{\text{Hinge}} &= 0 \\
F_H \times (0.00625 \text{ m}) - 1\,\text{m} \times F &= 0
\end{aligned}
$$

$$
\begin{aligned}
\text{But } F &= F_{\text{buoy}} - W \\
&= A(10 \text{ m} - \ell)\gamma_{\text{H}_2\text{O}} - 200 \\
&= \frac{\pi}{4}(0.25\,\text{m})^2(10-\ell)(9,810\,\text{N}/\,\text{m}^3) - 200\,\text{N} \\
&= 4815.5 \text{ N} - 481.5\ell \text{ N} - 200 \text{ N} \\
&= (4615.5 - 481.5\ell) \text{ N} \\
\text{where } \ell &= \text{length of chain}
\end{aligned}
$$

$$
\begin{aligned}
77,048\,\text{N} \times 0.00625\,\text{m} - 1\,\text{m} \times (4615.5 - 481.5\ell)\,\text{N} &= 0 \\
(481.55 - 4615.5 + 481.5\ell) \text{ N m} &= 0
\end{aligned}
$$

$$\boxed{\ell = 8.59 \text{ m}}$$

3.97: PROBLEM DEFINITION

Situation:

A balloon is used to carry instruments.

$z = 15000\,\text{ft}$, $W_{balloon} = 8.3$ psia.

$W_{\text{instruments}} = 10\,\text{lbf}$.

Find:

Diameter of spherical balloon.

Assumptions:

Standard atmospheric temperature condition.

Properties:

$p_{air} = 8.3$ psia.

PLAN

Apply buoyancy force and the ideal gas law.

SOLUTION

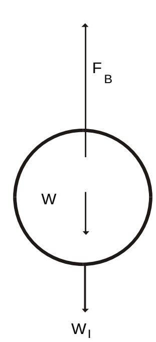
Temperature in the atmosphere

$$T = 533 - 3.221 \times 10^{-3} \times 15,000 = 485R$$

Ideal gas law

$$\begin{aligned}
\rho_{\text{air}} &= \frac{8.3\text{ psia} \times 144\,\text{in}^2/\,\text{ft}^2}{1,715\,\text{ft lbf}/\,\text{slug}\,^\circ\text{R} \times 485\,^\circ\text{R}} \\
&= 0.001437\text{ slugs/ft}^3 \\
\rho_{\text{He}} &= \frac{8.3\text{ psia} \times 144\,\text{in}^2/\,\text{ft}^2}{12,429\,\text{ft lbf}/\,\text{slug}\,^\circ\text{R} \times 485\,^\circ\text{R}} \\
&= 0.000198\text{ slugs/ft}^3
\end{aligned}$$

Equilibrium



$$
\begin{aligned}
\sum F &= 0 \\
&= F_B - W_{\text{helium}} - W_{\text{material}} - W_{\text{instruments}} \\
&= \frac{1}{6}\pi D^3 g(\rho_{\text{air}} - \rho_{\text{He}}) - \pi D^2 (0.01\,\text{lbf}/\,\text{ft}^2) - 10\,\text{lbf} \\
&= D^3 \times 16.88(14.37 - 1.98)10^{-4} - D^2 \times 3.14 \times 10^{-2} - 10 \\
& \boxed{D = 8.35 \text{ ft}}
\end{aligned}
$$

3.98: PROBLEM DEFINITION

Situation:

A helium weather balloon is made of flexible material.
$p_{balloon} = 10\,\text{kPa} + p_{atm}$.
$d_{sea} = 1\,\text{m}$, $m = 100\,\text{g}$.

Find:

Maximum altitude of balloon.

Assumptions:

$T_0 = 288$ K

SOLUTION

Initial Volume

$$\begin{aligned} \forall_0 &= \frac{\pi}{6} D_0^3 \\ &= \frac{\pi}{6} (1\,\text{m})^3 \\ &= 0.524\ \text{m}^3 \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_{0,\text{He}} &= \frac{p_{0,\text{He}}}{R_{\text{He}} T_0} \\ &= \frac{111,300\,\text{kPa}}{(2077\,\text{J/ kg K})(288\,\text{K})} \\ &= 0.186\ \text{kg/m}^3 \end{aligned}$$

Conservation of mass

$$m_0 = m_{\text{alt.}}$$
$$\forall_0 \rho_{0,\text{He}} = \forall_{\text{alt.}} \rho_{\text{He}}$$
$$\forall_{\text{alt.}} = \forall_0 \frac{\rho_{0,\text{He}}}{\rho_{\text{He}}}$$

Equilibrium

$$\begin{aligned} \sum F_z &= 0 \\ F_{\text{buoy.}} - W &= 0 \\ \forall_{\text{alt.}} \rho_{\text{air}} g - (mg + W_{\text{He}}) &= 0 \end{aligned}$$

Eliminate $\forall_{\text{alt.}}$

$$\left(\frac{\forall_0 \rho_0}{\rho_{\text{He}}} \right) \rho_{\text{air}} g = (mg + \forall_0 \rho_{0,\text{He}} g)$$

Eliminate ρ's with equation of state

$$\frac{(\forall_0 \rho_0)(p_{\text{alt.}}/R_{\text{air}}) g}{(p_{\text{alt.}} + 10,000)/(R_{\text{He}})} = (mg \ + \forall_0 \rho_0 g)$$

$$
\begin{aligned}
& \frac{(0.524\,\mathrm{m}^3)(0.186\,\mathrm{kg/\,m}^3)(9.81\,\mathrm{m/\,s}^2)(2077\,\mathrm{J/\,kg\,K})p_{\mathrm{alt.}}}{(p_{\mathrm{alt.}}+10,000)(287\,\mathrm{J/\,kg\,K})} \\
= \;& (0.1\,\mathrm{kg})(9.81\,\mathrm{m/\,s}^2)+(0.524\,\mathrm{m}^3)(0.186\,\mathrm{kg/\,m}^3)(9.81\,\mathrm{m/\,s}^2)
\end{aligned}
$$

Solve

$$p_{\mathrm{alt.}} = 3888 \text{ Pa}$$

Check to see if $p_{\mathrm{alt.}}$ is in the troposphere or stratosphere. Using Eq. (3.15) solve for pressure at top of troposphere.

$$
\begin{aligned}
p &= p_0\left[\frac{T_0-\alpha(z-z_0)}{T_0}\right]^{g/\alpha R} \\
&= 101,300\,\mathrm{Pa}\left[\frac{296\,\mathrm{K}-5.87\times10^{-3}\,\mathrm{K/\,m}(13,720\,\mathrm{m})}{296\,\mathrm{K}}\right]^{5.823} \\
&= 15,940 \text{ Pa}
\end{aligned}
$$

Because $p_{\mathrm{alt.}} < p_{\text{at top of troposphere}}$ we know that $p_{\mathrm{alt.}}$ occurs above the stratosphere. The stratosphere extends to 16.8 km where the temperature is constant at -57.5$^{\mathrm{o}}$C. The pressure at the top of the stratosphere is given by Eq. (3.16)

$$p = p_0 e^{-(z-z_0)g/RT}$$

$$= 15.9\,\mathrm{kPa}\exp\left[-(16,800-13,720)\,\mathrm{m}\times\frac{9.81\,\mathrm{m/\,s}^2}{287\,\mathrm{J/\,kg\,K}\times 215.5\,\mathrm{K}}\right]$$

$$= 9.75 \text{ kPa}$$

Thus the balloon is above the stratosphere where the temperature increases linearly at 1.387$^{\mathrm{o}}$C/km. In this region the pressure varies as

$$p = p_0\left[\frac{T_0+\alpha(z-z_0)}{T_0}\right]^{-g/\alpha R}$$

Using this equation to solve for the altitude, we have

$$
\begin{aligned}
\frac{3888\,\mathrm{kPa}}{9750\,\mathrm{kPa}} &= \left[\frac{215.5\,\mathrm{K}+1.387\,\mathrm{K/\,km}\times(z-16.8)\,\mathrm{km}}{215.5\,\mathrm{K}}\right]^{-9.81/(0.001387\times 287)} \\
0.399 &= [1+0.00644\times(z-16.8)]^{-24.6}
\end{aligned}
$$

$$\boxed{z = 22.8 \text{ km}}$$

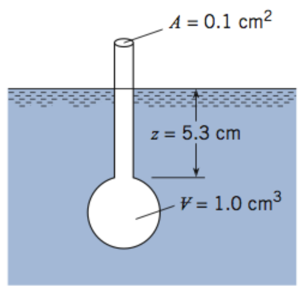
3.99: PROBLEM DEFINITION

Situation:

A hydrometer in water.

$\forall_{\text{bulb}} = 1\,\text{cm}^3$, $A_{\text{stem}} = 0.1\,\text{cm}^2$.

$z = 5.3\,\text{cm}$.



Find:

Weight of hydrometer.

Properties:

Water, $\gamma_W = 9810\,\text{N/m}^3$.

SOLUTION

Equilibrium

$$\begin{aligned} F_{\text{buoy.}} &= W \\ \forall_D \gamma_W &= W \end{aligned}$$

Calculations

$$\begin{aligned} (1\text{ cm}^3 + (5.3\text{ cm})(0.1\text{ cm}^2))(0.1\text{m})^3/\text{cm}^3(\gamma_W) &= W \\ (1.53\text{ cm}^3)(10^{-6}\text{ m}^3/\text{cm}^3)(9810\text{ N/m}^3) &= W \end{aligned}$$

$$\boxed{W = 1.50 \times 10^{-2}\text{ N}}$$

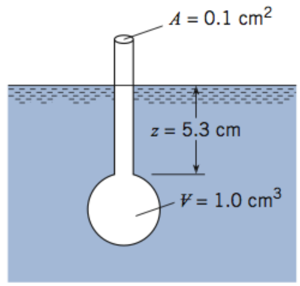
3.100: PROBLEM DEFINITION

Situation:

A hydrometer is floating in oil.

$\forall_{\text{bulb}} = 1\,\text{cm}^3$, $A_{\text{stem}} = 0.1\,\text{cm}^2$.

$z = 6.3\,\text{cm}$ (not 5.3 cm as shown in sketch), $W = 0.015\,\text{N}$.



Find:

Specific gravity of oil.

Properties:

$\gamma_W = 9810\,\text{N/m}^3$.

SOLUTION

1. Equilibrium

$$
\begin{aligned}
F_{\text{buoy.}} &= W \\
\forall_D \gamma_{\text{oil}} &= W
\end{aligned}
$$

2. Calculations

$$
\begin{aligned}
(1\ \text{cm}^3 + (6.3\ \text{cm})(0.1\ \text{cm}^2))(0.01^3)\ \text{m}^3/\text{cm}^3 \gamma_{\text{oil}} &= 0.015\ \text{N} \\
(1 + 0.63) \times 10^{-6}\ \text{m}^3 \gamma_{\text{oil}} &= 0.015\ \text{N} \\
\gamma_{\text{oil}} &= 9202\ \text{N/m}^3
\end{aligned}
$$

3. Definition of S

$$
\begin{aligned}
S &= \frac{\gamma_{\text{oil}}}{\gamma_{H_2O}} \\
&= \frac{9202\,\text{N/m}^3}{9810\,\text{N/m}^3}
\end{aligned}
$$

$$\boxed{S = 0.938}$$

3.101: PROBLEM DEFINITION

Situation:

A hydrometer is described in the problem statement.

Find:

Weight of each ball.

Properties:

$S_{10\%} = 1.012$, $S_{50\%} = 1.065$.

$\gamma_{water} = 9810\,\mathrm{N/\,m^3}$.

SOLUTION

Equilibrium (for a ball to just float, the buoyant force equals the weight)

$$F_B = W \tag{1}$$

Buoyancy force

$$F_B = \left(\frac{\pi D^3}{6}\right)\gamma_{\text{fluid}} \tag{2}$$

Combine Eq. (1) and (2) and let $D = 0.01\,\mathrm{m}$.

$$\begin{aligned} W &= \left(\frac{\pi D^3}{6}\right) S\gamma_{\text{water}} \\ &= \left(\frac{\pi\,(0.01)^3}{6}\right) S\,(9810) \\ &= 5.136\times10^{-3} S \end{aligned} \tag{3}$$

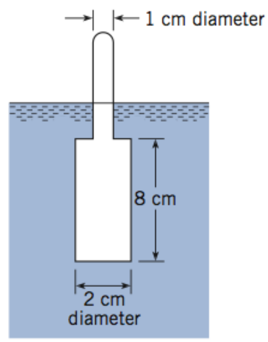
The following table (from Eq. 3) shows the weights of the balls needed for the required specific gravity intervals.

ball number	1	2	3	4	5	6
sp. gr.	1.01	1.02	1.03	1.04	1.05	1.06
weight (mN)	5.19	5.24	5.29	5.34	5.38	5.44

3.102: PROBLEM DEFINITION

Situation:

A hydrometer floats in a liquid.
Liquid levels range from btm to top of stem.
$d_1 = 1\,\text{cm}$, $d_2 = 2\,\text{cm}$.
$L_1 = 8\,\text{cm}$, $L_2 = 8\,\text{cm}$.
$W = 35\,\text{g}$.



Find:

Range of specific gravities.

Properties:

$\gamma_{H_2O} = 9810\,\text{N/m}^3$.

SOLUTION

When only the bulb is submerged

$$\begin{aligned} F_B &= W \\ \forall_D \gamma_{H_2O} &= W \\ \frac{\pi}{4}\left[(0.02\,\text{m})^2 \times 0.08\,\text{m}\right] \times 9810\,\text{N/m}^3 \times S &= 0.035\,\text{kg} \times 9.81\,\text{m/s}^2 \end{aligned}$$

$$S = 1.39$$

When the full stem is submerged

$$\frac{\pi}{4}\left[(0.02\,\text{m})^2 \times (0.08\,\text{m}) + (0.01\,\text{m})^2 \times (0.08\,\text{m})\right] 9,810\,\text{N/m}^3 \times S = 0.035\,\text{kg} \times 9.81\,\text{m/s}^2$$

$$S = 1.11$$

Thus, the range is

$$\boxed{1.11 \leq S \leq 1.39}$$

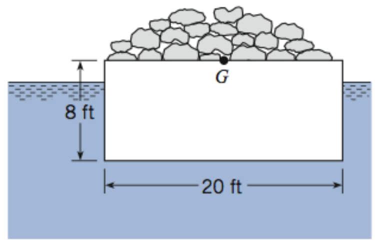
3.103: PROBLEM DEFINITION

Situation:

A barge is floating in water.

$l = 50\,\text{ft}$, $b = 20\,\text{ft}$.

$W = 400,000\,\text{lbf}$.



Find:

Stability of barge.

Properties:

$\gamma_{water} = 62.4\,\text{lbf/ft}^3$.

SOLUTION

$$\begin{aligned} \text{Draft} &= \frac{400000\,\text{lbf}}{50\,\text{ft} \times 20\,\text{ft} \times 62.4\,\text{lbf/ft}^3} \\ &= 6.41 \text{ ft} < 8 \text{ ft} \end{aligned}$$

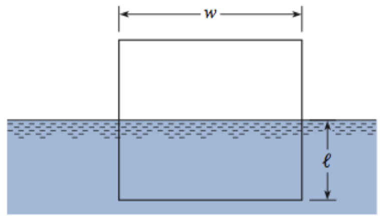
$$\begin{aligned} \text{GM} &= \frac{I_{00}}{V} - \text{CG} \\ &= \frac{\left(50\,\text{ft} \times (20\,\text{ft})^3 / 12\right)}{(6.41\,\text{ft} \times 50\,\text{ft} \times 20\,\text{ft})} - (8 - 3.205)\,\text{ft} \\ &= 0.40 \text{ ft} \end{aligned}$$

Will float stable

3.104: PROBLEM DEFINITION

Situation:

A floating body is in water.



Find:

Location of water line for stability.

Specific gravity of material.

SOLUTION

For neutral stability, the distance to the metacenter is zero. In other words

$$GM = \frac{I_{oo}}{V\!\!\!\!-} - GC = 0$$

where GC is the distance from the center of gravity to the center of buoyancy.

Moment of inertia at the waterline

$$I_{oo} = \frac{w^3 L}{12}$$

where L is the length of the body. The volume of liquid displaced is $\ell w L$ so

$$GC = \frac{w^3 L}{12 \ell w L} = \frac{w^2}{12\ell}$$

The value for GC is the distance from the center of buoyancy to the center of gravity, or

$$GC = \frac{w}{2} - \frac{\ell}{2}$$

So

$$\frac{w}{2} - \frac{\ell}{2} = \frac{w^2}{12\ell}$$

or

$$\left(\frac{\ell}{w}\right)^2 - \frac{\ell}{w} + \frac{1}{6} = 0$$

Solving for ℓ/w gives 0.789 and 0.211. The first root gives a physically unreasonable solution. Therefore

$$\boxed{\frac{\ell}{w} = 0.211}$$

The weight of the body is equal to the weight of water displaced.

$$\gamma_b \forall_b = \gamma_f \forall_f$$

Therefore

$$S = \frac{\gamma_b}{\gamma_f} = \frac{w\ell L}{w^2 L} = \frac{\ell}{w} = 0.211$$

$$\boxed{S = 0.211}$$

The specific gravity is smaller than this value, thus the body will be unstable (floats too high).

3.105: PROBLEM DEFINITION

Situation:
A block of wood.
$d = 1\,\text{m}$, $L = 1\,\text{m}$.

Find:
Stability.

Properties:
$\gamma_{wood} = 7500\,\text{N/m}^3$.

SOLUTION

$$\begin{aligned} \text{draft} &= \frac{1 \times 7500\,\text{N/m}^3}{9,810\,\text{N/m}^3} = 0.7645\text{ m} \\ c_{\text{from bottom}} &= \frac{0.7645\,\text{m}}{2} = 0.3823\,\text{m} \end{aligned}$$

Metacentric height

$$\begin{aligned} G &= 0.500\text{ m; CG } = 0.500 - 0.3823 = 0.1177\text{ m} \\ GM &= \frac{I}{\forall} - \text{CG} \\ &= \frac{\pi R^4/4}{0.7645 \times \pi R^2} - 0.1177 \\ &= 0.0818\text{ m} - 0.1177\text{ m (negative)} \end{aligned}$$

Thus, block is **unstable with axis vertical.**

3.106: PROBLEM DEFINITION

Situation: A block of wood.
$d = 1\,\text{m}$, $L = 1\,\text{m}$.

Find:
Stability.

Properties:
$\gamma_{wood} = 5000\,\text{N/m}^3$.

SOLUTION

$$\begin{aligned}\text{draft} &= 1\,\text{m} \times \frac{5000\,\text{N/m}^3}{9810\,\text{N/m}^3} \\ &= 0.5097\text{ m}\end{aligned}$$

Metacentric height

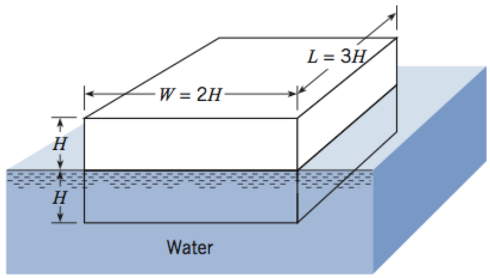
$$\begin{aligned}\text{GM} &= \frac{I_{00}}{\forall} - \text{CG} \\ &= \left[\frac{\pi \times (0.5\,\text{m})^4/4}{0.5097\,\text{m} \times \pi \times (0.5\,\text{m})^2}\right] - \left(0.5 - \frac{0.5097}{2}\right)\text{ m} \\ &= -0.122\text{ m, negative}\end{aligned}$$

So will not float stable with its ends horizontal.

3.107: PROBLEM DEFINITION

Situation:

A floating block is described in the problem statement.
$W = 2H$, $L = 3H$.



Find:

Stability.

SOLUTION

Analyze longitudinal axis

$$\begin{aligned} \text{GM} &= \frac{I_{00}}{\forall} - \text{CG} \\ &= \frac{3H(2H)^3}{12 \times H \times 2H \times 3H} - \frac{H}{2} \\ &= -\frac{H}{6} \end{aligned}$$

Not stable about longitudinal axis.

Analyze transverse axis.

$$\begin{aligned} \text{GM} &= \frac{2H \times (3H)^3}{12 \times H \times 2H \times 3H} - \frac{3H}{4} \\ &= 0 \end{aligned}$$

Neutrally stable about transverse axis.

Not stable