6.1: PROBLEM DEFINITION

Situation:

Inertial reference frame.

Find:

Definition of inertial reference frame.

SOLUTION

The inertial reference is any frame in which Newton's first and second laws are valid. It is any frame which is neither rotating nor accelerating with respect to the sun.

6.2: PROBLEM DEFINITION

Situation:

Centrifugal acceleration on the surface of earth.

$t = 24\,\text{h},\ D = 8000\,\text{mi}.$

Find:

Value of centrifugal acceleration on earth's surface and comparison to acceleration to gravity.

SOLUTION

The acceleration is

$$a_r = \omega^2 r$$

The angular velocity is

$$\omega = \frac{2\pi \text{ rad}}{24 \text{ hr} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}}} = 7.27 \times 10^{-5} \text{ rad/s}$$

Acceleration

$$a_r = (7.27 \times 10^{-5} \text{ rad/s})^2 \times 4000 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}}$$

$$\boxed{a_r = 0.112\,\text{ft/s}^2}$$

The acceleration due to gravity is 32.2 ft/s^2 so

$$\frac{a_r}{g_c} = \frac{0.112}{32.2}$$

$$\boxed{\frac{a_r}{g_c} = 0.0035}$$

or less than 0.5%

6.3: PROBLEM DEFINITION

Situation:

Interpretation of Newton's second law.

$F = \frac{d(mv)}{dt}$, $F = m\frac{dv}{dt} + v\frac{dm}{dt}$

Find:

Relationship between momentum and acceleration.

SOLUTION

Expressing Newton's second law as

$$F = \frac{d}{dt}(mv)$$

is correct. However, Newton's second law is valid only for a system of constant mass so differentiation yields

$$F = m\frac{dv}{dt}$$

In the differentiation by parts, the dm/dt term is zero.

6.4: PROBLEM DEFINITION

Situation:

Examples of jets and how used in practice.

Find:

Give 5 examples of jets and applications.

SOLUTION

1. Water jet from a fire hose - fire suppression
2. Ink jet in a printer - produce ink letters on page
3. High pressure water jet - used from cutting in manufacturing
4. Jet engine nozzle - produce thrust
5. nozzle on lawn sprinkler - used to distribute water for agricultural needs.

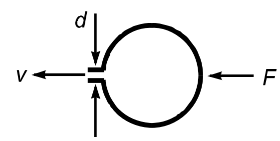
6.5: PROBLEM DEFINITION

Situation:

A balloon is held stationary by a force F.
$d = 10\,\text{mm}$, $v = 40\,\text{m/s}$.

Find: Force required to hold balloon stationary (N).

Sketch:



Assumptions:
Steady flow, constant density.

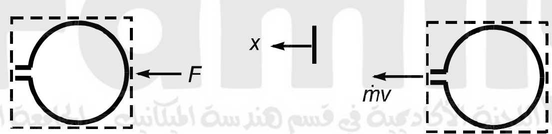
Properties:
$\rho = 1.2\,\text{kg/m}^3$.

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams (x-direction terms)



Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= \dot{m} v \\ &= \rho A v^2 \\ &= (1.2)\left(\frac{\pi \times 0.01^2}{4}\right)(40^2) \end{aligned}$$

$$\boxed{F = 0.151\ N}$$

6.6: PROBLEM DEFINITION

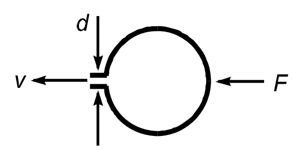
Situation:

A balloon is held stationary by a force.
$d = 1$ cm, $p = 8$ in H_2O.

Find:

x-component of force required to hold balloon stationary (N).
Exit velocity (m/s).

Sketch:



Assumptions:

Steady, irrotational, constant density flow.

Properties:

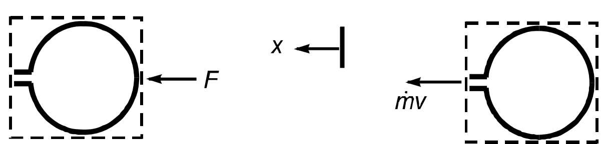
$\rho = 1.2\,\mathrm{kg/m^3}$.

PLAN

To find the exit velocity, apply the Bernoulli equation. To find the force, apply the momentum equation.

SOLUTION

Force and momentum diagrams (x-direction terms)



Bernoulli equation applied from inside the balloon to nozzle exit

$$\begin{aligned} p &= 8 \text{ in } H_2O = 1990\,\mathrm{Pa} \\ \frac{p}{\rho} &= \frac{v^2}{2} \\ v &= \sqrt{\frac{2p}{\rho}} = \sqrt{\frac{2 \times 1990\,\mathrm{Pa}}{1.2\,\mathrm{kg/m^3}}} \end{aligned}$$

$$\boxed{v = 57.6 \ \mathrm{m/s}}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
F &= \dot{m} v = \rho A v^2 = \left(1.2\,\mathrm{kg/\,m^3}\right)\left(\frac{\pi}{4} \times (0.01\,\mathrm{m})^2\right)(57.6\,\mathrm{m/\,s})^2 \\
& \boxed{F = 0.31\,\mathrm{N}}
\end{aligned}$$

6.7: PROBLEM DEFINITION

Situation:
A water jet is filling a tank.

$m = 20\,\text{kg}$, $\forall = 20$ L.

$d = 30$ mm, $v = 20$ m/s.

Find:
Force on the bottom of the tank (N).
Force acting on the stop block (N).

Assumptions:
Steady flow.

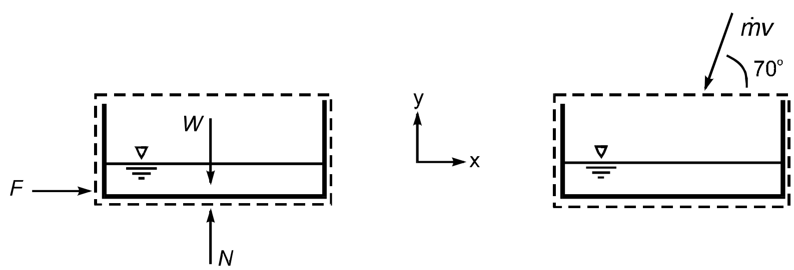
Properties:
Water (15 °C), Table A.5: $\rho = 999\,\text{kg/m}^3$, $\gamma = 9800\,\text{N/m}^3$.

PLAN

Apply the momentum equation in the x-direction and in the y-direction.

SOLUTION

Force and momentum diagrams



Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= -(-\dot{m}v\cos 70^o) \\ &= \rho A v^2 \cos 70^o\end{aligned}$$

Calculations

$$\begin{aligned}\rho A v^2 &= \left(999\,\text{kg/m}^3\right)\left(\frac{\pi \times (0.03\,\text{m})^2}{4}\right)(20\,\text{m/s})^2 \\ &= 282.5\ \text{N}\end{aligned}$$

$$\begin{aligned}F &= (282.5\ \text{N})(\cos 70^o) \\ &= 96.6\,\text{N}\end{aligned}$$

$$\boxed{F = 96.6\,\mathrm{N}\text{ acting to right}}$$

y-direction

$$\begin{aligned}
\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\
N - W &= -(-\dot{m} v \sin 70^o) \\
N &= W + \rho A v^2 \sin 70^o
\end{aligned}$$

Calculations:

$$\begin{aligned}
W &= W_{\text{tank}} + W_{\text{water}} \\
&= (20\,\mathrm{kg})\left(9.81\,\mathrm{m/\,s^2}\right) + (0.02\,\mathrm{m}^3)(9800\,\mathrm{N/\,m^3}) \\
&= 392.\,2\ \mathrm{N}
\end{aligned}$$

$$\begin{aligned}
N &= W + \rho A v^2 \sin 70^o \\
&= (392.2\ \mathrm{N}) + (282.5\ \mathrm{N}) \sin 70^o
\end{aligned}$$

$$\boxed{N = 658\ N\text{ acting upward}}$$

6.8: PROBLEM DEFINITION

Situation:

Water jet is filling a tank.
$m = 25$ lbm, $V\!\!\!/ = 5\,\text{gal}$.
$d = 2$ in., $v = 50$ ft/s.

Find:

Minimum coefficient of friction so force on stop block is zero.

Assumptions:

Steady flow, constant density, steady and irrotational flow.

Properties:

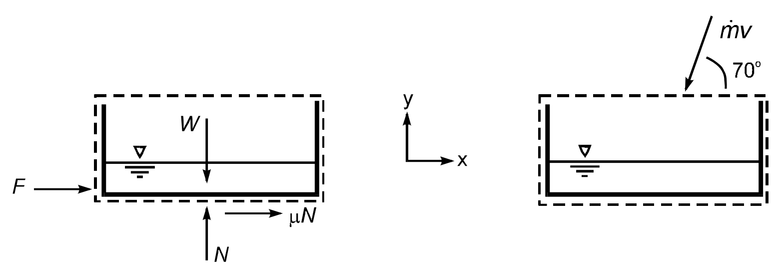
Water (70 °F), Table A.5: $\rho = 1.94\,\text{slug/ft}^3$, $\gamma = 62.4\,\text{lbf/ft}^3$.

PLAN

Apply the momentum equation in the x- and y-directions.

SOLUTION

Force and momentum diagrams



Momentum equation (y-direction)

$$\begin{aligned}\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= -(-\dot{m}v \sin 70^o) \\ N &= W + \rho A v^2 \sin 70^o\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}\mu N &= -(-\dot{m}v \cos 70^o) = \rho A v^2 \cos 70^o \\ \mu &= \frac{(\rho A v^2 \cos 70^o)}{N}\end{aligned}$$

Calculations

$$
\begin{aligned}
\rho A v^2 &= \left(1.94\,\mathrm{slug/\,ft^3}\right)\left(\pi \times \left(\frac{1}{12}\,\mathrm{ft}\right)^2\right)(50\,\mathrm{ft/\,s})^2 \\
&= 105.8 \text{ lbf} \\
W_{H20} &= \gamma \forall \\
&= \frac{(62.37\,\mathrm{lbf/\,ft^3})(5\,\mathrm{gal})}{7.481\,\mathrm{gal/\,ft^3}} \\
&= 41.75 \text{ lbf} \\
W &= (41.75 + 25) \text{ lbf} \\
&= 66.7 \text{ lbf} \\
N &= 66.7\,\mathrm{lbf} + 105.8\,\mathrm{lbf} \times \sin 70^o = \\
&\quad 166.1 \text{ lbf} \\
\mu &= \frac{105.8\,\mathrm{lbf} \times \cos 70^o}{166.1\,\mathrm{lbf}}
\end{aligned}
$$

$$\boxed{\mu = 0.218}$$

6.9: PROBLEM DEFINITION

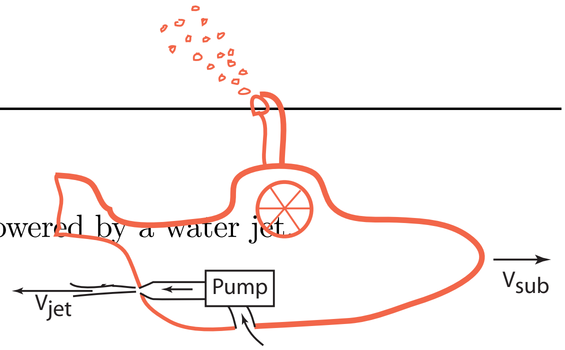
Situation:

A design contest features a submarine powered by a water jet.
$V_{\text{sub}} = 1.0\,\text{m/s}$, $D_1 = 25\,\text{mm}$.
$D_2 = 5\,\text{mm}$, $F_D = C_D \left(\frac{\rho V_{\text{sub}}^2}{2}\right) A_p$
$C_D = 0.3$, $A_p = 0.28\,\text{m}^2$.

Find:

Speed of the fluid jet (m/s).

Sketch:



Assumptions:

Assume steady flow so that the accumulation of momentum term is zero.

Properties:

Water ($15\,°\text{C}$), Table A.5: $\rho = 999\,\text{kg/m}^3$.

PLAN

The speed of the fluid jet can be found from the momentum equation because the drag force will balance with the net rate of momentum outflow.

SOLUTION

Momentum equation. Select a control volume that surrounds the sub. Select a reference frame located on the submarine. Let section 1 be the outlet (water jet) and section 2 be the inlet. The momentum equation is

$$\begin{aligned} \sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\ F_{\text{Drag}} &= \dot{m}_2 v_2 - \dot{m}_1 v_{1x} \end{aligned}$$

By continuity, $\dot{m}_1 = \dot{m}_2 = \rho A_{\text{jet}} V_{\text{jet}}$. The outlet velocity is $v_2 = V_{\text{jet}}$. The x-component of the inlet velocity is $v_{1x} = V_{\text{sub}}$. The momentum equation simplifies to

$$F_{\text{Drag}} = \rho A_{\text{jet}} V_{\text{jet}} (V_{\text{jet}} - V_{\text{sub}})$$

The drag force is

$$\begin{aligned} F_{\text{Drag}} &= C_D \left(\frac{\rho V_{\text{sub}}^2}{2} \right) A_p \\ &= 0.3 \left(\frac{(999\,\text{kg/m}^3)\,(1.0\,\text{m/s})^2}{2} \right) (0.28\,\text{m}^2) \\ &= 42.0\,\text{N} \end{aligned}$$

The momentum equation becomes

$$\begin{aligned} F_{\text{Drag}} &= \rho A_{\text{jet}} V_{\text{jet}} \left[V_{\text{jet}} - V_{\text{sub}} \right] \\ 42.0\,\text{N} &= (999\,\text{kg/m}^3)\,(1.96 \times 10^{-5}\,\text{m}^2)\, V_{\text{jet}} \left[V_{\text{jet}} - (1.0\,\text{m/s}) \right] \end{aligned}$$

Solving for the jet speed gives

$$\boxed{V_{\text{jet}} = 46.8\,\text{m/s}}$$

REVIEW

1. The jet speed (46.6 m/s) is above 100 mph. This present a safety issue. Also, this would require a pump that can produce a large pressure rise.
2. It is recommended that the design be modified to produce a lower jet velocity. One way to accomplish this goal is to increase the diameter of the jet.

6.10: PROBLEM DEFINITION

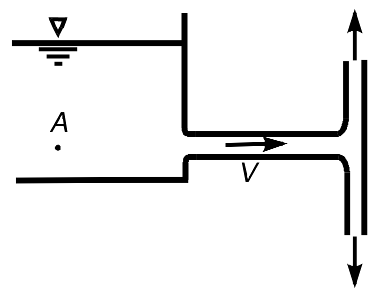
Situation:

Horizontal round jet strikes a plate.
$Q = 2$ cfs, $F_x = 200$ lbf.

Find:

Speed of water jet (ft/s).

Sketch:



Properties:

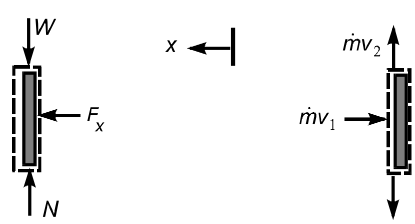
Water (70°F), Table A.5: $\rho = 1.94$ slug/ft^3.

PLAN

Apply the momentum equation to a control volume surrounding the plate.

SOLUTION

Force and momentum diagrams



Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= -\dot{m}v_{1x} \\
F_x &= -(-\dot{m}v_1) = \rho Q v_1 \\
v_1 &= \frac{F_x}{\rho Q} \\
&= \frac{200\,\text{lbf}}{1.94\,\text{slug/ft}^3 \times 2\,\text{ft}^3/\text{s}}
\end{aligned}$$

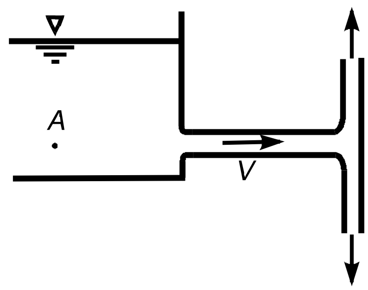
$$\boxed{v_1 = 51.5 \text{ ft/s}}$$

6.11: PROBLEM DEFINITION

Situation:
Horizontal round jet strikes a plate.
$F_x = 600$ lbf

Find:
Diameter of jet (ft).

Sketch:

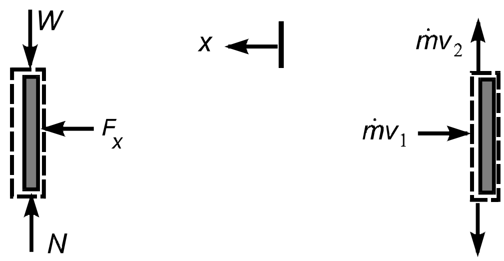


Properties:
$p_A = 25$ psig.
Water (70 ^{o}F), Table A.5: $\rho = 1.94$ slug/ft^3.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION



Force and momentum diagrams

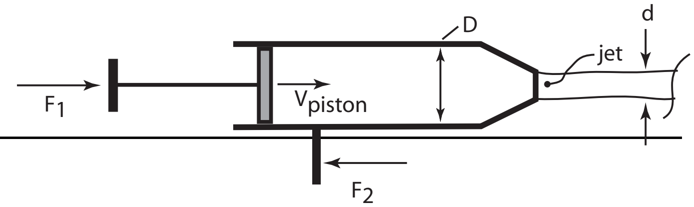
Bernoulli equation applied from inside of tank to nozzle exit

$$\begin{aligned}
\frac{p_A}{\rho} &= \frac{v_1^2}{2} \\
v_1 &= \sqrt{\frac{2p_A}{\rho}} \\
&= \sqrt{\frac{2 \times 25 \text{ psig} \times 144\,\text{in}^2/\,\text{ft}^2}{1.94\,\text{slug}/\,\text{ft}^3}} \\
&= 60.92 \text{ ft/s}
\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= -\dot{m}v_{1x} \\
F_x &= -(-\dot{m}v_1) = \rho A v_1^2 \\
A &= \frac{F_x}{\rho v_1^2} = \frac{600\,\text{lbf}}{1.94\,\text{slug}/\,\text{ft}^3 \times (60.92\,\text{ft}/\,\text{s})^2} \\
A &= 0.0833 \text{ ft}^2 \\
d &= \sqrt{\frac{4A}{\pi}} \\
&= \sqrt{\frac{4 \times 0.0833\,\text{ft}^2}{\pi}}
\end{aligned}$$

$$\boxed{d = 0.326\,\text{ft}}$$



6.12: PROBLEM DEFINITION

Situation:

An engineer is designing a toy to create a jet of water.
$D = 80\,\text{mm}$, $d = 15\,\text{mm}$.
$V_{\text{piston}} = 300\,\text{mm/s}$.

Find:

Which force (F_1 versus F_2) is larger? Explain your answer using concepts of the momentum equation.
Calculate F_1.
Calculate F_2.

Sketch:

Assumptions:

Neglect friction between the piston and the wall.
Assume the Bernoulli equation applies (neglect viscous effects; neglect unsteady flow effects).

Properties:

Table A.5 (water at 20 °C): $\rho = 998\,\text{kg/m}^3$.

PLAN

To find the larger force, recognize that the net force must be in the direction of acceleration. To solve the problem, apply the momentum equation, continuity equation, equilibrium equation, and the Bernoulli equation.

SOLUTION

Finding the larger force (F_1 versus F_2). Since the fluid is accelerating to the right the net force must act to the right. Thus, F_1 is larger than F_2. This can also be seen by application of the momentum equation.

Momentum equation (x-direction) applied to a control volume surrounding the toy.

$$
\begin{aligned}
\sum F_x &= \dot{m}v_{\text{out}} \\
F_1 - F_2 &= \dot{m}v_{\text{out}} \\
F_1 - F_2 &= \rho\left(\frac{\pi d^2}{4}\right)V_{\text{out}}^2 \qquad (1)
\end{aligned}
$$

Notice that Eq. (1) shows that $F_1 > F_2$.

Continuity equation applied to a control volume situated inside the toy.

$$\begin{aligned}
Q_{\text{in}} &= Q_{\text{out}} \\
\left(\frac{\pi D^2}{4}\right) V_{\text{piston}} &= \left(\frac{\pi d^2}{4}\right) V_{\text{out}} \\
V_{\text{out}} &= V_{\text{piston}} \frac{D^2}{d^2} \\
&= (0.3\,\text{m/s}) \left(\frac{80\,\text{mm}}{15\,\text{mm}}\right)^2 \\
V_{\text{out}} &= 8.533\,\text{m/s}
\end{aligned}$$

Bernoulli equation applied from inside the toy to the nozzle exit plane.

$$\begin{aligned}
p_{\text{inside}} + \frac{\rho V_{\text{piston}}^2}{2} &= \frac{\rho V_{\text{out}}^2}{2} \\
p_{\text{inside}} &= \frac{\rho \left(V_{\text{out}}^2 - V_{\text{piston}}^2\right)}{2} \\
&= \frac{(998\,\text{kg/m}^3)\left((8.533\,\text{m/s})^2 - (0.3\,\text{m/s})^2\right)}{2} \\
&= 36.29\,\text{kPa}
\end{aligned}$$

Equilibrium applied to the piston (the applied force F_1 balances the pressure force).

$$\begin{aligned}
F_1 &= p_{\text{inside}} \left(\frac{\pi D^2}{4}\right) \\
&= (36290\,\text{Pa}) \left(\frac{\pi\,(0.08\,\text{m})^2}{4}\right) \\
&\boxed{F_1 = 182\,\text{N}}
\end{aligned}$$

Momentum equation (Eq. 1)

$$\begin{aligned}
F_2 &= F_1 - \rho \left(\frac{\pi d^2}{4}\right) V_{\text{out}}^2 \\
&= 182\,\text{N} - \left(998\,\text{kg/m}^3\right) \left(\frac{\pi\,(0.015\,\text{m})^2}{4}\right) (8.533\,\text{m/s})^2 \\
&\boxed{F_2 = 169\,\text{N}}
\end{aligned}$$

REVIEW

1. The force F_1 is only slightly larger than F_2.

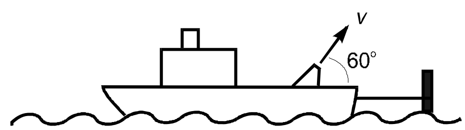
2. The forces (F_1 and F_2) are each about 40 lbf. This magnitude of force may be too large for users of a toy. Or, this magnitude of force may lead to material failure (it breaks!). It is recommended that the specifications for this product be modified.

6.13: PROBLEM DEFINITION

Situation:
Water jet from a fire hose on a boat.
$d = 3$ in, $V = 70$ mph $= 102.7$ ft/s.

Find:
Tension in cable (lbf).

Sketch:



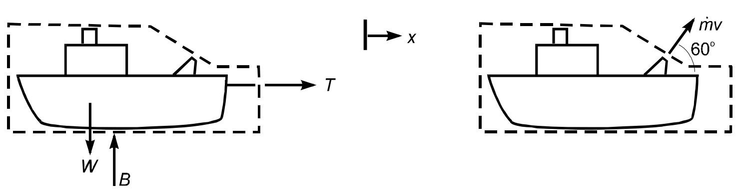
Properties:
Water (50 °F), Table A.5: $\rho = 1.94\,\text{slug}/\,\text{ft}^3$.

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Flow rate

$$\begin{aligned}\dot{m} &= \rho A V \\ &= \left(1.94\,\text{slug}/\,\text{ft}^3\right)\left(\pi \times (1.5/12\,\text{ft})^2\right)(102.7\,\text{ft}/\,\text{s}) \\ &= 9.78\,\text{slug}/\,\text{s}\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}\sum F &= \dot{m}\,(v_o)_x \\ T &= \dot{m}V\cos 60^o \\ T &= (9.78\,\text{slug}/\,\text{s})(102.7\,\text{ft}/\,\text{s})\cos 60^o \\ &= 502.\,2\,\text{lbf}\end{aligned}$$

$$\boxed{T = 502\ \text{lbf}}$$

6.14: PROBLEM DEFINITION

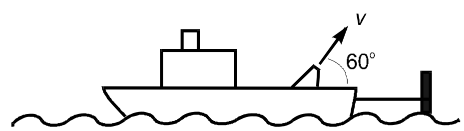
Situation:

Water jet from a fire hose on a boat.
$T = 5.0$ kN, $v = 50$ m/s.

Find:

Mass flow rate of jet (kg/s).
Diameter of jet (cm).

Sketch:



Properties:

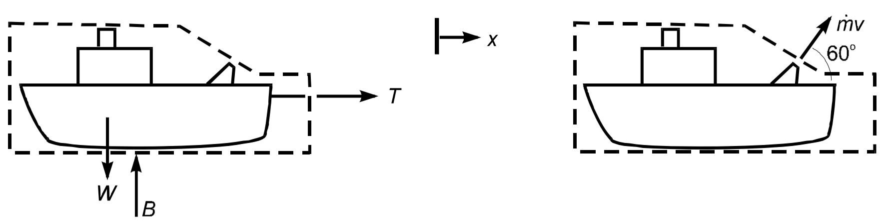
Water (5 °C), Table A.5: $\rho = 1000\,\mathrm{kg/\,m^3}$.

PLAN

Apply the momentum equation to find the mass flow rate. Then, calculate diameter using the flow rate equation.

SOLUTION

Force and momentum diagrams



Momentum equation (x-direction)

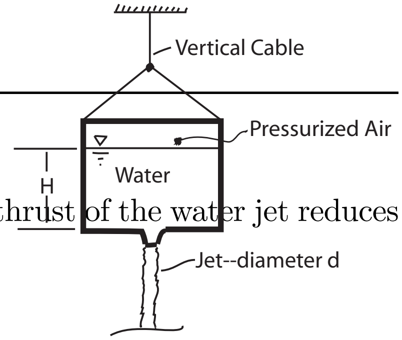
$$\begin{aligned}
\sum F &= \dot{m}\,(v_o)_x \\
T &= \dot{m}v\cos 60^o \\
\dot{m} &= \frac{T}{v\cos 60^o} = \frac{5000\,\mathrm{N}}{(50\times\cos 60^o)\,\mathrm{m/\,s}}
\end{aligned}$$

$$\boxed{\dot{m} = 200\,\mathrm{kg/\,s}}$$

Flow rate

$$
\begin{aligned}
\dot{m} &= \rho A v = \frac{\rho \pi d^2 v}{4} \\
d &= \sqrt{\frac{4\dot{m}}{\rho \pi v}} \\
&= \sqrt{\frac{4 \times 200\,\mathrm{kg/\,s}}{1000\,\mathrm{kg/\,m^3} \times \pi \times 50\,\mathrm{m/\,s}}} \\
&= 7.136 \times 10^{-2}\,\mathrm{m}
\end{aligned}
$$

$$
\boxed{d = 7.14\,\mathrm{cm}}
$$



6.15: PROBLEM DEFINITION

Situation:

Pressurized air drives a water jet out of a tank. The thrust of the water jet reduces the tension in a supporting cable.

$W = 200\,\text{N}$, $T = 10\,\text{N}$.

$d = 12\,\text{mm}$, $H = 425\,\text{mm}$.

Find:

The pressure in the air that is situated above the water.

Sketch:

Assumptions:

Assume that the Bernoulli equation can be applied (i.e. assume irrotational and steady flow).

Properties:

Water (15 °C), Table A.5: $\rho = 999\,\text{kg/m}^3$.

PLAN

Apply the momentum equation to find the exit velocity. Then, apply the Bernoulli equation to find the pressure in the air.

SOLUTION

Section area of jet

$$
\begin{aligned}
A_2 &= \frac{\pi d^2}{4} \\
&= \frac{\pi\,(0.012\,\text{m})^2}{4} \\
&= 1.131 \times 10^{-4}\,\text{m}^2
\end{aligned}
$$

Momentum equation (cv surrounding the tank; section 2 at the nozzle)

$$\begin{aligned} \sum \mathbf{F} &= \dot{m}_o \mathbf{v}_o \\ -T + W &= \dot{m} v_2 \\ (-10 + 200)\ \text{N} &= \rho A_2 v_2^2 \end{aligned}$$

Solve for exit speed (v_2)

$$\begin{aligned} 190\,\text{N} &= \left(999\,\text{kg/m}^3\right)\left(1.131 \times 10^{-4}\,\text{m}^2\right) v_2^2 \\ v_2 &= 41.01\,\text{m/s} \end{aligned}$$

Bernoulli equation (location 1 is on the water surface, location 2 is at the water jet).

$$p_{\text{air}} + \frac{\rho v_1^2}{2} + \rho g z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g z_2$$

Let $v_1 \approx 0$, $p_2 = 0$ gage and $\Delta z = 0.425\,\text{m}$.

$$\begin{aligned} p_{\text{air}} &= \frac{\rho v_2^2}{2} - \rho g \Delta z \\ &= \frac{(999\,\text{kg/m}^3)(41.01\,\text{m/s})^2}{2} - \left(999\,\text{kg/m}^3\right)(9.81\,\text{m/s}^2)(0.425\,\text{m}) \\ &= (835,900\,\text{Pa})\left(\frac{1.0\,\text{atm}}{101.3\,\text{kPa}}\right) \end{aligned}$$

$$\boxed{p_{\text{air}} = 8.25\,\text{atm}}$$

6.16: PROBLEM DEFINITION

Situation:

Free water jet from upper tank to lower tank, lower tank supported by scales A and B.

$Q = 2$ cfs, $d_1 = 4$ in.

$h = 1$ ft, $H = 9$ ft

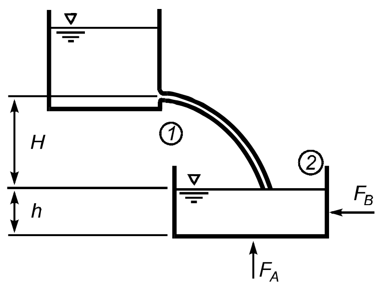
$W_T = 300$ lbf, $A_2 = 4\,\text{ft}^2$.

Find:

Force on scale A (lbf).

Force on scale B (lbf).

Sketch:



Properties:

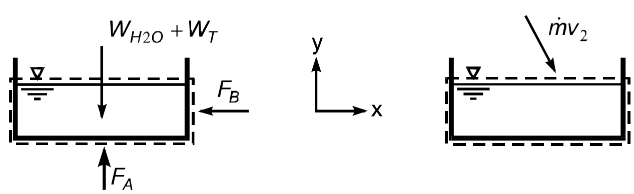
Water (60 ^{o}F): $\rho = 1.94$ slug/ft^3, $\gamma = 62.4$ lbf/ft^3.

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Flow rate

$$\begin{aligned}
\dot{m} &= \rho Q \\
&= 1.94\,\mathrm{slug/\,ft^3} \times 2.0\,\mathrm{ft^3/\,s} \\
&= 3.88 \text{ slug/s} \\
v_1 &= \frac{Q}{A_1} = \frac{4Q}{\pi D^2} \\
&= \frac{4 \times 2.0\,\mathrm{ft^3/\,s}}{\pi \times (4/12)^2\,\mathrm{ft^2}} \\
&= 22.9 \text{ ft/s}
\end{aligned}$$

Projectile motion equations

$$\begin{aligned}
v_{2x} &= v_1 = 22.9 \text{ ft/s} \\
v_{2y} &= \sqrt{2gH} \\
&= \sqrt{2 \times 32.2\,\mathrm{ft/\,s} \times 9\,\mathrm{ft}} \\
&= 24.1 \text{ ft/s}
\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\
-F_B &= -\dot{m}\,(v_{2x}) \\
-F_B &= -3.88\,\mathrm{slug/\,s} \times 22.9\,\mathrm{ft/\,s} \\
& \boxed{F_B = 88.9 \text{ lbf}}
\end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned}
\sum F_y &= \dot{m}\left[(v_o)_y - (v_i)_y\right] \\
F_A - W_{H2O} - W_T &= -\dot{m}\,(v_{2y}) \\
F_A &= W_{H2O} + W_T - \dot{m}\,(v_{2y}) \\
F_A &= (62.4\,\mathrm{lbf/\,ft^3} \times 4\,\mathrm{ft^2} \times 1\,\mathrm{ft}) + 300\,\mathrm{lbf} - (3.88\,\mathrm{slug/\,s} \times (-24.1\,\mathrm{ft/\,s})) \\
& \boxed{F_A = 643.0 \text{ lbf}}
\end{aligned}$$

6.17: PROBLEM DEFINITION

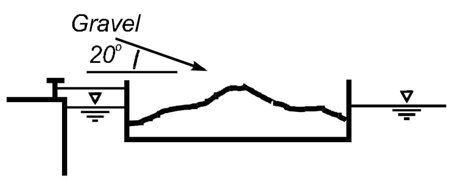
Situation:

Gravel flows into a barge that is secured with a hawser.
$Q = 50$ yd^3/min $= 22.5$ ft^3/s, $v = 10$ ft/s.

Find:

Tension in hawser: T

Sketch:



Assumptions:

Steady flow.

Properties:

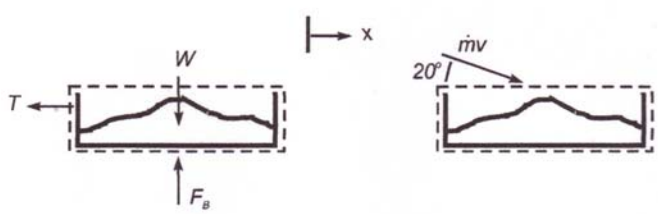
$\gamma = 120$ lbf/ft^3

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Momentum equation (x-direction)

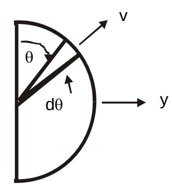
$$\begin{aligned}\sum F_x &= \dot{m}\left(v_o\right)_x - \dot{m}\left(v_i\right)_x \\ -T &= -\dot{m}(v\cos 20) = -(\gamma/g)Q(v\cos 20) \\ T &= \frac{120\,\mathrm{lbf}}{32.2\,\mathrm{ft/s^2}} \times 22.5\,\mathrm{ft^3/s} \times 10\,\mathrm{ft/s} \times \cos(20) = 788 \text{ lbf} \\ &\boxed{T = 788 \text{ lbf}}\end{aligned}$$

6.18: PROBLEM DEFINITION

Situation:
A hemispherical nozzle sprays a sheet of liquid through an arc.

Find:
An expression for the force in y-direction to hold the nozzle stationary.
$F_y = F_y(\rho, v, r, t)$.

Sketch:



PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (y-direction)

$$\begin{aligned} F_y &= \int_{cs} v_y \rho \mathbf{V} \cdot \mathbf{dA} \\ &= \int_0^{\pi} (v \sin\theta)\rho v(trd\theta) \\ &= \rho v^2 tr \int_0^{\pi} \sin\theta d\theta \end{aligned}$$

$$\boxed{F_y = 2\rho v^2 tr}$$

6.19: PROBLEM DEFINITION

Situation:
The design of a conical rocket nozzle.

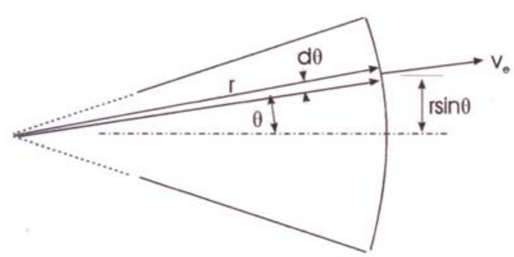
Find:
Show that $T = \dot{m}V_e \frac{1+\cos\alpha}{2}$.

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)



$$\begin{aligned}
\sum \mathbf{F} &= \int \mathbf{v}\rho\mathbf{v}\cdot\mathbf{dA} \\
T &= \int_0^\alpha v_e\cos\theta\rho v_e 2\pi r\sin\theta r d\theta \\
T &= 2\pi r^2\rho v_e^2\int_0^\alpha \cos\theta\sin\theta d\theta \\
&= 2\pi r^2\rho v_e^2\sin^2\alpha/2 \\
&= \rho v_e^2 2\pi r^2\frac{(1-\cos\alpha)(1+\cos\alpha)}{2}
\end{aligned}$$

Exit Area

$$A_e = \int_0^\alpha 2\pi r\sin\theta r d\theta = 2\pi r^2(1-\cos\alpha)$$

$$T = \rho v_e^2 A_e(1+\cos\alpha)/2$$

$$\boxed{T = \dot{m}v_e(1+\cos\alpha)/2}$$

6.20: PROBLEM DEFINITION

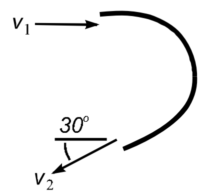
Situation:

A fixed vane in the horizontal plane.
$v_1 = 18$ m/s, $v_2 = 17$ m/s.
$Q = 0.15$ m^3/s, $S = 0.9$.

Find:

Components of force to hold vane stationary (kN).

Sketch:

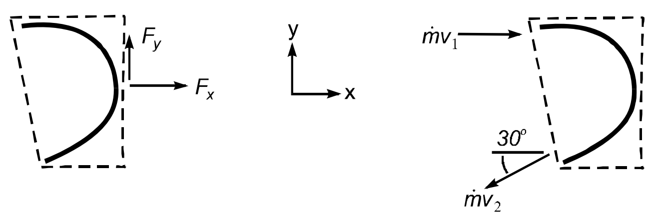


PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Mass flow rate

$$\begin{aligned} \dot{m} &= \rho Q \\ &= 0.9 \times 1000\,\mathrm{kg/\,m^3} \times 0.15\,\mathrm{m^3/\,s} \\ &= 135 \text{ kg/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}\,(v_o)_x - \dot{m}\,(v_i)_x \\ F_x &= \dot{m}(-v_2 \cos 30) - \dot{m}v_1 \\ F_x &= -135\,\mathrm{kg/\,s}(17\,\mathrm{m/\,s}\cos 30 + 18\,\mathrm{m/\,s}) \end{aligned}$$

$$\boxed{F_x = -4.42\,\mathrm{kN}\ \text{(acts to the left)}}$$

Momentum equation (y-direction)

$$\begin{aligned}
\sum F_y &= \dot{m}\,(v_o)_y - \dot{m}\,(v_i)_y \\
F_y &= \dot{m}\,(-v_2 \sin 30) \\
&= 135\,\mathrm{kg/\,s}\,(-17\,\mathrm{m/\,s}\sin 30) \\
&= -1.15\ \mathrm{kN}
\end{aligned}$$

$$\boxed{F_y = -1.15\,\mathrm{kN}\ \text{(acts downward)}}$$

6.21: PROBLEM DEFINITION

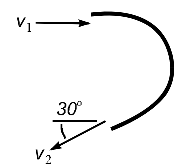
Situation:

A fixed vane in the horizontal plane.
$v_1 = 90$ ft/s, $v_2 = 85$ ft/s.
$Q = 2.0$ cfs, $S = 0.9$.

Find:

Components of force to hold vane stationary (lbf).

Sketch:

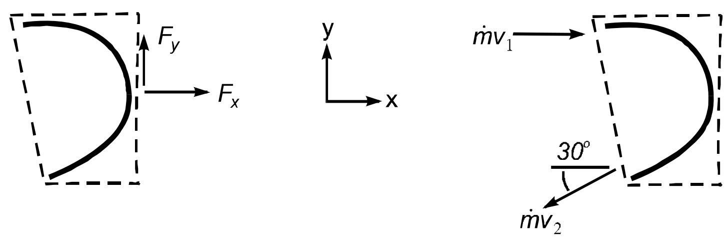


PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



Mass flow rate

$$\dot{m} = \rho Q = 0.9 \times 1.94\,\text{slug/ft}^3 \times 2.0\,\text{ft}^3/\text{s} = 3.49 \text{ slug/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}\,(v_o)_x - \dot{m}\,(v_i)_x \\
F_x &= \dot{m}(-v_2 \cos 30) - \dot{m}v_1 \\
F_x &= -3.49\,\text{slug/s}(85\,\text{ft/s} \cos 30 + 90\,\text{ft/s}) \\
&\boxed{F_x\text{=-571 lbf (acts to the left)}}
\end{aligned}$$

y-direction

$$\begin{aligned}
\sum F_y &= \dot{m}\,(v_o)_y - \dot{m}\,(v_i)_y \\
F_y &= \dot{m}\,(-v_2 \sin 30) = 3.49\,\mathrm{slug/\,s}\,(-85\,\mathrm{ft/\,s}\sin 30) = -148 \text{ lbf} \\
&\boxed{F_y = -148 \text{ lbf (acts downward)}}
\end{aligned}$$

6.22: PROBLEM DEFINITION

Situation:

A horizontal, two-dimensional water jet deflected by a fixed vane.
$v_1 = 40$ ft/s, $w_2 = 0.2$ ft, $w_3 = 0.1$ ft.

Find:

Components of force, per foot of width, to hold the vane stationary (lbf/ft).

Assumptions:

Neglect elevation changes.
Neglect viscous effects.

Properties:

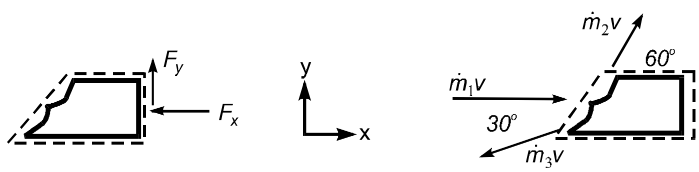
Water, Table A.5: $\rho = 1.94$ slug/ft^3.

PLAN

Apply the Bernoulli equation, the continuity equation, and finally the momentum equation.

SOLUTION

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v_3 = v = 40 \text{ ft/s}$$

Continuity equation

$$\begin{aligned} w_1 v_1 &= w_2 v_2 + w_3 v_3 \\ w_1 &= w_2 + w_3 = (0.2 + 0.1) = 0.3 \text{ ft} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\ -F_x &= \dot{m}_2 v \cos 60 + \dot{m}_3(-v \cos 30) - \dot{m}_1 v \\ F_x &= \rho v^2(-A_2 \cos 60 + A_3 \cos 30 + A_1) \\ F_x &= 1.94 \, \text{slug/ft}^3 \times (40 \, \text{ft/s})^2 \times (-0.2 \, \text{ft} \cos 60 + 0.1 \, \text{ft} \cos 30 + 0.3 \, \text{ft}) \end{aligned}$$

$$\boxed{F_x = 890 \text{ lbf/ft (acts to the left)}}$$

Momentum equation (y-direction)

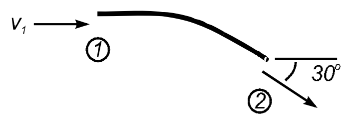
$$\begin{aligned}
\sum F_y &= \sum \dot{m}_o \left(v_o\right)_y \\
F_y &= \dot{m}_2 v \sin 60 + \dot{m}_3(-v \sin 30) \\
&= \rho v^2 (A_2 \sin 60 - A_3 \sin 30) \\
&= 1.94\,\text{slug}/\,\text{ft}^3 \times (40\,\text{ft}/\,\text{s})^2 \times (0.2\,\text{ft} \sin 60 - 0.1\,\text{ft} \sin 30) \\
&\boxed{F_y = 382 \text{ lbf/ft (acts upward)}}
\end{aligned}$$

6.23: PROBLEM DEFINITION

Situation:
A water jet is deflected by a fixed vane.
$v_1 = 20$ ft/s, $\dot{m} = 25$ lbm/s $= 0.776$ slug/s.

Find:
Force of the water on the vane (lbf).

Sketch:

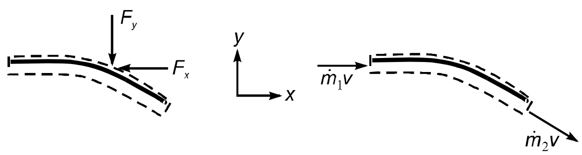


PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 20 \text{ ft/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\
-F_x &= \dot{m}v\cos 30 - \dot{m}v \\
F_x &= \dot{m}v(1-\cos 30) = 0.776\,\text{slug/s} \times 20\,\text{ft/s} \times (1-\cos 30) \\
F_x &= 2.08 \text{ lbf to the left}
\end{aligned}$$

y-direction

$$\begin{aligned}
\sum F_y &= \dot{m}_o (v_o)_y \\
-F_y &= \dot{m}(-v\cos 60) = -0.776\,\text{slug/s} \times 20\,\text{ft/s} \times \sin 30 \\
F_y &= 7.76 \text{ lbf downward}
\end{aligned}$$

Since the forces acting on the vane represent a state of equilibrium, the force of water on the vane is equal in magnitude & opposite in direction.

$$\begin{aligned}
\mathbf{F} &= -F_x\mathbf{i} - F_y\mathbf{j} \\
&= \boxed{(2.08 \text{ lbf})\,\mathbf{i} + (7.76 \text{ lbf})\,\mathbf{j}}
\end{aligned}$$

6.24: PROBLEM DEFINITION

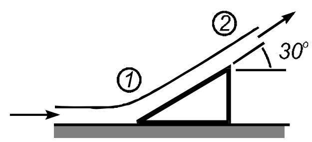
Situation:

A water jet strikes a block and the block is held in place by friction.
$v_1 = 10$ m/s, $\dot{m} = 1.5$ kg/s.
$\mu = 0.1$, $m = 1$ kg.

Find:

Will the block slip?
Force of the water jet on the block (N).

Sketch:



Assumptions:

Neglect weight of water.
Neglect elevation changes.
Neglect viscous forces.

Properties:

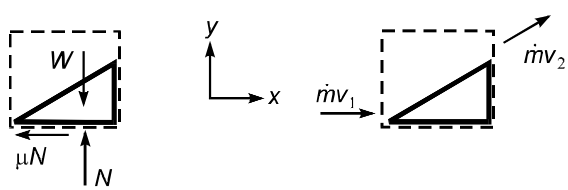
$\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 10 \text{ m/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\
-F_f &= \dot{m} v \cos 30 - \dot{m} v \\
F_f &= \dot{m} v (1 - \cos 30) \\
&= 1.5\,\text{kg/s} \times 10\,\text{m/s} \times (1 - \cos 30) \\
&\boxed{F_f = 2.01\,\text{N}}
\end{aligned}$$

y-direction

$$\begin{aligned}\sum F_y &= \dot{m}_o\,(v_o)_y \\ N - W &= \dot{m}(v\sin 30) \\ N &= mg + \dot{m}(v\sin 30) \\ &= 1.0\,\text{kg}\times 9.81\,\text{m/s}^2 + 1.5\,\text{kg/s}\times 10\,\text{m/s}\times\sin 30 \\ &\boxed{N = 17.3\ \text{N}}\end{aligned}$$

Analyze friction:

- F_f (required to prevent block from slipping) = 2.01 N
- F_f (maximum possible value) = $\mu N = 0.1 \times 17.3 = 1.73$ N

$$\boxed{\text{block will move}}$$

6.25: PROBLEM DEFINITION

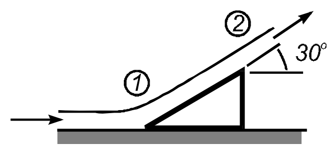
Situation:

A water jet strikes a block and the block is held in place by friction.
$\dot{m} = 1$ kg/s, $m = 1$ kg.
$\mu = 0.1$, $\theta = 30°$.

Find:

Maximum velocity such that the block will not slip.

Sketch:



Assumptions:

Neglect weight of water.

Properties:

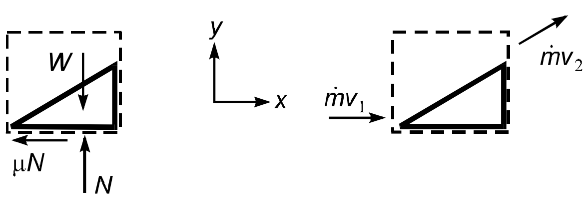
$\rho = 1000\,\text{kg/m}^3$.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v$$

Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ -\mu N &= \dot{m}v\cos 30 - \dot{m}v \\ N &= \dot{m}v\left(1 - \cos 30\right)/\mu\end{aligned}$$

y-direction

$$\begin{aligned}\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= \dot{m}(v\sin 30) \\ N &= mg + \dot{m}(v\sin 30)\end{aligned}$$

Combine previous two equations

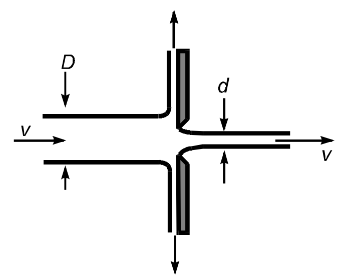
$$
\begin{aligned}
\frac{\dot{m}v\,(1-\cos 30)}{\mu} &= mg + \dot{m}(v \sin 30) \\
v &= \frac{mg}{[\dot{m}\,(1/\mu - \cos 30/\mu - \sin 30)]} \\
v &= \frac{1\,\mathrm{kg} \times 9.81\,\mathrm{m/\,s^2}}{[1.5\,\mathrm{kg/\,s} \times (1/0.1 - \cos 30/0.1 - \sin 30)]} \\
& \boxed{v = 7.79\ \ \mathrm{m/\,s}}
\end{aligned}
$$

6.26: PROBLEM DEFINITION

Situation:

A water jet strikes a plate with a sharp edged orifice at its center.

$v = 30$ m/s, $D = 5$ cm, $d = 2$ cm



Find:

Force required to hold plate stationary (N).

Assumptions:

Neglect gravity.

Properties:

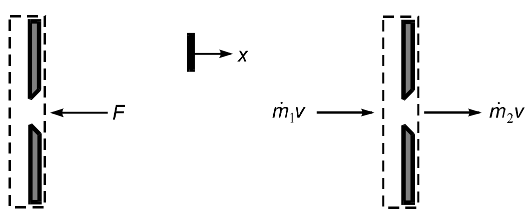
$\rho = 1000$ kg/m^3

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams (only x-direction vectors shown)



Momentum equation (x-direction)

$$\begin{aligned}
\sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\
-F &= \dot{m}_2 v - \dot{m}_1 v \\
F &= \rho A_1 v^2 - \rho A_2 v^2 \\
&= \rho v^2 \left(\frac{\pi}{4}\right)(D^2 - d^2) \\
&= 1000\,\mathrm{kg/\,m^3} \times (30\,\mathrm{m/\,s})^2 \times \frac{\pi}{4} \times ((0.05\,\mathrm{m})^2 - (0.02\,\mathrm{m})^2)
\end{aligned}$$

$$\boxed{F = 1.48 \text{ kN (to the left)}}$$

6.27: PROBLEM DEFINITION

Situation:

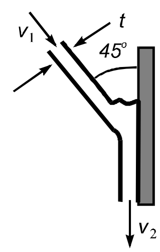
A 2D liquid jet impinges on a vertical wall.

$v_1 = v_2 = v$, $\theta = 45\,^\circ$.

Find:

Calculate the force acting on the wall.

Sketch and explain the shape of the liquid surface.



Assumptions:

Steady flow.

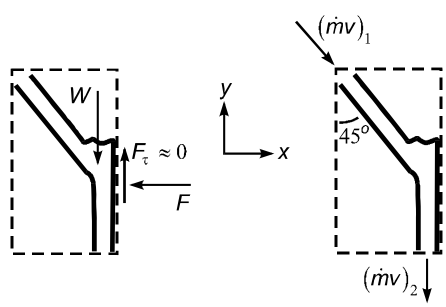
Force associated with shear stress is negligible.

PLAN

Apply the momentum equation.

SOLUTION

Let w = the width of the jet in the z-direction. Force and momentum diagrams



Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
-F &= -\dot{m} v_1 \sin 45^o \\
F &= \rho w t v^2 \sin 45^o
\end{aligned}$$

The force on that acts on the wall is in the opposite direction to force pictured on the force diagram, thus

$$\boxed{F/w = \rho t v^2 \sin 45^o \text{ (acting to the right)}}$$

y-direction

$$\begin{aligned}
\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\
-W &= \dot{m}\,(-v) - \dot{m}\,(-v)\cos 45^o \\
W &= \dot{m}v(1 - \cos 45^o)
\end{aligned}$$

REVIEW

Thus, weight provides the force needed to increase y-momentum flow. This weight is produced by the fluid swirling up to form the shape show in the above sketches.

6.28: PROBLEM DEFINITION

Situation:
A cone is supported by a vertical jet of water.
$W = 30\,\mathrm{N}$, $V_1 = 15\,\mathrm{m/s}$.
$d_1 = 2\,\mathrm{cm}$, $\theta = 60\,^\circ$.

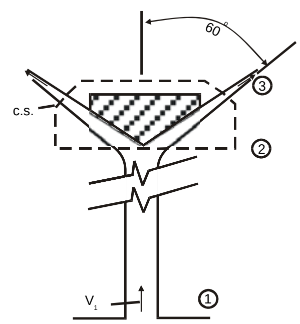
Find:
Height to which cone will rise (m).

Assumptions:
Speed of the fluid as it passes by the cone is constant $(V_2 = V_3)$.

PLAN

Apply the Bernoulli equation and the momentum equation.

SOLUTION



Bernoulli equation

$$\begin{aligned}\frac{V_1^2}{2g} + 0 &= \frac{V_2^2}{2g} + h \\ V_2^2 &= (V_1)^2 - 2gh \\ V_2^2 &= 225 - 19.62h\end{aligned}$$

Momentum equation (y-direction). Select a control volume surrounding the cone.

$$\begin{aligned}\sum F_y &= \dot{m}_o v_{oy} - \dot{m}_i v_{iy} \\ -W &= \dot{m}(v_{3y} - v_2) \\ -30\,\mathrm{N} &= 1000\,\mathrm{kg/m^3} \times 15\,\mathrm{m/s} \times \pi \times (0.01\,\mathrm{m})^2 (V_2 \sin 30^\circ - V_2)\end{aligned}$$

Solve for the V_2

$$V_2 = 12.73 \text{ m/s}$$

Complete the Bernoulli equation calculation

$$\begin{aligned} V_2^2 &= 225 - 19.62h \\ (12.73\,\mathrm{m/s})^2 &= 225 - 19.62h \end{aligned}$$

$$\boxed{h = 3.21\,\mathrm{m}}$$

6.29: PROBLEM DEFINITION

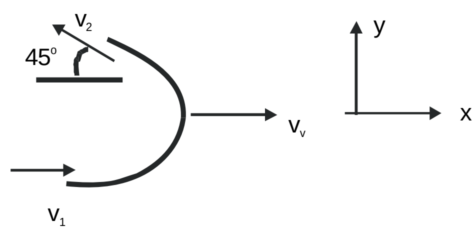
Situation:

A fluid jet strikes a vane that is moving at a speed.
$v_1 = 20\,\text{m/s}$, $v_v = 7$ m/s.
$D_1 = 6$ cm.

Find:

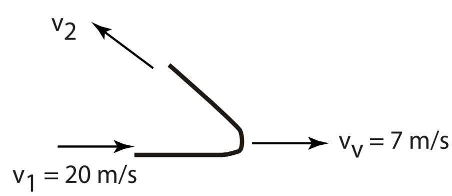
Force of the water on the vane.

Sketch:



SOLUTION

Force and momentum diagrams
Select a control volume surrounding and moving with the vane. Select a reference frame attached to the moving vane.



Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}v_{2x} - \dot{m}v_{1x} \\ -F_x &= -\dot{m}v_2 \cos 45^\circ - \dot{m}v_1 \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \dot{m}v_{2y} - \dot{m}v_{1y} \\ F_y &= \dot{m}v_2 \sin 45^\circ \end{aligned}$$

Velocity analysis

- v_1 is relative to the reference frame $= (20 - 7) = 13\,\text{m/s}$.

- in the term $\dot{m} = \rho A v$ use v which is relative to the control surface. In this case $v = (20 - 7) = 13$ m/s
- v_2 is relative to the reference frame $v_2 = v_1 = 13$ m/s

Mass flow rate

$$\begin{aligned} \dot{m} &= \rho A v \\ &= (1,000 \text{ kg})(\pi/4 \times \left(0.06\,\text{m}^2\right)^2)(13\,\text{m/s}) \\ &= 36.76 \text{ kg/s} \end{aligned}$$

Evaluate forces

$$\begin{aligned} F_x &= \dot{m} v_1 (1 + \cos 45) \\ &= 36.76\,\text{kg/s} \times 13\,\text{m/s}(1 + \cos 45) = 816 \text{ N} \end{aligned}$$

which is in the negative $x-$direction.

$$\begin{aligned} F_y &= \dot{m} v_2 \sin 45 \\ &= 36.76\,\text{kg/s} \times 13\,\text{m/s} \sin 45 = 338 \text{ N} \end{aligned}$$

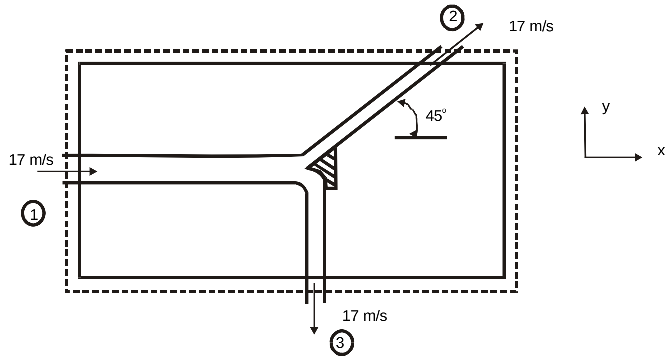
The force of the water on the vane is the negative of the force of the vane on the water. Thus the force of the water on the vane is

$$\boxed{\mathbf{F} = (816\mathbf{i} - 338\mathbf{j}) \text{ N}}$$

6.30: PROBLEM DEFINITION

Situation:

A cart is moving with steady speed.



Find:

Force exerted by the vane on the jet: **F**

PLAN

Apply the momentum equation.

SOLUTION

Make the flow steady by referencing all velocities to the moving vane and let the c.v. move with the vane as shown.

Momentum equation (x-direction)

$$\begin{aligned}
F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_1 \\
\dot{m} &= \rho A V = 1000\,\mathrm{kg/\,m^3} \times (\pi/4) \times (0.1\,\mathrm{m})^2 \times 17\,\mathrm{m/\,s} = 133.5\ \mathrm{kg/s} \\
F_x &= (\frac{\dot{m}}{2} v \cos 45^{\mathrm{o}} - \dot{m} v) = \dot{m} v (\frac{\cos 45^{\mathrm{o}}}{2} - 1) \\
&= 133.5\ \mathrm{kg/s} \times 17\ \mathrm{m/s} \times (0.3535 - 1) \\
&= -1470\ \mathrm{N}
\end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned}
F_y &= \dot{m}_2 v_{2y} - \dot{m} v_{3y} \\
&= \frac{\dot{m}}{2} v \sin 45^{\mathrm{o}} - \frac{\dot{m}}{2} v = \frac{\dot{m}}{2} v (\sin 45^{\mathrm{o}} - 1) \\
&= \frac{133.5\ \mathrm{kg/s}}{2} \times 17\ \mathrm{m/s} \times (0.707 - 1) \\
&= -332\ \mathrm{N}
\end{aligned}$$

$$\boxed{\mathbf{F}(\text{water on vane}) = (1470\mathbf{i} + 332\mathbf{j})\ \mathrm{N}}$$

6.31: PROBLEM DEFINITION

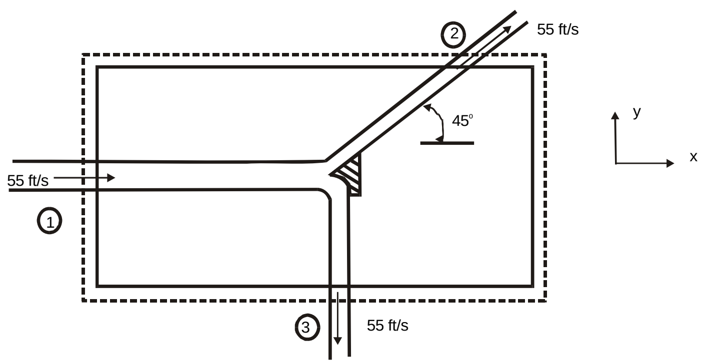
Situation:

A cart is moving with steady speed—additional details are provided in the problem statement.

Find:

Rolling resistance of the cart: $F_{rolling}$

SOLUTION Let the control surface surround the cart and let it move with the cart at 5 ft/s. Then we have a steady flow situation and the relative jet velocities are shown below.



Momentum equation (x-direction)

$$\sum F_x = \dot{m}_2 v_{2_x} - \dot{m}_1 v_1$$

Calculations (All calculations need o be fixed)

$$\begin{aligned}
\dot{m}_1 &= \rho A_1 V_1 \\
&= (1.94 \text{ slug/ft}^3)(\pi/4 \times (0.15^2 \text{ft}^2) \times 55 \text{ ft/s} \\
&= 1.886 \text{ slug/s} \\
\dot{m}_2 &= \dot{m}_3 = (1.886 \text{ slug/s})/2 \\
&= 0.943 \text{ slug/s} \\
F_{\text{rolling}} &= \dot{m}_1 v_1 - \dot{m}_2 v_2 \cos 45^\circ \\
&= 1.886 \, \text{slug/s} \times 55 \, \text{ft/s} - 0.943 \, \text{slug/s} \times 55 \, \text{ft/s} \cos 45^\circ \\
& \boxed{F_{\text{rolling}} = 67.1 \text{ lbf (acting to the left)}}
\end{aligned}$$

6.32: PROBLEM DEFINITION

Situation:

A water is deflected by a moving cone.

Speed of the water jet is 25 m/s (to the right). Speed of the cone is 13 m/s (to the left). Diameter of the jet is $D = 10\,\text{cm}$.

Angle of the cone is $\theta = 50^o$.

Find:

Calculate the external horizontal force needed to move the cone: F_x

Assumptions:

As the jet passes over the cone (a) assume the Bernoulli equation applies, and (b) neglect changes in elevation.

PLAN

Apply the momentum equation.

SOLUTION

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone. Section 1 is the inlet. Section 2 is the outlet.
Inlet velocity (relative to the reference frame and surface of the control volume).

$$\begin{aligned} v_1 &= V_1 = (25 + 13)\ \text{m/s} \\ & 38\,\text{m/s} \end{aligned}$$

Bernoulli equation. Pressure and elevation terms are zero, so

$$V_1 = V_2 = v_2 = 38\,\text{m/s}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= \dot{m}(v_{2x} - v_1) \\ &= \rho A_1 V_1 (v_2 \cos\theta - v_1) \\ &= \rho A_1 V_1^2 (\cos\theta - 1) \\ &= \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \times \left(\frac{\pi \times (0.1\,\text{m})^2}{4}\right) \times (38\,\text{m/s})^2 (\cos 50^o - 1) \\ &= -4.05\ \text{kN} \end{aligned}$$

$$\boxed{F_x = 4.05\ \text{kN (acting to the left)}}$$

6.33: PROBLEM DEFINITION

Situation:

A jet of water is deflected by a moving vane—additional details are provided in the problem statement.

Find:

Power (per foot of width of the jet) transmitted to the vane: P

PLAN

Apply the momentum equation.

SOLUTION

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone.

Density

$$\rho = \frac{62.4\,\mathrm{lbf/\,ft^3}}{32.2\,\mathrm{ft/\,s^2}} = 1.94 \text{ slug/ft}^3$$

Velocity analysis

$$\begin{aligned} v_1 &= V_1 = 40 \text{ ft/s} \\ v_2 &= 40 \text{ ft/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_1) \\ F_x &= 1.94\,\mathrm{slug/\,ft^3} \times 40\,\mathrm{ft/\,s} \times 0.3\,\mathrm{ft^2} \times (40\,\mathrm{ft/\,s}\cos 50 - 40\,\mathrm{ft/\,s}) \\ &= -332.6 \text{ lbf/ ft} \end{aligned}$$

Calculate power

$$\begin{aligned} P &= Fv \\ &= 332.6\,\mathrm{lbf/\,ft} \times 60\,\mathrm{ft/\,s} \\ &= \boxed{P = 19,956 \text{ ft-lbf/s/ft} = 36.3 \text{ hp/ft}} \end{aligned}$$

6.34: PROBLEM DEFINITION

Situation:

A sled of mass $m_s = 1000$ kg is decelerated by placing a scoop of width $w = 20$ cm into water at a depth $d = 8$ cm.

Find:

Deceleration of the sled: a_s

SOLUTION

Select a moving control volume surrounding the scoop and sled. Select a stationary reference frame.

Momentum equation (x-direction)

$$0 = \frac{d}{dt}(m_s v_s) + \dot{m}v_{2_x} - \dot{m}v_{1_x}$$

Velocity analysis

$$\begin{aligned} v_{1_x} &= 0 \\ V_1 &= 100 \text{ m/s} \\ V_2 &= 100 \text{ m/s} \\ \mathbf{v}_2 &= 100 \text{ m/s}[-\cos 60\mathbf{i} + \sin 60\mathbf{j}] + 100\mathbf{i} \text{ m/s} \\ v_{2x} &= 50 \text{ m/s} \end{aligned}$$

The momentum equation equation simplifies to

$$0 = m_s a_s + \dot{m}v_{2_x} \tag{1}$$

Flow rate

$$\begin{aligned} \dot{m} &= \rho A_1 V_1 \\ &= 1000 \,\text{kg/m}^3 \times 0.2 \,\text{m} \times 0.08 \,\text{m} \times 100 \,\text{m/s} \\ &= 1600 \text{ kg/s} \end{aligned}$$

From Eq. (1).

$$\begin{aligned} a_s &= -\frac{\dot{m}v_{2x}}{m_s} \\ &= \frac{(-1600 \,\text{kg/s})(50 \,\text{m/s})}{1000 \,\text{kg}} \\ &= \boxed{a_s = -80 \text{ m/s}^2} \end{aligned}$$

6.35: PROBLEM DEFINITION

Situation:

A snowplow is described in the problem statement.

Find:

Power required for snow removal: P

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)
Select a control volume surrounding the snow-plow blade. Attach a reference frame to the moving blade.(Snow is 4 in deep)

$$\sum F_x = \rho Q(v_{2x} - v_1)$$

Velocity analysis

$$\begin{aligned} V_1 &= v_1 = 40 \text{ ft/s} \\ v_{2_x} &= -40 \,\text{ft/s} \cos 60^\circ \cos 30^\circ \\ &= -17.32 \text{ ft/s} \end{aligned}$$

Calculations

$$\begin{aligned} \sum F_x &= \rho V dWS(v_{2x} - v_1) \\ &= 1.94 \,\text{slug/ft}^3 \times 0.2 \times 40 \,\text{ft/s} \times 2 \,\text{ft} \times \frac{1}{3} \,\text{ft}(-17.32 \,\text{ft/s} - 40 \,\text{ft/s}) \\ &= -593.1 \text{ lbf} \end{aligned}$$

Power

$$\begin{aligned} P &= FV \\ &= 593.1 \,\text{lbf} \times 40 \,\text{ft/s} \\ &= 23,723 \text{ ft-lbf/s} \\ & \boxed{P = 43.1 \text{ hp}} \end{aligned}$$

6.36: PROBLEM DEFINITION

Situation:

The flow over an airfoil is modeled as the flow in a circular stream tube which has a diameter equal to the wing span and is deflected by an angle of 2°.New

Find:

The lift and the drag forces.

Assumptions:

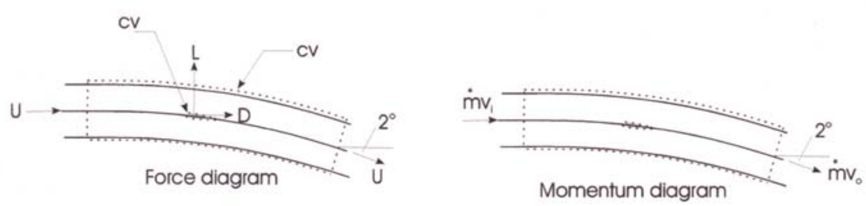
Assume the pressure is constant far from the airfoil.

PLAN

Apply the component form of the momentum equation.

SOLUTION

Draw an outer volume that encloses the airfoil far from the airfoil and one around the airfoil as shown in the diagram. The space between the two volumes is the control volume



The force diagram shows a lift force and drag force produced by the airfoil and act on the control surface. There is no net pressure force on the outer surface since the pressure is constant. The momentum diagram shows an inflow and outflow of momentum. From continuity, the mass flow rate in is equal to the mass flow rate out. The sum of the forces in the x-direction is

$$\sum F_x = D$$

and in the y-direction

$$\sum F_y = L$$

The component momentum equation in the x-direction for steady flow is

$$\sum F_x = \dot{m}\left(U\cos\theta - U\right)$$

The component momentum equation in the y-direction is

$$\sum F_y = \dot{m}(-U\sin\theta - 0)$$

The mass flow rate is

$$\dot{m} = \rho U A = \rho U \left(\frac{\pi b^2}{4} \right)$$

The density is obtained from the ideal gas law

$$\rho = \frac{p}{RT} = \frac{14.7 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2}{1716 \text{ ft-lbf/slug-R} \times (460 + 60) \text{ R}} = 0.00237 \text{ slug/ft}^3$$

The mass flow rate is

$$\dot{m} = 0.00237 \text{ slug/ft}^3 \times 300 \text{ ft/s} \times \frac{\pi \times 30^2 \text{ ft}^2}{4} = 502.6 \text{ slug/s}$$

Solving for the drag force

$$\begin{aligned} D &= \dot{m} U (\cos\theta - 1) \\ &= 502.6 \text{ slug/s} \times 300 \text{ ft/s} \times (\cos 2^{\text{o}} - 1) \\ &= -91.8 \text{ lbf} \end{aligned}$$

Solving for lift force

$$\begin{aligned} L &= -\dot{m} U \sin\theta \\ &= -502.6 \times 300 \times \sin 2^{\text{o}} \\ &= -5260 \text{ lbf} \end{aligned}$$

The values calculated are the force of the airfoil in the fluid. The force of the fluid on the airfoil which are the actual definitions of lift and drag would have the opposite sign so

$$\boxed{D = 91.8 \text{ lbf}}$$

$$\boxed{L = 5260 \text{ lbf}}$$

6.37: PROBLEM DEFINITION

Situation:

A clam shell thrust reverser is deployed on an aircraft engine.

Find:

(a) The thrust under normal operation.

(b) the reverse thrust.

Assumptions:

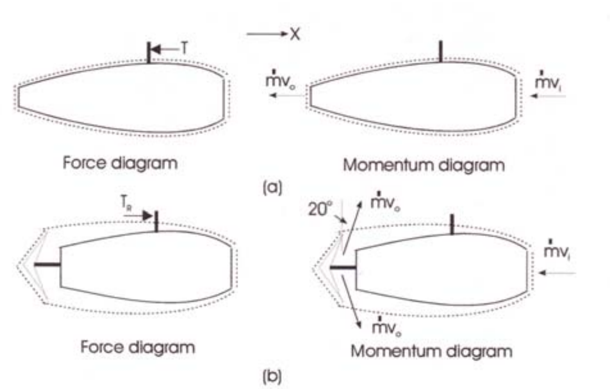
Engine is stationary.

Exit gas velocity unchanged at deployment.

Pressure is atmospheric at exhaust plane.

PLAN

Apply the component momentum equation.



SOLUTION

The control volumes for both cases are shown in the diagram. For case (a) the sum of the forces in the x-direction is

$$\sum F_x = -T$$

and for case (b)

$$\sum F_x = T_R$$

From the momentum diagrams for case (a) there is an influx and outflow of momentum in the same direction. For case (b), the outlet direction of the momentum is altered by the vane of the thrust reverser.

The component momentum equation in the x-direction is

$$\sum F_x = \frac{d}{dt}\int_{cv} \rho v d\forall + \sum \dot{m}_o v_{xo} - \sum \dot{m}_i v_{xi}$$

The motor is stationary so there is no unsteady term. Also the mass flow rate in is equal to the mass flow rate out, $\dot{m}_i = \dot{m}_o = \dot{m}$. For case (a)

$$\begin{aligned} -T &= \dot{m}[-U_o - (-U_i)] \\ T &= \dot{m}(U_o - U_i) \end{aligned}$$

The mass flow rate is

$$\dot{m} = 150 \text{ lbm/s} \frac{1 \text{ slug}}{32.2 \text{ lbm}} = 4.658 \text{ slug/s}$$

The thrust for case (a) is

$$\begin{aligned} T &= 4.658 \frac{\text{slug}}{\text{s}} \left(1400 \ \frac{\text{ft}}{\text{s}} - 300 \ \frac{\text{ft}}{\text{s}} \right) \\ &= \boxed{T = 5120 \text{ lbf}} \end{aligned}$$

For case (b)

$$\begin{aligned} T_R &= \dot{m}[U_o \sin 20^o - (-U_i)] \\ &= \dot{m}(U_o \sin 20^o + U_i) \end{aligned}$$

The reverse thrust is

$$\begin{aligned} T_R &= 4.658 \frac{\text{slug}}{\text{s}} (1400 \frac{\text{ft}}{\text{s}} \times \sin 20^o + 300 \frac{\text{ft}}{\text{s}}) \\ &= \boxed{T = 3630 \text{ lbf}} \end{aligned}$$

6.38: PROBLEM DEFINITION

Situation:

Hot gas flows through a return bend—additional details are provided in the problem statement.

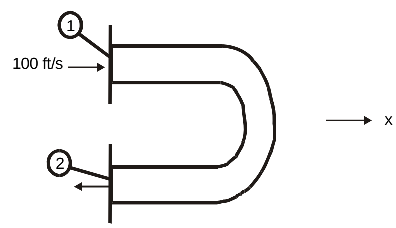
Find:

Force required to hold the bend in place: F_x

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION



$$\dot{m} = 1 \text{ lbm/s} = 0.0311 \text{ slugs/s}$$

At section (1):

$$\begin{aligned} v_1 &= 100 \text{ ft/s} \\ \rho_1 &= 0.02 \text{ lbm/ft}^3 = 0.000621 \text{ slugs/ft}^3 \end{aligned}$$

At section (2):

$$\rho_2 = 0.06 \text{ lbm/ft}^3 = 0.00186 \text{ slugs/ft}^3$$

Continuity equation

$$\begin{aligned} \rho_1 v_1 A_1 &= \rho_2 v_2 A_2 \\ v_2 &= \frac{\rho_1}{\rho_2}\left(\frac{A_1}{A_2}\right) v_1 \\ v_2 &= \left(\frac{0.02 \text{ lbm/ ft}^3}{0.06 \text{ lbm/ ft}^3}\right)\left(\frac{1}{1}\right) v_1 \\ &= 33.33 \text{ ft/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{\subset s} \dot{m}_o v_{o_x} - \sum_{cs} \dot{m}_i v_{i_x} \\ &= \dot{m}(v_2 - v_1) \\ F_x &= 0.0311 \, \text{slug/ s}(-33.33 \, \text{ft/ s} - 100 \, \text{ft/ s}) \\ & \boxed{F_x = -4.15 \text{ lbf}} \end{aligned}$$

6.39: PROBLEM DEFINITION

Situation:

Fluid (density ρ, discharge Q, and velocity V) flows through a 180^o pipe bend—additional details are provided in the problem statement.. Cross sectional area of pipe is A.

Find:

Magnitude of force required at flanges to hold the bend in place.

Assumptions:

Gage pressure is same at sections 1 and 2. Neglect gravity.

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ p_1 A_1 + p_2 A_2 + F_x &= \dot{m}(v_2 - v_1)\end{aligned}$$

thus

$$\boxed{F_x = -2pA - 2\dot{m}V}$$

$$\boxed{F_x = -2pA - 2\rho QV}$$

Correct choice is (d)

6.40: PROBLEM DEFINITION

Situation:

Water flows through a 180^o pipe bend—additional details are provided in the problem statement.

Find:

External force required to hold bend in place.

PLAN

Apply the momentum equation.

SOLUTION

Flow rate equation

$$v = \frac{Q}{A} = \frac{20\,\mathrm{ft}^3/\,\mathrm{s}}{\pi \times 0.5\,\mathrm{ft} \times 0.5\,\mathrm{ft}} = 25.5\ \mathrm{fps}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ p_1 A_1 + p_2 A_2 + F_x &= \dot{m}(v_2 - v_1) \end{aligned}$$

thus

$$\begin{aligned} F_x &= -2pA - 2\dot{m}v \\ &= -2(15\mathrm{psi} \times 144\,\mathrm{in}^2/\,\mathrm{ft}^2 \times \frac{\pi}{4} \times (1\,\mathrm{ft})^2) + 1.94\,\mathrm{slug}/\,\mathrm{ft}^3 \times 20\,\mathrm{ft}^3/\,\mathrm{s} \times 25.5\,\mathrm{ft}/\,\mathrm{s}) \\ &= -5,370\ \mathrm{lbf} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= 0 \\ -W_{\mathrm{bend}} - W_{H_2 0} + F_y &= 0 \\ F_y &= 200\,\mathrm{lbf} + 3\,\mathrm{ft}^3 \times 62.4\,\mathrm{lbf}/\,\mathrm{ft}^3 = 387.2\ \mathrm{lbf} \end{aligned}$$

Force required

$$\boxed{\mathbf{F} = (-5370\mathbf{i} + 387\mathbf{j})\ \mathrm{lbf}}$$

6.41: PROBLEM DEFINITION

Situation:

Water flows through a 180° pipe bend—additional details are provided in the problem statement.

Find:

Force that acts on the flanges to hold the bend in place.

PLAN

Apply the continuity and momentum equations.

SOLUTION

Flow rate

$$\begin{aligned} v_1 &= \frac{Q}{A} \\ &= \frac{4 \times 0.3\,\mathrm{m}^3/\,\mathrm{s}}{\pi \times (0.2\,\mathrm{m})^2} \\ &= 9.55 \text{ m/s} \end{aligned}$$

Continuity. Place a control volume around the pipe bend. Let section 2 be the exit and section 1 be the inlet

$$\begin{aligned} Q &= A_1 v_1 = A_2 v_2 \\ \text{thus } v_1 &= v_2 \end{aligned}$$

Momentum equation (x-direction). Place a control volume around the pipe bend. Let section 2 be the exit and section 1 be the inlet.

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ 2pA + F_x &= \rho Q\left(-v_2\right) - pQv_1 \\ F_x &= -2pA - 2\rho Qv \end{aligned}$$

Calculations

$$\begin{aligned} 2pA &= (2)(100,000\,\mathrm{Pa})(\frac{\pi}{4})(0.2\,\mathrm{m})^2 \\ &= 6283\,\mathrm{N} \\ 2\rho QV &= (2)(1000\,\mathrm{kg}/\,\mathrm{m}^3)(0.3\,\mathrm{m}^3/\,\mathrm{s})(9.55\,\mathrm{m}/\,\mathrm{s}) \\ &= 5730\,\mathrm{N} \\ F_x &= -\left(2pA + 2\rho Qv\right) \\ &= -\left(6283\,\mathrm{N} + 5730\,\mathrm{N}\right) \\ &= -12.0\,\mathrm{kN} \end{aligned}$$

Momentum equation (z-direction). There are no momentum flow terms so the momentum equation simplifies to

$$\begin{aligned} F_z &= W_{\text{bend}} + W_{\text{water}} \\ &= 500\,\text{N} + (0.1\,\text{m}^3)(9810\,\text{N}/\,\text{m}^3) \\ &= 1.481 \text{ kN} \end{aligned}$$

The force that acts on the flanges is

$$\boxed{\mathbf{F} = (-12.0\mathbf{i} + 0\mathbf{j} + 1.48\mathbf{k}) \text{ kN}}$$

6.42: PROBLEM DEFINITION

Situation:
A 90° pipe bend is described in the problem statement.

Find:
Force on the upstream flange to hold the bend in place.

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculation

$$v = \frac{Q}{A} = \frac{12\,\text{ft}^3/\,\text{s}}{\pi/4 \times (1.0\,\text{ft})^2} = 15.28 \text{ ft/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
pA + F_x &= \rho Q(0 - v) \\
F_x &= 1.94\,\text{slug}/\,\text{ft}^3 \times 12\,\text{ft}^3/\,\text{s}(0 - 15.28\,\text{ft}/\,\text{s}) - 4 \text{ psi} \times 144\,\text{in}^2/\,\text{ft}^2 \times \frac{\pi}{4} \times (1\,\text{ft})^2 = -808 \text{ lbf}
\end{aligned}$$

y-direction

$$\begin{aligned}
F_y &= \rho Q(-v - 0) \\
F_y &= -1.94\,\text{slug}/\,\text{ft}^3 \times 12\,\text{ft}^3/\,\text{s} \times 15.28\,\text{ft}/\,\text{s} = -356 \text{ lbf}
\end{aligned}$$

z-direction

$$\sum F_z = 0$$

$$-100\,\text{lbf} - 4\,\text{ft}^3 \times 62.4\,\text{lbf}/\,\text{ft}^3 + F_z = 0$$

$$F_z = +350 \text{ lbf}$$

The force is

$$\boxed{\mathbf{F} = (-808\mathbf{i} \; - 356\mathbf{j} \; + 350\mathbf{k}) \text{ lbf}}$$

6.43: PROBLEM DEFINITION

Situation:
A 90^0 pipe bend is described in the problem statement.

Find:
$x-$component of force applied to bend to hold it in place: F_x

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculation

$$v = \frac{Q}{A} = \frac{10\,\mathrm{m}^3/\,\mathrm{s}}{\pi/4 \times (1\,\mathrm{m})^2} = 12.73 \text{ m/s}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m} v_{ox} - \sum_{cs} \dot{m} v_{ix} \\ pA + F_x &= \rho Q (0 - v) \end{aligned}$$

$$\begin{aligned} 300,000\,\mathrm{Pa} \times \pi \times (0.5\,\mathrm{m})^2 + F_x &= 1000\,\mathrm{kg/\,m^3} \times 10\,\mathrm{m}^3/\,\mathrm{s} \times (0 - 12.73\,\mathrm{m/\,s}) \\ F_x &= -362,919 \text{ N} \\ & \boxed{F_x = -363 \text{ kN}} \end{aligned}$$

6.44: PROBLEM DEFINITION

Situation:
Water flows through a 30^o pipe bend—additional details are provided in the problem statement.

Find:
Vertical component of force exerted by the anchor on the bend: F_a

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v &= \frac{Q}{A} \\ &= \frac{31.4\,\mathrm{ft}^3/\,\mathrm{s}}{\pi \times 1\,\mathrm{ft} \times 1\,\mathrm{ft}} \\ &= 9.995 \text{ ft/sec} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ F_a - W_{\text{water}} - W_{\text{bend}} - p_2 A_2 \sin 30^\circ &= \rho Q(v \sin 30^\circ - v \sin 0^\circ) \\ F_a &= \pi \times 1\,\mathrm{ft} \times 1\,\mathrm{ft} \times 4\,\mathrm{ft} \times 62.4\,\mathrm{lbf}/\,\mathrm{ft}^3 + 300\,\mathrm{lbf} \\ &\quad +8.5 \times 144\,\mathrm{in}^2/\,\mathrm{ft}^2 \times \pi \times 1\,\mathrm{ft} \times 1\,\mathrm{ft} \times 0.5 \\ &\quad +1.94\,\mathrm{slug}/\,\mathrm{ft}^3 \times 31.4\,\mathrm{ft}^3/\,\mathrm{s} \times (9.995\,\mathrm{ft}/\,\mathrm{s} \times 0.5 - 0) \\ &\boxed{F_a = 3310 \text{ lbf}} \end{aligned}$$

6.45: PROBLEM DEFINITION

Situation:

Water flows through a 60^o pipe bend and jets out to atmosphere—additional details are provided in the problem statement.

Find:

Magnitude and direction of external force components to hold bend in place.

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Flow rate equation

$$\begin{aligned}\left(\frac{D_2}{D_1}\right)^2 v_2 &= \left(\frac{30\,\text{cm}}{60\,\text{cm}}\right)^2 10\,\text{m/s} = 2.5\,\text{m/s} \\ Q &= A_1 v_1 = \pi \times 0.3\,\text{m} \times 0.3\,\text{m} \times 2.5\,\text{m/s} = 0.707\ \text{m}^3/\text{s}\end{aligned}$$

Bernoulli equation

$$\begin{aligned}p_1 &= p_2 + \frac{\rho}{2}(v_2^2 - v_1^2) \\ &= 0 + \frac{1000\,\text{kg/m}^3}{2}(10\,\text{m} \times 10\,\text{m} - 2.5\,\text{m} \times 2.5\,\text{m}) \\ &= 46,875\ \text{Pa gage}\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}F_x + p_1 A_1 &= \rho Q(-v_2 \cos\ 60^\circ - v_1) \\ F_x &= -46,875\,\text{Pa} \times \pi \times 0.3\,\text{m} \times 0.3\,\text{m} \\ &\quad +1000\,\text{kg/m}^3 \times 0.707\,\text{m}^3/\text{s} \times (-10\,\text{m/s}\cos 60^\circ - 2.5\,\text{m/s}) \\ &= -18,560\ \text{N}\end{aligned}$$

y-direction

$$\begin{aligned}F_y &= \rho Q(-v_2 \sin 60^\circ - v_1) \\ F_y &= 1000\,\text{kg/m}^3 \times 0.707 \times (-10\,\text{m/s} \sin 60^\circ - 0) \\ &= -6120\ \text{N}\end{aligned}$$

z-direction

$$\begin{aligned}F_z - W_{\text{H}_20} - W_{\text{bend}} &= 0 \\ F_z &= (0.25\,\text{m}^3 \times 9,810\,\text{N/m}^3) + (250\,\text{kg} \times 9.81\,\text{m/s}^2) = 4,905\ \text{N}\end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-18.6\mathbf{i} - 6.12\mathbf{j} + 4.91\mathbf{k})\ \text{kN}}$$

6.46: PROBLEM DEFINITION

Situation:

Water flows through a nozzle—additional details are provided in the problem statement.

Find:

Vertical force applied to the nozzle at the flange: F_y

PLAN

Apply the continuity equation, then the Bernoulli equation, and then the momentum equation.

SOLUTION

Continuity equation

$$\begin{aligned} v_1A_1 &= v_2A_2 \\ v_1 &= v_2\frac{A_2}{A_1} = 65 \text{ ft/s} \\ Q &= v_2A_2 = (130 \text{ ft/s})(0.5 \text{ ft}^2) \\ &= 65 \text{ cfs} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 \\ \frac{p_1}{\gamma} &= 0 + \frac{(130\,\text{ft/s})^2}{2g} + 2 - \frac{(65\,\text{ft/s})^2}{2g} \\ p_1 &= 62.4\,\text{lbf/ft}^3(262.4 + 2 - 65.6)\,\text{ft} \\ p_1 &= 12,400 \text{ lbf/ft}^2 \end{aligned}$$

Momentum equation (y-direction)

$$p_1A_1 - W_{H_20} - W_{\text{nozzle}} + F_y = \rho Q(v_2 \sin 30^\circ - v_1) \qquad (1)$$

Momentum flow terms

$$\begin{aligned} \rho Q(v_2 \sin 30^\circ - v_1) &= (1.94\,\text{slug/ft}^3)(65\,\text{ft}^3/\text{s})\left[(130\,\text{ft/s}\sin 30^\circ) - 65\,\text{ft/s}\right] \\ &= 0 \text{ lbf} \end{aligned}$$

Thus, Eq. (1) becomes

$$\begin{aligned} F_y &= W_{H_20} + W_{\text{nozzle}} - p_1A_1 \\ &= (1.8\,\text{ft}^3 \times 62.4\,\text{lbf/ft}^3) + (100\,\text{lbf}) - (12400\text{psf} \times 1\,\text{ft}^2) \\ &= -12,190 \text{ lbf} \end{aligned}$$

$$\boxed{F_y = 12,200\,\text{lbf (acting downward)}}$$

6.47: PROBLEM DEFINITION

Situation:

Gasoline flows through a 135^o pipe bend—additional details are provided in the problem statement.

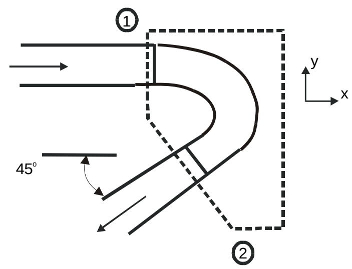
Find:

External force required to hold the bend: F

PLAN

Apply the momentum equation.

SOLUTION



Flow rate

$$\begin{aligned} Q &= vA = 20\,\text{ft/s} \times \pi/4 \times (1\,\text{ft})^2 \\ &= 15.71 \text{ cfs} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \rho Q(v_{2x} - v_{1x}) \\ p_1A_1 + p_2A_2\cos 45^\circ + F_x &= \rho Q(-v_2\cos 45^\circ - v_1) \end{aligned}$$

$$\begin{aligned} F_x &= -pA(1+\cos 45^\circ) - \rho Qv(1+\cos 45^\circ) \\ &= -(1440 \text{ psf}) \times (\pi/4 \times 1^2)(1+\cos 45^\circ) \\ &\quad -(0.8 \times 1.94\,\text{slug/ft}^3)(15.71\,\text{ft}^3/\text{s})(20\,\text{ft/s})(1+\cos 45^\circ) \\ &= -2760 \text{ lbf} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ p_2A_2\sin 45^\circ + F_y &= \rho Q(-v_2\sin 45^\circ - 0) \end{aligned}$$

$$
\begin{aligned}
F_y &= -pA\sin 45^\circ - \rho Q v_2 \sin 45^\circ \\
F_y &= -(1440\ \text{psf})(\pi/4 \times (1\,\text{ft})^2)\sin 45^\circ - (0.8 \times 1.94\,\text{slug}/\,\text{ft}^3)(11.78\,\text{ft}^3/\,\text{s})(15\,\text{ft}/\,\text{s})\sin 45^\circ \\
F_y &= -994\ \text{lbf}
\end{aligned}
$$

Net force

$$
\boxed{\mathbf{F} = (-2760\mathbf{i} - 994\mathbf{j})\ \text{lbf}}
$$

6.48: PROBLEM DEFINITION

Situation:

A 180^o pipe bend (6 in. diameter) carries water.

$Q = 6$ cfs, $p = 20$ psi gage

Find:

Force needed to hold the bend in place: F_x (the component of force in the direction parallel to the inlet flow)

Assumptions:

The weight acts perpendicular to the flow direction; the pressure is constant throughout the bend.

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ 2pA - F_x &= -2\dot{m}v\end{aligned}$$

Calculations

$$\begin{aligned}pA &= \left(20 \text{ psig} \times 144\, \text{in}^2/\,\text{ft}^2\right)\left(\pi/4 \times 0.5^2\right) = 565.5 \text{ lbf} \\ \dot{m}v &= \frac{\rho Q^2}{A} = \frac{1.94\,\text{slug}/\,\text{ft}^3 \times \left(6\,\text{ft}^3/\,\text{s}\right)^2}{\pi/4 \times (0.5\,\text{ft})^2} = 355.7 \text{ lbf} \\ F_x &= 2(pA + \dot{m}v) = 2 \times (565.5 + 355.7) \text{ lbf} \\ &\boxed{F_x\text{=1840 lbf (acting to the left, opposite of inlet flow)}}\end{aligned}$$

6.49: PROBLEM DEFINITION

Situation:

Gasoline flows through a 135^o pipe bend—additional details are provided in the problem statement.

Find:

External force required to hold the bend in place: F

PLAN

Apply the momentum equation.

SOLUTION

Discharge

$$\begin{aligned} Q &= 8 \times \pi/4 \times 0.15\,\text{m} \times 0.15\,\text{m} \\ &= 0.141\ \text{m}^3/\text{s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_{1x}) \\ p_1 A_1 + p_2 A_2 \cos 45^\circ + F_x &= \rho Q(-v_2 \cos 45^\circ - v_1) \\ F_x &= -pA(1 + \cos 45^0) - \rho Q v(1 + \cos 45^\circ) \\ &= -(100,000\,\text{Pa})(\pi/4 \times (0.15\,\text{m})^2)(1 + \cos 45^\circ) \\ &\quad -(1000\,\text{kg/m}^3 \times 0.8)(0.141\,\text{m}^3/\text{s})(8\,\text{m/s})(1 + \cos 45^\circ) \\ &= -4557\ \text{N} \end{aligned}$$

Momentum equation y-direction

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ p_2 A_2 \sin 45^\circ + F_y &= -\rho Q v_2 \sin 45^\circ \\ &= -(100,000\,\text{Pa})(\pi/4 \times (0.15\,\text{m})^2) \sin 45^\circ \\ &\quad -(1,000\,\text{kg/m}^3 \times 0.8)(0.141\,\text{m}^3/\text{s})(8\,\text{m/s}) \sin 45^\circ \\ &= -1,888\ \text{N} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-4.56\mathbf{i} - 1.89\mathbf{j})\ \text{kN}}$$

6.50: PROBLEM DEFINITION

Situation:
Water flows through a 60^o reducing bend—additional details are provided in the problem statement.

Find:
Horizontal force required to hold bend in place: F_x

PLAN

Apply the Bernoulli equation, then the momentum equation.

SOLUTION

Bernoulli equation

$$\begin{aligned} v_1 &= v_2 \frac{A_2}{A_1} \\ &= 50\,\text{m/s} \frac{1}{10} \\ &= 5 \text{ m/s} \\ p_1 + \frac{\rho v_1^2}{2} &= p_2 + \frac{\rho v_2^2}{2} \end{aligned}$$

Let $p_2 = 0$, then

$$\begin{aligned} p_1 &= -\left(\frac{1,000\,\text{kg/m}^3}{2}\right)(5\,\text{m/s})^2 + \left(\frac{1,000\,\text{kg/m}^3}{2}\right)(50\,\text{m/s})^2 \\ p_1 &= 1237 \text{ kPa} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_{1x}) \\ p_1 A_1 + F_x &= \rho A_2 v_2 (v_2 \cos 60^\circ - v_1) \\ F_x &= -1,237,000\,\text{Pa/m}^2 \times 0.001\,\text{m}^2 \\ &\quad +1,000\,\text{kg/m}^3 \times 0.0001\,\text{m}^2 \times 50\,\text{m/s}(50\,\text{m/s}\cos 60^\circ - 5\,\text{m/s}) \\ &\boxed{F_x = 1140 \text{ N}} \end{aligned}$$

6.51: PROBLEM DEFINITION

Situation:

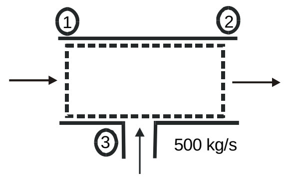
Water flows through a tee—additional details are provided in the problem statement.

Find:

Pressure difference between sections 1 and 2.

PLAN

Apply the continuity equation, then the momentum equation.



SOLUTION

Continuity equation

$$\begin{aligned}
\dot{m}_1 + 500 \text{ kg/s} &= \dot{m}_2 \\
\dot{m}_1 &= (10 \text{ m/s})(0.10 \text{ m}^2)(1000 \text{ kg/m}^3) = 1000 \text{ kg/s} \\
\dot{m}_2 &= 1000 \, \text{kg/s} + 500 \, \text{kg/s} = 1500 \text{ kg/s} \\
v_2 &= \frac{\dot{m}_2}{\rho A_2} = \frac{1500 \, \text{kg/s}}{1000 \, \text{kg/m}^3 \times 0.1 \, \text{m}^2} = 15 \text{ m/s}
\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_{1x} - \dot{m}_3 v_{3x} \\
p_1 A_1 + p_2 A_2 &= \dot{m}_2 v_2 - \dot{m}_1 v_1 - 0 \\
A(p_1 - p_2) &= (1500 \, \text{kg}/s)(15 \, \text{m/s}) - (1000 \, \text{kg/s})(10 \, \text{m/s}) \\
p_1 - p_2 &= \frac{(22,500 - 10,000) \, \text{Pa}}{0.10 \, \text{m}^2} \\
&= 125,000 \text{ Pa} \\
&\boxed{p_1 - p_2 = 125 \text{ kPa}}
\end{aligned}$$

6.52: PROBLEM DEFINITION

Situation:

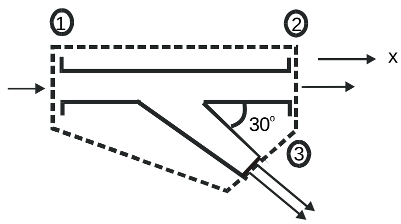
Water flows through a wye—additional details are provided in the problem statement.

Find:

$x-$component of force to hold wye in place.

PLAN

Apply the momentum equation.



Flow rate

$$\begin{aligned} v_1 &= \frac{Q_1}{A_1} = 20 \text{ ft/s} \\ v_2 &= \frac{Q_2}{A_2} = 12 \text{ ft/s} \\ Q_3 &= 20 - 12 = 8 \text{ ft}^3\text{/s} \\ v_3 &= \frac{Q_3}{A_3} = 32 \text{ ft/s} \end{aligned}$$

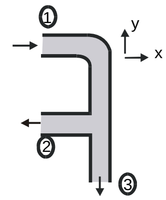
Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_2 v_2 + \dot{m}_3 v_3 \cos 30^\circ - \dot{m}_1 v_1 \\ F_x + p_1 A_1 - p_2 A_2 &= (20\rho)(-20) + (12\rho)(+12) + (32 \cos 30^\circ)(\rho)(8) \\ F_x + (1000 \text{ psfg})(1\,\text{ft}^2) - (900 \text{ psfg})(1\,\text{ft}^2) &= -400\rho + 144\rho + \rho(8\,\text{ft}^3/\,\text{s})(32\,\text{ft}/\,\text{s})(0.866) \\ F_x &= -100\,\text{lbf} + 1.94\,\text{slug}/\,\text{ft}^3(-34.3\,\text{ft}^4/\,\text{s}) \\ & \boxed{F_x = -167 \text{ lbf (acting to the left)}} \end{aligned}$$

6.53: PROBLEM DEFINITION

Situation:

Water flow through a horizontal bend and T section—additional details are provided in the problem statement.



$$
\begin{aligned}
\dot{m}_1 &= 12 \text{ lbm/s} \\
\dot{m}_2 &= \dot{m}_3 = 6 \text{ lbm/s} \\
A_1 &= A_2 = A_3 = 5 \text{ in}^2 \\
p_1 &= 5 \text{ psig} \\
p_2 &= p_3 = 0
\end{aligned}
$$

Find:

Horizontal component of force to hold fitting stationary: F_x

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculations

$$
\begin{aligned}
v_1 &= \frac{\dot{m}_1}{\rho A_1} \\
&= \frac{(12 \text{ lbm}/32.2 \text{ lbm/ slug})}{\left[(1.94 \,\text{slug/ ft}^3)(5 \,\text{in}^2/144 \,\text{in}^2/\text{ ft}^2)\right]} \\
&= 5.531 \text{ ft/s} \\
v_2 &= \frac{\dot{m}_2}{\rho A_2} \\
&= \frac{(6 \text{ lbm}/32.2 \text{ lbm/ slug})}{\left[(1.94 \,\text{slug/ ft}^3)(5 \,\text{in}^2/144 \,\text{in}^2/\text{ ft}^2)\right]}. \\
&= 2.766 \text{ ft/s}
\end{aligned}
$$

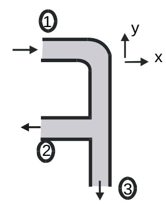
Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\
p_1 A_1 + F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\
F_x &= -p_1 A_1 - \dot{m}_2 v_2 - \dot{m}_1 v_1 \\
&= -(5 \text{ psig} \times 5\,\text{in}^2) - (6 \text{ lbm}/32.2 \text{ lbm}/\,\text{slug})(2.766\,\text{ft}/\,\text{s}) \\
&\quad -(12 \text{ lbm}/32.2 \text{ lbm}/\,\text{slug})(5.531\,\text{ft}/\,\text{s}) \\
&\quad \boxed{F_x = -27.6 \text{ lbf}}
\end{aligned}$$

6.54: PROBLEM DEFINITION

Situation:

Water flows through a horizontal bend and T section—additional details are provided in the problem statement.



$$\begin{aligned} v_1 &= 6 \text{ m/s} \qquad p_1 = 4.8 \text{ kPa} \\ v_2 &= v_3 = 3 \text{ m/s} \qquad p_2 = p_3 = 0 \\ A_1 &= A_2 = A_3 = 0.20 \text{ m}^2 \end{aligned}$$

Find:

Components of force (F_x, F_y) needed to hold bend stationary.

PLAN

Apply the momentum equation.

SOLUTION

Discharge

$$\begin{aligned} Q_1 &= A_1 v_1 = 0.2 \times 6 = 1.2 \text{ m}^3/\text{s} \\ Q_2 &= Q_3 = A_2 v_2 = 0.2 \times 3 = 0.6 \text{ m}^3/\text{s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\ p_1 A_1 + F_x &= -\rho(Q_2 v_2 + Q_1 v_1) \\ F_x &= -p_1 A_1 - \rho(Q_2 v_2 + Q_1 v_1) \\ &= -4800 \times 0.2 - 1000(0.6 \times 3 + 1.2 \times 6) \\ &\boxed{F_x = -9.96 \text{ kN (acts to the left)}} \end{aligned}$$

y-direction

$$\begin{aligned} \sum F_y &= \dot{m}_3(-v_3) \\ F_y &= -\rho Q_3 v_3 = -1000 \times 0.6 \times 3 \\ &\boxed{F_y\text{=-1.8 kN (acts downward)}} \end{aligned}$$

6.55: PROBLEM DEFINITION

Situation:

Water flows through a horizontal tee—additional details are provided in the problem statement.

Find:

Components of force (F_x, F_y) needed to hold the tee in place.

PLAN

Apply the momentum equation.

SOLUTION

Velocity calculations

$$\begin{aligned} V_1 &= \frac{0.25\,\mathrm{m^3/s}}{(\pi \times 0.075\,\mathrm{m} \times 0.075\,\mathrm{m})} \\ &= 14.15 \text{ m/s} \\ V_2 &= \frac{0.10\,\mathrm{m^3/s}}{(\pi \times 0.035\,\mathrm{m} \times 0.035\,\mathrm{m})} \\ &= 25.98 \text{ m/s} \\ V_3 &= \frac{(0.25 - 0.10)\,\mathrm{m^3/s}}{(\pi \times 0.075\,\mathrm{m} \times 0.075\,\mathrm{m})} \\ &= 8.49 \text{ m/s} \end{aligned}$$

Momentum equation (x-direction)

$$F_x + p_1A_1 - p_3A_3 = \dot{m}_3V_3 - \dot{m}_1V_1$$

$$F_x = -p_1A_1 + p_3A_3 + \rho V_3Q - \rho V_1Q$$

$$F_x = -(100,000\,\mathrm{Pa} \times \pi \times 0.075\,\mathrm{m} \times 0.075\,\mathrm{m}) + (80,000\,\mathrm{Pa} \times \pi \times 0.075\,\mathrm{m} \times 0.075\,\mathrm{m}) + \left(1000\,\mathrm{kg/m^3} \times 8.49\,\mathrm{m/s} \times 0.15\,\mathrm{m^3/s}\right) - \left(1000\,\mathrm{kg/m^3} \times 14.15\,\mathrm{m/s} \times 0.25\,\mathrm{m^3/s}\right)$$

$$F_x = -2617\,\mathrm{N}$$

Momentum equation y-direction

$$\begin{aligned} F_y + p_3A_3 &= -\rho V_3Q \\ F_y &= -\rho V_3Q - p_3A_3 \\ F_y &= -1000\,\mathrm{kg/m^3} \times 25.98\,\mathrm{m/s} \times 0.10\,\mathrm{m^3/s} - 70,000\,\mathrm{Pa} \times \pi \times 0.035\,\mathrm{m} \times 0.035\,\mathrm{m} \\ &= -2867 \text{ N} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-2.62\mathbf{i} - 2.87\mathbf{j}) \text{ kN}}$$

6.56: PROBLEM DEFINITION

Situation:

Information of fire hoses and nozzles

Find:

Information of operational conditions and typical hose sizes and nozzles.

SOLUTION

Information depends on source.

6.57: PROBLEM DEFINITION

Situation:

High speed water jets.

Find:

Estimate water speed for 60,000 psig pressure

Assumptions:

Inlet velocity is negligible and viscous effects are not important. Assume the exit pressure is atmospheric

PLAN

Apply the Bernoulli equation.

SOLUTION

The Bernoulli equation between the chamber and nozzle exit

$$p_o + \gamma z_o + \rho \frac{V_o^2}{2} = p_e + \gamma z_e + \rho \frac{V_e^2}{2}$$

The pressure difference is much larger than the pressure due to elevation change so

$$\begin{aligned} V_e^2 &= \frac{2p_c}{\rho} \\ V_e &= \sqrt{\frac{2 \times 60,000 \text{ psi} \times 144 \text{ in}^2/\text{ft}^2}{1.94 \text{ slug/ft}^3}} \\ &= 2980 \text{ ft/s} \end{aligned}$$

This velocity is less than the speed of sound in water ($\sim$ 5000 ft/s) so the exit velocity is subsonic and the exit pressure will equal the atmospheric pressure.

6.58: PROBLEM DEFINITION

Situation:

Water (60 ^{o}F) flows through a nozzle.
$d_1 = 3\,\text{in}$, $d_2 = 1$ in..
$p_1 = 2500$ psfg, $p_2 = 0$ psfg

Find:

(a) Speed at nozzle exit: v_2
(b) Force to hold nozzle stationary: F

Assumptions:

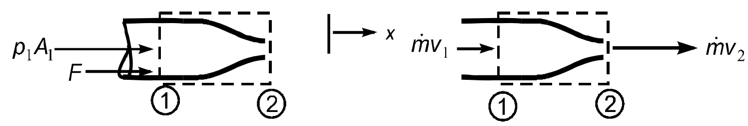
Neglect weight, steady flow.

PLAN

Apply the continuity equation, then the Bernoulli equation, and finally the momentum equation.

SOLUTION

Force and momentum diagrams



Continuity equation

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ v_1 &= v_2 \left(\frac{d_2}{d_1}\right)^2 \end{aligned} \tag{1}$$

Bernoulli equation applied from 1 to 2

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{v_2^2}{2} \tag{2}$$

Combining Eqs. (1) and (2)

$$\begin{aligned} p_1 &= \rho \left(\frac{v_2^2}{2}\right)\left(1 - \left(\frac{d_2}{d_1}\right)^4\right) \\ 2500 \text{ psfg} &= 1.94\,\text{slug}/\text{ft}^3 \times \left(\frac{v_2^2}{2}\right) \times \left(1 - \left(\frac{1}{3}\right)^4\right) \\ v_2 &= v_e = 51.1 \text{ ft/s} \end{aligned}$$

From Eq. (1)

$$\begin{aligned} v_1 &= v_2 \left(\frac{d_2}{d_1}\right)^2 \\ &= 51.1\,\text{ft/ s} \times \left(\frac{1}{3}\right)^2 \\ &= 5.675 \text{ ft/s} \end{aligned}$$

Flow rate

$$\begin{aligned} \dot{m}_1 &= \dot{m}_2 = \dot{m} \\ &= (\rho A v)_2 \\ &= 1.94\,\text{slug/ ft}^3 \times \left(\frac{\pi}{4} \times \left(\frac{1.0}{12}\,\text{ft}\right)^2\right) \times 51.08\,\text{ft/ s} \\ &= 0.5405 \text{ slug/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\ F + p_1 A_1 &= \dot{m}\,(v_2 - v_1) \\ F &= -p_1 A_1 + \dot{m}\,(v_2 - v_1) \\ F &= -(2500\,\text{lbf/ ft}^2) \times \left(\frac{\pi}{4} \times \left(\frac{3}{12}\right)^2\right) \text{ft}^2 \\ &\quad + (0.5405\,\text{slug/ s}) \times (51.08 - 5.675)\,\text{ft/ s} \\ &= -98.26 \text{ lbf} \end{aligned}$$

$$\boxed{\text{Force on nozzle} = 98.3 \text{ lbf to the left}}$$

6.59: PROBLEM DEFINITION

Situation:
Water (15 ^{o}C) flows through a nozzle.
$d_1 = 10$ cm., $d_2 = 2$ cm., $v_2 = 25$ m/s, $\rho = 999$ kg/m^3

Find:
(a)Pressure at inlet: p_1
(b)Force to hold nozzle stationary: F

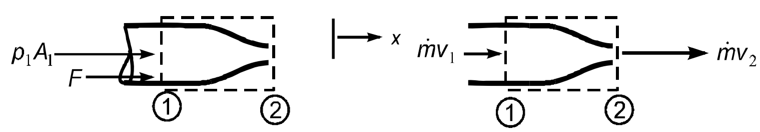
Assumptions:
Neglect weight, steady flow, $p_2 = 0$ kPa-gage.

PLAN

Apply the continuity equation, then the Bernoulli equation, and finally the momentum equation.

SOLUTION

Force and momentum diagrams



Continuity equation

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ v_1 &= v_2 \left(\frac{d_2}{d_1}\right)^2 \\ &= 25 \times \left(\frac{2}{10}\right)^2 \\ &= 1.0 \text{ m/s} \\ \dot{m}_1 &= \dot{m}_2 \\ &= (\rho A v)_2 \\ &= 999 \, \text{kg/m}^3 \times \left(\frac{\pi \times (0.02 \, \text{m})^2}{4}\right) \times 25 \, \text{m/s} \\ &= 7.85 \text{ kg/s} \end{aligned}$$

Bernoulli equation applied from 1 to 2

$$
\begin{aligned}
\frac{p_1}{\rho} + \frac{v_1^2}{2} &= \frac{v_2^2}{2} \\
p_1 &= \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \\
&= \frac{999\,\mathrm{kg/\,m^3}}{2}((25\,\mathrm{m/\,s})^2 - (1\,\mathrm{m/\,s})^2) \\
&= 3.\,117 \times 10^5\,\mathrm{Pa}
\end{aligned}
$$

$$
\boxed{p_1 = 312 \;\; \mathrm{kPa}}
$$

Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\
F + p_1 A_1 &= \dot{m}\left(v_2 - v_1\right) \\
F &= -p_1 A_1 + \dot{m}\left(v_2 - v_1\right) \\
F &= -(311.7 \times 10^3\,\mathrm{Pa})\left(\frac{\pi \times (0.1\,\mathrm{m})^2}{4}\right) + (7.85\,\mathrm{kg/\,s})\,(25 - 1)\,\mathrm{m/\,s} \\
&= -2259.\,7\,\mathrm{N}
\end{aligned}
$$

$$
\boxed{\text{Force on nozzle} = 2.26 \text{ kN to the left}}
$$

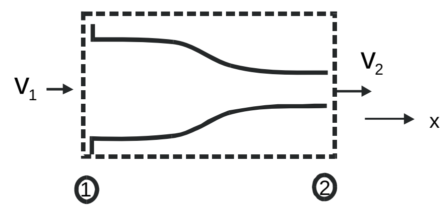
6.60: PROBLEM DEFINITION

The problem involves writing a program for the flow in a nozzle and applying it to problems 6.59 and 6.15. No solution is provided.

6.61: PROBLEM DEFINITION

Situation:

Water flows through a converging nozzle—additional details are provided in the problem statement.



Find:

Force at the flange to hold the nozzle in place: F

PLAN

Apply the Bernoulli equation to establish the pressure at section 1, and then apply the momentum equation to find the force at the flange.

SOLUTION

Continuity equation (select a control volume that surrounds the nozzle).

$$Q_1 = Q_2 = Q = 15\,\text{ft}^3/\,\text{s}$$

Flow rate equations

$$\begin{aligned} v_1 &= \frac{Q}{A_1} = \frac{4\times Q}{\pi D_1^2} = \frac{4\times\left(15\,\text{ft}^3/\,\text{s}\right)}{\pi\left(1\,\text{ft}\right)^2} \\ &= 19.099\ \text{ft}/\,\text{s} \\ v_2 &= \frac{Q}{A_2} = \frac{4\times Q}{\pi D_2^2} = \frac{4\times\left(15\,\text{ft}^3/\,\text{s}\right)}{\pi\left(9/12\,\text{ft}\right)^2} \\ &= 33.953\ \text{ft}/\,\text{s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1 + \frac{\rho v_1^2}{2} &= p_2 + \frac{\rho v_2^2}{2} \\ p_1 &= 0 + \frac{\rho(v_2^2 - v_1^2)}{2} \\ &= \frac{1.94\,\text{slug}/\,\text{ft}^3((33.953\,\text{ft}/\,\text{s})^2 - (19.099\,\text{ft}/\,\text{s})^2)}{2} \\ &= 764.4\ \text{lbf}/\,\text{ft}^2 \end{aligned}$$

Momentum equation (x-direction)

$$p_1 A_1 + F = \dot{m} v_2 - \dot{m} v_1$$

Calculations

$$\begin{aligned} p_1 A_1 &= (764.4\,\mathrm{lbf/\,ft^2})(\pi/4)(1\,\mathrm{ft})^2 \\ &= 600.4\,\mathrm{lbf} \\ \dot{m}v_2 - \dot{m}v_1 &= \rho Q\,(v_2 - v_1) \\ &= (1.94\,\mathrm{slug/\,ft^3})(15\,\mathrm{ft^3/\,s})\,(33.953 - 19.098)\ \mathrm{ft/\,s} \\ &= 432.3\,\mathrm{lbf} \end{aligned}$$

Substituting numerical values into the momentum equation

$$\begin{aligned} F &= -p_1 A_1 + (\dot{m}v_2 - \dot{m}v_1) \\ &= -600.4\,\mathrm{lbf} + 432.3\,\mathrm{lbf} \\ &= -168.1\,\mathrm{lbf} \end{aligned}$$

$$\boxed{F = -168\ \text{ lbf (acts to left)}}$$

6.62: PROBLEM DEFINITION

Situation:

Water flows through a converging nozzle—additional details are provided in the problem statement.

Find:

Force at the flange to hold the nozzle in place: F_x

PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v_1 &= \frac{0.3\,\text{m}^3}{\pi \times 0.15\,\text{m} \times 0.15\,\text{m}} = 4.244 \text{ m/s} \\ v_2 &= 4.244\,\text{m/s} \times 9 = 38.196 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_1 = 0 + \frac{1000\,\text{kg/m}^3}{2}(38.196^2 - 4.244^2)\,\text{m}^2/\text{s}^2 = 720 \text{ kPa}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= -720,000\,\text{Pa} \times \pi \times (0.15\,\text{m})^2 + 1,000\,\text{kg/m}^3 \times 0.3\,\text{m}^3/\text{s} \times (38.196 - 4.244)\,\text{m/s} \\ & \boxed{F_x = -40.7 \text{ kN (acts to the left)}} \end{aligned}$$

6.63: PROBLEM DEFINITION

Situation:

Water flows through a nozzle with two openings—additional details are provided in the problem statement

Find:

x-component of force through flange bolts to hold nozzle in place.

PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v_A &= v_B = \frac{16\,\mathrm{ft}^3/\,\mathrm{s} \times 144\,\mathrm{in}^2/\,\mathrm{ft}^2}{[(\pi/4)(4\,\mathrm{in} \times 4\,\mathrm{in} + 4.5\,\mathrm{in} \times 4.5\,\mathrm{in})]} \\ &= 80.93 \text{ fps} \\ v_1 &= \frac{16\,\mathrm{ft}^3/\,\mathrm{s}}{\pi \times 0.5\,\mathrm{ft} \times 0.5\,\mathrm{ft}} \\ &= 20.37 \text{ fps} \end{aligned}$$

Bernoulli equation

$$p_1 = 0 + \frac{1.94\,\mathrm{slug}/\,\mathrm{ft}^3}{2}(80.93\,\mathrm{ft}/\,\mathrm{s} \times 80.93\,\mathrm{ft}/\,\mathrm{s} - 20.37\,\mathrm{ft}/\,\mathrm{s} \times 20.37\,\mathrm{ft}/\,\mathrm{s}) = 5951 \text{ psf}$$

Momentum equation (x-direction)

$$F_x + \rho_1 A_1 \sin 30\,^\circ = -\dot{m}_A v_A - \dot{m}_B v_B \sin 30\,^\circ$$

$$F_x = -5,951 \text{ psf} \times \pi \times 0.5\,\mathrm{ft} \times 0.5\,\mathrm{ft} \times \sin 30^\circ - 80.93\,\mathrm{ft}/\,\mathrm{s} \times 1.94\,\mathrm{slug}/\,\mathrm{ft} \times 80.93\,\mathrm{ft}/\,\mathrm{s} \times \pi$$

$$\times \frac{2\,\mathrm{in} \times 2\,\mathrm{in}}{144\,\mathrm{ft}^2} - 20.37\,\mathrm{ft}/\,\mathrm{s} \times 1.94\,\mathrm{slug}/\,\mathrm{ft} \times 16.0\,\mathrm{ft}^3/\,\mathrm{s} \sin 30^\circ$$

$$\boxed{F_x = -3762 \text{ lbf}}$$

6.64: PROBLEM DEFINITION

Situation:

Water flows through a nozzle with two openings—additional details are provided in the problem statement.

Find:

x-component of force through flange bolts to hold nozzle in place: F_x

PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v_A &= v_B = \frac{0.5\,\mathrm{m^3/\,s}}{\pi \times 0.05\,\mathrm{m} \times 0.05\,\mathrm{m} + \pi \times 0.06\,\mathrm{m} \times 0.06\,\mathrm{m}} = 26.1 \text{ m/s} \\ v_1 &= \frac{0.5\,\mathrm{m^3/\,s}}{\pi \times 0.15\,\mathrm{m} \times 0.15\,\mathrm{m}} = 7.07 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_1 = \frac{1000\,\mathrm{slug/\,ft^3}}{2}((26.1\,\mathrm{m/\,s})^2 - (7.07\,\mathrm{m/\,s})^2) = 315,612 \text{ Pa}$$

Momentum equation (x-direction)

$$\sum F_x = \dot{m}_o v_{ox} - m_i v_{ix}$$

$$F_x + p_1 A_1 \sin 30 = -\dot{m} v_A - \dot{m} v_i \sin 30$$

$$F_x = -315,612\,\mathrm{Pa} \times \pi \times (0.15\,\mathrm{m})^2 \times \sin 30° - 26.1\,\mathrm{m/\,s} \times 1,000\,\mathrm{kg/\,m^3} \times 26.1\,\mathrm{m/\,s}$$

$$\times \pi \times (0.05\,\mathrm{m})^2 - 7.07\,\mathrm{m/\,s} \times 1000\,\mathrm{kg/\,m^3} \times 0.5\,\mathrm{m^3/\,s} \times \sin 30° = -18,270 \text{ N}$$

$$\boxed{F_x = -18.3 \text{ kN}}$$

6.65: PROBLEM DEFINITION

Situation:

A rocket nozzle is connected to a combustion chamber.

Mass flow: $\dot{m} = 220\,\text{kg/s}$. Ambient pressure: $p_o = 100\,\text{kPa}$.

Nozzle inlet conditions: $A_1 = 1\,\text{m}^2$, $u_1 = 100\,\text{m/s}$, $p_1 = 1.5\,\text{MPa-abs}$.

Nozzle exit condition? $A_2 = 2\,\text{m}^2$, $u_2 = 2000\,\text{m/s}$, $p_2 = 80\,\text{kPa-abs}$.

Assumptions:

The rocket is moving at a steady speed.

Find:

Force on the connection between the nozzle and the chamber.

PLAN

Apply the momentum equation to a control volume situated around the nozzle.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F + p_1 A_1 - p_2 A_2 &= \dot{m}(v_2 - v1) \end{aligned}$$

where F is the force carried by the material that connects the rocket nozzle to the rocket chamber.

Calculations (note the use of gage pressures).

$$\begin{aligned} F &= \dot{m}(v_2 - v1) + p_2 A_2 - p_1 A_1 \\ &= (220\,\text{kg/s})\,(2000 - 100)\,\text{m/s} + \left(-20,000\,\text{N/m}^2\right)\left(2\,\text{m}^2\right) \\ &\quad - \left(1,400,000\,\text{N/m}^2\right)\left(1\,\text{m}^2\right) \\ &= -1.022 \times 10^6\ \text{N} \\ &= -1.022\ \text{MN} \end{aligned}$$

The force on the connection will be

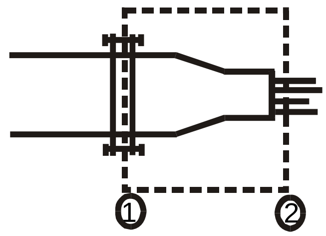
$$\boxed{F = 1.02\ \text{MN}}$$

The material in the connection is in tension.

6.66: PROBLEM DEFINITION

Water flows through a nozzle.
The nozzle is bolted to a pipe flange with 6 bolts.
$D_1 = 0.30\,\text{m}$, $D_2 = 0.15\,\text{m}$, $p_1 = 200\,\text{kPa}$ gage.

Sketch:

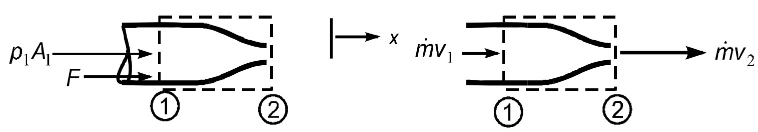


Find:
Tension in each bolt (in Newtons)

PLAN Since force is the goal, start with the momentum equation. Then, apply continuity and the Bernoulli equations to find terms needed to calculate force. The steps are.
1. Apply the momentum equation to relate force to properties at 1 and 2.
2. Relate v_2 and v_1 using continuity.
3. Solve for v_2, v_1, and Q using the Bernoulli equation and the flow rate equation.
4. Calculate force.

SOLUTION

1. Momentum equation (x-direction)



$$\begin{aligned} \sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F_{\text{bolts}} + p_1 A_1 &= \rho Q (v_2 - v_1) \end{aligned}$$

2. Continuity equation (apply to cv shown above; accumulation is zero).

$$v_2 = \frac{A_1}{A_2} v_1 = \left(\frac{0.30\,\text{m}}{0.15\,\text{m}}\right)^2 v_1 = 4 v_1$$

3. Bernoulli equation

$$\begin{aligned}
\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+z_1 &= \frac{p_2}{\gamma}+\frac{v_2^2}{2g}+z_2 \\
\frac{p_1}{\gamma}+\frac{v_1^2}{2g}+0 &= 0+\frac{16v_1^2}{2g}+0 \\
\frac{200000\,\mathrm{Pa}}{9810\,\mathrm{N/\,m^3}} &= \frac{15v_1^2}{2g} \\
v_1 &= 5.16\ \mathrm{m/s}
\end{aligned}$$

$$v_2 = 4v_1 = 20.66\ \mathrm{m/s}$$

Flow rate equation.

$$Q = A_1 v_1 = \frac{\pi\,(0.3\,\mathrm{m})^2}{4}\,(5.16\,\mathrm{m/\,s}) = 0.365\,\mathrm{m^3/\,s}$$

4. Calculate force

$$\begin{aligned}
F_{\mathrm{bolts}} &= -p_1 A_1 + \rho Q(v_2 - v_1) \\
F_{\mathrm{bolts}} &= -\,(200,000\,\mathrm{Pa})\,\pi\,(0.15\,\mathrm{m})^2 + \left(1000\,\mathrm{kg/\,m^3}\right)\left(0.365\,\mathrm{m^3/\,s}\right)(20.66\,\mathrm{m/\,s} - 5.16\,\mathrm{m/\,s}) \\
&= -8480\ \mathrm{N}
\end{aligned}$$

$$\boxed{\text{Force per bolt} = 1410\ \mathrm{N}}$$

6.67: PROBLEM DEFINITION

Situation:

Water jets out of a two dimensional slot.

Flow rate is $Q = 8$ cfs per ft of slot width. Slot spacing is $H = 8\,\text{in}$. Jet height is $b = 4\,\text{in}$.

Find:

(a)Pressure at the gage.

(b)Force (per foot of length of slot) of the water acting on the end plates of the slot.

PLAN

To find pressure at the centerline of the flow, apply the Bernoulli equation. To find the pressure at the gage (higher elevation), apply the hydrostatic equation. To find the force required to hold the slot stationary, apply the momentum equation.

SOLUTION

Continuity. Select a control volume surrounding the nozzle. Locate section 1 across the slot. Locate section 2 across the water jet.

$$Q_1 = Q_2 = Q = \frac{8\,\text{ft}^3/\,\text{s}}{\text{ft}}$$

Flow rate equations

$$\begin{aligned} V_1 &= \frac{Q}{A_1} = \frac{8\,\text{ft}^2/\,\text{s}}{(8/12)\ \text{ft}} \\ &= 12\,\text{ft}/\,\text{s} \\ V_2 &= \frac{Q}{A_2} = \frac{8\,\text{ft}^2/\,\text{s}}{(4/12)\ \text{ft}} \\ &= 24\,\text{ft}/\,\text{s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1 &= \frac{\rho}{2}(V_2^2 - V_1^2) \\ &= \frac{1.94\,\text{slug}/\,\text{ft}^3}{2}(24^2 - 12^2)\frac{\text{ft}^2}{\text{s}^2} \\ p_1 &= 419.0\ \text{lbf}/\,\text{ft}^2 \end{aligned}$$

Hydrostatic equation. Location position 1 at the centerline of the slot. Locate position 3 at the gage.

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_3}{\gamma} + z_3 \\ \frac{419.0\ \text{lbf}/\,\text{ft}^2}{62.4\,\text{lbf}/\,\text{ft}^3} + 0 &= \frac{p_3}{62.4\,\text{lbf}/\,\text{ft}^3} + \frac{(8/12)\ \text{ft}}{2} \\ p_3 &= 398.2\ \text{psf} \end{aligned}$$

$$\boxed{p_3 = 398.2 \text{ lbf/ft}^2 = 2.77 \text{ lbf/in}^2}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}V_2 - \dot{m}V_1 \\ F_x + p_1 A_1 &= \rho Q(V_2 - V_1) \\ F_x &= -p_1 A_1 + \rho Q(V_2 - V_1) \quad (1) \end{aligned}$$

Calculations

$$\begin{aligned} p_1 A_1 &= \left(419 \text{ lbf/ft}^2\right)(8/12 \text{ ft}) \\ &= 279.3 \text{ lbf/ft} \quad \text{(a)} \\ \rho Q(V_2 - V_1) &= \left(1.94 \text{ slug/ft}^3\right)\left(8 \text{ ft}^2/\text{s}\right)(24. \text{ ft/s} - 12. \text{ ft/s}) \\ &= 186.2 \text{ lbf/ft} \quad \text{(b)} \end{aligned}$$

Substitute (a) and (b) into Eq. (1)

$$\begin{aligned} F_x &= -(279.3 \text{ lbf/ft}) + 186.2 \text{ lbf/ft} \\ &= -93.1 \frac{\text{lbf}}{\text{ft}} \end{aligned}$$

The force acting on the end plates is equal in magnitude and opposite in direction (Newton's third law).

$$\boxed{F_{\text{water on the end plates}} = 93.1 \frac{\text{lbf}}{\text{ft}} \text{ acting to the right}}$$

6.68: PROBLEM DEFINITION

Situation:

Water is discharged from a two-dimensional slot—additional details are provided in the problem statement

Find:

(a)Pressure at the gage.

(b)Force (per foot of length of slot) on the end plates of the slot.

PLAN

Apply the Bernoulli equation, then the hydrostatic equation, and finally the momentum equation.

SOLUTION

Velocity calculation

$$\begin{aligned} v_b &= \frac{0.4\,\mathrm{m}^3/\,\mathrm{s}}{0.07\,\mathrm{m}^2} = 5.71 \text{ m/s} \\ v_B &= \frac{0.4\,\mathrm{m}^3/\,\mathrm{s}}{0.2\,\mathrm{m}^2} = 2.00 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_B = \frac{1000\,\mathrm{kg}/\,\mathrm{m}^3}{2}((5.71\,\mathrm{m}/\,\mathrm{s})^2 - (2.00\,\mathrm{m}/\,\mathrm{s})^2) = 14.302 \text{ kPa}$$

Hydrostatic equation

$$\begin{aligned} p_{\text{gage}} &= 14,302\,\mathrm{kPa} - 9810\,\mathrm{N}/\,\mathrm{m}^3 \times 0.1\,\mathrm{m}^2 \\ & \boxed{p_{\text{gage}} = 13.3 \text{ kPa}} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F_x + p_B A_B &= \rho Q(v_b - v_B) \end{aligned}$$

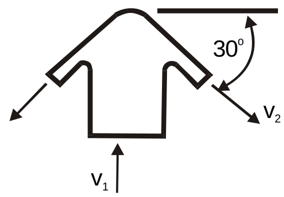
thus

$$\begin{aligned} F_x &= -14,302\,\mathrm{kPa} \times 0.2\,\mathrm{m}^2 + 1000\,\mathrm{kg}/\,\mathrm{m}^3 \times 0.4\,\mathrm{m}^3/\,\mathrm{s} \times (5.71 - 2.00)\,\mathrm{m}/\,\mathrm{s} \\ &= -1,376 \text{ N} \\ & \boxed{F_x = -1.38 \text{ kN/m}} \end{aligned}$$

6.69: PROBLEM DEFINITION

Situation:

Water flows through a spray head—additional details are provided in the problem statement.



Find:

Force acting through the bolts needed to hold the spray head on: F_y

PLAN

Apply the Bernoulli equation, and then the momentum equation.

SOLUTION

Velocity calculation

$$v_1 = \frac{Q}{A_1} = \frac{4\,\text{ft}^3/\,\text{s}}{\pi/4 \times (0.5\,\text{ft})^2} = 20.37 \text{ ft/s}$$

Bernoulli equation

$$\begin{aligned} p_1 &= \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \\ &= \frac{1.94\,\text{slug}/\,\text{ft}^3}{2}\left((65\,\text{ft}/\,\text{s})^2 - (20.37\,\text{ft}/\,\text{s})^2\right) \\ &= 3696 \text{ psf} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \dot{m}_o v_{oy} - \dot{m}_i v_{iy} \\ F_y + p_1 A_1 &= \rho Q(-v_2 \sin 30^\circ - v_1) \\ F_y &= (-3696 \text{ psf})(\pi/4 \times (0.5\,\text{ft})^2) + 1.94\,\text{slug}/\,\text{ft}^3 \times 4\,\text{ft}^3/\,\text{s} \times (-65\,\text{ft}/\,\text{s} \sin 30^\circ - 20.37\,\text{ft}/\,\text{s}) \\ & \boxed{F_y = -1140 \text{ lbf}} \end{aligned}$$

6.70: PROBLEM DEFINITION

Situation:

An unusual nozzle creates two jets of water.
$d = 1\,\text{in}, v_2 = v_3 = 80.2\,\text{ft/s}.$
$D = 4\,\text{in}. p = 43$ psig.

Find:

Force required at the flange to hold the nozzle in place: **F**

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$\begin{aligned} v_1 &= \frac{Q}{A} \\ &= \frac{2 \times 80.2\,\text{ft/s} \times \pi/4 \times (1\,\text{ft})^2}{\pi/4 \times (4\,\text{ft})^2} \\ &= 10.025 \text{ fps} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} \sum F_x &= \sum \dot{m}_{ox} - \dot{m}_i v_{ix} \\ p_1 A_1 + F_x &= \dot{m}_2 v_{2x} + \dot{m}_3 v_{3x} - \dot{m}_1 v_{1x} \\ F_x &= -43 \text{ psig} \times \pi \times (2\,\text{in})^2 + 1.94\,\text{slug/ft}^3 \times (80.2\,\text{ft/s})^2 \times \pi \times \frac{(0.5\,\text{ft})^2}{144\,\text{in}^2/\text{ft}^2} \\ &\quad -(1.94\,\text{slug/ft}^3 \times 80.2\,\text{ft/s} \times \pi \times \frac{(0.5\,\text{ft})^2}{144\,\text{in}^2/\text{ft}^2} \times 80.2\,\text{ft/s} \times \sin 30 \\ &\quad -(1.94\,\text{slug/ft}^3 \times 10.025\,\text{ft/s} \times \pi \times (0.1667\,\text{ft})^2) \times 10.025\,\text{ft/s} \\ &= -524.1 \text{ lbf} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \dot{m}_{oy} - \dot{m}_i v_{iy} \\ F_y &= \dot{m}_3 v_{3y} = \rho A v_3 (-v_3 \cos 30^\circ) \\ &= -1.94\,\text{slug/ft}^3 (\pi/4 \times (1/12)^2\,\text{ft}^2)\,(80.2\,\text{ft/s})^2 \cos 30^\circ \\ &= -58.94 \text{ lbf} \end{aligned}$$

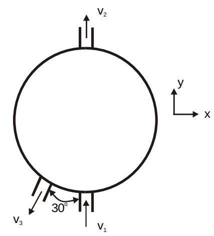
Net force

$$\boxed{\mathbf{F} = (-524\mathbf{i} - 58.9\mathbf{j}) \text{ lbf}}$$

6.71: PROBLEM DEFINITION

Situation:

Liquid flows through a "black sphere"—additional details are provided in the problem statement.



Find:

Force in the inlet pipe wall required to hold sphere stationary: **F**

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$\begin{aligned} A_1 v_1 &= A_2 v_2 + A_3 v_3 \\ v_3 &= v_1 \frac{A_1}{A_3} - v_2 \frac{A_2}{A_3} \\ &= 50\,\mathrm{ft/s}\left(\frac{2^2}{1^2}\right) - 100\,\mathrm{ft/s}\left(\frac{1^2}{1^2}\right) \\ &= 100\,\mathrm{ft/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= \dot{m}_3 v_{3x} \\ &= -\rho A_3 v_3^2 \sin 30^\circ \\ &= -(1.94\,\mathrm{slug/ft^3} \times 1.2)\left(\frac{\pi\left(1/12\right)^2\ \mathrm{ft}^2}{4}\right)(100\,\mathrm{ft/s})^2 \sin 30^\circ \\ &= -63.49\ \mathrm{lbf} \end{aligned}$$

y-direction

$$F_y - W + p_1 A_1 = \dot{m}_2 v_{2y} + \dot{m}_3 v_{3y} - \dot{m}_1 v_{1y}$$

thus

$$F_y = W - p_1 A_1 + \dot{m}_2 v_2 - m_3 v_3 \cos 30^\circ - \dot{m}_1 v_1$$

Calculations

$$
\begin{aligned}
W - p_1 A_1 &= 200 - 60 \times \pi \times 1^2 \\
&= 11.50 \text{ lbf} \\
\dot{m}_2 v_2 &= \rho A_2 v_2^2 \\
&= (1.2 \times 1.94 \, \text{slug/ ft}^3) \left(\frac{\pi \left(1/12\right)^2 \text{ ft}^2}{4} \right) (100 \, \text{ft/ s})^2 \\
&= 126.97 \text{ lbf} \\
\dot{m}_3 v_3 \cos 30^\circ &= \rho A_3 v_3^2 \cos 30^\circ \\
&= (1.2 \times 1.94 \, \text{slug/ ft}^3) \left(\frac{\pi \left(1/12\right)^2 \text{ ft}^2}{4} \right) (100 \, \text{ft/ s})^2 \cos 30^\circ \\
&= 109.96 \text{ lbf} \\
\dot{m}_1 v_1 &= \rho A_1 v_1^2 \\
&= (1.2 \times 1.94 \, \text{slug/ ft}^3) \left(\frac{\pi \left(2/12\right)^2 \text{ ft}^2}{4} \right) (50 \, \text{ft/ s})^2 \\
&= 126.97 \text{ lbf}
\end{aligned}
$$

thus,

$$
\begin{aligned}
F_y &= (W - p_1 A_1) + \dot{m}_2 v_2 - (m_3 v_3 \cos 30^\circ) - \dot{m}_1 v_1 \\
&= (11.50) \, \text{lbf} + 126.97 \, \text{lbf} - (109.96) \text{ lbf} - 126.97 \, \text{lbf} \\
&= -98.46 \text{ lbf}
\end{aligned}
$$

Net Force

$$
\boxed{\mathbf{F} = (-63.5\mathbf{i} - 98.5\mathbf{j}) \text{ lbf}}
$$

6.72: PROBLEM DEFINITION

Situation:

Liquid flows through a "black sphere"—additional details are provided in the problem statement.

Find:

Force required in the pipe wall to hold the sphere in place: **F**

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$\begin{aligned} v_3 &= \frac{10 \times 5^2 - 30 \times 2.5^2}{(2.5u)^2} \\ &= 10 \text{ m/s} \\ \rho &= S\rho_w = 1.5 \times 1000\,\text{kg/m}^3 = 1500\,\text{kg/m}^3 \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= -\dot{m}_j v_j \sin 30^\circ = -\rho_j A_j v_j^2 \sin 30^\circ \\ &= -10 \sin 30^\circ \times 1500\,\text{kg/m}^3 \times 10\,\text{m/s} \times \pi \times (0.0125\,\text{m})^2 \\ &= -36.8 \text{ N} \end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned} F_y &= -P_i A_i + W_t - \dot{m}_i v_i + \dot{m}_o v_o - \dot{m}_j v_j \cos 30^\circ \\ &= -400,000\,\text{Pa} \times \frac{\pi}{4} \times (0.025\,\text{m})^2 + 600\,\text{N} + (1500\pi) \\ &\quad \times (-(10\,\text{m/s})^2 \times (0.025\,\text{m})^2 + (30\,\text{m/s})^2 \times (0.0125\,\text{m})^2 \\ &\quad - (10\,\text{m/s})^2 \times (0.0125\,\text{m})^2 \cos 30^\circ) \\ &= 119 \text{ N} \end{aligned}$$

Net Force

$$\boxed{\mathbf{F} = (-36.8\mathbf{i} + 119\mathbf{j}) \text{ N}}$$

6.73: PROBLEM DEFINITION

To verify the analysis the quantities $Q, v_1, v_{2,}b, y_1, y_2$ and F_G will have to be measured. Since a laboratory is available for your experiment it is assumed that the laboratory has equipment to obtain Q. The width b can be measured by a suitable scale. The depths y_1 and y_2 can be measured by means of piezometer tubes attached to openings in the bottom of the channel or by means of point gages by which the actual level of the surface of the water can be determined. Then v_1 and v_2 can be calculated from $v = Q/A = Q/(by)$.

The force on the gate can be indirectly evaluated by measuring the pressure distribution on the face of the gate. This pressure may be sensed by piezometers or pressure transducer attached to small openings (holes) in the gate. The pressure taps on the face of the gate could all be connected to a manifold, and by appropriate valving the pressure at any particular tap could be sensed by a piezometer or pressure transducer. The pressures at all the taps should be measured for a given run. Then by integrating the pressure distribution over the surface of the gate one can obtain F_G. Then compare the measured F_G with the value obtained from the right hand side of Eq. (6.11). The design should be such that air bubbles can be purged from tubes leading to piezometer or transducer so that valid pressure readings are obtained.

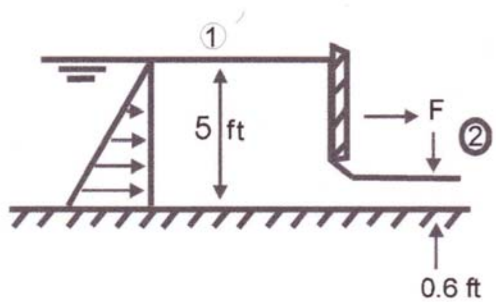
6.74: PROBLEM DEFINITION

Situation:
Water flows through a sluice gate——additional details are provided in the problem statement.

Find:
Force of water (per unit width) acting on the sluice gate.

PLAN

Apply the Bernoulli equation, and then the momentum equation.



SOLUTION

Bernoulli equation

$$\begin{aligned}
\frac{v_1^2}{2g} + z_1 &= \frac{v_2^2}{2g} + z_2 \\
\left(\frac{0.6}{5}\right)^2 \frac{v_2^2}{2g} + 5 &= \frac{v_2^2}{2g} + 0.6 \\
v_2 &= 16.96 \text{ fps} \\
v_1 &= 2.03 \\
Q &= 10.176 \text{ cfs/ft}
\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \rho Q(v_{2x} - v_{1x}) \\
& F_x + \overline{p}_1 A_1 - \overline{p}_2 A_2 = \rho Q(v_2 - v_1) \\
F_x &= -62.4 \,\text{lbf} \times \frac{5.0\,\text{ft} \times 5.0\,\text{ft}}{2} + 62.4\,\text{lbf} \times \frac{0.6\,\text{ft} \times 0.6\,\text{ft}}{2} + 1.94\,\text{slug}/\,\text{ft}^3 \times 10.176 \text{ cfs/ft} \\
& \times (16.96 - 2.03)\,\text{ft}/\,\text{s} \\
& \boxed{F_x = -474 \text{ lbf/ft}}
\end{aligned}$$

6.75: PROBLEM DEFINITION

Situation:

A flow in a pipe is laminar and fully developed—additional details are provided in the problem statement.

Find:

Derive a formula for the resisting shear force (F_τ) as a function of the parameters $D,\ p_1, p_2,\ \rho$, and U.

PLAN

Apply the momentum equation, then the continuity equation.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \int_{cs} \rho v(v \cdot dA) \\ p_1 A_1 - p_2 A_2 - F_\tau &= \int_{A_2} \rho u_2^2 dA - (\rho A u_1) u_1 \\ p_1 A - p_2 A - F_\tau &= -\rho u_1^2 A + \int_{A_2} \rho u_2^2 dA \qquad (1)\end{aligned}$$

Integration of momentum outflow term

$$\begin{aligned} u_2 &= u_{\max}(1-(r/r_0)^2)^2 \\ u_2^2 &= u_{\max}^2(1-(r/r_0)^2)^2 \\ \int_{A_2} \rho u_2^2 dA &= \int_0^{r_0} \rho u_{\max}^2 (1-(r/r_0)^2)^2 2\pi r dr \\ &= -\rho u_{\max}^2 \pi r_0^2 \int_0^{r_0} (1-(r/r_0)^2)^2 (-2r/r_0^2) dr \end{aligned}$$

To solve the integral, let

$$u = 1 - \left(\frac{r}{r_o}\right)^2$$

Thus

$$du = \left(-\frac{2r}{r_o^2}\right) dr$$

The integral becomes

$$\begin{aligned}
\int_{A_2} \rho u_2^2 dA &= -\rho u_{\max}^2 \pi r_0^2 \int_1^0 u^2 du \\
&= -\rho u_{\max}^2 \pi r_0^2 \left(\frac{u^3}{3} |_1^0 \right) \\
&= -\rho u_{\max}^2 \pi r_0^2 \left(0 - \frac{1}{3} \right) \\
&= \frac{+\rho u_{\max}^2 \pi r_0^2}{3} \qquad (2)
\end{aligned}$$

Continuity equation

$$\begin{aligned}
UA &= \int u dA \\
&= \int_0^{r_0} u_{\max}(1-(r/r_0)^2) 2\pi \text{ rdr} \\
&= -u_{\max}\pi r_0^2 \int_0^{r_0} (1-(r/r_0)^2)(-2r/r_0^2) \text{ dr} \\
&= -u_{\max}\pi r_0^2 (1-(r/r_0)^2)^2/2|_0^{r_0} \\
&= u_{\max}\pi r_0^2/2
\end{aligned}$$

Therefore

$$u_{\max} = 2U$$

Substituting back into Eq. 2 gives

$$\int_{A_2} \rho u_2^2 dA = 4\rho U^2 \pi r_0^2/3$$

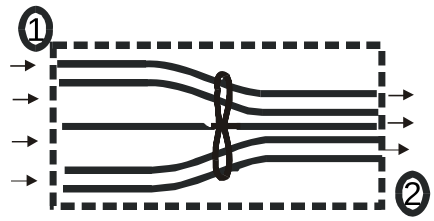
Finally substituting back into Eq. 1, and letting $u_1 = U$, the shearing force is given by

$$\boxed{F_\tau = \frac{\pi D^2}{4} \left[p_1 - p_2 - (1/3)\rho U^2 \right]}$$

6.76: PROBLEM DEFINITION

Situation:

A swamp boat is powered by a propeller—additional details are provided in the problem statement.



Find:

(a) Propulsive force when the boat is not moving.

(b) Propulsive force when the boat is moving at 30 ft/s.

Assumptions:

When the boat is stationary, neglect the inlet flow of momentum–that is, assume $v_1 \approx 0$.

PLAN

Apply the momentum equation.

SOLUTION

From Table A.3, $\rho = 0.00228$ slug/ft^3

a.) Boat is stationary

Momentum equation (x-direction) Select a control volume that surrounds the boat.

$$\begin{aligned} \sum F_x &= \dot{m}v_2 - \dot{m}v_1 \\ F_{\text{stop}} &\approx \dot{m}v_2 \end{aligned}$$

Mass flow rate

$$\begin{aligned} \dot{m} &= \rho A_2 v_2 \\ &= \left(0.00228\,\text{slug/ ft}^3\right)\left(\frac{\pi\,(3\,\text{ft})^2}{4}\right)(100\,\text{ft/ s}) \\ &= 1.612\,\text{slug/ s} \end{aligned}$$

Thus

$$\begin{aligned} F_{\text{stop}} &= \dot{m}v_2 \\ &= (1.612\,\text{slug/ s})\,(100\,\text{ft/ s}) \\ &= 161\,\text{lbf} \end{aligned}$$

$$\boxed{\text{Force (stationary boat)} = 161 \text{ lbf}}$$

b.) Boat is moving

Momentum equation (x-direction). Select a control volume that surrounds the boat and moves with the speed of the boat. The inlet velocity is $v_1 = 30$ ft/s

$$\begin{aligned} \sum F_x &= \dot{m}\,(v_2 - v_1) \\ &= (1.612\,\text{slug/s})\,(100 - 30)\ \text{ft/s} \\ &= 113\ \text{lbf} \end{aligned}$$

$$\boxed{\text{Force (moving boat)} = 113 \text{ lbf}}$$

6.77: PROBLEM DEFINITION

Situation:
Air flows through a windmill—additional details are provided in the problem statement.

Find:
Thrust on windmill.

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$v_2 = 10\,\mathrm{m/\,s} \times \left(\frac{3\,\mathrm{m}}{4.5\,\mathrm{m}}\right)^2 = 4.44\ \mathrm{m/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}(v_2 - v_1) \\
F_x &= \dot{m}(v_2 - v_1) \\
&= (1.2\,\mathrm{kg/\,s})(\pi/4 \times (3\,\mathrm{m})^3)(10\,\mathrm{m/\,s})(4.44 - 10)\,\mathrm{m/\,s} \\
F_x &= -1415.0\ \mathrm{N}\ \text{(acting to the left)} \\
& \boxed{T = 1415\ \mathrm{N}\ \text{(acting to the right)}}
\end{aligned}$$

6.78: PROBLEM DEFINITION

Situation:
A jet pump is described in the problem statement.

Find:
(a) Derive a formula for pressure increase across a jet pump.
(b) Evaluate the pressure change for water if $A_j/A_o = 1/3$, $v_j = 15$ m/s and $v_o = 2$ m/s.

PLAN

Apply the continuity equation, then the momentum equation.

SOLUTION

Continuity equation

$$v_1 = \frac{v_0 D_0^2}{D_0^2 - D_j^2} \tag{1}$$

$$v_2 = \frac{v_0 D_0^2 + v_j D_j^2}{D_0^2} \tag{2}$$

Momentum equation (x-direction)

$$\sum F_x = \dot{m}(v_2 - v_1)$$

$$(p_1 - p_2)\frac{\pi D_0^2}{4} = -\frac{\rho v_1^2 \pi (D_0^2 - D_j^2)}{4} - \frac{\rho v_j^2 \pi D_j^2}{4} + \frac{\rho v_2^2 \pi D_0^2}{4}$$

thus,

$$\boxed{(p_2 - p_1) = \frac{\rho v_1^2 (D_0^2 - D_j^2)}{D_0^2} + \rho v_j^2 \times \frac{D_j^2}{D_0^2} - \rho v_2^2} \tag{3}$$

Calculations

$$\begin{aligned} v_1 &= \frac{v_0}{1 - (D_j/D_0)^2} \\ &= \frac{2}{1 - (1/3)} \\ &= 3 \text{ m/s} \\ v_2 &= v_0 + v_j \frac{D_j^2}{D_0^2} \\ &= 2 + 15\left(\frac{1}{3}\right) \\ &= 7 \text{ m/s} \end{aligned}$$

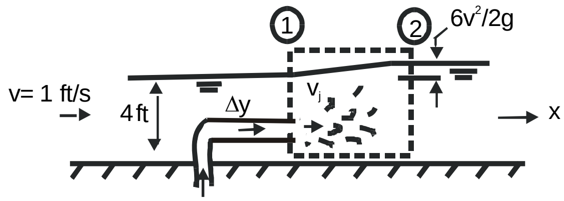
from Eq. (3)

$$\begin{aligned} p_2 - p_1 &= \rho \left[v_1^2 \left(1 - \frac{D_j^2}{D_0^2} \right) + v_j^2 \frac{D_j^2}{D_0^2} - v_2^2 \right] \\ &= 1000\,\mathrm{kg/\,m^3} \left[(3\,\mathrm{m/\,s})^2 \left(1 - \frac{1}{3} \right) + (15\,\mathrm{m/\,s})^2 \left(\frac{1}{3} \right) - (7\,\mathrm{m/\,s})^2 \right] \\ & \boxed{p_2 - p_1 = 32 \text{ kPa}} \end{aligned}$$

6.79: PROBLEM DEFINITION

Situation:

The problem statement describes a jet pump.



Find:

Develop a preliminary design by calculating basic dimensions for a jet pump.

PLAN

Apply the momentum equation, then the continuity equation.

SOLUTION

Momentum equation (x-direction)
Carry out the analysis for a section 1 ft wide (unit width) and neglect bottom friction.

$$\begin{aligned} \sum F_x &= \dot{m}_2 v_2 - \dot{m}_1 v_1 - \dot{m}_j v_j \\ \gamma y_1^2/2 - \gamma y_2^2/2 &= -1\rho(1\times(4-\Delta y)) - v_j\rho(v_j\Delta y) + v_2\rho(v_2 y_2) \qquad (1) \\ \text{but } y_2 &= 4 \text{ ft} + 6\ v^2/2g \\ &= 4 + 6/2g = 4.0932 \text{ ft} \end{aligned}$$

Continuity equation

$$\begin{aligned} v_2 y_2 &= v_1(4-\Delta y) + v_j \Delta y \\ v_2 &= v_1 \frac{(4-\Delta y)}{y_2} + \frac{v_j \Delta y}{y_2} \end{aligned}$$

Assume

$$\Delta y = 0.10 \text{ ft}$$

Then

$$v_2 = 1(3.9)/(4.093) + v_j \times 0.1/4.0392 = 0.9528 + 0.02476 v_j \qquad (2)$$

Combine Eqs. (1) and (2)

$$\begin{aligned} v_j^2 - (0.9528 + 0.02476 v_j^2 \times 40.932 &= 5g(y_2^2 - y_1^2) - 39.0 \\ &= 82.44 \text{ ft}^2/\text{s}^2 \end{aligned}$$

Solving:

$$v_j = 12.1 \text{ ft/s} \quad A_j = 0.10 \text{ ft}^2$$

If circular nozzles were used, then $A_j = (\pi/4)d_j^2$; $d_j = 4.28$ in. Therefore, one could use $\boxed{\text{8 nozzles of about 4.3 in.}}$ in diameter discharging water at $\boxed{12.1 \text{ ft/s}}$

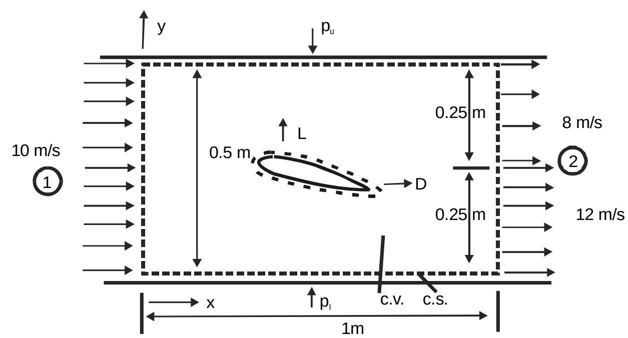
REVIEW

Like most design problems, this problem has more than one solution. That is, other combinations of d_j, v_j and the number of jets are possible to achieve the desired result.

6.80: PROBLEM DEFINITION

Situation:

Lift and drag forces are being measured on an airfoil that is situated in a wind tunnel—additional details are provided in the problem statement.



Find:

(a) Lift force: L

(b) Drag force: D

PLAN

Apply the momentum equation.

SOLUTION

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}v_0 - \dot{m}_1 v_1 \\
-D + p_1 A_1 - p_2 A_2 &= v_1(-\rho v_1 A) + v_a \frac{\rho v_a A}{2} + v_b \frac{\rho v_b A}{2} \\
-\frac{D}{A} &= p_2 - p_1 - \rho v_1^2 + \frac{\rho v_a^2}{2} + \frac{\rho v_b^2}{2}
\end{aligned}$$

where

$$\begin{aligned}
p_1 &= p_u(x=0) = p_\ell(x=0) = 100 \text{ Pa, gage} \\
p_2 &= p_u(x=1) = p_\ell(x=1) = 90 \text{ Pa, gage}
\end{aligned}$$

then

$$\begin{aligned}
-\frac{D}{A} &= 90\,\text{Pa} - 100\,\text{Pa} + 1.2\,\text{kg/}\,\text{m}^3 \times (-100 + 32 + 72)\,\text{m}^2/\,\text{s}^2 \\
-\frac{D}{A} &= -5.2 \\
D &= 5.2\,\text{Pa} \times (0.5\,\text{m})^2 \\
& \boxed{D = 1.3 \text{ N}}
\end{aligned}$$

Momentum equation (y-direction)

$$\sum F_y = 0$$

$$-L + \int_1^2 p_\ell B dx - \int_0^1 p_u B dx = 0 \text{ where } B \text{ is depth of tunnel}$$

$$-L + \int_0^1 (100 - 10x + 20x(1-x))0.5dx - \int_0^1 (100 - 10x - 20x(1-x))0.5dx = 0$$

$$-L + 0.5(100x - 5x^2 + 10x^2 - (20/3)x^3)|_0^1 - 0.5(100x - 5x^2 - 10x^2 + (20/3)x^3|_0^1 = 0$$

thus,

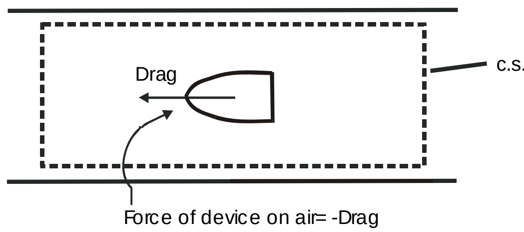
$$-L + 0.5 \times (20 - \frac{40}{3}) = 0$$

$$\boxed{L = 3.33 \text{ N}}$$

6.81: PROBLEM DEFINITION

Situation:

A torpedo-like device is being tested in a wind tunnel—additional details are provided in the problem statement.



Find:

(a) Mass rate of flow.

(b)Maximum velocity at the outlet section.

(c)Drag on the device and support vanes.

PLAN

Apply the momentum equation.

SOLUTION

Mass flow rate

$$\begin{aligned}\dot{m} &= \rho v A \\ &= \left(0.0026\,\text{slug}/\,\text{ft}^3\right) \times (120\,\text{ft}/\,\text{s}) \times \left(\frac{\pi(3.0\,\text{ft})^2}{4}\right) \\ &= 2.205\ \ \text{slug}/\,\text{s}\end{aligned}$$

$$\boxed{\dot{m} = 2.205\ \ \text{slug}/\,\text{s}}$$

At the outlet section

$$\int_0^0 vdA = Q$$

But v is linearly distributed, so $v = v_{\max}(r/r_0)$. Thus

$$\begin{aligned}\int_0^{r_0} \left(v_{\max}\frac{r}{r_o}\right) 2\pi r dr &= \overline{v}A \\ \frac{2v_{\max}r_0^2}{3} &= \overline{v}r_0^2 \\ v_{\max} &= \frac{3\overline{v}}{2} \\ &= \frac{3\,(120\,\text{ft}/\,\text{s})}{2} \\ v_{\max} &= 180\,\text{ft}/\,\text{s}\end{aligned}$$

$$\boxed{v_{\max} = 180\,\mathrm{ft/\,s}}$$

Momentum equation (x-direction)

$$\sum F_x = \int_0^{r_0} \rho v_2^2 dA - \dot{m} v_1 \tag{1}$$

a.) Forces analysis

$$\sum F_x = p_1 A_1 - p_2 A_2 - D \tag{a}$$

b.) Outlet velocity profile

$$\begin{aligned} v_2 &= v_{\max} \frac{r}{r_o} \\ &= \left(\frac{3\overline{v}}{2}\right)\left(\frac{r}{r_o}\right) \end{aligned} \tag{b}$$

c.) Outlet momentum flow

$$\begin{aligned} \int_0^{r_0} \rho v_2^2 dA &= \int_0^{r_0} \rho \left[\left(\frac{3\overline{v}}{2}\right)\left(\frac{r}{r_o}\right)\right]^2 2\pi r dr \\ &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \int_0^{r_0} \left(\frac{r}{r_o}\right)^2 r dr \\ &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) \end{aligned} \tag{c}$$

Substituting Eqns. (a) and (c) into the momentum equation (1) gives

$$\begin{aligned} \sum F_x &= \int_0^{r_0} \rho v_2^2 dA - \dot{m} v_1 \\ p_1 A_1 - p_2 A_2 - D &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) - \dot{m} v_1 \\ D &= p_1 A_1 - p_2 A_2 - 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) + \dot{m} v_1 \end{aligned} \tag{2}$$

Calculations (term by term)

$$
\begin{aligned}
p_1 A_1 &= (144 \times 0.24) \times \left(\frac{\pi \times 3^2}{4} \right) \\
&= 244.3\,\text{lbf} \\
p_2 A_2 &= (144 \times 0.1) \times \left(\frac{\pi \times 3^2}{4} \right) \\
&= 101.\,9\,\text{lbf} \\
\int_0^{r_0} \rho v_2^2 dA &= 2\pi\rho \left(\frac{3\overline{v}}{2} \right)^2 \left(\frac{r_o^2}{4} \right) \\
&= 2\pi\,(0.0026) \left(\frac{3\,(120)}{2} \right)^2 \left(\frac{1.5^2}{4} \right) \\
&= 297.\,7\,\text{lbf} \\
\dot{m} v_1 &= (2.205)\,(120) \\
&= 264.\,6\,\text{lbf}
\end{aligned}
$$

Substituting numerical values into Eq. (2)

$$
\begin{aligned}
D &= p_1 A_1 - p_2 A_2 - 2\pi\rho \left(\frac{3\overline{v}}{2} \right)^2 \left(\frac{r_o^2}{4} \right) + \dot{m} v_1 \\
&= 244.3\,\text{lbf} - 101.\,9\,\text{lbf} - 297.\,7\,\text{lbf} + 264.\,6\,\text{lbf} \\
&= 109.3\,\text{lbf}
\end{aligned}
$$

$$
\boxed{D = 109\,\text{lbf}}
$$

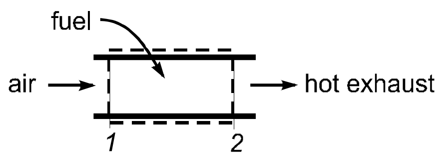
6.82: PROBLEM DEFINITION

Situation:

A jet engine (ramjet) takes in air, adds fuel, and then exhausts the hot gases produced by combustion.

$v_1 = 225$ m/s

$\rho_2 = 0.25$ kg/m^3, $A_2 = 0.5$ m^2



Find:

Thrust force produced by the ramjet: T

Assumptions:

Neglect the mass addition due to the fuel (that is, $\dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m} = 60$ kg/s).

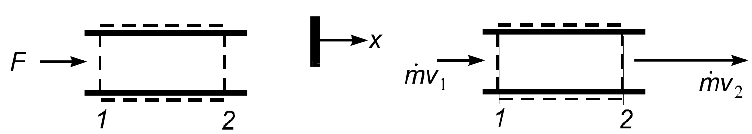
2.) Assume steady flow.

PLAN

Apply the momentum equation.

SOLUTION

Force and momentum diagrams



where F is the force required to hold the ramjet stationary.

Calculate exit velocity

$$\begin{aligned}
\dot{m}_2 &= \rho_2 A_2 v_2 \\
v_2 &= \frac{\dot{m}_2}{\rho_2 A_2} = \frac{60\,\text{kg/s}}{0.25\,\text{kg/m}^3 \times 0.5\,\text{m}^2} = 480 \text{ m/s}
\end{aligned}$$

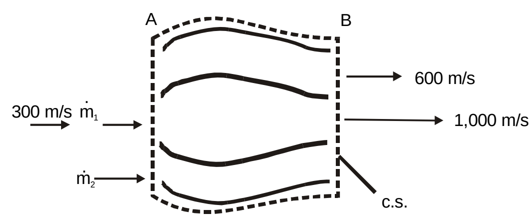
Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
F &= \dot{m}(v_2 - v_1) = 60\,\text{kg/s}(480\,\text{m/s} - 225\,\text{m/s}) \\
& \boxed{T = 15.3 \text{ kN (to the left)}}
\end{aligned}$$

6.83: PROBLEM DEFINITION

Situation:

Air flows through a turbofan engine. Inlet mass flow is 300 kg/ s.
Bypass ratio is 2.5. Speed of bypass air is 600 m/ s.
Speed of air that passes through the combustor is 1000 m/ s.



Additional details are given in the problem statement.

Find:

Thrust (T) of the turbofan engine.

Assumptions:

Neglect the mass flow rate of the incoming fuel.

PLAN

Apply the continuity and momentum equations.

SOLUTION

Continuity equation

$$\dot{m}_A = \dot{m}_B = 300\,\mathrm{kg/\,s}$$

also

$$\begin{aligned}\dot{m}_B &= \dot{m}_{\text{combustor}} + \dot{m}_{\text{bypass}} \\ &= \dot{m}_{\text{combustor}} + 2.5\dot{m}_{\text{combustor}} \\ \dot{m}_B &= 3.5\dot{m}_{\text{combustor}}\end{aligned}$$

Thus

$$\begin{aligned}\dot{m}_{\text{combustor}} &= \frac{\dot{m}_B}{3.5} = \frac{300\,\mathrm{kg/\,s}}{3.5} \\ &= 85.71\,\mathrm{kg/\,s} \\ \dot{m}_{\text{bypass}} &= \dot{m}_B - \dot{m}_{\text{combustor}} \\ &= 300\,\mathrm{kg/\,s} - 85.71\,\mathrm{kg/\,s} \\ &= 214.3\,\mathrm{kg/\,s}\end{aligned}$$

Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \sum \dot{m}v_{\text{out}} - \dot{m}v_{\text{in}} \\
F_x &= [\dot{m}_{\text{bypass}}V_{\text{bypass}} + \dot{m}_{\text{combustor}}V_{\text{combustor}}] - \dot{m}_A V_A \\
&= [(214.3\,\text{kg/s})\,(600\,\text{m/s}) + (85.71\,\text{kg/s})\,(1000\,\text{m/s})] - (300\,\text{kg/s})\,(300\,\text{m/s}) \\
&= 124,290\,\text{N}
\end{aligned}
$$

$$
\boxed{T = 124\ \text{kN}}
$$

6.84: PROBLEM DEFINITION

Maximum force occurs at the beginning; hence, the tank will accelerate immediately after opening the cap. However, as water leaves the tank the force will decrease, but acceleration may decrease or increase because mass will also be decreasing. In any event, the tank will go faster and faster until the last drop leaves, assuming no aerodynamic drag.

6.85: PROBLEM DEFINITION

Situation:

A tank of water rests on a sled—additional details are provided in the problem statement.

Find:

Acceleration of sled at time t

PLAN

Apply the momentum equation.

SOLUTION

This type of problem is directly analogous to the rocket problem except that the weight does not directly enter as a force term and $p_e = p_{\mathrm{atm}}$. Therefore, the appropriate equation is

$$\begin{aligned} M\ dv_s/dt &= \rho v_e^2 A_e - F_f \\ a &= (1/M)(\rho v_e^2 (\pi/4) d_e^2 - \mu W) \end{aligned}$$

where μ =coefficient of sliding friction and W is the weight

$$\begin{aligned} W &= 350 + 0.1 \times 1000 \times 9.81 = 1331 \text{ N} \\ a &= (g/W)(1,000 \times 25^2 (\pi/4) \times 0.015^2 - (1331 \times 0.05)) \\ &= (9.81/1,331)(43.90) \text{ m/s}^2 \\ &= \boxed{W = 0.324 \text{ m/s}^2} \end{aligned}$$

6.86: PROBLEM DEFINITION

Situation:

A cart is moving with a steady speed along a track.
Speed of cart is 5 m/s (to the right). Speed of water jet is 10 m/s.
Nozzle area is $A = 0.0012\,\text{m}^2$.

Find:

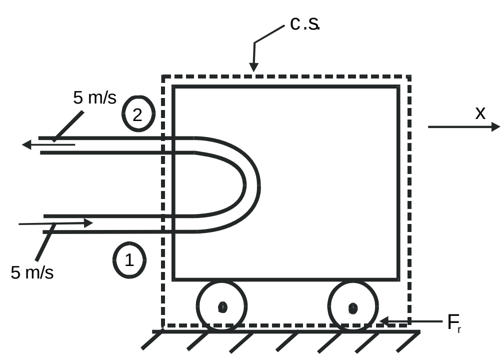
Resistive force on cart: F_r

PLAN

Apply the momentum equation.

SOLUTION

Assume the resistive force (F_r) is caused primarily by rolling resistance (bearing friction, etc.); therefore, the resistive force will act on the wheels at the ground surface. Select a reference frame fixed to the moving cart. The velocities and resistive force are shown below.



Velocity analysis

$$
\begin{aligned}
V_1 &= v_1 = v_2 = 5 \text{ m/s} \\
\dot{m} &= \rho A_1 V_1 \\
&= (1000\,\text{kg/m}^3)(0.0012\,\text{m}^2)(5\,\text{m/s}) \\
&= 6 \text{ kg/s}
\end{aligned}
$$

Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}(v_2 - v_1) \\
-F_r &= 6\,\text{kg/s}(-5\,\text{m/s} - 5\,\text{m/s}) = -60 \text{ N}
\end{aligned}
$$

$$\boxed{F_r = 60 \text{ N (acting to the left)}}$$

6.87: PROBLEM DEFINITION

Situation:

A jet with speed v_j strikes a cart ($M = 10$ kg), causing the cart to accelerate.
The deflection of the jet is normal to the cart [when cart is not moving].
Jet speed is $v_j = 10$ m/s. Jet discharge is $Q = 0.1$ m^3/s.

Find:

(a)Develop an expression for the acceleration of the cart.
(b)Calculate the acceleration when $v_c = 5$ m/s.

Assumptions:

Neglect rolling resistance.
Neglect mass of water within the cart.

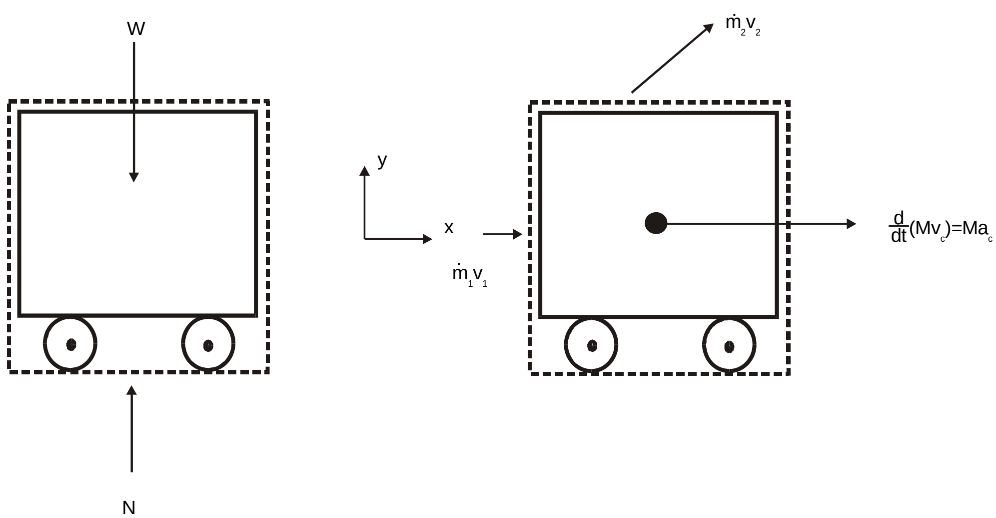
PLAN

Apply the momentum equation.

SOLUTION

Select a control surface surrounding the moving cart. Select a reference frame fixed to the nozzle. Note that a reference frame fixed to the cart would be non-inertial.

Force and momentum diagrams



Momentum equation (x-direction)

$$\sum F_x = \frac{d}{dt}(mv_c) + \dot{m}_2 v_{2_x} = -\dot{m}_1 v_1$$

Momentum accumulation

Note that the cart is accelerating. Thus,

$$\begin{aligned}\frac{d}{dt}\int_{cv} v_x \rho dV &= \frac{d}{dt} v_c \int_{cv} \rho dV = \frac{d}{dt}(M v_c) \\ &= m a_c\end{aligned}$$

Velocity analysis

$$\begin{aligned}V_1 &= v_j - v_c \text{ [relative to control surface]} \\ v_1 &= v_j \text{ [relative to reference frame (nozzle)]}\end{aligned}$$

from conservation of mass

$$\begin{aligned}v_{2y} &= (v_j - v_c) \\ v_{2x} &= v_c \\ \dot{m}_2 &= \dot{m}_1\end{aligned}$$

Combining terms

$$\begin{aligned}\sum F_x &= \frac{d}{dt}(M v_c) + \dot{m}(v_{2x} - v_1) \\ 0 &= M a_c + \rho A_1 (v_j - v_c)(v_c - v_j) \\ & \boxed{a_c = \frac{(\rho Q / v_j)(v_j - v_c)^2}{M}}\end{aligned}$$

Calculations

$$a_c = \frac{1,000 \times 0.1/10(10-5)^2}{10}$$

$$\boxed{a_c = 25 \text{ m/s}^2 \quad (\text{when } v_c = 5 \text{ m/s})}$$

6.88: PROBLEM DEFINITION

Situation:
A jet strikes a cart and accelerates the cart from zero to one-half the jet velocity.

Find:
Time (s) to accelerate to one-half jet velocity.

Assumptions:
No resistance to cart motion and mass of water jet moving with cart is negligible.

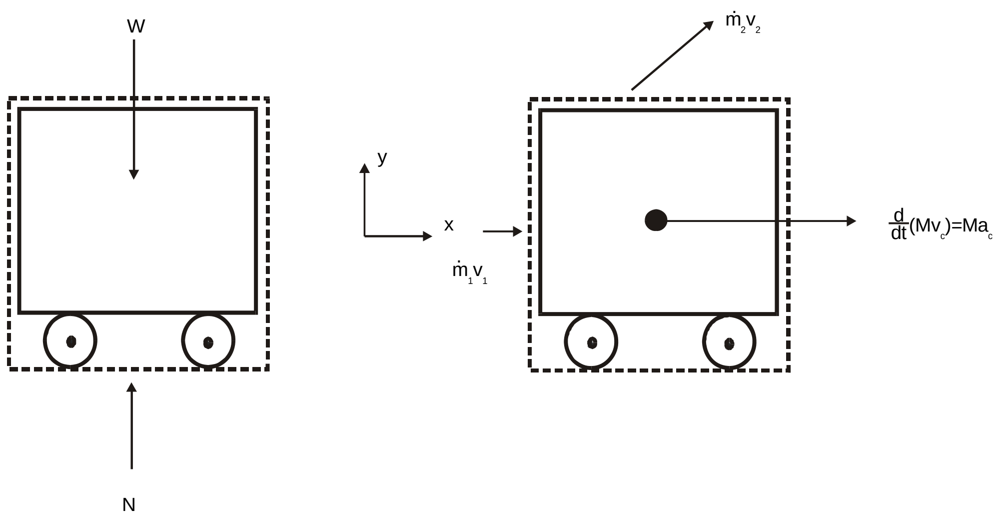
PLAN

Apply the momentum equation to obtain equation of motion for cart and integrate to obtain time.

SOLUTION

Select a control surface surrounding the moving cart. Select a reference frame fixed to the nozzle. Note that a reference frame fixed to the cart would be non-inertial.

Force and momentum diagrams



Momentum equation (x-direction)

$$\sum F_x = \frac{d}{dt}(mv_c) + \dot{m}_2 v_{2_x} = -\dot{m}_1 v_1$$

Momentum accumulation

Note that the cart is accelerating. Thus,

$$\begin{aligned} \frac{d}{dt}\int_{cv} v_x \rho dV &= \frac{d}{dt} v_c \int_{cv} \rho dV = \frac{d}{dt}(Mv_c) \\ &= M\frac{dv_c}{dt} \end{aligned}$$

where M is the mass of the cart (mass of water moving with cart is negligible)
From conservation of mass

$$\begin{aligned} v_{2y} &= (v_j - v_c) \\ v_{2_x} &= v_c \\ \dot{m}_2 &= \dot{m}_1 \end{aligned}$$

Combining terms

$$\begin{aligned} \sum F_x &= \frac{d}{dt}(Mv_c) + \dot{m}(v_{2x} - v_1) \\ 0 &= M\frac{dv_c}{dt} + \rho A_1(v_j - v_c)(v_c - v_j) \\ M\frac{dv_c}{dt} &= \rho A_1 v_j^2 (1 - \frac{v_c}{v_j})^2 \\ &= \dot{m} v_j (1 - \frac{v_c}{v_j})^2 \end{aligned}$$

Since the jet velocity is constant

$$\begin{aligned} \frac{d}{dt}\left(\frac{v_c}{v_j}\right) &= \frac{\dot{m}}{M}(1 - \frac{v_c}{v_j})^2 \\ \frac{d\left(\frac{v_v}{v_j}\right)}{(1 - \frac{v_c}{v_j})^2} &= \frac{\dot{m}}{M} dt \end{aligned}$$

Integrating and substituting in the limits, $v_c/v_j = 0$ at $t = 0$ and $v_c/v_j = 0.5$ at $t = \Delta t$ gives

$$\begin{aligned} \Delta t &= \frac{M}{\dot{m}} \\ &= \frac{100 \text{ kg}}{10 \text{ kg/s}} \\ &\boxed{\Delta t = 10 \text{ s}} \end{aligned}$$

6.89: PROBLEM DEFINITION

Situation:

A problem in rocket-trajectory analysis is described in the problem statement.

Find:

Initial mass of a rocket needed to place the rocket in orbit.

SOLUTION

$$
\begin{aligned}
M_0 &= M_f \exp\left(\frac{V_{b0}\lambda}{T}\right) \\
&= 50\,\text{kg} \exp\left(\frac{7200\,\text{m/s}}{3000\,\text{m/s}}\right) \\
& \boxed{M_0 = 551 \text{ kg}}
\end{aligned}
$$

6.90: PROBLEM DEFINITION

Situation:
A toy rocket is powered by a jet of water—additional details are provided in the problem statement.

Find:
Maximum velocity of the rocket.

Assumptions:
Neglect hydrostatic pressure; Inlet kinetic pressure is negligible.

SOLUTION

Newtons 2^{nd} law.

$$\begin{aligned} \sum F &= ma \\ T - W &= ma \end{aligned}$$

where T =thrust and W =weight

$$\begin{aligned} T &= \dot{m}v_e \\ \dot{m}v_e - mg &= m dv_R/dt \\ dv_R/dt &= (T/m) - g \\ &= (T/(m_i - \dot{m}t)) - g \\ dv_R &= ((Tdt)/(m_i - \dot{m}t)) - gdt \\ v_R &= (-T/\dot{m})\ell n(m_i - \dot{m}t) - gt + \text{const.} \end{aligned}$$

where $v_R = 0$ when $t = 0$. Then

$$\begin{aligned} \text{const.} &= (T/\dot{m})\ln(m_i) \\ v_R &= (T/\dot{m})\ln((m_i)/(m_i - \dot{m}t)) - gt \\ v_{R\max} &= (T/\dot{m})\ln(m_i/m_f) - gt_f \\ T/\dot{m} &= \dot{m}v_e/\dot{m} = v_e \end{aligned}$$

Bernoulli equation
(neglecting hydrostatic pressure)

$$p_i + \rho_f v_i^2/2 = p_e + \rho_f v_e^2/2$$

The exit pressure is zero (gage) and the inlet kinetic pressure is negligible. So

$$\begin{aligned} v_e^2 &= 2p_i/\rho_f \\ &= 2 \times 100 \times 10^3/1000 \\ &= 200 \text{ m}^2/\text{s}^2 \\ v_e &= 14.14 \text{ m/s} \\ \dot{m} &= \rho_e v_e A_e \\ &= 1000 \times 14.14 \times 0.1 \times 0.05^2 \times \pi/4 \\ &= 2.77 \text{ kg/s} \end{aligned}$$

Time for the water to exhaust:

$$
\begin{aligned}
t &= m_w/\dot{m} \\
&= 0.10/2.77 \\
&= 0.036s
\end{aligned}
$$

Thus

$$
\begin{aligned}
v_{\max} &= 14.14 \ln((100+50)/50) - (9.81)(0.036) \\
& \boxed{v_{\max} = 15.2 \text{ m/s}}
\end{aligned}
$$

6.91: PROBLEM DEFINITION

Situation:

A valve at the end of a gasoline pipeline is rapidly closed—additional details are provided in the problem statement.

Find:

Water hammer pressure rise: Δp

SOLUTION

Speed of sound

$$
\begin{aligned}
c &= \sqrt{E_v/\rho} \\
&= ((715)(10^6)/(680))^{0.5} \\
&= 1025 \text{ m/s}
\end{aligned}
$$

Pressure rise

$$
\begin{aligned}
\Delta p &= \rho v c \\
&= (680)(12)(1025) \\
& \boxed{\Delta p = 8.36 \text{ MPa}}
\end{aligned}
$$

6.92: PROBLEM DEFINITION

Situation:

A valve at the end of a long water pipeline is rapidly closed—additional details are provided in the problem statement.

Find:

Water hammer pressure rise: Δp

SOLUTION

$$\begin{aligned} c &= \sqrt{\frac{E_v}{\rho}} \\ &= \sqrt{\frac{2.2 \times 10^9 \,\text{Pa}}{1000 \,\text{kg/ m}^3}} \\ &= 1483 \text{ m/s} \\ t_{\text{crit}} &= \frac{2L}{c} \\ &= 2 \times 10,000 \,\text{m}/1483 \,\text{m/ s} \\ &= 13.5 \text{ s} > 10 \text{ s} \end{aligned}$$

Then

$$\begin{aligned} \Delta p &= \rho v c \\ &= 1000 \,\text{kg/ m}^3 \times 4 \,\text{m/ s} \times 1483 \,\text{m/ s} \\ &= 5,932,000 \text{ Pa} \\ &= \boxed{\Delta p = 5.93 \text{ MPa}} \end{aligned}$$

6.93: PROBLEM DEFINITION

Situation:

A valve at the end of a water pipeline is instantaneously closed—additional details are provided in the problem statement.

Find:

Pipe length: L

SOLUTION

Determine the speed of sound in water

$$\begin{aligned} c &= \sqrt{\frac{E_v}{\rho}} \\ &= \sqrt{\frac{2.2 \times 10^9}{1000}} \\ &= 1483 \text{ m/s} \end{aligned}$$

Calculate the pipe length

$$\begin{aligned} t &= 4L/c \\ 3 &= 4L/1483 \\ & \boxed{L = 1112 \text{ m}} \end{aligned}$$

6.94: PROBLEM DEFINITION

Situation:

A valve at the end of a water pipeline is closed during a time period of 10 seconds. Additional details are provided in the problem statement.

Find:

Maximum water hammer pressure: $\Delta p_{\max}$

SOLUTION

Determine the speed of sound in water

$$
\begin{aligned}
c &= \sqrt{\frac{E_v}{\rho}} \\
c &= \sqrt{\frac{320,000\,\mathrm{lbf/\,in^2} \times 144\,\mathrm{in^2/\,ft^2}}{1.94\,\mathrm{slug/\,ft^3}}} \\
&= 4874 \text{ ft/s}
\end{aligned}
$$

Determine the critical time of closure

$$
\begin{aligned}
t_{\text{crit}} &= 2L/c \\
&= 2 \times 5 \times 5280/4874 \\
&= 10.83 \text{ s} > 10 \text{ s}
\end{aligned}
$$

Pressure rise

$$
\begin{aligned}
\Delta p_{\max} &= \rho v c \\
&= 1.94 \times 8 \times 4874 \\
&= \boxed{\Delta p_{\max} = 75,644 \text{ psf} = 525 \text{ psi}}
\end{aligned}
$$

6.95: PROBLEM DEFINITION

Situation:

A valve at the end of a long water pipe is shut in 3 seconds—additional details are provided in the problem statement

Find:

Maximum force exerted on valve due to the water hammer pressure rise: F_{valve}

SOLUTION

$$\begin{aligned} t_{\text{crit}} &= \frac{2L}{c} \\ &= \frac{2 \times 4000}{1485.4} \\ &= 5.385 \text{ s} > 3 \text{ s} \\ F_{\text{valve}} &= A\Delta p \\ &= A\rho(Q/A)c \\ &= \rho Q c \\ &= 998 \times 0.03 \times 1483 \\ & \boxed{F_{\text{valve}} = 44.4\,\text{kN}} \end{aligned}$$

6.96: PROBLEM DEFINITION

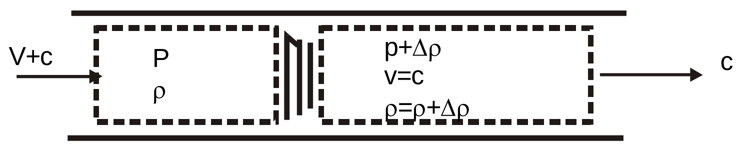
Situation:

The easy way to derive the equation for water hammer pressure rise is to use a moving control volume.

Find:

Derive the equation for water hammer pressure rise (Eq. 6.12).

SOLUTION



Continuity equation

$$
\begin{aligned}
(v+c)\rho &= c(\rho+\Delta\rho) \\
&\therefore \;\; \Delta\rho = v\rho/c
\end{aligned}
$$

Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \sum v_x \rho \mathbf{v}\cdot\mathbf{A} \\
pA-(p+\Delta p)A &= -(V+c)\rho(V+c)A + c^2(\rho+\Delta\rho)A \\
\Delta p &= 2\rho vc - c^2\Delta\rho + v^2\rho \\
&= 2\rho vc - c^2 v\rho/c + v^2\rho \\
&= \rho vc + \rho v^2
\end{aligned}
$$

Here ρv^2 is very small compared to ρvc

$$
\therefore \boxed{\Delta p = \rho vc}
$$

6.97: PROBLEM DEFINITION

Situation:

The problem statement describes a water hammer phenomena in a pipe.

Find:

Plot a pressure versus time trace at point B for a time period of 5 seconds.
Plot a pressure versus distance trace at $t = 1.5\,\mathrm{s}$.

SOLUTION

$$\begin{aligned} v &= 0.1\,\mathrm{m/s} \\ c &= 1483\,\mathrm{m/s} \end{aligned}$$

$$\begin{aligned} p_{\text{pipe}} &= 10\gamma - \rho v_{\text{pipe}}^2/2 \\ &\approx 98,000\ \text{Pa} \\ \Delta p &= \rho v c \\ &= 1000 \times 0.10 \times 1483 \\ \Delta p &= 148,000\ \text{Pa} \end{aligned}$$

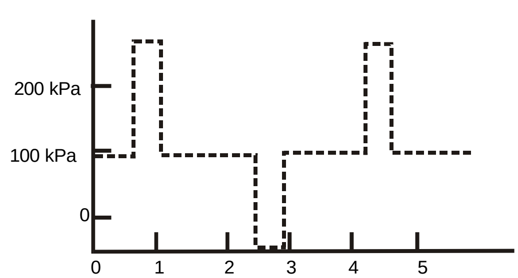
Thus

$$\begin{aligned} p_{\max} &= p + \Delta p \\ &= 98,000 + 148,000 \\ &= 246\ \text{k Pa- gage} \end{aligned}$$

$p_{\min} = p - \Delta p = -50$ kPa gage
The sequence of events are as follows:

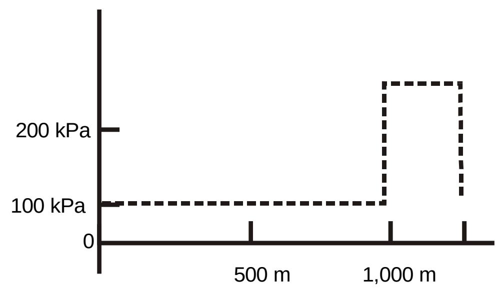
			Δt		$\Sigma\Delta t$	
Pressure wave reaches pt B.	1000/1483	=	0.674	s	0.67	s
Time period of high pressure at B	600/1483	=	0.405	s	1.08	s
Time period of static pressure at B	2000/1483	=	1.349	s	2.43	s
Time period of negative pressure at B	600/1483	=	0.405	s	2.83	s
Time period of static pressure at B	2000/1483	=	1.349	s	4.18	s
Time period of high pressure at B	600/1,483	=	0.405	s	4.59	s
Time period of static pressure atB	2000/1483	=	1.349	s	5.94	s

Results are plotted below:



At $t = 1.5$ s high pressure wave will have travelled to reservoir and static wave will be travelling toward valve.

Time period for wave to reach reservoir $= 1300/1483 = 0.877$ s. Then static wave will have travelled for $1.5 - 0.877$ s $= 0.623$ s. Distance static wave has travelled $= 0.623$ s $\times 1,483$ m/s $= 924$ m. The pressure vs. position plot is shown below:



6.98: PROBLEM DEFINITION

Situation:

A water hammer phenomenon occurs in a steel pipe—additional details are provided in the problem statement.

Find:

(a) The initial discharge.
(b) Length from A to B.

SOLUTION

$$
\begin{aligned}
c &= 1483 \text{ m/s} \\
\Delta p &= \rho \Delta v c \\
t &= L/c \\
L &= tc = 1.46 \text{ s } \times 1,483 \\
& \boxed{L = 2160 \text{ m}} \\
\Delta v &= \Delta p / \rho c \\
&= (2.5 - 0.2) \times 10^6 \text{ Pa}/1.483 \times 10^6 \text{ kg/m}^2\text{s} = 1.551 \text{ m/s} \\
Q &= vA = 1.551 \times \pi/4 \\
& \boxed{Q = 1.22 \text{ m}^3/\text{s}}
\end{aligned}
$$

6.99: PROBLEM DEFINITION

Situation:

Water is discharged from a slot in a pipe—additional details are provided in the problem statement.

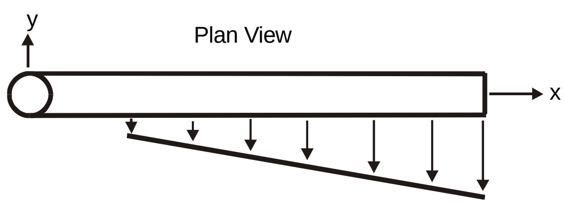
Find:

Reaction (Force and Moment) at station $A - A$

PLAN

Apply the momentum equation and the moment of momentum equation.

SOLUTION



$$v_y = -(3.1 + 3x) \text{ m/s}$$

Momentum equation (y-direction)

$$\begin{aligned}\sum F_y &= \int v_y \rho \mathbf{v} \cdot \mathbf{dA} \\ F_y &= -\int_{0.3}^{1.3} (3.1 + 3x) \times 1,000 \times (3.1 + 3x) \times 0.015 dx = -465 \text{ N} \\ R_y &= 465 \text{ N}\end{aligned}$$

Flow rate

$$\begin{aligned}Q &= \int v dA = 0.015 \int_{0.3}^{1.3} (3.1 + 3x) dx = 0.0825 \text{ m}^3/\text{s} \\ v_1 &= Q/A = 0.0825/(\pi \times 0.04^2) = 16.4 \text{ m/s}\end{aligned}$$

Momentum equation (z-direction)

$$\sum F_z = -\dot{m}_1 v_1$$

$$F_z - p_A A_A - W_f = -\dot{m} v_1$$

$$F_z = 30,000 \times \pi \times 0.04^2 + 0.08 \times \pi \times 0.04^2 \times 9,810$$

$$+1.3 \times \pi \times 0.025^2 \times 9,810 + 1000 \times 0.0825 \times 16.4$$

$$= 1530 \text{ N}$$

$$R_z = -1530 \text{ N}$$

Moment-of-momentum ($z-$direction)

$$\begin{aligned} T_z &= \int_{cs} rv\rho\mathbf{v}\cdot\mathbf{dA} \\ &= 15\int_{0.3}^{1.3}(3.1+3r)^2 r dr = 413.2\ \mathrm{N}\cdot\mathrm{m} \end{aligned}$$

Moment-of-momentum (y-direction)

$$T_y + Wr_{cm} = 0$$

where W=weight, r_{cm}=distance to center of mass

$$T_y = -1.3\pi\times 0.025^2\times 9810\times 0.65 = -16.28\ \mathrm{N}\cdot\mathrm{m}$$

Net reaction at A-A

$$\boxed{\mathbf{F} = (465\mathbf{j} - 1530\mathbf{k})\,\mathrm{N}}$$

$$\boxed{\mathbf{T} = (16.3\mathbf{j} - 413\mathbf{k})\,\mathrm{N}\cdot\ \mathrm{m}}$$

6.100: PROBLEM DEFINITION

Situation:

A helicopter rotor uses two small rockets motors—details are provided in the problem statement.

Find:

Power provided by rocket motors.

PLAN

Apply the momentum equation. Select a control volume that encloses one motor, and select a stationary reference frame.

SOLUTION

Velocity analysis

$$\begin{aligned} v_i &= 0 \\ V_i &= rw \\ &= 3.5 \times 2\pi \\ &= 21.991 \text{ m/ s} \\ V_0 &= 500 \text{ m/ s} \\ v_0 &= (500 - 21.99) \text{ m/ s} \\ &= 478.01 \text{ m/ s} \end{aligned}$$

Flow rate

$$\begin{aligned} \dot{m} &= \rho A_i V_i \\ &= 1.2 \times .002 \times 21.991 \\ &= 0.05278 \text{ kg/s} \end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned} F_x &= \dot{m}v_0 - \dot{m}v_i \\ &= \dot{m}v_0 \\ &= 0.05278 \times 478 \\ &= 25.23 \text{ N} \end{aligned}$$

Power

$$\begin{aligned} P &= 2Frw \\ &= 2 \times 25.23 \times 3.5 \times 2\pi \\ &= 1110 \text{ W} \end{aligned}$$

$$\boxed{P = 1.11 \text{ kW}}$$

6.101: PROBLEM DEFINITION

Situation:

A rotating lawn sprinkler is to be designed.

The design target is 0.25 in. of water per hour over a circle of 50-ft radius.

Find:

Determine the basic dimensions of the lawn sprinkler.

Assumptions:

The Bernoulli equation applies.

Assume mechanical friction is present.

PLAN

Apply the momentum equation.

SOLUTION

Flow rate. To supply water to a circle 50 ft. in diameter at a 1/4 inch per hour requires a discharge of

$$\begin{aligned} Q &= \dot{h}A \\ &= (1/48)\pi(50^2/4)/3600 \\ &= 0.011 \text{ cfs} \end{aligned}$$

Bernoulli equation. Assuming no losses between the supply pressure and the sprinkler head would give and exit velocity at the head of

$$\begin{aligned} V &= \sqrt{\frac{2p}{\rho}} \\ &= \sqrt{\frac{2 \times 50 \times 144}{1.94}} \\ &= 86 \text{ ft/s} \end{aligned}$$

If the water were to exit the sprinkler head at the angle for the optimum trajectory (45^o), the distance traveled by the water would be

$$s = \frac{V_e^2}{2g}$$

The velocity necessary for a 25 ft distance (radius of the spray circle) would be

$$\begin{aligned} V_e^2 &= 2gs = 2 \times 32.2 \times 25 = 1610 \\ V_e &= 40 \text{ ft/s} \end{aligned}$$

This means that there is ample pressure available to do the design. There will be losses which will affect the design. As the water spray emerges from the spray head, atomization will occur which produces droplets. These droplets will experience aerodynamic drag which will reduce the distance of the trajectory. The size distribution of droplets will lead to small droplets moving shorted distances and larger droplets farther which will contribute to a uniform spray pattern.

The sprinkler head can be set in motion by having the water exit at an angle with respect to the radius. For example if the arm of the sprinkler is 4 inches and the angle of deflection at the end of the arm is 10 degrees, the torque produced is

$$\begin{aligned} M &= \rho Q r V_e \sin\theta \\ &= 1.94 \times 0.011 \times 40 \times \sin 10 \\ &= 0.148 \text{ ft-lbf} \end{aligned}$$

The downward load on the head due to the discharge of the water is

$$\begin{aligned} F_y &= \rho Q V_e \sin 45 \\ &= 1.94 \times 0.011 \times 40 \times \sin 45 \\ &= 0.6 \text{ lbf} \end{aligned}$$

The moment necessary to overcome friction on a flat plate rotating on another flat plate is

$$M = (2/3)\mu F_n r_o$$

where μ is the coefficient of friction and r_o is the radius of the plate. Assuming a 1/2 inch radius, the limiting coefficient of friction would be

$$\begin{aligned} \mu &= \frac{3}{2}\frac{M}{F_n r_o} \\ &= \frac{3}{2}\frac{0.148}{0.6 \times (1/24)} \\ &= 8.9 \end{aligned}$$

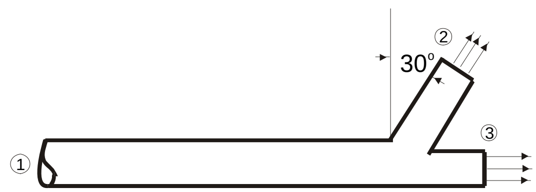
This is very high, which means there is adequate torque to overcome friction.

These are initial calculations showing the feasibility of the design. A more detailed design would now follow.

6.102: PROBLEM DEFINITION

Situation:

Water flows out a pipe with two exit nozzles—additional details are provided in the problem statement.



Find:

Reaction (Force and Moment) at section 1.

PLAN

Apply the continuity equation, then the momentum equation and the moment of momentum equation.

SOLUTION

Continuity equation equation

$$v_1 = (0.1 \times 50 + 0.2 \times 50)/0.6 = 25 \text{ ft/s}$$

Momentum equation (x-direction)

$$\sum F_x = \dot{m}_3 v_{3x} + \dot{m}_2 v_{2x}$$
$$F_x + p_1 A_1 = -\rho v_1^2 A_1 + \rho v_3^2 A_3 + \rho v_2^2 A_2 \cos 60^\circ$$
$$F_x = -20 \times 144 \times 0.6 - 1.94 \times 25^2 \times 0.6 + 1.94 \times 50^2 \times 0.2$$
$$+1.94 \times 50^2 \times 0.1 \times \cos 60^\circ = -1{,}243 \text{ lbf}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \dot{m}_2 v_{2y} \\ F_y &= 1.94 \times 50 \times 50 \times 0.1 \times \cos 30^\circ = 420 \text{ lbf} \end{aligned}$$

Moment-of-momentum ($z-$direction)

$$r_2 \dot{m}_2 v_{2y} = (36/12)(1.94 \times 0.1 \times 50)50 \sin 60^\circ = 1260 \text{ ft-lbf}$$

Reaction at section 1

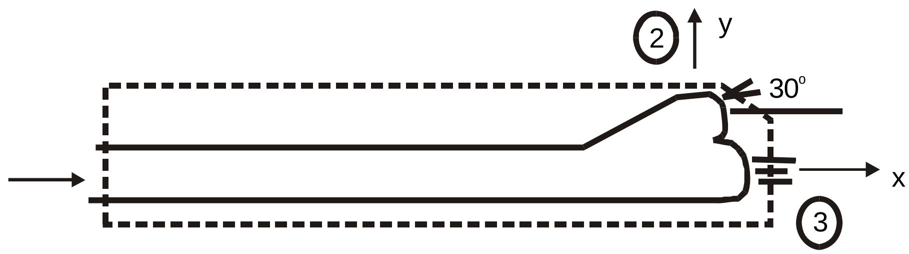
$$\boxed{\mathbf{F} = (1240\mathbf{i} - 420\mathbf{j})\text{lbf}}$$

$$\boxed{\mathbf{M} = (-1260\mathbf{k}) \text{ ft-lbf}}$$

6.103: PROBLEM DEFINITION

Situation:

Water flows out a pipe with two exit nozzles—additional details are provided in the problem statement.



Find:

Reaction (Force and Moment) at section 1.

PLAN

Apply the continuity equation, then the momentum equation and the moment of momentum equation.

SOLUTION

Continuity equation equation

$$V_1 = (0.01 \times 20 + 0.02 \times 20)/0.1 = 6 \text{ m/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum \dot{m}_o v_{ox} - \sum \dot{m}_i v_{ix} \\
F_x + p_1 A_1 &= \dot{m}_3 v_3 + \dot{m}_2 v_2 \cos 30 - \dot{m}_1 v_1 \\
F_x &= -200,000 \times 0.1 - 1000 \times 6^2 \\
&\quad \times 0.1 + 1000 \times 20^2 \times 0.02 \\
&\quad +1000 \times 20^2 \times 0.01 \times \cos 30^\circ \\
&= \boxed{F_x = \text{-}12,100 \text{ N}}
\end{aligned}$$

Momentum equation (y-direction)

$$F_y - W = \dot{m}_2 v_2 \sin 30^\circ$$

Weight

$$\begin{aligned}
W &= W_{\mathrm{H_2O}} + W_{\text{pipe}} \\
&= (0.1)(1)(9810) + 90 \\
&= 1071 \text{ N}
\end{aligned}$$

thus

$$\begin{aligned} F_y &= 1000 \times 20^2 \times 0.01 \times \sin 30^\circ + 1,071 \\ &= \boxed{F_y = 3070 \text{ N}} \end{aligned}$$

Moment-of-momentum ($z-$direction)

$$\begin{aligned} M_z - W r_{cm} &= r_2 \dot{m}_2 v_{2y} \\ M_z &= (1071 \times 0.5) + (1.0)(1000 \times 0.01 \times 20)(20 \sin 30^\circ) \\ &= 2535 \text{ N} \cdot \text{m} \end{aligned}$$

Reaction at section 1

$$\boxed{\mathbf{F} = (12.1\mathbf{i} - 3.1\mathbf{j})\,\text{kN}}$$

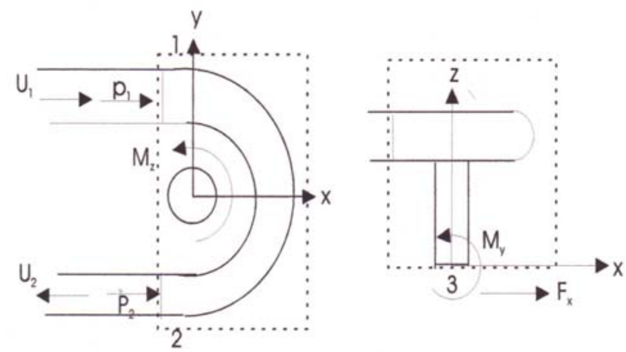
$$\boxed{\mathbf{M} = (-2.54\mathbf{k})\,\text{kN} \cdot \text{ m}}$$

6.104: PROBLEM DEFINITION

Situation:

A reducing pipe bend held in place by a pedestal. Water flow. No force transmission through the pipe at sections 1 and 2.

Assume irrotational flow. Neglect weight



Find:

(a) Force needed to hold bend stationary: **F**

(b) Moment needed to hold bend stationary: **M**

PLAN

Apply the Bernoulli equation, then the momentum equation, and then the moment of momentum equation.

SOLUTION

Bernoulli equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{v_1^2}{2g} &= \frac{p_2}{\gamma} + \frac{v_2^2}{2g} \\
v_1 &= Q/A_1 = 2/(\pi/4 \times 0.5^2) = 10.19 \text{ ft/s} \\
v_2 &= Q/A_1 = 2/(\pi/4 \times (4/12)^2) = 22.92 \text{ ft/s} \\
p_1 &= 20 \times 144 = 2,880 \text{ psf} \\
p_2 &= p_1 + \rho(v_1^2 - v_2^2)/2 \\
&= 2,880 + 1.94(10.19^2 - 22.92^2)/2 \\
&= 2,471 \text{ psf}
\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}
F_x + p_1 A_1 + p_2 A_2 &= \dot{m} v_{2_x} - \dot{m} v_{1_x} \\
F_x &= -p_1 A_1 - p_2 A_2 - \dot{m}(v_2 + v_1)
\end{aligned}$$

where

$$
\begin{aligned}
A_1 &= \pi/4 \times 0.5^2 = 0.196 \text{ ft}^2 \\
A_2 &= \pi/4 \times 0.333^2 = 0.0873 \text{ ft}^2 \\
\dot{m} &= \rho A_1 v_1 = 1.94 \times 0.196 \times 10.19 = 3.875 \text{ slug/s}
\end{aligned}
$$

thus

$$
F_x = -2,880 \times 0.196 - 2,471 \times 0.0873 - 3.875(10.19 + 22.92) = -908 \text{ lbf}
$$

Moment-of-momentum ($z-$direction)

$$
\begin{aligned}
m_z - rp_1A_1 + rp_2A_2 &= -r\dot{m}v_2 + r\dot{m}v_1 \\
m_z &= r(p_1A_1 - p_2A_2) - r\dot{m}(v_2 - v_1)
\end{aligned}
$$

where $r = 1.0$ ft.

$$
\begin{aligned}
M_z &= 1.0(2,880 \times 0.196 - 2,471 \times 0.08753) - 1.0 \times 3.875(22.92 - 10.19) \\
&= 299 \text{ ft-lbf}
\end{aligned}
$$

Moment-of-momentum (y-direction)

$$
M_y + p_1A_1r_3 + p_2A_2r_3 = -r_3\dot{m}v_2 - r_3\dot{m}_1v_1
$$

where $r_3 = 2.0$ ft.

$$
\begin{aligned}
M_y &= -r_3[p_1A_1 + p_2A_2 + \dot{m}(v_1 + v_2)] \\
&= -2.0 \times 908 \\
M_y &= -1816 \text{ ft-lbf}
\end{aligned}
$$

Net force and moment at 3

$$
\boxed{\mathbf{F} = -908\mathbf{i}\,\text{lbf}}
$$

$$
\boxed{\mathbf{M} = (-1820\mathbf{j} + 299\mathbf{k}) \text{ ft-lbf}}
$$

6.105: PROBLEM DEFINITION

Situation:
Centrifugal fan is used to pump air

Find:
Power (hp) required to operate fan

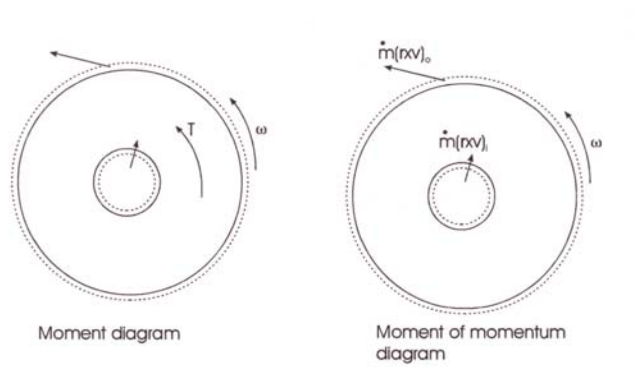
Assumptions:
Neglect the compressibility of air.

PLAN

Apply the moment of momentum equation between inlet and outlet.

SOLUTION

The control volume enclosed the rotor but does not rotate. The flow is steady within the control volume. Assume positive direction comes out of the page, the $\mathbf{e}_z$ direction.



The moment diagram shows one moment (torque)

$$\sum \mathbf{M} = T\mathbf{e}_z$$

There is no moment of momentum inflow because the inlet velocity is radial (or the fluid enters with zero radius). There is a moment of momentum outflow.
The moment of momentum equation is

$$\sum \mathbf{M} = \frac{d}{dt}\int_{cv} \rho(\mathbf{r}\times\mathbf{v})dV + \sum \dot{m}_o(\mathbf{r}\times\mathbf{v})_o - \sum \dot{m}_i(\mathbf{r}\times\mathbf{v})_i$$

Since the flow is steady and there is not inflow of moment of moment, the equation reduces to

$$T\mathbf{e}_z = \dot{m}_o(\mathbf{r}\times\mathbf{v})_o$$

The exit radial velocity is

$$v_r = \frac{Q}{\pi D \ell} = \frac{1500 \text{ cfm} \times \frac{1 \text{ min}}{60 \text{ sec}}}{\pi \times 1 \text{ ft} \times (2/12 \text{ ft})} = 47.75 \text{ ft/s}$$

The density of the air is

$$\rho = \frac{p}{RT} = \frac{14.7 \text{ psia} \times 144 \text{ in}^2/\text{ft}^2}{1716 \text{ ft-lbf/slug-R} \times 520 \text{ R}} = 0.00237 \text{ slug/ft}^3$$

At the outlet

$$(\mathbf{r} \times \mathbf{v})_o = \frac{D}{2}\omega\frac{D}{2}\mathbf{e}_z$$

The torque is

$$\begin{aligned} T &= \rho Q \omega \frac{D^2}{4} = 0.00237 \frac{\text{slug}}{\text{ft}^3} \times 1500 \frac{\text{ft}^3}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{3600 \times 2\pi \text{ rad/min}}{60 \text{ s/min}} \times \frac{1^2 \text{ft}^2}{4} \\ &= 5.584 \text{ ft-lbf} \end{aligned}$$

The power is

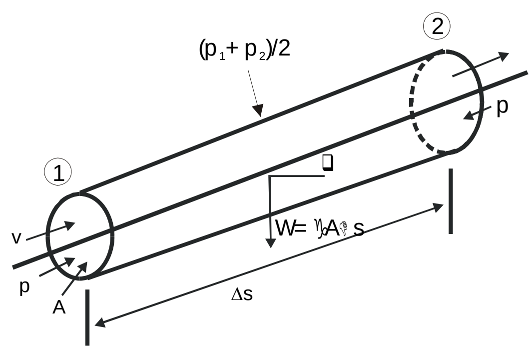
$$\begin{aligned} P &= T\omega = 5.584 \text{ ft-lbf} \times \frac{3600 \times 2\pi \text{ rad/min}}{60 \text{ s/min}} = 2105 \text{ ft-lbf/s} \\ &= 2105 \frac{\text{ft-lbf}}{\text{s}} \times \frac{1 \text{ hp}}{550 \text{ ft-lbf/s}} \\ & \boxed{P = 3.83 \text{ hp}} \end{aligned}$$

6.106: PROBLEM DEFINITION

Situation:
Arbitrary control volume with length Δs.

Find:
Derive Euler's equation using the momentum equation.

SOLUTION



Continuity equation

$$\frac{d}{dt}\int \rho dV + \dot{m}_o - \dot{m}_i = 0$$

For a control volume that is fixed in space

$$\int \frac{\partial \rho}{\partial t} dV + \dot{m}_o - \dot{m}_i = 0$$

For the control volume shown above the continuity equation is expressed as

$$\frac{\partial \rho}{\partial t}\bar{A}\Delta s + (\rho v A)_2 - (\rho v A)_1 = 0$$

where $\bar{A}$ is the average cross-sectional area between 1 and 2 and the volume of the control volume is $\bar{A}\Delta s$. Dividing by Δs and taking the limit as $\Delta s \to 0$ we have

$$A\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial s}(\rho v A) = 0$$

In the limit the average area becomes the local area of the stream tube.

The momentum equation for the control volume is

$$\sum F_s = \frac{d}{dt}\int \rho v dV + \dot{m}_o v_o - \dot{m}_i v_i$$

For a control volume fixed in space, the accumulation term can be written as

$$\frac{d}{dt}\int \rho v dV = \int \frac{\partial}{\partial t}(\rho v) dV$$

The forces are due to pressure and weight

$$\sum F_s = p_1 A_1 - p_2 A_2 + (\frac{p_1 + p_2}{2})(A_2 - A_1) - \gamma \bar{A} \Delta s \sin\theta$$

where the third term on the right is the pressure force on the sloping surface and θ is the orientation of control volume from the horizontal. The momentum equation for the control volume around the stream tube becomes

$$\frac{\partial}{\partial t}(\rho v)\bar{A}\Delta s + \rho A v_2 v_2 - \rho A v_1 v_1 = (p_1 - p_2)\bar{A} - \gamma \bar{A}\Delta s \sin\theta$$

Dividing by Δs and taking limit as $\Delta s \to 0$, we have

$$A\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial s}(\rho A v^2) = -\frac{\partial p}{\partial s} A - \gamma A \sin\theta$$

By differentiating product terms the left side can be written as

$$A\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial s}(\rho A v^2) = v[A\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial s}(\rho v A)] + A\rho\frac{\partial v}{\partial t} + A\rho v\frac{\partial v}{\partial s}$$

The first term on the right is zero because of the continuity equation. Thus the momentum equation becomes

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial s} = -\frac{\partial p}{\partial s} - \gamma \sin\theta$$

But $\sin\theta = \partial z/\partial s$ and $\partial v/\partial t + v\partial v/\partial s = a_s$, the acceleration along the path line. Thus the equation becomes

$$\rho a_s = -\frac{\partial}{\partial s}(p + \gamma z)$$

which is Euler's equation.

6.107: PROBLEM DEFINITION

Following the same development in the text done for the planar case, there will be another term added for the two additional faces in the z-direction. The rate of change of momentum in the control volume plus the net efflux through the surfaces becomes

$$\frac{1}{\Delta x \Delta y \Delta z}\int_{cv}\frac{\partial}{\partial t}(\rho u)dV + \frac{\rho\,\mathrm{u}u_{x+\Delta x/2} - \rho\,\mathrm{u}u_{x-\Delta x/2}}{\Delta x}$$
$$+\frac{\rho u v_{y+\Delta y/2} - \rho u v_{y-\Delta y/2}}{\Delta y} + \frac{\rho u w_{z+\Delta z/2} - \rho u w_{z-\Delta z/2}}{\Delta z}$$

where w is the velocity in the $z-$direction and Δz is the size of the control volume in the $z-$direction. Taking the limit as Δx, Δy, and $\Delta z \rightarrow 0$ results in

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho\,\mathrm{u}u) + \frac{\partial}{\partial y}(\rho u v) + \frac{\partial}{\partial z}(\rho u w)$$

In the same way, accounting for the pressure and shear stress forces on the three-dimensional control volume leads to an additional shear stress term on the z-face. There is no additional pressure force because there can only be a force due to pressure on the faces normal to the x-direction. The force terms on the control volume become

$$\frac{p_{x-\Delta x/2} - p_{x+\Delta x/2}}{\Delta x} + \frac{\tau_{xx}\mid_{x+\Delta x/2} - \tau_{xx}\mid_{x-\Delta x/2}}{\Delta x}$$
$$+\frac{\tau_{yx}\mid_{y+\Delta y/2} - \tau_{yx}\mid_{y-\Delta y/2}}{\Delta y} + \frac{\tau_{zx}\mid_{z+\Delta z/2} - \tau_{zx}\mid_{z-\Delta z/2}}{\Delta z}$$

Taking the limit as Δx, Δy, and $\Delta z \rightarrow 0$ results in

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

The body force in the x-direction is

$$\frac{\rho g_x \Delta V\!\!\!/}{\Delta x \Delta y \Delta z} = \rho g_x$$

6.108: PROBLEM DEFINITION

Substituting in the constitutive relations gives

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial}{\partial y}(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) + \mu \frac{\partial}{\partial z}(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x})$$

This can be written as

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}) + \mu \frac{\partial}{\partial x}(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z})$$

The last term is equal to zero from the Continuity equation equation for an incompressible flow, so

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = \mu(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2})$$

6.109: PROBLEM DEFINITION

Situation:

Apply the Navier-Stokes equation to direction normal to a rectilinear flow

Find:

Pressure variation in direction normal to flow

PLAN

Write the Navier-Stokes equation in direction normal to flow and reduce for rectilinear flow

SOLUTION

The Navier-Stokes equation in the y-direction is

$$\rho\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \mu(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) + \rho g_y$$

For a rectilinear flow as shown, $v = 0$, so the equation reduces to

$$-\frac{\partial p}{\partial y} + \rho g_y = 0$$

The component of the gravitational force in the y-direction is

$$g_y = -g\frac{\Delta z}{\Delta y} = -g\frac{\partial z}{\partial y}$$

so

$$\frac{\partial}{\partial y}(p + \rho z) = 0$$

and

$$p + \gamma z = \text{ const}$$