

Chapter (1), self reading.

{Chapter(2), :- Basic dimensions and concepts:-

Dimensions & units :-

SI

US

Basic :- mass (kg), length (m),
time (s), Temp ($^{\circ}\text{C}$) / (K).

Basic :- length (ft), time (s),
Temp ($^{\circ}\text{F}$) / (R), Force (lbf)

Derived :- Force (N).

* Power = J/s
 $\text{W} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3}$ Watt, Horse power = 746 W

* Force = ma
 $= \text{kg} \cdot \text{m}/\text{s}^2$ Newton, slug = 32.2 lbm
 $= \text{kg} \cdot \text{m}/\text{s}^2$ Newton, 1 lbm = 0.45 kg

* Energy = F.d
 $= \text{N} \cdot \text{m}$ Joule, 1 ft = 0.3048 m
 $= \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}$ Joule, 1 ft = 12 inch

$T(^{\circ}\text{C}) = T(\text{K}) - 273.15$

$T(^{\circ}\text{F}) = T(^{\circ}\text{R}) - 459.67$

$T(^{\circ}\text{F}) = 32 + 1.8^{\circ}\text{C}$

Chapter (1)
review

Examples Convert 60 km/hr to m/s ---

$$60 \frac{\text{km}}{\text{hr}} \times \frac{1 \text{ hr}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}}$$

$$= \frac{60 \times 1000}{3600} \text{ m/s}$$

$$= 16.67 \text{ m/s}$$

Chapter (2): Fluid Properties:-

- Fluid: It's a substance (liquid & gas), that cannot resist any shear force.

i.e., it will continue to deform under the action of shear force no matter how small this force is. (it will flow).

- Fluid Mechanics: It's the science that deals with the action of forces on fluid.

Flow classifications:

[1] Hydrodynamics.

[2] Gas dynamics.

①* Hydrodynamics:- We study the flow of fluid that have no change in its density.

ex:- liquids, / gases if the change of Pressure is small.

②* Gas dynamics / Study of the fluid that have significant change in density.

$\frac{dP}{dx}, \frac{dP}{dy}, \frac{dP}{dz}, \frac{dP}{dt} \Rightarrow$ if (not) change $\rightarrow$ Hydrodynamics.
 $\frac{dP}{dx}, \frac{dP}{dy}, \frac{dP}{dz}, \frac{dP}{dt} \Rightarrow$ if it changes $\rightarrow$ Gas dynamics.

ex:- Gases under high speed & high changes in ΔP , is an example on Gas dynamics.

$\Rightarrow$ There is another type of flow that we call it Aerodynamics:- which deals with air only.

Extensive Properties & the value depends on the mass, (Volume, Energy, Weight, mass, Enthalpy, internal energy).

Extensive / m $\Rightarrow$ Intensive

$\downarrow$
Capital letter

$\downarrow$ $\rightarrow$ the coming sheet
Small letter

Intensive Properties:-

This value doesn't depend on the mass.

(Temperature, Pressure, density, Viscosity, thermal conductivity)

$$\frac{E}{m} = e \text{ (specific Energy).}$$

$$\frac{V}{m} = v \text{ (specific volume).}$$

$$\frac{H}{m} = h \text{ (specific enthalpy).}$$

Mass Density = ρ ; is the mass per unit volume.

$$\rho = \frac{m}{V} = \frac{\left\{ \begin{matrix} \text{Kg} \\ \text{m}^3 \end{matrix} \right\}}{\text{volume}} = \frac{\text{mass}}{\text{volume}}$$

Specific Weight = γ ; is the weight per unit volume.

$$\gamma = \frac{W}{V} = \frac{mg}{V} = \rho g = \left[\frac{\text{N}}{\text{m}^3} \right]$$

Specific Gravity (S') is the ratio of (i) specific weight of a given fluid to that of standard fluid @ given standard temperature.

- Standard liquid (H_2O).

- Standard gas (Air).

$$S' = \frac{\gamma_{\text{Fluid}}}{\gamma_{\text{Standard}}} = \frac{\rho_{\text{g. Fluid}}}{\rho_{\text{g. Standard}}}$$

S' or SG
the same

$$S' = \frac{\rho_{\text{Fluid}}}{\rho_{\text{Standard}}}$$

For liquid (S') = $\frac{\rho_{\text{liquid}}}{\rho_{H_2O}}$ @ standard temp

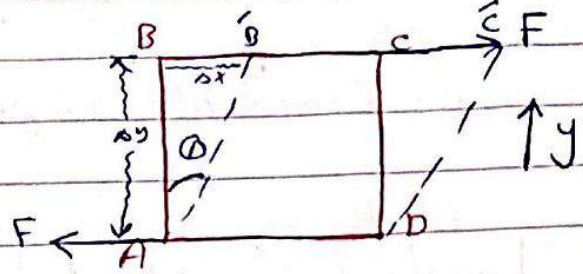
For Gases (S') = $\frac{\rho_{\text{gas}}}{\rho_{\text{Air}}}$ @ standard temp.

Remarks Maximum density of H_2O

Occurs @ $4^\circ C$.

Viscosity :- is the fluid property that characterizes resistance to flow.

* For Solids :-



Shear Force $\tau \propto$ Strain.

مثال: خرد و عار من عسل نطلب
جهد رکن من خرد و عار من عسل
من لزوجة العسل أكبر بكثير

* For liquids :-

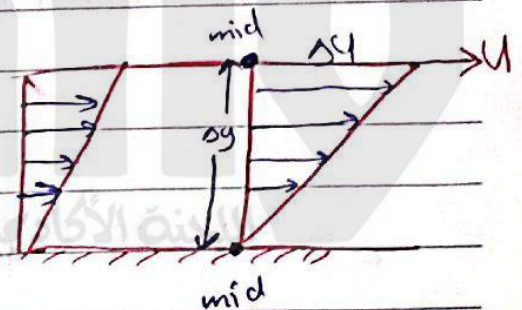
Shear force $\tau \propto$ time rate of shear strain.

$$\text{Shear Strain} \Rightarrow \theta = \frac{\Delta x}{\Delta y}$$

$$\text{Rate of shear strain} = \frac{\Delta x}{\Delta y t} = \frac{\Delta x}{t} \cdot \frac{1}{\Delta y} = \frac{\Delta u}{\Delta y} = \frac{du}{dy}$$

$$\vec{U} = U_x \hat{i} + U_y \hat{j} + U_z \hat{k} \\ = \vec{U}_i + \vec{U}_j + \vec{U}_k \Rightarrow$$

$$\tau \propto \frac{du}{dy}$$



$$\tau = \text{Constant} \cdot \frac{du}{dy}$$

@ mid point, $U = \text{Zero}$

$$\tau = \mu \frac{du}{dy}$$

Where: (μ) is the Viscosity of the fluid.

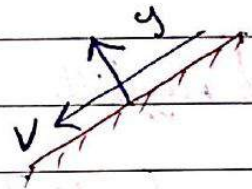
$\frac{du}{dy}$ = time rate of Shear strain

$\frac{du}{dy}$ = Velocity change with respect to some Coordinate axis.

$\Rightarrow$ = Velocity gradient (du/dy)

y = coordinate axis $\perp$ to the wall.

$$\mu = \frac{\tau}{du/dy}$$



$$[\mu] = \frac{N/m^2}{m/s.m} = \frac{N}{m^2 \cdot (s/m)} = \frac{N \cdot s}{m^2} = \text{Pa} \cdot s$$

$$= \frac{Kg \cdot m \cdot s}{s^2 \cdot m^2} = \frac{Kg \cdot m \cdot s}{m^2 \cdot s^2}$$

- a common unit of viscosity is the [POISE]

$$1 \text{ Poise} = 1 \frac{\text{dyne} \cdot s}{\text{cm}^2}$$

$$1 \text{ Newton} = 10^5 \text{ dyne}$$

$$\text{cm}^2 \leftarrow (\text{cm}^2)$$

$$= 0.1 \frac{N \cdot s}{m^2}$$

$[\mu]$ = Viscosity = is the property of the fluid which determine its resistance to shear force. ($\tau = \mu \frac{du}{dy}$)

Kinematic Viscosity: $[\nu] = \text{new}$

$$\nu = \frac{\mu}{\rho} ; [\nu] = \frac{Kg \cdot m^3}{m^2 \cdot s \cdot Kg} = \frac{m^2}{s}$$

Units 8

* in SI units :- $[M] = \frac{N \cdot s}{m^2} = Pa \cdot s = \frac{kg}{m \cdot s}$

* in English Engineering System [EES]

$$[M] = \frac{lb_f \cdot s}{ft^2} \text{ or } \frac{slug}{ft \cdot s}$$

$$\frac{1 \text{ slug}}{ft \cdot s} = \frac{(32.174) \text{ lbm}}{ft \cdot s}$$

* In Centimeter, gram, Second (cm, g, s)

[M] units Poise $\rightarrow$ (وحدة كبيرة)

$$\text{Poise} = \frac{\text{dyne} \cdot s}{cm^2} \text{ or } \frac{g}{cm \cdot s}$$

Poise is relatively large ; centipoise = 0.01 Poise

To convert Poise to SI ;

$$1 \text{ Poise} = 0.1 \text{ Pa} \cdot s = 0.1 \frac{N \cdot s}{m^2}$$

To convert $\frac{lb_f \cdot s}{ft^2}$ to $\frac{N \cdot s}{m^2}$; multiply by 47.88

$$\frac{lbm}{ft \cdot s} \text{ to } \frac{N \cdot s}{m^2} ; \text{ multiply by } 1.488$$

$$[V] = \frac{\mu}{\rho} = \frac{m^2}{s}$$

CGS $\frac{cm^2}{s} = \text{Stoke}$, Centistoke = 0.01 Stoke

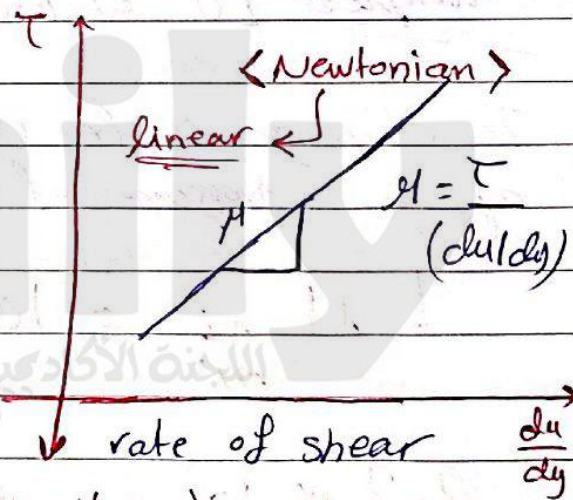
To convert $\frac{ft^2}{s}$ to $\frac{m^2}{s}$, multiply by 0.0929

To convert $\frac{ft^2}{s}$ to Stoke ($\frac{cm^2}{s}$), multiply by (30.48)²

Newtonian & Non-Newtonian fluid:-

in this course, we deal with Newtonian fluids.

$$\tau \propto du/dy$$



if $\tau = \mu \cdot du/dy$; this is form for all fluids that can be considered Newtonian fluids. (water, oil, ...)

if $\tau \neq \mu \cdot du/dy$; the fluid is classified as Non-Newtonian fluids. (meted plastic, ketchup, ...)

Elasticity of fluid is related to the change of the volume of the fluid, resulting from a change of the applied pressure.

$dP \propto \frac{dV}{V}$; V is Volume. All fluids are compressible...

$$dP = -E_V \left(\frac{dV}{V} \right); \text{Where:}$$

We use minus because (dV/V) is a negative value. $* dP =$ change in the applied pressure
 $* dV =$ " " " resulting volume
 $* V =$ Original Volume

$$dV = V_2 - V_1 = -$$

$* E_V =$ Bulk modulus of Elasticity

Example E_V of water = $2.2 \times 10^9 \text{ N/m}^2$ (GPa):

To change water volume by 0.05%, we need to apply a pressure of :-

$$1 \text{ N/m}^2 = 10^6 \text{ N/m}^2$$

E_V unit is Pascal

$$2.2 \times 10^9 \text{ N/m}^2 \times \frac{0.05}{100}$$

$$= 1.1 \times 10^6 \text{ N/m}^2$$

assume that the total $V = 1 \text{ m}^3 = 100\%$

$$= 1.1 \text{ Mega Pascal}$$

For gases:

$$M = P \cdot V = \text{constant} = \text{mass}$$

$$dM = P dV + V dP = 0 \quad ; \quad (dm = 0, \text{ because } m = \text{const})$$

$$P dV = -V dP \quad (\text{inverse relationship}).$$

$$\frac{dV}{V} = - \frac{dP}{P} \quad ; \quad E_V = - \frac{dP}{(dV/V)}$$

$$= + \frac{dP}{(dP/P)}$$

$\Rightarrow$ to be
continued

For ideal Gas law:

$$PV = nRT$$

; if the process is iso-
thermal

$$P dV + V dP = nR dT$$

$$P dV + V dP = 0$$

$$E_V = - \frac{dP}{-dP/P} = P$$

$\Rightarrow$

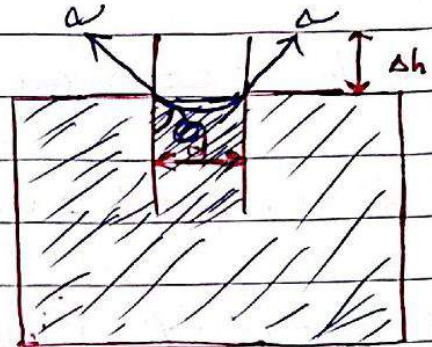
$$P dV = -V dP \Rightarrow \frac{dV}{V}$$

$$= - \frac{dP}{P}$$

$E_V = P$ For ideal gas
law if the process
is isothermal

Surface tension = (التوتر السطحي)

It is the result of molecular attraction near a free surface causing the surface to act like a stretched membrane.



σ unit is N/m

كلما قل قطر، زاد ارتفاع

$$\sum F_y = 0 \quad (m\ddot{a})$$

$$F_{\sigma} - W = 0$$

$$F_{\sigma} = W$$

$$\sigma \cos \theta \pi d = mg, \quad p \text{ (حجم، الكثافة)}$$

$$\sigma \cos \theta = \rho h g$$

$$\sigma \cos \theta (\pi d) = \rho A \Delta h g$$

$$\sigma \cos \theta \pi d = \rho \frac{\pi d^2}{4} \Delta h g$$

$$\Delta h = \frac{4 \sigma \cos \theta}{\rho g d} = \boxed{\frac{4 \sigma \cos \theta}{\rho g d}} \quad ; \quad \cos \theta \approx 1$$

كلما قل قطر، زاد ارتفاع

Vapor Pressure: The vapor pressure @ given temp is the pressure @ which liquid boils.

$$P_{atm} = 101.325 \text{ kPa}, \quad [P] = \text{Pa, kPa, MPa, Bar, Psi}$$

Homework ; chapter (2) :- 10th edition:-

2.59 ; 2.38 ; 2.36 ; 2.31 ; 2.14 . (5 - problems).

* التَّسْلِيمُ فِي رُكُومِ الدَّعَاةِ (2014 / 10 / 11)

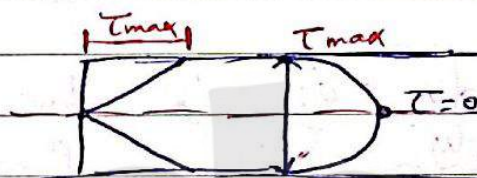
* 10 / 11 ، 10 / 13 ، 10 / 15 ، 10 / 17 ، 10 / 19 ، 10 / 21 ، 10 / 23 ، 10 / 25 ، 10 / 27 ، 10 / 29 ، 10 / 31



Examples $U = 100y(0.1 - y)$

$\tau = ??$

((نَسْتَعِينُ))



$$\tau = M \frac{du}{dy} \quad ; \quad = M [10 - 200y]$$

$$\tau|_{y=0}$$

Notices

$$\text{Torque} = F \cdot r = T$$

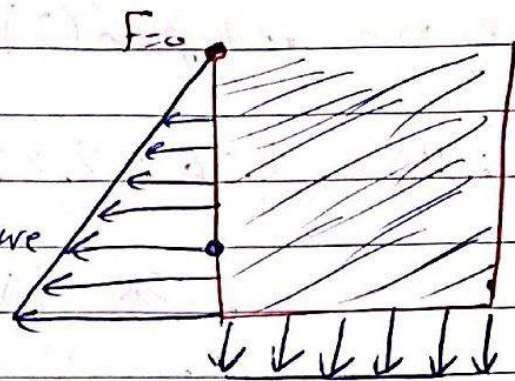
$$T = M \frac{du}{dy}$$

$$\frac{F}{A} = M \frac{du}{dy}$$

Fluid in Statics:-

.(1/3) center of pressure

$F = \text{max}$



Defn it is the study of properties when the relative motion is zero!!

$$\frac{dV}{d\gamma} = 0$$

; For this case, only normal Pressure force exist.

For fluid in Motion $\Rightarrow \frac{dV}{d\gamma} \neq 0$

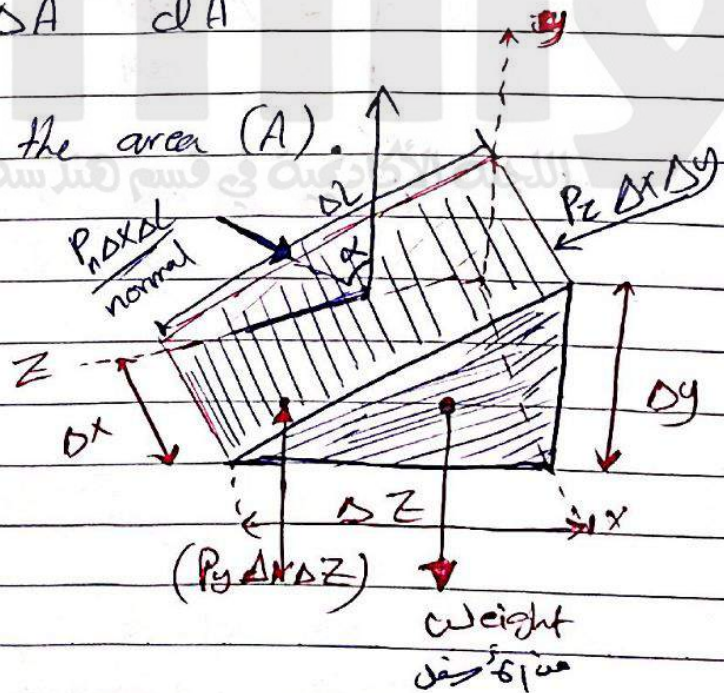
Two types of forces exist:-

1. Pressure or normal forces.
2. Shear forces.

$$\# P = \frac{\text{Force}}{\text{Area}} = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}, \text{ where if } =$$

Normal force acting on the area (A).

$$Pgh$$



$$\rho_{air} = 1.2 \text{ kg/m}^3$$

$$\rho_{H_2O} = 1000 \text{ kg/m}^3$$

$$\Rightarrow P_z \Delta x \Delta y = P_n \sin \alpha \Delta x \Delta L$$

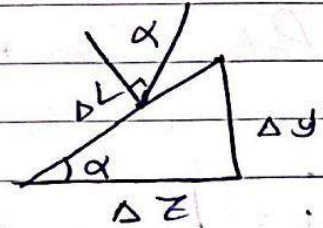
$$P_z = \Delta x \Delta L \sin \alpha = P_n \sin \alpha \Delta x \Delta L$$

$$P_z = P_n \cos \alpha$$

z-direction
 $\Sigma F_z = 0$

y-direction:-

$$\sin \alpha = \frac{\Delta y}{\Delta z}$$



$$\Sigma F_y = 0$$

$$P_y \Delta x \Delta z = W + P_n \Delta x \Delta L \cos \alpha$$

$$\# W = mg$$

$$= \rho V g$$

$$= \rho g \left(\frac{\Delta x \Delta y \Delta z}{2} \right)$$



implies

$$P_y \Delta x \Delta z = \frac{1}{2} \rho g \Delta x \Delta y \Delta z + P_n \Delta x \Delta L \cos \alpha$$

$$P_y = \frac{1}{2} \rho g \Delta y + P_n \cos \alpha ; \text{ if } \Delta L \rightarrow 0$$

$$P_y = P_n$$

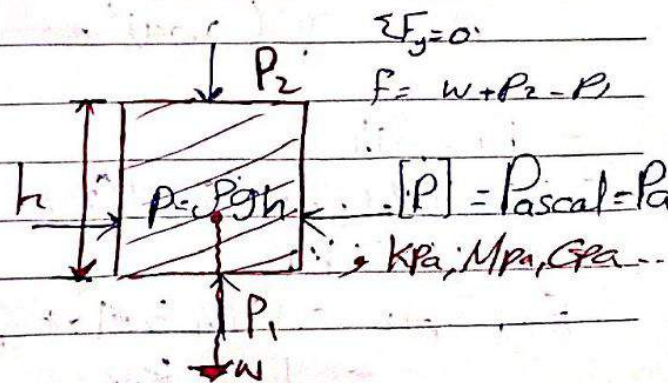
In fluid in Statics, pressure is equal in all directions.

$$P = \frac{F}{A} = \frac{W}{A}$$

$$= \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho \rho h g}{A}$$

$$= \rho g h$$

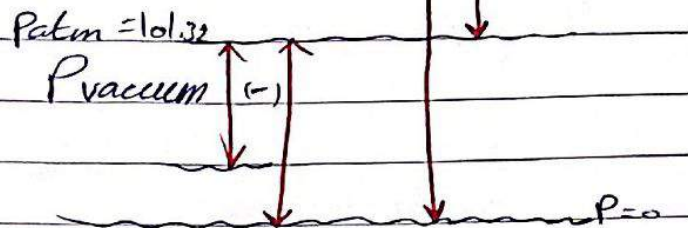
①



$$\Rightarrow 1 \text{ atm} = 101.325 \text{ kPa} = 760 \text{ mm Hg } P_{\text{abs}}$$

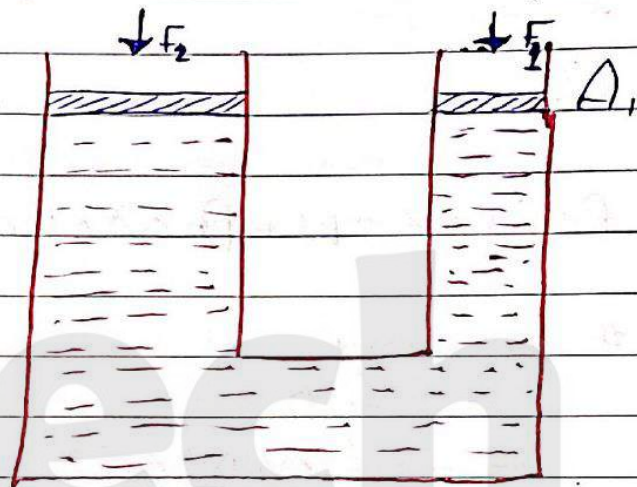
$$1 \text{ bar} = 10^5 \text{ Pa}$$

$$[P] = 14.7 \text{ Psi}$$



$$\# P_{\text{abs}} = P_{\text{atm}} + P_{\text{gage}}$$

$$F_1 = \frac{P_1}{A_1} \cdot A_2, \quad F_2 = \frac{P_2}{A_1} \cdot A_1$$



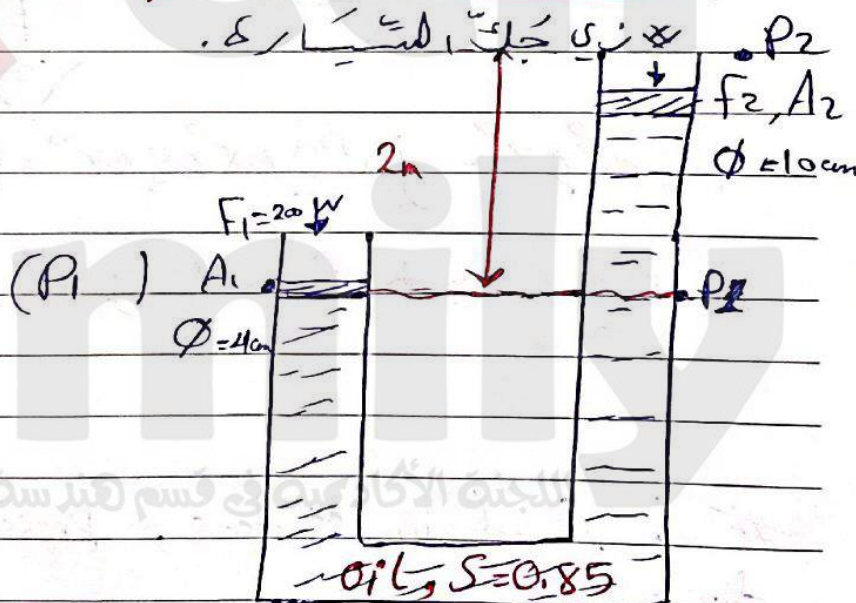
@ same elevation, $P_1 = P_2$

$$\frac{F_1}{F_2} = \frac{A_1}{A_2}$$

Prob (3.17) e

$$P_1 = P_2 + \rho g h$$

$$P_2 = P_1 - \rho g h$$



$$P_1 = \frac{F_1}{A_1} = \frac{200 \text{ N}}{\frac{\pi (0.04)^2}{4} \text{ m}^2}$$

not same elevation

$S = \text{specific gravity}$

$$= P_1 - \rho g h$$

$$= 159.15 \text{ kPa} - (850)(9.81)(2)$$

$$= 142.5 \text{ kPa}$$

$$S = \frac{\rho_{\text{oil}}}{\rho_{\text{H}_2\text{O}}} = \frac{0.85(1000)}{1000}$$

$$\rho_{\text{H}_2\text{O}} = 1000 \text{ kg/m}^3$$

$$F = P_2 A_2 = 142.5 \times \left(\frac{\pi}{4} (6.1)^2 \right)$$

$$= 1.4 \text{ kN}$$

Variation of Temperature and Pressure in the Atmosphere:

Troposphere:

- From sea-level (Zero), to 10,769 m.

10,000

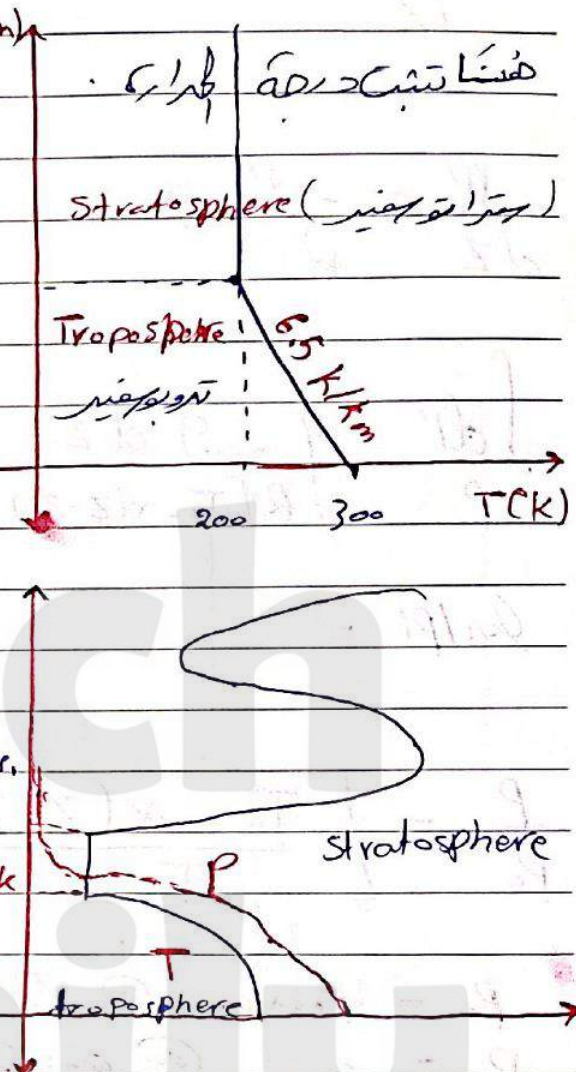
- For this layer, the temperature decreases @ a rate of 6.5 K/km

- Real atmosphere: $0 \rightarrow 7600 \text{ m}$ @ the poles.

1800 m @ the equator. 30 K

- equator: 30°C

10 K



Stratosphere:

- From $Z = 10,769 \text{ m} \rightarrow 32.3 \text{ km}$
- for this layer, T is constant.

- Pressure variation in the Troposphere:-

$$T = T_0 - \alpha(Z - Z_0)$$

$\rightarrow$ Temp @ Zero

α : Given.

$$\Rightarrow \frac{dp}{dz} = -\rho g$$

$$P \cdot V = nRT$$

$$\frac{dp}{dz} = -\frac{P}{RT} g$$

$$P = \frac{nRT}{V}$$

$$\int \frac{dp}{P} = \int -\frac{g dz}{R(T_0 - \alpha(z - z_0))}$$

$$P = PRT$$

$$P = \frac{P}{RT}$$

const

$$\ln(P) =$$

$$\frac{P}{P_0} = \left[\frac{T_0 - \alpha(z - z_0)}{T_0} \right]^{g/\alpha R}$$

$P_0 = \text{Pressure @ } z=0$, $T_0 = \text{Temp @ } z=0$
 $P = \text{ " @ any } (z)$, $T = \text{ " @ any } (z)$

Pressure Variation in the Stratospheres

$$\frac{dp}{dz} = -\rho g, \quad \rho = \frac{P}{RT}$$

$$= -\frac{P}{RT} \cdot g, \quad \text{+ , } R, g = \text{Constant.}$$

$$\int \frac{dp}{P} = \int -\frac{g dz}{RT} \Rightarrow \ln P = -\frac{g z}{RT} + C$$

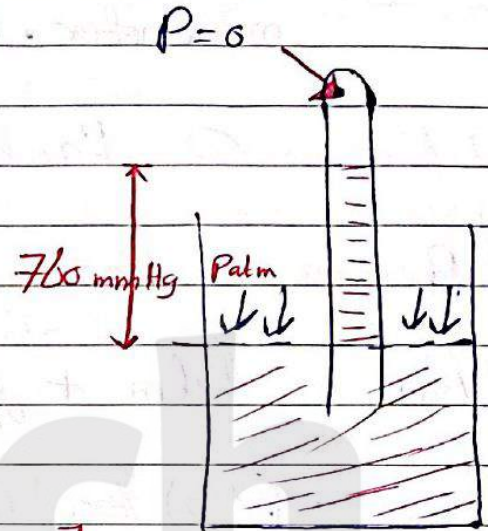
①

$$\frac{P}{P_0} = \exp \left[\frac{-(z - z_0)g}{RT} \right]$$

exponential

Pressure Measurements :-

- i) Value of the measured Pressure.
- ii) Type of fluid.
- iii) Location of measured value.



① Manometer & Barometer.

② Pressure gage; [Burdon tube gage].

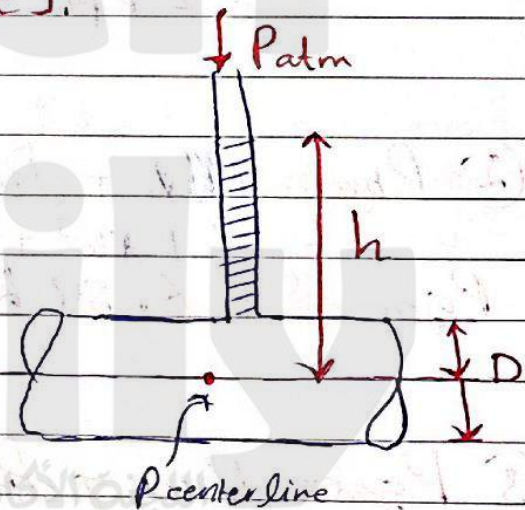
③ Pressure transducer.

Manometers

i) Simple manometer :-

$$P_{c.l} = P_{atm} + \rho g h$$

$$P_{c.l. (gage)} = \rho g h = \delta h$$

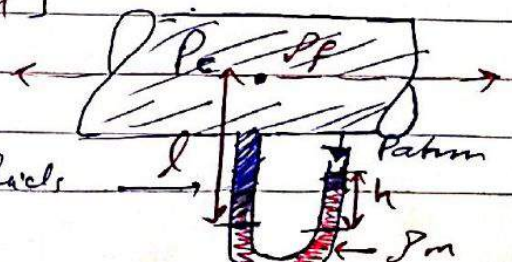


2) U-tube Manometer :- [Differential]

$$P_{c.l} + \rho g h = P_{atm} + \rho_m g h$$

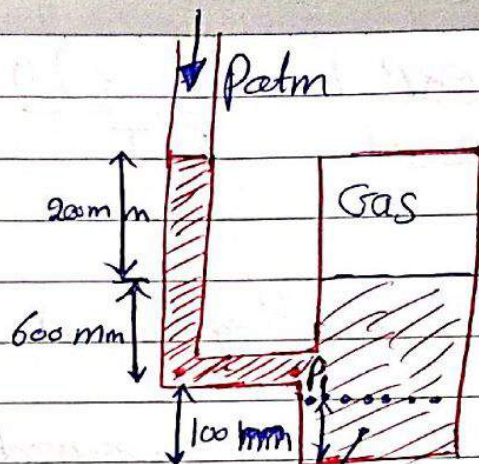
نوعان من المقاييس التفاضلية

diff. fluids



$$\Rightarrow P_{\text{rel (gage)}} = \gamma_m h - \gamma_f h$$

Examples Consider the shown manometer, Find:-



$\gamma = 0.78$
Kerosen

$$\rho_{\text{oil}} = \gamma \times \rho_{\text{H}_2\text{O}} = 780 \text{ kg/m}^3$$

$$\textcircled{1} P_{\text{entry}} = P_{\text{atm}} + \rho_K g (800)$$

$$P_{\text{abs}} = P_{\text{atm}} + P_{\text{gage}}$$

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}}$$

$$P_{\text{gage}} = (780)(1000)(0.8)(9.81) = 6.1 \text{ kPa}$$

$$\textcircled{2} P_{\text{base}} = P_{\text{atm}} + \rho_K g h$$

$$= 6.1 \text{ kPa} + 780(9.81)(0.1)$$

$$P_{\text{gage}} = 6.89 \text{ kPa}$$

gage air ni is

je Patm "is

... (air) "is

is) (air) "is

Pressure (kPa)

$$\textcircled{3} P_{\text{gas}} = P_{\text{atm}} + \rho_K g h$$

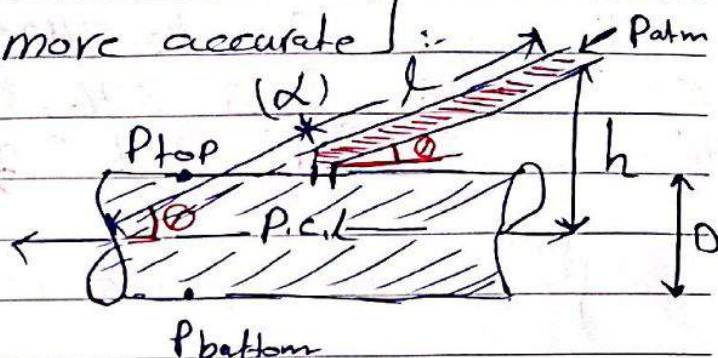
$$= P_{\text{atm}} + \rho_K g (6200)$$

$$P_{\text{gas (gage)}} = 1.53 \text{ kPa}$$

(ii) Inclined Manometer:- [more accurate] :-

$$P_{c.l. (gage)} = \gamma h = \gamma \alpha \sin \theta$$

$$P_{top} = P_{c.l.} - \gamma \frac{D}{2}$$



$$P_{bottom} = P_{c.l.} + \gamma \frac{D}{2}$$

α = from $l \rightarrow$ center line.

* We use ($P_{c.l.}$ & P_{top} & P_{bottom})

because we determine that $\sin \theta = \frac{h}{\alpha}$
The pressure is different from point to point in the same device.

* Example:-

$$D = 100 \text{ mm}$$

$$\alpha = 350 \text{ mm}$$

$$\theta = 30^\circ \text{ from horizontal}$$

$$\text{Fluid, water } \rho = 1000 \text{ kg/m}^3$$

$$* P_{c.l.} = \gamma h_{c.l.} = \gamma (\alpha \sin \theta)$$

$$= 9,810 \times (0.35) (\sin 30^\circ)$$

$$= 1.72 \text{ kPa}$$

* Find 1) $P_{c.l.}$

2) P_{top} , 3) P_{bottom}

* (عبر المركز) (عبر الأعلى)

$$* P_{top} = P_{c.l.} - \gamma \frac{D}{2}$$

$$= 1.72 \times 10^3 - 9,810 (0.1/2)$$

$$= 1.23 \text{ kPa}$$

$$* P_{bottom} = P_{c.l.} + \gamma \frac{D}{2}$$

$$= 1720 + 9,810 (0.1/2)$$

$$= 2.21 \text{ kPa}$$

Homework 8 - Due to 8/10/2014 - (#2);
 10th :- 3.15, 3.23, 3.26, 3.48, 3.45. (VIP Section)
 4th Probs: 3.11, 3.18, 3.21, 3.23, 3.30.

⇒ Hydrostatic forces on submerged Surfaces:-

① Plane Surface:-

- Horizontal Surface

② Curved Surface:-

- vertical

- incline surface

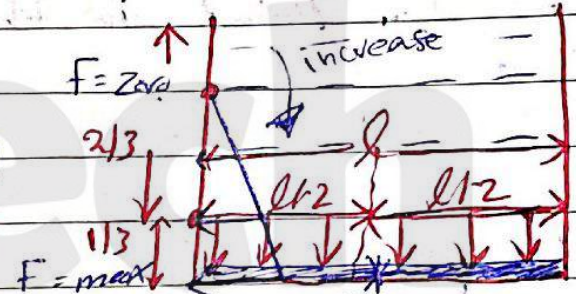
(1), Plane Surface:-

- Force on the vertical Surface

$$F = PA = phW = \rho g h^2 W$$

W = width (depth)

Zero



$$F = \bar{P} \cdot A$$

(centroid)

- Pressure

$$P = \rho g h$$

$$F = P_0 A$$

When F is Variable

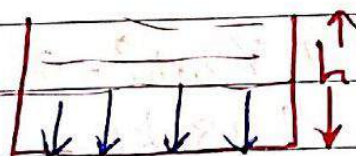
$$dF = dP \cdot A$$

$$= \rho g dh (h \times \text{width})$$

$$\int dF = \int \rho g h dh$$

$$F = \frac{\rho g h^2}{2} \Big|_0^W = \frac{\rho g H^2}{2}$$

* Horizontal plane Surfaces:-



- Pressure is Uniform (equal).

$$\text{Resultant Force} = \int P dA = \rho g h \int dA = \rho g h A$$

$$= h \times \rho g A$$

⇒ See the book for more clearness:-

$$dF = P dA \quad (\text{for a part}).$$

$$= \gamma (\underbrace{y \sin \alpha}_{\text{vertical distance}}) dA$$

Total force acting on the area --- :-

$$F = \int_A P dA = \int_A \gamma (y \sin \alpha) dA$$

$$F = \gamma \sin \alpha \int_A y dA$$

First moment of area $\bar{y} \cdot A$

$$F = \gamma \sin \alpha \bar{y} A$$

اذا كان سطح مائل، فإن مركزه الجاذبي (centroid) يقع على عمق $\bar{y}$ من السطح.

$$\# \bar{y} : \text{centroid} = \frac{\int y dA}{A}$$

$$\# \bar{x} : \frac{\int x dA}{A}$$

A = which effected by the fluid.

→ Pressure @ the centroid of the surface.

$$\# F = P \cdot A \rightarrow \text{Area of the Surface.}$$

↳ resultant of hydro static forces

* Vertical location of line of action of resultant Hydrostatic force:-

- This line lies below the centroid because the Pressure increases with depth. (y_{cp})

Let y_{cp} = y coordinate of the center of Pressure

⇒ Derived the Expression for y_{cp} :-

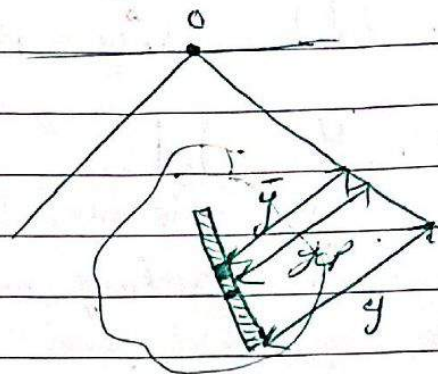
$$F \cdot y_{cp} = \int y \cdot df$$

$$= \int p y dA$$

$$= \int \rho (\sin \alpha) y dA$$

$$= \rho \sin \alpha \left\{ \int y^2 dA \right\} \Rightarrow \text{Second moment of Area (moment of inertia)}$$

$$\int y^2 dA = I_o$$



$$F \cdot y_{cp} = \rho \sin \alpha I_o, \text{ where } I_o = \bar{I} + \bar{y}^2 A$$

Parallel axis theorem

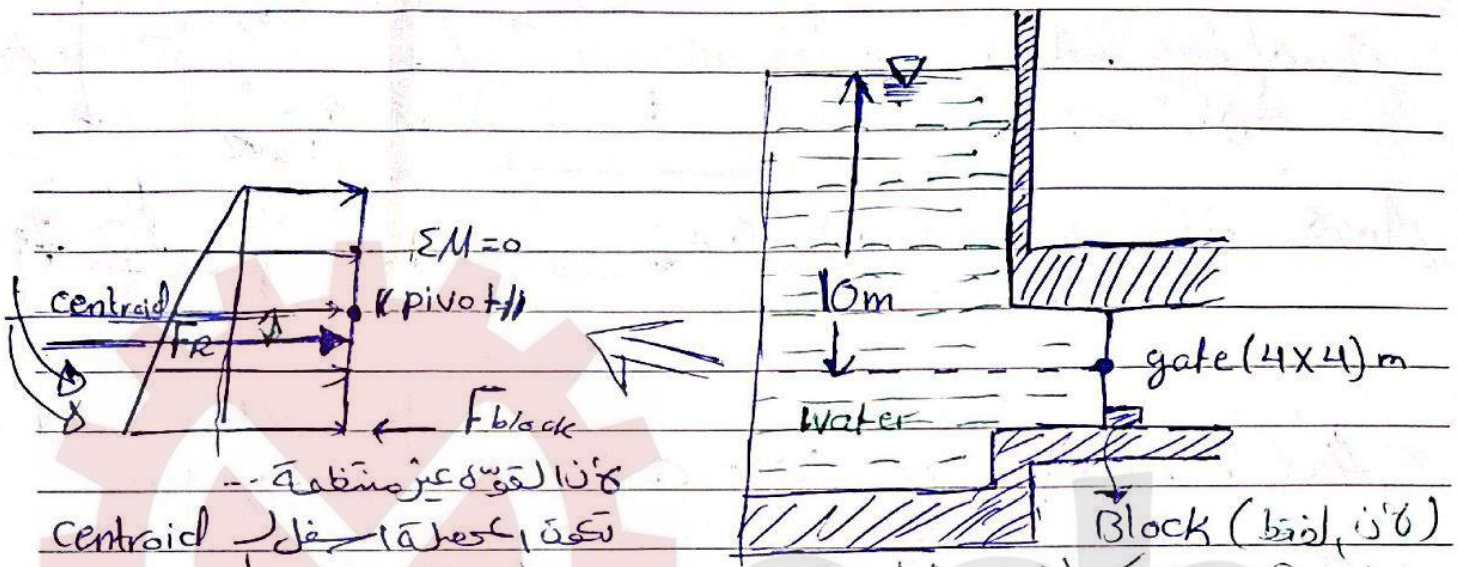
$$F \cdot y_{cp} = \rho \sin \alpha (\bar{I} + \bar{y}^2 A), \text{ But } F = \bar{p} \cdot A = \bar{y} \rho \sin \alpha A$$

$$(\bar{y} \rho \sin \alpha A) y_{cp} = \rho \sin \alpha (\bar{I} + \bar{y}^2 A) \quad \text{--- @ centroid of } (A)$$

$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y} A} \Rightarrow y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y} A} \Rightarrow \textcircled{1}$$

$$F = \bar{p} \cdot A \Rightarrow \textcircled{2}$$

Prob 3.70 :-



① Find the Force of the block. (Block رفع)

$$F_R = \bar{p} \cdot A$$

$$= \gamma \bar{y} \cdot A$$

$$= 9810 (10) (4 \times 4)$$

$$= 1.5696 \times 10^6 \text{ N}$$

* نأخذ مرجع عادي سطح المياه
حيث أن قوة سارية للمر

بوابة، مفصل، $\Sigma M = 0$, ΔH_{eng}

$$\textcircled{2} y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y} A}, \quad \bar{I} = \frac{bh^3}{12} = \frac{4 \times 4^3}{12}$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y} A}$$

$$y_{cp} = 10 + 0.133 = 10.133 \text{ m}$$

$$= \frac{4 \times 4^3}{12}$$

$$\Rightarrow 0.133 \text{ m}$$

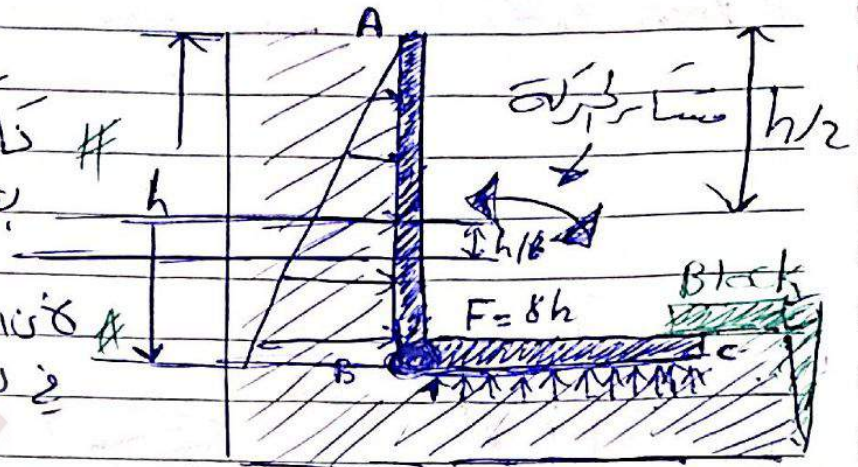
$$F_{\text{block}} = 104,378 \text{ N}$$

$$\Sigma M_{\text{pivot}} = 0, \quad F_{\text{block}} \times 2 = F_R \times 0.133$$

①

Example (خ، پ. پی)

ناخذ دالة المساحة المحورية
بفرض ينظر عند



كانا (نظرة من أعلى) (نظرة من أسفل)
في نفس السائل

Find h , when the gate automatically open.

Block من جهة اليمين، لتفتح البوابة، ونفس البوابة في (B)

$$\begin{aligned} \textcircled{1} F_{AB} &= \bar{P} \cdot A \\ &= \bar{P}_{AB} \cdot A_{AB} = 8 \left(\frac{h}{2} \right) (h \times 1) \\ &= \frac{8h^2}{2} \end{aligned}$$

centroid (Z) (القلب) (القلب)

(الفرضية = 1) (القلب)

$$\textcircled{2} F_{BC} = \bar{P}_{BC} \cdot A = 8h \cdot A_{BC} = 8h (4 \times 1)$$

في (CB)، $\bar{y} = \bar{y}(CP)$ ، لأن لفة متساوية على

$$\bar{y}_{CP} - \bar{y} = \frac{\bar{I}}{\bar{y}_A} = \frac{bh^3}{12 \cdot (h/2) (h \times b)} = \frac{b}{6}$$

$$\bar{I} = \frac{bh^3}{12}, \quad h = \text{elevation}, \quad b = \text{depth.}$$

$$\Rightarrow \Sigma M_B = 0, \quad F_{AB} (h/2 - h/6) = 48h(2)$$

$$\frac{8h^2}{2} (h/2 - h/6) = 48h(2) \quad \text{Be careful}$$

$$\frac{h}{2} \cdot \frac{h}{3} = 8, \quad h^2 = 48$$

$$\therefore h = \sqrt{48} \approx 6.93 \text{ ft}$$

Homework-(3), Due to Monday, 20/10/2014

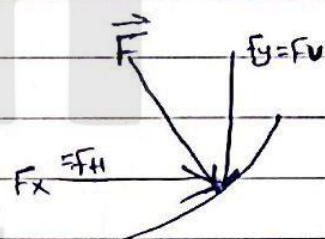
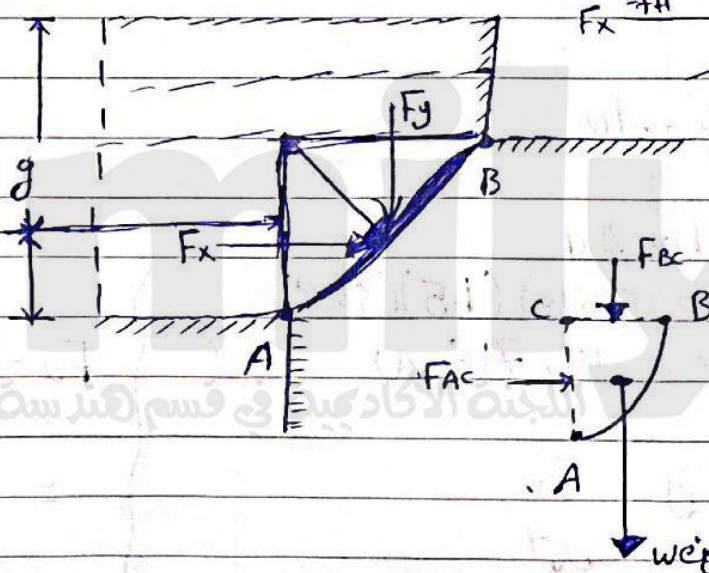
9th Edition:- 3.58, 3.63, 3.64, 3.67, 3.75

Hydrostatic Forces On Curved Surfaces :-

$$F_x = F_H = F_{Ac} = \bar{P}_v \cdot A_v$$

((Centroid))

$$y_{cp} = \bar{y}_v = \frac{I}{\bar{y}_v A_v}$$



$$\# F_y = \gamma V = W$$

= Weight of the liquid above the surface AB

y = @ the centroid
adding ...

$$F = \sqrt{F_x^2 + F_y^2}$$

كامل القوى



ساحة السطح

$$\theta = \tan^{-1} \frac{F_y}{F_x}$$

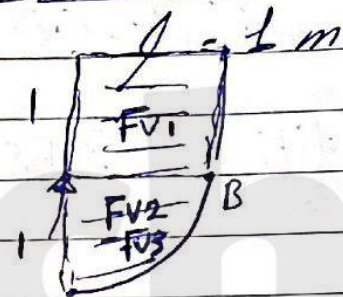
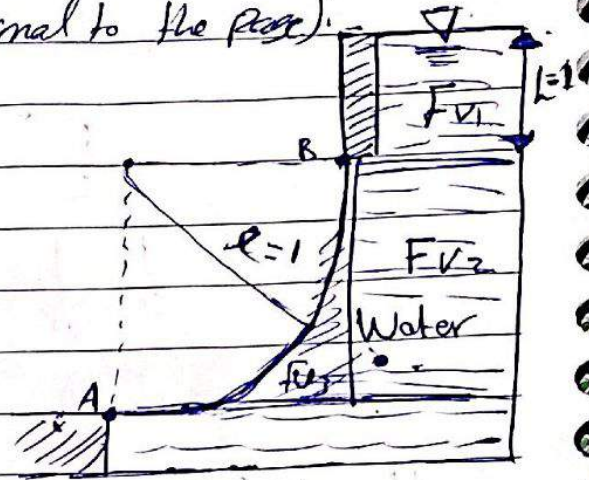
①

$$F = P A = \rho g h A = \gamma V$$

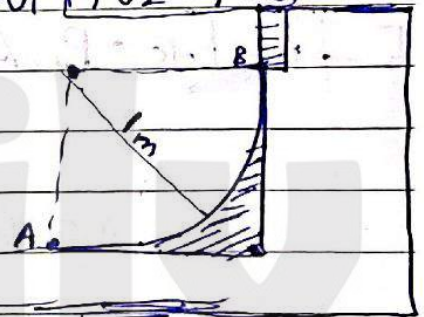
Surface is one meter long (normal to the page).
"Depth = 1m".

- F_v & line of action
- F_H , " " "
- F_R

line of action for 2 components
occurs between them.



$$F_v = F_{v1} + F_{v2} - F_{v3}$$



$$\textcircled{2} F_H = \bar{P}_c \cdot A_v$$

$$= \rho g h_v \cdot A_v$$

$$= 1000 (9.810) (1.5) (1 \times 1)$$

$$= 14,715 \text{ N}$$

$$y_{cp} = \bar{y} + \frac{I_v}{\bar{y} A}$$

$$= 1.5 + \frac{(1)(1)^3/12}{(1.5)(1 \times 1)}$$

$$= 1.555 \text{ m}$$

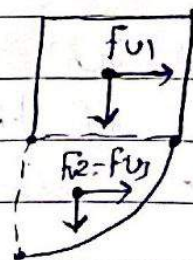
$$y_{cp} - \bar{y} = 0.555 \text{ m}$$

①

$$F_v = \delta V$$

$$= (9.810)(1 \times 1 \times 1) + \left(\frac{\pi}{4} \times 1^2 \times 1 \right)$$

$$= 19,515 \text{ N}$$



* Take Moment : then

$$F \cdot x = F_{v1} x_1 + F_{v2} x_2$$

$$x = \frac{F_{v1} x_1 + F_{v2} x_2}{F_v}$$

$$= 9810 (0.5) + \int_0^1 F_{v2-v3} dx$$

$$= 9810 (0.5) + 9,810 \int_0^1 \sqrt{1-x^2} x dx$$

$$\Rightarrow X = 0.467 \text{ m}$$

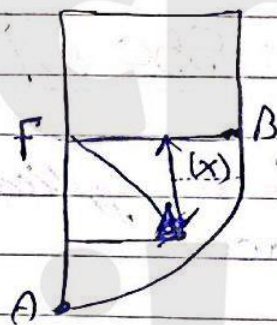


$$\theta = 40^\circ 2$$

$$F_R = \sqrt{14,745^2 + 17,515^2} = 22,878 \text{ N}$$

* integration Not in course.

X = distance of the resultant force.



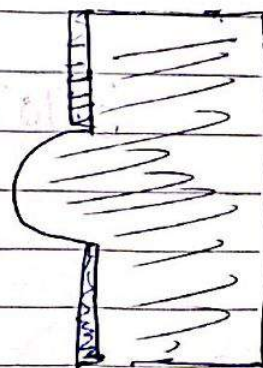
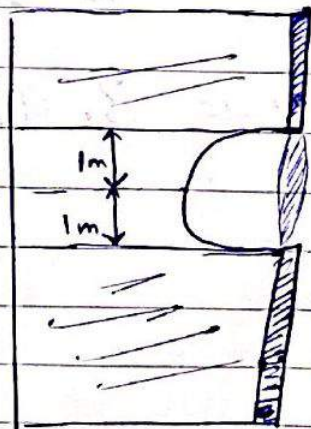
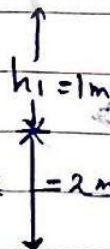
#Prob: (3.89)

① Hemispherical dome.

① F_v ,

② F_H

centroid



$$F_H = P_v \cdot A_v$$

$$= \rho g h_v \cdot A_v$$

$$= \rho g v \cdot A_v$$

$$v = 1 + 1 = 2 \text{ m}$$

①

$$A_v = \pi r^2 = \pi (1)^2 = \pi \text{ m}^2$$

$$F_x = 9,810 (2) (\pi) = 61,640 \text{ N} = 61.64 \text{ kN}$$

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}_v A_v}$$

$$\bar{I}_v = 2 \left(\frac{\pi r^4}{8} \right)$$

$$y_{cp} = 2 + \frac{\pi (1)^4}{4 (\pi) (2) (1)^2}$$

$$y_{cp} = \bar{y}_v = 0.125 \text{ m}$$

$$= 9810 \left(\frac{4 \pi r^3}{3} \right) \times \frac{1}{2}$$

(r=1)

$$= \boxed{20,550 \text{ N}} \Rightarrow 20.55 \text{ kN}$$

Force to be applied downward to hold the cone in place.

Homework (4) :- Due to 5/11/2014

Ninth Edition :-

3.107, 3.99, 3.93, 3.82, 3.77

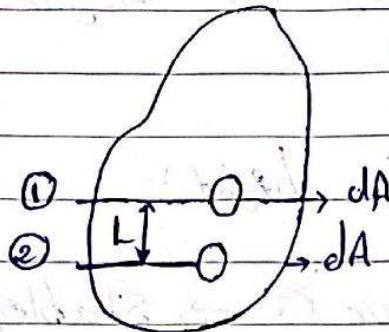
Difference in force between 1 & 2?

$$\Delta F = (P_2 - P_1) dA$$

$$= \rho_F h dA$$

$$= \rho_F dV$$

$$F = \rho_F V$$



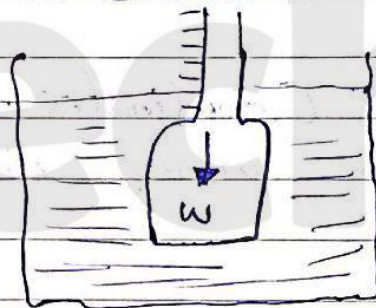
$$W = mg$$

$$= \rho g h$$

$$= \rho b V$$

Hydrometry:-

The hydrometer will float deeper or shallower depending on the specific weight of the liquid.



Stability of immersed bodies:-



Neutral



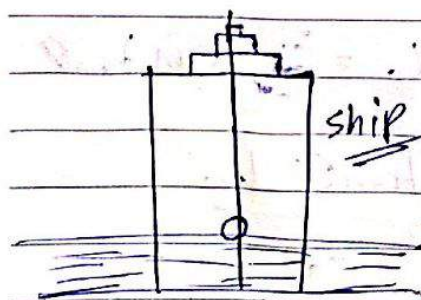
Stable



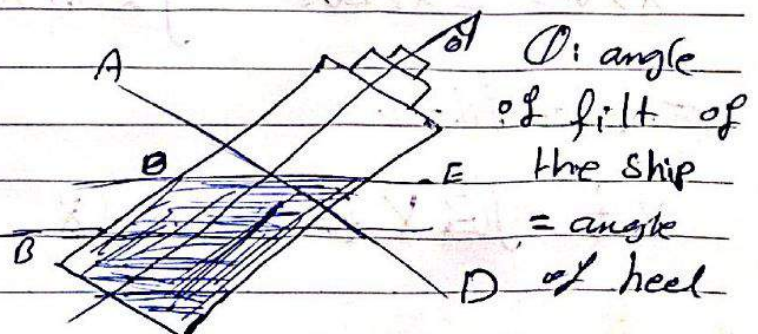
unstable

C = Center of buoyancy
G = Center of gravity

Stability of Floating bodies:-



①



θ : angle of tilt of the ship
= angle of heel

M: metacenter: the point of intersection of the line of the action of the buoyancy force before and after heel (tilt).

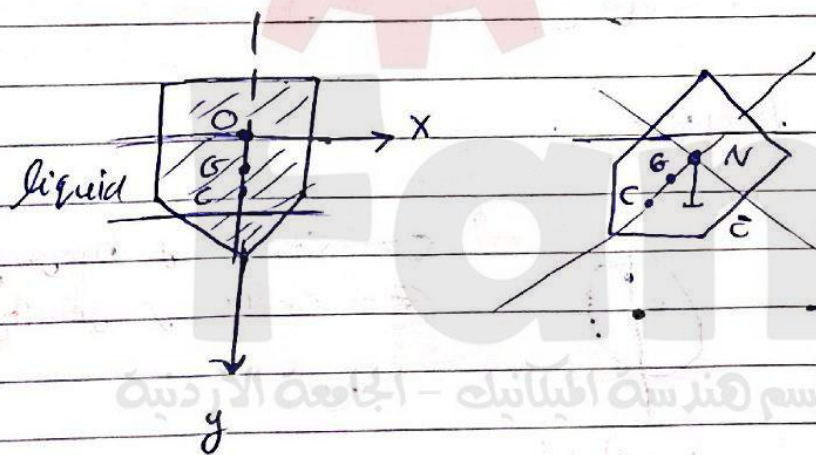
GM = Metacentric height :-

If M is above G $\Rightarrow$ Stable, i.e. GM is positive.

If M is below G $\Rightarrow$ unstable, i.e. GM is Negative.

Calculation of metacenter height CM

- Select Coordinates through the origin O.
- axis are moving with the ship.



before tilt :-

$$X_2 \bar{V} = \sum X_2 \Delta V_2 = \int z dA X_2$$

After tilt :-

$$X_2 \bar{V} = \int (z + X \tan \phi) dA dX$$

$$\int (X - X_0) = \tan \phi \int x^2 dA$$

I_{00} = Second Moment of Inertia

$$X - X_0 = \bar{C} = \bar{C}M \cdot \tan \phi$$

$$= \frac{\tan \phi I_{00}}{\bar{C}}$$

$$\bar{C}M \tan \theta = \frac{\tan \theta I_{oo}}{\bar{A}}$$

$$\bar{C}M = \frac{I_{oo}}{\bar{A}}, \text{ But } \bar{C}M = \bar{C}M - \bar{C}\bar{G}, \text{ So } \bar{C}\bar{G} = \frac{I_{oo}}{\bar{A}}$$

$$\boxed{\bar{C}M = \bar{C}M - \bar{C}\bar{G}} \quad \& \quad \bar{C}M = \frac{I_{oo}}{\bar{A}} - \bar{C}\bar{G}$$

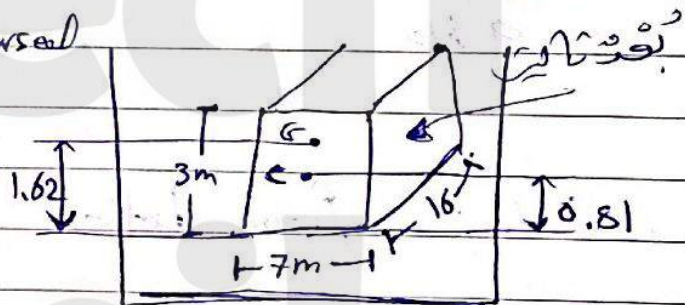
Problem (3.103)

Weight = Buoyancy force

$$1,780,000 = 98100 \times V_{\text{immersed}}$$

$$1,780,000 = 9810 (7 \times 16 \times h)$$

$$h = 1.62 \text{ m}$$



$$W = 1,780,000 \text{ N}$$

$$\bar{C}\bar{G} = 3 - 0.81 = 2.19 \text{ m}$$

$$\bar{A} = (16 \times 7 \times 1.62) = 181.4 \text{ m}^3$$

$$I_{oo} = \frac{bh^3}{12} = \frac{7 \times (16)^3}{12} = 15,775$$

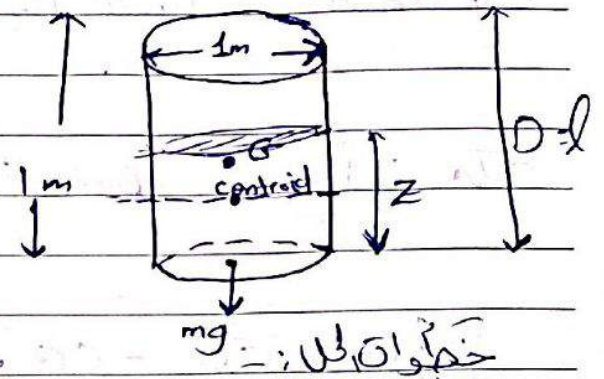
$$\bar{C}\bar{M} = \frac{15,775}{181.4} - 2.19 = 2.913$$

Example 8

$$\# \text{ Diameter} = 1 \text{ m}$$

$$\# \text{ length} = 1 \text{ m}$$

$$\# \rho_{\text{wood}} = 8000 \frac{\text{N}}{\text{m}^3} = (8)$$

Solutions

$$\pi R^2 L (8000) = \pi R^2 (z) (9,810)$$

center of gravity (9,810)
(3)

$$z = \frac{8000 \times 1}{9,810} = 0.8155 \text{ meter} \quad [\text{الجزء المفقود}]$$

$$\# \text{ Centroid from bottom} = \frac{0.8155}{2} = 0.4077 \text{ m}$$

$$\# \text{ Gravity from bottom} = 0.5$$

$$\# CG = 0.5 - 0.4077 \quad (\text{centroid of G من المركز})$$

$$\# BM = CM - CG \quad \left\{ \begin{array}{l} GM = 0.077 - 0.0922 \\ \Rightarrow \text{Negative} \\ \Rightarrow \text{Unstable} \end{array} \right.$$

$$= \frac{I_{oo}}{V} - CG$$

$$CM = \frac{I_{oo}}{V} = \frac{(\pi R^4 / 4)}{\pi R^2 (0.8)} \rightarrow (\text{المساحة المفقودة})$$

$$= 0.077 \text{ m}$$

$\therefore$ The block is unstable with axis vertical.

Chapter 4

الأسئلة / 50

Fluids in Motion

• Classification of Flow:-

- Uniform Flow $\Rightarrow$ The velocity at a given instant is the same in magnitude and direction at every point in the field ($\partial V / \partial s = 0$)



- Non uniform Flow $\Rightarrow$ the velocity at a given instant changes from point to point ($\partial V / \partial s \neq 0$)

- Steady Flow $\Rightarrow$ it is one in which the velocity, pressure and cross section of the stream may vary from point to point but do not change with time, ($\frac{\partial V}{\partial t} = 0$)

- Unsteady Flow $\Rightarrow$ if at a given point, condition do change with time, ($\frac{\partial V}{\partial t} \neq 0$)

• Therefore, there are 4 possible type of Flow

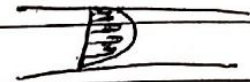
- 1) steady uniform Flow
- 2) steady nonuniform Flow
- 3) unsteady uniform Flow
- 4) unsteady nonuniform Flow

• Laminar and Turbulent Flow

- Laminar flow has a very smooth appearance

② Flowing in layer above each other

③ No mixing between layers



- Turbulent: ① Violent motion

② eddy motion

③ High mixing



- Transitions Flow: Flow between laminar and Turbulent

- Reynolds Number: $[Re]$ مقدار جريان

$$Re = \frac{\text{Inertia Force (قوة الدفع)}}{\text{Viscous Force (قوة اللزوجة, الاحتكاك)}}$$

$$= \frac{\rho V L_c}{\mu}$$

where:- ρ = fluid density

V = fluid velocity

μ = fluid viscosity

L_c = characteristic length

(d) if the flow passage is Tube

(L) if the flow passage is Flat plate

$$= \frac{V L_c}{\nu}$$

- For tube flow: laminar flow if $Re \leq 2300$

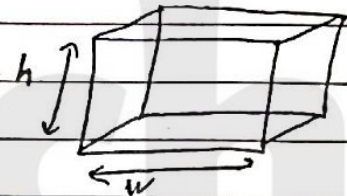
For plate flow: laminar flow if $Re \leq 5 \times 10^5$

• Define Hydraulic Diameter: D_H

$$D_H = \frac{4 \text{ Flow area}}{\text{Perimeter (length)}}$$

(Re) critical (L_c) for laminar flow is $Re \leq 2300$ for all flow as in diagrams

$$D_H = \frac{4A}{P} = \frac{4(H \times W)}{2(W + H)} = \frac{2HW}{W + H}$$



• Real and ideal fluid $\Rightarrow$

- ideal fluid: No viscosity (No shear stress)

shear stress is developed between layers of fluid moving with different flow velocities



ideal

Boundary layer



Real

- outside the B.L ($y \geq 0.99U_m$), in a real fluid, the effect of shear stresses due to the Boundary can be ignored and the fluid can be treated as if it were an ideal fluid.

• Compressible and incompressible :

$$\text{Mach Number} = M_a = \frac{V}{C}$$

if $M_a < 0.3$ incompressible

- all fluids are compressible, so that their density will change with pressure
- incompressible / $P = \text{const}$ (liquid under steady flow)
- compressible / $P \neq \text{const}$ (Gases are compressible except when the changes of pressure, and therefore density are very small)

• One - two and three - Dimensional Flow :

1-D : if the parameters such as velocity, pressure describing the flow at a given instant vary only along the direction of the flow.

2-D : The flow parameters may vary in the direction of flow and in one direction at right angles

• Rotational and irrotational flow:-

- Rotational flow: Fluid parameters rotate about its center as it flows

- Irrotational flow: Fluid parameters does not rotate about its center as it flows.

• Velocity:-

$$V = u\hat{i} + v\hat{j} + w\hat{k}$$

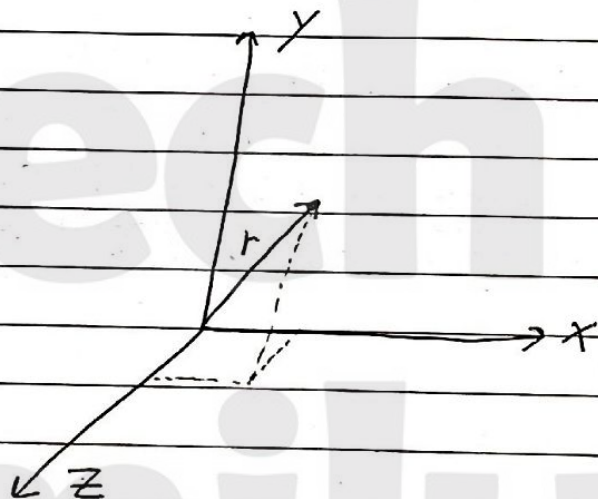
$$V = V_r\hat{e}_r + V_\theta\hat{e}_\theta + V_z\hat{k}$$

In general

$$u = f(x, y, z, t)$$

$$v = f(x, y, z, t)$$

$$w = f(x, y, z, t)$$



• Two approaches exist to describe V

a) Lagrangian Approach

Select a fluid particle and follow the motion of this particle. At any instant:

$$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{V}(t) = \frac{\partial \vec{r}}{\partial t} = \frac{\partial x}{\partial t}\hat{i} + \frac{\partial y}{\partial t}\hat{j} + \frac{\partial z}{\partial t}\hat{k}$$

b) Eulerian Approach

select any point (x, y, z) in the flow field and observe the fluid flow passing through that point

$$u = f(x, y, z, t)$$

$$v = f(x, y, z, t)$$

$$w = f(x, y, z, t)$$

$$V = \sqrt{u^2 + v^2 + w^2}$$

• Streamlines and Flow Patterns

Construction of these lines to indicate the speed and direction of flow is called Flow Patterns.
streamline, streakline, path line.

- Streamline: is a line that is everywhere tangent to the velocity field, it is often used in Analytical Approach

For 2-D flow: $\frac{dy}{dx} = \frac{v}{u} = \frac{y \cdot \frac{d\psi}{dy}}{x \cdot \frac{d\psi}{dx}}$

- Streakline: it consists of all particles in a flow that have previously passed through a common point, it is more of a laboratory tool than an analytical tool.

- Pathline: is the line traced out by a given particle as it flows from one point to another, The pathline is a Lagrangian concept that can be produced in a laboratory.

⇒ Note: For steady flow ⇒ pathlines, streaklines and streamlines are the same.

• Methods for developing flow patterns:

Analytical, Numerical and experimental used to produce flow patterns.

• Acceleration

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$u = f(x, y, z, t)$$

$$v = f(x, y, z, t)$$

$$w = f(x, y, z, t)$$

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial t} \hat{i} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial t} \hat{j} + \frac{\partial u}{\partial z} \frac{\partial z}{\partial t} \hat{k} + \frac{\partial u}{\partial t} \frac{\partial t}{\partial t}$$

$$\vec{a} = u \frac{\partial u}{\partial x} \hat{i} + v \frac{\partial u}{\partial y} \hat{j} + w \frac{\partial u}{\partial z} \hat{k} + \frac{\partial u}{\partial t}$$

$$a_x = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial t}$$

$$= u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = \frac{\partial v}{\partial t} = \frac{\partial v}{\partial t}$$

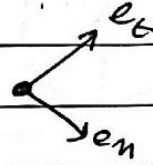
$$= \frac{\partial v}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial v}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial v}{\partial z} \frac{\partial z}{\partial t} + \frac{\partial v}{\partial t} \frac{\partial t}{\partial t}$$

$$= u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = \frac{\partial w}{\partial t} = \frac{\partial w}{\partial t} = u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

• Normal and Tangential Components

$$V = V(s, t) \mathbf{e}_t$$



$$a = \frac{\partial V}{\partial t} = \left(\frac{\partial V}{\partial t} \right) \mathbf{e}_t + V \frac{d\mathbf{e}_t}{dt}$$

$$a_t = \frac{\partial V}{\partial t} = \frac{\partial V}{\partial s} \frac{ds}{dt} + \frac{\partial V}{\partial t}$$

$$a_t = V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$$

$$a = a_t \mathbf{e}_t + a_n \mathbf{e}_n$$

$$= \left(V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right) \mathbf{e}_t + \frac{V^2}{r} \mathbf{e}_n$$

• Local acceleration (تسارع موضعي)

$$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$$

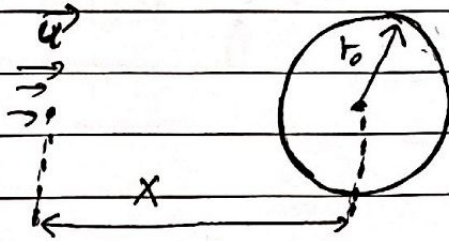
• Convective Acceleration

$$u \frac{\partial u}{\partial x}, v \frac{\partial v}{\partial x}$$

- Example

$$u = -u_0 \left(1 - \frac{r_0^3}{x^3}\right)$$

$$u_x = ??$$



Sol:-

$$u_x = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$= u \frac{\partial u}{\partial x} = -u_0 \left[1 - \frac{r_0^3}{x^3}\right] \cdot \left[-3u_0 \frac{r_0^3}{x^4}\right]$$

$$= 3u_0 \left(1 - \frac{r_0^3}{x^3}\right) \left(\frac{u_0 r_0^3}{x^4}\right)$$

• Flow rate (discharge)

$$\dot{V} = \frac{\partial V}{\partial t} = \frac{\partial (L \cdot A)}{\partial t}$$

$$\dot{V} = Q = VA$$

$$\vec{Q} = \vec{A} \cdot \vec{v}$$

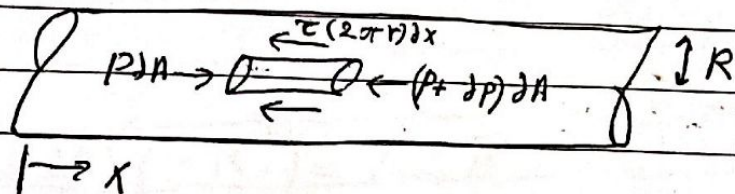
$$Q = \int \delta Q = \int_A v \delta A$$

$$\dot{m} = \rho Q = \rho VA$$

Volume Flow Rate	إذا كانت
$Q = \dot{V} = A \cdot V$	السرعة المتوسطة
Mass Flow Rate	
$\rho Q = \rho VA$	

إذا كانت السرعة متغيرة (السرعة لا تساوي صفرًا)

$$\dot{m} = \rho \sum_{i=1}^n v_i \delta A_i$$



$$\sum F_x = 0$$

$$p(\pi r^2) - (p + dp)\pi r^2 - \tau(2\pi r)dx = 0$$

$$-dp\pi r^2 - \tau(2\pi r)dx = 0$$

$$-\frac{dp}{dx}r = 2\tau$$

$$\tau = \mu \frac{\partial v}{\partial r}$$

$$-\frac{dp}{dx}r = 2\mu \frac{\partial v}{\partial r}$$

$$\text{at } r=R, v=0$$

$$\left(\frac{-dp}{dx}\right) \frac{1}{2\mu} r dr = dv$$

$$v = \frac{1}{2\mu} \left(\frac{-dp}{dx}\right) \left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$v = \frac{1}{4\mu} \left(\frac{-dp}{dx}\right) (r^2 - R^2)$$

$$Q = \int dQ = \int v dA = \int_{r=R}^{r=0} \frac{1}{4\mu} \left(\frac{-dp}{dx}\right) (r^2 - R^2) 2\pi r dr$$

$$Q = \frac{\pi}{24} \left(\frac{-dP}{dx} \right) \left(\frac{-R^4}{4} \right) = V_{avg} A$$

$$V_{max} = 2 V_{avg}$$

$$\bullet V_{avg} = \frac{1}{2} V_{max} \text{ (laminar)}$$

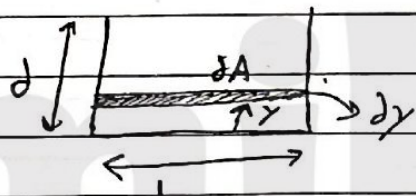


$$\bullet V_{avg} = \frac{3}{4} V_{max} \text{ (Turbulent)}$$



- Example:-

$$u = U_{max} \left(\frac{y}{\delta} \right)^n$$



$$U_{max} = 3 \text{ m/s}$$

$$n = 1/6$$

$$\delta = 1.2 \text{ m}$$

$$Q = ??, \quad m = ??$$

$$Q = V_{avg} \cdot A$$

$$V_{avg} = \frac{Q}{A}$$

$$Q = \int_{y=0}^{\delta} u \, dA$$

$$= \int_{y=0}^{\delta} U_{max} \left(\frac{y}{\delta} \right)^n dy \cdot 1$$

$$= \frac{3.09}{1.2 \times 1.2} = 2.57 \text{ m/s}$$

$$= \int_0^{1.2} 3 \left(\frac{y}{1.2} \right)^{1/6} dy = 3.09 \text{ m/s}$$

$$m = \rho \cdot Q$$

Conservation principles:-

1) Mass conservation

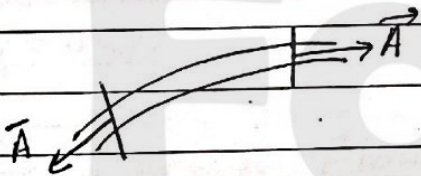
Continuity equation (1-eqn)

2) Momentum conservation

Navier-Stokes eqs. (3-eqns)

3) Conservation of energy (1-eqn)

-Area is considered as a vector $\vec{A}$ where it's always pointing out of the Control Volume



• Continuity eqn.

$$Q_1 = V_1 A_1$$

$$Q_1 = Q_2$$

$$V_1 A_1 = V_2 A_2$$

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

- Example:

$$\rho_1 = 20 \text{ kg/m}^3$$

$$D = 1.2 \text{ m}$$

$$V_1 = 15 \text{ m/s}$$

$$\rho_2 = 1.5 \text{ kg/m}^3$$

$$d = 0.6 \text{ m}$$

$$V_2 = ?? \text{ (2)}$$

For a steady flow of gas in the conduit shown, what is V_2 ?

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

$$V_2 = \frac{\rho_1 V_1 A_1}{\rho_2 A_2} = \frac{20 \times 15 \times \frac{\pi}{4} (1.2)^2}{1.5 \times \frac{\pi}{4} (0.6)^2}$$

$$V_2 = 80 \text{ m/s}$$

- if the flow is unsteady, such that

Flow rate in $\neq$ Flow Rate out

$$-\frac{\partial}{\partial t} \int_{CV} \rho dV = \sum_{CS} \rho \vec{V} \cdot \vec{A}$$

or

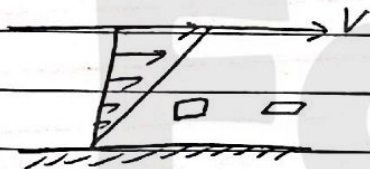
For steady Flow

$$\sum_{CS} \rho \vec{V} \cdot \vec{A} = 0$$

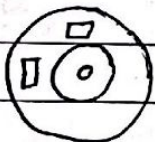
$$-\frac{\partial}{\partial t} \int_{CV} \rho dV = \sum_{CS} \rho \vec{V} \cdot \vec{A}$$

$$\frac{dM}{dt} = (\sum \dot{m}_c - \sum \dot{m}_p)$$

• Rotation of fluid &



Rotation of fluid element in a flow between moving and stationary parallel plate.



Rotation of fluid element in a rotating tank of fluid.

$$\Delta \theta_B = (\Delta y_B - \Delta y_A) / \Delta x$$

$$u = \frac{\partial x}{\partial t} \Rightarrow \Delta x = u \Delta t$$

$$v = \frac{\partial y}{\partial t} \Rightarrow \Delta y = v \Delta t$$

$$\Delta \theta_B = \frac{(v + \frac{\partial v}{\partial x} \Delta x - u) \Delta t}{\Delta x} = \frac{\partial v}{\partial x} \Delta t$$

$$\omega_{AB} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta_B}{\Delta t} = \text{Rate of Rotation of AB}$$

$$\omega_{AB} = \lim_{\Delta t \rightarrow 0} \frac{\frac{\partial v}{\partial x} \Delta t}{\Delta t} = \frac{\partial v}{\partial x}$$

$$\omega_{AC} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta_C}{\Delta t} = \text{Rate of Rotation of AC}$$

Similarity

$$\omega_{AC} = -\frac{\partial u}{\partial y}$$

$$\omega_{AZ} = \frac{\omega_{AB} + \omega_{AC}}{2} = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

Rate of Rotation about Z-axis

if Rotation about z-axis exist,

$$\omega_z \neq 0$$

such that $\frac{\partial v}{\partial x} \neq \frac{\partial u}{\partial y}$

The Rate of rotation about x and y-axis are-

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)$$

$$\omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)$$

$$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

$$= \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \hat{i} + \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \hat{j} + \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k}$$

$$\text{Vorticity} = \zeta = 2\omega$$

• Rotation of Fluid

$$\omega = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

$$= \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \hat{i} + \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \hat{j} + \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \hat{k}$$

• Vorticity $= \zeta = 2\vec{\omega}$ (observed)

• Two Types of vortex:-

1) Forced vortex

$V = K r$ → The distance from the center of flow
 ↳ const
 ↳ velocity
 To the point where the velocity is V_{max}

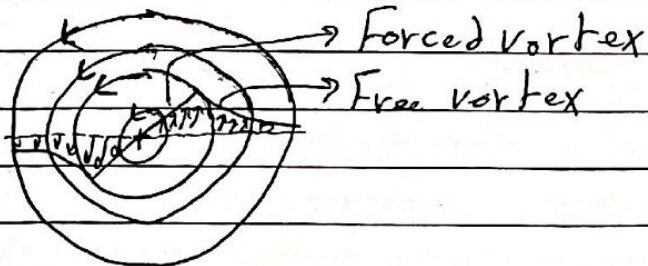
V increases with r

2) Free (Natural) vortex

$$V = \frac{k_2}{r}$$

V decreases with r

Tornadoes and whirlpools are often a combination of Free and Forced Vortex



• Continuity eqn

$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$$

- special case ($\rho = \text{const}$)

$$\frac{\partial (\rho u_i)}{\partial x_i} + \frac{\partial \rho}{\partial t} = 0 \quad i=1, 2, 3$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u_i}{\partial x_i} = 0, \quad i=1, 2, 3$$

- Ex: The velocity component for 2D flow are given by

$$u = -\frac{x}{x^2 + y^2}, \quad v = -\frac{y}{x^2 + y^2}$$

① Is the flow possible (is the continuity eqn satisfied)?

② Is the flow rotational

③ Find $a_x(1,1)$

sol.

④ if $\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \stackrel{?}{=} 0$

$$\frac{\partial \psi}{\partial x} = \frac{(x^2+y^2)(-1) + x(2x)}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$\frac{\partial \psi}{\partial y} = \frac{(x^2+y^2)(-1) + y(2y)}{(x^2+y^2)^2} = \frac{-(x^2-y^2)}{x^2+y^2}$$

$$\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = \frac{(x^2-y^2) - (x^2-y^2)}{(x^2+y^2)^2} = 0 \Rightarrow \text{continuity is valid}$$

⑤ $w_z = \frac{1}{2} \left(\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} \right)$

$$\frac{\partial \psi}{\partial x} = \frac{+2xy}{(x^2+y^2)^2}, \quad \frac{\partial \psi}{\partial y} = \frac{+2xy}{(x^2+y^2)^2}$$

$$w_z = \frac{1}{2} \left(\frac{2xy - 2xy}{(x^2+y^2)^2} \right) = 0 \quad \text{Flow is irrotational}$$

⑥ $q_x = \frac{\partial \psi}{\partial t} + u \frac{\partial \psi}{\partial x} + v \frac{\partial \psi}{\partial y} + w \frac{\partial \psi}{\partial z}$

$$= u \frac{\partial \psi}{\partial x} + v \frac{\partial \psi}{\partial y}$$

$$= \frac{-x}{x^2+y^2} \left[\frac{x^2-y^2}{(x^2+y^2)^2} \right] - \frac{y}{(x^2+y^2)} \left[\frac{2xy}{(x^2+y^2)^2} \right] \Rightarrow \frac{-1}{2}(0) + \frac{1}{2} \frac{2}{y} = \frac{1}{y} \text{ m/s}^2$$

Five Apple

• Note :-

m_i

$\Rightarrow$

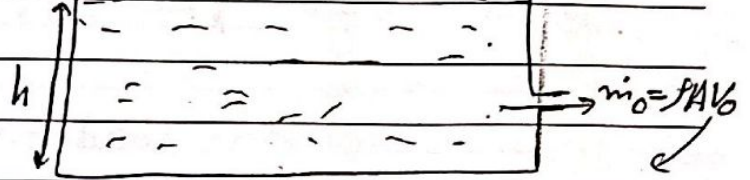
$$\Sigma M = - \frac{\partial M}{\partial t}$$

$$M = \rho A h$$

$$= \rho A h$$

$$m_o - m_i = - \frac{\partial M}{\partial t} = - \frac{\partial}{\partial t} \rho A h$$

$$\rho A v_o = - \rho A \frac{\partial h}{\partial t}$$



$$m_o = - \frac{\partial M}{\partial t}$$

$$V = v_o + g t \quad v_o = \sqrt{2 g h}$$

$$v_o^2 = v_o^2 + 2 g h$$

$$v_o = \sqrt{2 g h}$$

$$\rho A_n v_o = - \frac{\partial}{\partial t} (\rho A_T h)$$

$$\rho A_n v_o = - \rho A_T \frac{\partial h}{\partial t}$$

$$\partial t = \frac{- \rho A_T}{\rho A_n v_o} \partial h$$

$$t = \frac{- A_T}{A_n} \int_{h_1}^{h_2} \frac{\partial h}{\sqrt{2 g h}}$$

لحساب الزمن اللازم
للماء السريان

Pressure Variation in Flowing Fluids:-

① Pressure varies with Elevation due to gravity
($\Delta P = \gamma \Delta z$)

② Acceleration of Fluids, $\sum F_x = m a_x$ ~~$P_1 > P_2$~~ P_2

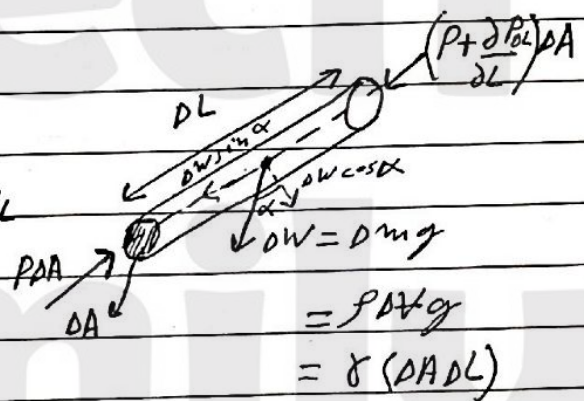
③ Viscous resistance and shear stress.

$$F_{sh} = \tau \cdot A = \mu \left(\frac{\partial v}{\partial y} \right) A$$

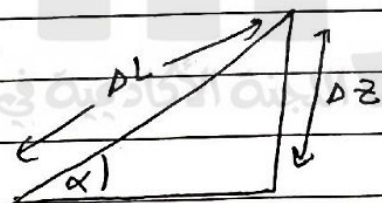
- Pressure Variation due to weight and acceleration

$$\sum F_L = M a_L$$

$$P \Delta A - \left(P + \frac{\partial P}{\partial L} \Delta L \right) \Delta A - \gamma (\Delta A \Delta L) \frac{\Delta z}{\Delta L} = \rho \Delta A \Delta L a_L$$



$$-\frac{\partial P}{\partial L} - \gamma \frac{\Delta z}{\Delta L} = \rho a_L$$



$$-\frac{\partial P}{\partial L} - \gamma \frac{\Delta z}{\Delta L} = \rho a_L$$

$$\sin \alpha = \frac{\Delta z}{\Delta L} = \frac{\partial z}{\partial L}$$

$$\boxed{-\frac{\partial (P + \gamma z)}{\partial L} = \rho a_L} \text{ Euler eqn}$$

-special cases:-

1) Constant velocity : $a_L = 0$

$$-\frac{\partial}{\partial L} (p + \gamma z) = 0$$

$$p + \gamma z = \text{const}$$

$$p_2 - p_1 = \gamma(z_1 - z_2)$$

$$dp = -\gamma(z_2 - z_1)$$

$$p_1 + \gamma z_1 = p_2 + \gamma z_2 = \text{const}$$

$$dp = -\gamma dz$$

$$dp = -\gamma dz$$

2) Uniform Acceleration : $a_L = \text{const}$

Select L-direction as shown

$$\frac{\partial p}{\partial L} = 0$$

at the bottom
of the tank

$$-\frac{\partial}{\partial L} (p + \gamma z) = \rho a_L \quad \frac{\partial}{\partial x} (p + \gamma z) = \rho a_x$$

$$-\frac{\partial}{\partial L} (\gamma z) = \rho a_L$$

$$-\frac{\partial p}{\partial x} = \rho a_x$$

$$\frac{\partial p}{\partial x} = -\rho a_x$$

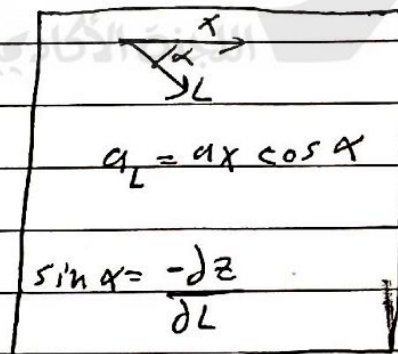
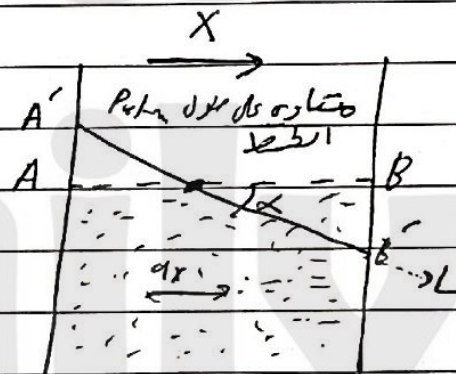
$$-\gamma \frac{\partial z}{\partial L} = \rho a_x \cos \alpha$$

$$\Delta p = (-\rho a_x) \Delta x$$

$$\rho g \sin \alpha = \rho a_x \cos \alpha$$

The pressure must decrease in the
direction of the acceleration

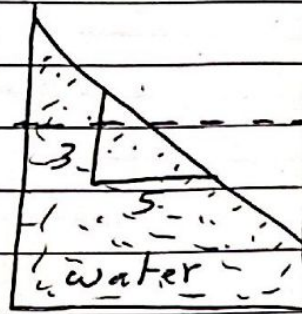
$$\frac{a_x}{g} = \tan \alpha$$



- problem (1)

acceleration of tank?

$$\tan \alpha = \frac{a_x}{g}$$



$$a_x = \tan \alpha (g)$$

$$= \frac{3}{5} \times 9.81$$

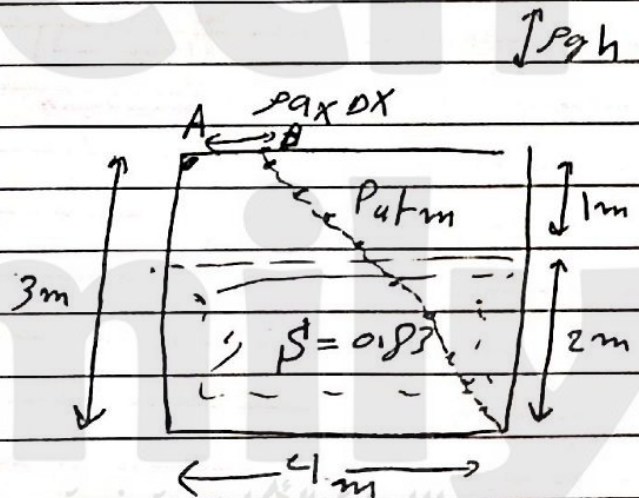
$$= 5.886 \text{ m/s}^2$$

- problem (2)

$$a_x = 2g = 19.62 \text{ m/s}^2$$

Find $p_{\max}$??

$$\tan \alpha = \frac{a_x}{g} = \frac{19.62}{9.81} = 2$$



$$\alpha = 63.43^\circ$$

$$55.6 \text{ kPa}$$

Apply Euler eqn for vertical line

$$\frac{\partial}{\partial z} (p + \gamma z) = f_{ay} = 0$$

$$p + \gamma z = \text{const} \Rightarrow p_{\max} + \gamma z_1 = p_{\text{top}} + \gamma z_2$$

$$p_{\max} = p_{\text{top}} + \gamma (3)$$

Apply Euler eq. for line AB

$$-\frac{d}{dx}(P + \gamma z) = \rho a_x$$

$$-\frac{dP}{dx} = \rho a_x$$

$$P = \rho a_x x + C$$

-Ex:

$$\rho = 0.8$$

$$D = 1 \text{ ft}$$

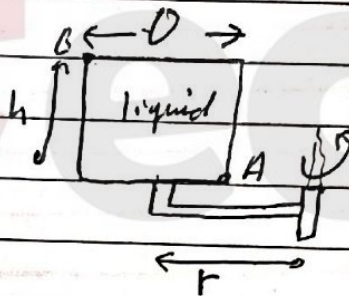
$$h = 1 \text{ ft}$$

$$r = 2 \text{ ft}$$

$$V_A = 20 \text{ ft/s}$$

$$P_A = 25 \text{ psf}$$

$$P_B = ??$$



$$P_A + \gamma z_A - \frac{\rho V_A^2}{2} = P_B + \gamma z_B - \frac{\rho V_B^2}{2}$$

$$\frac{P_A}{\gamma} + z_A - \frac{V_A^2}{2g} = \frac{P_B}{\gamma} + z_B - \frac{V_B^2}{2g}$$

$$V_A = \omega r_A, V_B = \omega r_B$$

$$P_B = 526.6 \text{ psf}$$

$$\omega_A = \omega_B, r_B = 2.5 \text{ ft}$$

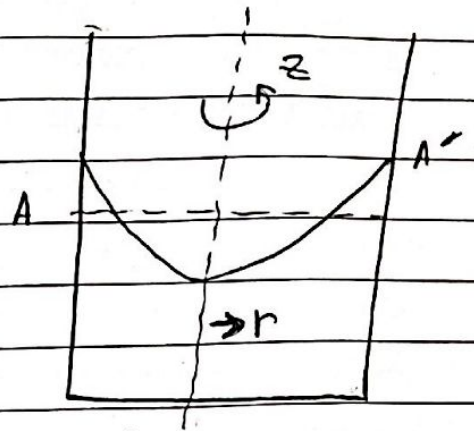
$$L = 13.33$$

• Pure Rotation of Tank of liquid

Const. Rotation

$$-\frac{d}{dr}(p + \gamma z) = \rho a_r$$

where $\frac{V_r}{\text{any } r} = \omega r$



Normal acceleration = a_n

$$a_n = \frac{V^2}{r} = \omega^2 r$$

$$= a_r \text{ (away from center)}$$

$$a_r = -\frac{V^2}{r} \text{ (away from the center)}$$

$$-\frac{d}{dr}(p + \gamma z) = \rho(-\omega^2 r)$$

Flow is steady and function only of r

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$$\frac{d}{dr}(p + \gamma z) = \rho r \omega^2$$

$$p + \gamma z = \frac{\rho r^2 \omega^2}{2} + C$$

$$= \frac{\rho V^2}{2} + C$$

$$\boxed{p + \gamma z - \frac{\rho V^2}{2} = \text{const}}$$

• Bernoulli equation :-

It is the integration of Euler equation for steady flow of an incompressible inviscid and irrotational flow.

$$\frac{-\partial}{\partial x} (p + \rho z) = \rho a_x$$

$$\frac{-\partial}{\partial x} \left(\frac{p}{\rho} + z \right) = \frac{\rho a_x}{\rho}$$

$$\frac{-\partial}{\partial x} (h) = \frac{a_x}{g}$$

Consider 2D and steady flow

$$a_x = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

$$\frac{-\partial h}{\partial x} = \frac{1}{g} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \rightarrow$$

Consider the flow is irrotational
such that $\omega_z = 0$ $\left(\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y} \right)$

$$\frac{-\partial h}{\partial x} = \frac{1}{g} \left[u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial x} \right]$$

$$u \frac{\partial u}{\partial x} = \frac{\partial}{\partial x} (u^2/2)$$

$$v \frac{\partial v}{\partial x} = \frac{\partial}{\partial x} (v^2/2)$$

$$-\frac{\partial h}{\partial x} = \frac{1}{g} \left[\frac{\partial}{\partial x} (u^2/2) + \frac{\partial}{\partial x} (v^2/2) \right]$$

$$= \frac{1}{g} \left[\frac{\partial}{\partial x} \left(\frac{u^2 + v^2}{2} \right) \right], \text{ but } V = \sqrt{u^2 + v^2}$$

$$= \frac{1}{g} \left[\frac{\partial}{\partial x} \left(\frac{V^2}{2} \right) \right]$$

$$\frac{\partial}{\partial x} \left[\frac{V^2}{2g} + h \right] = 0$$

$$\frac{V^2}{2g} + h = \text{const}$$

$$\frac{V^2}{2g} + \frac{p}{\gamma} + z = \text{const}$$

• pressure head

• velocity head

• potential head

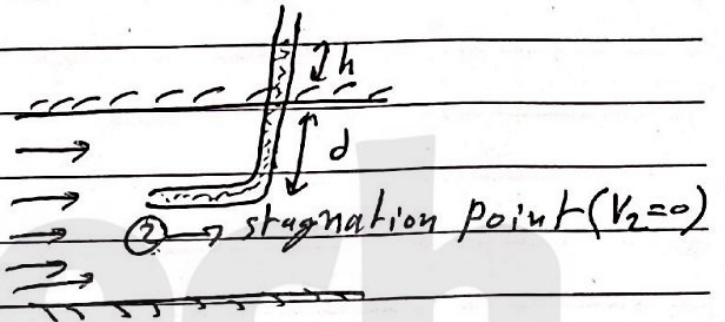
$$\boxed{\frac{p}{\gamma} + \frac{V^2}{2g} + z = \text{const.}} \quad \text{Bernoulli's eqn}$$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \text{const.}$$

• 2D, steady, inviscid, irrotational
incompressible.

• Application of the Bernoulli eqn:-

① stagnation tube



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

ch. 7, 10

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

Major
 $\frac{f L V^2}{D 2g}$
Minor
 $\sum K \frac{V^2}{2g}$

Ch. 7

الأنابيب ١٥/٣

- Energy equation :-

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

• إذا كان في ملاحظة

$$\text{power pump} = \Delta p \cdot Q$$

$$\text{power pump} = \rho g D h \cdot Q$$

$$\text{power pump} = \dot{m} g h_p$$

ضرب h_p

للجزء السائل في المعادلة

• إذا كان في تسريع

ضرب h_e

للجزء الصلب في المعادلة

$$h_p = \frac{\text{Power}}{\dot{m} g} = \text{head for pump}$$

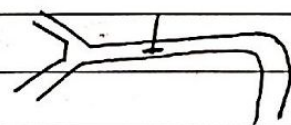
Ch. 10

Flow in conduits and pipes

Major losses (pipes)

h_L

Minor Losses (valves, fittings, Elbows)



$$-h_{L\text{major}} = f \frac{L}{D} \frac{V^2}{2g} \quad \text{where}$$

f = Friction factor (Moody chart)

Function in $(Re, \text{surface Ruffness } \frac{\epsilon}{D})$

L = pipe length

D = pipe diameter

V = flow velocity

$$-h_{L\text{minor}} = \sum_{i=1}^n K_i \frac{V^2}{2g}$$

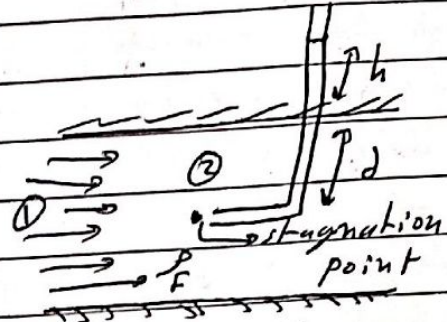
$$h_L = h_{L\text{major}} + h_{L\text{minor}}$$

اللجنة الأكاديمية في قسم هندسة الميكانيك - الجامعة الأردنية

• Application of the Bernoulli equations

① Stagnation Tube

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$



$$\frac{P_1 - P_2}{\gamma} = \frac{V_1^2}{2g} \Rightarrow V_1 = \sqrt{\frac{2 \Delta P}{\rho}}$$

$$V_1 = \sqrt{2gh}$$

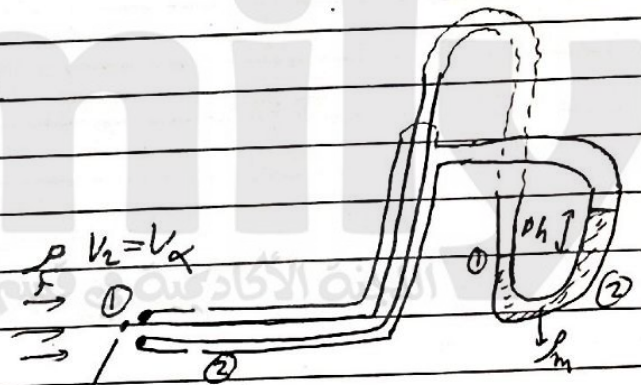
$$P_1 = \rho g d$$

$$P_2 = \rho g (d + h)$$

② pitot tube

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{P_1}{\rho g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$



stagnation point ($V=0$)

$$P_1 = \rho g h_1$$

$$P_2 = \rho g h_2$$

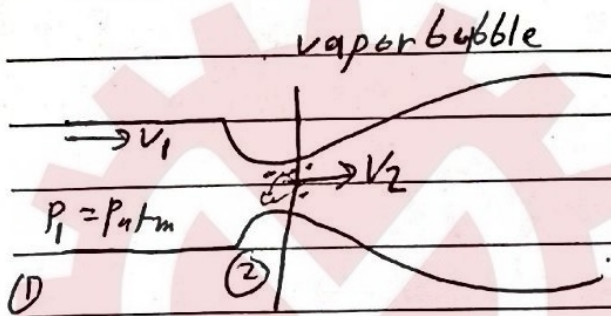
$$\Delta P = \rho g \Delta h$$

$$V_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$= \sqrt{\frac{2 \rho_m 2 \Delta h_m}{\rho}}$$

• Cavitation



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \text{const}$$

if the pressure decreases by increasing $V_2 \Rightarrow$ The flow evaporate (boil) at Low Temp.