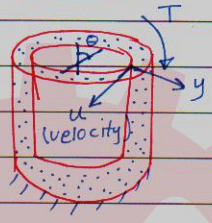
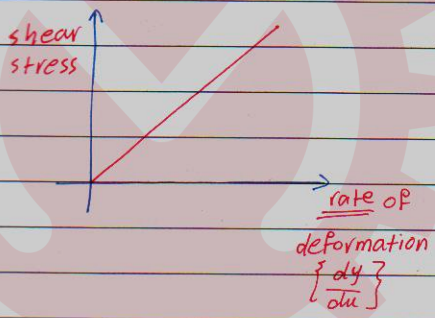
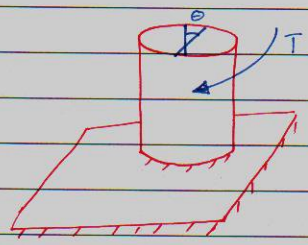
Fluid Machanics

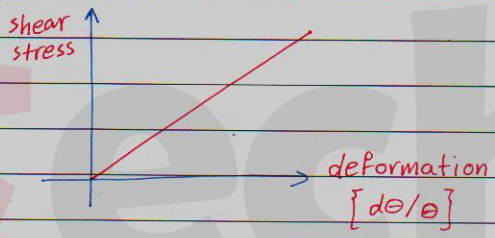
Fluid

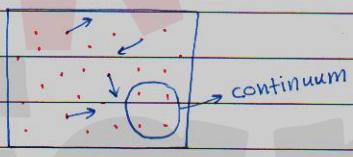




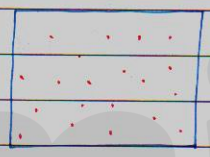
solid







In the Fluid the particle in motion.



In the solid the particle in static.

Fluid

- Liquid — const. distance
- Gas — distance in variable.

* continuum :-

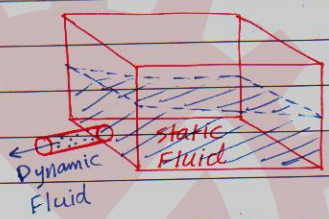
هو ان تكون جزيئات المادة قريبة من بعضها وتتصادم مع بعضها ، اما على اطراف الغلاف الجوي X يتكون continuum لأن المسافة بين الجزيئات هائلة .

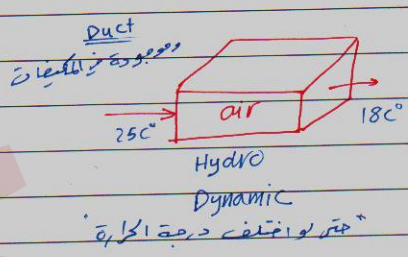
Fluid Mechanics

- Fluid statics
- Fluid Dynamics

Fluid Mechanics

- Hydro dynamic [no change in density]
- Gas dyna. [There is change in ρ]





* Mach Number :-

mach No. :- $M = \frac{\text{speed of flow}}{\text{speed of sound}}$

$M < 1$: subsonic flow

$M = 1$: Tran sonic

$M > 1$: supper sonic

Chapter 2

* Units :-

we have two types :-

① Primary or base

Mass [M], length [L], time [t], tempreture [T]

② secondary or derived

Force [F], velocity [v], acceleration [a], pressure [P]

* Units systems :-

SI system	British system	USCS (customary)
Kg		slug = 32.2 lbm
m		ft = 0.305 m
sec		sec
$K = C^\circ + 273.15^\circ$		Rankine ← $R = F^\circ + 460^\circ$
N/m^2 [Pa]		lbf/ft^2 = Psi

⇐ ملاحظة: اذا كانت الوحدة تدل على اسم عالم يكتب الحرف capital مثل [N, Pa]
غير ذلك تكتب الحرف small ، مثل kPa
small ↲ ↳ capital

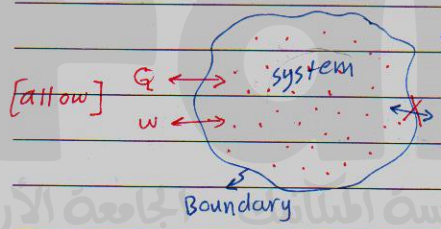
* Newton's 2nd low :-

$F \propto ma$

$F = \frac{ma}{g_c}$, g_c : is constant & $g_c = 1$ [in both SI system & USCS]

$\therefore F = ma$

2.2 Systems, Extensive & Intensive properties :-



2.3 properties involving mass & weight :-

	SI	British type II
density, ρ	kg/m^3	$slug/ft^3$
→ for water	$1000\ kg/m^3$	$1.94\ slug/ft^3$
specific wieght, γ	$\frac{N}{m^3}$	Lbf/ft^3

~~we~~ specific wieght : weight per unit Volume.

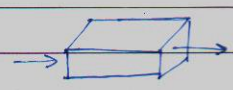
$\gamma = 9810\ N/m^3$ [SI system]

$\gamma = 62.4$ [British system]

$$\boxed{\gamma = \rho g}$$

For Liquids, ρ is usually const.

Gas, ρ is constant at law speeds $V < 100\ m/s$



specific gravity S = weight of certen volume of material / " " similar " " water {dimension less}

$$S = \frac{\gamma}{\gamma_w} = \frac{\rho g}{\rho_w g} = \frac{\rho}{\rho_w}$$

specific volume $v = \frac{1}{\rho} = \frac{m^3}{kg} = \frac{ft^3}{slug}$

2.5 Viscosity :- "اللزوجة"

we have two type :-

1) Dynamic, or absolute, μ

2) Kinematic, ν

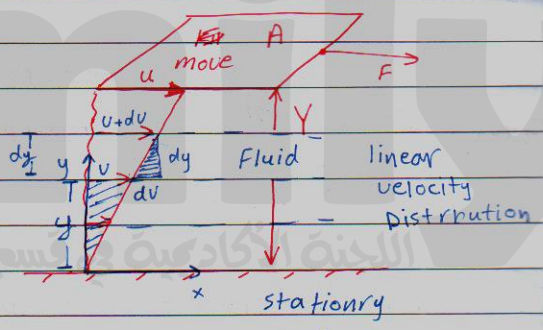
$$\boxed{\mu = \rho \nu}$$

physical meaning :-

The Force that required to pull an Area with velocity that have y (distance) from stationary surface.

$$F \propto \frac{A U}{Y} \Rightarrow \frac{F}{A} \propto \frac{U}{Y}$$

Shear stress, $\tau \propto \frac{U}{Y}$



من تشابه المثلثات

$$\frac{U}{Y} = \frac{v}{y} = \frac{du}{dy}$$

$$\boxed{\therefore\ \tau = \mu \frac{du}{dy}}$$ Newton's low of viscosity.

→ units of Viscosity :-

Note :-

1) τ is max at the wall ($y = 0$)

2) $\tau = 0$ (min) at midway ($Y/2$)

3) $V = 0$, at $y = 0$ {No slip condition}

in SI system:-

$$[\mu] = \frac{N/m^2}{\frac{m/s}{m}} = \frac{N \cdot s}{m^2} = \frac{\frac{kg\,m}{s^2} \cdot s}{m^2} = kg/m \cdot s$$

1 poise = 0.1 kg/m.s

1 centipoise = 0.01 poise = 0.001 kg/m.s

For water : at 20 C° ⇒ $\mu = 1 \times 10^{-3} \frac{N \cdot s}{m^2} = 1 \times 10^{-6}\ m^2/s$

Now, $\nu = \frac{\mu}{\rho}$

$$[\nu] = \frac{kg/m \cdot s}{kg/m^3} = m^2/s$$

1 stokes = 0.0001 m^2/s

1 centi stokes = 0.001 stokes

* Relation between viscosity & temp.

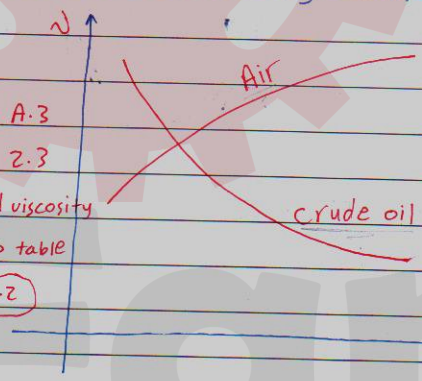


Fig A.3

Fig 2.3

to find viscosity

back to table

Fig A.2

في السوائل تقل اللزوجة مع درجة الحرارة ، لأن عند زيادة درجة حرارته تبتعد الـ particle عن بعضها فيقل التصادم بين الـ particle (Force)

اذن سوف تقل اللزوجة لانها بالأصل علاقة بين القوة والمساحة

في الغاز الجزيئات بعيدة عن بعضها فعند التسخين تزداد سرعة الجزيئات فيؤدي ذلك الى رفع الضغط ، اذن كلما زاد T السرعة زاد التصادم بين الجزيئات لذلك تزيد اللزوجة

Note:-

$\nu_{liquid} < \nu_{gases}$ $\qquad \mu = \rho \nu$

$\mu_{liquid} > \mu_{gases}$

SAE number

society ↓ ↓ Engineers.

Automotive

For Example : SAE 10 W — small viscosity ⇒ winter

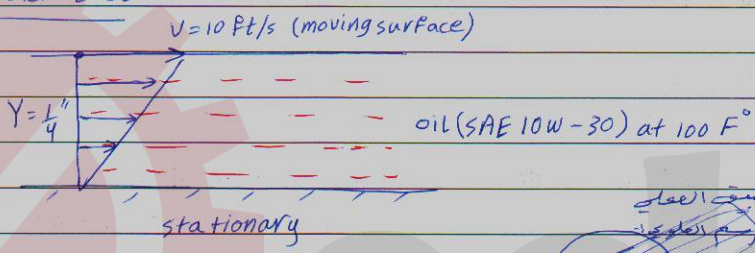
SAE 20 W

SAE 40 — High viscosity ⇒ summer

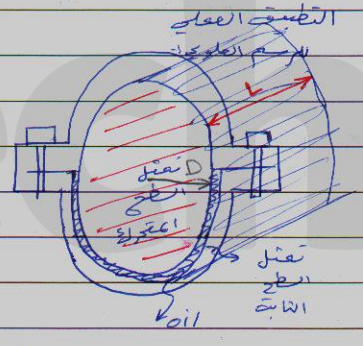
SAE 10 W 40 — Multigrade oil

SAE 90 — suitable for gears.

Example : problem 2.28



* Find τ in oil ??



sol :-

$$\tau = \mu \frac{dV}{dy}$$

since viscosity profile is linear

$$\frac{dV}{dy} = \frac{V}{Y}$$

$\mu = 1.4 \times 10^{-3} \frac{lb.s}{ft^2}$, by using Apendex A , Figure

$$\tau = \frac{1.4 \times 10^{-3} \times 10}{(1/4)/12} = 0.672 \ lbf/ft^2$$

Extra $F = \tau \times A$

$= 0.672 \times \pi D L$

H.W on Ch.2 :-

7th 2.31, 33, 38, 40

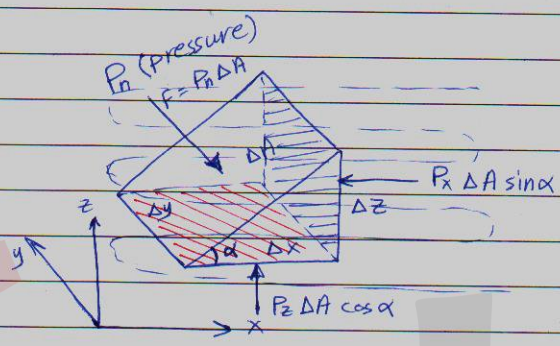
6th 2.19, 21, 26

9th 2.35, 37, 39, 40

Chapter 3

Fluid statics :-

3.1 Pressure :-



Force = pressure × Area

Force balance :-

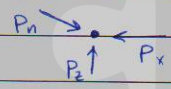
in x-direction

$P_n \Delta A \sin\alpha - P_x \Delta A \sin\alpha = 0$

$$\boxed{P_n = P_x}$$

$$\boxed{P_n = P_z}$$

* الضغط متساوي على الجسم في اي اتجاه كان



$P_n = P_z = P_x$

$P_n = P_x = P_z$; the presure is independent of director.

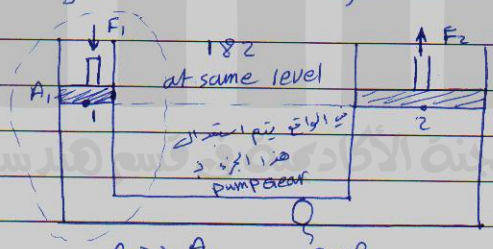
* Pressure Transmission (Pascal's principle) :-

in a closed system, a pressure change produced at one point in the system will be transmitted throughout the entire system.

$P_1 = P_2$

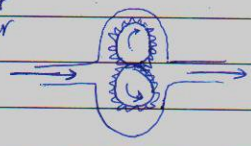
$\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$F_1 = \frac{A_1}{A_2} F_2$ } Pascal's Principle



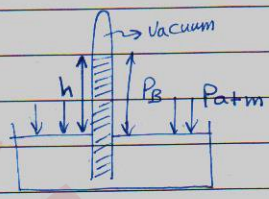
$A_2 >> A_1$

$F_2 >> F_1$

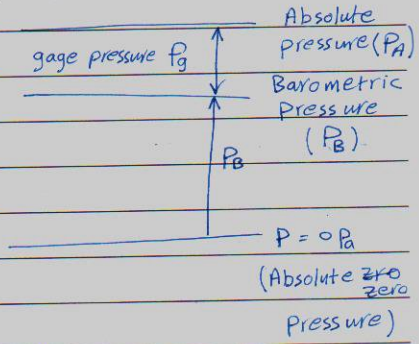


* Pressure Measurements :-

$P_{atm} = \rho_{Hg}\, h$



"Barameter"

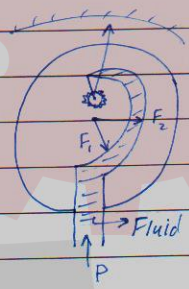


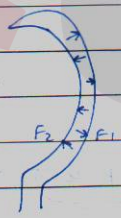
$$\boxed{P_A = P_g + P_B}$$

$P_B = 101.325$ kPa at sea level.

* The method whic use to calculate gage pressure :-

Bourdon Tube :-





$F = P \times A$

$F_1 > F_2$.

Pressure Measurments :-
- Manometers
  - peisometer.
  - differntial manometer.
- Bourdon Tube
- pressure Transduce

① Manometers :-

Peisometer :-

اصل انواع Param. ال

$P_1 - P_2 = \rho g h$

if P_2 is gage pressure

$\therefore P_2 = 0$

$$\boxed{\therefore P_1 = \rho h g}$$

* we use peisometer for small pressure $\simeq 10$ kPa

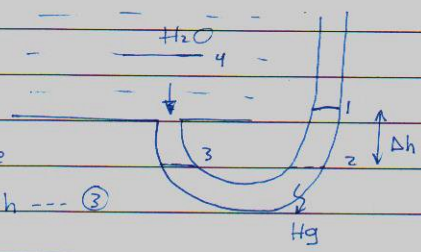
Differential Manometers :-

1- equilibrium at the interface between any two media

$P_3 - P_4 = \rho_w g L$ --- ①

$P_3 = P_2$ --- ②

2. applying the rule No.1 on the right branch ⇒ $P_2 - P_1 = \rho_m g \Delta h$ --- ③



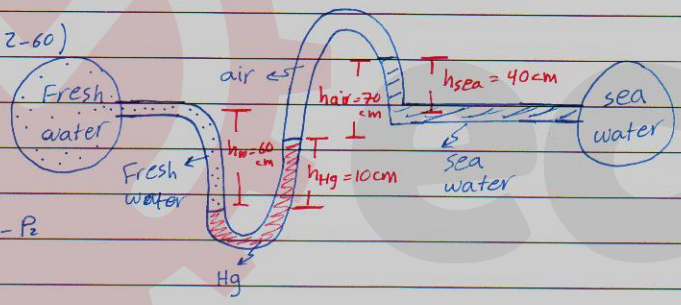
$\therefore P_4 = \rho_m g \Delta h - \rho_w g L$

U- Tube Manometer

Ex:- (problem 2-60)

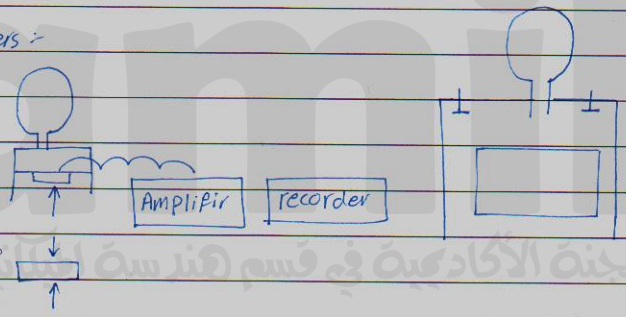
we will find ΔP in next lecture

- calculate $P_1 - P_2$



② Bourdon Tube : [تم شرحه]

③ Pressure Transducers :-

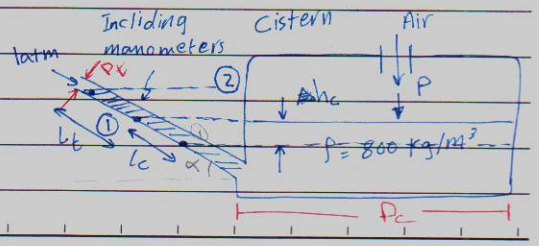


هذه القطعة اذا تعرضت لضغط يتولد تيار كهربائي.

Example, [Problem 3.31] :-

find P in the cistern ??

$\frac{D_c}{D_t} = 8$,



$P = \rho g L \sin\alpha$

الهدف من Inclined tube هو تضخيم القراءة

$L = L_t + L_c$

$= L_t + \frac{h_c}{\sin\alpha}$

Now: $V_c = V_t$

$\frac{\pi D_c^2}{4} h_c = \frac{\pi D_t^2}{4} \times L_t$

$h_c = L_t \frac{D_t^2}{D_c^2} = 50\left(\frac{1}{8}\right)^2 = 0.718$ cm

$\therefore P = \rho g \left(L_t + \frac{h_c}{\sin\alpha}\right) \sin\alpha$

$= 800 \times 9.81 \left(0.5 + \frac{0.781 \times 10^{-2}}{\sin 10}\right) \sin 10$

$L_t = 0.5$ m, $\alpha = 10°$ } معطى

$= 743$ Pa

Proble 2.60 :-

sol:-

$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \cancel{\rho_{air} g h_{air}} + \rho_{sea} g h_{sea}$

neglect compared to ρ_w & ρ_{Hg}

$P_1 - P_2 = -\rho_w g h_w + \rho_{Hg} g h_{Hg} - \rho_{sea} g h_{sea} = g(\rho_{Hg} h_{Hg} - \rho_w h_w - \rho_{sea} h_{sea})$

$= 9.81(13{,}600 \times 0.1 - 1000 \times 0.6 - 1035 \times 0.4)$

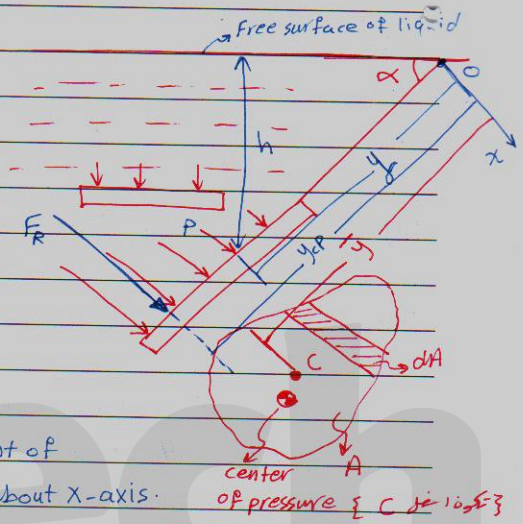
$= 3{,}394$ Pa $/1000$ → devided by 1000 to convert Pa to kPa.

$= 3.39$ kPa

نقطة تأثير الـ F_R هو
center of pressure.

3.4 Hydrostatic Force on plane surface :-

Horizantal surface ⇒ $F = P \cdot A$



Inclinded surface:-

Force on elemantal area dA

$dF = P\,dA$

$= \gamma\, y \sin\alpha \, dA \quad \{\gamma = \rho g\}$

$\{h = y \sin\alpha\}$

$F = \int \gamma\, y \sin\alpha \, dA$

$= \gamma \sin\alpha \int_A y \cdot dA$ ⇝ ~~distan~~ First moment of Plate Area about X-axis.

Put :

$\bar{y} = \frac{\int_A y\, dA}{A}$, Distance of center of gravity (centroid) from O.

مهم جداً جداً ... البعد العمودي بين سطح الماء و centroid

$F = \underbrace{\gamma\, \bar{y} \sin\alpha}_{P'} \, A$

P', Pressure at the centroid

$$\boxed{F = \bar{P} A}$$

If P is absolute pressure

$F_{abs} = P_{at} A + \bar{P} A$

Line of action of force

Moments @ O

$y_{cp} F = \int_A y\, dF = \int_A y P\, dA = \int_A y \gamma \sin\alpha \, dA$

$= \gamma \sin\alpha \int_A y^2 \cdot dA$ ⇝ 2nd mom. of Area @ x axis or moment of Inertia I_{xx}

$I_{xx} = \bar{I} + \bar{y}^2 A$ → from fig A.1

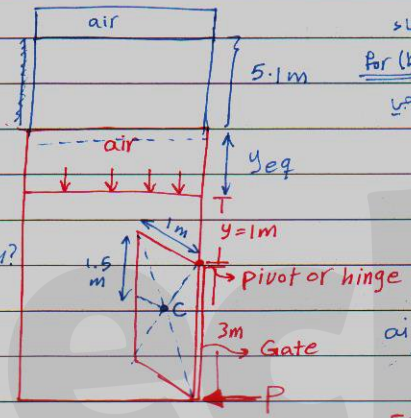
القوة مقدارها تحسب على اساس ال centroid مهم جداً جداً...

أما اتجاهها تحسب على اساس ال center of press البُعد المائل بين سطح الماء وال centroid.

$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A}$

$X_{cp} = \frac{\gamma \sin\alpha}{F} I_{xy}$ product of inertia.

Example [problem 3-96]



تم اضافة ارتفاع الماء ليصبح الضغط for (b) الداخلي يساوي الخارجي

$P_{air} = 1atm$

$S = 0.8$

Find : a) Force P , if the $P_{air} = 1atm$?

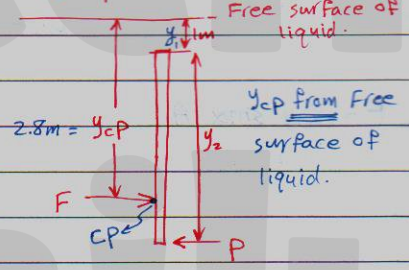
b) if $P_{air} = 40\ kPa$?

* في الفرع الاول الضغط الداخلي والخارجي = P_{atm}

* الفرع الثاني تم اضافة الضغط بمقدار 40 kPa

sol :-

Free Body diagram on hinge



M about hinge :-

$F(y_{cp} - y_1) = P y_2$ ---- (1)

$F = \gamma \bar{y} \sin\alpha A$

$= \gamma (\frac{1}{2} y_2 + y_1)(y_2 \times 1)$

$= 9810 \times 0.8 (1.5 + 1)(3) = 58,860 N.$

$y_{cp} = \bar{y} + \frac{I}{\bar{y}A} = \bar{y} + \frac{1 \cdot L^3/12}{\bar{y}A} = 2.5 + \frac{1 \cdot (3)^3/(12)}{2.5(3 \times 1)} = 2.8 m.$

from (1) $P = \frac{F(y_{cp} - y)}{y_2} = \frac{58,560(2.5 - 1)}{3} = 35,316 N$

$$y_{eq} = \frac{P_{air}}{\gamma_w} = \frac{40{,}000}{0.8 \times 9810} = 5.1\,m$$

$$P = \rho g h$$

$$h = \frac{P}{\rho g}$$

Now: $\bar{y} = \frac{1}{2}y_2 + y_1 + y_{eq} = 7.6\,m$

$$F = 0.8 \times 9810 \times 7.6 \times 3 = 178{,}934.4\,N.$$

Other Method {supper position method} :-

$$F = F_{atm} + F_{hydrostatic}$$

$$F_{atm} = P_{atm} A$$

$$= 40{,}000\,(3)(1) = 120{,}000$$

$$F = 120\,000 + 58\,860 = 178\,934.4\,N$$

$\boxed{3.5}$ Hydrostatic forces on curved surface :-

$$dF = P\,dA$$

$$\underset{dF_x}{\swarrow}\ \underset{dF_y}{\searrow} = \gamma y L\,ds$$

عمق الجسم داخل الموج

$$dF_x = \gamma y L\,ds \sin\alpha$$

$$= \gamma y\,dA_v$$

integrating

$$F_x = \bar{P} A_v$$

① Horizantal force

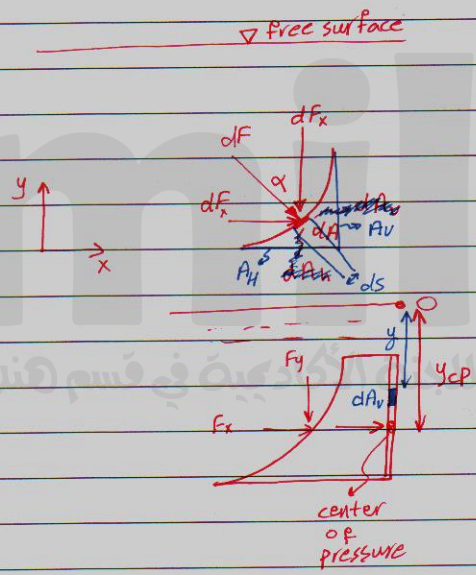
$$F_x = \bar{P} A_v$$

line of action

moment around O :

$$F_x\, y_{cp} = \int y\,dF_x$$

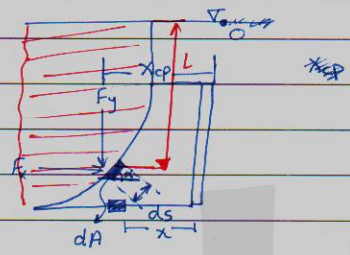
$$\gamma \bar{y}_v A_v\, y_{cp} = \int y\,\gamma y\,dA_v$$



$\bar{y}_{cp}\, \bar{y}_v A_v = \int_A y^2\, dA_v$ Iov mom. of Inertia of vertical projection of Area Av about o.

$I_{ov} = I_v + \bar{y}^2 A_v$

$$y_{cp} = \bar{y}_v + \frac{I_v}{\bar{y}_v A_v}$$



② vertical direction of force :-

$dF_y = \gamma y L\, ds \cos\alpha$ (width)

force, depth, proj. area of base

$dF_y = \gamma\, d\forall \quad \{ d\forall = y L\, ds \cos\alpha \}$

$F_y = \int_{\forall} \gamma \cdot d\forall = \gamma \forall$

$$\boxed{F_y = \gamma \forall}$$

Line of action :-

$F_y \times X_{cp} = \int x \gamma\, d\forall$

$$X_{cp} = \frac{\int x\, d\forall}{\forall}$$

Example : { problem 3.75 } :-

$\gamma = \rho g = 1000 \cdot 9.81 = 9810\ N/m^3$

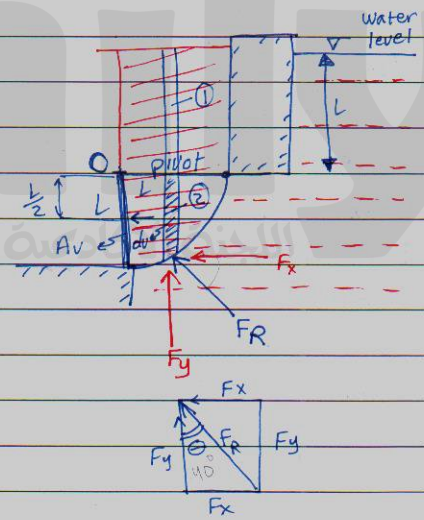
width (داخل مستوى الورق) = 1m.

Find :-

a) F_y, X_{cp}

b) F_x, y_{cp}

c) F_R, θ



a) $F_y = \gamma \forall = \gamma \left[L^2 \times 1 + \frac{1}{4}\pi L^3 \times 1 \right]$

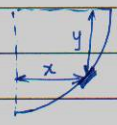
$= 9810 \left[1^3 + \frac{1}{4}\pi 1^3 \right] = 17,515\ N$

- upper part ①

$X_{cp} = \frac{L}{2}$ @ the centroid

- lower part ②

$$\int_0^1 \frac{x\, d\forall}{\forall} = \int_0^1 \frac{x\,(y \times 1 \times dx)}{\frac{1}{4}\pi L^2}$$



Put $L^2 = x^2 + y^2$

$y^2 = L^2 - x^2 \Rightarrow y = \sqrt{L^2 - x^2}$

$y = \sqrt{1 - x^2}$

$$X_{cp} = \frac{\int_0^1 x\sqrt{1-x^2}\,.dx}{\frac{1}{4}\pi} = \frac{4}{\pi}\left[-\frac{1}{3}\sqrt{(1-x^2)^3} \right]_0^1 = \frac{4}{\pi}\left(\frac{1}{3}\sqrt{1}\right) = 0.42$$

b) $F_x = \bar{P} A_v = 9810\,(1+0.5)(1)(1) = 14,715\ N.$

$$y_{cp} = \bar{y}_v + \frac{\bar{I}_v}{\bar{y}_v A_v} = 1.5 + \frac{1 \times 1^3/12}{1.5 \times (1)^2} = 1.555\ m$$

c) $F_R = \sqrt{F_x^2 + F_y^2} = 22876\ N$

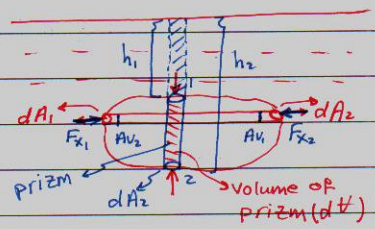
$\tan\theta = \frac{F_x}{F_y} = \frac{14,715}{17,515} = 40^\circ$

H.w #2 on Chapter 2 :-

7th ed.	9th ed.
8	13 ✓
27	X
30	33 ✓
78	62
137	78
142	100

3.6 Buoyancy [قوة الطفو] :-

$F_{x_1} = F_{x_2}$ [at same level]



Force on area 1 (vertical):-

$F_1 = P_x \, dA_1$

$= \rho g h_1 \, dA_1$

$= \gamma \, d\forall_1$

F_2 on area 2 (vertical)

$F_2 = \gamma \, d\forall_2 = \rho g h_2 \, dA_2$

Net force on prizm:-

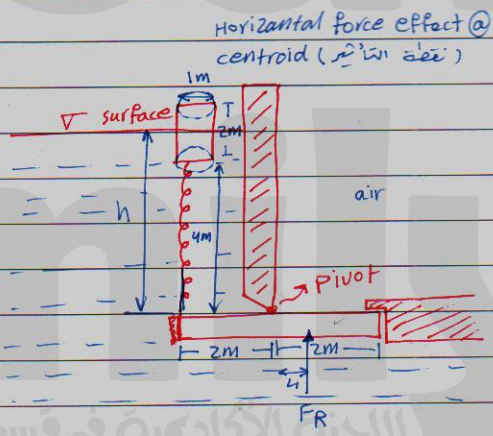
$dF_v = \gamma \underbrace{(d\forall_2 - d\forall_1)}_{d\forall} = \gamma \, d\forall$

$$\boxed{F_v = \int_{\forall} \gamma \, d\forall} \rightsquigarrow F_B = \gamma \forall$$

↳ is the sub-merged volume

Example:-

Find h to open the gate ??



sol:-

Moment about pivot:-

$F_R \times L_1 = T \times L_2$

$T = \frac{F_R}{2}$

$T = F_v = \rho g \forall \Rightarrow T = \frac{F_R}{2}$

$\forall = \frac{F_R}{2} \frac{1}{\rho g}$

$F_R = \rho g h A_{gate}$

$\forall = A_{gate} (h-4)$

$(h-4) A_{gate} = \rho g h A_{gate}$

$(h-4)\pi(1)^2 = 0.5\,h \times 2$

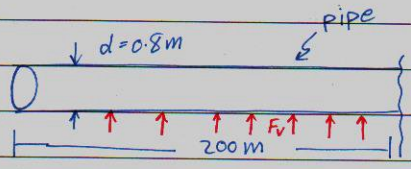
$\boxed{h = 5.872\,m}$

Example:-

$M_{pipe} = 80{,}000\ kg$

calculate F_u??

will the pipe float or not ?



$\forall = \frac{\pi(0.8)^2}{4} \times 200 = 100.53\ m^3$

$F_u = \gamma \forall = 9810 \times 100.53 = 986{,}199.3\ N$

pipe wieght $= 80{,}000 \times 9.81 = 784{,}800\ N$

$\therefore$ Yes, will be float $F_y >$ pipe wight

20/6/2013

Chapter 4

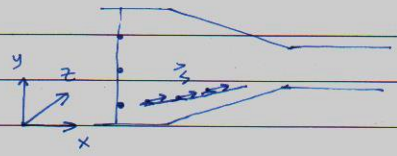
Fluid in Motion :-

4.1 velocity and Flow Visualization :-

a) Lagragian description:

$\bar{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$

$\bar{v}(t) = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$



$\vec{a}(t) = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$

b) Eulerian Description :-

$v(t) = f(x, y, z, t)$

* Stream lines :-

Line drawn such that the velocity vector is tangent to it.

stream line :-

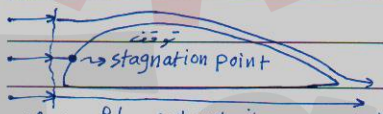
X تتقاطع , لكن عند turbin , pump , shaft

تتغير ال streamline .

* stream line coordinates :-

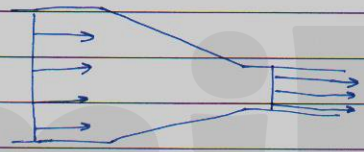
$\bar{v} = \bar{v}(\bar{s}, t)$

* Air foil :-



uniform flow (all velocity are equil)

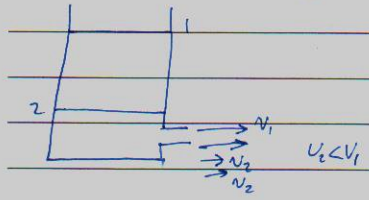
$\frac{dv}{ds} = 0$



Non uniform - flow $\frac{dv}{ds} \neq 0$

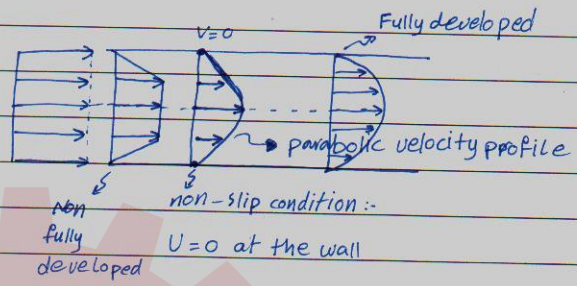
* steady flow :- $\frac{dv}{dt} = 0$, But nonsteady flow $\frac{dv}{ds} \neq 0$

Example for Non-steady flow

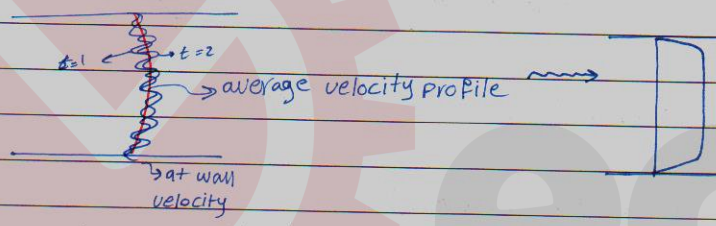


* Laminar and Turbulent flow :-

طبقة



IF Turbulent is non steady:-



* Reynolds Number (Re) :-

$$Re = \frac{\rho U D}{\mu} \lessgtr 2000$$

D: Diameter

U: velocity

ρ: Density

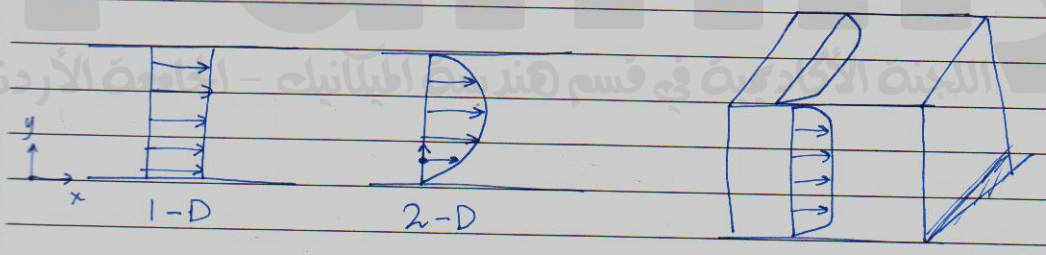
μ: Dynamic viscosity.

Laminar $Re < 2000$

Turbulent $Re > 2000$

* 1-D, 2-D, 3-D Flows :-

The number (1, 2, 3) depends on the direction of velocity or Number of components of velocity.



23/6/2013

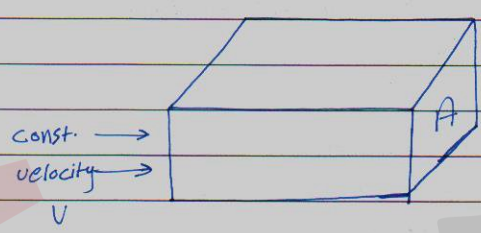
4.2 Rate of flow

- constant V
- variable V
- average V

- const. velocity :-

flow throught (thru) A

$$\Delta \forall = \overbrace{V \cdot \Delta t}^{\text{distance}} \cdot A$$



$$\frac{\Delta \forall}{\Delta t} = V \cdot A$$

$$\boxed{\dot{\forall} \equiv Q = V \cdot A}$$ volumetric Rate of flow. (m^3/s)

$$\boxed{\dot{m} = \rho V A}$$ mass flow rate (kg/s)

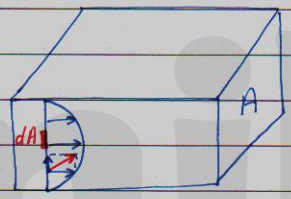
ρ: kg/m^3, V: m/s, A: m^2

- Variable velocity :-

$$dQ = V \cdot dA$$

integrating on the wole area

$$Q = \int_A V \cdot dA$$



* mass flow rate :-

$$\dot{m} = \int_A \rho V \cdot dA.$$

<u>Note</u>

$$Q = \int_A \vec{u} \cdot \vec{dA} = \int_A V \cos\theta \cdot dA$$

or, in vectorial form :

$$Q = \int_A \vec{V} \cdot \vec{dA}$$

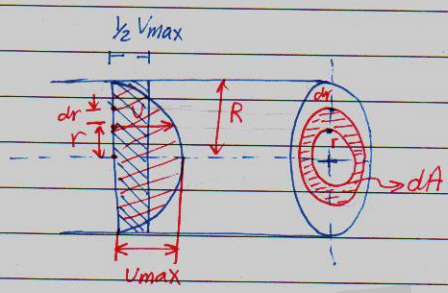
- Average Velocity :-

diff :

$$\bar{V} = \frac{Q}{A} = \frac{\int_A \vec{V}\cdot d\vec{A}}{A}$$

Example:-

The velocity distribution in a circular duct is given as:



$$V = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

calculate the avg. velocity ??!

$$dA = 2\pi r\,dr$$

$$\bar{V} = \frac{Q}{A} = \frac{\int_0^2 V_{max}\left(1-\frac{r^2}{R^2}\right)2\pi r\,dr}{\pi R^2} = \frac{2V_{max}}{R^2}\left[\int_0^R r\,dr - \frac{1}{R^2}\int_0^R r^3.dr\right]$$

total

$$= \frac{2V_{max}}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$

$$\boxed{\bar{V} = \tfrac{1}{2}V_{max}}$$

4.3 Accelaration :-

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

in Eulerian discription : $u = f(x, y, z, t)$

$$a_x = \frac{du}{dt},\quad a_y = \frac{dv}{dt},\quad a_z = \frac{dw}{dt}$$

$$a_x = \frac{du}{dx}\cdot\frac{dx}{dt} + \frac{du}{dy}\cdot\frac{dy}{dt} + \frac{du}{dz}\cdot\frac{dz}{dt}$$

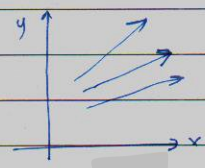
and since $\frac{dx}{dt} = u$, $\frac{dy}{dt} = v$, $\frac{dz}{dt} = w$, then

$$a_x = \boxed{u\frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz} + \frac{du}{dt}}$$

$$a_y = \underbrace{u\frac{dv}{dx} + v\frac{dv}{dy} + w\frac{dv}{dz}}_{\text{convective accelaration}} + \underbrace{\frac{dv}{dt}}_{\text{Local accelaration}}$$

$$a_z = \underbrace{u\frac{dw}{dx} + v\frac{dw}{dy} + w\frac{dw}{dz}}_{\text{convective accelaration}} + \underbrace{\frac{dw}{dt}}_{\text{Local accelaration}}$$

Example, problem 4.43 (on accelaration) :-



A Flow field has the following velocity components :-

$u = xt + 2y$, $v = xt^2 - yt$, $w = 0$, what is the total accelaration at a point $x = 1m$, $y = 1m$ at time $t = 2sec$

sol :-

$$a_x = u\frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz} + \frac{du}{dt}$$

$$= (xt + 2y)(t) + (xt^2 - yt)(2) + 0 + x$$

$$= (1(2) + 2(1))(2) + (1(4) - 1(2))(2) + 0 + 1$$

$$= 13\ m/s^2$$

$$a_y = u\frac{dv}{dx} + v\frac{dv}{dy} + w\frac{dv}{dz} + \frac{dv}{dt}$$

$$= (xt + 2y)(t^2) + (xt^2 - yt)(-t) + 0 + (2xt - y)$$

$$= (2 + 2)(4) + (4 - 2)(-2) + (4 - 1)$$

$$= 15\ m/s^2$$

$$\boxed{\vec{a_t} = 13\hat{i} + 15\hat{j}}, \quad a_t = \sqrt{(13)^2 + (15)^2}$$

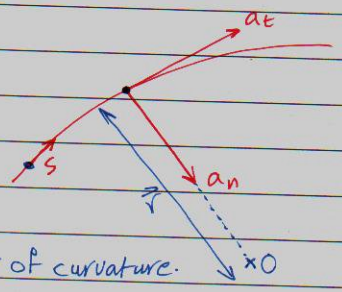
Accelaration in...

24/6/2013

* stream line coordinates

$$a_t = \frac{dv_s}{ds} \cdot \frac{ds}{dt} + \frac{dv_s}{dt}$$

$$\boxed{a_t = v_s \frac{dv_s}{ds} + \frac{dv_s}{dt}}$$



$$\boxed{a_n = \frac{v_s^2}{r}}$$, where r is the raduis of curvature.

4.4 Basic control volume approach :-

let B represent any extensive property, such ~~that~~ as : mass (M) momentum $M\vec{V}$ and Energy (E)

let b represent any intensive property such as :-

$\frac{M}{M} = 1$, for mass

$\frac{M\vec{V}}{M} = \vec{V}$, for momentum

$\frac{E}{M} = e$, Energy

i.e $\frac{B}{M}$ b

Extensive prop.

intensive prop.

for a differential mass (dm), and a volume $d\forall$

$$\boxed{B = \int_{\forall} b \cdot dm = \int b \rho \, d\forall} \quad \text{---- (4-23)}$$

* Applying this for flow in to & out of control volume :-

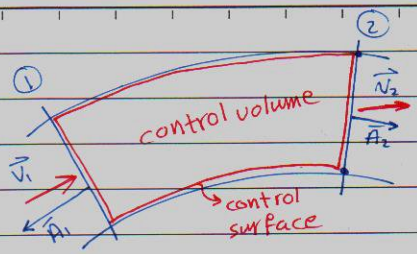
1-D

Volumetric flow rate:

$Q = VA$

in a vector form:

$Q = \vec{V} \cdot \vec{A}$



$\vec{A}$ is the Area vector

→ take station 1

$\vec{V} \cdot \vec{A} = V_1 A_1 \cos(180)$

$= -V_1 A_1$

→ take station 2

$\vec{V} \cdot \vec{A} = V_2 A_2 \cos(0) = V_2 A_2$

Net rate of flow OUT of control volume $= \sum_{c.s} \vec{V} \cdot \vec{A} = \underline{V_2 A_2 - V_1 A_1}$

(+) Net flow out

(−) Net flow in

0 no net flow

Usually $V_2 A_2 - V_1 A_1 = 0$

$\therefore Q = V_2 A_2 = V_1 A_1$

Mass flow rate OUT of C.V :-

$\dot{m} = \sum_{c.s} \rho \vec{V} \cdot \vec{A} = \rho V_2 A_2 - \rho V_1 A_1$

↳ control surface

⇒ similarly for any Extensive property :-

{ Intesive prop
x تشتق }

recall: $B = bM$

The flow rate of this property becomes

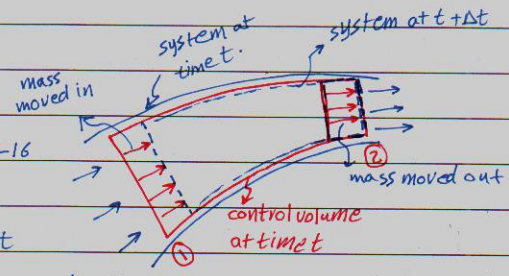
$\dot{B} = b\dot{M}$

$\dot{B} = \sum_{c.s} b \cdot \dot{m}$

$\dot{B} = \sum_{c.s} b(\rho \vec{V} \cdot \vec{A})$ - - - - - 4-26 1-D, steady, (const. velocity)

$\dot{B} = \int_{c.s} b \rho V \cdot \overline{dA}$ - - - - 4-27 1-D, steady, velocity is variable over the cross section.

1-D Non-steady flow :-



let B the extensive property

$$\frac{dB_{sys}}{dt} = \lim_{\Delta t \to 0}\left[\frac{B_{t+\Delta t} - B_t}{\Delta t}\right] \quad \text{--- 4-16}$$

The mass of the system at time $t+\Delta t$

= mass with in C.V at $t+\Delta t$ + Mass moved out during Δt − mass moved in during Δt

$$M_{sys,t+\Delta t} = M_{c.v,t+\Delta t} + \Delta M_{out} - \Delta M_{in}$$

similarly for any extensive property

$$B_{sys,t+\Delta t} = B_{c.v,t+\Delta t} + \Delta B_{out} - \Delta B_{in}$$

hence
$$\frac{dB_{sys}}{dt} = \lim\left[\frac{(B_{c.v,t+\Delta t} + \Delta B_{out} - \Delta B_{in}) - B_{c.v,t}}{\Delta t}\right]$$

$$= \lim_{\Delta t \to 0}\left[\frac{B_{c.v,t+\Delta t} - B_{c.v,t}}{\Delta t}\right] + \lim_{\Delta t \to 0}\left[\frac{\Delta B_{out} - \Delta B_{in}}{\Delta t}\right]$$

$$= \frac{d B_{c.v}}{dt} + \dot{B}_{out} - \dot{B}_{in}$$

The control volume eqn becomes

$$\boxed{\frac{d B_{sys}}{dt} = \underbrace{\frac{d}{dt}\int_{c.v} b\rho\, d\forall}_{\text{from eq 4-23}} + \underbrace{\sum_{c.s}(b\rho\vec{V}\cdot\vec{A})}_{\text{from eq 4-26}}} \quad \text{4-33 a}$$

1-D

Non-steady

if the velocity is variable along the cross section (2-D) flow

The control volume eqn. becomes :-

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{c.v} b\rho\, d\forall + \int_{c.s} b\rho\,\bar{V}\cdot d\bar{A} \quad \text{---- 4-33 b}$$

If the flow is steady $\frac{d}{dt} = 0$

$\frac{dB_{sys}}{dt} = \int b\rho\,\bar{V}\,d\bar{A}$, which is the same eqn 4-15 for 1-D steady

4.5 Continuity eqn:-

Put B_{sys} as M and $b = 1$ in eq 4-33 b

$$\boxed{\left.\frac{dM}{dt}\right)_{sys} = \frac{d}{dt}\int_{c.v} \rho \, d\forall + \int_{c.s} \rho \vec{V} \cdot d\vec{A}}$$

Eq. 4-35
cont. eq in integral form

$$\int_{c.s} \rho \vec{V} d\vec{A} = -\frac{d}{dt}\int_{c.v} \rho \, d\vec{\forall} \quad ----- \quad 4-36$$

if the velocity is const over the cross section

$$\sum_{c.s} \rho \vec{V} \cdot d\vec{A} = -\frac{d}{dt}\int_{c.v} \rho \cdot d\forall \quad ----- \quad 4-37$$

for steady flow: $\frac{d}{dt} = 0$

$$\boxed{\sum_{c.s} \rho \vec{V} \cdot \vec{A} = 0}$$

Example, Problem (4-69):-

Find Q in the branshes 2, 3
, if the velocity same at all branches ??

for steady flow:-

$$\sum_{c.s} \rho \vec{V} \cdot d\vec{A} = 0$$

for one inlet and one outlet

$$-\rho_1 V_1 A_1 + \rho_2 V_2 A_2 = 0$$

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

for constant density

$$V_1 A_1 = V_2 A_2 \rightsquigarrow Q_1 = Q_2$$

For more than one inlet and one outlet

$$\sum \text{flow,in} = \sum \text{flow,out}$$

$$Q_1 = Q_2 + Q_3$$

$$= V A_2 + V A_3$$

$$= V\left[A_2 + A_3\right]$$

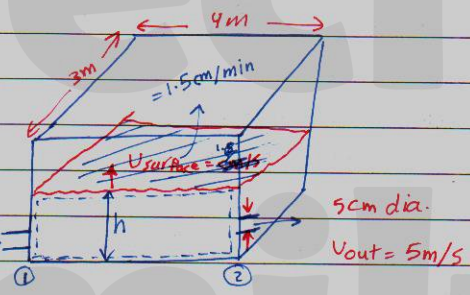
$$V = \frac{Q_1}{A_2 + A_3} = \frac{0.3}{\frac{\pi}{4}\left(0.2^2 + 0.15^2\right)} = 6.11 \text{ m/s}$$

$$Q_2 = V A_2 = 6.11\left(\frac{\pi}{4}(0.4)^2\right) = 0.192 \text{ m}^3/\text{s}$$

$$Q_3 = V A_3 = 6.11\left(\frac{\pi}{4}(0.15)^2\right) = 0.108 \text{ m}^3/\text{s}$$

Example on non-steady flow :-

Find volumetric flowrate $\dot{V}_1$ at the inlet of pool?



Method I :- {Fixed control volume}

Eq. 4-35

$$\left.\frac{dm}{dt}\right)_{sys} = \frac{d}{dt}\int_{c.v} \rho \, d\forall + \int_{c.s} \rho \, \bar{V} \cdot d\bar{A}$$

since the mass inside C.V is fixed.

$$\therefore \left.\frac{dm}{dt}\right)_{sys} = 0$$

$$\int_{c.s} \rho \, \bar{V} \cdot d\bar{A} = -\frac{d}{dt}\int_{c.v} \rho \cdot d\forall$$

and assuming the flow on 1-D flow

$$\sum_{c.s} \rho \cdot \bar{V} \cdot \bar{A} = -\rho \frac{d}{dt}\int_{c.v} d\forall = -\rho \frac{d\forall}{dt}$$

$$\rho\left(\bar{V}_1 \cdot \bar{A}_1 - \bar{V}_2 \bar{A}\right) = -\rho \frac{d\forall}{dt}$$

$\dot{m}_{in} = \rho \dot{V}_{in}$

$\dot{m}_{out} = \rho V_{out} A_{out} = \rho (5)\left(\frac{\pi}{4}(0.5)^2\right)$

$-\rho \frac{d\forall}{dt} = -\rho \frac{d}{dt}(h A_{surface})$

$= -\rho A_{surface} \frac{dh}{dt} = -\rho A_{surf.}(\ominus V_{surface}) = \rho A_{surf.} V_{surf.}$

لأنه الماء خارج من C.V اذاً (فقد) (سالب)

$\cancel{\rho} \dot{V}_{in} - \cancel{\rho} V_{out} A_{out} = \cancel{\rho} A_{surf.} V_{surf.}$

$\dot{V}_{in} = V_{out} A_{out} + A_{surf} V_{surf.}$

$= 5 \times \frac{\pi (0.05)^2}{4} + (4 \times 3) \times \frac{1.5 \times 10^{-2}}{60}$

$\boxed{\dot{V}_{in} = 0.0128 \ m^3/s}$

Method II :- {Moving C.V}

$\frac{dm_{sys}}{dt} = \cancel{\frac{d}{dt}\int_{C.V} \rho \, d\forall}^{\,0} + \int_{C.S} \rho \vec{V} \cdot d\vec{A}$

$\frac{dm_{sys}}{dt} = \dot{m}_{in} - \dot{m}_{out}$

$= \rho \dot{V}_1 - \rho V_{out} A_{out}$

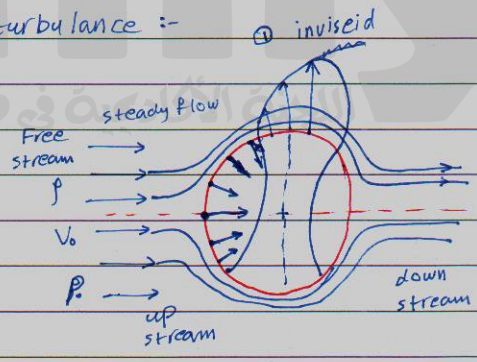
$\left.\frac{dm}{dt}\right)_{sys} = \frac{d}{dt}(\rho \forall_{sys}) = \rho \left(\frac{d\forall_{sys}}{dt}\right) = \rho \frac{d(h_{surf} \times A_{surf})}{dt}$

$= \rho A_{surf} \frac{dh_{surf}}{dt} = \rho A_{surf.} V_{surface.}$

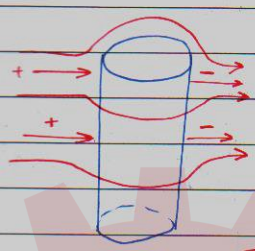
4.7 Seperation, Vortices and turbulance :-

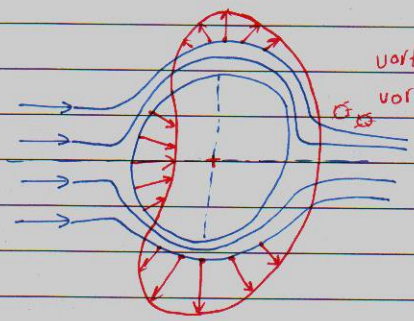
Example :- flow around acylinder

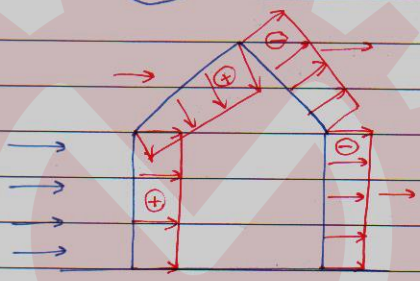
pressure coefficient $C_p = \frac{P - P_\infty}{\rho V_\infty^2 / 2}$



② Real Flow (Viscouse)



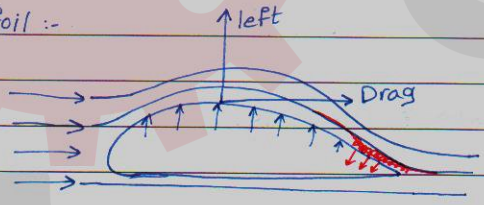




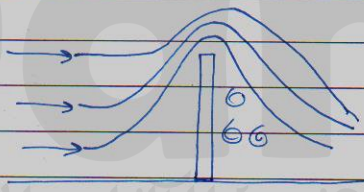
- Drag forces.

* Applications of seperation :-

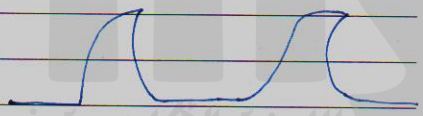
- air foil :-



- sand formations :-



sand Dunes :

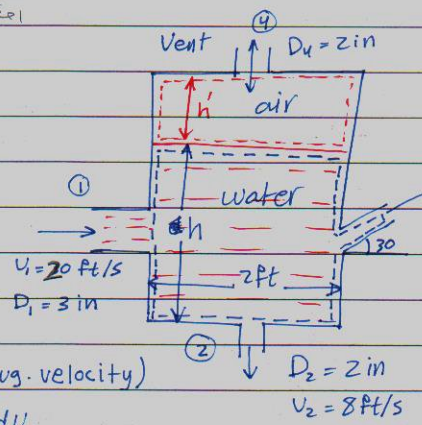


Exampels:- امتحان 2013 فصل ثاني

a) what is dh/dt

b) is the water level rising or falling

c) what is the velocity of air through ④ assuming $\rho_{air} = const.$



a) cont. eqn :- (4-35, with const. avg. velocity)

$$\left.\frac{dm}{dt}\right)_{sys} = \sum_{c.s} \rho \cdot \bar{V} \cdot \bar{A} + \frac{d}{dt}\int_{c.v} \rho \cdot dV$$

دائماً يساوي zero

اول خطوة نعوض

$$0 = -V_1 A_1 + V_2 A_2 + V_3 A_3 + \rho A_T \frac{dh}{dt}$$

رامية ρ لانه قسمنا المعادلة على ρ

$$\frac{dh}{dt} = (V_1 A_1 - V_2 A_2 - V_3 A_3) \times \frac{1}{A_T}$$

$$= \left[20\frac{(3/12)^2 \pi}{4} - 8\frac{\pi (2/12)^2}{4} - 10\frac{\pi (2.5/12)^2}{4}\right] \frac{1}{\pi \frac{2^2}{4}} = 0.1482$$

b) since $\frac{dh}{dt}$ positive, the water level is rising

c) $\frac{dh'}{dt} = \frac{-dh}{dt}$

$$0 = \sum_{c.s} \rho \bar{V} \cdot \bar{A} + \frac{d}{dt}\int_{c.v} \rho \cdot dV$$

$$= V_4 A_4 + A_T \frac{dh'}{dt}$$

$$V_4 = -\frac{A_T}{A_4}\frac{dh'}{dt} = \frac{A_T}{A_4}\frac{dh}{dt}$$

$$= \frac{\pi (2^2/4)}{\pi \frac{(2/12)^2}{4}} \times 0.1482$$

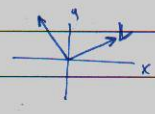
$$= 21.34 \text{ ft/s}$$

suggested problems on Ch. 4 :-

7.th	9.th
4.66	5.52
4.23	5.11
4.57	5.49
4.72	5.58
	EX 5.6
	5.39

27/6/2013

Chapter 5



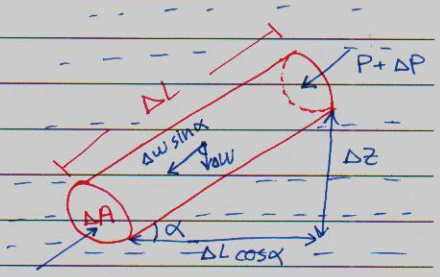
* Pressure Variations in Moving Fluids :-

pressure variation occurs become of :-

① Elevation (ch.3)

② weight and accelaration (ch.5)

③ Viscous effects (friction) (ch.7)



$\Sigma F_L = m a_L$

$P\Delta A - (P+\Delta P)\Delta A - w\sin\alpha = m a_L$

$P\Delta A - (P+\Delta P)\Delta A - w\sin\alpha = \rho \Delta A \Delta L a_L$

put $\Delta w = \gamma \Delta A \Delta L$

$$\frac{-\Delta P}{\Delta L} - \gamma \sin\alpha = \rho a_L$$

$$-\frac{dP}{dL} - \gamma \frac{dz}{dL} = \rho a_L$$

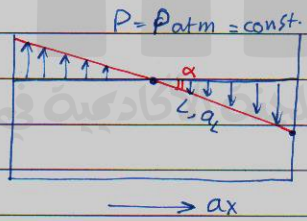
for γ = const.

$$\boxed{-\frac{d}{dL}(P+\gamma z) = \rho a_L} \quad \text{---- (5-3)}$$

Euler equation for motion of a fluid.

* Applications :-

Accelerometer



$$\frac{dP}{dL} = 0$$

$a_L = a_x \cos\alpha$

$$-\frac{d}{dL}(\gamma z) = \rho a_x \cos\alpha$$

for γ = const.

$$\frac{dz}{dl} = -\frac{a_x \cos\alpha}{g}$$

and since $\frac{dz}{dl} = -\sin\alpha$

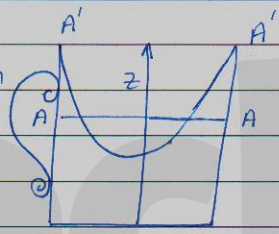
$$\boxed{\tan\alpha = \frac{a_x}{g}}$$

Accelerometer

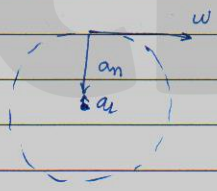
Rotation:-

Find pressure variation due to accelaration at w = const.

apply eq.n 5.3 cylindrecal coord.



$$-\frac{d}{dr}(P+\gamma z) = \rho\left(-\frac{v^2}{r}\right)$$



$$\frac{d}{dr}(P+\gamma z) = \rho r w^2$$

integrating:-

$$P+\gamma z = \rho\frac{v^2 w^2}{z} + \text{const.}$$

$$\boxed{\frac{P}{\gamma} + z - \frac{V^2}{2g} = \text{const.}} \quad ---- \quad (5-8)$$

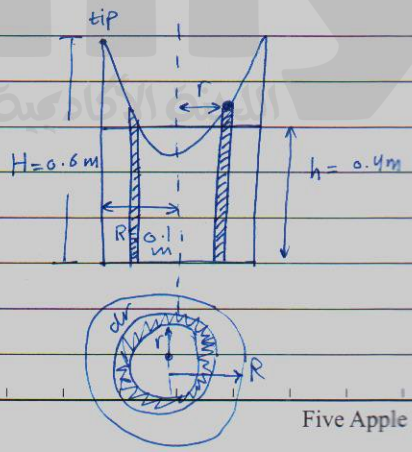
Example:-

Find w required for water to spill from the edge of the cylinder ?!?

applying eq. 5-8 on the free surface:-

$$\frac{P}{\gamma} + z - \frac{V^2}{2g} = \text{const}$$

substute at two differant points.



- at the tip:-

$Z = H$, $r = R$, $P = P_{atm}$

$$\frac{P_a}{\gamma} + H - \frac{w^2 R^2}{2g} = C \quad \text{---- (1)}$$

- at any point on the free surface

$$\frac{P_a}{\gamma} + Z - \frac{w^2 r^2}{2g} = C \quad \text{---- (2)}$$

$$Z = H - \frac{w^2}{2g}(R^2 - r^2) \quad \text{---- eq. of the surface}$$

Volume of water at spilling = initial volume of water $= \pi R^2 h_1$

$$\int_0^R 2\pi r Z \cdot dr = 2\pi \int_0^R H - \frac{w^2}{2g}(R^2 - r^2)\, r\, dr$$

$$= \pi R^2 \left[H - \frac{w^2 R^2}{4g} \right] = \pi R^2 h_1$$

solving for w:-

$$w = \frac{\sqrt{4g(H - h_1)}}{R} = \frac{\sqrt{4 \times 9.8\,(0.6 - 0.4)}}{0.1} = 28.01 \text{ rad/s}$$

Homework:- [Chapter 5]

7th	9th
5.28	4.45
51	59
69	67
78	73
85	79

5.3 Bernoulli's eq:-

Back to Euler's eq.n 5.3 :-

$$-\frac{d}{dl}(p+\gamma z) = \rho a_l$$

In Cartezian coord's :-

$$-\frac{d}{dx}(p+\gamma z) = \rho a_x \quad \text{(in the x-direction)}$$

$$-\frac{d}{dx}\left(\frac{p}{\gamma}+z\right) = \frac{a_x}{g}$$

Put $\frac{p}{\gamma}+z = h$

$$-\frac{dh}{dx} = \frac{a_x}{g}$$

Put $a_x = u\frac{du}{dx} + v\frac{du}{dy}$ (steady 2-D)

$$-\frac{dh}{dx} = \frac{1}{g}\left(u\frac{du}{dx} + v\frac{du}{dy}\right)$$

similarly; in the y-Direction:-

$$-\frac{dh}{dx} = \frac{1}{g}\left(u\frac{dv}{dx} + v\frac{dv}{dy}\right)$$

Put $u\frac{du}{dx} = \frac{d(u^2/2)}{dx}$ & $v\frac{dv}{dx} = \frac{d(v^2/2)}{dx}$

$$\frac{d}{dx}\left(h+\frac{u^2+v^2}{2g}\right) = 0$$

Put $V_s^2 = u^2+v^2$

$$\frac{d}{dx}\left(h+\frac{V_s^2}{2g}\right) = 0$$

similarly :-

$$\frac{d}{dy}\left(h+\frac{V_s^2}{2g}\right) = 0$$

C → because $\frac{d}{dy} = 0$

$$h + \frac{V_s^2}{2} = C$$

$$\boxed{\underbrace{\frac{P}{\gamma}}_{\text{Static head}} + \underbrace{Z}_{\text{Elevation head}} + \underbrace{\frac{V_s^2}{2g}}_{\text{velocity head}} = C}$$

Bernoulli eq.

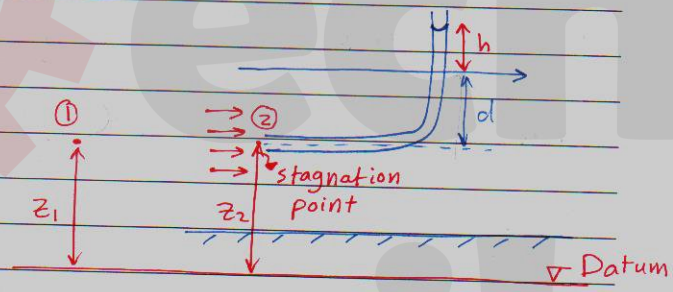
- steady
- Non viscous
- Irrotational.
- Incompresible (ρ. const).
- Flow along astream line.

$$P + \gamma Z + \rho \frac{V^2}{2} = C$$

other form for Bernoulli eq.

5.4 Applications on Bernoulli's eq :-

Stagnation tube :-



Bernoulli Between two points :-

$$\frac{P_1}{\gamma} + \cancel{Z_1} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \cancel{Z_2} + \frac{V_2^2}{2g}$$ { point ① & ② at same level }

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + 0$$ { Point ② ⇝ stagnation point → ∴ $V_2 = 0$ }

$$\frac{V_1^2}{2g} = \frac{P_2}{\gamma} - \frac{P_1}{\gamma}$$

$$= \frac{\cancel{\gamma}(h+d)}{\cancel{\gamma}} - \frac{\cancel{\gamma} d}{\cancel{\gamma}} = h$$

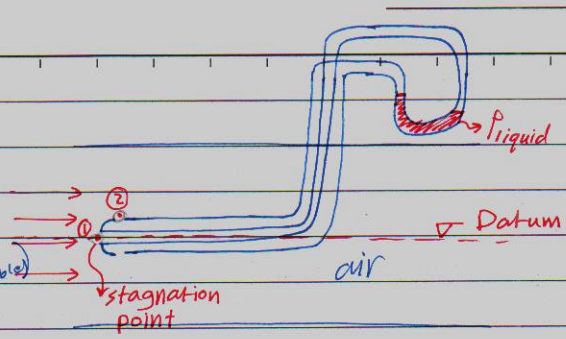
$$\boxed{V_1 = \sqrt{2gh}}$$

* Pitot Tube :-

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}}^{\,0} + \cancel{Z_1} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{Z_2}^{\,0}$$

(stag. point)

(small, ngelegable)



$$\frac{V_2^2}{2g} = \left(\frac{P_1}{\gamma} - \frac{P_2}{\gamma}\right)$$

$$\boxed{V_2 = \sqrt{2\,\Delta P/\rho_{air}} = \sqrt{2\,\frac{\rho_{liquid}}{\rho_{Air}}\,gh}}$$

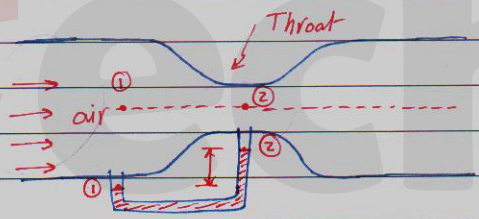
$\frac{\Delta P}{\rho} = gh$

$\frac{2\Delta P}{\rho_{air}}$

* Venturi tube (OR venturi meter)

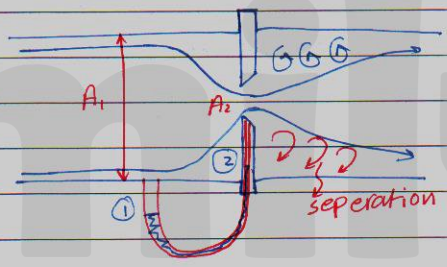
$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1}^{\,0} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{Z_2}^{\,0}$$

$$\cancel{\rho} V_1 A_1 = \cancel{\rho} V_2 A_2$$

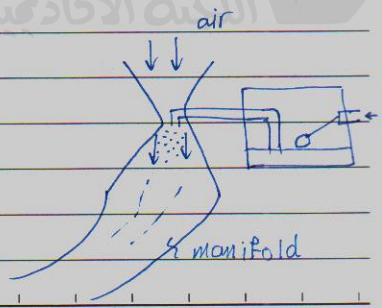


* Orifice :-

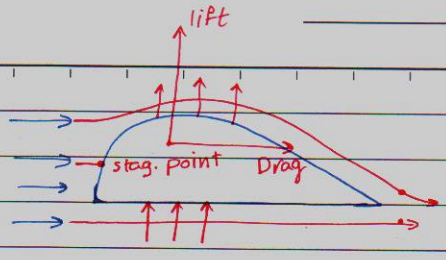
$$Q = \underbrace{C}_{\text{Discharge coeff.}} A_2 \sqrt{\frac{2}{\rho}\left[\frac{P_1 - P_2}{1 - A_2^2/A_1^2}\right]}$$



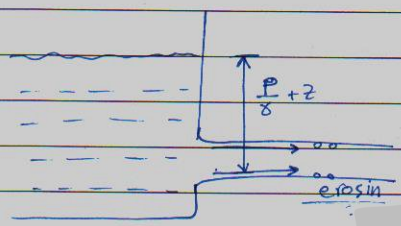
* Carburettor :-



* Air Foil :-



* Cavitation :-

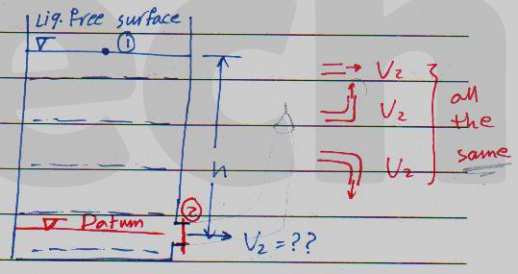


Flow out of a reservoir :-

$$\cancel{\frac{P_1}{\gamma}}^{\,0} + \cancel{\frac{V_1^2}{2g}}^{\,0} + z_1 = \cancel{\frac{P_2}{\gamma}}^{\,0} + \frac{V_2^2}{2g} + \cancel{z_2}^{\,0}$$

gage — 0, free surface of reservior — gage

$$V_2 = \sqrt{2gh}$$

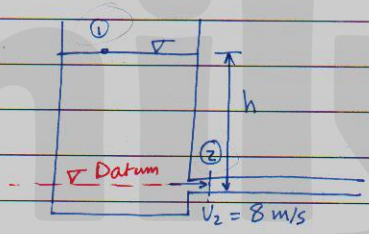


Example, Problem 5-47 :-

Find the gage pressure at ② ?!

$$\frac{P_{atm}}{\gamma} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2}$$

free surface — Datum



$$\underbrace{\frac{P_2 - P_{atm}}{\gamma}}_{P_{gage}} = z_1 - \frac{V_2^2}{2g}$$

$$P_{gage} = \gamma\left(z_1 - \frac{V_2^2}{2g}\right) = 9810\left(15 - \frac{64}{2 \times 9.81}\right) = 115{,}150\ Pa = 115.150\ kPa$$

Chapter 6

Momentum principle :-

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{c.v} b\,\rho\,d\forall + \int_{c.s} b\,\rho\,\bar{V}\cdot d\bar{A} \quad \text{----- } 4\text{-}33b$$

Put B = momentum (MV)

$b = V$

$$\frac{d(MV)_{sys}}{dt} = \frac{d}{dt}\int_{c.v} V\rho\,d\forall + \int_{c.s} V\rho\,\bar{V}\cdot d\bar{A} \quad \text{---- } 6.4$$

$$\frac{d(MV)_{sys}}{dt} = \sum \vec{F}_{external}$$

$$= \sum F_s + \sum F_B \longrightarrow \text{Mass}, \text{Charge}$$

Force effecting surface : pressure

Reaction

friction

forces on the fluid

$$\boxed{\sum F_s + \sum F_B = \int_{c.s} \bar{V}\rho\vec{V}\cdot d\vec{A} + \frac{d}{dt}\int_{c.v} \bar{V}\rho\cdot d\forall}$$

→ Establish a control volume

Establish a reference frame

عامودية $\bar{V}\cdot d\bar{A}$ is referenced to the c.v

مائلة $\vec{V}$ is referenced to inertial refrence frame

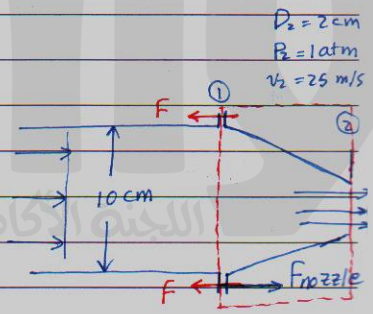
Example, Problem 6.12 :-

calculate

$V_1 = ?!$

$P_1 = ?!$; if $\rho = 999\ kg/m^3$

Force by the nozzle on the Fluid.



F: Reaction Force for bolt

continuity :-

$$V_1 A_1 = V_2 A_2$$

$$V_1 = V_2 \frac{A_2}{A_1} = V_2 \left(\frac{2}{10}\right)^2 = 0.04(25) = 1\ m/s$$

Bernoulli :-

$$P_1 + \rho \frac{V_1^2}{2} = P_2 + \rho \frac{V_2^2}{2}$$

$$V_2^2 - (0.04\, V_2)^2 = \frac{2}{\rho}(P_1 - P_2)$$

$$V_2 = \frac{1}{0.9984}\sqrt{\frac{2}{\rho}(P_1 - P_2)}$$

$$\frac{2}{\rho}(P_1 - P_2) = (0.9984)^2 (25)^2 = 623\ m^2/s^2$$

$$P_1 - P_2 = \frac{\rho}{2}(623) = 311.2\ kPa \qquad \rho = 999\ kg/m^3$$

$$\Sigma F_s + \underbrace{\cancel{\Sigma F_B}}_{\substack{0 \\ \text{neglect} \\ \text{mass} \\ \text{inside} \\ \text{nozzle}}} = \int_{c.s} v \rho \bar{V} \cdot d\bar{A} + \underbrace{\cancel{\frac{d}{dt}\int_{c.v} v \rho\, dV}}_{0\ \text{steady}}$$

$$\Sigma F_s = \sum_{c.s} \bar{v} \cdot \rho \bar{V} \cdot dA \qquad (\text{2-D, velocity const. at } ① \,\&\, ②)$$

F_s : pressure $(P_1 A)$

Reaction (F_{nozzle})

friction (neglect)

$$P_1 A_1 + F_{nozzle} = -V_1 \rho_1 V_1 A_1 + V_2 \rho V_2 A_2$$

$$P_1 V_1 A_1 = \rho V_2 A_2 = \dot{m}$$

$$= 999 \times 25 \times \pi \frac{(0.02)^2}{4} = 7.846\ kg/s$$

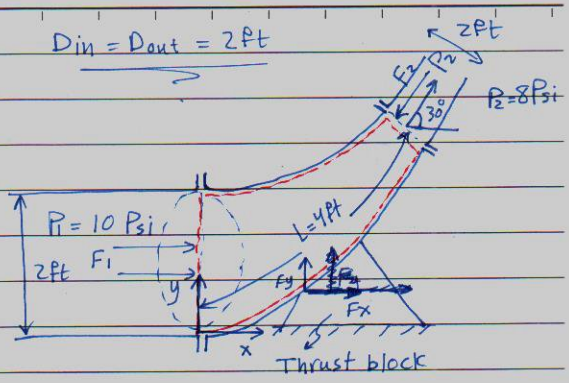
$$F_{nozzle} = -P_1 A_1 + \dot{m}(V_2 - V_1)$$

$$= -311{,}188.5 \times \pi \frac{(0.1)^2}{4} + 7.846\,(25-1) = -2{,}256\ N$$

$Psi = \frac{Lbf}{In^2}$

Example, Problem (6-37) :-

$D_{in} = D_{out} = 2ft$



$Q = 31.4 \ ft^3/s$

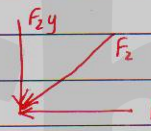
w of empty bend = 300 lb.f

① what is the Force Fy That holds the Bend in position ?

Extra ② what is the Force Fx ?

1. select C.V as shown
2. select Ref.
3. apply Mom eq. in the y-direction taking $\frac{d}{dt} = 0$

$$\cancel{F_{1y}}^{0} - F_{2y} + F_y - W = -\cancel{U_{1y}\rho V_1 A_1}^{0} + V_{2y}\rho V_2 A_2$$



$$0 - P_2 A_2 \sin\alpha + F_y - W = V_2 \sin\alpha (\rho Q)$$

solving for Fy

$$F_y = \rho Q (V_2 \sin\alpha) + P_2 A_2 \sin\alpha + W$$

$$V_2 = \frac{Q}{A_2} = \frac{31.4}{\pi \frac{2^2}{4}} = 9.995 \ ft/s$$

$$L \times \frac{\pi D^2}{4} = V$$

$$V \times \rho = m$$

$$m \times g = W$$

$$W = L \frac{\pi D^2}{4} \times \rho \times g \ \#$$

$$W = 300 + \rho g L \frac{\pi D^2}{4} = 549.5 \ lbf$$

$\rho_{water} = 1.94 \ slug/ft^3$

??

$$\therefore F_y = 1.94 \times 31.4 (9.995) \sin 30 + 8 (144) \times \frac{\pi (2)^2}{4} \sin 30 + 549.5 = \boxed{3,198 \ lbf}$$

② clculate Fx ?!

$$\sum F_x = \sum_{c.s} V_x (\rho \bar{V} \cdot \bar{A}) + \underbrace{\frac{d}{dt} \int_{c.v} V_x \rho \, d\forall}_{0 \text{ steady}}$$

$$F_1 - F_{2x} + F_x = V_1 (-\rho \bar{V} \cdot \bar{A}) + V_{2x} (\rho V_2 A_2)$$

$$P_1 A_1 - P_2 A_2 \cos\alpha + F_x = -V_1 \rho Q + V_2 \cos\alpha \, \rho (Q)$$

from continuity :- $V_1 = V_2 = V$, $A_1 = A_2 = A$

solving for Fx :-

$$F_x = V\rho Q (\cos\alpha - 1) - A (P_1 - P_2 \cos\alpha)$$

From previous section:- $V = 9.995$ ft/s

$$F_x = 9.995 \times 1.94 \times 31.4 (\cos 30 - 1) - \frac{\pi}{4} z^2 (10 \times 144 - 8 \times 144 \cos 30)$$

$$\boxed{F_x = -1,471 \text{ lbf}}$$

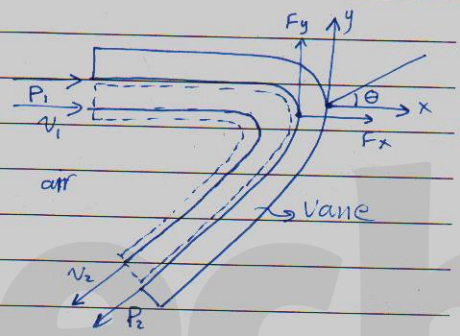
الزاوية اللي : θ
بتحرف فيها الـ vane الـ jet

* Defflection of a jet by a fixed Vane :-

① Establish a control volume

② a coord. system.

Find exerted force by vane on jet



assumptionts:-

$P_1 = P_2 = P$

$V_1 = V_2 = V$

steady

X Apply x-mom. eq-n :-

حكينا $P_2 A_2$ بدون اعتبار الزاوية
لانه اخترنا V ك عامودي
على section 2.

$$P_1 A_1 + F_x + \cancel{W_g} - P_2 A_2 = V_{1x} \rho (-V_1 A_1) + V_{2x} (\rho V_2 A_2)$$

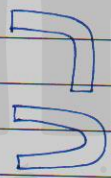
$$\underbrace{\cancel{P_1 A_1 - P_2 A_2}}_{0} + F_x = V_{1x} \rho (-V_1 A_1) + V_{2x} (\rho V_2 A_2)$$

$$F_x = -V_{1x} \rho V_1 A_1 + V_{2x} \rho V_2 A$$

$$F_x = \dot{m} V (\cos\theta - 1)$$

if $\theta = 90$: $F_x = -\dot{m} V$

$\theta = 180$: $F_x = -2 \dot{m} V$



similarly - in the y-direction

$$0 + F_y - \cancel{W} = \cancel{V_{1y} \rho (-V_1 A_1)} + \underbrace{V_{2y} \rho (V_2 A_2)}_{\dot{m}}$$

$$F_y = \dot{m} V \sin\theta$$

Sugg. problem :-

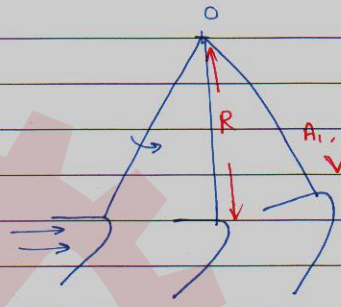
7th : 17, 23, 29, 38, 48, 81

9th : 20, 26, 28, 45, 55, 87

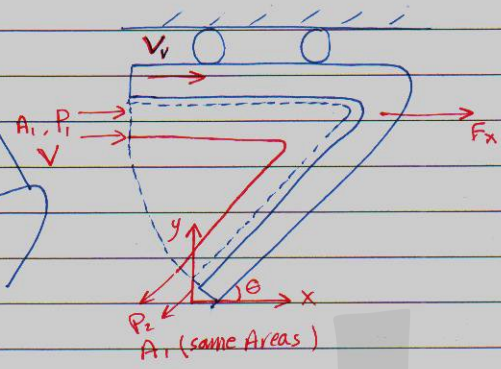
2nd Exam 21/7/2013

Ch. 5, 6, 7

Moving Vane with constant velocity :-



Turbine wheel.



X-moment. eq-n :-

$P_1 = P_2 = P$

$A_1 = A_2 = A$

$$0 + F_x = \sum_{C.V} \dot{m}\, \bar{V}\cdot\bar{A} = \overbrace{(V-V_v)\rho\left[-(V-V_v)A\right]}^{in} + \overbrace{(V-V_v)\cos\theta\,\rho\,(V-V_v)A}^{out}$$

pressure force are the same on the both sides of C.V

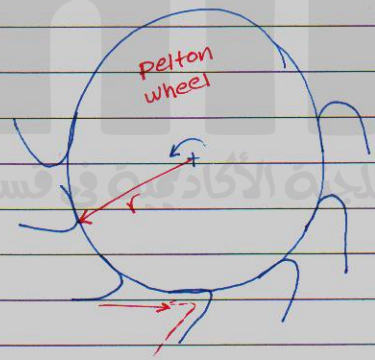
$F_x = -\rho\,(V-V_v)^2 A(1-\cos\theta)$

$F_x = \rho\,(V-V_v)^2 A(\cos\theta - 1)$

→ Force On the Fluid.

Moment $= F_x \times r$

power $= F_x \times \bar{r} \times \bar{w}$



Chapter 7

* The Energy principle :-

7.1 The Energy Equation :-

Recall eqn : 4.21a

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{c.v} b\rho\, d\forall + \sum_{c.s} b\rho\, \bar{V}\cdot\bar{A}$$

replace B_{sys} with E, $b = \frac{E}{m} = e$

$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{c.v} e\rho\, d\forall + \sum_{c.s} e\rho\, \bar{V}\cdot\bar{A}$$

1st law :-

$$\frac{dE_{sys}}{dt} = \underbrace{\dot{Q}}_{\text{rate of heat in}} - \underbrace{\dot{W}}_{\text{rate of work out}}$$

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{c.v} e\rho\, d\forall + \sum_{c.s} e\rho\, \bar{V}\cdot\bar{A}$$

$$= \frac{d}{dt}\int_{c.v} (e_k + e_p + u)\rho\, d\forall + \sum_{c.s} (e_k + e_p + u)\rho\, \bar{V}\cdot\bar{A}$$

$$= \frac{d}{dt}\int_{c.v} \left(\frac{V^2}{2} + gz + u\right)\rho\, d\forall + \sum_{c.s} \left(\frac{V^2}{2} + gz + u\right)\rho\, \bar{V}\cdot\bar{A}$$

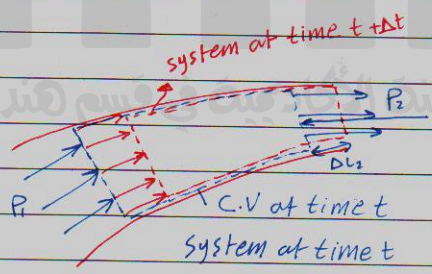
Flow work :-

External pressure force on Right hand side $= P_2 A_2$

distance travelled by fluid on R.H.S $= \Delta L_2 = V_2 \Delta t$

work done on sorrounding fluid

$= \Delta W_{f_2} = P_2 A_2 V_2 \Delta t$



$$\dot{W}_{F2} = \frac{P_2 V_2 A_2 \Delta t}{\Delta t} = V_2 P_2 A_2 = P_2 \bar{V}_2 \bar{A}_2$$

similarly, the rate of work on the L.H.S :-

$$\dot{W}_{F1} = P_1 \bar{V}_1 \bar{A}_1$$

∴ Flow power :-

$$\boxed{\dot{W}_F = \sum_{c.s} P \vec{V} \cdot \vec{A}}$$

Shaft work :-

any work other flow work

$$\dot{W} = \dot{W}_s + \dot{W}_F$$

Now;

$$\dot{Q} - \dot{W}_s - \sum_{c.s} \frac{P}{\rho} \rho \vec{V} \cdot \vec{A} = \frac{d}{dt} \int_{c.v} \left(\frac{v^2}{2} + gz + u\right) \rho \, d\forall + \int_{c.s} \left(\frac{v^2}{2} + gz + u\right) \rho \bar{V} \cdot \bar{A}$$

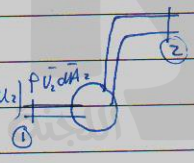
for 2-D case :- (∫ نضع اشارة ∑ بدل كل اشارة ∑)

$$\dot{Q} - \dot{W}_s - \sum_{c.s} \frac{P}{\rho} \rho \bar{V} \cdot \bar{A} = \frac{d}{dt} \int_{c.v} \left(\frac{v^2}{2} + gz + u\right) \rho \, d\forall + \sum_{c.s} \left(\frac{v^2}{2} + gz + u\right) \rho \bar{V} \cdot \bar{A}$$

simplifications :-

~~steady state~~ steady state [2-D]

$$\dot{Q} - \dot{W}_s + \int_{A_1} \left(\frac{P_1}{\rho} + \frac{v_1^2}{2} + gz_1 + u_1\right) \rho V_1 \, dA_1 = \int_{A_2} \left(\frac{P_2}{\rho} + \frac{v_2^2}{2} + gz_2 + u_2\right) \rho V_2 \, dA_2$$



$$\dot{Q} - \dot{W}_s + \int_{A_1} \left(\frac{P_1}{\rho} + gz_1 + u_1\right) \rho \bar{V}_1 \cdot d\bar{A}_1 + \int_{A_1} \rho \frac{\vec{V}_1^3}{2} d\bar{A}_1$$

$$= \int_{A_2} \left(\frac{P_2}{\rho} + gz_2 + u_2\right) \rho V_2 \, dA_2 + \int_{A_2} \rho \frac{\vec{V}_2^3}{2} d\bar{A}_2$$

For uniform flow:-

$$\left(\frac{P}{\rho} + gz\right) = C$$

if T is const. $\Rightarrow$ u is const.

$$\dot{Q} - \dot{W}_s + \left(\frac{P_1}{\rho} + gz_1 + u_1\right)\int_{A_1} \rho \bar{V}_1^3 \, dA_1 + \int_{A_1} \rho \frac{V_1^3}{2} \, dA_1 = \alpha_1 \rho \frac{\bar{V}_1^3}{2} A_1$$

$$= \left(\frac{P_2}{\rho} + gz_2 + u_2\right)\int_{A_2} \rho \bar{V}_2 \cdot d\bar{A}_2 + \int_{A_2} \rho \frac{V_2^3}{2} \, dA_2 = \alpha_2 \rho \frac{\bar{V}_2^3}{2} A_2$$

Define α :-

$\alpha \equiv$ kinetic energy correction Factor

$\alpha = 1$ for uniform flow

$\alpha \geq 1$ for Non-uniform flow

$\alpha = 2$ for laminar flow

$\alpha = 1.05$ for Turb. flow

Take $\alpha = 1$ for all Turb. flow as aproximation.

$$\left(\frac{\dot{Q} - \dot{W}_s}{\dot{m}}\right) + \left(\frac{P_1}{\rho} + gz_1 + u_1 + \frac{V_1^2}{2}\right) = \frac{P_2}{\rho} + gz_2 + u_2 + \frac{V_2^2}{2}$$

put :

$W_s = W_t - W_p$

t: turbin

p: pump

$$\frac{\dot{W}_p}{\dot{m}g} + \frac{P_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{\dot{W}_t}{\dot{m}g} + \frac{P_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \underbrace{\left[\frac{u_2 - u_1}{g} - \frac{\dot{Q}}{\dot{m}g}\right]}_{\text{head loss, } h_L}$$

pump head, h_p — الارتفاع الذي يعطيه الماء عندما يضخه

Turbine head, h_t

$$[h_p] = \frac{(kg \cdot m/s^2)\, m/s}{(kg/s)\, m/s^2} = m$$

هو الارتفاع اللازم للماء حتى يسقط على الموتور ويحوله تيار كهربائي

$$\boxed{\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_t + h_L}$$ Energy Eqn.

h_L = for stright pipe $= f \frac{L}{D} \frac{V^2}{2g}$

for fittings $= \frac{C V^2}{2g}$

Chapter 7

* The Energy principle :-

7.1 The Energy Equation :-

Recall eqn : 4.21a

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{c.v} b\rho\, d\forall + \sum_{c.s} b\rho\, \bar{V}\cdot\bar{A}$$

replace B_{sys} with E, $b = \frac{E}{m} = e$

$$\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{c.v} e\rho\, d\forall + \sum_{c.s} e\rho\, \bar{V}\cdot\bar{A}$$

1st law :-

$$\frac{dE_{sys}}{dt} = \underbrace{\dot{Q}}_{\text{rate of heat in}} - \underbrace{\dot{W}}_{\text{rate of work out}}$$

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int_{c.v} e\rho\, d\forall + \sum_{c.s} e\rho\, \bar{V}\cdot\bar{A}$$

$$= \frac{d}{dt}\int_{c.v} (e_k + e_p + u)\rho\, d\forall + \sum_{c.s} (e_k + e_p + u)\rho\, \bar{V}\cdot\bar{A}$$

$$= \frac{d}{dt}\int_{c.v} \left(\frac{V^2}{2} + gz + u\right)\rho\, d\forall + \sum_{c.s} \left(\frac{V^2}{2} + gz + u\right)\rho\, \bar{V}\cdot\bar{A}$$

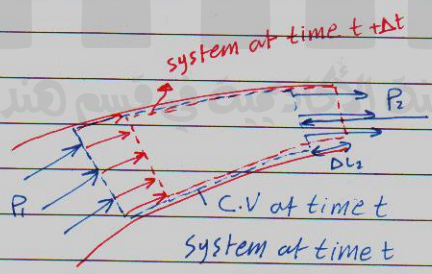
Flow work :-

External pressure force on Right hand side $= P_2 A_2$

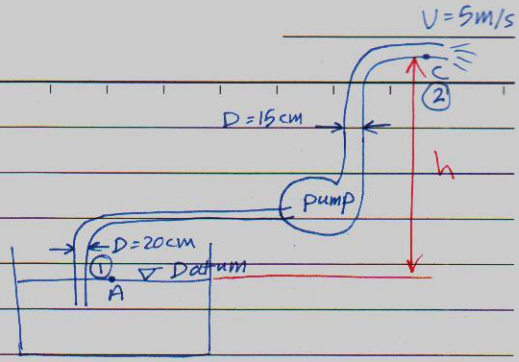
distance travelled by fluid on R.H.S $= \Delta L_2 = V_2 \Delta t$

work done on sorrounding fluid

$= \Delta W_{f_2} = P_2 A_2 V_2 \Delta t$



Example, prob 7-34



Given:

Power delivered to pump 35 kw

$\eta_p = 70\%$

$h_L = 2\frac{V_c^2}{2g}$

Find h ?!

Energy eq-n between 1 & 2

$$\cancel{\frac{P_1}{\gamma}}^{0} + \cancel{\frac{V_1^2}{2g}}^{0} + \cancel{Z_1}^{0} + h_p = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + Z_2 + \cancel{h_t}^{0} + h_L$$

$$h_p = \frac{V_2^2}{2g} + h + 2\frac{V_c^2}{2g}$$

$$\therefore h = h_p - 3\frac{V_2^2}{2g}$$

$h_t \rightarrow \eta$ نقسم على

$h_p \rightarrow \eta$ نضرب بـ

$$= \frac{\dot{w}_p}{\dot{m}g} \times \eta_p - 3\frac{V_2^2}{2g}$$

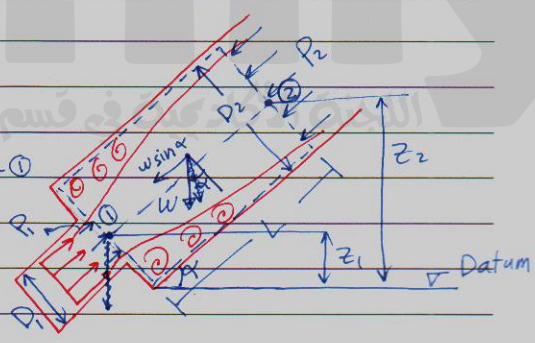
$$\left[\dot{m} = \rho V A = 1000 \times 5 \times \frac{\pi (0.15)^2}{4} = 88.36 \text{ kg/s}\right]$$

$$= \frac{35 \times 1000}{88.36 \times 9.81} \times 0.7 - \frac{3(5)^2}{2(9.81)}$$

$$= 28.26\text{m} - \frac{3(25)}{2(9.81)} = 24.4\text{m}$$

7.3 Other applications on Energy, momentum, and continuity eqn :-

Sudden expansion



① Energy Eqn

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + h_L \quad \text{---} ①$$

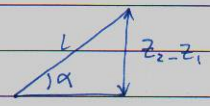
→ this eq-n called "modified Bernoulli eq-n"

② Mom. eq-n :-

$$\sum_{c.s} F = \sum_{c.s} V \rho \bar{V} \cdot \bar{A}$$

$$P_1 A_1 - P_2 A_2 - W \sin\alpha = -\rho V_1^2 A_1 + \rho V_2^2 A_2 \quad ---- ②$$

$$W = \gamma \forall = \gamma A_2 L$$



$$\frac{P_1}{\gamma} + \frac{P_2}{\gamma} - \underbrace{L \sin\alpha}_{z_2 - z_1} = \frac{V_2^2}{g} - \frac{V_1^2}{g} \frac{A_1}{A_2}$$

③ continuity :-

$$\dot{m} = \rho V_1 A_1 = \rho V_2 A_2 \rightarrow \frac{A_1}{A_2} = \frac{V_2}{V_1}$$

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

sugg. problem :-

7th_ed.

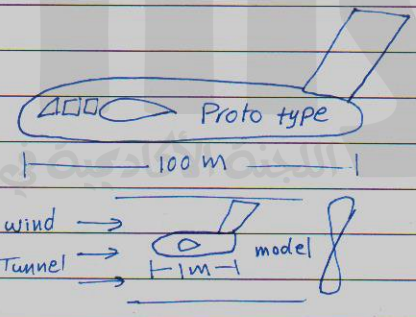
7.15, 28, 37, 47, 53, 68.

9th_ed

7.27, 33, 50, 60, 65, 80.

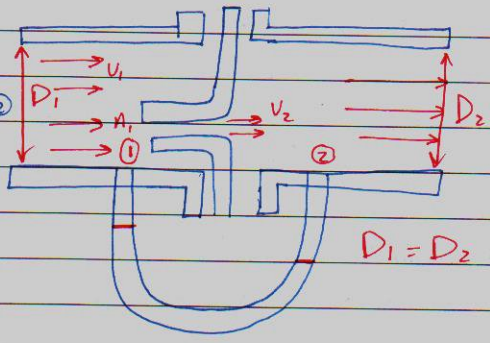
Chapter 8

- Dimensial Analysis and Similitude
- Modeling
- Correlation of experemantal data



Example:-

$P_1 - P_2 \propto V_1$



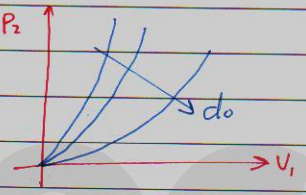
Apply Bernoulli equation between ①&②

$$P_1 + \rho \frac{V_1^2}{2} = P_2 + \rho \frac{V_2^2}{2}$$

OR $$\frac{P_1 - P_2}{\rho \frac{V_1^2}{2}} = \frac{V_2^2}{V_1^2} - 1$$

$$\underbrace{\left[\frac{P_1 - P_2}{\rho V_1^2/2}\right]}_{C_p} = \frac{N/m^2}{\frac{kg}{m^3} \cdot \frac{m^2}{s^2}} = \text{Dimension less}$$

C_p: pressure coefficient.

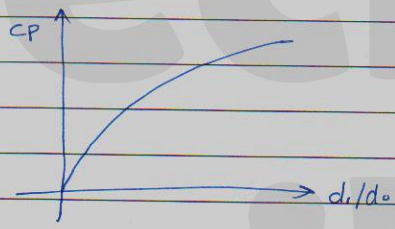


continuity eq. $\frac{V_2}{V_1} = \frac{A_1}{A_2}$

$$\left[\frac{V_2}{V_1}\right] = f\left(\frac{d_1}{d_0}\right)^2$$

$$\frac{P_2 - P_1}{\rho V_1^2/2} = \left[f\left(\frac{d_1}{d_0}\right)^2\right]^2 - 1$$

$$C_p = f(d_1/d_0)$$



$\Delta P, d_1, d_0, V_1, \rho$ (5 Variables) $\Rightarrow \left(C_p, \frac{d_1}{d_0}\right)$ 2 variables

Buckingham's Theorem

If the number of variables involved in a process is n and the number of basic dimensions that form those variables is m, then the number of dimensionless groups needed to correlate those variables is (n-m)

m : mass | T : time
L : length | F : Force

Two method's :-

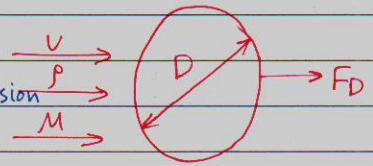
① step by step method (inspection method) → m L T (basic dimensions)
② Exponent method. → F L T

Examples:- problem [8-2]

Using the step by step method, find the dimensionless groups that expresses drag force F_D in terms of the physical parameters which are V, ρ, μ, D for flow about a sphere.

sol.

Find Dimensionless for each variables in terms of the system of Basic dimension MLT



$$\left[F_D = \frac{ML}{T^2}\right]$$

$$[V] = \frac{L}{T}$$

$$[\rho] = \frac{M}{L^3}$$

$$[\mu] = \frac{M}{LT}$$

$$[D] = L$$

$$F_D = f(V, \rho, \mu, D)$$

dependent variable — Independent variable

- Eliminate the Basic dimenions of the dependant variables one by one

$$\frac{F_D}{\rho} = f_2\left(V, \frac{\mu}{\rho}, D\right)$$

$$\left[\frac{F_D}{\rho}\right] = \frac{\frac{ML}{T^2}}{\frac{M}{T^3}} = \frac{L^4}{T^2}$$

$$\left[\frac{\mu}{\rho}\right] = \frac{\frac{M}{L.T}}{\frac{M}{L^3}} = \frac{L^2}{T}$$

Eliminate Time dimension

$$\frac{F_D}{\rho V^2} = f_3\left(\frac{\mu}{\rho V}, D\right)$$

$$\left[\frac{F_D}{\rho V^2}\right] = \frac{L^4/T^2}{L^2/T^2} = L^2$$

$$\left[\frac{\mu}{\rho V}\right] = \frac{L^2/T}{L/T} = L$$

Eliminate length dimension by dividing by D

$$\left[\frac{F_D}{\rho V^2 D}\right] \quad \text{Dimensionless}$$

$$\frac{F_D}{\rho V^2 D^2} = f_4\left(\frac{\mu}{\rho V D}\right)$$

$$\frac{F_D}{\rho V^2 D^2} = f_4 \frac{1}{Re}$$

* The exponent Method:-

solve Example 8.2 using this method using FLT as primary variables ?!

- functional relation ship

$$F = f(V, \rho, \mu, D) \quad \{\text{based on intuition}\}$$

$n = 4$, $m = 3$

- put the function in the form of a power series

$$F = f(V^a, \rho^b, \mu^c, D^d) \qquad [\rho] = \frac{[F]/[a]}{L^3} = \frac{F/\frac{L}{T^2}}{L^3} = \frac{F\,T^2}{L^4}$$

$$F = \left(\frac{L}{T}\right)^a \left(\frac{FT^2}{L^4}\right)^b \left(\frac{FT}{L^2}\right)^c (L)^d \qquad [\mu] = \frac{[N]\cdot S}{L^2} = \frac{FT}{L^2}$$

- collect similar terms:-

$$F = \frac{L^{(a-4b-2c+d)}\, F^{b+c}}{T^{a-2b-c}}$$

- collect similar exponents :

for F: $b + c = 1$ --- ①

for L : $a - 4b - 2c + d = 0$ --- ②

for T : $a - 2b - c = 0$ --- ③

solve ① & ② & ③ in terms (n-m) unknowns ; i.e in terms of 1 unknown.

Take the one that appears most in the equations, such as C :

put the eq.ns in matrix form

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 1 & -2 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ d \end{pmatrix} = \begin{pmatrix} 1-c \\ 2c \\ c \end{pmatrix}$$

check the determinate of the matrix is not zero

$0(0+2) - 1(0-1) + 0(-2+4) = 1$, ok :P

hence, a unique solution is obtainable.

from ①

$b = 1 - c$

from ③

$a = 2(1-c) + c = 2 - c$

from ③

$d = -2 + c + 4(1-c) + 2c = 2 - c \Rightarrow a = d$

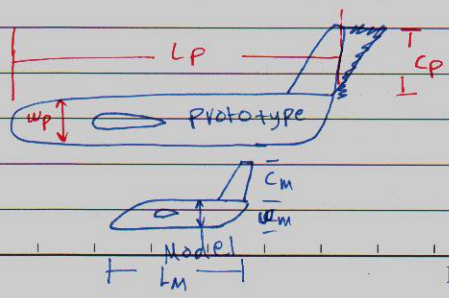
$$F = f\left(V^{(2-c)}, \rho^{(1-c)}, \mu^{c}, D^{(2-c)}\right)$$

$$F = V^2 \rho D^2 f_1\left(\frac{\mu}{V \rho D}\right)$$

$$\underbrace{\frac{F}{\rho V^2 D^2}}_{C_p} = f_1\underbrace{\left(\frac{\mu}{\rho V D}\right)}_{\frac{1}{Re}}$$

$\boxed{8.6}$ Similitude:-

Geometrical Dynamic



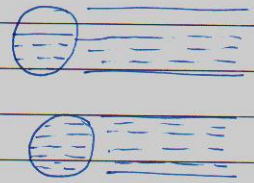
* Geometrical similarity :-

$\frac{l_m}{l_p} = \frac{w_m}{w_p} = \frac{C_m}{C_p} = L_p$ scale ratio

$\frac{A_m}{A_p} = A_r = L_r^2$

$\frac{V_m}{V_p} = V_r = L_r^3$

* Dynamic similarity :-
  - partially submerged
  - Fully submerged

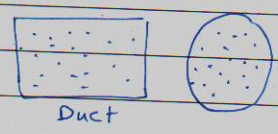


$\frac{F_m}{F_p} = \text{const.}$

spill way of a dam

Chapter 10

* Flow in conduits [pipes]



recall modified Bernoulli Eq.n :-

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + Z_1 = \frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + Z_2 + h_L \quad \text{if } [D_1 = D_2]$$

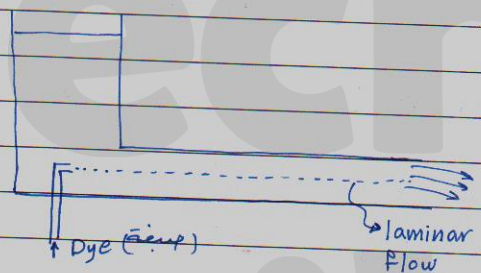
$$\frac{P_1}{\gamma} + Z_1 = \frac{P_2}{\gamma} + Z_2 + h_L$$

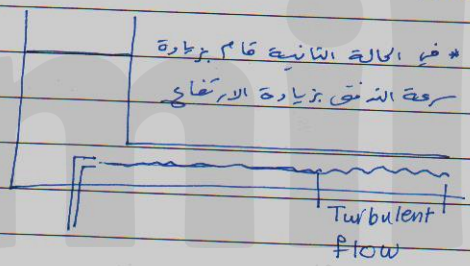
Laminar Turbulent

Osbunn Reynold (1883) Experiment

if $\frac{\rho U D}{\mu} < 2000$ laminar (Reynolds Number)

$2000 < \frac{\rho U D}{\mu} < 3000$ Transition





* في الحالة الثانية قام بزيادة سرعة التدفق بزيادة الارتفاع

Darcy - wiesbach :-

friction factor, f → lam. $f = \frac{64}{Re}$

$h_L \equiv h_f = f \frac{L}{D} \frac{V^2}{2g}$ → Turb. $\frac{1}{\sqrt{f}} = 2 \log (Re \sqrt{f}) - 0.8$ {implicit eq.n}

Petukhov (explicit)

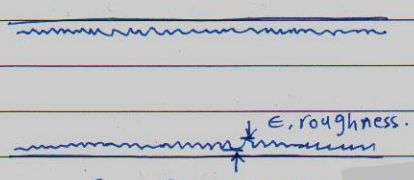
$f = (0.79 \ln Re_D - 1.64)^{-2}$ $3000 \le Re_D \le 5 \times 10^6$

Colebrook - white (explicit)

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

Define : Relative Roughness

$$= \frac{\text{Roughness}}{\text{Diameter}}$$



$\frac{\epsilon}{D}$ = Relative roughness.

* Minor losses [in fitting, valves, ...] :-

$$h = \frac{K V^2}{2g}$$

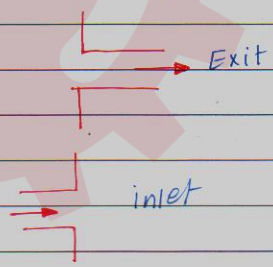
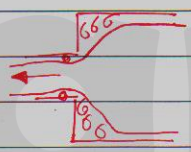
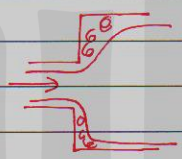


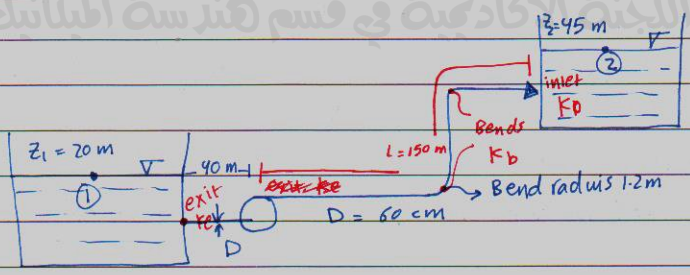
Table 14-3

$$h = \alpha \frac{V^2}{2g}$$





Example, Problem 10-96 :-



Calculate the pump, P, if

$Q = 1\ m^3/s$, Fuel oil $(S.G = 0.94)$, $\eta_p = 70\%$

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + Z_1 + h_p = \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + Z_2 + \sum h_L$$

gage, free surface reser.

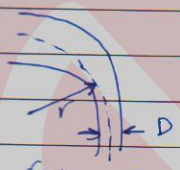
$\sum h_L$ → major, minor → exit, bends, inlet

$$20 + h_p = Z_2 + \sum h_L$$

$$h_p = 25 + f\frac{L}{D}\frac{V^2}{2g} + \frac{V^2}{2g}\left(K_e + 2K_b + K_o\right)$$

$$= 25 + \frac{V^2}{2g}\left(f\frac{L}{D} + K_e + 2K_b + K_o\right)$$

دائماً تأخذ على أنها ①.



$r/d = 0.19$

$$25 + \frac{V^2}{2g}\left(f\frac{L}{D} + 0.5 + 2\times 0.19 + 1\right)$$

~~K_s = raghnous = 0.054~~

$$V = \frac{Q}{A} = \frac{1}{\frac{\pi}{4}(0.6)^2} = 3.54\ m/s$$

$$Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{3.54\times 0.6}{5\times 10^{-5}} = 4.25\times 10^4 \text{ ; Turb}$$

K_s = raghnous $= 0.054$

relative raghnous $= \frac{K_s}{D} = \frac{0.054}{0.6\times 10^3} = 0.00009$

from Mody chart, bisect, $Re = 4.25\times 10^4$ with 0.00009

find $f = 0.021$

$$h_p = 25 + 0.637(0.5 + 0.38 + 1 + 6.65)$$
$$= 30.4\ m$$

$$P = \frac{Q\gamma h_p}{\eta} = \frac{1\times 0.94\times 9{,}810\times 30.4}{0.7} = 400\ kW.$$

⇐ متى نجد الـ f بدون استخدام الـ chart :

* colebrook - white eq.n expressed by Swamee & Jain:-

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{K_s}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

Three types of problems are usually encountered

1) Determine h_f, given L, D & Q are known → previous example 10.96.
2) " " Q, " L, D & h_f " " .
3) " " D, " L, Q & h_f " " .

Case 2:-

- solve for V the expression of h_f

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

$$V = \sqrt{(2gh_f/L)\left(\frac{D}{f}\right)}$$

since:- $Re = \frac{VD}{\nu} = \frac{D^{3/2}}{\nu f^{1/2}} \left(\frac{2g h_f}{L}\right)^{\frac{1}{2}}$

$$Re f^{1/2} = \frac{D^{3/2}}{\nu} \left(\frac{2g h_f}{L}\right)^{\frac{1}{2}}$$

case 3:-

Determine D, Given L, h_f, Q are known.

- Guess f
- calculate $h_f = f \frac{L}{D} \frac{V^2}{2g}$

$$= f \frac{L}{D} \frac{(Q/A)^2}{2g} = \frac{f L Q^2}{2g(\pi/4)^2 D^5}$$

solve $D^5 = \frac{f L Q^2}{(0.785)^2 (2gh_f)}$

Check f :-

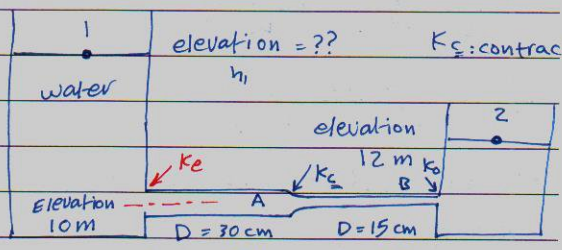
if not the same ase assumed, say f_1

$$\frac{D_1^5}{D^5} = \frac{f_1}{f}$$

* pipes in series with different diameter

Example, Prob 10.97 :-

if the discharge of the system is $Q = 0.15\ m^3/s$, find h_1 ??



$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + Z_1 = \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + Z_2 + \Sigma h_L$$

$$h_1 = 12 + \frac{V_A^2}{2g}\left(K_e + f\frac{L}{D}\right) + \frac{V_B^2}{2g}\left(K_c + f\left(\frac{L_2}{D_2}\right) + K_o\right)$$

$$V_A = \frac{Q}{A_A} = \frac{0.15}{\frac{\pi}{4}(0.3)^2} = 2.12\ m/s$$

$$\therefore \frac{V_A^2}{2g} = 0.229\ m$$

$$V_B = \frac{Q}{A_B} = \frac{0.15}{\frac{\pi}{4}(0.15)^2} = 8.488\ m/s$$

$$\therefore \frac{V_B^2}{2g} = 3.67\ m$$

$$\frac{D_A}{D_B} = \frac{15}{30} = 0.5$$

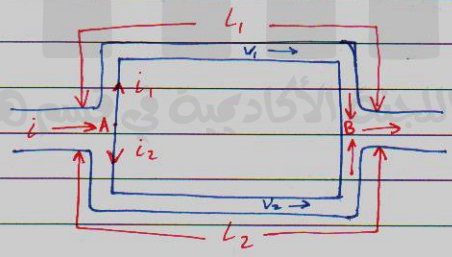
from Table 10.3, find by interpolation $K_c = 0.37$.

$$Z_1 = 12 + 0.229\left[0.5 + 0.02 \times \frac{20}{0.3}\right] + 3.67\left[0.37 + 0.02 \times \frac{10}{0.15} + 1\right] = 22.3\ m$$

* Pipes in Parallel :-

Apply Modified Bernoulli eqn :-

Between A & B at the upper branch :-



$$\frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} + Z_A = \frac{P_B}{\gamma} + \cancel{\frac{V_B^2}{2g}} + Z_B + h_{L_1}$$

similarly at the lower branch

$$\frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} + Z_A = \frac{P_B}{\gamma} + \cancel{\frac{V_B^2}{2g}} + Z_B + h_{L_2}$$

$h_{L_1} = h_{L_2}$

$$f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g}$$

$$\frac{V_1^2}{V_2^2} = \left(\frac{f_2}{f_1} \cdot \frac{L_2}{L_1} \cdot \frac{D_1}{D_2} \right)$$

Example, Based on prob. 10.106

assuming $L_1 = L_2$

$D_1 = D_2$

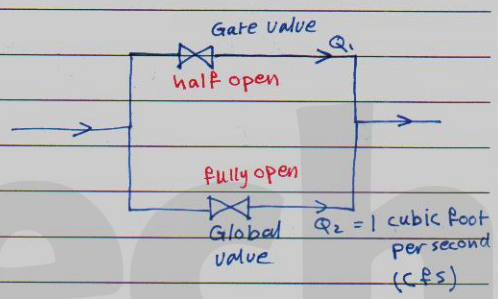
$f_1 = 4 f_2$

$V_1^2 = 4 V_2^2$

$V_1 = 2 V_2$

$Q \propto V$

$Q_1 = 2 Q_2 \Rightarrow Q_1 = 2$ cfs



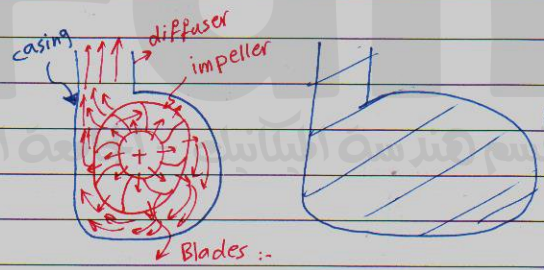
* Pumps :- two types :-

① centrifugal

② positive displacement

centrifugal

casing (stator) ← → Impeller (rotor)

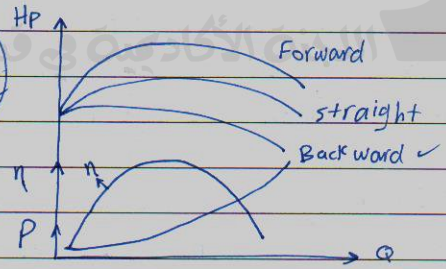


① Forward - curved (

② Backward - curved)

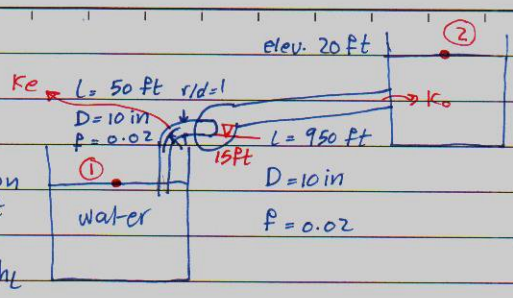
③ Radial (or straight) |

Pump characteristics :-



Example, problem 10.59

Find Q = ??



Apply Energy eq.n between ① & ②

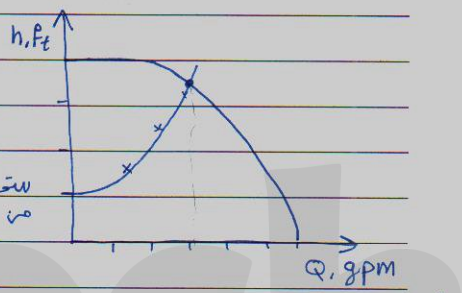
$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 + h_p = \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + z_2 + \Sigma h_L$$

$$10 + h_p = 20 + \frac{V_2^2}{2g}\left(K_e + K_o + f\frac{L}{D}\right)$$

$$h_p = 10 + \frac{Q^2}{2gA^2}\left(0.1 + 1.0 + 0.02\,\frac{1000}{10/12}\right)$$

ft ، in

$$A = \frac{\pi}{4}(10/12)^2 = 0.545\ ft^2$$



$$h_p = 10 + 1.31\,Q^2$$

$$h_p = 10 + 1.31\,\frac{Q^2}{(449)^2}$$

Q in cfs = 449 gpm

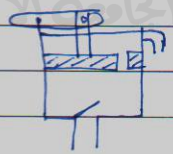
$$h_p = 10 + 6.51\times10^{-6}\,Q^2 \rightarrow Q \text{ in gpm}$$

put Q =	1000	2000	3000
h_p =	16.5	36	68.6

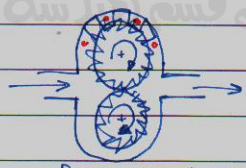
Operating point at Q = 2,950 gpm.

* Positive displacement pumps :-

Piston



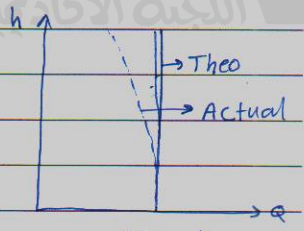
Gear



for highly viscous fluids

fuel, Oil, Diesel

~~screw~~



positive disp.