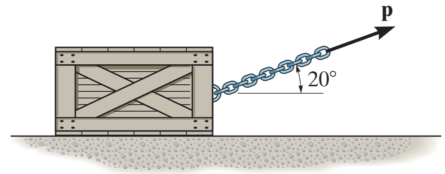
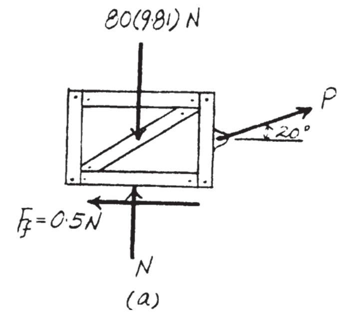
13–1.

The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. If the magnitude of **P** is increased until the crate begins to slide, determine the crate's initial acceleration if the coefficient of static friction is $\mu_s = 0.5$ and the coefficient of kinetic friction is $\mu_k = 0.3$.



SOLUTION

Equations of Equilibrium: If the crate is on the verge of slipping, $F_f = \mu_s N = 0.5N$. From FBD(a),



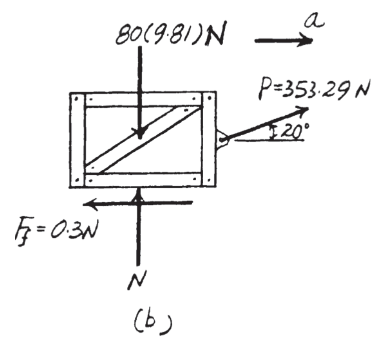
$+\uparrow \Sigma F_y = 0; \qquad N + P \sin 20° - 80(9.81) = 0$ **(1)**

$\xrightarrow{+} \Sigma F_x = 0; \qquad P \cos 20° - 0.5N = 0$ **(2)**

Solving Eqs.(1) and (2) yields

$$P = 353.29 \text{ N} \qquad N = 663.97 \text{ N}$$

Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),



$+\uparrow \Sigma F_y = ma_y; \qquad N - 80(9.81) + 353.29 \sin 20° = 80(0)$

$$N = 663.97 \text{ N}$$

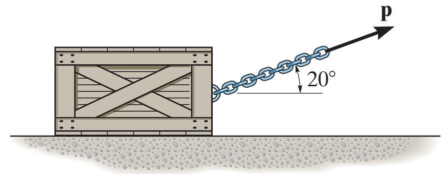
$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 353.29 \cos 20° - 0.3(663.97) = 80a$

$$a = 1.66 \text{ m/s}^2$$ **Ans.**

Ans:
$a = 1.66 \text{ m/s}^2$

13–2.

The crate has a mass of 80 kg and is being towed by a chain which is always directed at 20° from the horizontal as shown. Determine the crate's acceleration in $t = 2$ s if the coefficient of static friction is $\mu_s = 0.4$, the coefficient of kinetic friction is $\mu_k = 0.3$, and the towing force is $P = (90t^2)$ N, where t is in seconds.



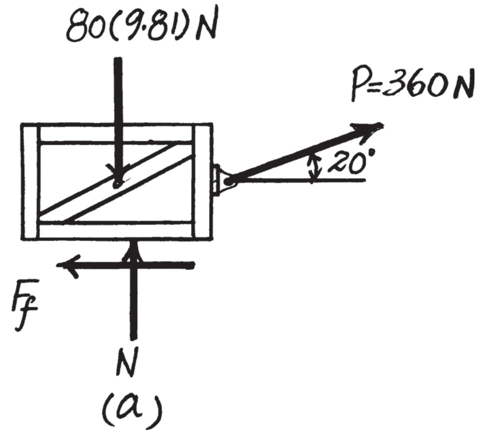
SOLUTION

Equations of Equilibrium: At $t = 2$ s, $P = 90\left(2^2\right) = 360$ N. From FBD(a)

$+\uparrow \Sigma F_y = 0; \quad N + 360 \sin 20° - 80(9.81) = 0 \quad N = 661.67 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \quad 360 \cos 20° - F_f = 0 \quad F_f = 338.29 \text{ N}$

Since $F_f > (F_f)_{\text{max}} = \mu_s N = 0.4(661.67) = 264.67$ N, the crate accelerates.



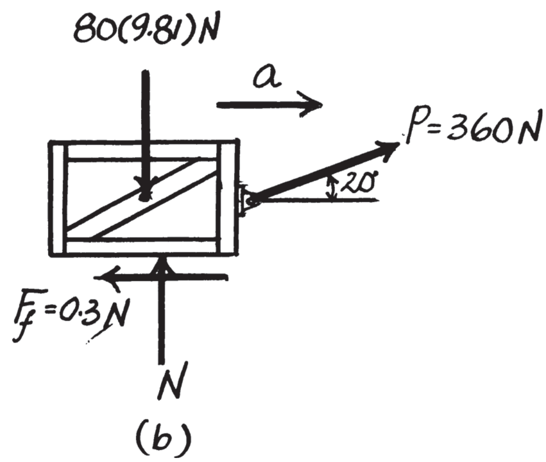
Equations of Motion: The friction force developed between the crate and its contacting surface is $F_f = \mu_k N = 0.3N$ since the crate is moving. From FBD(b),

$$+\uparrow \Sigma F_y = ma_y; \quad N - 80(9.81) + 360 \sin 20° = 80(0)$$

$$N = 661.67 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 360 \cos 20° - 0.3(661.67) = 80a$$

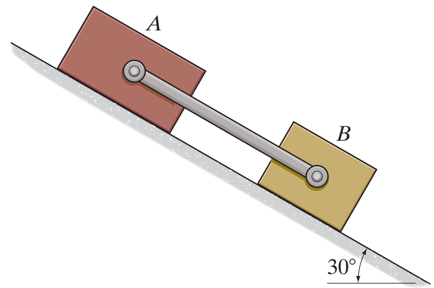
$$a = 1.75 \text{ m/s}^2$$ **Ans.**



Ans:
$a = 1.75 \text{ m/s}^2$

13–3.

If blocks A and B of mass 10 kg and 6 kg, respectively, are placed on the inclined plane and released, determine the force developed in the link. The coefficients of kinetic friction between the blocks and the inclined plane are $\mu_A = 0.1$ and $\mu_B = 0.3$. Neglect the mass of the link.



SOLUTION

Free-Body Diagram: Here, the kinetic friction $(F_f)_A = \mu_A N_A = 0.1N_A$ and $(F_f)_B = \mu_B N_B = 0.3N_B$ are required to act up the plane to oppose the motion of the blocks which are down the plane. Since the blocks are connected, they have a common acceleration **a**.

Equations of Motion: By referring to Figs. (a) and (b),

$+\nearrow \Sigma F_{y'} = ma_{y'}$; $\quad N_A - 10(9.81)\cos 30° = 10(0)$

$$N_A = 84.96 \text{ N}$$

$\searrow + \Sigma F_{x'} = ma_{x'}$; $\quad 10(9.81)\sin 30° - 0.1(84.96) - F = 10a$

$$40.55 - F = 10a \quad \textbf{(1)}$$

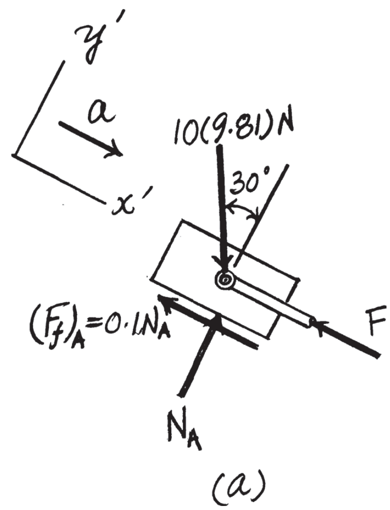
and

$+\nearrow \Sigma F_{y'} = ma_{y'}$; $\quad N_B - 6(9.81)\cos 30° = 6(0)$

$$N_B = 50.97 \text{ N}$$

$\searrow + \Sigma F_{x'} = ma_{x'}$; $\quad F + 6(9.81)\sin 30° - 0.3(50.97) = 6a$

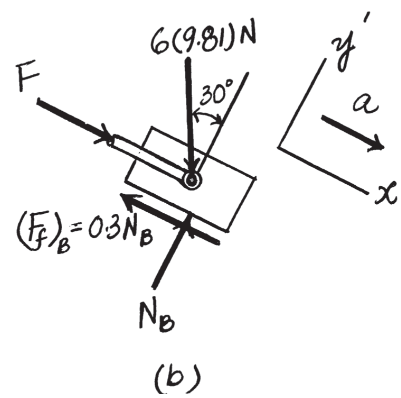
$$F + 14.14 = 6a \quad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$a = 3.42 \text{ m/s}^2$$

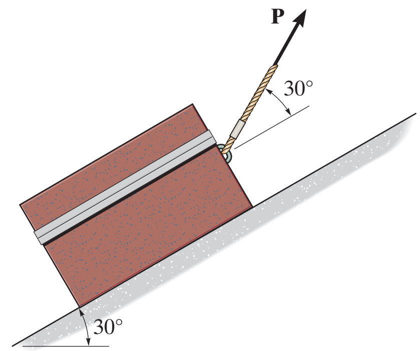
$$F = 6.37 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$F = 6.37$ N

***13–4.**

If $P = 400$ N and the coefficient of kinetic friction between the 50-kg crate and the inclined plane is $\mu_k = 0.25$, determine the velocity of the crate after it travels 6 m up the plane. The crate starts from rest.



SOLUTION

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.25N$ is required to be directed down the plane to oppose the motion of the crate which is assumed to be directed up the plane. The acceleration **a** of the crate is also assumed to be directed up the plane, Fig. *a*.

Equations of Motion: Here, $a_{y'} = 0$. Thus,

$\Sigma F_{y'} = ma_{y'};$ $\quad N + 400 \sin 30° - 50(9.81) \cos 30° = 50(0)$

$$N = 224.79 \text{ N}$$

Using the result of **N**,

$\Sigma F_{x'} = ma_{y'};$ $\quad 400 \cos 30° - 50(9.81) \sin 30° - 0.25(224.79) = 50a$

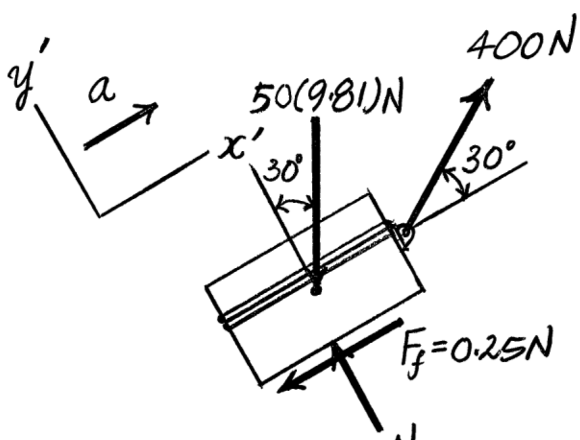
$$a = 0.8993 \text{ m/s}^2$$

Kinematics: Since the acceleration **a** of the crate is constant,

$$v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$v^2 = 0 + 2(0.8993)(6 - 0)$$

$$v = 3.29 \text{ m/s}$$ **Ans.**

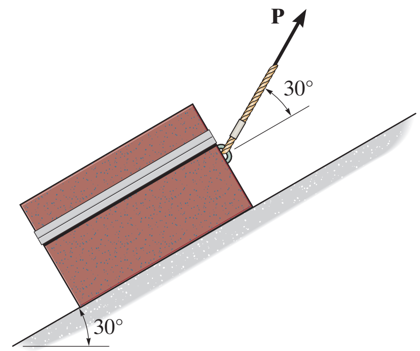


(a)

Ans:
$v = 3.29$ m/s

13–5.

If the 50-kg crate starts from rest and travels a distance of 6 m up the plane in 4 s, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.25$.



SOLUTION

Kinematics: Here, the acceleration **a** of the crate will be determined first since its motion is known.

$$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$6 = 0 + 0 + \frac{1}{2}a(4^2)$$

$$a = 0.75 \text{ m/s}^2$$

Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.25N$ is required to be directed down the plane to oppose the motion of the crate which is directed up the plane, Fig. *a*.

Equations of Motion: Here, $a_{y'} = 0$. Thus,

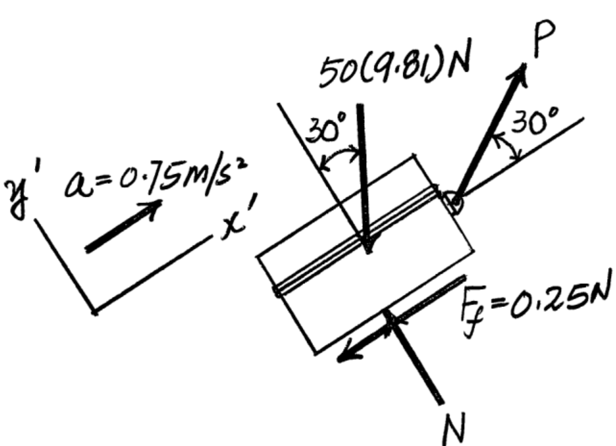
$\Sigma F_{y'} = ma_{y'};$ $\quad N + P\sin 30° - 50(9.81)\cos 30° = 50(0)$

$$N = 424.79 - 0.5P$$

Using the results of **N** and **a**,

$\Sigma F_{x'} = ma_{x'};$ $\quad P\cos 30° - 0.25(424.79 - 0.5P) - 50(9.81)\sin 30° = 50(0.75)$

$P = 392$ N **Ans.**

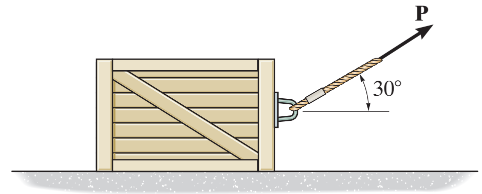


(a)

Ans:
$P = 392$ N

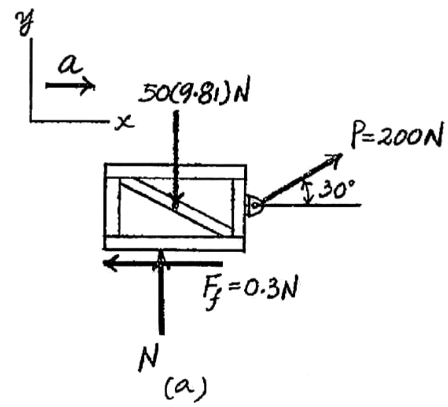
13–6.

If the coefficient of kinetic friction between the 50-kg crate and the ground is $\mu_k = 0.3$, determine the distance the crate travels and its velocity when $t = 3$ s. The crate starts from rest, and $P = 200$ N.



SOLUTION

Free-Body Diagram: The kinetic friction $F_f = \mu_k N$ is directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.



Equations of Motion: Here, $a_y = 0$. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad N - 50(9.81) + 200 \sin 30^\circ = 0$$

$$N = 390.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = ma_x; \quad 200 \cos 30^\circ - 0.3(390.5) = 50a$$

$$a = 1.121 \text{ m/s}^2$$

Kinematics: Since the acceleration **a** of the crate is constant,

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t$$

$$v = 0 + 1.121(3) = 3.36 \text{ m/s}$$ **Ans.**

and

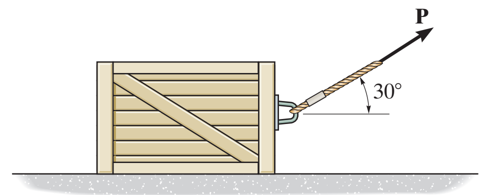
$$(\xrightarrow{+}) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2}(1.121)\left(3^2\right) = 5.04 \text{ m}$$ **Ans.**

Ans:
$v = 3.36$ m/s
$s = 5.04$ m

13–7.

If the 50-kg crate starts from rest and achieves a velocity of $v = 4\text{ m/s}$ when it travels a distance of 5 m to the right, determine the magnitude of force **P** acting on the crate. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.3$.



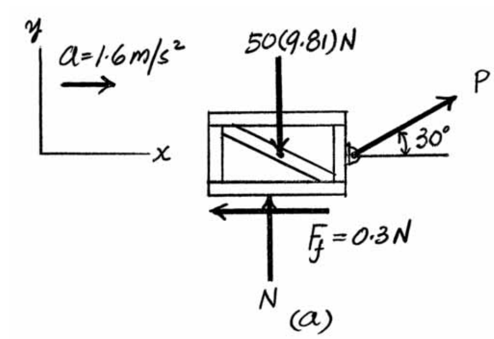
SOLUTION

Kinematics: The acceleration **a** of the crate will be determined first since its motion is known.

$(\xrightarrow{+})$ $\quad v^2 = v_0{}^2 + 2a_c(s - s_0)$

$$4^2 = 0^2 + 2a(5 - 0)$$

$$a = 1.60\text{ m/s}^2 \rightarrow$$



Free-Body Diagram: Here, the kinetic friction $F_f = \mu_k N = 0.3N$ is required to be directed to the left to oppose the motion of the crate which is to the right, Fig. *a*.

Equations of Motion:

$+\uparrow\Sigma F_y = ma_y;$ $\quad N + P\sin 30° - 50(9.81) = 50(0)$

$$N = 490.5 - 0.5P$$

Using the results of **N** and **a**,

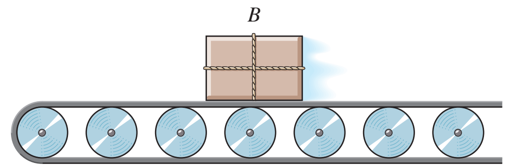
$\xrightarrow{+}\Sigma F_x = ma_x;$ $\quad P\cos 30° - 0.3(490.5 - 0.5P) = 50(1.60)$

$$P = 224\text{ N}$$ **Ans.**

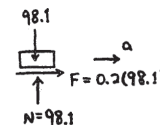
Ans:
$P = 224\text{ N}$

***13–8.**

The conveyor belt is moving at 4 m/s. If the coefficient of static friction between the conveyor and the 10-kg package B is $\mu_s = 0.2$, determine the shortest time the belt can stop so that the package does not slide on the belt.



SOLUTION



$\xrightarrow{+} \Sigma F_x = ma_x;$ $\quad 0.2(98.1) = 10\,a$

$a = 1.962 \text{ m/s}^2$

$(\xrightarrow{+})\, v = v_0 + a_c t$

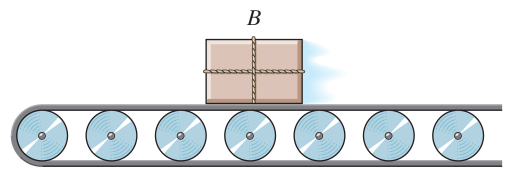
$4 = 0 + 1.962\,t$

$t = 2.04 \text{ s}$ **Ans.**

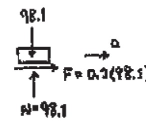
Ans:
$t = 2.04 \text{ s}$

13–9.

The conveyor belt is designed to transport packages of various weights. Each 10-kg package has a coefficient of kinetic friction $\mu_k = 0.15$. If the speed of the conveyor is 5 m/s, and then it suddenly stops, determine the distance the package will slide on the belt before coming to rest.



SOLUTION



$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 0.15\, m(9.81) = ma$

$a = 1.4715 \text{ m/s}^2$

$(\overset{+}{\rightarrow})\; v^2 = v_0^2 + 2a_c(s - s_0)$

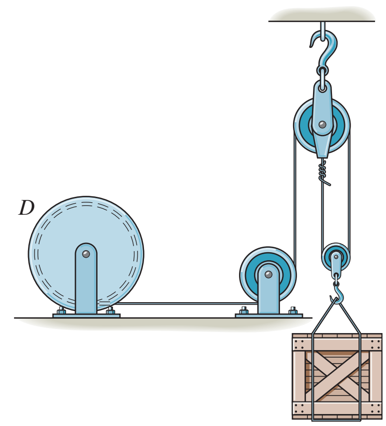
$0 = (5)^2 + 2(-1.4715)(s - 0)$

$s = 8.49 \text{ m}$ **Ans.**

Ans:
$s = 8.49 \text{ m}$

13–10.

The winding drum D is drawing in the cable at an accelerated rate of $5\ \text{m/s}^2$. Determine the cable tension if the suspended crate has a mass of 800 kg.



SOLUTION

$$s_A + 2\,s_B = l$$

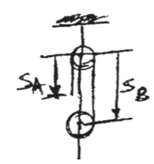
$$a_A = -2\,a_B$$

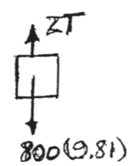
$$5 = -2\,a_B$$

$$a_B = -2.5\ \text{m/s}^2 = 2.5\ \text{m/s}^2 \uparrow$$

$+\uparrow \Sigma F_y = ma_y;$ $\quad 2T - 800(9.81) = 800(2.5)$

$$T = 4924\ \text{N} = 4.92\ \text{kN}$$ **Ans.**

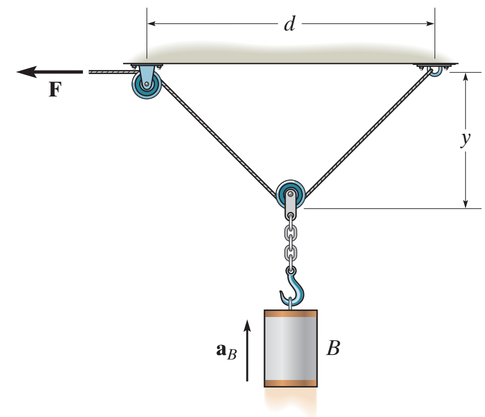




Ans:
$T = 4.92\ \text{kN}$

13–11.

Cylinder B has a mass m and is hoisted using the cord and pulley system shown. Determine the magnitude of force $\mathbf{F}$ as a function of the block's vertical position y so that when $\mathbf{F}$ is applied, the block rises with a constant acceleration $\mathbf{a}_B$. Neglect the mass of the cord and pulleys.

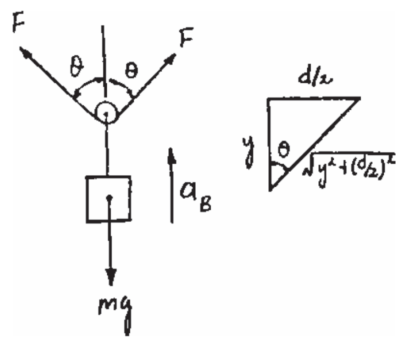


SOLUTION

$+\uparrow \Sigma F_y = ma_y;\qquad 2F\cos\theta - mg = ma_B \qquad \text{where } \cos\theta = \dfrac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}$

$$2F\left(\frac{y}{\sqrt{y^2 + \left(\frac{d}{2}\right)^2}}\right) - mg = ma_B$$

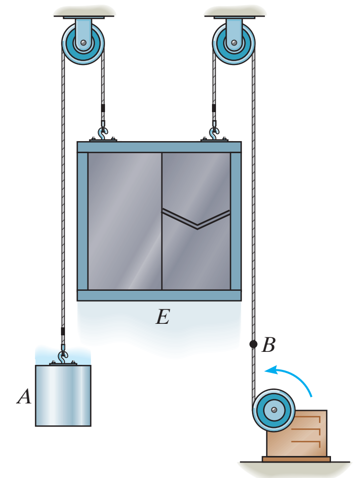
$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$ **Ans.**



Ans:

$$F = \frac{m(a_B + g)\sqrt{4y^2 + d^2}}{4y}$$

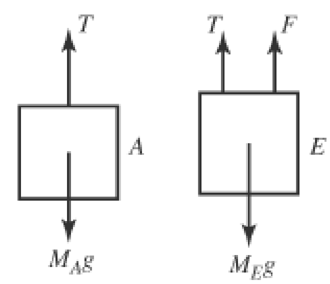
***13–12.** The elevator E has a mass of 500 kg and the counterweight at A has a mass of 150 kg. If the elevator attains a speed of 10 m/s after it rises 40 m, determine the constant force developed in the cable at B. Neglect the mass of the pulleys and cable.



SOLUTION

Guesses $T = 1\text{ kN}$ $F = 1\text{ kN}$ $a = 1\text{ m/s}^2$

Given $T - M_A g = -M_A a$ $F + T - M_E g = M_E a$ $v^2 = 2ah$



$M_E = 500\text{ kg}$

$M_A = 150\text{ kg}$

$v = 10\text{ m/s}$

$h = 40\text{ m}$

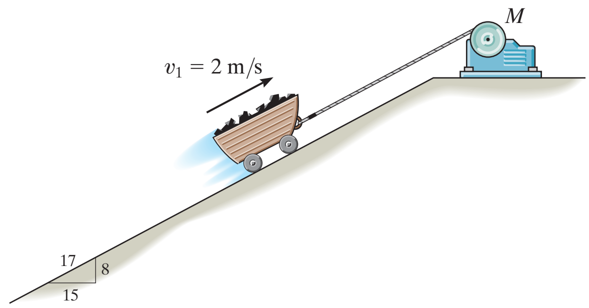
$\begin{pmatrix} F \\ T \\ a \end{pmatrix} = \text{Find}(F, T, a)$ $a = 1.250\text{ m/s}^2$ $T = 1.28\text{ kN}$ $F = 4.24\text{ kN}$ **Ans.**

Ans:

$F = 4.24\text{ kN}$

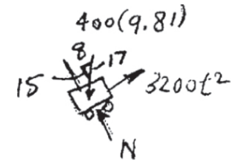
13–13.

The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s when $t = 0$, determine its velocity when $t = 2$ s.



SOLUTION

$$\nearrow + \Sigma F_{x'} = ma_{x'}; \qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$$



$$dv = adt$$

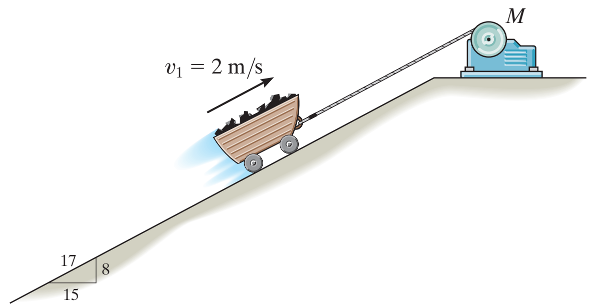
$$\int_2^v dv = \int_0^2 (8t^2 - 4.616)\, dt$$

$$v = 14.1 \text{ m/s}$$ **Ans.**

Ans:
$v = 14.1$ m/s

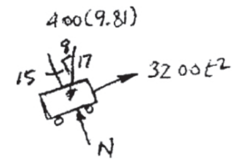
13–14.

The 400-kg mine car is hoisted up the incline using the cable and motor M. For a short time, the force in the cable is $F = (3200t^2)$ N, where t is in seconds. If the car has an initial velocity $v_1 = 2$ m/s at $s = 0$ and $t = 0$, determine the distance it moves up the plane when $t = 2$ s.



SOLUTION

$\nearrow + \Sigma F_{x'} = ma_{x'};\qquad 3200t^2 - 400(9.81)\left(\frac{8}{17}\right) = 400a \qquad a = 8t^2 - 4.616$



$$dv = a\,dt$$

$$\int_2^v dv = \int_0^t (8t^2 - 4.616)\,dt$$

$$v = \frac{ds}{dt} = 2.667t^3 - 4.616t + 2$$

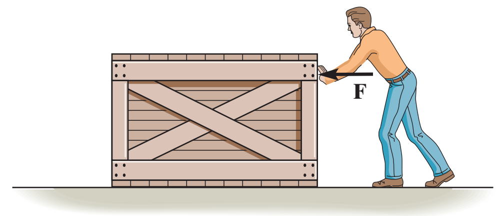
$$\int_0^s ds = \int_0^2 (2.667\,t^3 - 4.616t + 2)\,dt$$

$$s = 5.43 \text{ m}$$ **Ans.**

Ans:
$s = 5.43$ m

13–15.

The 75-kg man pushes on the 150-kg crate with a horizontal force **F**. If the coefficients of static and kinetic friction between the crate and the surface are $\mu_s = 0.3$ and $\mu_k = 0.2$, and the coefficient of static friction between the man's shoes and the surface is $\mu_s = 0.8$, show that the man is able to move the crate. What is the greatest acceleration the man can give the crate?



SOLUTION

Equation of Equilibrium. Assuming that the crate is on the verge of sliding $(F_f)_C = \mu_s N_C = 0.3N_C$. Referring to the FBD of the crate shown in Fig. *a*,

$+\uparrow \Sigma F_y = 0; \quad N_C - 150(9.81) = 0 \quad N_C = 1471.5 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 0.3(1471.5) - F = 0 \quad F = 441.45 \text{ N}$

Referring to the FBD of the man, Fig. *b*,

$+\uparrow \Sigma F_y = 0; \quad N_m - 75(9.81) = 0 \quad N_B = 735.75 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 441.45 - (F_f)_m = 0 \quad (F_f)_m = 441.45 \text{ N}$

Since $(F_f)_m < \mu'_s N_m = 0.8(735.75) = 588.6$ N, **the man is able to move the crate**.

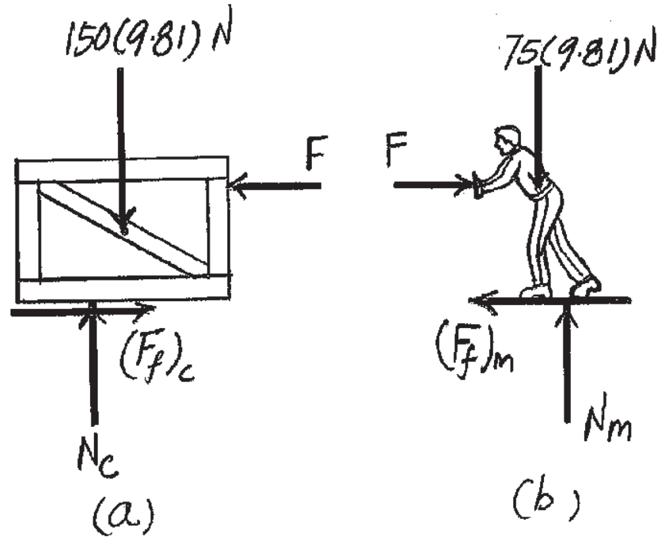
Equation of Motion. The greatest acceleration of the crate can be produced when the man is on the verge of slipping. Thus, $(F_f)_m = \mu'_s N_m = 0.8(735.75) = 588.6$ N.

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F - 588.6 = 0 \quad F = 588.6 \text{ N}$

Since the crate slides, $(F_f)_C = \mu_k N_C = 0.2(1471.5) = 294.3$ N. Thus,

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \quad 588.6 - 294.3 = 150a$

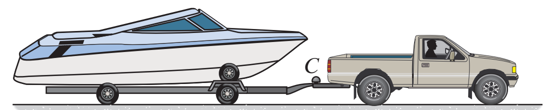
$a = 1.962 \text{ m/s}^2 = 1.96 \text{ m/s}^2$ **Ans.**



Ans:
$a = 1.96 \text{ m/s}^2$

***13–16.**

The 2-Mg truck is traveling at 15 m/s when the brakes on all its wheels are applied, causing it to skid for a distance of 10 m before coming to rest. Determine the constant horizontal force developed in the coupling C, and the frictional force developed between the tires of the truck and the road during this time. The total mass of the boat and trailer is 1 Mg.



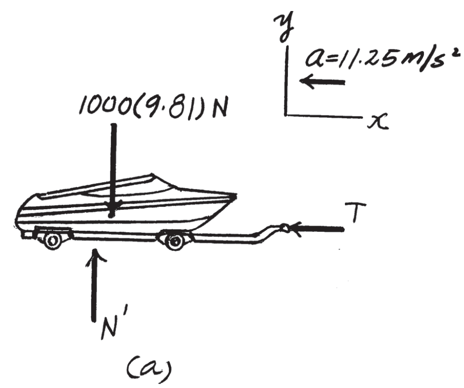
SOLUTION

Kinematics: Since the motion of the truck and trailer is known, their common acceleration **a** will be determined first.

$$\left(\overset{+}{\rightarrow}\right) \quad v^2 = v_0{}^2 + 2a_c(s - s_0)$$

$$0 = 15^2 + 2a(10 - 0)$$

$$a = -11.25 \text{ m/s}^2 = 11.25 \text{ m/s}^2 \leftarrow$$



Free-Body Diagram: The free-body diagram of the truck and trailer are shown in Figs. (a) and (b), respectively. Here, **F** representes the frictional force developed when the truck skids, while the force developed in coupling C is represented by **T**.

Equations of Motion: Using the result of **a** and referrning to Fig. (a),

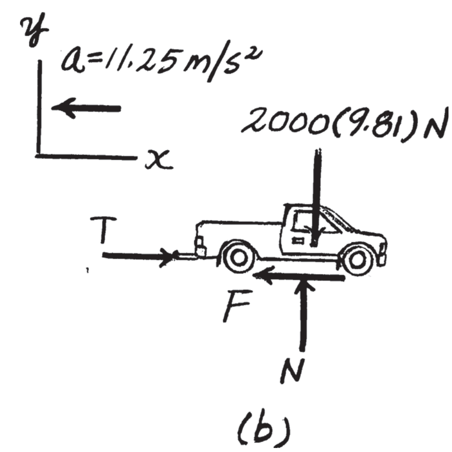
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -T = 1000(-11.25)$$

$$T = 11\,250 \text{ N} = 11.25 \text{ kN} \qquad \textbf{Ans.}$$

Using the results of **a** and **T** and referring to Fig. (b),

$$+\uparrow \Sigma F_x = ma_x; \qquad 11\,250 - F = 2000(-11.25)$$

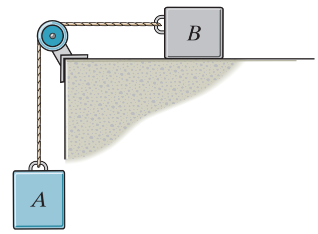
$$F = 33\,750 \text{ N} = 33.75 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$T = 11.25$ kN
$F = 33.75$ kN

13–17.

Determine the acceleration of the blocks when the system is released. The coefficient of kinetic friction is μ_k, and the mass of each block is m. Neglect the mass of the pulleys and cord.



SOLUTION

Free Body Diagram. Since the pulley is smooth, the tension is constant throughout the entire cord. Since block B is required to slide, $F_f = \mu_k N$. Also, blocks A and B are attached together with inextensible cord, so $a_A = a_B = a$. The FBDs of blocks A and B are shown in Figs. a and b, respectively.

Equations of Motion. For block A, Fig. a,

$$+\uparrow \Sigma F_y = ma_y; \qquad T - mg = m(-a) \qquad \textbf{(1)}$$

For block B, Fig. b,

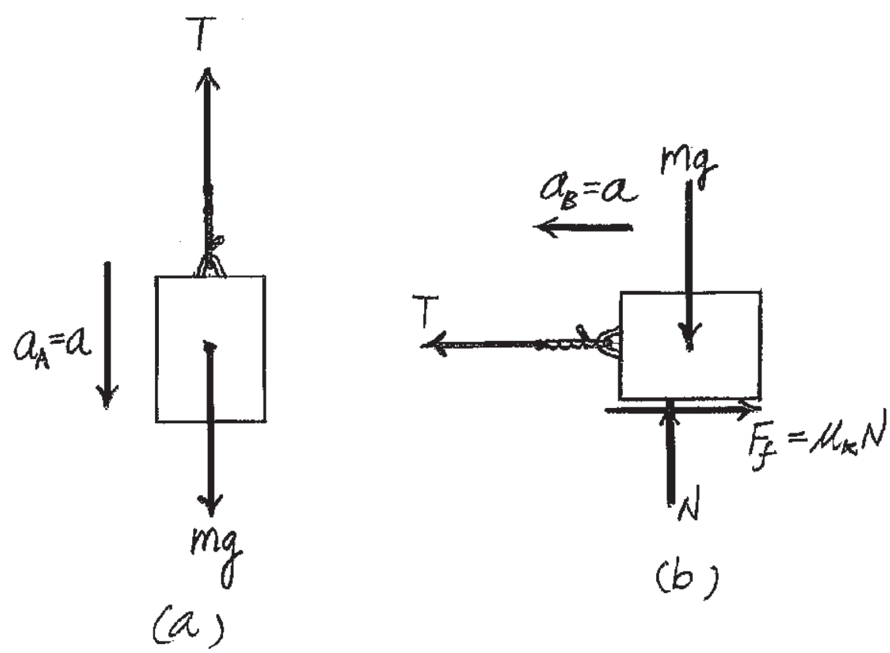
$$+\uparrow \Sigma F_y = ma_y; \qquad N - mg = m(0) \qquad N = mg$$

$$(\overset{+}{\leftarrow})\, \Sigma F_x = ma_x; \quad T - \mu_k mg = ma \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$a = \frac{1}{2}(1 - \mu_k)\, g \qquad \textbf{Ans.}$$

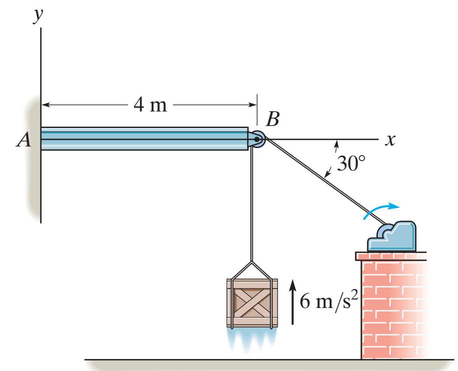
$$T = \frac{1}{2}(1 + \mu_k)\, mg$$



Ans:
$a = \frac{1}{2}(1 - \mu_k)\, g$

13–18.

The motor lifts the 50-kg crate with an acceleration of 6 m/s^2. Determine the components of force reaction and the couple moment at the fixed support A.



SOLUTION

Equation of Motion. Referring to the FBD of the crate shown in Fig. *a*,

$+\uparrow \Sigma F_y = ma_y; \qquad T - 50(9.81) = 50(6) \qquad T = 790.5 \text{ N}$

Equations of Equilibrium. Since the pulley is smooth, the tension is constant throughout entire cable. Referring to the FBD of the pulley shown in Fig. *b*,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 790.5 \cos 30° - B_x = 0 \qquad B_x = 684.59 \text{ N}$

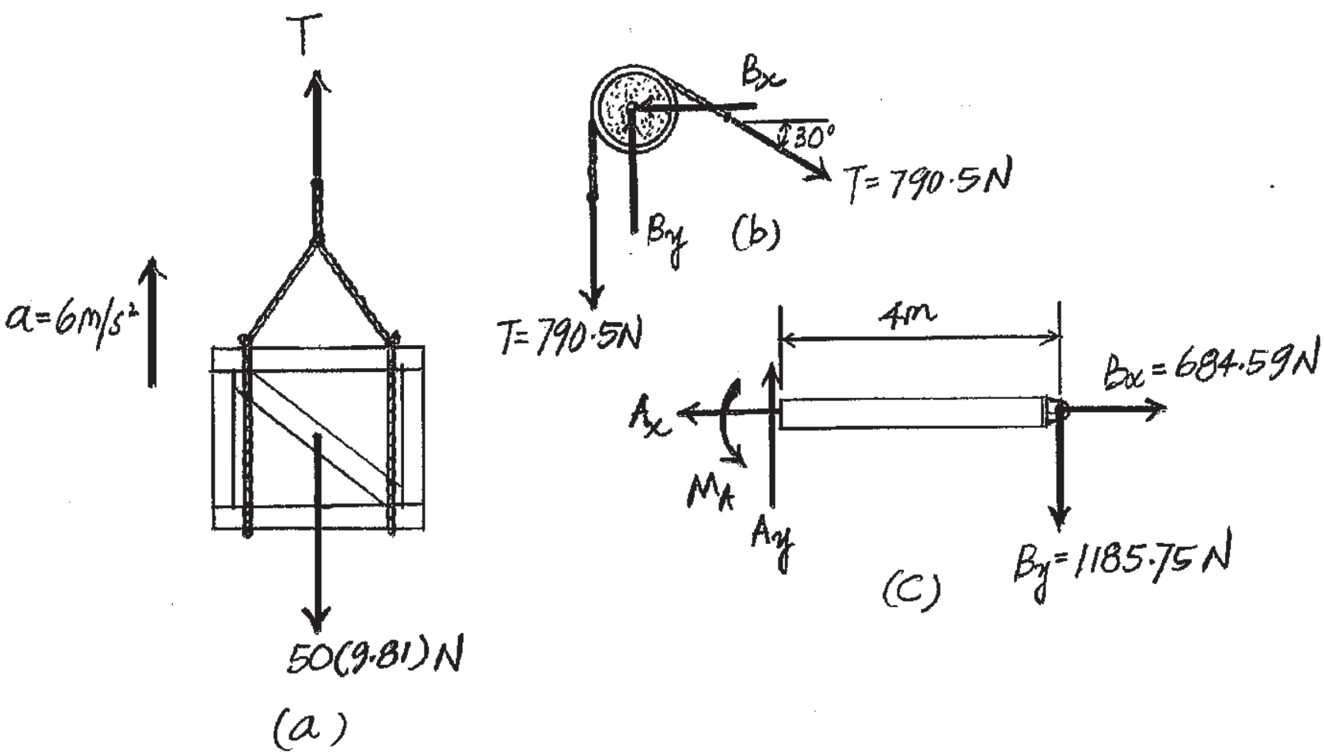
$+\uparrow \Sigma F_y = 0; \qquad B_y - 790.5 - 790.5 \sin 30° = 0 \qquad B_y = 1185.75 \text{ N}$

Consider the FBD of the cantilever beam shown in Fig. *c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 684.59 - A_x = 0 \qquad A_x = 684.59 \text{ N} = 685 \text{ N}$ **Ans.**

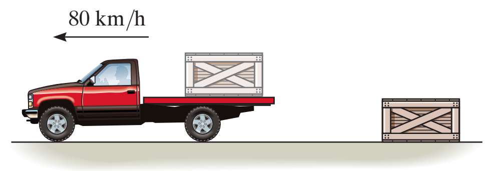
$+\uparrow \Sigma F_y = 0; \qquad A_y - 1185.75 = 0 \qquad A_y = 1185.75 \text{ N} = 1.19 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - 1185.75(4) = 0 \qquad M_A = 4743 \text{ N} \cdot \text{m} = 4.74 \text{ kN} \cdot \text{m}$ **Ans.**



Ans:
$A_x = 685 \text{ N}$
$A_y = 1.19 \text{ kN}$
$M_A = 4.74 \text{ kN} \cdot \text{m}$

13–19. A crate having a mass of 60 kg falls horizontally off the back of a truck which is traveling at 80 km/h. Determine the coefficient of kinetic friction between the road and the crate if the crate slides 45 m on the ground with no tumbling along the road before coming to rest. Assume that the initial speed of the crate along the road is 80 km/h.

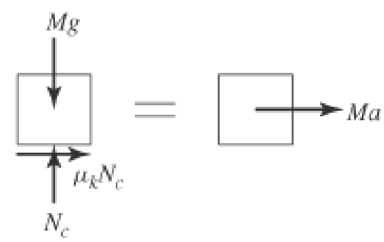


SOLUTION

$N_C - Mg = 0 \qquad N_C = Mg$

$\mu_k N_C = Ma \qquad a = \mu_k g$

$\frac{v^2}{2} = ad = \mu_k g d$



Given:

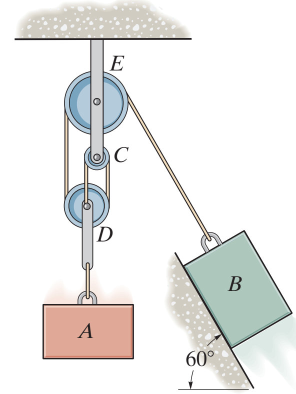
$M = 60 \text{ kg} \qquad d = 45 \text{ m}$

$v = 80 \text{ km/h} \qquad g = 9.81 \text{ m/s}^2$

$\mu_k = \frac{v^2}{2gd} \qquad \mu_k = 0.559$ **Ans.**

Ans:
$\mu_k = 0.559$

***13–20.** Determine the required mass of block A so that when it is released from rest it moves the 5-kg block B 0.75 m up along the smooth inclined plane in $t = 2$ s. Neglect the mass of the pulleys and cords.



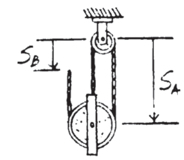
SOLUTION

Kinematic: Applying equation $s = s_0 + v_0 t + \frac{1}{2} a_c t^2$, we have

$$(\nwarrow +) \qquad 0.75 = 0 + 0 + \frac{1}{2} a_B \left(2^2\right) \qquad a_B = 0.375 \text{ m/s}^2$$

Establishing the position - coordinate equation, we have

$$2s_A + (s_A - s_B) = l \qquad 3s_A - s_B = l$$

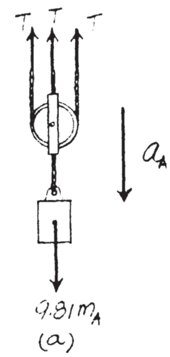


Taking time derivative twice yields

$$3a_A - a_B = 0 \tag{1}$$

From Eq.(1),

$$3a_A - 0.375 = 0 \qquad a_A = 0.125 \text{ m/s}^2$$



Equation of Motion: The tension T developed in the cord is the same throughout the entire cord since the cord passes over the smooth pulleys. From FBD(b),

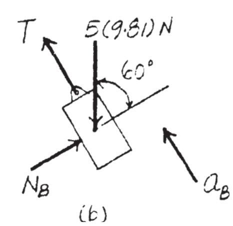
$$\nwarrow + \Sigma F_{y'} = ma_{y'}; \qquad T - 5(9.81)\sin 60° = 5(0.375)$$

$$T = 44.35 \text{ N}$$

From FBD(a),

$$+\uparrow \Sigma F_y = ma_y; \qquad 3(44.35) - 9.81m_A = m_A(-0.125)$$

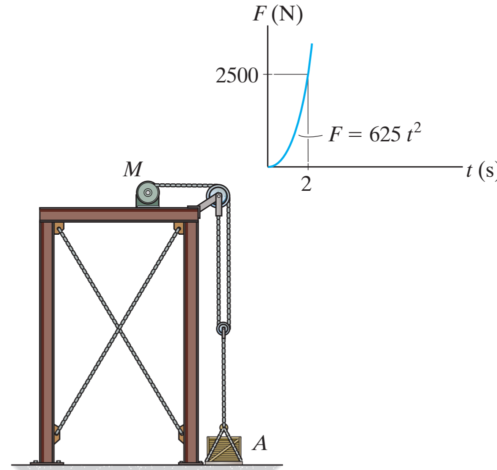
$$m_A = 13.7 \text{ kg} \qquad \textbf{Ans.}$$



Ans:
$m_A = 13.7$ kg

13–21.

The force of the motor M on the cable is shown in the graph. Determine the velocity of the 400-kg crate A when $t = 2$ s.



SOLUTION

Free-Body Diagram: The free-body diagram of the crate is shown in Fig. *a*.

Equilibrium: For the crate to move, force 2**F** must overcome its weight. Thus, the time required to move the crate is given by

$+\uparrow \Sigma F_y = 0; \qquad 2(625t^2) - 400(9.81) = 0$

$$t = 1.772 \text{ s}$$

Equations of Motion: $F = \left(625t^2\right)$ N. By referring to Fig. *a*,

$+\uparrow \Sigma F_y = ma_y; \qquad 2\left(625t^2\right) - 400(9.81) = 400a$

$$a = (3.125t^2 - 9.81) \text{ m/s}^2$$

Kinematics: The velocity of the crate can be obtained by integrating the kinematic equation, $dv = a\,dt$. For $1.772 \text{ s} \le t < 2 \text{ s}$, $v = 0$ at $t = 1.772$ s will be used as the lower integration limit. Thus,

$(+\uparrow) \qquad \int dv = \int a\,dt$

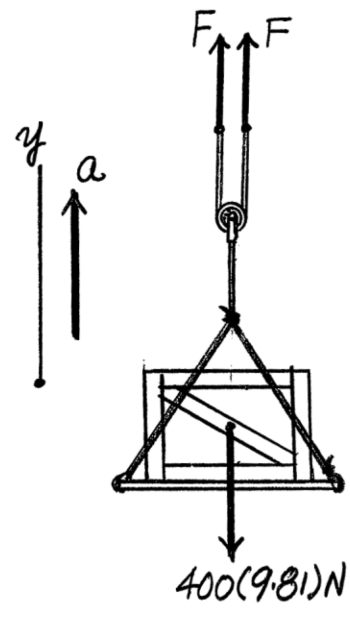
$$\int_0^v dv = \int_{1.772 \text{ s}}^t \left(3.125t^2 - 9.81\right)dt$$

$$v = \left(1.0417t^3 - 9.81t\right)\Big|_{1.772 \text{ s}}^t$$

$$= \left(1.0417t^3 - 9.81t + 11.587\right) \text{ m/s}$$

When $t = 2$ s,

$$v = 1.0417(2^3) - 9.81(2) + 11.587 = 0.301 \text{ m/s} \qquad \textbf{Ans.}$$

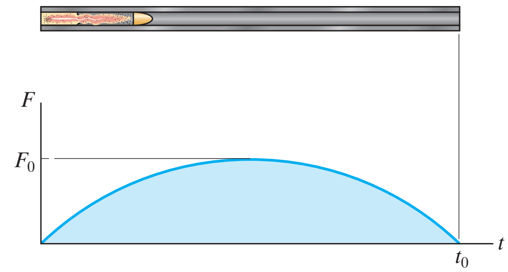


(a)

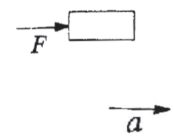
Ans:
$v = 0.301$ m/s

13–22.

The bullet of mass m is given a velocity due to gas pressure caused by the burning of powder within the chamber of the gun. Assuming this pressure creates a force of $F = F_0 \sin(\pi t/t_0)$ on the bullet, determine the velocity of the bullet at any instant it is in the barrel. What is the bullet's maximum velocity? Also, determine the position of the bullet in the barrel as a function of time.



SOLUTION



$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 \sin\left(\frac{\pi t}{t_0}\right) = ma$$

$$a = \frac{dv}{dt} = \left(\frac{F_0}{m}\right)\sin\left(\frac{\pi t}{t_0}\right)$$

$$\int_0^v dv = \int_0^t \left(\frac{F_0}{m}\right)\sin\left(\frac{\pi t}{t_0}\right)dt \qquad v = -\left(\frac{F_0 t_0}{\pi m}\right)\cos\left(\frac{\pi t}{t_0}\right)\Bigg]_0^t$$

$$v = \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

v_{max} occurs when $\cos\left(\frac{\pi t}{t_0}\right) = -1$, or $t = t_0$.

$$v_{max} = \frac{2F_0 t_0}{\pi m}$$ **Ans.**

$$\int_0^s ds = \int_0^t \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)dt$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left[t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right]_0^t$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left(t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

Ans:

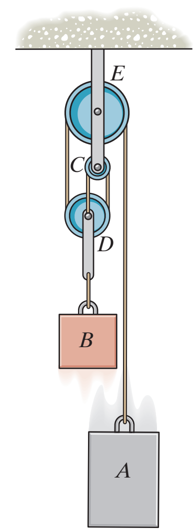
$$v = \left(\frac{F_0 t_0}{\pi m}\right)\left[1 - \cos\left(\frac{\pi t}{t_0}\right)\right]$$

$$v_{max} = \frac{2F_0 t_0}{\pi m}$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left[t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right]$$

13–23.

The 50-kg block *A* is released from rest. Determine the velocity of the 15-kg block *B* in 2 s.



SOLUTION

Kinematics. As shown in Fig. *a*, the position of block *B* and point *A* are specified by s_B and s_A respectively. Here the pulley system has only one cable which gives

$$s_A + s_B + 2(s_B - a) = l$$

$$s_A + 3s_B = l + 2a \qquad \textbf{(1)}$$

Taking the time derivative of Eq. (1) twice,

$$a_A + 3a_B = 0 \qquad \textbf{(2)}$$

Equations of Motion. The FBD of blocks *B* and *A* are shown in Fig. *b* and *c*. To be consistent to those in Eq. (2), $\mathbf{a}_A$ and $\mathbf{a}_B$ are assumed to be directed towards the positive sense of their respective position coordinates s_A and s_B. For block *B*,

$$+\uparrow \Sigma F_y = ma_y; \qquad 3T - 15(9.81) = 15(-a_B) \qquad \textbf{(3)}$$

For block *A*,

$$+\uparrow \Sigma F_y = ma_y; \qquad T - 50(9.81) = 50(-a_A) \qquad \textbf{(4)}$$

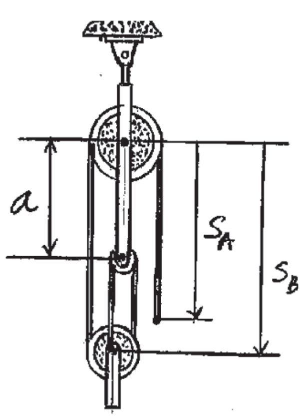
Solving Eqs. (2), (3) and (4),

$$a_B = -2.848 \text{ m/s}^2 = 2.848 \text{ m/s}^2 \uparrow \qquad a_A = 8.554 \text{ m/s}^2 \qquad T = 63.29 \text{ N}$$

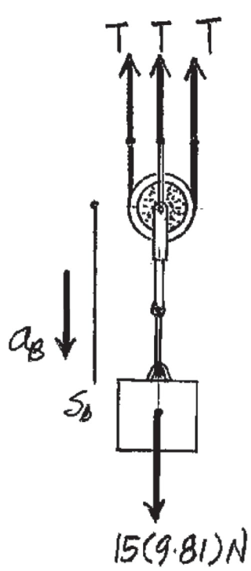
The negative sign indicates that $\mathbf{a}_B$ acts in the sense opposite to that shown in FBD. The velocity of block *B* can be determined using

$$+\uparrow \quad v_B = (v_A)_0 + a_B t; \qquad v_B = 0 + 2.848(2)$$

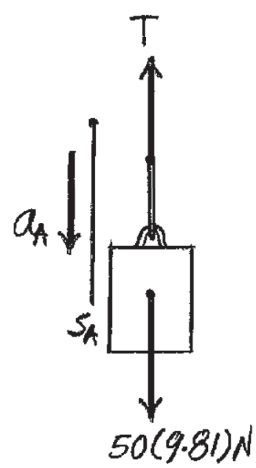
$$v_B = 5.696 \text{ m/s} = 5.70 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$



(a)



(b)

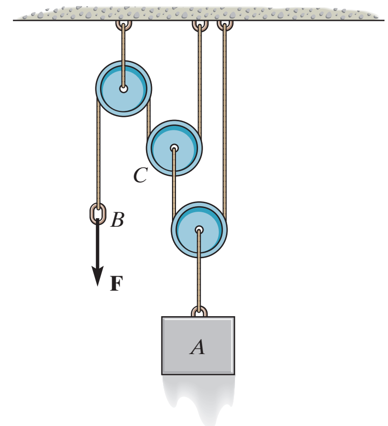


(c)

Ans:
$v_B = 5.70 \text{ m/s} \uparrow$

***13–24.**

If the supplied force $F = 150$ N, determine the velocity of the 50-kg block A when it has risen 3 m, starting from rest.



SOLUTION

Equations of Motion. Since the pulleys are smooth, the tension is constant throughout each entire cable. Referring to the FBD of pulley C, Fig. a, of which its mass is negligible.

$+\uparrow \Sigma F_y = 0; \qquad 150 + 150 - T = 0 \qquad T = 300 \text{ N}$

Subsequently, considered the FBD of block A shown in Fig. b,

$+\uparrow \Sigma F_y = ma_y; \qquad 300 + 300 - 50(9.81) = 50a$

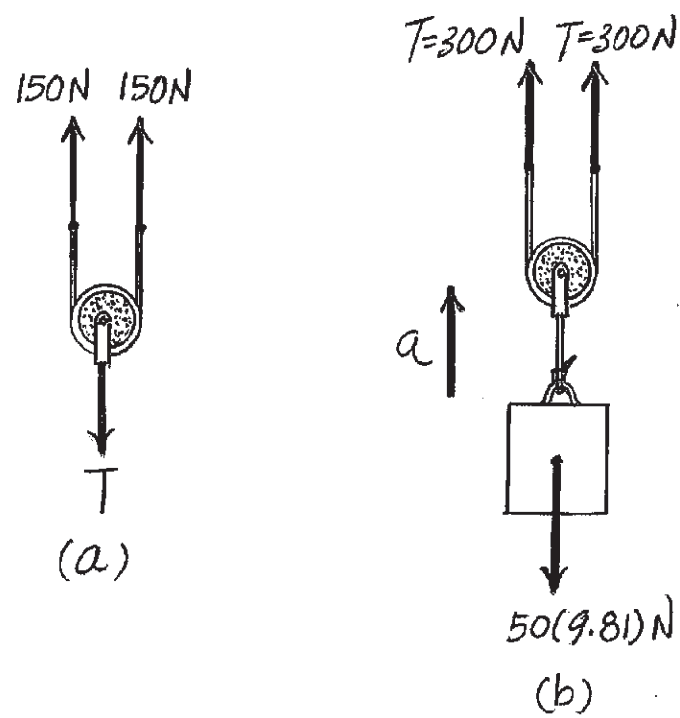
$$a = 2.19 \text{ m/s}^2 \uparrow$$

Kinematics. Using the result of **a**,

$(+\uparrow) \quad v^2 = v_0^2 + 2a_c s;$

$$v^2 = 0^2 + 2(2.19)(3)$$

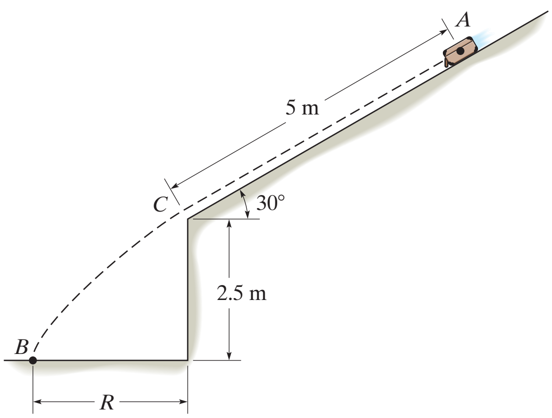
$$v = 3.6249 \text{ m/s} = 3.62 \text{ m/s}$$ **Ans.**



Ans:
$v = 3.62 \text{ m/s}\uparrow$

13–25.

A 60-kg suitcase slides from rest 5 m down the smooth ramp. Determine the distance R where it strikes the ground at B. How long does it take to go from A to B?



SOLUTION

Equation of Motion. Referring to the FBD of the suitcase shown in Fig. *a*

$+\swarrow \Sigma F_{x'} = ma_{x'};\qquad 60(9.81)\sin 30° = 60a \qquad a = 4.905\ \text{m/s}^2$

Kinematics. From A to C, the suitcase moves along the inclined plane (straight line).

$(+\swarrow)\ v^2 = v_0^2 + 2a_c s;\qquad v^2 = 0^2 + 2(4.905)(5)$

$$v = 7.0036\ \text{m/s}$$

$(+\swarrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2;\qquad 5 = 0 + 0 + \frac{1}{2}(4.905)t_{AC}^2$

$$t_{AC} = 1.4278\ \text{s}$$

From C to B, the suitcase undergoes projectile motion. Referring to x–y coordinate system with origin at C, Fig. *b*, the vertical motion gives

$(+\downarrow)\quad s_y = (s_0)_y + v_y t + \frac{1}{2}a_y t^2;$

$$2.5 = 0 + 7.0036\sin 30°\, t_{CB} + \frac{1}{2}(9.81)t_{CB}^2$$

$$4.905\, t_{CB}^2 + 3.5018\, t_{CB} - 2.5 = 0$$

Solve for positive root,

$$t_{CB} = 0.4412\ \text{s}$$

Then, the horizontal motion gives

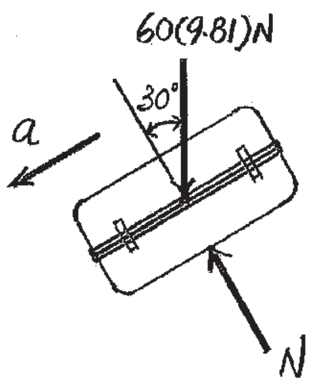
$(\overset{+}{\leftarrow})\quad s_x = (s_0)_x + v_x t;$

$$R = 0 + 7.0036\cos 30°\,(0.4412)$$

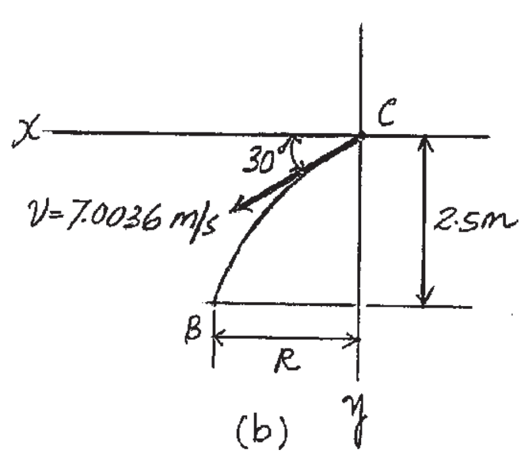
$$= 2.676\ \text{m} = 2.68\ \text{m}$$ **Ans.**

The time taken from A to B is

$$t_{AB} = t_{AC} + t_{CB} = 1.4278 + 0.4412 = 1.869\ \text{s} = 1.87\ \text{s}$$ **Ans.**



(a)

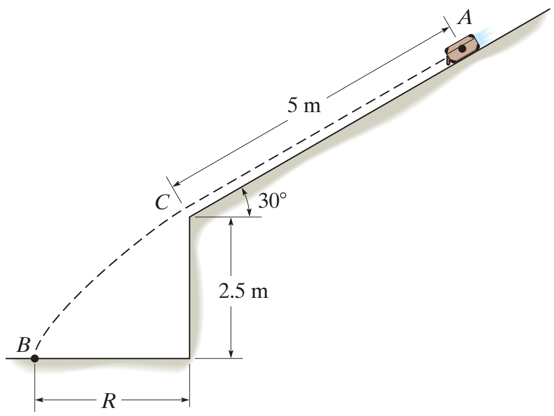


(b)

Ans:
$(\overset{+}{\leftarrow})\quad s_x = 2.68$ m
$t_{AB} = 1.87$ s

13–26.

Solve Prob. 13–25 if the suitcase has an initial velocity down the ramp of $v_A = 2\text{ m/s}$, and the coefficient of kinetic friction along AC is $\mu_k = 0.2$.



SOLUTION

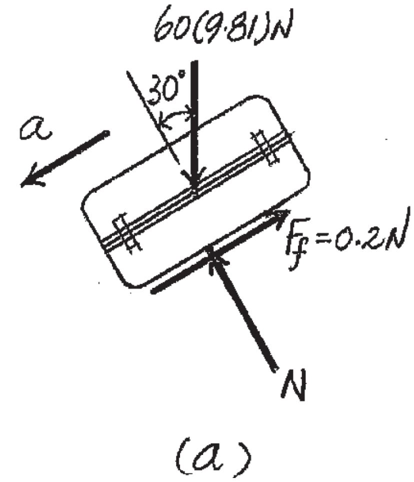
Equations of Motion. The friction is $F_f = \mu_k N = 0.2N$. Referring to the FBD of the suitcase shown in Fig. a

$$\nwarrow^{+} \Sigma F_{y'} = ma_{y'}; \qquad N - 60(9.81)\cos 30^\circ = 60(0)$$

$$N = 509.74\text{ N}$$

$$+\swarrow \Sigma F_{x'} = ma_{x'}; \qquad 60(9.81)\sin 30^\circ - 0.2(509.74) = 60\,a$$

$$a = 3.2059\text{ m/s}^2 \swarrow$$



Kinematics. From A to C, the suitcase moves along the inclined plane (straight line).

$$(+\swarrow) \qquad v^2 = v_0^2 + 2a_c s; \qquad v^2 = 2^2 + 2(3.2059)(5)$$

$$v = 6.0049\text{ m/s} \swarrow$$

$$(+\swarrow) \qquad s = s_0 + v_0 t + \frac{1}{2}a_c t^2; \qquad 5 = 0 + 2t_{AC} + \frac{1}{2}(3.2059)t_{AC}^2$$

$$1.6029\,t_{AC}^2 + 2t_{AC} - 5 = 0$$

Solve for positive root,

$$t_{AC} = 1.2492\text{ s}$$

From C to B, the suitcase undergoes projectile motion. Referring to x–y coordinate system with origin at C, Fig. b, the vertical motion gives

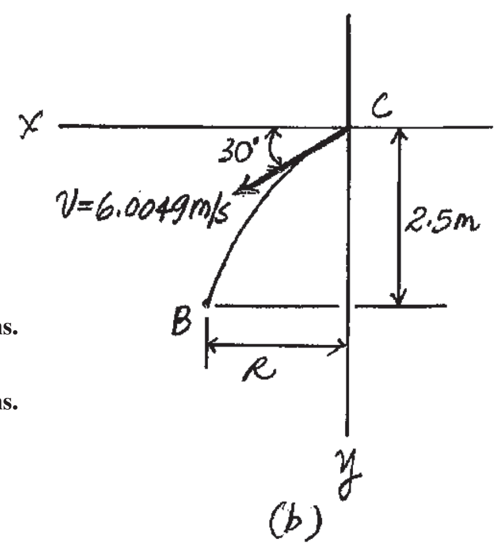
$$(+\downarrow) \qquad s_y = (s_0)_y + v_y t + \frac{1}{2}a_y t^2;$$

$$2.5 = 0 + 6.0049\sin 30^\circ\, t_{CB} + \frac{1}{2}(9.81)t_{CB}^2$$

$$4.905\,t_{CB}^2 + 3.0024\,t_{CB} - 2.5 = 0$$

Solve for positive root,

$$t_{CB} = 0.4707\text{ s}$$



Then, the horizontal motion gives

$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t;$$

$$R = 0 + 6.0049\cos 30^\circ\,(0.4707)$$

$$= 2.448\text{ m} = 2.45\text{ m} \qquad \textbf{Ans.}$$

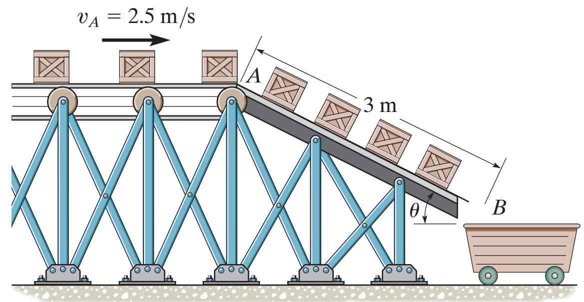
The time taken from A to B is

$$t_{AB} = t_{AC} + t_{CB} = 1.2492 + 0.4707 = 1.7199\text{ s} = 1.72\text{ s} \qquad \textbf{Ans.}$$

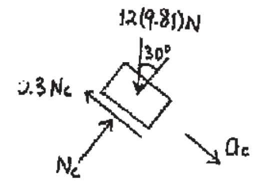
Ans:
$R = 2.45\text{ m}$
$t_{AB} = 1.72\text{ s}$

13–27.

The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5$ m/s, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the speed at which each crate slides off the ramp at B. Assume that no tipping occurs. Take $\theta = 30°$.



SOLUTION



$\nearrow + \Sigma F_y = ma_y;\qquad N_C - 12(9.81)\cos 30° = 0$

$N_C = 101.95\text{ N}$

$+\searrow \Sigma F_x = ma_x;\qquad 12(9.81)\sin 30° - 0.3(101.95) = 12\,a_C$

$a_C = 2.356\text{ m/s}^2$

$(+\searrow)\qquad v_B^2 = v_A^2 + 2\,a_C(s_B - s_A)$

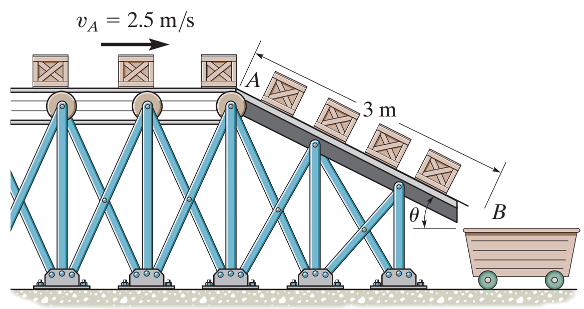
$v_B^2 = (2.5)^2 + 2(2.356)(3 - 0)$

$v_B = 4.5152 = 4.52\text{ m/s}$ **Ans.**

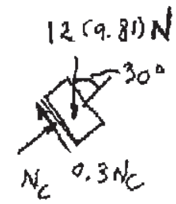
Ans:
$v_B = 4.52$ m/s

***13–28.**

The conveyor belt delivers each 12-kg crate to the ramp at A such that the crate's speed is $v_A = 2.5\text{ m/s}$, directed down *along* the ramp. If the coefficient of kinetic friction between each crate and the ramp is $\mu_k = 0.3$, determine the smallest incline θ of the ramp so that the crates will slide off and fall into the cart.



SOLUTION



$(+\searrow)\quad v_B^2 = v_A^2 + 2a_C(s_B - s_A)$

$$0 = (2.5)^2 + 2(a_C)(3 - 0)$$

$$a_C = 1.0417$$

$\nearrow+\Sigma F_y = ma_y;\qquad N_C - 12(9.81)\cos\theta = 0$

$$N_C = 117.72\cos\theta$$

$+\searrow\Sigma F_x = ma_x;\qquad 12(9.81)\sin\theta - 0.3(N_C) = 12\,(1.0417)$

$$117.72\sin\theta - 35.316\cos\theta - 12.5 = 0$$

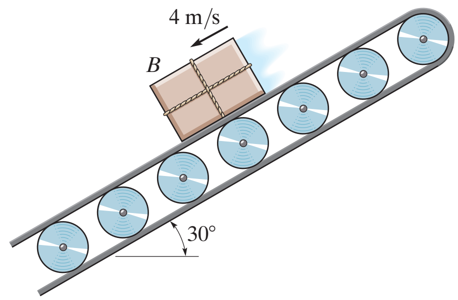
Solving,

$\theta = 22.6°$ **Ans.**

Ans:
$\theta = 22.6°$

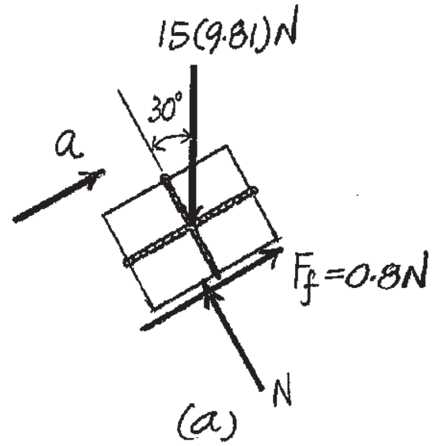
13–29.

The conveyor belt is moving downward at 4 m/s. If the coefficient of static friction between the conveyor and the 15-kg package B is $\mu_s = 0.8$, determine the shortest time the belt can stop so that the package does not slide on the belt.



SOLUTION

Equations of Motion. It is required that the package is on the verge to slide. Thus, $F_f = \mu_s N = 0.8N$. Referring to the FBD of the package shown in Fig. *a*,



$+\nwarrow \Sigma F_{y'} = ma_{y'};\quad N - 15(9.81)\cos 30° = 15(0)\quad N = 127.44\text{ N}$

$+\nearrow \Sigma F_{x'} = ma_{x'};\quad 0.8(127.44) - 15(9.81)\sin 30° = 15\,a$

$$a = 1.8916\text{ m/s}^2 \nearrow$$

Kinematic. Since the package is required to stop, $v = 0$. Here $v_0 = 4$ m/s.

$(+\swarrow)\quad v = v_0 + a_0 t;$

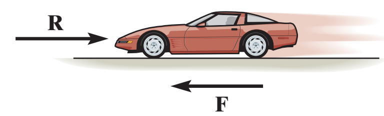
$$0 = 4 + (-1.8916)\,t$$

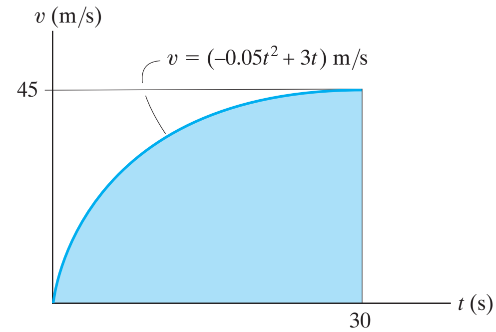
$$t = 2.1146\text{ s} = 2.11\text{ s}$$ **Ans.**

Ans:
$t = 2.11$ s

13–30.

The 1.5 Mg sports car has a tractive force of $F = 4.5$ kN. If it produces the velocity described by v-t graph shown, plot the air resistance R versus t for this time period.





SOLUTION

Kinematic. For the v–t graph, the acceleration of the car as a function of t is

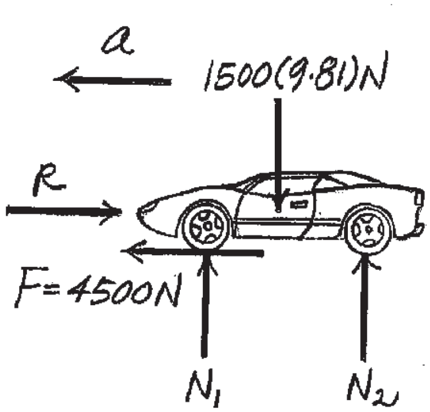
$$a = \frac{dv}{dt} = \{-0.1t + 3\}\ \text{m/s}^2$$

Equation of Motion. Referring to the FBD of the car shown in Fig. a,

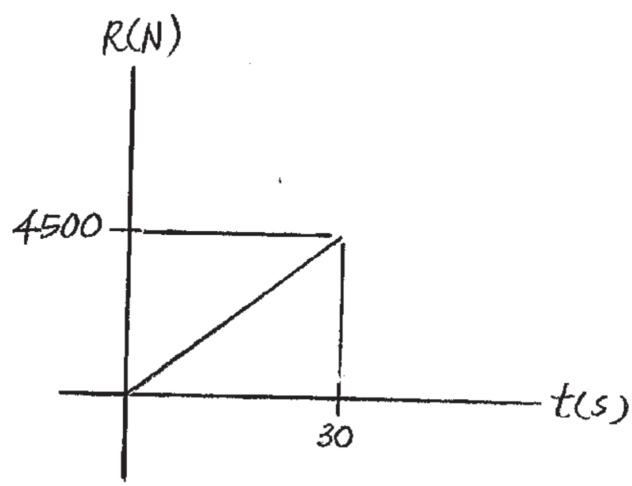
$$(\overset{+}{\leftarrow})\ \Sigma F_x = ma_x; \qquad 4500 - R = 1500(-0.1t + 3)$$

$$R = \{150t\}\ \text{N}$$

The plot of R vs t is shown in Fig. b



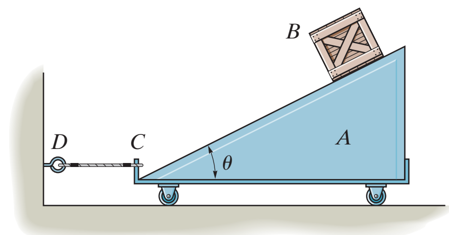
(a)



Ans:
$R = \{150t\}$ N

13–31.

Crate B has a mass m and is released from rest when it is on top of cart A, which has a mass $3m$. Determine the tension in cord CD needed to hold the cart from moving while B is sliding down A. Neglect friction.



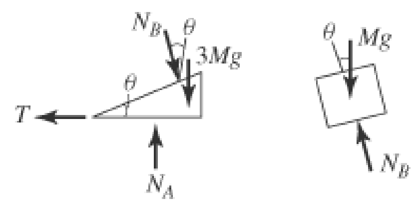
SOLUTION

Block B:

$$N_B - m g \cos(\theta) = 0$$

$$N_B = m g \cos(\theta)$$

Cart :



$$-T + N_B \sin(\theta) = 0$$

$$T = m g \sin(\theta) \cos(\theta)$$

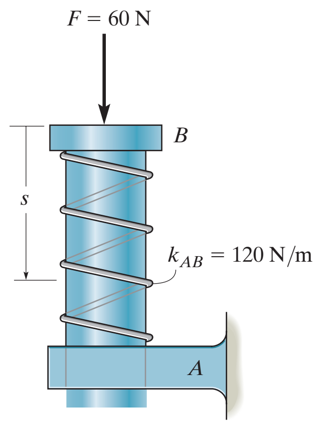
$$T = \left(\frac{mg}{2}\right)\sin(2\theta) \qquad \textbf{Ans.}$$

Ans:

$$T = \left(\frac{mg}{2}\right)\sin(2\theta)$$

***13–32.**

The 4-kg smooth cylinder is supported by the spring having a stiffness of $k_{AB} = 120$ N/m. Determine the velocity of the cylinder when it moves downward $s = 0.2$ m from its equilibrium position, which is caused by the application of the force $F = 60$ N.

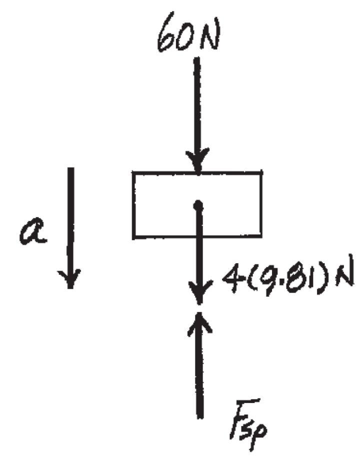


SOLUTION

Equation of Motion. At the equilibrium position, realizing that $F_{sp} = kx_0 = 120x_0$ the compression of the spring can be determined from

$$+\uparrow \Sigma F_y = 0; \qquad 120x_0 - 4(9.81) = 0 \qquad x_0 = 0.327 \text{ m}$$

Thus, when 60 N force is applied, the compression of the spring is $x = s + x_0 = s + 0.327$. Thus, $F_{sp} = kx = 120(s + 0.327)$. Then, referring to the FBD of the collar shown in Fig. *a*,



(a)

$$+\uparrow \Sigma F_y = ma_y; \qquad 120(s + 0.327) - 60 - 4(9.81) = 4(-a)$$

$$a = \{15 - 30\,s\} \text{ m/s}^2$$

Kinematics. Using the result of **a** and integrate $\int v dv = a ds$ with the initial condition $v = 0$ at $s = 0$,

$$\int_0^v v dv = \int_0^s (15 - 30\,s) ds$$

$$\frac{v^2}{2} = 15\,s - 15\,s^2$$

$$v = \left\{\sqrt{30(s - s^2)}\right\} \text{ m/s}$$

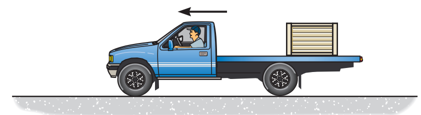
At $s = 0.2$ m,

$$v = \sqrt{30(0.2 - 0.2^2)} = 2.191 \text{ m/s} = 2.19 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 2.19$ m/s

13–33.

The coefficient of static friction between the 200-kg crate and the flat bed of the truck is $\mu_s = 0.3$. Determine the shortest time for the truck to reach a speed of 60 km/h, starting from rest with constant acceleration, so that the crate does not slip.



SOLUTION

Free-Body Diagram: When the crate accelerates with the truck, the frictional force F_f develops. Since the crate is required to be on the verge of slipping, $F_f = \mu_s N = 0.3N$.

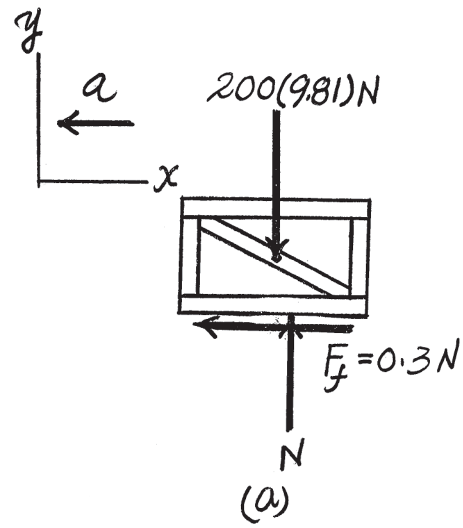
Equations of Motion: Here, $a_y = 0$. By referring to Fig. *a*,

$$+\uparrow \Sigma F_y = ma_y; \qquad N - 200(9.81) = 200(0)$$

$$N = 1962 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -0.3(1962) = 200(-a)$$

$$a = 2.943 \text{ m/s}^2 \leftarrow$$



Kinematics: The final velocity of the truck is $v = \left(60\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1000 \text{ m}}{1 \text{ km}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right) =$ 16.67 m/s. Since the acceleration of the truck is constant,

$$(\overset{+}{\leftarrow}) \qquad v = v_0 + a_c t$$

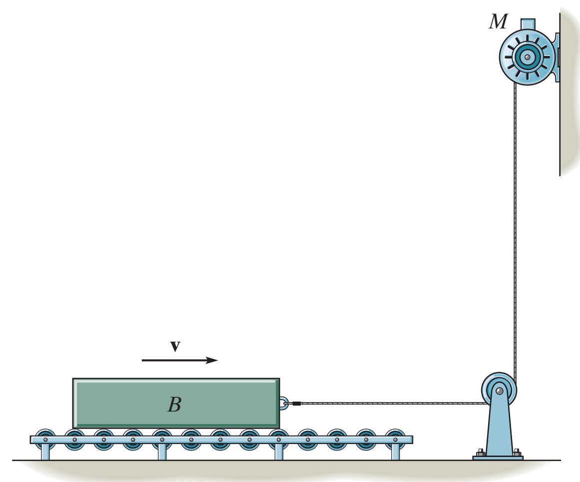
$$16.67 = 0 + 2.943t$$

$$t = 5.66 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t = 5.66$ s

13–34.

The 300-kg bar B, originally at rest, is being towed over a series of small rollers. Determine the force in the cable when $t = 5$ s, if the motor M is drawing in the cable for a short time at a rate of $v = (0.4t^2)$ m/s, where t is in seconds $(0 \leq t \leq 6 \text{ s})$. How far does the bar move in 5 s? Neglect the mass of the cable, pulley, and the rollers.



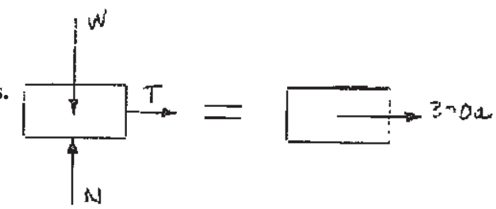
SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad T = 300a$

$v = 0.4t^2$

$$a = \frac{dv}{dt} = 0.8t$$

When $t = 5$ s, $a = 4 \text{ m/s}^2$



$T = 300(4) = 1200 \text{ N} = 1.20 \text{ kN}$ **Ans.**

$ds = v\,dt$

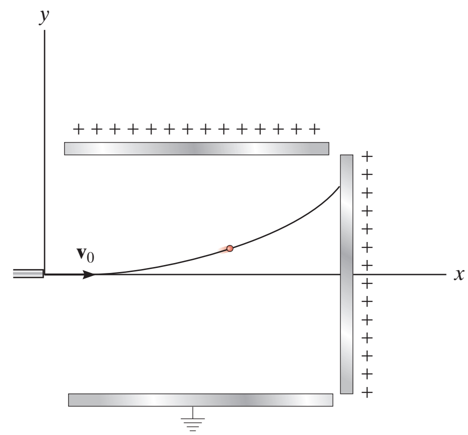
$$\int_0^s ds = \int_0^5 0.4t^2\,ds$$

$$s = \left(\frac{0.4}{3}\right)(5)^3 = 16.7 \text{ m}$$ **Ans.**

Ans:
$s = 16.7$ m

13–35.

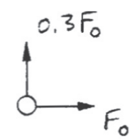
An electron of mass m is discharged with an initial horizontal velocity of $\mathbf{v}_0$. If it is subjected to two fields of force for which $F_x = F_0$ and $F_y = 0.3F_0$, where F_0 is constant, determine the equation of the path, and the speed of the electron at any time t.



SOLUTION

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 = ma_x$

$+\uparrow \Sigma F_y = ma_y; \qquad 0.3\,F_0 = ma_y$



Thus,

$$\int_{v_0}^{v_x} dv_x = \int_0^t \frac{F_0}{m}\,dt$$

$$v_x = \frac{F_0}{m}t + v_0$$

$$\int_0^{v_y} dv_y = \int_0^t \frac{0.3F_0}{m}\,dt \qquad v_y = \frac{0.3F_0}{m}t$$

$$v = \sqrt{\left(\frac{F_0}{m}t + v_0\right)^2 + \left(\frac{0.3F_0}{m}t\right)^2}$$

$$= \frac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2} \qquad \textbf{Ans.}$$

$$\int_0^x dx = \int_0^t \left(\frac{F_0}{m}t + v_0\right)dt$$

$$x = \frac{F_0t^2}{2m} + v_0\,t$$

$$\int_0^y dy = \int_0^t \frac{0.3F_0}{m}\,t\,dt$$

$$y = \frac{0.3F_0t^2}{2m}$$

$$t = \left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

$$x = \frac{F_0}{2m}\left(\frac{2m}{0.3F_0}\right)y + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}}$$

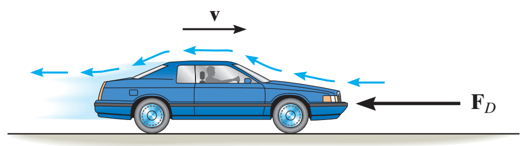
$$x = \frac{y}{0.3} + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{\frac{1}{2}} \qquad \textbf{Ans.}$$

Ans:

$$v = \frac{1}{m}\sqrt{1.09F_0^2t^2 + 2F_0tmv_0 + m^2v_0^2}$$

$$x = \frac{y}{0.3} + v_0\left(\sqrt{\frac{2m}{0.3F_0}}\right)y^{1/2}$$

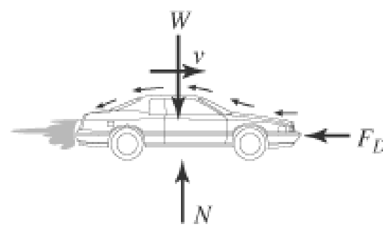
***13–36.** A car of mass m is traveling at a slow velocity v_0. If it is subjected to the drag resistance of the wind, which is proportional to its velocity, i.e., $F_D = kv$, determine the distance and the time the car will travel before its velocity becomes $0.5v_0$. Assume no other frictional forces act on the car.



SOLUTION

$$-F_D = ma$$

$$-kv = ma$$



Find time $\qquad a = \frac{d}{dt}v = \frac{-k}{m}v$

$$\frac{-k}{m}\int_0^t 1\,dt = \int_{v_0}^{0.5v_0} \frac{1}{v}\,dv$$

$$t = \frac{m}{k}\ln\left(\frac{v_0}{0.5v_0}\right) \qquad t = \frac{m}{k}\ln(2) \qquad t = 0.693\frac{m}{k} \qquad \textbf{Ans.}$$

Find distance $\qquad a = v\frac{d}{dx}v = \frac{-k}{m}v$

$$-\int_0^x k\,dx = \int_{v_0}^{0.5v_0} m\,dv \qquad x = \frac{m}{k}(0.5v_0) \qquad x = 0.5\frac{mv_0}{k} \qquad \textbf{Ans.}$$

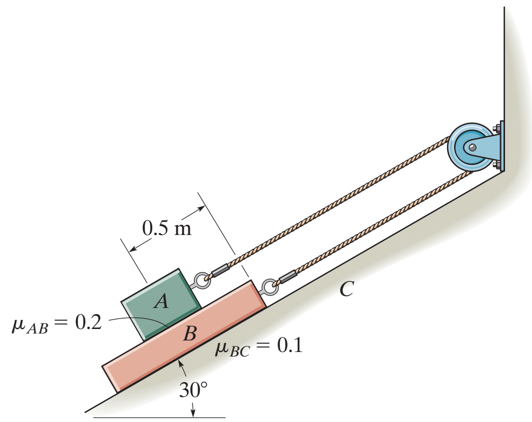
Ans:

$t = 0.693\frac{m}{k}$

$x = 0.5\frac{mv_0}{k}$

13–37.

The 10-kg block A rests on the 50-kg plate B in the position shown. Neglecting the mass of the rope and pulley, and using the coefficients of kinetic friction indicated, determine the time needed for block A to slide 0.5 m *on the plate* when the system is released from rest.



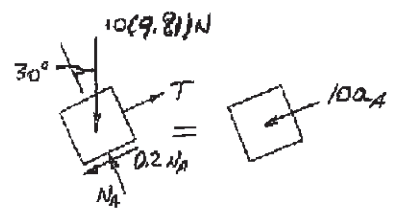
SOLUTION

Block A:

$+\nwarrow \Sigma F_y = ma_y;\quad N_A - 10(9.81)\cos 30° = 0 \qquad N_A = 84.96\text{ N}$

$+\swarrow \Sigma F_x = ma_x;\quad -T + 0.2(84.96) + 10(9.81)\sin 30° = 10a_A$

$T - 66.04 = -10a_A \qquad$ **(1)**



Block B:

$+\nwarrow \Sigma F_y = ma_y;\quad N_B - 84.96 - 50(9.81)\cos 30° = 0$

$N_B = 509.7\text{ N}$

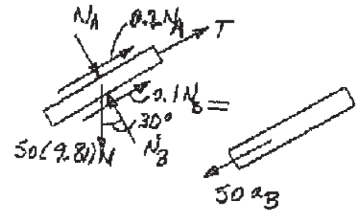
$+\swarrow \Sigma F_x = ma_x;\quad -0.2(84.96) - 0.1(509.7) - T + 50(9.81\sin 30°) = 50a_B$

$177.28 - T = 50a_B \qquad$ **(2)**

$s_A + s_B = l$

$\Delta s_A = -\Delta s_B$

$a_A = -a_B \qquad$ **(3)**



Solving Eqs. (1) – (3):

$a_B = 1.854\text{ m/s}^2$

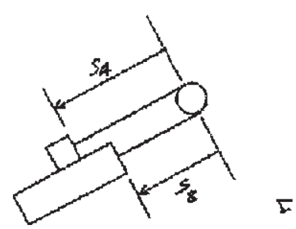
$a_A = -1.854\text{ m/s}^2 \qquad T = 84.58\text{ N}$

In order to slide 0.5 m along the plate the block must move 0.25 m. Thus,

$(+\swarrow)\qquad s_B = s_A + s_{B/A}$

$-\Delta s_A = \Delta s_A + 0.5$

$\Delta s_A = -0.25\text{ m}$



$(+\swarrow)\qquad s_A = s_0 + v_0 t + \frac{1}{2}a_A t^2$

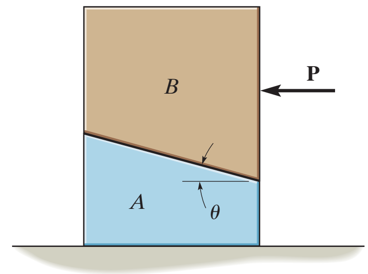
$-0.25 = 0 + 0 + \frac{1}{2}(-1.854)t^2$

$t = 0.519\text{ s} \qquad$ **Ans.**

Ans:
$t = 0.519\text{ s}$

13–38.

Block A and B each have a mass m. Determine the largest horizontal force $\mathbf{P}$ which can be applied to B so that it will not slide on A. Also, what is the corresponding acceleration? The coefficient of static friction between A and B is μ_s. Neglect any friction between A and the horizontal surface.



SOLUTION

Equations of Motion. Since block B is required to be on the verge to slide on A, $F_f = \mu_s N_B$. Referring to the FBD of block B shown in Fig. a,

$+\uparrow \Sigma F_y = ma_y; \qquad N_B \cos\theta - \mu_s N_B \sin\theta - mg = m(0)$

$$N_B = \frac{mg}{\cos\theta - \mu_s \sin\theta} \tag{1}$$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - N_B \sin\theta - \mu_s N_B \cos\theta = ma$

$$P - N_B(\sin\theta + \mu_s \cos\theta) = ma \tag{2}$$

Substitute Eq. (1) into (2),

$$P - \left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) mg = ma \tag{3}$$

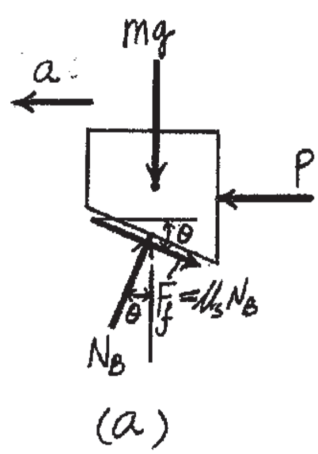
Referring to the FBD of blocks A and B shown in Fig. b

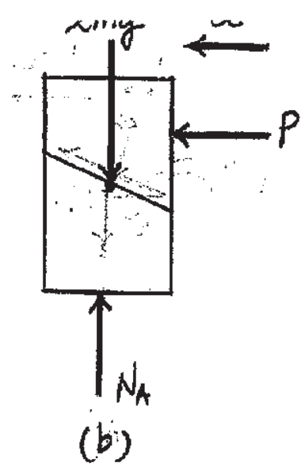
$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P = 2\,ma \qquad (4)$

Solving Eqs. (2) into (3),

$$P = 2mg\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) \qquad \textbf{Ans.}$$

$$a = \left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) g \qquad \textbf{Ans.}$$





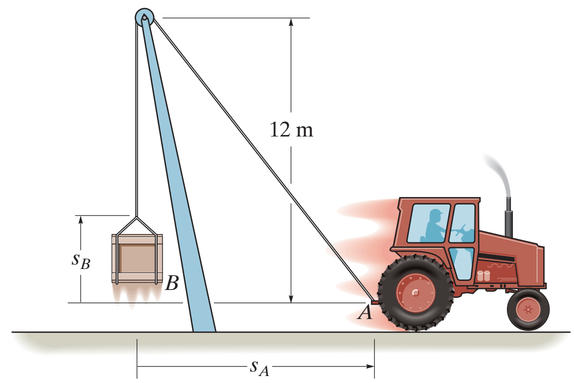
Ans:

$P = 2mg\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)$

$a = \left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right) g$

13–39.

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right at a constant speed of 4 m/s, determine the tension in the rope when $s_A = 5$ m. When $s_A = 0$, $s_B = 0$.



SOLUTION

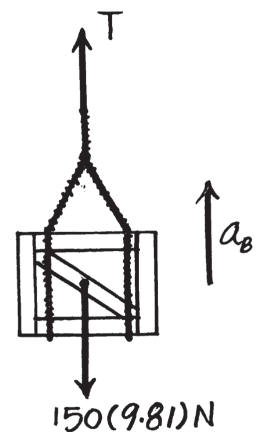
$$12 - s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2 \dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{\left((5)^2 + 144\right)^{\frac{3}{2}}} - \frac{(4)^2 + 0}{\left((5)^2 + 144\right)^{\frac{1}{2}}}\right] = 1.0487 \text{ m/s}^2$$



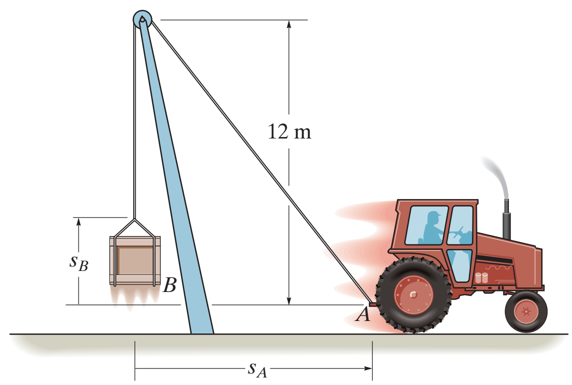
$+\uparrow \Sigma F_y = m a_y; \qquad T - 150(9.81) = 150(1.0487)$

$T = 1.63$ kN **Ans.**

Ans:
$T = 1.63$ kN

***13–40.**

The tractor is used to lift the 150-kg load B with the 24-m-long rope, boom, and pulley system. If the tractor travels to the right with an acceleration of 3 m/s^2 and has a velocity of 4 m/s at the instant $s_A = 5$ m, determine the tension in the rope at this instant. When $s_A = 0$, $s_B = 0$.



SOLUTION

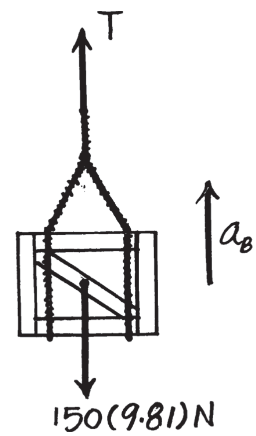
$$12 = s_B + \sqrt{s_A^2 + (12)^2} = 24$$

$$-\dot{s}_B + \frac{1}{2}\left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(2 s_A \dot{s}_A\right) = 0$$

$$-\ddot{s}_B - \left(s_A^2 + 144\right)^{-\frac{3}{2}}\left(s_A \dot{s}_A\right)^2 + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(\dot{s}_A^2\right) + \left(s_A^2 + 144\right)^{-\frac{1}{2}}\left(s_A \ddot{s}_A\right) = 0$$

$$\ddot{s}_B = -\left[\frac{s_A^2\,\dot{s}_A^2}{\left(s_A^2 + 144\right)^{\frac{3}{2}}} - \frac{\dot{s}_A^2 + s_A \ddot{s}_A}{\left(s_A^2 + 144\right)^{\frac{1}{2}}}\right]$$

$$a_B = -\left[\frac{(5)^2(4)^2}{\left((5)^2 + 144\right)^{\frac{3}{2}}} - \frac{(4)^2 + (5)(3)}{\left((5)^2 + 144\right)^{\frac{1}{2}}}\right] = 2.2025 \text{ m/s}^2$$



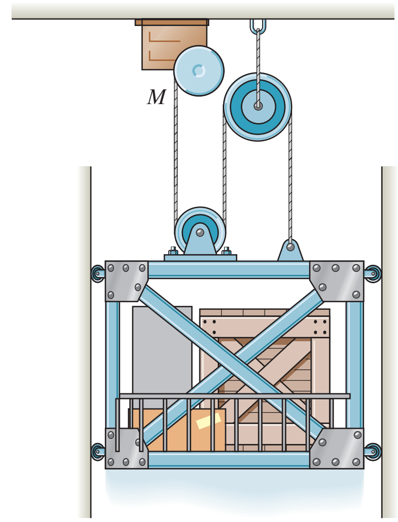
$+\uparrow \Sigma F_y = m a_y;\qquad T - 150(9.81) = 150(2.2025)$

$T = 1.80$ kN **Ans.**

Ans:
$T = 1.80$ kN

13–41.

A freight elevator, including its load, has a mass of 1 Mg. It is prevented from rotating due to the track and wheels mounted along its sides. If the motor M develops a constant tension $T = 4$ kN in its attached cable, determine the velocity of the elevator when it has moved upward 6 m starting from rest. Neglect the mass of the pulleys and cables.



SOLUTION

Equation of Motion. Referring to the FBD of the freight elevator shown in Fig. *a*,

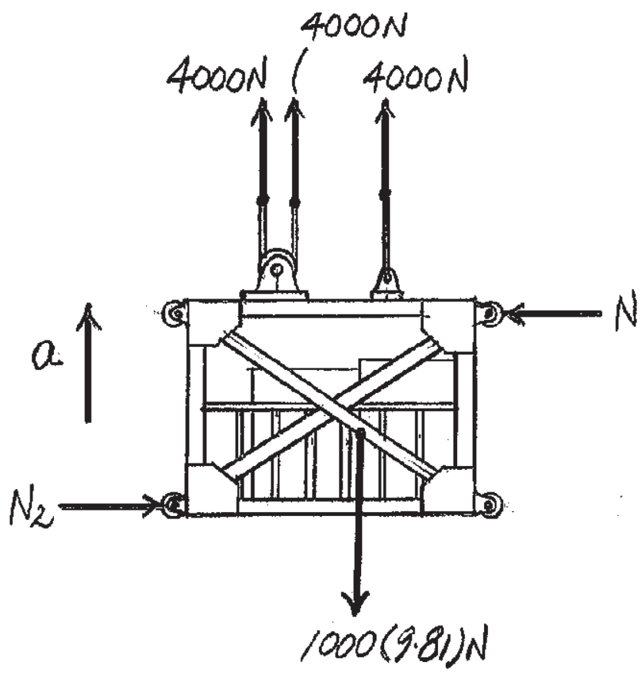
$$+\uparrow \Sigma F_y = ma_y; \qquad 3(4000) - 1000(9.81) = 1000a$$

$$a = 2.19 \text{ m/s}^2 \uparrow$$

Kinematics. Using the result of **a**,

$$(+\uparrow) \qquad v^2 = v_0^2 + 2as; \qquad v^2 = 0^2 + 2(2.19)(6)$$

$$v = 5.126 \text{ m/s} = 5.13 \text{ m/s} \qquad \textbf{Ans.}$$

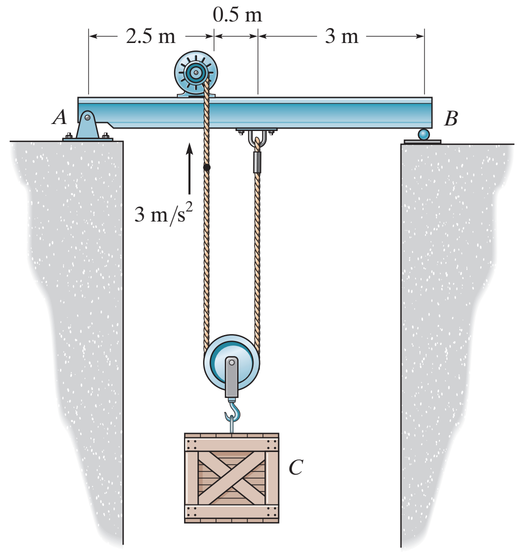


(a)

Ans:
$v = 5.13$ m/s

13–42.

If the motor draws in the cable with an acceleration of $3\,\text{m/s}^2$, determine the reactions at the supports A and B. The beam has a uniform mass of 30 kg/m, and the crate has a mass of 200 kg. Neglect the mass of the motor and pulleys.



SOLUTION

$S_c + (S_c - S_p)$

$2v_c = v_p$

$2a_c = a_p$

$2a_c = 3\text{ m/s}^2$

$a_c = 1.5\text{ m/s}^2$

$+\uparrow \Sigma F_y = ma_y \qquad 2T - 1962 = 200(1.5)$

$T = 1{,}131\text{ N}$

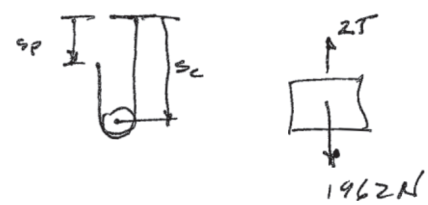
$\zeta + \Sigma M_A = 0; \qquad B_y(6) - (1765.8 + 1{,}131)3 - (1{,}131)(2.5) = 0$

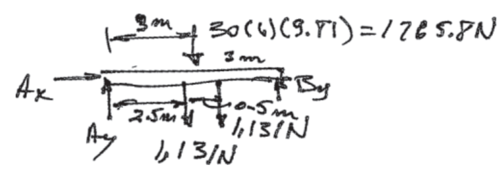
$B_y = 1919.65\text{ N} = 1.92\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad A_y - 1765.8 - (2)(1{,}131) + 1919.65 = 0$

$A_y = 2108.15\text{ N} = 2.11\text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**





Ans:

$B_y = 1.92\text{ kN}$
$A_y = 2.11\text{ kN}$
$A_x = 0$

13–43.

If the force exerted on cable AB by the motor is $F = (100t^{3/2})$ N, where t is in seconds, determine the 50-kg crate's velocity when $t = 5$ s. The coefficients of static and kinetic friction between the crate and the ground are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. Initially the crate is at rest.

SOLUTION

Free-Body Diagram: The frictional force $\mathbf{F}_f$ is required to act to the left to oppose the motion of the crate which is to the right.

Equations of Motion: Here, $a_y = 0$. Thus,

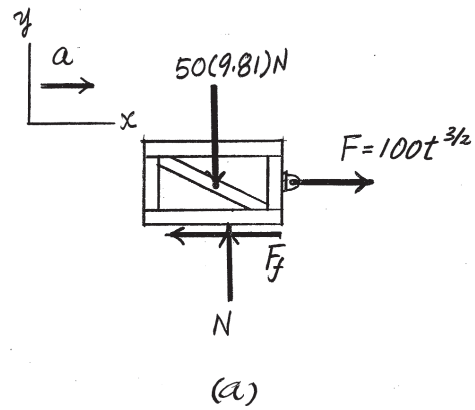
$$+\uparrow \Sigma F_y = ma_y; \qquad N - 50(9.81) = 50(0)$$

$$N = 490.5 \text{ N}$$

Realizing that $F_f = \mu_k N = 0.3(490.5) = 147.15$ N,

$$+\uparrow \Sigma F_x = ma_x; \qquad 100t^{3/2} - 147.15 = 50a$$

$$a = \left(2t^{3/2} - 2.943\right) \text{ m/s}$$



Equilibrium: For the crate to move, force $\mathbf{F}$ must overcome the static friction of $F_f = \mu_s N = 0.4(490.5) = 196.2$ N. Thus, the time required to cause the crate to be on the verge of moving can be obtained from.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 100t^{3/2} - 196.2 = 0$$

$$t = 1.567 \text{ s}$$

Kinematics: Using the result of **a** and integrating the kinematic equation $dv = a\,dt$ with the initial condition $v = 0$ at $t = 1.567$ as the lower integration limit,

$$(\overset{+}{\rightarrow}) \qquad \int dv = \int a\,dt$$

$$\int_0^v dv = \int_{1.567\text{ s}}^t \left(2t^{3/2} - 2.943\right)dt$$

$$v = \left(0.8t^{5/2} - 2.943t\right)\Big|_{1.567\text{ s}}^t$$

$$v = \left(0.8t^{5/2} - 2.943t + 2.152\right) \text{ m/s}$$

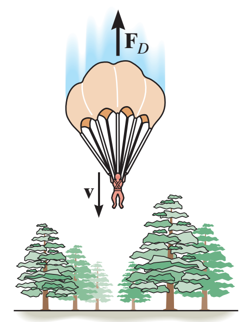
When $t = 5$ s,

$$v = 0.8(5)^{5/2} - 2.943(5) + 2.152 = 32.16 \text{ m/s} = 32.2 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 32.2$ m/s

***13–44.**

A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.



SOLUTION

$+\downarrow \Sigma F_z = ma_z;\qquad mg - kv^2 = m\frac{dv}{dt}$

$$m\int_0^v \frac{m\,dv}{(mg - kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k} - v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$e^{2t\sqrt{\frac{mg}{k}}} = \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$\sqrt{\frac{mg}{k}}e^{2t\sqrt{\frac{mg}{k}}} - ve^{2t\sqrt{\frac{mg}{k}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{mg}{k}}} - 1}{e^{2t\sqrt{\frac{mg}{k}}} + 1}\right] \qquad \textbf{Ans.}$$

When $t \rightarrow \infty \qquad v_t = \sqrt{\frac{mg}{k}}$ **Ans.**

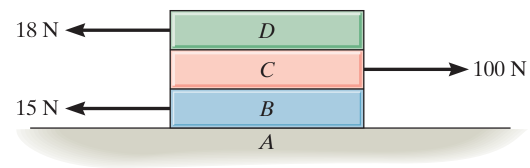
Ans:

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{mg/k}} - 1}{e^{2t\sqrt{mg/k}} + 1}\right]$$

$$v_t = \sqrt{\frac{mg}{k}}$$

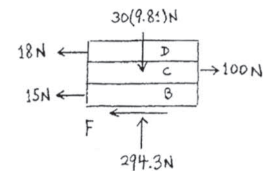
13–45.

Each of the three plates has a mass of 10 kg. If the coefficients of static and kinetic friction at each surface of contact are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the acceleration of each plate when the three horizontal forces are applied.



SOLUTION

Plates *B, C* and *D*

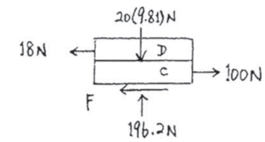


$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 100 - 15 - 18 - F = 0$

$F = 67\text{ N}$

$F_{max} = 0.3(294.3) = 88.3\text{ N} > 67\text{ N}$

Plate *B* will not slip.



$a_B = 0$ **Ans.**

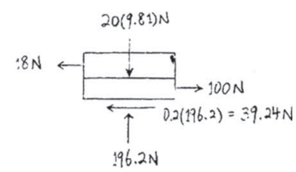
Plates *D* and *C*

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 100 - 18 - F = 0$

$F = 82\text{ N}$

$F_{max} = 0.3(196.2) = 58.86\text{ N} < 82\text{N}$

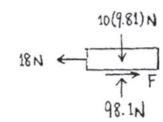
Slipping between *B* and *C*.



Assume no slipping between *D* and *C*,

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 100 - 39.24 - 18 = 20\,a_x$

$a_x = 2.138\text{ m/s}^2 \rightarrow$

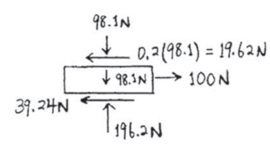


Check slipping between *D* and *C*.

$\overset{+}{\rightarrow} \Sigma F_x = m\,a_x; \qquad F - 18 = 10(2.138)$

$F = 39.38\text{ N}$

$F_{max} = 0.3(98.1) = 29.43\text{ N} < 39.38\text{ N}$



Slipping between *D* and *C*.

Plate *C*:

$\overset{+}{\rightarrow} \Sigma F_x = m\,a_x; \qquad 100 - 39.24 - 19.62 = 10\,a_c$

$a_C = 4.11\text{ m/s}^2 \rightarrow$ **Ans.**

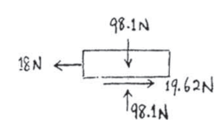


Plate *D*:

$\overset{+}{\rightarrow} \Sigma F_x = m\,a_x; \qquad 19.62 - 18 = 10\,a_D$

$a_D = 0.162\text{m/s}^2 \rightarrow$ **Ans.**

Ans:

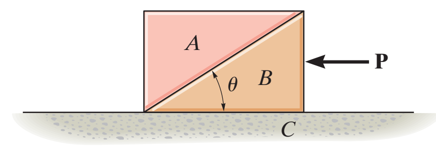
$a_B = 0$

$a_C = 4.11\text{ m/s}^2 \rightarrow$

$a_D = 0.162\text{ m/s}^2 \rightarrow$

13–46.

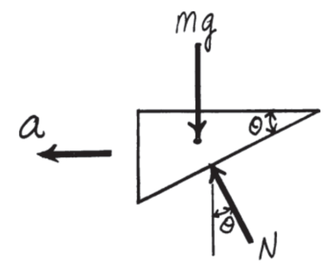
Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not move relative to B. All surfaces are smooth.



SOLUTION

Require

$a_A = a_B = a$



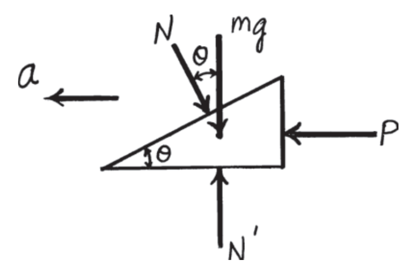
Block A:

$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N\sin\theta = ma$

$a = g\tan\theta$

Block B:



$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - N\sin\theta = ma$

$P - mg\tan\theta = mg\tan\theta$

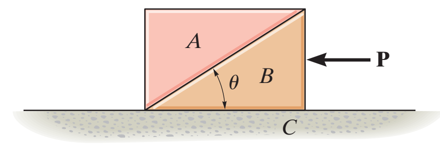
$P = 2mg\tan\theta$ **Ans.**

Ans:

$P = 2mg\tan\theta$

13–47.

Blocks A and B each have a mass m. Determine the largest horizontal force **P** which can be applied to B so that A will not slip on B. The coefficient of static friction between A and B is μ_s. Neglect any friction between B and C.

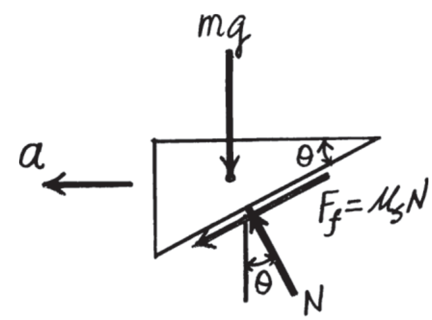


SOLUTION

Require

$a_A = a_B = a$

Block A:



$+\uparrow \Sigma F_y = 0; \qquad N\cos\theta - \mu_s N \sin\theta - mg = 0$

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad N\sin\theta + \mu_s N\cos\theta = ma$

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

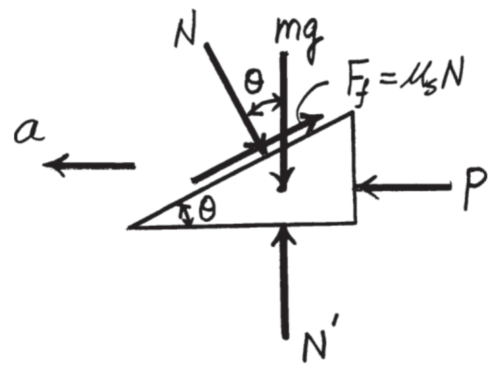
$$a = g\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

Block B:

$\overset{+}{\leftarrow} \Sigma F_x = ma_x; \qquad P - \mu_s N\cos\theta - N\sin\theta = ma$

$$P - mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right) = mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$ **Ans.**

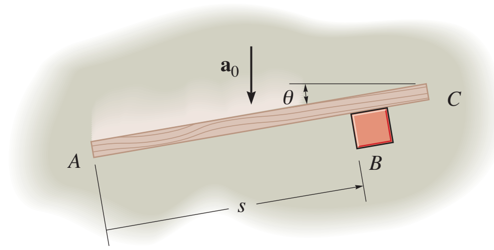


Ans:

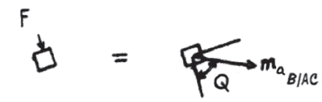
$$P = 2mg\left(\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)$$

***13–48.**

The smooth block B of negligible size has a mass m and rests on the horizontal plane. If the board AC pushes on the block at an angle θ with a constant acceleration $\mathbf{a}_0$, determine the velocity of the block along the board and the distance s the block moves along the board as a function of time t. The block starts from rest when $s = 0$, $t = 0$.



SOLUTION



$\nearrow + \Sigma F_x = m\,a_x;$ $\quad 0 = m\,a_B \sin\phi$

$$\mathbf{a}_B = \mathbf{a}_{AC} + \mathbf{a}_{B/AC}$$

$$\mathbf{a}_B = \mathbf{a_0} + \mathbf{a}_{B/AC}$$

$\nearrow +$ $\quad a_B \sin\phi = -a_0 \sin\theta + a_{B/AC}$

Thus,

$$0 = m(-a_0 \sin\theta + a_{B/AC})$$

$$a_{B/AC} = a_0 \sin\theta$$

$$\int_0^{v_{B/AC}} dv_{B/AC} = \int_0^t a_0 \sin\theta\, dt$$

$$v_{B/AC} = a_0 \sin\theta\, t \qquad \textbf{Ans.}$$

$$s_{B/AC} = s = \int_0^t a_0 \sin\theta\, t\, dt$$

$$s = \frac{1}{2} a_0 \sin\theta\, t^2 \qquad \textbf{Ans.}$$

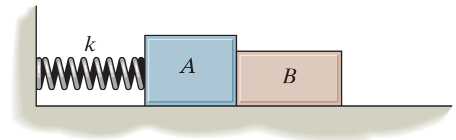
Ans:

$v_{B/AC} = a_0 \sin\theta\, t$

$s = \frac{1}{2} a_0 \sin\theta\, t^2$

13–49.

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, determine the distance both blocks slide on the smooth surface before they begin to separate. What is their velocity at this instant?



SOLUTION

Block A:

$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad -k(x-d) - N = m_A a_A$

Block B:

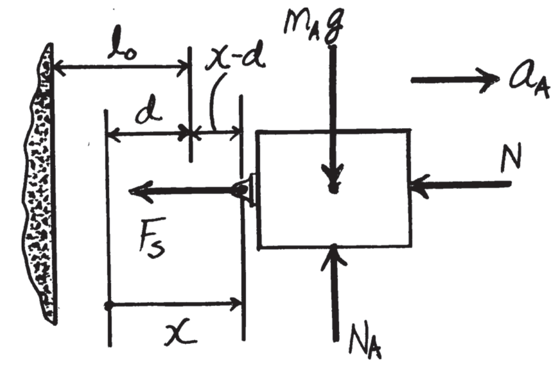
$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad N = m_B a_B$

Since $a_A = a_B = a$,

$-k(x-d) - m_B a = m_A a$

$$a = \frac{k(d-x)}{(m_A + m_B)} \qquad N = \frac{k m_B (d-x)}{(m_A+m_B)}$$

$N = 0$ when $\quad d - x = 0, \quad$ or $\qquad x = d$ **Ans.**

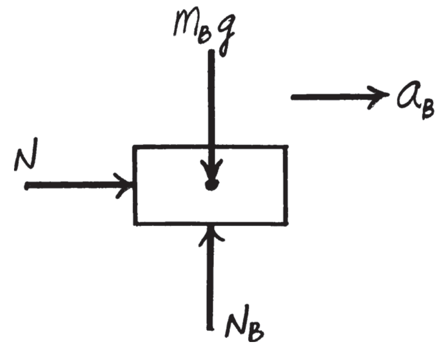


$v\,dv = a\,dx$

$$\int_0^v v\,dv = \int_0^d \frac{k(d-x)}{(m_A+m_B)}\,dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A+m_B)}\left[(d)x - \frac{1}{2}x^2\right]_0^d = \frac{1}{2}\frac{kd^2}{(m_A+m_B)}$$

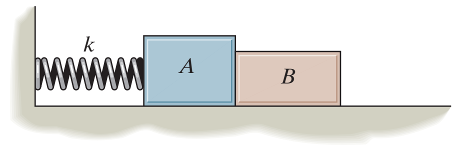
$$v = \sqrt{\frac{kd^2}{(m_A+m_B)}}$$ **Ans.**



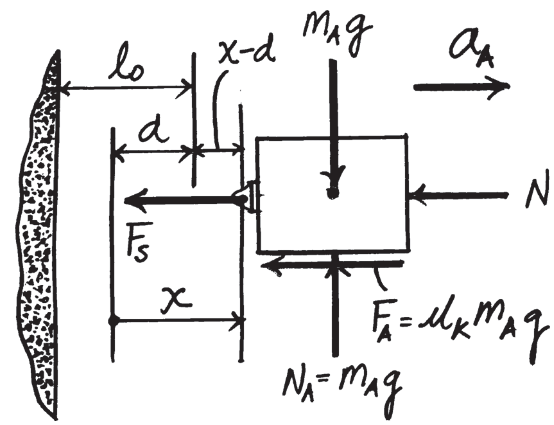
Ans:
$x = d$
$v = \sqrt{\dfrac{kd^2}{m_A + m_B}}$

13–50.

Block A has a mass m_A and is attached to a spring having a stiffness k and unstretched length l_0. If another block B, having a mass m_B, is pressed against A so that the spring deforms a distance d, show that for separation to occur it is necessary that $d > 2\mu_k g(m_A + m_B)/k$, where μ_k is the coefficient of kinetic friction between the blocks and the ground. Also, what is the distance the blocks slide on the surface before they separate?



SOLUTION



Block A:

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -k(x-d) - N - \mu_k m_A g = m_A a_A$$

Block B:

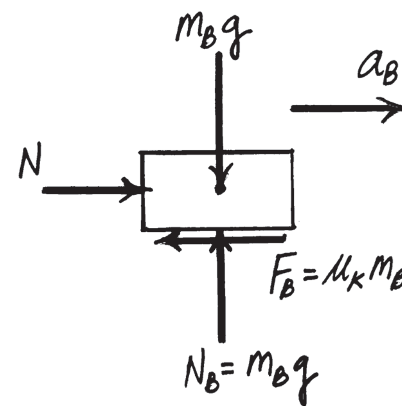
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad N - \mu_k m_B g = m_B a_B$$

Since $a_A = a_B = a$,

$$a = \frac{k(d-x) - \mu_k g(m_A + m_B)}{(m_A + m_B)} = \frac{k(d-x)}{(m_A + m_B)} - \mu_k g$$

$$N = \frac{k m_B (d-x)}{(m_A + m_B)}$$

$N = 0$, then $x = d$ for separation. **Ans.**



At the moment of separation:

$$v\,dv = a\,dx$$

$$\int_0^v v\,dv = \int_0^d \left[\frac{k(d-x)}{(m_A + m_B)} - \mu_k g\right] dx$$

$$\frac{1}{2}v^2 = \frac{k}{(m_A + m_B)}\left[(d)x - \frac{1}{2}x^2 - \mu_k g\,x\right]_0^d$$

$$v = \sqrt{\frac{kd^2 - 2\mu_k g(m_A + m_B)d}{(m_A + m_B)}}$$

Require $v > 0$, so that

$$kd^2 - 2\mu_k g(m_A + m_B)d > 0$$

Thus,

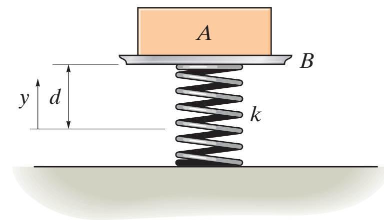
$$kd > 2\mu_k g(m_A + m_B)$$

$$d > \frac{2\mu_k g}{k}(m_A + m_B)$$ **Q.E.D.**

Ans:
$x = d$ for separation.

13–51.

The block A has a mass m_A and rests on the pan B, which has a mass m_B. Both are supported by a spring having a stiffness k that is attached to the bottom of the pan and to the ground. Determine the distance d the pan should be pushed down from the equilibrium position and then released from rest so that separation of the block will take place from the surface of the pan at the instant the spring becomes unstretched.

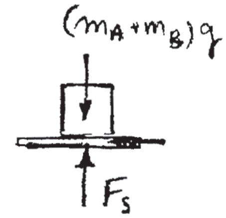


SOLUTION

For Equilibrium

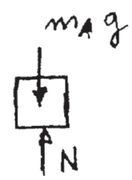
$$+\uparrow \Sigma F_y = ma_y; \qquad F_s = (m_A + m_B)g$$

$$y_{eq} = \frac{F_s}{k} = \frac{(m_A + m_B)g}{k}$$



Block:

$$+\uparrow \Sigma F_y = ma_y; \qquad -m_A g + N = m_A a$$



Block and pan

$$+\uparrow \Sigma F_y = ma_y; \qquad -(m_A + m_B)g + k(y_{eq} + y) = (m_A + m_B)a$$

Thus,

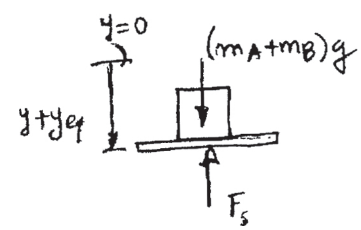
$$-(m_A + m_B)g + k\left[\left(\frac{m_A + m_B}{k}\right)g + y\right] = (m_A + m_B)\left(\frac{-m_A g + N}{m_A}\right)$$

Require $y = d, N = 0$

$$kd = -(m_A + m_B)g$$

Since d is downward,

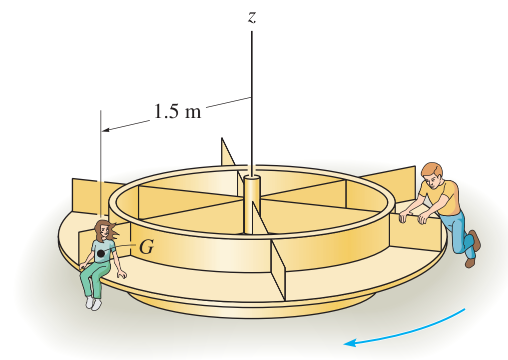
$$d = \frac{(m_A + m_B)g}{k} \qquad \textbf{Ans.}$$



Ans:
$d = \dfrac{(m_A + m_B)g}{k}$

***13–52.**

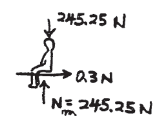
A girl having a mass of 25 kg sits at the edge of the merry-go-round so her center of mass G is at a distance of 1.5 m from the axis of rotation. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which she can have before she begins to slip off the merry-go-round. The coefficient of static friction between the girl and the merry-go-round is $\mu_s = 0.3$.



SOLUTION

$$\xrightarrow{+} \Sigma F_n = ma_n; \qquad 0.3(245.25) = 25\left(\frac{v^2}{1.5}\right)$$

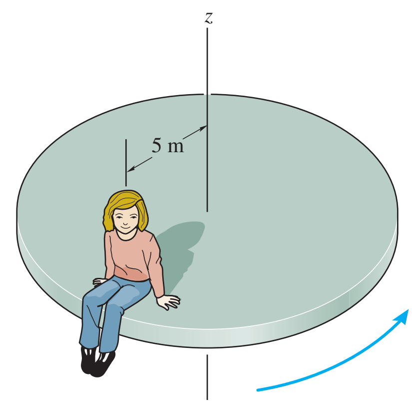
$$v = 2.10 \text{ m/s}$$ **Ans.**



Ans:
$v = 2.10$ m/s

13–53.

A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.



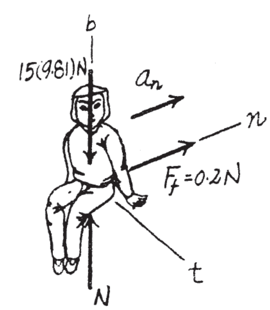
SOLUTION

Equation of Motion: Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13–7, we have

$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15 \text{ N}$$

$$\Sigma F_n = ma_n; \qquad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$$

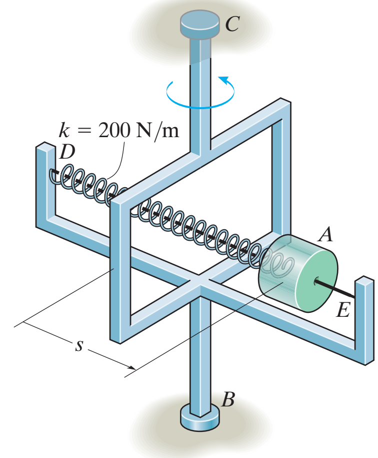
$$v = 3.13 \text{ m/s}$$ **Ans.**



Ans:
$v = 3.13 \text{ m/s}$

13–54.

The collar A, having a mass of 0.75 kg, is attached to a spring having a stiffness of $k = 200$ N/m. When rod BC rotates about the vertical axis, the collar slides outward along the smooth rod DE. If the spring is unstretched when $s = 0$, determine the constant speed of the collar in order that $s = 100$ mm. Also, what is the normal force of the rod on the collar? Neglect the size of the collar.



SOLUTION

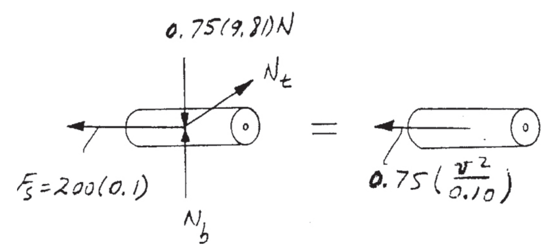
$\Sigma F_b = 0;$ $\quad N_b - 0.75(9.81) = 0 \quad N_b = 7.36$

$\Sigma F_n = ma_n;$ $\quad 200(0.1) = 0.75\left(\frac{v^2}{0.10}\right)$

$\Sigma F_t = ma_t;$ $\quad N_t = 0$

$v = 1.63$ m/s **Ans.**

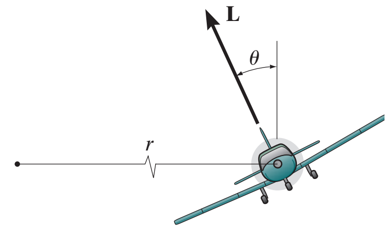
$N = \sqrt{(7.36)^2 + (0)} = 7.36$ N **Ans.**



Ans:
$v = 1.63$ m/s
$N = 7.36$ N

13–55.

A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path of radius $r = 3000$ m. Determine the uplift force **L** acting on the airplane and the banking angle θ. Neglect the size of the airplane.



SOLUTION

Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

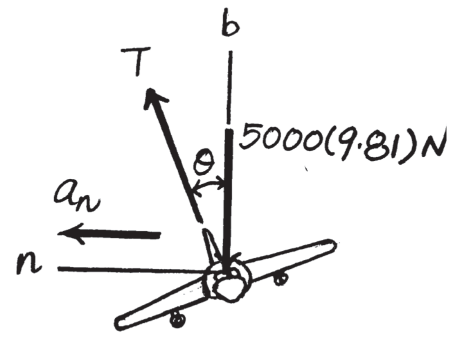
Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 97.22$ m/s. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{3000} = 3.151\text{ m/s}^2$ and referring to Fig. (a),

$+\uparrow \Sigma F_b = 0;$ $\qquad T\cos\theta - 5000(9.81) = 0$ **(1)**

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\qquad T\sin\theta = 5000(3.151)$ **(2)**

Solving Eqs. (1) and (2) yields

$\theta = 17.8^\circ \qquad T = 51517.75 = 51.5\text{ kN}$ **Ans.**



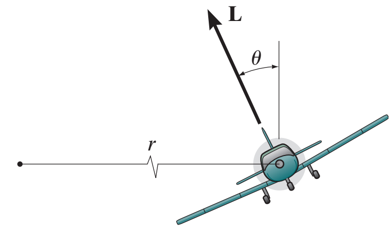
(a)

Ans:

$T = 51.5$ kN

***13–56.**

A 5-Mg airplane is flying at a constant speed of 350 km/h along a horizontal circular path. If the banking angle $\theta = 15°$, determine the uplift force **L** acting on the airplane and the radius r of the circular path. Neglect the size of the airplane.



SOLUTION

Free-Body Diagram: The free-body diagram of the airplane is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).

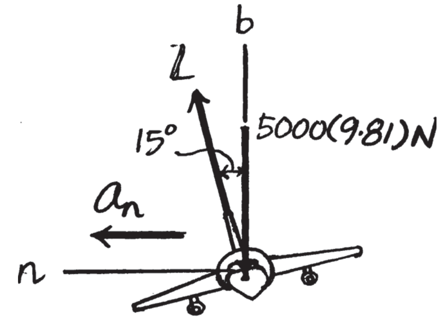
Equations of Motion: The speed of the airplane is $v = \left(350\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 97.22\text{ m/s}$. Realizing that $a_n = \frac{v^2}{\rho} = \frac{97.22^2}{r}$ and referring to Fig. (a),

$+\uparrow \Sigma F_b = 0;$ $\quad L\cos 15° - 5000(9.81) = 0$

$L = 50780.30\text{ N} = 50.8\text{ kN}$ **Ans.**

$\overset{+}{\leftarrow} \Sigma F_n = ma_n;$ $\quad 50780.30\sin 15° = 5000\left(\frac{97.22^2}{r}\right)$

$r = 3595.92\text{ m} = 3.60\text{ km}$ **Ans.**

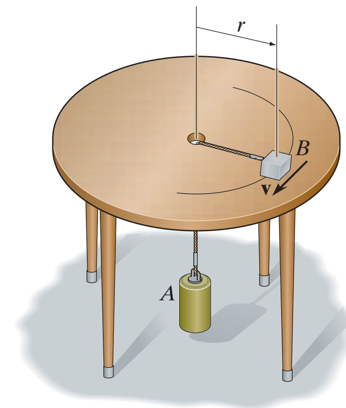


(a)

Ans:
$L = 50.8\text{ kN}$
$r = 3.60\text{ km}$

13–57.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block is given a speed of $v = 10\,\text{m/s}$, determine the radius r of the circular path along which it travels.



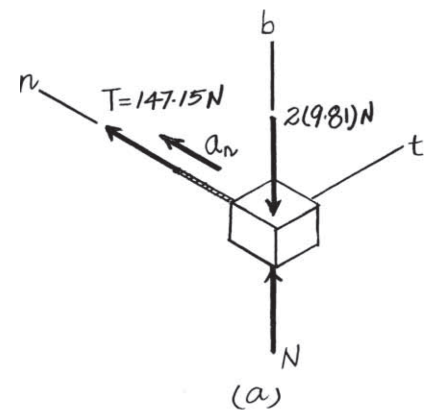
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{10^2}{r}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{10^2}{r}\right)$$

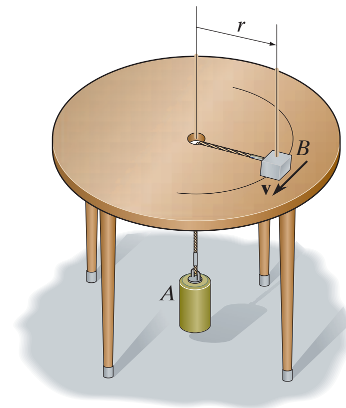
$$r = 1.36\text{ m} \qquad \textbf{Ans.}$$



Ans:
$r = 1.36\text{ m}$

13–58.

The 2-kg block B and 15-kg cylinder A are connected to a light cord that passes through a hole in the center of the smooth table. If the block travels along a circular path of radius $r = 1.5\,\text{m}$, determine the speed of the block.



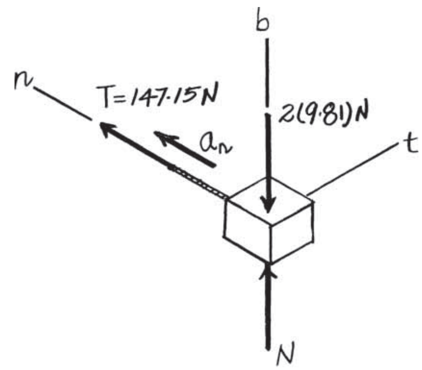
SOLUTION

Free-Body Diagram: The free-body diagram of block B is shown in Fig. (a). The tension in the cord is equal to the weight of cylinder A, i.e., $T = 15(9.81)\text{ N} = 147.15\text{ N}$. Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: Realizing that $a_n = \dfrac{v^2}{r} = \dfrac{v^2}{1.5}$ and referring to Fig. (a),

$$\Sigma F_n = ma_n; \qquad 147.15 = 2\left(\frac{v^2}{1.5}\right)$$

$$v = 10.5\text{ m/s} \qquad \textbf{Ans.}$$

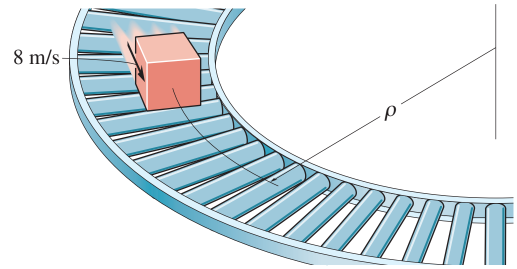


(a)

Ans:
$v = 10.5\text{ m/s}$

13–59.

Cartons having a mass of 5 kg are required to move along the assembly line at a constant speed of 8 m/s. Determine the smallest radius of curvature, ρ, for the conveyor so the cartons do not slip. The coefficients of static and kinetic friction between a carton and the conveyor are $\mu_s = 0.7$ and $\mu_k = 0.5$, respectively.



SOLUTION

$+\uparrow \Sigma F_b = m\,a_b;\qquad N - W = 0$

$N = W$

$F_x = 0.7W$

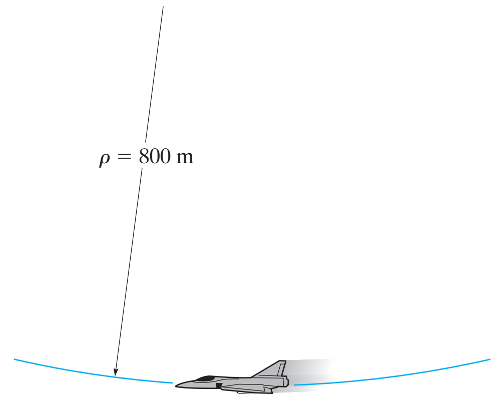
$\overset{+}{\leftarrow} \Sigma F_n = m a_n;\qquad 0.7W = \dfrac{W}{9.81}\left(\dfrac{8^2}{\rho}\right)$

$\rho = 9.32\text{ m}$ **Ans.**

Ans:
$\rho = 9.32\text{ m}$

***13–60.**

Determine the maximum constant speed at which the pilot can travel around the vertical curve having a radius of curvature $\rho = 800$ m, so that he experiences a maximum acceleration $a_n = 8g = 78.5\ \text{m/s}^2$. If he has a mass of 70 kg, determine the normal force he exerts on the seat of the airplane when the plane is traveling at this speed and is at its lowest point.



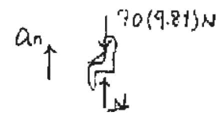
SOLUTION

$$a_n = \frac{v^2}{\rho}; \qquad 78.5 = \frac{v^2}{800}$$

$v = 251$ m/s **Ans.**

$+\uparrow \Sigma F_n = ma_n; \qquad N - 70(9.81) = 70(78.5)$

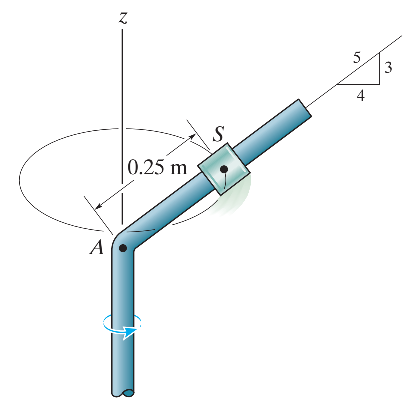
$N = 6.18$ kN **Ans.**



Ans:
$N = 6.18$ kN

13–61.

The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the minimum constant speed the spool can have so that it does not slip down the rod.



SOLUTION

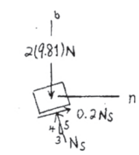
$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2\text{ m}$$

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $$N_s\left(\frac{3}{5}\right) - 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$$

$+\uparrow \Sigma F_b = m\,a_b;$ $$N_s\left(\frac{4}{5}\right) + 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$$

$$N_s = 21.3\text{ N}$$

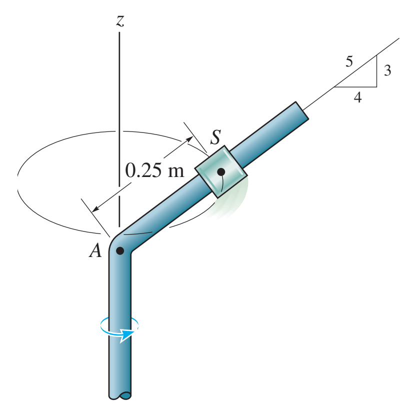
$$v = 0.969\text{ m/s}$$ **Ans.**



Ans:
$v = 0.969\text{ m/s}$

13–62.

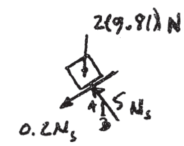
The 2-kg spool S fits loosely on the inclined rod for which the coefficient of static friction is $\mu_s = 0.2$. If the spool is located 0.25 m from A, determine the maximum constant speed the spool can have so that it does not slip up the rod.



SOLUTION

$$\rho = 0.25\left(\frac{4}{5}\right) = 0.2 \text{ m}$$

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;\qquad N_s\left(\frac{3}{5}\right) + 0.2N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$



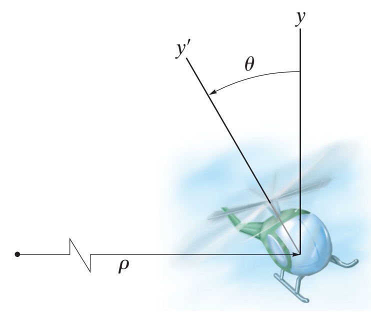
$+\uparrow \Sigma F_b = m\,a_b;\qquad N_s\left(\frac{4}{5}\right) - 0.2N_s\left(\frac{3}{5}\right) - 2(9.81) = 0$

$$N_s = 28.85 \text{ N}$$

$$v = 1.48 \text{ m/s}$$ **Ans.**

Ans:
$v = 1.48$ m/s

13–63. The 1.40-Mg helicopter is traveling at a constant speed of 40 m/s along the horizontal curved path while banking at $\theta = 30^\circ$. Determine the force acting normal to the blade, i.e., in the y' direction, and the radius of curvature of the path.



SOLUTION

Guesses $F_N = 1 \text{ kN}$ $\rho = 1 \text{ m}$

Given $F_N \cos(\theta) - Mg = 0$

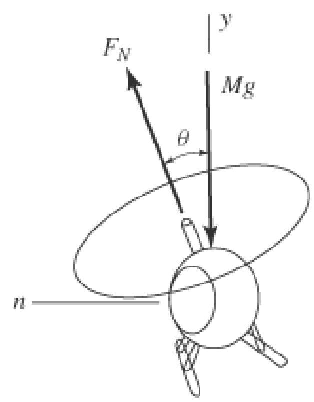
$$F_N \sin(\theta) = M\left(\frac{v^2}{\rho}\right)$$

$v = 40 \text{ m/s}$

$M = 1.4 \times 10^3 \text{ kg}$

$\theta = 30^\circ$

$g = 9.81 \text{ m/s}^2$

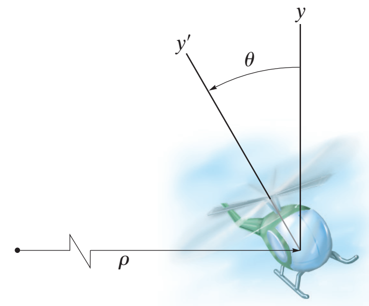


$$\begin{pmatrix} F_N \\ \rho \end{pmatrix} = \text{Find}(F_N, \rho)$$ $F_N = 15.86 \text{ kN}$ **Ans.**

$\rho = 282 \text{ m}$ **Ans.**

Ans:
$F_N = 15.86 \text{ kN}$
$\rho = 282 \text{ m}$

***13–64.** The 1.40-Mg helicopter is traveling at a constant speed of 33 m/s along the horizontal curved path having a radius of curvature of $\rho = 300$ m. Determine the force the blade exerts on the frame and the bank angle θ.



SOLUTION

Guesses $F_N = 1\ \text{kN}$ $\theta = 1°$

Given $F_N\cos(\theta) - Mg = 0$

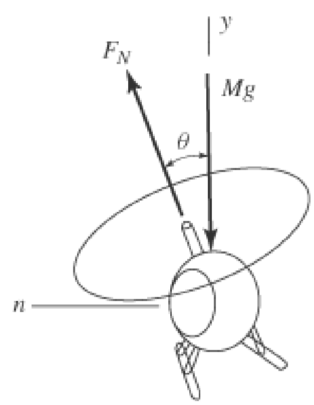
$$F_N\sin(\theta) = M\left(\frac{v^2}{\rho}\right)$$

$v = 33\ \text{m/s}$

$M = 1.4 \times 10^3\ \text{kg}$

$\rho = 300\ \text{m}$

$g = 9.81\ \text{m/s}^2$



$$\begin{pmatrix} F_N \\ \theta \end{pmatrix} = \text{Find}(F_N, \theta)$$

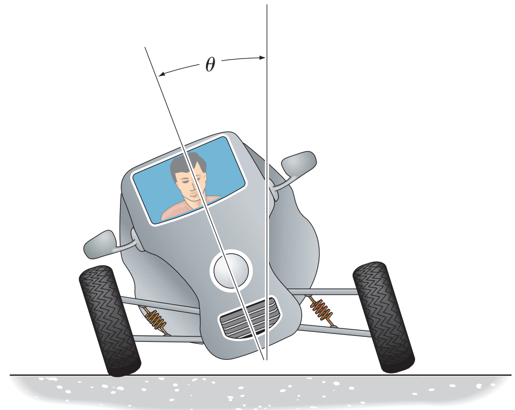
$F_N = 14.64\ \text{kN}$ **Ans.**

$\theta = 20°$ **Ans.**

Ans:
$F_N = 14.64\ \text{kN}$
$\theta = 20°$

13–65.

The vehicle is designed to combine the feel of a motorcycle with the comfort and safety of an automobile. If the vehicle is traveling at a constant speed of 80 km/h along a circular curved road of radius 100 m, determine the tilt angle θ of the vehicle so that only a normal force from the seat acts on the driver. Neglect the size of the driver.



SOLUTION

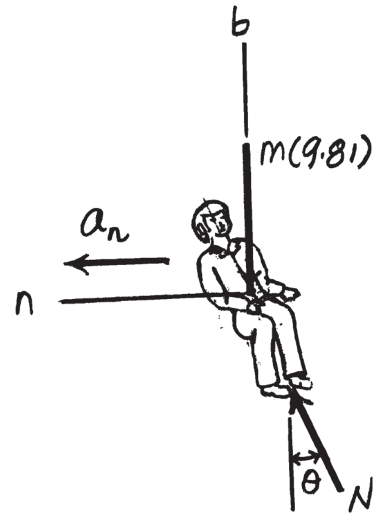
Free-Body Diagram: The free-body diagram of the passenger is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of the circular path (positive n axis).

Equations of Motion: The speed of the passenger is $v = \left(80\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$ $= 22.22$ m/s. Thus, the normal component of the passenger's acceleration is given by $a_n = \frac{v^2}{\rho} = \frac{22.22^2}{100} = 4.938\text{ m/s}^2$. By referring to Fig. (a),

$$+\uparrow \Sigma F_b = 0; \qquad N\cos\theta - m(9.81) = 0 \qquad N = \frac{9.81m}{\cos\theta}$$

$$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad \frac{9.81m}{\cos\theta}\sin\theta = m(4.938)$$

$$\theta = 26.7^\circ$$ **Ans.**

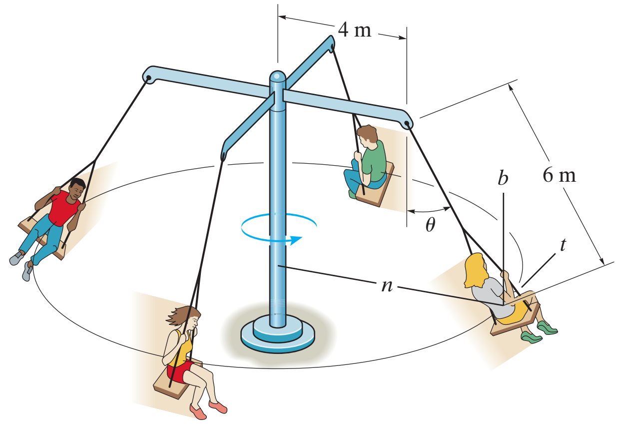


(a)

Ans:
$\theta = 26.7^\circ$

13–66.

Determine the constant speed of the passengers on the amusement-park ride if it is observed that the supporting cables are directed at $\theta = 30°$ from the vertical. Each chair including its passenger has a mass of 80 kg. Also, what are the components of force in the *n, t*, and *b* directions which the chair exerts on a 50-kg passenger during the motion?



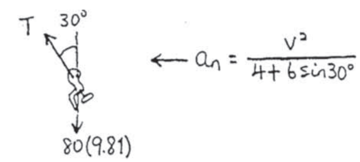
SOLUTION

$\overset{+}{\leftarrow} \Sigma F_n = m\,a_n;$ $\quad T \sin 30° = 80\left(\dfrac{v^2}{4 + 6\sin 30°}\right)$

$+\uparrow \Sigma F_b = 0;$ $\quad T \cos 30° - 80(9.81) = 0$

$T = 906.2 \text{ N}$

$v = 6.30 \text{ m/s}$ **Ans.**

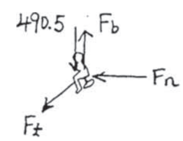


$\Sigma F_n = m\,a_n;$ $\quad F_n = 50\left(\dfrac{(6.30)^2}{7}\right) = 283 \text{ N}$ **Ans.**

$\Sigma F_t = m\,a_t;$ $\quad F_t = 0$ **Ans.**

$\Sigma F_b = m\,a_b;$ $\quad F_b - 490.5 = 0$

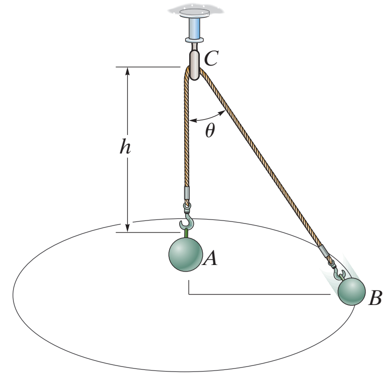
$F_b = 490 \text{ N}$ **Ans.**



Ans:
$v = 6.30 \text{ m/s}$
$F_n = 283 \text{ N}$
$F_t = 0$
$F_b = 490 \text{ N}$

13–67.

Bobs A and B of mass m_A and m_B $(m_A > m_B)$ are connected to an inextensible light string of length l that passes through the smooth ring at C. If bob B moves as a conical pendulum such that A is suspended a distance of h from C, determine the angle θ and the speed of bob B. Neglect the size of both bobs.



SOLUTION

Free-Body Diagram: The free-body diagram of bob B is shown in Fig. a. The tension developed in the string is equal to the weight of bob A, i.e., $T = m_A g$. Here, $\mathbf{a}_n$ must be directed towards the center of the horizontal circular path (positive n axis).

Equations of Motion: The radius of the horizontal circular path is $r = (l - h)\sin\theta$. Thus, $a_n = \dfrac{v^2}{\rho} = \dfrac{v_B{}^2}{(l - h)\sin\theta}$. By referring to Fig. a,

$+\uparrow \Sigma F_b = 0; \qquad m_A g\cos\theta - m_B g = 0$

$$\theta = \cos^{-1}\left(\frac{m_B}{m_A}\right) \qquad \textbf{Ans.}$$

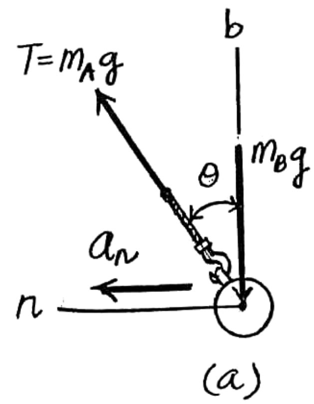
$\overset{+}{\leftarrow} \Sigma F_n = ma_n; \qquad m_A g\sin\theta = m_B\left[\dfrac{v_B{}^2}{(l - h)\sin\theta}\right]$

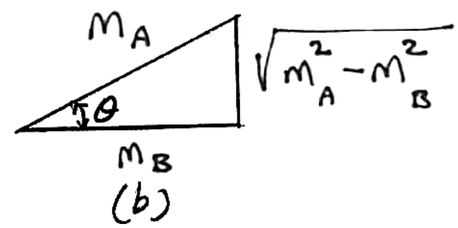
$$v_B = \sqrt{\frac{m_A g(l - h)}{m_B}}\sin\theta \qquad \textbf{(1)}$$

From Fig. b, $\sin\theta = \dfrac{\sqrt{m_A{}^2 - m_B{}^2}}{m_A}$. Substituting this value into Eq. (1),

$$v_B = \sqrt{\frac{m_A g(l - h)}{m_B}}\left(\frac{\sqrt{m_A{}^2 - m_B{}^2}}{m_A}\right)$$

$$= \sqrt{\frac{g(l - h)(m_A{}^2 - m_B{}^2)}{m_A m_B}} \qquad \textbf{Ans.}$$





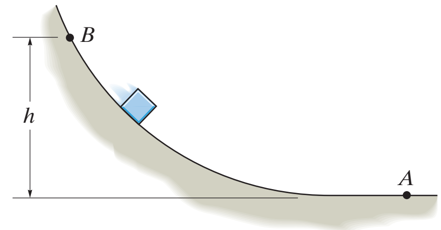
Ans:

$\theta = \cos^{-1}\left(\dfrac{m_B}{m_A}\right)$

$v_B = \sqrt{\dfrac{g(l - h)(m_A^2 - m_B^2)}{m_A m_B}}$

***13–68.**

Prove that if the block is released from rest at point B of a smooth path of *arbitrary shape*, the speed it attains when it reaches point A is equal to the speed it attains when it falls freely through a distance h; i.e., $v = \sqrt{2gh}$.



SOLUTION

$+\searrow \Sigma F_t = ma_t; \qquad mg \sin\theta = ma_t \qquad a_t = g\sin\theta$

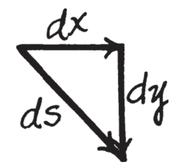
$v\,dv = a_t\,ds = g\sin\theta\,ds \qquad$ However $\quad dy = ds\sin\theta$

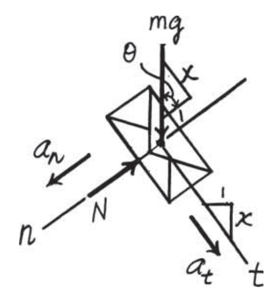
$$\int_0^v v\,dv = \int_0^h g\,dy$$

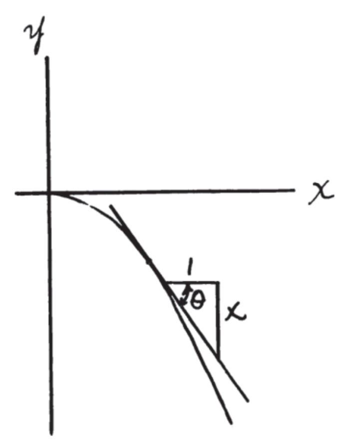
$$\frac{v^2}{2} = gh$$

$$v = \sqrt{2gh}$$

Q.E.D.

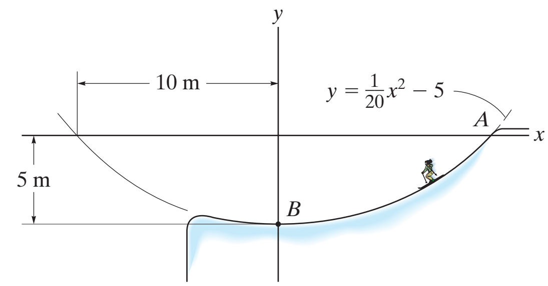






13–69.

The skier starts from rest at $A(10\text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force the ground exerts on the skier at the instant she arrives at point B. Neglect the size of the skier.



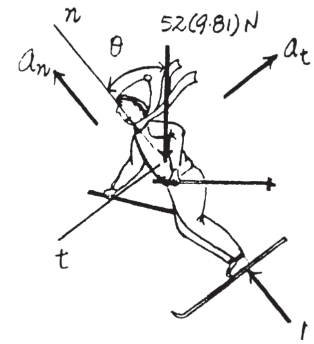
SOLUTION

Geometry: Here, $\dfrac{dy}{dx} = \dfrac{1}{10}x$ and $\dfrac{d^2y}{dx^2} = \dfrac{1}{10}$. The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=0\text{ m}} = 0 \qquad \theta = 0°$$

and the radius of curvature at point B is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{|1/10|}\right|_{x=0\text{ m}} = 10.0\text{ m}$$



Equations of Motion:

$+\swarrow\Sigma F_t = ma_t;$ $\quad 52(9.81)\sin\theta = -52a_t \qquad a_t = -9.81\sin\theta$

$+\nwarrow\Sigma F_n = ma_n;$ $\quad N - 52(9.81)\cos\theta = m\left(\dfrac{v^2}{\rho}\right)$ **(1)**

Kinematics: The speed of the skier can be determined using $v\,dv = a_t\,ds$. Here, a_t must be in the direction of positive ds. Also, $ds = \sqrt{1 + (dy/dx)^2}dx = \sqrt{1 + \frac{1}{100}x^2}dx$

Here, $\tan\theta = \dfrac{1}{10}x$. Then, $\sin\theta = \dfrac{x}{10\sqrt{1 + \frac{1}{100}x^2}}$.

$$(+\ ) \qquad \int_0^v v\,dv = -9.81\int_{10\text{ m}}^{0}\left(\frac{x}{10\sqrt{1 + \frac{1}{100}x^2}}\right)\left(\sqrt{1 + \tfrac{1}{100}x^2}dx\right)$$

$$v^2 = 9.81\text{ m}^2/\text{s}^2$$

Substituting $v^2 = 98.1\text{ m}^2/\text{s}^2$, $\theta = 0°$, and $\rho = 10.0$ m into Eq.(1) yields

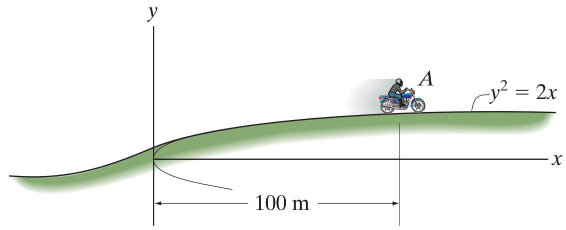
$$N - 52(9.81)\cos 0° = 52\left(\frac{98.1}{10.0}\right)$$

$$N = 1020.24\text{ N} = 1.02\text{ kN}$$ **Ans.**

Ans:
$N = 1.02$ kN

13–70.

The 800-kg motorbike travels with a constant speed of 80 km/h up the hill. Determine the normal force the surface exerts on its wheels when it reaches point A. Neglect its size.



SOLUTION

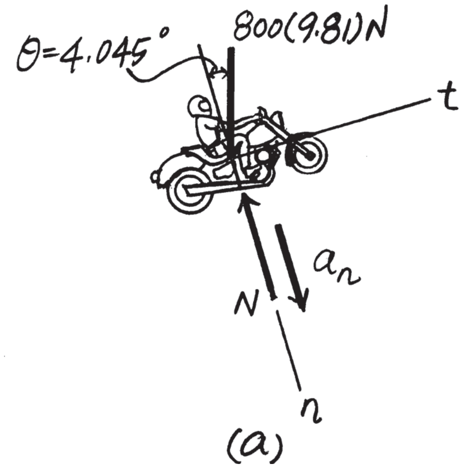
Geometry: Here, $y = \sqrt{2}x^{1/2}$. Thus, $\frac{dy}{dx} = \frac{\sqrt{2}}{2x^{1/2}}$ and $\frac{d^2y}{dx^2} = -\frac{\sqrt{2}}{4x^{3/2}}$. The angle that the hill slope at A makes with the horizontal is

$$\theta = \tan^{-1}\left(\frac{dy}{dx}\right)\bigg|_{x=100\text{ m}} = \tan^{-1}\left(\frac{\sqrt{2}}{2x^{1/2}}\right)\bigg|_{x=100\text{ m}} = 4.045°$$

The radius of curvature of the hill at A is given by

$$\rho_A = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}\Bigg|_{x=100\text{ m}} = \frac{\left[1+\left(\frac{\sqrt{2}}{2(100^{1/2})}\right)^2\right]^{3/2}}{\left|-\frac{\sqrt{2}}{4(100^{3/2})}\right|} = 2849.67\text{ m}$$

Free-Body Diagram: The free-body diagram of the motorcycle is shown in Fig. (a). Here, $\mathbf{a}_n$ must be directed towards the center of curvature (positive n axis).



Equations of Motion: The speed of the motorcycle is

$$v = \left(80\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 22.22\text{ m/s}$$

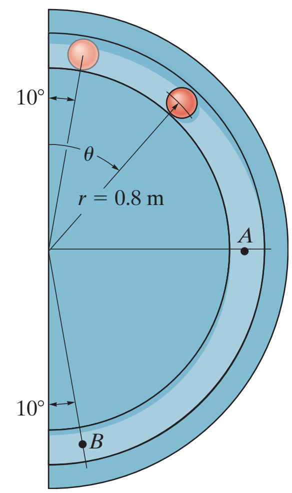
Thus, $a_n = \frac{v^2}{\rho_A} = \frac{22.22^2}{2849.67} = 0.1733\text{ m/s}^2$. By referring to Fig. (a),

$\searrow + \Sigma F_n = ma_n;$ $\quad 800(9.81)\cos 4.045° - N = 800(0.1733)$

$N = 7689.82\text{ N} = 7.69\text{ kN}$ **Ans.**

Ans:
$N = 7.69\text{ kN}$

13–71. A ball having a mass 2 kg and negligible size moves within a smooth vertical circular slot. If it is released from rest when $\theta = 10°$, determine the force of the slot on the ball when the ball arrives at points A and B.



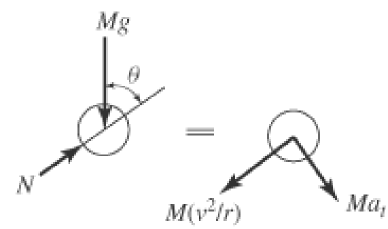
SOLUTION

Given:

$$M = 2\ \text{kg} \qquad \theta = 90° \qquad \theta_1 = 10°$$

$$g = 9.81\ \text{m/s}^2 \qquad r = 0.8\ \text{m}$$

$$Mg\sin(\theta) = Ma_t \qquad a_t = g\sin(\theta)$$



At A $\qquad \theta_A = 90°$

$$v_A = \sqrt{2g\left(\int_{\theta_1}^{\theta_A} \sin(\theta)\, r\, d\theta\right)}$$

$$N_A - Mg\cos(\theta_A) = -M\left(\frac{v_A^{\ 2}}{r}\right)$$

$$N_A = Mg\cos(\theta_A) - M\left(\frac{v_A^{\ 2}}{r}\right) \qquad N_A = -38.6\ \text{N} \qquad \textbf{Ans.}$$

At B $\qquad \theta_B = 180° - \theta_1$

$$v_B = \sqrt{2g\left(\int_{\theta_1}^{\theta_B} \sin(\theta)\, r\, d\theta\right)}$$

$$N_B - Mg\cos(\theta_B) = -M\left(\frac{v_B^{\ 2}}{r}\right)$$

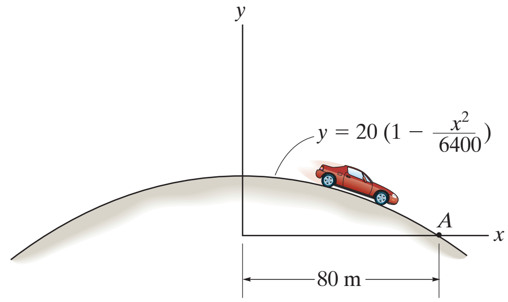
$$N_B = Mg\cos(\theta_B) - M\left(\frac{v_B^{\ 2}}{r}\right) \qquad N_B = -96.6\ \text{N} \qquad \textbf{Ans.}$$

Ans:

$N_A = -38.6\ \text{N}$
$N_B = -96.6\ \text{N}$

***13–72.**

The 0.8-Mg car travels over the hill having the shape of a parabola. If the driver maintains a constant speed of 9 m/s, determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at the instant it reaches point A. Neglect the size of the car.



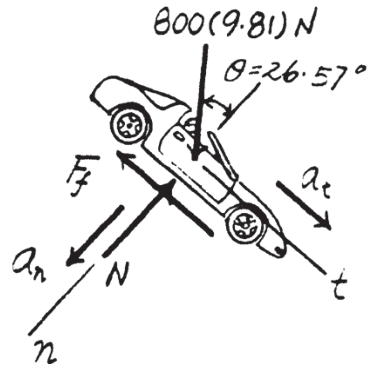
SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$

and the radius of curvature at point A is

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{[1 + (-0.00625x)^2]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$



Equations of Motion: Here, $a_t = 0$. Applying Eq. 13–7 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$$\Sigma F_t = ma_t; \qquad 800(9.81)\sin 26.57^\circ - F_f = 800(0)$$

$$F_f = 3509.73\text{ N} = 3.51\text{ kN} \qquad \textbf{Ans.}$$

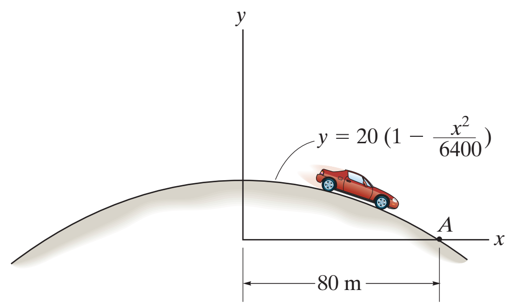
$$\Sigma F_n = ma_n; \qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$$

$$N = 6729.67\text{ N} = 6.73\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_f = 3.51$ kN
$N = 6.73$ kN

13–73.

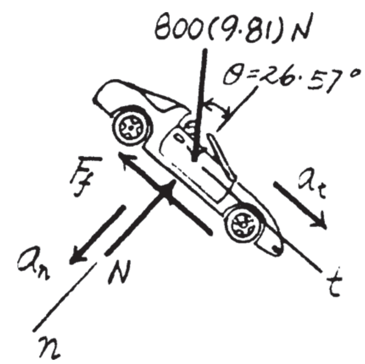
The 0.8-Mg car travels over the hill having the shape of a parabola. When the car is at point A, it is traveling at 9 m/s and increasing its speed at 3 m/s^2. Determine both the resultant normal force and the resultant frictional force that all the wheels of the car exert on the road at this instant. Neglect the size of the car.



SOLUTION

Geometry: Here, $\frac{dy}{dx} = -0.00625x$ and $\frac{d^2y}{dx^2} = -0.00625$. The slope angle θ at point A is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=80\text{ m}} = -0.00625(80) \qquad \theta = -26.57^\circ$$



and the radius of curvature at point A is

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + (-0.00625x)^2\right]^{3/2}}{|-0.00625|}\right|_{x=80\text{ m}} = 223.61\text{ m}$$

Equation of Motion: Applying Eq. 13–7 with $\theta = 26.57^\circ$ and $\rho = 223.61$ m, we have

$\Sigma F_t = ma_t;$ $\qquad 800(9.81)\sin 26.57^\circ - F_f = 800(3)$

$F_f = 1109.73\text{ N} = 1.11\text{ kN}$ **Ans.**

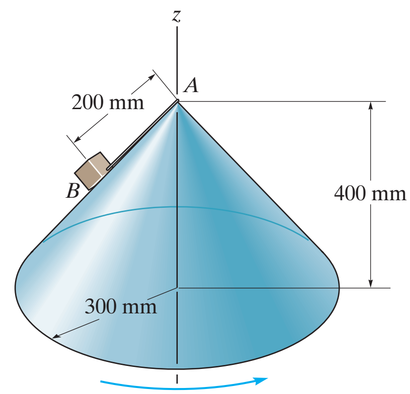
$\Sigma F_n = ma_n;$ $\qquad 800(9.81)\cos 26.57^\circ - N = 800\left(\frac{9^2}{223.61}\right)$

$N = 6729.67\text{ N} = 6.73\text{ kN}$ **Ans.**

Ans:

$F_f = 1.11$ kN
$N = 6.73$ kN

13–74. The block B, having a mass of 0.2 kg, is attached to the vertex A of the right circular cone using a light cord. The cone is rotating at a constant angular rate about the z axis such that the block attains a speed of 0.5 m/s. At this speed, determine the tension in the cord and the reaction which the cone exerts on the block. Neglect the size of the block and the effect of friction.



SOLUTION

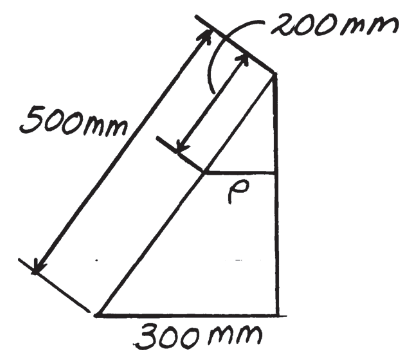
$$\frac{\rho}{200} = \frac{300}{500}; \qquad \rho = 120 \text{ mm} = 0.120 \text{ m}$$

$$+\nearrow \Sigma F_y = ma_y; \qquad T - 0.2(9.81)\left(\frac{4}{5}\right) = \left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{3}{5}\right)$$

$$T = 1.82 \text{ N} \qquad \textbf{Ans.}$$

$$+\nwarrow \Sigma F_x = ma_x; \qquad N_B - 0.2(9.81)\left(\frac{3}{5}\right) = -\left[0.2\left(\frac{(0.5)^2}{0.120}\right)\right]\left(\frac{4}{5}\right)$$

$$N_B = 0.844 \text{ N} \qquad \textbf{Ans.}$$



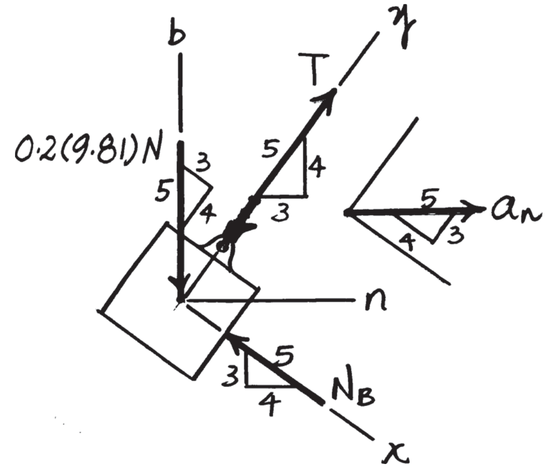
Also,

$$\xrightarrow{+} \Sigma F_n = ma_n; \qquad T\left(\frac{3}{5}\right) - N_B\left(\frac{4}{5}\right) = 0.2\left(\frac{(0.5)^2}{0.120}\right)$$

$$+\uparrow \Sigma F_b = 0; \qquad T\left(\frac{4}{5}\right) + N_B\left(\frac{3}{5}\right) - 0.2(9.81) = 0$$

$$T = 1.82 \text{ N} \qquad \textbf{Ans.}$$

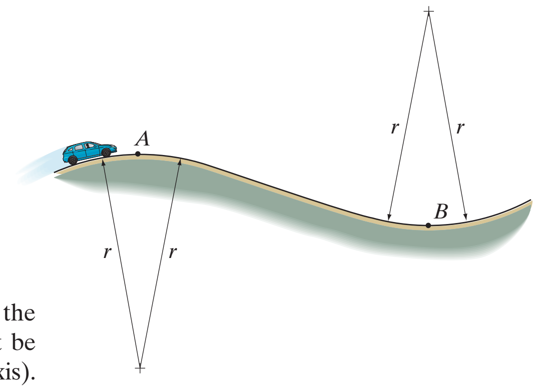
$$N_B = 0.844 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$T = 1.82$ N
$N_B = 0.844$ N

13–75.

Determine the maximum speed at which the car with mass m can pass over the top point A of the vertical curved road and still maintain contact with the road. If the car maintains this speed, what is the normal reaction the road exerts on the car when it passes the lowest point B on the road?



SOLUTION

Free-Body Diagram: The free-body diagram of the car at the top and bottom of the vertical curved road are shown in Figs. (a) and (b), respectively. Here, $\mathbf{a}_n$ must be directed towards the center of curvature of the vertical curved road (positive n axis).

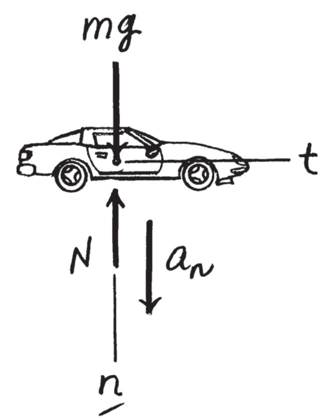
Equations of Motion: When the car is on top of the vertical curved road, it is required that its tires are about to lose contact with the road surface. Thus, $N = 0$. Realizing that $a_n = \dfrac{v^2}{\rho} = \dfrac{v^2}{r}$ and referring to Fig. (a),

$$+\downarrow \Sigma F_n = ma_n; \qquad mg = m\left(\frac{v^2}{r}\right) \qquad v = \sqrt{gr} \qquad \textbf{Ans.}$$

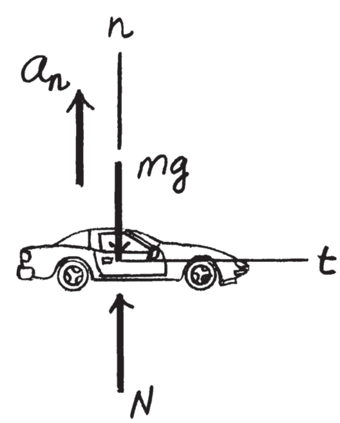
Using the result of v, the normal component of car acceleration is $a_n = \dfrac{v^2}{\rho} = \dfrac{gr}{r} = g$ when it is at the lowest point on the road. By referring to Fig. (b),

$$+\uparrow \Sigma F_n = ma_n; \qquad N - mg = mg$$

$$N = 2mg \qquad \textbf{Ans.}$$



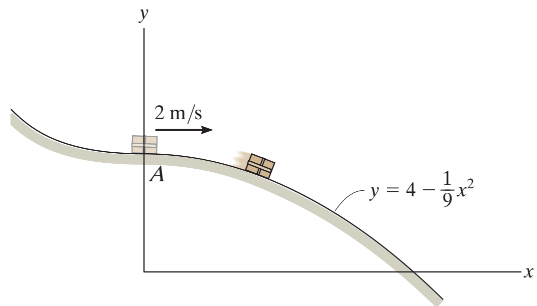
(a)



(b)

Ans:
$v = \sqrt{gr}$
$N = 2mg$

***13–76.** The 35-kg box has a speed 2 m/s when it is at A on the smooth ramp. If the surface is in the shape of a parabola, determine the normal force on the box at the instant $x = 3$ m. Also, what is the rate of increase in its speed at this instant?



SOLUTION

Given:

$M = 35\ \text{kg} \qquad a = 4\ \text{m}$

$v_0 = 2\ \text{m/s} \qquad b = \dfrac{1}{9}\ \text{m}^{-1}$

$x_1 = 3\ \text{m}$

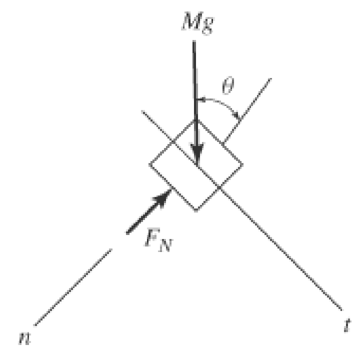
$y(x) = a - bx^2$

$y'(x) = -2bx$

$y''(x) = -2b$

$$\rho(x) = \frac{\sqrt{\left(1 + y'(x)^2\right)^3}}{y''(x)}$$

$\theta(x) = \text{atan}(y'(x))$



Find the velocity

$$v_1 = \sqrt{v_0^2 + 2g\left(y(0\ \text{m}) - y(x_1)\right)} \qquad v_1 = 4.859\ \text{m/s}$$

Guesses $\quad F_N = 1\ \text{N} \qquad v' = 1\ \text{m/s}^2$

Given $\quad F_N - Mg\cos\left(\theta(x_1)\right) = M\left(\dfrac{v_1^2}{\rho(x_1)}\right) \qquad -Mg\sin\left(\theta(x_1)\right) = Mv'$

$$\begin{pmatrix} F_N \\ v' \end{pmatrix} = \text{Find}\left(F_N, v'\right) \qquad F_N = 179.8\ \text{N} \qquad v' = 5.44\ \text{m/s}^2 \qquad \textbf{Ans.}$$

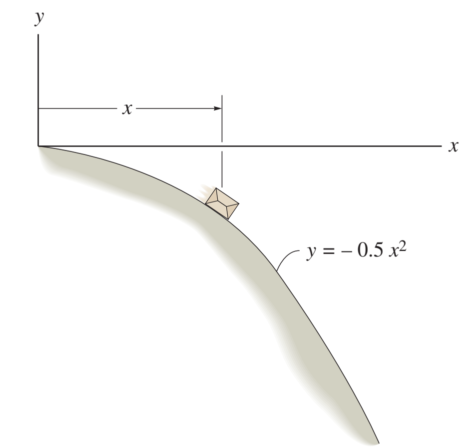
Ans:

$F_N = 179.8\ \text{N}$

$v' = 5.44\ \text{m/s}^2$

13–77.

The box has a mass m and slides down the smooth chute having the shape of a parabola. If it has an initial velocity of v_0 at the origin, determine its velocity as a function of x. Also, what is the normal force on the box, and the tangential acceleration as a function of x?



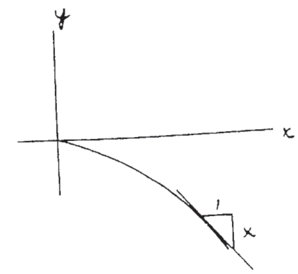
SOLUTION

$$y = -\frac{1}{2}x^2$$

$$\frac{dy}{dx} = -x$$

$$\frac{d^2y}{dx^2} = -1$$

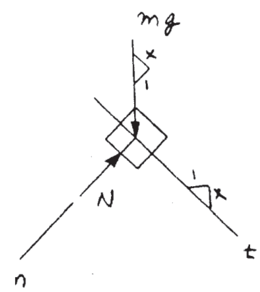
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + x^2\right]^{\frac{3}{2}}}{|-1|} = \left(1 + x^2\right)^{\frac{3}{2}}$$



$+\swarrow \Sigma F_n = ma_n;\qquad mg\left(\frac{1}{\sqrt{1 + x^2}}\right) - N = m\left(\frac{v^2}{(1 + x^2)^{\frac{3}{2}}}\right)$ **(1)**

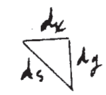
$$+\searrow \Sigma F_t = ma_t;\qquad mg\left(\frac{x}{\sqrt{1 + x^2}}\right) = ma_t$$

$a_t = g\left(\frac{x}{\sqrt{1 + x^2}}\right)$ **Ans.**



$$v\,dv = a_t\,ds = g\left(\frac{x}{\sqrt{1 + x^2}}\right)ds$$

$$ds = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}dx = \left(1 + x^2\right)^{\frac{1}{2}}dx$$



$$\int_{v_0}^{v} v\,dv = \int_0^x gx\,dx$$

$$\frac{1}{2}v^2 - \frac{1}{2}v_0^2 = g\left(\frac{x^2}{2}\right)$$

$v = \sqrt{v_0^2 + gx^2}$ **Ans.**

From Eq. (1):

$N = \frac{m}{\sqrt{1 + x^2}}\left[g - \frac{(v_0^2 + gx^2)}{(1 + x^2)}\right]$ **Ans.**

Ans:

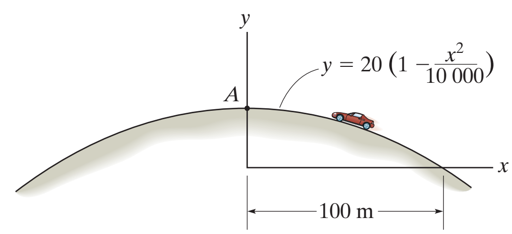
$$a_t = g\left(\frac{x}{\sqrt{1 + x^2}}\right)$$

$$v = \sqrt{v_0^2 + gx^2}$$

$$N = \frac{m}{\sqrt{1 + x^2}}\left[g - \frac{v_0^2 + gx^2}{1 + x^2}\right]$$

13–78.

Determine the maximum constant speed at which the 2-Mg car can travel over the crest of the hill at A without leaving the surface of the road. Neglect the size of the car in the calculation.



SOLUTION

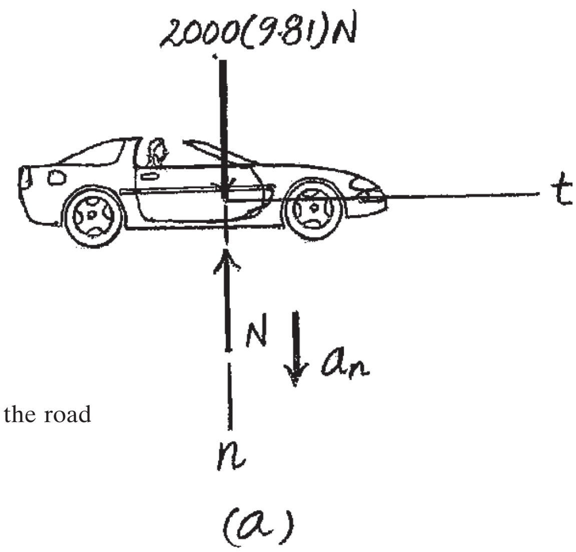
Geometry. The radius of curvature of the road at A must be determined first. Here

$$\frac{dy}{dx} = 20\left(-\frac{2x}{10000}\right) = -0.004x$$

$$\frac{d^2y}{dx^2} = -0.004$$

At point A, $x = 0$. Thus, $\left.\frac{dy}{dx}\right|_{x=0} = 0$. Then

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + 0^2)^{3/2}}{0.004} = 250 \text{ m}$$



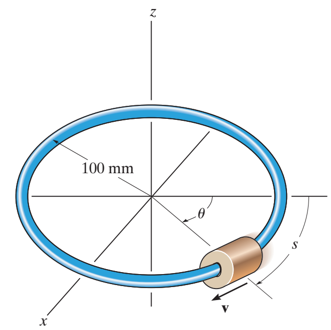
Equation of Motion. Since the car is required to be on the verge to leave the road surface, $N = 0$.

$$\Sigma F_n = ma_n; \qquad 2000(9.81) = 2000\left(\frac{v^2}{250}\right)$$

$$v = 49.52 \text{ m/s} = 49.5 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v = 49.5$ m/s

13–79. A collar having a mass 0.75 kg and negligible size slides over the surface of a horizontal circular rod for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the collar is given a speed of 4 m/s and then released at $\theta = 0°$, determine how far, s, it slides on the rod before coming to rest.



SOLUTION

$$N_{Cz} - Mg = 0$$

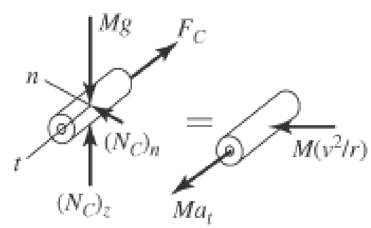
$$N_{Cn} = M\left(\frac{v^2}{r}\right)$$

$$N_C = \sqrt{N_{Cz}^{\ 2} + N_{Cn}^{\ 2}}$$

$$F_C = \mu_k N_C = -M a_t$$

$$a_t(v) = -\mu_k \sqrt{g^2 + \frac{v^4}{r^2}}$$

Given:



$M = 0.75$ kg $\quad r = 100$ mm

$\mu_k = 0.3 \quad g = 9.81\ \text{m/s}^2$

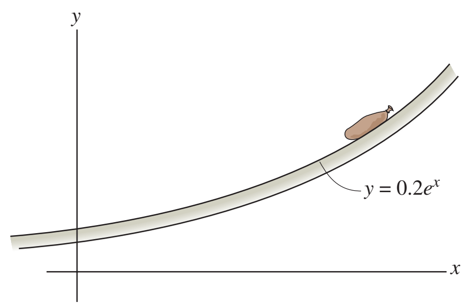
$v_1 = 4$ m/s

$$d = \int_{v_1}^{0} \frac{v}{a_t(v)}\, dv \qquad d = 0.581 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$d = 0.581$ m

***13–80.**

The 8-kg sack slides down the smooth ramp. If it has a speed of 1.5 m/s when $y = 0.2$ m, determine the normal reaction the ramp exerts on the sack and the rate of increase in the speed of the sack at this instant.



SOLUTION

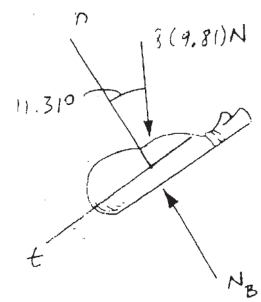
$y = 0.2 \qquad x = 0$

$y = 0.2e^x$

$$\frac{dy}{dx} = 0.2e^x \Big|_{x=0} = 0.2$$

$$\frac{d^2y}{dx^2} = 0.2e^x \Big|_{x=0} = 0.2$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1 + (0.2)^2\right]^{\frac{3}{2}}}{|0.2|} = 5.303$$



$\theta = \tan^{-1}(0.2) = 11.31°$

$+\nwarrow \Sigma F_n = ma_n; \qquad N_B - 8(9.81)\cos 11.31° = 8\left(\frac{(1.5)^2}{5.303}\right)$

$N_B = 80.4$ N **Ans.**

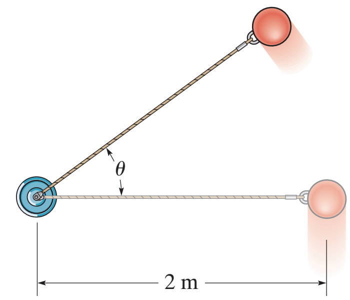
$+\swarrow \Sigma F_t = ma_t; \qquad 8(9.81)\sin 11.31° = 8a_t$

$a_t = 1.92 \text{ m/s}^2$ **Ans.**

Ans:
$N_B = 80.4$ N
$a_t = 1.92 \text{ m/s}^2$

13–81.

The 2-kg pendulum bob moves in the vertical plane with a velocity of 8 m/s when $\theta = 0°$. Determine the initial tension in the cord and also at the instant the bob reaches $\theta = 30°$. Neglect the size of the bob.



SOLUTION

Equations of Motion. Referring to the FBD of the bob at position $\theta = 0°$, Fig. *a*,

$$\Sigma F_n = ma_n; \qquad T = 2\left(\frac{8^2}{2}\right) = 64.0 \text{ N} \qquad \textbf{Ans.}$$

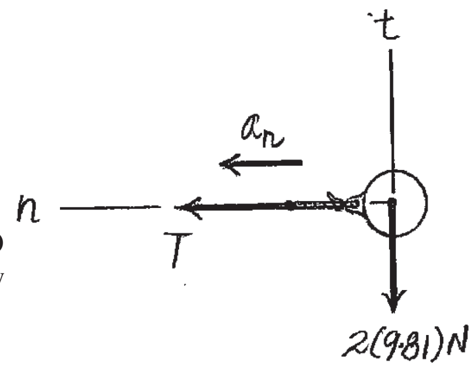
For the bob at an arbitrary position θ, the FBD is shown in Fig. *b*.

$$\Sigma F_t = ma_t; \qquad -2(9.81)\cos\theta = 2a_t$$

$$a_t = -9.81\cos\theta$$

$$\Sigma F_n = ma_n; \qquad T + 2(9.81)\sin\theta = 2\left(\frac{v^2}{2}\right)$$

$$T = v^2 - 19.62\sin\theta \qquad \textbf{(1)}$$



(a)

Kinematics. The velocity of the bob at the position $\theta = 30°$ can be determined by integrating $vdv = a_t\,ds$. However, $ds = rd\theta = 2d\theta$.
Then,

$$\int_{8\text{ m/s}}^{v} vdv = \int_{0°}^{30°} -9.81\cos\theta(zd\theta)$$

$$\left.\frac{v^2}{2}\right|_{8\text{ m/s}}^{v} = -19.62\sin\theta\Big|_{0°}^{30°}$$

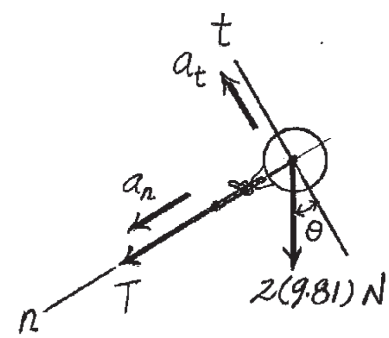
$$\frac{v^2}{2} - \frac{8^2}{2} = -19.62(\sin 30° - 0)$$

$$v^2 = 44.38 \text{ m}^2/\text{s}^2$$

Substitute this result and $\theta = 30°$ into Eq. (1),

$$T = 44.38 - 19.62\sin 30°$$

$$= 34.57 \text{ N} = 34.6 \text{ N} \qquad \textbf{Ans.}$$

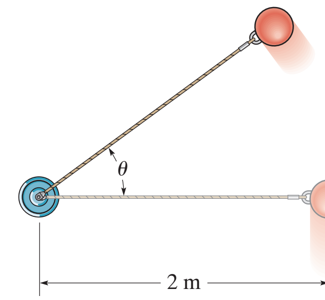


(b)

Ans:
$T = 64.0$ N
$T = 34.6$ N

13–82.

The 2-kg pendulum bob moves in the vertical plane with a velocity of 6 m/s when $\theta = 0°$. Determine the angle θ where the tension in the cord becomes zero.



SOLUTION

Equation of Motion. The FBD of the bob at an arbitrary position θ is shown in Fig. *a*. Here, it is required that $T = 0$.

$$\Sigma F_t = ma_t; \qquad -2(9.81)\cos\theta = 2a_t$$

$$a_t = -9.81\cos\theta$$

$$\Sigma F_n = ma_n; \qquad 2(9.81)\sin\theta = 2\left(\frac{v^2}{2}\right)$$

$$v^2 = 19.62\sin\theta \qquad \textbf{(1)}$$

Kinematics. The velocity of the bob at an arbitrary position θ can be determined by integrating $v\,dv = a_t\,ds$. However, $ds = r\,d\theta = 2\,d\theta$.
Then

$$\int_{6\text{ m/s}}^{v} v\,dv = \int_{0°}^{\theta} -9.81\cos\theta(2\,d\theta)$$

$$\left.\frac{v^2}{2}\right|_{6\text{ m/s}}^{v} = \left.-19.62\sin\theta\right|_{0°}^{\theta}$$

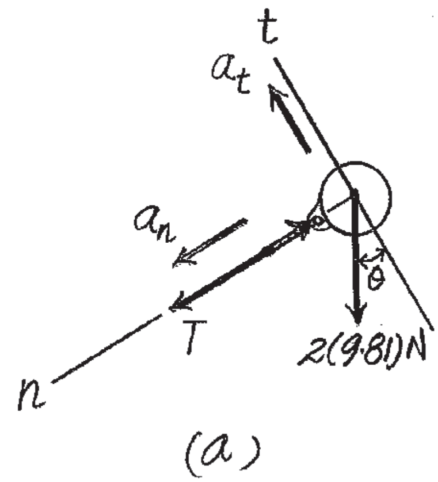
$$v^2 = 36 - 39.24\sin\theta \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2)

$$19.62\sin\theta = 36 - 39.24\sin\theta$$

$$58.86\sin\theta = 36$$

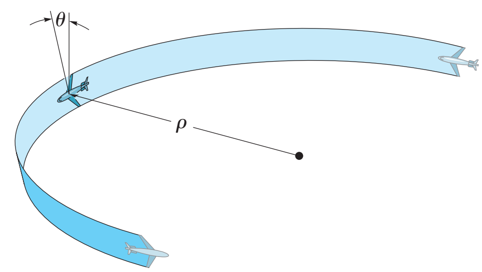
$$\theta = 37.71° = 37.7° \qquad \textbf{Ans.}$$



Ans:
$\theta = 37.7°$

13–83.

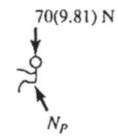
The airplane, traveling at a constant speed of 50 m/s, is executing a horizontal turn. If the plane is banked at $\theta = 15°$, when the pilot experiences only a normal force on the seat of the plane, determine the radius of curvature ρ of the turn. Also, what is the normal force of the seat on the pilot if he has a mass of 70 kg.



SOLUTION

$+\uparrow \sum F_b = ma_b; \qquad N_P \sin 15° - 70(9.81) = 0$

$N_P = 2.65 \text{ kN}$ **Ans.**



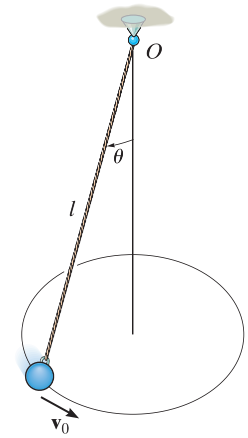
$\overset{+}{\leftarrow} \sum F_n = ma_n; \qquad N_P \cos 15° = 70\left(\frac{50^2}{\rho}\right)$

$\rho = 68.3 \text{ m}$ **Ans.**

Ans:
$N_P = 2.65 \text{ kN}$
$\rho = 68.3 \text{ m}$

***13–84.**

The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta\sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.



SOLUTION

$\overset{+}{\rightarrow}\Sigma F_n = ma_n;\qquad T\sin\theta = m\left(\frac{v_0^2}{r}\right)$

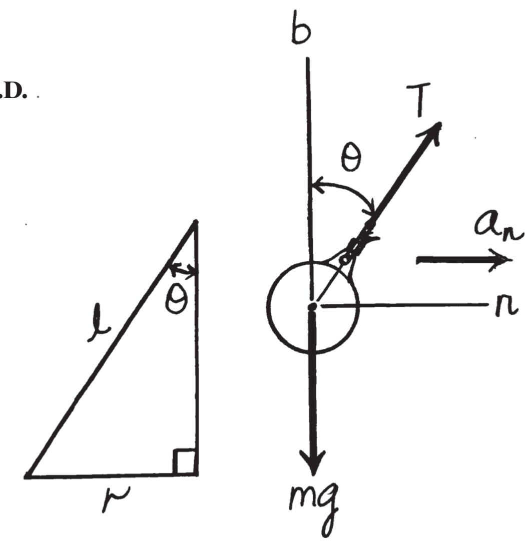
$+\uparrow\Sigma F_b = 0;\qquad T\cos\theta - mg = 0$

Since $r = l\sin\theta \qquad T = \dfrac{mv_0^2}{l\sin^2\theta}$

$$\left(\frac{mv_0^2}{l}\right)\left(\frac{\cos\theta}{\sin^2\theta}\right) = mg$$

$$\tan\theta\sin\theta = \frac{v_0^2}{gl}$$

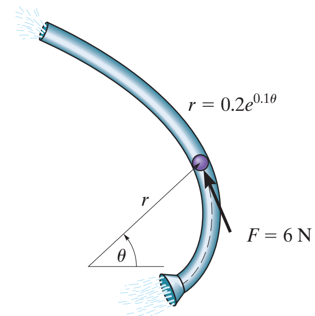
Q.E.D.



Ans:

$\tan\theta\sin\theta = \dfrac{v_0^2}{gl}$

13–85. Using air pressure, the 0.5-kg ball is forced to move through the tube lying in the *horizontal plane* and having the shape of a logarithmic spiral. If the tangential force exerted on the ball due to the air is 6 N, determine the rate of increase in the ball's speed at the instant $\theta = \pi/2$. What direction does it act in?



SOLUTION

Given:

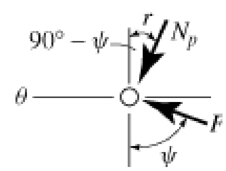
$$M = 0.5 \text{ kg}$$

$$a = 0.2 \text{ m}$$

$$b = 0.1$$

$$F = 6 \text{ N}$$

$$\theta_1 = \frac{\pi}{2}$$

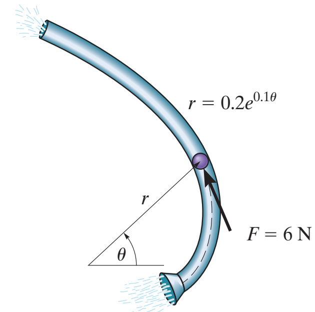


$$\tan(\psi) = \frac{r}{\frac{d}{d\theta}r} = \frac{a e^{b\theta}}{a b e^{b\theta}} = \frac{1}{b} \qquad \psi = \text{atan}\left(\frac{1}{b}\right) \qquad \psi = 84.29° \qquad \textbf{Ans.}$$

$$F = Mv' \qquad v' = \frac{F}{M} \qquad v' = 12.00 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$\psi = 84.29°$
$v' = 12.00 \text{ m/s}^2$

13–86. Solve Prob. 13–85 if the tube lies in a *vertical plane*.



SOLUTION

Given:

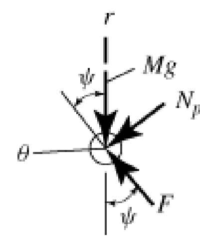
$M = 0.5 \text{ kg}$

$a = 0.2 \text{ m}$

$b = 0.1$

$F = 6 \text{ N}$

$\theta_1 = \dfrac{\pi}{2}$



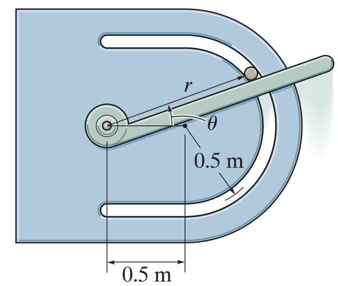
$$\tan(\psi) = \frac{r}{\frac{d}{d\theta}r} = \frac{a e^{b\theta}}{a b e^{b\theta}} = \frac{1}{b} \qquad \psi = \operatorname{atan}\left(\frac{1}{b}\right) \qquad \psi = 84.29° \qquad \textbf{Ans.}$$

$$F - Mg\cos(\psi) = Mv' \qquad v' = \frac{F}{M} - g\cos(\psi) \qquad v' = 11.02 \text{ m/s}^2 \quad \textbf{Ans.}$$

Ans:

$\psi = 84.29°$
$v' = 11.02 \text{ m/s}^2$

13–87. The 0.75-kg smooth can is guided along the circular path using the arm guide. If the arm has an angular velocity $\dot{\theta} = 2$ rad/s and an angular acceleration $\ddot{\theta} = 0.4$ rad/s^2 at the instant $\theta = 30°$, determine the force of the guide on the can. Motion occurs in the *horizontal plane.*

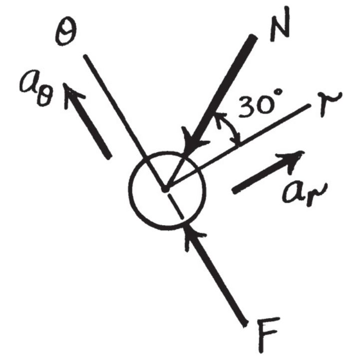


SOLUTION

$$r = \cos\theta\big|_{\theta=30°} = 0.8660 \text{ m}$$

$$\dot{r} = -\sin\theta\dot{\theta}\big|_{\theta=30°} = -1.00 \text{ m/s}$$

$$\ddot{r} = -(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})\big|_{\theta=30°} = -3.664 \text{ m/s}^2$$



Using the above time derivative, we obtain

$$a_r = \ddot{r} - r\dot{\theta}^2 = -3.664 - 0.8660(2^2) = -7.128 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(4) + 2(-1)(2) = -0.5359 \text{ m/s}^2$$

Equations of Motion: By referring to Fig. (a),

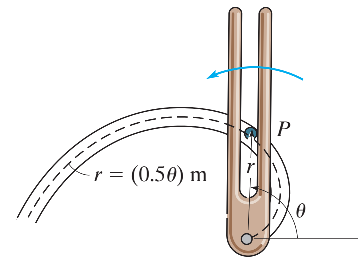
$\Sigma F_r = ma_r;$ $\quad -N\cos 30° = 0.75(-7.128)$ $\quad N = 5.346$ N

$\Sigma F_\theta = ma_\theta;$ $\quad F - 5.346\sin 30° = 0.75(-0.5359)$ $\quad F = 2.271$ N $\quad$ **Ans.**

Ans:
$F = 2.271$ N

***13–88.**

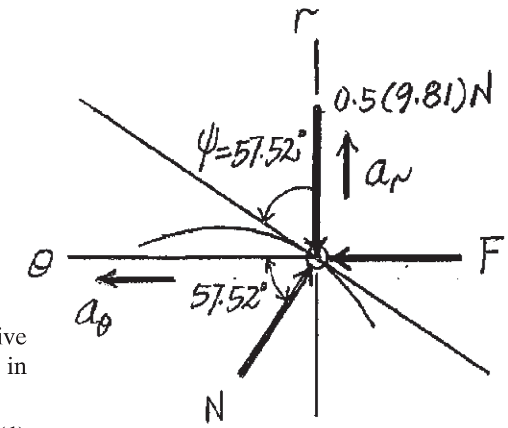
Using a forked rod, a 0.5-kg smooth peg P is forced to move along the *vertical slotted* path $r = (0.5\theta)$ m, where θ is in radians. If the angular position of the arm is $\theta = (\frac{\pi}{8}t^2)$ rad, where t is in seconds, determine the force of the rod on the peg and the normal force of the slot on the peg at the instant $t = 2$ s. The peg is in contact with only *one edge* of the rod and slot at any instant.



SOLUTION

Equation of Motion. Here, $r = 0.5\theta$. Then $\frac{dr}{d\theta} = 0.5$. The angle ψ between the extended radial line and the tangent can be determined from

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{0.5\theta}{0.5} = \theta$$



At the instant $t = 25$, $\theta = \frac{\pi}{8}(2^2) = \frac{\pi}{2}$ rad

$$\tan\psi = \frac{\pi}{2} \qquad \psi = 57.52°$$

The positive sign indicates that ψ is measured from extended radial line in positive sense of θ (counter clockwise) to the tangent. Then the FBD of the peg shown in Fig. *a* can be drawn.

$$\Sigma F_r = ma_r; \quad N \sin 57.52° - 0.5(9.81) = 0.5a_r \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \quad F - N\cos 57.52° = 0.5a_\theta \qquad \textbf{(2)}$$

Kinematics. Using the chain rule, the first and second derivatives of r and θ with respect to t are

$$r = 0.5\theta = 0.5\left(\frac{\pi}{8}t^2\right) = \frac{\pi}{16}t^2 \qquad \theta = \frac{\pi}{8}t^2$$

$$\dot{r} = \frac{\pi}{8}t \qquad \dot{\theta} = \frac{\pi}{4}t$$

$$\ddot{r} = \frac{\pi}{8} \qquad \ddot{\theta} = \frac{\pi}{4}$$

When $t = 2$ s,

$$r = \frac{\pi}{16}(2^2) = \frac{\pi}{4}\text{ m} \qquad \theta = \frac{\pi}{8}(2^2) = \frac{\pi}{2}\text{ rad}$$

$$\dot{r} = \frac{\pi}{8}(2) = \frac{\pi}{4}\text{ m/s} \qquad \dot{\theta} = \frac{\pi}{4}(2) = \frac{\pi}{2}\text{ rad/s}$$

$$\ddot{r} = \frac{\pi}{8}\text{ m/s}^2 \qquad \ddot{\theta} = \frac{\pi}{4}\text{ rad/s}^2$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{\pi}{8} - \frac{\pi}{4}\left(\frac{\pi}{2}\right)^2 = -1.5452\text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{\pi}{4}\left(\frac{\pi}{4}\right) + 2\left(\frac{\pi}{4}\right)\left(\frac{\pi}{2}\right) = 3.0843\text{ m/s}^2$$

Substitute these results in Eqs. (1) and (2)

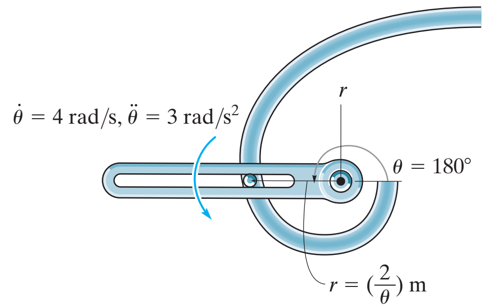
$$N = 4.8987\text{ N} = 4.90\text{ N} \qquad \textbf{Ans.}$$

$$F = 4.173\text{ N} = 4.17\text{ N} \qquad \textbf{Ans.}$$

Ans:
$N = 4.90$ N
$F = 4.17$ N

13–89.

The arm is rotating at a rate of $\dot{\theta} = 4$ rad/s when $\ddot{\theta} = 3$ rad/s^2 and $\theta = 180°$. Determine the force it must exert on the 0.5-kg smooth cylinder if it is confined to move along the slotted path. Motion occurs in the horizontal plane.



SOLUTION

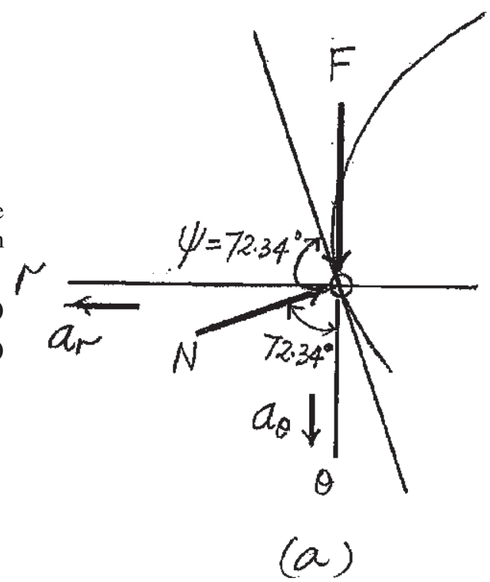
Equation of Motion. Here, $r = \frac{2}{\theta}$. Then $\frac{dr}{d\theta} = -\frac{2}{\theta^2}$. The angle ψ between the extended radial line and the tangent can be determined from

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{2/\theta}{-2/\theta^2} = -\theta$$

At $\theta = 180° = \pi$ rad,

$$\tan\psi = -\pi \qquad \psi = -72.34°$$

The negative sign indicates that ψ is measured from extended radial line in the negative sense of θ (clockwise) to the tangent. Then, the FBD of the peg shown in Fig. *a* can be drawn.



$$\Sigma F_r = ma_r; \qquad -N\sin 72.34° = 0.5a_r \qquad \textbf{(1)}$$

$$\Sigma F_\theta = ma_\theta; \qquad F - N\cos 72.34° = 0.5a_\theta \qquad \textbf{(2)}$$

Kinematics. Using the chain rule, the first and second time derivatives of r are

$$r = 2\theta^{-1}$$

$$\dot{r} = -2\theta^{-2}\dot{\theta} = -\left(\frac{2}{\theta^2}\right)\dot{\theta}$$

$$\ddot{r} = -2\left(-2\theta^{-3}\dot{\theta}^2 + \theta^{-2}\ddot{\theta}\right) = \frac{2}{\theta^3}\left(2\dot{\theta}^2 - \theta\ddot{\theta}\right)$$

When $\theta = 180° = \pi$ rad, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 3$ rad/s^2. Thus

$$r = \frac{2}{\pi}\text{ m} = 0.6366 \text{ m}$$

$$\dot{r} = -\left(\frac{2}{\pi^2}\right)(4) = -0.8106 \text{ m/s}$$

$$\ddot{r} = \frac{2}{\pi^3}\left[2(4^2) - \pi(3)\right] = 1.4562 \text{ m/s}^2$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 1.4562 - 0.6366(4^2) = -8.7297 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6366(3) + 2(-0.8106)(4) = -4.5747 \text{ m/s}^2$$

Substitute these result into Eqs. (1) and (2),

$$N = 4.5807 \text{ N}$$

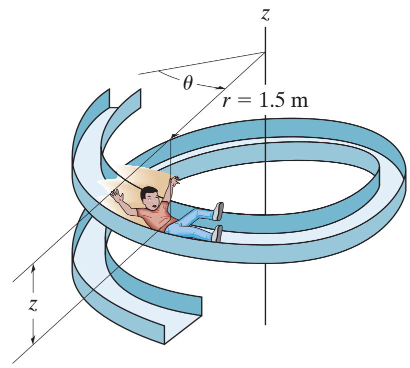
$$F = -0.8980 \text{ N} = -0.898 \text{ N} \qquad \textbf{Ans.}$$

The negative sign indicates that **F** acts in the sense opposite to that shown in the FBD.

Ans:
$F = -0.898$ N

13–90.

The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components $r = 1.5$ m, $\theta = (0.7t)$ rad, and $z = (-0.5t)$ m, where t is in seconds. Determine the components of force $\mathbf{F}_r$, $\mathbf{F}_\theta$, and $\mathbf{F}_z$ which the slide exerts on him at the instant $t = 2$ s. Neglect the size of the boy.

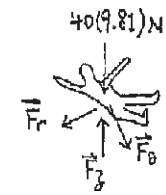


SOLUTION

$r = 1.5 \qquad \theta = 0.7t \qquad z = -0.5t$

$\dot{r} = \ddot{r} = 0 \qquad \dot{\theta} = 0.7 \qquad \dot{z} = -0.5$

$\ddot{\theta} = 0 \qquad \ddot{z} = 0$



$a_r = \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$a_z = \ddot{z} = 0$

$\Sigma F_r = ma_r; \qquad F_r = 40(-0.735) = -29.4\text{ N}$ **Ans.**

$\Sigma F_\theta = ma_\theta; \qquad F_\theta = 0$ **Ans.**

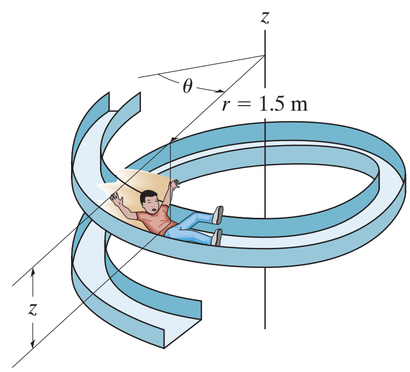
$\Sigma F_z = ma_z; \qquad F_z - 40(9.81) = 0$

$F_z = 392\text{ N}$ **Ans.**

Ans:
$F_r = -29.4\text{ N}$
$F_\theta = 0$
$F_z = 392\text{ N}$

13–91.

The 40-kg boy is sliding down the smooth spiral slide such that $z = -2$ m/s and his speed is 2 m/s. Determine the r, θ, z components of force the slide exerts on him at this instant. Neglect the size of the boy.



SOLUTION

$r = 1.5$ m

$\dot{r} = 0$

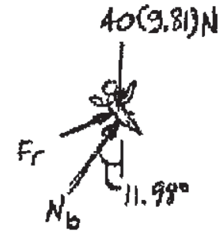
$\ddot{r} = 0$

$v_\theta = 2 \cos 11.98° = 1.9564$ m/s

$v_z = -2 \sin 11.98° = -0.41517$ m/s

$v_\theta = r\dot{\theta}; \qquad 1.9564 = 1.5\,\dot{\theta}$

$\dot{\theta} = 1.3043$ rad/s



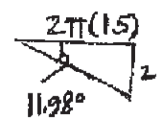
$\Sigma F_r = ma_r; \qquad -F_r = 40(0 - 1.5(1.3043)^2)$

$F_r = 102$ N **Ans.**

$\Sigma F_\theta = ma_\theta; \qquad N_b \sin 11.98° = 40(a_\theta)$

$\Sigma F_z = ma_z; \qquad -N_b \cos 11.98° + 40(9.81) = 40a_z$

Require $\tan 11.98° = \dfrac{a_z}{a_\theta}, \qquad a_\theta = 4.7123a_z$



Thus,

$a_z = 0.423$ m/s^2

$a_\theta = 1.99$ m/s^2

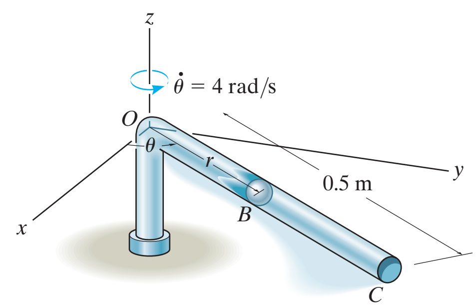
$N_b = 383.85$ N

$N_z = 383.85 \cos 11.98° = 375$ N **Ans.**

$N_\theta = 383.85 \sin 11.98° = 79.7$ N **Ans.**

Ans:
$F_r = 102$ N
$F_z = 375$ N
$F_\theta = 79.7$ N

***13–92.** The tube rotates in the horizontal plane at a constant rate of $\dot{\theta} = 4$ rad/s. If a 0.2-kg ball B starts at the origin O with an initial radial velocity $\dot{r} = 1.5$ m/s and moves outward through the tube, determine the radial and transverse components of the ball's velocity at the instant it leaves the outer end at C, $r = 0.5$ m. *Hint:* Show that the equation of motion in the r direction is $\ddot{r} - 16r = 0$. The solution is of the form $r = Ae^{-4t} + Be^{4t}$. Evaluate the integration constants A and B, and determine the time t when $r = 0.5$ m. Proceed to obtain v_r and v_θ.



SOLUTION

$$0 = M\left(r'' - r\theta'^2\right)$$

$$r(t) = A\,e^{\theta' t} + B\,e^{-\theta' t}$$

$$r'(t) = \theta'\left(A\,e^{\theta' t} - B\,e^{-\theta' t}\right)$$

Guess $\quad A = 1\ \text{m} \qquad B = 1\ \text{m}$

$\qquad\qquad t = 1\ \text{s}$

Given $\quad 0 = A + B \qquad r'_0 = \theta'(A - B) \qquad r_1 = A\,e^{\theta' t} + B\,e^{-\theta' t}$

Given:

$\theta' = 4\ \text{rad/s} \qquad M = 0.2\ \text{kg} \qquad r'_0 = 1.5\ \text{m/s} \qquad r_1 = 0.5\ \text{m}$

$$\begin{pmatrix} A \\ B \\ t_1 \end{pmatrix} = \text{Find}(A, B, t) \qquad \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0.188 \\ -0.188 \end{pmatrix}\text{m} \qquad t_1 = 0.275\ \text{s}$$

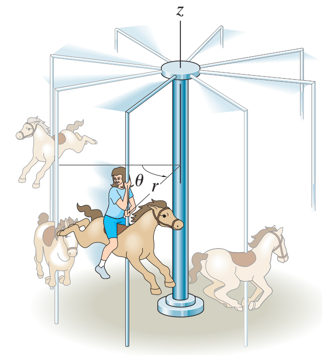
$$r(t) = A\,e^{\theta' t} + B\,e^{-\theta' t} \qquad r'(t) = \theta'\left(A\,e^{\theta' t} - B\,e^{-\theta' t}\right)$$

$$v_r = r'(t_1) \qquad v_\theta = r(t_1)\,\theta'$$

$$\begin{pmatrix} v_r \\ v_\theta \end{pmatrix} = \begin{pmatrix} 2.5 \\ 2 \end{pmatrix}\text{m/s} \qquad \textbf{Ans.}$$

Ans:
$v_r = 2.5$ m/s
$v_\theta = 2$ m/s

13–93. The girl has a mass of 50 kg. She is seated on the horse of the merry-go-round which undergoes constant rotational motion $\dot{\theta} = 1.5$ rad/s. If the path of the horse is defined by $r = 4$ m, $z = (0.5 \sin \theta)$ m, determine the maximum and minimum force F_z the horse exerts on her during the motion.



SOLUTION

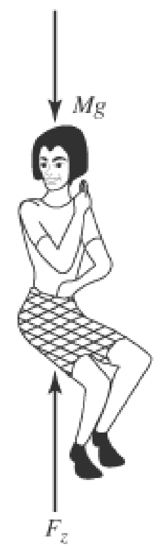
Given:

$$M = 50 \text{ kg}$$

$$\theta' = 1.5 \text{ rad/s}$$

$$r_0 = 4 \text{ m}$$

$$b = 0.5 \text{ m}$$



$$z = b\sin(\theta)$$

$$z' = b\cos(\theta)\theta'$$

$$z'' = -b\sin(\theta)\,\theta'^2$$

$$F_z - Mg = Mz''$$

$$F_z = M\left(g - b\sin(\theta)\,\theta'^2\right)$$

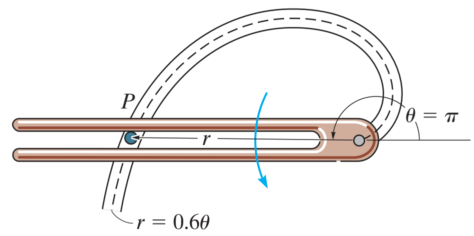
$F_{zmax} = M\left(g + b\theta'^2\right)$ $\quad$ $F_{zmax} = 547\text{ N}$ $\quad$ **Ans.**

$F_{zmin} = M\left(g - b\theta'^2\right)$ $\quad$ $F_{zmin} = 434\text{ N}$ $\quad$ **Ans.**

Ans:
$F_{zmax} = 547$ N
$F_{zmin} = 434$ N

13–94.

Using a forked rod, a smooth cylinder P, having a mass of 0.4 kg, is forced to move along the *vertical slotted* path $r = (0.6\theta)$ m, where θ is in radians. If the cylinder has a constant speed of $v_C = 2$ m/s, determine the force of the rod and the normal force of the slot on the cylinder at the instant $\theta = \pi$ rad. Assume the cylinder is in contact with only *one edge* of the rod and slot at any instant. *Hint:* To obtain the time derivatives necessary to compute the cylinder's acceleration components a_r and a_θ, take the first and second time derivatives of $r = 0.6\theta$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$, to determine $\ddot{\theta}$.



SOLUTION

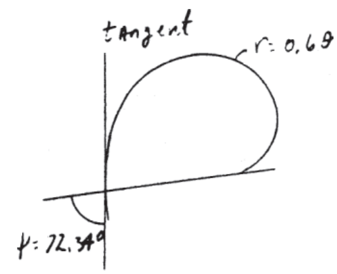
$r = 0.6\theta \qquad \dot{r} = 0.6\dot{\theta} \qquad \ddot{r} = 0.6\ddot{\theta}$

$v_r = \dot{r} = 0.6\dot{\theta} \qquad v_\theta = r\dot{\theta} = 0.6\theta\dot{\theta}$

$v^2 = \dot{r}^2 + \left(r\dot{\theta}\right)^2$

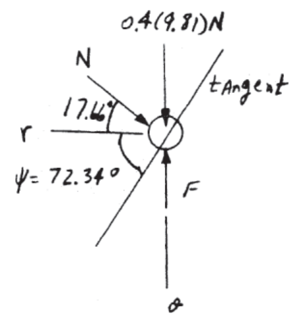
$2^2 = \left(0.6\dot{\theta}\right)^2 + \left(0.6\theta\dot{\theta}\right)^2 \qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\theta^2}}$

$0 = 0.72\dot{\theta}\ddot{\theta} + 0.36\left(2\theta\dot{\theta}^3 + 2\theta^2\dot{\theta}\ddot{\theta}\right) \qquad \ddot{\theta} = -\dfrac{\theta\dot{\theta}^2}{1+\theta^2}$



At $\theta = \pi$ rad, $\qquad \dot{\theta} = \dfrac{2}{0.6\sqrt{1+\pi^2}} = 1.011 \text{ rad/s}$

$\qquad \ddot{\theta} = -\dfrac{(\pi)(1.011)^2}{1+\pi^2} = -0.2954 \text{ rad/s}^2$



$r = 0.6(\pi) = 0.6\,\pi \text{ m} \qquad \dot{r} = 0.6(1.011) = 0.6066 \text{ m/s}$

$\ddot{r} = 0.6(-0.2954) = -0.1772 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.1772 - 0.6\pi(1.011)^2 = -2.104 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6\pi(-0.2954) + 2(0.6066)(1.011) = 0.6698 \text{ m/s}^2$

$\tan\psi = \dfrac{r}{dr/d\theta} = \dfrac{0.6\theta}{0.6} = \theta = \pi \qquad \psi = 72.34°$

$\overset{+}{\leftarrow} \Sigma F_r = ma_r; \qquad -N\cos 17.66° = 0.4(-2.104) \qquad N = 0.883 \text{ N}$ **Ans.**

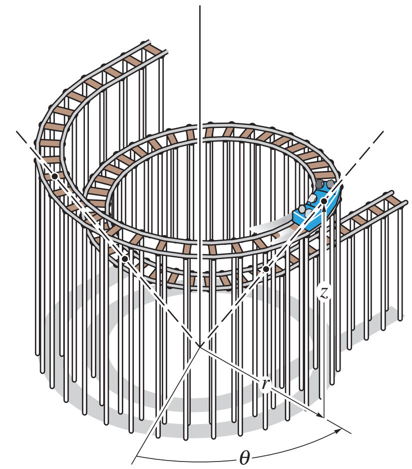
$+\downarrow \Sigma F_\theta = ma_\theta; \qquad -F + 0.4(9.81) + 0.883\sin 17.66° = 0.4(0.6698)$

$F = 3.92 \text{ N}$ **Ans.**

Ans:
$N = 0.883$ N
$F = 3.92$ N

13–95.

A car of a roller coaster travels along a track which for a short distance is defined by a conical spiral, $r = \frac{3}{4}z$, $\theta = -1.5z$, where r and z are in meters and θ in radians. If the angular motion $\dot{\theta} = 1$ rad/s is always maintained, determine the r, θ, z components of reaction exerted on the car by the track at the instant $z = 6$ m. The car and passengers have a total mass of 200 kg.



SOLUTION

$$r = 0.75z \qquad \dot{r} = 0.75\dot{z} \qquad \ddot{r} = 0.75\ddot{z}$$

$$\theta = -1.5z \qquad \dot{\theta} = -1.5\dot{z} \qquad \ddot{\theta} = -1.5\ddot{z}$$

$$\dot{\theta} = 1 = -1.5\dot{z} \qquad \dot{z} = -0.6667 \text{ m/s} \qquad \ddot{z} = 0$$

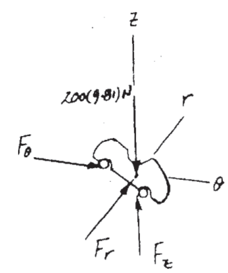
At $z = 6$ m,

$$r = 0.75(6) = 4.5 \text{ m} \qquad \dot{r} = 0.75(-0.6667) = -0.5 \text{ m/s} \qquad \ddot{r} = 0.75(0) = 0 \qquad \ddot{\theta} = 0$$

$$a_t = \ddot{r} - r\dot{\theta}^2 = 0 - 4.5(1)^2 = -4.5 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.5(0) + 2(-0.5)(1) = -1 \text{ m/s}^2$$

$$a_z = \ddot{z} = 0$$



$\Sigma F_t = ma_r;$ $\quad F_r = 200(-4.5)$ $\quad F_r = -900$ N **Ans.**

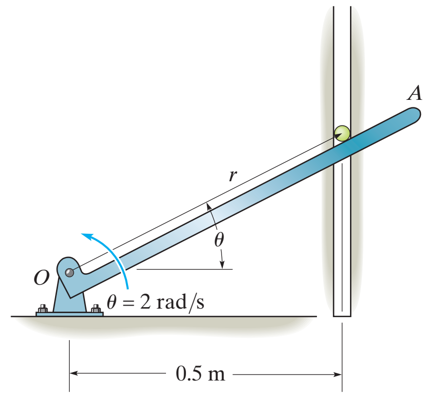
$\Sigma F_\theta = ma_\theta;$ $\quad F_\theta = 200(-1)$ $\quad F_\theta = -200$N **Ans.**

$\Sigma F_z = ma_z;$ $\quad F_z - 200(9.81) = 0$ $\quad F_z = 1962 \text{ N} = 1.96 \text{ kN}$ **Ans.**

Ans:
$F_r = -900$ N
$F_\theta = -200$ N
$F_z = 1.96$ kN

***13–96.**

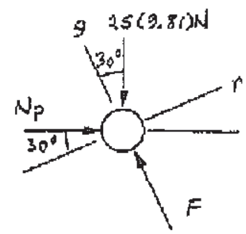
The particle has a mass of 0.5 kg and is confined to move along the smooth vertical slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



SOLUTION

$r = \dfrac{0.5}{\cos\theta} = 0.5 \sec\theta, \qquad \dot{r} = 0.5 \sec\theta \tan\theta\dot{\theta}$

$\ddot{r} = 0.5 \sec\theta \tan\theta\ddot{\theta} + 0.5 \sec^3\theta\dot{\theta}^2 + 0.5 \sec\theta \tan^2\theta\dot{\theta}^2$



At $\theta = 30°$.

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 0.5774$ m

$\dot{r} = 0.6667$ m/s

$\ddot{r} = 3.8490$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = 3.8490 - 0.5774(2)^2 = 1.5396$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(0.6667)(2) = 2.667$ m/s^2

$+\nearrow \Sigma F_r = ma_r; \qquad N_P \cos 30° - 0.5(9.81)\sin 30° = 0.5(1.5396)$

$N_P = 3.7208 = 3.72$ N **Ans.**

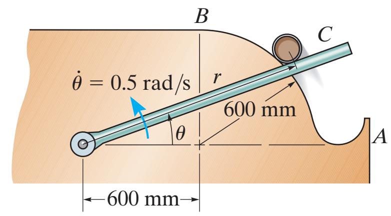
$+\nwarrow \Sigma F_\theta = ma_\theta; \qquad F - 3.7208 \sin 30° - 0.5(9.81)\cos 30° = 0.5(2.667)$

$F = 7.44$ N **Ans.**

Ans:
$N_P = 3.72$ N
$F = 7.44$ N

13–97.

A smooth can C, having a mass of 3 kg, is lifted from a feed at A to a ramp at B by a rotating rod. If the rod maintains a constant angular velocity of $\dot{\theta} = 0.5$ rad/s, determine the force which the rod exerts on the can at the instant $\theta = 30^\circ$. Neglect the effects of friction in the calculation and the size of the can so that $r = (1.2 \cos \theta)$ m. The ramp from A to B is circular, having a radius of 600 mm.



SOLUTION

$r = 2(0.6 \cos \theta) = 1.2 \cos \theta$

$\dot{r} = -1.2 \sin \theta \dot{\theta}$

$\ddot{r} = -1.2 \cos \theta \dot{\theta}^2 - 1.2 \sin \theta \ddot{\theta}$

At $\theta = 30^\circ, \dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$

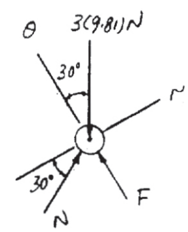
$r = 1.2 \cos 30^\circ = 1.0392$ m

$\dot{r} = -1.2 \sin 30^\circ(0.5) = -0.3$ m/s

$\ddot{r} = -1.2 \cos 30^\circ(0.5)^2 - 1.2 \sin 30^\circ(0) = -0.2598 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -0.2598 - 1.0392(0.5)^2 = -0.5196 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.0392(0) + 2(-0.3)(0.5) = -0.3 \text{ m/s}^2$



$+\nearrow \Sigma F_r = ma_r;$ $\quad N \cos 30^\circ - 3(9.81) \sin 30^\circ = 3(-0.5196) \quad N = 15.19$ N

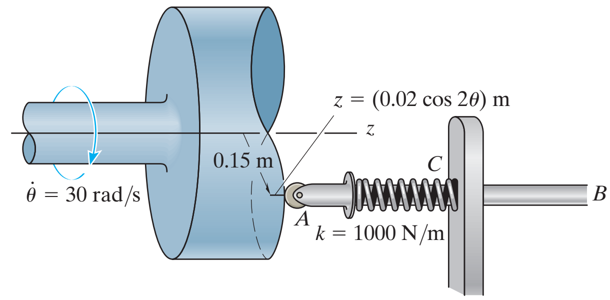
$+\nwarrow \Sigma F_\theta = ma_\theta;$ $\quad F + 15.19 \sin 30^\circ - 3(9.81) \cos 30^\circ = 3(-0.3)$

$F = 17.0$ N **Ans.**

Ans:
$F = 17.0$ N

13–98.

The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the force component F_z at the end A of the follower when $\theta = 30°$. The spring is uncompressed when $\theta = 90°$. Neglect friction at the bearing C.



SOLUTION

Kinematics. Using the chain rule, the first and second time derivatives of z are

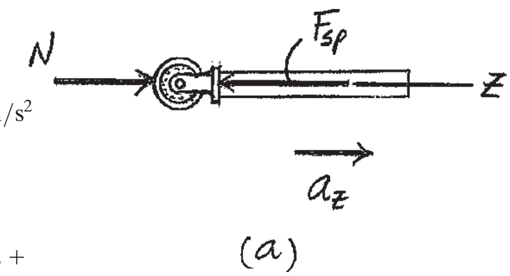
$$z = (0.02 \cos 2\theta) \text{ m}$$

$$\dot{z} = 0.02[-\sin 2\theta(2\dot{\theta})] = [-0.04(\sin 2\theta)\dot{\theta}] \text{ m/s}$$

$$\ddot{z} = -0.04[\cos 2\theta(2\dot{\theta})\dot{\theta} + (\sin 2\theta)\ddot{\theta}] = [-0.04(2 \cos 2\theta(\dot{\theta})^2 + \sin 2\theta(\ddot{\theta}))] \text{ m/s}^2$$

Here, $\dot{\theta} = 30$ rad/s and $\ddot{\theta} = 0$. Then

$$\ddot{z} = -0.04[2 \cos 2\theta(30^2) + \sin 2\theta(0)] = (-72 \cos 2\theta) \text{ m/s}^2$$



Equation of Motion. When $\theta = 30°$, the spring compresses $x = 0.02 + 0.02 \cos 2(30°) = 0.03$ m. Thus, $F_{sp} = kx = 1000(0.03) = 30$ N. Also, at this position $a_z = \ddot{z} = -72 \cos 2(30°) = -36.0 \text{ m/s}^2$. Referring to the FBD of the follower, Fig. a,

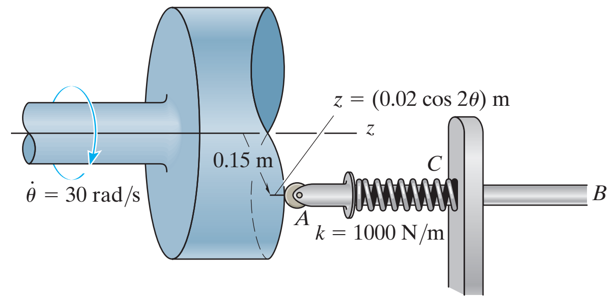
$$\Sigma F_z = ma_z; \qquad N - 30 = 0.5(-36.0)$$

$$N = 12.0 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$N = 12.0$ N

13–99.

The spring-held follower AB has a mass of 0.5 kg and moves back and forth as its end rolls on the contoured surface of the cam, where $r = 0.15$ m and $z = (0.02 \cos 2\theta)$ m. If the cam is rotating at a constant rate of 30 rad/s, determine the maximum and minimum force components F_z the follower exerts on the cam if the spring is uncompressed when $\theta = 90°$.



SOLUTION

Kinematics. Using the chain rule, the first and second time derivatives of z are

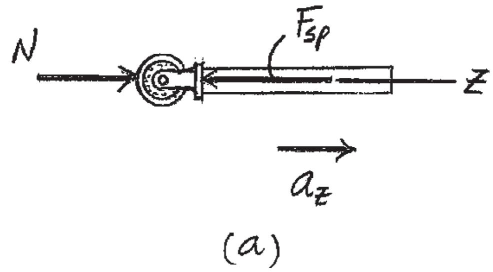
$$z = (0.02 \cos 2\theta) \text{ m}$$

$$\dot{z} = 0.02[-\sin 2\theta(2\dot{\theta})] = (-0.04 \sin 2\theta\dot{\theta}) \text{ m/s}$$

$$\ddot{z} = -0.04[\cos 2\theta(2\dot{\theta})\dot{\theta} + \sin 2\theta\ddot{\theta}] = [-0.04(2 \cos 2\theta(\dot{\theta})^2 + \sin 2\theta(\ddot{\theta}))] \text{ m/s}^2$$

Here $\dot{\theta} = 30$ rad/s and $\ddot{\theta} = 0$. Then,

$$\ddot{z} = -0.04[2 \cos 2\theta(30^2) + \sin 2\theta(0)] = (-72 \cos 2\theta) \text{ m/s}^2$$



Equation of Motion. At any arbitrary θ, the spring compresses $x = 0.02(1 + \cos 2\theta)$. Thus, $F_{sp} = kx = 1000[0.02(1 + \cos 2\theta)] = 20\,(1 + \cos 2\theta)$. Referring to the FBD of the follower, Fig. *a*,

$$\Sigma F_z = ma_z; \qquad N - 20(1 + \cos 2\theta) = 0.5(-72 \cos 2\theta)$$

$$N = (20 - 16 \cos 2\theta) \text{ N}$$

N is maximum when $\cos 2\theta = -1$. Then

$$(N)_{max} = 36.0 \text{ N} \qquad \textbf{Ans.}$$

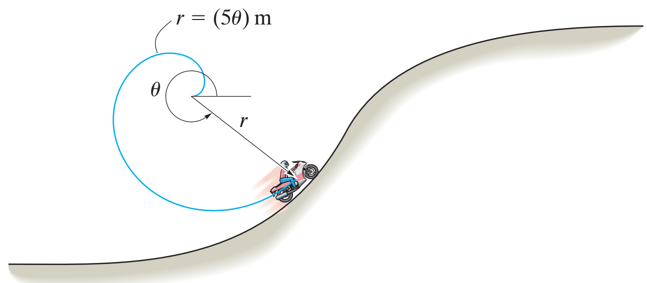
N is minimum when $\cos 2\theta = 1$. Then

$$(N)_{min} = 4.00 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$(N)_{max} = 36.0$ N
$(N)_{min} = 4.00$ N

***13–100.**

Determine the normal and frictional driving forces that the partial spiral track exerts on the 200-kg motorcycle at the instant $\theta = \frac{5}{3}\pi$ rad, $\dot{\theta} = 0.4$ rad/s, and $\ddot{\theta} = 0.8$ rad/s^2. Neglect the size of the motorcycle.



SOLUTION

$$\theta = \left(\frac{5}{3}\pi\right) = 300^\circ \qquad \dot{\theta} = 0.4 \qquad \ddot{\theta} = 0.8$$

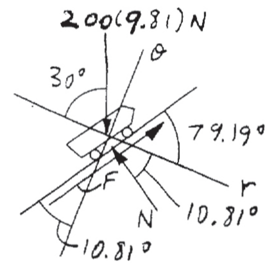
$$r = 5\theta = 5\left(\frac{5}{3}\pi\right) = 26.18$$

$$\dot{r} = 5\dot{\theta} = 5(0.4) = 2$$

$$\ddot{r} = 5\ddot{\theta} = 5(0.8) = 4$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4 - 26.18(0.4)^2 = -0.1888$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 26.18(0.8) + 2(2)(0.4) = 22.54$$



$$\tan\psi = \frac{r}{dr/d\theta} = \frac{5\left(\frac{5}{3}\pi\right)}{5} = 5.236 \qquad \psi = 79.19^\circ$$

$+\searrow\Sigma F_r = ma_r;\qquad F\sin 10.81^\circ - N\cos 10.81^\circ + 200(9.81)\cos 30^\circ = 200(-0.1888)$

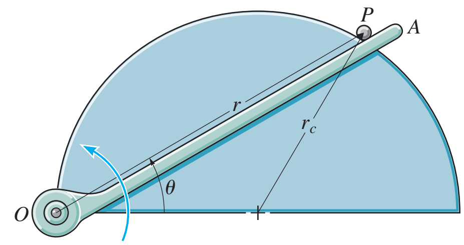
$+\nearrow\Sigma F_\theta = ma_\theta;\qquad F\cos 10.81^\circ - 200(9.81)\sin 30^\circ + N\sin 10.81^\circ = 200(22.54)$

$F = 5.07$ kN **Ans.**

$N = 2.74$ kN **Ans.**

Ans:
$F = 5.07$ kN
$N = 2.74$ kN

13–101. The 0.5-kg ball is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has an angular velocity $\dot{\theta} = 0.4$ rad/s and an angular acceleration $\ddot{\theta} = 0.8$ rad/s^2 at the instant $\theta = 30°$, determine the force of the arm on the ball. Neglect friction and the size of the ball. Set $r_c = 0.4$ m.



SOLUTION

Kinematics. Using the chain rule, the first and second time derivative of r are

$$r = 2(0.4)\cos\theta = 0.8\cos\theta \text{ m}$$

$$\dot{r} = -0.8\sin\theta\dot{\theta}$$

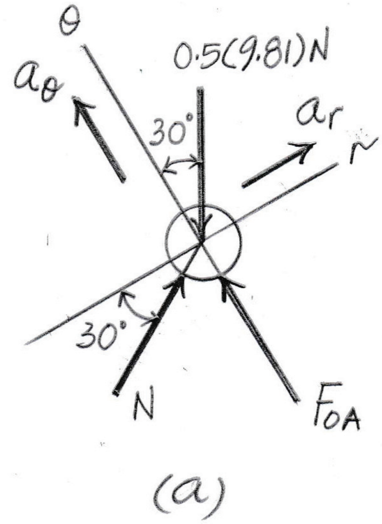
$$\ddot{r} = -0.8(\cos\theta\dot{\theta}^2 + \sin\theta\ddot{\theta})$$

At $\theta = 30°$, $\dot{\theta} = 0.4$ rad/s and $\ddot{\theta} = 0.8$ rad/s^2.

$$r = 0.8\cos 30° = 0.6928 \text{ m}$$

$$\dot{r} = -0.8\sin 30°(0.4) = -0.16 \text{ m/s}$$

$$\ddot{r} = -0.8\,[(\cos 30°)(0.4^2) + \sin 30°(0.8)] = -0.4309 \text{ m/s}^2$$



Then

$$a_r = \ddot{r} - r\dot{\theta}^2 = -0.4309 - 0.6928(0.4^2) = -0.5417 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.6928)(0.8) + 2(-0.16)(0.4) = 0.4263 \text{ m/s}^2$$

Equation of motion. Referring to the FBD of the ball, Fig. *a*,

$\Sigma F_r = ma_r;\quad N\cos 30° - 0.5(9.81)\sin 30° = 0.5(-0.5417)\quad N = 2.5192$ N

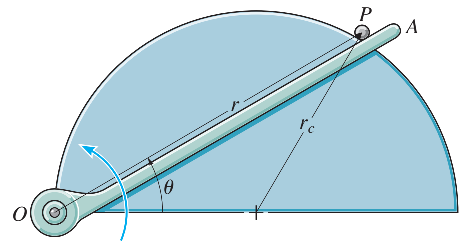
$\Sigma F_\theta = ma_\theta;\quad F_{OA} + 2.5192\sin 30° - 0.5(9.81)\cos 30° = 0.5(0.4263)$

$$F_{OA} = 3.2014 \text{ N} = 3.20 \text{ N}$$ **Ans.**

Ans:
$F_{OA} = 3.20$ N

13–102.

The ball of mass m is guided along the vertical circular path $r = 2r_c \cos\theta$ using the arm OA. If the arm has a constant angular velocity $\dot{\theta}_0$, determine the angle $\theta \leq 45°$ at which the ball starts to leave the surface of the semicylinder. Neglect friction and the size of the ball.



SOLUTION

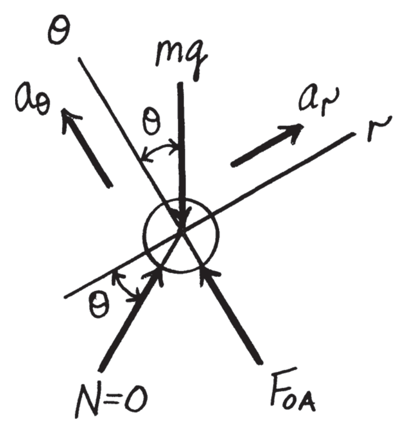
$r = 2r_c \cos\theta$

$\dot{r} = -2r_c \sin\theta\dot{\theta}$

$\ddot{r} = -2r_c \cos\theta\dot{\theta}^2 - 2r_c \sin\theta\ddot{\theta}$

Since $\dot{\theta}$ is constant, $\ddot{\theta} = 0$.

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2r_c \cos\theta\dot{\theta}_0^2 - 2r_c \cos\theta\dot{\theta}_0^2 = -4r_c \cos\theta\dot{\theta}_0^2$$



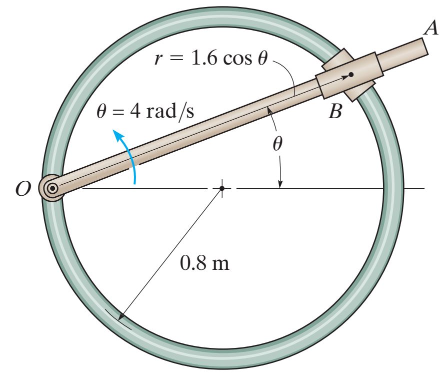
$+\nearrow\Sigma F_r = ma_r; \qquad -mg\sin\theta = m(-4r_c\cos\theta\dot{\theta}_0^2)$

$$\tan\theta = \frac{4r_c\dot{\theta}_0^2}{g} \qquad \theta = \tan^{-1}\left(\frac{4r_c\dot{\theta}_0^2}{g}\right)$$ **Ans.**

Ans:
$\theta = \tan^{-1}\left(\frac{4r_c\dot{\theta}_0^2}{g}\right)$

13–103.

Rod OA rotates counterclockwise at a constant angular rate $\dot{\theta} = 4$ rad/s. The double collar B is pin-connected together such that one collar slides over the rotating rod and the other collar slides over the circular rod described by the equation $r = (1.6 \cos \theta)$ m. If *both* collars have a mass of 0.5 kg, determine the force which the circular rod exerts on one of the collars and the force that OA exerts on the other collar at the instant $\theta = 45°$. Motion is in the horizontal plane.



SOLUTION

$r = 1.6 \cos \theta$

$\dot{r} = -1.6 \sin \theta \dot{\theta}$

$\ddot{r} = -1.6 \cos \theta \dot{\theta}^2 - 1.6 \sin \theta \ddot{\theta}$

At $\theta = 45°, \dot{\theta} = 4$ rad/s and $\ddot{\theta} = 0$

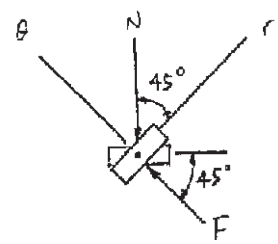
$r = 1.6 \cos 45° = 1.1314$ m

$\dot{r} = -1.6 \sin 45°(4) = -4.5255$ m/s

$\ddot{r} = -1.6 \cos 45°(4)^2 - 1.6 \sin 45°(0) = -18.1019$ m/s^2

$a_r = \ddot{r} - r\dot{\theta}^2 = -18.1019 - 1.1314(4)^2 = -36.20$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.1314(0) + 2(-4.5255)(4) = -36.20$ m/s^2



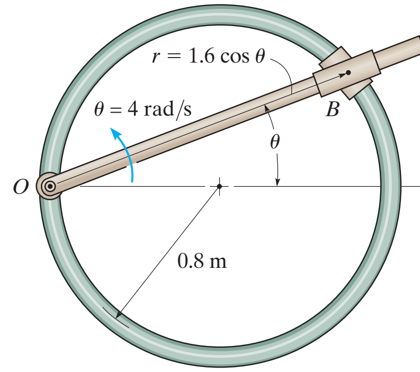
$\nearrow + \Sigma F_r = ma_r$; $\quad -N_C \cos 45° = 0.5(-36.20) \quad N_C = 25.6$ N **Ans.**

$+\nwarrow \Sigma F_\theta = ma_\theta$; $\quad F_{OA} - 25.6 \sin 45° = 0.5(-36.20) \quad F_{OA} = 0$ **Ans.**

Ans:
$N_C = 25.6$ N
$F_{OA} = 0$

***13–104.**

Solve Prob. 13–103 if motion is in the vertical plane.



SOLUTION

$r = 1.6 \cos \theta$

$\dot{r} = -1.6 \sin \theta \dot{\theta}$

$\ddot{r} = -1.6 \cos \theta \dot{\theta}^2 - 1.6 \sin \theta \ddot{\theta}$

At $\theta = 45°$, $\dot{\theta} = 4$ rad/s and $\ddot{\theta} = 0$

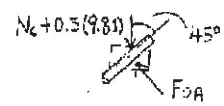
$r = 1.6 \cos 45° = 1.1314$ m

$\dot{r} = -1.6 \sin 45°(4) = -4.5255$ m/s

$\ddot{r} = -1.6 \cos 45°(4)^2 - 1.6 \sin 45°(0) = -18.1019 \text{ m/s}^2$

$a_r = \ddot{r} - r\dot{\theta}^2 = -18.1019 - 1.1314(4)^2 = -36.20 \text{ m/s}^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 1.1314(0) + 2(-4.5255)(4) = -36.20 \text{ m/s}^2$



$+\swarrow \Sigma F_r = ma_r;\quad -N_C \cos 45° - 4.905 \cos 45° = 0.5(-36.204)$

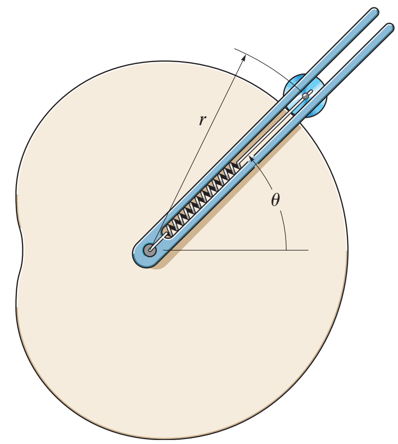
$+\nwarrow \Sigma F_\theta = ma_\theta;\quad F_{OA} - N_C \sin 45° - 4.905 \sin 45° = 0.5(-36.204)$

$N_C = 20.7$ N **Ans.**

$F_{OA} = 0$ **Ans.**

Ans:
$N_C = 20.7$ N
$F_{OA} = 0$

13–105. The smooth surface of the vertical cam is defined in part by the curve $r = (0.2\cos\theta + 0.3)$ m. The forked rod is rotating with an angular acceleration of $\ddot{\theta} = 2\ \text{rad/s}^2$, and when $\theta = 45°$ the angular velocity is $\dot{\theta} = 6\ \text{rad/s}$. Determine the force the cam and the rod exert on the 2-kg roller at this instant. The attached spring has a stiffness $k = 100$ N/m and an unstretched length of 0.1 m.



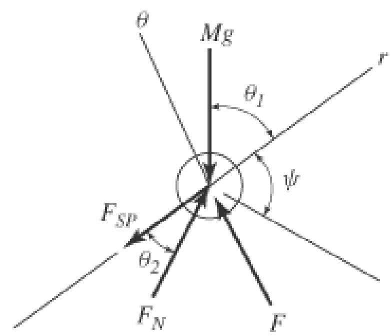
SOLUTION

Given:

$a = 0.2\ \text{m}$ $\quad k = 100\ \frac{\text{N}}{\text{m}}$ $\quad \theta = 45°$

$b = 0.3\ \text{m}$ $\quad l = 0.1\ \text{m}$ $\quad \theta' = 6\ \text{rad/s}$

$g = 9.81\ \text{m/s}^2$ $\quad M = 2\ \text{kg}$ $\quad \theta'' = 2\ \text{rad/s}^2$



$$r = a\cos(\theta) + b$$

$$r' = -(a)\sin(\theta)\theta'$$

$$r'' = -(a)\cos(\theta)\theta'^2 - (a)\sin(\theta)\theta''$$

$$\psi = \text{atan}\left(\frac{r\theta'}{r'}\right) + \pi$$

Guesses $\quad F_N = 1\ \text{N} \quad F = 1\ \text{N}$

Given

$$F_N\sin(\psi) - Mg\sin(\theta) - k(r - l) = M\left(r'' - r\theta'^2\right)$$

$$F - F_N\cos(\psi) - Mg\cos(\theta) = M(r\theta'' + 2r'\theta')$$

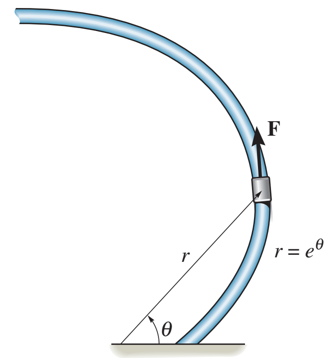
$$\begin{pmatrix} F \\ F_N \end{pmatrix} = \text{Find}(F, F_N) \qquad \begin{pmatrix} F \\ F_N \end{pmatrix} = \begin{pmatrix} -6.483 \\ 5.76 \end{pmatrix} \text{N} \quad \textbf{Ans.}$$

Ans:

$F = -6.483$ N
$F_N = 5.76$ N

13–106.

The collar has a mass of 2 kg and travels along the smooth horizontal rod defined by the equiangular spiral $r = (e^{\theta})$ m, where θ is in radians. Determine the tangential force F and the normal force N acting on the collar when $\theta = 45°$, if the force F maintains a constant angular motion $\dot{\theta} = 2$ rad/s.



SOLUTION

$r = e^{\theta}$

$\dot{r} = e^{\theta}\,\dot{\theta}$

$\ddot{r} = e^{\theta}(\dot{\theta})^2 + e^{\theta}\ddot{\theta}$

At $\theta = 45°$

$\dot{\theta} = 2$ rad/s

$\ddot{\theta} = 0$

$r = 2.1933$

$\dot{r} = 4.38656$

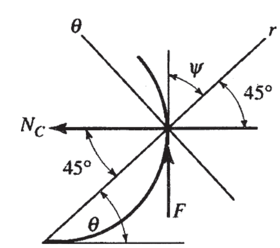
$\ddot{r} = 8.7731$

$a_r = \ddot{r} - r(\dot{\theta})^2 = 8.7731 - 2.1933(2)^2 = 0$

$a_{\theta} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(4.38656)(2) = 17.5462 \text{ m/s}^2$

$$\tan\psi = \frac{r}{\left(\dfrac{dr}{d\theta}\right)} = e^{\theta}/e^{\theta} = 1$$

$\psi = \theta = 45°$



$\nearrow + \sum F_r = ma_r;\qquad -N_C\cos 45° + F\cos 45° = 2(0)$

$+\nwarrow \sum F_{\theta} = ma_{\theta};\qquad F\sin 45° + N_C\sin 45° = 2(17.5462)$

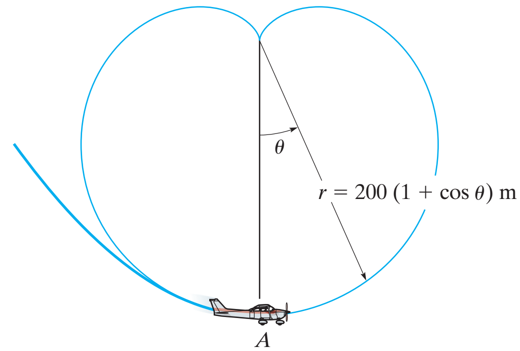
$N = 24.8$ N **Ans.**

$F = 24.8$ N **Ans.**

Ans:
$N = 24.8$ N
$F = 24.8$ N

13–107.

The pilot of the airplane executes a vertical loop which in part follows the path of a cardioid, $r = 200(1 + \cos\theta)$ m, where θ is in radians. If his speed at A is a constant $v_p = 85$ m/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He has a mass of 80 kg. *Hint:* To determine the time derivatives necessary to calculate the acceleration components a_r and a_θ, take the first and second time derivatives of $r = 200(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$.



SOLUTION

Kinematic. Using the chain rule, the first and second time derivatives of r are

$$r = 200(1 + \cos\theta)$$

$$\dot{r} = 200(-\sin\theta)(\dot{\theta}) = -200(\sin\theta)\dot{\theta}$$

$$\ddot{r} = -200[(\cos\theta)(\dot{\theta})^2 + (\sin\theta)(\ddot{\theta})]$$

When $\theta = 0°$,

$$r = 200(1 + \cos 0°) = 400 \text{ m}$$

$$\dot{r} = -200(\sin 0°)\,\dot{\theta} = 0$$

$$\ddot{r} = -200\left[(\cos 0°)(\dot{\theta})^2 + (\sin 0°)(\ddot{\theta})\right] = -200\dot{\theta}^2$$

Using Eq. 12–26

$$v = \sqrt{\dot{r}^2 + (r\dot{\theta})^2}$$

$$v^2 = \dot{r}^2 + (r\dot{\theta})^2$$

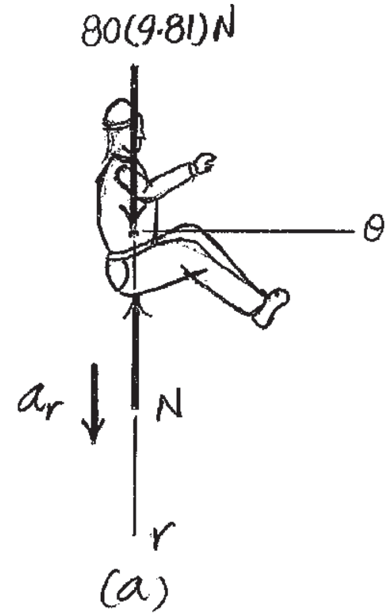
$$85^2 = 0^2 + (400\dot{\theta})^2$$

$$\dot{\theta} = 0.2125 \text{ rad/s}$$

Thus,

$$a_r = \ddot{r} - r\dot{\theta}^2 = -200\left(0.2125^2\right) - 400\left(0.2125^2\right) = -27.09 \text{ m/s}^2$$

Equation of Motion. Referring to the FBD of the pilot, Fig. *a*,

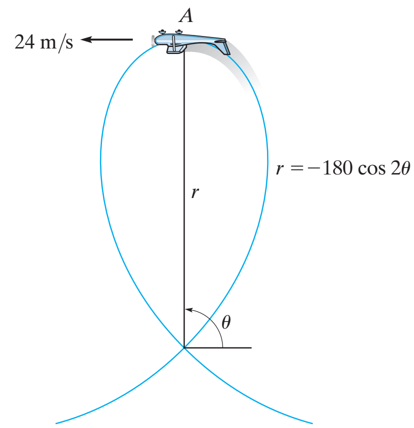


$+\downarrow \Sigma F_r = ma_r;\qquad 80(9.81) - N = 80(-27.09)$

$$N = 2952.3 \text{ N} = 2.95 \text{ kN}$$ **Ans.**

Ans:
$N = 2.95$ N

***13–108.** The pilot of an airplane executes a vertical loop which in part follows the path of a "four-leaved rose," $r = (-180 \cos 2\theta)$ m, where θ is in radians. If his speed at A is a constant $v_p = 24$ m/s, determine the vertical reaction the seat of the plane exerts on the pilot when the plane is at A. He weights 650 N. *Hint:* To determine the time derivatives necessary to compute the acceleration components a_r and a_θ, take the first and second time derivatives of $r = -180(1 + \cos\theta)$. Then, for further information, use Eq. 12–26 to determine $\dot{\theta}$. Also, take the time derivative of Eq. 12–26, noting that $\dot{v}_C = 0$ to determine $\ddot{\theta}$.



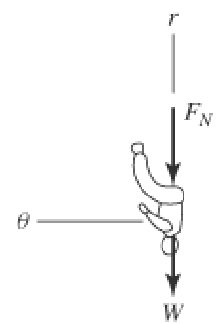
SOLUTION

$$\theta = 90°$$

$$r = (a)\cos(2\theta)$$

Guesses

$$r' = 1 \text{ m/s} \qquad r'' = 1 \text{ m/s}^2 \qquad \theta' = 1 \text{ rad/s} \qquad \theta'' = 1 \text{ rad/s}^2$$



Given Note that v_p is constant so $dv_p/dt = 0$

$$r' = -(a)\sin(2\theta)2\theta' \qquad r'' = -(a)\sin(2\theta)2\theta'' - (a)\cos(2\theta)4\theta'^2$$

$$v_p = \sqrt{r'^2 + (r\theta')^2} \qquad 0 = \frac{r'r'' + r\theta'(r\theta'' + r'\theta')}{\sqrt{r'^2 + (r\theta')^2}}$$

Given:

$$a = -180 \text{ m} \qquad W = 650 \text{ N}$$

$$v_p = 24 \text{ m/s} \qquad g = 9.81 \text{ m/s}^2$$

$$\begin{pmatrix} r' \\ r'' \\ \theta' \\ \theta'' \end{pmatrix} = \text{Find}(r', r'', \theta', \theta'') \qquad r' = 0 \text{ m/s} \qquad r'' = -12.8 \text{ m/s}^2$$

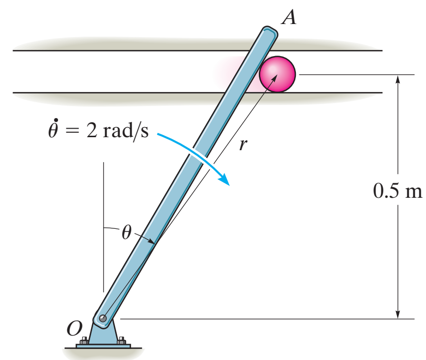
$$\theta' = 0.133 \text{ rad/s} \qquad \theta'' = 0 \text{ rad/s}^2$$

$$-F_N - W = M(r'' - r\theta'^2) \qquad F_N = -W - \left(\frac{W}{g}\right)(r'' - r\theta'^2) \qquad F_N = 410.1 \text{ N} \quad \textbf{Ans.}$$

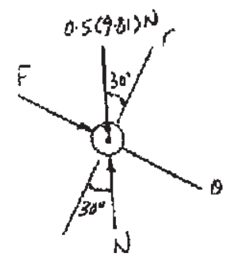
Ans:
$F_N = 410.1$ N

13–109.

The particle has a mass of 0.5 kg and is confined to move along the smooth horizontal slot due to the rotation of the arm OA. Determine the force of the rod on the particle and the normal force of the slot on the particle when $\theta = 30°$. The rod is rotating with a constant angular velocity $\dot{\theta} = 2$ rad/s. Assume the particle contacts only one side of the slot at any instant.



SOLUTION



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta})\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}]$$

When $\theta = 30°$, $\dot{\theta} = 2$ rad/s and $\ddot{\theta} = 0$

$$r = 0.5\sec 30° = 0.5774 \text{ m}$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667 \text{ m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(0)\right]$$

$$= 3.849 \text{ m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 3.849 - 0.5774(2)^2 = 1.540 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(0) + 2(0.6667)(2) = 2.667 \text{ m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;$ $\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(1.540)$

$N = 5.79$ N **Ans.**

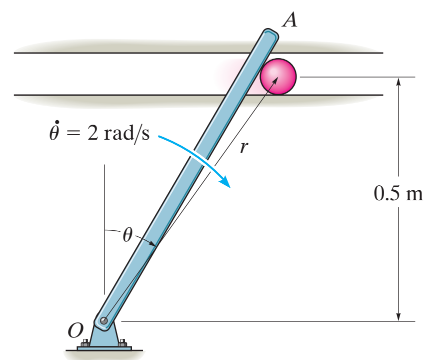
$+\searrow \Sigma F_\theta = ma_\theta;$ $\quad F + 0.5(9.81)\sin 30° - 5.79\sin 30° = 0.5(2.667)$

$F = 1.78$ N **Ans.**

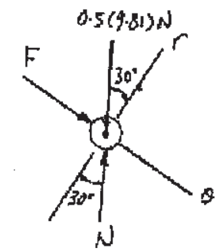
Ans:
$F_r = 1.78$ N
$N_s = 5.79$ N

13–110.

Solve Prob. 13–109 if the arm has an angular acceleration of $\ddot{\theta} = 3\ \text{rad/s}^2$ when $\dot{\theta} = 2\ \text{rad/s}$ at $\theta = 30°$.



SOLUTION



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\dot{\theta}$$

$$\ddot{r} = 0.5\left\{\left[(\sec\theta\tan\theta\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\dot{\theta})\right]\dot{\theta} + \sec\theta\tan\theta\ddot{\theta}\right\}$$

$$= 0.5\left[\sec\theta\tan^2\theta\dot{\theta}^2 + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan\theta\ddot{\theta}\right]$$

When $\theta = 30°, \dot{\theta} = 2\ \text{rad/s}$ and $\ddot{\theta} = 3\ \text{rad/s}^2$

$$r = 0.5\sec 30° = 0.5774\ \text{m}$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667\ \text{m/s}$$

$$\ddot{r} = 0.5\left[\sec 30°\tan^2 30°(2)^2 + \sec^3 30°(2)^2 + \sec 30°\tan 30°(3)\right]$$

$$= 4.849\ \text{m/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2)^2 = 2.5396\ \text{m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.5774(3) + 2(0.6667)(2) = 4.3987\ \text{m/s}^2$$

$\nearrow + \Sigma F_r = ma_r;\quad N\cos 30° - 0.5(9.81)\cos 30° = 0.5(2.5396)$

$N = 6.3712 = 6.37\ \text{N}$ **Ans.**

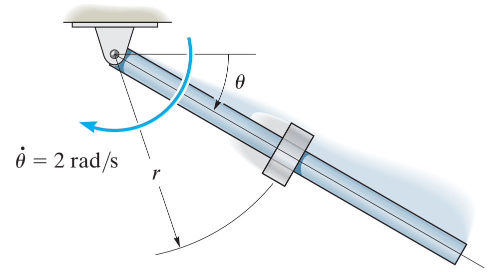
$+\searrow \Sigma F_\theta = ma_\theta;\quad F + 0.5(9.81)\sin 30° - 6.3712\sin 30° = 0.5(4.3987)$

$F = 2.93\ \text{N}$ **Ans.**

Ans:
$F_r = 2.93\ \text{N}$
$N_s = 6.37\ \text{N}$

13–111.

A 0.2-kg spool slides down along a smooth rod. If the rod has a constant angular rate of rotation $\dot{\theta} = 2$ rad/s in the vertical plane, show that the equations of motion for the spool are $\ddot{r} - 4r - 9.81 \sin\theta = 0$ and $0.8\dot{r} + N_s - 1.962\cos\theta = 0$, where N_s is the magnitude of the normal force of the rod on the spool. Using the methods of differential equations, it can be shown that the solution of the first of these equations is $r = C_1 e^{-2t} + C_2 e^{2t} - (9.81/8)\sin 2t$. If r, $\dot{r}$, and θ are zero when $t = 0$, evaluate the constants C_1 and C_2 to determine r at the instant $\theta = \pi/4$ rad.

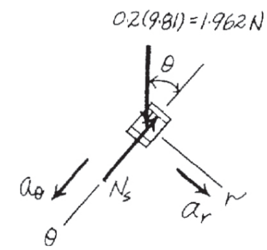


SOLUTION

Kinematic: Here, $\dot{\theta}. = 2$ rad/s and $\dot{\theta} = 0$. Applying Eqs. 12–29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = \ddot{r} - r(2^2) = \ddot{r} - 4r$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r(0) + 2\dot{r}(2) = 4\dot{r}$$



Equation of Motion: Applying Eq. 13–8, we have

$\Sigma F_r = ma_r;$ $\qquad 1.962\sin\theta = 0.2(\ddot{r} - 4r)$

$$\ddot{r} - 4r - 9.81\sin\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(1)}$$

$\Sigma F_\theta = ma_\theta;$ $\qquad 1.962\cos\theta - N_s = 0.2(4\dot{r})$

$$0.8\dot{r} + N_s - 1.962\cos\theta = 0 \qquad \textbf{(Q.E.D.)} \qquad \textbf{(2)}$$

Since $\theta. = 2$ rad/s, then $\int_0^\theta \dot{\theta} = \int_0^1 2dt, \theta = 2t$. The solution of the differential equation (Eq.(1)) is given by

$$r = C_1\, e^{-2t} + C_2\, e^{2t} - \frac{9.81}{8}\sin 2t \qquad \textbf{(3)}$$

Thus,

$$\dot{r} = -2\,C_1 e^{-2t} + 2C_2\, e^{2t} - \frac{9.81}{4}\cos 2t \qquad \textbf{(4)}$$

At $t = 0, r = 0$. From Eq.(3) $0 = C_1(1) + C_2(1) - 0$ **(5)**

At $t = 0, \dot{r} = 0$. From Eq.(4) $0 = -2\,C_1(1) + 2C_2(1) - \frac{9.81}{4}$ **(6)**

Solving Eqs. (5) and (6) yields

$$C_1 = -\frac{9.81}{16} \qquad C_2 = \frac{9.81}{16}$$

Thus,

$$r = -\frac{9.81}{16}e^{-2t} + \frac{9.81}{16}e^{2t} - \frac{9.81}{8}\sin 2t$$

$$= \frac{9.81}{8}\left(\frac{-e^{-2t} + e^{2t}}{2} - \sin 2t\right)$$

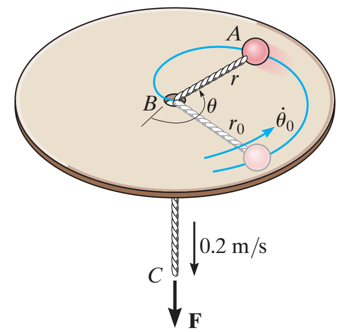
$$= \frac{9.81}{8}(\sin h\, 2t - \sin 2t)$$

At $\theta = 2t = \frac{\pi}{4}$, $\qquad r = \frac{9.81}{8}\left(\sin h\frac{\pi}{4} - \sin\frac{\pi}{4}3\right) = 0.198$ m **Ans.**

Ans:
$r = 0.198$ m

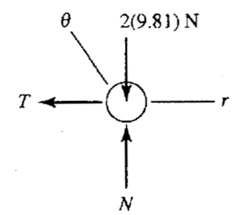
***13–112.**

The ball has a mass of 2 kg and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0 = 0.5$ m such that the angular rate of rotation is $\dot{\theta}_0 = 1$ rad/s. If the attached cord ABC is drawn down through the hole at a constant speed of 0.2 m/s, determine the tension the cord exerts on the ball at the instant $r = 0.25$ m. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)(d(r^2\dot{\theta})/dt) = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



SOLUTION

$$\sum F_\theta = ma_\theta; \qquad 0 = m[r\ddot{\theta} + 2\dot{r}\dot{\theta}] = m\left[\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\right] = 0$$



Thus,

$$d(r^2\dot{\theta}) = 0$$

$$r^2\dot{\theta} = C$$

$$(0.5)^2(1) = C = (0.25)^2\dot{\theta}$$

$$\dot{\theta} = 4.00 \text{ rad/s}$$ **Ans.**

Since $\dot{r} = -0.2$ m/s, $\quad \ddot{r} = 0$

$$a_r = \ddot{r} - \dot{r}(\theta)^2 = 0 - 0.25(4.00)^2 = -4 \text{ m/s}^2$$

$$\sum F_r = ma_r; \quad -T = 2(-4)$$

$$T = 8 \text{ N}$$ **Ans.**

Ans:
$\dot{\theta} = 4.00$ rad/s
$T = 8$ N

13–113.

The earth has an orbit with eccentricity $e = 0.0167$ around the sun. Knowing that the earth's minimum distance from the sun is $146(10^6)$ km, find the speed at which the earth travels when it is at this distance. Determine the equation in polar coordinates which describes the earth's orbit about the sun.

SOLUTION

$$e = \frac{Ch^2}{GM_S} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_S r_0}\left(1 - \frac{GM_S}{r_0 v_0^2}\right)(r_0 v_0)^2 \qquad e = \left(\frac{r_0 v_0^2}{GM_S} - 1\right) \qquad \frac{r_0 v_0^2}{GM_S} = e + 1$$

$$v_0 = \sqrt{\frac{GM_S(e+1)}{r_0}}$$

$$= \sqrt{\frac{66.73(10^{-12})(1.99)(10^{30})(0.0167+1)}{146(10^9)}} = 30409 \text{ m/s} = 30.4 \text{ km/s} \qquad \textbf{Ans.}$$

$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{GM_S}{r_0 v_0^2}\right)\cos\theta + \frac{GM_S}{r_0^2 v_0^2}$$

$$\frac{1}{r} = \frac{1}{146(10^9)}\left(1 - \frac{66.73(10^{-12})(1.99)(10^{30})}{151.3(10^9)(30409)^2}\right)\cos\theta + \frac{66.73(10^{-12})(1.99)(10^{30})}{\left[146(10^9)\right]^2(30409)^2}$$

$$\frac{1}{r} = 0.348(10^{-12})\cos\theta + 6.74(10^{-12}) \qquad \textbf{Ans.}$$

Ans:
$v_o = 30.4$ km/s
$\frac{1}{r} = 0.348\left(10^{-12}\right)\cos\theta + 6.74\left(10^{-12}\right)$

13–114.

A communications satellite is in a circular orbit above the earth such that it always remains directly over a point on the earth's surface. As a result, the period of the satellite must equal the rotation of the earth, which is approximately 24 hours. Determine the satellite's altitude h above the earth's surface and its orbital speed.

SOLUTION

The period of the satellite around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$T = \frac{2\pi r_0}{v_s}$$

$$24(3600) = \frac{2\pi\left[h + 6.378(10^6)\right]}{v_s}$$

$$v_s = \frac{2\pi\left[h + 6.378(10^6)\right.}{86.4(10^3)} \qquad \textbf{(1)}$$

The velocity of the satellite orbiting around the circular orbit of radius $r_0 = h + r_e = \left[h + 6.378(10^6)\right]$ m is given by

$$v_S = \sqrt{\frac{GM_e}{r_0}}$$

$$v_S = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{h + 6.378(10^6)}} \qquad \textbf{(2)}$$

Solving Eqs.(1) and (2),

$$h = 35.87(10^6)\text{ m} = 35.9\text{ Mm} \qquad v_S = 3072.32\text{ m/s} = 3.07\text{ km/s} \quad \textbf{Ans.}$$

Ans:
$h = 35.9$ mm
$v_s = 3.07$ km/s

13–115.

The speed of a satellite launched into a circular orbit about the earth is given by Eq. 13–24. Determine the speed of a satellite launched parallel to the surface of the earth so that it travels in a circular orbit 800 km from the earth's surface.

SOLUTION

For a 800-km orbit

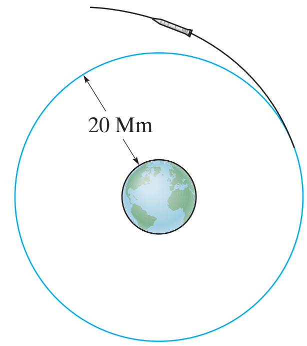
$$v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})}{(800 + 6378)(10^3)}}$$

$$= 7453.6 \text{ m/s} = 7.45 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v_0 = 7.45 \text{ km/s}$

***13–116.**

The rocket is in circular orbit about the earth at an altitude of 20 Mm. Determine the minimum increment in speed it must have in order to escape the earth's gravitational field.



SOLUTION

The speed of the rocket in circular orbit is

$$v_c = \sqrt{\frac{GM_e}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[5.976(10^{24})\right]}{20(10^6) + 6378(10^3)}} = 3888.17 \text{ m/s}$$

To escape the earth's gravitational field, the rocket must enter the parabolic trajectory, which require its speed to be

$$v_e = \sqrt{\frac{2GM_e}{r_0}} = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]}{20(10^6) + 6378(10^3)}} = 5498.70 \text{ m/s}$$

The required increment in speed is

$$\Delta v = v_e - v_c = 5498.70 - 3888.17$$

$$= 1610.53 \text{ m/s}$$

$$= 1.61(10^3) \text{ m/s}$$ **Ans.**

Ans:
$\Delta v = 1.61(10^3)$ m/s

13–117.

Prove Kepler's third law of motion. *Hint:* Use Eqs. 13–18, 13–27, 13–28, and 13–30.

SOLUTION

From Eq. 13–18,

$$\frac{1}{r} = C\cos\theta + \frac{GM_s}{h^2}$$

For $\theta = 0°$ and $\theta = 180°$,

$$\frac{1}{r_p} = C + \frac{GM_s}{h^2}$$

$$\frac{1}{r_a} = -C + \frac{GM_s}{h^2}$$

Eliminating C, from Eqs. 13–27 and 13–28,

$$\frac{2a}{b^2} = \frac{2GM_s}{h^2}$$

From Eq. 13–30,

$$T = \frac{\pi}{h}(2a)(b)$$

Thus,

$$b^2 = \frac{T^2h^2}{4\pi^2a^2}$$

$$\frac{4\pi^2a^3}{T^2h^2} = \frac{GM_s}{h^2}$$

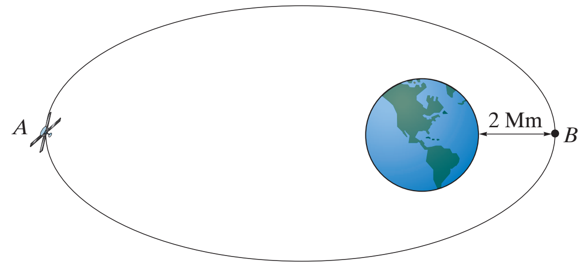
$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$ **Q.E.D.**

Ans:

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right)a^3$$

13–118.

The satellite is moving in an elliptical orbit with an eccentricity $e = 0.25$. Determine its speed when it is at its maximum distance A and minimum distance B from the earth.



SOLUTION

$$e = \frac{Ch^2}{GM_e}$$

where $C = \dfrac{1}{r_0}\left(1 - \dfrac{GM_e}{r_0 v_0^2}\right)$ and $h = r_0\, v_0$.

$$e = \frac{1}{GM_e\, r_0}\left(1 - \frac{GM_e}{r_0\, v_0^2}\right)(r_0\, v_0)^2$$

$$e = \left(\frac{r_0\, v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0\, v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e\,(e + 1)}{r_0}}$$

where $r_0 = r_p = 2\left(10^6\right) + 6378\left(10^3\right) = 8.378\left(10^6\right)$ m.

$$v_B = v_0 = \sqrt{\frac{66.73(10^{-12})(5.976)(10^{24})(0.25 + 1)}{8.378(10^6)}} = 7713 \text{ m/s} = 7.71 \text{ km/s} \qquad \textbf{Ans.}$$

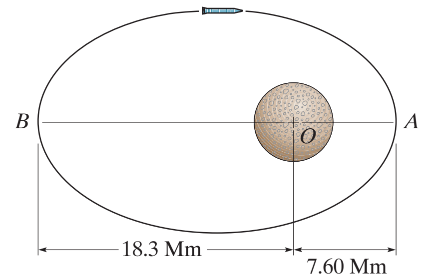
$$r_a = \frac{r_0}{\dfrac{2GM_e}{r_0\, v_0} - 1} = \frac{8.378(10^6)}{\dfrac{2(66.73)(10^{-12})(5.976)(10^{24})}{8.378(10^6)(7713)^2} - 1} = 13.96\left(10^6\right) \text{ m}$$

$$v_A = \frac{r_p}{r_a}\, v_B = \frac{8.378(10^6)}{13.96(10^6)}(7713) = 4628 \text{ m/s} = 4.63 \text{ km/s} \qquad \textbf{Ans.}$$

Ans:
$v_B = 7.71$ km/s
$v_A = 4.63$ km/s

13–119.

The rocket is traveling in free flight along the elliptical orbit. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's speed when it is at A and at B.



SOLUTION

Applying Eq. 13–26,

$$r_a = \frac{r_p}{\left(2GM/r_p v_p^2\right) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM\, r_a}{r_p(r_p + r_a)}}$$

The elliptical orbit has $r_p = 7.60(10^6)$ m, $r_a = 18.3(10^6)$ m and $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73\left(10^{-12}\right)\right]\left[0.6(5.976)\left(10^{24}\right)\right]\left[18.3\left(10^6\right)\right]}{7.60\left(10^6\right)\left[7.60\left(10^6\right) + 18.3\left(10^6\right)\right]}}$$

$$= 6669.99 \text{ m/s} = 6.67\left(10^3\right) \text{ m/s}$$ **Ans.**

In this case,

$$h = r_p v_A = r_a v_B$$

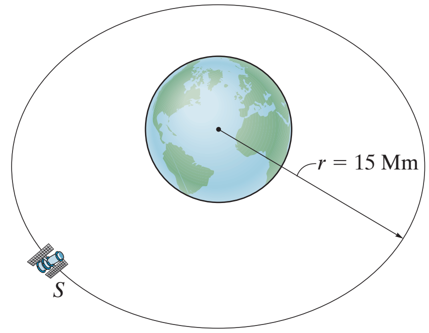
$$7.60\left(10^6\right)(6669.99) = 18.3\left(10^6\right)v_B$$

$$v_B = 2770.05 \text{ m/s} = 2.77\left(10^3\right) \text{ m/s}$$ **Ans.**

Ans:
$v_A = 6.67\left(10^3\right)$ m/s
$v_B = 2.77\left(10^3\right)$ m/s

***13–120.**

Determine the constant speed of satellite S so that it circles the earth with an orbit of radius $r = 15$ Mm. *Hint:* Use Eq. 13–1.



SOLUTION

$F = G\dfrac{m_s m_e}{r^2}$ Also $F = m_s\left(\dfrac{v_s^2}{r}\right)$ Hence

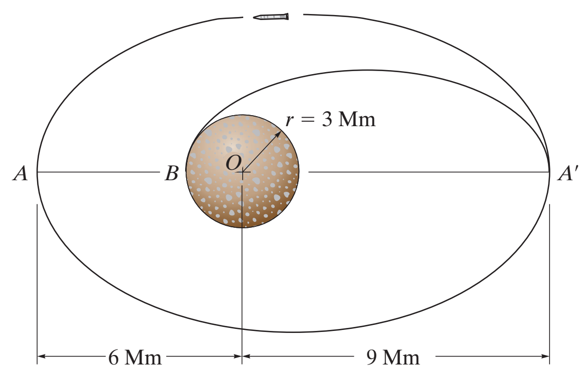
$$m_s\left(\frac{v_0^2}{r}\right) = G\frac{m_s m_e}{r^2}$$

$$v = \sqrt{G\frac{m_e}{r}} = \sqrt{66.73(10^{-12})\left(\frac{5.976(10^{24})}{15(10^6)}\right)} = 5156 \text{ m/s} = 5.16 \text{ km/s}$$ **Ans.**

Ans:
$v = 5.16$ km/s

13–121.

The rocket is in free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.70 times that of the earth. If the rocket has an apoapsis and periapsis as shown in the figure, determine the speed of the rocket when it is at point A.



SOLUTION

Central-Force Motion: Use $r_a = \dfrac{r_0}{\left(2\,GM/r_0\,v_0^2\right)-1}$, with $r_0 = r_p = 6\left(10^6\right)$ m and $M = 0.70M_e$, we have

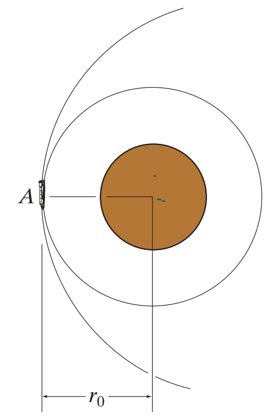
$$9\left(10^6\right) = \frac{6(10)^6}{\left(\dfrac{2(66.73)\,(10^{-12})\,(0.7)\,[5.976(10^{24})]}{6(10^6)v_P^2}\right) - 1}$$

$$v_A = 7471.89 \text{ m/s} = 7.47 \text{ km/s}$$ **Ans.**

Ans:
$v_A = 7.47$ km/s

13–122.

The Viking Explorer approaches the planet Mars on a parabolic trajectory as shown. When it reaches point A its velocity is 10 Mm/h. Determine r_0 and the required velocity at A so that it can then maintain a circular orbit as shown. The mass of Mars is 0.1074 times the mass of the earth.



SOLUTION

When the Viking explorer approaches point A on a parabolic trajectory, its velocity at point A is given by

$$v_A = \sqrt{\frac{2GM_M}{r_0}}$$

$$\left[10(10^6)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \sqrt{\frac{2(66.73)(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{r_0}}$$

$$r_0 = 11.101(10^6)\text{ m} = 11.1\text{ Mm} \qquad \textbf{Ans.}$$

When the explorer travels along a circular orbit of $r_0 = 11.101(10^6)$ m, its velocity is

$$v_{A'} = \sqrt{\frac{GM_r}{r_0}} = \sqrt{\frac{66.73(10^{-12})\left[0.1074(5.976)(10^{24})\right]}{11.101(10^6)}}$$

$$= 1964.19\text{ m/s}$$

Thus, the required sudden decrease in the explorer's velocity is

$$\Delta v_A = v_A - v_{A'}$$

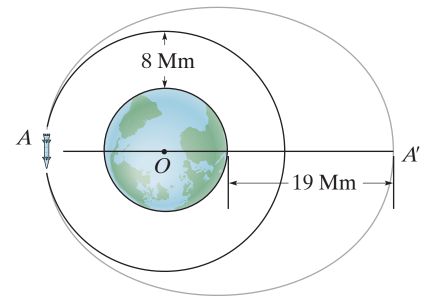
$$= 10(10^6)\left(\frac{1}{3600}\right) - 1964.19$$

$$= 814\text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$r_0 = 11.1$ Mm
$\Delta v_A = 814$ m/s

13–123.

The rocket is initially in free-flight circular orbit around the earth. Determine the speed of the rocket at A. What change in the speed at A is required so that it can move in an elliptical orbit to reach point A'?



SOLUTION

The required speed to remain in circular orbit containing point A of which $r_0 = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m can be determined from

$$(v_A)_C = \sqrt{\frac{GM_e}{r_0}}$$

$$= \sqrt{\frac{[66.73(10^{-12})][5.976(10^{24})]}{14.378(10^6)}}$$

$$= 5266.43 \text{ m/s} = 5.27(10^3) \text{ m/s}$$ **Ans.**

To more from A to A', the rocket has to follow the elliptical orbit with $r_p = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m and $r_a = 19(10^6) + 6378(10^3) = 25.378(10^6)$ m. The required speed at A to do so can be determined using Eq. 13–26.

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here, $v_p = (v_A)_e$. Then

$$(v_A)_e = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][25.378(10^6)]}{14.378(10^6)[14.378(10^6) + 25.378(10^6)]}}$$

$$= 5950.58 \text{ m/s}$$

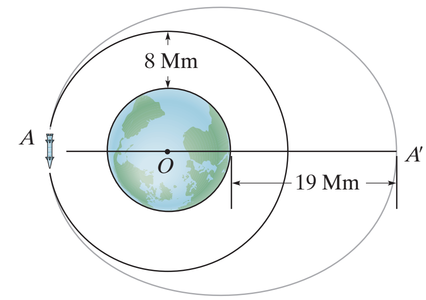
Thus, the required change in speed is

$$\Delta v = (v_A)_e - (v_A)_c = 5950.58 - 5266.43 = 684.14 \text{ m/s} = 684 \text{ m/s}$$ **Ans.**

Ans:
$(v_A)_C = 5.27(10^3)$ m/s
$\Delta v = 684$ m/s

***13–124.**

The rocket is in free-flight circular orbit around the earth. Determine the time needed for the rocket to travel from the inner orbit at A to the outer orbit at A'.



SOLUTION

To move from A to A', the rocket has to follow the elliptical orbit with $r_p = 8(10^6) + 6378(10^3) = 14.378(10^6)$ m and $r_a = 19(10^6) + 6378(10^3) = 25.378(10^6)$ m. The required speed at A to do so can be determined using Eq. 13–26.

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here, $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]\left[25.378(10^6)\right]}{14.378(10^6)\left[14.378(10^6) + 25.378(10^6)\right]}} = 5950.58 \text{ m/s}$$

Then

$$h = v_A r_p = 5950.58\left[14.378(10^6)\right] = 85.5573(10^9) \text{ m}^2\text{/s}$$

The period of this elliptical orbit can be determined using Eq. 13–30.

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{85.5573(10^9)}\left[14.378(10^6) + 25.378(10^6)\right]\sqrt{\left[14.378(10^6)\right]\left[25.378(10^6)\right]}$$

$$= 27.885(10^3) \text{ s}$$

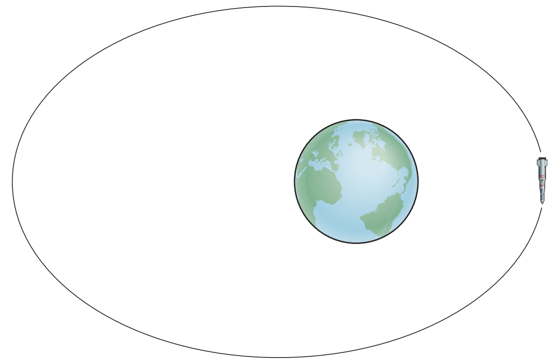
Thus, the time required to travel from A to A' is

$$t = \frac{T}{2} = \frac{27.885(10^3)}{2} = 13.94(10^3) \text{ s} = 3.87 \text{ h}$$ **Ans.**

Ans:
$t = 3.87$ h

13–125.

A rocket is in a free-flight elliptical orbit about the earth such that the eccentricity of its orbit is e and its perigee is r_0. Determine the minimum increment of speed it should have in order to escape the earth's gravitational field when it is at this point along its orbit.



SOLUTION

To escape the earth's gravitational field, the rocket has to make a parabolic trajectory.

ParabolicTrajectory:

$$v_e = \sqrt{\frac{2GM_e}{r_0}}$$

Elliptical Orbit:

$$e = \frac{Ch^2}{GM_e} \qquad \text{where } C = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right) \text{ and } h = r_0 v_0$$

$$e = \frac{1}{GM_e r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)(r_0 v_0)^2$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1 \qquad v_0 = \sqrt{\frac{GM_e(e+1)}{r_0}}$$

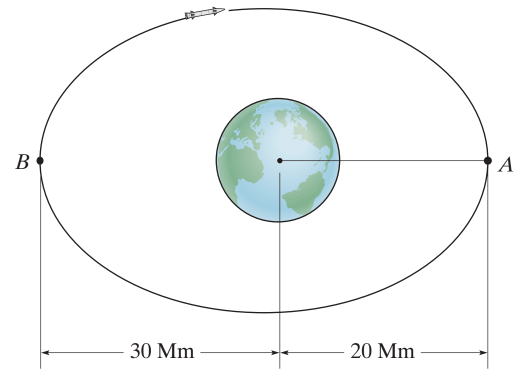
$$\Delta v = \sqrt{\frac{2GM_e}{r_0}} - \sqrt{\frac{GM_e(e+1)}{r_0}} = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right) \qquad \textbf{Ans.}$$

Ans:

$$\Delta v = \sqrt{\frac{GM_e}{r_0}}\left(\sqrt{2} - \sqrt{1+e}\right)$$

13–126.

The rocket is traveling around the earth in free flight along the elliptical orbit. If the rocket has the orbit shown, determine the speed of the rocket when it is at A and at B.



SOLUTION

Here $r_p = 20(10^6)$ m and $r_a = 30(10^6)$ m. Applying Eq. 13–26,

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_P = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here $v_p = v_A$. Then

$$v_A = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][30(10^6)]}{20(10^6)[20(10^6) + 30(10^6)]}}$$

$$= 4891.49 \text{ m/s} = 4.89(10^3) \text{ m/s}$$ **Ans.**

For the same orbit h is constant. Thus,

$$h = r_p v_p = r_a v_a$$

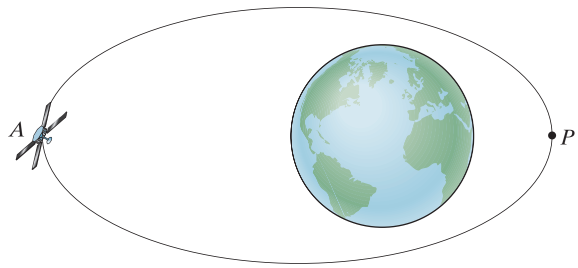
$$[20(10^6)](4891.49) = [30(10^6)]v_B$$

$$v_B = 3261.00 \text{ m/s} = 3.26(10^3) \text{ m/s}$$ **Ans.**

Ans:
$v_A = 4.89(10^3)$ m/s
$v_B = 3.26(10^3)$ m/s

13–127.

An elliptical path of a satellite has an eccentricity $e = 0.130$. If it has a speed of 15 Mm/h when it is at perigee, P, determine its speed when it arrives at apogee, A. Also, how far is it from the earth's surface when it is at A?



SOLUTION

$$e = 0.130$$

$$v_p = v_0 = 15 \text{ Mm/h} = 4.167 \text{ km/s}$$

$$e = \frac{Ch^2}{GM_e} = \frac{1}{r_0}\left(1 - \frac{GM_e}{r_0 v_0^2}\right)\left(\frac{r_0^2 v_0^2}{GM_e}\right)$$

$$e = \left(\frac{r_0 v_0^2}{GM_e} - 1\right)$$

$$\frac{r_0 v_0^2}{GM_e} = e + 1$$

$$r_0 = \frac{(e + 1)GM_e}{v_0^2}$$

$$= \frac{1.130(66.73)(10^{-12})(5.976)(10^{24})}{\left[4.167(10^3)\right]^2}$$

$$= 25.96 \text{ Mm}$$

$$\frac{GM_e}{r_0 v_0^2} = \frac{1}{e + 1}$$

$$r_A = \frac{r_0}{\frac{2GM_e}{r_0 v_0^2} - 1} = \frac{r_0}{\left(\frac{2}{e + 1}\right) - 1}$$

$$r_A = \frac{r_0(e + 1)}{1 - e}$$

$$= \frac{25.96(10^6)(1.130)}{0.870}$$

$$= 33.71(10^6) \text{ m} = 33.7 \text{ Mm}$$

$$v_A = \frac{v_0 r_0}{r_A}$$

$$= \frac{15(25.96)(10^6)}{33.71(10^6)}$$

$$= 11.5 \text{ Mm/h}$$ **Ans.**

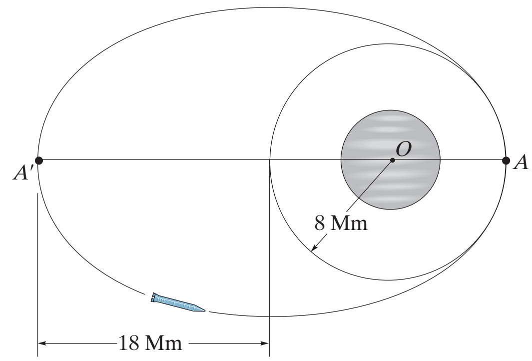
$$d = 33.71(10^6) - 6.378(10^6)$$

$$= 27.3 \text{ Mm}$$ **Ans.**

Ans:
$v_A = 11.5$ Mm/h
$d = 27.3$ Mm

***13–128.**

A rocket is in free-flight elliptical orbit around the planet Venus. Knowing that the periapsis and apoapsis of the orbit are 8 Mm and 26 Mm, respectively, determine (a) the speed of the rocket at point A', (b) the required speed it must attain at A just after braking so that it undergoes an 8-Mm free-flight circular orbit around Venus, and (c) the periods of both the circular and elliptical orbits. The mass of Venus is 0.816 times the mass of the earth.



SOLUTION

a) $$M_\nu = 0.816(5.976(10^{24})) = 4.876(10^{24})$$

$$OA' = \frac{OA}{\left(\frac{2GM\nu}{OA\,\nu_A^2} - 1\right)}$$

$$26(10)^6 = \frac{8(10^6)}{\left(\frac{2(66.73)(10^{-12})4.876(10^{24})}{8(10^6)\nu_A^2} - 1\right)}$$

$$\frac{81.35(10^6)}{\nu_A^2} = 1.307$$

$$\nu_A = 7887.3\text{ m/s} = 7.89\text{ km/s}$$

$$\nu_A = \frac{OA\,\nu_A}{OA'} = \frac{8(10^6)(7887.3)}{26(10^6)} = 2426.9\text{ m/s} = 2.43\text{ m/s}$$ **Ans.**

b) $$\nu_{A''} = \sqrt{\frac{GM_\nu}{OA'}} = \sqrt{\frac{66.73(10^{-12})4.876(10^{24})}{8(10^6)}}$$

$$\nu_{A''} = 6377.7\text{ m/s} = 6.38\text{ km/s}$$ **Ans.**

c) Circular orbit:

$$T_c = \frac{2\pi OA}{\nu_{A''}} = \frac{2\pi 8(10^6)}{6377.7} = 7881.41\text{ s} = 2.19\text{ h}$$ **Ans.**

Elliptic orbit:

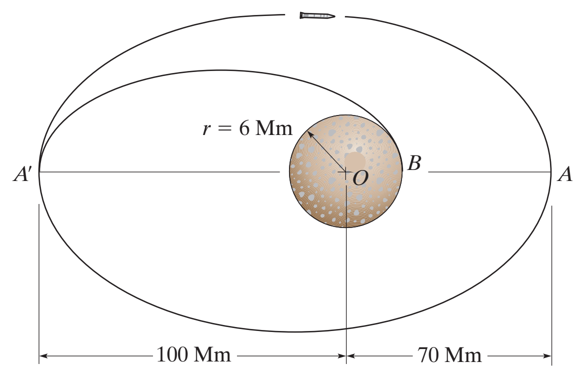
$$T_e = \frac{\pi}{OA\nu_A}(OA + OA')\sqrt{(OA)(OA')} = \frac{\pi}{8(10^6)(7886.8)}(8 + 26)\left(10^6\right)\left(\sqrt{(8)(26)}\right)\left(10^6\right)$$

$$T_e = 24414.2\text{ s} = 6.78\text{ h}$$ **Ans.**

Ans:
$\nu_A = 2.43$ m/s
$\nu_{A''} = 6.38$ km/s
$T_c = 2.19$ h
$T_e = 6.78$ h

13–129.

The rocket is traveling in a free flight along an elliptical trajectory $A'A$. The planet has no atmosphere, and its mass is 0.60 times that of the earth. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.



SOLUTION

Applying Eq. 13–26,

$$r_a = \frac{r_p}{(2GM/r_p v_p^2) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GMr_a}{r_p(r_p + r_a)}}$$

The rocket is traveling around the elliptical orbit with $r_p = 70(10^6)$ m, $r_a = 100(10^6)$ m and $v_p = v_A$. Then

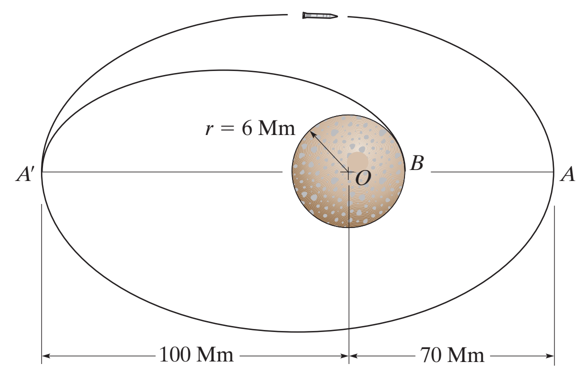
$$v_A = \sqrt{\frac{2[66.73(10^{-12})][0.6(5.976)(10^{24})][100(10^6)]}{70(10^6)[70(10^6) + 100(10^6)]}}$$

$$= 2005.32 \text{ m/s} = 2.01(10^3) \text{ m/s}$$ **Ans.**

Ans:
$v_A = 2.01(10^3)$ m/s

13–130.

If the rocket is to land on the surface of the planet, determine the required free-flight speed it must have at A' so that the landing occurs at B. How long does it take for the rocket to land, going from A' to B? The planet has no atmosphere, and its mass is 0.6 times that of the earth.



SOLUTION

Applying Eq. 13–26,

$$r_a = \frac{r_p}{\left(2GM/r_p v_p^2\right) - 1}$$

$$\frac{2GM}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GMr_a}{r_p(r_p + r_a)}}$$

To land on B, the rocket has to follow the elliptical orbit $A'B$ with $r_p = 6(10^6)$, $r_a = 100(10^6)$ m and $v_p = v_B$.

$$v_B = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[0.6(5.976)(10^{24})\right]\left[100(10^6)\right]}{6(10^6)\left[6(10^6) + 100(10^6)\right]}} = 8674.17 \text{ m/s}$$

In this case

$$h = r_p v_B = r_a v_{A'}$$

$$6(10^6)(8674.17) = 100(10^6)v_{A'}$$

$$v_{A'} = 520.45 \text{ m/s} = 521 \text{ m/s}$$ **Ans.**

The period of the elliptical orbit can be determined using Eq. 13–30.

$$T = \frac{\pi}{h}(r_p + r_a)\sqrt{r_p r_a}$$

$$= \frac{\pi}{6(10^6)(8674.17)}\left[6(10^6) + 100(10^6)\right]\sqrt{\left[6(10^6)\right]\left[100(10^6)\right]}$$

$$= 156.73(10^3) \text{ s}$$

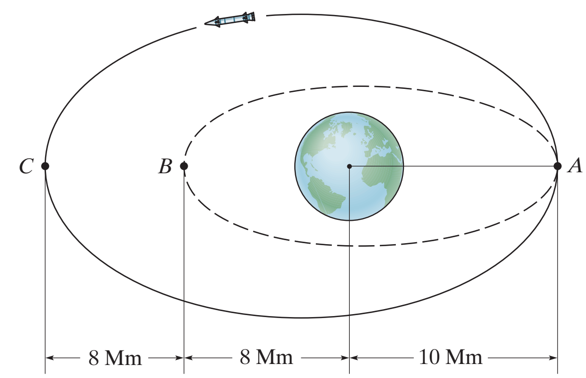
Thus, the time required to travel from A' to B is

$$t = \frac{T}{2} = 78.365(10^3) \text{ s} = 21.8 \text{ h}$$ **Ans.**

Ans:
$v_{A'} = 521$ m/s
$t = 21.8$ h

13–131.

The rocket is traveling around the earth in free flight along an elliptical orbit AC. If the rocket has the orbit shown, determine the rocket's velocity when it is at point A.



SOLUTION

For orbit AC, $r_p = 10(10^6)$ m and $r_a = 16(10^6)$ m. Applying Eq. 13–26

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_p v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

Here $v_p = v_A$. Then

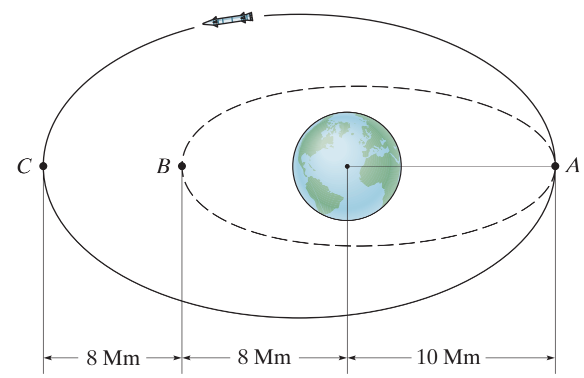
$$v_A = \sqrt{\frac{2[66.73(10^{-12})][5.976(10^{24})][16(10^6)]}{10(10^6)[10(10^6) + 16(10^6)]}}$$

$$= 7005.74 \text{ m/s} = 7.01(10^3) \text{ m/s}$$ **Ans.**

Ans:
$v_A = 7.01(10^3)$ m/s

***13–132.**

The rocket is traveling around the earth in free flight along the elliptical orbit AC. Determine its change in speed when it reaches A so that it travels along the elliptical orbit AB.



SOLUTION

Applying Eq. 13–26,

$$r_a = \frac{r_p}{(2GM_e/r_p v_p^2) - 1}$$

$$\frac{2GM_e}{r_p v_p^2} - 1 = \frac{r_p}{r_a}$$

$$\frac{2GM_e}{r_a v_p^2} = \frac{r_p + r_a}{r_a}$$

$$v_p = \sqrt{\frac{2GM_e r_a}{r_p(r_p + r_a)}}$$

For orbit AC, $r_p = 10(10^6)$ m, $r_a = 16(10^6)$ m and $v_p = (v_A)_{AC}$. Then

$$(v_A)_{AC} = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]\left[16(10^6)\right]}{10(10^6)\left[10(10^6) + 16(10^6)\right]}} = 7005.74 \text{ m/s}$$

For orbit AB, $r_p = 8(10^6)$ m, $r_a = 10(10^6)$ m and $v_p = v_B$. Then

$$v_B = \sqrt{\frac{2\left[66.73(10^{-12})\right]\left[5.976(10^{24})\right]\left[10(10^6)\right]}{8(10^6)\left[8(10^6) + 10(10^6)\right]}} = 7442.17 \text{ m/s}$$

Since h is constant at any position of the orbit,

$$h = r_p v_p = r_a v_a$$

$$8(10^6)(7442.17) = 10(10^6)(v_A)_{AB}$$

$$(v_A)_{AB} = 5953.74 \text{ m/s}$$

Thus, the required change in speed is

$$\Delta v = (v_A)_{AB} - (v_A)_{AC} = 5953.74 - 7005.74$$

$$= -1052.01 \text{ m/s} = -1.05 \text{ km/s} \qquad \textbf{Ans.}$$

The negative sign indicates that the speed must be decreased.

Ans:
$\Delta v = -1.05$ km/s