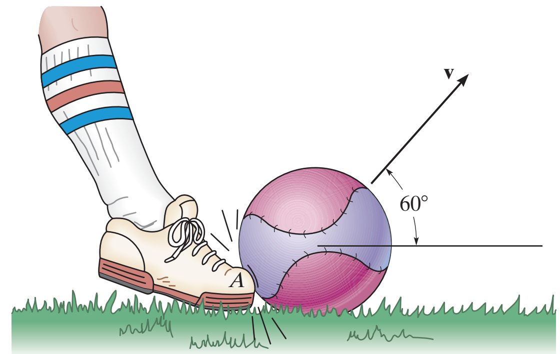
15–1.

A man kicks the 150-g ball such that it leaves the ground at an angle of 60° and strikes the ground at the same elevation a distance of 12 m away. Determine the impulse of his foot on the ball at A. Neglect the impulse caused by the ball's weight while it's being kicked.



SOLUTION

Kinematics. Consider the vertical motion of the ball where

$(s_0)_y = s_y = 0, (v_0)_y = v \sin 60° \uparrow$ and $a_y = 9.81 \text{ m/s}^2 \downarrow$,

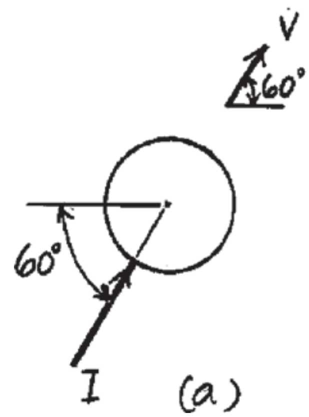
$$(+\uparrow) \quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}a_y t^2; \quad 0 = 0 + v \sin 60° t + \frac{1}{2}(-9.81)t^2$$

$$t(v \sin 60° - 4.905t) = 0$$

Since $t \neq 0$, then

$$v \sin 60° - 4.905t = 0$$

$$t = 0.1766\, v \qquad \textbf{(1)}$$



Then, consider the horizontal motion where $(v_0)_x = v \cos 60°$, and $(s_0)_x = 0$,

$$(\overset{+}{\rightarrow}) \quad s_x = (s_0)_x + (v_0)_x t; \quad 12 = 0 + v \cos 60° t$$

$$t = \frac{24}{v} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2)

$$0.1766\, v = \frac{24}{v}$$

$$v = 11.66 \text{ m/s}$$

Principle of Impulse and Momentum.

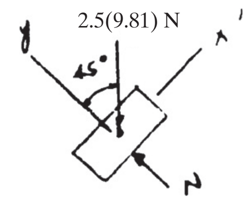
$$(+\nearrow) \quad mv_1 + \Sigma\int_{t_1}^{t_2} F dt = mv_2$$

$$0 + I = 0.15\,(11.66)$$

$$I = 1.749 \text{ N}\cdot\text{s} = 1.75 \text{ N}\cdot\text{s} \qquad \textbf{Ans.}$$

Ans:
$I = 1.75 \text{ N}\cdot\text{s}$

15–2. A 2.5-kg block is given an initial velocity of 3 m/s up a 45° smooth slope. Determine the time for it to travel up the slope before it stops.



SOLUTION

$$(\nearrow +) \qquad m(v_{x'})_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_{x'})_2$$

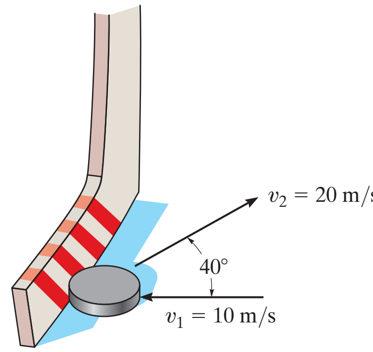
$$2.5(3) + (-2.5(9.81)\sin 45^\circ)t = 0$$

$t = 0.432$ s **Ans.**

Ans:
$t = 0.432$ s

15–3.

A hockey puck is traveling to the left with a velocity of $v_1 = 10$ m/s when it is struck by a hockey stick and given a velocity of $v_2 = 20$ m/s as shown. Determine the magnitude of the net impulse exerted by the hockey stick on the puck. The puck has a mass of 0.2 kg.



SOLUTION

$v_1 = \{-10\mathbf{i}\}$ m/s

$v_2 = \{20 \cos 40°\mathbf{i} + 20 \sin 40°\mathbf{j}\}$ m/s

$\mathbf{I} = m\Delta v = (0.2)\,\{[20 \cos 40° - (-10)]\mathbf{i} + 20 \sin 40°\mathbf{j}\}$

$= \{5.0642\mathbf{i} + 2.5712\mathbf{j}\}\ \text{kg} \cdot \text{m/s}$

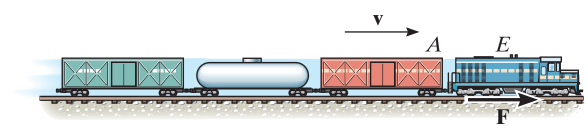
$\mathbf{I} = \sqrt{(5.0642)^2 + (2.5712)^2}$

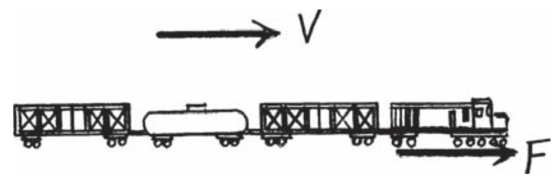
$= 5.6795 = 5.68\ \text{kg} \cdot \text{m/s}$ **Ans.**

Ans:
$I = 5.68\ \text{N} \cdot \text{s}$

***15–4.**

A train consists of a 50-Mg engine and three cars, each having a mass of 30 Mg. If it takes 80 s for the train to increase its speed uniformly to 40 km/h, starting from rest, determine the force T developed at the coupling between the engine E and the first car A. The wheels of the engine provide a resultant frictional tractive force $\mathbf{F}$ which gives the train forward motion, whereas the car wheels roll freely. Also, determine F acting on the engine wheels.





SOLUTION

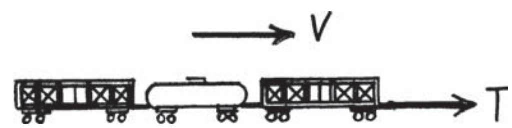
$(v_x)_2 = 40 \text{ km/h} = 11.11 \text{ m/s}$

Entire train:

$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + F(80) = [50 + 3(30)]\left(10^3\right)(11.11)$$

$$F = 19.4 \text{ kN} \qquad \textbf{Ans.}$$



Three cars:

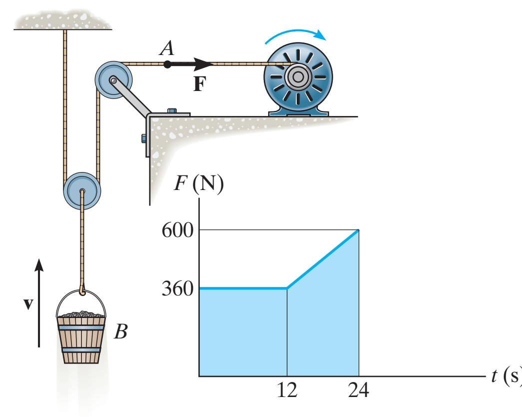
$$\left(\xrightarrow{+}\right) \qquad m(v_x)_1 + \Sigma \int F_x\, dt = m(v_x)_2$$

$$0 + T(80) = 3(30)\left(10^3\right)(11.11) \qquad T = 12.5 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F = 19.4$ kN
$T = 12.5$ kN

15–5.

The winch delivers a horizontal towing force **F** to its cable at A which varies as shown in the graph. Determine the speed of the 70-kg bucket when $t = 18$ s. Originally the bucket is moving upward at $v_1 = 3$ m/s.



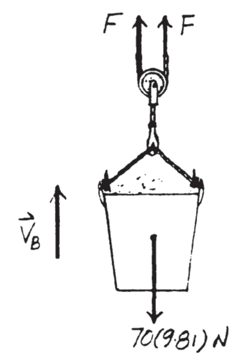
SOLUTION

Principle of Linear Impulse and Momentum: For the time period $12\text{ s} \le t < 18\text{ s}$, $\dfrac{F - 360}{t - 12} = \dfrac{600 - 360}{24 - 12}$, $F = (20t + 120)$ N. Applying Eq. 15–4 to bucket B, we have

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$(+\uparrow) \qquad 70(3) + 2\left[360(12) + \int_{12\text{ s}}^{18\text{ s}} (20t + 120)dt\right] - 70(9.81)(18) = 70v_2$$

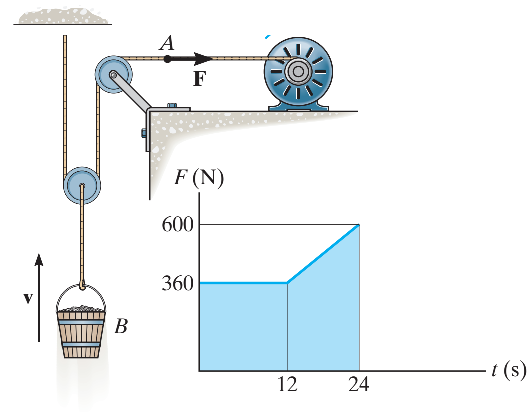
$$v_2 = 21.8 \text{ m/s}$$ **Ans.**



Ans:
$v_2 = 21.8$ m/s

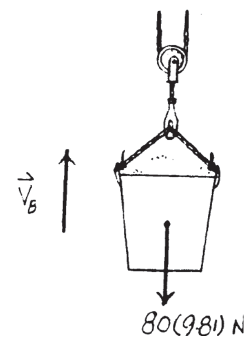
15–6.

The winch delivers a horizontal towing force **F** to its cable at A which varies as shown in the graph. Determine the speed of the 80-kg bucket when $t = 24$ s. Originally the bucket is released from rest.



SOLUTION

Principle of Linear Impulse and Momentum: The total impluse exerted on bucket B can be obtained by evaluating the area under the F–t graph. Thus, $I = \Sigma \int_{t_1}^{t_2} F_y\,dt = 2\left[360(12) + \frac{1}{2}(360 + 600)(24 - 12)\right] = 20160\ \text{N} \cdot \text{s}$. Applying Eq. 15–4 to the bucket B, we have



$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

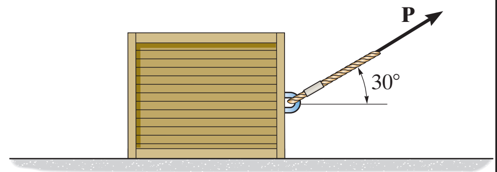
$$(+\uparrow) \qquad 80(0) + 20160 - 80(9.81)(24) = 80v_2$$

$$v_2 = 16.6\ \text{m/s}$$ **Ans.**

Ans:
$v_2 = 16.6$ m/s

15–7.

The 50-kg crate is pulled by the constant force **P**. If the crate starts from rest and achieves a speed of 10 m/s in 5 s, determine the magnitude of **P**. The coefficient of kinetic friction between the crate and the ground is $\mu_k = 0.2$.



SOLUTION

Impulse and Momentum Diagram: The frictional force acting on the crate is $F_f = \mu_k N = 0.2N$.

Principle of Impulse and Momentum:

$(+\uparrow)$ $$m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$0 + N(5) + P(5)\sin 30^\circ - 50(9.81)(5) = 0$$

$$N = 490.5 - 0.5P \qquad \textbf{(1)}$$

$(\overset{+}{\rightarrow})$ $$m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

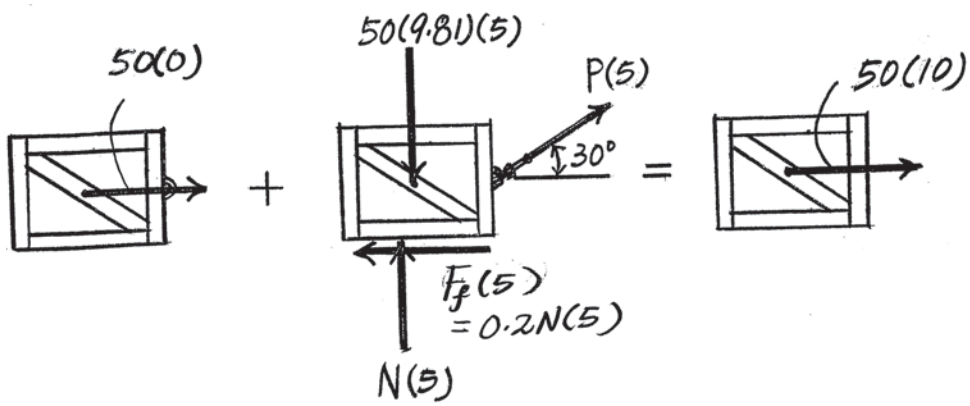
$$50(0) + P(5)\cos 30^\circ - 0.2N(5) = 50(10)$$

$$4.3301P - N = 500 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$$N = 387.97\text{ N}$$

$$P = 205\text{ N} \qquad \textbf{Ans.}$$

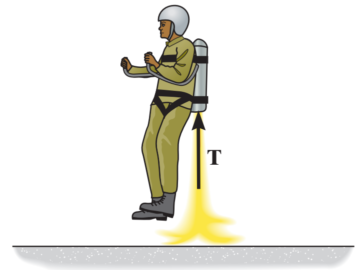


(a)

Ans:
$P = 205$ N

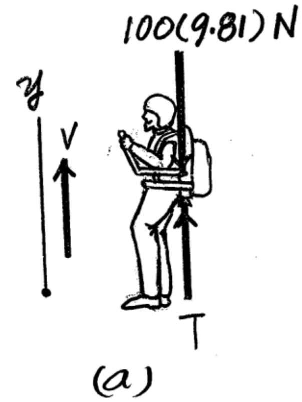
***15–8.**

If the jets exert a vertical thrust of $T = (500t^{3/2})$N, where t is in seconds, determine the man's speed when $t = 3$ s. The total mass of the man and the jet suit is 100 kg. Neglect the loss of mass due to the fuel consumed during the lift which begins from rest on the ground.



SOLUTION

Free-Body Diagram: The thrust **T** must overcome the weight of the man and jet before they move. Considering the equilibrium of the free-body diagram of the man and jet shown in Fig. *a*,



$+\uparrow \Sigma F_y = 0; \qquad 500t^{3/2} - 100(9.81) = 0 \qquad t = 1.567 \text{ s}$

Principle of Impulse and Momentum: Only the impulse generated by thrust **T** after $t = 1.567$ s contributes to the motion. Referring to Fig. *a*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$100(0) + \int_{1.567 \text{ s}}^{3 \text{ s}} 500t^{3/2} dt - 100(9.81)(3 - 1.567) = 100v$$

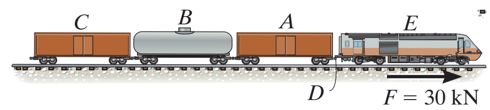
$$\left(200t^{5/2}\right)\Big|_{1.567 \text{ s}}^{3 \text{ s}} - 1405.55 = 100v$$

$v = 11.0$ m/s **Ans.**

Ans:
$v = 11.0$ m/s

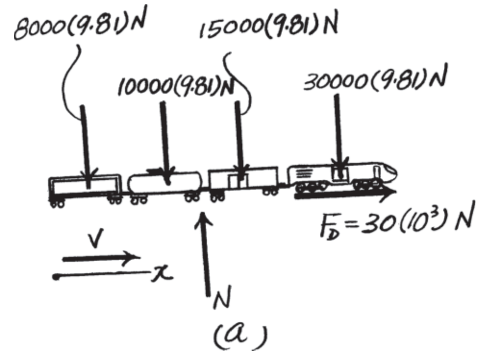
15–9.

The train consists of a 30-Mg engine E, and cars A, B, and C, which have a mass of 15 Mg, 10 Mg, and 8 Mg, respectively. If the tracks provide a traction force of $F = 30$ kN on the engine wheels, determine the speed of the train when $t = 30$ s, starting from rest. Also, find the horizontal coupling force at D between the engine E and car A. Neglect rolling resistance.



SOLUTION

Principle of Impulse and Momentum: By referring to the free-body diagram of the entire train shown in Fig. *a*, we can write



$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$63\,000(0) + 30(10^3)(30) = 63\,000v$$

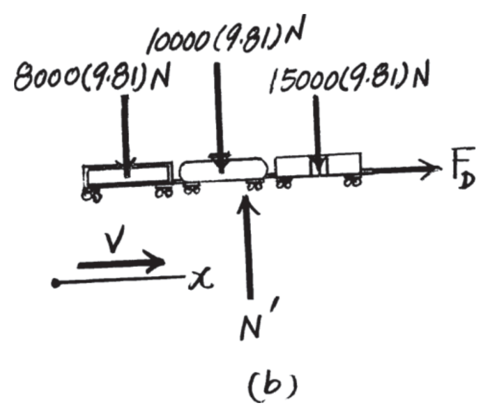
$$v = 14.29 \text{ m/s} \qquad \textbf{Ans.}$$

Using this result and referring to the free-body diagram of the train's car shown in Fig. *b*,

$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\, dt = m(v_2)_x$$

$$33000(0) + F_D(30) = 33\,000(14.29)$$

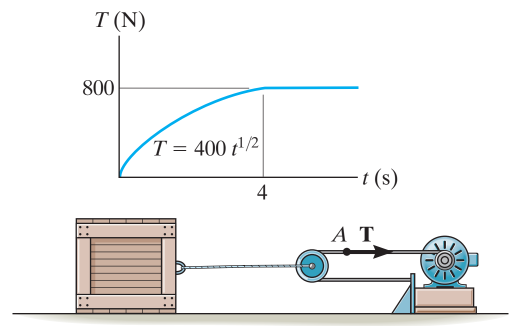
$$F_D = 15\,714.29 \text{ N} = 15.7 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$v = 14.29$ m/s
$F_D = 15.7$ kN

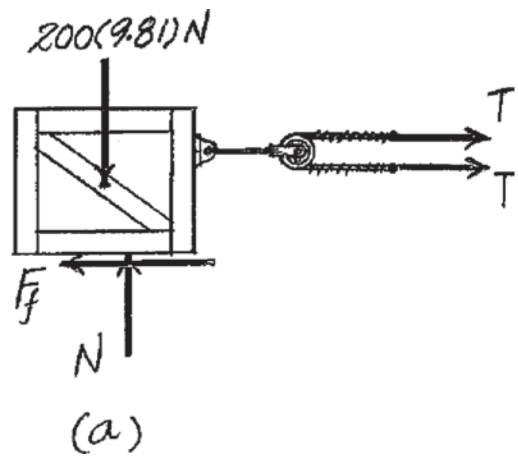
15–10.

The 200-kg crate rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force $\boldsymbol{T}$ to its cable at A which varies as shown in the graph. Determine the speed of the crate when $t = 4$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the crate.



SOLUTION

Equilibrium. The time required to move the crate can be determined by considering the equilibrium of the crate. Since the crate is required to be on the verge of sliding, $F_f = \mu_s N = 0.5$ N. Referring to the FBD of the crate, Fig. *a*,



$$+\uparrow \Sigma F_y = 0; \quad N - 200(9.81) = 0 \quad N = 1962 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 2\left(400t^{\frac{1}{2}}\right) - 0.5(1962) = 0 \quad t = 1.5037 \text{ s}$$

Principle of Impulse and Momentum. Since the crate is sliding, $F_f = \mu_k N = 0.4(1962) = 784.8$ N. Referring to the FBD of the crate, Fig. *a*

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

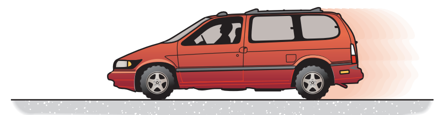
$$0 + 2\int_{1.5037 \text{ s}}^{4 \text{ s}} 400t^{\frac{1}{2}}\, dt - 784.8(4 - 1.5037) = 200v$$

$$v = 6.621 \text{ m/s} = 6.62 \text{ m/s}$$ **Ans.**

Ans:
$v = 6.62$ m/s

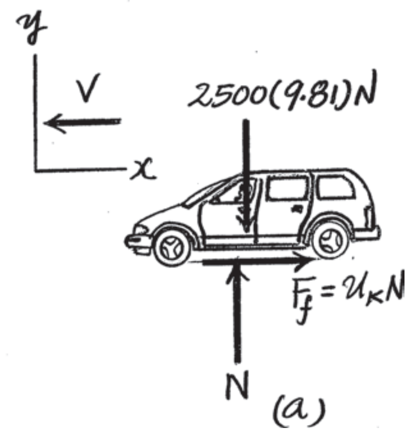
15–11.

The 2.5-Mg van is traveling with a speed of 100 km/h when the brakes are applied and all four wheels lock. If the speed decreases to 40 km/h in 5 s, determine the coefficient of kinetic friction between the tires and the road.



SOLUTION

Free-Body Diagram: The free-body diagram of the van is shown in Fig. *a*. The frictional force is $F_f = \mu_k N$ since all the wheels of the van are locked and will cause the van to slide.



Principle of Impulse and Momentum: The initial and final speeds of the van are $v_1 = \left[100(10^3)\,\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 27.78\text{ m/s}$ and $v_2 = \left[40(10^3)\,\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 11.11\text{ m/s}$. Referring to Fig. *a*,

$$(+\uparrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_2)_y$$

$$2500(0) + N(5) - 2500(9.81)(5) = 2500(0)$$

$$N = 24\,525\text{ N}$$

$$(\overset{+}{\leftarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_2)_x$$

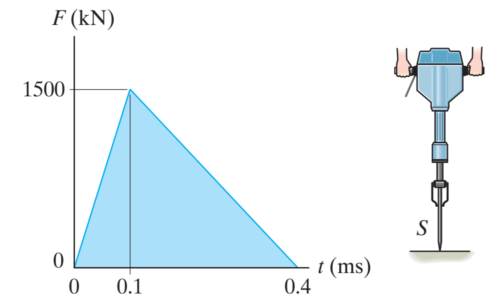
$$2500(27.78) + [-\mu_k(24525)(5)] = 2500(11.1)$$

$$\mu_k = 0.340 \qquad \textbf{Ans.}$$

Ans:
$\mu_k = 0.340$

***15–12.**

During operation the jack hammer strikes the concrete surface with a force which is indicated in the graph. To achieve this the 2-kg spike S is fired into the surface at 90 m/s. Determine the speed of the spike just after rebounding.



SOLUTION

Principle of Impulse and Momentum. The impulse of the force $\boldsymbol{F}$ is equal to the area under the F–t graph. Referring to the FBD of the spike, Fig. *a*

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$2(-90) + \frac{1}{2}\left[0.4\left(10^{-3}\right)\right]\left[1500\left(10^{3}\right)\right] = 2v$$

$$v = 60.0 \text{ m/s} \uparrow$$ **Ans.**

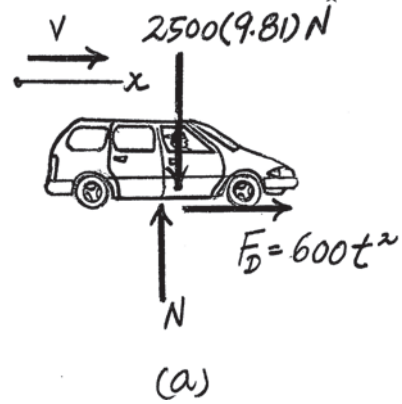
Ans:
$v = 60.0$ m/s

15–13.

For a short period of time, the frictional driving force acting on the wheels of the 2.5-Mg van is $F_D = (600t^2)$ N, where t is in seconds. If the van has a speed of 20 km/h when $t = 0$, determine its speed when $t = 5$ s.

SOLUTION

Principle of Impulse and Momentum: The initial speed of the van is $v_1 = \left[20(10^3)\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right] = 5.556$ m/s. Referring to the free-body diagram of the van shown in Fig. a,



$$(\overset{+}{\rightarrow}) \qquad m(v_1)_x + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

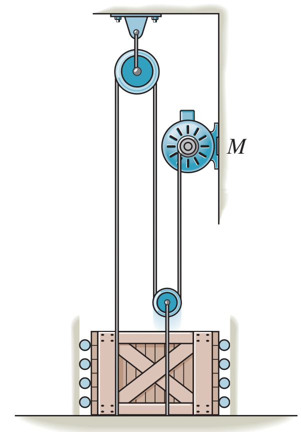
$$2500(5.556) + \int_0^{5\text{ s}} 600t^2 dt = 2500\, v_2$$

$$v_2 = 15.6 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_2 = 15.6$ m/s

15–14.

The motor, M, pulls on the cable with a force $F = (10t^2 + 300)$ N, where t is in seconds. If the 100 kg crate is originally at rest at $t = 0$, determine its speed when $t = 4$ s. Neglect the mass of the cable and pulleys. *Hint:* First find the time needed to begin lifting the crate.



SOLUTION

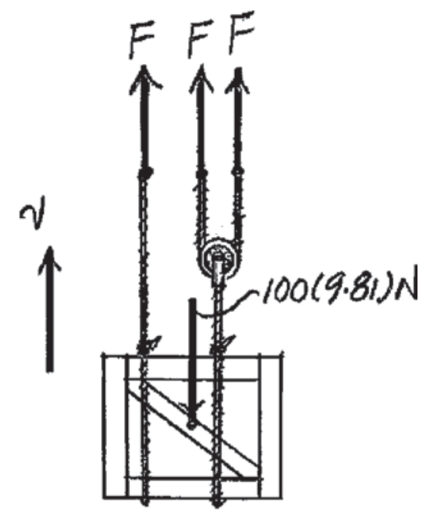
Principle of Impulse and Momentum. The crate will only move when $3(10t^2 + 300) = 100(9.81)$. Thus, this instant is $t = 1.6432$ s. Referring to the FBD of the crate, Fig. *a*,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$0 + \int_{1.6432\text{ s}}^{4\text{ s}} 3(10t^2 + 300)\, dt - 100(9.81)(4 - 1.6432) = 100v$$

$$3\left(\frac{10t^3}{3} + 300t\right)\Bigg|_{1.6432\text{ s}}^{4\text{ s}} - 2312.05 = 100v$$

$$v = 4.047 \text{ m/s} = 4.05 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

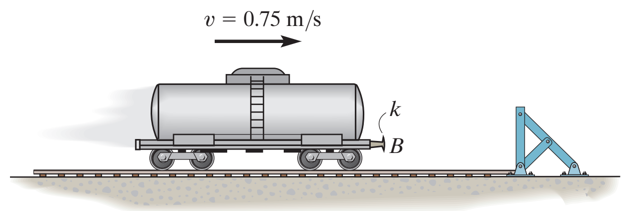


(a)

Ans:
$v = 4.05$ m/s

15–15.

A tankcar has a mass of 20 Mg and is freely rolling to the right with a speed of 0.75 m/s. If it strikes the barrier, determine the horizontal impulse needed to stop the car if the spring in the bumper B has a stiffness (a) $k \rightarrow \infty$ (bumper is rigid), and (b) $k = 15$ kN/m.



SOLUTION

a) b) $\quad (\overset{+}{\rightarrow}) \quad mv_1 + \Sigma \int F\,dt = mv_2$

$$20(10^3)(0.75) - \int F\,dt = 0$$

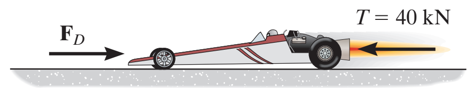
$$\int F\,dt = 15 \text{ kN} \cdot \text{s} \qquad \textbf{Ans.}$$

The impulse is the same for both cases. For the spring having a stiffness $k = 15$ kN/m, the impulse is applied over a longer period of time than for $k \rightarrow \infty$.

Ans:
$I = 15$ kN · s in both cases.

***15–16.**

Under a constant thrust of $T = 40$ kN, the 1.5-Mg dragster reaches its maximum speed of 125 m/s in 8 s starting from rest. Determine the average drag resistance $\mathbf{F}_D$ during this period of time.



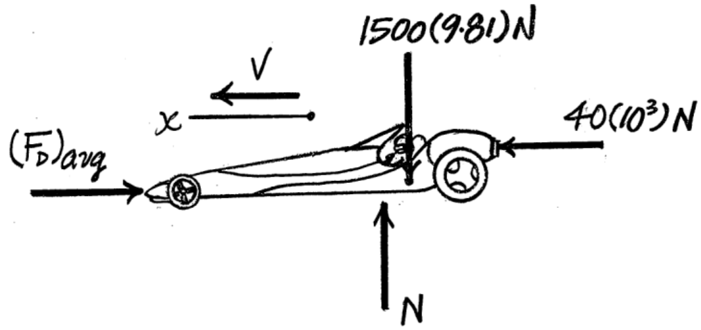
SOLUTION

Principle of Impulse and Momentum: The final speed of the dragster is $v_2 = 125$ m/s. Referring to the free-body diagram of the dragster shown in Fig. *a*,

$$(\overset{+}{\leftarrow}) \qquad m(v_1)_x + \Sigma \int_{t_1}^{t_2} F_x dt = m(v_2)_x$$

$$1500(0) + 40(10^3)(8) - (F_D)_{\text{avg}}(8) = 1500(125)$$

$$(F_D)_{\text{avg}} = 16\,562.5 \text{ N} = 16.6 \text{ kN} \qquad \textbf{Ans.}$$



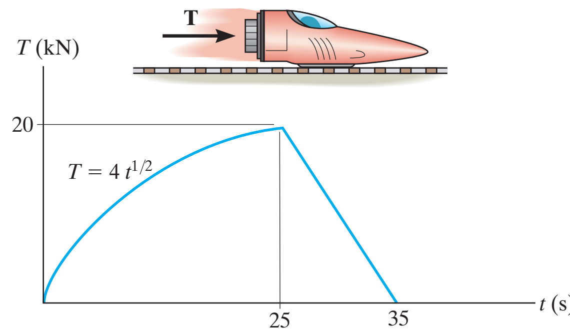
(a)

Ans:

$(F_D)_{\text{avg}} = 16.6$ kN

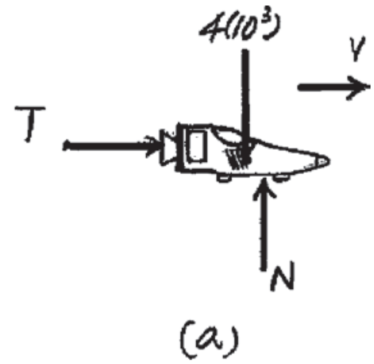
15–17.

The thrust on the 4-Mg rocket sled is shown in the graph. Determine the sleds maximum velocity and the distance the sled travels when $t = 35$ s. Neglect friction.



SOLUTION

Principle of Impulse And Momentum. The FBD of the rocket sled is shown in Fig. *a*. For $0 \le t < 25$ s,



$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + \int_0^t 4(10^3)t^{\frac{1}{2}}dt = 4(10^3)\,v$$

$$4(10^3)\left(\frac{2}{3}t^{\frac{3}{2}}\right)\Bigg|_0^t = 4(10^3)\,v$$

$$v = \left\{\frac{2}{3}t^{\frac{3}{2}}\right\}\text{ m/s}$$

At $t = 25$ s,

$$v = \frac{2}{3}(25)^{\frac{3}{2}} = 83.33\text{ m/s}$$

For $25\text{ s} < t < 35$ s, $\dfrac{T - 0}{t - 35} = \dfrac{20(10^3) - 0}{25 - 35}$ or $T = 2(10^3)(35 - t)$.

Here, $(v_x)_1 = 83.33$ m/s and $t_1 = 25$ s.

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$4(10^3)(83.33) + \int_{25\text{ s}}^{t} 2(10^3)(35 - t)dt = 4(10^3)\,v$$

$$v = \{-0.25t^2 + 17.5t - 197.9167\}\text{ m/s}$$

The maximum velocity occurs at $t = 35$ s, Thus,

$$v_{max} = -0.25(35^2) + 17.5(35) - 197.9167$$

$$= 108.33\text{ m/s} = 108\text{ m/s} \qquad \textbf{Ans.}$$

15–17. Continued

Kinematics. The displacement of the sled can be determined by integrating $ds = vdt$. For $0 \le t < 25$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t \frac{2}{3} t^{\frac{3}{2}}\, dt$$

$$s\Big|_0^s = \frac{2}{3}\left(\frac{2}{5}\right) t^{\frac{5}{2}}\Big|_0^t$$

$$s = \left\{\frac{4}{15} t^{\frac{5}{2}}\right\} \text{ m}$$

At $t = 25$ s,

$$s = \frac{4}{15}(25)^{\frac{5}{2}} = 833.33 \text{ m}$$

For $25 < t \le 35$ s, the initial condition is $s = 833.33$ at $t = 25$ s.

$$\int_{833.33 \text{ m}}^{S} ds = \int_{25 \text{ s}}^{t} (-0.25t^2 + 17.5t - 197.9167)\, dt$$

$$s\Big|_{833.33 \text{ m}}^{S} = \left(-0.08333t^3 + 8.75t^2 - 197.9167t\right)\Big|_{25 \text{ s}}^{t}$$

$$s = \{-0.08333t^3 + 8.75t^2 - 197.9167t + 1614.58\} \text{ m}$$

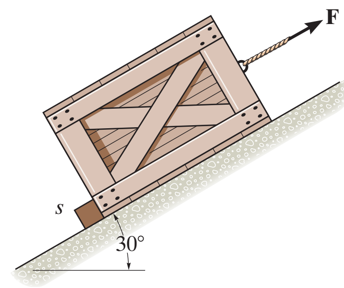
At $t = 35$ s,

$$s = -0.08333(35^3) + 8.75(35^2) - 197.9167(35) + 1614.58$$

$$= 1833.33 \text{ m} = 1833 \text{ m}$$ **Ans.**

Ans:
$v_{max} = 108$ m/s
$s = 1.83$ km

15–18. A 50-kg crate rests against a stop block *s*, which prevents the crate from moving down the plane. If the coefficients of static and kinetic friction between the plane and the crate are $\mu_s = 0.3$ and $\mu_k = 0.2$, respectively, determine the time needed for the force **F** to give the crate a speed of 2 m/s up the plane. The force always acts parallel to the plane and has a magnitude of $F = (300t)$ N, where t is in seconds. *Hint*: First determine the time needed to overcome static friction and start the crate moving.



SOLUTION

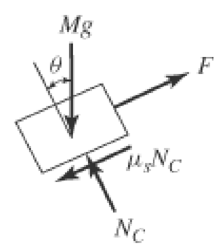
Guesses $t_1 = 1$ s $\quad N_C = 1$ N $\quad t_2 = 1$ s

Given:

$M = 50$ kg $\quad \theta = 30°$ $\quad g = 9.81$ m/s^2

$v = 2$ m/s $\quad \mu_s = 0.3$

$a = 300$ N/s $\quad \mu_k = 0.2$



Given $\quad N_C - Mg\cos(\theta) = 0$

$$a t_1 - \mu_s N_C - Mg\sin(\theta) = 0$$

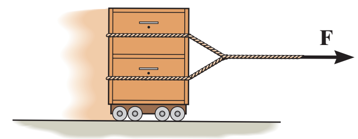
$$\int_{t_1}^{t_2} \left(a t - Mg\sin(\theta) - \mu_k N_C\right) \mathrm{d}t = Mv$$

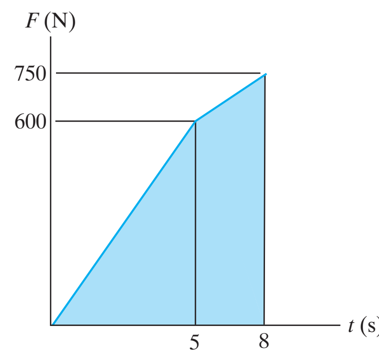
$$\begin{pmatrix} t_1 \\ t_2 \\ N_C \end{pmatrix} = \text{Find}(t_1, t_2, N_C) \qquad t_1 = 1.242 \text{ s} \qquad t_2 = 1.929 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$t_1 = 1.242$ s
$t_2 = 1.929$ s

15–19.

The towing force acting on the 400-kg safe varies as shown on the graph. Determine its speed, starting from rest, when $t = 8$ s. How far has it traveled during this time?





SOLUTION

Principle of Impulse and Momentum. The FBD of the safe is shown in Fig. *a*. For $0 \leq t < 5$ s, $F = \frac{600}{5}t = 120t$.

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$0 + \int_0^t 120t\, dt = 400v$$

$$v = \{0.15t^2\} \text{ m/s}$$

At $t = 5$ s,

$$v = 0.15(5^2) = 3.75 \text{ m/s}$$

For $5 \text{ s} < t \leq 8$ s, $\frac{F - 600}{t - 5} = \frac{750 - 600}{8 - 5}$, $F = 50t + 350$. Here,

$(v_x)_1 = 3.75$ m/s and $t_1 = 5$ s.

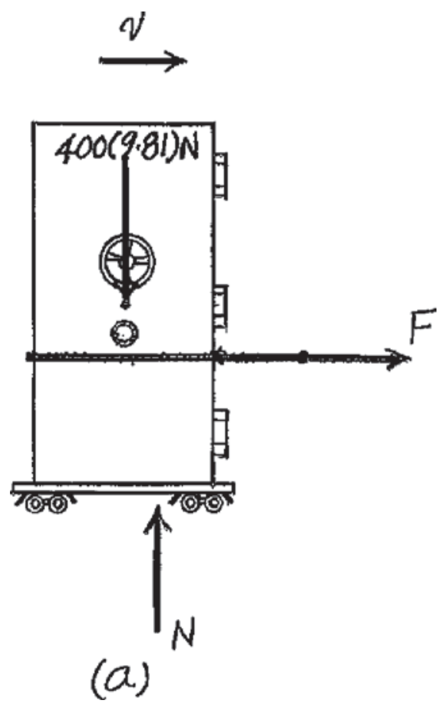
$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$400(3.75) + \int_{5\text{ s}}^t (50t + 350)\, dt = 400v$$

$$v = \{0.0625t^2 + 0.875t - 2.1875\} \text{ m/s}$$

At $t = 8$ s,

$$v = 0.0625(8^2) + 0.875(8) - 2.1875 = 8.8125 \text{ m/s} = 8.81 \text{ m/s} \quad \textbf{Ans.}$$



15–19. Continued

Kinematics. The displacement of the safe can be determined by integrating $ds = v\,dt$. For $0 \le t < 5$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t 0.15t^2\,dt$$

$$s = \{0.05t^3\}\text{ m}$$

At $t = 5$ s,

$$s = 0.05(5^3) = 6.25\text{ m}$$

For $5\text{ s} < t \le 8$ s, the initial condition is $s = 6.25$ m at $t = 5$ s.

$$\int_{6.25\text{ m}}^s ds = \int_{5\text{ s}}^t \left(0.0625t^2 + 0.875\,t - 2.1875\right) dt$$

$$s - 6.25 = \left(0.02083t^3 + 0.4375t^2 - 2.1875t\right)\Big|_{5\text{ s}}^t$$

$$s = \{0.02083t^3 + 0.4375t^2 - 2.1875t + 3.6458\}\text{ m}$$

At $t = 8$ s,

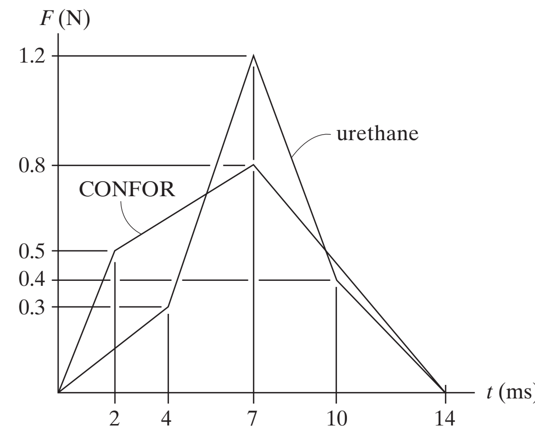
$$s = 0.02083(8^3) + 0.4375(8^2) - 2.1875(8) + 3.6458$$

$$= 24.8125\text{ m} = 24.8\text{ m}$$ **Ans.**

Ans:
$v = 8.81$ m/s
$s = 24.8$ m

***15–20.**

The choice of a seating material for moving vehicles depends upon its ability to resist shock and vibration. From the data shown in the graphs, determine the impulses created by a falling weight onto a sample of urethane foam and CONFOR foam.



SOLUTION

CONFOR foam:

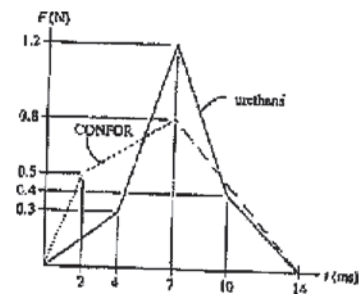
$$I_c = \int F\,dt = \left[\frac{1}{2}(2)(0.5) + \frac{1}{2}(0.5 + 0.8)(7 - 2) + \frac{1}{2}(0.8)(14 - 7)\right](10^{-3})$$

$= 6.55\ \text{N}\cdot\text{ms}$ **Ans.**

Urethane foam:

$$I_v = \int F\,dt = \left[\frac{1}{2}(4)(0.3) + \frac{1}{2}(1.2 + 0.3)(7 - 4) + \frac{1}{2}(1.2 + 0.4)(10 - 7) + \frac{1}{2}(14 - 10)(0.4)\right](10^{-3})$$

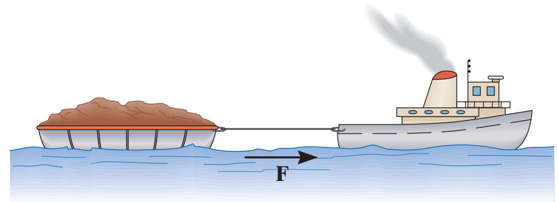
$= 6.05\ \text{N}\cdot\text{ms}$ **Ans.**



Ans:
$I_c = 6.55\ \text{N}\cdot\text{ms}$
$I_v = 6.05\ \text{N}\cdot\text{ms}$

15–21.

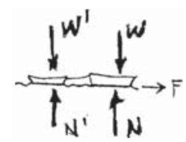
If it takes 35 s for the 50-Mg tugboat to increase its speed uniformly to 25 km/h, starting from rest, determine the force of the rope on the tugboat. The propeller provides the propulsion force **F** which gives the tugboat forward motion, whereas the barge moves freely. Also, determine F acting on the tugboat. The barge has a mass of 75 Mg.



SOLUTION

$$25\left(\frac{1000}{3600}\right) = 6.944 \text{ m/s}$$

System:

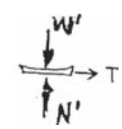


$(\overset{+}{\rightarrow})\quad mv_1 + \Sigma\int F\,dt = mv_2$

$$[0+0] + F(35) = (50 + 75)(10^3)(6.944)$$

$F = 24.8$ kN **Ans.**

Barge:



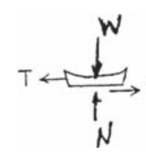
$(\overset{+}{\rightarrow})\quad mv_1 + \Sigma\int F\,dt = mv_2$

$$0 + T(35) = (75)(10^3)(6.944)$$

$T = 14.881 = 14.9$ kN **Ans.**

Also, using this result for T,

Tugboat:



$(\overset{+}{\rightarrow})\quad mv_1 + \Sigma\int F\,dt = mv_2$

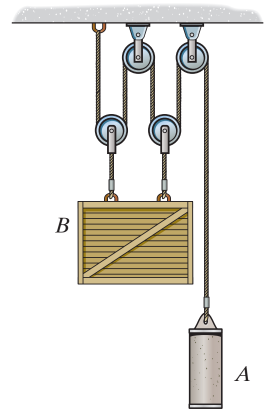
$$0 + F(35) - (14.881)(35) = (50)(10^3)(6.944)$$

$F = 24.8$ kN **Ans.**

Ans:
$T = 14.9$ kN
$F = 24.8$ kN

15–22.

The crate B and cylinder A have a mass of 200 kg and 75 kg, respectively. If the system is released from rest, determine the speed of the crate and cylinder when $t = 3$ s. Neglect the mass of the pulleys.



SOLUTION

Free-Body Diagram: The free-body diagrams of cylinder A and crate B are shown in Figs. b and c. $\mathbf{v}_A$ and $\mathbf{v}_B$ must be assumed to be directed downward so that they are consistent with the positive sense of s_A and s_B shown in Fig. a.

Principle of Impulse and Momentum: Referring to Fig. b,

$$(+\downarrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$75(0) + 75(9.81)(3) - T(3) = 75v_A$$

$$v_A = 29.43 - 0.04T \qquad \textbf{(1)}$$

From Fig. b,

$$(+\downarrow) \qquad m(v_1)_y + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$200(0) + 2500(9.81)(3) - 4T(3) = 200v_B$$

$$v_B = 29.43 - 0.06T \qquad \textbf{(2)}$$

Kinematics: Expressing the length of the cable in terms of s_A and s_B and referring to Fig. a,

$$s_A + 4s_B = l \qquad \textbf{(3)}$$

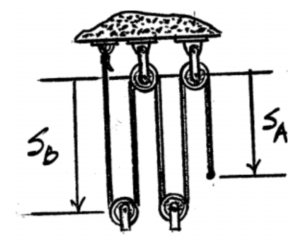
Taking the time derivative,

$$v_A + 4v_B = 0 \qquad \textbf{(4)}$$

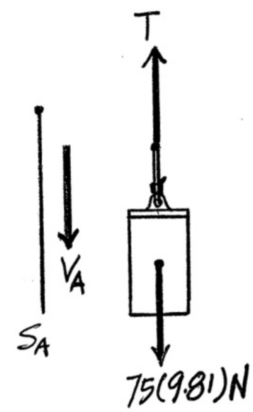
Solving Eqs. (1), (2), and (4) yields

$$v_B = -2.102 \text{ m/s} = 2.10 \text{ m/s} \uparrow \qquad v_A = 8.409 \text{ m/s} = 8.41 \text{ m/s} \downarrow \quad \textbf{Ans.}$$

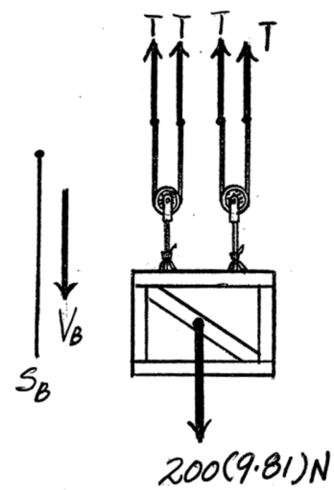
$$T = 525.54 \text{ N}$$



(a)



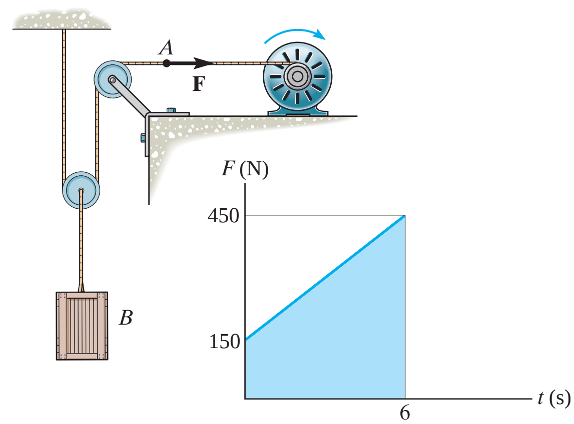
(b)



(c)

15–23.

The motor exerts a force $\boldsymbol{F}$ on the 40-kg crate as shown in the graph. Determine the speed of the crate when $t = 3$ s and when $t = 6$ s. When $t = 0$, the crate is moving downward at 10 m/s.



SOLUTION

Principle of Impulse and Momentum. The impulse of force $\boldsymbol{F}$ is equal to the area under the F–t graph. At $t = 3$ s, $\dfrac{F - 150}{3 - 0} = \dfrac{450 - 150}{6 - 0}$ $F = 300$ N. Referring to the FBD of the crate, Fig. a

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$40(-10) + 2\left[\frac{1}{2}(150 + 300)(3)\right] - 40(9.81)(3) = 40v$$

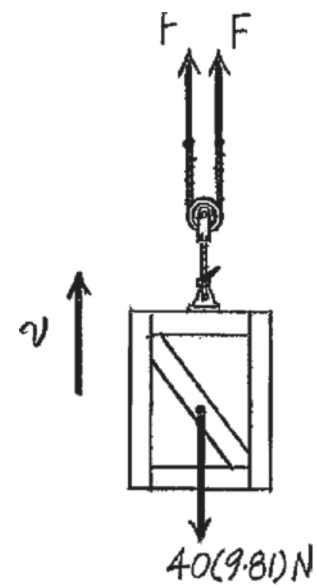
$$v = -5.68 \text{ m/s} = 5.68 \text{ m/s} \downarrow \qquad \textbf{Ans.}$$

At $t = 6$ s,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y\, dt = m(v_y)_2$$

$$40(-10) + 2\left[\frac{1}{2}(450 + 150)(6)\right] - 40(9.81)(6) = 40v$$

$$v = 21.14 \text{ m/s} = 21.1 \text{ m/s} \uparrow \qquad \textbf{Ans.}$$

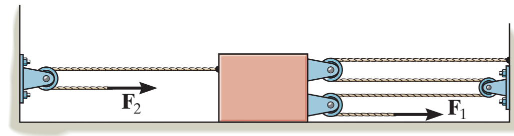


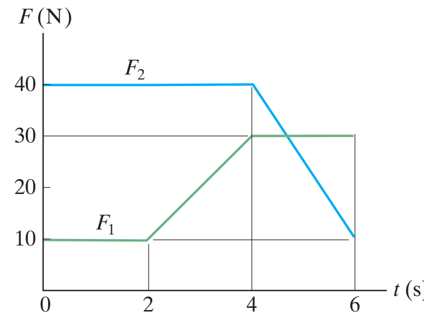
(a)

Ans:
$v|_{t=3\text{ s}} = 5.68 \text{ m/s} \downarrow$
$v|_{t=6\text{ s}} = 21.1 \text{ m/s} \uparrow$

***15–24.**

The 30-kg slider block is moving to the left with a speed of 5 m/s when it is acted upon by the forces $\mathbf{F}_1$ and $\mathbf{F}_2$. If these loadings vary in the manner shown on the graph, determine the speed of the block at $t = 6$ s. Neglect friction and the mass of the pulleys and cords.





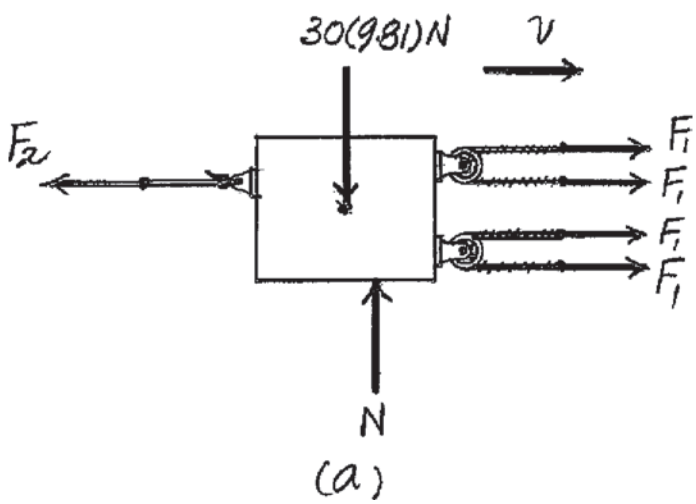
SOLUTION

Principle of Impulse and Momentum. The impulses produced by $\mathbf{F}_1$ and $\mathbf{F}_2$ are equal to the area under the respective F–t graph. Referring to the FBD of the block Fig. *a*,

$$(\overset{+}{\rightarrow}) \quad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$

$$-30(5) + 4\left[10(2) + \frac{1}{2}(10 + 30)(4 - 2) + 30(6 - 4)\right] + \left[-40(4) - \frac{1}{2}(10 + 40)(6 - 4)\right] = 30v$$

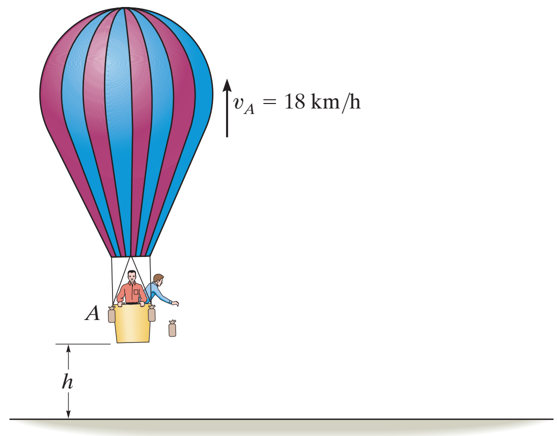
$$v = 4.00 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



Ans:
$v = 4.00$ m/s

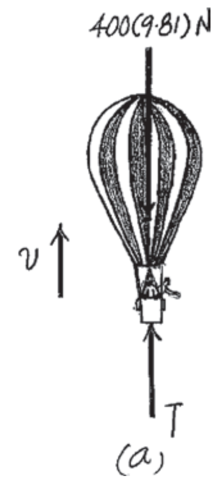
15–25.

The balloon has a total mass of 400 kg including the passengers and ballast. The balloon is rising at a constant velocity of 18 km/h when $h = 10$ m. If the man drops the 40-kg sand bag, determine the velocity of the balloon when the bag strikes the ground. Neglect air resistance.



SOLUTION

Kinematic. When the sand bag is dropped, it will have an upward velocity of $v_0 = \left(18\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 5\text{ m/s}\uparrow$. When the sand bag strikes the ground $s = 10\text{ m}\downarrow$. The time taken for the sand bag to strike the ground can be determined from



$$(+\uparrow)\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2;$$

$$-10 = 0 + 5t + \frac{1}{2}\left(-9.81t^2\right)$$

$$4.905t^2 - 5t - 10 = 0$$

Solve for the positive root,

$$t = 2.0258\text{ s}$$

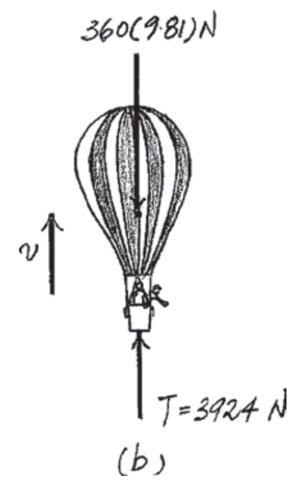
Principle of Impulse and Momentum. The FBD of the ballon when the ballon is rising with the constant velocity of 5 m/s is shown in Fig. *a*

$$(+\uparrow)\quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$400(5) + T(t) - 400(9.81)t = 400(5)$$

$$T = 3924\text{ N}$$

When the sand bag is dropped, the thrust $T = 3924$ N is still maintained as shown in the FBD, Fig. *b*.



$$(+\uparrow)\quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

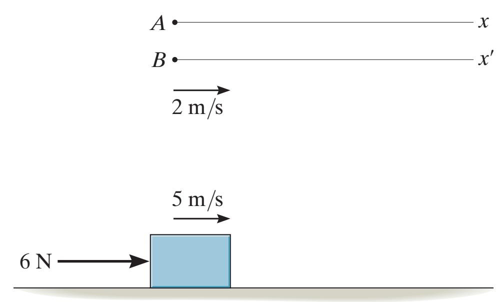
$$360(5) + 3924(2.0258) - 360(9.81)(2.0258) = 360v$$

$$v = 7.208\text{ m/s} = 7.21\text{ m/s}\uparrow \qquad \textbf{Ans.}$$

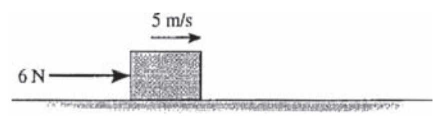
Ans:
$v = 7.21\text{ m/s}\uparrow$

15–26.

As indicated by the derivation, the principle of impulse and momentum is valid for observers in *any* inertial reference frame. Show that this is so, by considering the 10-kg block which slides along the smooth surface and is subjected to a horizontal force of 6 N. If observer A is in a *fixed* frame x, determine the final speed of the block in 4 s if it has an initial speed of 5 m/s measured from the fixed frame. Compare the result with that obtained by an observer B, attached to the x' axis that moves at a constant velocity of 2 m/s relative to A.



SOLUTION



Observer A:

$$(\overset{+}{\rightarrow}) \qquad m\,v_1 + \sum \int F\,dt = m\,v_2$$

$$10(5) + 6(4) = 10v$$

$$v = 7.40 \text{ m/s}$$ **Ans.**

Observer B:

$$(\overset{+}{\rightarrow}) \qquad m\,v_1 + \sum \int F\,dt = m\,v_2$$

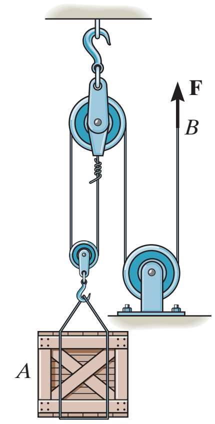
$$10(3) + 6(4) = 10v$$

$$v = 5.40 \text{ m/s}$$ **Ans.**

Ans:
Observer A: $v = 7.40$ m/s
Observer B: $v = 5.40$ m/s

15–27.

The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where t is in seconds. Determine the speed of the crate when $t = 3$ s, starting from rest.



SOLUTION

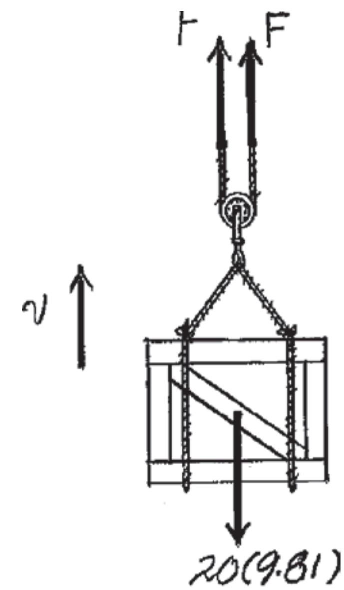
Principle of Impulse and Momentum. At $t = 0$, $F = 100$ N. Since at this instant, $2F = 200\text{ N} > W = 20(9.81) = 196.2$ N, the crate will move the instant force **F** is applied. Referring to the FBD of the crate, Fig. a,

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + 2\int_0^{3\text{ s}} (100 + 5t^2)dt - 20(9.81)(3) = 20v$$

$$2\left(100t + \frac{5}{3}t^3\right)\Bigg|_0^{3\text{ s}} - 588.6 = 20v$$

$$v = 5.07\text{ m/s}$$ **Ans.**

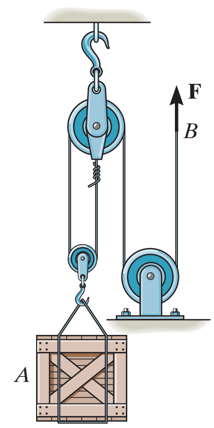


(a)

Ans:
$v = 5.07$ m/s

***15–28.**

The 20-kg crate is lifted by a force of $F = (100 + 5t^2)$ N, where t is in seconds. Determine how high the crate has moved upward when $t = 3$ s, starting from rest.



SOLUTION

Principle of Impulse and Momentum. At $t = 0$, $F = 100$ N. Since at this instant, $2F = 200\text{ N} > W = 20(9.81) = 196.2$ N, the crate will move the instant force F is applied. Referring to the FBD of the crate, Fig. *a*

$$(+\uparrow) \quad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + 2\int_0^t (100 + 5t^2)dt - 20(9.81)t = 20v$$

$$2\left(100t + \frac{5}{3}t^3\right)\Big|_0^t - 196.2t = 20v$$

$$v = \{0.1667t^3 + 0.19t\}\text{ m/s}$$

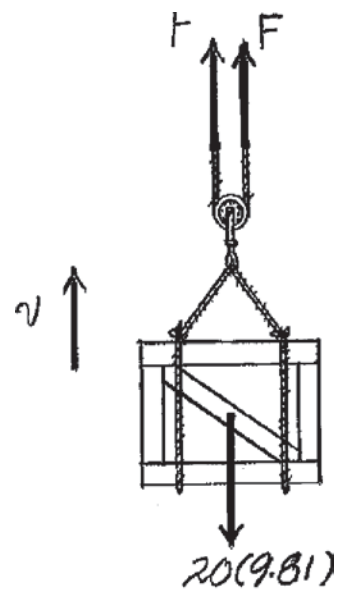
Kinematics. The displacement of the crate can be determined by integrating $ds = v\,dt$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t (0.1667t^3 + 0.19t)\,dt$$

$$s = \{0.04167t^4 + 0.095t^2\}\text{ m}$$

At $t = 3$ s,

$$s = 0.04167(3^4) + 0.095(3^2) = 4.23\text{ m} \qquad \textbf{Ans.}$$

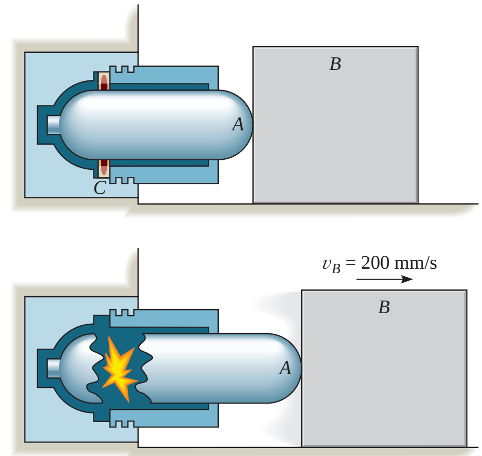


(a)

Ans:
$s = 4.23$ m

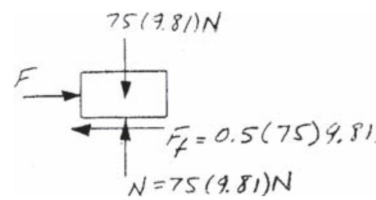
15–29.

In case of emergency, the gas actuator is used to move a 75-kg block B by exploding a charge C near a pressurized cylinder of negligible mass. As a result of the explosion, the cylinder fractures and the released gas forces the front part of the cylinder, A, to move B forward, giving it a speed of 200 mm/s in 0.4 s. If the coefficient of kinetic friction between B and the floor is $\mu_k = 0.5$, determine the impulse that the actuator imparts to B.



SOLUTION

Principle of Linear Impulse and Momentum: In order for the package to rest on top of the belt, it has to travel at the same speed as the belt. Applying Eq. 15–4, we have



$$m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

$$(+\uparrow) \qquad 6(0) + Nt - 6(9.81)\,t = 6(0)$$

$$N = 58.86 \text{ N}$$

$$m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 6(3) + [-0.2(58.86)t] = 6(1)$$

$$t = 1.02 \text{ s}$$ **Ans.**

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

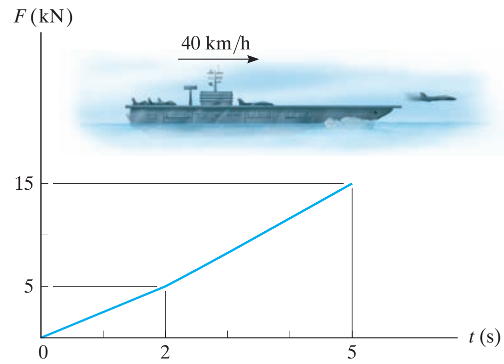
$$0 + \int F\,dt - (0.5)(9.81)(75)(0.4) = 75(0.2)$$

$$\int F\,dt = 162 \text{ N}\cdot\text{s}$$ **Ans.**

Ans:
$t = 1.02$ s
$I = 162$ N·s

15–30.

A jet plane having a mass of 7 Mg takes off from an aircraft carrier such that the engine thrust varies as shown by the graph. If the carrier is traveling forward with a speed of 40 km/h, determine the plane's airspeed after 5 s.

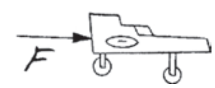


SOLUTION

The impulse exerted on the plane is equal to the area under the graph.

$v_1 = 40 \text{ km/h} = 11.11 \text{ m/s}$

$$(\overset{+}{\rightarrow}) \qquad m(v_x)_1 + \Sigma \int F_x\,dt = m(v_x)_2$$

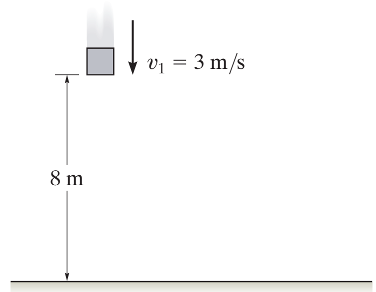


$$(7)(10^3)(11.11) - \frac{1}{2}(2)(5)(10^3) + \frac{1}{2}(15 + 5)(5 - 2)(10^3) = 7(10^3)_{v_2}$$

$v_2 = 16.1 \text{ m/s}$ **Ans.**

Ans:
$v = 16.1 \text{ m/s}$

15–31. The 6-kg block is moving downward at $v_1 = 3$ m/s when it is 8 m from the sandy surface. Determine the impulse of the sand on the block necessary to stop its motion. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.



SOLUTION

Given:

$M = 6$ kg

$v_1 = 3$ m/s

$h = 8$ m

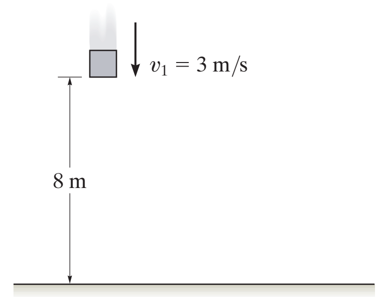
$g = 9.81$ m/s^2

Just before impact $\quad v_2 = \sqrt{v_1^2 + 2gh} \quad v_2 = 12.88$ m/s

Collision $\quad Mv_2 - I = 0 \quad I = Mv_2 \quad I = 77.3$ N·s **Ans.**

Ans:
$I = 77.3$ N·s

***15–32.** The 6-kg block is falling downward at $v_1 = 3$ m/s when it is 8 m from the sandy surface. Determine the average impulsive force acting on the block by the sand if the motion of the block is stopped in time 1.2 s once the block strikes the sand. Neglect the distance the block dents into the sand and assume the block does not rebound. Neglect the weight of the block during the impact with the sand.



SOLUTION

Given:

$M = 6$ kg

$v_1 = 3$ m/s

$\Delta t = 0.9$ s

$h = 8$ m

$g = 9.81$ m/s^2

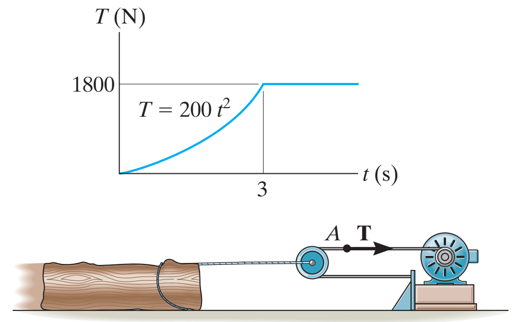
Just before impact $\quad v_2 = \sqrt{v_1^2 + 2gh} \quad v_2 = 12.88$ m/s

Collision $\quad M v_2 - F\Delta t = 0 \quad F = \dfrac{M v_2}{\Delta t} \quad F = 64.4$ N **Ans.**

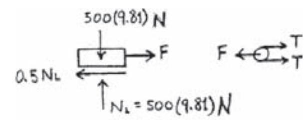
Ans:
$F = 64.4$ N

15–33.

The log has a mass of 500 kg and rests on the ground for which the coefficients of static and kinetic friction are $\mu_s = 0.5$ and $\mu_k = 0.4$, respectively. The winch delivers a horizontal towing force T to its cable at A which varies as shown in the graph. Determine the speed of the log when $t = 5$ s. Originally the tension in the cable is zero. *Hint:* First determine the force needed to begin moving the log.



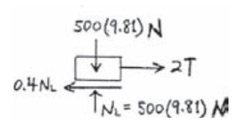
SOLUTION



$\xrightarrow{+} \Sigma F_x = 0; \qquad F - 0.5(500)(9.81) = 0$

$F = 2452.5 \text{ N}$

Thus,



$2T = F$

$2(200t^2) = 2452.5$

$t = 2.476$ s to start log moving

$$(\xrightarrow{+}) \qquad m v_1 + \Sigma \int F\,dt = m v_2$$

$$0 + 2\int_{2.476}^{3} 200t^2\,dt + 2(1800)(5-3) - 0.4(500)(9.81)(5-2.476) = 500v_2$$

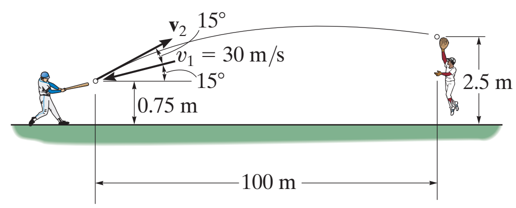
$$400\left(\frac{t^3}{3}\right)\Big|_{2.476}^{3} + 2247.91 = 500v_2$$

$v_2 = 7.65$ m/s **Ans.**

Ans:
$v = 7.65$ m/s

15–34.

The 0.15-kg baseball has a speed of $v = 30$ m/s just before it is struck by the bat. It then travels along the trajectory shown before the outfielder catches it. Determine the magnitude of the average impulsive force imparted to the ball if it is in contact with the bat for 0.75 ms.



SOLUTION

Kinematics. First, we will consider the horizontal motion. Here, the horizontal component of the initial velocity is $(v_0)_x = v_2 \cos 30° = \frac{\sqrt{3}}{2} v_2$ and the initial and final horizontal positions are $(s_0)_x = 0$ and $s_x = 100$ m

$$(\overset{+}{\rightarrow})\ S_x = (S_0)_x + (v_0)_x t$$

$$100 = 0 + \left(\frac{\sqrt{3}}{2} v_2\right)t$$

$$v_2 = \frac{200}{\sqrt{3}t} \qquad (1)$$

For the vertical motion, the vertical component of the initial velocity is $(v_0)_y = v_2 \sin 30° = \frac{1}{2} v_2$ and the initial and find positions are $(s_0)_y = 0.75$ m and $s_y = 2.5$ m.

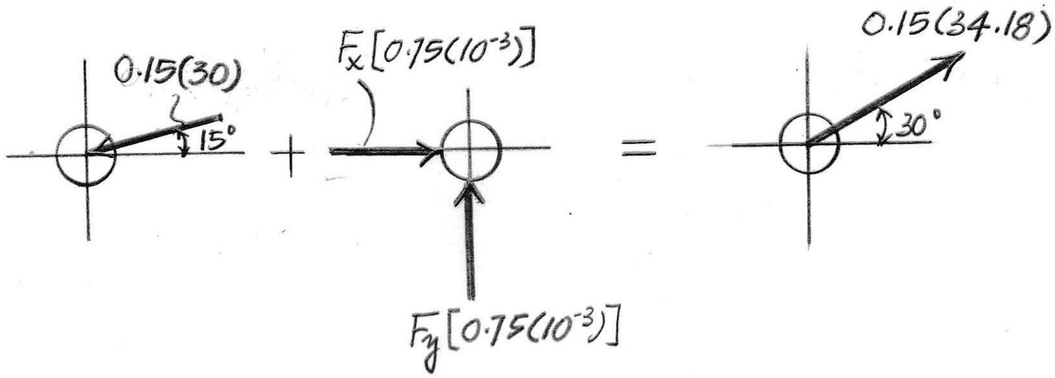
$$(+\uparrow) S_y = (S_0)_y + (v_0)_y\, t + \frac{1}{2}(a_c)_y t^2$$

$$2.5 = 0.75 + \left(\frac{1}{2} v_2\right)t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 0.5v_2 t + 1.75 = 0 \qquad (2)$$

Solving Eqs. 1 and 2,

$v_2 = 34.18$ m/s $\quad t = 3.378$ s



(a)

15–34. Continued

Principle of Impulse and Momentum. Referring to the impulse and momentum diagram shown in Fig. *a*,

$$M(v_1)_x + \Sigma \int_{t_1}^{t_z} F_x dt = m(v_2)_x$$

$$(\overset{+}{\rightarrow}) \; -[0.15(30)] \cos 15° + F_x[0.75(10^{-3})] = [0.15(34.18)] \cos 30°$$

$$F_x = 11.715(10^3)\text{N} = 11.715 \text{ kN}$$

$$M(v_1)_y + \Sigma \int_{t_1}^{t_z} F_y dt = m(v_2)_y$$

$$(+\uparrow) \; -[0.15(30)] \sin 15° + F_y[0.75(10^3)] = [0.15(34.18)] \sin 30°$$

$$F_y = 4.971(10^3)\text{N} = 4.971 \text{ kN}$$

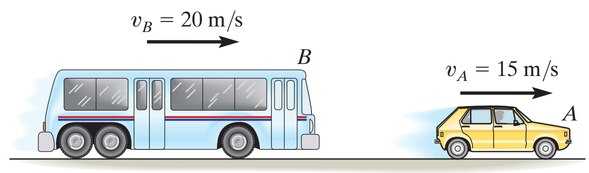
Thus, the magnitude of the average impulsive force on the ball is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{11.715^2 + 4.971^2} = 12.73 \text{ kN} = 12.7 \text{ kN}$$ **Ans.**

Ans:
$F = 12.7$ kN

15–35.

The 5-Mg bus B is traveling to the right at 20 m/s. Meanwhile a 2-Mg car A is traveling at 15 m/s to the right. If the vehicles crash and become entangled, determine their common velocity just after the collision. Assume that the vehicles are free to roll during collision.



SOLUTION

Conservation of Linear Momentum.

$$(\xrightarrow{+}) \qquad m_A v_A + m_B v_B = (m_A + m_B)v$$

$$\left[5(10^3)\right](20) + \left[2(10^3)\right](15) = \left[5(10^3) + 2(10^3)\right]v$$

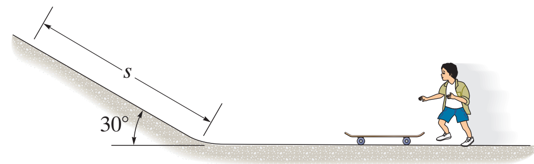
$$v = 18.57 \text{ m/s} = 18.6 \text{ m/s} \rightarrow$$ **Ans.**

Ans:

$v = 18.6 \text{ m/s} \rightarrow$

***15–36.**

The 50-kg boy jumps on the 5-kg skateboard with a horizontal velocity of 5 m/s. Determine the distance s the boy reaches up the inclined plane before momentarily coming to rest. Neglect the skateboard's rolling resistance.



SOLUTION

Free-Body Diagram: The free-body diagram of the boy and skateboard system is shown in Fig. *a*. Here, $\mathbf{W}_b$, $\mathbf{W}_{sb}$, and $\mathbf{N}$ are nonimpulsive forces. The pair of impulsive forces $\mathbf{F}$ resulting from the impact during landing cancel each other out since they are internal to the system.

Conservation of Linear Momentum: Since the resultant of the impulsive force along the x axis is zero, the linear momentum of the system is conserved along the x axis.

$$(\overset{+}{\leftarrow}) \qquad m_b(v_b)_1 + m_{sb}(v_{sb})_1 = (m_b + m_{sb})v$$

$$50(5) + 5(0) = (50 + 5)v$$

$$v = 4.545 \text{ m/s}$$

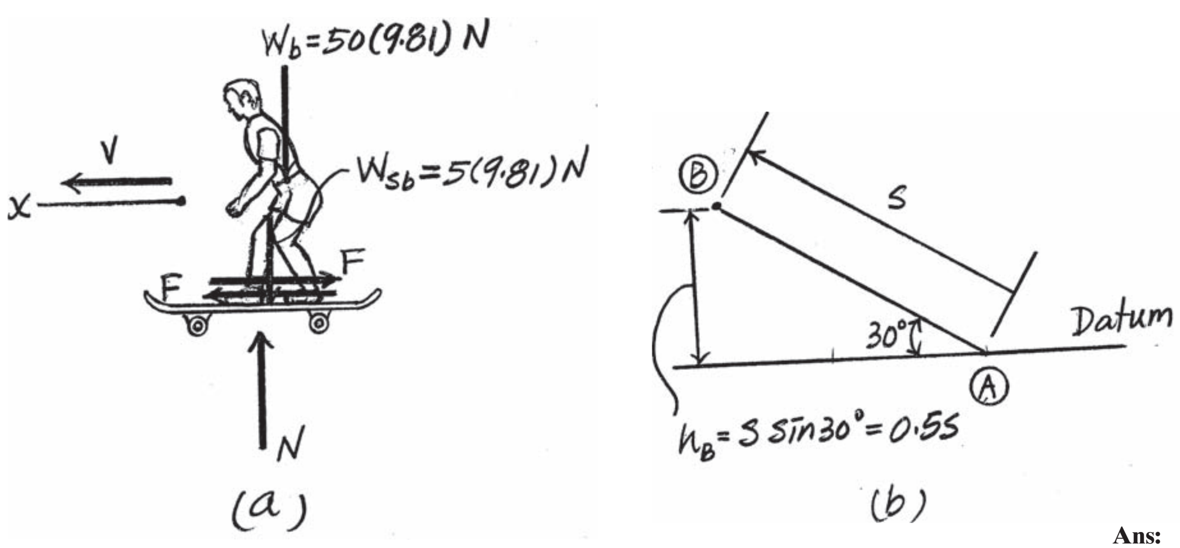
Conservation of Energy: With reference to the datum set in Fig. b, the gravitational potential energy of the boy and skateboard at positions A and B are $\left(V_g\right)_A = (m_b + m_{sb})gh_A = 0$ and $\left(V_g\right)_B = (m_b + m_{sb})gh_B = (50 + 5)(9.81)(s \sin 30°) = 269.775s$.

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(m_b + m_{sb})v_A{}^2 + \left(V_g\right)_A = \frac{1}{2}(m_b + m_{sb})v_B{}^2 + \left(V_g\right)_B$$

$$\frac{1}{2}(50 + 5)\left(4.545^2\right) + 0 = 0 + 269.775s$$

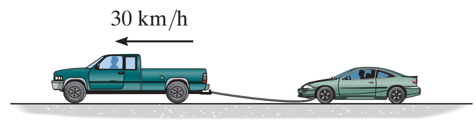
$$s = 2.11 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$s = 2.11$ m

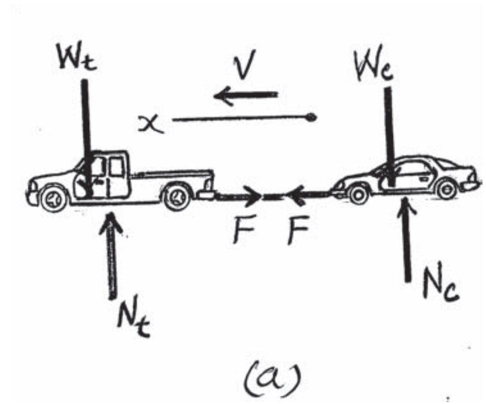
15–37.

The 2.5-Mg pickup truck is towing the 1.5-Mg car using a cable as shown. If the car is initially at rest and the truck is coasting with a velocity of 30 km/h when the cable is slack, determine the common velocity of the truck and the car just after the cable becomes taut. Also, find the loss of energy.



SOLUTION

Free-Body Diagram: The free-body diagram of the truck and car system is shown in Fig. *a*. Here, $\mathbf{W}_t, \mathbf{W}_C$, $\mathbf{N}_t$, and $\mathbf{N}_C$ are nonimpulsive forces. The pair of impulsive forces **F** generated at the instant the cable becomes taut are internal to the system and thus cancel each other out.



Conservation of Linear Momentum: Since the resultant of the impulsive force is zero, the linear momentum of the system is conserved along the x axis. The initial speed of the truck is $(v_t)_1 = \left[30(10^3)\frac{\text{m}}{\text{h}}\right]\left[\frac{1 \text{ h}}{3600 \text{ s}}\right] = 8.333$ m/s.

$$(\overset{+}{\leftarrow}) \qquad m_t(v_t)_1 + m_C(v_C)_1 = (m_t + m_C)v_2$$

$$2500(8.333) + 0 = (2500 + 1500)v_2$$

$$v_2 = 5.208 \text{ m/s} = 5.21 \text{ m/s} \leftarrow$$ **Ans.**

Kinetic Energy: The initial and final kinetic energy of the system is

$$T_1 = \frac{1}{2}m_t(v_t)_1^2 + \frac{1}{2}m_C(v_C)_1^2$$

$$= \frac{1}{2}(2500)(8.333^2) + 0$$

$$= 86\,805.56 \text{ J}$$

and

$$T_2 = (m_t + m_C)v_2^2$$

$$= \frac{1}{2}(2500 + 1500)(5.208^2)$$

$$= 54\,253.47$$

Thus, the loss of energy during the impact is

$$\Delta T = T_1 - T_2 = 86\,805.56 - 54\,253.47 = 32.55(10^3)\text{ J} = 32.6 \text{ kJ}$$ **Ans.**

Ans:
$v = 5.21$ m/s $\leftarrow$
$\Delta T = -32.6$ kJ

15–38.

A railroad car having a mass of 15 Mg is coasting at 1.5 m/s on a horizontal track. At the same time another car having a mass of 12 Mg is coasting at 0.75 m/s in the opposite direction. If the cars meet and couple together, determine the speed of both cars just after the coupling. Find the difference between the total kinetic energy before and after coupling has occurred, and explain qualitatively what happened to this energy.

SOLUTION

$(\overset{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$$15\,000(1.5) - 12\,000(0.75) = 27\,000(v_2)$$

$v_2 = 0.5 \text{ m/s}$ **Ans.**

$$T_1 = \frac{1}{2}(15\,000)(1.5)^2 + \frac{1}{2}(12\,000)(0.75)^2 = 20.25 \text{ kJ}$$

$$T_2 = \frac{1}{2}(27\,000)(0.5)^2 = 3.375 \text{ kJ}$$

$$\Delta T = T_2 - T_1$$

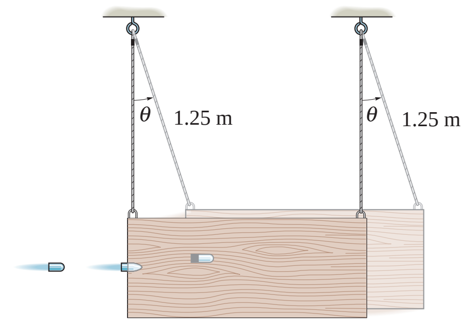
$= 3.375 - 20.25 = -16.9 \text{ kJ}$ **Ans.**

This energy is dissipated as noise, shock, and heat during the coupling.

Ans:
$v = 0.5 \text{ m/s}$
$\Delta T = -16.9 \text{ kJ}$

15–39.

A ballistic pendulum consists of a 4-kg wooden block originally at rest, $\theta = 0°$. When a 2-g bullet strikes and becomes embedded in it, it is observed that the block swings upward to a maximum angle of $\theta = 6°$. Estimate the speed of the bullet.



SOLUTION

Just after impact:

Datum at lowest point.

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(4 + 0.002)(v_B)_2^2 + 0 = 0 + (4 + 0.002)(9.81)(1.25)(1 - \cos 6°)$$

$$(v_B)_2 = 0.3665 \text{ m/s}$$

For the system of bullet and block:

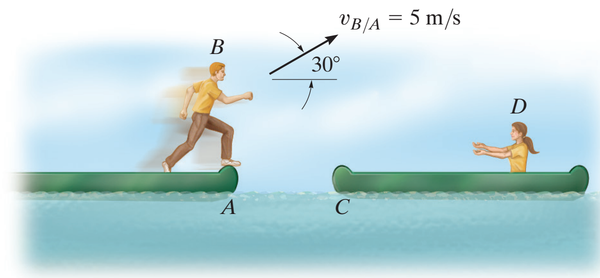
$$(\overset{+}{\rightarrow}) \qquad \Sigma mv_1 = \Sigma mv_2$$

$$0.002(v_B)_1 = (4 + 0.002)(0.3665)$$

$$(v_B)_1 = 733 \text{ m/s}$$ **Ans.**

Ans:
$v = 733$ m/s

***15–40.** The boy B jumps off the canoe at A with a velocity 5 m/s relative to the canoe as shown. If he lands in the second canoe C, determine the final speed of both canoes after the motion. Each canoe has a mass of 40 kg. The boy's mass is 30 kg, and the girl D has a mass of 25 kg. Both canoes are originally at rest.



SOLUTION

Guesses $\quad v_A = 1\,\frac{\text{m}}{\text{s}} \qquad v_C = 1\,\frac{\text{m}}{\text{s}}$

Given $\quad 0 = M_c v_A + M_B(v_A + v_{BA}\cos(\theta))$

$$M_B(v_A + v_{BA}\cos(\theta)) = (M_c + M_B + M_D)v_C$$

Given:

$M_c = 40\ \text{kg}$

$M_B = 30\ \text{kg}$

$M_D = 25\ \text{kg}$

$v_{BA} = 5\ \text{m/s}$

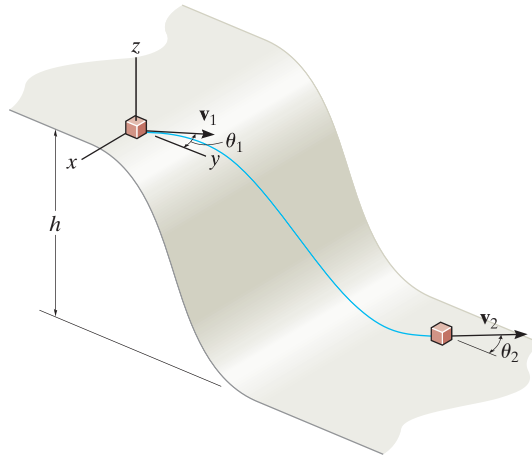
$\theta = 30°$

$$\begin{pmatrix} v_A \\ v_C \end{pmatrix} = \text{Find}(v_A, v_C) \qquad \begin{pmatrix} v_A \\ v_C \end{pmatrix} = \begin{pmatrix} -1.856 \\ 0.781 \end{pmatrix}\ \text{m/s}$$ **Ans.**

Ans:
$v_A = -1.856$ m/s
$v_C = 0.781$ m/s

15–41.

The block of mass m is traveling at v_1 in the direction θ_1 shown at the top of the smooth slope. Determine its speed v_2 and its direction θ_2 when it reaches the bottom.



SOLUTION

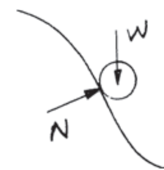
There are no impulses in the v direction:

$$mv_1 \sin \theta_1 = mv_2 \sin \theta_2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} mv_1^2 + mgh = \frac{1}{2} mv_2^2 + 0$$

$$v_2 = \sqrt{v_1^2 + 2gh}$$ **Ans.**



$$\sin \theta_2 = \frac{v_1 \sin \theta_1}{\sqrt{v_1^2 + 2gh}}$$

$$\theta_2 = \sin^{-1}\left(\frac{v_1 \sin \theta_1}{\sqrt{v_1^2 + 2gh}}\right)$$ **Ans.**

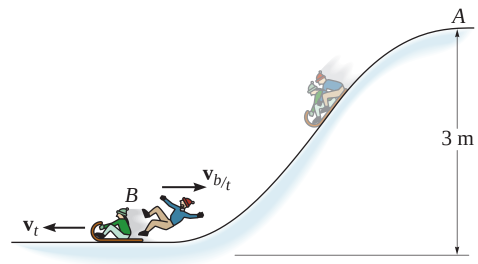
Ans:

$v_2 = \sqrt{v_1^2 + 2gh}$

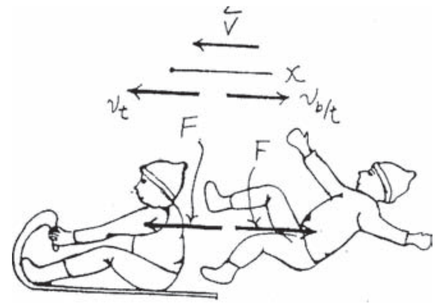
$\theta_2 = \sin^{-1}\left(\frac{v_1 \sin\theta}{\sqrt{v_1^2 + 2gh}}\right)$

15–42.

A toboggan having a mass of 10 kg starts from rest at A and carries a girl and boy having a mass of 40 kg and 45 kg, respectively. When the toboggan reaches the bottom of the slope at B, the boy is pushed off from the back with a horizontal velocity of $v_{b/t} = 2$ m/s, measured relative to the toboggan. Determine the velocity of the toboggan afterwards. Neglect friction in the calculation.



SOLUTION



Conservation of Energy: The datum is set at the lowest point B. When the toboggan and its rider is at A, their position is 3 m *above* the datum and their gravitational potential energy is $(10 + 40 + 45)(9.81)(3) = 2795.85\ \text{N}\cdot\text{m}$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2795.85 = \frac{1}{2}(10 + 40 + 45)\,v_B^2 + 0$$

$$v_B = 7.672\ \text{m/s}$$

Relative Velocity: The relative velocity of the falling boy with respect to the toboggan is $v_{b/t} = 2$ m/s. Thus, the velocity of the boy falling off the toboggan is

$$v_b = v_t + v_{b/t}$$

$$(\overset{+}{\leftarrow}) \qquad v_b = v_t - 2 \qquad \textbf{[1]}$$

Conservation of Linear Momentum: If we consider the tobbogan and the riders as a system, then the impulsive force caused by the push is *internal* to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the x axis.

$$m_T v_B = m_b\, v_b + \left(m_t + m_g\right) v_t$$

$$(\overset{+}{\leftarrow}) \qquad (10 + 40 + 45)(7.672) = 45 v_b + (10 + 40)\, v_t \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

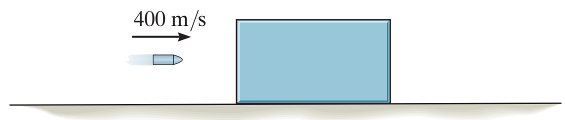
$$v_t = 8.62\ \text{m/s} \qquad \textbf{Ans.}$$

$$v_b = 6.619\ \text{m/s}$$

Ans:
$v_t = 8.62$ m/s

15–43.

The 20-g bullet is traveling at 400 m/s when it becomes embedded in the 2-kg stationary block. Determine the distance the block will slide before it stops. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$.



SOLUTION

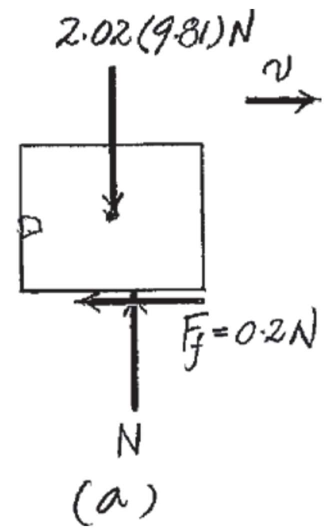
Conservation of Momentum.

$$(\xrightarrow{+}) \qquad m_b v_b + m_B v_B = (m_b + m_B)v$$

$$0.02(400) + 0 = (0.02 + 2)v$$

$$v = 3.9604 \text{ m/s}$$

Principle of Impulse and Momentum. Here, friction $F_f = \mu_k N = 0.2$ N. Referring to the FBD of the blocks, Fig. *a*,



$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y dt = m(v_y)_2$$

$$0 + N(t) - 2.02(9.81)(t) = 0$$

$$N = 19.8162 \text{ N}$$

$$(\xrightarrow{+}) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x dt = m(v_x)_2$$

$$2.02(3.9604) + [-0.2(19.8162)t] = 2.02v$$

$$v = \{3.9604 - 1.962t\} \text{ m/s}$$

Thus, the stopping time can be determined from

$$0 = 3.9604 - 1.962t$$

$$t = 2.0186 \text{ s}$$

Kinematics. The displacement of the block can be determined by integrating $ds = v\,dt$ with the initial condition $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t (3.9604 - 1.962t)\,dt$$

$$s = \{3.9604t - 0.981t^2\} \text{ m}$$

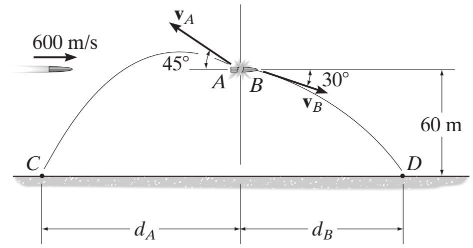
The block stopped at $t = 2.0186$ s. Thus

$$s = 3.9604(2.0186) - 0.981(2.0186^2)$$

$$= 3.9971 \text{ m} = 4.00 \text{ m}$$ **Ans.**

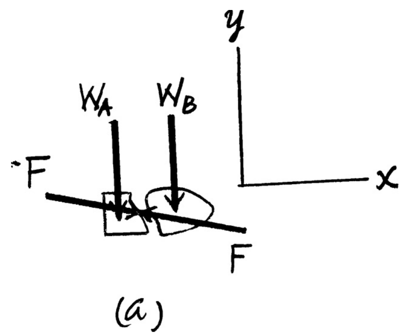
Ans:
$s = 4.00$ m

***15–44** A 4-kg projectile travels with a horizontal velocity of 600 m/s before it explodes and breaks into two fragments A and B of mass 1.5 kg and 2.5 kg, respectively. If the fragments travel along the parabolic trajectories shown, determine the magnitude of velocity of each fragment just after the explosion and the horizontal distance d_A where segment A strikes the ground at C.



SOLUTION

Conservation of Linear Momentum: By referring to the free-body diagram of the projectile just after the explosion shown in Fig. *a*, we notice that the pair of impulsive forces **F** generated during the explosion cancel each other since they are internal to the system. Here, $\mathbf{W}_A$ and $\mathbf{W}_B$ are non-impulsive forces. Since the resultant impulsive force along the x and y axes is zero, the linear momentum of the system is conserved along these two axes.



$(\overset{+}{\rightarrow})$ $\quad mv_x = m_A(v_A)_x + m_B(v_B)_x$

$$4(600) = -1.5v_A \cos 45° + 2.5v_B \cos 30°$$

$$2.165v_B - 1.061v_A = 2400 \qquad \textbf{(1)}$$

$(+\uparrow)$ $\quad mv_y = m_A(v_A)_y + m_B(v_B)_y$

$$0 = 1.5v_A \sin 45° - 2.5v_B \sin 30°$$

$$v_B = 0.8485v_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_A = 3090.96 \text{ m/s} = 3.09(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

$$v_B = 2622.77 \text{ m/s} = 2.62(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

By considering the x and y motion of segment A,

$(+\uparrow)$ $\quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} a_y t^2$

$$-60 = 0 + 3090.96 \sin 45° \, t_{AC} + \frac{1}{2}(-9.81)t_{AC}^2$$

$$4.905t_{AC}^2 - 2185.64t_{AC} - 60 = 0$$

Solving for the positive root of this equation,

$$t_{AC} = 445.62 \text{ s}$$

and

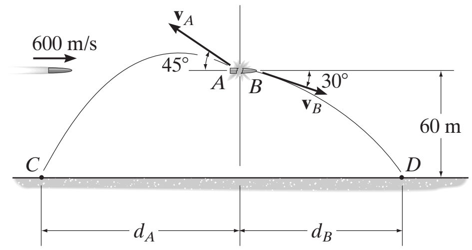
$(\overset{+}{\leftarrow})$ $\quad s_x = (s_0)_x + (v_0)_x t$

$$d_A = 0 + 3090.96 \cos 45°(445.62)$$

$$= 973.96(10^3) \text{ m} = 974 \text{ km} \qquad \textbf{Ans.}$$

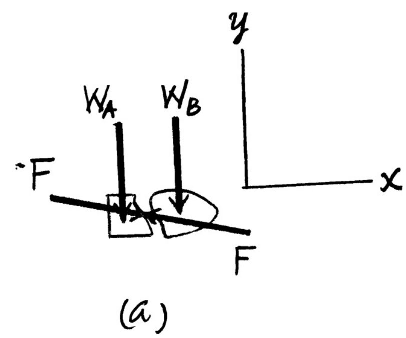
Ans:
$v_A = 3.09(10^3)$ m/s
$v_B = 2.62(10^3)$ m/s
$d_A = 974$ km

15–45. A 4-kg projectile travels with a horizontal velocity of 600 m/s before it explodes and breaks into two fragments A and B of mass 1.5 kg and 2.5 kg, respectively. If the fragments travel along the parabolic trajectories shown, determine the magnitude of velocity of each fragment just after the explosion and the horizontal distance d_B where segment B strikes the ground at D.



SOLUTION

Conservation of Linear Momentum: By referring to the free-body diagram of the projectile just after the explosion shown in Fig. *a*, we notice that the pair of impulsive forces **F** generated during the explosion cancel each other since they are internal to the system. Here, $\mathbf{W}_A$ and $\mathbf{W}_B$ are non-impulsive forces. Since the resultant impulsive force along the x and y axes is zero, the linear momentum of the system is conserved along these two axes.



$(\overset{+}{\rightarrow})$ $\quad m v_x = m_A (v_A)_x + m_B (v_B)_x$

$$4(600) = -1.5 v_A \cos 45^\circ + 2.5 v_B \cos 30^\circ$$

$$2.165 v_B - 1.061 v_A = 2400 \qquad \textbf{(1)}$$

$(+\uparrow)$ $\quad m v_y = m_A (v_A)_y + m_B (v_B)_y$

$$0 = 1.5 v_A \sin 45^\circ - 2.5 v_B \sin 30^\circ$$

$$v_B = 0.8485 v_A \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$v_A = 3090.96 \text{ m/s} = 3.09(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

$$v_B = 2622.77 \text{ m/s} = 2.62(10^3) \text{ m/s} \qquad \textbf{Ans.}$$

By considering the x and y motion of segment B,

$(+\uparrow)$ $\quad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} a_y t^2$

$$-60 = 0 - 2622.77 \sin 30^\circ \, t_{BD} + \frac{1}{2}(-9.81) t_{BD}{}^2$$

$$4.905 t_{BD}{}^2 + 1311.38 t_{BD} - 60 = 0$$

Solving for the positive root of the above equation,

$$t_{BD} = 0.04574 \text{ s}$$

and

$(\overset{+}{\rightarrow})$ $\quad s_x = (s_0)_x + (v_0)_x t$

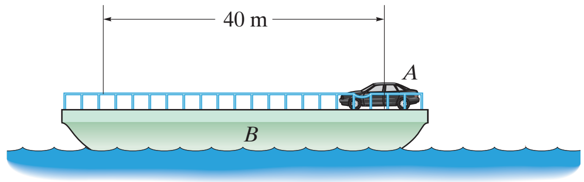
$$d_B = 0 + 2622.77 \cos 30^\circ (0.04574)$$

$$= 103.91 \text{ m} = 104 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$v_A = 3.09(10^3)$ m/s
$v_B = 2.62(10^3)$ m/s
$d_B = 104$ m

15–46.

The 10-Mg barge B supports a 2-Mg automobile A. If someone drives the automobile to the other side of the barge, determine how far the barge moves. Neglect the resistance of the water.



SOLUTION

Conservation of Momentum. Assuming that $\mathbf{V}_B$ is to the left,

$$(\overset{+}{\leftarrow}) \quad m_A v_A + m_B v_B = 0$$

$$2(10^3)v_A + 10(10^3)v_B = 0$$

$$2\,v_A + 10\,v_B = 0$$

Integrate this equation,

$$2\,s_A + 10\,s_B = 0 \qquad \textbf{(1)}$$

Kinematics. Here, $s_{A/B} = 40\text{ m} \leftarrow$, using the relative displacement equation by assuming that $\mathbf{s}_B$ is to the left,

$$(\overset{+}{\leftarrow}) \quad \mathbf{s}_A = \mathbf{s}_B + \mathbf{s}_{A/B}$$

$$\mathbf{s}_A = \mathbf{s}_B + 40 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2),

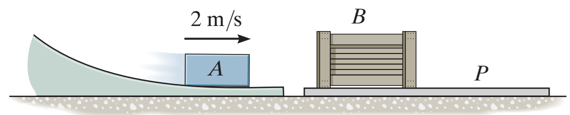
$$s_B = -6.6667\text{ m} = 6.67\text{ m} \rightarrow \qquad \textbf{Ans.}$$

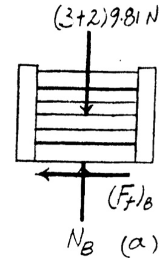
$$s_A = 33.33\text{ m} \leftarrow$$

The negative sign indicates that $\mathbf{s}_B$ is directed to the right which is opposite to that of the assumed.

Ans:
$s_B = 6.67\text{ m} \rightarrow$

15–47. Block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box B has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu'_k = 0.4$. Also, the coefficient of static friction between the plate and the floor is $\mu'_s = 0.5$.



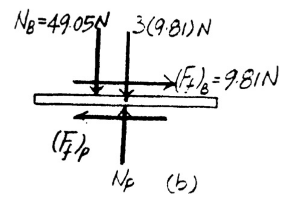


SOLUTION

Equations of Equilibrium: From FBD(a).

$$+\uparrow \Sigma F_y = 0; \qquad N_B - (3 + 2)(9.81) = 0 \qquad N_B = 49.05 \text{ N}$$

When box B slides on top of plate P, $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b).



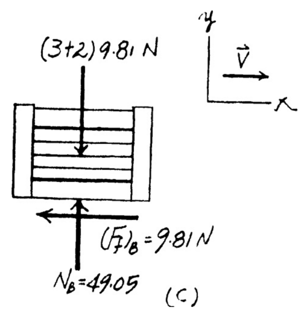
$$+\uparrow \Sigma F_y = 0; \qquad N_P - 49.05 - 3(9.81) = 0 \qquad N_P = 78.48 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 9.81 - (F_f)_P = 0 \qquad (F_f)_P = 9.81 \text{ N}$$

Since $(F_f)_P < \left[(F_f)_P\right]_{\max} = \mu_s' N_P = 0.5(78.48) = 39.24$ N, plate P does not move. Thus

$$s_P = 0 \qquad \textbf{Ans.}$$

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along the x axis.



$$m_A (v_A)_1 + m_R (v_R)_1 = (m_A + m_R) v_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(2) + 0 = (2 + 3) v_2$$

$$v_2 = 0.800 \text{ m/s} \rightarrow$$

Principle of Linear Impulse and Momentum: Applying Eq. 15–4, we have

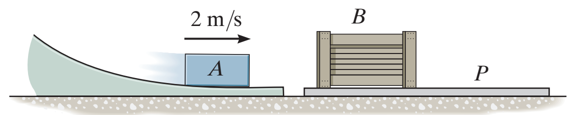
$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 5(0.8) + \left[-9.81(t)\right] = 5(0)$$

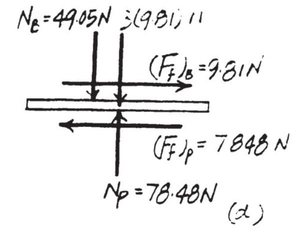
$$t = 0.408 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$s_P = 0$
$t = 0.408$ s

***15–48.** Block A has a mass of 2 kg and slides into an open ended box B with a velocity of 2 m/s. If the box B has a mass of 3 kg and rests on top of a plate P that has a mass of 3 kg, determine the distance the plate moves after it stops sliding on the floor. Also, how long is it after impact before all motion ceases? The coefficient of kinetic friction between the box and the plate is $\mu_k = 0.2$, and between the plate and the floor $\mu'_k = 0.1$. Also, the coefficient of static friction between the plate and the floor is $\mu'_s = 0.12$.



SOLUTION



Equations of Equilibrium: From FBD(a),

$$+\uparrow \Sigma F_x = 0; \qquad N_B - (3 + 2)(9.81) = 0 \qquad N_B = 49.05 \text{ N}$$

When box B slides on top of plate P. $(F_f)_B = \mu_k N_B = 0.2(49.05) = 9.81$ N. From FBD(b).

$$+\uparrow \Sigma F_y = 0; \qquad N_P - 49.05 - 3(9.81) = 0 \qquad N_P = 78.48 \text{ N}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 9.81 - (F_f)_P = 0 \qquad (F_f)_P = 9.81 \text{ N}$$

Since $(F_f)_P > \left[(F_f)_P\right]_{\max} = \mu_s{}' N_P = 0.12(78.48) = 9.418\text{N}$, plate P slides. Thus, $(F_f)_P = \mu_k{}' N_P = 0.1(78.48) = 7.848$ N.

Conservation of Linear Momentum: If we consider the block and the box as a system, then the impulsive force caused by the impact is *internal* to the system. Therefore, it will cancel out. As the result, linear momentum is conserved along x axis.

$$m_A (v_A)_1 + m_R (v_R)_1 = (m_A + m_R) v_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(2) + 0 = (2 + 3) v_2$$

$$v_2 = 0.800 \text{ m/s} \rightarrow$$

Principle of Linear Impulse and Momentum: In order for box B to stop sliding on plate P, both box B and plate P must have same speed v_3. Applying Eq. 15–4 to box B (FBD(c)], we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 5(0.8) + \left[-9.81(t_1)\right] = 5v_3 \qquad \textbf{[1]}$$

Applying Eq. 15–4 to plate P[FBD(d)], we have

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$(\overset{+}{\rightarrow}) \qquad 3(0) + 9.81(t_1) - 7.848(t_1) = 3v_3 \qquad \textbf{[2]}$$

Solving Eqs. [1] and [2] yields

$$t_1 = 0.3058 \text{ s} \qquad v_3 = 0.200 \text{ m/s}$$

Equation of Motion: From FBD(d), the acceleration of plate P when box B still slides on top of it is given by

$$\overset{+}{\rightarrow} \Sigma Fx = ma_x; \qquad 9.81 - 7.848 = 3(a_P)_1 \qquad (a_P)_1 = 0.654 \text{ m/s}^2$$

***15–48. Continued**

When box B stop slid ling on top of box B, $(F_f)_B = 0$. From this instant onward plate P and box B act as a unit and slide together. From FBD(d), the acceleration of plate P and box B is given by

$$\overset{+}{\rightarrow} \Sigma Fx = ma_x; \qquad -7.848 = 8(a_P)_2 \qquad (a_P)_2 = -0.981 \text{ m/s}^2$$

Kinematics: Plate P travels a distance s_1 before box B stop sliding.

$$(\overset{+}{\rightarrow}) \qquad s_1 = (v_0)_P\, t_1 + \frac{1}{2}(a_P)_1\, t_1^2$$

$$= 0 + \frac{1}{2}(0.654)\left(0.3058^2\right) = 0.03058 \text{ m}$$

The time t_2 for plate P to stop after box B stop slidding is given by

$$(\overset{+}{\rightarrow}) \qquad v_4 = v_3 + (a_P)_2\, t_2$$

$$0 = 0.200 + (-0.981)t_2 \qquad t_2 = 0.2039 \text{ s}$$

The distance s_2 traveled by plate P after box B stop sliding is given by

$$(\overset{+}{\rightarrow}) \qquad v_4^2 = v_3^2 + 2(a_P)_2\, s_2$$

$$0 = 0.200^2 + 2(-0.981)s_2 \qquad s_2 = 0.02039 \text{ m}$$

The total distance travel by plate P is

$$s_P = s_1 + s_2 = 0.03058 + 0.02039 = 0.05097 \text{ m} = 51.0 \text{ mm} \qquad \textbf{Ans.}$$

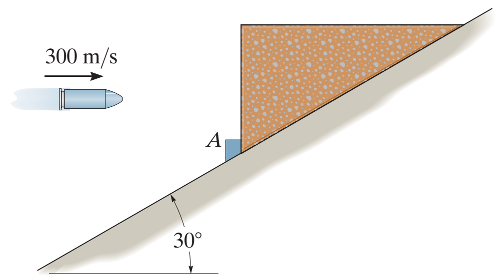
The total time taken to cease all the motion is

$$t_{\text{Tot}} = t_1 + t_2 = 0.3058 + 0.2039 = 0.510 \text{ s} \qquad \textbf{Ans.}$$

Ans:
$s_P = 51.0$ mm
$t_{\text{Tot}} = 0.510$ s

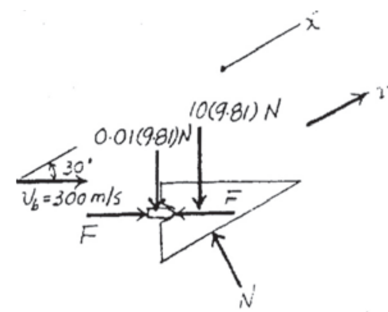
15–49.

The 10-kg block is held at rest on the smooth inclined plane by the stop block at A. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.



SOLUTION

Conservation of Linear Momentum: If we consider the block and the bullet as a system, then from the FBD, the *impulsive* force F caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the x' axis.



$$m_b(v_b)_{x'} = (m_b + m_B)\,v_{x'}$$

$$0.01(300 \cos 30°) = (0.01 + 10)\,v$$

$$v = 0.2595 \text{ m/s}$$

Conservation of Energy: The datum is set at the blocks initial position. When the block and the embedded bullet is at their highest point they are h *above* the datum. Their gravitational potential energy is $(10 + 0.01)(9.81)h = 98.1981h$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10 + 0.01)\left(0.2595^2\right) = 0 + 98.1981h$$

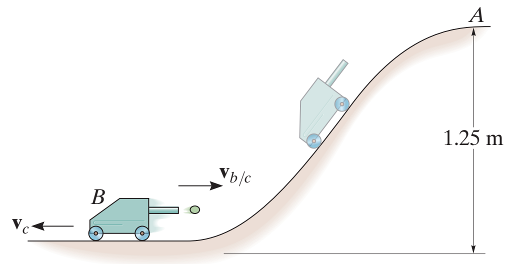
$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = 3.43 / \sin 30° = 6.87 \text{ mm}$$ **Ans.**

Ans:
$d = 6.87$ mm

15–50.

The cart has a mass of 3 kg and rolls freely down the slope. When it reaches the bottom, a spring loaded gun fires a 0.5-kg ball out the back with a horizontal velocity of $v_{b/c} = 0.6$ m/s, measured relative to the cart. Determine the final velocity of the cart.



SOLUTION

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$0 + (3 + 0.5)(9.81)(1.25) = \frac{1}{2}(3 + 0.5)(v_B)_2^2 + 0$$

$$v_B = 4.952 \text{ m/s}$$

$(\overset{+}{\leftarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$(3 + 0.5)(4.952) = (3)v_c - (0.5)v_b \qquad \textbf{(1)}$$

$(\overset{+}{\leftarrow})$ $\quad v_b = v_c + v_{b/c}$

$$-v_b = v_c - 0.6 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

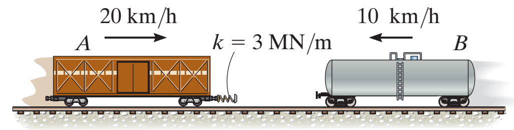
$$v_c = 5.04 \text{ m/s} \leftarrow \qquad \textbf{Ans.}$$

$$v_b = -4.44 \text{ m/s} = 4.44 \text{ m/s} \leftarrow$$

Ans:
$v_c = 5.04 \text{ m/s} \leftarrow$

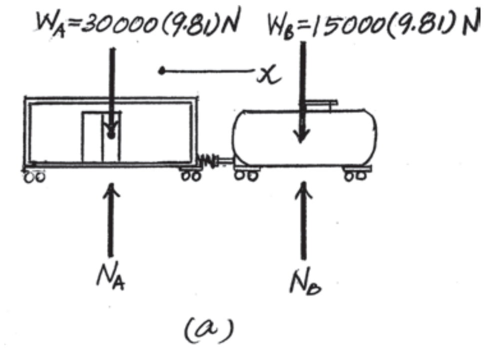
15–51.

The 30-Mg freight car A and 15-Mg freight car B are moving towards each other with the velocities shown. Determine the maximum compression of the spring mounted on car A. Neglect rolling resistance.



SOLUTION

Conservation of Linear Momentum: Referring to the free-body diagram of the freight cars A and B shown in Fig. a, notice that the linear momentum of the system is conserved along the x axis. The initial speed of freight cars A and B are $(v_A)_1 = \left[20(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 5.556\text{ m/s}$ and $(v_B)_1 = \left[10(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 2.778\text{ m/s}$. At this instant, the spring is compressed to its maximum, and no relative motion occurs between freight cars A and B and they move with a common speed.



$$(\overset{+}{\rightarrow}) \qquad m_A(v_A)_1 + m_B(v_B)_1 = (m_A + m_B)v_2$$

$$30(10^3)(5.556) + \left[-15(10^3)(2.778)\right] = \left[30(10^3) + 15(10^3)\right]v_2$$

$$v_2 = 2.778\text{ m/s} \rightarrow$$

Conservation of Energy: The initial and final elastic potential energy of the spring is $(V_e)_1 = \frac{1}{2}ks_1^2 = 0$ and $(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(3)(10^6)s_{max}^2 = 1.5(10^6)s_{max}^2$.

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$\left[\frac{1}{2}m_A(v_A)_1^2 + \frac{1}{2}m_B(v_B)_1^2\right] + (V_e)_1 = \frac{1}{2}(m_A + m_B)v_2^2 + (V_e)_2$$

$$\frac{1}{2}(30)\left(10^3\right)(5.556^2) + \frac{1}{2}(15)\left(10^3\right)\left(2.778^2\right) + 0$$

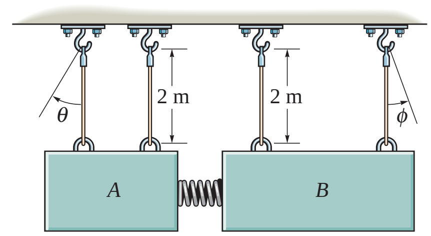
$$= \frac{1}{2}\left[30\left(10^3\right) + 15\left(10^3\right)\right]\left(2.778^2\right) + 1.5\left(10^6\right)s_{max}^2$$

$$s_{max} = 0.4811\text{ m} = 481\text{ mm}$$ **Ans.**

Ans:
$s_{max} = 481$ mm

***15–52.**

The two blocks A and B each have a mass of 5 kg and are suspended from parallel cords. A spring, having a stiffness of $k = 60\ \text{N/m}$, is attached to B and is compressed 0.3 m against A and B as shown. Determine the maximum angles θ and ϕ of the cords when the blocks are released from rest and the spring becomes unstretched.



SOLUTION

$(\overset{+}{\rightarrow})$ $\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 + 0 = -5v_A + 5v_B$$

$$v_A = v_B = v$$

Just before the blocks begin to rise:

$$T_1 + V_1 = T_2 + V_2$$

$$(0 + 0) + \frac{1}{2}(60)(0.3)^2 = \frac{1}{2}(5)(v)^2 + \frac{1}{2}(5)(v)^2 + 0$$

$$v = 0.7348\ \text{m/s}$$

For A or B:

Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

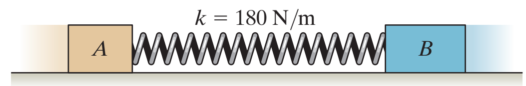
$$\frac{1}{2}(5)(0.7348)^2 + 0 = 0 + 5(9.81)(2)(1 - \cos\theta)$$

$\theta = \phi = 9.52°$ **Ans.**

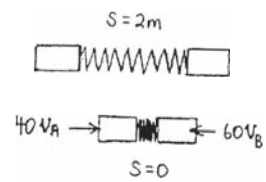
Ans:
$\theta = \phi = 9.52°$

15–53.

Blocks A and B have masses of 40 kg and 60 kg, respectively. They are placed on a smooth surface and the spring connected between them is stretched 2 m. If they are released from rest, determine the speeds of both blocks the instant the spring becomes unstretched.



SOLUTION



$(\overset{+}{\rightarrow})$ $\Sigma m v_1 = \Sigma m v_2$

$0 + 0 = 40\, v_A - 60\, v_B$

$T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}(180)(2)^2 = \frac{1}{2}(40)(v_A)^2 + \frac{1}{2}(60)(v_B)^2$

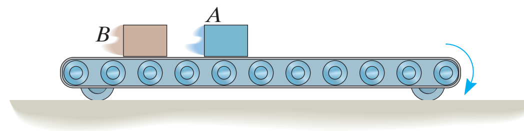
$v_A = 3.29$ m/s **Ans.**

$v_B = 2.19$ m/s **Ans.**

Ans:
$v_A = 3.29$ m/s
$v_B = 2.19$ m/s

15–54.

Two boxes A and B, each having a mass of 80 kg, sit on the 250-kg conveyor which is free to roll on the ground. If the belt starts from rest and begins to run with a speed of 1 m/s, determine the final speed of the conveyor if (a) the boxes are not stacked and A falls off then B falls off, and (b) A is stacked on top of B and both fall off together.



SOLUTION

a) Let v_b be the velocity of A and B.

$(\overset{+}{\rightarrow})\qquad \Sigma m v_1 = \Sigma m v_2$

$$0 = (160)(v_b) - (250)(v_c)$$

$(\overset{+}{\rightarrow})\qquad v_b = v_c + v_{b/c}$

$$v_b = -v_c + 1$$

Thus, $v_b = 0.610 \text{ m/s} \rightarrow \qquad v_c = 0.390 \text{ m/s} \leftarrow$

When a box falls off, it exerts no impulse on the conveyor, and so does not alter the momentum of the conveyor. Thus,

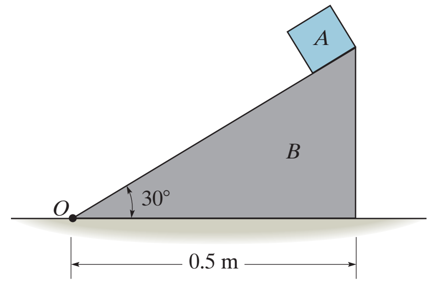
a) $v_c = 0.390 \text{ m/s} \leftarrow$ **Ans.**

b) $v_c = 0.390 \text{ m/s} \leftarrow$ **Ans.**

Ans:
$v_C = 0.390 \text{ m/s} \leftarrow$
$v_C = 0.390 \text{ m/s} \leftarrow$

15–55.

Block A has a mass of 5 kg and is placed on the smooth triangular block B having a mass of 30 kg. If the system is released from rest, determine the distance B moves from point O when A reaches the bottom. Neglect the size of block A.

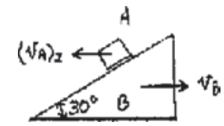


SOLUTION

$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$$0 = 30v_B - 5(v_A)_x$$

$$(v_A)_x = 6v_B$$



$$v_B = v_A + v_{B/A}$$

$(\overset{+}{\rightarrow})$ $\quad v_B = -(v_A)_x + (v_{B/A})_x$

$$v_B = -6v_B + (v_{B/A})_x$$

$$(v_{B/A})_x = 7v_B$$

Integrate

$$(s_{B/A})_x = 7s_B$$

$$(s_{B/A})_x = 0.5 \text{ m}$$

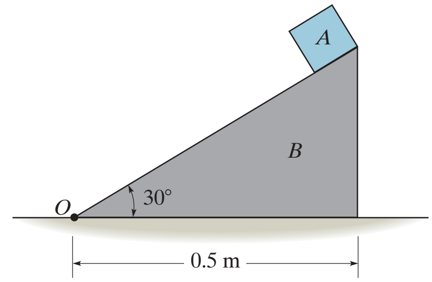
Thus,

$$s_B = \frac{0.5}{7} = 0.0714 \text{ m} = 71.4 \text{ mm} \rightarrow$$ **Ans.**

Ans:
$s_B = 71.4 \text{ mm} \rightarrow$

***15–56.**

Solve Prob. 15–55 if the coefficient of kinetic friction between A and B is $\mu_k = 0.3$. Neglect friction between block B and the horizontal plane.



SOLUTION

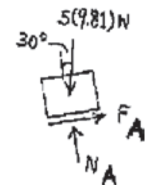
$+\nwarrow \Sigma F_y = 0; \qquad N_A - 5(9.81)\cos 30° = 0$

$N_A = 42.4785 \text{ N}$

$\nearrow + \Sigma F_x = 0; \qquad F_A - 5(9.81)\sin 30° = 0$

$F_A = 24.525 \text{ N}$

$F_{max} = \mu N_A = 0.3(42.4785) = 12.74 \text{ N} < 24.525 \text{ N}$



Block indeed slides.

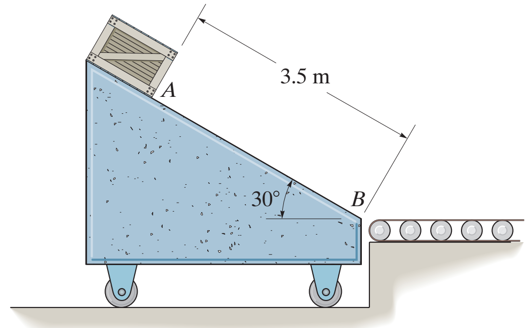
Solution is the same as in Prob. 15–55. Since F_A is internal to the system.

$s_B = 71.4 \text{ mm} \rightarrow$ **Ans.**

Ans:
$s_B = 71.4 \text{ mm} \rightarrow$

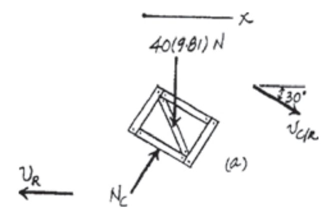
15–57.

The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at A and slides down 3.5 m to point B. If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches B. Also, what is the velocity of the crate?



SOLUTION

Conservation of Energy: The datum is set at lowest point B. When the crate is at point A, it is $3.5 \sin 30^\circ = 1.75$ m *above* the datum. Its gravitational potential energy is $10(9.81)(1.75) = 171.675\ \text{N}\cdot\text{m}$. Applying Eq. 14–21, we have

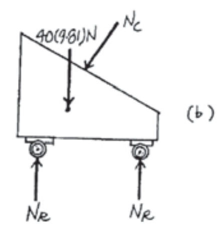


$$T_1 + V_1 = T_2 + V_2$$

$$0 + 171.675 = \frac{1}{2}(10)v_C^2 + \frac{1}{2}(40)v_R^2$$

$$171.675 = 5v_C^2 + 20v_R^2 \qquad \textbf{(1)}$$

Relative Velocity: The velocity of the crate is given by



$$\mathbf{v}_C = \mathbf{v}_R + \mathbf{v}_{C/R}$$

$$= -v_R\mathbf{i} + (v_{C/R}\cos 30^\circ\mathbf{i} - v_{C/R}\sin 30^\circ\mathbf{j})$$

$$= (0.8660\,v_{C/R} - v_R)\mathbf{i} - 0.5\,v_{C/R}\mathbf{j} \qquad \textbf{(2)}$$

The magnitude of v_C is

$$v_C = \sqrt{(0.8660\,v_{C/R} - v_R)^2 + (-0.5\,v_{C/R})^2}$$

$$= \sqrt{v_{C/R}^2 + v_R^2 - 1.732\,v_R v_{C/R}} \qquad \textbf{(3)}$$

Conservation of Linear Momentum: If we consider the crate and the ramp as a system, from the FBD, one realizes that the normal reaction N_C (*impulsive force*) is *internal* to the system and will cancel each other. As the result, the linear momentum is conserved along the x axis.

$$0 = m_C(v_C)_x + m_R v_R$$

$$(\overset{+}{\rightarrow}) \qquad 0 = 10(0.8660\,v_{C/R} - v_R) + 40(-v_R)$$

$$0 = 8.660\,v_{C/R} - 50\,v_R \qquad \textbf{(4)}$$

Solving Eqs. (1), (3), and (4) yields

$$v_R = 1.101\ \text{m/s} = 1.10\ \text{m/s} \quad v_C = 5.43\ \text{m/s} \qquad \textbf{Ans.}$$

$$v_{C/R} = 6.356\ \text{m/s}$$

From Eq. (2)

$$\mathbf{v}_C = [0.8660(6.356) - 1.101]\mathbf{i} - 0.5(6.356)\mathbf{j} = \{4.403\mathbf{i} - 3.178\mathbf{j}\}\ \text{m/s}$$

Thus, the directional angle ϕ of v_C is

$$\phi = \tan^{-1}\frac{3.178}{4.403} = 35.8^\circ \quad ⦬\phi \qquad \textbf{Ans.}$$

Ans:
$v_{C/R} = 6.356\ \text{m/s}$
$\phi = 35.8^\circ$ ⦬

15–58.

Disk A has a mass of 250 g and is sliding on a *smooth* horizontal surface with an initial velocity $(v_A)_1 = 2\text{ m/s}$. It makes a direct collision with disk B, which has a mass of 175 g and is originally at rest. If both disks are of the same size and the collision is perfectly elastic ($e = 1$), determine the velocity of each disk just after collision. Show that the kinetic energy of the disks before and after collision is the same.

SOLUTION

$(\overset{+}{\rightarrow})\quad (0.250)(2) + 0 = (0.250)(v_A)_2 + (0.175)(v_B)_2$

$(\overset{+}{\rightarrow})\quad e = 1 = \dfrac{(v_B)_2 - (v_A)_2}{2 - 0}$

Solving

$(v_A)_2 = 0.353\text{ m/s}$ **Ans.**

$(v_B)_2 = 2.35\text{ m/s}$ **Ans.**

$$T_1 = \frac{1}{2}(0.25)(2)^2 = 0.5\text{ J}$$

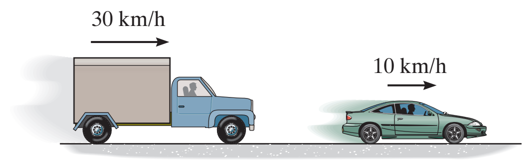
$$T_2 = \frac{1}{2}(0.25)(0.353)^2 + \frac{1}{2}(0.175)(2.35)^2 = 0.5\text{ J}$$

$T_1 = T_2$ **QED**

Ans:
$(v_A)_2 = 0.353\text{ m/s}$
$(v_B)_2 = 2.35\text{ m/s}$

15–59.

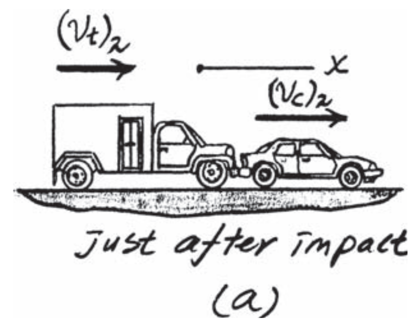
The 5-Mg truck and 2-Mg car are traveling with the free-rolling velocities shown just before they collide. After the collision, the car moves with a velocity of 15 km/h to the right *relative* to the truck. Determine the coefficient of restitution between the truck and car and the loss of energy due to the collision.



SOLUTION

Conservation of Linear Momentum: The linear momentum of the system is conserved along the x axis (line of impact).

The initial speeds of the truck and car are $(v_t)_1 = \left[30\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 8.333\text{ m/s}$ and $(v_c)_1 = \left[10\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 2.778\text{ m/s}$.



By referring to Fig. *a*,

$$(\overset{+}{\rightarrow}) \qquad m_t(v_t)_1 + m_c(v_c)_1 = m_t(v_t)_2 + m_c(v_c)_2$$

$$5000(8.333) + 2000(2.778) = 5000(v_t)_2 + 2000(v_c)_2$$

$$5(v_t)_2 + 2(v_c)_2 = 47.22 \qquad \textbf{(1)}$$

Coefficient of Restitution: Here, $(v_{c/t}) = \left[15\left(10^3\right)\dfrac{\text{m}}{\text{h}}\right]\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 4.167\text{ m/s} \rightarrow$.
Applying the relative velocity equation,

$$(\mathbf{v}_c)_2 = (\mathbf{v}_t)_2 + (\mathbf{v}_{c/t})_2$$

$$(\overset{+}{\rightarrow}) \qquad (v_c)_2 = (v_t)_2 + 4.167$$

$$(v_c)_2 - (v_t)_2 = 4.167 \qquad \textbf{(2)}$$

Applying the coefficient of restitution equation,

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_c)_2 - (v_t)_2}{(v_t)_1 - (v_c)_1}$$

$$e = \frac{(v_c)_2 - (v_t)_2}{8.333 - 2.778} \qquad \textbf{(3)}$$

15–59. Continued

Substituting Eq. (2) into Eq. (3),

$$e = \frac{4.167}{8.333 - 2.778} = 0.75 \qquad \textbf{Ans.}$$

Solving Eqs. (1) and (2) yields

$$(v_t)_2 = 5.556 \text{ m/s}$$

$$(v_c)_2 = 9.722 \text{ m/s}$$

Kinetic Energy: The kinetic energy of the system just before and just after the collision are

$$T_1 = \frac{1}{2} m_t (v_t)_1{}^2 + \frac{1}{2} m_c (v_c)_1{}^2$$

$$= \frac{1}{2}(5000)(8.333^2) + \frac{1}{2}(2000)(2.778^2)$$

$$= 181.33\left(10^3\right) \text{ J}$$

$$T_2 = \frac{1}{2} m_t (v_t)_2{}^2 + \frac{1}{2} m_c (v_c)_2{}^2$$

$$= \frac{1}{2}(5000)(5.556^2) + \frac{1}{2}(2000)(9.722^2)$$

$$= 171.68\left(10^3\right) \text{J}$$

Thus,

$$\Delta T = T_1 - T_2 = 181.33\left(10^3\right) - 171.68\left(10^3\right)$$

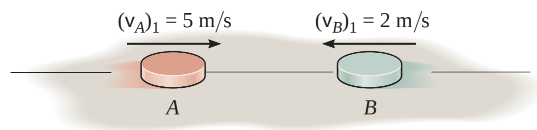
$$= 9.645\left(10^3\right) \text{ J}$$

$$= 9.65 \text{ kJ} \qquad \textbf{Ans.}$$

Ans:
$e = 0.75$
$\Delta T = -9.65$ kJ

***15–60.**

Disk A has a mass of 2 kg and is sliding forward on the *smooth* surface with a velocity $(v_A)_1 = 5$ m/s when it strikes the 4-kg disk B, which is sliding towards A at $(v_B)_1 = 2$ m/s, with direct central impact. If the coefficient of restitution between the disks is $e = 0.4$, compute the velocities of A and B just after collision.

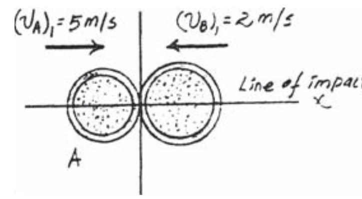


SOLUTION

Conservation of Momentum :

$$m_A\,(v_A)_1 + m_B\,(v_B)_1 = m_A\,(v_A)_2 + m_B\,(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad 2(5) + 4(-2) = 2(v_A)_2 + 4(v_B)_2 \qquad \textbf{(1)}$$



Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.4 = \frac{(v_B)_2 - (v_A)_2}{5 - (-2)} \qquad \textbf{(2)}$$

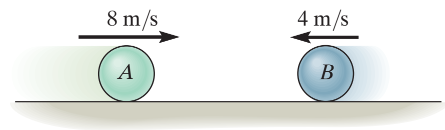
Solving Eqs. (1) and (2) yields

$$(v_A)_2 = -1.53 \text{ m/s} = 1.53 \text{ m/s} \leftarrow \qquad (v_B)_2 = 1.27 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

Ans:
$(v_A)_2 = 1.53$ m/s $\leftarrow$
$(v_B)_2 = 1.27$ m/s $\rightarrow$

15–61.

Ball A has a mass of 3 kg and is moving with a velocity of 8 m/s when it makes a direct collision with ball B, which has a mass of 2 kg and is moving with a velocity of 4 m/s. If $e = 0.7$, determine the velocity of each ball just after the collision. Neglect the size of the balls.



SOLUTION

Conservation of Momentum. The velocity of balls A and B before and after impact are shown in Fig. a

$$(\overset{+}{\rightarrow}) \quad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$3(8) + 2(-4) = 3v_A + 2v_B$$

$$3v_A + 2v_B = 16 \qquad \textbf{(1)}$$

Coefficient of Restitution.

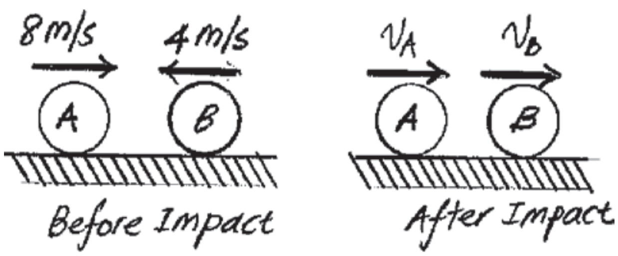
$$(\overset{+}{\rightarrow}) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \quad 0.7 = \frac{v_B - v_A}{8 - (-4)}$$

$$v_B - v_A = 8.4 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$v_B = 8.24 \text{ m/s} \rightarrow$ **Ans.**

$v_A = -0.16 \text{ m/s} = 0.160 \text{ m/s} \leftarrow$ **Ans.**

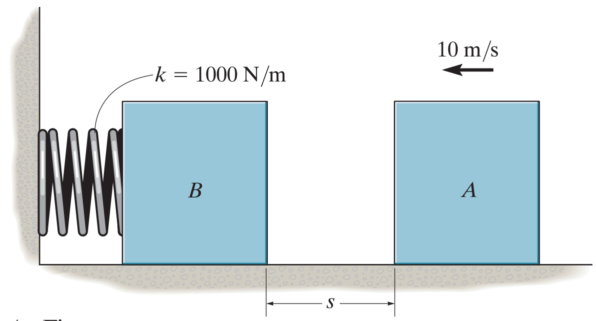


(a)

Ans:
$v_B = 8.24 \text{ m/s} \rightarrow$
$v_A = 0.160 \text{ m/s} \leftarrow$

15–62.

The 15-kg block A slides on the surface for which $m_k = 0.3$. The block has a velocity $v = 10$ m/s when it is $s = 4$ m from the 10-kg block B. If the unstretched spring has a stiffness $k = 1000$ N/m, determine the maximum compression of the spring due to the collision. Take $e = 0.6$.



SOLUTION

Principle of Work and Energy. Referring to the FBD of block A, Fig. a, motion along the y axis gives $N_A = 15(9.81) = 147.15$ N. Thus the friction is $F_f = \mu_k N_A = 0.3(147.15) = 44.145$ N.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(15)(10^2) + (-44.145)(4) = \frac{1}{2}(15)(v_A)_1^2$$

$$(v_A)_1 = 8.7439 \text{ m/s} \leftarrow$$

Conservation of Momentum.

$$(\overset{+}{\leftarrow}) \quad m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$15(8.7439) + 0 = 15(v_A)_2 + 10(v_B)_2$$

$$3(v_A)_2 + 2(v_B)_2 = 26.2317 \qquad \textbf{(1)}$$

Coefficient of Restitution.

$$(\overset{+}{\leftarrow}) \quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.6 = \frac{(v_B)_2 - (v_A)_2}{8.7439 - 0}$$

$$(v_B)_2 - (v_A)_2 = 5.2463 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

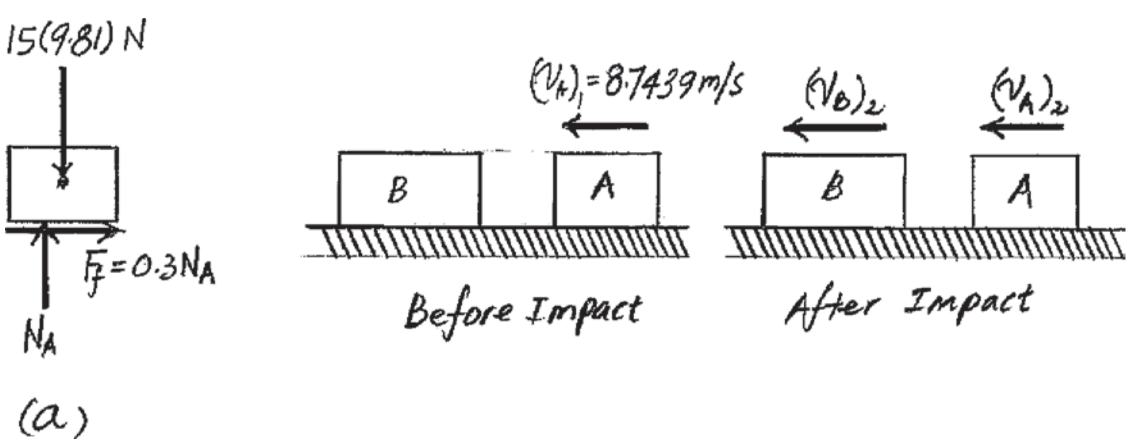
$$(v_B)_2 = 8.3942 \text{ m/s} \leftarrow \quad (v_A)_2 = 3.1478 \text{ m/s} \leftarrow$$

Conservation of Energy. When block B stops momentarily, the compression of the spring is maximum. Thus, $T_2 = 0$.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(10)(8.3942^2) + 0 = 0 + \frac{1}{2}(1000)x_{max}^2$$

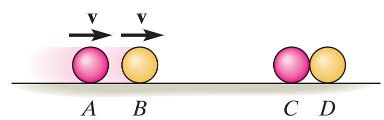
$$x_{max} = 0.8394 \text{ m} = 0.839 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$x_{max} = 0.839$ m

15–63.

The four smooth balls each have the same mass m. If A and B are rolling forward with velocity $\mathbf{v}$ and strike C, explain why after collision C and D each move off with velocity $\mathbf{v}$. Why doesn't D move off with velocity $2\mathbf{v}$? The collision is elastic, $e = 1$. Neglect the size of each ball.



SOLUTION

Collision will occur in the following sequence;

B strikes C

$(\overset{+}{\rightarrow})\quad mv = -mv_B + mv_C$

$v = -v_B + v_C$

$(\overset{+}{\rightarrow})\quad e = 1 = \dfrac{v_C + v_B}{v}$

$v_C = v, \quad v_B = 0$

C strikes D

$(\overset{+}{\rightarrow})\quad mv = -mv_C + mv_D$

$(\overset{+}{\rightarrow})\quad e = 1 = \dfrac{v_D + v_C}{v}$

$v_C = 0, \quad v_D = v$ **Ans.**

A strikes B

$(\overset{+}{\rightarrow})\quad mv = -mv_A + mv_B$

$(\overset{+}{\rightarrow})\quad e = 1 = \dfrac{v_B + v_A}{v}$

$v_B = v, \quad v_A = 0$ **Ans.**

Finally, B strikes C

$(\overset{+}{\rightarrow})\quad mv = -mv_B + mv_C$

$(\overset{+}{\rightarrow})\quad e = 1 = \dfrac{v_C + v_B}{v}$

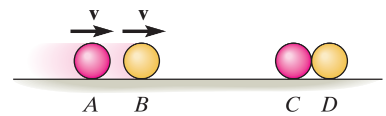
$v_C = v, \quad v_B = 0$ **Ans.**

Note: If D rolled off with twice the velocity, its kinetic energy would be twice the energy available from the original two A and B: $\left(\frac{1}{2}mv^2 + \frac{1}{2}mv^2 \neq \frac{1}{2}(2v)^2\right)$

Ans:
$v_C = 0, v_D = v$
$v_B = v, v_A = 0$
$v_C = v, v_B = 0$

***15–64.**

The four balls each have the same mass m. If A and B are rolling forward with velocity **v** and strike C, determine the velocity of each ball after the first three collisions. Take $e = 0.5$ between each ball.



SOLUTION

Collision will occur in the following sequence;

B strikes C

$(\overset{+}{\rightarrow})$ $$mv = mv_B + mv_c$$

$$v = v_B + v_C$$

$(\overset{+}{\rightarrow})$ $$e = 0.5 = \frac{v_C - v_B}{v}$$

$$v_C = 0.75v \rightarrow, \qquad v_B = 0.25v \rightarrow$$

C strikes D

$(\overset{+}{\rightarrow})$ $$m(0.75v) = mv_C + mv_D$$

$(\overset{+}{\rightarrow})$ $$e = 0.5 = \frac{v_D - v_C}{0.75v}$$

$$v_C = 0.1875v \rightarrow$$ **Ans.**

$$v_D = 0.5625v \rightarrow$$ **Ans.**

A strikes B

$(\overset{+}{\rightarrow})$ $$mv + m(0.25v) = mv_A + mv_B$$

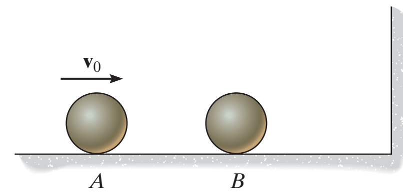
$(\overset{+}{\rightarrow})$ $$e = 0.5 = \frac{v_B - v_A}{(v - 0.25v)}$$

$$v_B = 0.8125v \rightarrow, \qquad v_A = 0.4375v \rightarrow$$ **Ans.**

Ans:
$v_C = 0.1875v \rightarrow$
$v_D = 0.5625v \rightarrow$
$v_B = 0.8125v \rightarrow$
$v_A = 0.4375v \rightarrow$

15–65.

Two smooth spheres A and B each have a mass m. If A is given a velocity of v_0, while sphere B is at rest, determine the velocity of B just after it strikes the wall. The coefficient of restitution for any collision is e.



SOLUTION

Impact: The first impact occurs when sphere A strikes sphere B. When this occurs, the linear momentum of the system is conserved along the x axis (line of impact). Referring to Fig. a,

$$(\overset{+}{\rightarrow}) \qquad m_A v_A + m_B v_B = m_A (v_A)_1 + m_B (v_B)_1$$

$$m v_0 + 0 = m(v_A)_1 + m(v_B)_1$$

$$(v_A)_1 + (v_B)_1 = v_0 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_1 - (v_A)_1}{v_A - v_B}$$

$$e = \frac{(v_B)_1 - (v_A)_1}{v_0 - 0}$$

$$(v_B)_1 - (v_A)_1 = e v_0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

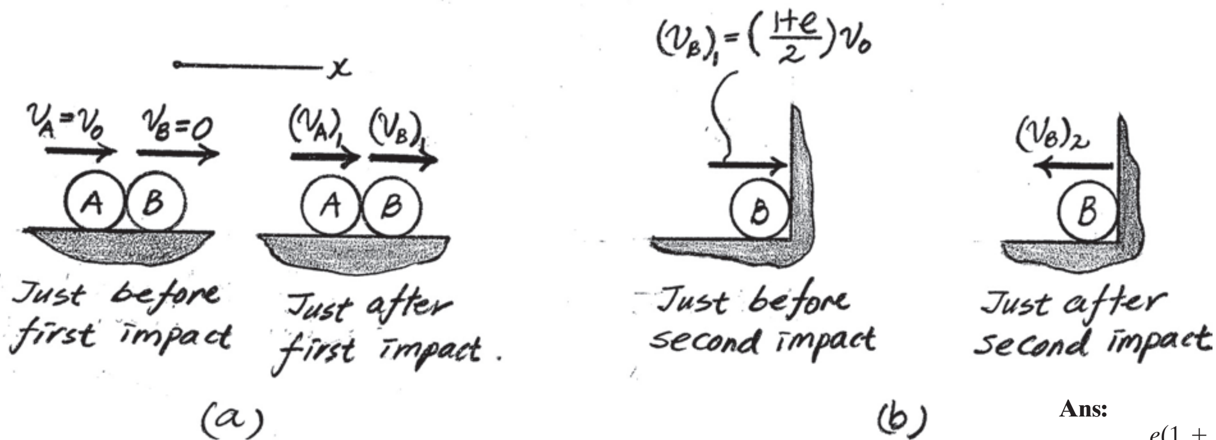
$$(v_B)_1 = \left(\frac{1+e}{2}\right) v_0 \rightarrow \qquad (v_A)_1 = \left(\frac{1-e}{2}\right) v_0 \rightarrow$$

The second impact occurs when sphere B strikes the wall, Fig. b. Since the wall does not move during the impact, the coefficient of restitution can be written as

$$(\overset{+}{\rightarrow}) \qquad e = \frac{0 - \left[-(v_B)_2\right]}{(v_B)_1 - 0}$$

$$e = \frac{0 + (v_B)_2}{\left[\frac{1+e}{2}\right] v_0 - 0}$$

$$(v_B)_2 = \frac{e(1+e)}{2} v_0 \qquad \textbf{Ans.}$$

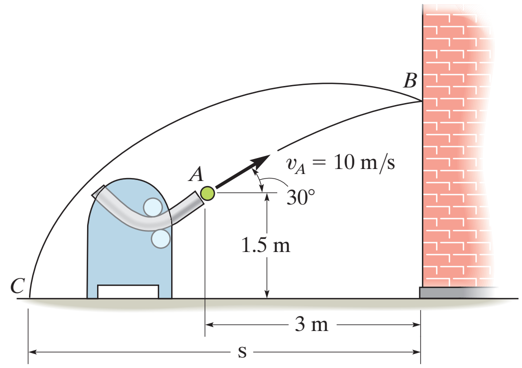


Ans:

$$(v_B)_2 = \frac{e(1+e)}{2} v_0$$

15–66.

A pitching machine throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10$ m/s as shown. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



SOLUTION

(a)

$(v_B)_{x1} = 10 \cos 30^\circ = 8.660 \text{ m/s} \rightarrow$

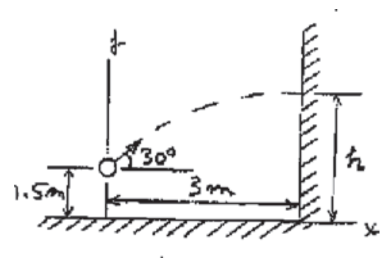
$(\overset{+}{\rightarrow})$ $\quad s = s_0 + v_0 t$

$3 = 0 + 10 \cos 30^\circ t$

$t = 0.3464$ s

$(+\uparrow)$ $\quad v = v_0 + a_c t$

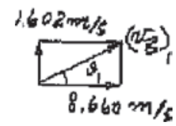
$(v_B)_{y\,t} = 10 \sin 30^\circ - 9.81(0.3464) = 1.602 \text{ m/s} \uparrow$



$(+\uparrow)$ $\quad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

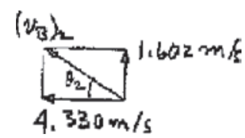
$h = 1.5 + 10 \sin 30^\circ(0.3464) - \frac{1}{2}(9.81)(0.3464)^2$

$= 2.643$ m



$(v_B)_1 = \sqrt{(1.602)^2 + (8.660)^2} = 8.81$ m/s **Ans.**

$\theta_1 = \tan^{-1}\left(\frac{1.602}{8.660}\right) = 10.5^\circ$ ↗ **Ans.**



(b)

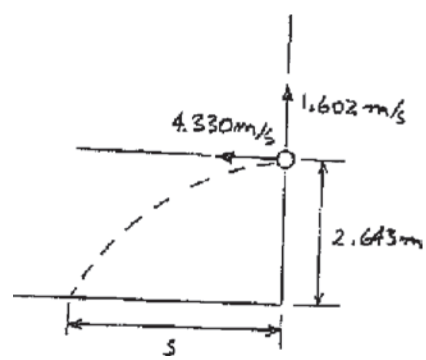
$(\overset{+}{\rightarrow})$ $\quad e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}; \qquad 0.5 = \frac{(v_{Bx})_2 - 0}{0 - (8.660)}$

$(v_{Bx})_2 = 4.330 \text{ m/s} \leftarrow$

$(v_{By})_2 = (v_{By})_1 = 1.602 \text{ m/s} \uparrow$

$(v_B)_2 = \sqrt{(4.330)^2 + (1.602)^2} = 4.62$ m/s **Ans.**

$\theta_2 = \tan^{-1}\left(\frac{1.602}{4.330}\right) = 20.3^\circ$ ↖ **Ans.**



(c)

$(+\uparrow)$ $\quad s = s_0 + v_{Bt} + \frac{1}{2} a_c t^2$

$-2.643 = 0 + 1.602(t) - \frac{1}{2}(9.81)(t)^2$

$t = 0.9153$ s

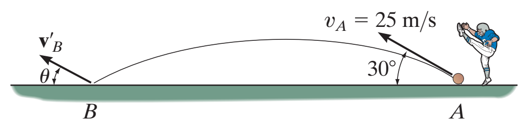
$(\overset{+}{\leftarrow})$ $\quad s = s_0 + v_0 t$

$s = 0 + 4.330(0.9153) = 3.96$ m **Ans.**

Ans:
$(v_B)_1 = 8.81$ m/s
$\theta_1 = 10.5^\circ$ ↗
$(v_B)_2 = 4.62$ m/s
$\theta_2 = 20.3^\circ$ ↖
$s = 3.96$ m

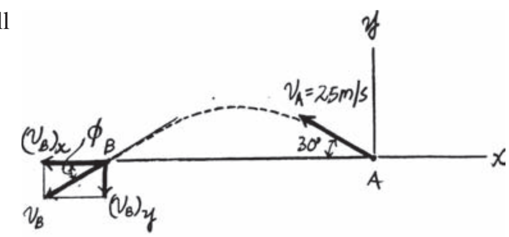
15–67.

A 300-g ball is kicked with a velocity of $v_A = 25$ m/s at point A as shown. If the coefficient of restitution between the ball and the field is $e = 0.4$, determine the magnitude and direction θ of the velocity of the rebounding ball at B.



SOLUTION

Kinematics: The parabolic trajectory of the football is shown in Fig. *a*. Due to the symmetrical properties of the trajectory, $v_B = v_A = 25$ m/s and $\phi = 30°$.



(a)

Conservation of Linear Momentum: Since no impulsive force acts on the football along the x axis, the linear momentum of the football is conserved along the x axis.

$$(\overset{+}{\leftarrow}) \qquad m(v_B)_x = m(v'_B)_x$$

$$0.3(25\cos 30°) = 0.3(v'_B)_x$$

$$(v'_B)_x = 21.65 \text{ m/s} \leftarrow$$

Coefficient of Restitution: Since the ground does not move during the impact, the coefficient of restitution can be written as

$$(+\uparrow) \qquad e = \frac{0 - (v'_B)_y}{(v_B)_y - 0}$$

$$0.4 = \frac{-(v'_B)_y}{-25\sin 30°}$$

$$(v'_B)_y = 5 \text{ m/s} \uparrow$$

Thus, the magnitude of $\mathbf{v}'_B$ is

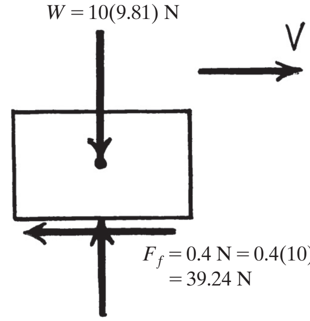
$$v'_B = \sqrt{(v'_B)_x + (v'_B)_y} = \sqrt{21.65^2 + 5^2} = 22.2 \text{ m/s} \qquad \textbf{Ans.}$$

and the angle of $\mathbf{v}'_B$ is

$$\theta = \tan^{-1}\left[\frac{(v'_B)_y}{(v'_B)_x}\right] = \tan^{-1}\left(\frac{5}{21.65}\right) = 13.0° \qquad \textbf{Ans.}$$

Ans:
$v'_B = 22.2$ m/s
$\theta = 13.0°$

***15–68.** The 1-kg ball A is traveling horizontally at 20 m/s when it strikes a 10-kg block B that is at rest. If the coefficient of restitution between A and B is $e = 0.6$, and the coefficient of kinetic friction between the plane and the block is $\mu_k = 0.4$, determine the distance block B slides on the plane before it stops sliding.



SOLUTION

$(\overset{+}{\rightarrow})$

$$\Sigma m_1 v_1 = \Sigma m_2 v_2$$

$$(1)(20) + 0 = (1)(v_A)_2 + (10)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 20$$

$(\overset{+}{\rightarrow})$

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$0.6 = \frac{(v_B)_2 - (v_A)_2}{20 - 0}$$

$$(v_B)_2 - (v_A)_2 = 12$$

Thus,

$(v_B)_2 = 2.909 \text{ m/s} \rightarrow$

$(v_A)_2 = -9.091 \text{ m/s} = 9.091 \text{ m/s} \leftarrow$

Block B:

$$T_1 + \Sigma U_{1-2} = T_2$$

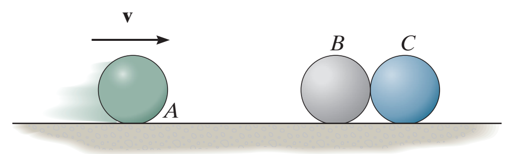
$$\frac{1}{2}(10)(2.909)^2 - 39.24d = 0$$

$d = 1.078$ m **Ans.**

Ans:
$d = 1.078$ m

15–69.

The three balls each have a mass m. If A has a speed v just before a direct collision with B, determine the speed of C after collision. The coefficient of restitution between each pair of balls e. Neglect the size of each ball.



SOLUTION

Conservation of Momentum: When ball A strikes ball B, we have

$$m_A\,(v_A)_1 + m_B\,(v_B)_1 = m_A\,(v_A)_2 + m_B\,(v_B)_2$$

$$(\overset{+}{\rightarrow}) \qquad mv + 0 = m(v_A)_2 + m(v_B)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_2 - (v_A)_2}{v - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$(v_A)_2 = \frac{v(1 - e)}{2} \qquad (v_B)_2 = \frac{v(1 + e)}{2}$$

Conservation of Momentum: When ball B strikes ball C, we have

$$m_B\,(v_B)_2 + m_C\,(v_C)_1 = m_B\,(v_B)_3 + m_C\,(v_C)_2$$

$$(\overset{+}{\rightarrow}) \qquad m\left[\frac{v(1 + e)}{2}\right] + 0 = m(v_B)_3 + m(v_C)_2 \qquad \textbf{(3)}$$

Coefficient of Restitution:

$$e = \frac{(v_C)_2 - (v_B)_3}{(v_B)_2 - (v_C)_1}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_C)_2 - (v_B)_3}{\frac{v(1 + e)}{2} - 0} \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$(v_C)_2 = \frac{v(1 + e)^2}{4} \qquad \textbf{Ans.}$$

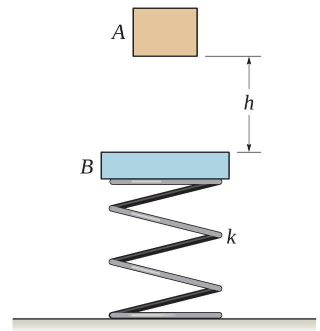
$$(v_B)_3 = \frac{v(1 - e^2)}{4}$$

Ans:

$$(v_C)_2 = \frac{v(1 + e)^2}{4}$$

15–70.

Block A, having a mass m, is released from rest, falls a distance h and strikes the plate B having a mass $2m$. If the coefficient of restitution between A and B is e, determine the velocity of the plate just after collision. The spring has a stiffness k.



SOLUTION

Just before impact, the velocity of A is

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}mv_A^2 - mgh$$

$$v_A = \sqrt{2gh}$$

$$(+\downarrow) \qquad e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}$$

$$e\sqrt{2gh} = (v_B)_2 - (v_A)_2 \qquad \textbf{(1)}$$

$$(+\downarrow) \qquad \Sigma mv_1 = \Sigma mv_2$$

$$m(v_A) + 0 = m(v_A)_2 + 2m(v_B)_2 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) for $(v_B)_2$ yields;

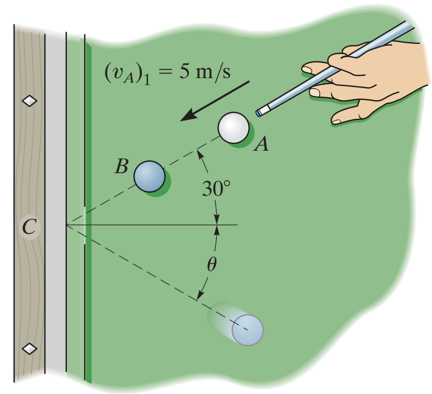
$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}(1 + e) \qquad \textbf{Ans.}$$

Ans:

$(v_B)_2 = \frac{1}{3}\sqrt{2gh}\,(1 + e)$

15–71.

The cue ball A is given an initial velocity $(v_A)_1 = 5$ m/s. If it makes a direct collision with ball B ($e = 0.8$), determine the velocity of B and the angle θ just after it rebounds from the cushion at C ($e' = 0.6$). Each ball has a mass of 0.4 kg. Neglect their size.



SOLUTION

Conservation of Momentum: When ball A strikes ball B, we have

$$m_A(v_A)_1 + m_B(v_B)_1 = m_A(v_A)_2 + m_B(v_B)_2$$

$$0.4(5) + 0 = 0.4(v_A)_2 + 0.4(v_B)_2 \quad \textbf{(1)}$$

Coefficient of Restitution:

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.8 = \frac{(v_B)_2 - (v_A)_2}{5 - 0} \quad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$(v_A)_2 = 0.500$ m/s $\quad (v_B)_2 = 4.50$ m/s

Conservation of "y" Momentum: When ball B strikes the cushion at C, we have

$$m_B(v_{B_y})_2 = m_B(v_{B_y})_3$$

$$(+\downarrow) \qquad 0.4(4.50 \sin 30°) = 0.4(v_B)_3 \sin\theta$$

$$(v_B)_3 \sin\theta = 2.25 \quad \textbf{(3)}$$

Coefficient of Restitution (x):

$$e = \frac{(v_C)_2 - (v_{B_x})_3}{(v_{B_x})_2 - (v_C)_1}$$

$$(\overset{+}{\leftarrow}) \qquad 0.6 = \frac{0 - [-(v_B)_3\ \cos\theta]}{4.50 \cos 30° - 0} \quad \textbf{(4)}$$

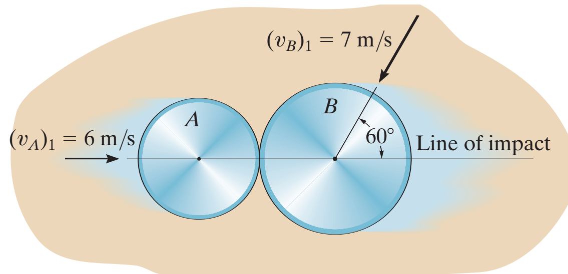
Solving Eqs. (1) and (2) yields

$(v_B)_3 = 3.24$ m/s $\quad \theta = 43.9°$ **Ans.**

Ans:
$(v_B)_3 = 3.24$ m/s
$\theta = 43.9°$

***15–72.**

The two disks A and B have a mass of 3 kg and 5 kg, respectively. If they collide with the initial velocities shown, determine their velocities just after impact. The coefficient of restitution is $e = 0.65$.



SOLUTION

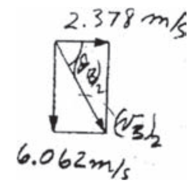
$(v_{Ax}) = 6$ m/s $\quad (v_{Ay})_1 = 0$

$(v_{Bx})_1 = -7\cos 60° = -3.5$ m/s $\quad (v_{By})_1 = -7\cos 60° = -6.062$ m/s

$(\overset{+}{\rightarrow})\quad m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$

$3(6) - 5(3.5) = 3(v_A)_{x2} + 5(v_B)_{x2}$

$(\overset{+}{\rightarrow})\quad e = \dfrac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1};\qquad 0.65 = \dfrac{(v_{Bx})_2 - (v_{Ax})_2}{6-(-3.5)}$



$(v_{Bx})_2 - (v_{Ax})_2 = 6.175$

Solving,

$(v_{Ax})_2 = -3.80$ m/s $\quad (v_{Bx})_2 = 2.378$ m/s

$(+\uparrow)\quad m_A(v_{Ay})_1 + m_A(v_{Ay})_2$

$(v_{Ay})_2 = 0$

$(+\uparrow)\quad m_B(v_{By})_1 + m_B(v_{Ay})_2$

$(v_{By})_2 = -6.062$ m/s

$(v_A)_2 = \sqrt{(3.80)^2 + (0)^2 =} = 3.80 \text{ m/s} \leftarrow$ **Ans.**

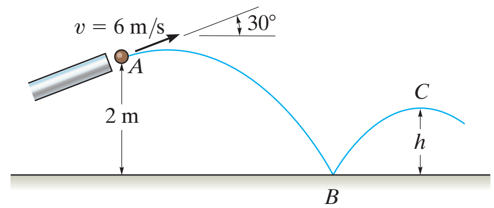
$(v_B)_2 = \sqrt{(2.378)^2 + (-6.062)^2} = 6.51$ m/s **Ans.**

$(\theta_B)_2 = \tan^{-1}\left(\dfrac{6.062}{2.378}\right) = 68.6°$ **Ans.**

Ans:
$(v_A)_2 = 3.80 \text{ m/s} \leftarrow$
$(v_B)_2 = 6.51$ m/s
$(\theta_B)_2 = 68.6°$

15–73.

The 0.5-kg ball is fired from the tube at A with a velocity of $v = 6\text{ m/s}$. If the coefficient of restitution between the ball and the surface is $e = 0.8$, determine the height h after it bounces off the surface.



SOLUTION

Kinematics. Consider the vertical motion from A to B.

$$(+\uparrow)\quad (v_B)_y^2 = (v_A)_y^2 + 2a_y[(s_B)_y - (s_A)_y]\,;$$

$$(v_B)_y^2 = (6\sin 30°)^2 + 2(-9.81)(-2 - 0)$$

$$(v_B)_y = 6.9455\text{ m/s}\downarrow$$

Coefficient of Restitution. The y-component of the rebounding velocity at B is $(v'_B)_y$ and the ground does not move. Then

$$(+\uparrow)\quad e = \frac{(v_g)_2 - (v'_B)_y}{(v_B)_y - (v_g)_1}\,;\qquad 0.8 = \frac{0 - (v'_B)_y}{-6.9455 - 0}$$

$$(v'_B)_y = 5.5564\text{ m/s}\uparrow$$

Kinematics. When the ball reach the maximum height h at C, $(v_c)_y = 0$.

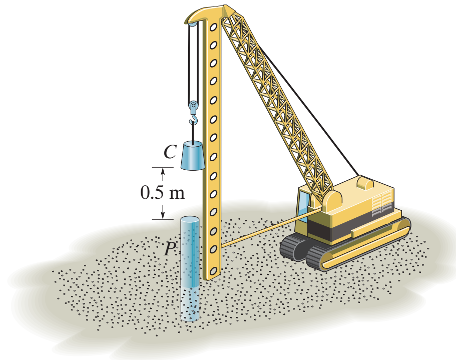
$$(+\uparrow)\quad (v_c)_y^2 = (v'_B)_y^2 + 2a_c[(s_c)_y - (s_B)_y]\,;$$

$$0^2 = 5.5564^2 + 2(-9.81)(h - 0)$$

$$h = 1.574\text{ m} = 1.57\text{ m}$$ **Ans.**

Ans:
$h = 1.57\text{ m}$

15–74. The pile P has a mass of 800 kg and is being driven into *loose sand* using the 300-kg hammer C which is dropped a distance of 0.5 m from the top of the pile. Determine the initial speed of the pile just after it is struck by the hammer. The coefficient of restitution between the hammer and the pile is $e = 0.1$. Neglect the impulses due to the weights of the pile and hammer and the impulse due to the sand during the impact.



SOLUTION

The force of the sand on the pile can be considered nonimpulsive, along with the weights of each colliding body. Hence,

Counter weight: Datum at lowest point.

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300(9.81)(0.5) = \frac{1}{2}(300)(v)^2 + 0$$

$$v = 3.1321 \text{ m/s}$$

System:

$$(+\downarrow) \qquad \Sigma m v_1 = \Sigma m v_2$$

$$300(3.1321) + 0 = 300(v_C)_2 + 800(v_P)_2$$

$$(v_C)_2 + 2.667(v_P)_2 = 3.1321$$

$$(+\downarrow) \qquad e = \frac{(v_P)_2 - (v_C)_2}{(v_C)_1 - (v_P)_1}$$

$$0.1 = \frac{(v_P)_2 - (v_C)_2}{3.1321 - 0}$$

$$(v_P)_2 - (v_C)_2 = 0.31321$$

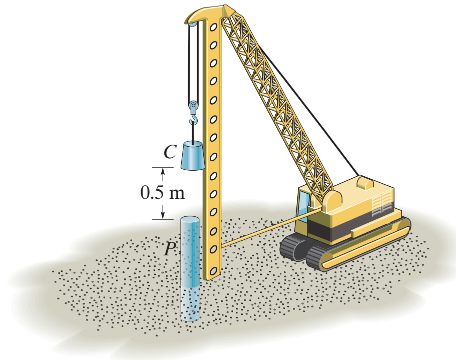
Solving:

$(v_P)_2 = 0.940$ m/s **Ans.**

$(v_C)_2 = 0.626$ m/s

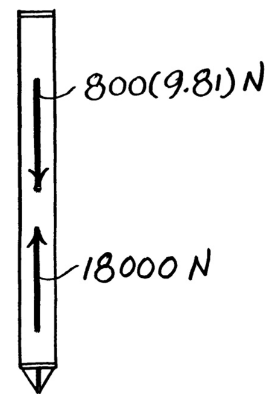
Ans:
$(v_P)_2 = 0.940$ m/s

15–75. The pile P has a mass of 800 kg and is being driven into *loose sand* using the 300-kg hammer C which is dropped a distance of 0.5 m from the top of the pile. Determine the distance the pile is driven into the sand after one blow if the sand offers a frictional resistance against the pile of 18 kN. The coefficient of restitution between the hammer and the pile is $e = 0.1$. Neglect the impulses due to the weights of the pile and hammer and the impulse due to the sand during the impact.



SOLUTION

The force of the sand on the pile can be considered nonimpulsive, along with the weights of each colliding body. Hence,



Counter weight: Datum at lowest point,

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 300(9.81)(0.5) = \frac{1}{2}(300)(v)^2 + 0$$

$$v = 3.1321 \text{ m/s}$$

System:

$$(+\downarrow) \qquad \Sigma m\, v_1 = \Sigma m\, v_2$$

$$300(3.1321) + 0 = 300(v_C)_2 + 800(v_P)_2$$

$$(v_C)_2 + 2.667(v_P)_2 = 3.1321$$

$$(+\downarrow) \qquad e = \frac{(v_P)_2 - (v_C)_2}{(v_C)_1 - (v_P)_1}$$

$$0.1 = \frac{(v_P)_2 - (v_C)_2}{3.1321 - 0}$$

$$(v_P)_2 - (v_C)_2 = 0.31321$$

Solving:

$$(v_P)_2 = 0.9396 \text{ m/s}$$

$$(v_C)_2 = 0.6264 \text{ m/s}$$

Pile:

$$T_2 + \Sigma U_{2-3} = T_3$$

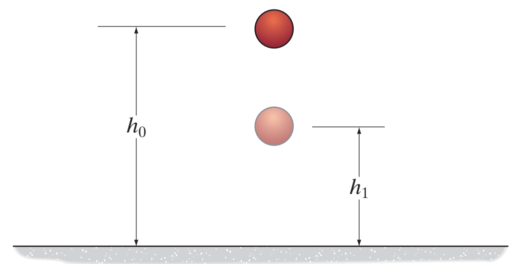
$$\frac{1}{2}(800)(0.9396)^2 + 800(9.81)d - 18\,000d = 0$$

$$d = 0.0348 \text{ m} = 34.8 \text{ mm}$$ **Ans.**

Ans:
$d = 34.8$ mm

***15–76.**

A ball of mass m is dropped vertically from a height h_0 above the ground. If it rebounds to a height of h_1, determine the coefficient of restitution between the ball and the ground.



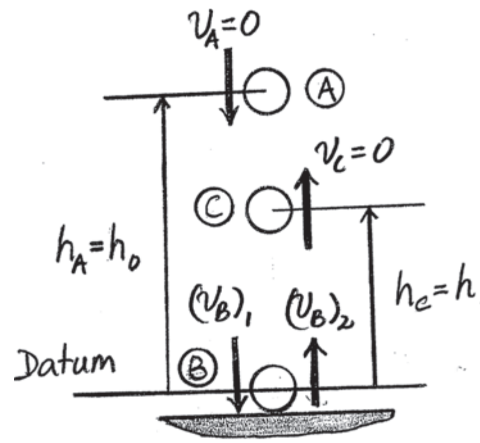
SOLUTION

Conservation of Energy: First, consider the ball's fall from position A to position B. Referring to Fig. a,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}mv_A{}^2 + (V_g)_A = \frac{1}{2}mv_B{}^2 + (V_g)_B$$

$$0 + mg(h_0) = \frac{1}{2}m(v_B)_1^{\,2} + 0$$



(a)

Subsequently, the ball's return from position B to position C will be considered.

$$T_B + V_B = T_C + V_C$$

$$\frac{1}{2}mv_B{}^2 + (V_g)_B = \frac{1}{2}mv_C{}^2 + (V_g)_C$$

$$\frac{1}{2}m(v_B)_2^{\,2} + 0 = 0 + mgh_1$$

$$(v_B)_2 = \sqrt{2gh_1}\uparrow$$

Coefficient of Restitution: Since the ground does not move,

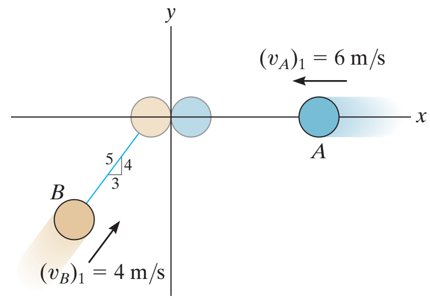
$$(+\uparrow) \qquad e = -\frac{(v_B)_2}{(v_B)_1}$$

$$e = -\frac{\sqrt{2gh_1}}{-\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}}$$ **Ans.**

Ans:
$e = \sqrt{\frac{h_1}{h_0}}$

15–77.

Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine their final velocities just after collision. The coefficient of restitution is $e = 0.75$.



SOLUTION

$(\overset{+}{\rightarrow})\quad \Sigma m v_1 = \Sigma m v_2$

$$0.5(4)\left(\frac{3}{5}\right) - 0.5(6) = 0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$(\overset{+}{\rightarrow})\quad e = \dfrac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$

$$0.75 = \frac{(v_A)_{2x} - (v_B)_{2x}}{4\left(\frac{3}{5}\right) - (-6)}$$

$$(v_A)_{2x} = 1.35 \text{ m/s} \rightarrow$$

$$(v_B)_{2x} = 4.95 \text{ m/s} \leftarrow$$

$(+\uparrow)\quad m v_1 = m v_2$

$$0.5\left(\frac{4}{5}\right)(4) = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$v_A = 1.35 \text{ m/s} \rightarrow$ **Ans.**

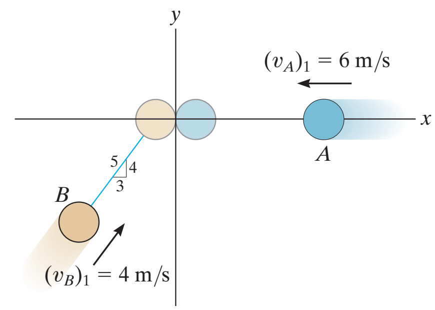
$v_B = \sqrt{(4.59)^2 + (3.20)^2} = 5.89 \text{ m/s}$ **Ans.**

$\theta = \tan^{-1}\dfrac{3.20}{4.95} = 32.9$ ⦣ **Ans.**

Ans:
$v_A = 1.35 \text{ m/s} \rightarrow$
$v_B = 5.89 \text{ m/s}$
$\theta = 32.9°$ ⦣

15–78.

Two smooth disks A and B each have a mass of 0.5 kg. If both disks are moving with the velocities shown when they collide, determine the coefficient of restitution between the disks if after collision B travels along a line, 30° counterclockwise from the y axis.



SOLUTION

$$\Sigma mv_1 = \Sigma mv_2$$

$(\overset{+}{\rightarrow})$ $\quad 0.5(4)(\frac{3}{5}) - 0.5(6) = -0.5(v_B)_{2x} + 0.5(v_A)_{2x}$

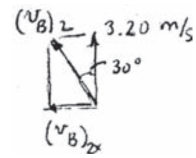
$$-3.60 = -(v_B)_{2x} + (v_A)_{2x}$$

$(+\uparrow)$ $\quad 0.5(4)(\frac{4}{5}) = 0.5(v_B)_{2y}$

$$(v_B)_{2y} = 3.20 \text{ m/s} \uparrow$$

$$(v_B)_{2x} = 3.20 \tan 30° = 1.8475 \text{ m/s} \leftarrow$$

$$(v_A)_{2x} = -1.752 \text{ m/s} = 1.752 \text{ m/s} \leftarrow$$



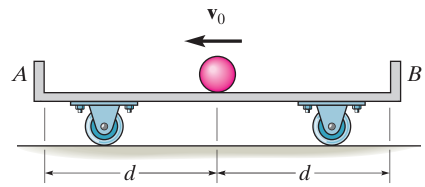
$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$

$$e = \frac{-1.752-(-1.8475)}{4(\frac{3}{5})-(-6)} = 0.0113$$ **Ans.**

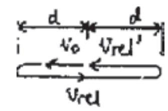
Ans:
$e = 0.0113$

15–79.

A ball of negligible size and mass m is given a velocity of $\mathbf{v}_0$ on the center of the cart which has a mass M and is originally at rest. If the coefficient of restitution between the ball and walls A and B is e, determine the velocity of the ball and the cart just after the ball strikes A. Also, determine the total time needed for the ball to strike A, rebound, then strike B, and rebound and then return to the center of the cart. Neglect friction.



SOLUTION



After the first collision;

$$(\overset{+}{\leftarrow})\qquad \Sigma m v_1 = \Sigma m v_2$$

$$0 + m v_0 = m v_b + M v_c$$

$$(\overset{+}{\leftarrow})\qquad e = \frac{v_c - v_b}{v_0}$$

$$m v_0 = m v_b + \frac{M}{m} v_c$$

$$e v_0 = v_c - v_b$$

$$v_0(1 + e) = \left(1 + \frac{M}{m}\right) v_c$$

$$v_c = \frac{v_0(1 + e)m}{(m + M)} \qquad \textbf{Ans.}$$

$$v_b = \frac{v_0(1 + e)m}{(m + M)} - e v_0$$

$$= v_0\left[\frac{m + me - em - eM}{m + M}\right]$$

$$= v_0\left(\frac{m - eM}{m + M}\right) \qquad \textbf{Ans.}$$

The relative velocity on the cart after the first collision is

$$e = \frac{v_{\text{ref}}}{v_0}$$

$$v_{\text{ref}} = e v_0$$

Similarly, the relative velocity after the second collision is

$$e = \frac{v_{\text{ref}}}{e v_0}$$

$$v_{ref} = e^2 v_0$$

Total time is

$$t = \frac{d}{v_0} + \frac{2d}{e v_0} + \frac{d}{e^2 v_0}$$

$$= \frac{d}{v_0}\left(1 + \frac{1}{e}\right)^2 \qquad \textbf{Ans.}$$

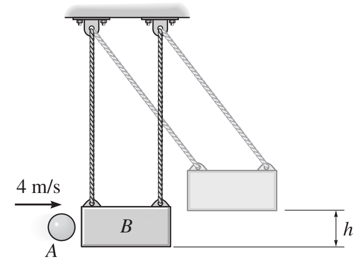
Ans:

$$v_c = \frac{v_0(1 + e)m}{(m + M)}$$

$$v_b = v_0\left(\frac{m - eM}{m + M}\right)$$

$$t = \frac{d}{v_0}\left(1 + \frac{1}{e}\right)^2$$

***15–80.** The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the coefficient of restitution between the ball and the block is $e = 0.8$, determine the maximum height h to which the block will swing before it momentarily stops.



SOLUTION

System:

$(\overset{+}{\rightarrow})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.8 = \frac{(v_B)_2 - (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$

Solving:

$(v_A)_2 = -2.545 \text{ m/s}$

$(v_B)_2 = 0.6545 \text{ m/s}$

Block:

Datum at lowest point.

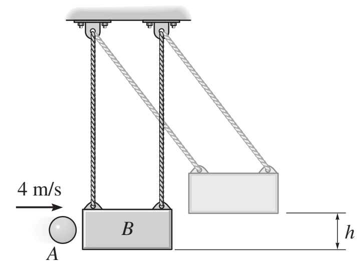
$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}(20)(0.6545)^2 + 0 = 0 + 20(9.81)h$

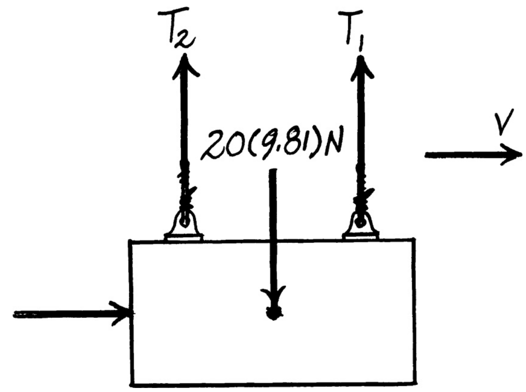
$h = 0.0218 \text{ m} = 21.8 \text{ mm}$ **Ans.**

Ans:
$h = 21.8 \text{ mm}$

15–81. The 2-kg ball is thrown at the suspended 20-kg block with a velocity of 4 m/s. If the time of impact between the ball and the block is 0.005 s, determine the average normal force exerted on the block during this time. Take $e = 0.8$.



SOLUTION



System:

$(\overset{+}{\rightarrow})$ $\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$(2)(4) + 0 = (2)(v_A)_2 + (20)(v_B)_2$$

$$(v_A)_2 + 10(v_B)_2 = 4$$

$(\overset{+}{\rightarrow})$ $\quad e = \dfrac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

$$0.8 = \frac{(v_B)_2 + (v_A)_2}{4 - 0}$$

$$(v_B)_2 - (v_A)_2 = 3.2$$

Solving:

$(v_A)_2 = -2.545$ m/s

$(v_B)_2 = 0.6545$ m/s

Block:

$(\overset{+}{\rightarrow})$ $\quad m v_1 + \Sigma \int F\,dt = m v_2$

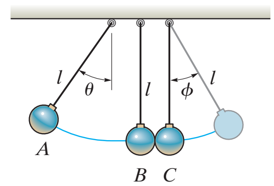
$$0 + F(0.005) = 20(0.6545)$$

$F = 2618$ N $= 2.62$ kN **Ans.**

Ans:
$F = 2.62$ kN

15-82.

The three balls each have the same mass m. If A is released from rest at θ, determine the angle ϕ to which C rises after collision. The coefficient of restitution between each ball is e.



SOLUTION

Energy

$$0 + l(1 - \cos(\theta))\, m g = \frac{1}{2} m v_A^{\,2}$$

$$v_A = \sqrt{2(1 - \cos(\theta))\, g l}$$

Collision of ball A with B:

$$m v_A + 0 = m v'_A + m v'_B \qquad e v_A = v'_B - v'_A \qquad v'_B = \frac{1}{2}(1 + e)\, v'_B$$

Collision of ball B with C:

$$m v'_B + 0 = m v''_B + m v''_C \qquad e v'_B = v''_C - v''_B \qquad v''_C = \frac{1}{4}(1 + e)^2 v_A$$

Energy

$$\frac{1}{2} m v''^{\,2}_c + 0 = 0 + l(1 - \cos(\phi))\, m g$$

$$\frac{1}{2}\left(\frac{1}{16}\right)(1 + e)^4 (2)(1 - \cos(\theta)) = (1 - \cos(\phi))$$

$$\left(\frac{1 + e}{2}\right)^4 (1 - \cos(\theta)) = 1 - \cos(\phi)$$

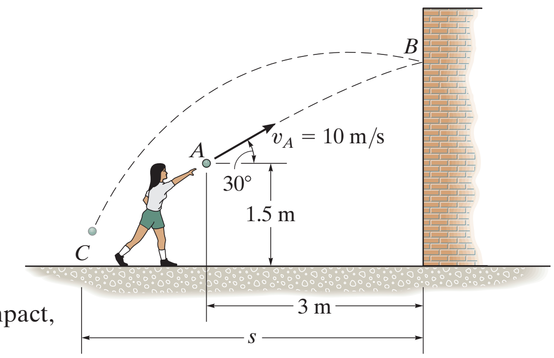
$$\phi = \operatorname{acos}\left[1 - \left(\frac{1 + e}{2}\right)^4 (1 - \cos(\theta))\right] \qquad \textbf{Ans.}$$

Ans:

$$\phi = \operatorname{acos}\left[1 - \left(\frac{1 + e}{2}\right)^4 (1 - \cos(\theta))\right]$$

15–83.

The girl throws the 0.5-kg ball toward the wall with an initial velocity $v_A = 10$ m/s. Determine (a) the velocity at which it strikes the wall at B, (b) the velocity at which it rebounds from the wall if the coefficient of restitution $e = 0.5$, and (c) the distance s from the wall to where it strikes the ground at C.



SOLUTION

Kinematics: By considering the horizontal motion of the ball before the impact, we have

$$(\overset{+}{\rightarrow}) \qquad s_x = (s_0)_x + v_x t$$

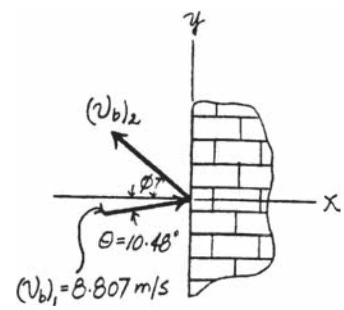
$$3 = 0 + 10\cos 30° t \qquad t = 0.3464 \text{ s}$$

By considering the vertical motion of the ball before the impact, we have

$$(+\uparrow) \qquad v_y = (v_0)_y + (a_c)_y t$$

$$= 10\sin 30° + (-9.81)(0.3464)$$

$$= 1.602 \text{ m/s}$$



The vertical position of point B above the ground is given by

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$(s_B)_y = 1.5 + 10\sin 30°(0.3464) + \frac{1}{2}(-9.81)\left(0.3464^2\right) = 2.643 \text{ m}$$

Thus, the magnitude of the velocity and its directional angle are

$$(v_b)_1 = \sqrt{(10\cos 30°)^2 + 1.602^2} = 8.807 \text{ m/s} = 8.81 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{1.602}{10\cos 30°} = 10.48° = 10.5° \qquad \textbf{Ans.}$$

Conservation of "y" Momentum: When the ball strikes the wall with a speed of $(v_b)_1 = 8.807$ m/s, it rebounds with a speed of $(v_b)_2$.

$$m_b\left(v_{b_y}\right)_1 = m_b\left(v_{b_y}\right)_2$$

$$(\overset{+}{\leftarrow}) \qquad m_b(1.602) = m_b\left[(v_b)_2\sin\phi\right]$$

$$(v_b)_2\sin\phi = 1.602 \qquad \textbf{(1)}$$

Coefficient of Restitution (x):

$$e = \frac{(v_w)_2 - \left(v_{b_x}\right)_2}{\left(v_{b_x}\right)_1 - (v_w)_1}$$

$$(\overset{+}{\rightarrow}) \qquad 0.5 = \frac{0 - \left[-(v_b)_2\cos\phi\right]}{10\cos 30° - 0} \qquad \textbf{(2)}$$

15–83. Continued

Solving Eqs. (1) and (2) yields

$$\phi = 20.30° = 20.3° \qquad (v_b)_2 = 4.617 \text{ m/s} = 4.62 \text{ m/s} \qquad \textbf{Ans.}$$

Kinematics: By considering the vertical motion of the ball after the impact, we have

$$(+\uparrow) \qquad s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$-2.643 = 0 + 4.617 \sin 20.30° t_1 + \frac{1}{2}(-9.81)t_1^2$$

$$t_1 = 0.9153 \text{ s}$$

By considering the horizontal motion of the ball after the impact, we have

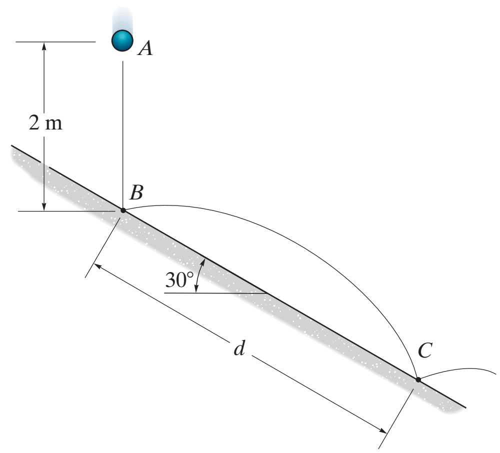
$$(\overset{+}{\leftarrow}) \qquad s_x = (s_0)_x + v_x t$$

$$s = 0 + 4.617 \cos 20.30°(0.9153) = 3.96 \text{ m} \qquad \textbf{Ans.}$$

Ans:
(a) $(v_B)_1 = 8.81 \text{ m/s}, \theta = 10.5°$ ⦢
(b) $(v_B)_2 = 4.62 \text{ m/s}, \phi = 20.3°$ ⦣
(c) $s = 3.96 \text{ m}$

***15–84.**

The 1-kg ball is dropped from rest at point A, 2 m above the smooth plane. If the coefficient of restitution between the ball and the plane is $e = 0.6$, determine the distance d where the ball again strikes the plane.



SOLUTION

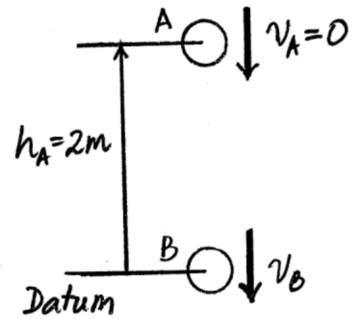
Conservation of Energy: By considering the ball's fall from position (1) to position (2) as shown in Fig. *a*,

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}m_A v_A^2 + (V_g)_A = \frac{1}{2}m_B v_B^2 + (V_g)_B$$

$$0 + 1(9.81)(2) = \frac{1}{2}(1)v_B^2 + 0$$

$$v_B = 6.264 \text{ m/s} \downarrow$$



(a)

Conservation of Linear Momentum: Since no impulsive force acts on the ball along the inclined plane (x' axis) during the impact, linear momentum of the ball is conserved along the x' axis. Referring to Fig. *b*,

$$m_B(v_B)_{x'} = m_B(v'_B)_{x'}$$

$$1(6.264)\sin 30° = 1(v'_B)\cos\theta$$

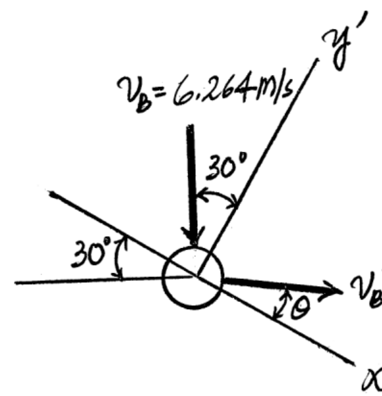
$$v'_B \cos\theta = 3.1321 \qquad \textbf{(1)}$$

Coefficient of Restitution: Since the inclined plane does not move during the impact,

$$e = \frac{0 - (v'_B)_{y'}}{(v'_B)_{y'} - 0}$$

$$0.6 = \frac{0 - v'_B \sin\theta}{-6.264\cos 30° - 0}$$

$$v'_B \sin\theta = 3.2550 \qquad \textbf{(2)}$$



(b)

Solving Eqs. (1) and (2) yields

$$\theta = 46.10° \qquad v'_B = 4.517 \text{ m/s}$$

Kinematics: By considering the x and y motion of the ball after the impact, Fig. *c*,

$(\overset{+}{\rightarrow})$ $$s_x = (s_0)_x + (v'_B)_x t$$

$$d\cos 30° = 0 + 4.517\cos 16.10° t$$

$$t = 0.1995d \qquad \textbf{(3)}$$

$(+\uparrow)$ $$s_y = (s_0)_y + (v'_B)_y t + \frac{1}{2}a_y t^2$$

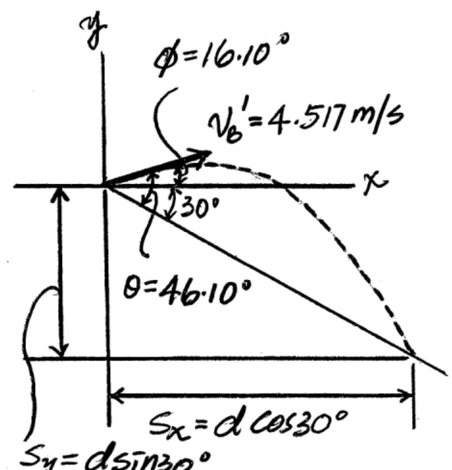
$$-d\sin 30° = 0 + 4.517\sin 16.10° t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - 1.2528t - 0.5d = 0 \qquad \textbf{(4)}$$

Solving Eqs. (3) and (4) yields

$$d = 3.84 \text{ m} \qquad \textbf{Ans.}$$

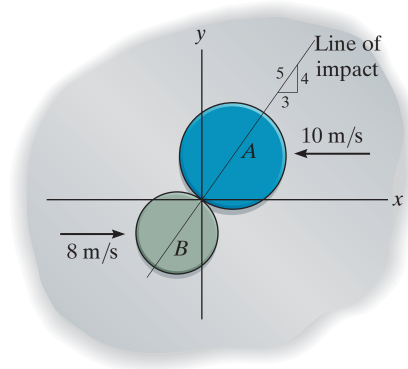
$$t = 0.7663 \text{ s}$$



(c)

15–85.

Disks A and B have a mass of 15 kg and 10 kg, respectively. If they are sliding on a smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.8$.



SOLUTION

Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. *a*, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$+\nearrow \quad m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v_A'\right)_n + m_B\left(v_B'\right)_n$$

$$15(10)\left(\frac{3}{5}\right) - 10(8)\left(\frac{3}{5}\right) = 15v_A' \cos\phi_A + 10v_B' \cos\phi_B$$

$$15v_A' \cos\phi_A + 10v_B' \cos\phi_B = 42 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis (tangent to? plane of impact). Thus,

$$+\nwarrow \quad m_A\left(v_A\right)_t = m_A\left(v_A'\right)_t$$

$$15(10)\left(\frac{4}{5}\right) = 15v_A' \sin\phi_A$$

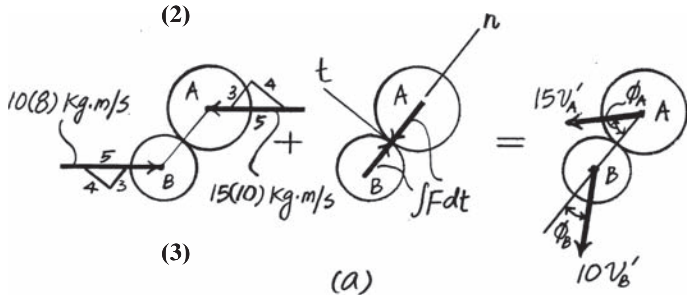
$$v_A' \sin\phi_A = 8 \qquad \textbf{(2)}$$

and

$$+\nwarrow \quad m_B\left(v_B\right)_t = m_B\left(v_B'\right)_t$$

$$10(8)\left(\frac{4}{5}\right) = 10\, v_B' \sin\phi_B$$

$$v_B' \sin\phi_B = 6.4 \qquad \textbf{(3)}$$



Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$+\nearrow \quad e = \frac{\left(v_B'\right)_n - \left(v_A'\right)_n}{\left(v_A\right)_n - \left(v_B\right)_n}$$

$$0.8 = \frac{v_B' \cos\phi_B - v_A' \cos\phi_A}{10\left(\frac{3}{5}\right) - \left[-8\left(\frac{3}{5}\right)\right]}$$

$$v_B' \cos\phi_B - v_A' \cos\phi_A = 8.64 \qquad \textbf{(4)}$$

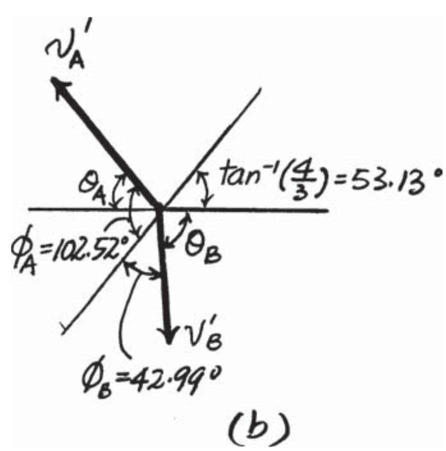
Solving Eqs. (1), (2), (3), and (4), yeilds

$$v_A' = 8.19 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_A = 102.52°$$

$$v_B' = 9.38 \text{ m/s} \qquad \textbf{Ans.}$$

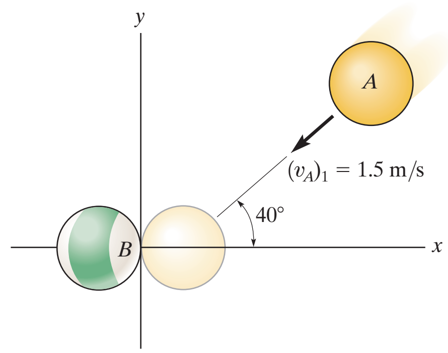
$$\phi_B = 42.99°$$



Ans:
$(v_A)_2 = 8.19 \text{ m/s}$
$(v_B)_2 = 9.38 \text{ m/s}$

15–86.

Two smooth billiard balls A and B each have a mass of 200 g. If A strikes B with a velocity $(v_A)_1 = 1.5\text{ m/s}$ as shown, determine their final velocities just after collision. Ball B is originally at rest and the coefficient of restitution is $e = 0.85$. Neglect the size of each ball.



SOLUTION

$(v_{A_x})_1 = -1.5\cos 40° = -1.1491\text{ m/s}$

$(v_{A_y})_1 = -1.5\sin 40° = -0.9642\text{ m/s}$

$(\overset{+}{\rightarrow})\quad m_A(v_{A_x})_1 + m_B(v_{B_x})_1 = m_A(v_{A_x})_2 + m_B(v_{B_x})_2$

$-0.2(1.1491) + 0 = 0.2(v_{A_x})_2 + 0.2(v_{B_x})_2$

$(\overset{+}{\rightarrow})\quad e = \dfrac{(v_{A_x})_2 - (v_{B_x})_2}{(v_{B_x})_1 - (v_{A_x})_1};\qquad 0.85 = \dfrac{(v_{A_x})_2 - (v_{B_x})_2}{1.1491}$

Solving,

$(v_{A_x})_2 = -0.08618\text{ m/s}$

$(v_{B_x})_2 = -1.0629\text{ m/s}$

For A:

$(+\downarrow)\quad m_A(v_{A_y})_1 = m_A(v_{A_y})_2$

$(v_{A_y})_2 = 0.9642\text{ m/s}$

For B:

$(+\uparrow)\quad m_B(v_{B_y})_1 = m_B(v_{B_y})_2$

$(v_{B_y})_2 = 0$

Hence.

$(v_B)_2 = (v_{B_x})_2 = 1.06\text{ m/s} \leftarrow$ **Ans.**

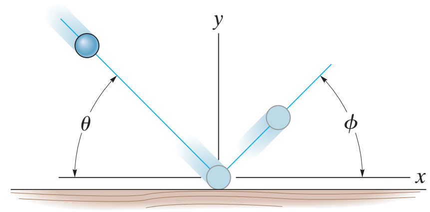
$(v_A)_2 = \sqrt{(-0.08618)^2 + (0.9642)^2} = 0.968\text{ m/s}$ **Ans.**

$(\theta_A)_2 = \tan^{-1}\left(\dfrac{0.08618}{0.9642}\right) = 5.11°$ ∡ **Ans.**

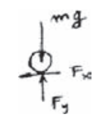
Ans:
$(v_B)_2 = 1.06\text{ m/s} \leftarrow$
$(v_A)_2 = 0.968\text{ m/s}$
$(\theta_A)_2 = 5.11°$ ∡

15–87.

A ball is thrown onto a rough floor at an angle θ. If it rebounds at an angle ϕ and the coefficient of kinetic friction is μ, determine the coefficient of restitution e. Neglect the size of the ball. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



SOLUTION



$(+\downarrow)$ $$e = \frac{0 - [-v_2 \sin\phi]}{v_1 \sin\theta - 0} \qquad e = \frac{v_2 \sin\phi}{v_1 \sin\theta} \qquad \textbf{(1)}$$

$(\overset{+}{\rightarrow})$ $$m(v_x)_1 + \int_{t_1}^{t_2} F_x\,dx = m(v_x)_2$$

$$mv_1 \cos\theta - F_x \Delta t = mv_2 \cos\phi$$

$$F_x = \frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} \qquad \textbf{(2)}$$

$(+\downarrow)$ $$m(v_y)_1 + \int_{t_1}^{t_2} F_y\,dx = m(v_y)_2$$

$$mv_1 \sin\theta - F_y \Delta t = -mv_2 \sin\phi$$

$$F_y = \frac{mv_1 \sin\theta + mv_2 \sin\phi}{\Delta t} \qquad \textbf{(3)}$$

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

$$\frac{mv_1 \cos\theta - mv_2 \cos\phi}{\Delta t} = \frac{\mu(mv_1 \sin\theta + mv_2 \sin\phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi} \qquad \textbf{(4)}$$

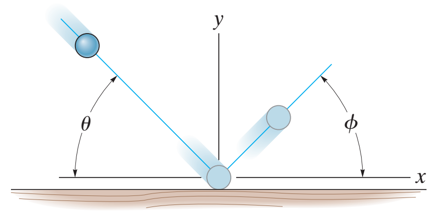
Substituting Eq. (4) into (1) yields:

$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi}\right) \qquad \textbf{Ans.}$$

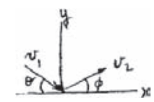
Ans:
$$e = \frac{\sin\phi}{\sin\theta}\left(\frac{\cos\theta - \mu \sin\theta}{\mu \sin\phi + \cos\phi}\right)$$

***15–88.**

A ball is thrown onto a rough floor at an angle of $\theta = 45°$. If it rebounds at the same angle $\phi = 45°$, determine the coefficient of kinetic friction between the floor and the ball. The coefficient of restitution is $e = 0.6$. *Hint:* Show that during impact, the average impulses in the x and y directions are related by $I_x = \mu I_y$. Since the time of impact is the same, $F_x \Delta t = \mu F_y \Delta t$ or $F_x = \mu F_y$.



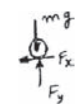
SOLUTION



$(+\downarrow)$ $$e = \frac{0 - [-v_2 \sin \phi]}{v_1 \sin \theta - 0} \qquad e = \frac{v_2 \sin \phi}{v_1 \sin \theta} \qquad \textbf{(1)}$$

$(\xrightarrow{+})$ $$m(v_x)_1 + \int_{t_1}^{t_2} F_x dx = m(v_x)_2$$

$$mv_1 \cos \theta - F_x \Delta t = mv_2 \cos \phi$$



$$F_x = \frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} \qquad \textbf{(2)}$$

$(+\uparrow)$ $$m(v_y)_1 + \int_{t_1}^{t_2} F_y dx = m(v_y)_2$$

$$mv_1 \sin \theta - F_y \Delta t = -mv_2 \sin \phi$$

$$F_y = \frac{mv_1 \sin \theta + mv_2 \sin \phi}{\Delta t} \qquad \textbf{(3)}$$

Since $F_x = \mu F_y$, from Eqs. (2) and (3)

$$\frac{mv_1 \cos \theta - mv_2 \cos \phi}{\Delta t} = \frac{\mu(mv_1 \sin \theta + mv_2 \sin \phi)}{\Delta t}$$

$$\frac{v_2}{v_1} = \frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi} \qquad \textbf{(4)}$$

Substituting Eq. (4) into (1) yields:

$$e = \frac{\sin \phi}{\sin \theta}\left(\frac{\cos \theta - \mu \sin \theta}{\mu \sin \phi + \cos \phi}\right)$$

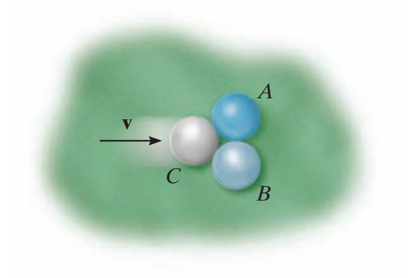
$$0.6 = \frac{\sin 45°}{\sin 45°}\left(\frac{\cos 45° - \mu \sin 45°}{\mu \sin 45° + \cos 45°}\right)$$

$$0.6 = \frac{1-\mu}{1+\mu} \qquad \mu = 0.25$$ **Ans.**

Ans:
$\mu_k = 0.25$

15–89.

The two billiard balls A and B are originally in contact with one another when a third ball C strikes each of them at the same time as shown. If ball C remains at rest after the collision, determine the coefficient of restitution. All the balls have the same mass. Neglect the size of each ball.

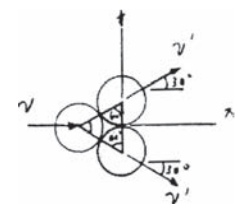


SOLUTION

Conservation of "x" momentum:

$$(\overset{+}{\rightarrow}) \qquad mv = 2mv' \cos 30°$$

$$v = 2v' \cos 30° \qquad \textbf{(1)}$$

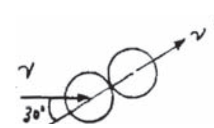


Coefficient of restitution:

$$(+\nearrow) \qquad e = \frac{v'}{v \cos 30°} \qquad \textbf{(2)}$$

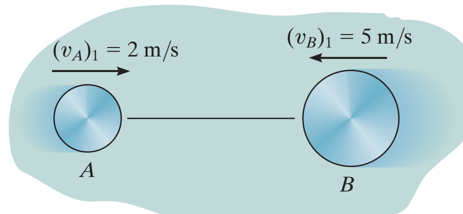
Substituting Eq. (1) into Eq. (2) yields:

$$e = \frac{v'}{2v' \cos^2 30°} = \frac{2}{3} \qquad \textbf{Ans.}$$



Ans:
$e = \frac{2}{3}$

15–90. Disks A and B have masses of 2 kg and 4 kg, respectively. If they have the velocities shown, and $e = 0.4$, determine their velocities just after direct central impact.



SOLUTION

Guesses $\quad v_{A2} = 1 \text{ m/s} \quad v_{B2} = 1 \text{ m/s}$

Given:

$M_A = 2 \text{ kg}$

$M_B = 4 \text{ kg}$

$e = 0.4$

$v_{A1} = 2 \text{ m/s}$

$v_{B1} = 5 \text{ m/s}$

Given $\quad M_A v_{A1} - M_B v_{B1} = M_A v_{A2} + M_B v_{B2}$

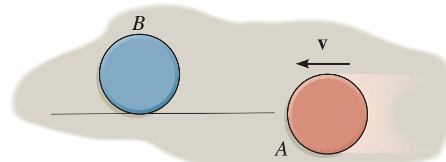
$$e(v_{A1} + v_{B1}) = v_{B2} - v_{A2}$$

$$\begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \text{Find}(v_{A2}, v_{B2}) \qquad \begin{pmatrix} v_{A2} \\ v_{B2} \end{pmatrix} = \begin{pmatrix} -4.533 \\ -1.733 \end{pmatrix} \text{m/s}$$ **Ans.**

Ans:
$v_{A2} = -4.533 \text{ m/s}$
$v_{B2} = -1.733 \text{ m/s}$

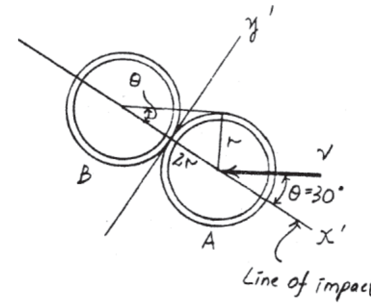
15–91.

If disk A is sliding along the tangent to disk B and strikes B with a velocity **v**, determine the velocity of B after the collision and compute the loss of kinetic energy during the collision. Neglect friction. Disk B is originally at rest. The coefficient of restitution is e, and each disk has the same size and mass m.



SOLUTION

Impact: This problem involves *oblique impact* where the *line of impact* lies along x' axis (line jointing the mass center of the two impact bodies). From the geometry $\theta = \sin^{-1}\left(\frac{r}{2r}\right) = 30°$. The x' and y' components of velocity for disk A just before impact are



$$(v_{A_{x'}})_1 = -v\cos 30° = -0.8660v \qquad (v_{A_{y'}})_1 = -v\sin 30° = -0.5\,v$$

Conservation of "x′" Momentum:

$$m_A\left(v_{A_{x'}}\right)_1 + m_B\left(v_{B_{x'}}\right)_1 = m_A\left(v_{A_{x'}}\right)_2 + m_B\left(v_{B_{x'}}\right)_2$$

$$(\searrow +) \qquad m(-0.8660v) + 0 = m\left(v_{A_x}\right)_2 + m\left(v_{B_{x'}}\right)_2 \qquad \textbf{(1)}$$

Coefficient of Restitution (x′):

$$e = \frac{\left(v_{B_{x'}}\right)_2 - \left(v_{A_{x'}}\right)_2}{\left(v_{A_{x'}}\right)_1 - \left(v_{B_{x'}}\right)_1}$$

$$e = \frac{\left(v_{B_{x'}}\right)_2 - \left(v_{A_{x'}}\right)_2}{-0.8660v - 0} \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$\left(v_{B_{x'}}\right)_2 = -\frac{\sqrt{3}}{4}(1+e)\,v \qquad \left(v_{A_{x'}}\right)_2 = \frac{\sqrt{3}}{4}(e-1)\,v$$

Conservation of "y′" Momentum: The momentum is conserved along y' axis for both disks A and B.

$$(+\nearrow) \qquad m_B\left(v_{B_{y'}}\right)_1 = m_B\left(v_{B_{y'}}\right)_2; \qquad \left(v_{B_{y'}}\right)_2 = 0$$

$$(+\nearrow) \qquad m_A\left(v_{A_{y'}}\right)_1 = m_A\left(v_{A_{y'}}\right)_2; \qquad \left(v_{A_{y'}}\right)_2 = -0.5\,v$$

Thus, the magnitude the velocity for disk B just after the impact is

$$(v_B)_2 = \sqrt{(v_{B_{x'}})_2^2 + (v_{B_{y'}})_2^2}$$

$$= \sqrt{\left(-\frac{\sqrt{3}}{4}(1+e)\,v\right)^2 + 0} = \frac{\sqrt{3}}{4}(1+e)\,v \qquad \textbf{Ans.}$$

and directed toward **negative x' axis**. **Ans.**

15–91. continued

The magnitude of the velocity for disk A just after the impact is

$$(v_A)_2 = \sqrt{(v_{A_{x'}})_2^2 + (v_{A_{y'}})_2^2}$$

$$= \sqrt{\left[\frac{\sqrt{3}}{4}(e - 1)\,v\right]^2 + (-0.5\,v)^2}$$

$$= \sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}$$

Loss of Kinetic Energy: Kinetic energy of the system before the impact is

$$U_k = \frac{1}{2}mv^2$$

Kinetic energy of the system after the impact is

$$U_k{}' = \frac{1}{2}m\left[\sqrt{\frac{v^2}{16}(3e^2 - 6e + 7)}\right]^2 + \frac{1}{2}m\left[\frac{\sqrt{3}}{4}(1 + e)\,v\right]^2$$

$$= \frac{mv^2}{32}\left(6e^2 + 10\right)$$

Thus, the kinetic energy loss is

$$\Delta U_k = U_k - U_k{}' = \frac{1}{2}mv^2 - \frac{mv^2}{32}(6e^2 + 10)$$

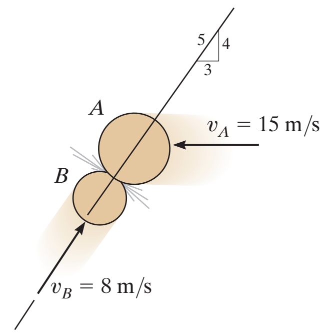
$$= \frac{3mv^2}{16}(1 - e^2) \qquad \textbf{Ans.}$$

Ans:

$(v_B)_2 = \frac{\sqrt{3}}{4}(1 + e)v$,
Negative x' axis.
$\Delta U_k = \frac{3mv^2}{16}(1 - e^2)$

***15–92.**

Two smooth disks A and B have the initial velocities shown just before they collide. If they have masses $m_A = 4$ kg and $m_B = 2$ kg, determine their speeds just after impact. The coefficient of restitution is $e = 0.8$.



SOLUTION

Impact. The line of impact is along the line joining the centers of disks A and B represented by y axis in Fig. a. Thus

$$[(v_A)_1]_y = 15\left(\frac{3}{5}\right) = 9 \text{ m/s} \swarrow \qquad [(v_A)_1]_x = 15\left(\frac{4}{5}\right) = 12 \text{ m/s} \nwarrow$$

$$[(v_B)_1]_y = 8 \text{ m/s} \nearrow \qquad [(v_B)_1]_x = 0$$

Coefficient of Restitution. Along the line of impact (y axis),

$$(+\nearrow) \qquad e = \frac{[(v_B)_2]_y - [(v_A)_2]_y}{[(v_A)_1]_y - [(v_B)_1]_y}; \qquad 0.8 = \frac{[(v_B)_2]_y - [(v_A)_2]_y}{-9 - 8}$$

$$[(v_A)_2]_y - [(v_B)_2]_y = 13.6 \qquad \textbf{(1)}$$

Conservation of 'y' Momentum.

$$(+\nearrow) \qquad m_A[(v_A)_1]_y + m_B[(v_B)_1]_y = m_A[(v_A)_2]_y + m_B[(v_B)_2]_y$$

$$4(-9) + 2(8) = 4[(v_A)_2]_y + 2[(v_B)_2]_y$$

$$2[(v_A)_2]_y + [(v_B)_2]_y = -10 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$[(v_A)_2]_y = 1.20 \text{ m/s} \nearrow \qquad [(v_B)_2]_y = -12.4 \text{ m/s} = 12.4 \text{ m/s} \swarrow$$

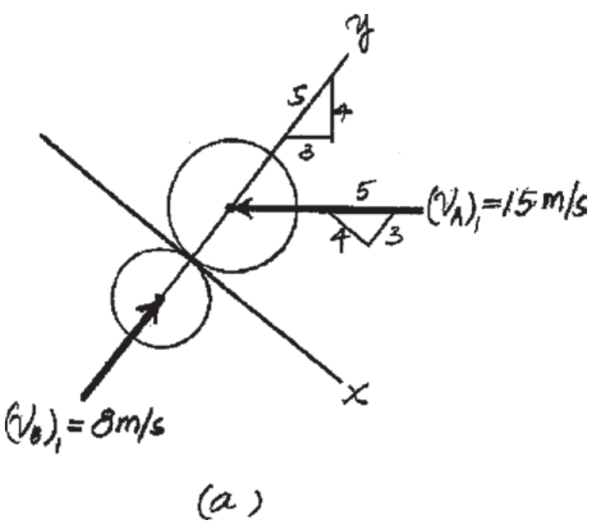
Conservation of 'x' Momentum. Since no impact occurs along the x axis, the component of velocity of each disk remain constant before and after the impact. Thus

$$[(v_A)_2]_x = [(v_A)_1]_x = 12 \text{ m/s} \nwarrow \qquad [(v_B)_2]_x = [(v_B)_1]_x = 0$$

Thus, the magnitude of the velocity of disks A and B just after the impact is

$$(v_A)_2 = \sqrt{[(v_A)_2]_x^2 + [(v_A)_2]_y^2} = \sqrt{12^2 + 1.20^2} = 12.06 \text{ m/s} = 12.1 \text{ m/s} \qquad \textbf{Ans.}$$

$$(v_B)_2 = \sqrt{[(v_B)_2]_x^2 + [(v_B)_2]_y^2} = \sqrt{0^2 + 12.4^2} = 12.4 \text{ m/s} \qquad \textbf{Ans.}$$

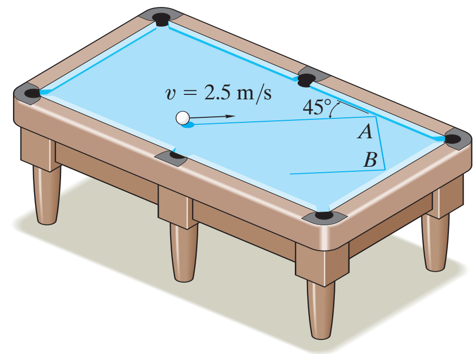


(a)

Ans:
$(v_A)_2 = 12.1$ m/s
$(v_B)_2 = 12.4$ m/s

15–93.

The 200-g billiard ball is moving with a speed of 2.5 m/s when it strikes the side of the pool table at A. If the coefficient of restitution between the ball and the side of the table is $e = 0.6$, determine the speed of the ball just after striking the table twice, i.e., at A, then at B. Neglect the size of the ball.



SOLUTION

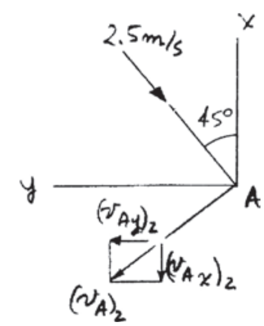
At A:

$$(v_A)_{y1} = 2.5(\sin 45°) = 1.7678 \text{ m/s} \rightarrow$$

$$e = \frac{(v_{A_y})_2}{(v_{A_y})_1}; \qquad 0.6 = \frac{(v_{A_y})_2}{1.7678}$$

$$(v_{A_y})_2 = 1.061 \text{ m/s} \leftarrow$$

$$(v_{A_x})_2 = (v_{A_x})_1 = 2.5 \cos 45° = 1.7678 \text{ m/s} \downarrow$$

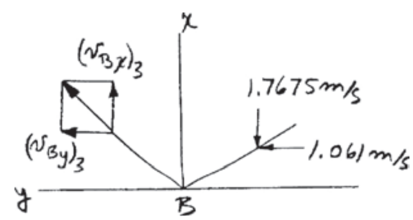


At B:

$$e = \frac{(v_{B_x})_3}{(v_{B_x})_2}; \qquad 0.6 = \frac{(v_{B_x})_3}{1.7678}$$

$$(v_{B_x})_3 = 1.061 \text{ m/s}$$

$$\left(v_{B_y}\right)_3 = \left(v_{A_y}\right)_2 = 1.061 \text{ m/s}$$



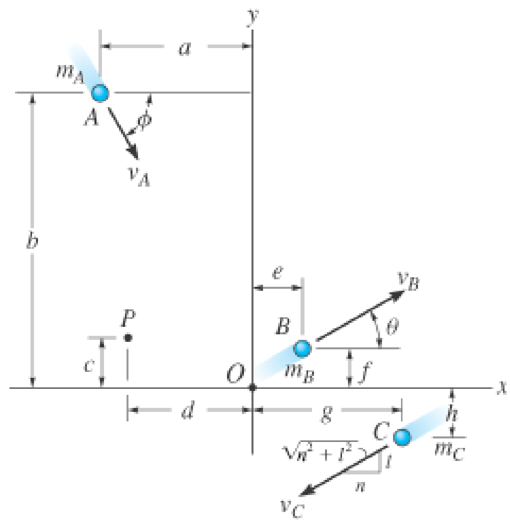
Hence,

$$(v_B)_3 = \sqrt{(1.061)^2 + (1.061)^2} = 1.50 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$(v_B)_3 = 1.50$ m/s

15-94.

Determine the angular momentum $\mathbf{H_O}$ of each of the particles about point O.



SOLUTION

Given: $\theta = 30°$ $\quad \phi = 60°$

$m_A = 6$ kg $\quad c = 2$ m

$m_B = 4$ kg $\quad d = 5$ m

$m_C = 2$ kg $\quad e = 2$ m

$v_A = 4$ m/s $\quad f = 1.5$ m

$v_B = 6$ m/s $\quad g = 6$ m

$v_C = 2.6$ m/s $\quad h = 2$ m

$a = 8$ m $\quad l = 5$

$b = 12$ m $\quad n = 12$

$$\mathbf{H_{AO}} = a m_A v_A \sin(\phi) - b m_A v_A \cos(\phi)$$

$\mathbf{H_{AO}} = 22.3 \text{ kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$$\mathbf{H_{BO}} = -f m_B v_B \cos(\theta) + e m_B v_B \sin(\theta)$$

$\mathbf{H_{BO}} = -7.18 \text{ kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$\mathbf{H_{CO}} = -21.60 \text{ kg}\cdot\text{m}^2/\text{s}$ **Ans.**

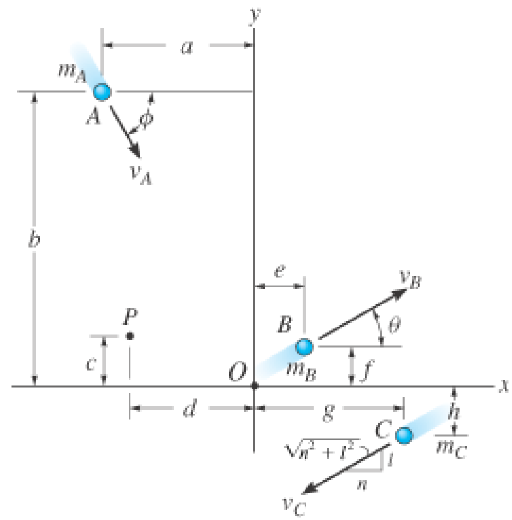
$$\mathbf{H_{CO}} = -h m_C \left(\frac{n}{\sqrt{l^2+n^2}}\right) v_C - g m_C \left(\frac{l}{\sqrt{l^2+n^2}}\right) v_C$$

Ans:

$\mathbf{H_{AO}} = 22.3 \text{ kg}\cdot\text{m}^2/\text{s}$
$\mathbf{H_{BO}} = -7.18 \text{ kg}\cdot\text{m}^2/\text{s}$
$\mathbf{H_{CO}} = -21.60 \text{ kg}\cdot\text{m}^2/\text{s}$

15-95.

Determine the angular momentum $\mathbf{H_P}$ of each of the particles about point P.



SOLUTION

Given:

$\theta = 30°$ $\phi = 60°$

$m_A = 6$ kg $c = 2$ m

$m_B = 4$ kg $d = 5$ m

$m_C = 2$ kg $e = 2$ m

$v_A = 4$ m/s $f = 1.5$ m

$v_B = 6$ m/s $g = 6$ m

$v_C = 2.6$ m/s $h = 2$ m

$a = 8$ m $l = 5$

$b = 12$ m $n = 12$

$$\mathbf{H_{AP}} = m_A v_A \sin(\phi)(a-d) - m_A v_A \cos(\phi)(b-c)$$

$\mathbf{H_{AP}} = -57.6\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$$\mathbf{H_{BP}} = m_B v_B \cos(\theta)(c-f) + m_B v_B \sin(\theta)(d+e)$$

$\mathbf{H_{BP}} = 94.4\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

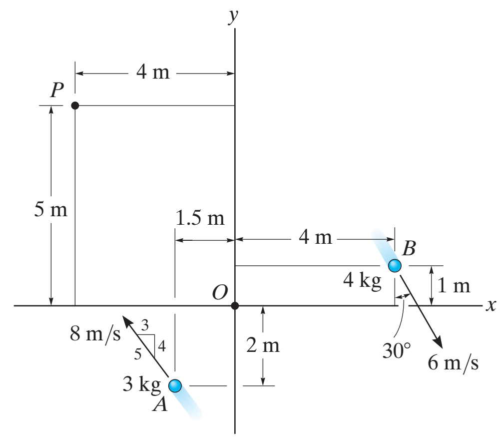
$$\mathbf{H_{CP}} = -m_C\left(\frac{n}{\sqrt{l^2+n^2}}\right)v_C(c+h) - m_C\left(\frac{l}{\sqrt{l^2+n^2}}\right)v_C(d+g)$$

$\mathbf{H_{CP}} = -41.2\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

Ans:
$\mathbf{H_{AP}} = -57.6\ \text{kg}\cdot\text{m}^2/\text{s}$
$\mathbf{H_{BP}} = 94.4\ \text{kg}\cdot\text{m}^2/\text{s}$
$\mathbf{H_{CP}} = -41.2\ \text{kg}\cdot\text{m}^2/\text{s}$

***15–96.**

Determine the angular momentum $\mathbf{H}_o$ of each of the two particles about point O.



SOLUTION

$$\circlearrowleft + (\boldsymbol{H}_A)_O = (-1.5)\left[3(8)\left(\frac{4}{5}\right)\right] - (2)\left[3(8)\left(\frac{3}{5}\right)\right] = -57.6\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\circlearrowleft + (\boldsymbol{H}_B)_O = (-1)[4(6\sin 30^\circ)] - (4)[4\,(6\cos 30^\circ)] = -95.14\ \text{kg}\cdot\text{m}^2/\text{s}$$

Thus

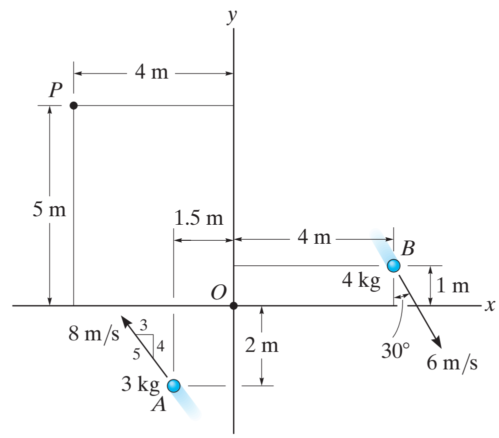
$$(\boldsymbol{H}_A)_O = \{-57.6\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

$$(\boldsymbol{H}_B)_O = \{-95.1\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Ans:
$(\boldsymbol{H}_A)_O = \{-57.6\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$
$(\boldsymbol{H}_B)_O = \{-95.1\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$

15–97.

Determine the angular momentum $\mathbf{H}_p$ of each of the two particles about point P.



SOLUTION

$$\zeta + (\boldsymbol{H}_A)_p = (2.5)\left[3(8)\left(\frac{4}{5}\right)\right] - (7)\left[3(8)\left(\frac{3}{5}\right)\right] = -52.8\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\zeta + (\boldsymbol{H}_B)_p = (4)[4(6\sin 30^\circ)] - 8[4\,(6\cos 30^\circ)] = -118.28\ \text{kg}\cdot\text{m}^2/\text{s}$$

Thus,

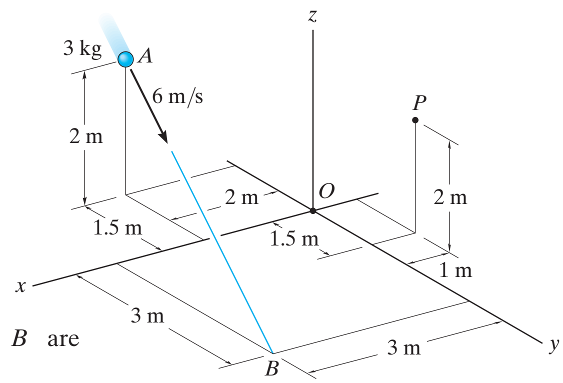
$(\boldsymbol{H}_A)_p = \{-52.8\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

$(\boldsymbol{H}_B)_p = \{-118\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$ **Ans.**

Ans:
$(H_A)_P = \{-52.8\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$
$(H_B)_P = \{-118\mathbf{k}\}\ \text{kg}\cdot\text{m}^2/\text{s}$

15–98.

Determine the angular momentum $\mathbf{H}_O$ of the 3-kg particle about point O.



SOLUTION

Position and Velocity Vectors. The coordinates of points A and B are $A(2, -1.5, 2)$ m and $B(3, 3, 0)$.

$$\mathbf{r}_{OB} = \{3\mathbf{i} + 3\mathbf{j}\}\text{ m} \qquad \mathbf{r}_{OA} = \{2\mathbf{i} - 1.5\mathbf{j} + 2\mathbf{k}\}\text{ m}$$

$$V_A = v_A\left(\frac{r_{AB}}{r_{AB}}\right) = (6)\left[\frac{(3-2)\mathbf{i} + [3-(-1.5)]\mathbf{j} + (0.2)\mathbf{k}}{\sqrt{(3-2)^2 + [3-(-1.5)]^2 + (0-2)^2}}\right]$$

$$= \left\{\frac{6}{\sqrt{25.25}}\mathbf{i} + \frac{27}{\sqrt{25.25}}\mathbf{j} - \frac{12}{\sqrt{25.25}}\mathbf{k}\right\}\text{ m/s}$$

Angular Momentum about Point O. Applying Eq. 15

$$\mathbf{H}_O = \mathbf{r}_{OB} \times mV_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 3 & 0 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{-21.4928\mathbf{i} + 21.4928\mathbf{j} + 37.6124\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\}\text{ kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

Also,

$$\mathbf{H}_O = \mathbf{r}_{OA} \times mV_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1.5 & 2 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

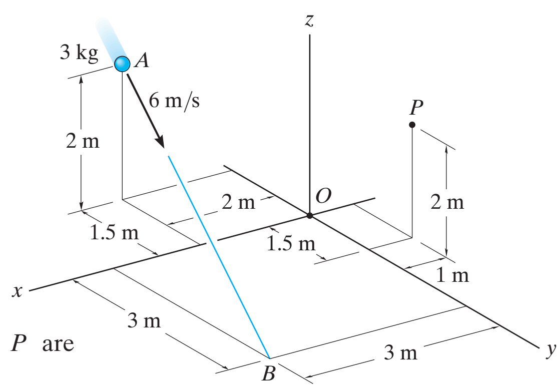
$$= \{-21.4928\mathbf{i} + 21.4928\mathbf{j} + 37.6124\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\mathbf{k}\}\text{ kg}\cdot\text{m}^2/\text{s} \qquad \textbf{Ans.}$$

Ans:
$\{-21.5\mathbf{i} + 21.5\mathbf{j} + 37.6\}\text{ kg}\cdot\text{m}^2/\text{s}$

15–99.

Determine the angular momentum $\mathbf{H}_P$ of the 3-kg particle about point P.



SOLUTION

Position and Velocity Vectors. The coordinates of points A, B and P are $A(2, -1.5, 2)$ m, $B(3, 3, 0)$ m and $P(-1, 1.5, 2)$ m.

$$\mathbf{r}_{PA} = [2-(-1)]\mathbf{i} + (-1.5-1.5)\mathbf{j} + (2-2)\mathbf{k} = \{3\mathbf{i} - 3\mathbf{j}\} \text{ m}$$

$$\mathbf{r}_{PB} = [3-(-1)]\mathbf{i} + (3-1.5)\mathbf{j} + (0-2)\mathbf{k} = \{4\mathbf{i} + 1.5\mathbf{j} - 2\mathbf{k}\} \text{ m}$$

$$\mathbf{V}_A = v_A\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 6\left[\frac{(3-2)\mathbf{i} + [3-(-1.5)]\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(3-2)^2 + [3-(-1.5)]^2 + (0-2)^2}}\right]$$

$$= \left\{\frac{6}{\sqrt{25.25}}\mathbf{i} + \frac{27}{\sqrt{25.25}}\mathbf{j} - \frac{12}{\sqrt{25.25}}\mathbf{k}\right\} \text{ m/s}$$

Angular Momentum about Point P. Applying Eq. 15

$$\mathbf{H}_p = \mathbf{r}_{pA} \times m\mathbf{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 0 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

$$= \{21.4928\mathbf{i} + 21.4928\mathbf{j} + 59.1052\mathbf{k}\} \text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k}\} \text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Also,

$$\mathbf{H}_p = \mathbf{r}_{PB} \times m\mathbf{V}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 1.5 & -2 \\ 3\left(\frac{6}{\sqrt{25.25}}\right) & 3\left(\frac{27}{\sqrt{25.25}}\right) & 3\left(-\frac{12}{\sqrt{25.25}}\right) \end{vmatrix}$$

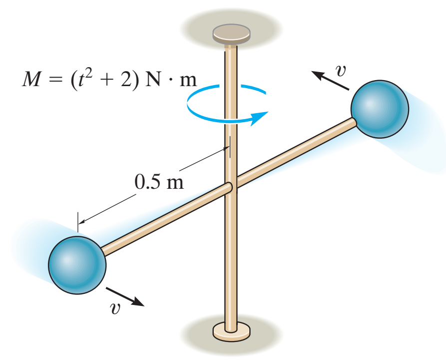
$$= \{21.4928\mathbf{i} + 21.4928\mathbf{j} + 59.1052\mathbf{k}\} \text{ kg}\cdot\text{m}^2/\text{s}$$

$$= \{21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k}\} \text{ kg}\cdot\text{m}^2/\text{s}$$ **Ans.**

Ans:
$\{21.5\mathbf{i} + 21.5\mathbf{j} + 59.1\mathbf{k}\}$ kg·m^2/s

***15–100.**

Each ball has a negligible size and a mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (t^2 + 2)\ \text{N}\cdot\text{m}$, where t is in seconds, determine the speed of each ball when $t = 3$ s. Each ball has a speed $v = 2$ m/s when $t = 0$.



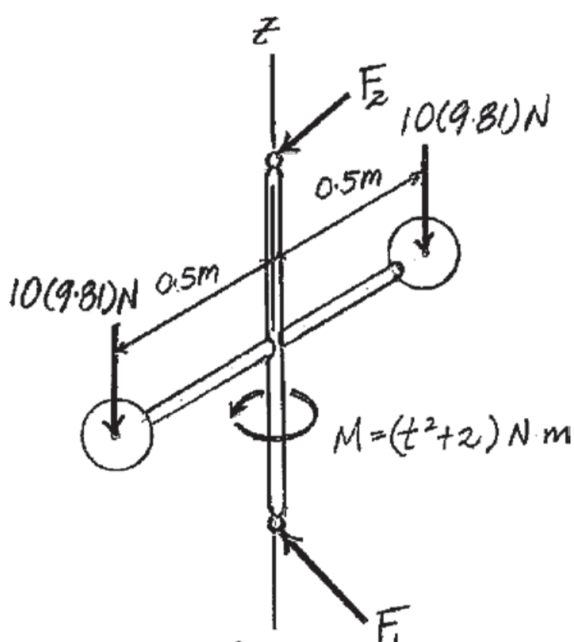
SOLUTION

Principle of Angular Impulse and Momentum. Referring to the FBD of the assembly, Fig. *a*

$$(H_Z)_1 + \Sigma \int_{t_1}^{t_2} M_Z\, dt = (H_Z)_2$$

$$2[0.5(10)(2)] + \int_0^{3\text{ s}} (t^2 + 2)dt = 2[0.5(10v)]$$

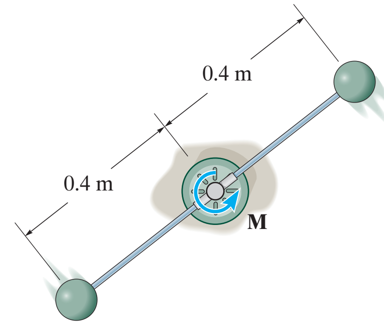
$$v = 3.50 \text{ m/s} \qquad \textbf{Ans.}$$



(a)

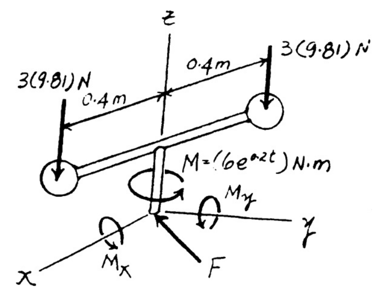
Ans:
$v = 3.50$ m/s

15–101. The two spheres each have a mass of 3 kg and are attached to the rod of negligible mass. If a torque $M = (6e^{0.2t})$ N · m, where t is in seconds, is applied to the rod as shown, determine the speed of each of the spheres in 2 s, starting from rest.



SOLUTION

Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have



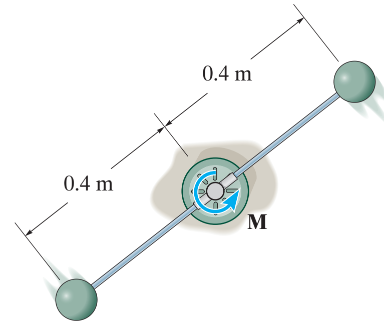
$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z \, dt = (H_z)_2$$

$$2[0.4\,(3)\,(0)] + \int_0^{2\,\text{s}} \left(6e^{0.2t}\right) dt = 2\,[0.4\,(3)\,v]$$

$$v = 6.15 \text{ m/s}$$ **Ans.**

Ans:
$v = 6.15$ m/s

15–102. The two spheres each have a mass of 3 kg and are attached to the rod of negligible mass. Determine the time the torque $M = (8t)\ \text{N} \cdot \text{m}$, where t is in seconds, must be applied to the rod so that each sphere attains a speed of 3 m/s starting from rest.



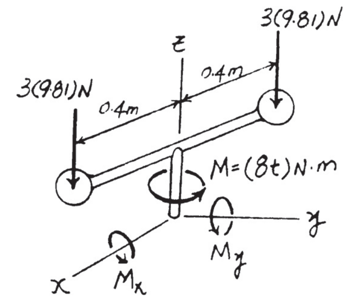
SOLUTION

Principle of Angular Impluse and Momentum: Applying Eq. 15–22, we have

$$(H_z)_1 + \Sigma \int_{t_1}^{t_2} M_z\, dt = (H_z)_2$$

$$2[0.4\,(3)\,(0)] + \int_0^t (8t)\, dt = 2[0.4\,(3)\,(3)]$$

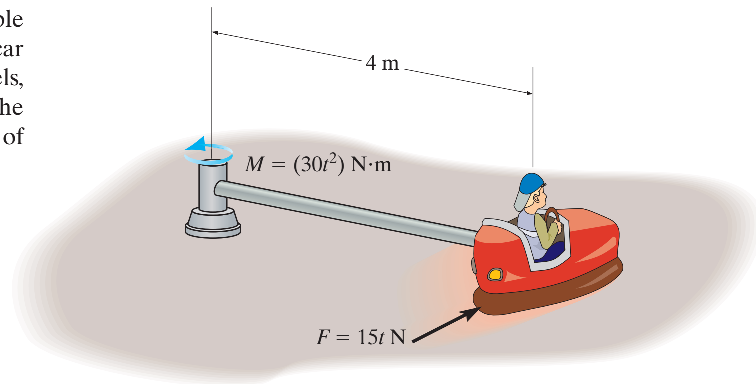
$$t = 1.34 \text{ s}$$ **Ans.**



Ans:
$t = 1.34$ s

15–103.

If the rod of negligible mass is subjected to a couple moment of $M = (30t^2)$ N·m and the engine of the car supplies a traction force of $F = (15t)$ N to the wheels, where t is in seconds, determine the speed of the car at the instant $t = 5$ s. The car starts from rest. The total mass of the car and rider is 150 kg. Neglect the size of the car.



SOLUTION

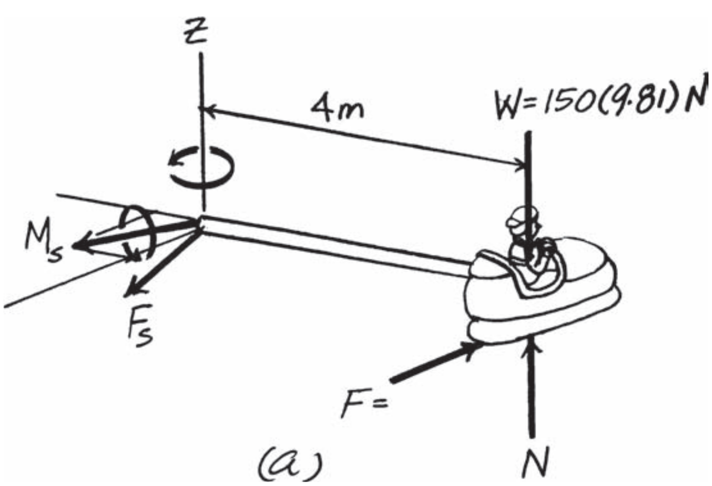
Free-Body Diagram: The free-body diagram of the system is shown in Fig. *a*. Since the moment reaction $\mathbf{M}_S$ has no component about the z axis, the force reaction $\mathbf{F}_S$ acts through the z axis, and the line of action of **W** and **N** are parallel to the z axis, they produce no angular impulse about the z axis.

Principle of Angular Impulse and Momentum:

$$(H_1)_z + \Sigma \int_{t_2}^{t_1} M_z\, dt = (H_2)_z$$

$$0 + \int_0^{5\text{ s}} 30t^2\, dt + \int_0^{5\text{ s}} 15t(4)dt = 150v(4)$$

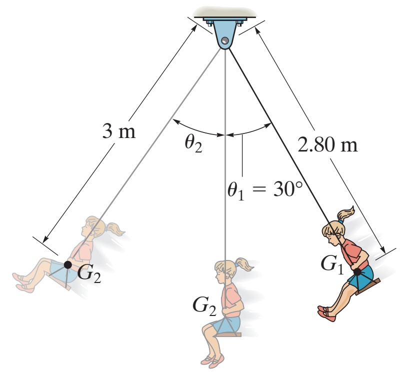
$v = 3.33$ m/s **Ans.**



Ans:
$v = 3.33$ m/s

***15–104.**

A child having a mass of 50 kg holds her legs up as shown as she swings downward from rest at $\theta_1 = 30°$. Her center of mass is located at point G_1. When she is at the bottom position $\theta = 0°$, she *suddenly* lets her legs come down, shifting her center of mass to position G_2. Determine her speed in the upswing due to this sudden movement and the angle θ_2 to which she swings before momentarily coming to rest. Treat the child's body as a particle.



SOLUTION

First before $\theta = 30°$;

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.80(1 - \cos 30°)(50)(9.81) = \frac{1}{2}(50)(v_1)^2 + 0$$

$$v_1 = 2.713 \text{ m/s}$$

$$H_1 = H_2$$

$$50(2.713)(2.80) = 50(v_2)(3)$$

$$v_2 = 2.532 = 2.53 \text{ m/s}$$ **Ans.**

Just after $\theta = 0°$;

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(50)(2.532)^2 + 0 = 0 + 50(9.81)(3)(1 - \cos\theta_2)$$

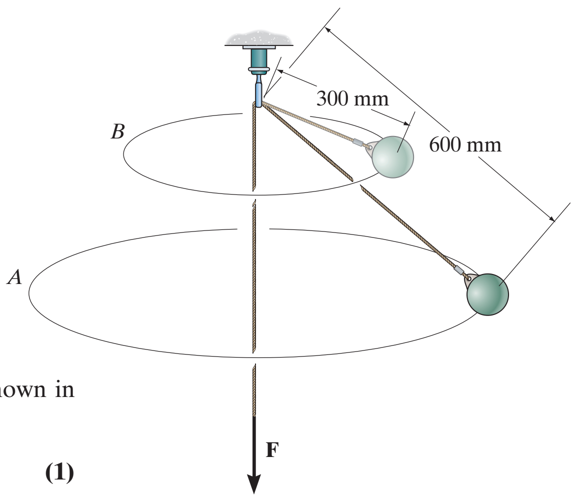
$$0.1089 = 1 - \cos\theta_2$$

$$\theta_2 = 27.0°$$ **Ans.**

Ans:
$v_2 = 2.53$ m/s
$\theta_2 = 27.0°$

15–105.

When the 2-kg bob is given a horizontal speed of 1.5 m/s, it begins to rotate around the horizontal circular path A. If the force $\mathbf{F}$ on the cord is increased, the bob rises and then rotates around the horizontal circular path B. Determine the speed of the bob around path B. Also, find the work done by force $\mathbf{F}$.



SOLUTION

Equations of Motion: By referring to the free-body diagram of the bob shown in Fig. a,

$$+\uparrow \Sigma F_b = 0; \qquad F\cos\theta - 2(9.81) = 0 \qquad \textbf{(1)}$$

$$\leftarrow \Sigma F_n = ma_n; \qquad F\sin\theta = 2\left(\frac{v^2}{l\sin\theta}\right) \qquad \textbf{(2)}$$

Eliminating F from Eqs. (1) and (2) yields

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{9.81l}$$

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v^2}{9.81l} \qquad \textbf{(3)}$$

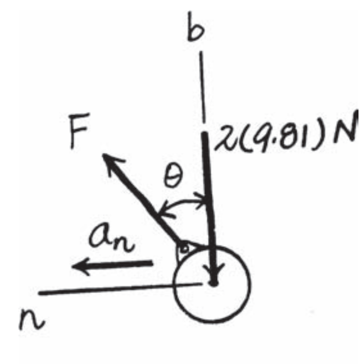
When $l = 0.6$ m, $v = v_1 = 5$ m/s. Using Eq. (3), we obtain

$$\frac{1-\cos^2\theta_1}{\cos\theta_1} = \frac{1.5^2}{9.81(0.6)}$$

$$\cos^2\theta_1 + 0.3823\cos\theta_1 - 1 = 0$$

Solving for the root < 1, we obtain

$$\theta_1 = 34.21^\circ$$



(a)

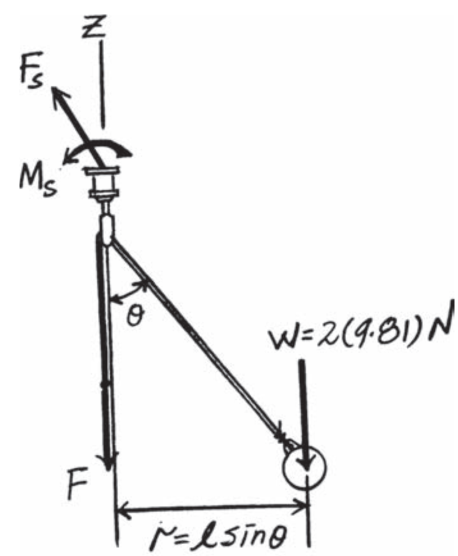
Conservation of Angular Momentum: By observing the free-body diagram of the system shown in Fig. b, notice that $\mathbf{W}$ and $\mathbf{F}$ are parallel to the z axis, $\mathbf{M}_S$ has no z component, and $\mathbf{F}_S$ acts through the z axis. Thus, they produce no angular impulse about the z axis. As a result, the angular momentum of the system is conserved about the z axis. When $\theta = \theta_1 = 34.21^\circ$ and $\theta = \theta_2$, $r = r_1 = 0.6\sin 34.21^\circ = 0.3373$ m and $r = r_2 = 0.3\sin\theta_2$. Thus,

$$(H_z)_1 = (H_z)_2$$

$$r_1 m v_1 = r_2 m v_2$$

$$0.3373(2)(1.5) = 0.3\sin\theta_2\,(2)v_2$$

$$v_2\sin\theta_2 = 1.6867 \qquad \textbf{(4)}$$



(b)

15–105. Continued

Substituting $l = 0.3$ and $\theta = \theta_2$ $v = v_2$ into Eq. (3) yields

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{v^2}{9.81(0.3)}$$

$$\frac{1 - \cos^2\theta_2}{\cos\theta_2} = \frac{{v_2}^2}{2.943} \qquad \textbf{(5)}$$

Eliminating v_2 from Eqs. (4) and (5),

$$\sin^3\theta_2 \tan\theta_2 - 0.9667 = 0$$

Solving the above equation by trial and error, we obtain

$$\theta_2 = 57.866°$$

Substituting the result of θ_2 into Eq. (4), we obtain

$$v_2 = 1.992 \text{ m/s} = 1.99 \text{ m/s} \qquad \textbf{Ans.}$$

Principle of Work and Energy: When θ changes from θ_1 to θ_2, **W** displaces vertically upward $h = 0.6 \cos 34.21° - 0.3 \cos 57.866° = 0.3366$ m. Thus, **W** does negatives work.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}m{v_1}^2 + U_F + \left(-Wh\right) = \frac{1}{2}m{v_2}^2$$

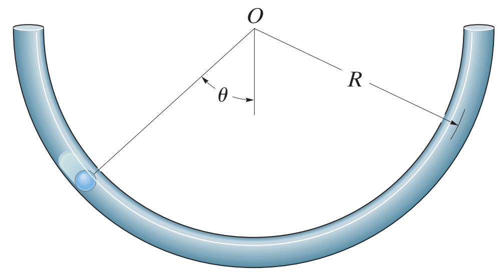
$$\frac{1}{2}(2)(1.5^2) + U_F - 2(9.81)(0.3366) = \frac{1}{2}(2)(1.992)^2$$

$$U_F = 8.32 \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

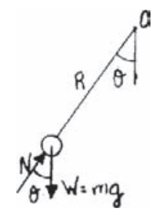
Ans:
$v_2 = 1.99$ m/s
$U_F = 8.32$ N · m

15–106.

A small particle having a mass m is placed inside the semicircular tube. The particle is placed at the position shown and released. Apply the principle of angular momentum about point O ($\Sigma M_O = \dot{H}_O$), and show that the motion of the particle is governed by the differential equation $\ddot{\theta} + (g/R)\sin\theta = 0$.



SOLUTION



$$\circlearrowleft + \Sigma M_O = \frac{dH_O}{dt}; \qquad -Rmg\sin\theta = \frac{d}{dt}(mvR)$$

$$g\sin\theta = -\frac{dv}{dt} = -\frac{d^2s}{dt^2}$$

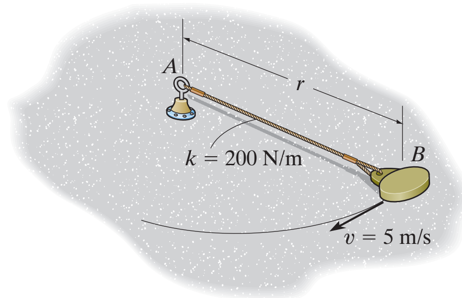
But, $s = R\theta$

Thus, $g\sin\theta = -R\ddot{\theta}$

or, $\ddot{\theta} + \left(\frac{g}{R}\right)\sin\theta = 0$ **Q.E.D.**

Ans:
$\ddot{\theta} + \left(\frac{g}{R}\right)\sin\theta = 0$

15–107. At the instant $r = 1.5$ m, the 5-kg disk is given a speed of $v = 5$ m/s, perpendicular to the elastic cord. Determine the speed of the disk and the rate of shortening of the elastic cord at the instant $r = 1.2$ m. The disk slides on the smooth horizontal plane. Neglect its size. The cord has an unstretched length of 0.5 m.



SOLUTION

Conservation of Energy: The intial and final stretch of the elastic cord is $s_1 = 1.5 - 0.5 = 1$ m and $s_2 = 1.2 - 0.5 = 0.7$ m. Thus,

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mv_1{}^2 + \frac{1}{2}ks_1{}^2 = \frac{1}{2}mv_2{}^2 = \frac{1}{2}ks_2{}^2$$

$$\frac{1}{2}5(5^2) + \frac{1}{2}(200)(1^2) = \frac{1}{2}(5)v_2{}^2 + \frac{1}{2}(200)(0.7^2)$$

$$v_2 = 6.738 \text{ m/s}$$ **Ans.**

Conservation of Angular Momentum: Since no angular impulse acts on the disk about an axis perpendicular to the page passing through point O, its angular momentum of the system is conserved about this z axis. Thus,

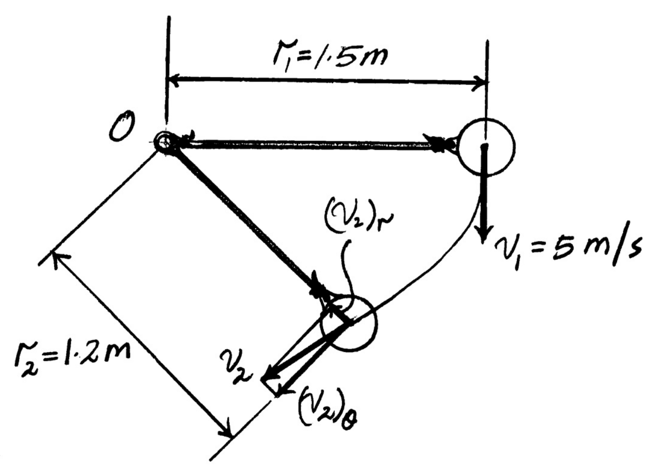
$$(H_O)_1 = (H_O)_2$$

$$r_1 m v_1 = r_2 m (v_2)_\theta$$

$$(v_2)_\theta = \frac{r_1 v_1}{r_2} = \frac{1.5(5)}{1.2} = 6.25 \text{ m/s}$$

Since $v_2{}^2 = (v_2)_\theta{}^2 + (v_2)_r{}^2$, then

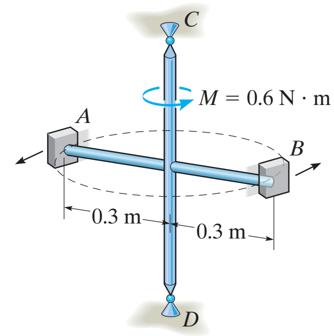
$$(v_2)_r = \sqrt{v_2{}^2 - (v_2)_\theta{}^2} = \sqrt{6.738^2 - 6.25^2} = 2.52 \text{ m/s}$$ **Ans.**



Ans:
$v_2 = 6.738$ m/s
$(v_2)_r = 2.52$ m/s

***15–108.**

The two blocks A and B each have a mass of 400 g. The blocks are fixed to the horizontal rods, and their initial velocity along the circular path is 2 m/s. If a couple moment of $M = (0.6)\ \text{N}\cdot\text{m}$ is applied about CD of the frame, determine the speed of the blocks when $t = 3$ s. The mass of the frame is negligible, and it is free to rotate about CD. Neglect the size of the blocks.



SOLUTION

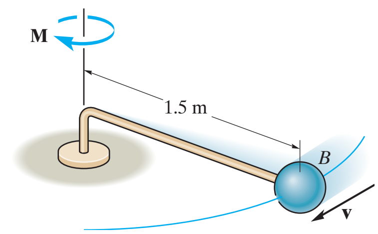
$$(H_o)_1 + \Sigma \int_{t_1}^{t_2} M_o dt = (H_o)_2$$

$$2[0.3(0.4)(2)] + 0.6(3) = 2[0.3(0.4)v]$$

$$v = 9.50 \text{ m/s}$$ **Ans.**

Ans:
$v = 9.50$ m/s

15–109. The ball B has mass of 10 kg and is attached to the end of a rod whose mass may be neglected. If the rod is subjected to a torque $M = (3t^2 + 5t + 2)$ N · m, where t is in seconds, determine the speed of the ball when $t = 2$ s. The ball has a speed $v = 2$ m/s when $t = 0$.



SOLUTION

Principle of angular impulse momentum

Given:

$M = 10$ kg

$a = 3$ N·m/s^2

$b = 5$ N·m/s

$c = 2$ N·m

$t_1 = 2$ s

$v_0 = 2$ m/s

$L = 1.5$ m

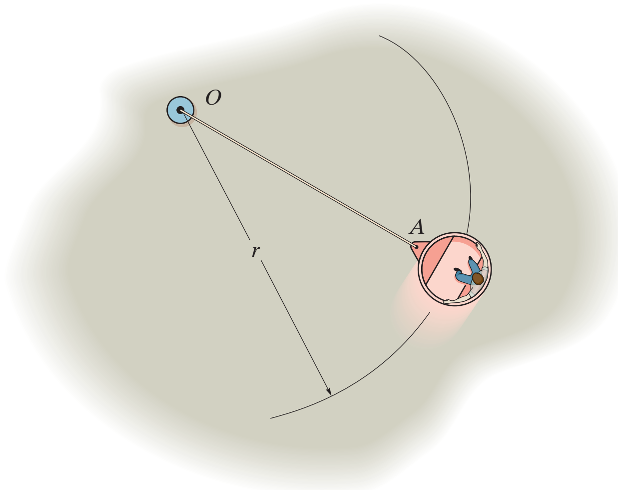
$$M v_0 L + \int_0^{t_1} a t^2 + b t + c \, dt = M v_1 L$$

$$v_1 = v_0 + \frac{1}{ML} \int_0^{t_1} a t^2 + b t + c \, dt \qquad v_1 = 3.47 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$v_1 = 3.47$ m/s

15–110.

The amusement park ride consists of a 200-kg car and passenger that are traveling at 3 m/s along a circular path having a radius of 8 m. If at $t = 0$, the cable OA is pulled in toward O at 0.5 m/s, determine the speed of the car when $t = 4$ s. Also, determine the work done to pull in the cable.



SOLUTION

Conservation of Angular Momentum. At $t = 4$ s,
$r_2 = 8 - 0.5(4) = 6$ m.

$$(H_0)_1 = (H_0)_2$$

$$r_1 m v_1 = r_2 m (v_2)_t$$

$$8[200(3)] = 6[200(v_2)_t]$$

$$(v_2)_t = 4.00 \text{ m/s}$$

Here, $(v_2)_t = 0.5$ m/s. Thus

$$v_2 = \sqrt{(v_2)_t^2 + (v_2)_r^2} = \sqrt{4.00^2 + 0.5^2} = 4.031 \text{ m/s} = 4.03 \text{ m/s} \qquad \textbf{Ans.}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

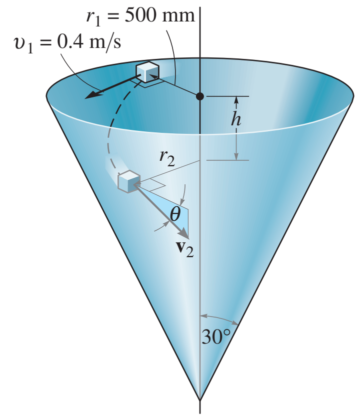
$$\frac{1}{2}(200)(3^2) + \Sigma U_{1-2} = \frac{1}{2}(200)(4.031)^2$$

$$\Sigma U_{1-2} = 725 \text{ J} \qquad \textbf{Ans.}$$

Ans:
$v_2 = 4.03$ m/s
$\Sigma U_{1-2} = 725$ J

15–111.

A small block having a mass of 0.1 kg is given a horizontal velocity of $v_1 = 0.4$ m/s when $r_1 = 500$ mm. It slides along the smooth conical surface. Determine the distance h it must descend for it to reach a speed of $v_2 = 2$ m/s. Also, what is the angle of descent θ, that is, the angle measured from the horizontal to the tangent of the path?

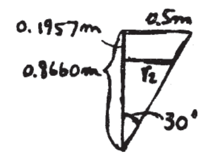


SOLUTION

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}(0.1)(0.4)^2 + 0.1(9.81)(h) = \frac{1}{2}(0.1)(2)^2$$

$h = 0.1957 \text{ m} = 196 \text{ mm}$ **Ans.**



From similar triangles

$$r_2 = \frac{(0.8660 - 0.1957)}{0.8660}(0.5) = 0.3870 \text{ m}$$

$(H_0)_1 = (H_0)_2$

$0.5(0.1)(0.4) = 0.3870(0.1)(v_2')$

$v_2' = 0.5168$ m/s

$v_2 = \cos\theta = v_2'$

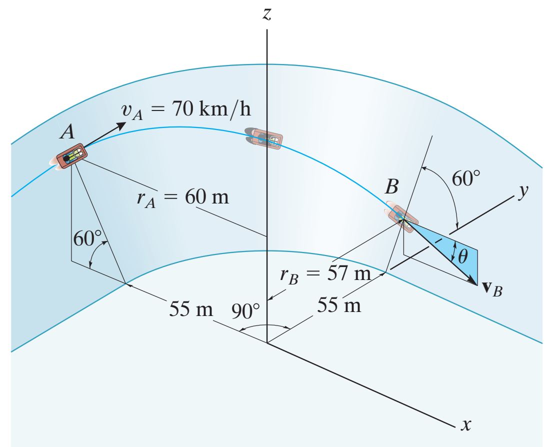
$2\cos\theta = 0.5168$

$\theta = 75.0°$ **Ans.**

Ans:
$h = 196$ mm
$\theta = 75.0°$

***15–112.**

A toboggan and rider, having a total mass of 150 kg, enter horizontally tangent to a 90° circular curve with a velocity of $v_A = 70$ km/h. If the track is flat and banked at an angle of 60°, determine the speed v_B and the angle θ of "descent," measured from the horizontal in a vertical x–z plane, at which the toboggan exists at B. Neglect friction in the calculation.



SOLUTION

$$v_A = 70 \text{ km/h} = 19.44 \text{ m/s}$$

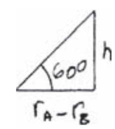
$$(H_A)_z = (H_B)_z$$

$$150(19.44)(60) = 150(v_B)\cos\theta(57) \qquad \textbf{(1)}$$

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2}(150)(19.44)^2 + 150(9.81)h = \frac{1}{2}(150)(v_B)^2 + 0 \qquad \textbf{(2)}$$



Since $h = (r_A - r_B)\tan 60° = (60 - 57)\tan 60° = 5.196$

Solving Eq. (1) and Eq (2):

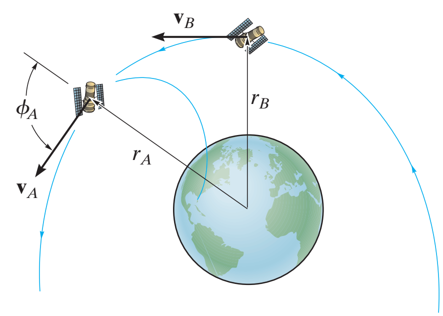
$$v_B = 21.9 \text{ m/s} \qquad \textbf{Ans.}$$

$$\theta = 20.9 \qquad \textbf{Ans.}$$

Ans:
$v_B = 21.9$ m/s
$\theta = 20.9$

15–113.

An earth satellite of mass 700 kg is launched into a free-flight trajectory about the earth with an initial speed of $v_A = 10\ \text{km/s}$ when the distance from the center of the earth is $r_A = 15\ \text{Mm}$. If the launch angle at this position is $\phi_A = 70°$, determine the speed v_B of the satellite and its closest distance r_B from the center of the earth. The earth has a mass $M_e = 5.976(10^{24})\ \text{kg}$. *Hint:* Under these conditions, the satellite is subjected only to the earth's gravitational force, $F = GM_e m_s/r^2$, Eq. 13–1. For part of the solution, use the conservation of energy.



SOLUTION

$(H_O)_1 = (H_O)_2$

$m_s\,(v_A \sin \phi_A) r_A = m_s\,(v_B) r_B$

$$700[10(10^3)\sin 70°](15)(10^6) = 700(v_B)(r_B) \qquad \textbf{(1)}$$

$T_A + V_A = T_B + V_B$

$$\frac{1}{2} m_s\,(v_A)^2 - \frac{GM_e m_s}{r_A} = \frac{1}{2} m_s\,(v_B)^2 - \frac{GM_e m_s}{r_B}$$

$$\frac{1}{2}(700)[10(10^3)]^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{[15(10^6)]} = \frac{1}{2}(700)(v_B)^2 - \frac{66.73(10^{-12})(5.976)(10^{24})(700)}{r_B} \qquad \textbf{(2)}$$

Solving,

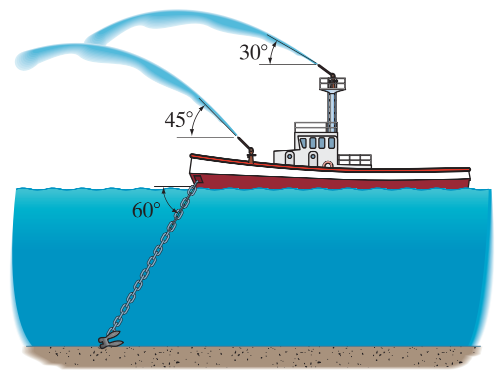
$v_B = 10.2\ \text{km/s}$ **Ans.**

$r_B = 13.8\ \text{Mm}$ **Ans.**

Ans:
$v_B = 10.2\ \text{km/s}$
$r_B = 13.8\ \text{Mm}$

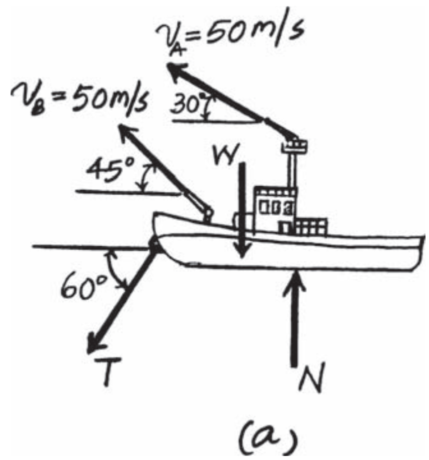
15–114.

The fire boat discharges two streams of seawater, each at a flow of $0.25\ m^3/s$ and with a nozzle velocity of 50 m/s. Determine the tension developed in the anchor chain needed to secure the boat. The density of seawater is $\rho_{sw} = 1020\ kg/m^3$.



SOLUTION

Steady Flow Equation: Here, the mass flow rate of the sea water at nozzles A and B are $\frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho sw\, Q = 1020(0.25) = 225\ kg/s$. Since the sea water is collected from the larger reservoir (the sea), the velocity of the sea water entering the control volume can be considered zero. By referring to the free-body diagram of the control volume (the boat),



$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm_A}{dt}(v_A)_x + \frac{dm_B}{dt}(v_B)_x;$$

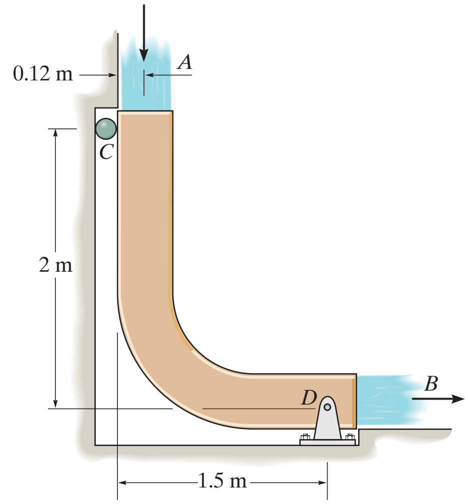
$$T \cos 60° = 225(50 \cos 30°) + 225(50 \cos 45°)$$

$$T = 40\,114.87\ N = 40.1\ kN \qquad \textbf{Ans.}$$

Ans:
$T = 40.1\ kN$

15–115.

The chute is used to divert the flow of water, $Q = 0.6\ \text{m}^3/\text{s}$. If the water has a cross-sectional area of $0.05\ \text{m}^2$, determine the force components at the pin D and roller C necessary for equilibrium. Neglect the weight of the chute and weight of the water on the chute. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

Equations of Steady Flow: Here, the flow rate $Q = 0.6\ \text{m}^2/\text{s}$. Then, $v = \dfrac{Q}{A} = \dfrac{0.6}{0.05} = 12.0\ \text{m/s}$. Also, $\dfrac{dm}{dt} = \rho_w Q = 1000\,(0.6) = 600\ \text{kg/s}$. Applying Eqs. 15–26 and 15–28, we have

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}(d_{DB}\, v_B - d_{DA}\, v_A);$$

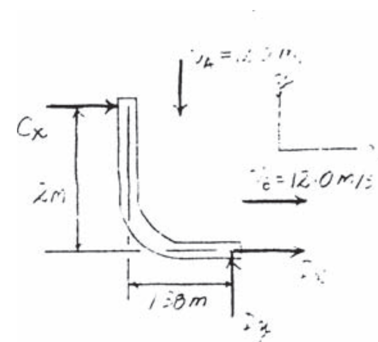
$$-C_x\,(2) = 600\,[0 - 1.38(12.0)] \qquad C_x = 4968\ \text{N} = 4.97\ \text{kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = \frac{dm}{dt}\left(v_{B_x} - v_{A_x}\right);$$

$$D_x + 4968 = 600\,(12.0 - 0) \qquad D_x = 2232\text{N} = 2.23\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \Sigma \frac{dm}{dt}\left(v_{\text{out}_y} - v_{\text{in}_y}\right);$$

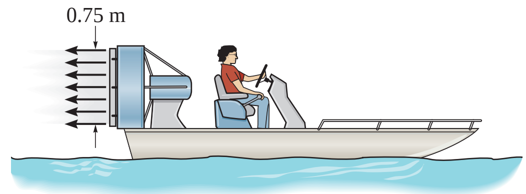
$$D_y = 600[0 - (-12.0)] \qquad D_y = 7200\ \text{N} = 7.20\ \text{kN} \qquad \textbf{Ans.}$$



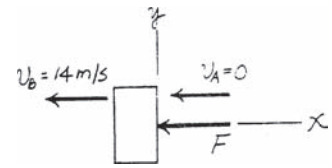
Ans:
$C_x = 4.97\ \text{kN}$
$D_x = 2.23\ \text{kN}$
$D_y = 7.20\ \text{kN}$

***15–116.**

The 200-kg boat is powered by the fan which develops a slipstream having a diameter of 0.75 m. If the fan ejects air with a speed of 14 m/s, measured relative to the boat, determine the initial acceleration of the boat if it is initially at rest. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$ and that the entering air is essentially at rest. Neglect the drag resistance of the water.

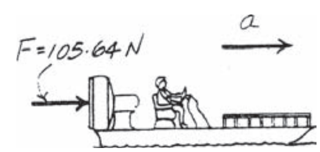


SOLUTION



Equations of Steady Flow: Initially, the boat is at rest hence $v_B = v_{a/b} = 14\ \text{m/s}$. Then, $Q = v_B A = 14\left[\frac{\pi}{4}\left(0.75^2\right)\right] = 6.185\ \text{m}^3/\text{s}$ and $\frac{dm}{dt} = \rho_a Q = 1.22(6.185) = 7.546\ \text{kg/s}$. Applying Eq. 15–25, we have

$$\Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x}); \qquad -F = 7.546(-14 - 0) \qquad F = 105.64\ \text{N}$$

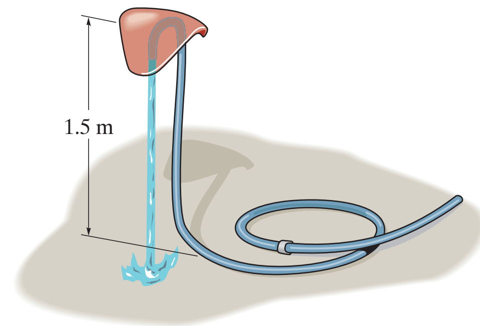


Equation of Motion :

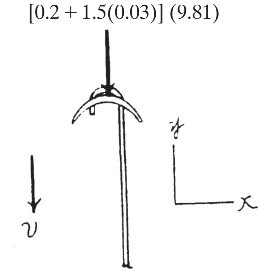
$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad 105.64 = 200a \qquad a = 0.528\ \text{m/s}^2$$ **Ans.**

Ans:
$a = 0.528\ \text{m/s}^2$

15–117. The toy sprinkler for children consists of a 0.2-kg cap and a hose that has a mass per length of 30 g/m. Determine the required rate of flow of water through the 5-mm-diameter tube so that the sprinkler will lift 1.5 m from the ground and hover from this position. Neglect the weight of the water in the tube. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION



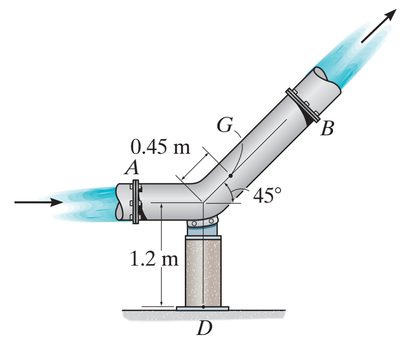
Equations of Steady Flow: Here, $v = \dfrac{Q}{\frac{\pi}{4}(0.005^2)} = \dfrac{Q}{6.25(10^{-6})\pi}$ and $\dfrac{dm}{dt} = \rho_w Q = 1000Q$. Applying Eq. 15–25, we have

$$\Sigma F_y = \frac{dm}{dt}\left(v_{B_y} - v_{A_y}\right); \quad -[0.2 + 1.5\,(0.03)]\,(9.81) = 1000Q\left(-\frac{Q}{6.25\,(10^{-6})\,\pi} - 0\right)$$

$$Q = 0.217\left(10^{-3}\right)\text{m}^3/\text{s} \qquad \textbf{Ans.}$$

Ans:
$Q = 0.217\left(10^{-3}\right)\text{m}^3/\text{s}$

15-118. The bend is connected to the pipe at flanges A and B as shown. If the diameter of the pipe is 0.3 m and it carries a discharge of 1.35 m^3/s, determine the horizontal and vertical components of force reaction and the moment reaction exerted at the fixed base D of the support. The total weight of the bend and the water within it is 2500 N, with a mass center at point G. The gauge pressure of the water at the flanges at A and B are 120 kN/m^2 and 96 kN/m^2, respectively. Assume that no force is transferred to the flanges at A and B. The specific weight of water is $\gamma_w = 10\ kN/m^3$.



SOLUTION

Free-Body Diagram: The free-body of the control volume is shown in Fig. *a*. The force exerted on sections A and B due to the water pressure is $F_A = P_A A_A = 120\left[\frac{\pi}{4}(0.3^2)\right] = 8.4823$ kN and $F_B = P_B A_B = 96\left[\frac{\pi}{4}(0.3^2)\right] = 6.7858$ kN. The speed of the water at, sections A and B are $v_A = v_B = \frac{Q}{A} = \frac{1.35}{\frac{\pi}{4}(0.3^2)} = 19.10$ m/s. Also, the mass flow rate at these two sections are $\frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho_W Q = \frac{10(10^3)}{9.81}(1.35) = 1376.1$ kg/s.

Steady Flow Equation: The moment steady flow equation will be written about point D to eliminate D_x and D_y.

$$\zeta + \quad \Sigma M_D = \frac{dm_B}{dt}dv_B - \frac{dm_A}{dt}dv_A;$$

$$M_D + 6.7858(10^3)\cos 45°(1.2) - 2500(0.45\cos 45°) - 8.4823(10^3)(1.2)$$

$$= -1376.1(1.2)(19.10\cos 45°) - [-1376.1(1.2)(19.10)]$$

$$M_D = 14\,454.23\ \text{N}\cdot\text{m} = 14.45\ \text{kN}\cdot\text{m}$$ **Ans.**

Writing the force steady flow equation along the x and y axes,

$$(+\uparrow)\ \Sigma F_y = \frac{dm}{dt}\left[(v_B)_y - (v_A)_y\right];$$

$$D_y - 2500 - 6.7858(10^3)\sin 45° = 1376.1(19.10\sin 45° - 0)$$

$$D_y = 25\,883.53\ \text{N} = 25.88\ \text{kN}$$ **Ans.**

$$(\overset{+}{\rightarrow})\ \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right];$$

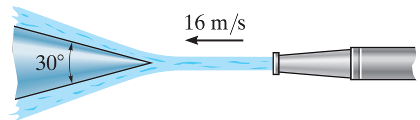
$$8.4823(10^3) - 6.7858(10^3)\cos 45° - D_x = 1376.1[19.1\cos 45° - 19.1]$$

$$D_x = 11382.28\ \text{N} = 11.38\ \text{kN}$$ **Ans.**

Ans:
$M_D = 14.45\ \text{kN}\cdot\text{m}$
$D_y = 25.88$ kN
$D_x = 11.38$ kN

15-119.

Water is discharged at speed v against the fixed cone diffuser. If the opening diameter of the nozzle is d, determine the horizontal force exerted by the water on the diffuser.



SOLUTION

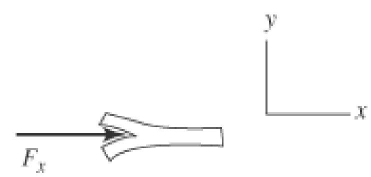
Given:

$v = 16\ \text{m/s}$

$d = 40\ \text{mm}$

$\theta = 30°$

$\rho_w = 1\ \text{Mg/m}^3$



$Q = \frac{\pi}{4} d^2 v$

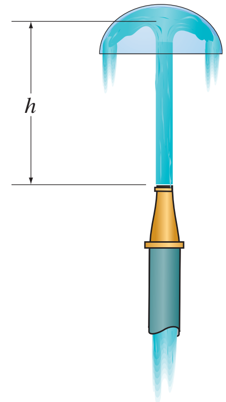
$m' = \rho_w Q$

$F_x = m'\left(-v\cos\left(\frac{\theta}{2}\right) + v\right)$

$F_x = 11.0\ \text{N}$ **Ans.**

Ans:
$F_x = 11.0\ \text{N}$

***15–120.** The hemispherical bowl of mass m is held in equilibrium by the vertical jet of water discharged through a nozzle of diameter d. If the discharge of the water through the nozzle is Q, determine the height h at which the bowl is suspended. The water density is ρ_w. Neglect the weight of the water jet.



SOLUTION

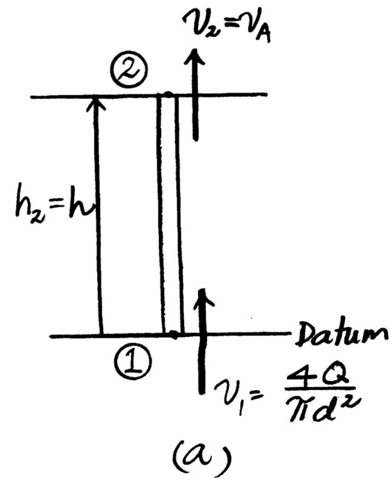
Conservation of Energy: The speed at which the water particle leaves the nozzle is $v_1 = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$. The speed of particle v_A when it comes in contact with the bowl can be determined using conservation of energy. With reference to the datum set in Fig. *a*,

$$T_1 + V_1 = T_2 + V_2 0$$

$$\frac{1}{2}mv_1{}^2 + \left(V_g\right)_1 = \frac{1}{2}mv_2{}^2 + \left(V_g\right)_2$$

$$\frac{1}{2}m\left(\frac{4Q}{\pi d^2}\right)^2 + 0 = \frac{1}{2}mv_A{}^2 + mgh$$

$$v_A = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$$



Steady Flow Equation: The mass flow rate of the water jet that enters the control volume at A is $\frac{dm_A}{dt} = \rho_w Q$, and exits from the control volume at B is $\frac{dm_B}{dt} = \frac{dm_A}{dt} = \rho_w Q$. Thus, $v_B = v_A = \sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}$. Here, the vertical force acting on the control volume is equal to the weight of the bowl. By referring to the free - body diagram of the control volume, Fig. *b*,

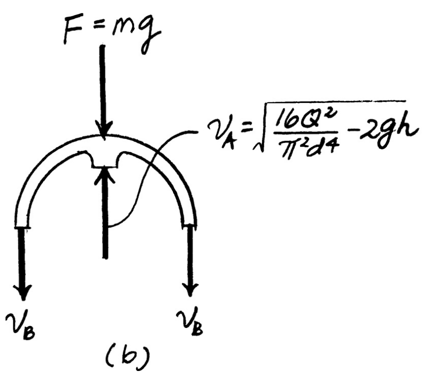
$$+\uparrow \Sigma F_y = 2\frac{dm_B}{dt}v_B - \frac{dm_A}{dt}v_A;$$

$$-mg = -(\rho_w Q)\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right) - \rho_w Q\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right)$$

$$mg = 2\rho_w Q\left(\sqrt{\frac{16Q^2}{\pi^2 d^4} - 2gh}\right)$$

$$m^2g^2 = 4\rho_w{}^2Q^2\left(\frac{16Q^2}{\pi^2 d^4} - 2gh\right)$$

$$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w{}^2 Q^2} \qquad \textbf{Ans.}$$

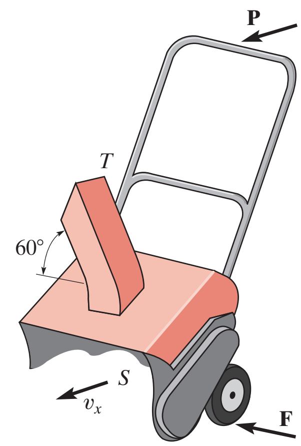


Ans:

$h = \frac{8Q^2}{\pi^2 d^4 g} - \frac{m^2 g}{8\rho_w{}^2 Q^2}$

15–121.

A snowblower having a scoop S with a cross-sectional area of $A_s = 0.12\text{ m}^3$ is pushed into snow with a speed of $v_s = 0.5\text{ m/s}$. The machine discharges the snow through a tube T that has a cross-sectional area of $A_T = 0.03\text{ m}^2$ and is directed 60° from the horizontal. If the density of snow is $\rho_s = 104\text{ kg/m}^3$, determine the horizontal force P required to push the blower forward, and the resultant frictional force F of the wheels on the ground, necessary to prevent the blower from moving sideways. The wheels roll freely.



SOLUTION

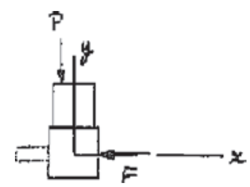
$$\frac{dm}{dt} = \rho v_s A_s = (104)(0.5)(0.12) = 6.24\text{ kg/s}$$

$$v_s = \frac{dm}{dt}\left(\frac{1}{\rho A_r}\right) = \left(\frac{6.24}{104(0.03)}\right) = 2.0\text{ m/s}$$

$$\Sigma F_x = \frac{dm}{dt}(v_{T_2} - v_{S_2})$$

$$-F = 6.24(-2\cos 60° - 0)$$

$$F = 6.24\text{ N}$$ **Ans.**



$$\Sigma F_y = \frac{dm}{dt}(v_{T_2} - v_{S_2})$$

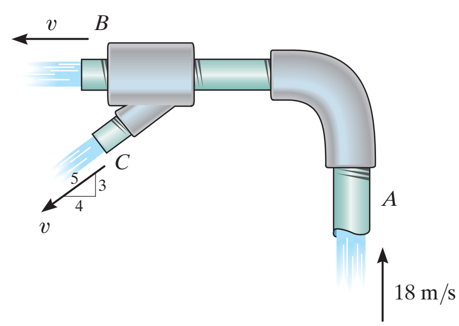
$$-P = 6.24(0 - 0.5)$$

$$P = 3.12\text{ N}$$ **Ans.**

Ans:
$F = 6.24\text{ N}$
$P = 3.12\text{ N}$

15–122.

The gauge pressure of water at A is 150.5 kPa. Water flows through the pipe at A with a velocity of 18 m/s, and out the pipe at B and C with the same velocity v. Determine the horizontal and vertical components of force exerted on the elbow necessary to hold the pipe assembly in equilibrium. Neglect the weight of water within the pipe and the weight of the pipe. The pipe has a diameter of 50 mm at A, and at B and C the diameter is 30 mm. $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

Continuity. The flow rate at B and C are the same since the pipe have the same diameter there. The flow rate at A is

$$Q_A = v_A A_A = (18)[\pi(0.025^2)] = 0.01125\pi\ \text{m}^3/\text{s}$$

Continuity negatives that

$$Q_A = Q_B + Q_C; \qquad 0.01125\pi = 2Q$$

$$Q = 0.005625\pi\ \text{m}^3/\text{s}$$

Thus,

$$v_c = v_B = \frac{Q}{A} = \frac{0.005625\pi}{\pi(0.015^2)} = 25\ \text{m/s}$$

Equation of Steady Flow. The force due to the pressure at A is $P = \rho_A A_A = (150.5)(10^3)[\pi(0.025^2)] = 94.0625\pi$ N. Here, $\dfrac{dm_A}{dt} = \rho_w Q_A = 1000(0.01125\pi) = 11.25\pi$ kg/s and $\dfrac{dm_A}{dt} = \dfrac{dM_c}{dt} = \rho_w Q = 1000(0.005625\pi) = 5.625\pi$ kg/s.

$$\overset{+}{\leftarrow}\Sigma F_x = \frac{dm_B}{dt}(v_B)_x + \frac{dm_c}{dt}(v_c)_x - \frac{dm_A}{dt}(v_A)_x;$$

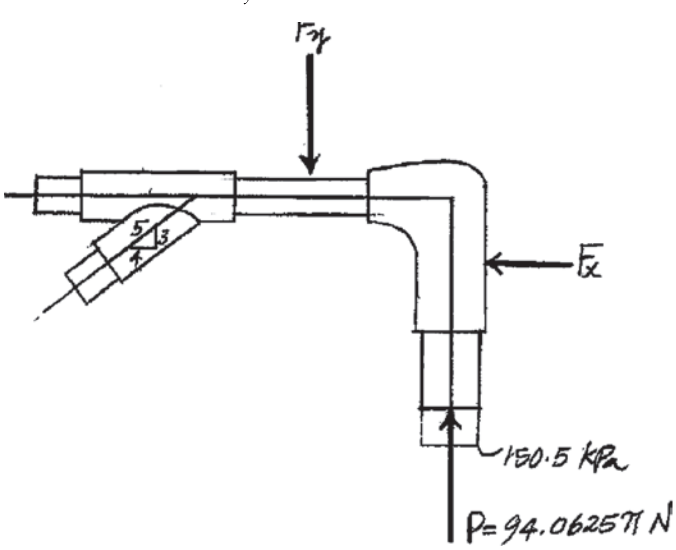
$$F_x = (5.625\pi)(25) + (5.625\pi)\left[25\left(\frac{4}{5}\right)\right] - (11.25\pi)(0)$$

$$= 795.22\ \text{N} = 795\ \text{N} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = \frac{dm_B}{dt}(v_B)_y + \frac{dm_C}{dt}(v_C)_y - \frac{dm_A}{dt}(v_A)_y;$$

$$94.0625\pi - F_y = (5.625\pi)(0) + (5.625\pi)\left[-25\left(\frac{3}{5}\right)\right] - (11.25\pi)(18)$$

$$F_y = 1196.75\ \text{N} = 1.20\ \text{kN} \qquad \textbf{Ans.}$$



Ans:
$F_x = 795$ N
$F_y = 1.20$ kN

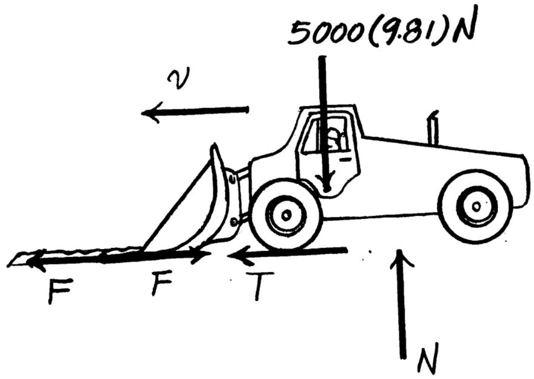
15–123. A scoop in front of the tractor collects snow at a rate of 200 kg/s. Determine the resultant traction force **T** that must be developed on all the wheels as it moves forward on level ground at a constant speed of 5 km/h. The tractor has a mass of 5 Mg.

SOLUTION

Here, the tractor moves with the constant speed of $v = \left[5\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left[\frac{1\text{ h}}{3600\text{ s}}\right]$ $= 1.389$ m/s. Thus, $v_{D/s} = v = 1.389$ m/s since the snow on the ground is at rest. The rate at which the tractor gains mass is $\frac{dm_s}{dt} = 200$ kg/s. Since the tractor is moving with a constant speeds $\frac{dv}{dt} = 0$. Referring to Fig. *a*,

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad T = 0 + 1.389(200)$$

$$T = 278 \text{ N} \qquad \textbf{Ans.}$$

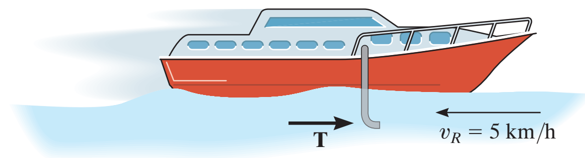


(a)

Ans:
$T = 278$ N

***15–124.**

The boat has a mass of 180 kg and is traveling forward on a river with a constant velocity of 70 km/h, measured *relative* to the river. The river is flowing in the opposite direction at 5 km/h. If a tube is placed in the water, as shown, and it collects 40 kg of water in the boat in 80 s, determine the horizontal thrust T on the tube that is required to overcome the resistance due to the water collection and yet maintain the constant speed of the boat. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

$$\frac{dm}{dt} = \frac{40}{80} = 0.5\ \text{kg/s}$$

$$v_{D/t} = (70)\left(\frac{1000}{3600}\right) = 19.444\ \text{m/s}$$

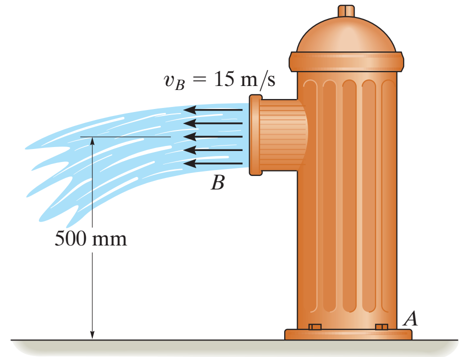
$$\Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

$$T = 0 + 19.444(0.5) = 9.72\ \text{N}$$ **Ans.**

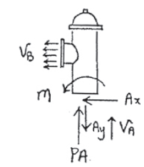
Ans:
$T = 9.72$ N

15–125.

Water is flowing from the 150-mm-diameter fire hydrant with a velocity $v_B = 15$ m/s. Determine the horizontal and vertical components of force and the moment developed at the base joint A, if the static (gauge) pressure at A is 50 kPa. The diameter of the fire hydrant at A is 200 mm. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION



$$\frac{dm}{dt} = \rho v_A A_B = 1000(15)(\pi)(0.075)^2$$

$$\frac{dm}{dt} = 265.07\ \text{kg/s}$$

$$v_A = \left(\frac{dm}{dt}\right)\frac{1}{\rho A_A} = \frac{265.07}{1000(\pi)(0.1)^2}$$

$$v_A = 8.4375\ \text{m/s}$$

$$\overset{+}{\leftarrow} \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

$A_x = 265.07(15-0) = 3.98$ kN **Ans.**

$$+\uparrow \Sigma F_y = \frac{dm}{dt}(v_{By} - v_{Ay})$$

$$-A_y + 50(10^3)(\pi)(0.1)^2 = 265.07(0-8.4375)$$

$A_y = 3.81$ kN **Ans.**

$$\circlearrowleft + \Sigma M_A = \frac{dm}{dt}(d_{AB}\,v_B - d_{AA}\,v_A)$$

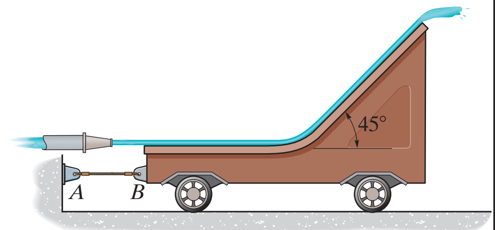
$$M = 265.07(0.5(15) - 0)$$

$M = 1.99\ \text{kN}\cdot\text{m}$ **Ans.**

Ans:
$A_x = 3.98$ kN
$A_y = 3.81$ kN
$M = 1.99\ \text{kN}\cdot\text{m}$

15–126.

Water is discharged from a nozzle with a velocity of 12 m/s and strikes the blade mounted on the 20-kg cart. Determine the tension developed in the cord, needed to hold the cart stationary, and the normal reaction of the wheels on the cart. The nozzle has a diameter of 50 mm and the density of water is $\rho_w = 1000\ \text{kg/m}^3$.



SOLUTION

Steady Flow Equation: Here, the mass flow rate at sections A and B of the control volume is $\dfrac{dm}{dt} = \rho_W Q = \rho_W A v = 1000\left[\dfrac{\pi}{4}(0.05^2)\right](12) = 7.5\pi\ \text{kg/s}$

Referring to the free-body diagram of the control volume shown in Fig. a,

$\overset{+}{\rightarrow} \Sigma F_x = \dfrac{dm}{dt}[(v_B)_x - (v_A)_x];\qquad -F_x = 7.5\pi(12\cos 45° - 12)$

$$F_x = 82.81\ \text{N}$$

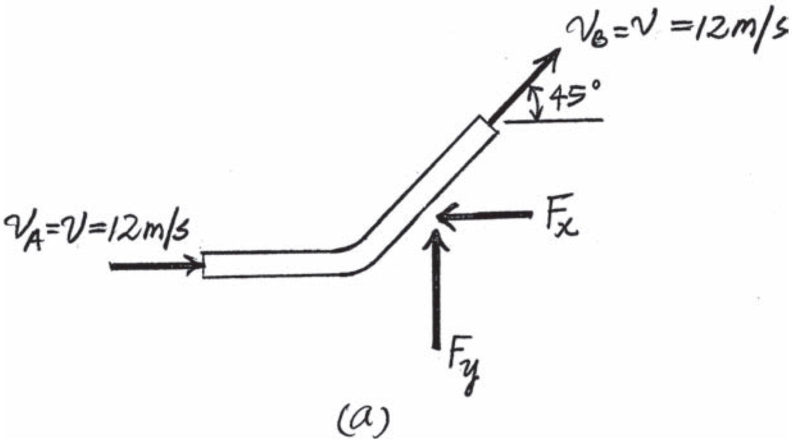
$+\uparrow \Sigma F_y = \dfrac{dm}{dt}[(v_B)_y - (v_A)_y];\qquad F_y = 7.5\pi(12\sin 45° - 0)$

$$F_y = 199.93\ \text{N}$$

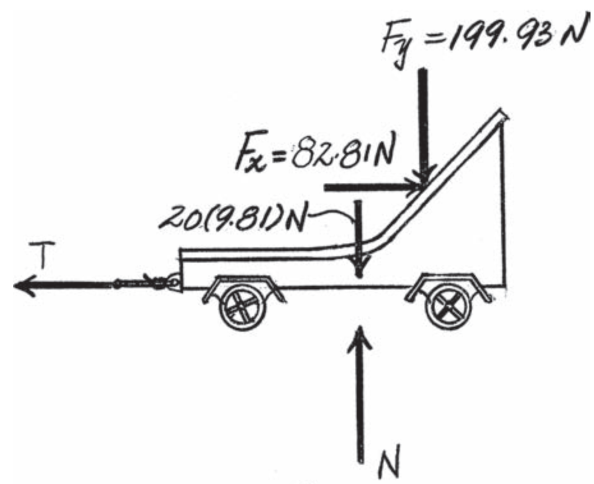
Equilibrium: Using the results of $\mathbf{F}_x$ and $\mathbf{F}_y$ and referring to the free-body diagram of the cart shown in Fig. b,

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 82.81 - T = 0\qquad T = 82.8\ \text{N}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad N - 20(9.81) - 199.93 = 0\qquad N = 396\ \text{N}$ **Ans.**



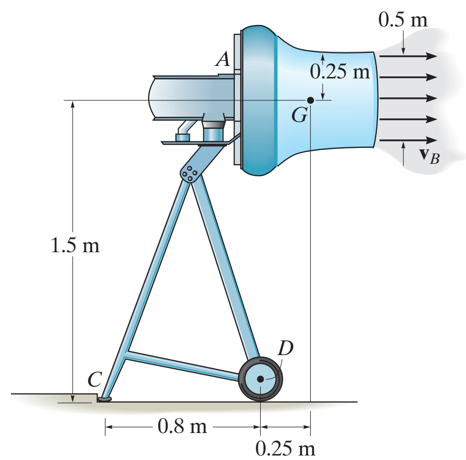
(a)



(b)

Ans:
$T = 82.8$ N
$N = 396$ N

15–127. When operating, the air-jet fan discharges air with a speed of $v_B = 20$ m/s into a slipstream having a diameter of 0.5 m. If air has a density of 1.22 kg/m^3, determine the horizontal and vertical components of reaction at C and the vertical reaction at each of the two wheels, D, when the fan is in operation. The fan and motor have a mass of 20 kg and a center of mass at G. Neglect the weight of the frame. Due to symmetry, both of the wheels support an equal load. Assume the air entering the fan at A is essentially at rest.



SOLUTION

$$\frac{dm}{dt} = \rho v A = 1.22(20)(\pi)(0.25)^2 = 4.791 \text{ kg/s}$$

$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}(v_{B_x} - v_{A_x})$$

$$C_x = 4.791(20 - 0)$$

$C_x = 95.8$ N **Ans.**

$$+\uparrow \Sigma F_y = 0; \qquad C_y + 2D_y - 20(9.81) = 0$$

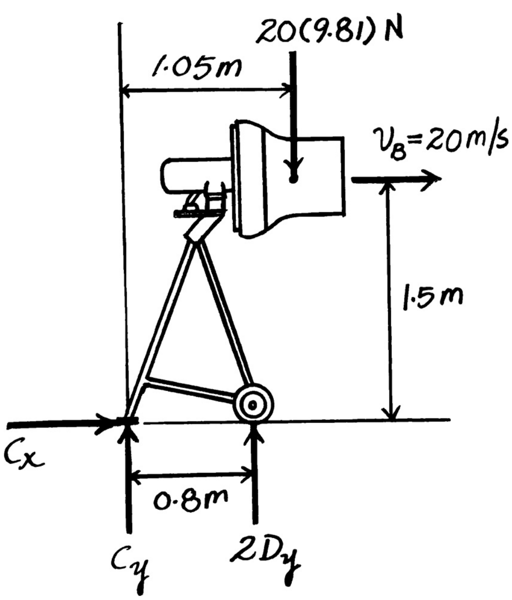
$$\zeta + \Sigma M_C = \frac{dm}{dt}(d_{CG}\, v_B - d_{CG}\, v_A)$$

$$2D_y(0.8) - 20(9.81)(1.05) = 4.791(-1.5(20) - 0)$$

Solving:

$D_y = 38.9$ N **Ans.**

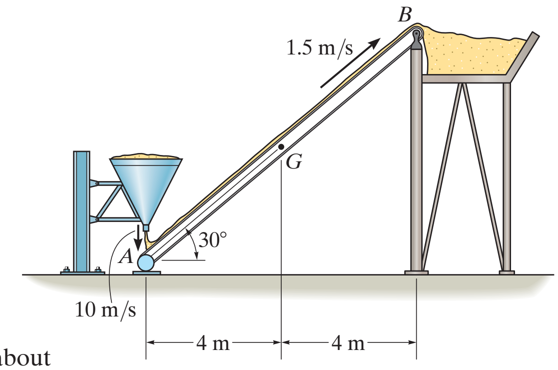
$C_y = 118$ N **Ans.**



Ans:
$C_x = 95.8$ N
$D_y = 38.9$ N
$C_y = 118$ N

***15–128.**

Sand is discharged from the silo at A at a rate of 50 kg/s with a vertical velocity of 10 m/s onto the conveyor belt, which is moving with a constant velocity of 1.5 m/s. If the conveyor system and the sand on it have a total mass of 750 kg and center of mass at point G, determine the horizontal and vertical components of reaction at the pin support B roller support A. Neglect the thickness of the conveyor.



SOLUTION

Steady Flow Equation: The moment steady flow equation will be written about point B to eliminate $\mathbf{B}_x$ and $\mathbf{B}_y$. Referring to the free-body diagram of the control volume shown in Fig. a,

$$+\Sigma M_B = \frac{dm}{dt}(dv_B - dv_A); \qquad 750(9.81)(4) - A_y(8) = 50[0 - 8(5)]$$

$$A_y = 4178.5 \text{ N} = 4.18 \text{ kN} \qquad \textbf{Ans.}$$

Writing the force steady flow equation along the x and y axes,

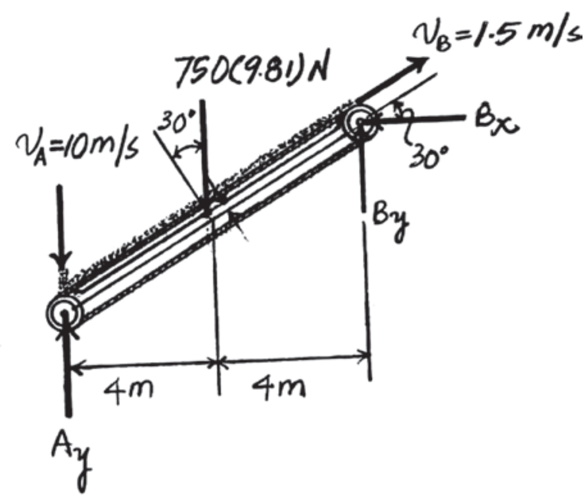
$$\xrightarrow{+} \Sigma F_x = \frac{dm}{dt}[(v_B)_x - (v_A)_x]; \qquad -B_x = 50(1.5 \cos 30° - 0)$$

$$B_x = |-64.95 \text{ N}| = 65.0 \text{ N} \rightarrow \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}[(v_B)_y - (v_A)_y]; \qquad B_y + 4178.5 - 750(9.81)$$

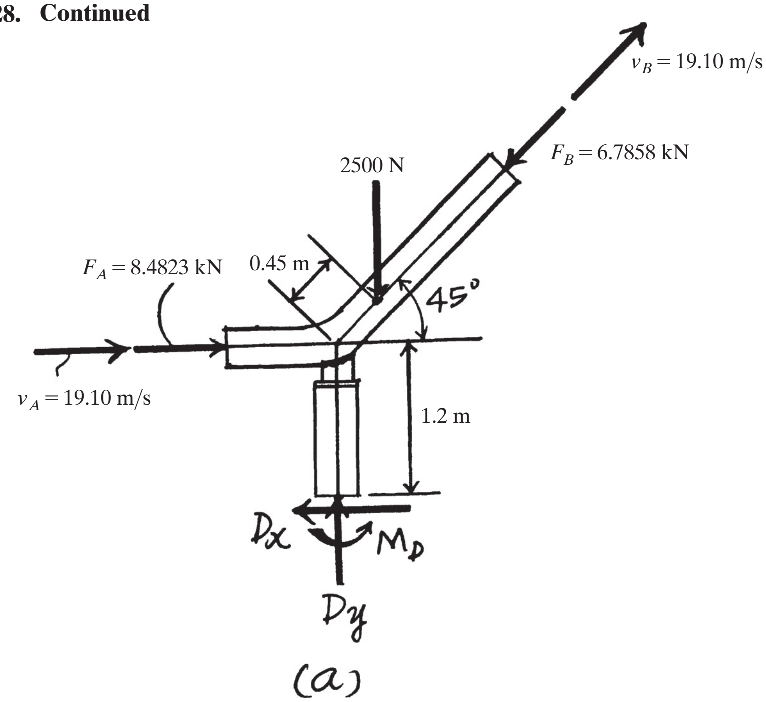
$$= 50[1.5 \sin 30° - (-10)]$$

$$B_y = 3716.25 \text{ N} = 3.72 \text{ kN} \uparrow \qquad \textbf{Ans.}$$



(a)

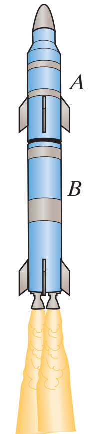
***15–128. Continued**



Ans:
$A_y = 4.18$ kN
$B_x = 65.0$ N →
$B_y = 3.72$ kN ↑

15–129.

Each of the two stages A and B of the rocket has a mass of 2 Mg when their fuel tanks are empty. They each carry 500 kg of fuel and are capable of consuming it at a rate of 50 kg/sand eject it with a constant velocity of 2500 m/s, measured with respect to the rocket. The rocket is launched vertically from rest by first igniting stage B. Then stage A is ignited immediately after all the fuel in B is consumed and A has separated from B. Determine the maximum velocity of stage A. Neglect drag resistance and the variation of the rocket's weight with altitude.



SOLUTION

The mass of the rocket at any instant t is $m = (M + m_0) - qt$. Thus, its weight at the same instant is $W = mg = [(M + m_0) - qt]g$.

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \quad -[(M + m_0) - qt]g = [(M + m_0) - qt]\frac{dv}{dt} - v_{D/e}q$$

$$\frac{dv}{dt} = \frac{v_{D/e}q}{(M + m_0) - qt} - g$$

During the first stage, $M = 4000$ kg, $m_0 = 1000$ kg, $q = 50$ kg/s, and $v_{D/e} = 2500$ m/s. Thus,

$$\frac{dv}{dt} = \frac{2500(50)}{(4000 + 1000) - 50t} - 9.81$$

$$\frac{dv}{dt} = \left(\frac{2500}{100 - t} - 9.81\right) \text{m/s}^2$$

The time that it take to complete the first stage is equal to the time for all the fuel in the rocket to be consumed, i.e., $t = \frac{500}{50} = 10$ s. Integrating,

$$\int_0^{v_1} dv = \int_0^{10\text{ s}} \left(\frac{2500}{100 - t} - 9.81\right)dt$$

$$v_1 = \left[-2500\ln(100 - t) - 9.81t\right]\Big|_0^{10\text{ s}}$$

$$= 165.30 \text{ m/s}$$

During the second stage of launching, $M = 2000$ kg, $m_0 = 500$ kg, $q = 50$ kg/s, and $v_{D/e} = 2500$ m/s. Thus, Eq. (1) becomes

$$\frac{dv}{dt} = \frac{2500(50)}{(2000 + 500) - 50t} - 9.81$$

$$\frac{dv}{dt} = \left(\frac{2500}{50 - t} - 9.81\right) \text{m/s}^2$$

The maximum velocity of rocket A occurs when it has consumed all the fuel. Thus, the time taken is given by $t = \frac{500}{50} = 10$ s. Integrating with the initial condition $v = v_1 = 165.30$ m/s when $t = 0$ s,

$$\int_{165.30\text{ m/s}}^{v_{max}} dv = \int_0^{10\text{ s}} \left(\frac{2500}{50 - t} - 9.81\right)dt$$

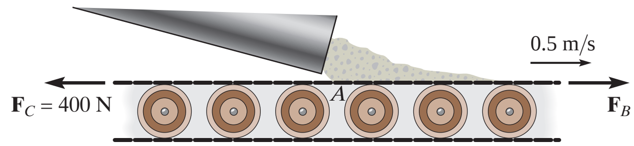
$$v_{max} - 165.30 = \left[-2500\ln(50 - t) - 9.81t\right]\Big|_0^{10\text{ s}}$$

$$v_{max} = 625 \text{ m/s}$$ **Ans.**

Ans:
$v_{max} = 625$ m/s

15–130.

Sand is deposited from a chute onto a conveyor belt which is moving at 0.5 m/s. If the sand is assumed to fall vertically onto the belt at A at the rate of 4 kg/s, determine the belt tension F_B to the right of A. The belt is free to move over the conveyor rollers and its tension to the left of A is $F_C = 400$ N.



SOLUTION

$$(\xrightarrow{+})\ \Sigma F_x = \frac{dm}{dt}(v_{Bx} - v_{Ax})$$

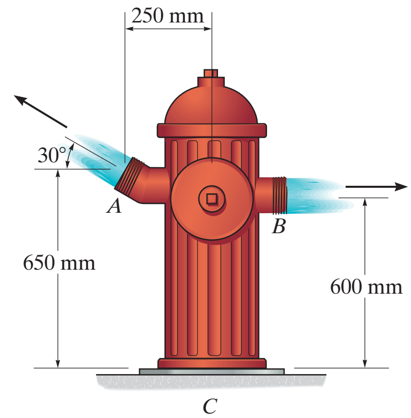
$$F_B - 400 = 4(0.5 - 0)$$

$$F_B = 2 + 400 = 402 \text{ N}$$ **Ans.**

Ans:
$F_B = 402$ N

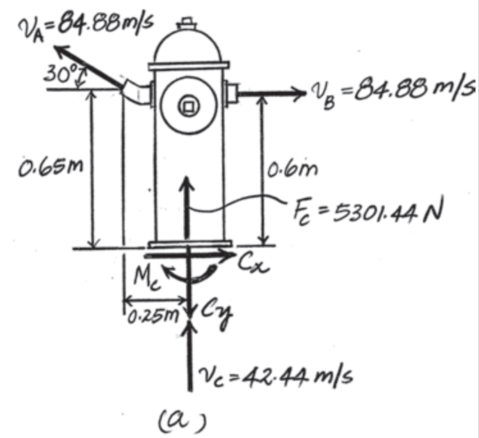
15–131.

The water flow enters below the hydrant at C at the rate of $0.75\ \text{m}^3/\text{s}$. It is then divided equally between the two outlets at A and B. If the gauge pressure at C is 300 kPa, determine the horizontal and vertical force reactions and the moment reaction on the fixed support at C. The diameter of the two outlets at A and B is 75 mm, and the diameter of the inlet pipe at C is 150 mm. The density of water is $\rho_w = 1000\ \text{kg/m}^3$. Neglect the mass of the contained water and the hydrant.



SOLUTION

Free-Body Diagram: The free-body diagram of the control volume is shown in Fig. *a*. The force exerted on section A due to the water pressure is $F_C = p_C A_C = 300(10^3)\left[\frac{\pi}{4}(0.15^2)\right] = 5301.44$ N. The mass flow rate at sections A, B, and C, are $\frac{dm_A}{dt} = \frac{dm_B}{dt} = \rho_W\left(\frac{Q}{2}\right) = 1000\left(\frac{0.75}{2}\right) = 375$ kg/s and $\frac{dm_C}{dt} = \rho_W Q = 1000(0.75) = 750$ kg/s.



The speed of the water at sections A, B, and C are

$$v_A = v_B = \frac{Q/2}{A_A} = \frac{0.75/2}{\frac{\pi}{4}(0.075^2)} = 84.88\ \text{m/s} \qquad v_C = \frac{Q}{A_C} = \frac{0.75}{\frac{\pi}{4}(0.15^2)} = 42.44\ \text{m/s}.$$

Steady Flow Equation: Writing the force steady flow equations along the x and y axes,

$$\xrightarrow{+} \Sigma F_x = \frac{dm_A}{dt}(v_A)_x + \frac{dm_B}{dt}(v_B)_x - \frac{dm_C}{dt}(v_C)_x;$$

$$C_x = -375(84.88\cos 30^\circ) + 375(84.88) - 0$$

$$C_x = 4264.54\ \text{N} = 4.26\ \text{kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = \frac{dm_A}{dt}(v_A)_y + \frac{dm_B}{dt}(v_B)_y - \frac{dm_C}{dt}(v_C)_y;$$

$$-C_y + 5301.44 = 375(84.88\sin 30^\circ) + 0 - 750(42.44)$$

$$Cy = 21\,216.93\ \text{N} = 2.12\ \text{kN} \qquad \textbf{Ans.}$$

Writing the steady flow equation about point C,

$$+\Sigma M_C = \frac{dm_A}{dt}dv_A + \frac{dm_B}{dt}dv_B - \frac{dm_C}{dt}dv_C;$$

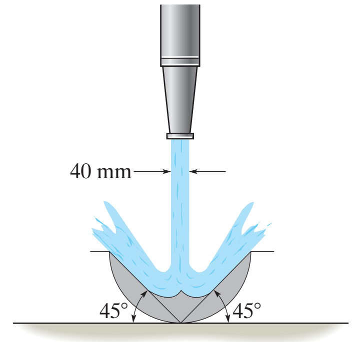
$$-M_C = 375(0.65)(84.88\cos 30^\circ) - 375(0.25)(84.88\sin 30^\circ) + [-375(0.6)(84.88)] - 0$$

$$M_C = 5159.28\ \text{N}\cdot\text{m} = 5.16\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$C_x = 4.26$ kN
$C_y = 2.12$ kN
$M_C = 5.16$ kN·m

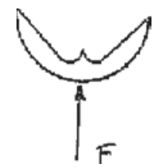
***15–132.**

The nozzle has a diameter of 40 mm. If it discharges water uniformly with a downward velocity of 20 m/s against the fixed blade, determine the vertical force exerted by the water on the blade. $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

$$\frac{dm}{dt} = \rho v A = (1000)(20)(\pi)(0.02)^2 = 25.13\ \text{kg/s}$$



$$+\uparrow \Sigma F_y = \frac{dm}{dt}\left(v_{By} - v_{Ay}\right)$$

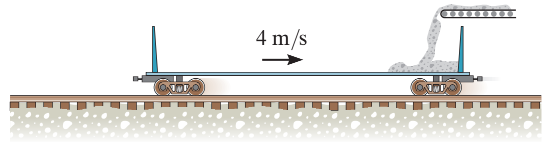
$$F = (25.13)(20 \sin 45° - (-20))$$

$$F = 858\ \text{N}$$ **Ans.**

Ans:
$F = 858\ \text{N}$

15–133.

Sand drops onto the 2-Mg empty rail car at 50 kg/s from a conveyor belt. If the car is initially coasting at 4 m/s, determine the speed of the car as a function of time.



SOLUTION

Gains Mass System. Here the sand drops vertically onto the rail car. Thus $(v_i)_x = 0$. Then

$$\mathbf{V}_D = \mathbf{V}_i + \mathbf{V}_{D/i}$$

$$(\overset{+}{\rightarrow}) \quad v = (v_i)_x + (v_{D/i})_x$$

$$v = 0 + (v_{D/i})_x$$

$$(v_{D/i})_x = v$$

Also, $\dfrac{dm_i}{dt} = 50$ kg/s and $m = 2000 + 50t$

$$\Sigma F_x = m\frac{dv}{dt} + (v_{D/i})_x\frac{dm_i}{dt};$$

$$0 = (2000 + 50t)\frac{dv}{dt} + v(50)$$

$$\frac{dv}{v} = -\frac{50\,dt}{2000 + 50t}$$

Integrate this equation with initial condition $v = 4$ m/s at $t = 0$.

$$\int_{4\text{ m/s}}^{v}\frac{dv}{v} = -50\int_0^t\frac{dt}{2000 + 50t}$$

$$\ln v\Big|_{4\text{ m/s}}^{v} = -\ln(2000 + 50t)\Big|_0^t$$

$$\ln\frac{v}{4} = \ln\left(\frac{2000}{2000 + 50t}\right)$$

$$\frac{v}{4} = \frac{2000}{2000 + 50t}$$

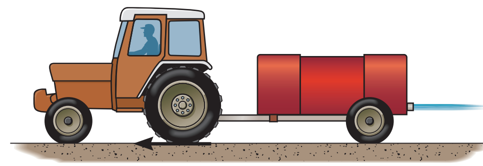
$$v = \left\{\frac{8000}{2000 + 50t}\right\}\text{ m/s} \qquad \textbf{Ans.}$$

Ans:

$$v = \left\{\frac{8000}{2000 + 50t}\right\}\text{ m/s}$$

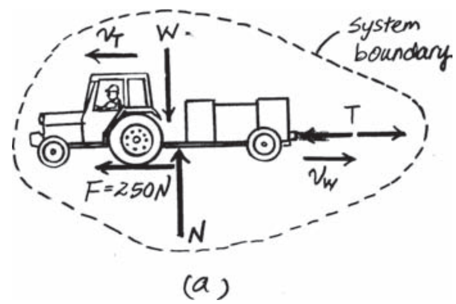
15–134.

The tractor together with the empty tank has a total mass of 4 Mg. The tank is filled with 2 Mg of water. The water is discharged at a constant rate of 50 kg/s with a constant velocity of 5 m/s, measured relative to the tractor. If the tractor starts from rest, and the rear wheels provide a resultant traction force of 250 N, determine the velocity and acceleration of the tractor at the instant the tank becomes empty.



SOLUTION

The free-body diagram of the tractor and water jet is shown in Fig. *a*. The pair of thrust **T** cancel each other since they are internal to the system. The mass of the tractor and the tank at any instant *t* is given by $m = (4000 + 2000) - 50t = (6000 - 50t)$kg.



$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad 250 = (6000 - 50t)\frac{dv}{dt} - 5(50)$$

$$a = \frac{dv}{dt} = \frac{10}{120 - t} \qquad \textbf{(1)}$$

The time taken to empty the tank is $t = \frac{2000}{50} = 40$ s. Substituting the result of *t* into Eq. (1),

$$a = \frac{10}{120 - 40} = 0.125\text{m/s}^2 \qquad \textbf{Ans.}$$

Integrating Eq. (1),

$$\int_0^v dv = \int_0^{40\text{ s}} \frac{10}{120 - t}dt$$

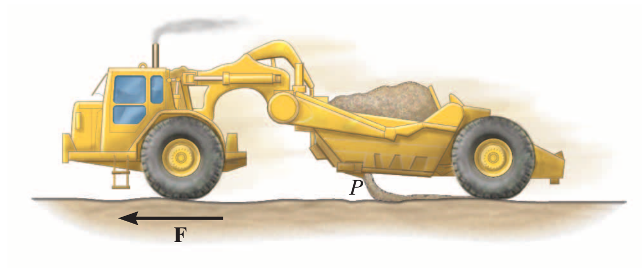
$$v = -10\ln(120 - t)\Big|_0^{40\text{ s}}$$

$$= 4.05 \text{ m/s} \qquad \textbf{Ans.}$$

Ans:
$a = 0.125 \text{ m/s}^2$
$v = 4.05 \text{ m/s}$

15 135.

The earthmover initially carries 10 m^3 of sand having a density of 1520 kg/m^3. The sand is unloaded horizontally through a 2.5 m^2 dumping port P at a rate of 900 kg/s measured relative to the port. Determine the resultant tractive force **F** at its front wheels if the acceleration of the earthmover is 0.1 m/s^2 when half the sand is dumped. When empty, the earthmover has a mass of 30 Mg. Neglect any resistance to forward motion and the mass of the wheels. The rear wheels are free to roll.



SOLUTION

When half the sand remains,

$$m = 30\,000 + \frac{1}{2}(10)(1520) = 37\,600 \text{ kg}$$

$$\frac{dm}{dt} = 900 \text{ kg/s} = \rho\, v_{D/e}\, A$$

$$900 = 1520(v_{D/e})(2.5)$$

$$v_{D/e} = 0.237 \text{ m/s}$$

$$a = \frac{dv}{dt} = 0.1$$

$$\overset{+}{\leftarrow} \Sigma F_s = m\frac{dv}{dt} - \frac{dm}{dt}v$$

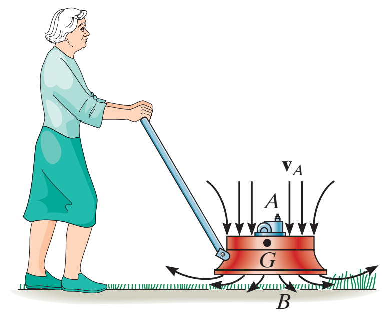
$$F = 37\,600(0.1) - 900(0.237)$$

$$F = 3.55 \text{ kN}$$ **Ans.**

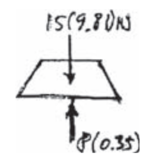
Ans:
$F = 3.55$ kN

***15–136.**

A power lawn mower hovers very close over the ground. This is done by drawing air in at a speed of 6 m/s through an intake unit A, which has a cross-sectional area of $A_A = 0.25\ \text{m}^2$, and then discharging it at the ground, B, where the cross-sectional area is $A_B = 0.35\ \text{m}^2$. If air at A is subjected only to atmospheric pressure, determine the air pressure which the lawn mower exerts on the ground when the weight of the mower is freely supported and no load is placed on the handle. The mower has a mass of 15 kg with center of mass at G. Assume that air has a constant density of $\rho_a = 1.22\ \text{kg/m}^3$.



SOLUTION



$$\frac{dm}{dt} = \rho\, A_A\, v_A = 1.22(0.25)(6) = 1.83\ \text{kg/s}$$

$$+\uparrow \Sigma F_y = \frac{dm}{dt}((v_B)_y - (v_A)_y)$$

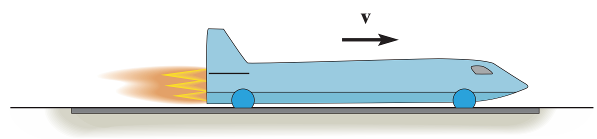
$$\text{pressure} = (0.35) - 15(9.81) = 1.83(0 - (-6))$$

$$\text{pressure} = 452\ \text{Pa}$$ **Ans.**

Ans:
452 Pa

15–137.

The rocket car has a mass of 2 Mg (empty) and carries 120 kg of fuel. If the fuel is consumed at a constant rate of 6 kg/s and ejected from the car with a relative velocity of 800 m/s, determine the maximum speed attained by the car starting from rest. The drag resistance due to the atmosphere is $F_D = (6.8v^2)$ N, where v is the speed in m/s.



SOLUTION

$$\Sigma F_s = m\frac{dv}{dt} - v_{[\text{placeholder}]}[\text{placeholder}]$$

At time t_1 the mass of the car is $m_0 - ct_1$ where $c = \dfrac{dm_c}{dt} = 6$ kg/s

Set $F = kv^2$, then

$$-kv^2 = (m_0 - ct)\frac{dv}{dt} - v_{D/e}c$$

$$\int_0^t \frac{dv}{(cv_{D/e} - kv^2)} = \int_0^t \frac{dt}{(m_0 - ct)}$$

$$\left(\frac{1}{2\sqrt{cv_{D/e}k}}\right)\ln\left[\frac{\sqrt{\dfrac{cv_{D/e}}{k}} + v}{\sqrt{\dfrac{cv_{D/e}}{k}} - v}\right]_0^v = -\frac{1}{c}\ln(m_0 - ct)\Big|_0^t$$

$$\left(\frac{1}{2\sqrt{cv_{D/e}k}}\right)\ln\left(\frac{\sqrt{\dfrac{cv_{D/e}}{k}} + v}{\sqrt{\dfrac{cv_{D/e}}{k}} - v}\right) = -\frac{1}{c}\ln\left(\frac{m_0 - ct}{m_0}\right)$$

Maximum speed occurs at the instant the fuel runs out

$$t = \frac{120}{6} = 20 \text{ s}$$

Thus,

$$\left(\frac{1}{2\sqrt{(6)(800)(6.8)}}\right)\ln\left(\frac{\sqrt{\dfrac{(6)(800)}{6.8}} + v}{\sqrt{\dfrac{(6)(800)}{6.8}} - v}\right) = -\frac{1}{6}\ln\left(\frac{2120 - 6(20)}{2120}\right)$$

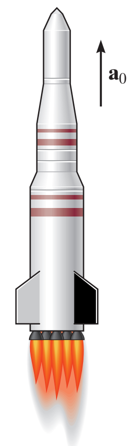
Solving,

$v = 25.0$ m/s **Ans.**

Ans:
$v = 25.0$ m/s

15-138.

The rocket has an initial mass m_0, including the fuel. For practical reasons desired for the crew, it is required that it maintain a constant upward acceleration a_0. If the fuel is expelled from the rocket at a relative speed v_{er}, determine the rate at which the fuel should be consumed to maintain the motion. Neglect air resistance, and assume that the gravitational acceleration is constant.



SOLUTION

$$a_0 = \frac{d}{dt}v$$

$$+\uparrow \Sigma F_s = m\frac{d}{dt}v - v_{er}\frac{d}{dt}m_e$$

$$-mg = ma_o - v_{er}\frac{d}{dt}m$$

$$v_{er}\frac{dm}{m} = (a_0 + g)dt$$

Since v_{er} is constant, integrating, with $t = 0$ when $m = m_0$ yields

$$v_{er}\ln\left(\frac{m}{m_0}\right) = (a_0 + g)t \qquad \frac{m}{m_0} = e^{\left(\frac{a_0+g}{v_{er}}\right)t}$$

The time rate fuel consumption is determined from Eq.[1]

$$\frac{d}{dt}m = m\frac{a_0 + g}{v_{er}} \qquad \frac{d}{dt}m = m_0\left(\frac{a_0 + g}{v_{er}}\right)e^{\left(\frac{a_0+g}{v_{er}}\right)t}$$ **Ans.**

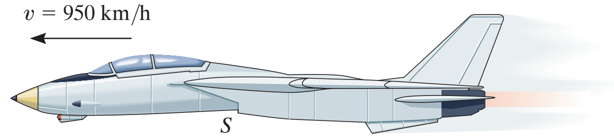
Note : v_{er} must be considered a negative quantity.

Ans:

$$\frac{d}{dt}m = m_0\left(\frac{a_0 + g}{v_{er}}\right)e^{\left(\frac{a_0+g}{v_{er}}\right)t}$$

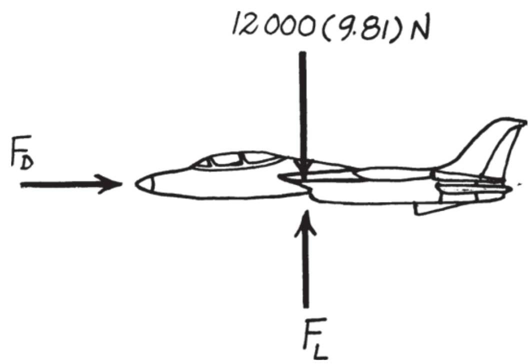
15-139.

The 12-Mg jet airplane has a constant speed of 950 km/h when it is flying along a horizontal straight line. Air enters the intake scoops S at the rate of 50 m^3/s. If the engine burns fuel at the rate of 0.4 kg/s and the gas (air and fuel) is exhausted relative to the plane with a speed of 450 m/s, determine the resultant drag force exerted on the plane by air resistance. Assume that air has a constant density of 1.22 kg/m^3. *Hint:* Since mass both enters and exits the plane, Eqs. 15–28 and 15–29 must be combined to yield $\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$.



SOLUTION

$$\Sigma F_s = m\frac{dv}{dt} - \frac{dm_e}{dt}(v_{D/E}) + \frac{dm_i}{dt}(v_{D/i}) \qquad \textbf{(1)}$$



$$v = 950 \text{ km/h} = 0.2639 \text{ km/s}, \qquad \frac{dv}{dt} = 0$$

$$v_{D/E} = 0.45 \text{ km/s}$$

$$v_{D/t} = 0.2639 \text{ km/s}$$

$$\frac{dm_t}{dt} = 50(1.22) = 61.0 \text{ kg/s}$$

$$\frac{dm_e}{dt} = 0.4 + 61.0 = 61.4 \text{ kg/s}$$

Forces T and R are incorporated into Eq. (1) as the last two terms in the equation.

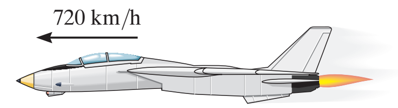
$$(\overset{+}{\leftarrow}) \quad -F_D = 0 - (0.45)(61.4) + (0.2639)(61)$$

$$F_D = 11.5 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_D = 11.5$ kN

***15–140.**

The jet is traveling at a speed of 720 km/h. If the fuel is being spent at 0.8 kg/s, and the engine takes in air at 200 kg/s, whereas the exhaust gas (air and fuel) has a relative speed of 12 000 m/s, determine the acceleration of the plane at this instant. The drag resistance of the air is $F_D = (55\ v^2)$, where the speed is measured in m/s. The jet has a mass of 7 Mg.



SOLUTION

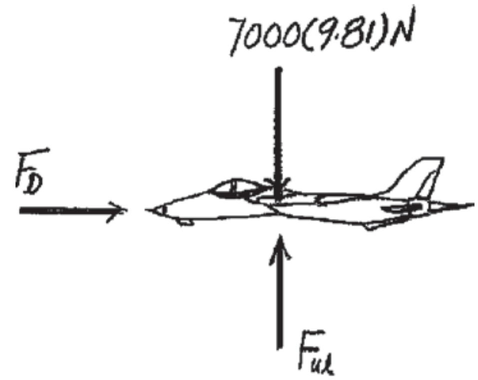
Since the mass enters and exits the plane at the same time, we can combine Eqs. 15-29 and 15-30 which resulted in

$$\Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt}$$

Here $m = 7000$ kg, $\frac{dv}{dt} = a$, $v_{D/e} = 12000$ m/s, $\frac{dm_e}{dt} = 0.8 + 200 = 200.8$ kg/s

$v = \left(720\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 200$ m/s, $v_{D/i} = v = 200$ m/s,

$\frac{dm_i}{dt} = 200$ kg/s



(a)

and $F_D = 55(200^2) = 2.2(10^6)$ N. Referring to the FBD of the jet, Fig. *a*

$$(\overset{+}{\leftarrow})\ \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt} + v_{D/i}\frac{dm_i}{dt};$$

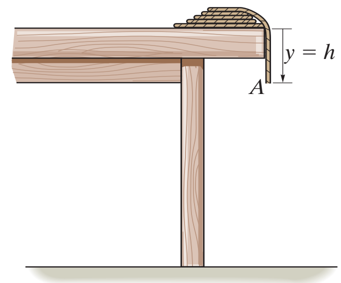
$$-2.2(10^6) = 7000a - 12000(200.8) + 200(200)$$

$$a = 24.23\text{ m/s}^2 = 24.2\text{ m/s}^2$$ **Ans.**

Ans:
$a = 24.2\text{ m/s}^2$

15–141.

The rope has a mass m' per unit length. If the end length $y = h$ is draped off the edge of the table, and released, determine the velocity of its end A for any position y, as the rope uncoils and begins to fall.



SOLUTION

$$+\downarrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time t, $m = m'y$ and $\frac{dm_i}{dt} = \frac{m'dy}{dt} = m'v$. Here, $v_{D/i} = v$, $\frac{dv}{dt} = g$.

$$m'gy = m'y\frac{dv}{dt} + v(m'v)$$

$$gy = y\frac{dv}{dt} + v^2 \qquad \text{since } v = \frac{dy}{dt}, \text{ then } dt = \frac{dy}{v}$$

$$gy = vy\frac{dv}{dy} + v^2$$

Multiply both sides by $2ydy$

$$2gy^2\,dy = 2vy^2\,dv + 2yv^2\,dy$$

$$\int 2gy^2\,dy = \int d\left(v^2y^2\right)$$

$$\tfrac{2}{3}gy^3 + C = v^2y^2$$

$$v = 0 \text{ at } y = h \qquad \tfrac{2}{3}gh^3 + C = 0 \qquad C = -\tfrac{2}{3}gh^3$$

$$\tfrac{2}{3}gy^3 - \tfrac{2}{3}gh^3 = v^2y^2$$

$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)}$$ **Ans.**

Ans:

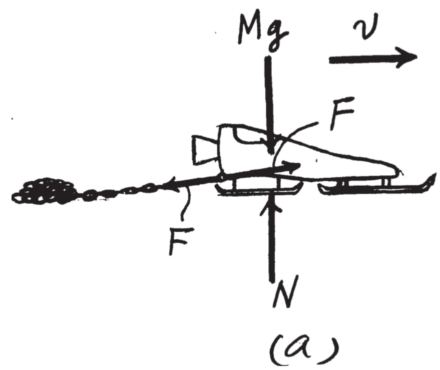
$$v = \sqrt{\frac{2}{3}g\left(\frac{y^3 - h^3}{y^2}\right)}$$

15–142.

A coil of heavy open chain is used to reduce the stopping distance of a sled that has a mass M and travels at a speed of v_0. Determine the required mass per unit length of the chain needed to slow down the sled to $(1/2)v_0$ within a distance $x = s$ if the sled is hooked to the chain at $x = 0$. Neglect friction between the chain and the ground.

SOLUTION

Observing the free-body diagram of the system shown in Fig. a, notice that the pair of forces **F**, which develop due to the change in momentum of the chain, cancel each other since they are internal to the system. Here, $v_{D/s} = v$ since the chain on the ground is at rest. The rate at which the system gains mass is $\frac{dm_s}{dt} = m'v$ and the mass of the system is $m = m'x + M$. Referring to Fig. a,



$$(\xrightarrow{+}) \quad \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad 0 = \left(m'x + M\right)\frac{dv}{dt} + v\left(m'v\right)$$

$$0 = \left(m'x + M\right)\frac{dv}{dt} + m'v^2 \qquad \textbf{(1)}$$

Since $\frac{dx}{dt} = v$ or $dt = \frac{dx}{v}$,

$$\left(m'x + M\right)v\frac{dv}{dx} + m'v^2 = 0$$

$$\frac{dv}{v} = -\left(\frac{m'}{m'x + M}\right)dx \qquad \textbf{(2)}$$

Integrating using the limit $v = v_0$ at $x = 0$ and $v = \frac{1}{2}v_0$ at $x = s$,

$$\int_{v_0}^{\frac{1}{2}v_0} \frac{dv}{v} = -\int_0^s \left(\frac{m'}{m'x + M}\right)dx$$

$$\ln v\Big|_{v_0}^{\frac{1}{2}v_0} = -\ln\left(m'x + M\right)\Big|_0^s$$

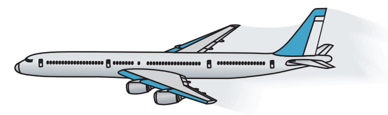
$$\frac{1}{2} = \frac{M}{m's + M}$$

$$m' = \frac{M}{s} \qquad \textbf{Ans.}$$

Ans:
$m' = \frac{M}{s}$

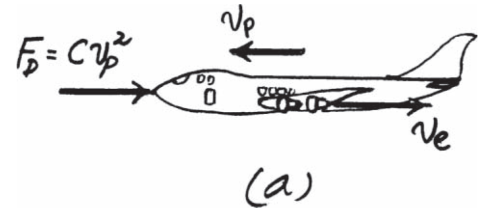
15–143.

A four-engine commercial jumbo jet is cruising at a constant speed of 800 km/h in level flight when all four engines are in operation. Each of the engines is capable of discharging combustion gases with a velocity of 775 m/s relative to the plane. If during a test two of the engines, one on each side of the plane, are shut off, determine the new cruising speed of the jet. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. Neglect the loss of mass due to fuel consumption.



SOLUTION

Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is



$$\left(\overset{+}{\rightarrow}\right) \qquad v_e + v_p + v_{e/p}$$

When the four engines are in operation, the airplane has a constant speed of $v_p = \left[800(10^3)\,\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 222.22$ m/s. Thus,

$$\left(\overset{+}{\rightarrow}\right) \qquad v_e = -222.22 + 775 = 552.78 \text{ m/s} \rightarrow$$

Referring to the free-body diagram of the airplane shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = \frac{dm}{dt}\left[\left(v_B\right)_x - \left(v_A\right)_x\right]; \qquad C(222.22^2) = 4\,\frac{dm}{dt}(552.78 - 0)$$

$$C = 0.044775\,\frac{dm}{dt}$$

When only two engines are in operation, the exit speed of the air is

$$\left(\overset{+}{\rightarrow}\right) \qquad v_e = -v_p + 775$$

Using the result for C,

$$\overset{+}{\rightarrow}\Sigma F_x = \frac{dm}{dt}\left[\left(v_B\right)_x - \left(v_A\right)_x\right]; \quad \left(0.044775\,\frac{dm}{dt}\right)\left(v_p{}^2\right) = 2\frac{dm}{dt}\left[-v_p + 775\right) - 0\right]$$

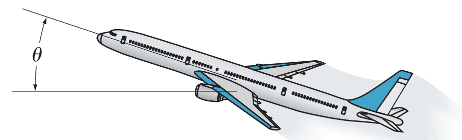
$$0.044775v_p{}^2 + 2v_p - 1550 = 0$$

Solving for the positive root,

$v_p = 165.06 \text{ m/s} = 594 \text{ km/h}$ **Ans.**

Ans:
$v_P = 594$ km/h

***15–144.** A commercial jet aircraft has a mass of 150 Mg and is cruising at a constant speed of 850 km/h in level flight ($\theta = 0°$). If each of the two engines draws in air at a rate of 1000 kg/s and ejects it with a velocity of 900 m/s, relative to the aircraft, determine the maximum angle of inclination θ at which the aircraft can fly with a constant speed of 750 km/h. Assume that air resistance (drag) is proportional to the square of the speed, that is, $F_D = cv^2$, where c is a constant to be determined. The engines are operating with the same power in both cases. Neglect the amount of fuel consumed.



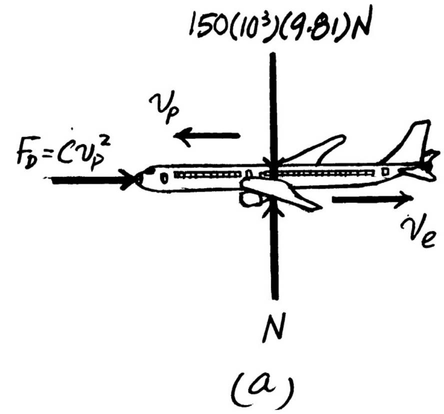
SOLUTION

Steady Flow Equation: Since the air is collected from a large source (the atmosphere), its entrance speed into the engine is negligible. The exit speed of the air from the engine is given by

$$v_e = v_p + v_{e/p}$$

When the airplane is in level flight, it has a constant speed of $v_p = \left[850(10^3)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 236.11\text{m/s}$. Thus,

$$(\overset{+}{\rightarrow}) \qquad v_e = -236.11 + 900 = 663.89\text{ m/s} \rightarrow$$



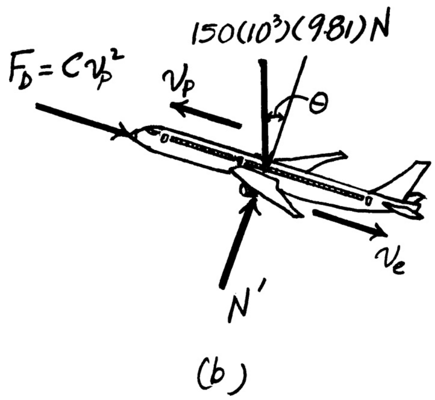
By referring to the free-body diagram of the airplane shown in Fig. *a*,

$$(\overset{+}{\rightarrow})\ \Sigma F_x = \frac{dm}{dt}\left[(v_B)_x - (v_A)_x\right]; \qquad C(236.11^2) = 2(1000)(663.89 - 0)$$

$$C = 23.817\text{ kg}\cdot\text{s/m}$$

When the airplane is in the inclined position, it has a constant speed of $v_p = \left[750\left(10^3\right)\frac{\text{m}}{\text{h}}\right]\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 208.33\text{ m/s}$. Thus,

$$v_e = -208.33 + 900 = 691.67\text{ m/s}$$



By referring to the free-body diagram of the airplane shown in Fig. *b* and using the result of *C*, we can write

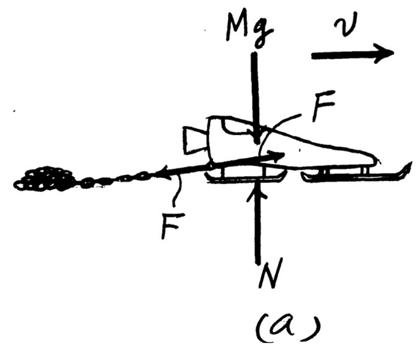
$$\nwarrow + \Sigma F_{x'} = \frac{dm}{dt}\left[(v_B)_{x'} - (v_A)_{x'}\right]; \qquad 23.817\left(208.33^2\right) + 150\left(10^3\right)(9.81)\sin\theta = 2(1000)(691.67 - 0)$$

$$\theta = 13.7° \qquad \textbf{Ans.}$$

Ans:
$\theta = 13.7°$

15–145. A coil of heavy open chain is used to reduce the stopping distance of a sled that has a mass M and travels at a speed of v_0. Determine the required mass per unit length of the chain needed to slow down the sled to $(1/4)v_0$ within a distance $x = s$ if the sled is hooked to the chain at $x = 0$. Neglect friction between the chain and the ground.

SOLUTION



Observing the free-body diagram of the system shown in Fig. *a*, notice that the pair of forces **F,** which develop due to the change in momentum of the chain, cancel each other since they are internal to the system. Here, $v_{D/s} = v$ since the chain on the ground is at rest. The rate at which the system gains mass is $\frac{dm_s}{dt} = m'v$ and the mass of the system is $m = m'x + M$. Referring to Fig. *a*,

$$(\overset{+}{\rightarrow}) \quad \Sigma F_s = m\frac{dv}{dt} + v_{D/s}\frac{dm_s}{dt}; \qquad 0 = \left(m'x + M\right)\frac{dv}{dt} + v\left(m'v\right)$$

$$0 = \left(m'x + M\right)\frac{dv}{dt} + m'v^2 \qquad \textbf{(1)}$$

Since $\frac{dx}{dt} = v$ or $dt = \frac{dx}{v}$,

$$\left(m'x + M\right)v\frac{dv}{dx} + m'v^2 = 0$$

$$\frac{dv}{v} = -\left(\frac{m'}{m'x + M}\right)dx \qquad \textbf{(2)}$$

Integrating using the limit $v = v_0$ at $x = 0$ and $v = \frac{1}{4}v_0$ at $x = s$,

$$\int_{v_0}^{\frac{1}{4}v_0}\frac{dv}{v} = -\int_0^s\left(\frac{m'}{m'x + M}\right)dx$$

$$1\text{n}\, v\Big|_{v_0}^{\frac{1}{4}v_0} = -1\text{n}\left(m'x + M\right)\Big|_0^s$$

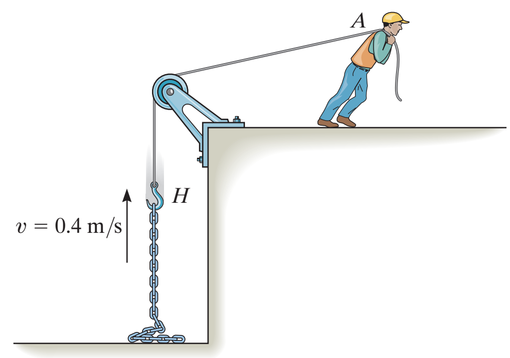
$$\frac{1}{4} = \frac{M}{m's + M}$$

$$m' = \frac{3M}{s} \qquad \textbf{Ans.}$$

Ans:
$m' = \frac{3M}{s}$

15–146.

Determine the magnitude of force **F** as a function of time, which must be applied to the end of the cord at A to raise the hook H with a constant speed $v = 0.4$ m/s. Initially the chain is at rest on the ground. Neglect the mass of the cord and the hook. The chain has a mass of 2 kg/m.

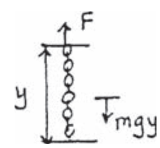


SOLUTION

$$\frac{dv}{dt} = 0, \qquad y = vt$$

$$m_i = my = mvt$$

$$\frac{dm_i}{dt} = mv$$



$$+\uparrow \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\left(\frac{dm_i}{dt}\right)$$

$$F - mgvt = 0 + v(mv)$$

$$F = m(gvt + v^2)$$

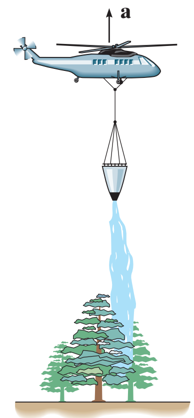
$$= 2[9.81(0.4)t + (0.4)^2]$$

$$F = (7.85t + 0.320)\text{ N}$$ **Ans.**

Ans:
$F = \{7.85t + 0.320\}$ N

15–147.

The 10-Mg helicopter carries a bucket containing 500 kg of water, which is used to fight fires. If it hovers over the land in a fixed position and then releases 50 kg/s of water at 10 m/s, measured relative to thehelicopter, determine the initial upward accelerationthe helicopter experiences as the water is being released.



SOLUTION

$$+\uparrow \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}$$

Initially, the bucket is full of water, hence $m = 10(10^3) + 0.5(10^3) = 10.5(10^3)$ kg

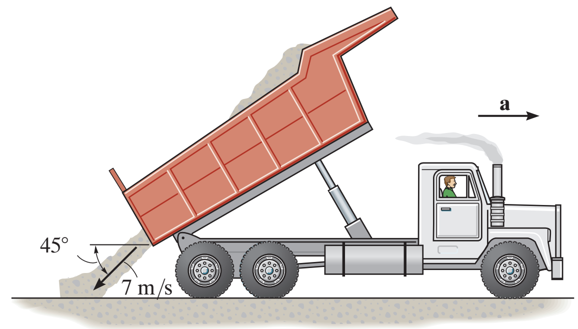
$0 = 10.5(10^3)\,a - (10)(50)$

$a = 0.0476\ \text{m/s}^2$ **Ans.**

Ans:
$a = 0.0476\ \text{m/s}^2$

***15–148.**

The truck has a mass of 50 Mg when empty. When it is unloading 5 m^3 of sand at a constant rate of 0.8 m^3/s, the sand flows out the back at a speed of 7 m/s, measured relative to the truck, in the direction shown. If the truck is free to roll, determine its initial acceleration just as the load begins to empty. Neglect the mass of the wheels and any frictional resistance to motion. The density of sand is $\rho_s = 1520 \text{ kg/m}^3$.



SOLUTION

A System That Loses Mass: Initially, the total mass of the truck is $m = 50(10^3) + 5(1520) = 57.6(10^3)$ kg and $\frac{dm_e}{dt} = 0.8(1520) = 1216$ kg/s. Applying Eq. 15–28, we have

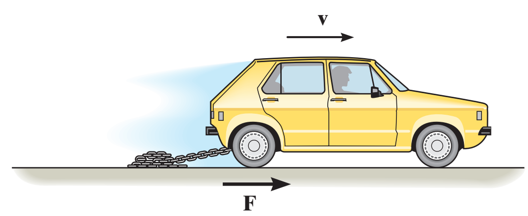
$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} - v_{D/e}\frac{dm_e}{dt}; \qquad 0 = 57.6(10^3)a - (0.8 \cos 45°)(1216)$$

$$a = 0.104 \text{ m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$a = 0.104 \text{ m/s}^2$

15–149.

The car has a mass m_0 and is used to tow the smooth chain having a total length l and a mass per unit of length m'. If the chain is originally piled up, determine the tractive force F that must be supplied by the rear wheels of the car, necessary to maintain a constant speed v while the chain is being drawn out.



SOLUTION

$$\xrightarrow{+} \Sigma F_s = m\frac{dv}{dt} + v_{D/i}\frac{dm_i}{dt}$$

At a time t, $m = m_0 + ct$, where $c = \dfrac{dm_i}{dt} = \dfrac{m'dx}{dt} = m'v$.

Here, $v_{D/i} = v$, $\quad \dfrac{dv}{dt} = 0$.

$$F = (m_0 - m'v)(0) + v(m'v) = m'v^2$$ **Ans.**

Ans:
$F = m'v^2$