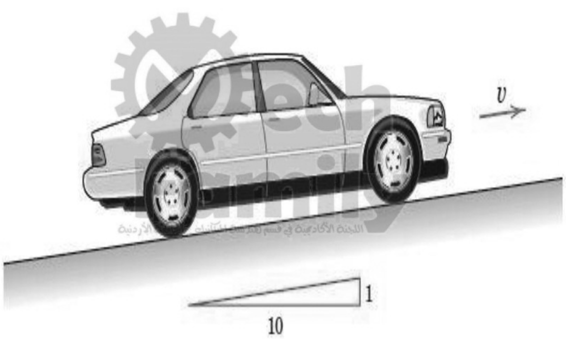
Dynamics second exam 2014/2015
The University of Jordan
Mech-Family

Question (1):-

The 1500-kg car has a velocity of 30 km/h up the 10-percent grade when the driver applies more power for 8 s to bring the car up to a speed of 60 km/h. Calculate the time average F of the total force tangent to the road exerted on the tires during the 8 s. Treat the car as a particle and neglect air resistance.

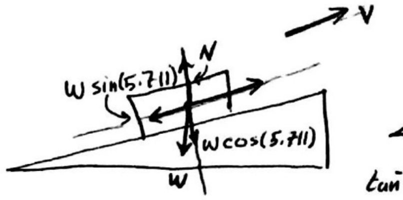


solu

$V_0 = 30\ Km/h = \frac{30*1000}{3600} = 8.333\ m/s$

$V = 60\ Km/h = \frac{60*1000}{3600} = 16.667\ m/s$

$\Delta t = 8s$



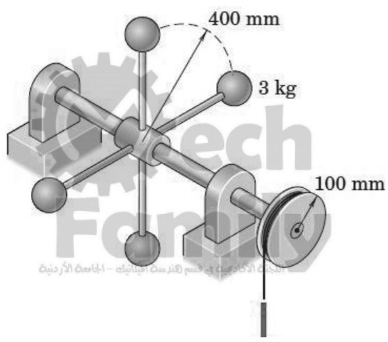
$\tan^{-1}\left(\frac{1}{10}\right) = 5.711^\circ$

$m\vec{V_0} + \int_1^2 \vec{F}\,dt = m\vec{V}$

$\nearrow^+ \; (1500)(8.333) + \int_0^8 F - \underbrace{W}_{(1500)(9.81) = 14715}\sin(5.711)\,dt = (1500)(16.667) \Rightarrow F = 3026.93\,N$

Question (2):-

The assembly starts from rest and reaches an angular speed of 150 rev/min under the action of a 20-N force T applied to the string for t seconds. Determine t. Neglect friction and all masses except those of the four 3-kg spheres, which may be treated as particles.



solu

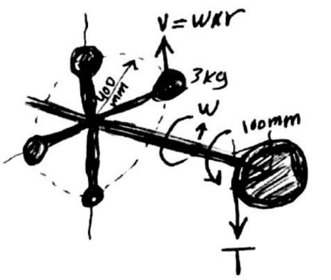
$V_0 = 0$

$W = 150\ \frac{rev}{min} = 150 * \frac{2\pi}{60} = 15.708\ rad/s$

$M = r_p \times T = (0.1\,m)(20) = 2\ N.m$

$m = 3\ kg$

$V = W \times r_{rod} = (15.708)(0.4) = 6.283\ m/s$

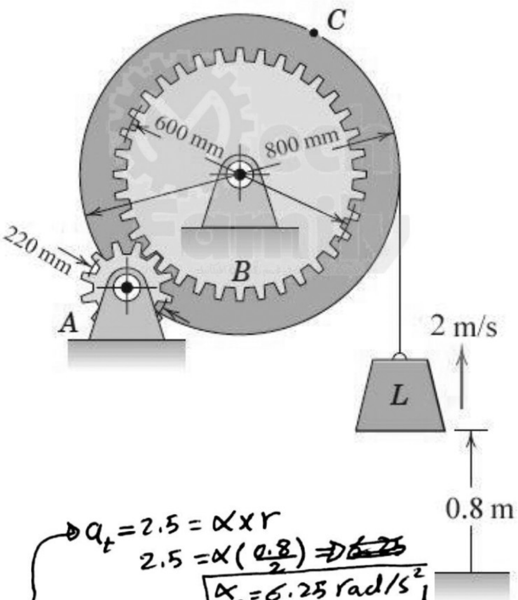


$m\vec{r_0}\times\vec{V_0}^{\,0} + \int_1^2 \underbrace{\vec{r}\times\vec{F}}_{\vec{r}\times\vec{T}} + M^{\,0}\,dt = 4\, m\vec{r}\times\vec{V}$

$\circlearrowleft^+ \; \int_0^{t_2} (0.1)(20)\,dt = \underset{\text{عدد الكرات}}{4}(3 * 0.4 * 6.283) \Rightarrow t = 15.0792\,s$

Question (3&4):-

The pinion A of the hoist motor drives gear B, which is attached to the hoisting drum. The load L is lifted from its rest position and acquires an upward velocity of 2 m/s in a vertical rise of 0.8 m with constant acceleration. As the load passes this position, compute (a) the acceleration of point C on the cable in contact with the drum and (b) the angular velocity and angular acceleration of the pinion A.



soln

$S_o = 0,\ v_o = 0$
$v = 2\,m/s \uparrow$
$S = 0.8\,m$
$a =$ constant

constant acceleration
$+\uparrow v^2 = v_o^2 + 2a\Delta S$
$(2)^2 = 0 + 2a(0.8)$
$a_t = 2.5\,m/s^2$
→ tangential acceleration

$a \begin{cases} a_t = 2.5 = \alpha \times r \\ 2.5 = \alpha\left(\frac{0.8}{2}\right) \Rightarrow \cancel{6.25} \\ \boxed{\alpha_c = 6.25\ rad/s^2} \\ a_n = v^2/r = (2)^2/\left(\frac{0.8}{2}\right) \\ = 10\ m/s^2 = w^2 r \end{cases}$

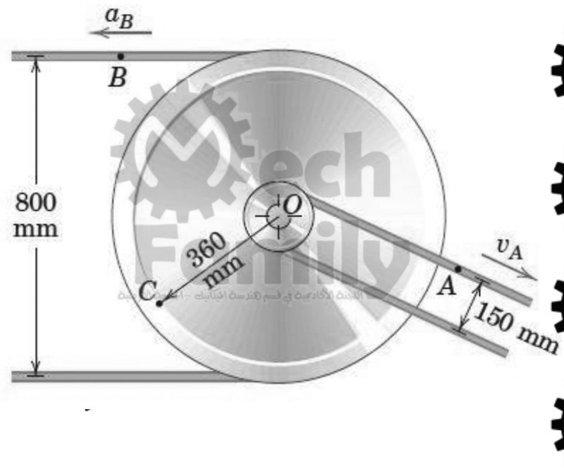
(A) $a_c = \sqrt{(a_t)^2 + (a_n)^2} = \sqrt{(2.5)^2 + (10)^2} = 10.3078\ m/s^2$ $\quad \boxed{w_c = 5\ rad/s^2}$

(B) $w_B = w_c$ $\quad w_B r_B = w_A r_A$
$(5)(0.3) = w_A(0.11)$
$w_A = 13.636\ rad/s$

$\alpha_B = \alpha_c \rightarrow \alpha_B r_B = \alpha_A r_A$
$(6.25)(0.3) = \alpha_A(0.11)$
$\alpha_A = 17.045\ rad/s^2$

Question (5):

The two V-belt pulleys form an integral unit and rotate about the fixed axis at O. At a certain instant, point A on the belt of the smaller pulley has a velocity $v_A = 1.5$ m/s, and point B on the belt of the larger pulley has an acceleration $a_B = 45$ m/s^2 as shown. For this instant determine the magnitude of th acceleration $\mathbf{a}_C$ of point C.



solution

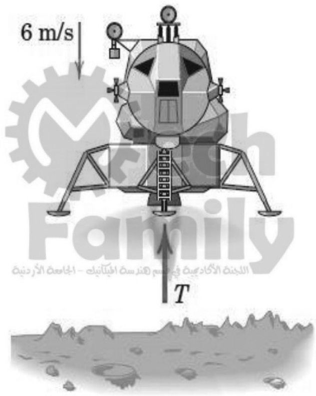
$v_A = 1.5\,m/s \Rightarrow v_A = w \times r_A \Rightarrow (1.5) = w\left(\frac{0.15}{2}\right) \Rightarrow w = 20\ rad/s$

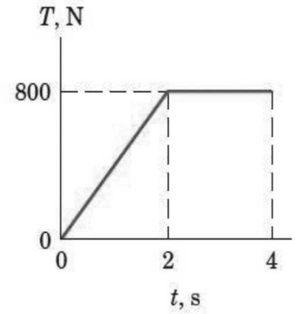
$\underset{tangential}{(a_B)} = 45\,m/s \Rightarrow a_B = \alpha \times r_B \Rightarrow 45 = \alpha\left(\frac{0.8}{2}\right) \Rightarrow \alpha = 112.5\ rad/s^2$

$a_c = \begin{cases} a_t = \alpha r_c = (112.5)(0.36) = 40.5\ m/s^2 \\ a_n = w^2 r_c = (20)^2(0.36) = 144\ m/s^2 \end{cases}$ $\quad a_c = \sqrt{(40.5)^2 + (144)^2}$
$= 149.6\ m$

Question (6):-

The 200-kg lunar lander is descending onto the moon's surface with a velocity of 6 m/s when its retro-engine is fired. If the engine produces a thrust T for 4 s which varies with time as shown and then cuts off, calculate the velocity of the lander when $t = 5$ s, assuming that it has not yet landed. Gravitational acceleration at the moon's surface is 1.62 m/s^2.





Solu

$m = 200\,kg$

$v_0 = 6\,m/s \downarrow$

T from $t_0 = 0$ to $t = 4$

$v|_{t=5s} = ??$

$g = 1.62\,m/s$

F.B.D

[1] $+\uparrow$ $m_0 \vec{v}_0 + \int_{t_1}^{t_2} F\,dt = m\vec{v}$

$-(200)(6) + \int_0^4 T - (200)(1.62)\,dt = (200)(v)$

$T\Delta t = \frac{1}{2} * 2 * 800 + 2 * 800 = 2400\ N.s$

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$-(200)(6) + 2400 - (200)(1.62)(4) = 200\,v$

$v = -0.48\,m/s = 0.48\,m/s \downarrow$

[2] F.BD

$m\vec{v} + \int^2 \vec{F}\,dt = m\vec{v}'$

$+\uparrow$ $-(200)(0.48) - (200)(1.62)(5-4) = 200\,v'$

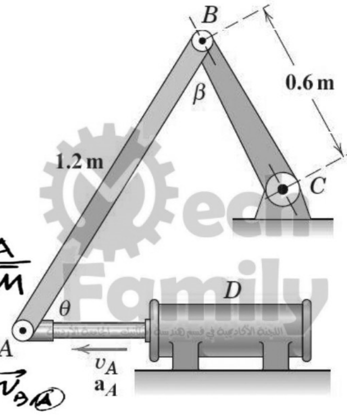
$v' = -2.1\,m/s = 2.1\,m/s \downarrow$

Question (7&8):-

For an interval of its motion the piston rod of the hydraulic cylinder has a velocity $v_A = 4$ m/s which is increasing by 3 m/s^2 each second as shown. At a certain instant $\theta = \beta = 60°$.

(a) For this instant determine the angular velocity ω_{BC} of link BC.

(b) For this instant determine the angular acceleration of link BC.



Solu

$v_A = 4\,m/s \leftarrow$

$a_A = 3\,m/s \leftarrow$

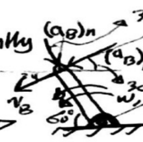
$\theta = \beta = 60^\circ$

$\omega_{BC} = ?$

$\alpha_{BC} = ?$

link BC

Rotation about fixed axis



$v_B = \omega_{BC}(0.6)$ 30

$(a_B)_t = \alpha_{BC}(0.6)$ 30

$(a_B)_n = \omega_{BC}^2(0.6)$ 30

link BA

G.B.M

Velocity

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$\omega_{BC}(0.6)$ 30 $= 3 \leftarrow + \omega_{AB}(1.2)$ 30

H:- $-\omega_{BC}(0.6)\cos 30 = -3 + -\omega_{AB}(1.2)\cos 30$

V:- $-\omega_{BC}(0.6)\sin 30 = 0 + \omega_{AB}(1.2)\sin 30$

الآلة الحاسبة Calculat → MODE → 5 → 1

$\omega_{BC} = 2.887\,rad/s \Rightarrow$ ↻

$\omega_{AB} = -1.443\,rad/s \Rightarrow$ ↷

acceleration

$\vec{a}_B = \vec{a}_A + \vec{a}_{A/B}$

$\alpha_{BC}(0.6)$ 30 $+ \omega_{BC}^2(0.6)$ 30 $= 3 \leftarrow + \alpha_{AB}(1.2)$ 30 $+ \omega_{AB}^2(1.2)$ 60

H:- $-\alpha_{BC}(0.6)\cos 30 + (2.887)^2(0.6)\sin 30 = -3 + -\alpha_{AB}(1.2)\cos 30 - (-1.443)^2(1.2)\cos 60$

$-0.5196\alpha_{BC} + 2.5 = -3 - 1.039\alpha_{AB} - 1.2493$

$\boxed{-0.5196\alpha_{BC} + 1.039\alpha_{AB} = -6.7443}$

V:- $-\alpha_{BC}(0.6)\sin(30) - (2.887)^2(0.6)\cos 30 = 0 + \alpha_{AB}(1.2)\sin 30 + -(-1.443)^2(1.2)\sin 60$

$-0.3\alpha_{BC} - 4.331 = 0.6\alpha_{AB} - 2.164$

$\boxed{-0.3\alpha_{BC} - 0.6\alpha_{AB} = -2.164 + 4.331 = 2.167}$

$\alpha_{BC} = 2.884\,rad/s^2 \rightarrow$ ↻

$\alpha_{AB} = -5.0537\,rad/s^2 \rightarrow$ ↺