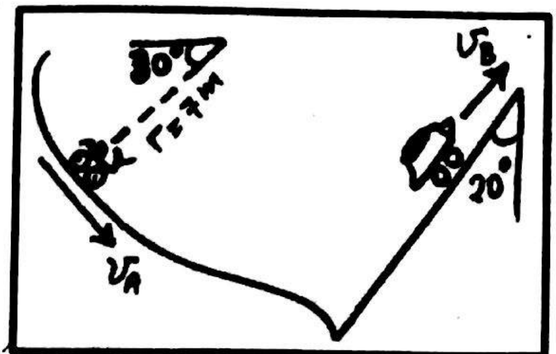
Q1) Bike A moves along a path in a curvilinear motion with a velocity v_A= 4 m/s. If the bike speed is decreasing by a constant rate of 2 m/s^2, while a car B is increasing its speed with a constant rate of 2.5 m/s^2. The magnitude of the relative acceleration of car B with respect to bike A in [m/s^2] is:

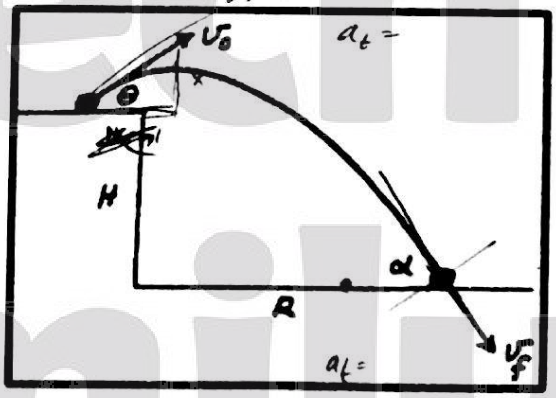


a) 1.2381 b) 1.0384 c) 0.5402
d) 0.3423 e) 0.7890 f) None

$v_A = 4$
$a_A = -2$
$a_B = 2.5$
$v_A = v_B + v_{A/B}$
$a_B = a_A + a_{B/A}$
$= -2 + a_{B/A}$
$a_A = \rho = 50$
$\frac{16}{7}$
2.507

Q7 $v_0 = 0$, $t = 0$
$a = (\sqrt{2v+1})^{-1/2}$ $\frac{1}{\sqrt{2v+1}}$
$\frac{dv}{dt} = \frac{1}{\sqrt{2v+1}}$
$\int_0^9 \sqrt{2v+1}\, dv = \int_0^t dt$
$\frac{(2v+1)^{3/2}}{3/2}$

Q2) A ball is thrown upward with an angle θ and initial velocity v_0 = 30 m/s. If the ball falls and attacks the ground with an angle α= 30° and a velocity v_f = 25 m/s. The throwing angle θ is:



a) 61.2° b) 43.8° c) 64.3°
d) 57.2° e) 51.8° f) None

$v_f^2 = v_0^2 + 2a_x \Delta x$
$v_0 = 30$ $v_f = 25$
$v_f^2 = v_0^2 - 2(a)(\Delta x)$ 9.81
$a =$ Δy 14.01
$v_f^2 = v_0^2 + 2$

Q3) A car starts moving from rest with an acceleration $a = (2v+1)^{-0.5}$ [in m/s^2]. How long [in seconds] the car needs to reach a speed of 9 m/s:

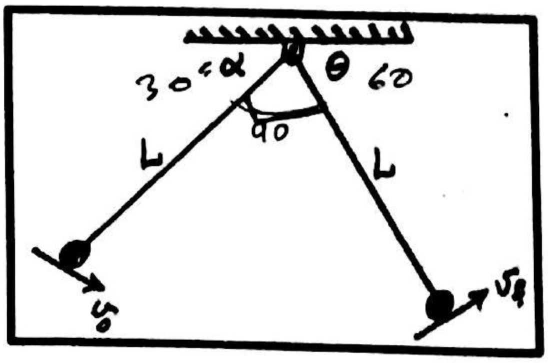
a) 27.27 b) 23.03 c) 19.03
d) 15.29 e) 11.83 f) None

$\frac{dv}{dt} = \sqrt{2v+1}$
$dv = \sqrt{2v+1}\, dt$
$\int_0^9 = \int_0^t \sqrt{2v+1}$ $(2v+1)^{1/2} = (2v+1)^{1/2} \cdot 2$
$\int_0^9 2\sqrt{2v+1} = t$
$9 =$
$2(4.35 - 2)$

$v_0 = 0$
$v = 9$
$a = \sqrt{2v+1}$
$\frac{dv}{dt} = 9\sqrt{2v+1}$
$\int_0^t \frac{dv}{\sqrt{2v+1}} = \quad 9$
$(2v+1)^{-1/2} =$

$a = \sqrt{2v+1}$
$\frac{dv}{dt} = \sqrt{2v+1}$
$\int_0^9 \frac{dv}{\sqrt{2v+1}} = \int_0^t$
$(2v+1)^{-1/2} - 0 \quad \frac{(2v+1)^{1/2}}{\frac{1}{2}} \cdot 2$
$4\sqrt{2v+1}\Big]_0^9$

Q4) A simple pendulum starts moving downward with a velocity v_0 = 2 m/s and α= 30°. On the opposite side, it reaches a velocity v_f = 4 m/s and θ= 60°. The cable length L of the pendulum [in meter] is:

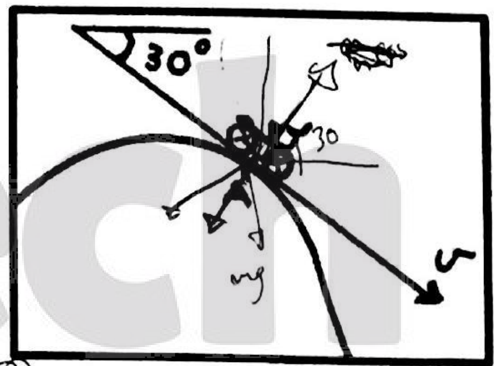


a) 2.74 b) 1.17 c) 1.67
d) 2.09 e) 1.38 f) None

$a_n = \frac{v^2}{\rho}$

~~$v^2 = v_1^2 + 2a\Delta s$~~

Q5) When it is at point A, a bike is moving down a hill with a speed v = 6 m/s. For what radius of curvature [in meter] the bike will lose the contact with the ground at point A:



a) 16.95 b) 26.48 c) 38.14
d) 9.53 e) 4.24 f) None

$\rho = s\theta$

$a_n = \frac{v^2}{\rho}$

$\rho = s\theta$

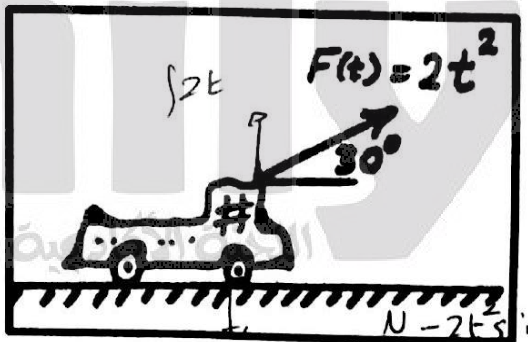
$\Sigma f = ma$

$a_n = \frac{v^2}{\rho}$

$N = 0$

$N \quad mg\cos 30 = 0$

Q6) A truck 2500 kg is drawn by a variable force F= $2t^2$, where F in kilo-Newtons and t in seconds. If the truck starts moving from rest along a horizontal rough road with kinetic coefficient of friction μ= 0.35. The truck velocity [in m/s] will be in 5 seconds:



$\int 2t$

$N - 2t^2 \sin 30 - mg$

$N = 2\sin 30 \int_0^5 + mg$

a) 14.30 b) 11.06 c) 17.53
d) 24.01 e) 20.77 f) None

$F_y = \mu N$

mg

$\mu_k N \Delta s + \int_0^{} 2t^2 \cos 30 = \frac{1}{2} 2500 v^2$

$\int_0^5 \frac{2}{3} t^3 \quad \frac{2 \cdot 125}{3} = 83.3$

$\Sigma f = ma$

$\Delta s = \cancel{v_0 t} + \frac{1}{2} a t^2$

$-0.35(mg) + \int 2t^2 \cos 30 = ma$

$\frac{2}{3} t^3 \Big]_0^5$

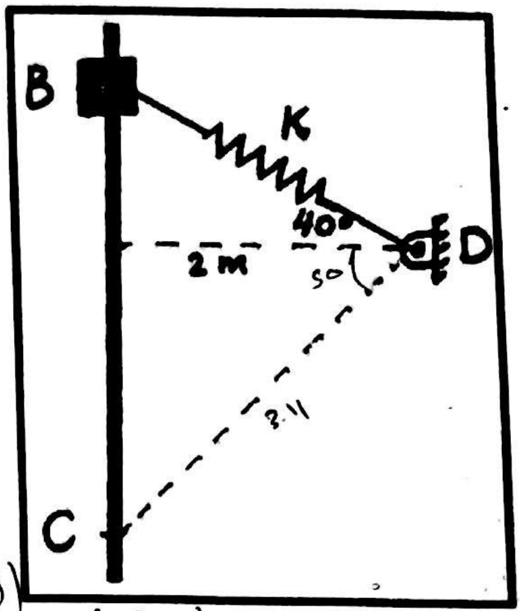
-3.4

$v_2 = 0 - at$

$\frac{2(125)}{3}$

$v_2 = 0 + at$

Q7) A collar 8 kg starts to move from rest from point B and falls 4 m down to reach point C. If the spring has a stiffness of k = 50 N/m and an unstretched length of 1.2 m. The collar velocity while reaching point C in [m/s] is:



a) 8.32 b) 6.43 c) 2.05
d) 4.77 e) 7.37 f) None

$\cancel{\frac{1}{2}m_1v_1^2} - W\Delta H - \frac{1}{2}k\delta = \frac{1}{2}mv^2$

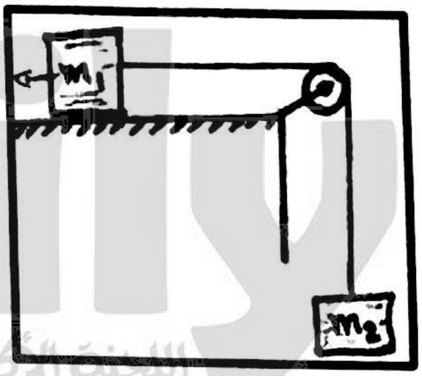
$-8\times 9.81\times 4 - \frac{1}{2}(50)\left((2.61-1.2)^2-(3.11-1.2)^2\right) = \frac{1}{2}8v^2$

$\tan 40 = \frac{x}{2}$ $\boxed{=1.678}$

$\cos 40 = \frac{2}{x}$

$\boxed{x = 2.61}$

Q8) Two blocks of masses m_1 and m_2 are connected together over a frictionless pulley of negligible mass. The block of m_1 is on a rough horizontal table with static coefficient of friction μ_s. In terms of m_1 and m_2, what value of μ_s will prevent the blocks from moving:



a) m_2 / m_1 b) m_1 / m_2 c) $m_1 / (m_1 + m_2)$
d) $m_2 / (m_1 + m_2)$ e) $(m_1 m_2) / (m_1 + m_2)$ f) None

$m_1 g = \frac{F_y}{\mu_s}$ $m_1 = \frac{F_y}{g\mu_s}$ $F = \mu_k N$

m m_2

$m_1 g = m_2 g$

$\cancel{\mu_k(m_1 g) = m_2}$

$m_2 - \mu N = 0$

$\frac{m_2}{N}$ $\mu_s m_1 g = m_2 g +$

$\mu_s =$ $m_2 - \mu_k(m_1 g) =$

$\frac{m_2}{F_y}$

$\mu_k = \frac{m_2}{m_1 g} \div \frac{\frac{N}{g}}{m_1 g}$

$\frac{N}{m_1 g^2} =$

N

$m_2 g = N$

...ntal Equations of Dynamics

KINEMATICS

...icle Rectilinear Motion

...riable a	Constant $a = a_c$
$a = \frac{dv}{dt}$	$v = v_0 + a_c t$
$v = \frac{ds}{dt}$	$s = s_0 + v_0 t + \frac{1}{2}a_c t^2$
$a\,ds = v\,dv$	$v^2 = v_0^2 + 2a_c(s - s_0)$

Particle Curvilinear Motion

x, y, z Coordinates		*r, θ, z Coordinates*	
$v_x = \dot{x}$	$a_x = \ddot{x}$	$v_r = \dot{r}$	$a_r = \ddot{r} - r\dot{\theta}^2$
$v_y = \dot{y}$	$a_y = \ddot{y}$	$v_\theta = r\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
$v_z = \dot{z}$	$a_z = \ddot{z}$	$v_z = \dot{z}$	$a_z = \ddot{z}$

n, t, b Coordinates

$v = \dot{s}$ | $a_t = \dot{v} = v\frac{dv}{ds}$

$a_n = \frac{v^2}{\rho}$ $\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

Relative Motion

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Rigid Body Motion About a Fixed Axis

Variable α	*Constant $\alpha = \alpha_c$*
$\alpha = \frac{d\omega}{dt}$	$\omega = \omega_0 + \alpha_c t$
$\omega = \frac{d\theta}{dt}$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$
$\omega\,d\omega = \alpha\,d\theta$	$\omega = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P

$s = \theta r$ $v = \omega r$ $a_t = \alpha r$ $a_n = \omega^2 r$

Relative General Plane Motion — Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(\text{pin})}$ $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(\text{pin})}$

Relative General Plane Motion — Trans. and Rot. Axis

$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_{B/A}) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia $I = \int r^2\,dm$

Parallel-Axis Theorem $I = I_G + md^2$

Radius of Gyration $k = \sqrt{\frac{I}{m}}$

Equations of Motion

Particle	$\Sigma\mathbf{F} = m\mathbf{a}$
Rigid Body (Plane Motion)	$\Sigma F_x = m(a_G)_x$ $\Sigma F_y = m(a_G)_y$ $\Sigma M_G = I_G\alpha$ or $\Sigma M_P = \Sigma(\mathcal{M}_k)_P$

Principle of Work and Energy

$T_1 + U_{1-2} = T_2$

Kinetic Energy

Particle	$T = \frac{1}{2}mv^2$
Rigid Body (Plane Motion)	$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

Variable force	$U_F = \int F\cos\theta\,ds$
Constant force	$U_F = (F_c\cos\theta)\,\Delta s$
Weight	$U_W = -W\,\Delta y$
Spring	$U_s = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$
Couple moment	$U_M = M\,\Delta\theta$

Power and Efficiency

$P = \frac{dU}{dt} = \mathbf{F}\cdot\mathbf{v}$ $\epsilon = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{U_{\text{out}}}{U_{\text{in}}}$

Conservation of Energy Theorem

$T_1 + V_1 = T_2 + V_2$

Potential Energy

$V = V_g + V_e$, where $V_g = \pm Wy$, $V_e = +\frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

Particle	$m\mathbf{v}_1 + \Sigma\int\mathbf{F}\,dt = m\mathbf{v}_2$
Rigid Body	$m(\mathbf{v}_G)_1 + \Sigma\int\mathbf{F}\,dt = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(\text{syst. } m\mathbf{v})_1 = \Sigma(\text{syst. } m\mathbf{v})_2$

Coefficient of Restitution $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

Particle	$(\mathbf{H}_O)_1 + \Sigma\int\mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = (d)(mv)$
Rigid Body (Plane motion)	$(\mathbf{H}_G)_1 + \Sigma\int\mathbf{M}_G\,dt = (\mathbf{H}_G)_2$ where $H_G = I_G\omega$ $(\mathbf{H}_O)_1 + \Sigma\int\mathbf{M}_O\,dt = (\mathbf{H}_O)_2$ where $H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(\text{syst. } \mathbf{H})_1 = \Sigma(\text{syst. } \mathbf{H})_2$

Rod ; $\bar{I} = \frac{1}{12}mL^2$, Plate $\bar{I} = \frac{1}{12}m(a^2 + b^2)$