1st Exam sheet

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \quad (2.17)$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (2.19)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \quad (2.24)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \quad (2.27)$$

TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2}=0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$
Temperature distribution	$T_{s,1}-\Delta T\frac{x}{L}$	$T_{s,2}+\Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1}-\Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,cond}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a]The critical radius of insulation is $r_{cr}=k/h$ for the cylinder and $r_{cr}=2k/h$ for the sphere.

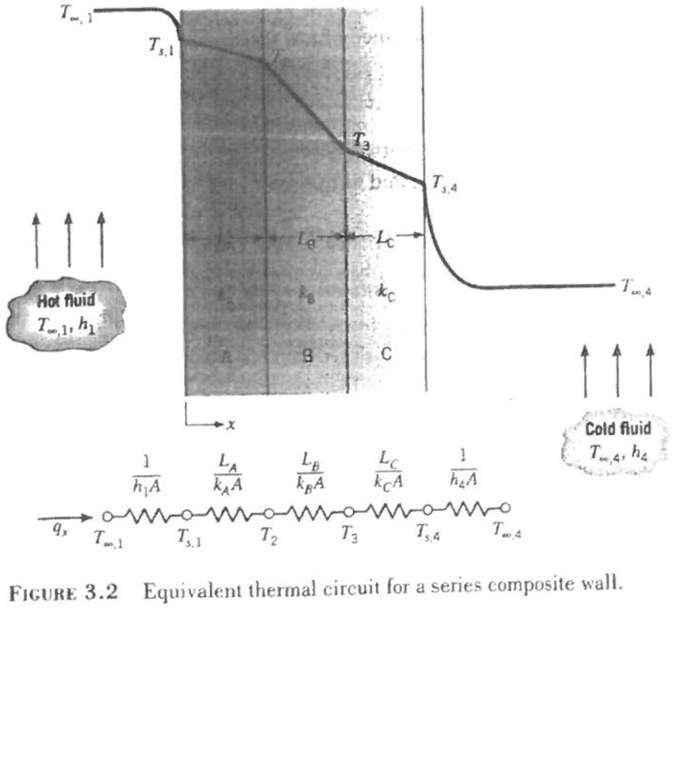


FIGURE 3.2 Equivalent thermal circuit for a series composite wall.

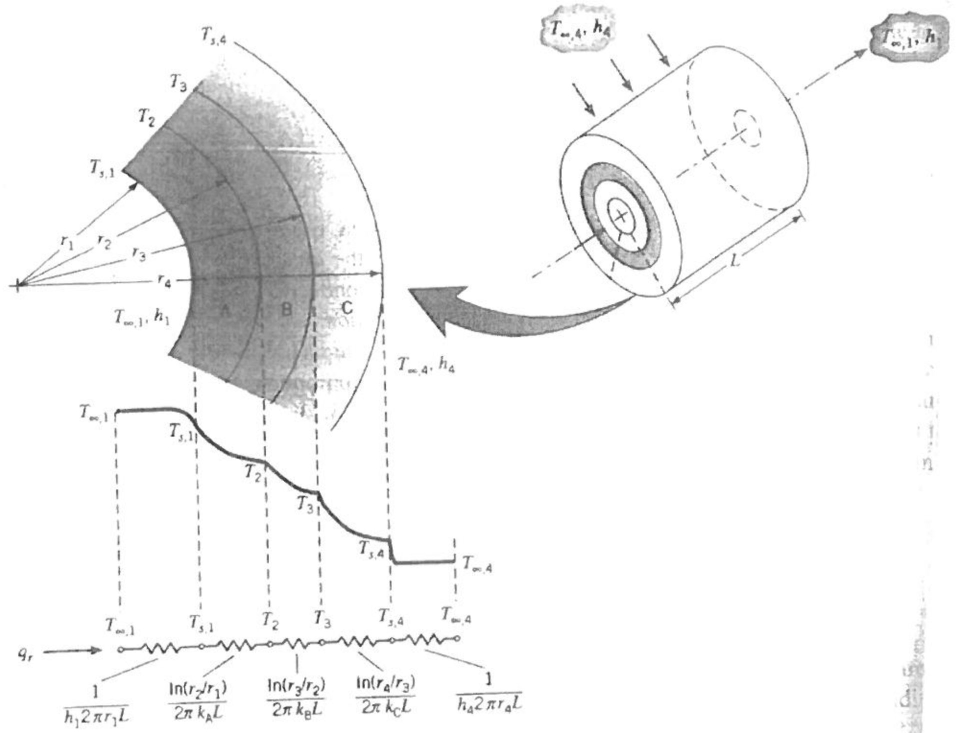


FIGURE 3.7 Temperature distribution for a composite cylindrical wall.

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b		Fin Heat Transfer Rate q_f	
A	Convection heat transfer: $h\theta(L) = -k\,d\theta/dx\rvert_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	(3.70)	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$	(3.72)
B	Adiabatic $d\theta/dx\rvert_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	(3.75)	$M \tanh mL$	(3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	(3.77)	$M\frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$	(3.78)
D	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx}	(3.79)	M	(3.80)

$\theta \equiv T - T_\infty$ $\qquad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\qquad M \equiv \sqrt{hPkA_c}\,\theta_b$

Lumped capacitance method:

$$-\dot{E}_{out} = \dot{E}_{st}$$

or

$$-hA_s(T - T_\infty) = \rho V c \frac{dT}{dt}$$

Introducing the temperature difference

$$\theta \equiv T - T_\infty$$

and recognizing that $(d\theta/dt) = (dT/dt)$ if T_∞ is constant, it follows that

$$\frac{\rho V c}{hA_s}\frac{d\theta}{dt} = -\theta$$

Separating variables and integrating from the initial condition, for which t
$T(0) = T_i$, we then obtain

$$\frac{\rho V c}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} = -\int_0^t dt$$

where

$$\theta_i \equiv T_i - T_\infty$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c}\right)t\right]$$

$$\tau_t = \left(\frac{1}{hA_s}\right)(\rho V c) = R_t C_t$$

$$Q = (\rho V c)\theta_i\left[1 - \exp\left(-\frac{t}{\tau_t}\right)\right]$$

The quantity Q is, of course, related to the change in the internal energy c
and from Equation 1.11b

$$-Q = \Delta E_{st}$$

$$q = \bar{h}A_s(T_s - T_\infty)$$

Equating Equations 6.11 and 6.12, it follows that the average and loc
coefficients are related by an expression of the form

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h\,dA_s$$

Note that for the special case of flow over a flat plate (Figure 6.4*b*),
with the distance x from the leading edge and Equation 6.13 reduces to

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx$$

$$\delta = \frac{5.0}{\sqrt{u_\infty/\nu x}} = \frac{5x}{\sqrt{Re_x}}$$

From Equation 7.19 it is clear that δ increases with increasing x and ν
with increasing u_∞ (the larger the free stream velocity, the *thinner*
layer). In addition, from Equation 7.15 the wall shear stress may be ex

$$\tau_s = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \mu u_\infty\sqrt{u_\infty/\nu x}\,\frac{d^2 f}{d\eta^2}\bigg|_{\eta=0}$$

Hence from Table 7.1

$$\tau_s = 0.332 u_\infty\sqrt{\rho\mu u_\infty/x}$$

The *local* friction coefficient is then

$$C_{f,x} \equiv \frac{\tau_{s,x}}{\rho u_\infty^2/2} = 0.664 Re_x^{-1/2}$$

External flow: Ch. 7

$$Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3} \qquad Pr \gtrsim 0.6$$

$$\overline{Nu}_x \equiv \frac{\bar{h}_x x}{k} = 0.664 Re_x^{1/2}\, Pr^{1/3} \qquad Pr \gtrsim 0.6$$

$$Nu_x = \frac{0.3387 Re_x^{1/2}\, Pr^{1/3}}{[1 + (0.0468/Pr)^{2/3}]^{1/4}} \qquad Pe_x \gtrsim 100$$

External flow: ch 7

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{\text{lam}}\, dx + \int_{x_c}^{L} h_{\text{turb}}\, dx\right)$$

where it is assumed that transition occurs abruptly at $x = x_c$. Substituting from Equations 7.23 and 7.36 for h_{lam} and h_{turb}, respectively, we obtain

$$\bar{h}_L = \left(\frac{k}{L}\right)\left[0.332\left(\frac{u_\infty}{\nu}\right)^{1/2}\int_0^{x_c}\frac{dx}{x^{1/2}} + 0.0296\left(\frac{u_\infty}{\nu}\right)^{4/5}\int_{x_c}^{L}\frac{dx}{x^{1/5}}\right] Pr^{1/3}$$

Integrating, we then obtain

$$\overline{Nu}_L = (0.037\, Re_L^{4/5} - A)\, Pr^{1/3}$$

$$\left[\begin{matrix} 0.6 \lesssim Pr \lesssim 60 \\ Re_{x,c} \lesssim Re_L \lesssim 10^8 \end{matrix}\right]$$

where the bracketed relations indicate the range of applicability and the co is determined by the value of the critical Reynolds number, $Re_{x,c}$. That is,

$$A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$$

Similarly, the average friction coefficient may be found using the expressio

$$\overline{C}_{f,L} = \frac{1}{L}\left(\int_0^{x_c} C_{f,x,\text{lam}}\, dx + \int_{x_c}^{L} C_{f,x,\text{turb}}\, dx\right)$$

Substituting expressions for $C_{f,x,\text{lam}}$ and $C_{f,x,\text{turb}}$ from Equations 7.20 a respectively, and carrying out the integration provides an expression of the

$$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - \frac{2A}{Re_L}$$

$$[Re_{x,c} \lesssim Re_L \lesssim 10^8]$$

It is also possible to have a uniform surface heat flux, rather than a unifor ature, imposed at the plate. For laminar flow, it may be shown that [5]

$$Nu_x = 0.453 Re_x^{1/2} Pr^{1/3} \qquad Pr \gtrsim 0.6$$

while for turbulent flow

$$Nu_x = 0.0308 Re_x^{4/5} Pr^{1/3} \qquad 0.6 \lesssim Pr \lesssim 60$$

$$T_s(x) = T_\infty + \frac{q_s''}{h_x}$$

Since the total heat rate is readily determined from the product o flux and the surface area, $q = q_s'' A_s$, it is not necessary to introduce an vection coefficient for the purpose of determining q. However, one ma determine an *average surface temperature* from an expression of the f

$$\overline{(T_s - T_\infty)} = \frac{1}{L}\int_0^L (T_s - T_\infty)\, dx = \frac{q_s''}{L}\int_0^L \frac{x}{k\, Nu_x}\, dx$$

where Nu_x is obtained from the appropriate convection correlation. from Equation 7.45, it follows that

$$\overline{(T_s - T_\infty)} = \frac{q_s'' L}{k Nu_L}$$

where

$$\overline{Nu}_L = 0.680 Re_L^{1/2} Pr^{1/3}$$

$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$Re_D \equiv \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu} \tag{8.1}$$

where u_m is the mean fluid velocity over the tube cross section and D is the tube diameter. In a fully developed flow, the critical Reynolds number corresponding to the *onset* of turbulence is

$$Re_{D,c} \approx 2300 \tag{8.2}$$

although much larger Reynolds numbers ($Re_D \approx 10{,}000$) are needed to achieve fully turbulent conditions. The transition to turbulence is likely to begin in the developing boundary layer of the entrance region.

For laminar flow ($Re_D \lesssim 2300$), the hydrodynamic entry length may be obtained from an expression of the form [1]

$$\left(\frac{x_{\text{fd},h}}{D}\right)_{\text{lam}} \approx 0.05\, Re_D \tag{8.3}$$

This expression is based on the presumption that fluid enters the tube from a rounded converging nozzle and is hence characterized by a nearly uniform velocity profile at the entrance (Figure 8.1). Although there is no satisfactory general expression for the entry length in turbulent flow, we know that it is approximately independent of Reynolds number and that, as a first approximation [2],

$$10 \lesssim \left(\frac{x_{\text{fd},h}}{D}\right)_{\text{turb}} \lesssim 60 \tag{8.4}$$

For the purposes of this text, we shall assume fully developed turbulent flow for $(x/D) > 10$.

$$\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} \neq f(x)$$

Integrating from $x = 0$, it follows that

$$T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m} c_p} x \qquad q_s'' = \text{constant}$$

$$\frac{\Delta T_o}{\Delta T_i} = \frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = \exp\left(-\frac{\overline{U} A_s}{\dot{m} c_p}\right)$$

$$q = \overline{U} A_s \, \Delta T_{\text{lm}}$$

$$\Delta T_{\text{lm}} \equiv \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}$$

or

$$\overline{h}_L = 0.943 \left[\frac{g \rho_l (\rho_l - \rho_v) k_l^3 h_{fg}'}{\mu_l (T_{\text{sat}} - T_s) L}\right]^{1/4}$$

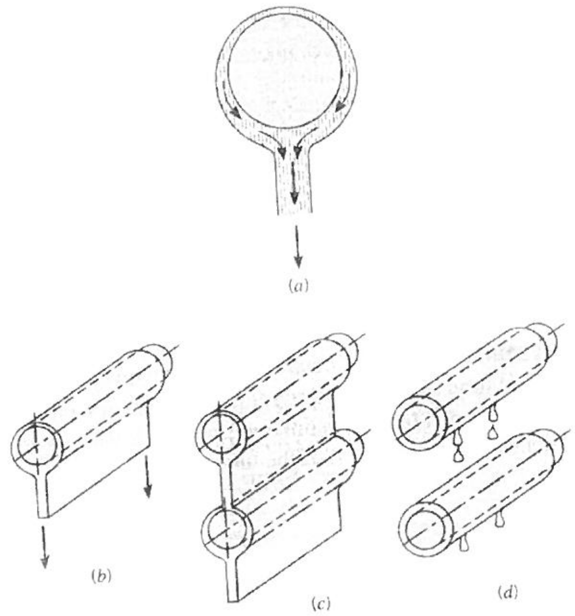
The average Nusselt number then has the form

$$\overline{Nu}_L = \frac{\overline{h}_L L}{k_l} = 0.943 \left[\frac{\rho_l g (\rho_l - \rho_v) h_{fg}' L^3}{\mu_l k_l (T_{\text{sat}} - T_s)}\right]^{1/4}$$

$$\dot{m} = \frac{q}{h_{fg}'} = \frac{\overline{h}_L A (T_{\text{sat}} - T_s)}{h_{fg}'}$$

... applicable to any surface geo

$$\bar{h}_D = C\left[\frac{g\rho_l(\rho_l - \rho_v)k_l^3 h'_{fg}}{\mu_l(T_{sat} - T_s)D}\right]^{1/4}$$



where $C = 0.826$ for the sphere [48] and 0.729 for the tube [44]. The
in this equation and the one below are evaluated as explained bene
tion 10.32.

For a vertical tier of N horizontal tubes, Figure 10.14c, the average c
coefficient (over the N tubes) may be expressed as

$$\bar{h}_{D,N} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)k_l^3 h'_{fg}}{N\mu_l(T_{sat} - T_s)D}\right]^{1/4}$$

$$\bar{h}_{dc} = 51{,}104 + 2044T_{sat}(°C) \qquad 22°C \leq T_{sat} \leq 100°C$$

$$\bar{h}_{dc} = 255{,}510 \qquad 100°C \leq T_{sat}$$