

Chapter Three

Matrix Algebra

- Addition, Subtraction and Multiplication
- Matrix Determinant
- Matrix Types and Properties
- Matrix Transpose
- Matrix Inverse/ Cofactor Method
- Eigenvalues and Eigenvectors

Ch3 Matrix Algebra

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Linear Algebra

Matrix: an array of variables or numbers

$$\begin{array}{c} \text{rows (i)} \\ \rightarrow \end{array}
 \begin{array}{|c|c|c|} \hline a & b & c \\ \hline d & e & f \\ \hline g & h & i \\ \hline \end{array}
 \begin{array}{c} \downarrow \text{columns} \\ \text{(j)} \end{array}$$

$$\begin{array}{|c|} \hline 3 \\ \hline 2 \\ \hline 1 \\ \hline 7 \\ \hline \end{array}$$

4x1

4x1 $\rightarrow$ Rows x Columns

Row col

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array}
 +
 \begin{array}{|c|c|} \hline e & f \\ \hline g & h \\ \hline \end{array}
 =
 \begin{array}{|c|c|} \hline a+e & b+f \\ \hline c+g & d+h \\ \hline \end{array}$$

$\begin{array}{cc} m \times n & m \times n \end{array}$

ex.

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}
 +
 \begin{array}{|c|c|} \hline 5 & 6 \\ \hline 7 & 8 \\ \hline \end{array}
 =
 \begin{array}{|c|c|} \hline 6 & 8 \\ \hline 10 & 12 \\ \hline \end{array}$$

A

B

$$A_{ij} + B_{ij} \rightarrow A_{11} + B_{11}$$

$$A_{21} + B_{21} \quad \text{etc.}$$

Scalar multiplication

a	b		$a \times x$	$b \times x$		$a \times$	$b \times$
c	d	$\times x =$	$c \times x$	$d \times x$	$=$	$c \times$	$d \times$

1	2		x	$2x$
3	4	$\times x =$	$3x$	$4x$

"Matrix Algebra"

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الجمع والطرح

Addition & Subtraction.

عدد الصفوف في (A) = عدد الصفوف في (B)

عدد الأعمدة في (A) = عدد الأعمدة في (B)

(Same size)

ex)

$$A = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 4 \\ -1 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & 2 \\ 4 & 3 \\ 3 & -1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 5 & 8 \\ -2 & 1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$A + C \equiv$ undefined غير معرفة

$$A + 4B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 8 & 16 \\ -4 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 20 \\ -5 & -2 \end{bmatrix}$$

Matrix x matrix

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline e & f \\ \hline \end{array} \times \begin{array}{|c|} \hline x \\ \hline y \\ \hline \end{array} = \begin{array}{|c|} \hline ax+by \\ \hline cx+dy \\ \hline ex+fy \\ \hline \end{array}$$

must be the same

$$m \times n \times n \times k = m \times k$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & 6 \\ \hline \end{array} \times \begin{array}{|c|} \hline 7 \\ \hline 8 \\ \hline \end{array} = \begin{array}{|c|} \hline (1)(7) + (2)(8) \\ \hline (3)(7) + (4)(8) \\ \hline (5)(7) + (6)(8) \\ \hline \end{array} = \begin{array}{|c|} \hline 23 \\ \hline 53 \\ \hline 83 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline e & f \\ \hline \end{array} \times \begin{array}{|c|c|} \hline w & x \\ \hline y & z \\ \hline \end{array} = \begin{array}{|c|c|} \hline aw+by & ax+bz \\ \hline cw+dy & cx+dz \\ \hline ew+fy & ex+fz \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline 3 & 4 \\ \hline 5 & 6 \\ \hline \end{array} \times \begin{array}{|c|c|} \hline 7 & 8 \\ \hline 9 & 10 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 \times 7 + 2 \times 9 & 1 \times 8 + 2 \times 10 \\ \hline 3 \times 7 + 4 \times 9 & 3 \times 8 + 4 \times 10 \\ \hline 5 \times 7 + 6 \times 9 & 5 \times 8 + 6 \times 10 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 25 & 28 \\ \hline 57 & 64 \\ \hline 89 & 100 \\ \hline \end{array}$$

$3 \times 2 \quad \quad 2 \times 2 \quad \quad \quad 3 \times 2$

Just forget $A \times B \neq B \times A$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \neq \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

* Multiplication القرب

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حيث يتم القرب عن طريق ضرب صف بعמוד

$$\begin{bmatrix} 2 & 4 & -2 \\ -1 & 3 & 1 \\ 5 & 0 & -2 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 3 \\ -2 & -2 & 1 \end{bmatrix}_{3 \times 3}$$

$$= \begin{bmatrix} \boxed{4+4+4} & \boxed{-2+0+4} & \boxed{6+12-2} \\ \boxed{-2+3-2} & \boxed{1+0-2} & \boxed{-3+9+1} \\ \boxed{10+0+4} & \boxed{-5+0+4} & \boxed{15+0-2} \end{bmatrix}_{3 \times 3}$$

12 2 16

-1 -1 7

14 -1 13

$$\begin{bmatrix} 2 & 4 & 3 \\ -2 & 1 & -1 \\ -1 & 2 & 1 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} \boxed{6-16+15} \\ \boxed{-6-4-5} \\ \boxed{-3-8+5} \end{bmatrix}_{3 \times 1}$$

5 -15 -6

$$\begin{array}{c}
 A \times (B \times C) = (A \times B) \times C \\
 \begin{array}{ccc} m \times n & n \times k & k \times p \\ \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} \\ m \times k & & m \times k \\ \underbrace{\hspace{1.5cm}} & & \underbrace{\hspace{1.5cm}} \\ & m \times p & m \times p \end{array}
 \end{array}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad G = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\det(A) = ad - bc \quad 1 \times 4 - 2 \times 3 = -2$$

$$X = \begin{bmatrix} 5 & 3 \\ 1 & 7 \end{bmatrix} \quad \det(X) = 5 \times 7 - 1 \times 3 = 32$$

$$3 \times G = \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix}$$

$$\begin{aligned}
 \frac{G}{3} &= G \times \frac{1}{3} \\
 &= \frac{1}{3} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ 1 & \frac{4}{3} \end{bmatrix}
 \end{aligned}$$

$$A^{-1} = \frac{1}{\det A} A \quad A \times A^{-1} = I$$

$$I = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & -1 \\ -1 & 5 & 2 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 2 & -4 & 5 \\ 6 & -1 & 7 \end{bmatrix}_{2 \times 3}$$

عملیه ضرب غیر مجاز است
و نیز معرفه

و لا يجوز ابرایها اشتقاق

$$\begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & -1 \\ -1 & 5 & 2 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 2 & 6 \\ -4 & -1 \\ 5 & 7 \end{bmatrix}_{3 \times 2}$$

$$= \begin{bmatrix} \square & \cup \\ \square & \cup \\ \square & \square \end{bmatrix}_{3 \times 2}$$

الضرب Multiplication

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عدد الأعمدة في المصفوفة = عدد الصفوف في المصفوفة

$$\begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}_{m \times n} \begin{bmatrix} ? & ? & ? \end{bmatrix}_{n \times l}$$

بشرط أن $n = k$

ex) $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 4 \end{bmatrix}_{2 \times 3}$ $B = \begin{bmatrix} 2 & 1 \\ 4 & 5 \\ 1 & + \end{bmatrix}_{3 \times 2}$

$$C = A \cdot B$$

$$= \begin{bmatrix} 6+8+1 & 3+10+7 \\ 4+4+4 & 2+5+28 \end{bmatrix} = \begin{bmatrix} 15 & 20 \\ 12 & 35 \end{bmatrix}_{2 \times 2}$$

Find $B \cdot A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \\ 1 & 7 \end{bmatrix}_{3 \times 2} \cdot \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 4 \end{bmatrix}_{2 \times 3}$

$$A \cdot B \neq B \cdot A$$

$$= \begin{bmatrix} 8 & 5 & 6 \\ 22 & 13 & 24 \\ 17 & 9 & 29 \end{bmatrix}_{3 \times 3}$$

* المحدد Determinant

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$$A = \begin{bmatrix} 2 & 3 \\ -2 & 4 \end{bmatrix}$$

* $|A| = \begin{matrix} \text{عنصر في الصف الأول، عمود 1} & \text{عنصر في الصف الثاني، عمود 2} \\ (2)(4) & - (8)(-2) \end{matrix}$
 $= 8 - -6 = 8 + 6 = 14$

$$B = \begin{bmatrix} 4 & 8 \\ 10 & -2 \end{bmatrix}$$

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دعسة ميكانيكية

$$|B| = (4)(-2) - (8)(10)$$

$$= -8 - 80 = -88$$

$$C = \begin{bmatrix} 6 & 4 \\ 3 & 2 \end{bmatrix}$$

$$|C| = (6)(2) - (4)(3)$$

$$= 12 - 12 = 0$$

$$\text{ex)} A = \begin{bmatrix} \textcircled{+} 2 & \textcircled{-} 4 & \textcircled{+} 1 \\ -2 & 5 & 1 \\ -3 & 2 & -1 \end{bmatrix}$$

$$|A| = +2 \left[\begin{array}{c} 5(-1) - (1)(2) \\ = \textcircled{-7} \end{array} \right]$$

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نسخة ميكانيكية

$$\textcircled{-} 4 \left[\begin{array}{c} (-2)(-1) - (1)(-3) \\ 2 - -3 = \textcircled{5} \end{array} \right]$$

$$+1 \left[\begin{array}{c} -2(2) - 5(-3) \\ -4 + 15 = \textcircled{11} \end{array} \right]$$

$$= 2(-7) - 4(5) + 1(11)$$

$$= -14 - 20 + 11 = -23$$

$$* B = \begin{bmatrix} -2 & 4 & 3 \\ 1 & 0 & -1 \\ 2 & 1 & 1 \end{bmatrix}$$

$$|B| = -2 \begin{bmatrix} 1 \end{bmatrix} - 4 \begin{bmatrix} 3 \end{bmatrix} + 3 \begin{bmatrix} 1 \end{bmatrix}$$

$$= -2 - 12 + 3 = -11$$

$$* C = \begin{bmatrix} -1 & -2 & -4 \\ 2 & 0 & 3 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\Rightarrow |C| = -1(3) + 2(4) - 4(-2) = -3 + 8 + 8 = \underline{\underline{13}}$$

Ex) $A = \begin{bmatrix} \overset{+}{2} & \overset{-}{-2} & \overset{+}{4} & \overset{-}{3} \\ 1 & 2 & 0 & -2 \\ -1 & -2 & 3 & 0 \\ 0 & 1 & 0 & 4 \end{bmatrix}$

$$|A| = 2 \begin{vmatrix} 2 & 0 & -2 \\ -2 & 3 & 0 \\ 1 & 0 & 4 \end{vmatrix} \rightarrow 2(12) + 0 - 2(-3) = \textcircled{30}$$

$$+2 \begin{vmatrix} 1 & 0 & -2 \\ -1 & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix} \rightarrow 1(12) + 0 - 2(0) = \textcircled{12}$$

$$+4 \begin{vmatrix} 1 & 2 & -2 \\ -1 & -2 & 0 \\ 0 & 1 & 4 \end{vmatrix} \rightarrow 1(-8) - 2(-4) - 2(-1) = \textcircled{2}$$

$$-3 \begin{vmatrix} 1 & 2 & 0 \\ -1 & -2 & 3 \\ 0 & 1 & 0 \end{vmatrix} \rightarrow 1(-3) - 2(0) + 0 = \textcircled{-3}$$

$$= 2(30) + 2(12) + 4(2) - 3(-3)$$

$$= 60 + 24 + 8 + 9$$

$$= \underline{\underline{101}}$$

* $|A^{-1}| = \frac{1}{|A|}$ بعض خصائص المحددات :-

* $|A^T| = |A|$

* if $A = B \cdot C \Rightarrow |A| = |B| \cdot |C|$

* if $A = B \pm C \Rightarrow |A| \neq |B| \pm |C|$

* Diagonal Matrix $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix} \Rightarrow |A| = (2)(4)(6)(-2) = -96$

* Upper diagonal Matrix $B = \begin{bmatrix} 2 & 9 & 2 & 6 \\ 0 & -2 & 3 & 2 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix} \Rightarrow |B| = (2)(-2)(4)(3) = -48$

* lower diagonal Matrix $C = \begin{bmatrix} 4 & & & \\ & -2 & & \\ & & \frac{1}{2} & \\ & & & 6 \end{bmatrix}$

$|C| = (4)(-2)(\frac{1}{2})(6) = -24$

* Find the determinant of :-

$A = \begin{bmatrix} 2 & -3 & 1 & 3 & 6 \\ -2 & 4 & 3 & -6 & 1 \\ 4 & 3 & -4 & 2 & 1 \\ -2 & 4 & 3 & -6 & 1 \\ 2 & -4 & 4 & 3 & 6 \end{bmatrix}$

$\Rightarrow |A| = 0$

إذا كانت صفين أو عمودين
تتبع الخطية للمحدد صفراً

$$B = \begin{bmatrix} 2 & -4 & 0 & 3 & 5 \\ 2 & 5 & 0 & 4 & -3 \\ 2 & -4 & 0 & -3 & 5 \\ -2 & 5 & 0 & 4 & -3 \\ -2 & 5 & 0 & 4 & 3 \end{bmatrix} \Rightarrow (|B| = 0)$$

إذا كان كاس
القاعد له
أو له صا
عنان، عتده لكون
صفه

$$A = \begin{bmatrix} 2 & 4 \\ 6 & 5 \end{bmatrix} \Rightarrow |A| = 10 - 24 = -14 \neq 0$$

$|A| \neq 0 \Rightarrow$ * Non singular
* Reversible

$$B = \begin{bmatrix} 3 & 5 \\ 6 & 10 \end{bmatrix} \Rightarrow |B| = 30 - 30 = 0$$

$|B| = 0 \Rightarrow$ * Singular
* IRReversible

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* Matrix Traspose

الترتيب

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$$A = \begin{bmatrix} 4 & 3 & -2 & 5 \\ 6 & 7 & 8 & -1 \\ -2 & 3 & 4 & 0 \\ 5 & -1 & -1 & -3 \end{bmatrix} \Rightarrow |A| = 132$$

محدد

$$A^T = \begin{bmatrix} 4 & 6 & -2 & 5 \\ 3 & 7 & 3 & -1 \\ -2 & 8 & 4 & -1 \\ 5 & -1 & 0 & -3 \end{bmatrix} \Rightarrow |A^T| = 132$$

القياس

قلب الصفوف

قاعدة

$$|A| = |A^T|$$

عكس

والعكس

صحيح

$$A = \begin{bmatrix} 6 & 4 & 3 & 2 \\ 0 & -2 & 4 & 1 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

مصفوفة فوقية

Upper Matrix

$$\Rightarrow |A| = (6)(-2)(-1)(-3) = -36$$

المحدد

$$B = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix}$$

Diagonal Matrix

مصفوفة قطرية

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$$|B| = (3)(5)(-2)(4)(6) = \checkmark$$

Ex) $A = \begin{bmatrix} 2 & -2 & 4 & 3 \\ 1 & 2 & 0 & -2 \\ -1 & -2 & 3 & 0 \\ 0 & 1 & 0 & 4 \end{bmatrix} \Rightarrow |A| = 101$

Find $B = \begin{bmatrix} -1 & -2 & 3 & 0 \\ 1 & 2 & 0 & -2 \\ 2 & -2 & 4 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix} \Rightarrow |B| = 101$

Find: $|B| = 101$

تغيير
بالاين

او كاعود
مطابقة كاعود

تبدل صف مكان صف

* Transpose الاستبدال

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$$A = \begin{bmatrix} 3 & -2 & 1 & 2 \\ 4 & 3 & -1 & 3 \\ 5 & -2 & 4 & 4 \\ -1 & 2 & 1 & 5 \end{bmatrix}$$

مصفوفة مربعة (استبدال)
العناصر في صف
والعناصر في عمود

Find $A^T = \begin{bmatrix} 3 & 4 & 5 & -1 \\ -2 & 3 & -2 & 2 \\ 1 & -1 & 4 & 1 \\ 2 & 3 & 4 & 5 \end{bmatrix}$

$$B = \begin{bmatrix} 5 & -6 & 3 \\ -6 & 2 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

نصف مربعة "مربعة"

Find $B^T = \begin{bmatrix} 5 & -6 & 3 \\ -6 & 2 & 4 \\ 3 & 4 & 7 \end{bmatrix}$

$$\Rightarrow \boxed{B = B^T}$$

B is Symmetric Matrix

مصفوفة متماثلة

$$C = \begin{bmatrix} 0 & -5 & 6 \\ 5 & 0 & -4 \\ -6 & 4 & 0 \end{bmatrix}$$

نصف مربعة "مربعة"

$$\boxed{C = -C^T}$$

$$C^T = \begin{bmatrix} 0 & 5 & -6 \\ -5 & 0 & 4 \\ 6 & -4 & 0 \end{bmatrix}$$

C is Skew-Symmetric
مصفوفة متماثلة

Matrix Inverse (2x2)

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$$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} \quad \det(A) = 8 - 3 = 5$$

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 4/5 & -1/5 \\ -3/5 & 2/5 \end{bmatrix} = \begin{matrix} x_1 & x_2 \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{matrix}$$

Matrix Inverse

مصفوفة معكوسة

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$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

"Cofactor Method"

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

مصفوفة معكوسة
Transpose

T

$$\Rightarrow A^{-1} = \frac{1}{|A|} \begin{bmatrix} |e f| & -|d f| & |d e| \\ -|b c| & |a c| & -|a b| \\ |b c| & -|a c| & |a b| \\ |e f| & -|d f| & |d e| \end{bmatrix}$$

example

$$A = \begin{bmatrix} 2 & 4 \\ -2 & 1 \end{bmatrix}$$

① Find A^{-1} ? العكوس

② Find A^T ? التباديل

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الحل

① $|A| = (2)(1) - 4(-2)$
 $= 2 + 8 = 10$

$$A^{-1} = \frac{1}{10} \begin{bmatrix} 1 & -4 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0.1 & -0.4 \\ 0.2 & 0.2 \end{bmatrix}$$

② $\Rightarrow A^T = \begin{bmatrix} 2 & -2 \\ 4 & 1 \end{bmatrix}$

* Inverse A^{-1} wagt!

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$$A = \begin{bmatrix} -3 & -4 \\ +3 & +6 \end{bmatrix} \Rightarrow |A| = -18 - 12 = -6$$

$$A^{-1} = \frac{1}{-6} \begin{bmatrix} 6 & 4 \\ -3 & -3 \end{bmatrix} = \begin{bmatrix} \frac{6}{-6} & \frac{4}{-6} \\ \frac{-3}{-6} & \frac{-3}{-6} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1 & -\frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$B = \begin{bmatrix} -4 & -2 \\ 3 & 5 \end{bmatrix} \Rightarrow |B| = -20 - -6 = -14$$

$$B^{-1} = \frac{-1}{14} \begin{bmatrix} 5 & 2 \\ -3 & -4 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Matrix
Inverse
(3x3)

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المستند
T

$$B^{-1} = \frac{1}{|B|}$$

عناصر الخالصي
عكس مكنونية

$$\begin{bmatrix} \boxed{\begin{matrix} e & f \\ h & i \end{matrix}} & \boxed{\begin{matrix} d & f \\ g & i \end{matrix}} & \boxed{\begin{matrix} d & e \\ g & h \end{matrix}} \\ \boxed{\begin{matrix} b & c \\ h & i \end{matrix}} & \boxed{\begin{matrix} a & c \\ g & i \end{matrix}} & \boxed{\begin{matrix} a & b \\ g & h \end{matrix}} \\ \boxed{\begin{matrix} b & c \\ e & f \end{matrix}} & \boxed{\begin{matrix} a & c \\ d & f \end{matrix}} & \boxed{\begin{matrix} a & b \\ d & e \end{matrix}} \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ 5 & 0 & 3 \end{bmatrix} \Rightarrow |B| = 2(-3) + 3(-7) + 4(5) = -6 - 21 + 20 = -7$$

$$\Rightarrow B^{-1} = \frac{-1}{-7} \begin{bmatrix} -3 & +7 & 5 \\ +9 & -14 & -15 \\ -2 & 0 & 1 \end{bmatrix}^T$$

$$B^{-1} = \frac{-1}{-7} \begin{bmatrix} -3 & 9 & -2 \\ 7 & -14 & 0 \\ 5 & -15 & 1 \end{bmatrix}$$

Find $|B^{-1}|$?

$= \frac{-1}{-7}$

=====

EX

$$A = \begin{bmatrix} 2 & -2 & 4 \\ 1 & 3 & -1 \\ -1 & 0 & 4 \end{bmatrix}$$

111

$$|A| = \underset{\text{determinant}}{2(12) + 2(3) + 4(3)} = \underline{\underline{42}}$$

$$A^{-1} = \frac{1}{42} \begin{bmatrix} 12 & \underline{-3} & 3 \\ +8 & 12 & +2 \\ -10 & +6 & 8 \end{bmatrix} \textcircled{T}$$

النتيجة

$$A^{-1} = \frac{1}{42} \begin{bmatrix} 12 & 8 & -10 \\ -3 & 12 & 6 \\ 3 & 2 & 8 \end{bmatrix}$$

Eigenvalues and Eigenvectors

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Matrix

$A \Rightarrow$ Find the eigenvalues (λ) and eigenvectors (x) .

$$(A - \lambda I)(x) = 0$$

$$\det(A - \lambda I) = 0$$

مثال: إذا كانت A مصفوفة 2×2 ← فالمعادلة تكون كالتالي

$$\begin{array}{l} \text{أو بالتكامل} \\ \text{أو بالمميز} \end{array} \left[\begin{array}{l} \text{ex: } \lambda^2 + 2\lambda + 1 = 0 \\ \text{ex: } 2\lambda^2 - \lambda + 4 = 0 \\ \text{ex: } \lambda^2 - 6 = 0 \end{array} \right] \parallel \begin{array}{l} \text{ex: } \lambda^2 - 2\lambda = 0 \\ \text{ex: } 4\lambda^2 + 4 = 0 \end{array}$$

مثال: إذا كانت A مصفوفة 3×3 ← فالمعادلة تكون كالتالي

$$\begin{array}{l} \text{أو بالتكامل} \\ \text{أو بالمميز} \\ \text{أو بالتكامل} \end{array} \left[\begin{array}{l} \text{ex: } \lambda^3 - 2\lambda^2 + 4\lambda + 7 = 0 \\ \text{ex: } \lambda^3 - 2\lambda^2 = 0 \\ \text{ex: } \lambda^3 - 2\lambda^2 + 4 = 0 \\ \text{ex: } 2\lambda^3 - 4\lambda + 6 = 0 \\ \text{ex: } 3\lambda^3 + 7 = 0 \end{array} \right]$$

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ex III

Find the eigenvalues and
eigenvectors :

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$$\begin{aligned}6x - 3y &= 0 \\ 2x + y &= 0\end{aligned}$$

$$\begin{bmatrix} 6-\lambda & -3 \\ 2 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(6-\lambda)(1-\lambda) + 6 = 0$$

$$\Rightarrow 6 - 7\lambda + \lambda^2 + 6 = 0$$

$$\Rightarrow 12 - 7\lambda + \lambda^2 = 0$$

$$\Rightarrow \lambda^2 - 7\lambda + 12 = 0$$

$$(\lambda - 3)(\lambda - 4) = 0$$

$\lambda_1 = 3$
$\lambda_2 = 4$

Two eigenvalues

عوض في المعادلات

$$\underline{\underline{\lambda_1 = 3}}$$

$$\begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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Dependent $\begin{cases} 3x - 3y = 0 \Rightarrow x - y = 0 \Rightarrow x = y \\ 2x - 2y = 0 \Rightarrow x - y = 0 \end{cases}$

$$u_{\lambda=3} = \text{Eigen vector} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\underline{\underline{\lambda_2 = 4}}$$

$$\begin{bmatrix} 2 & -3 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} 2x - 3y &= 0 \Rightarrow 2x = 3y \\ 2x - 3y &= 0 \Rightarrow x = \frac{3}{2}y \end{aligned}$$

Eigen
vector

$$u_{\lambda=4} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

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Ex 11

Find the eigenvalues and eigenvectors of the matrix (system)

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$$A = \begin{bmatrix} 2 & 1 \\ -4 & -3 \end{bmatrix}$$

Sol.

$$(A - \lambda I)(x) = (0)$$

$$\begin{bmatrix} 2-\lambda & 1 \\ -4 & -3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(2-\lambda)(-3-\lambda) + 4 = 0$$

$$-6 + \lambda + \lambda^2 + 4 = 0$$

$$\Rightarrow \lambda^2 + \lambda - 2 = 0$$

$$(\lambda - 1)(\lambda + 2) = 0$$

eigen
values.

$$\begin{array}{|c|} \hline \lambda_1 = 1 \\ \hline \lambda_2 = -2 \\ \hline \end{array}$$

OR

$$a\lambda^2 + b\lambda + c = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-1 \pm \sqrt{1 - 4(-2)}}{2(1)}$$

$$\lambda = \frac{-1 \pm \sqrt{9}}{2}$$

$$\lambda = \frac{-1 \pm 3}{2} = \underline{\underline{1, -2}}$$

$$\underline{\underline{\lambda_1 = 1}}$$

$$\begin{bmatrix} 1 & 1 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$$\begin{aligned} x + y &= 0 \\ -4x - 4y &= 0 \end{aligned} \Rightarrow x = -y$$

eigen vector

$$u_{\lambda=1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\underline{\underline{\lambda_2 = -2}}$$

$$\begin{bmatrix} 4 & 1 \\ -4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

eigen vector

$$\begin{aligned} 4x + y &= 0 \\ -4x - y &= 0 \end{aligned} \Rightarrow y = -4x$$

$$u_{\lambda=-2} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

ex Eigenvalue and Eigenvector
For 3x3 Matrix.

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Solve the for $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}_{3 \times 3}$

Sol. $[A - \lambda I] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\det(A - \lambda I) = 0$

$\Rightarrow (2-\lambda) [(3-\lambda)(2-\lambda) - 2]$

$- 2 [(2-\lambda) - 1]$

$+ 1 [2 - (3-\lambda)] = 0$

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$\Rightarrow (2-\lambda) [6 - 5\lambda + \lambda^2 - 2] - 2[1-\lambda] + [-1+\lambda] = 0$

$\Rightarrow (2-\lambda) [\lambda^2 - 5\lambda + 4] - 2 + 2\lambda - 1 + \lambda = 0$

$\Rightarrow 2\lambda^2 - 10\lambda + 8 - \lambda^3 + 5\lambda^2 - 4\lambda - 3 + \lambda = 0$

$\Rightarrow -\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$

$$x^3 - 7x^2 + 11x - 5 = 0$$

نیمی صی دور، لی، ثابت

$$\begin{aligned} -5 &= 1 \times -5 \\ &= -1 \times 5 \end{aligned}$$

$$\boxed{x=1}$$

$$\Rightarrow 1 - 7 + 11 - 5 = 0$$

الفه، ایلو، علی $x-1$

$$\begin{array}{r} x^2 - 6x + 5 \\ x-1 \overline{) x^3 - 7x^2 + 11x - 5} \\ \underline{-x^3 + x^2} \\ -6x^2 + 11x - 5 \\ \underline{+6x^2 - 6x} \\ 5x - 5 \\ \underline{5x - 5} \\ 0 \end{array}$$

$$\lambda^2 - 6\lambda + 5 = 0$$

$$\Rightarrow (\lambda - 5)(\lambda - 1)$$

$$\boxed{\lambda = 5}$$

$$\boxed{\lambda = 1}$$

$$\boxed{\lambda = 1}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x + 2y + z = 0$$

$$\text{for } x = m \Rightarrow z = x - 2y$$

$$y = k \Rightarrow z = m - 2k$$

Eigen vector
for
 $\lambda = 1$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} m \\ k \\ m - 2k \end{bmatrix}$$

$$\boxed{\lambda = 5}$$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\Rightarrow -3x + 2y + z = 0 \quad \text{--- (1)}$$

$$\Rightarrow x - 2y + z = 0 \quad \text{--- (2)}$$

$$\Rightarrow x + 2y - 3z = 0 \quad \text{--- (3)}$$

$$\begin{array}{r} 3x - 2y - z = 0 \\ x - 2y + z = 0 \\ \hline 4x - 4y = 0 \end{array} \Rightarrow \boxed{x = y}$$

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$$\begin{array}{r} 3x - 6y + 3z = 0 \\ x + 2y - z = 0 \\ \hline 4x - 4y = 0 \end{array} \Rightarrow \boxed{x = y}$$

$$z = 2y - x$$

Eigen
vector
for
 $\lambda = 5$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$