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The University of Jordan
Mathematics Department

Eng. Mathematics (II)

First exam
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Lecture time: 8 → 9:30

Q1. (6 points)

If $f(x,y,z) = x^3 + y^2 + z$, $\vec{F}(x,y,z) = (xyz)\hat{i} - (x^2y^2z^2)\hat{j} - (x^2 + y^3 + z^4)\hat{k}$,

$\vec{G}(x,y,z) = (y^2)\hat{i} + (z^2)\hat{j} + (x^2)\hat{k}$

then find

a) $\nabla(\nabla \cdot \vec{F})$

$\nabla \cdot \vec{F} = yz - 2yx^2z^2 - (0 + 0 + 4z^3)$

$= (yz - 2yx^2z^2 - 4z^3)$

$\therefore \nabla(\nabla \cdot \vec{F}) =$

$(0 - 4yxz^2 - 0)i +$

$(z - 2x^2z^2 - 0)j +$

$(y - 4yzx^2 - 12z^2)K$

$\nabla(\nabla \cdot \vec{F}) = [-4yxz^2, z - 2x^2z^2, y - 4yzx^2 - 12z^2]$

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b) $\nabla^2 f + \nabla \cdot (\nabla \times (\nabla f))$ Zero

div (curl $*$) = 0

$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \Rightarrow 6x + 2 + 0$

$= 6x + 2$

c) $\nabla f \cdot (\nabla \times \vec{G})$

$\nabla f = [3x^2, 2y, 1]$

$$\nabla \times \vec{G} = \begin{vmatrix} i & j & K \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & z^2 & x^2 \end{vmatrix}$$

$= i(0 - 2z) - j(2x - 0) + \hat{k}(0 - 2y)$

$[-2z, -2x, -2y]$

$\therefore \nabla f \cdot (\nabla \times G) =$

$= [3x^2, 2y, 1] \cdot [-2z, -2x, -2y]$

$= -6x^2z - 4yx - 2y$

Q2. (6 points) Evaluate $\int_C \vec{F}\cdot d\vec{r}$, where

$\vec{F} = \dfrac{1}{\sqrt{x^2+y^2+z^2-x-y+2z}}\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$, and C is the line segment from (1,1,1) to (4,4,4).

$$\vec{r}(t) = \vec{r}_0 + (\vec{r}_1 - \vec{r}_0)t$$

$$= (1,1,1) + (3,3,3)t$$

$$= \left[\underbrace{3t+1}_{x},\ \underbrace{3t+1}_{y},\ \underbrace{3t+1}_{z}\right]$$

$$= \frac{1}{\sqrt{3(3t+1)^2 \cancel{+2(3t+1)} \cancel{+2(3t+1)}}} = \frac{1}{\sqrt{3}} * \frac{1}{3t+1}$$

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$$\vec{F}(\vec{r}(t)) = \left[\frac{\cancel{(3t+1)}}{\sqrt{3}*\cancel{(3t+1)}},\ \frac{\cancel{(3t+1)}}{\sqrt{3}*\cancel{(3t+1)}},\ \frac{1}{\sqrt{3}} * \frac{\cancel{(3t+1)}}{\cancel{(3t+1)}}\right]$$

$$\vec{F}(\vec{r}(t)) = \left[\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}}\right]$$

$$\vec{r}\,'(t) = [3,\ 3,\ 3]$$

limits, when $x=1$

$3t+1=1$

$3t=0$

$\boxed{t=0}$

when $x=4$

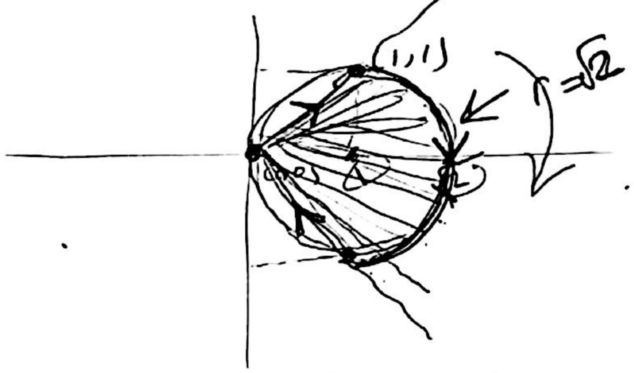
$3t+1=4 \Rightarrow \boxed{t=1}$

$$\vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t) = \left[\frac{3}{\sqrt{3}}+\frac{3}{\sqrt{3}}+\frac{3}{\sqrt{3}}\right] = \left[\sqrt{3}+\sqrt{3}+\sqrt{3}\right]$$

$= 3\sqrt{3}$

$$\int_0^1 \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\cdot dt = \int_0^1 3\sqrt{3}\cdot dt = \boxed{3\sqrt{3}}$$

Q3.(6 points) Evaluate $\oint_C (y^2 \cos x)dx + (x^2 + 2y\sin x)dy$, where C consists of the line segment from (0,0) to (1,1), the part of the circle $x^2 + y^2 = 2$ from (1,1) to (1,-1) and the line segment from (1,-1) to (0,0).



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curl ⇒
$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2\cos x & x^2+2y\sin x & 0 \end{vmatrix}$$

$$\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = (2x + \cancel{2y\cos(x)}) - (\cancel{2y\cos(x)})$$
$$= 2x$$

* by green's thorm *

$$\iint \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) \cdot dx\,dy = \int \vec{F} \cdot d\vec{r}$$

by supstituting
$x = r\cos\theta$
$dx\,dy = r\,dr\,d\theta$

$$\iint_R (2x) \cdot dx\,dy =$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int 2r^2\cos\theta * dr\,d\theta$$

negtive oreinted

$$\sin\theta \Big|_0^{\frac{\pi}{4}} * 2 * (2) * \left(\frac{r^3}{3}\right)\Big|_0^{\sqrt{2}}$$

$$\frac{\sqrt{3}}{\cancel{2}} * \cancel{2} * \frac{2}{3} * (\sqrt{2})^3 = \frac{2\sqrt{3}\,\sqrt{2}^3}{3} \Rightarrow -\frac{2\sqrt{3}\,\sqrt{2}^3}{3}$$

Q4. (6 points) Given $\int_{(0,0,0)}^{(1,-1,0)} (2x\cos z)dx + zdy + (y - x^2 \sin z)dz = \vec{F}$

a) Show that the given integral is independent of the path.

b) Evaluate the above integral.

if the integral is path independent, then:

$$\text{curl}(\vec{F}) = 0$$

where $\vec{F} = [2x\cos z, z, y - x^2 \sin z]$

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a)

$$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x\cos z & z & y - x^2 \sin z \end{vmatrix}$$

$$= \hat{i}\underbrace{(1-1)}_{\text{zero}} - \hat{j}\underbrace{(-2x\sin z + 2x\sin z)}_{\text{zero}} + \hat{k}\underbrace{(0-0)}_{0}$$

$$= \boxed{\vec{0}} \quad \therefore \#$$

b) by $\Rightarrow \vec{F} = \nabla f \iff \int \vec{F}\cdot dr = f(B) - f(A)$

$\frac{\partial f_1}{\partial x} = 2x\cos z \Rightarrow \boxed{f_1 = x^2 \cos z}$

$\frac{\partial f_2}{\partial y} = z \Longrightarrow \boxed{f_2 = zy}$

$\frac{\partial f_3}{\partial z} = y - x^2 \sin z \Rightarrow \boxed{f_3 = yz + x^2\cos z}$

$1 \geq x \geq 0$

$-1 \leq y \leq 0$

~~zero~~ $\boxed{z=0}$

$\therefore \boxed{f = x^2\cos z + zy + g}$

$\int \vec{F}\cdot dr = f(B) - f(A) = ((1)^2 * \cos(0) + (0)(-1) + \cancel{g}) - (0+0+0+\cancel{g})$

$= \boxed{1}$

Q5. (6 points)

Evaluate $\iint_S \vec{F}\cdot\hat{n}\,dA$, where $\vec{F}(x,y,z)=(x+y)\hat{i}+(z-y)\hat{j}+(x-z)\hat{k}$, and S is the portion of the plane $4x+2y+z=4$ in the first octant. Normal vectors point in the positive z-direction.

$\vec{r}(x,y) = (x, y, \boxed{4-4x-2y})$

$\vec{N} = grad(4x+2y+z-4) = [4,2,1]$

$\iint_R \vec{F}(\vec{r}(t))\cdot\vec{N}\,dA$

$\vec{F}(\vec{r}(t)) = [x+y, 4-4x-y, 4-5x-2y]$

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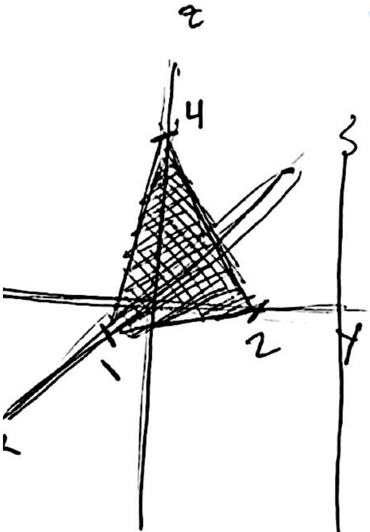
$\vec{N} = [4,2,1]$



$\vec{F}(\vec{r}(t))\cdot\vec{N} = [4(x+y)+8-8x-2y+4-5x-2y]$

$= 4x+\cancel{4y}+8-8x\cancel{-2y}+4-5x\cancel{-2y}$

$= x(4-8-5)+y(0)+12$

$= -10x+12$

$\iint \vec{F}(r(t))\cdot\vec{n}\,dA = \int_0^1\int_0^{(-2x+2)} -10x+12\,dy\,dx$

$= \int_0^1 (-10x+12)(-2x+2)\,dx$

$= \int_0^1 +20x^2-20x-24x+24$

$= \int_0^1 20x^2-48x+24\,dx$

$= 20\frac{x^3}{3}\Big|_0^1 - 48\left(\frac{x^2}{2}\right)\Big|_0^1 + 24(x)\Big|_0^1$

$= \frac{20}{3} - \cancel{24} + \cancel{24} = 0 \quad \boxed{\frac{20}{3}}$

$1\ge x\ge 0$
$2\ge y\ge 0$
$4\ge z\ge 0$

$mx+b$

$y = -2x+2$

$\frac{(2-0)}{(0-1)} = -2$