Strength

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1st semester 2019

Mech Family

كن أنت التغيير

Notebooks

* External Forces:-
  1) Affect of a Body in another Body.
  2) weight
  3) Reactions . " Newton's third Law"

* Internal Forces:-
  (inside the Body)

Pages (13 + 14) in Book.

Diff.

Forces ocures inside the material due to external Forces.

* types of internal forces:-

1. Normal forces. (axial Forces).
   perpendicular to the area
2. trasverse shear forces.
   (2 Forces)
   parallel to the area.
3. Bending Moments.
   (2 Moments) → الطعج (عزم الانحناء)
4. Torsional Moments.
   عزم العصر (اللي).

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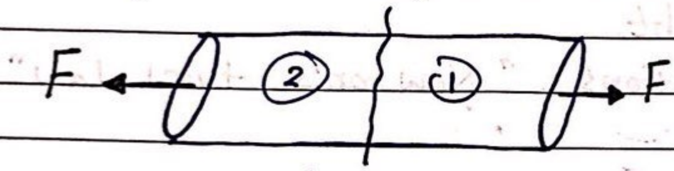
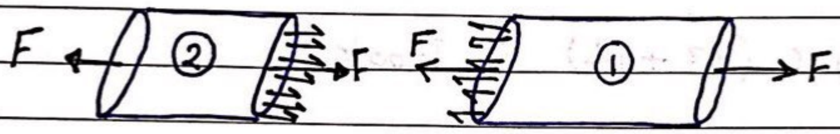
① Normal Forces :- ch2
(axial Force)



imagenary
cut



Perpendicular ~~two~~ the area.

Just one Force.

② Shear Force :: ch4 + ch5 + ch2
parallel to the area.

two Forces.

③ Bending Moment :- ch4 + ch5
parallel to the area
ثني الجسم

two Forces.

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Subject:

④ Torshing Moment :- Ch3

perpendicular to the area.

σ : Normal Stress.

τ : Shear Stress.

Sec 1.3 :: Normal Stress and Strain.

sigma $\sigma = \frac{F}{A}$; $(N/m^2 \equiv Pa)$

; F is always Normal to the area.

أنواعه :

١) سحب (استطالة)

2) ضغط (قِصر)

① compressive (−)

directed into the area.

② Tensile :- (+)

directed out of the area.

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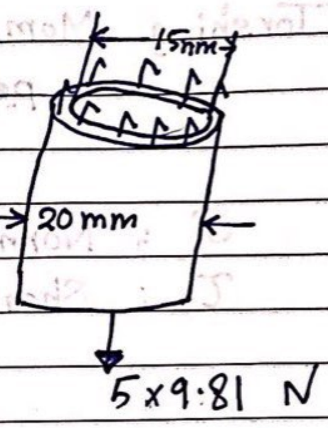
Subject: ______ / /

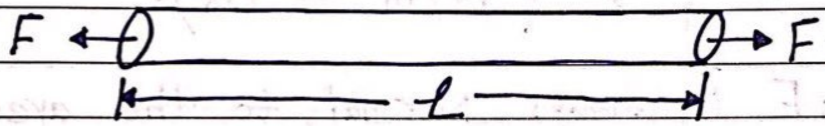
Ex ::

$$\sigma = \frac{49.05}{\frac{\pi}{4}(0.02^2 - 0.015^2)}$$

$$= 0.356871 \text{ MPa}$$



Strain ::



$$\varepsilon = \frac{\delta_\ell}{\ell} \longrightarrow \text{longitudinal Strain}$$

$$\underset{\text{tensile}}{\sigma > 0} \iff \underset{\text{positive Strain}}{\varepsilon > 0} \longrightarrow \delta_\ell > 0$$

$$\varepsilon_{\text{lotiral}} = \frac{\delta_d}{d}$$

Negative when $\varepsilon > 0$.

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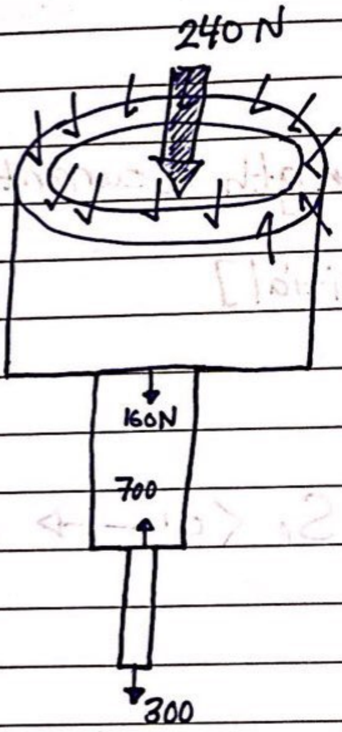
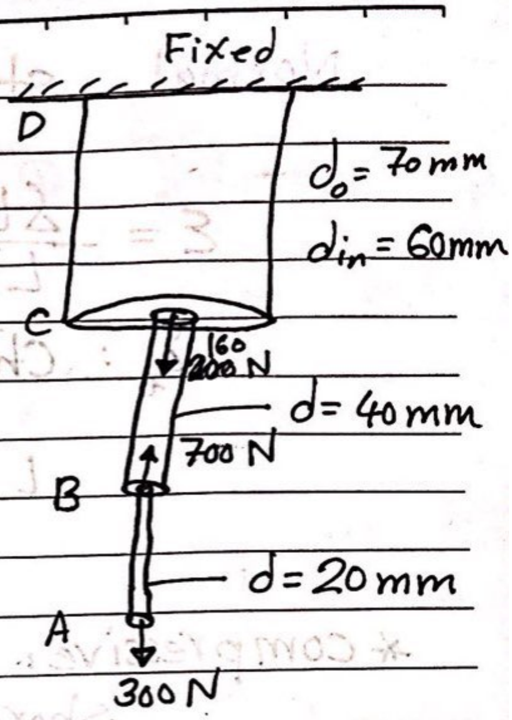
Subject:

Example:

Find σ_{AB} , σ_{BC} , σ_{CD}





$$\sigma_{AB} = \frac{300}{\frac{\pi}{4}(0.02)^2} =$$

$$\sigma_{BC} = \frac{-400}{\frac{\pi}{4}(0.04)^2} =$$

$$\sigma_{CD} = \frac{-240}{\frac{\pi}{4}(0.07^2 - 0.06^2)} =$$

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Subject: 19 / 9 / 2018

Normal strain :- ε (epsilon)

$$\varepsilon = \frac{\delta L}{L}$$

δ_L : Change in length [current - initial]

L = length [initial]

* compressive :-

shortining $\rightarrow$ $\delta_L < 0$ $\rightarrow$ $\varepsilon < 0$

* Tensile :-

elongation $\rightarrow$ $\delta_e > 0$ $\rightarrow$ $\varepsilon > 0$

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Subject: Sec 1.8 24 / 9 / 2018

* Shear Stress and Strain :-

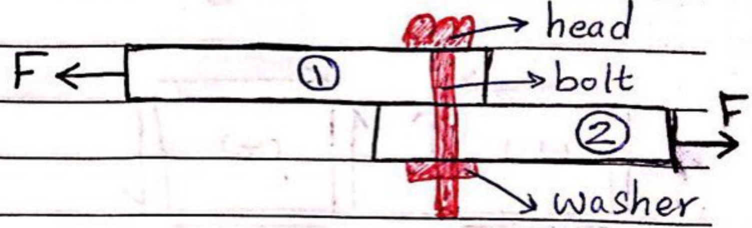
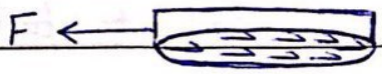


fig 1-1

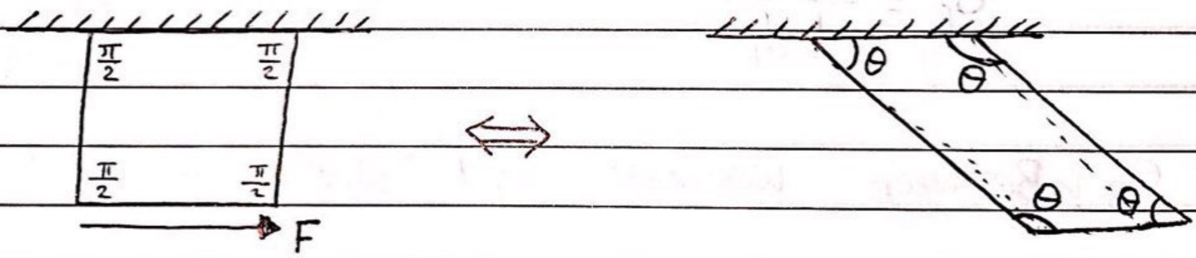
①



$\tau = \frac{F}{A}$; where $(F \parallel A)$

②

$\tau = \frac{F}{A}$; where $(F \parallel A)$



مع العلم أن الزوايا
غير متساوية .

* $\gamma = \frac{\pi}{2} - \theta$

* Hook's law :-

$\tau = G\gamma$

$\hookrightarrow$ Shear of elasticity.

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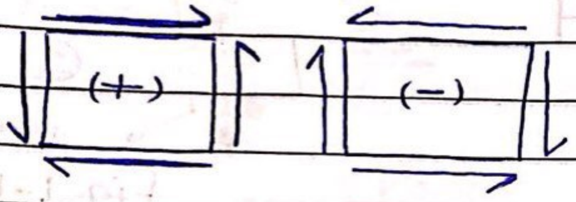
* Shear sign :



نسمي القانون
head to head
tail to tail

* Bearing Stress : (σ_B)
(contact stress)

σ_B : is a Normal Stress.

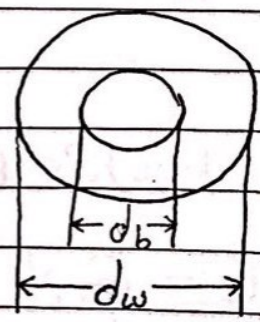
From the last figure (fig 1-1) :

* σ_B Between plate ① and the bolt ?

$$\sigma_B = \frac{F}{t\, d_b}$$

* σ_B Between washer and the plate ② ?

the washer :



$$\sigma_B = \frac{F}{\frac{\pi}{4}\left(d_w^2 - d_b^2\right)}$$

لا تضرب بـ t لأن المساحة
دائرة مفرغة .

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Subject: 26 / 9 / 2018

* Factor of Softy:-

$$F.S. = \frac{\sigma_{yeild}}{\sigma_{allowable}} , \quad F.S. = \frac{\tau_{yeild}}{\tau_{allowable}}$$

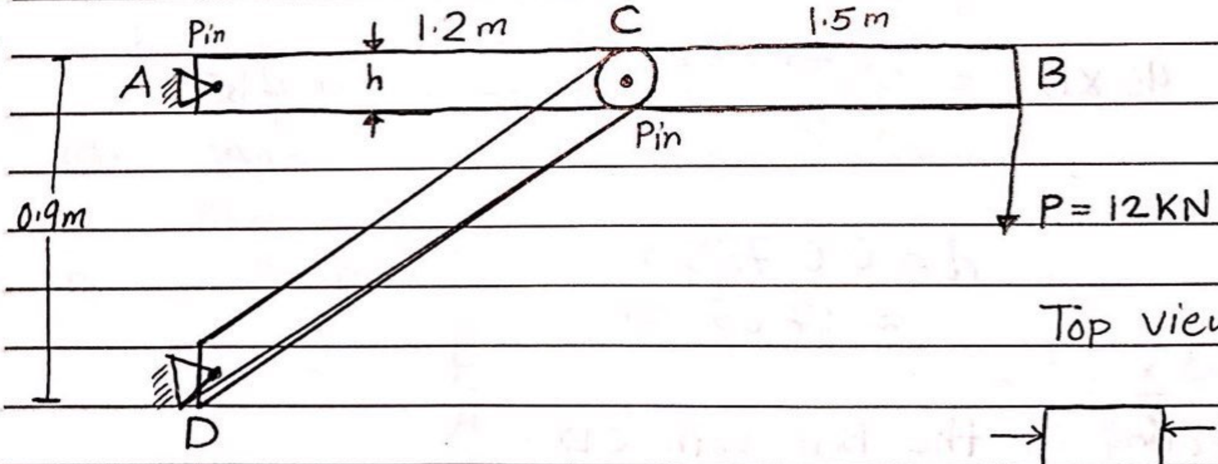
Ex:- $\tau_{allowable} = 40\ MPa$, $F = 1000\ N$

Find the Area:-

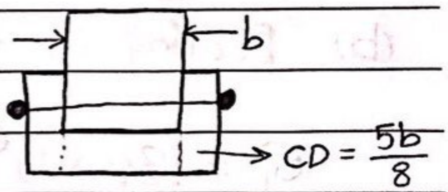
$$Area = \frac{F}{\tau_{allowable}} = \frac{1000}{40 \times 10^6}$$

$$A = 2.5 \times 10^{-5}\ m.$$

Ex (1.10.3) :-



Top view:-



$h = 200\ mm$ $\quad \tau_{allow} = 90\ MPa$

$b = 19\ mm$

$P = 12\ KN$

(a) Find the minimum diameter Required of the bolt?

(b) σ_b (allowable) $= 130\ MPa$ (in the bolt)

what is the minimum diameter?

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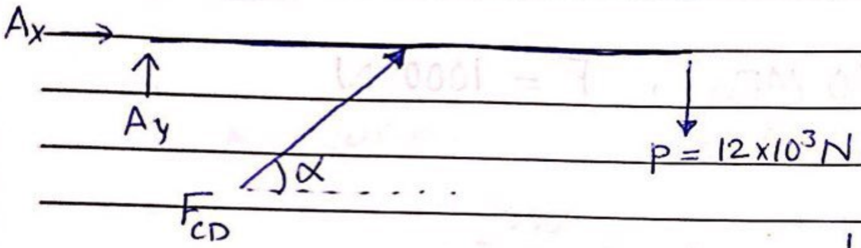
(a)

$$90 \times 10^6 = \frac{F_{CD}}{2\ \text{Area}} = \frac{F_{CD}}{2\ \frac{\pi}{4}(d)^2}$$

F. B. D. :



$$\tan\alpha = \frac{0.9}{1.2} \Rightarrow \alpha = 36.8^\circ$$

$$\Sigma M_A = 0$$

$$(F_{CD}\sin\alpha)(1.2) - 12\times10^3(2.7) = 0$$

$$\rightarrow F_{CD} = 45.073\times10^3\ N$$

$$90\times10^6 = \frac{45.073\times10^3}{\frac{\pi}{2}\ d^2}$$ ..... حل المعادلة جبراً

$$d = 0.01785\ m$$
$$\approx 17.85\ mm$$

(b) Bearing in the Bolt with CD :

$$\sigma_b = 130\times10^6 = \frac{F_{CD}}{2(d)(\frac{5B}{8})} = \frac{45.073\times10^3}{2(d)(\frac{5\times0.019}{8})}$$

$$\longrightarrow d = 0.0146\ m$$

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Subject:

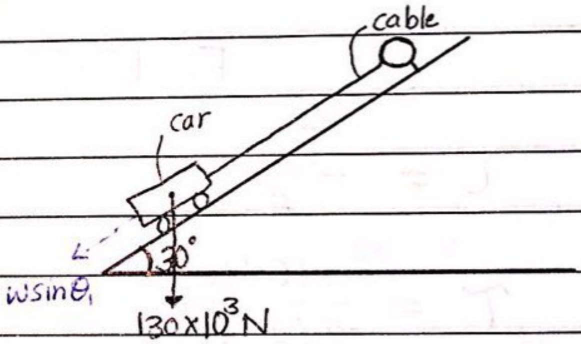
Bearing in the Bolt with AB :

$$\sigma_b = 130\times10^6 = \frac{F_{CD}}{d\,(b)} = \frac{45.073\times10^3}{d\,(0.019)}$$

$\therefore\; d = 0.0182\,m$

في الأقطار دائماً ما يكون أكبر قطر ناتج هو
القطر الأنسب لنا والأكثر تحملاً.
فهنا نختار القطر
$d = 0.0182\,m$
أو قطراً أكبر.

Ex (1.4.8) :-



has affective Area is $490\,mm^2$

(a) Calculate the tensile Stress. (Normal stress)

(b) $\sigma_{allow} = 150\,MPa$ Find θ_{max}

Sol

(a) $$\sigma = \frac{F}{A} = \frac{W\sin\theta_1}{A} = \frac{130\times10^3\times\sin 30}{490\times10^{-6}}$$

$$\sigma = 132.653\,MPa$$

(b)
$$150\times10^6 = \frac{130\times10^3\,\sin\theta}{490\times10^{-6}}$$

$$\theta_{max} = 34.3^\circ$$

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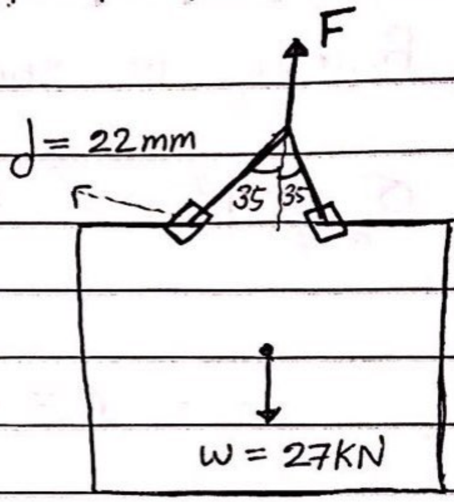
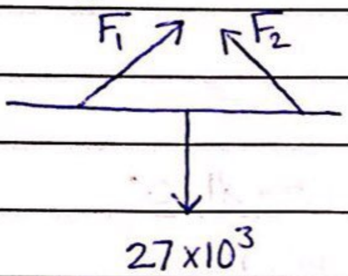
Ex (R.1.6):



Find $\tau_{pin} = ??$

F.B.D.



$\Sigma f_x = 0 \longrightarrow F_{1x} = F_{2x}$

$\Sigma f_y = 0$

$2F\cos 35 = W \longrightarrow$ By Simmetry

$F = 16\,480$ N

$$\tau = \frac{F}{2A} \longrightarrow \tau = \frac{2F}{4A} \longleftarrow \text{الأصل}$$

$$\tau = \frac{164800}{2\left(\frac{\pi}{4}\right)(0.022)^2}$$

$$\tau = 21.67 \text{ MPa}$$

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Subject:

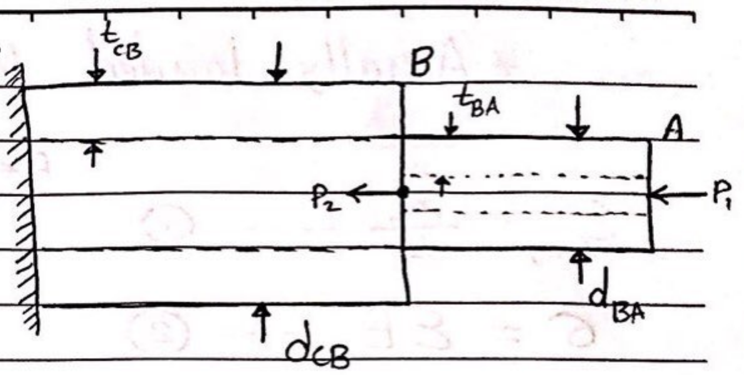
Ex:-

Given

$P_1 = 120\ kN$ $t_{BA} = 12mm$

$P_2 = 100\ kN$ $d_{CB} = 70mm$

$d_{BA} = 38\ mm$ $t_{AB} = 10mm$

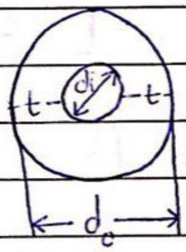


$E_{CA} = 110\ GPa$

(A) The wall thickness t_{CB} increases By 0.0036mm

Find D?

$\sigma = \varepsilon E$



$d_o - d_i = 2t$

$$D = \frac{-\varepsilon_{lat}}{\varepsilon_{long}}$$

Hook's Law:-

$$\varepsilon_{long} = \frac{\sigma}{E} = \frac{\left(\frac{-220 \times 10^3}{\frac{\pi}{4}(0.03^2 - 0.05^2)}\right)}{110 \times 10^9} = -1.06 \times 10^{-3}$$

Negative Because it's compressive.

$$\varepsilon_{late} = \frac{\delta_d}{d} = \frac{0.0038}{10} = 0.000368$$

$$D = \frac{-\varepsilon_{late}}{\varepsilon_{long}} = 0.34$$

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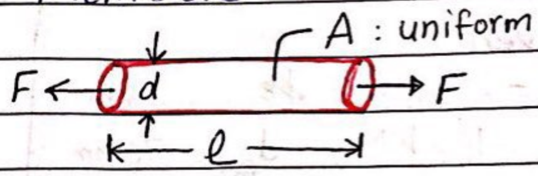
Subject: Chapter 2 1 / 10 / 2018

* Axially loaded Members :-



$$\varepsilon_{long} = \frac{\delta_L}{L} \quad \text{------} \; (1)$$

$$\sigma = \varepsilon E \quad \text{-----} \; (2)$$

$$\sigma = \frac{F}{A} \quad \text{------} \; (3)$$

$$\delta_L = L\,\varepsilon_{long} = L\,\frac{\sigma}{E} = \frac{LF}{EA}$$

* is used in the elastic region.

$$* \; \delta = \frac{FL}{EA} \longrightarrow \text{الإستطالة}$$

* $\sigma > 0 \longrightarrow \varepsilon > 0 \longrightarrow \delta > 0 \longrightarrow$ Tension.

* $\sigma < 0 \longrightarrow \varepsilon < 0 \longrightarrow \delta < 0 \longrightarrow$ compression.

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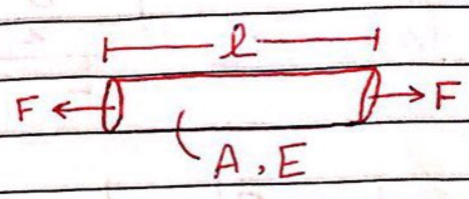
Laws :-



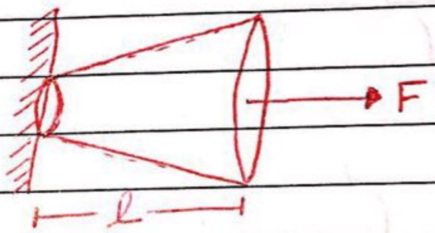
① $\delta = \dfrac{FL}{EA}$

لو كانت لولبية

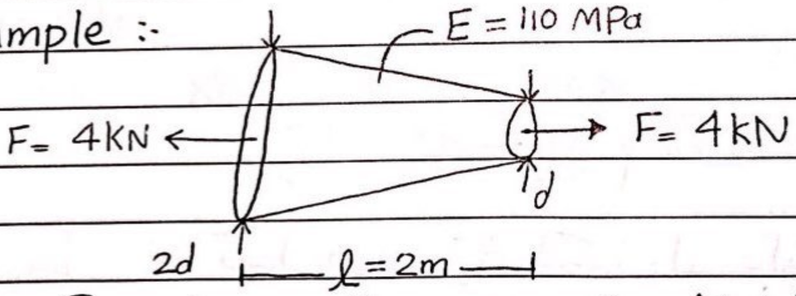
② $\delta_{tot} = \sum_{i=1}^{N} \dfrac{F_i L_i}{E_i A_i}$

③

$$\delta = \int_0^L \frac{F(x)}{E\,A(x)}\,dx$$



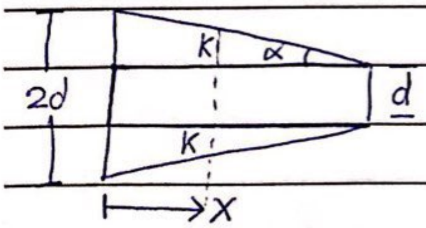
Example :-



Find δ where the area is $A(x)$?

Sol :-

side view



; $0 \leq x \leq L$

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$$\tan\alpha = \frac{d-\frac{d}{2}}{L} = \frac{K}{L-x} \quad \cdots\cdots\cdots (1)$$

$$d_x = 2K + d = 2\left[\frac{d}{2L}(L-x)\right] + d$$

$$d_x = \frac{d}{L}(L-x) + d \quad \cdots\cdots\cdots (2)$$

$x=0$ $\quad$ $x=L$ $\quad$ $A(x) = \frac{\pi}{4} d_x^{\,2}$

$d_x = 2d$ $\quad$ $d_x = d$

$$= \frac{\pi}{4}\left[\frac{d}{L}(L-x)+d\right]$$

$$\delta = \int_0^L \frac{F(x)}{E \cdot A(x)}\, dx$$

$$= \int_0^L \frac{4\times10^3}{110\times10^9\left(\frac{\pi}{4}\left(\frac{d}{L}(L-x)+d\right)^2\right)}\, dx$$

$$= \int_0^L \frac{4\times10^3}{110\times10^9\left(\frac{\pi}{4}\left(\left(d-\frac{dx}{L}\right)+d\right)^2\right)}\, dx$$

تحل بطريقة التكامل بالتعويض .

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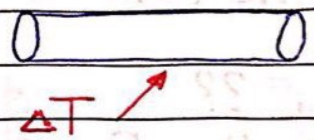
Subject: 2.5 Thermal Effect 8 / 10 / 2018

$\varepsilon = \alpha(\Delta T)$



; $\Delta T = T_2 - T_1$

if

① $T_2 > T_1 \longleftrightarrow \varepsilon > 0$ elongation

② $T_2 < T_1 \longleftrightarrow \varepsilon < 0$ Shortening

α : is called the coefficient of Thermal Expansion.

it's unit : $1/C°$

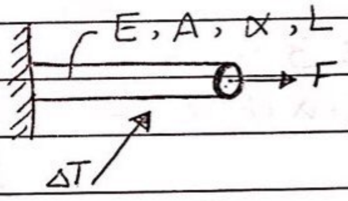
The Stress :-

$$\sigma = \varepsilon E = \alpha(\Delta T)E$$

$$\delta = \alpha(\Delta T)L$$

Example :-

Find $\delta_{tot} = ??$



$$\delta_{total} = \delta_T + \delta_F$$

$$= \alpha(\Delta T)L + \frac{FL}{EA}$$

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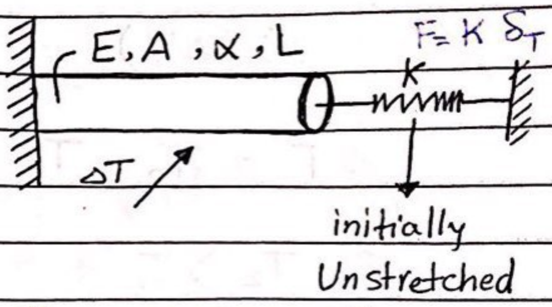
Example (2) :-



① $\delta_{total} = ??$

② What's the Reaction at the Support ?

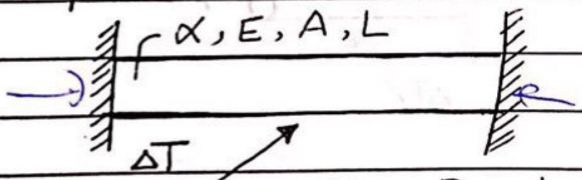
* the Reaction is only the spring Force.

~~$F_s = K(\delta_T)$~~

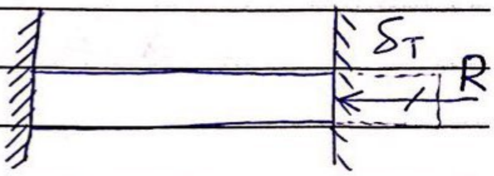
$$\delta_{total} = \delta_T - \delta_s$$

$$= \alpha(\Delta T)L - \frac{(K\delta_s)L}{EA}$$

Example 3 :-



what is the Reaction?



$$\delta_R = \delta_T$$

$$\frac{RL}{EA} = \alpha(\Delta T)L$$

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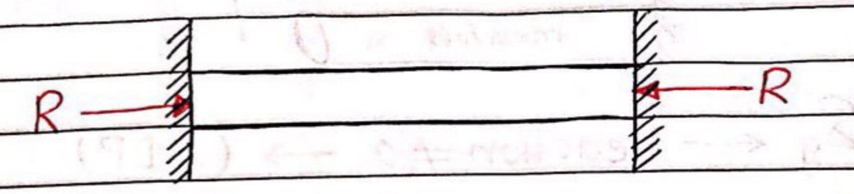
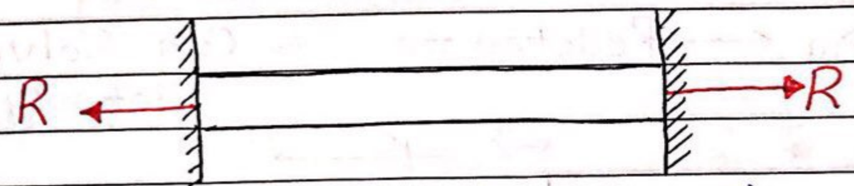
Subject:

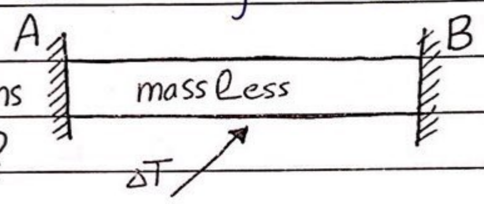




* (تلغي ال Reactions بعضها)

Sec 2.4:- Statically Indeterminat form:-

Find the Reactions of A and B?



In Statics:-

$R_A + R_B = 0 \quad \cdots\cdots (1)$

since we have two variables with one eq.
then the problem is statically Indeterminat.

In Strength:-

Combatibility equation

$$\delta_{tot} = \delta_{AB} = 0$$

$$\delta_T + \delta_{R_B} = 0 \quad \cdots\cdots (2)$$

$$R_B = \frac{-\alpha(\Delta T) L E A}{L}$$

Negative $\longleftrightarrow$ wrong direction.

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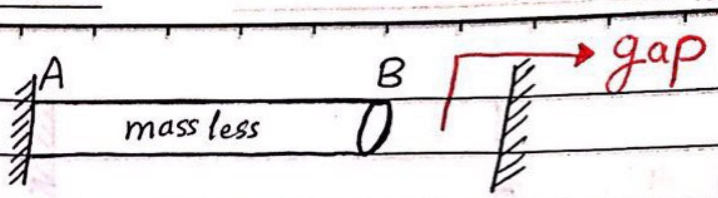
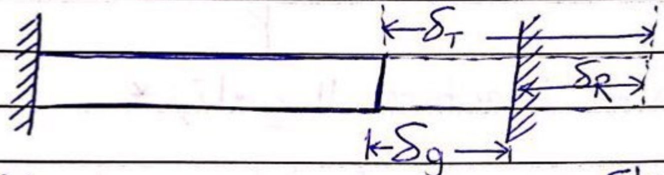
Subject:

Example 2 :-



$\delta_T > \delta_g \longleftarrow \text{Reaction} \neq 0 \longrightarrow$ (SIP)

$\delta_T \ll \delta_g \longleftarrow \text{Reaction} = 0 \longrightarrow$ Can Solve Statically.



Statics :-

$R_A = R_B$ ------ (1)

Strength :-

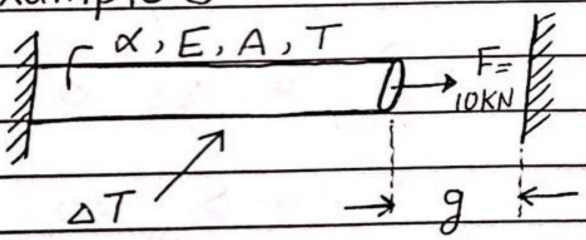
Combatibility Equation.

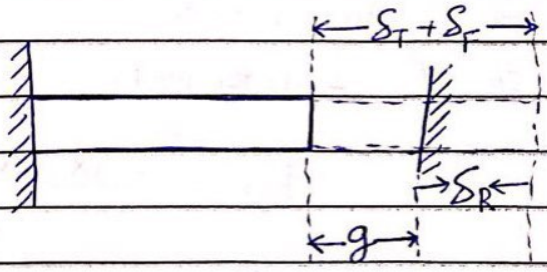
$\delta_T = \delta_R + \delta_g$

$\alpha (\Delta T) L = \dfrac{R_B L}{EA} + \delta_g$

$R_B =$ -----

Example 3 :-





Combatibilit Equation :-

$\delta_T + \delta_F = \delta_R + g$

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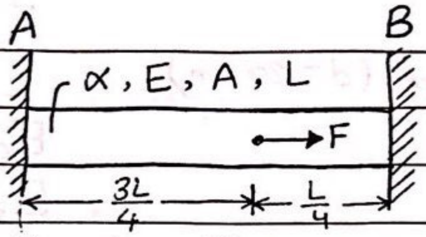
Subject: ______ / /

Example 4 :-



F = 12 KN

Find the Reaction ?

Statics :-

$R_A + 12\times10^3 = R_B$ ----- ①

$R_A = 9000 - 12000$

$\boxed{R_A = -3000\ N}$

Comb. Eq. :-

$\delta_F + \delta_{R_B} = 0$

$$\frac{F\left(\frac{3}{4}\cancel{L}\right)}{\cancel{EA}} = \frac{R_B\,\cancel{(L)}}{\cancel{EA}}$$

$\frac{3}{4}F = R_B$

$\boxed{R_B = 9000\ N}$

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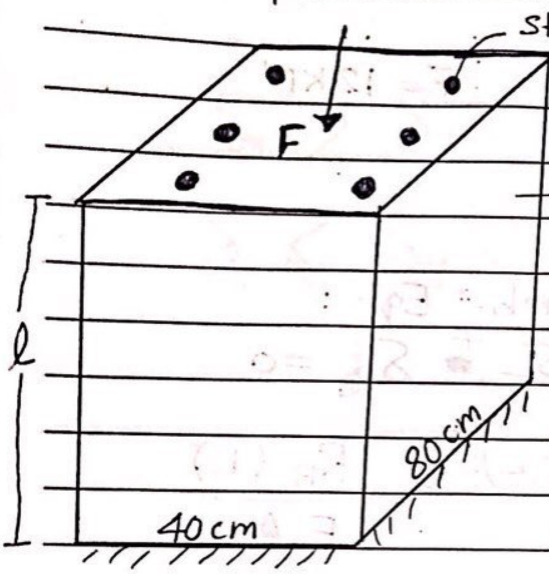
Example. (1) :-

$F = 300\ kN$

$\ell = 3m$

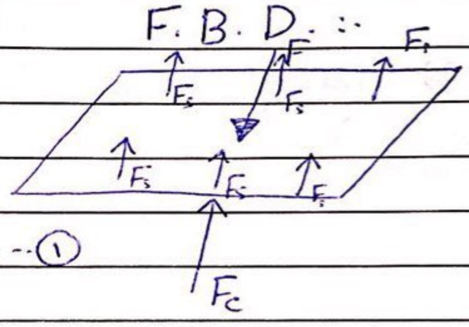
$E_c = 30\ GPa$

$E_s = 200\ GPa$



Find σ in concrete and in Steel ?

F. B. D. :-



① Statics :-

$$6F_s + F_c = 300 \times 10^3 \quad \text{---- ①}$$

② Strength :-

$$\delta_s = \delta_c$$

$$\frac{F_s L}{E_s A_s} = \frac{F_c L}{E_c A_c} \quad \text{------ ②}$$

$$A_c = (0.4)(0.8) - A_s$$

$$A_s = 6\frac{\pi}{4}(0.02)^2$$

$$\frac{F_c}{30 \times 10^9 (A_c)} = \frac{F_s}{200 \times 10^9 (A_s)}$$

$$\sigma_s = \frac{F_s}{A_s}$$

$$\sigma_c = \frac{F_c}{A_c}$$

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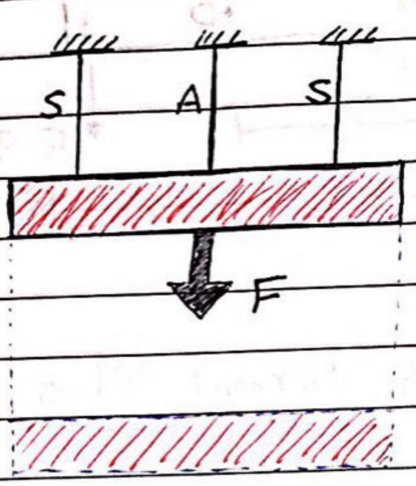
Subject:

Example (2) :

All is Rigid

(a)

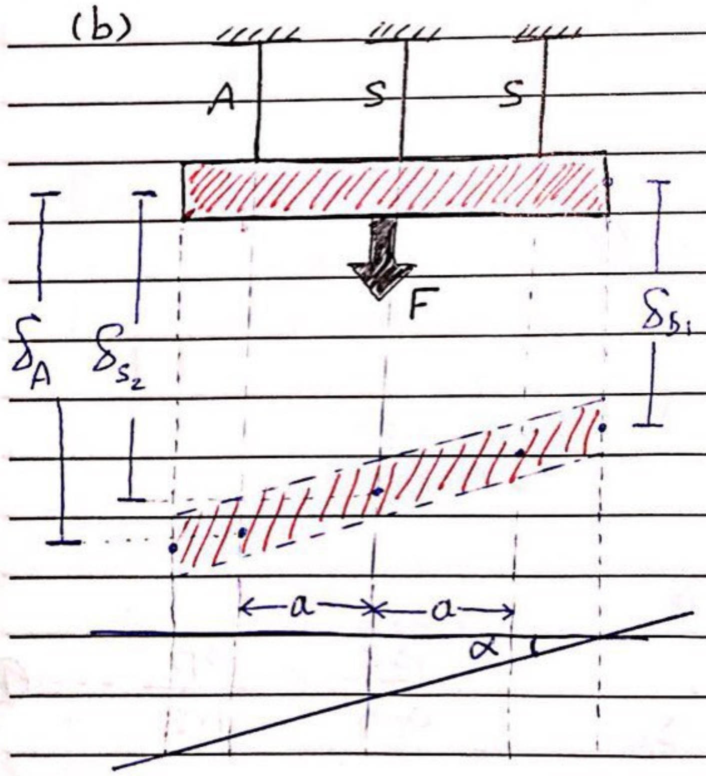


الجسم في حالة تماثل

$$F = 2F_s + F_A \text{ ------- } ①$$

$$\delta_s = \delta_A \text{ ------- } ②$$

(b)



$$F_A + F_{s_1} + F_{s_2} = F \text{ ------- } ①$$

$$\tan\alpha = \frac{\delta_A - \delta_{s_1}}{2a}$$

$$\delta_A + \delta_{s_1} = 2\delta_{s_2} \text{ ------ } ②$$

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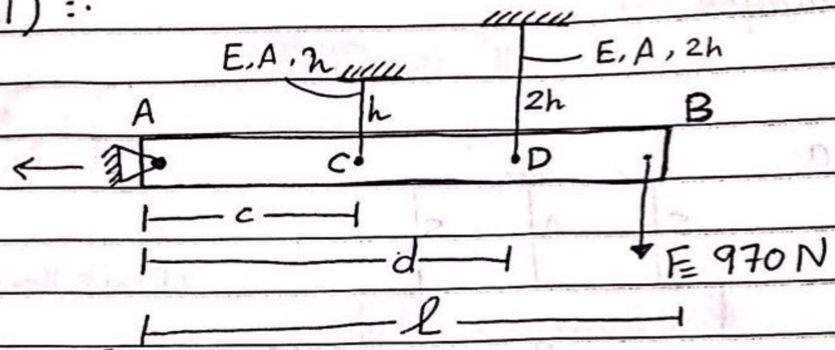
Subject: ______

Ex(2.4.21):

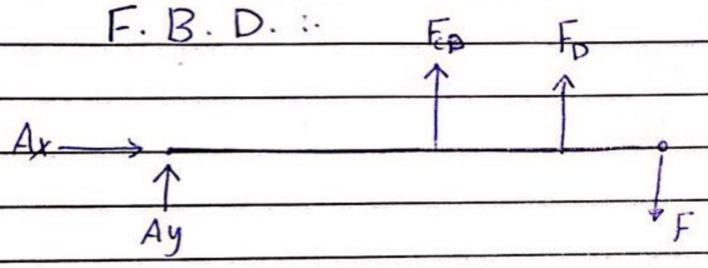
double shear connection
$d = 30\ mm$



$\ell = 1.6\,m$ $\quad A = 16\ mm^2$
$d = 1.2\,m$ $\quad E = 200\ GPa$
$c = 0.5\,m$
$h = 0.4\,m$

Find the Normal Stress in the wires?

F.B.D.:



Statics:

$\sum f_x = 0 \longrightarrow A_x = 0$ ---- (1)

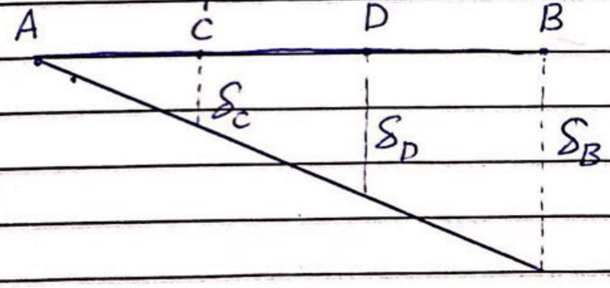
$\sum f_y = 0$

$A_y + F_C + F_D = F = 970$ ---- (2)

$\sum M_A = 0$

$-970(1.6) + F_D(1.2) + F_C(0.5) = 0$ ---- (3)

* Comb. Equation:



$$\frac{\delta_C}{0.5} = \frac{\delta_D}{1.2} \text{ ---- (4)}$$

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$$(2.4)\ \frac{F_C L_C}{\cancel{E_C}\cancel{A_C}} = \frac{F_D L_D}{\cancel{E_D}\cancel{A_D}}$$

$$2.4\,(F_C)(0.4) = F_D(0.8)$$

$$F_D = 1.2\,F_C \text{ ------ } (4)$$

$$\left.\begin{aligned} F_C &= 800\ N \\ F_D &= 960\ N \end{aligned}\right\} \text{Reactions.}$$

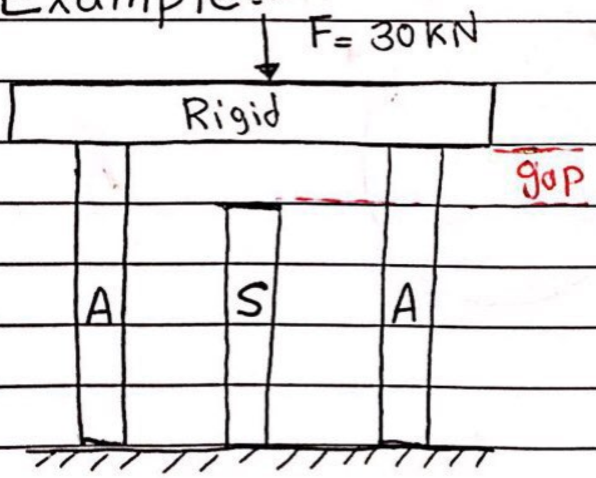
$$\sigma_C = \frac{F_C}{A_C} = \frac{800}{16\times10^{-6}} = 50\ MPa$$

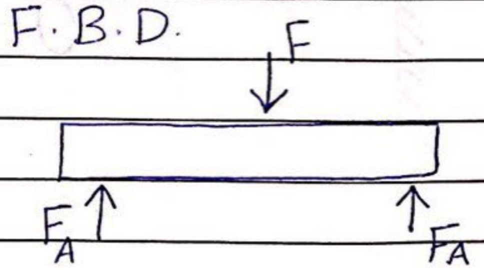
$$\sigma_D = \frac{F_D}{A_D} = \frac{960}{16\times10^{-6}} = 60\ MPa$$

$$\tau_{pin} = \frac{790}{2\left(\frac{\pi}{4}\right)(0.03)^2} = 558.8\ kPa \quad ; \ A_y = 970$$

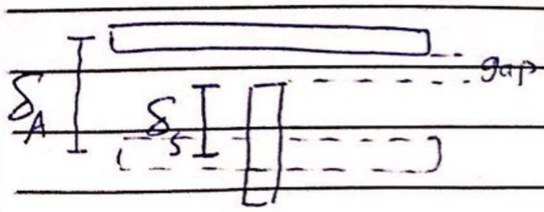
وهي المحصلة.

Example:-



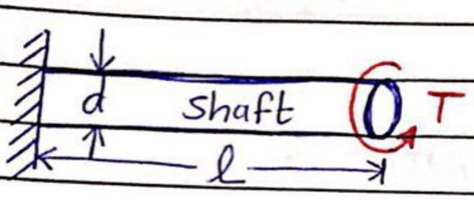


Statics $\quad F = F_s + 2F_A$ ---- (1)



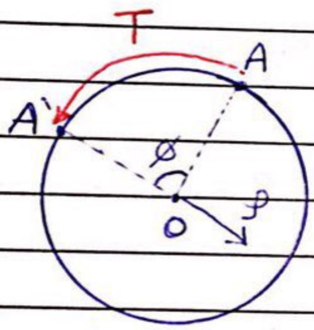
$$\delta_s + gap = \delta_A \text{ ------ } (2)$$

Subject: Chapter 3 :- Torsion 15 / 10 / 2018



* Shear Modulus of elasticity:-

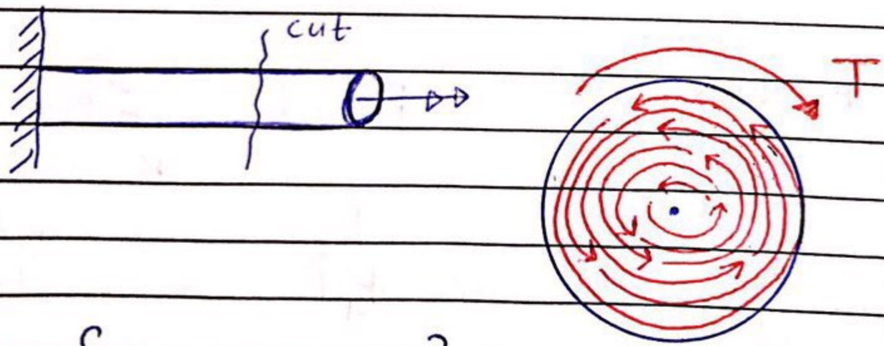
$$G = \frac{E}{2(1+\nu)}$$, T ⟹ torque ⊥ Area.



Ø ≡ Angle of twist.

⟹ $AA' = r\varnothing$

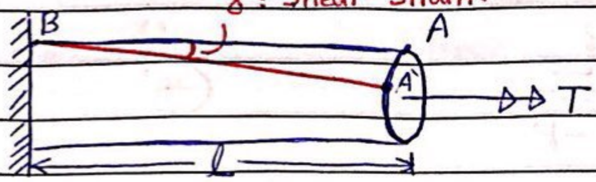
* if we have a cut:-



$\{0 \leq \rho \leq r\}$

Subject: ______________ / /

γ : Shear Strain.



$$AA` = L\gamma$$

$$\checkmark\ r\phi = L\gamma \quad \text{------} \quad (1)$$

from CH 1 :

Hook's Law

$$\checkmark\ \tau = G\gamma \quad \text{------} \quad (2)$$

ملاحظة:

نقطة الأصل (o) لا تتأثر أبداً أو تتحرك.

$$\gamma_{max} = \frac{r\phi}{L} \qquad \tau = G\gamma$$

$$\gamma = \frac{\rho\phi}{L} \qquad \tau_{max} = G\gamma_{max}$$

$$* \quad \rho\phi = L\left(\frac{\tau}{G}\right)$$

$$\checkmark\ \phi = \frac{\tau L}{G\rho} \quad \text{------} \quad (3)$$

$$* \quad dT = \rho\,\tau\,dA \rightarrow \tau = \frac{\phi\rho G}{L}$$

$$dT = \frac{\phi\rho^2 G}{L}\,dA \rightarrow \text{Polar coordinate} \quad \rho\,d\rho\,d\theta$$

$$T = \frac{\phi G}{L}\int_0^{2\pi}\int_0^{r} \rho^3\,d\rho\,d\theta \rightarrow$$

تكامل خاص بالدائرة فقط.

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$$\phi = \frac{TL}{G I_p} \quad (*)$$

(للدائرة) $$I_p = \int_0^{2\pi}\int_0^{r} \rho^3 \, d\rho \, d\theta = \int_0^{2\pi} \left[\frac{\rho^4}{4}\right]_0^r d\rho$$

$$= \frac{r^4}{4}(2\pi) = \frac{\pi r^4}{2}$$

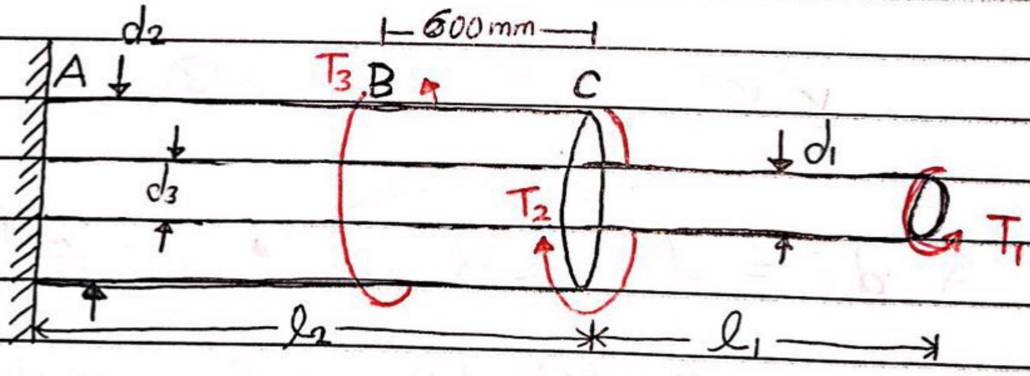
$$= \frac{\pi}{32} d^4$$

* τ

$$\tau = \frac{\phi \rho G}{L} = \frac{T \cancel{L}}{\cancel{G} I_p} \cdot \frac{\rho \cancel{G}}{\cancel{L}}$$

$$\tau = \frac{T\rho}{I_p} \quad (*)$$

Ex :-



$l_1 = 800\,mm$ $\quad T_1 = 300\,N\cdot m$

$l_2 = 1200\,mm$ $\quad T_2 = 300\,N\cdot m$

$G = 70\,GPa$ $\quad T_3 = 900\,N\cdot m$

$d_1 = 20\,mm$

$d_2 = 60\,mm$

$d_3 = 40\,mm$

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Subject: 17 / 10 / 2018

① Find the total Angle of twist?

② Find the max Shear Stress in AB, BC and DC?

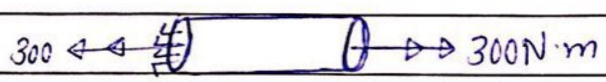
③ Find the minimum Shear Stress at the inner surface of AB and BC?

Sol:

①

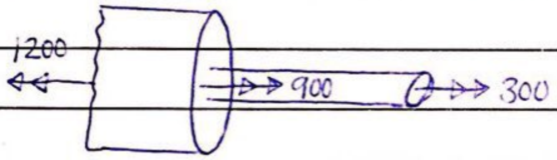
$$\phi_{tot} = \sum_{n=1}^{3} \frac{T_i L_i}{G_i I_{pi}}$$

* First cut:



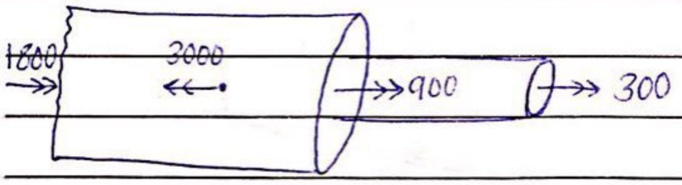
$\phi > 0$, $T > 0$

* Second cut:



$\phi > 0$, $T > 0$

* Third cut:



$\phi < 0$, $T < 0$

$$\phi = \frac{300\ (0.9)}{70\times10^9 \left(\frac{\pi}{32}\right)(0.03)^4} + \frac{1200\ (0.7)}{70\times10^9 \left(\frac{\pi}{32}\right)(0.07^4 - 0.05^4)} - \frac{1800\ (0.6)}{70\times10^9 \left(\frac{\pi}{32}\right)(0.07^4 - 0.06}$$

$\phi = \ ....$ rad

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Subject: ______ / /

(2)

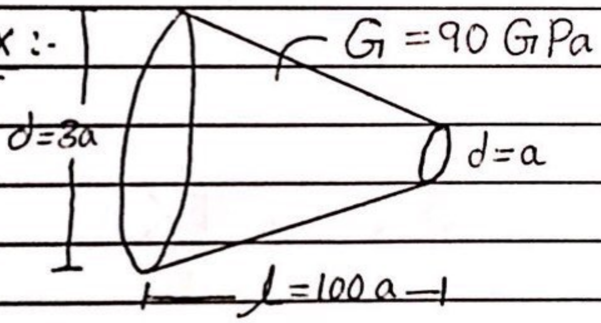
$$\tau_{CD}\Big|_{max} = \frac{300\,(0.015)}{\frac{\pi}{32}(0.03)^4} = 56.59\ MPa$$

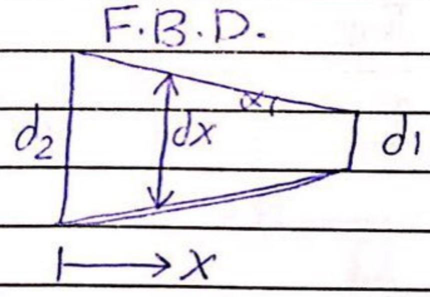
$$\tau_{BC}\Big|_{max} = \frac{1200\,(0.035)}{\frac{\pi}{32}(0.07^4 - 0.05^4)} = 24.08\ MPa$$

$$\tau_{AB}\Big|_{max} = \frac{1800\,(0.035)}{\frac{\pi}{32}(0.07^4 - 0.05^4)} = 36.1\ MPa$$

The Max Shear Stress is on CD.

Ex :-





$$d_x = \frac{d_2 - d_1}{L}(L - x) + d_2$$

$$\phi = \int_0^L \frac{T}{G\,I_p}\,dx = \int_0^L \frac{3\times10^3}{(90\times10^9)\frac{\pi}{32}(d^4)}\,dx$$

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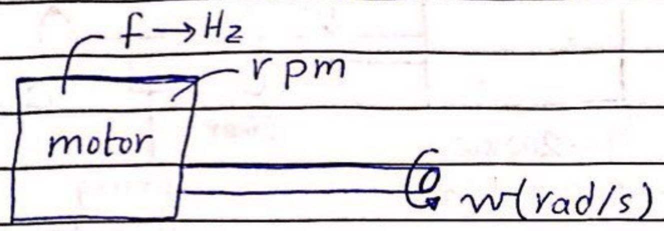
Subject: ______________ / /

Sec 3.7: transmisson of power by Circular Shaft :



*** Power :**

$$P = T \; w$$

torque ← T ; w → angular velocity (rad/s) → $f \rightarrow Hz$; rpm

$$w = rpm \left(\frac{2\pi}{rev}\right)\left(\frac{min}{60\,s}\right) \Rightarrow rpm\left[\frac{2\pi}{60}\right] \rightarrow rad/s$$

$$w = f\,(2\pi) = rad/s$$

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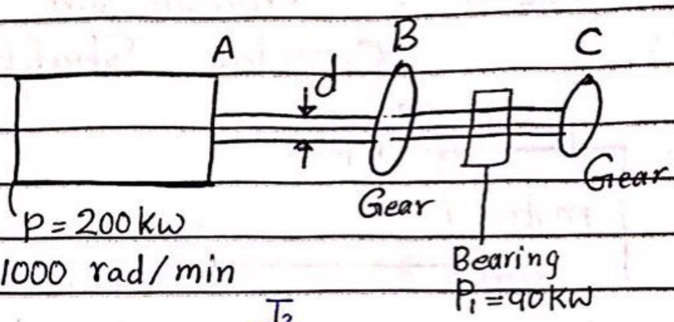
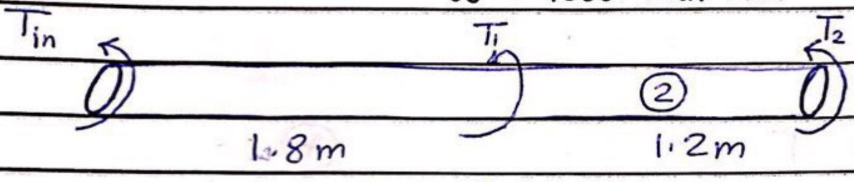
Ex (3.7.9) :-

Find d_{min} if $\tau_{allowable}$ is 50 Mpa and $\phi_{AC} = 1.5°$



$\omega = 1000$ rad/min



$P = P_1 + P_2$

$T_{in} = T_1 + T_2$

$$\omega = 1000 * \frac{2\pi}{60} (rad/s) = 104.7 \text{ rad/s}$$

$$T_{input} = \frac{200 \times 10^3}{\omega} = \frac{200 \times 10^3}{104.7} = 1910.22 \text{ N.m}$$

max Shaft.

$$T_1 = \frac{90 \times 10^3}{\omega} = 859.8 N$$

$$T_2 = \frac{110 \times 10^3}{\omega} = 1050.6 \rightsquigarrow \text{min Shaft}$$

$$\tau_{allow} = 50 \times 10^6 = \frac{T_{input} \ (d/2)}{\frac{\pi}{32} d^4} = \frac{1910.22 * 16}{\pi d^3}$$

$d = 0.013$m.

First cut :.

$T_{in} \leftarrow\!\!\leftarrow$ [①] $\rightarrow\!\!\rightarrow T_{input}$

موجب ϕ

Second cut

$T_2 \leftarrow$ [ ] $\rightarrow T_2$

T_{input} , T_2 → are internal torque and T_1 is the difference bt. T_{input} and T_2

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Subject:

$$\phi_{AC} = \phi_{AB} + \phi_{BC}$$

$$(1.5)\left(\frac{\pi}{180}\right) = \frac{1910.22\,(1.8)}{80\times10^{9}\ I_p} + \frac{(1050)\ (1.2)}{80\times10^{9}\ I_p}$$

$$\longrightarrow I_p = 2.24\times10^{-6}$$

$$I_p = \frac{\pi}{32} d^4 \dashrightarrow d = 0.069\ \text{m}$$

$$0.069 > 0.0139$$

we select the bigger d always.

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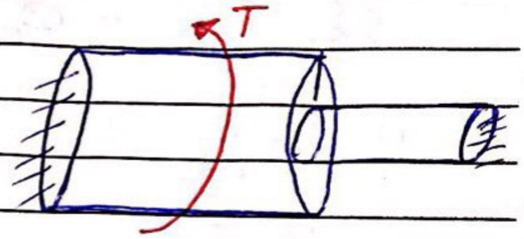
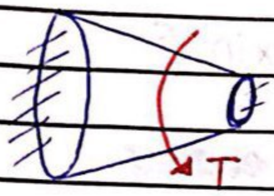
Subject: Sec 3.8 24 / 10 / 2018

* Statically Indeterminate Torsional Members :-

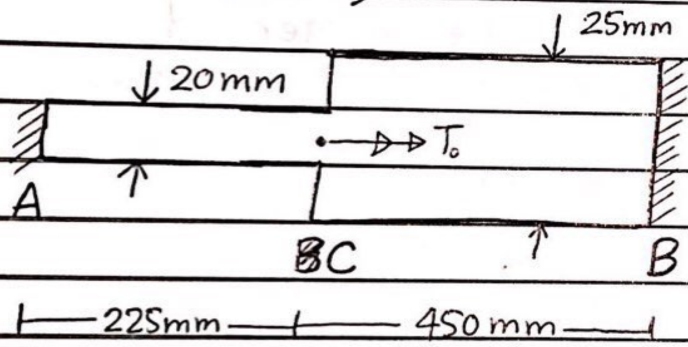




Combatibility Equation :.

$$\phi_{tot} = 0$$

Ex (3.8.6)



(a) if $\tau_{allow} = 43\ MPa$ Find $T_{o(max)} = ??$

(b) if $\phi_{max} = 1.85$ and $G = 85\ GPa$ Find $P_{o(max)}$?

(c) For (a) and (b) Find $P_{o(max)}$?

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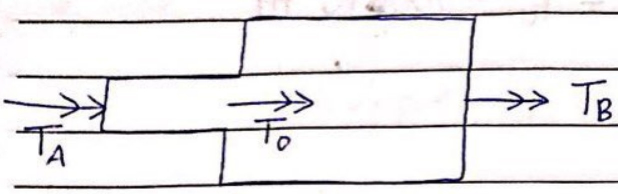
Subject: ____________ / /

Sol:- F.B.D.



Statics:

$T_A + T_B + T_o = 0 \quad \cdots\cdots (1)$

Stregth:

$\phi_{tot} = \phi_1 + \phi_2$

(a)

$$\phi_{tot} = \frac{-T_A\,(0.225)}{G\,\frac{\pi}{32}(0.02)^4} + \frac{T_B\,(0.45)}{G\,\frac{\pi}{32}(0.025)^4} = 0$$

$$\frac{2\,T_B}{(0.025)^4} = \frac{T_A}{(0.02)^4}$$

$$T_A = 0.8192\,T_B \quad \cdots\cdots (2)$$

$$\tau_{allow} = 43\times10^6 = \frac{T_{max}\,\rho}{I_P} \quad ; \quad \text{Assume } T_{max} = T_A$$

$$\tau_{allow} = \frac{T_A\,(0.01)}{\frac{\pi}{32}(0.02)^4} = 43\times10^6$$

$T_A = 67.5$ N·m

$T_B = 82.47$ N·m

$T_o = -149.97$ N·m

(b) $$1.85\left(\frac{\pi}{180}\right) = \frac{T_A\,(0.225)}{G\left(\frac{\pi}{32}\right)(0.02)^4}$$

$T_A = -191.6$ N·m

$T_B = -233.74$ N·m

$(T_o)_{max} = 425.35$ N·m

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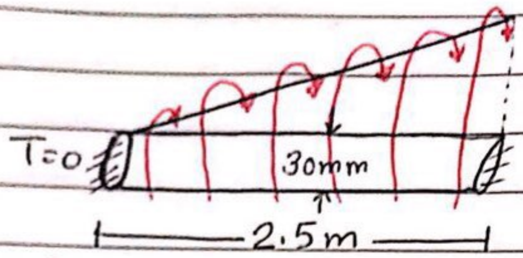
Subject: / /

Ex:-



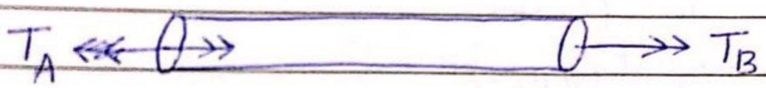
$T = t_0 = 1000\ N.m$

$G = 70\ GPa$

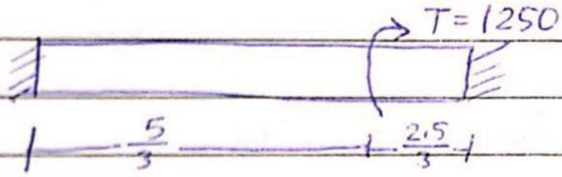
$\longmapsto X$

معادلة الخط

$T = 400X$



$$T_A + T_B = \frac{1}{2}(\cancel{1000})(1000)(2.5) \text{ - - - - - - - } (1)$$



$$\int_0^L \left(\frac{(400x - T_A)}{G\ I_p}\right) dx = 0 \text{ - - - - - } (2)$$

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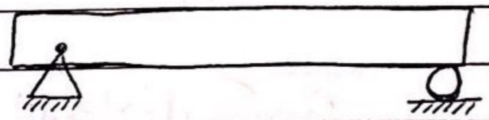
Subject: Chapter 4 :- 29 / 10 / 2018

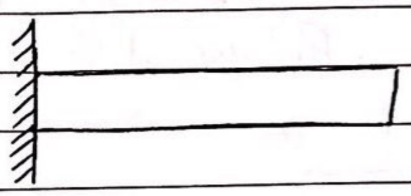
* Shear Force and Bending Moments :-

* Types of Beams:-

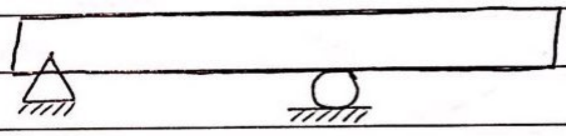
1. Simply Supporting Beam.



2. Cantilever Beam.



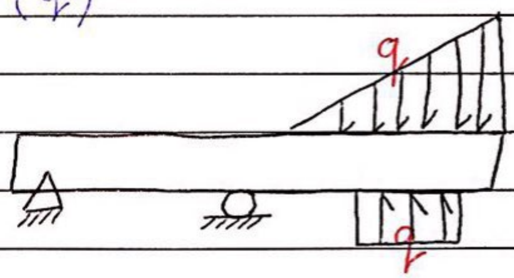
3. Overhanged Beam.



* Types of loading :-

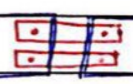
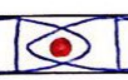
1. Binding Moment in z-direction.
2. Concentrated force. (in the xy-plane).
3. Distributed load. (q)

Note :-
Weight is a uniformaly distributed load.

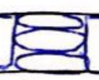


* Internal Release :- (على الأغلب مش معنا)

شكلها الواقعي

ترسم هيكذا




⇓ Shear Force = 0

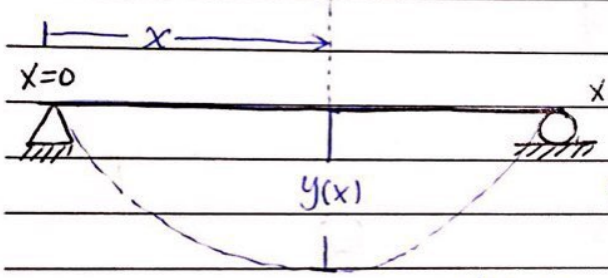
⇓ Moment = 0

Subject: ____________ / /

* Look at Fig 4.4 in your textbook.

n. Supports − n. Releases = 3

* تضاف الـ Release لأن المجاهيل تكون
أكثر من المعادلات وعند إضافة الـ Releases
تتشكل لدينا معادلة جديدة لكل Release.



y(x) ≡ Beam Defflection.

$y'(x)$ = Slope
$y''(x)$ = Moment
$y'''(x)$ = Shear
$y^{(4)}(x)$ = distributed load

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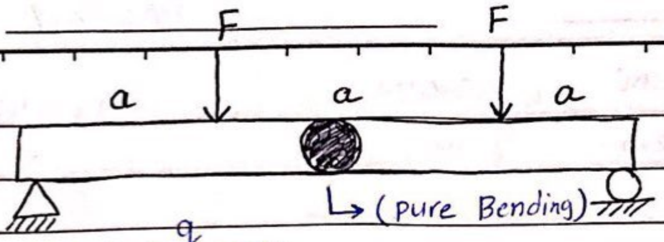
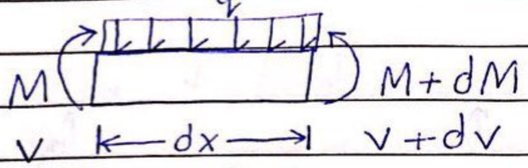
Subject: ______ / /





from Statics:-

$$\Sigma M = 0$$

$$M + dM - M - V\,dx + q\,dx\left(\frac{dx}{2}\right) = 0$$

Now Assume that

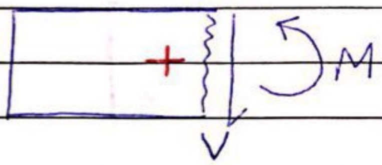
$(dx)^2 = 0 \rightsquigarrow$ Because it's very Small

$$V = \frac{dM}{dx} \quad \text{-------} \ *$$

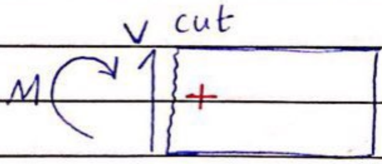
$$\frac{dM}{dx} = V \rightarrow dM = V\,dx$$

$$M = \int V\,dx$$

* Sign Convension:-



* العزم بانحناء أعلى موجب
والعزم بانحناء أسفل سالب.



$$\frac{dV}{dx} = -q \quad \text{-------} \ *$$

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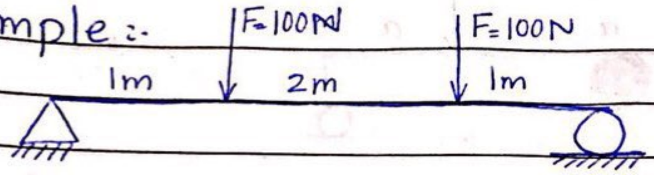
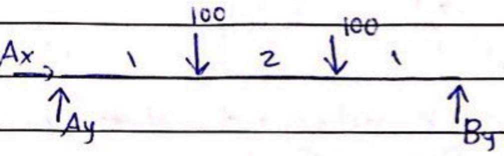
Subject: ____________ / /

Example :-



Step ① :- Find the Reactions



$\Sigma f_x = 0$

$\boxed{A_x = 0}$

$\Sigma f_y = 0$

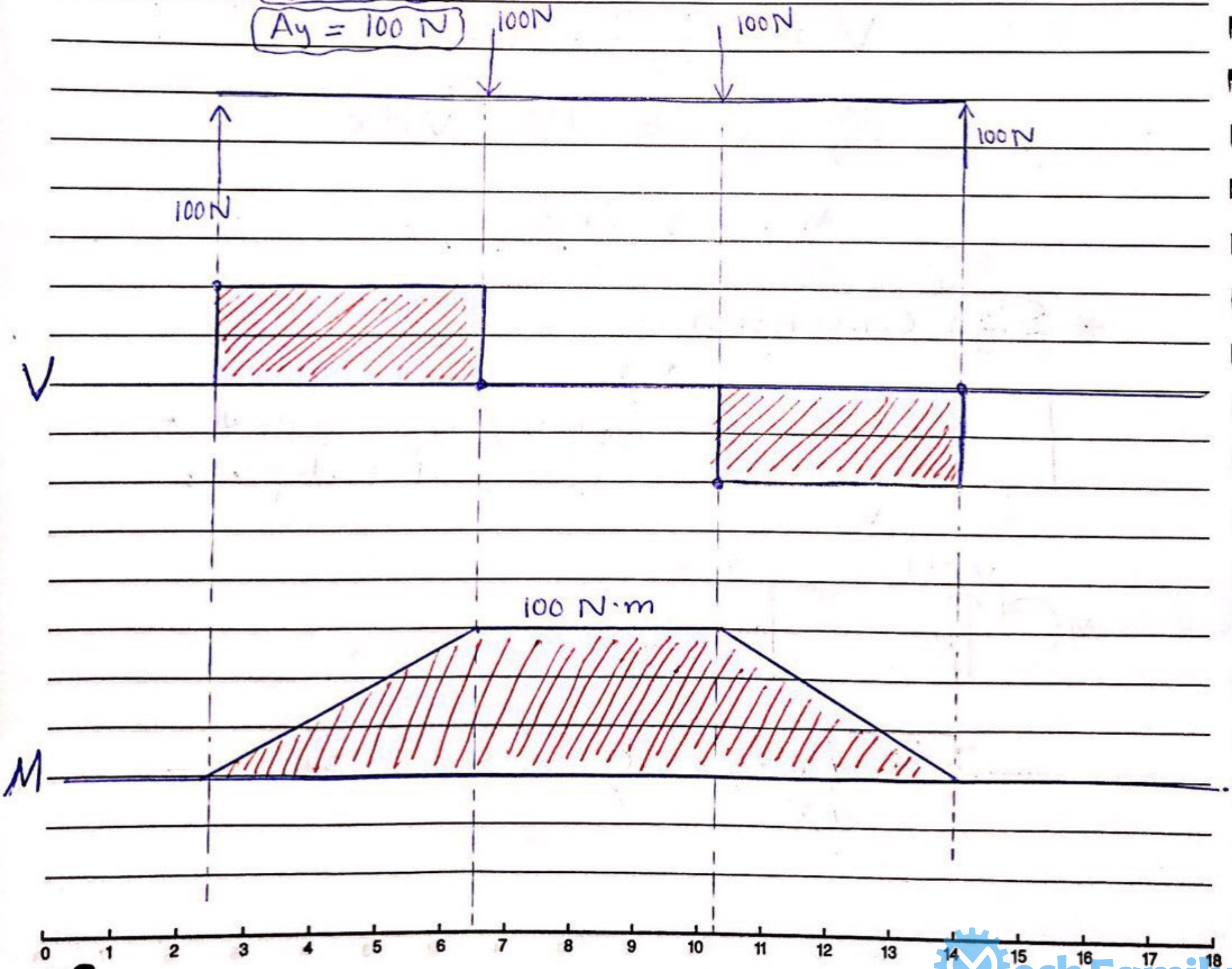
$A_y + B_y = 200\,N$

$\Sigma M_A = 0$ +↺

$4B_y - 3(100) - 100(.1) = 0$

$\boxed{B_y = 100\ N}$

$\boxed{A_y = 100\ N}$



Bim

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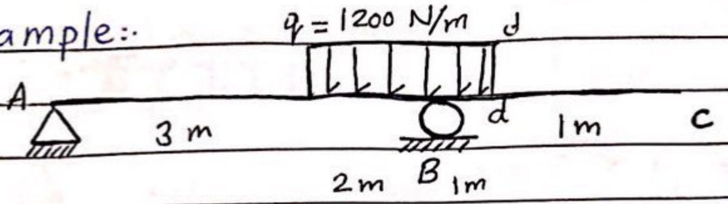
Subject: Chapter 4 31 / 10 / 2018

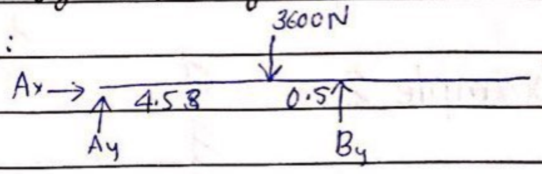
Example:



① Draw the Shear force - Bending Moment diagrams.

② Along Section d-d Find V_d and M_d.

Step①: Find the Reactions:



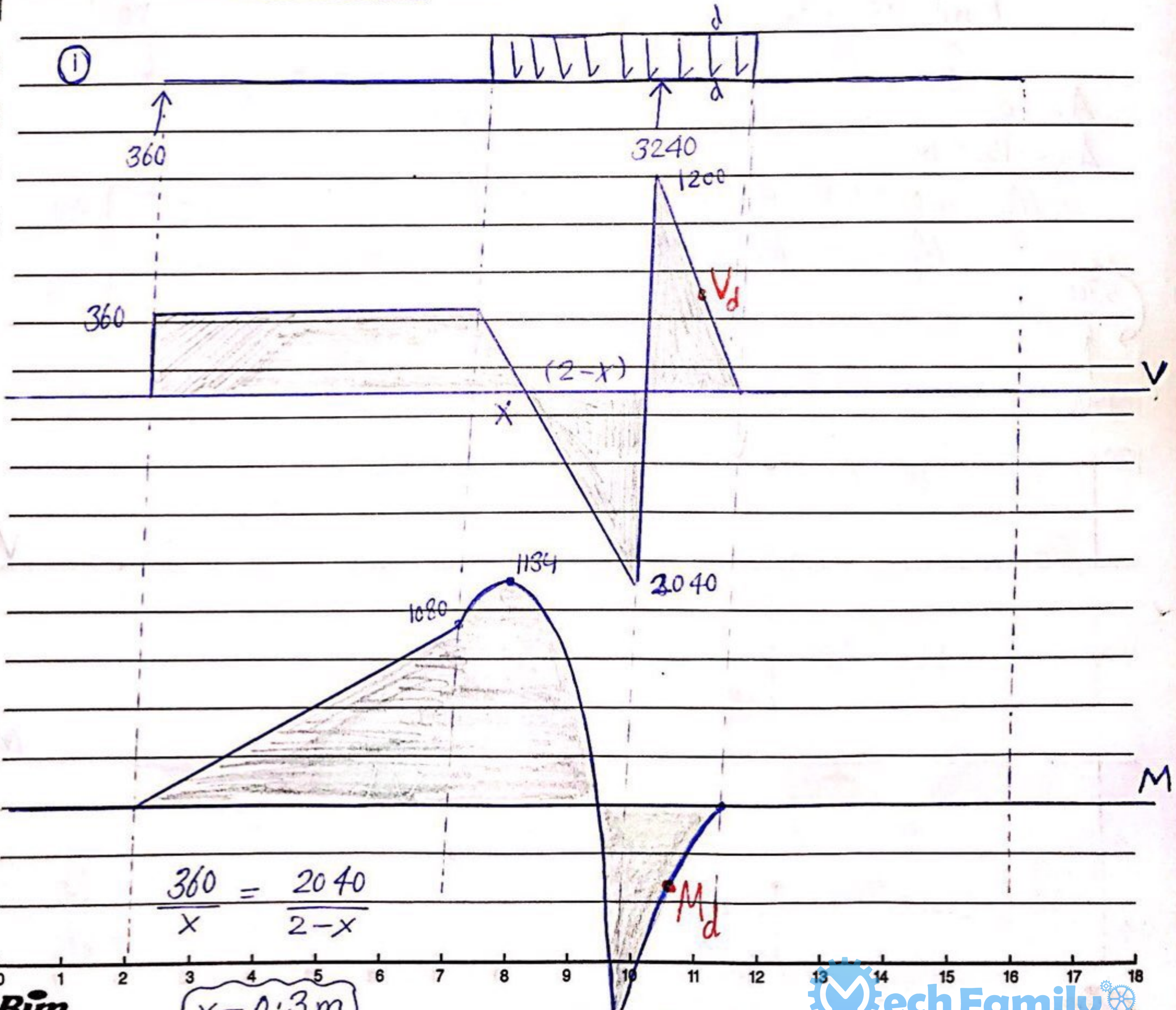
$\sum F_x = 0 \longrightarrow A_x = 0$

$\sum F_y = 0 \longrightarrow A_y + B_y = 3600$

$+\circlearrowleft \sum M_A = 0 \longrightarrow B_y(5) - 3600(4.5) = 0$

$\boxed{B_y = 3240\ N}$ $\boxed{A_y = 360\ N}$

①



$$\frac{360}{x} = \frac{2040}{2-x}$$

$$\boxed{x = 0.3\ m}$$

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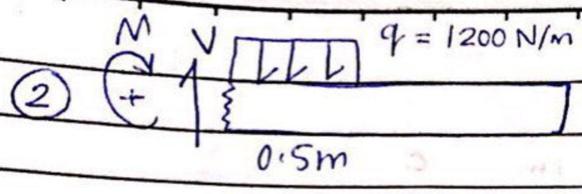
Subject: / /



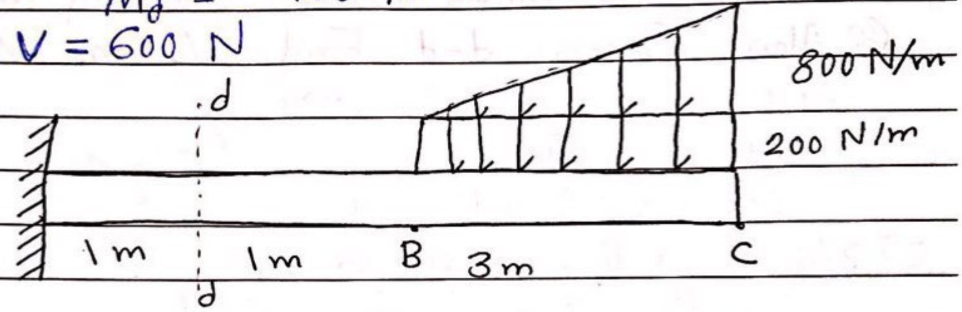
$$M_d + 600(0.25) = 0$$

$$M_d = -150 \text{ N.m}$$

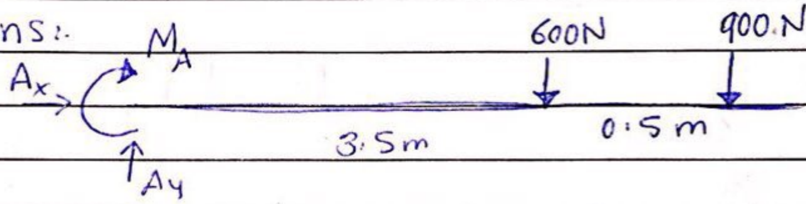
$$V = 600 \text{ N}$$

Example 2 :-

نفس السؤال السابق



Find Reactions:-

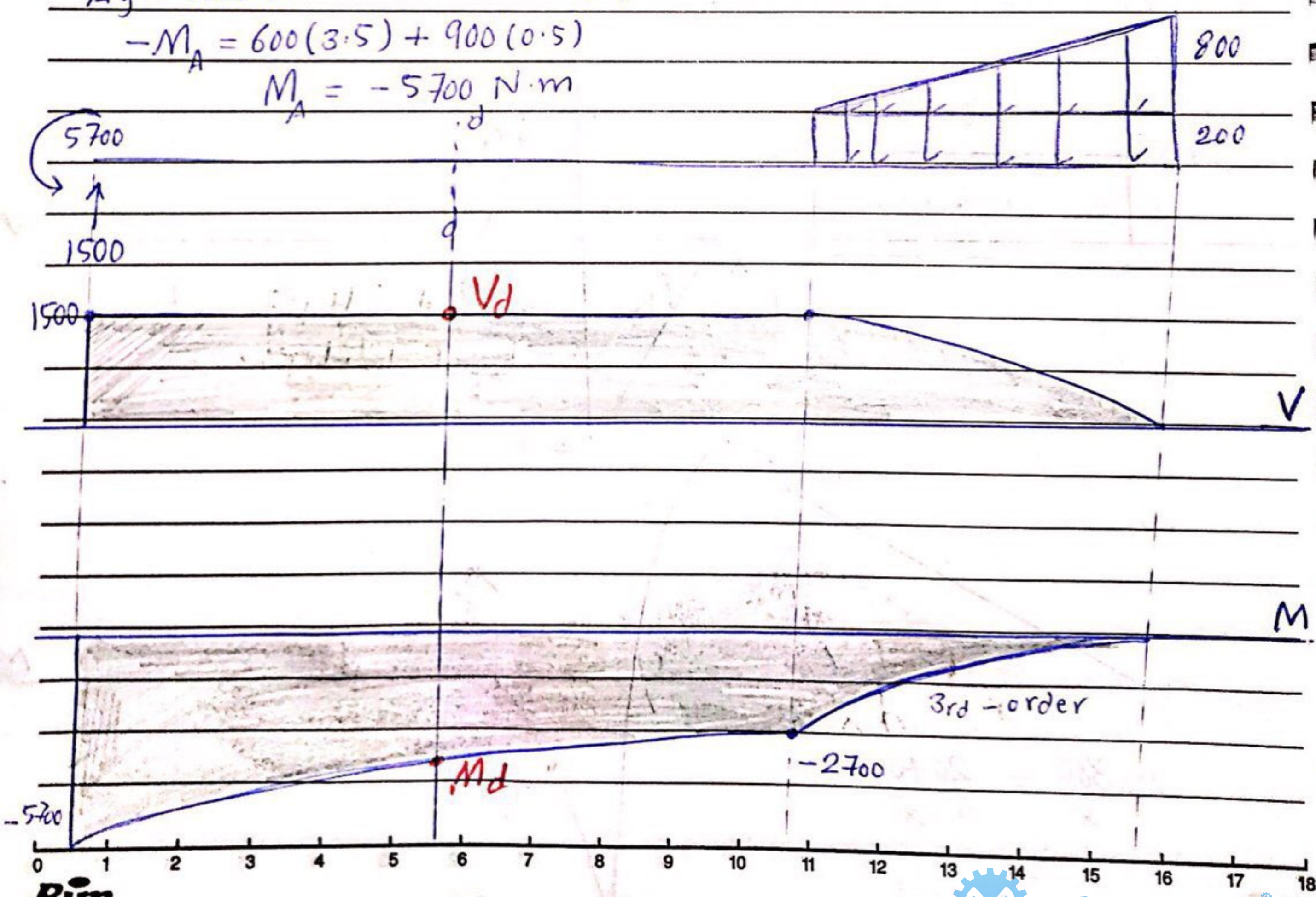


$A_x = 0$

$A_y = 1500$ N

$$-M_A = 600(3.5) + 900(0.5)$$

$$M_A = -5700 \text{ N.m}$$



$V_d = 1500$ $M_d = -4200$ N.m

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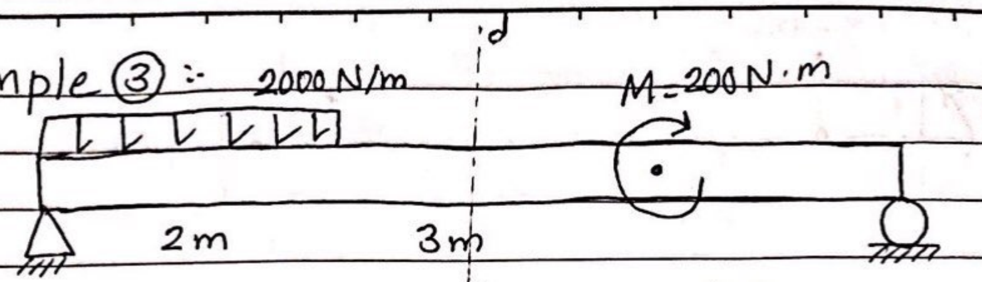
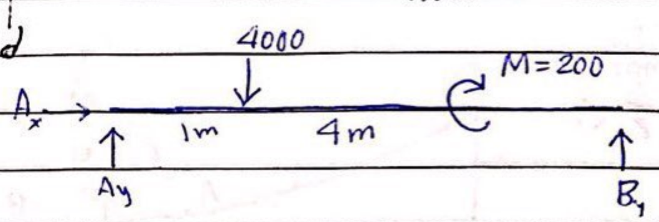
Subject: / /

Example ③ :





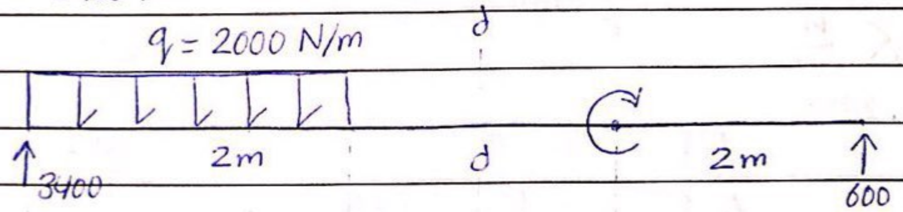
$A_x = 0$

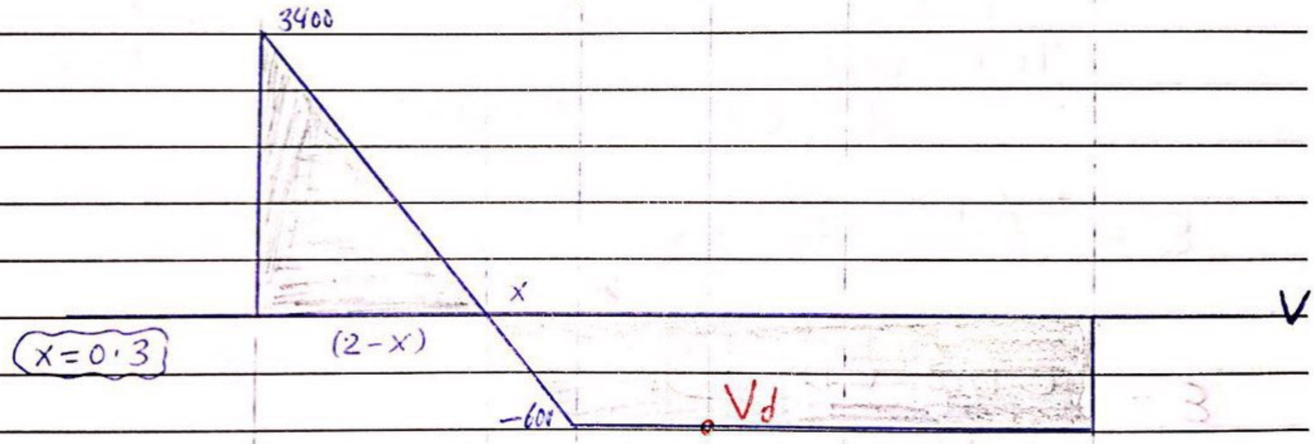
$A_y + B_y = 4000\ N$

$200 + 7B_y - 4000(1) = 0$

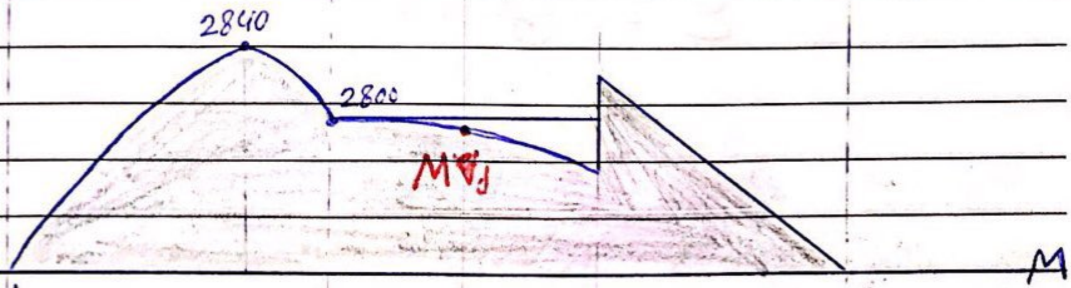
$B_y = 600\ N$

$A_y = 3400\ N$





$x = 0.3$



$V_d = -600\ N$

$M_d = 2200\ N\cdot m$

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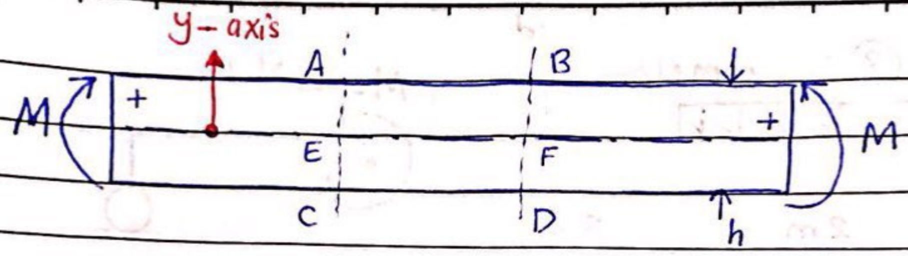
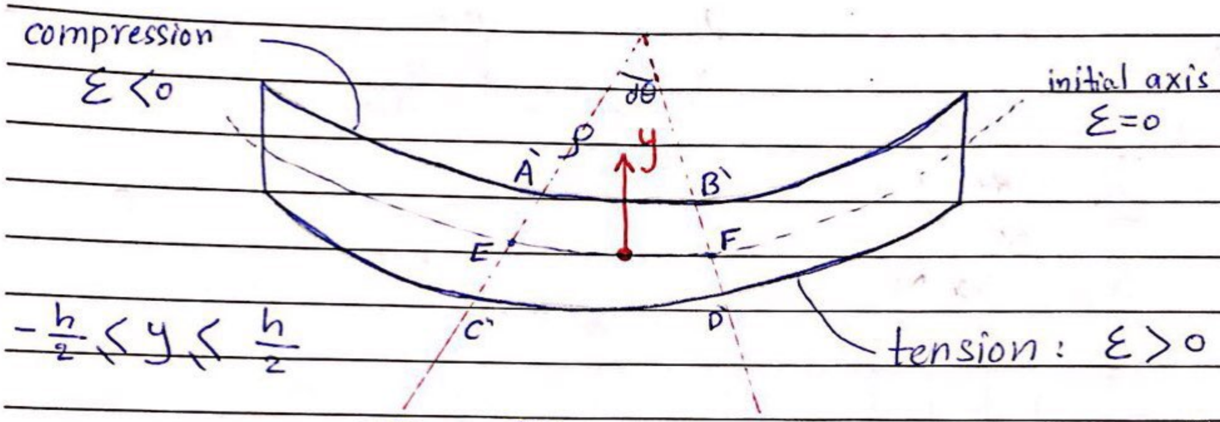
Subject: Chapter 5: Stresses in Beams. 5 / 11 / 2018





$-\frac{h}{2} \leq y \leq \frac{h}{2}$

$$EF = \rho \, d\theta$$

$\rho \equiv$ radius of carvature.

$$A'B' = \left(\rho - \frac{h}{2}\right) d\theta$$

$$C'D' = \left(\rho - \left(\frac{-h}{2}\right)\right) d\theta$$

In General :

$$A'B' = C'D' = (\rho - y)\, d\theta$$

$$\varepsilon = \frac{\overline{A'B'} - \overline{AB}}{\overline{AB}}$$

$$\varepsilon = \frac{\overline{C'D'} - \overline{CD}}{\overline{CD}}$$

$$= \frac{(\rho - y)\, d\theta - \rho \, d\theta}{\rho \, d\theta}$$

$$\varepsilon = \frac{-y}{\rho} \quad \text{--------- } *$$

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Subject: / /

Hook's Law:

$$\sigma = \frac{-yE}{\rho}$$

See text:

$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

اشتقاقه موجود بالكتاب وغير مطلوب .

; $I \equiv 2^{nd}$ Moment of inertia.

$$I = \iint y^2 \, dA$$

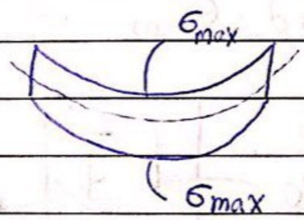
أهم القوانين :

$$\sigma = \frac{-My}{I}$$ ...... is called Flexure Formela.

$M \uparrow$, $\sigma \uparrow$

$y \uparrow$, $\sigma \uparrow$

$I \uparrow$, $\sigma \nearrow$

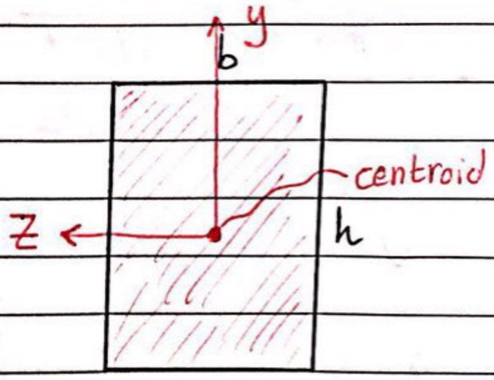


$M > 0$ and $y > 0$ then $\sigma < 0$

$M > 0$ and $y < 0$ then $\sigma > 0$

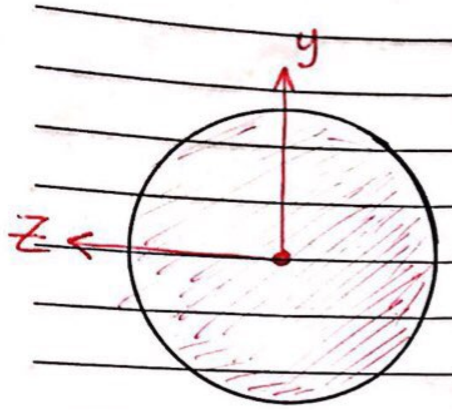
$M < 0$ and $y > 0$ then $\sigma > 0$

$M < 0$ and $y < 0$ then $\sigma < 0$



$$I = \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_{0}^{b} y^2 \, dx \, dy = \ldots$$

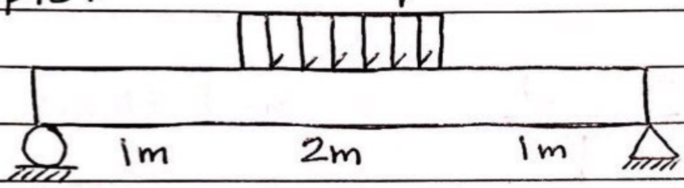
$$I = \frac{1}{12} bh^3$$



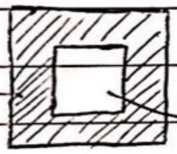
$$I = \int_0^{2\pi}\int_0^{R} (\rho \sin\theta)^2 \, \rho \, d\rho \, d\theta = \cdots$$

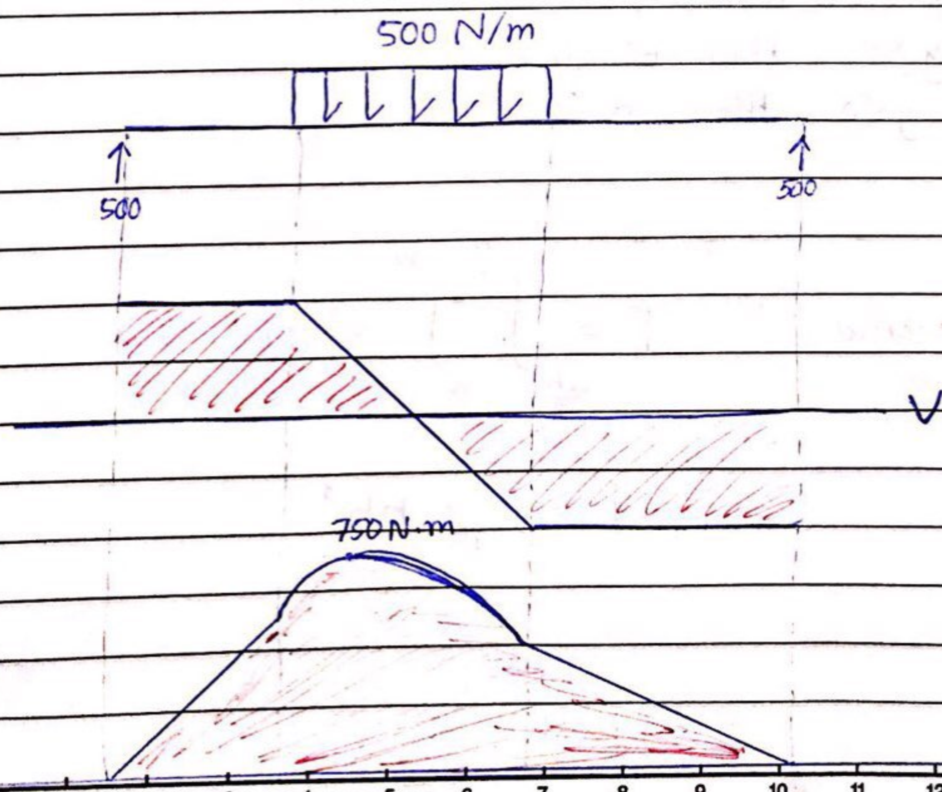
$$\boxed{I = \frac{\pi}{64} d^4}$$

Example :-



Find σ_{max} in the Beam and it's location. if

(a)  cross-section.



$$\sigma_{max} = -\frac{M_{max} \cdot y_{max}}{I}$$

$$\sigma_{max} = -\frac{750\,(0.02)}{2\times10^{-7}}$$

$$= 75 \text{ MPa}$$

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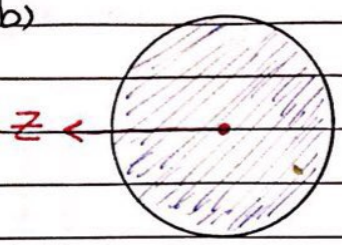
Subject: ______ / /

(b)



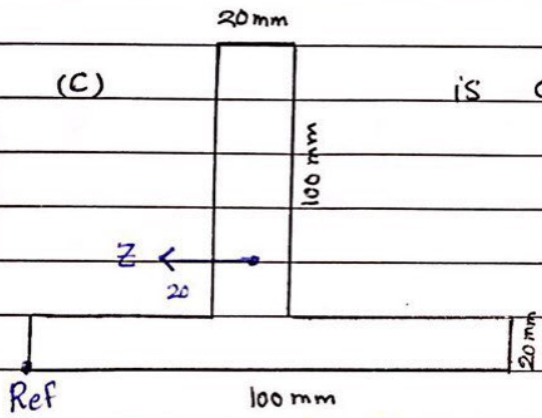
is cross-section.

$d = 30$ mm.

$$I = \frac{\pi}{64}(0.03)^4 = 3.97 \times 10^{-8}$$

$$\sigma_{max} = \frac{750\,(0.015)}{3.97 \times 10^{-8}} = 282.94 \text{ MPa}.$$

(C)



is C.S.

$$\bar{y} = \frac{\sum A_i \tilde{y}_i}{\sum A_i}$$

$$= \frac{20(100)\,70 + 20(100)(10)}{20 \times 100 \times 2}$$

$$= 40 \text{ mm}$$

$$I_z = \sum I_{z_i} + A_i d_i^2$$

$$= (I_{z_1} + A_1 d_1^2) + (I_{z_2} + A_2 d_2^2)$$

$$= \frac{1}{12}(20)(100)^3 + 100(20)(30)^2 + \frac{1}{12}(100)(20)^3 + 100(20)(30)^2$$

$$= 5.3 \times 10^6 \text{ mm}^4 = 5.3 \times 10^{-6} \text{ m}^4$$

$$\sigma_{max} = \frac{750\,(0.08)}{5.3 \times 10^{-6}} = 11.3 \text{ MPa}.$$

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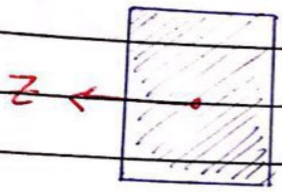
Subject: Strength 7 / 11 / 2018

* cross-sections:

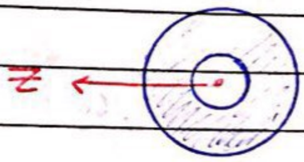
(1) symm. about z and y.

shape: I:

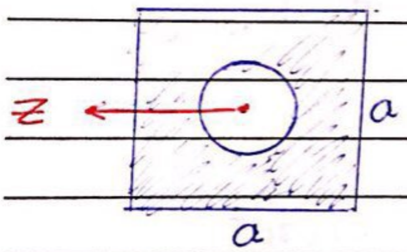
$$\frac{\pi}{64} d^4$$



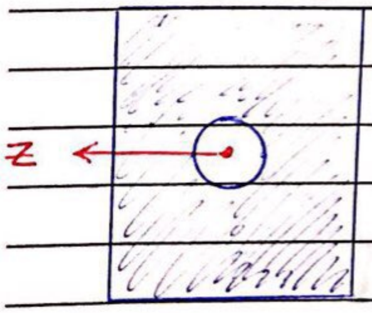
$$\frac{1}{12} b h^3$$



$$\frac{\pi}{64} \left(d_o^4 - d_i^4 \right)$$



$$\frac{a^4}{12} - \frac{\pi}{64} d^4$$



$$\frac{1}{12} b h^3 - \frac{\pi}{64} d^4$$

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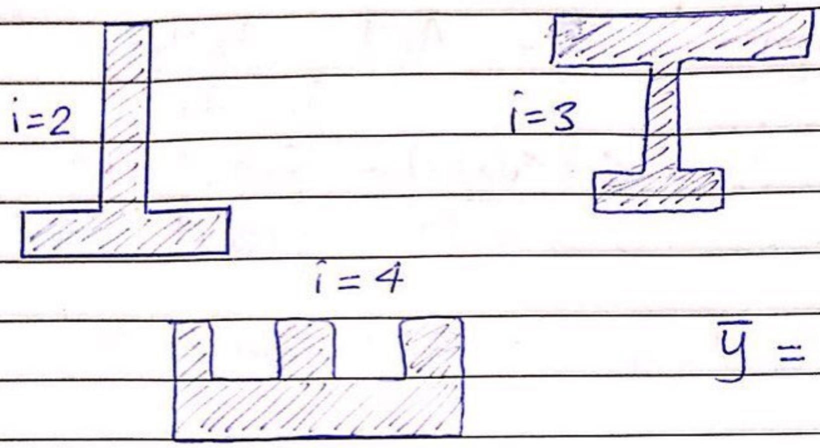
Subject: ______ / /

(2) Symm. about y-axis:-

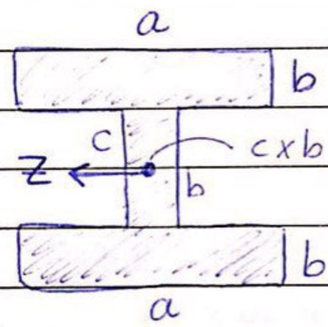


$$\bar{y} = \frac{\sum A_i \tilde{y}_i}{\sum A_i}$$

$\tilde{y} \equiv$ distance between Ref. to the local centroid.

$\bar{y} \equiv$ distance between Ref. and the Global Centroid.

$$I_z = \sum (I_{z_i} + A_i d_i^2)$$



$$I = \sum_{i=1}^{3} I_i + A_i d_i^2$$

distance between the Global centroid of $\bar{y}$ and local centroid.

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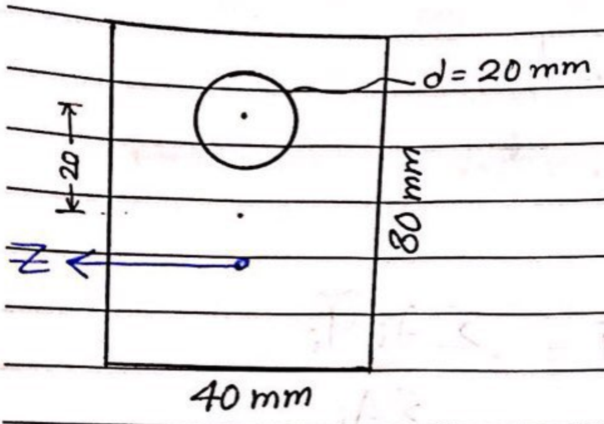
Example:- Find I



$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2}$$

$$= \frac{40(40 \times 80) - \frac{\pi}{4}(20)^2 \times 60}{40 \times 80 - \frac{\pi}{4}(20)^2}$$

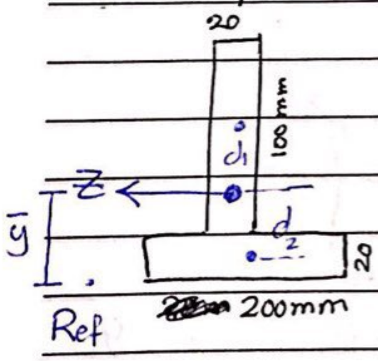
$$= 37.82 \text{ mm}$$

$$I = \left(\frac{1}{12}(40)(80)^3 + 40 \times 80 \times (2.18)^2\right) - \left(\frac{\pi}{64}(20)^4 + \frac{\pi}{4}(20)^2 (22.18)^2\right)$$

$$= 1.01 \times 10^{-5} \text{ m}^4$$

$$1 \text{m}^4 = 10^{12} \text{mm}^4$$

Example:-



$$\bar{y} = \frac{A_1 \tilde{y}_1 + A_2 \tilde{y}_2}{A_1 + A_2}$$

$$= \frac{20 \times 100 \times 70 + 20 \times 200 \times 10}{2 \times 100 + 20 \times 200}$$

$$= 30 \text{ mm}.$$

$$I = (I_1 + A_1 d_1^2) + (I_2 + A_2 d_2^2)$$

$$= \left(\frac{1}{12}(20)(100)^3 + 20 \times 100 (40)^2\right) + \left(\frac{200(20)^3}{12} + 200 \times 20 \times (20)^2\right)$$

$$= 6.6 \times 10^{-6} \text{ m}^4$$

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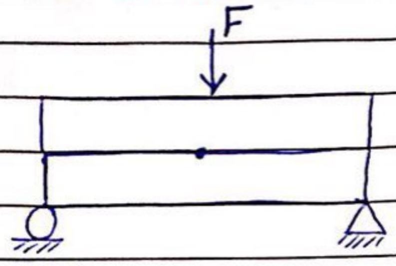
Subject: / /

* Shear Stress:-

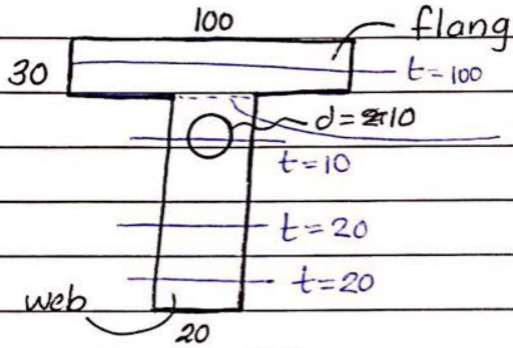
Sec 5.5 } غير
Se 5.7 } مطلوب



$$\tau = \frac{VQ}{I\,t} \dashrightarrow \tau_{max} = \frac{V_{max}\;Q_{max}}{I\;\;t_{max}}$$

$V \equiv$ Shear Force ---- CH 4

$t \equiv$ thickness of the cross-section.



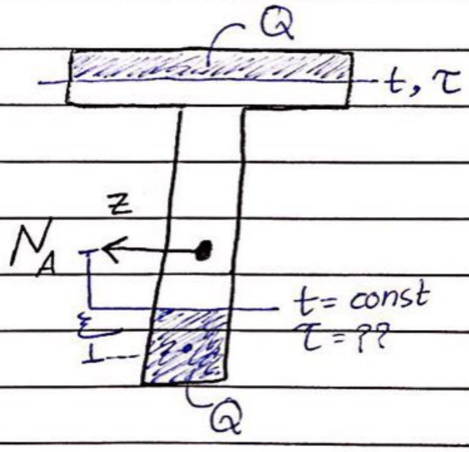
$t_w = 20$

$t_f = 100$

$\therefore$ $t \equiv$ depth of the x sectional Area when τ to be calculated.

$$Q = \iint_{shaded} y \, dA$$

$Q \equiv 1^{st}$ moment of inertia area.



ال Q هي المساحة مابعد القطع.

$$Q = A_{shaded} \cdot \xi$$

↳ unit is m^3

$$Q_{max} = Q_{N_A}$$

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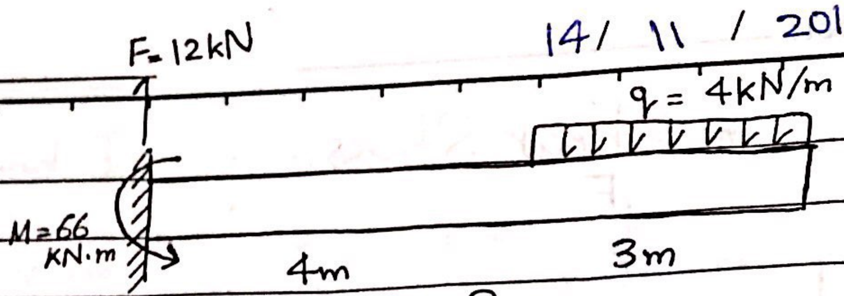
Subject: CH5 14/ 11 / 2018

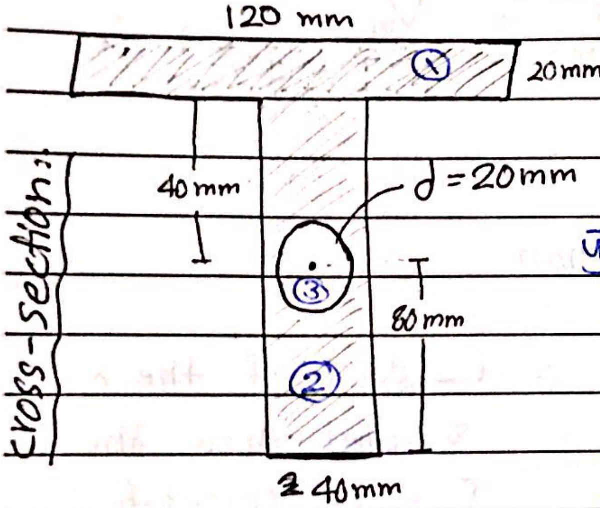
Example :-



a) Find σ_{max} and τ_{max} in the beam ?

b) Find σ_{max} and τ_{max} at the top of the hollow circle ?

c) Find σ_{max} and τ_{max} between web & flange ?



$$\bar{y} = \frac{\sum A_i \tilde{y}}{\sum A_i} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$$

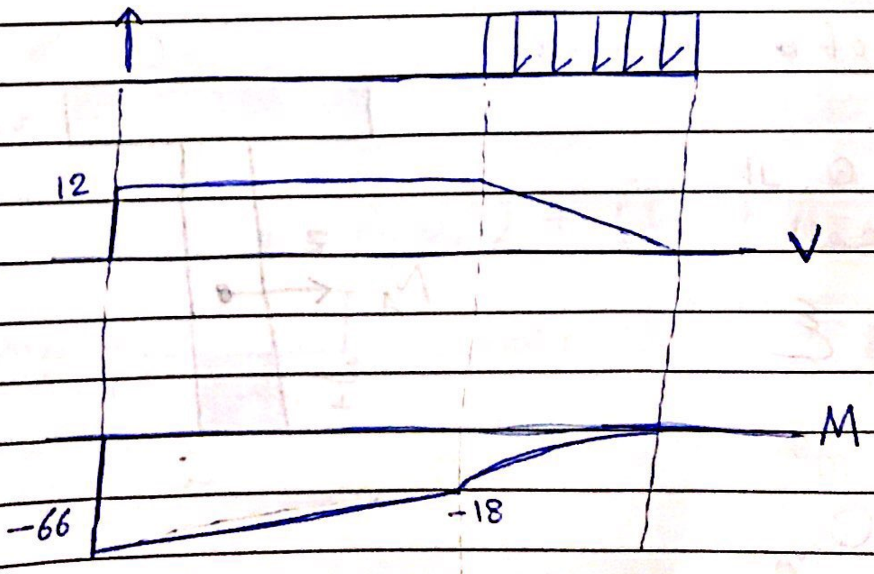
$$\bar{y} = \frac{120(20)(130) + 120(40)(60) - \frac{\pi}{4}(20)^2(80)}{20(120) + 120(40) - \frac{\pi}{4}(20)^2}$$

$$\bar{y} = 83.48 \text{ mm}$$

$$I = \sum_{i=1}^{i=3} (I_i + A_i d_i^2)$$

$$\left[\frac{120 \times 20^3}{12} + 120(20)(46.6)^2\right] + \left[40 \times 120 \times (23.4)^2 + \frac{40(120)^3}{12}\right] - \left[\frac{\pi}{64}(20)^4 + \frac{\pi}{4}(20)^2 (3.4)^2\right]$$

$$I = 13.5 \times 10^{-6} \text{ m}^4$$



Subject: ______

ⓐ $$\sigma_{max} = -\frac{(-66\times10^3)(-0.0834)}{13.5\times10^{-6}}$$

$$\sigma_{max} = -407.73 \text{ MPa}$$

$$Q_{N.A} = \sum [A_i d_i] = A_1 d_1 + A_2 d_2 - A_3 d_3$$

$$= [20(120)(46.6)] + [(40)(36.6)(18.3)] - [(r^2(\alpha - \sin\alpha\cos\alpha))((r - \frac{2r}{3})(\frac{\sin^3\alpha}{\alpha - \sin\alpha\cos\alpha}))]$$

$\therefore$ M_{max} is at fixed support.

$$M_{max} = 66 \text{ kN}\cdot\text{m}$$

$$V_{max} = 12 \text{ kN}$$

ⓑ $$\sigma_{max} = -\frac{(-66\times10^3)(6.6)}{13.5\times10^{-6}} = 32.26 \text{ MPa}$$

$$Q = A_1 d_1 + A_2 d_2 = 30\times40\times21.6 + 120\times20\times46.6$$

$$Q = 0.1378\times10^{-3} \text{ m}^3$$

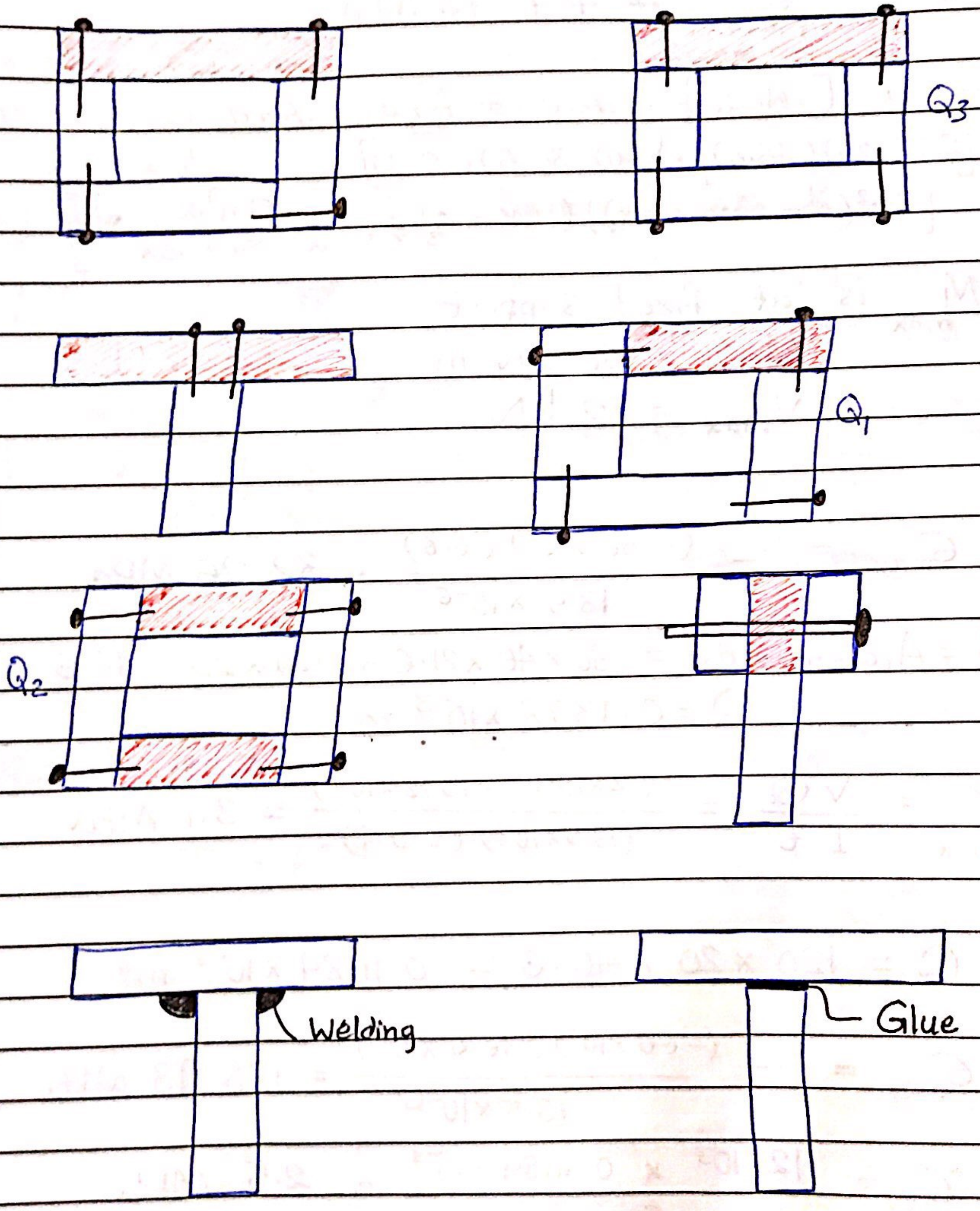
$$\tau_{max} = \frac{VQ}{It} = \frac{(12\times10^3)(0.1378\times10^{-3})}{(13.5\times10^{-6})(0.04)} = 3.1 \text{ MPa}.$$

ⓒ $$Q = 120\times20\times40.6 = 0.11184\times10^{-3} \text{ m}^3$$

$$\sigma_{max} = -\frac{(-66\times10^3)(36.6\times10^{-3})}{13.5\times10^{-6}} = 178.93 \text{ MPa}$$

$$\tau_{max} = \frac{12\times10^3 \times 0.11184\times10^{-3}}{13.5\times10^{-6} \times 0.04} = 2.5 \text{ MPa}$$

* Build-up structures and Shear flow :



* المساحة المحددة باللون الأحمر هي (Q)

Subject:

Shear flow :- $(f) \rightsquigarrow (N/m)$

$$f = \tau t = \frac{VQ}{I}$$

* يتكون هذا ال flow في أماكن التلامس بين الأجزاء المربوطة بواسطة المسامير أو البراغي .

it may be :-

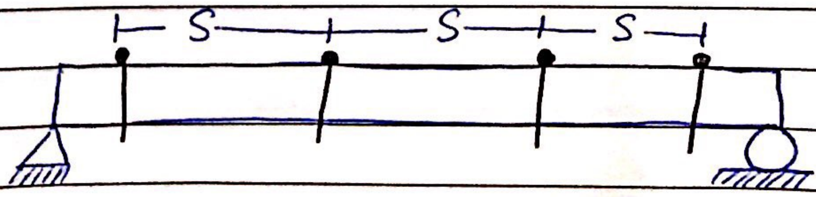
① Nail
② ~~skrew~~ screw
③ bolt

Ultimate force :-
هي القوة التي يتحملها إمّا المسمار أو البرغي .

$$\tau_{allow} = \frac{f}{t}$$

* Spacing between (Nails or ~~skrew~~ screw or bolt)

$$S = \frac{(\text{\# of Nails})^{N} \, (\text{Allowable Shear force})^{\text{for each (Nail or bolt)}}}{f_{(\text{Shear flow})}}$$



Assume that :-

$$Q_1 < Q_2 < Q_3$$

* Which One is the best ?

$Q \uparrow \quad f \uparrow$

$f \uparrow \quad S \downarrow$

$S \uparrow \quad Q \downarrow$

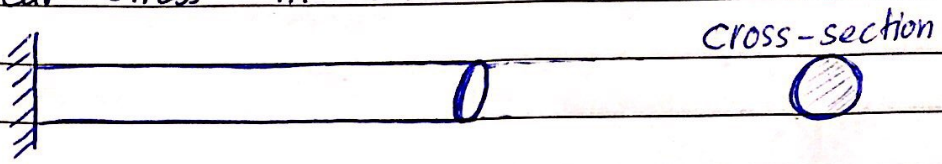
* الأفضل هي التي لها أقل Q لأن S ستكون أكبر ما يمكن وبالتالي سنحتاج إلى عدد أقل من المسامير أو البراغي .

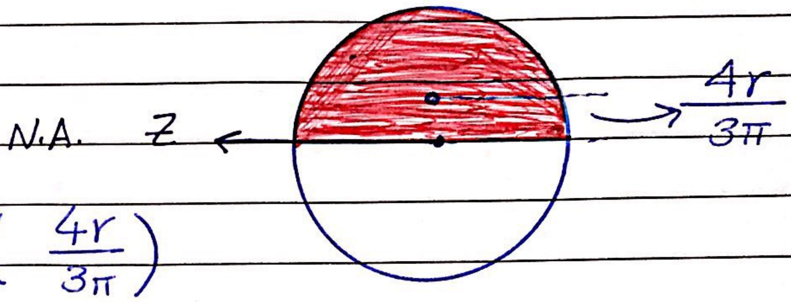
Welding And Gluing :-

f = (# of welds)(Allowable force per unit length)

Subject: ______

* Shear-Stress in circular Beams :-





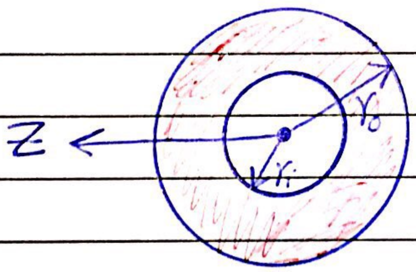
$$Q_{N.A} = \left(\frac{\pi}{8}d^2\right)\left(\frac{4r}{3\pi}\right)$$

$$= \frac{\pi}{8}(2r)^2\left(\frac{4r}{3\pi}\right)$$

$$= \frac{2}{3}r^3$$

$$I = \frac{\pi}{64}d^4$$

* if we have a hollow :-

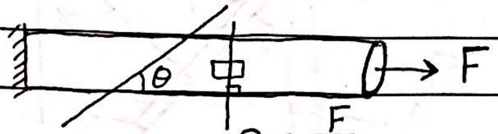


$$Q_{NA} = \frac{2}{3}\left(r_o^3 - r_i^3\right)$$

$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right)$$

Subject: Chapter 7 :.   /   /

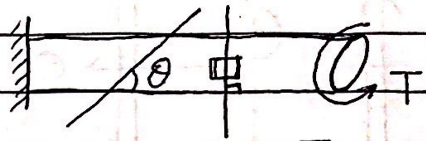
Analysis of Stress :.

① 

$\sigma = \frac{F}{A}$

$\sigma = ??$

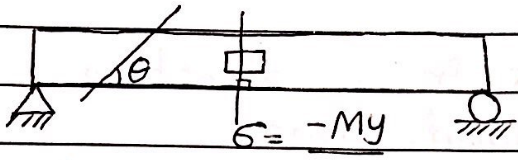
$\tau = ??$

② 

$\sigma = ??$

$\tau = ??$

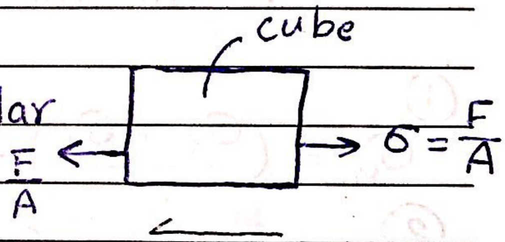
$\tau = \frac{T\,r}{I_p}$

③ 

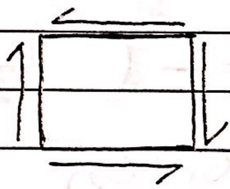
$\sigma = \frac{-My}{I}$

$\tau = \frac{VQ}{It}$

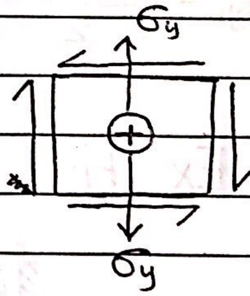
* Element for first Drawing :.
  - The normal stress is perpindicular to the cut.

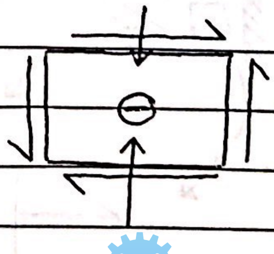


* Element for second Drawing :.
  - It only has Shear stress.

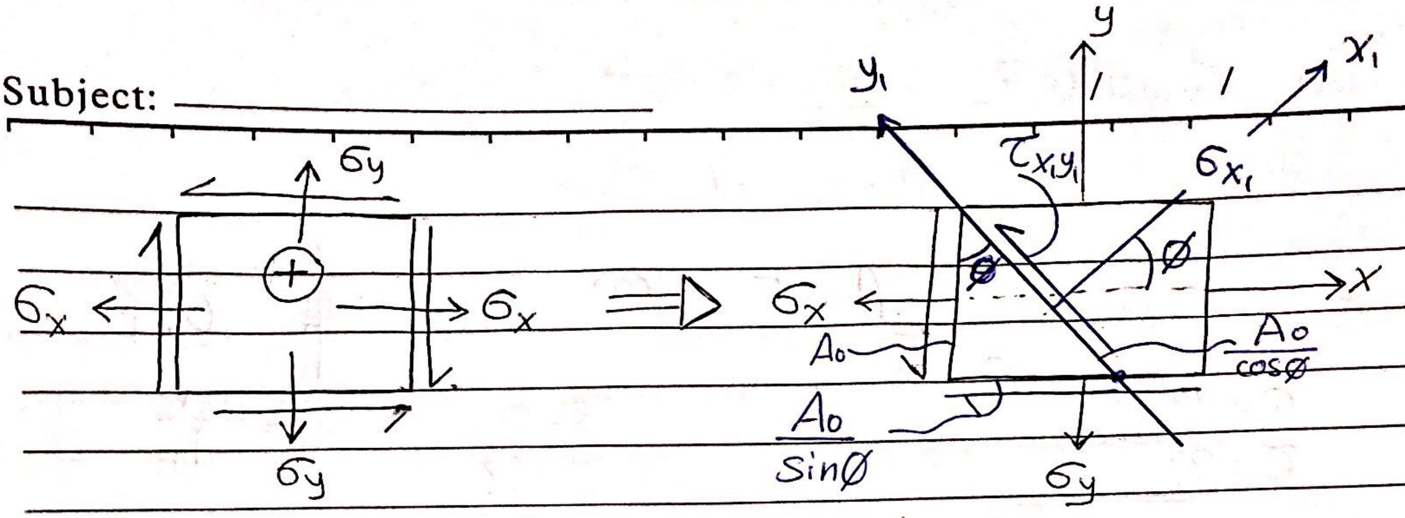


* Element for third Drawing :.
  - The plus sign (+) means that the stress is tensile stress, while negative σ_y sign means compressive for all stresses.





Subject:



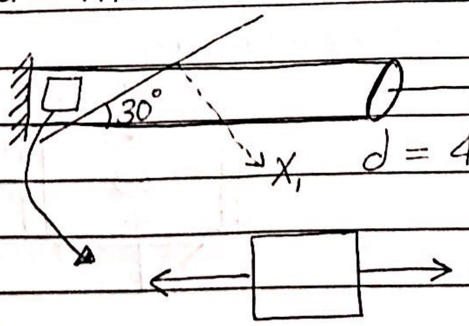
* σ_{x_1} is the Normal stress in x_1-direction which is ⊥ to the inclined plane.
* $\tau_{x_1y_1}$ is Shear stress along the inclined plane.
  Ø is measured from x-axis to the ⊥ to the inclined plane.

① $$\sigma_{x_1} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cdot \cos 2\phi + \tau_{xy} \sin 2\phi$$

② $$\tau_{x_1y_1} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\phi + \tau_{xy} \cos 2\phi$$

③ $$\sigma_{y_1} = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\phi - \tau_{xy} \sin 2\phi$$

Ex: Find the Stresses along the inclined plane.



F = 1000 N

d = 40 mm

$$\sigma_x = \frac{1000}{\frac{\pi}{4}(0.04)^2}$$

$$\sigma = \sigma_x = 7.957 \times 10^5$$

$\sigma_y = 0$ ⟹ $\phi = \frac{-\pi}{3} = -60°$

$\tau_{xy} = 0$

From Equation (1):-

$$\sigma_{x_1} = \frac{\sigma_x}{2} + \left(\frac{\sigma_x}{2}\right)\cos(2\phi)$$

$$\sigma_{x_1} = \frac{7.957\times10^5}{2}\left(1+\cos(-120)\right)$$

$$\sigma_{x_1} = 1.988\times10^5\ Pa$$

* Mohr's Circle:-

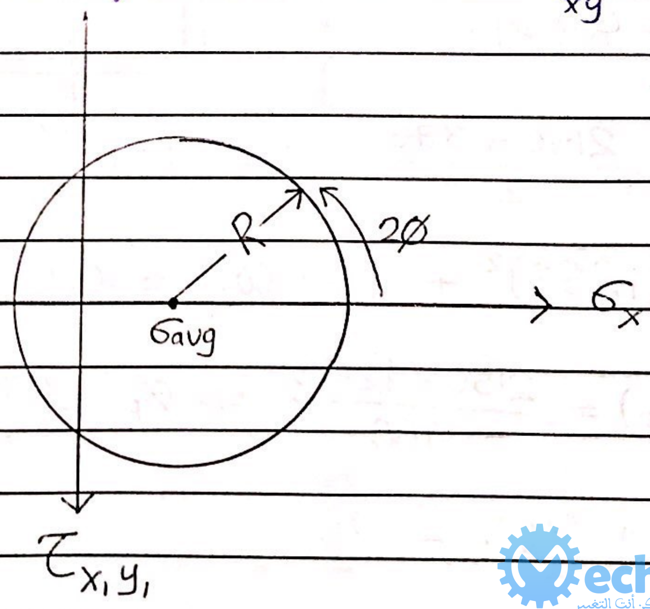
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} ;$$

$$\underbrace{(\sigma_{x_1} - \sigma_{avg})^2}_{x} + \underbrace{(\tau_{x_1 y_1})^2}_{y} = \underbrace{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}_{R^2}$$

$$(x - x_0)^2 + (y - 0)^2 = R^2$$

Hint:- * σ_{avg} ≡ center of the circle.

** $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$



Same Example:-

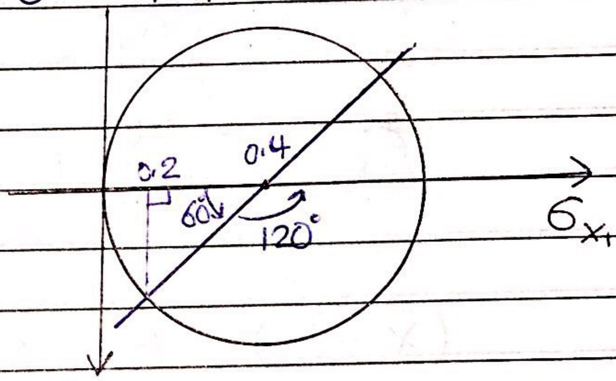
$\sigma_x = 0.7957\ MPa$

$\sigma_{avg} = \frac{0.7957}{2} \approx 0.4\ MPa$

$R = \sqrt{\left(\frac{0.8}{2}\right)^2} = 0.4\ m$

$\theta = -60°$, $\sigma_{x_1} = 0.4 - 0.4\cos(60)$

$\sigma_{x_1} = 0.2\ MPa$

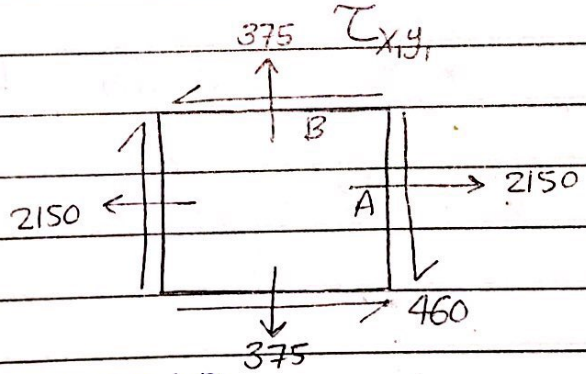


Example (7.3.18)

$\sigma_x = 2150\ kPa$

$\sigma_y = 376\ kPa$

$\tau_{xy} = -460\ kPa$



$\sigma_{avg} = \left(\frac{2150 + 376}{2}\right) = 1263\ kPa$

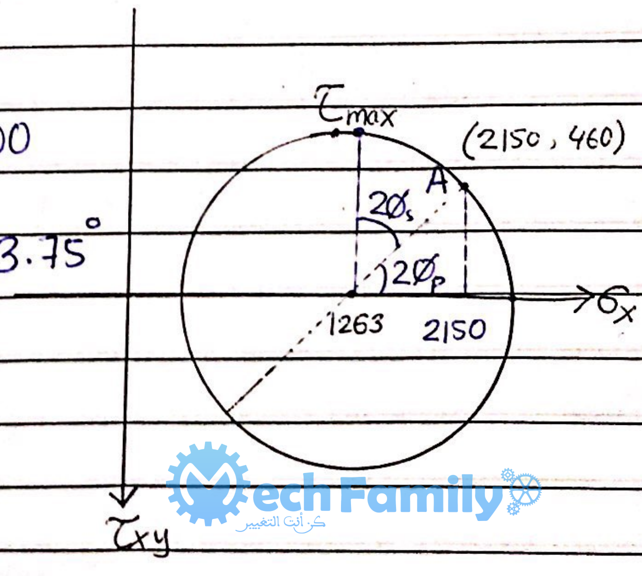
$R = \sqrt{(1263)^2 + (-460)^2} = 1000$

$\cot(2\phi_p) = \frac{2150 - 1263}{460} \Rightarrow \phi_p = 13.75°$

$2\phi_p + 2\phi_s = \frac{\pi}{2}$

$\therefore \phi_s = 31.25°$

$\tau_{max} = R$

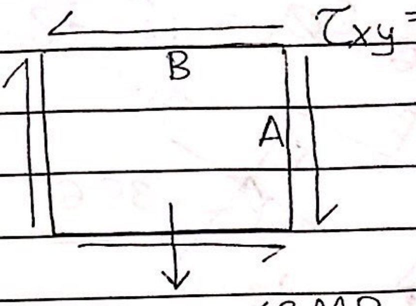


Subject: Chapter 7 :- 28 / 11 / 2018

Example :- 7.4.10.

As shown in text Book.

(a) Find σ_{x_1} , $\tau_{x_1y_1}$ at $\theta = 22.5°$ counter clock wise (ccw).



(b) Find max σ (σ_1 and σ_2) and τ_{max} and sketch them on a properly Oriented elements.

$\sigma_x = 0$, $\sigma_y = 68$ MPa , $\tau_{xy} = -100$ MPa

$$R = \sqrt{\left(\frac{-68}{2}\right)^2 + (-100)^2} = 105.6 \text{ MPa}$$

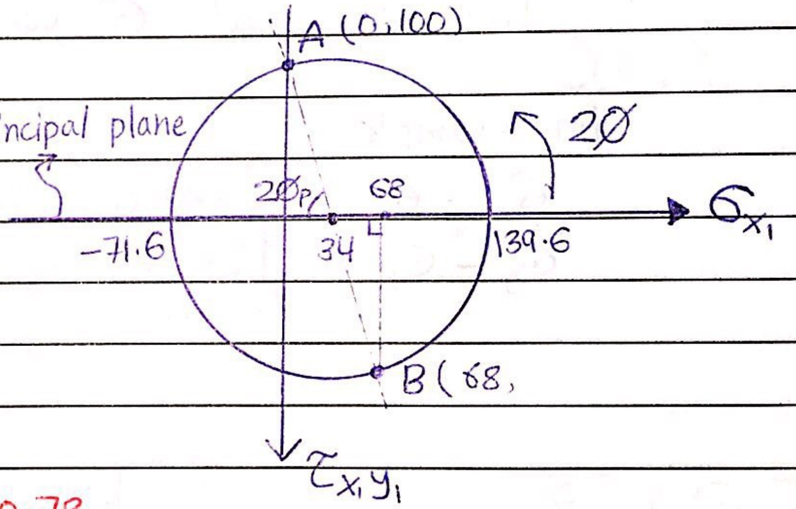
$$\sigma_{avg} = \frac{0 + 68}{2} = 34 \text{ MPa}$$

$$\sin(2\theta_p) = \frac{100}{105.6}$$

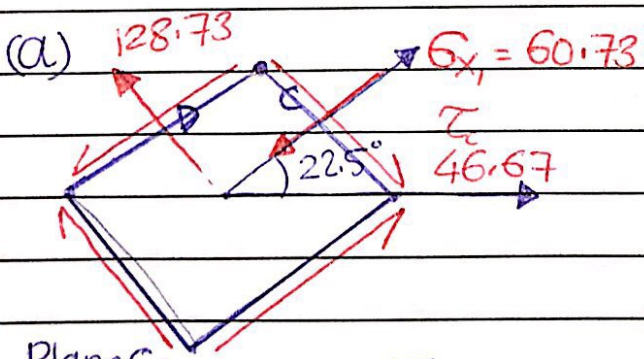
$$2\theta_p = \sin^{-1}\left(\frac{100}{105.6}\right)$$

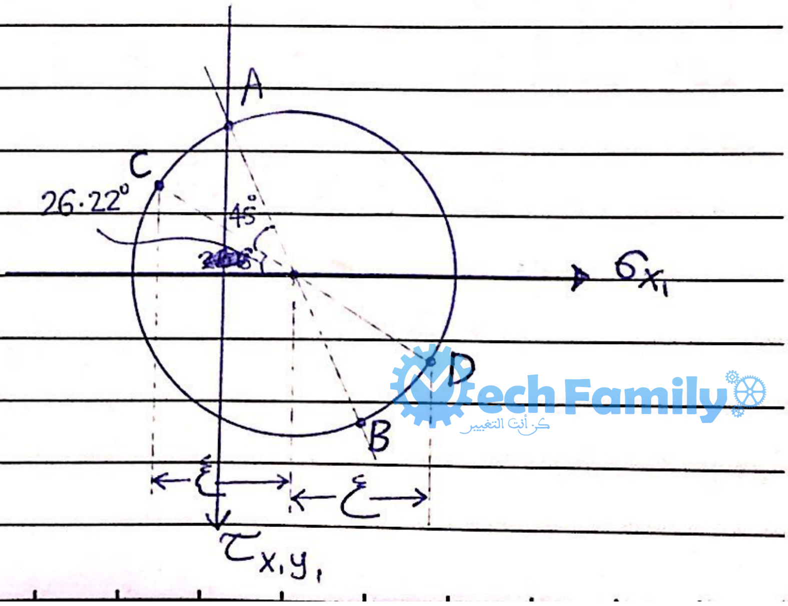
$$2\theta_p = 71.22°$$

$$\theta_p = 35.61°$$



(a)





Plane C :-

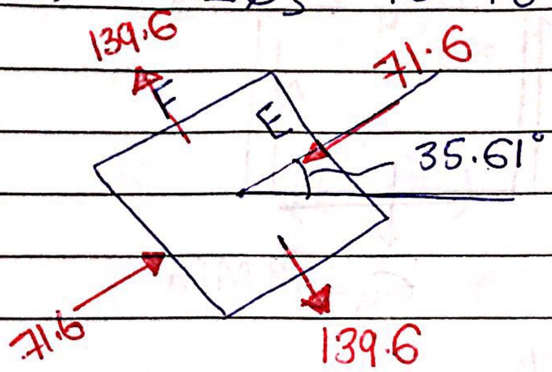
$$\sin 26.22 = \frac{\tau_c}{R}$$

$$\cos 26.22 = \frac{\varepsilon}{R}$$

$\tau_c = 46.67$ MPa

$\varepsilon = 94.73$ MPa

(b) $2\phi_s = 18.78^\circ \Rightarrow \phi_s = 9.39^\circ$ CW



\# **Rule:**

Always $\Sigma\sigma$ in All angles at any point is equal to the original σ we have.

for example

$\sigma_y - \sigma_x = 139.6 - 71.6 = 68$ MPa

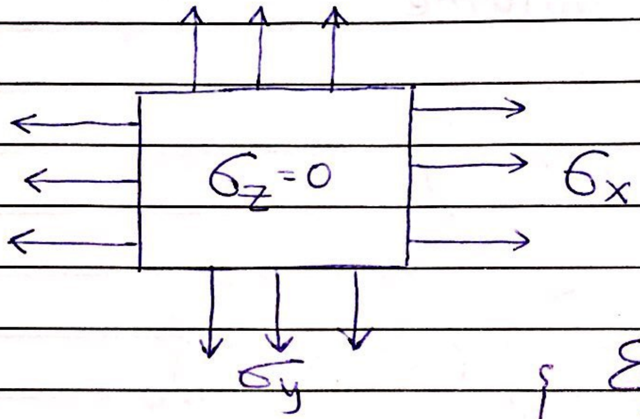
$\sigma_y - \sigma_x = 128.73 - 60.73 = 68$ MPa

* Hook's Law for plane Stress :. (2D - elements)

(Hook's Law for 1-D elements :.)

$$\sigma = \varepsilon E$$

$$\tau = \gamma G$$



Strain - Stress Relations

$$\left\{\begin{aligned} \varepsilon_x &= \frac{1}{E}(\sigma_x - \nu\sigma_y) \\ \varepsilon_y &= \frac{1}{E}(\sigma_y - \nu\sigma_x) \\ \gamma_{xy} &= \frac{1}{G}(\tau_{xy}) \end{aligned}\right.$$

$$G = \frac{E}{2(1+\nu)} \quad ; \quad \nu = -\frac{\varepsilon_{late}}{\varepsilon_{long}}$$

Matrix Form :.

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E} & \frac{-\nu}{E} & 0 \\ \frac{-\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + 1)\varepsilon_y$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\tau_{xy} = G\gamma_{xy}$$

$*$ Dilatation :

$$e = \frac{\Delta V}{V_o} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

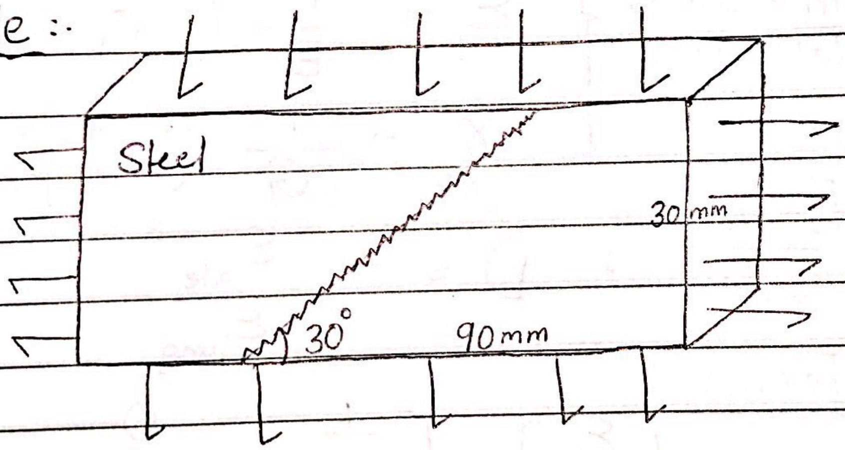
$V \equiv$ Volume.

$V_o \equiv$ Orginal volume.

$$\varepsilon_z = \frac{-\nu}{E} (\sigma_x + \sigma_y)$$

$$= \left(\frac{1-2\nu}{E}\right)(\sigma_x + \sigma_y)$$

Example :



(A) Find ε_x , ε_y , ε_z .

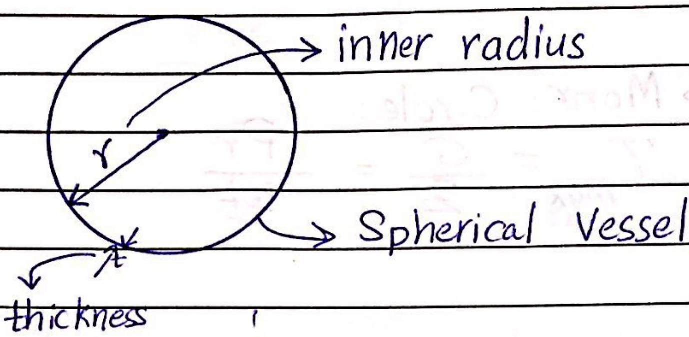
(B) Find Change in theckness.

(C) Find the Normal and Shear Stress.

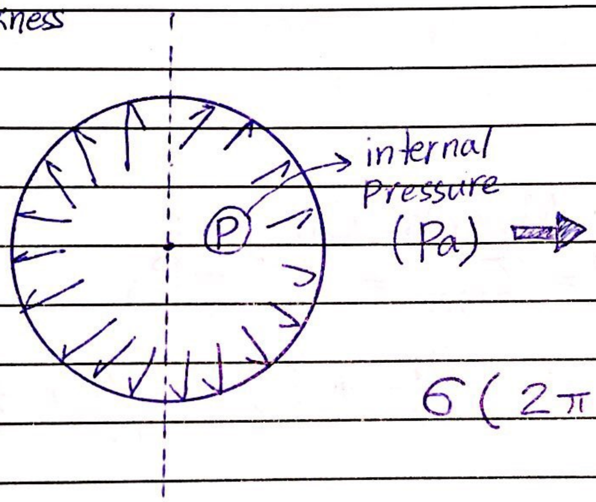
σ_w , τ_w to be calculated.

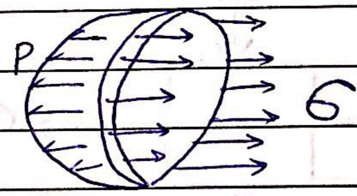
Subject: Chapter 8 :- 3/12/2018

* Applications of plane stress (pressure Vessels, Beams with a combined loading.)



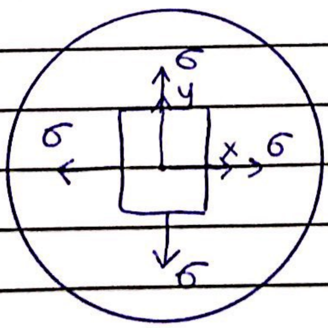
$\frac{r}{t} \geq 10$ for thin Vessels.



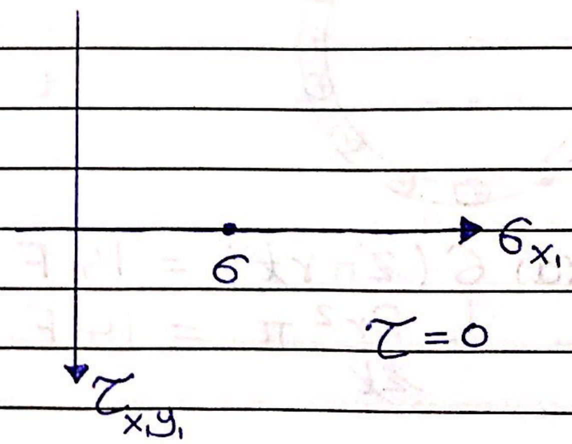


$$\sigma(2\pi r t) = \underbrace{P(\pi r)}_{\text{project Area.}}$$

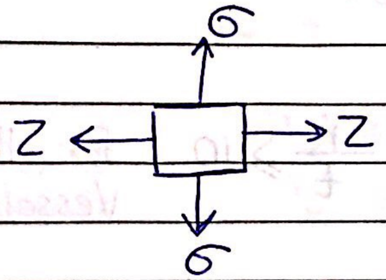
$$\rightarrow \sigma = \frac{Pr}{2t} \quad \text{--------} \quad *$$



Mohr's circle :-



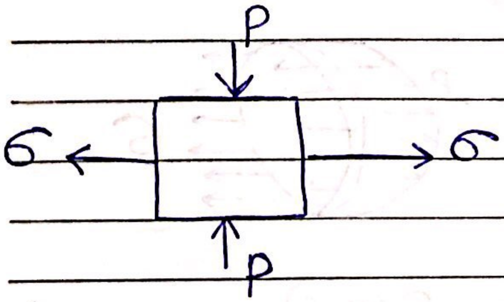
(A) Outer Surface:



$\Rightarrow$ Mohr's Circle:

$$\tau_{max} = \frac{\sigma}{2} = \frac{Pr}{4t}$$

(B) inner Surface

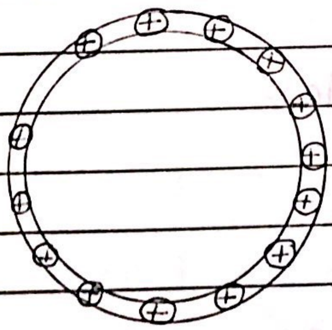


$\Rightarrow$ Mohr's Circle:

$$R = \tau_{max} = \left(\frac{\sigma + P}{2}\right)$$

$$= \frac{1}{2}\left(\frac{Pr}{2t} + P\right) = \frac{Pr}{4t} + \frac{P}{2}$$

Example: Prob. 8.2.5



internal pressure = 575 kPa

14 bolts.

$r = 350$ mm

$t = 32$ mm

(a) $\sigma(\cancel{2}\pi r \cancel{t}) = 14F$

$\hookrightarrow \frac{Pr^2}{\cancel{2t}}\pi = 14F \rightarrow F = 221.286$ kN

(b) σ_y (each bolt) $= 345\ MPa$, $F.S. = 3$

Find $d = ??$

$$\frac{345 \times 10^6}{3} = \frac{F}{\frac{\pi}{4} d^2}$$

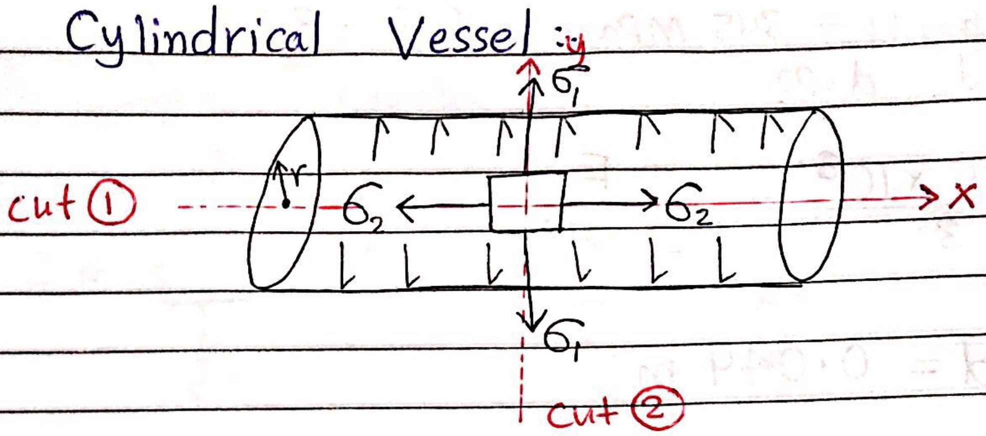
$$d = 0.049\ m$$

(c) $\sigma = 3 \times 10^6 = P(\pi r^2)$

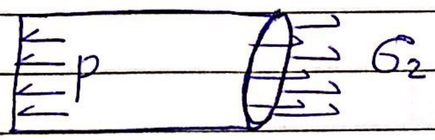
$$\Rightarrow r^2 = 1.66\ m$$

Subject: 5 / 12 / 2018

Cylindrical Vessel :



cut ②

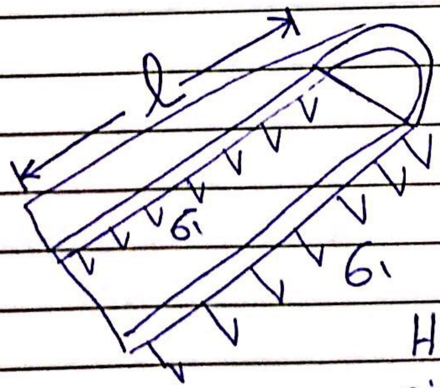


$$\sigma_2 (2\pi r t) = P (\pi r^2)$$

$$\sigma_2 = \frac{Pr}{2t}$$ ------- similar to spherical Vessel

is called longitudenal OR axial Stress.

cut ①



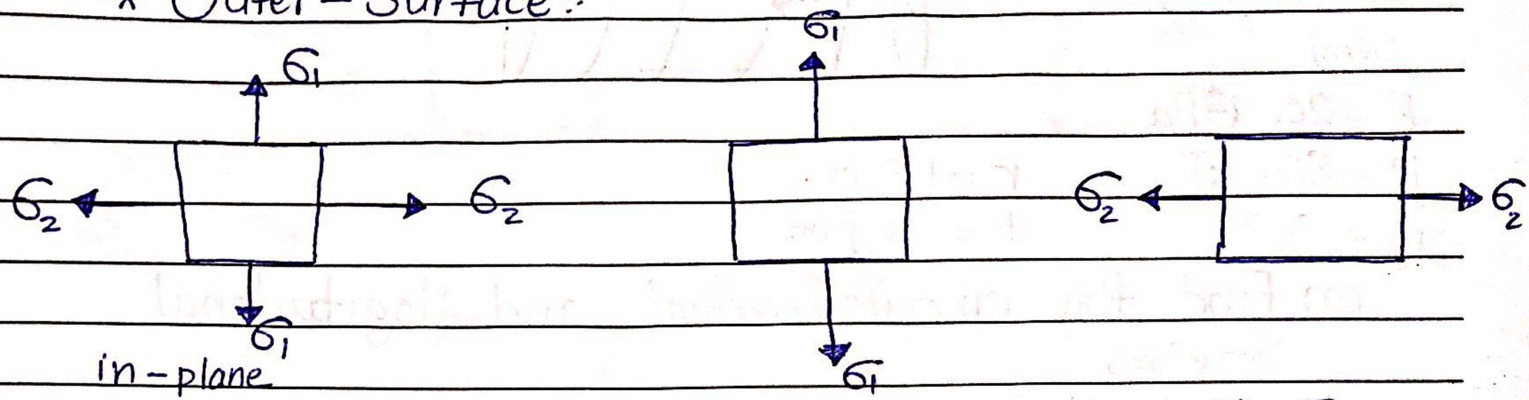
$$\sigma_1 (2Lt) = P(Lr)$$

$$\sigma_1 = \frac{Pr}{t}$$

Hopp-Stress

circumferential Stress.

* Outer-Surface :-

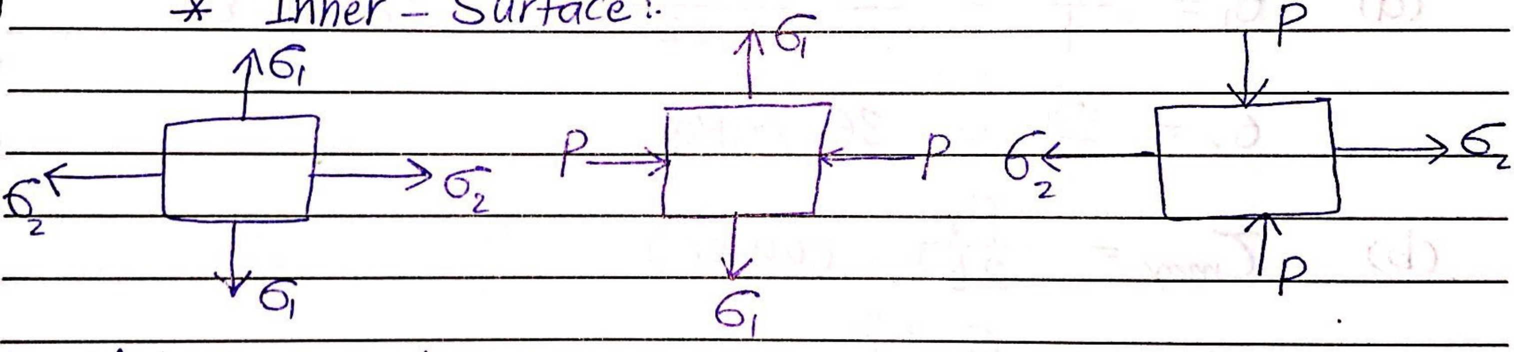


in-plane
x−y　　　　　　y−z　　　　　　x−z

Mohr's circle :-

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{max} = \frac{\sigma_1}{2} = \frac{Pr}{2t} \qquad \tau_{max} = \frac{\sigma_2}{2} = \frac{Pr}{4t}$$

* Inner - Surface :-



Mohr's circle :-

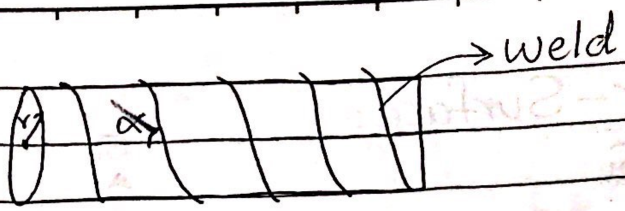
$$\tau_{max} = \frac{Pr}{4t} \qquad \tau_{max} = \frac{\sigma_1 + P}{2} \qquad \tau_{max} = \frac{\sigma_2 + P}{2}$$

Subject: ________ / /

Example:-

$\alpha = 55^\circ$

Steel



$E = 200\ GPa$

$P = 800\ kPa$ $\qquad r = 1.8\ m$

$\nu = 0.3$ $\qquad t = 20\ mm$

(a) Find the circumferential and logitudenal Stresses.

(b) Find the max Shear Stress.

(c) Find ε_x and ε_y

(d) Find τ_w and σ_w (along the weld)

(e) Find the Change of th thickness

Sol.

(a) $$\sigma_1 = \frac{P r}{t} = \frac{(800 \times 10^3)(1.8)}{0.02} = 72\ MPa$$

$$\sigma_2 = \frac{\sigma_1}{2} = 36\ MPa$$

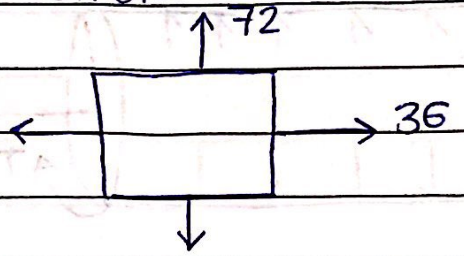
(b) $$\tau_{max} = \frac{Pr}{2t} \quad \text{(outer)}$$

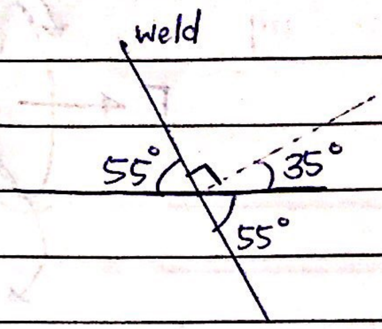
$$\tau_{max} = \frac{\sigma_1 + P}{2} \quad \text{(inner)}$$

(c) $$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu \sigma_y) = \frac{1}{E}[\sigma_2 - 0.3\sigma_1]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_1 - 0.3\sigma_2]$$

(d) Mohr's circle.



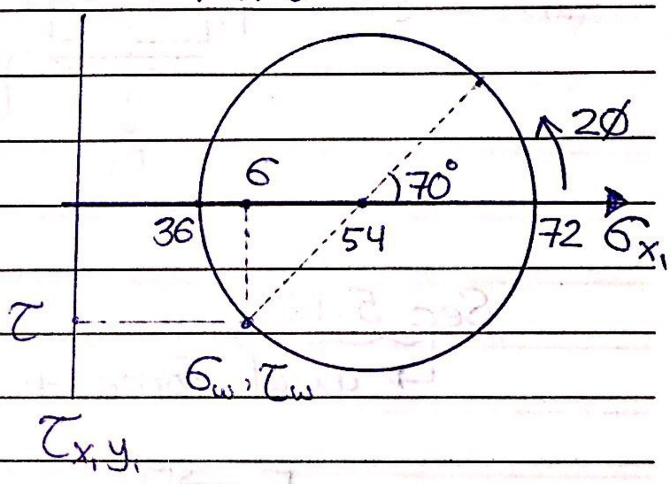


$R = 18$

$\sigma_{avg} = 54$

$$\sigma_w = 54 - 18\cos 70 = 47.8 \text{ MPa}$$

$$\tau_w = 18 \sin 70 = 16.7 \text{ MPa}$$

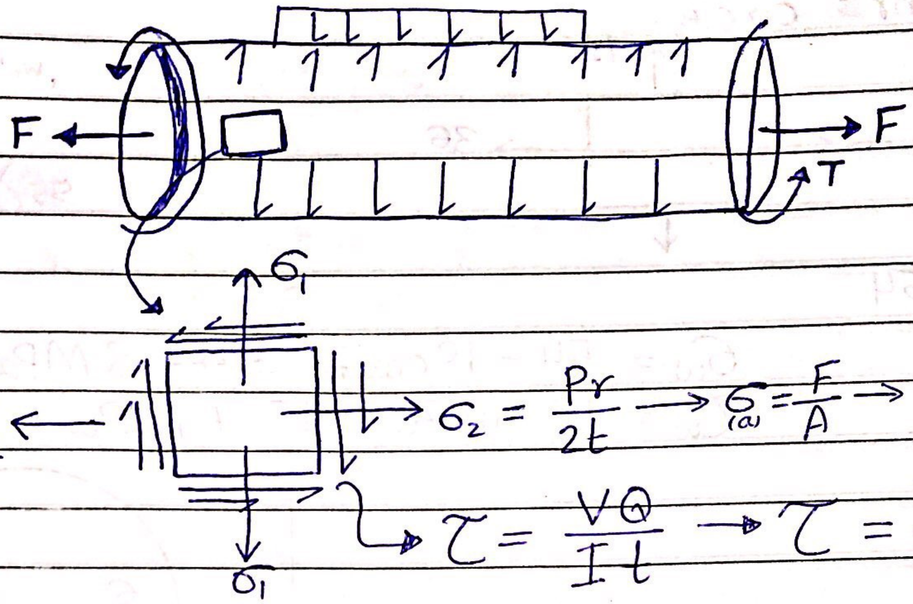


(e) $$\varepsilon_z = \frac{\delta t}{t} = \frac{-\nu}{E}(\sigma_1 + \sigma_2)$$

$$\delta t = -(\varepsilon_z)(0.02)$$

Subject: Sec 5.12 & Sec 8.5

Combind loading:



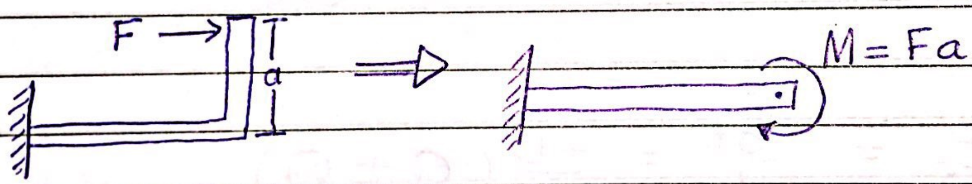
$\sigma_{(b)} \leftarrow \sigma_{(a)} \leftarrow \sigma_2 \leftarrow$

$\rightarrow \sigma_2 = \frac{Pr}{2t} \rightarrow \sigma_{(a)} = \frac{F}{A} \rightarrow \sigma_{(b)} = \frac{-My}{I}$

$\rightarrow \tau = \frac{VQ}{It} \rightarrow \tau = \frac{Tr}{I_p}$

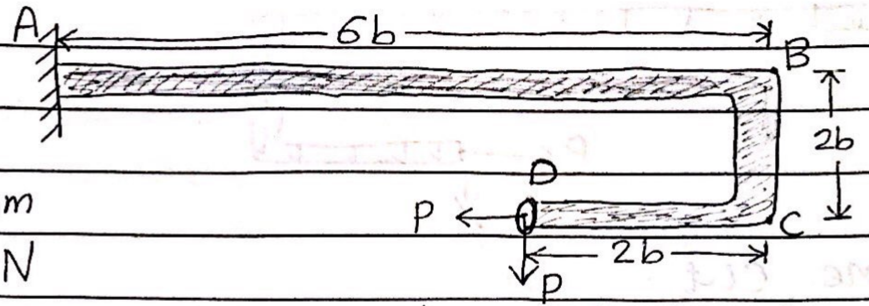
Sec 5.12

↳ axial Force + Bending Moment Only.



Subject: ______ / /

Example: Prob. 5.12.8



$b = 240\ mm$

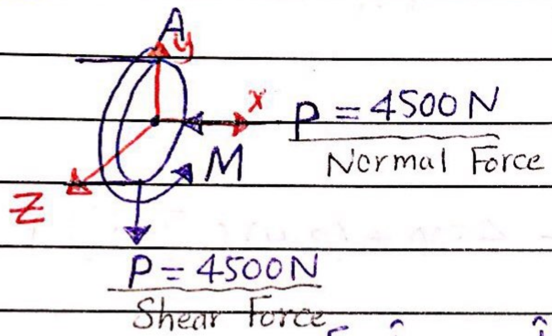
$P = 4.5\ kN$

(a) Find the minimum diameter if $\sigma_{allow} = 110\ MPa$.

(b) Repeat part (a) considering the weight of the pipe if $\rho = 7.7\ kN/m^3$.

Sol:

(a)



$$M = \vec{r} \times \vec{F} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4b & -2b & 0 \\ -P & -P & 0 \end{bmatrix}$$

$$M = (-P(4b)) - (-P)(-2b) \quad \hat{k}$$

$$= -6\ P\ b$$

$$= -6\ (4.5 \times 10^3)(0.24)$$

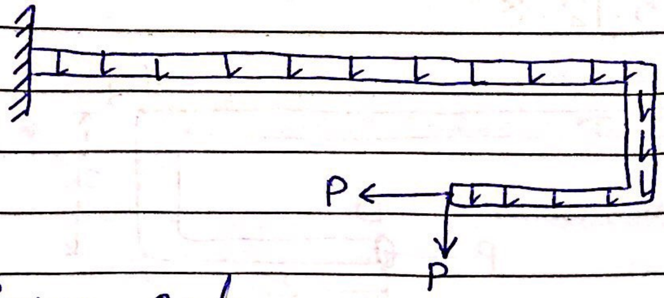
$$= -6480\ N \cdot m\ \hat{k}$$

$$\boxed{\sigma_{max} = \pm \frac{F}{A} \pm \frac{My}{I}}$$

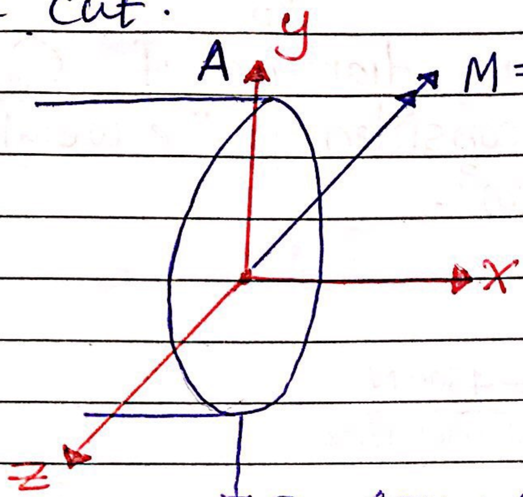
$$\sigma_{max} = 110 \times 10^6 = -\frac{4500}{\frac{\pi}{4} d^2} + \frac{6480\left(\frac{d}{2}\right)}{\frac{\pi}{64}(d)^4}$$

$\therefore \quad d = \text{......}\ m$

(b)



The Same cut.



$M = 6480 + 3b\,W_1 + 6b\,W_2 + 5b\,W_3$

$P = 4500 + (2.4)\left(\frac{\pi}{4} d^2\right) \rho$

$W_1 = 6(b)\left(\frac{\pi}{4} d^2\right) \rho$

$W_2 = 2(b)\left(\frac{\pi}{4} d^2\right) \rho$

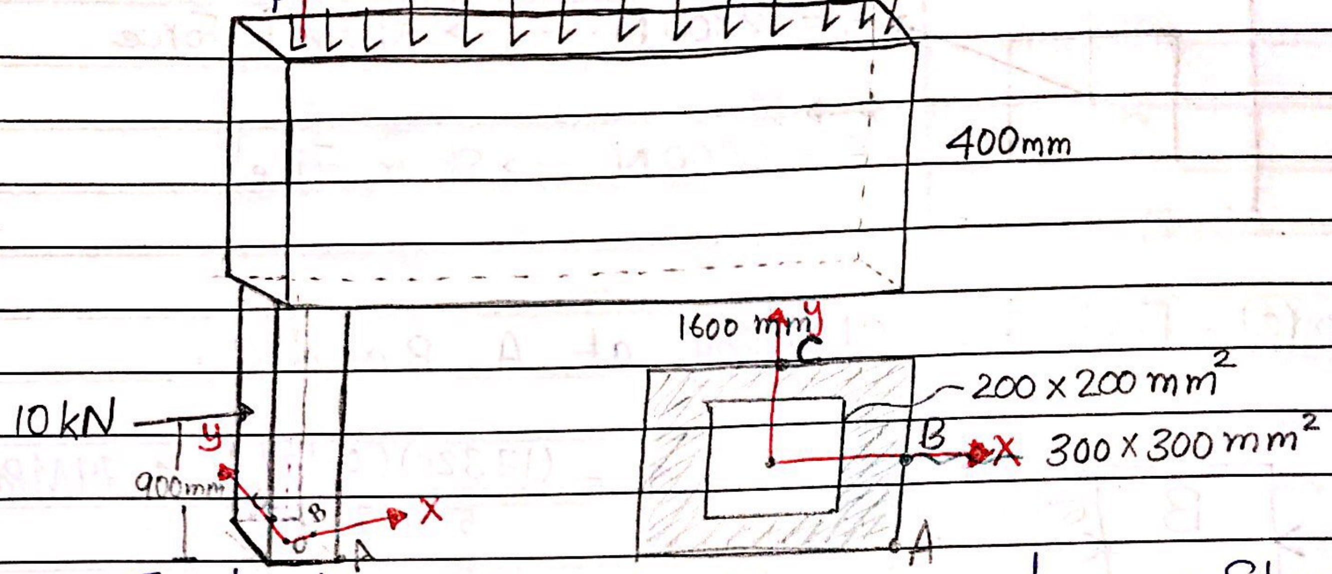
$W_3 = 2(b)\left(\frac{\pi}{4} d^2\right) \rho$

بعد ذلك نعوض بالقانون نفسه
لإيجاد قيمة d .

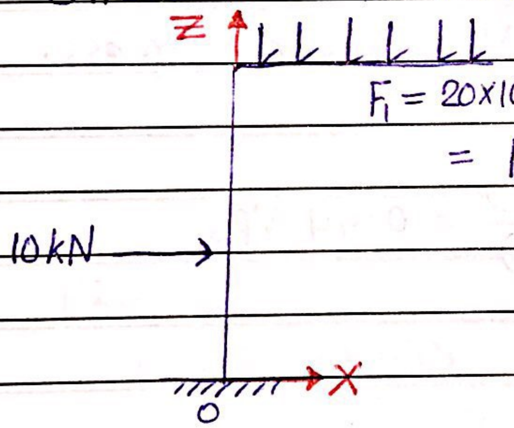
$$\sigma_{max} = \pm \frac{F}{A} \pm \frac{My}{I}$$

Subject: ____________ 10 / 12 / 2018

Example :



* Find the principal Stresses and max Shear Stress at A , B and C .



$F_1 = 20\times10^3 \times 0.4 \times 1.6$

$= 12800\ N$

Step (1) : Find the Moment

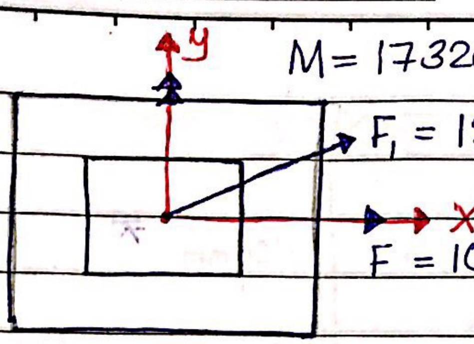
$$M_o = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2}$$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.65 & 0 & L \\ 0 & 0 & -12.8 \end{bmatrix} + \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0.9 \\ 10 & 0 & 0 \end{bmatrix}$$

$$= -(-12.8\times10^3(0.65))\,\hat{j} - (-(0.9)(10\times10^3))\,\hat{j}$$

$$= 17320\,\hat{j}\ (N\cdot m)$$

Subject: ______________ / /

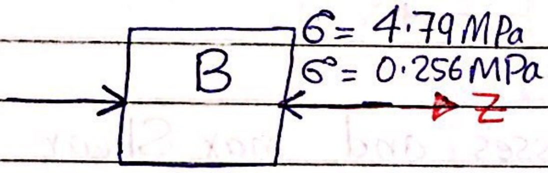


$M = 17320\ N\cdot m$ -------> Bending Moment.

$F_1 = 12800\ N$ ------> Normal Force.

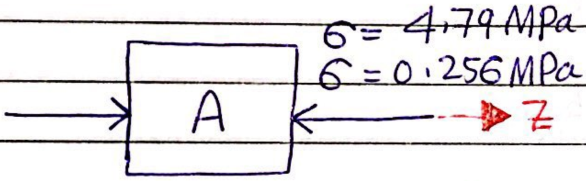
$F = 10000\ N$ ---> Shear Force.

Step (2) :- Find the Stressing at A, B and C.

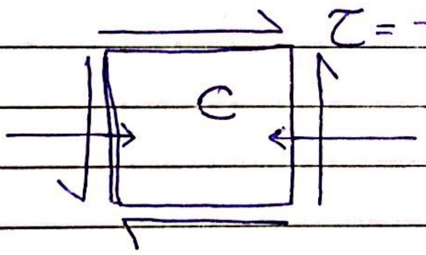


$$\sigma_1 = \frac{(17320)(0.15)}{5.417\times10^{-4}} = 4.79\,MPa$$

$$\sigma_2 = \frac{12800}{0.05} = 0.256\ MPa$$



No Shear Stress.



$$\tau = \frac{(10000)(2.375\times10^{-3})}{(5.417\times10^{-4})(0.1)} = 0.44\ MPa$$

Where :-

$$I = \frac{1}{12}\left(0.3^4 - 0.2^4\right) = 5.417\times10^{-4}\ m^4$$

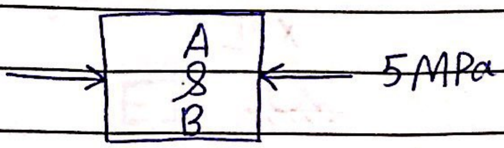
$$A = (0.3^2 - 0.2^2) = 0.05\ m^2$$

$$Q = (0.15)(0.3)(0.075) - (0.1)(0.2)(0.05)$$

$$= 2.375\times10^{-3}\ m^3$$

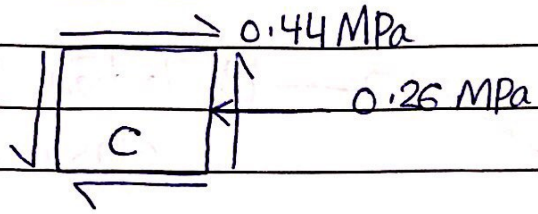
Subject: ______________ / /

Step (3) :- Use Mohr's circle



$\sigma_{avg} = 2.5\,MPa$

$R = 2.5$



$\sigma_{avg} = 0.13\,MPa$

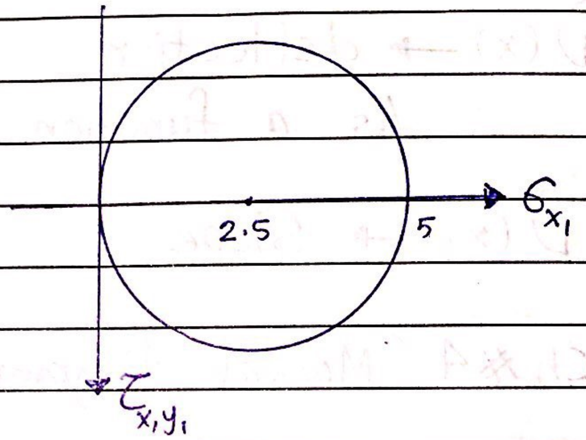
$R = \sqrt{(0.13)^2 + (0.44)^2}$
$= 0.458$

For A & B :-

$\sigma_1 = 5\,MPa$

$\sigma_2 = 0$

$\tau_{max} = 2.5\,MPa$

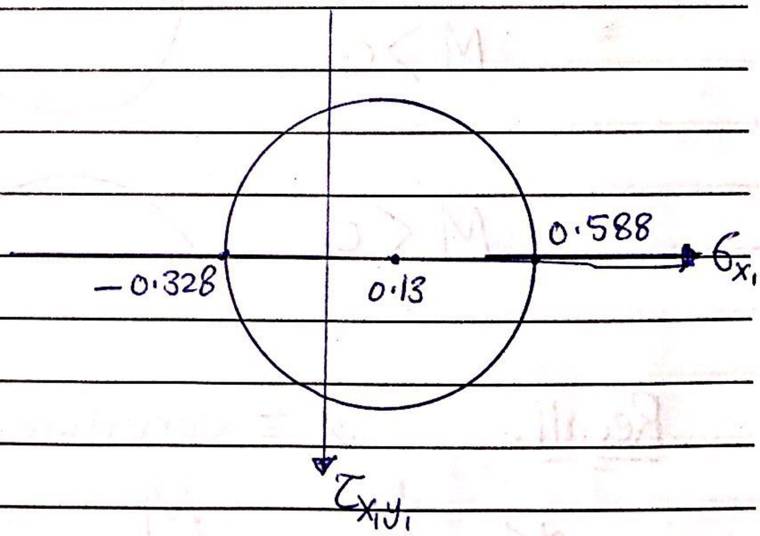


For C :-

$\sigma_1 = 0.588\,MPa$

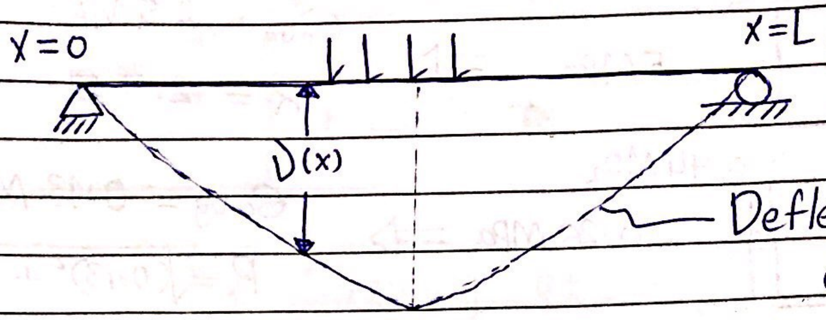
$\sigma_2 = 0.528\,MPa$

$\tau_{max} = 0.458\,MPa$



Subject: Chapter 9

Beam Deflection :-

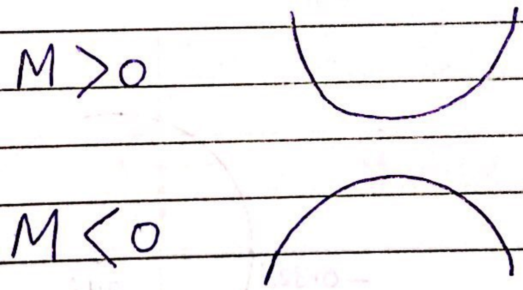


$\nu(x) \rightarrow$ deflection

As a function of "x"

$\nu'(x) \rightarrow$ Slope

Ch #4 "Moment diagram"



Recall :- $\kappa \equiv$ carvature.

$$\kappa = \frac{1}{\rho} = \frac{M}{EI} \quad \text{------- ch \#5}$$

$$\kappa = \frac{1}{\rho} = \frac{d^2\nu}{dx^2} \quad \text{------- Ch\#9}$$

$$*\,*\,* \rightarrow \frac{M}{EI} = \frac{d^2\nu}{dx^2}$$

Subject: ____________ / /

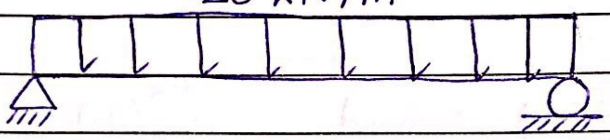
$v(x) \longrightarrow$ deflection

$v'(x) \Longrightarrow$ Slope

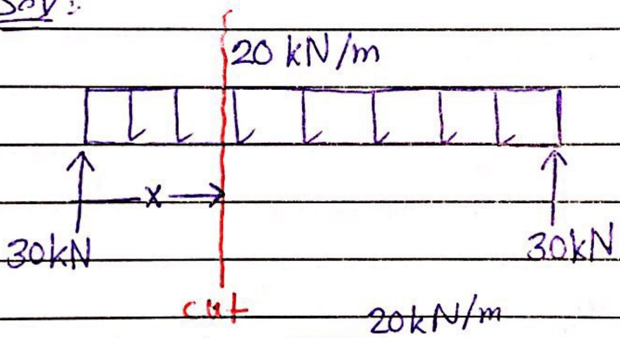
$\therefore \quad EI\, v''(x) \Longrightarrow$ Moment.

$EI\, v'''(x) \Longrightarrow$ Shear Force.

Example: Find M as M(x).

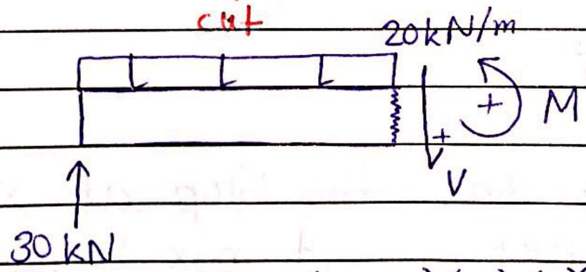


Sol:



$\Longrightarrow \quad v(0) = 0$

$v(L) = 0$



$$M + (20000)(x)\left(\frac{x}{2}\right) - (30000)(x) = 0$$

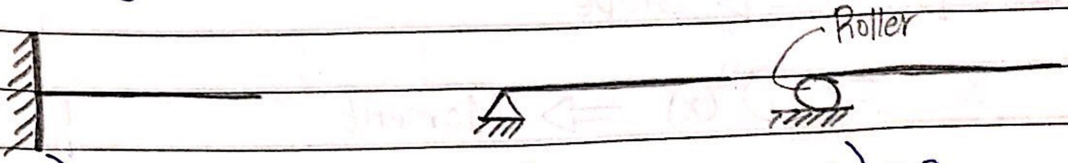
$$M = 30000x - 10000x^2$$

$$EI\frac{d^2v}{dx^2} = 30\,000x - 10\,000x^2$$

$$EI\frac{dv}{dx} = 15000x^2 - \frac{10000}{3}x^3 + C_1$$

$$EI\, v = 5000x^3 - \frac{2500}{3}x^4 + C_1x + C_2$$

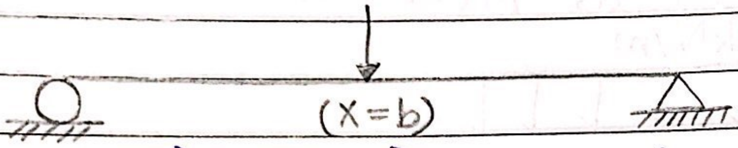
* Boundary Conditions :-



$v = 0$ $v = 0$ $v = 0$

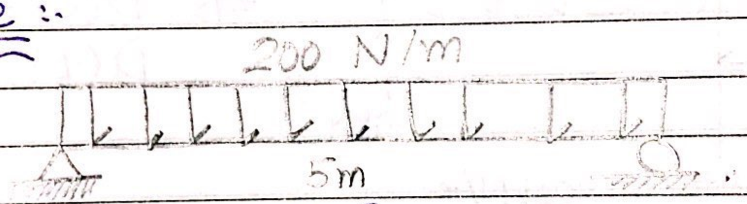
$v' = 0$

*Continuity Conditions:-



$v_1(b) = v_2(b)$ and $v_1'(b) = v_2'(b)$

Example :-

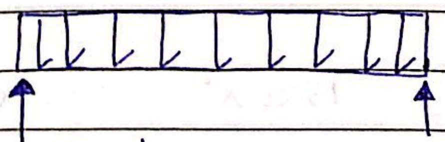


$EI = 1234567.0 \ N \cdot m^2$

(a) Find the deflection and the Slop at $x = 2m$.

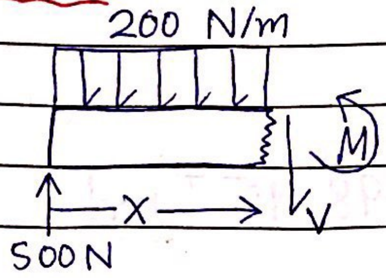
(b) Find the max deflection and max Slope of the Beam.

Step(1) :- Find the Reactions.



Subject: ____________ / /

Step(2) :- Find the moment as a function of "x"



$$M + 200(x)\left(\frac{x}{2}\right) - 500x = 0$$

$$M = 500x - 100x^2$$

valid for $0 \leq x \leq 5$

Step(3) :- Apply (*) with the BC's :-

$\upsilon(0) = 0$

$\upsilon(5) = 0$

$$EI\frac{d^2\upsilon}{dx^2} = 500x - 100x^2$$

$$EI\frac{d\upsilon}{dx} = 250x^2 - \frac{100}{3}x^3 + C_1$$

$$EI\,\upsilon(x) = \frac{250}{3}x^3 - \frac{25}{3}x^4 + C_1x + C_2$$

$$\upsilon(0) = 0 \implies \boxed{C_2 = 0}$$

$\upsilon(5) = 0$

$$0 = \frac{250}{3}(5)^3 - \frac{25}{3}(5)^4 + 5C_1$$

$$\boxed{C_1 = -1041.67}$$

$$\upsilon(x) = \frac{1}{1234567}\left[\frac{250}{3}x^3 - \frac{25}{3}x^4 - 1041.67x\right]$$

$$\theta(x) = \frac{1}{1234567}\left[250x^2 - \frac{100}{3}x^3 - 1041.67\right]$$

Subject: ______ / /

(a) نعوض كل "x" بـ 2
في كل قانون .

$\nu(2) = -0.0025\ m$

$Q(\cancel{x}\ 2) = \nu'(2) = -2.498 \times 10^{-4}\ rad$

(b) Max moment $\Rightarrow$ Max ν

$\therefore$ at $x = 2.5\ m$

$\nu_{max}(2.5) = -1.32 \times 10^{-3}\ m$

Max Slope $\Rightarrow$ Min. Moment

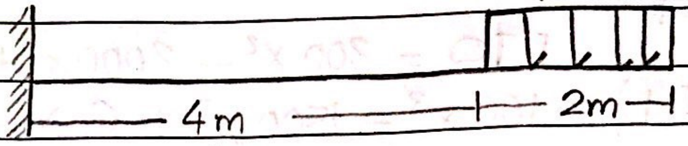
$\therefore$ at $x = 0$

OR $x = 5$ (But the sign will be (-))

$\nu'_{max}(0) = -8.437 \times 10^{-4}\ rad$

Subject: ________ / /

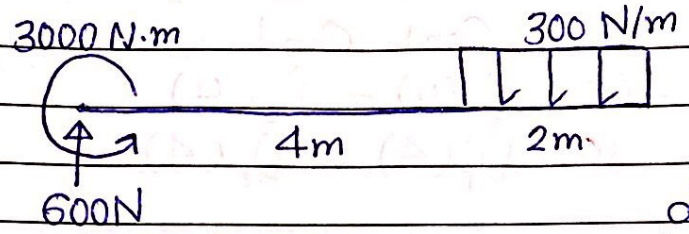
Example:



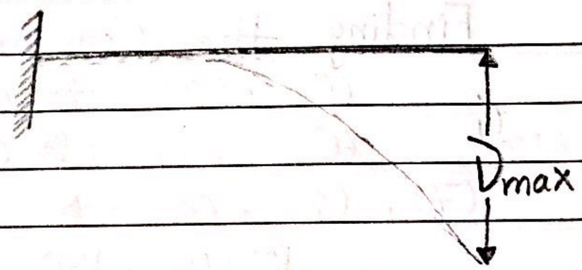
$EI = 7654321\ N\cdot m^2$

* Find max Slope and deflection at the Free End.

Step(1): Reactions:

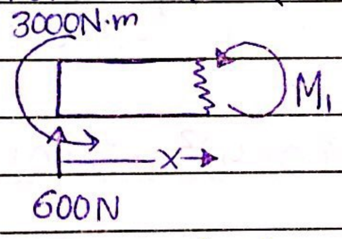


deflection:



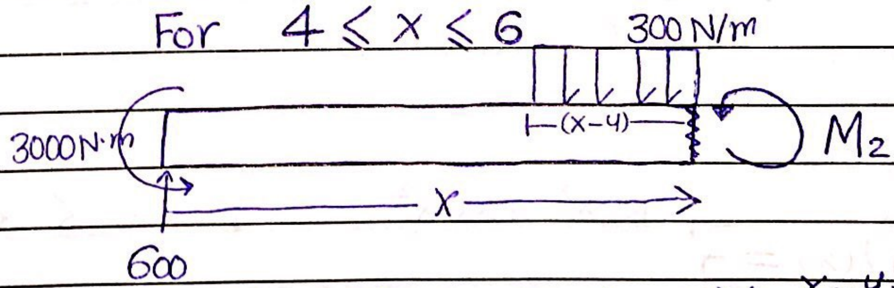
Step(2): $M(x) = ??$

For $0 \leq x \leq 4$



$\Rightarrow M_1 = 600x - 3000$

For $4 \leq x \leq 6$



$$M_2 + 3000 + 300(x-4)\left(\frac{x-4}{2}\right) - 600x = 0$$

$$\Rightarrow M_2 = 600x - 3000 - 1500(x-4)^2$$

Subject:

Step (3):-

$$EI\theta_1 = 300x^2 - 3000x + C_1 \quad \text{------} (1)$$

$$EIv_1 = 100x^3 - 1500x^2 + C_1 x + C_2 \quad \text{------} (2)$$

$$EI\theta_2 = 300x^2 - 3000x - 50(x-4)^3 + C_3 \quad \text{---} (3)$$

$$EIv_2 = 100x^3 - 1500x^2 - 12.5(x-4)^4 + C_3 x + C_4 \quad \text{----} (4)$$

BC's:- Cont. Cond. :-

$v_1(0) = 0 \quad \text{...} (5)$ $\qquad v_1(4) = v_2(4) \quad \text{-----} (7)$

$v_1'(0) = 0 \quad \text{---} (6)$ $\qquad v_1'(4) = v_2'(4) \quad \text{----} (8)$

Finding the Constants:-

(5), (2) $\Longrightarrow C_2 = 0$

(6), (1) $\Longrightarrow C_1 = 0$

(7), (2), (4) $\Longrightarrow$

$$100(4)^3 - 1500(4)^2 = 100(4)^3 - 1500(4)^2 + 4C_3 + C_4$$

(1), (3), (8) $\Longrightarrow$

$$300(4)^2 - 3000(4) = 300(4)^2 - 3000(4) + C_3$$

$\Longrightarrow C_3 = 0$

$C_4 = 0$

$$v(x) = \begin{cases} 100x^3 - 1500x^2 & ; \quad 0 \leq x \leq 4 \\ 100x^3 - 1500x^2 - 12.5(x-4)^4 & ; \quad 4 \leq x \leq 6 \end{cases}$$