

 **Mech
Family**

نقدہ

دفتر

STRENGTH

اعداد احمد جبر

** Mechanics Engineering **

{ Behavior of material under loads.

For:

Stress, strain, deflections, stability...

(Material)

Properties

geometry



Chemical

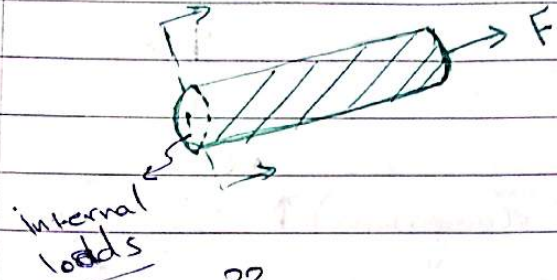
Physical



Strength

thermal

- To find the external load make a section in the (Material).



??

P



(external)

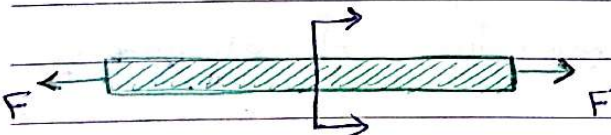
- By applying statics equ.

$$\sum F_x = 0 \Rightarrow (F = P) \#$$

- The internal force (P) is distributed over the cross-section area. (... $\sigma_x dA$)

** Stress & strain **

① Stress:



(external force)

- [internal force ^{Produce} → Stress]

then
Stress:

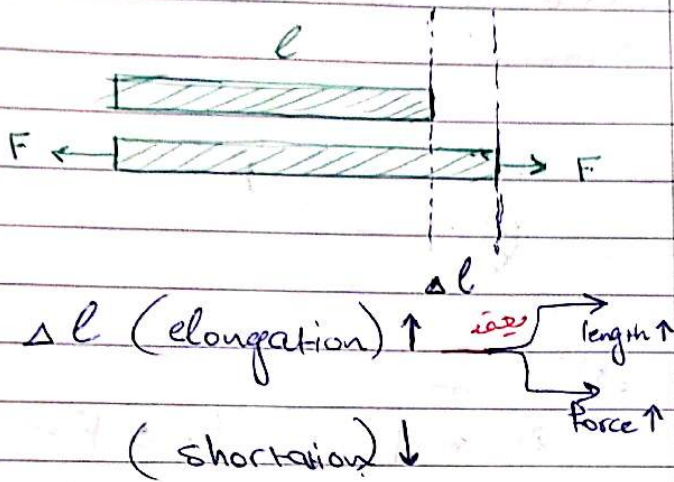
Is the distribution of the internal loads over the cross-section per unit area.

- external force ^{Prod.} → internal loads

$$\sigma = \frac{F}{A}$$

N. Stress

② Strain:-



* How do we relate σ, ϵ ??

$$(\sigma \propto \epsilon)$$

- to find this relation, a (tensile test) is performed for a given material.

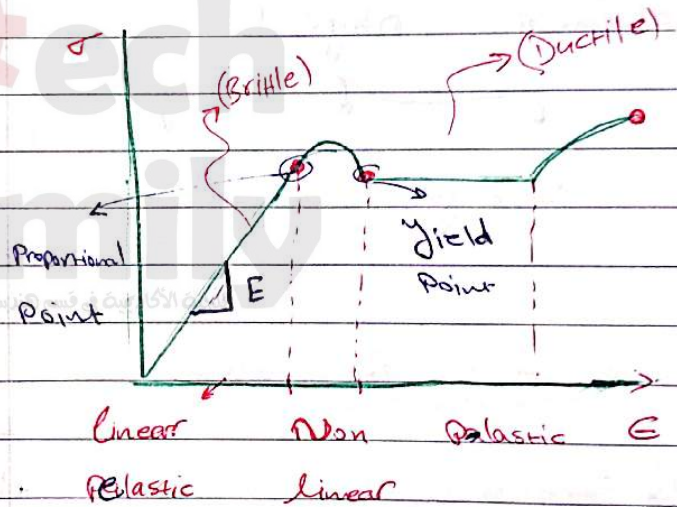
F	l	Δl	σ	ϵ
F_1	l_1	$l_1 - l_0$	$\frac{F_1}{A}$	$\frac{\Delta l}{l}$
...	...	...	...	...
F_n	...	...	...	...

Note:-

Δl :- تكون قيمتها أكبر كلما زادت (زاد) طول القطعة (المادة) وتقل إذا كانت = قصيرة وذلك (تأثير) Internal Force على مسافة ممتدة

$$\text{Strain} = \frac{\Delta l}{l} = \epsilon$$

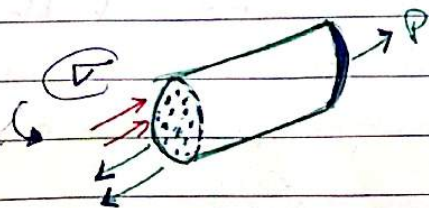
= elongation per unit length



stress = force per unit area
= σ (Normal stress)

E:- modulus of elasticity

$$E = \frac{\sigma}{\epsilon} \text{ (Slope)}$$



σ is a normal stress $(\oplus)$ when it is (outward) from the surface and $(\ominus)$ when (inward) surface

elastic $\Rightarrow$ $\sigma(F)$ always plastic $\Rightarrow$

(Perpendicular on surface) $\sigma \Rightarrow$ Normal stress

(Parallel to surface) $\tau \Rightarrow$ Shear =

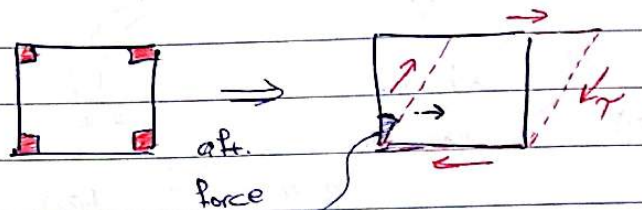
$\rightarrow$ in the elastic zone the material goes back to its original shape and dimension after removing the loads



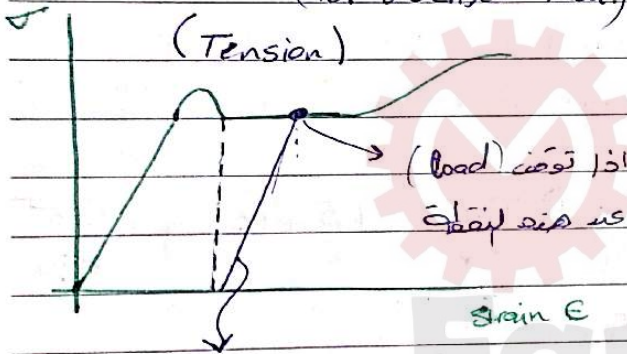
this distribution of the internal

- Force per unit area $\Rightarrow$ stress parallel to the surface called [shear stress] (τ)

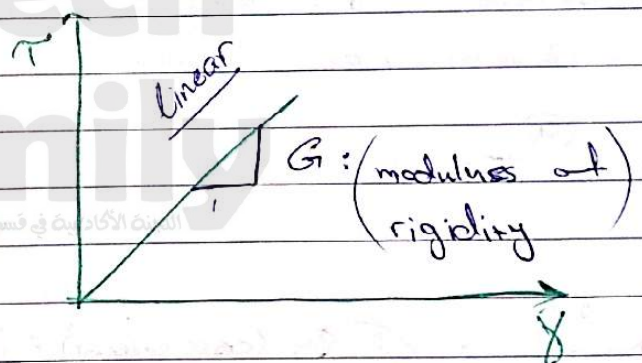
$\rightarrow$ in the plastic zone the material goes under permanent deformation



(for ductile mat)

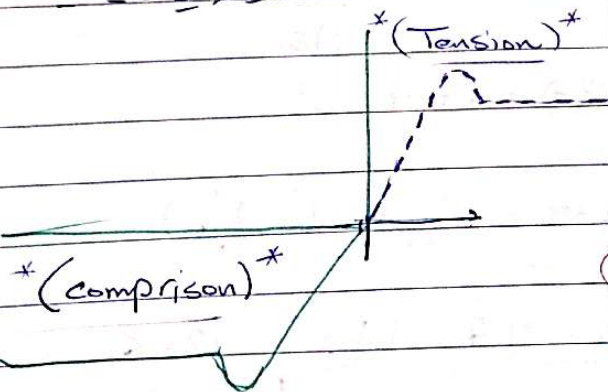


(deformation) تشوه



(linear elastic zone) المنطقة المرنة الخطية

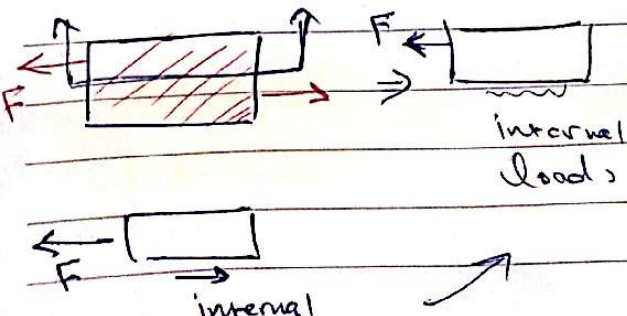
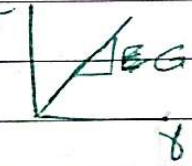
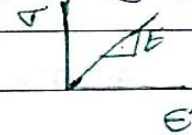
المادة الأكاديمية في قسم هندسة الميكانيك - الجامعة الأردنية



- In linear elastic zone:

$$\sigma = E \epsilon$$

$$\tau = G \gamma$$

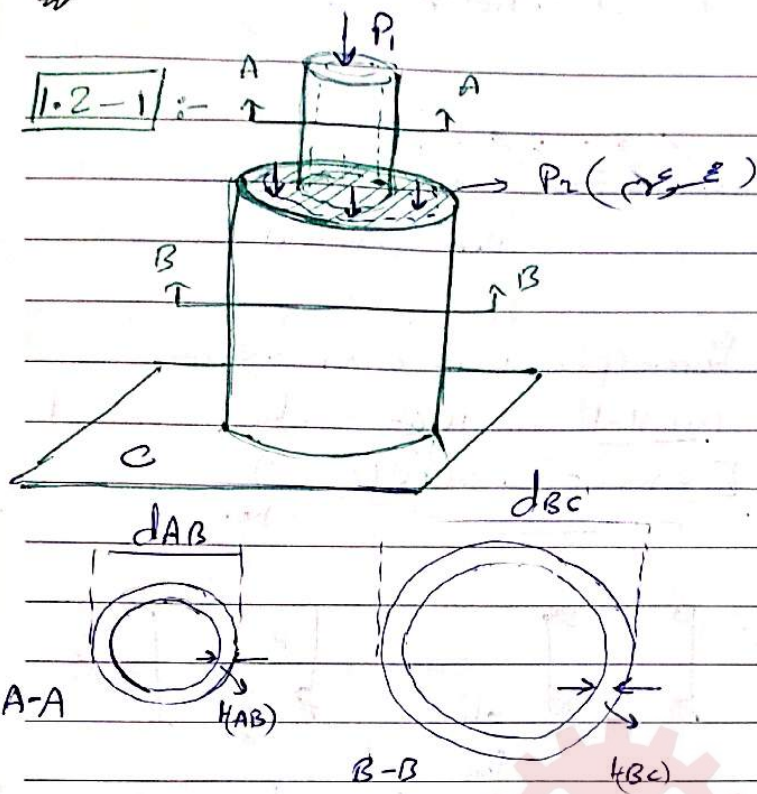


E : modulus of elasticity

G : = rigidity

$$[P, \tau] = \text{Pascal}$$

$$\frac{N}{m^2}$$



b) (what is the stress (P_2) if)

$$\sigma_A = (\sigma_B)$$

$$\sigma_B = \frac{F_2}{A} = \sigma_A$$

$$\sigma_A = \frac{F_2}{\left(\frac{\pi}{4}(57)^2 - \frac{\pi}{4}(39)^2\right)}$$

$$F_2 = P_1 + P_2$$

$$F_2 = 13.4 \text{ KN}$$

$$P_2 = F_2 - P_1 = 5.9 \text{ KN}$$

c) what (t_{BC}) if $P_2 = 10$ and P_1 remain 7.5 ??

$$P_1 = 7.5 \text{ KN}$$

$$d_{AB} = 32 \text{ mm} \quad d_{BC} = 57 \text{ mm}$$

$$t_{AB} = 12 \text{ mm} \quad t_{BC} = 9 \text{ mm}$$

$$F = P_1 + P_2 = 7.5 + 10 = 17.5 \text{ KN}$$

$$\sigma_{BC} = \frac{F}{A}$$

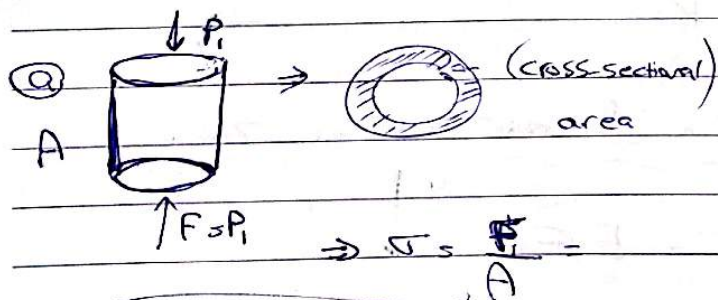
$$9.95 \text{ MPa} = \frac{17.5 \times 10^3}{A_{BC}}$$

$$A_{BC} = 1759 \text{ mm}^2$$

$$1759 = \frac{\pi}{4} (57^2 - d_1^2)$$

$$d_1 = 31.7 \quad t_{BC} = 12.6 \text{ mm}$$

$$\sigma_{BC} (P) \text{ and } \sigma_{AB}$$



$$A = \frac{\pi}{4} (32^2) - \left(\frac{\pi}{4}\right) (8)^2$$

$$A = 754 \text{ mm}^2$$

$$\sigma = \frac{7.5 \times 10^3}{754 \times 10^{-6}} = 9.9 \text{ MPa}$$

$$\tau = E \cdot \epsilon \quad \& \quad \sigma = \frac{P}{A}$$

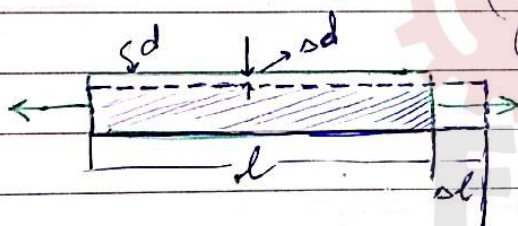
$$E = \frac{\Delta l}{l}$$

$$\left(\frac{P}{A} = E \frac{\Delta l}{l} \right)$$

$$\boxed{\Delta l = \frac{P l}{EA}} \quad \times \quad \left\{ \begin{array}{l} \text{نقص في الطول} \\ \text{انحراف الى الداخل} \end{array} \right.$$

$$\tau = \gamma G \quad \& \quad \left(\tau = \frac{F}{A} \right)$$

* Poisson's ratio :-



النقص في العرض
نقص في العرض
انحراف الى الداخل

$$\boxed{\nu = - \frac{\epsilon'}{\epsilon}}$$

ϵ' : lateral strain

ϵ : axial strain

$$\left(\epsilon = \frac{\Delta l}{l} \right) \quad \left(\epsilon' = - \frac{\Delta d}{d} \right)$$

انحراف في العرض

$$\boxed{G = \frac{E}{2(1+\nu)}}$$

(modulus of rigidity)

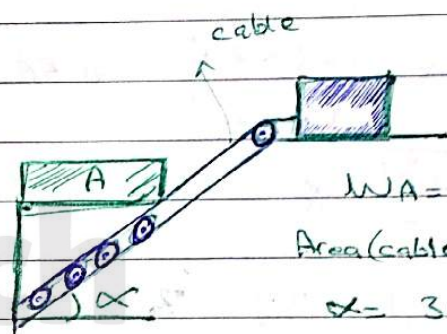
* Factor of safety :-

- Yield strength $\Rightarrow$ permanent deformation
- Ultimate $\Rightarrow$ Fracture

$$\Rightarrow F.S. = \frac{\text{Strength}}{\text{Stress}} = (n)$$

$n < 1$ ~~safe~~
 $n > 1$
- deformation
- fracture

Problem 1.2.6



$$W_A = 130 \text{ kN}$$

$$\text{Area (cable)} = 490 \text{ mm}^2$$

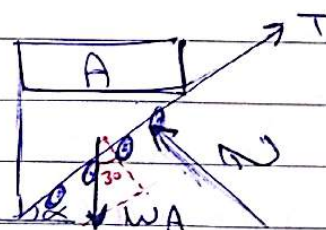
$$\alpha = 30^\circ$$

- calculate (σ_T) ??

(tensile stress)

in the cable

الحل :-
(F.B.D)



$$N = W_A \cos 30^\circ$$

$$T = W_A \sin 30^\circ = 65 \text{ kN}$$

$$P \leftarrow \boxed{} \rightarrow T = 65 \quad T = P = 65$$

$$\sigma_T = \frac{P}{A} = \frac{65 \times 10^3}{490 \times 10^{-6}} = \boxed{13.3 \times 10^8} \text{ Pa} = 133 \text{ MPa}$$

~~1.3.3~~

(b) if $\sigma = 150 \text{ MPa}$ $\alpha = ??$

Sol:

$$T = 130 \sin \alpha$$

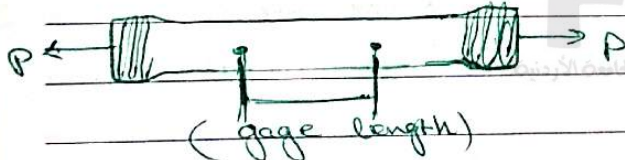
$$\sigma = \frac{P}{A} \Rightarrow 150 \times 10^6 = \frac{P}{490 \times 10^{-6}}$$

$$P = 73.5 \text{ kN}, (T) = W \sin \alpha$$

$$\sin \alpha = \frac{73.5 \times 10^3}{130 \times 10^3} = 0.56$$

$$\alpha = 34.9^\circ$$

Problem (1.4.3) (3 materials)
(A, B, C)



$$\phi = 12 \text{ mm}$$

$$\text{gauge } (L) = 50 \text{ mm}$$

$$\begin{aligned} L_{FA} &= 54.2 & D_A &= 11.46 \\ L_{FB} &= 63.2 & D_B &= 9.48 \\ L_{FC} &= 69.4 & D_C &= 8.06 \end{aligned}$$

$\Rightarrow$ Find: (a) % elongation
(b) %

(c) type of material.

$\rightarrow$ (A) Brittle (B, C) ductile

$$\% \Delta l = \frac{L_f - l}{L} \times 100\%$$

$$\% A = \frac{A - A_f}{A} \times 100\%$$

(c)

$P_{or}(A)$

$$\% \Delta l \Rightarrow \frac{54.2 - 50}{50} \times 100\% \Rightarrow 8.4$$

(B, C) ductile

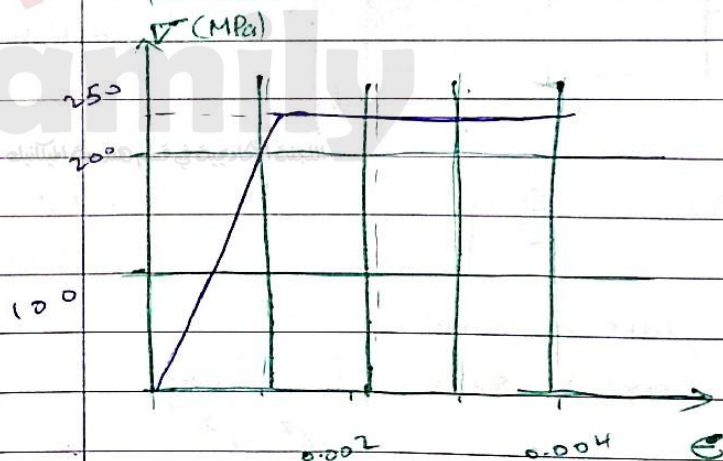
(c)

A $\rightarrow$ Brittle

B, C $\rightarrow$ ductile

[A: l (Pailar) قسمة التواء]
[B, C: ل (تقوى كافي) (B, C) د]

1.5-2



$$L = 2 \text{ m}$$

σ_y = yield strength = (250 MPa)

$$E = 200 \text{ GPa}$$

$$\Delta l = 6.5 \quad (P > L)$$

$\Rightarrow$ (دالة التواء) (تقوى كافي)
% load elongation

1.6-5 / 1.7-3 / 1.7-11 / 1.7-16 / 1.8-10
1.9-10
1.9-13

Sol: (we will find ϵ)

if $\epsilon > \epsilon_y \Rightarrow$ deformation

$\epsilon < \epsilon_y \Rightarrow$ No change

$$\Rightarrow \epsilon = \frac{\Delta l}{l} = \frac{6.5}{2} = \boxed{3.25} \times 10^{-3}$$

$$\Rightarrow \epsilon_y = ??$$

$$\hookrightarrow \sigma_y = \epsilon_y \cdot E$$

$$\epsilon_y = \frac{\sigma_y}{E} = \frac{250 \times 10^6}{200 \times 10^9}$$

$$\boxed{\epsilon_y \Rightarrow 1.25 \times 10^{-3}}$$

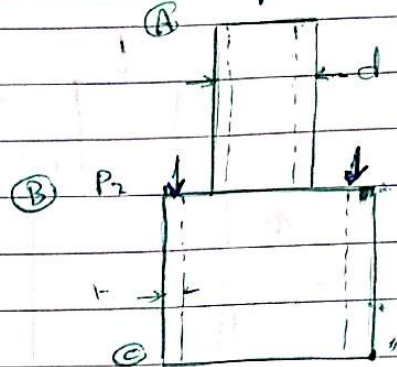
$$\epsilon > \epsilon_y$$

$\hookrightarrow \epsilon$ after removing the load

$$\Rightarrow \epsilon - \epsilon_y = \boxed{2 \times 10^{-3}}$$

$$2 \times 10^{-3} = \frac{\Delta l}{l} \Rightarrow \boxed{\Delta l = 4 \text{ mm}}$$

$\Rightarrow 1.5-7$



$$P_1 = (118 \text{ kN})$$

$$P_2 = (98 \text{ kN})$$

$$d_{AB} = 31 \text{ mm}$$

$$t_{AB} = 12 \text{ mm}$$

$$d_{BC} = 57 \text{ mm}$$

$$t_{BC} = 9 \text{ mm}$$

$$E = 96 \text{ GPa}$$

$$\Delta t_{BC} = 5 \times 10^{-3} \text{ mm (P is applied)}$$

comp.

$\rightarrow$ Find the increase in inner diameter (BC)

$\hookrightarrow$ 95

$\hookrightarrow \Delta t_{AB}$

$\hookrightarrow$ increase in the inner dia (AB)

Sol:

$$\frac{\Delta t_{BC}}{t_{BC}} = \frac{\Delta d_{in}(BC)}{d_{in}(BC)}$$

$$\frac{5 \times 10^{-3}}{9 \times 10^{-3}} = \frac{\Delta d}{57 - 2(9)}$$

$$\boxed{\Delta d = 0.0217 \text{ mm}} \Rightarrow 21.7 \text{ } \mu\text{m}$$

(b) we need to find: (ϵ', ϵ) .

$$\nu = -\frac{\epsilon'}{\epsilon}$$

$$\epsilon = \frac{\sigma}{E}$$

$$(P_1 + P_2) = \epsilon E$$

$$\epsilon = \frac{216}{96 \times 10^9} \Rightarrow 2.25 \times 10^{-9}$$

$$(P_1 + P_2) = F$$

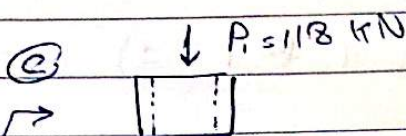
$$\sigma = \frac{F}{A} \Rightarrow \frac{216 \times 10^3}{\frac{\pi}{4} (57^2 - 39^2)}$$

$$\sigma = 159 \text{ MPa}$$

$$\epsilon = \frac{\sigma}{E} \Rightarrow \frac{159 \times 10^6}{96 \times 10^9} \Rightarrow 1.66 \times 10^{-3} \quad (\text{comp.})$$

$$\Rightarrow \nu = -\frac{\epsilon'}{\epsilon} = \frac{\Delta L_{BC}}{L_{BC}} = -1.66 \times 10^{-3}$$

$$\nu = + \frac{\frac{5 \times 10^{-3}}{9}}{1.66 \times 10^{-3}} \Rightarrow 0.3$$



$$\sigma_{AB} = -\frac{F}{A} = -\frac{118 \times 10^3}{\frac{\pi}{4} (31^2 - 7^2)}$$

$$\sigma_{AB} = -165 \text{ MPa}$$

$$\epsilon = \frac{-165 \text{ MPa}}{96 \text{ GPa}} = -1.72 \times 10^{-3}$$

$$\nu = -\frac{\epsilon'}{\epsilon} = 0.33 = -\frac{\epsilon'}{-1.72 \times 10^{-3}}$$

$$\epsilon' = 5.6 \times 10^{-4}$$

$$\epsilon' = \frac{\Delta L_{AB}}{L_{AB}}$$

$$5.6 \times 10^{-4} = \frac{\Delta L_{AB}}{12}$$

$$\Delta L_{AB} = 6.8 \times 10^{-3} \text{ mm}$$

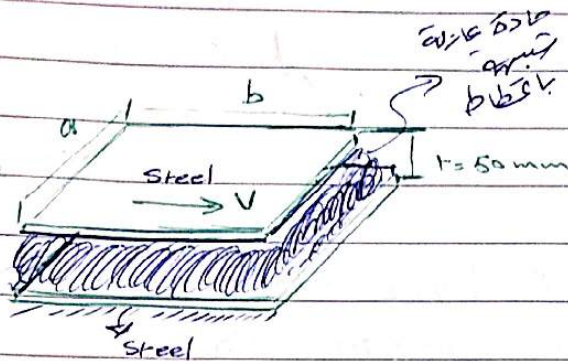
(d) $\Delta d_{in}(AB)$

$$\epsilon' = \frac{\Delta d_{in}(AB)}{d_{AB}}$$

$$5.67 \times 10^{-4} = \frac{\Delta d}{7}$$

$$\Delta d = 4 \times 10^{-3} \text{ mm}$$

* 1.6-8



$$a = 125 \text{ mm}$$

$$b = 240 \text{ mm}$$

$$V = 12 \text{ kN}$$

$$\text{lateral disp} = (8 \text{ mm})$$

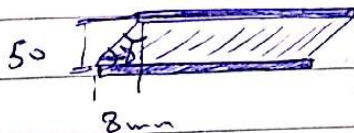
Find G ??

Sol:

$$G = \frac{\tau}{\gamma}$$



(a.f.) load



$$\gamma = \tan^{-1} \frac{8}{50} \Rightarrow \gamma = 9.1^\circ = 0.16 \text{ rad}$$

$$\gamma = \tan^{-1} \frac{8}{50} = 9.1^\circ = 0.16 \text{ rad}$$

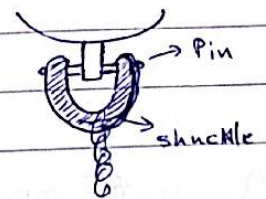
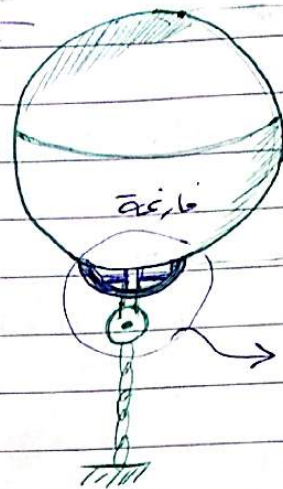
$$\tau = \frac{V}{A} = \frac{12 \times 10^3}{(125)(240) \times 10^{-6}}$$

$$\tau = 0.4 \text{ MPa}$$

$$\Rightarrow G = \frac{0.4 \times 10^6}{0.16} \Rightarrow 2.5 \text{ MPa}$$

* 1.6-10

(Water)



$$d(\text{pin}) = 13 \text{ mm}$$

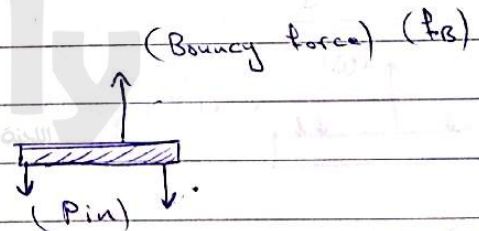
$$t(s) = 6 \text{ mm}$$

$$W(\text{sphere}) = 8 \text{ kN [on land]}$$

Determine $\tau_{\text{avg}}(\text{Pin})$

$\tau(\text{bearing})$

Sol:

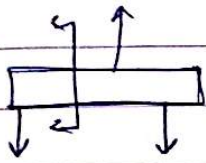


(shackle force) (F_s)

$$F_B = W(\text{sphere}) \times f \times g - W$$

$$F_B = \frac{4}{3} \pi (r^3) \times 1000 \times 9.81 - 8 \times 10^3$$

$$F_B = 9.3 \text{ kN} \Rightarrow 2134 \text{ N}$$



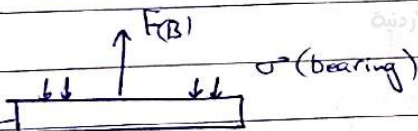
$\uparrow V = F_{\text{shear}} = F_B/2$
 $V = \frac{9.3}{2}$
 $V = 4.65 \text{ kN}$

$$\tau = \frac{V}{A}$$

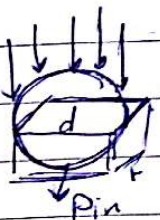
$$\tau = \frac{4.65 \times 10^3}{\frac{\pi}{4} (13)^2 \times 10^{-6}}$$

$$\tau_{\text{avg}} = 35 \text{ MPa} \quad \#$$

⑤ $\sigma(\text{bearing})$ (تحت الضغط)



$$\sigma(\text{bearing}) = \frac{F}{A(\text{projection})}$$

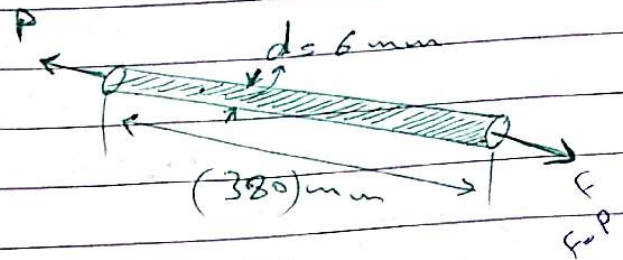


$A(\text{proj}) = d \cdot t$
 $= (13)(6) \times 10^{-6}$
 $= 78 \times 10^{-6} \quad \#$

$$\sigma(\text{bearing}) = \frac{4.65 \times 10^3}{78 \times 10^{-6}}$$

$$\sigma_b = 60 \text{ MPa} \quad \#$$

$$\times 1.8 - 1$$



$E = (42.2) \text{ GPa}$
 $\tau_{\text{all}} = (89.6) \text{ MPa}$
 $\Delta l = 0.08$

- Find $(P)_{\text{allowable}}$?

Sol:

- For (τ_{all})

$$\tau = \frac{F}{A} = \frac{P}{\frac{\pi}{4} (6)^2 \times 10^{-6}} = 89.6 \times 10^6$$

$$P = 2.53 \text{ kN} \quad \#$$

- For Δl_{max}

$$E = \frac{\Delta l}{l} \quad E = \frac{\sigma}{\epsilon}$$

$$\Rightarrow \Delta l = \frac{P}{A} \cdot \frac{l}{E}$$

$$P = \frac{(\Delta l)(A)(E)}{l}$$

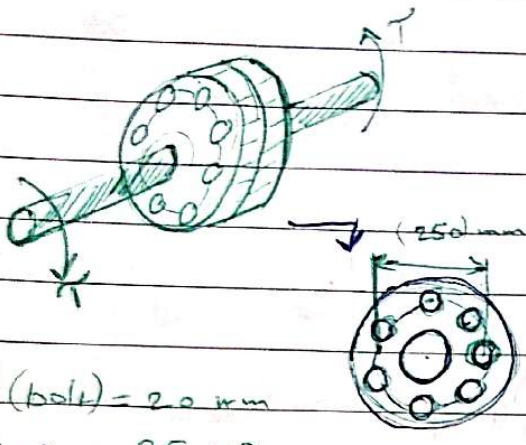
$$P = \frac{(0.08 \times 10^{-3}) \left(\frac{\pi}{4} \right) (6)^2 \times 10^{-6} (42.2 \times 10^9)}{380 \times 10^{-3}}$$

$$P = 0.25 \text{ kN} \quad \#$$

Then $(P_{\text{all}}) = 0.25 \text{ kN} \quad \#$

Beam $\Rightarrow$ bending load
Truss $\Rightarrow$ Tension

* 1.8-2 :- (تجهيز المحور)



$d(\text{bolt}) = 20 \text{ mm}$

$\tau_{all} = 85 \text{ MPa}$

$\# \text{ bolts} = 10$



- find $T_{(max)}$??
(Torque) (on the bolts)

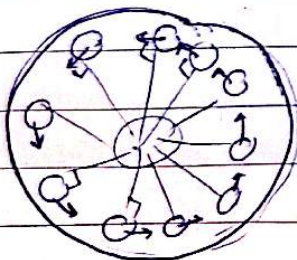
Sol:

$$\tau_{all} = \frac{V}{A}$$

$$V = (\tau_{all})(A)$$

$$= (85 \times 10^6) \left(\frac{\pi}{4} \right) (20)^2 \times 10^{-6}$$

$$V = 2.67 \text{ kN} = [F] \checkmark$$

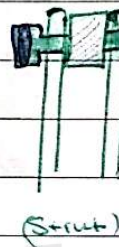
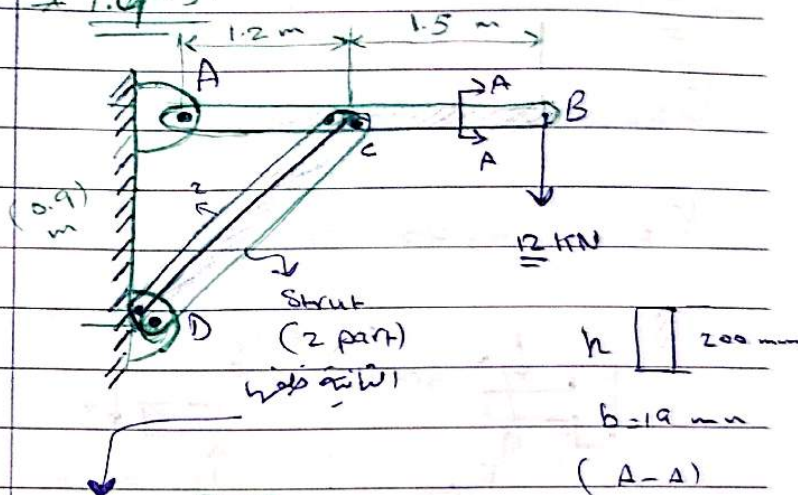


$$T = F r$$

$$= (2.67 \times 10^3) (10) \left(\frac{250}{2} \right) \times 10^{-3}$$

$\# \text{ bolts} \quad T \Rightarrow 3337.5 \text{ N.m}$

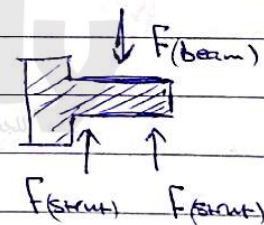
* 1.9-3



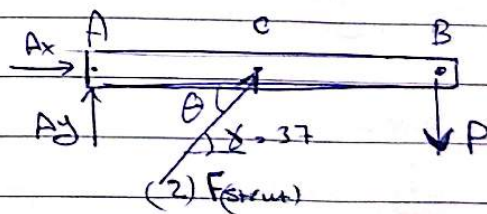
- find d_{min} bolt ??

Sol:

F.B.D for the bolt :



- F.B.D for the beam :



$$\theta = \gamma = \tan^{-1} \frac{0.9}{1.2} \Rightarrow 37^\circ$$

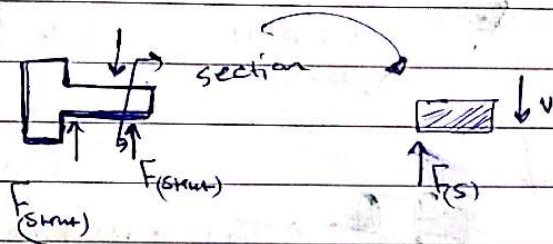
$$M_A = -(P)(2.7) + 2 F_{strut} \sin 37 (1.2) = 0$$

$$\Rightarrow F_{strut} = 22.5 \text{ kN}$$

- Now:

$$F_{\text{beam}} = 2 F_{\text{strut}}$$

$$F_{\text{beam}} = (45 \text{ kN})$$

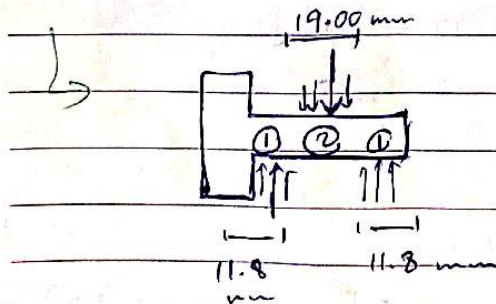
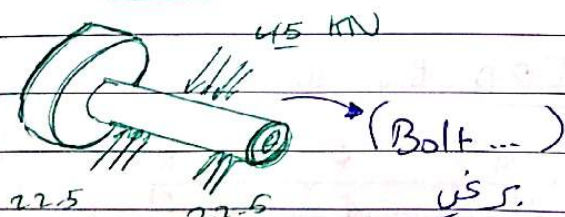


$$- T_{\text{all}} = \frac{V}{A}$$

$$A = \frac{22.5 \times 10^3}{\cancel{\pi d^2} \times 90 \times 10^6} = \frac{\pi d^2}{4}$$

$$[d = 17.8] \text{ mm} \#$$

- if $\sigma_{\text{bearing (all)}} = 130 \text{ MPa}$
find d (min) bolt?



$$F_{\text{bearing}} = \frac{F}{A(\text{proj})}$$

$$A_{\text{proj} \text{ (1)}} = t_{\text{strut}} d_{\text{bolt}} = (11.8)$$

$$130 \times 10^6 = \frac{22.5 \times 10^3}{A_1 \left(\frac{\pi}{4} d^2 \right)}$$

$$[d_1 = 14.7] \text{ mm} \# \text{ (for } t_{\text{strut}})$$

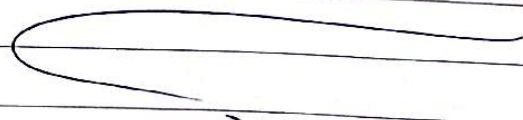
$$- A(\text{proj}) \text{ (2)} = t_{\text{beam}} d_{\text{bolt}}$$

$$\sigma = \frac{F_{\text{beam}}}{A_2 \left(\frac{\pi}{4} d^2 \right)}$$

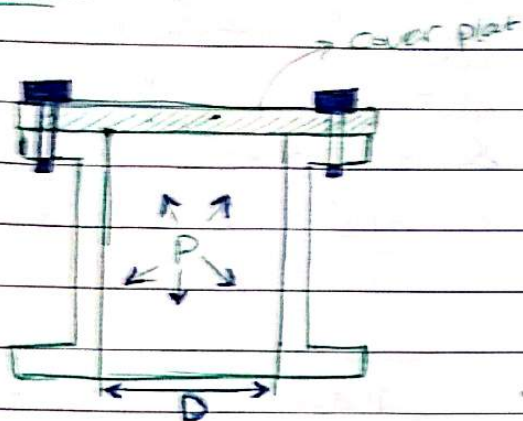
$$130 \times 10^6 = \frac{45 \times 10^3}{\frac{\pi}{4} d^2}$$

$$\Rightarrow [d = 18.2] \text{ mm} \# \text{ (for beam)}$$

قوة الضغط في
الزوايا (d) يجب أن تكون
(45) كحد أقصى



1.9-9



$$P = (1900) \text{ kPa}$$

$$D = 250 \text{ mm}$$

$$d_{\text{bolt}} = 12 \text{ mm}$$

$$\tau_{\text{all (bolt)}} = 70 \text{ MPa}$$

Find # of bolts ??

Sol:



$$F_p = PA$$

$$= (1900 \times 10^6) \left(\frac{\pi}{4} \right) (250)^2 \times 10^{-6}$$

$$F_p = 933 \text{ kN}$$

$$F_{\text{bolt}} = \tau A$$

$$= (70 \times 10^6) \left(\frac{\pi}{4} \right) (12)^2 \times 10^{-6}$$

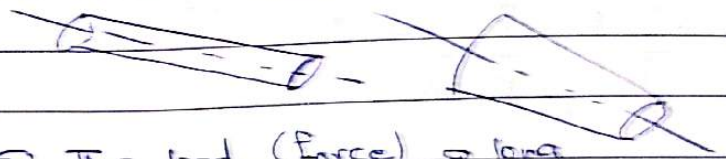
$$F_{\text{bolt}} = 7.9 \text{ kN}$$

$$\# \text{ bolts} = \frac{93.3 \times 10^3}{7.9 \times 10^3}$$

$$\# = 12 \text{ bolts}$$

Axially Loaded Members

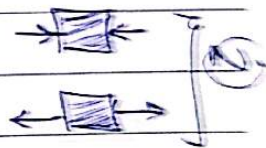
① Member that are axially symmetric



② The load (force) is along the axis

- Tensile load

- compressive load



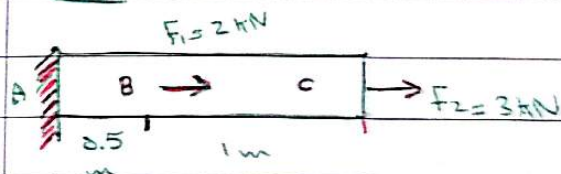
stress $\Rightarrow$ Normal stress

$$\text{Normal strain} \Rightarrow \epsilon = \frac{\Delta l}{l} = \frac{\sigma}{E}$$

$$\Rightarrow \frac{\Delta l}{l} = \frac{P}{A E} \Rightarrow \Delta l = \frac{Pl}{AE}$$

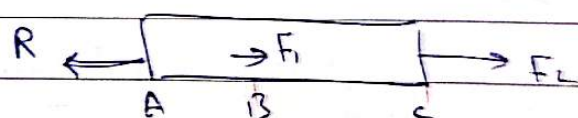
l : the length on which (P) is effective

Exp:



Find: δ_c (elongation @ c) ??

Sol:

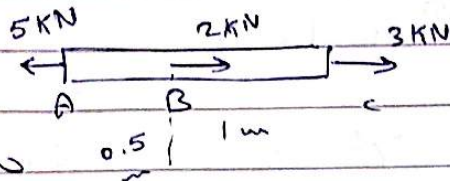


$$- R = ?$$

$$\sum F_x = 0$$

$$-R + F_L + F_R = 0$$

$$R = 5 \text{ kN}$$



$$\Delta_c = \Delta_{B/A} + \Delta_{c/B}$$

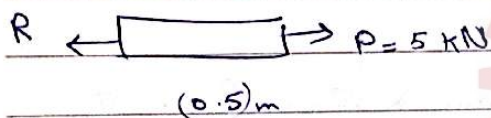
$$= 1.6 + 1.9$$

$$\Delta_c = 3.5 \times 10^{-3} \text{ mm}$$

$$\Delta = \frac{PL}{AE} \Rightarrow \text{لا بد من معرفة طول (P) في مقطع واحد}$$

* في حالة تغير المقاطع (A) في (P)

→ Section (AB)



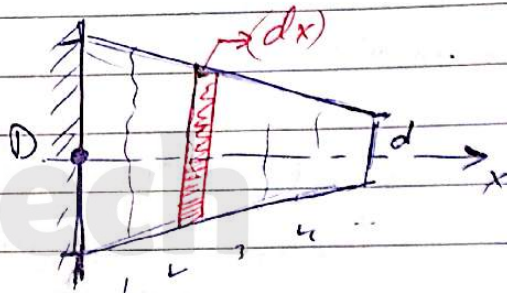
$$P = 5 \text{ kN}$$

$$l = 0.5$$

$$A = \left(\frac{\pi}{4}\right)(100)^2 \times 10^{-6} = 7.85 \times 10^{-3}$$

$$E = 200 \text{ GPa}$$

$$\Delta_{AB} = \frac{(5 \times 10^3)(0.5)}{(7.85 \times 10^{-3})(200 \times 10^9)}$$

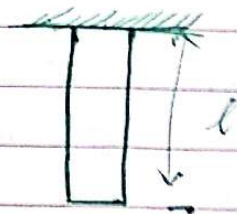


$$\Delta = \frac{PL}{AE} \quad d\Delta = \frac{P(x) \cdot dx}{AE}$$

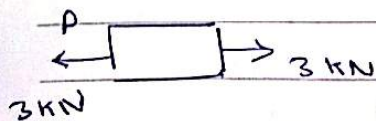
$$\Delta = \int_0^l \frac{P(x)}{A(x)E} dx$$

* Ex: (suspended rod)

- weight
- length (l)
- Area (A)
- specific gravity p



→ Section (BC)



$$P_{BC} = 3 \times 10^3$$

$$l = 1 \text{ m}$$

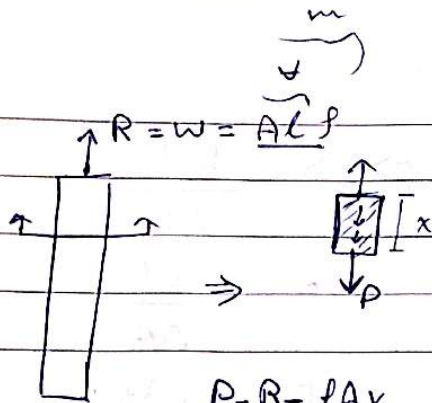
$$A = 7.85 \times 10^{-3}$$

$$\Delta_{BC} = \frac{(3 \times 10^3)(1 \times 10^3)}{(7.85 \times 10^{-3})(200 \times 10^9)}$$

$$\Delta_{BC} = 1.9 \times 10^{-3} \text{ mm}$$

→ Find: elongation ??

Sol:



$$P = R - \int A x$$

$$P(x) = \int A (l - x)$$

$$\delta = \int_0^l \frac{P(x)}{A E} dx$$

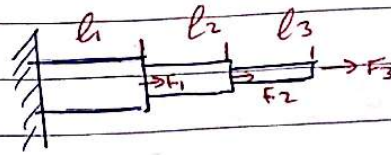
$$\delta = \int_0^l \frac{\int A (l - x)}{A E} dx$$

$$\delta = \frac{\int}{E} \int_0^l (l - x) dx$$

$$\delta = \frac{\int}{E} \left(lx - \frac{x^2}{2} \right) \Big|_0^l$$

$$\delta = \frac{\int}{E} \left(l^2 - \frac{l^2}{2} \right)$$

$$\delta = \frac{\int l^2}{2E}$$



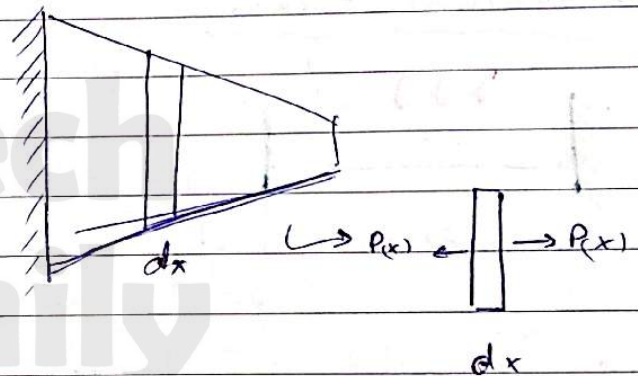
$$\delta_{AD} = \sum_{i=1}^3 \frac{P_i l_i}{E_i A_i}$$

→ Bar (1) is fixed

$$P_1 = F_1 + F_2 + F_3$$

$$P_2 = F_2 + F_3$$

$$P_3 = F_3$$



$$d\delta = \frac{P(x) dx}{E A(x)}$$

Thermal strain

$$\epsilon_T = \alpha \Delta T$$

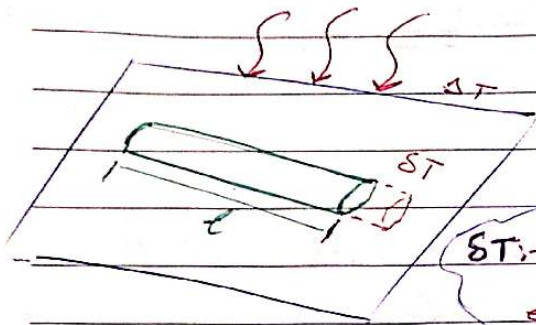
Thermal strain

α : Thermal coeff.

ΔT : Change in Temp.

$$[\alpha] = \frac{1}{^\circ C}$$

$$[\epsilon_T] = \text{unitless}$$



δT - thermal elongation

$$e_T = \frac{\delta T}{l} = \alpha \Delta T$$

$$\delta T = \alpha \Delta T l$$

$$\delta = \frac{FL}{AE}$$

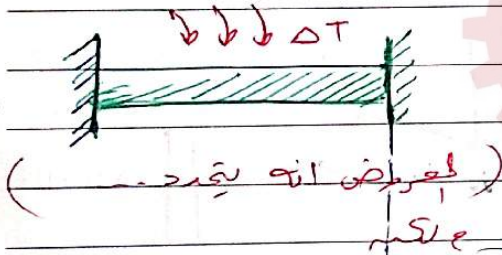
$$\delta T = \frac{FL}{AE}$$

$$F = \frac{\delta T AE}{l}$$

$$\sigma_T = \frac{-F}{A}$$

$$= - \frac{\delta T AE}{Al}$$

* Thermal stress



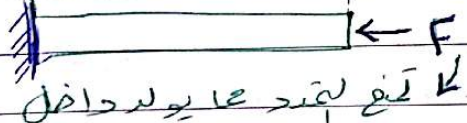
$$\sigma_T = - \frac{\delta T E}{l}$$

$$\sigma_T = - \frac{(\alpha \Delta T) E l}{l}$$

$$\therefore \sigma_T = - \alpha \Delta T E$$

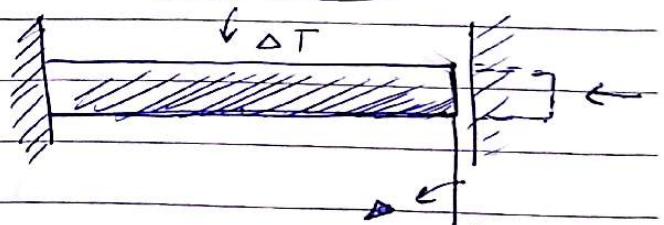
* اذا حدث لي التمدد

فـ F



(rod) (Thermal stress)
 σ_T

$$-F = \frac{(\sigma_T - \Delta) AE}{l}$$

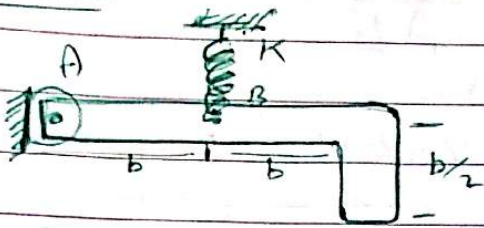


(التمدد)
يتمدد

(F ⇒ make $\delta T = 0$)

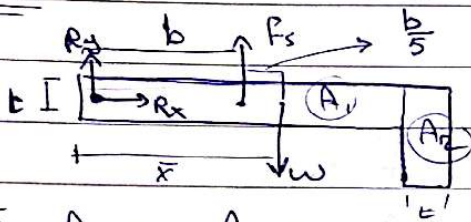
$$\sigma_T = \frac{F}{A}$$

2.2-1



- find spring elongation??

Sol:



$$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$\bar{x} = \frac{(2b)(1)(b) + (b/2)(1)(2b)}{2b + b/2}$$

$$\bar{x} = \frac{6b}{5}$$

$$+ \sum M_A = 0$$

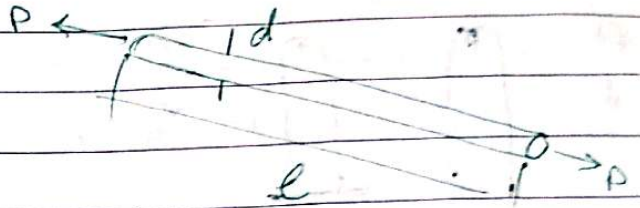
$$F_s(b) - w\left(\frac{6b}{5}\right) = 0$$

$$F_s = \frac{6w}{5} = Kx$$

$$x = \frac{6w}{5K}$$

طالو دىل
(Thermal) ب
(spring) دىل

2.2-9 (wire)



$$d = 2 \text{ mm} \quad \Delta l_{\text{max}} = 3 \text{ mm}$$

$$l = 3.8 \text{ m} \quad \sigma_{\text{all}} = 60 \text{ MPa}$$

$$E = 72 \text{ GPa}$$

find P_{max} ??

Sol:

① From Δl

$$\Delta l = \frac{Pl}{AE}$$

$$3 \times 10^{-3} = P(3.8)$$

$$\frac{\pi}{4} (2 \times 10^{-3})^2 (72) \times 10^9$$

$$P = 178 \text{ N} \quad \text{--- ①}$$

② From σ_{all}

$$\sigma_{\text{all}} = \frac{P}{A}$$

$$P = \sigma_{\text{all}} A$$

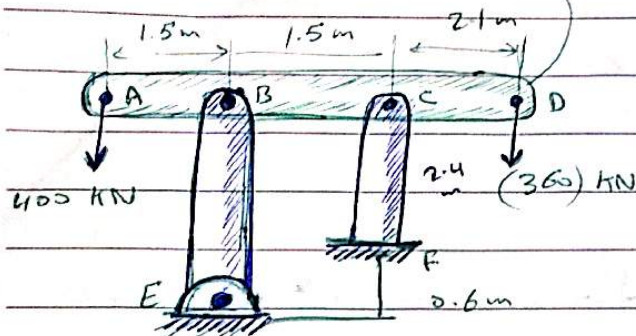
$$= (60 \times 10^6) \left(\frac{\pi}{4} \right) (2 \times 10^{-3})^2$$

$$P = 188 \text{ N} \quad \text{--- ②}$$

$$\text{then } P = P_1$$

2.2-12

Rigid



(E) For BE, CF $\Rightarrow$ (200) GPa

$$\delta_{BE} = \frac{F_{BE} l}{AE}$$

$$= \frac{(296 \times 10^3)(2.4 + 0.6)}{(11100 \times 10^{-6})(200 \times 10^9)}$$

$$\delta_{BE} = 0.4 \text{ mm}$$

$$\delta_{BE} = 0.4 \text{ mm} \quad \#$$

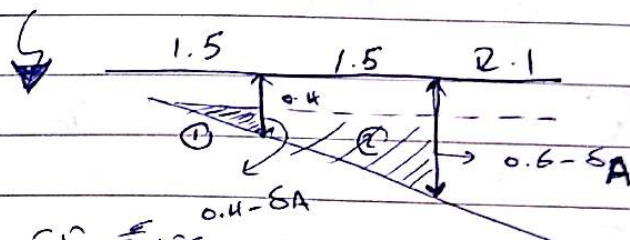
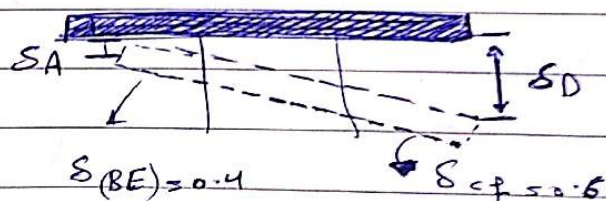
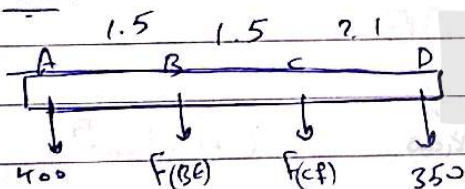
$$\delta_{CF} = \frac{F l}{AE}$$

$$\Rightarrow \frac{(464 \times 10^3)(2.4)}{(9280 \times 10^{-6})(200 \times 10^9)}$$

$$\delta_{CF} = 0.6 \text{ mm}$$

$$\delta_{CF} = 0.6 \text{ mm} \quad \#$$

Sol:



$$F_{CF} = 464 \text{ kN} \quad \#$$

$$F_{BE} = 400 + 360 - 464$$

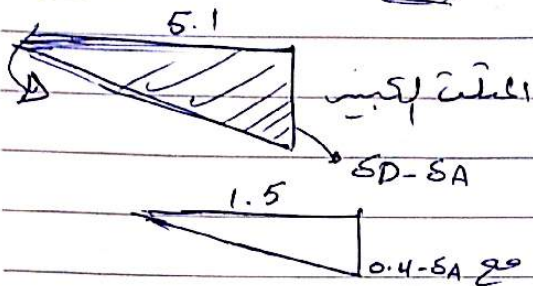
$$= 296 \text{ kN} \quad \#$$

(1,2) ratio of δ

$$\frac{0.4 - \delta_A}{1.5} = \frac{0.6 - \delta_A}{3}$$

$$\Rightarrow \delta_A = 0.2 \text{ mm}$$

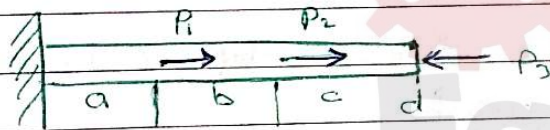
Now to find (δ_D)



$$\frac{0.2}{1.5} = \frac{\delta_D - 0.2}{5.1}$$

$$\delta_D = 0.88 \text{ mm} \quad \#$$

2.3-3



$$A = 260 \text{ mm}^2$$

$$P_1 = 12 \times 10^3 \text{ N} \quad E = 200 \text{ GPa}$$

$$P_2 = 8 \times 10^3 \text{ N}$$

$$P_3 = 6 \times 10^3 \text{ N}$$

$$a = 1.5 \text{ m}$$

$$b = 0.6 \text{ m}$$

$$c = 0.9 \text{ m}$$

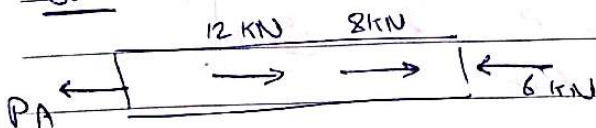
(a) Find:

$$\delta_D/A$$

(b) P_3 , so that

$$\delta_D/A = 0$$

Sol:

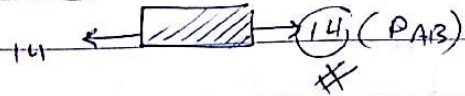


$$\sum F = -P_A + 12 + 8 - 6 = 0$$

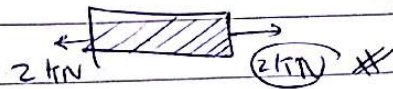
$$P_A = 14 \text{ kN} \quad \#$$

(a)

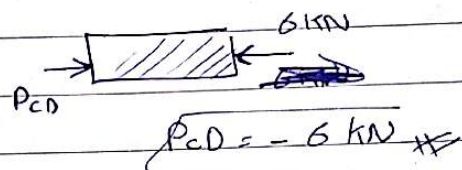
A → b



B → c



c → D



Now:

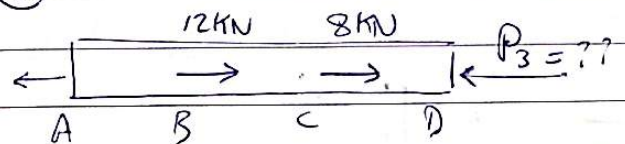
$$\delta_D/A = \sum \frac{P_i l_i}{A_i E}$$

$$= \frac{1}{(260 \times 10^{-6})(200 \times 10^9)} \left[(14 \times 10^3)(1.5) + (2 \times 10^3)(0.6) + (-6 \times 10^3)(0.9) \right]$$

$$\Rightarrow \delta_D/A = 0.323 \text{ mm} \quad \#$$

$$P = 8 - P_3$$

(b)



$$P_{AB} = (20 - P_3) \text{ kN}$$

$$P_{BC} = (8 - P_3) \text{ kN}$$

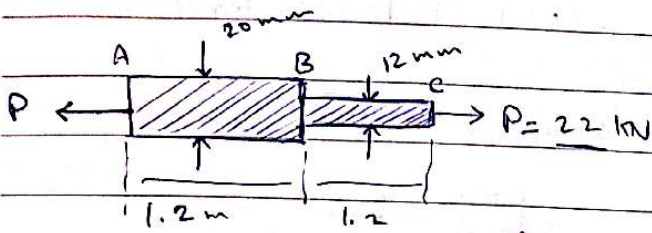
$$P_{CD} = -P_3$$

when $\delta_D/A = 0$

$$0 = \frac{(20 - P_3) \times 10^3}{(260 \times 10^{-6})(200 \times 10^9)} (1.5) + \frac{(8 - P_3) \times 10^3}{(260 \times 10^{-6})(200 \times 10^9)} (0.6) - \frac{P_3 \times 10^3}{(260 \times 10^{-6})(200 \times 10^9)} (0.9)$$

$$\Rightarrow P_3 = 11.6 \text{ kN} \quad \#$$

2.3-7



$$E = 205 \text{ GPa}$$

القيمة المطلوبة

Find:

① δ

② Plot (σ)

Sol:



$\vec{F} \rightarrow$ Friction force

$\hookrightarrow$ per unit length.

$$= \frac{P}{L}$$

Internal force @ depth (x)

$$\Rightarrow P_i(x) = P - f_x$$

$$= P - \left(\frac{P}{L}\right)(x)$$

$$\hookrightarrow d\delta = \frac{P(x) dx}{AE}$$

$$\delta = \int_0^L \frac{P}{L} \frac{(L-x)}{AE} dx$$

$$\delta = \frac{P}{LAE} \int_0^L (L-x) dx$$

$$= \frac{P}{LAE} \left(Lx - \frac{x^2}{2} \right) \Big|_0^L$$

Find δ

Sol:

$$\delta_{C/A} = \delta_{B/A} + \delta_{C/B}$$

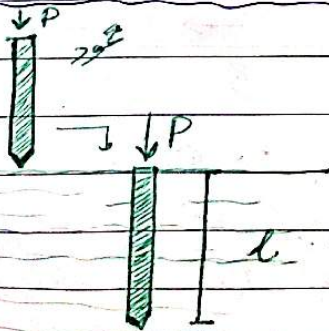
$$\Rightarrow \frac{(22 \times 10^3)(1.2)}{\left(\frac{\pi}{4}\right)(20)^2 \times 10^{-4} (205) \times 10^9}$$

+

$$\frac{(22 \times 10^3)(1.2)}{\frac{\pi}{4} (12)^2 \times 10^{-4} (205) \times 10^9}$$

$$\delta \Rightarrow 1.56 \times 10^{-3} \text{ m}$$

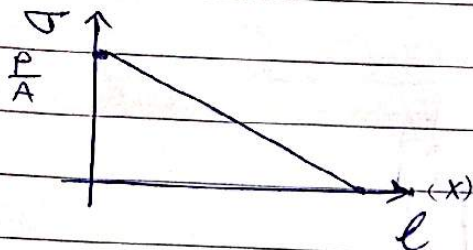
2.3-9



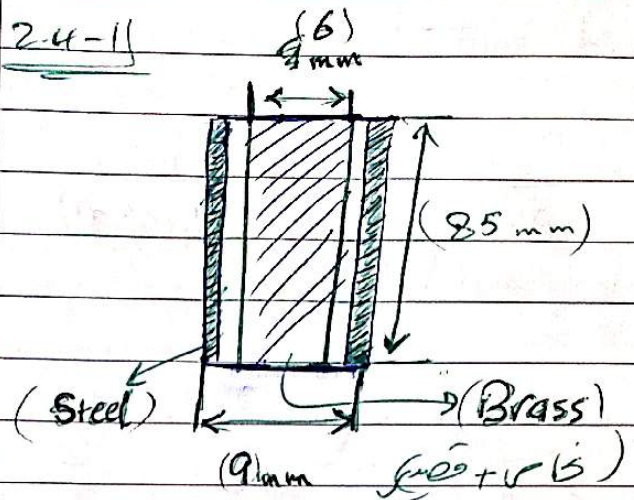
(Soil)
تراب

هذه تقاوم نزول الجدران الاسفلتية مع التربة
تزداد المقاومة مع التربة
تزداد المقاومة مع التربة

② $\sigma = \frac{P_i}{A}$
 $= \frac{P}{AL} (l-x)$



2.4-11



2.4-11

Statically indeterminate structure

$E_b = 100 \text{ GPa}$ (for brass)

$E_s = 200 \text{ GPa}$ (for steel)

* of reactions is greater than static equilibrium equations

Find:

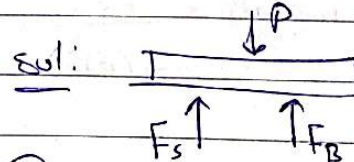
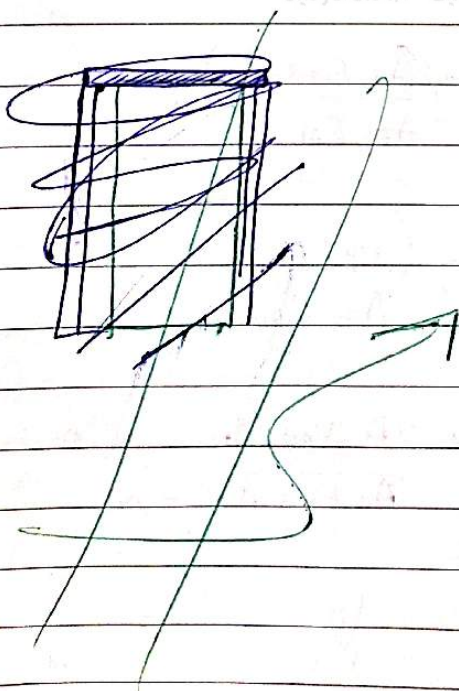
(a) P , $\delta = 0.1 \text{ mm}$

(b) $\sigma_{\text{steel}} = 180 \text{ MPa}$

$\sigma_{\text{brass}} = 140 \text{ MPa}$

Constraint equation are used to solve the system deflection constraints

Find P_{all} ??



(a) $\delta_{\text{steel}} = \delta_{\text{brass}} = \delta = 0.1 \text{ mm}$

$\delta_{\text{brass}} = \frac{F_B l}{A E}$, $\delta_{\text{steel}} = \frac{F_B l}{A E}$

$0.1 = \frac{F_B (0.85) \times 10^{-3}}{(\frac{\pi}{4})(6)^2 \times 10^{-6} (E)}$; $0.1 = \frac{F_B (0.85) \times 10^{-3}}{(\frac{\pi}{4})(9-6)^2 \times 10^{-6} (E)}$

$F_B = 3.3$

$F_B = 1.8$

$P = 4.9$

Find P_{all} ??

$$\sigma_{sr} = \frac{F_s}{A_s} = 180 \text{ MPa} = \frac{F_s}{\frac{\pi}{4}(9^2 - 7^2)}$$

$$F_s = 4.5 \text{ kN}$$

by (constraint equation) :-

$$\frac{F_s l}{A_s E_s} = \frac{F_B l}{A_E E_s}$$

$$\frac{4.5 \text{ kN}}{\frac{\pi}{4}(9^2 - 7^2)(200 \text{ GPa})} = \frac{F_B}{(\frac{\pi}{4})(6)^2(100 \text{ GPa})}$$

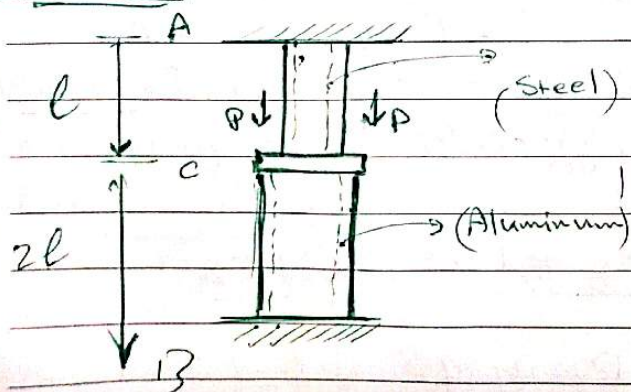
$$F_B = 2.5 \text{ kN}$$

Now (from σ_B)

$$\sigma_B = \frac{F_B}{A} \Rightarrow 140 \text{ MPa} = \frac{F_B}{\frac{\pi}{4}(6)^2}$$

$$F_B =$$

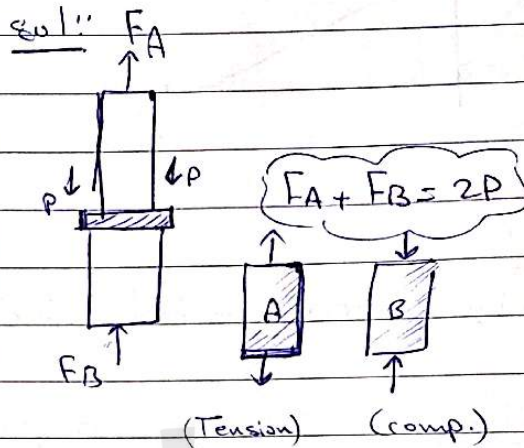
2.4-9



Find:

(a) σ_A , σ_S

(b)



(نصف السؤال)
But: $[\delta_s = \delta_{Al}]$ (constraint)

$$\frac{F_A l_s}{A_s E_s} = \frac{F_B l_{Al}}{A_{Al} E_{Al}}$$

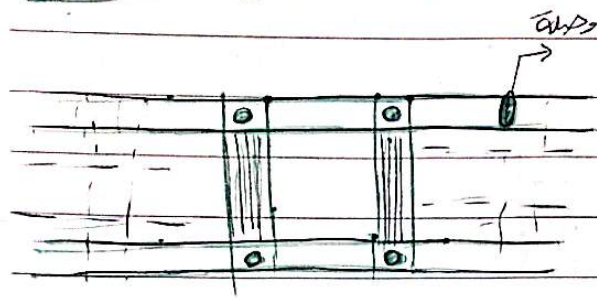
$$F_A = \left(\frac{A_s E_s}{A_{Al} E_{Al}} \right) (F_B)$$

$$2P = \left(\frac{A_s E_s}{A_{Al} E_{Al}} + 1 \right) F_B$$

$$F_B = \frac{2P A_{Al} E_{Al}}{A_s E_s + A_{Al} E_{Al}}$$

$$\sigma_A = \frac{F_A}{A_s} \quad \sigma_B = \frac{F_B}{A_{Al}}$$

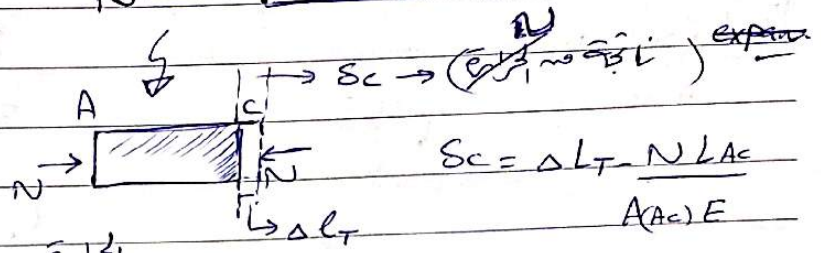
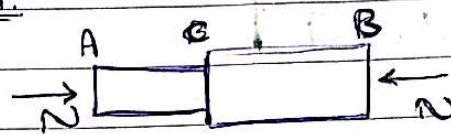
2.5-1] (مسألة)



Find:-

- (a) compressive force N
- (b) σ_c , δ_c

Sol:-

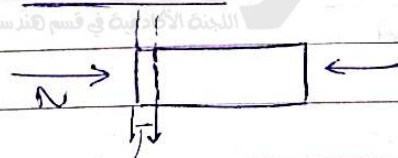


$$\delta_c = \Delta L_T - \frac{N L_{AC}}{A_{AC} E}$$

$$\delta_c = \alpha \Delta T L_{AC} - \frac{L_{AC} N}{A_{AC} E}$$

$$\delta_c = (100 \times 10^{-6})(30)(225 \text{ mm}) - \frac{(N)(225 \text{ mm})}{\frac{\pi}{4}(5 \text{ mm})^2(6)}$$

From (b)



$$\delta_c = \frac{N L_{CB}}{A_{CB} E} - \alpha \Delta T L_{CB}$$

$$\delta_c = \frac{(N)(300 \text{ mm})}{\frac{\pi}{4}(75 \text{ mm})^2(6)} - (100 \times 10^{-6})(30)(300 \text{ mm})$$

From (1, 2)

$$\sigma_c = \frac{N}{A_{AC}}$$

(A) 24
200

$$N = 52 \text{ kN}$$

$$\sigma_c = \frac{N}{A_{AC}} \Rightarrow 2$$

What is the σ_T ?

When:

$$\alpha = 12 \times 10^{-6}$$

$$T_1 = 10^\circ \text{C} \quad T_2 = 52$$

المادة 1

$$E = 200 \text{ GPa}$$

Sol:-

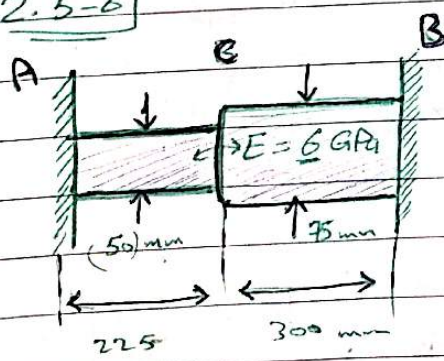
$$\epsilon_T = \alpha \Delta T$$

$$\sigma_T = E \epsilon_T$$

$$= (200 \times 10^9)(12 \times 10^{-6})(52 - 10)$$

$$\sigma_T = 101 \text{ MPa} \quad \#$$

2.5-6]



$$\alpha = 100 \times 10^{-6} / ^\circ \text{C}$$

$$\Delta T = 30^\circ \text{C}$$

From (1, 2)

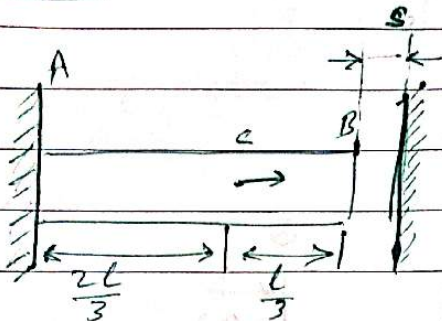
$$\sigma_c = \frac{N}{A_{AC}}$$

(A) 24
200

$$N = 52 \text{ kN}$$

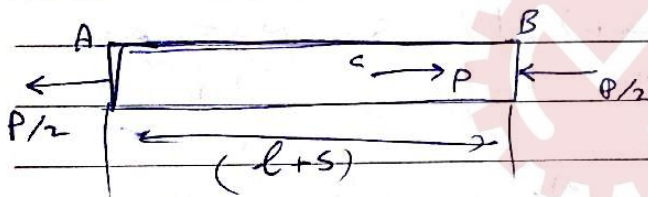
$$\sigma_c = \frac{N}{A_{AC}} \Rightarrow 2$$

2.5-14



Find (s) so that the two reactions are equal?

Sol:

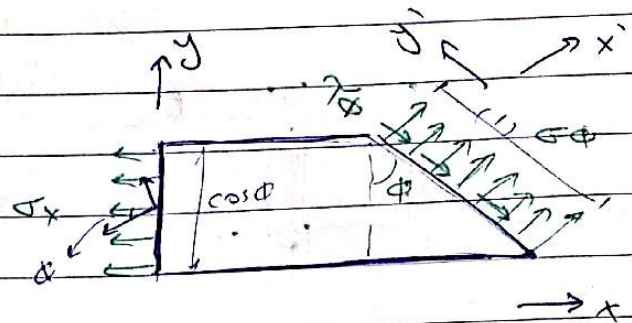


$$\delta A_c + \delta B = s$$

$$\frac{(P/2)l_{AC}}{EA} + \frac{(P/2)l_{CB}}{EA} = s$$

$$s = \frac{PL}{6EA} \quad \#$$

XX STRESSES in inclined planes



$$\begin{aligned} \sum F_x &= 0 & \sum F_{x'} &= 0 \\ \sum F_y &= 0 & \sum F_{y'} &= 0 \end{aligned}$$

$$\sum F_{x'} = \sigma_\phi (1 \times 1) - (\sigma_x \cos \phi) (\cos \phi \times 1) = 0$$

$$\sigma_\phi = \sigma_x \cos^2 \phi \quad \#$$

$$\sum F_{y'} = 0$$

$$\Rightarrow -\tau_\phi (1 \times 1) + (\sigma_x \sin \phi) (\cos \phi \times 1) = 0$$

$$\tau_\phi = \frac{\sigma_x \sin 2\phi}{2} \quad \#$$

$$\hookrightarrow \text{Max @ } \phi = \frac{\pi}{4}$$

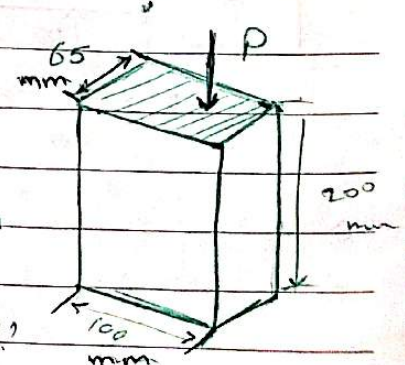
2-6-3

الضغط في الزوايا

$$\tau_{alt} = 8 \text{ MPa}$$

$$\sigma_{alt} = 26 \text{ MPa}$$

- find P max??



Sol:

$$-T_{alt} = T_{max} = \frac{\sigma_x}{2} \quad \rightarrow \phi = 45^\circ$$

$$\Rightarrow 8 \text{ MPa} = \frac{P}{A} \quad (A = (65)(100))$$

$$P = 104 \text{ kN} \quad \text{--- (1)}$$

$$-T_{max} = \sigma_{alt} = \sigma_x \quad \rightarrow \phi = 0$$

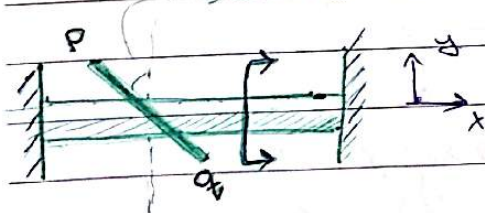
$$26 \text{ MPa} = \frac{P}{A}$$

$$P = 169 \text{ kN} \quad \text{--- (2)}$$

$$\therefore (P_{max}) = P_1 = 104 \text{ kN}$$

* 2.6-12 :-

$$\rightarrow \theta = 55^\circ$$



$$\sigma_{all} = -60 \text{ MPa (comp.)}$$

$$\tau_{all} = 30 \text{ MPa}$$

$$\text{--- (a) } \Delta T \quad (\alpha = 17 \times 10^{-6}, E = 120 \text{ GPa})$$

$$\text{(b) @ } \Delta T_{max} \text{ find } (\sigma)$$

Sol: ??

$$\sigma_x = \sigma_T = \alpha \Delta T E$$

(To find σ_x)

$$\text{(1) For } (\sigma_\theta)$$

$$-60 \text{ MPa} = -\sigma_x \cos^2 \phi$$

$$-60 \text{ MPa} = \sigma_x (\cos^2 (55))$$

$$\sigma_x = -118.2 \text{ MPa} \quad \text{--- (1)}$$

$$\text{(2) For } T_\theta$$

$$30 \text{ MPa} = \frac{\sigma_x}{2} \sin 2\phi$$

$$30 \text{ MPa} = \frac{\sigma_x}{2} \sin (110)$$

$$\sigma_x = 63.8 \text{ MPa} \quad \text{--- (2)}$$

$$(\sigma_x = \sigma_{x2} = 63.8 \text{ MPa})$$

!!! دقق !!

$$\hookrightarrow \sigma_x = \alpha \Delta T E$$

$$63.8 \times 10^6 = (17 \times 10^{-6}) (\Delta T) (120 \times 10^9)$$

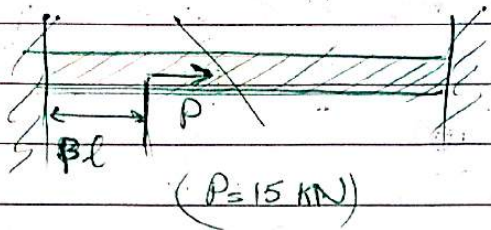
$$\Delta T = 31.4^\circ \text{C} \quad \#$$

$$\text{(b)}$$

$$\sigma_{(55)} = \sigma_x \cos^2 55$$

$$\sigma = 64 \cos^2 55 = 12 \text{ MPa}$$

المسألة ١٠
 $\Delta T = 28^\circ C$



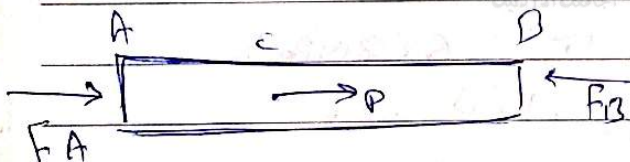
Find: β

so that: $(\sigma_{all} = 75 \text{ MPa})$
 $(\tau_{all} = 35 \text{ MPa})$

Sol:

$$\sigma_{\phi(max)} = \sigma_x (\phi = 0)$$

$$\tau_{\phi(max)} = \frac{\sigma_x}{2} (\phi = 45^\circ)$$



$$\delta_{A/B} = \delta_{A/A} + \delta_{C/B} = 0$$

$$\delta_{A/C} = -\frac{R_A (\beta l)}{AE} + \alpha \Delta T (\beta l)$$

$$\delta_{B/C} = -\frac{R_B (l(1-\beta))}{AE} + \alpha \Delta T (l-\beta l)$$

$$\delta_{C/A} + \delta_{C/B} = 0$$

$$-\frac{R_A (\beta l)}{(18)(40)(120)} + (17 \times 10^{-6})(28)(\beta l)$$

$$-\frac{(R_A + 15 \text{ kN})}{R_B} \frac{(l-\beta l) + (17 \times 10^{-6})(28)}{(18)(40)(120)} \frac{(l-\beta l)}{(28)}$$

$$= 0$$

$$\text{But } \sigma_x = 70$$

$$R_A + P = \sigma_x A$$

$$= (70 \times 10^6)(40)(18) \times 10^{-6}$$

$$= 50.4 \text{ kN}$$

$$R_A = 50.4 - 15 \text{ kN}$$

$$= 35.4 \text{ kN} \quad \#$$

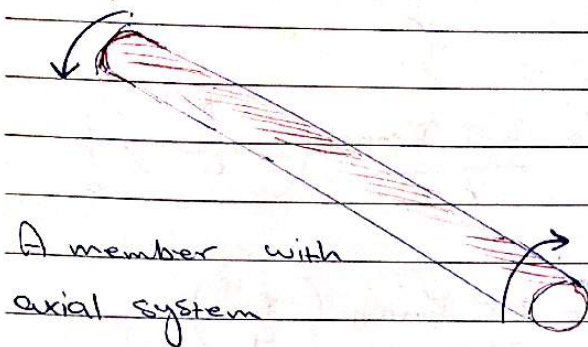
نقص في المواد

(B) في

(l) ~~هو~~
 ↑ ~~تزيد~~

**CH.3 →

* Torsion :-



A member with axial system

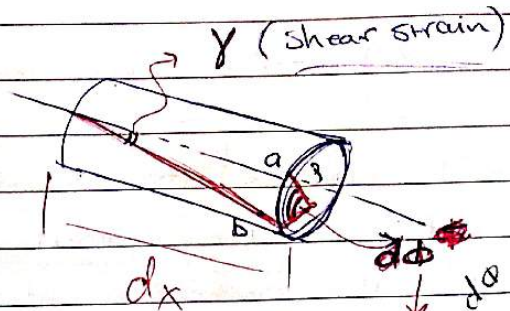
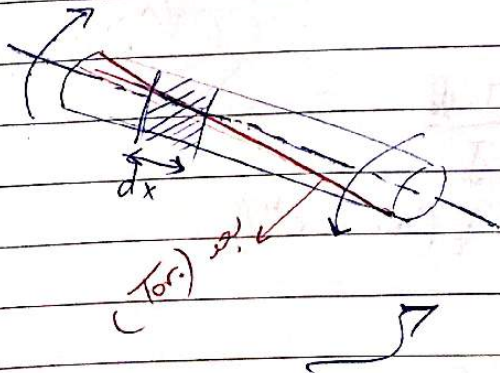
Particularly member with circular cross section (solid or hollow)



When these members are subjected to a couple, where axis is along the member axis, the load is called

[Torsional load]

⇒ Stress due to the torsional load:



(اینجا مورفاتی تغییر)
که ما زیاد (طول) زیاد (ϕ)

a (before loading)
 b (after $\hookrightarrow$)

$$\text{arc}(ab) = \gamma dx = r d\phi$$

$$\hookrightarrow (\gamma dx = r d\phi)$$

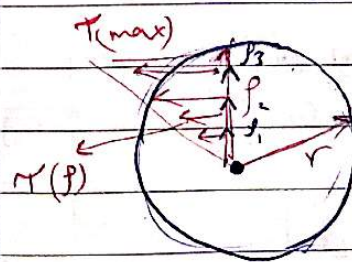
$$\frac{\gamma}{r} = \frac{d\phi}{dx} \quad \text{--- (*)}$$

$$(\tau = \gamma G \Rightarrow \gamma = \frac{\tau}{G})$$

$$\hookrightarrow \tau = \rho G \frac{d\phi}{dx} \left[\frac{d\phi}{dx} = c \right]$$

$$\tau = \rho G c$$

↳ (Shear stress distribution is linear with respect to r)



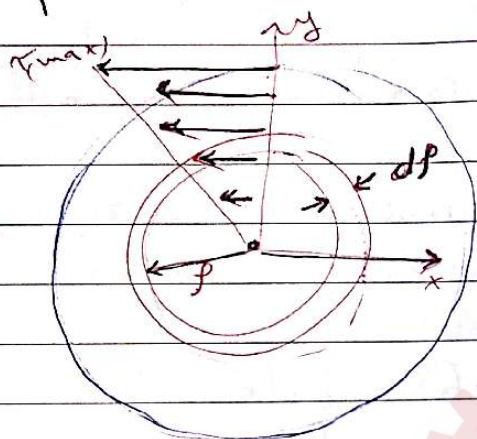
$$\tau_{max} = r G c = \tau(r)$$

$$\tau(r) = \frac{r}{R} \tau_{max}$$

① $\hookrightarrow$

- Relate $\tau(p)$ to the internal Torque

↳ The resultant effect of the shear stress is the internal torque @ the center.



$$\tau(p) = \frac{p}{r} \tau_{max}$$

$$\Rightarrow dF = \tau(p) dA$$

$$= \left(\frac{p}{r} \tau_{max} \right) (2\pi p dp)$$

$$dF = \frac{2\pi p^2 \tau_{max}}{r} dp$$

$$dF \Rightarrow dT (\text{diff. torque})$$

$$dT = p dF$$

$$dT = \frac{2\pi p^3}{r} \tau_{max} dp$$

$$\hookrightarrow T = \int_0^r \frac{2\pi p^3}{r} dp$$

$$\rightarrow T = \frac{2\pi \tau_{max}}{r} \int_0^r p^3 dp$$

$$T = \frac{2\pi \tau_{max}}{r} \left(\frac{p^4}{4} \right) \Big|_0^r$$

$$T = \frac{2\pi \tau_{max}}{r} \left(\frac{r^4}{4} \right)$$

(Torque) (N.m) → Max (R)

$$\tau_{max} = \frac{T r}{\frac{\pi}{2} r^4}$$

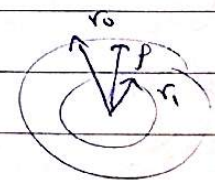
↳ (J): Polar moment area of inertia

$$\tau_{max} = \frac{T r}{J} \quad \# \text{ --- ②}$$

$$\hookrightarrow \tau(p) = \left(\frac{p}{r} \right) \left(\frac{T r}{J} \right)$$

$$\tau(p) = \frac{T p}{J} \quad \# \text{ --- ③}$$

for (hollow)

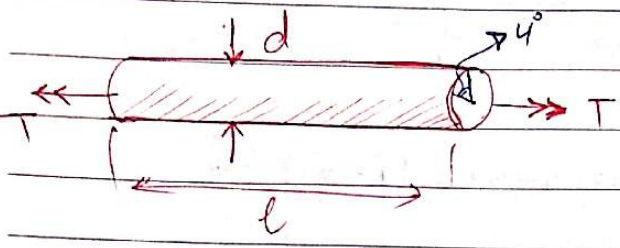


$$\tau = \frac{T p}{J}$$

$$J = \left(\frac{\pi}{2} (r_o^4 - r_i^4) \right)$$

$$\phi = \frac{T L}{J G} \quad \# \text{ --- ④}$$

* 3.2-1



$l = 610 \text{ mm}$

$\phi_{\max} = 4^\circ$

$\gamma_{\max} = 0.0008 \text{ rad}$

Find: $d_{\min}$??

Sol:

$\gamma dx = l d\phi$

$\frac{\gamma}{\rho} = \frac{d\phi}{dx} = \frac{\phi_{\max}}{L}$

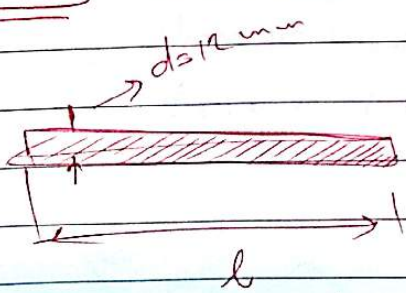
$0.0008 \text{ rad} = \frac{(4^\circ) \left(\frac{\pi}{180} \right)}{0.61}$

$\downarrow$

ρ

$\Rightarrow d = 14 \text{ mm}$

* 3.3-5



$T_{\max} = 300 \text{ MPa}$ $\phi_{\max} = 22.5^\circ$

$G = 80 \text{ GPa}$

Find: $L_{\min}$??

Sol:

$\gamma_{\max} = \frac{T_{\max}}{G} = \frac{300 \times 10^6}{8 \times 10^{10}}$

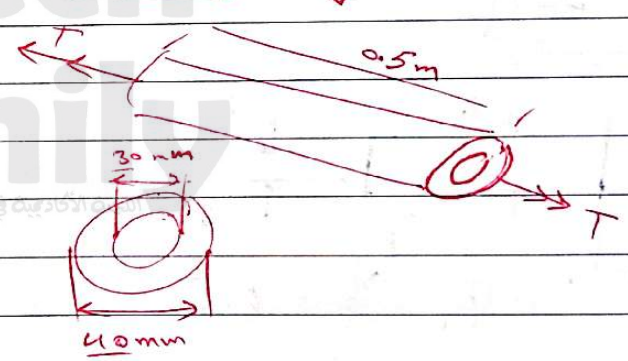
$\gamma_{\max} = 3.75 \times 10^{-3}$

$\frac{\gamma_{\max}}{\rho} = \frac{\phi}{L}$

$\frac{3.75 \times 10^{-3}}{d_{\min}} = \frac{22.5}{L}$

$L_{\min} = 628.3 \text{ mm}$

* 3.3-7 Tube (Aluminum)



$\phi = 3.57^\circ$ @ $T = 600 \text{ N.m}$

Find: $(\gamma_{\max}, G, \gamma_{\max})$??

Sol:

$J = \frac{\pi}{2} ((20)^4 - (15)^4) \Rightarrow 17.2 \times 10^4 \text{ mm}^4$

$\tau_{\max} = \frac{T \rho}{J} \Rightarrow \frac{(600)(20)}{17.2 \times 10^4}$

$\tau_{\max} = 70 \text{ MPa}$

$$\frac{\gamma_{\max}}{\rho} = \frac{\phi}{L}$$

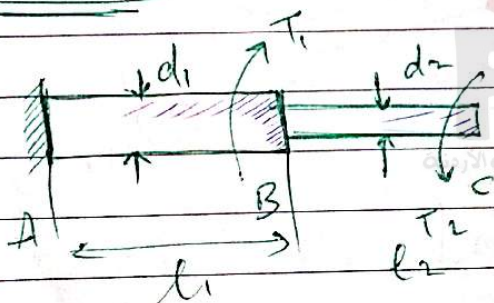
$$\frac{\gamma_{\max}}{20 \times 10^{-3}} = \frac{3.57}{0.5}$$

$$\gamma_{\max} = 143 \quad \#$$

$$G = \frac{\tau_{\max}}{\gamma_{\max}} \Rightarrow \frac{70 \times 10^6}{143}$$

$$G = 0.5 \text{ MPa} \quad \#$$

* 3.4-11



$$d_1 = 58 \text{ mm} \quad T_1 = 2300$$

$$l_1 = 760 \text{ mm} \quad T_2 = 900$$

$$d_2 = 45 \text{ mm}$$

$$l_2 = 510 \text{ mm}$$

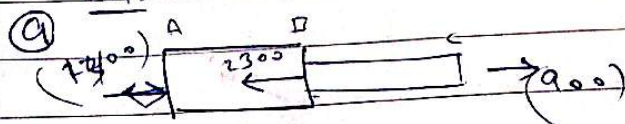
$$G = 76.6 \text{ GPa}$$

* find:

(a) $\tau_{\max}$

(b) ϕ_c (degrees)

Sol:



$$\sum T = 0$$

$$2300 - 900 = 1400 \rightarrow$$

$$\tau_{\max AB} = \frac{T_{AB} \cdot \frac{d_1}{2}}{\frac{\pi}{32} d^4} \Rightarrow \rho$$

$$\left(\frac{\pi}{2} r^4 \right)$$

$$\tau_{\max AB} = \frac{(1400) \left(\frac{58}{2} \right)}{\frac{\pi}{32} (58)^4}$$

$$\tau_{AB} = 36.54 \text{ MPa} \quad \#$$

Now @ Bc: ($T = 900$)

$$\tau_{\max BC} = \frac{T d_2 / 2}{\frac{\pi}{32} (d)^4}$$

$$= \frac{(900) (45/2) \times 10^{-3}}{\frac{\pi}{32} (45)^4 \times 10^{-12}}$$

$$\frac{\pi}{32} (45)^4 \times 10^{-12}$$

$$\tau_{\max BC} = 50 \text{ MPa} \quad \#$$

$$\phi_c = \frac{T_1 l_1}{G J_1} + \frac{T_2 l_2}{G J_2}$$



2.4-11 / 2.5-8 / 2.5-10 / 2.6-6
2.6-12 /

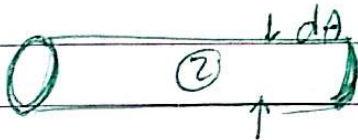
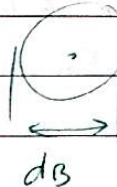
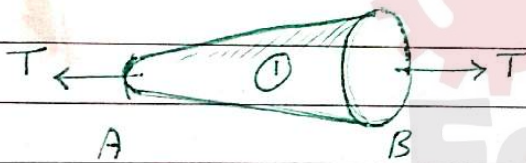
① $\Phi_{C/A} = \Phi_{B/A} + \Phi_{C/B}$

$\Rightarrow \frac{T_{AB} l_{AB}}{G J_{AB}} + \frac{T_{BC} l_{BC}}{G J_{BC}}$

$\Rightarrow \frac{(1400)(760) \times 10^{-3}}{(766 \times 10^9) \left(\frac{\pi}{32} (58)^4 \right)} + \frac{(900)(510) \times 10^{-3}}{(766 \times 10^9) \left(\frac{\pi}{32} (45)^4 \right)}$
 $\leftarrow (T_{BC} \sim 1)$

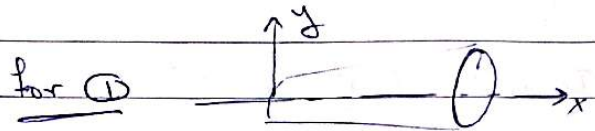
$\Phi = -2.4 \text{ rad}$
 $\Rightarrow -0.138^\circ$

* 3.4-8 /



Find: $\frac{dB}{dA}$ so that $\Phi_{AB1} = \frac{1}{2} \Phi_{AB2}$

Sol:



$dx = dA + \frac{dB - dA}{L} x$

$\Phi_{AB1} = \int_0^L \frac{T dx}{G J(x)}$
 $J(x) = \frac{\pi}{32} (dA + \frac{dB - dA}{L} x)^4$

$\Rightarrow \frac{T}{32 G} \int_0^L \frac{dx}{(dA + \frac{dB - dA}{L} x)^4}$

$\Rightarrow \frac{32 TL}{3 \pi G (dB - dA)} \left(\frac{1}{dA^2} - \frac{1}{dB^2} \right)$
 جدول اشتقاق في آخر الكتاب

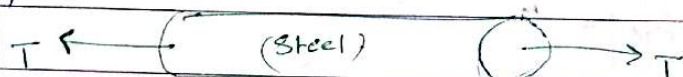
for ②:-

$\Phi_{AB2} = \frac{TL}{\frac{\pi dA^4}{32} G} = \frac{32 TL}{\pi dA^4 G}$

$\frac{\Phi_{AB1}}{\Phi_{AB2}} = \frac{1}{2}$

dA/dB مطلوب

* 3.5-9 /



$G = 81 \text{ GPa}$ $T = 900 \text{ N.m}$
 $d = 50 \text{ mm}$

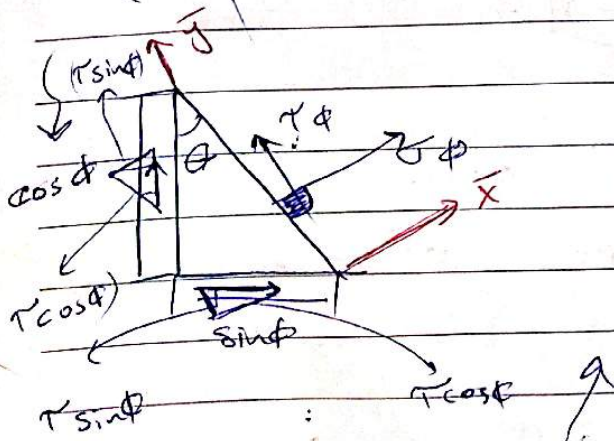
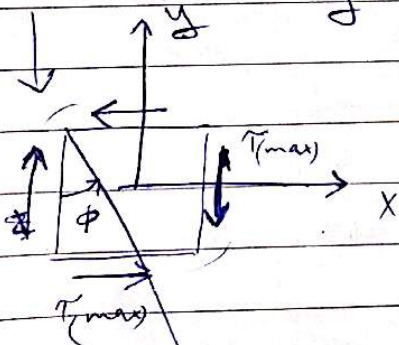
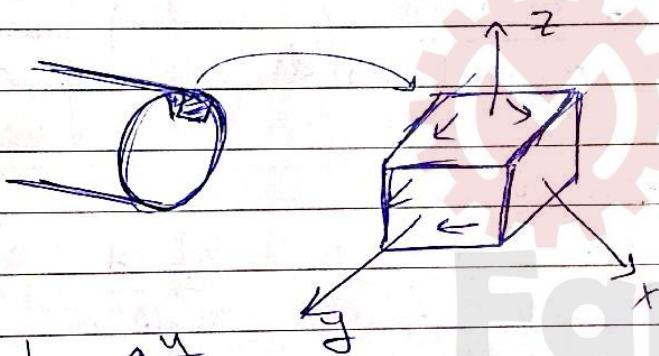
* Find: ① T_{max} , ② σ_{max}

$$a) \tau_{max} = \frac{Tr}{J}$$

$$= \frac{(900)(50/2 \times 10^{-3})}{\frac{\pi}{2} (50/2 \times 10^{-3})^4}$$

$$\tau_{max} = 36.7 \text{ MPa} \quad \#$$

$$\gamma = \frac{\tau}{G} = \frac{36.7 \times 10^6}{81 \times 10^9} = 4.5 \times 10^{-5} \text{ rad}$$



$$\rightarrow \sum F_x = 0$$

$$\Rightarrow \sigma_x(1 \times 1) + \tau \sin \phi \cos \phi \times 1 + \tau \cos \phi \sin \phi \times 1 = 0$$

$$\sigma_x = -2\tau \sin \phi \cos \phi$$

$$= -2\tau \frac{\sin 2\phi}{2}$$

$$= -\tau \sin 2\phi$$

$$\sigma_x = -\tau \sin 2\phi$$

$$@ \phi = 45^\circ \quad \sigma_{max} = -\tau$$

$$@ \phi = 135^\circ \quad \sigma_{max} = \tau$$

Now

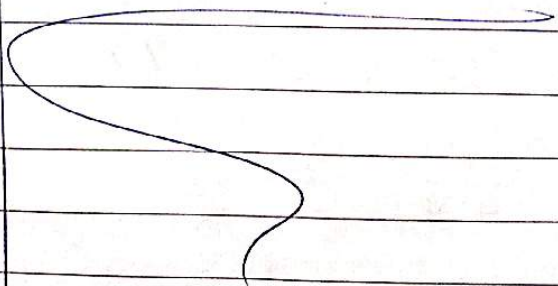
$$\tau_{max} = 36.7 \text{ MPa}$$

$$\tau_{max} = \tau = 36.7 \text{ MPa}$$

$$@ 135^\circ$$

$$\sigma_{comp} = -\tau = 36.7 \text{ MPa}$$

$$@ 45^\circ$$



** Power transmission shafts:

$$\text{Power} = \frac{dw}{dt}$$

$$\text{Work} = T(\Delta\theta)$$

(Torque) (Rotation)

$$\rightarrow \text{Power} = P_s T \frac{\Delta\theta}{\Delta t}$$

$\Delta\theta \rightarrow$ rotational disp.
 $\Delta t \rightarrow$ change in time

But $W = \frac{\Delta\theta}{\Delta t}$

$$\boxed{P_s \cdot T \cdot W} \neq [P] = \text{Watt} = \text{Joule/s}$$

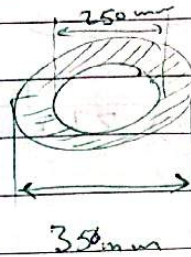
(n: revolutions per minute)
- rpm -

$$n = \frac{2\pi \text{ rad} \cdot \text{min}}{\text{min} \cdot (60/s)} = \frac{2\pi}{60} \text{ rad/s}$$

$$\boxed{\text{rad/s} = 2\pi \text{ Hz}}$$

$$\boxed{W = \frac{\pi}{30} \text{ rad}}$$

* 3.7 - 3 (Shaft)



$$\tau_{\max} = 62 \text{ MPa}$$

$$\textcircled{a} n_s = 500 \text{ (rpm)}$$

Find $P_{\max} \rightarrow$ Power

$$\textcircled{b} n = 2 n_s$$

Find (τ) ??

Sol:

$$\textcircled{a} P = T \cdot W$$

$$\tau_{\max} = \frac{T \cdot r}{J}$$

$$62 \times 10^6 = \frac{T \cdot (35/2) \times 10^{-3}}{\frac{\pi}{4} ((350)^4 - (250)^4) \times 10^{-12}}$$

$$\boxed{T = 386 \times 10^3 \text{ N.m}}$$

$$W = \left(n \times \frac{2\pi}{60} \right) = (500) \left(\frac{2\pi}{60} \right)$$

$$\boxed{W = 26.16}$$

$$P = T \cdot W = (386 \times 10^3) (26.16) =$$

$$\boxed{P = 20.2 \text{ MW}}$$

Note: House wat. - 743 wat.

$$\rightarrow P_s \frac{20.2 \times 10^6}{743} \Rightarrow \sqrt{\frac{15}{27.2 \text{ Hp}}}$$

(b)

$$n = (2)(500)$$

$n \approx 1000 \text{ rpm}$

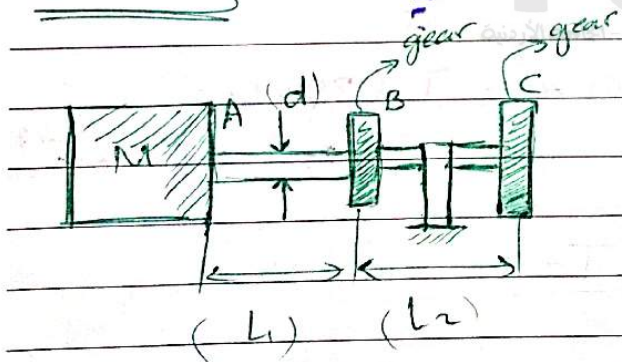
T (سوف نقل للضيف)

Trax 118

$$T = \frac{386 \text{ kN.m}}{2} = 193 \text{ kN.m}$$

$$\tau_{\max} = \frac{1}{2} (62 \text{ MPa}) = 31 \text{ MPa}$$

* 3.7-91: Motor



$$P = 200 \text{ Kw} \quad (P_B = 90 \text{ Kw})$$

$n = 1000 \text{ (rpm)}$ $P_c = 110 \text{ Kw}$

* Find: (d) if $T_{all} = 50 \text{ MPa}$

$$\Phi_c(A) = 1.5^\circ$$

$G = 20 \text{ GPa}$

L1 = 1.8 L2 = 1.2 m

(rad) $\rightarrow$

Sol:

① for (Tail)

~~(A-15)~~

مسئله (90) لا جواب است
 $(A \rightarrow B) \rightarrow (M \rightarrow B)$
 و با فرض B

$$T_{AB} = \frac{P}{\omega} = \frac{200 \times 10^2}{(1000) \left(\frac{2\pi}{60} \right)}$$

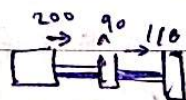
$$|T_{AB} = 1.91 \text{ kN.m}|$$

$$\tau_{\text{all}} = \frac{T r}{\frac{\pi}{32} (d^4) \rightarrow J}$$

$$50 \times 10^6 = \frac{(1910) (d_r^2)}{\frac{\pi}{4} (d^4)}$$

$d = 58 \text{ m}$ for # ①

② For Φ



$$T_{bc} = \frac{P}{\omega} = \frac{110 \times 10^3}{(1000) \left(\frac{\pi}{720} \right)}$$

$$T_{BC} = 1.05 \text{ kN.m}$$

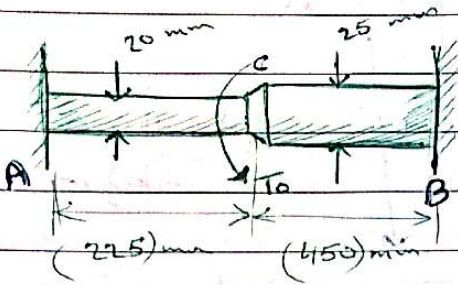
Now

~~$$\Phi_{C/A} = \Phi_{B/A} + \Phi_{C/B}$$~~

$$\frac{1.5 \times \pi}{180} = \frac{(T_{AB})(L_1)}{JG} + \frac{(T_{BC})(L_2)}{JG}$$

~~$\frac{\pi}{4} (d)^4 \Rightarrow d = 69 \text{ m}$~~ (2)

3.8-6 (stepped shaft)



$$\tau_{(all)} = 43 \text{ MPa}$$

* Find T_0 max ??

Sol:

$$\phi_{B/A} = \phi_{C/A} + \phi_{B/C} = 0$$

$$\Rightarrow \frac{T_{CA} L_{CA}}{J} + \frac{T_{CB} L_{CB}}{J} = 0 \quad (1)$$



T_{CA}

$$\sum T_x = -T_{CA} + T_{co} + T_{CB} = 0$$

$$T_{CA} = T_{co} + T_{CB} \quad (2)$$

(Sub. (2) in (1))

$$\frac{T_{co} (225 \times 10^{-3})}{\frac{\pi}{32} (20)^4 \times 10^{-12}} + \frac{T_{CB} (450 \times 10^{-3})}{\frac{\pi}{32} (25)^4 \times 10^{-12}} = 0$$

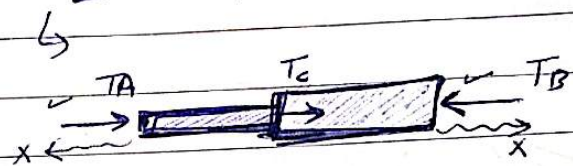
$$\Rightarrow (0.014)(T_{co} + T_{CB}) + 0.0117 T_{CB} = 0$$

$$0.0275 T_{CB} = -0.014 T_{co}$$

$$T_{CB} = -1.83 T_{co} \quad \#$$

$$T_{CA} = -0.83 T_{co} \quad \#$$

على العرض لا يصح الوباء



$$\tau_{(all)} = \frac{T_{AC} (16)}{\pi d_{ac}^3} = 43 \text{ MPa}$$

$$43 \text{ MPa} \leq \frac{(16)(0.83 T_0)}{\pi (20)^3}$$

$$T_0 = 217 \text{ N.m}$$

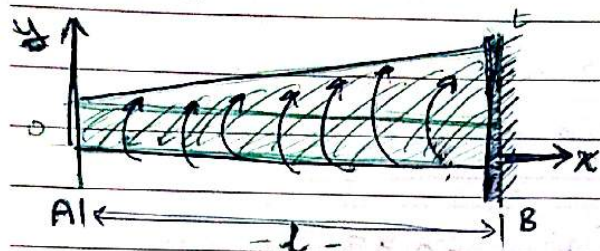
on (C-B)

$$43 \text{ MPa} \leq \frac{16 T_{CB}}{\pi d_{CB}^3}$$

$$T_0 = 72 \text{ MPa} \quad \#$$

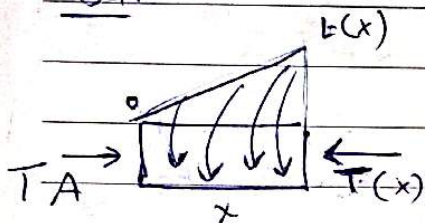
في هذا الحد

* 3.8-8)



$\Rightarrow t(x) \Rightarrow$ distribute torque
 * find: (T_A, T_B) ??

Sol:



$$T(x) = T_A + \frac{t(x) \cdot x}{2}$$

cut to find

$$\int_0^x t(x) dx$$

$$t(x) = \frac{t_0}{L} x$$

$$T_x = T_A + \int_0^x \frac{t_0}{L} x dx$$

$$T_x = T_A + \frac{t_0}{L} \frac{x^2}{2}$$

$$T_x = T_A + \frac{x^2}{2} \frac{t_0}{L}$$

$$\phi = \frac{TL}{GJ} \Rightarrow d\phi = \frac{T(x) dx}{GJ}$$

$$\phi = \frac{1}{GJ} \int_0^L T(x) dx = 0$$

(fixed) $\phi = 0$
 !! $\phi = 0$

$$0 = \frac{1}{GJ} \int_0^L \left(T_A + \frac{t_0 x^2}{2L} \right) dx$$

$$0 = \int_0^L T_A + \frac{t_0 x^2}{2L} dx$$

$$0 \Rightarrow T_A L + \frac{t_0 L^3}{6L}$$

$$T_A = -\frac{t_0 L}{6} \quad \#$$

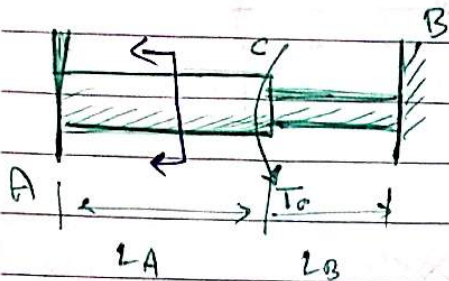
$$* T_A + \int_0^L t(x) dx - T_B = 0$$

$$T_A + \int_0^L \frac{t_0}{L} x dx - T_B = 0$$

$$T_B = \quad \#$$

$$\left. \begin{array}{l} \ominus \phi \text{ is } \text{---} \\ \oplus \phi \text{ is } \text{---} \end{array} \right\} =$$

* 3.9 - 7



$$\phi_{B/A} = \frac{-T_A L_A}{GJ} + \frac{(-T_A - T_0) L_B}{GJ} = 0$$

$$T_A = \dots$$

$$\phi_{C/A} = T_A \text{ (clockwise for } \phi_{C/A} \text{)} \quad \#$$

$$\phi_{B/A} = 0$$

⇒ Find: $(\phi_{C/A})$??

Sol:

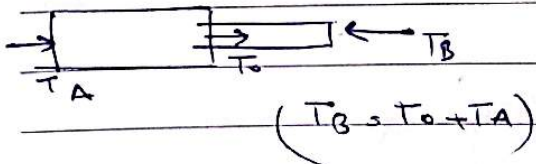
$$\phi_{B/A} = 0$$

$$0 = \phi_{C/A} + \phi_{B/C}$$



Internal torque in (A-C) is $(-T_A)$

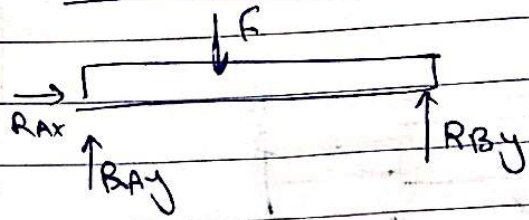
$$\phi_{C/A} = \frac{-T_A L_A}{GJ}$$



$$\phi_{B/C} = \frac{-T_B L_B}{GJ} \Rightarrow \frac{(-T_A - T_0) L_B}{GJ}$$

**CH.4 → Shear Force & Bending

① find the reaction:



$$\sum F_x = 0$$

$$(R_{Ax} = 0) \quad \#$$

$$\sum F_y = 0$$

$$(R_{Ay} + R_{By} - F = 0)$$

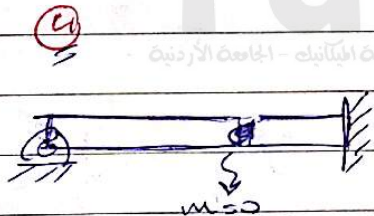
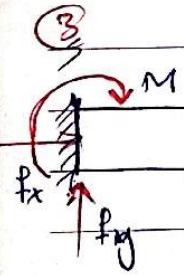
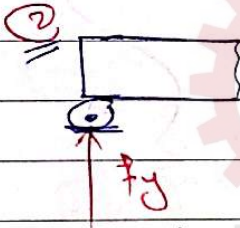
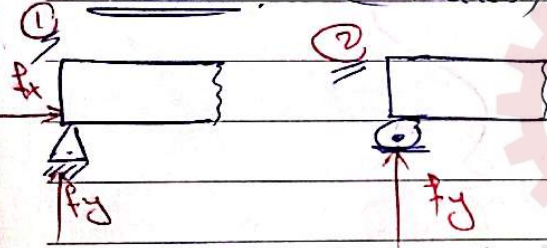
$$\sum M_A = 0$$

$$-FL_1 + R_{By}(L_1 + L_2) = 0$$

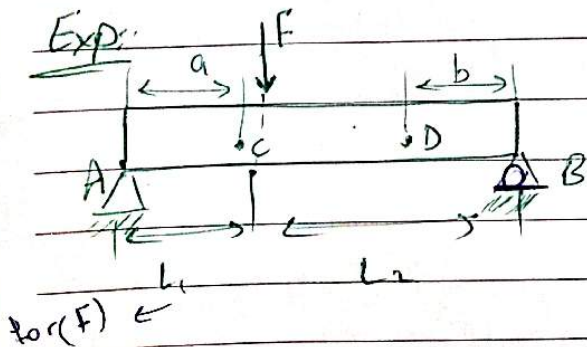
$$R_{By} = \left(\frac{FL_1}{L_1 + L_2} \right)$$

$$R_{Ay} = \left(\frac{FL_2}{L_1 + L_2} \right)$$

* Review: (statics)

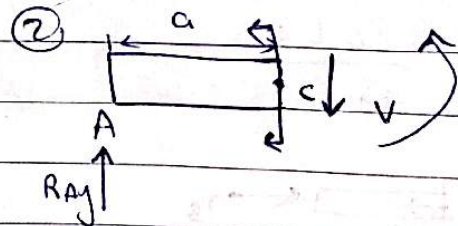


Exp:



for (F) ←

Sd/1

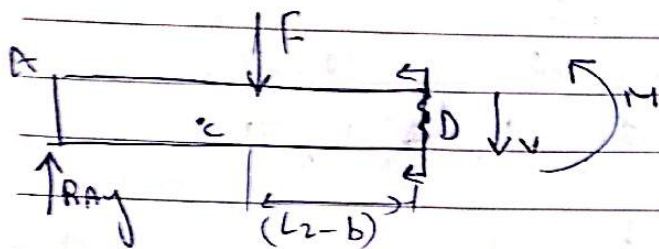


$$V = R_{Ay} \quad \#$$

$$\sum M_C = 0$$

$$M = R_{Ay} a \quad \#$$

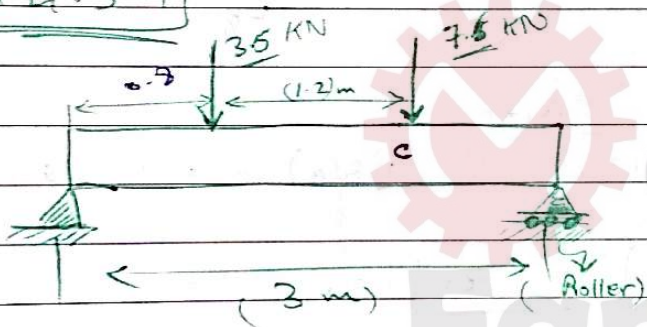
@ D



$$V_D = R_{Ay} - F$$

$$M_D = F(L2 - b) - R_{Ay}(L1 + L2 - b)$$

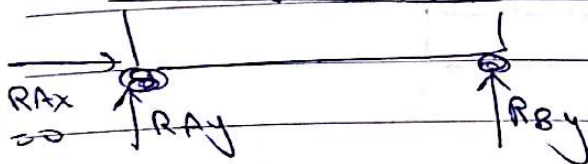
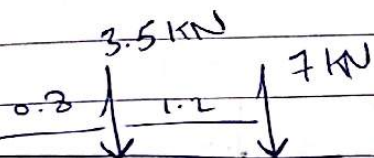
* 4.3-11



* Find:

(V, M) to the left of (7 kN)?

Sol:



$$\sum M_A = 0$$

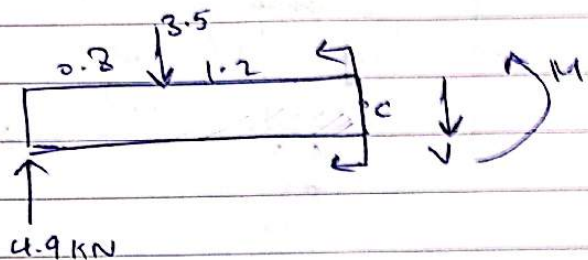
$$-(3.5)(0.8) - 7(2) + R_{By}(3) = 0$$

$$R_{By} = 5.6 \text{ kN}$$

$$\sum F_y = 0$$

$$R_{Ay} = 4.9 \text{ kN}$$

Now shear @ (7) kN (c)

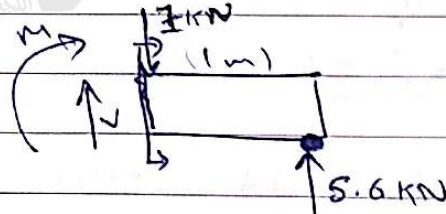


$$V_c = 4.9 - 3.5 = 1.4 \text{ kN}$$

$$M_c = 4.9(2) + 3.5(1.2)$$

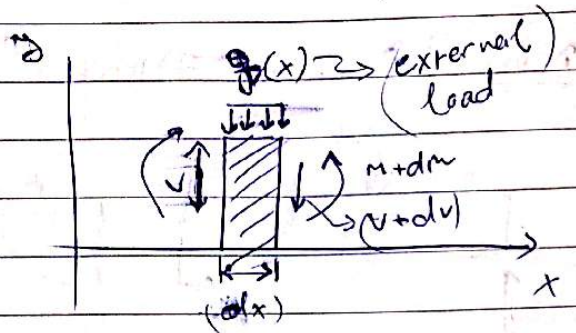
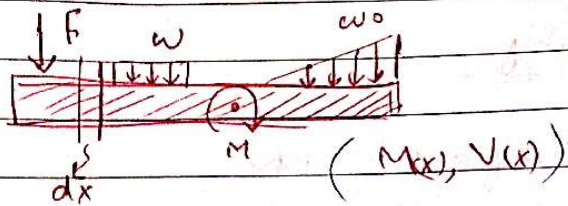
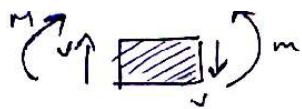
$$M_c = 5.6$$

2) (V, M) to the right of (7 kN)



$$V = -7 + 5.6 = -1.4 \text{ kN}$$

$$M = (5.6)(1) = 5.6$$



$$\sum F_y = 0$$

$$V(x) - q(x)dx - (V+dV) = 0$$

$$-q(x)dx = dV$$

$$\left(q(x) = -\frac{dV}{dx} \right) \quad \#$$

$$\sum M = 0$$

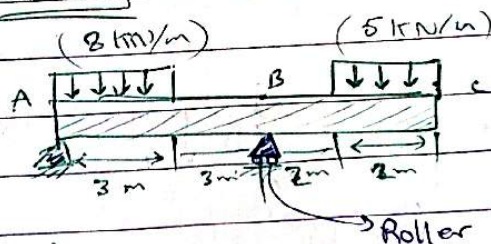
$$-M(x) - V(x)dx + M(x) + dM = 0$$

$$\left(V(x) = \frac{dM}{dx} \right) \quad \#$$

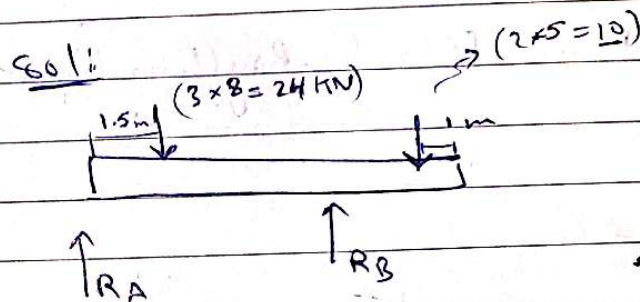
$$* \quad q(x) = -\frac{dV}{dx}$$

$$* \quad V(x) = \frac{dM}{dx}$$

4.3-5



Find:
VD, MD @ 5m from A



$$\sum M_A = 0$$

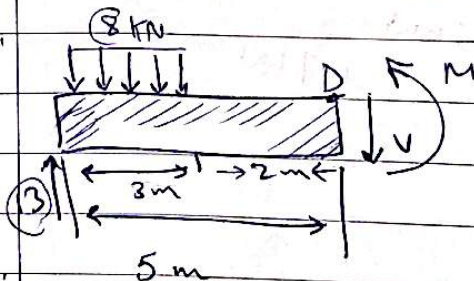
$$-24(1.5) + R_B(5) - 10(4) = 0$$

$$R_B = 21 \text{ kN}$$

$$\sum F_y = 0$$

$$R_A = 13$$

Now (Section @ 5m)



$$\sum F_y = 0$$

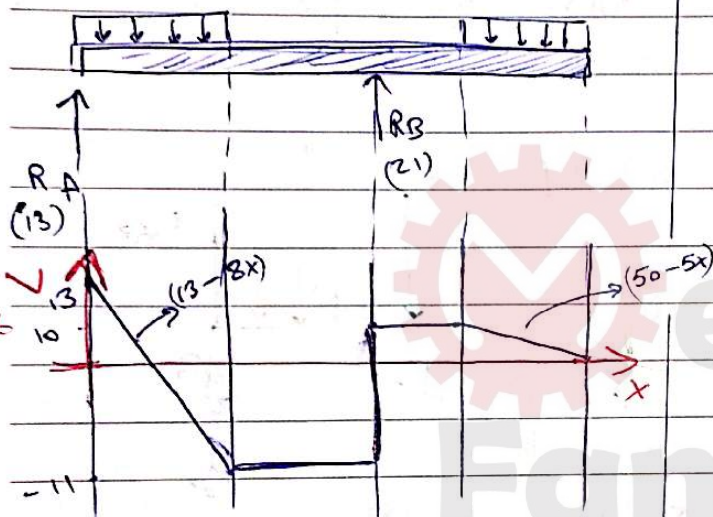
$$-V - 24 + 13 = 0$$

$$V = -11 \text{ kN}$$

$$M_D = -(13)(5) + (21)(35)$$

$$M_D = (-19 \text{ kN} \cdot \text{m})$$

To find shear force & bending moment diagram :-



$$0 \leq x < 3$$

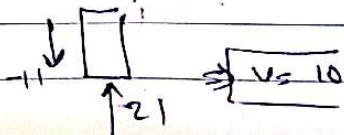
$$q(x) = 8 \text{ kN}$$

$$V = V_0 + \int_0^x -q(x) dx$$

$$= 13 - \int_0^x 8 dx$$

$$\Rightarrow (13 - 8x) \Rightarrow 13 - 8(3) \Rightarrow (-11)$$

$$(3-6)$$



$$(8-10) \quad 8 < x < 10$$

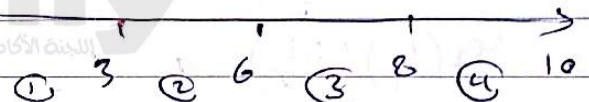
$$N = V_0 - \int_8^{10} q(x) dx$$

$$V = 10 - \int_8^{10} 5 dx$$

$$V = (30 - 5x) (50 - 5x)$$

Now:- (Moment dia)

$$M(x) = M_0 + \int_0^x V(x) dx$$



$$\textcircled{1} \quad 0 \leq x \leq 3$$

$$M(x) = 0 + \int_0^x (13 - 8x) dx$$

$$M(x) = 13x - \frac{8x^2}{2}$$

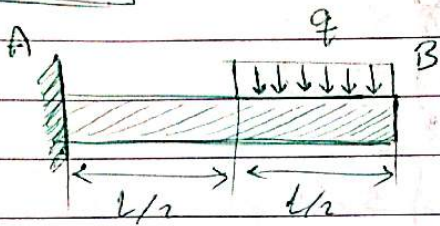
$$\Rightarrow M(3) = 13(3) - \frac{8(9)}{2}$$

$$\textcircled{4} \quad 8 < x < 10$$

$$M = -10 + \int_8^{10} (50 - 5x) dx$$

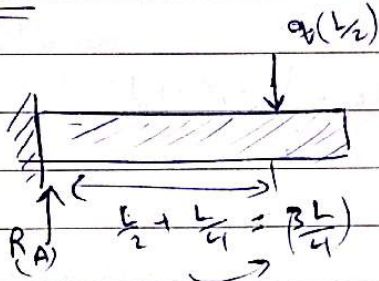
$$M = (-250 + 50x - 2.5x^2)$$

4.5-3 :-



⇒ Draw shear and bending moment diagram ??

Sol:

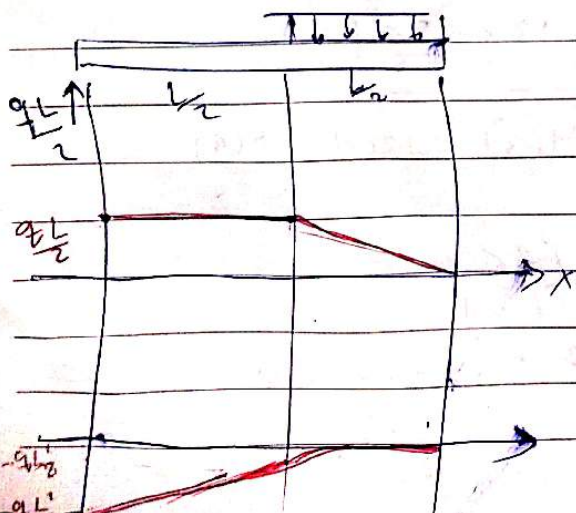


$$+\circlearrowleft \sum M_A = 0$$

$$M_A - q\left(\frac{L}{2}\right)\left(\frac{3L}{4}\right) = 0$$

$$M_A = -q\frac{3L^3}{8}$$

$$R_A = \frac{qL}{2}$$



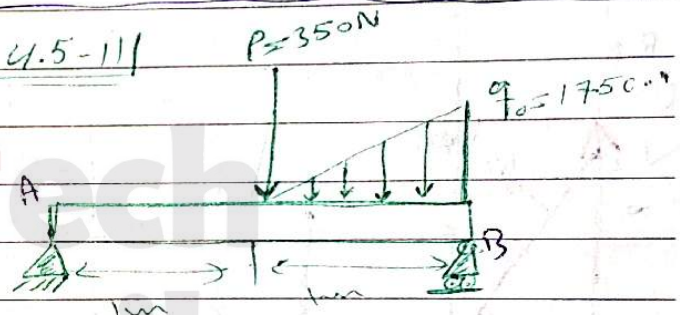
$$\left(\frac{L}{2} - L\right)$$

$$V(x) = V_0 + \int_{\frac{L}{2}}^x -q(x) dx$$

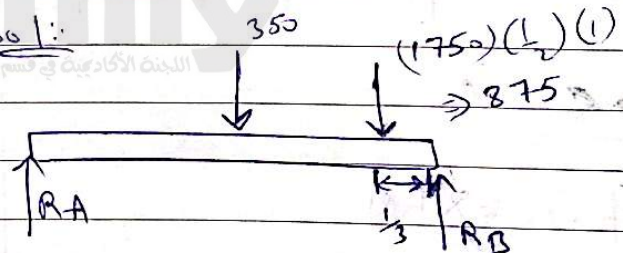
$$V(x) = \frac{qL}{2} - \int_{\frac{L}{2}}^x q \frac{L}{2} dx$$

$$V(x) = \frac{qL}{2} + q\left(x - \frac{L}{2}\right)$$

4.5-11



Sol:



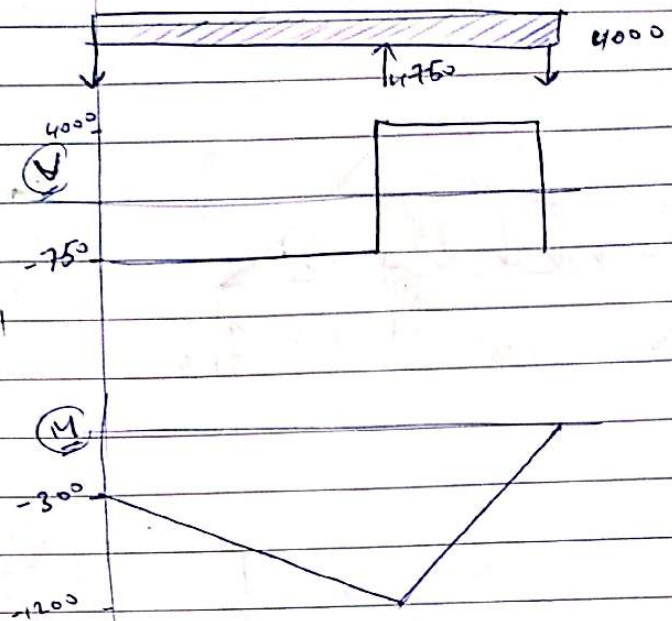
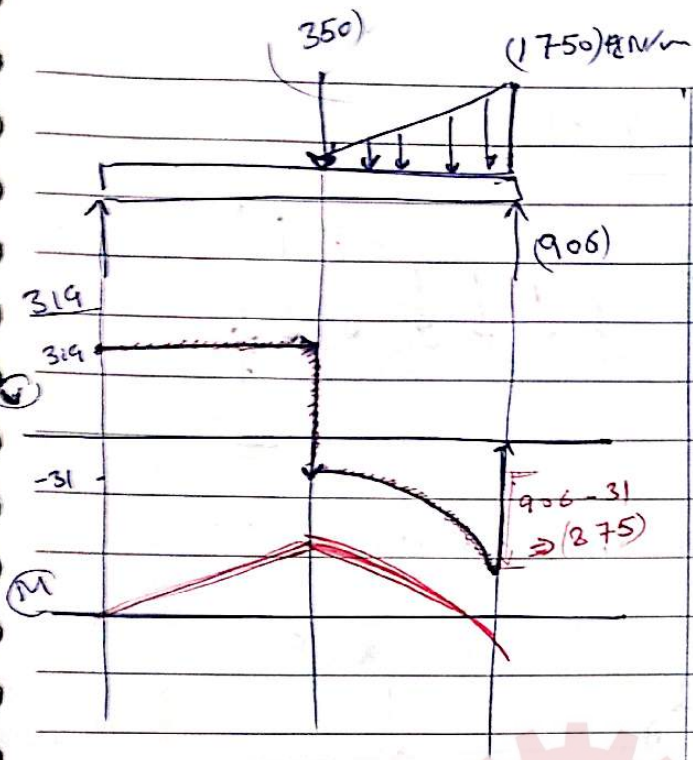
$$\sum M_A = 0$$

$$-350(1) - 875\left(1 + \frac{2}{3}\right) + R_B(2) = 0$$

$$R_B = 906 \text{ N}$$

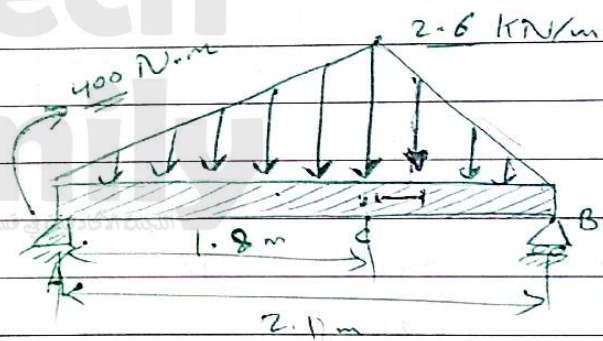
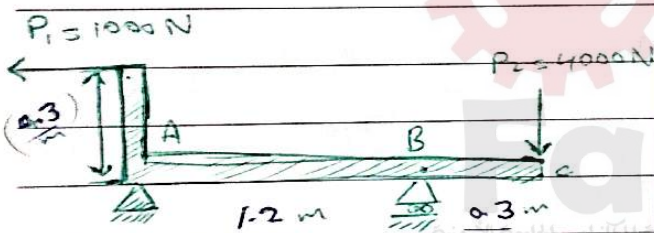
$$R_A = 319$$

$$\left[\sum F_y = 0 \right]$$



Ex 4.9 - 11

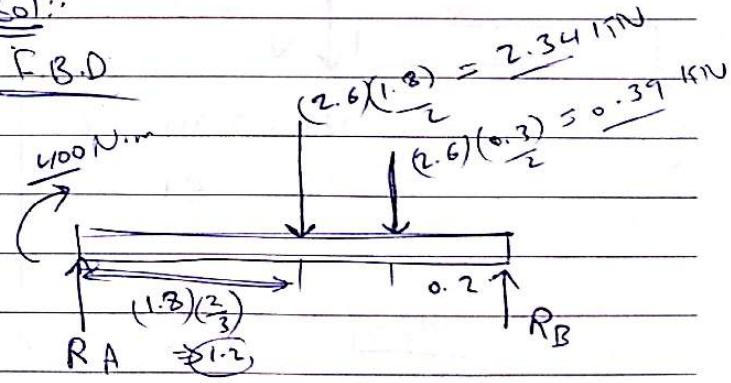
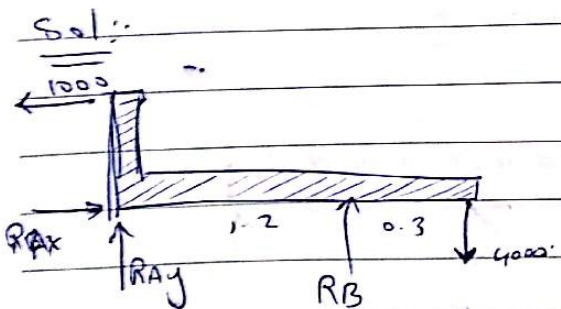
Ex 4.9 - 27



⇒ Draw (Shear & M) Diagram

Sol:

F.B.D



$$R_B = 4750 \text{ N}$$

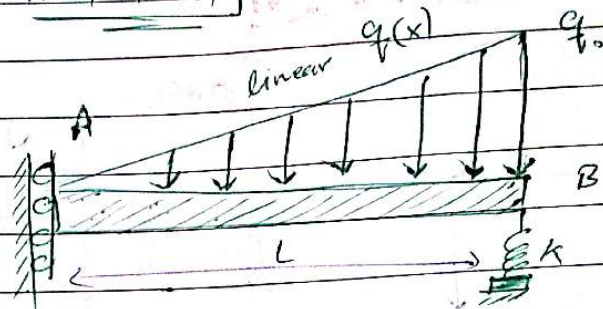
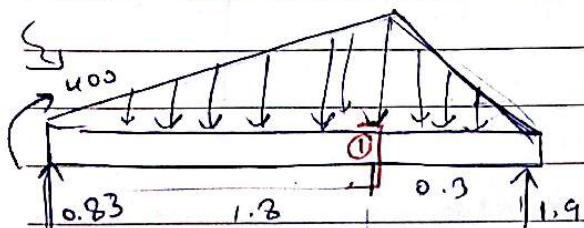
$$R_{Ay} = 750$$

$$\sum M_A = 0 \rightarrow 400 - (2.34 \times 10^3)(1.2) - 0.39 \times 10^3(1.9) + R_B(2.1) = 0$$

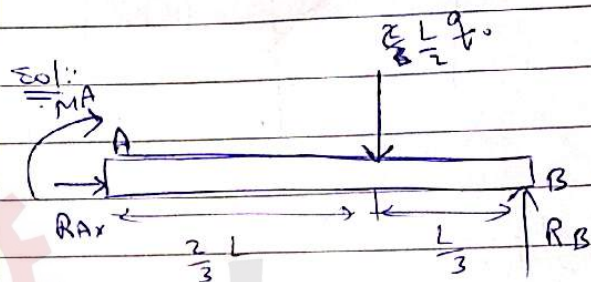
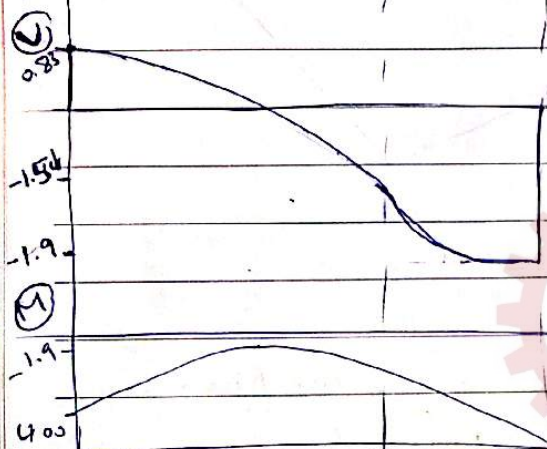
$$R_B = 1.9 \text{ kN}$$

$$R_A = 0.83 \text{ kN} \quad (\Sigma F_x = 0)$$

$$* 4.9 - 31$$



Plot V, M diagrams ??



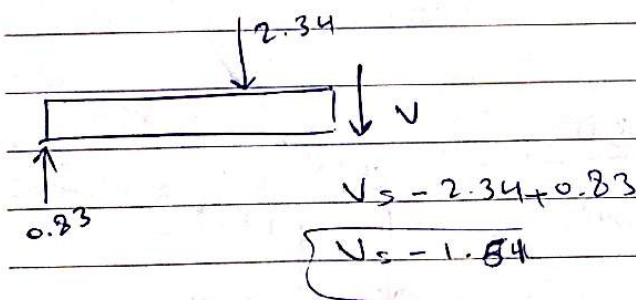
$$\Sigma M_A = 0$$

$$-\left(\frac{q_0 L}{2}\right)\left(\frac{2}{3}L\right) + R_B(L) = 0 \quad \text{--- (1)}$$

@ (1) Section (بج)

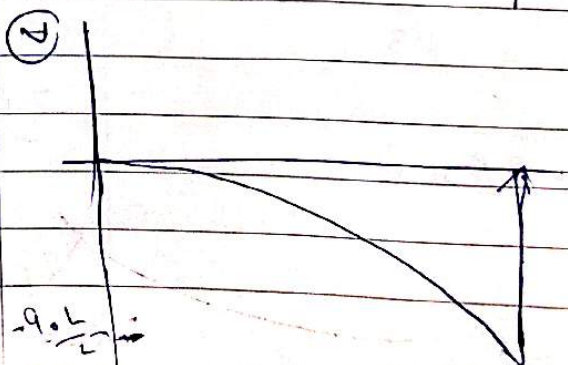
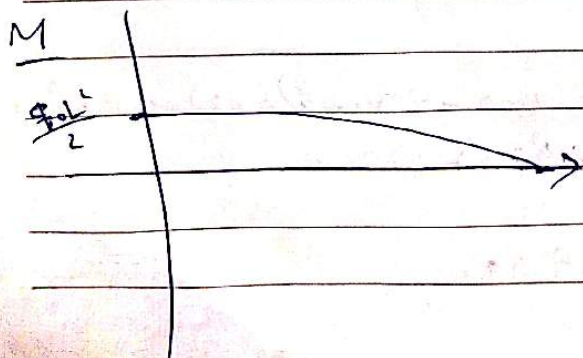
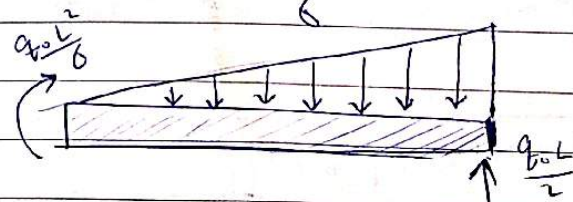
$$\Sigma F_y = 0$$

$$R_B = \frac{q_0 L}{2} \quad \text{--- (2)}$$



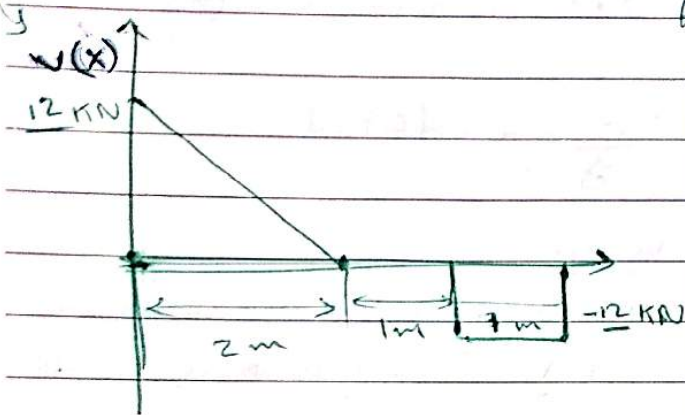
(1, 2)

$$M_A = \frac{q_0 L^2}{6}$$

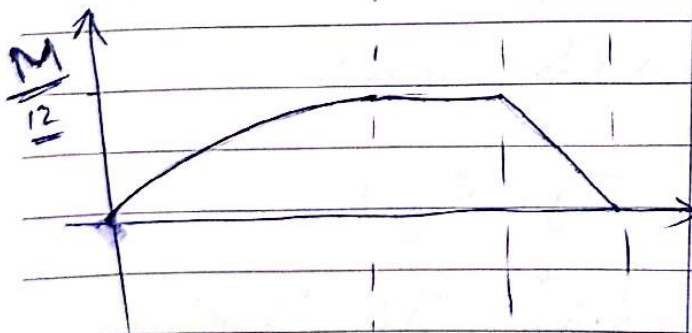
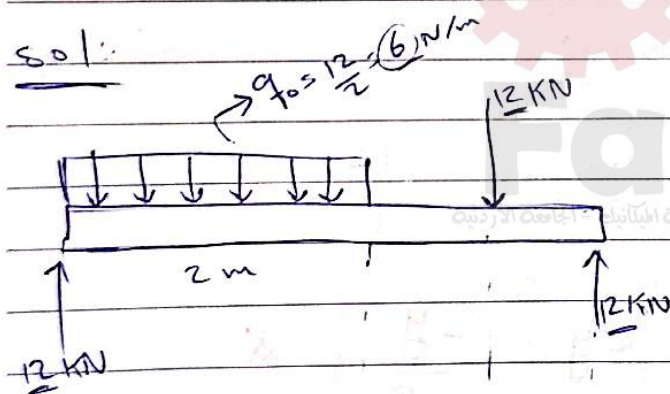


* 4.9-32 - (Shear Force diagram)

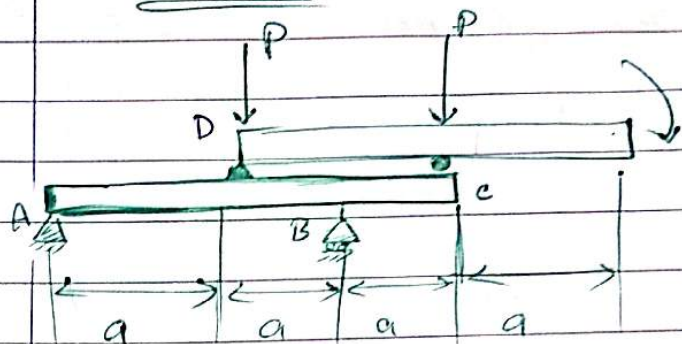
(القص على طول المسار)



⇒ Find the ① loads
② M. diagram

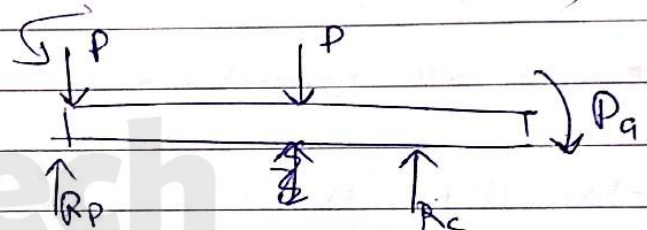


* 4.8-81



Sol:

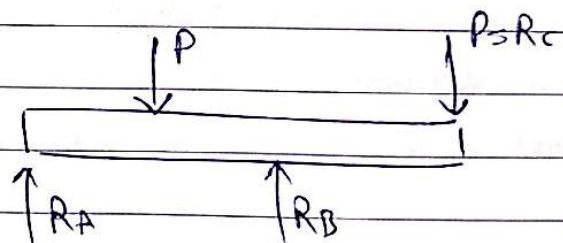
F.B.D (for upper Beam)



$\sum M_D = 0$

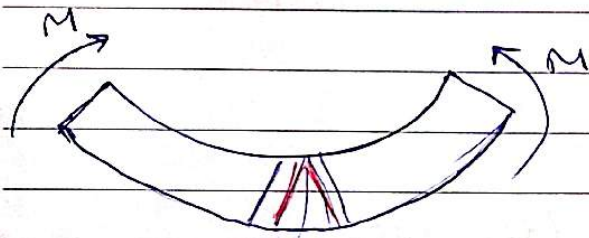
$$R_c = P \quad R$$

F.B.D (ABC)



* CH.5

⇒ Normal stress in beam



⇒ ~~Stresses~~ nonuniform.

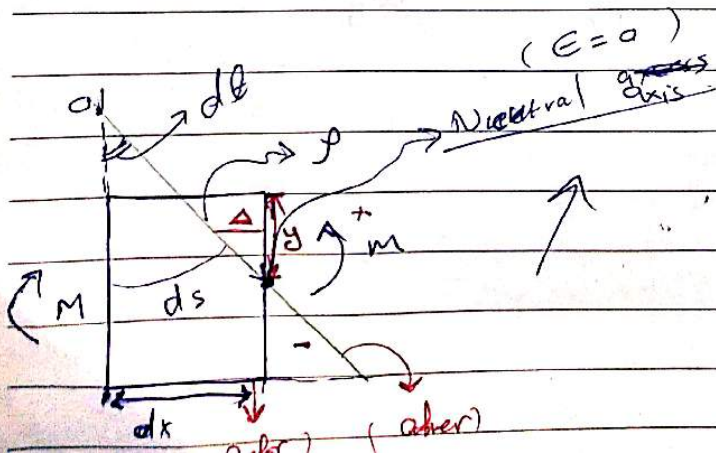
- Relat (M, σ)

(Bending Moment) (Stress)

-(σ) depends on moment
(Internal moment)

2. Cross-section

3. location on x, y, z



→ for small deflection:

$$ds = dx$$

$$R d\theta = ds = dx \quad \text{--- (*)}$$

$$\left(\frac{\Delta}{y} = \tan d\theta \right) = d\theta$$

where:

Δ :- shorting @ a distance
(y) from (n. axis)

$$\epsilon_y = \frac{\Delta}{y} = \frac{d\theta}{dx} \Rightarrow \frac{-y d\theta}{dx}$$

(Shorting)

$$\left(\text{from (*)} \frac{d\theta}{dx} = \frac{1}{R} \right)$$

$$\epsilon_y = \frac{-y}{R} \quad \text{---}$$

⇒ (for stress)

$$\sigma_y = \epsilon_y \cdot E$$

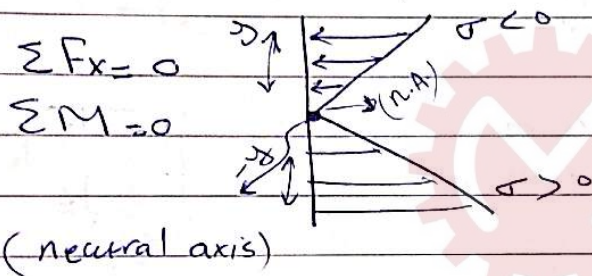
$$\sigma_y = \frac{-y E}{R} \quad \text{---}$$

⇒ Now: I need to know:

- location on (C-S) for y_{50}
(neutral axis)
location
- f (curvature)



The deflection segment must be in equilibrium.



$$\sigma_y = -\frac{E y}{\rho}$$

$$F = 0 \Rightarrow \int \sigma_y dA = 0$$

$$\int \sigma_y dA = 0 \quad \text{--- (1)}$$

$$\int -\frac{y E}{\rho} dA = 0 \quad \text{--- (2)}$$

$\int y dA = 0$ 1st moment of area.

⇒ The 1st moment of area about the neutral axis is (zero)

↳ Therefore, the neutral axis is the centroid

- The moment produced by the stress distribution is equivalent to the bending moment

$$dM = y \overbrace{\sigma_y}^{\text{force}} dA$$

$$= -y \left(-\frac{y E}{\rho} \right) dA$$

$$dM = \frac{y^2 E}{\rho} dA$$

$$\int dM = \int \frac{y^2 E}{\rho} dA$$

$$\Rightarrow \frac{E}{\rho} \int y^2 dA$$

Second area moment of inertia about (N.A)

- neutral axis

$$M = \frac{E I}{\rho}$$

$$\rho = \frac{E I}{M}$$

$$\sigma_y = -y \frac{E}{\rho}$$

$$E = -\frac{y}{\rho}$$

$$\sigma_y = -y \frac{E}{\frac{EI}{M}}$$

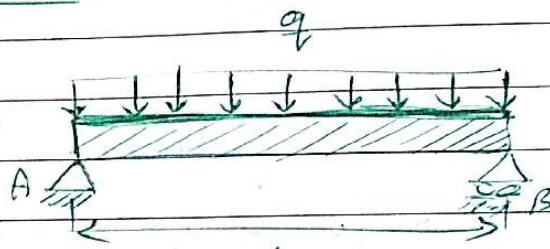
$$= -\frac{1.6}{500 + \frac{1.6}{2}} = -1.6 \times 10^{-3}$$

$$\sigma_y = -\frac{My}{I}$$

5.5-41 ✓

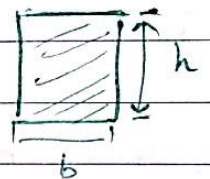
c: distance from the neutral axis to the upper fibers.

$$\sigma_{max} = -\frac{Mc}{I}$$



$$L = 4 \text{ m}$$

$$q = 5.8 \text{ kN/m}$$



$S \Rightarrow \frac{I}{c}$: section modulus

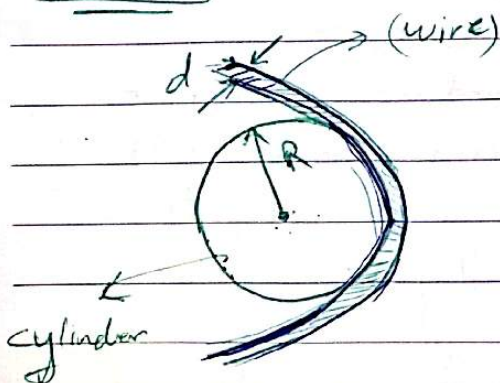
$$b = 140 \text{ mm}$$

$$h = 240 \text{ mm}$$

$$\sigma_{(max)} = -\frac{M}{S}$$

find: $\sigma_{(max)}$?

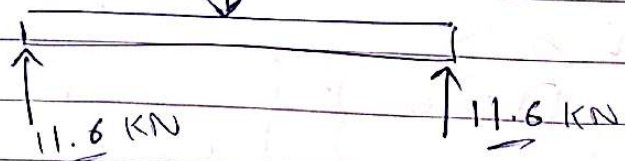
* 5.4-11



$$\sigma = \frac{My}{I}$$

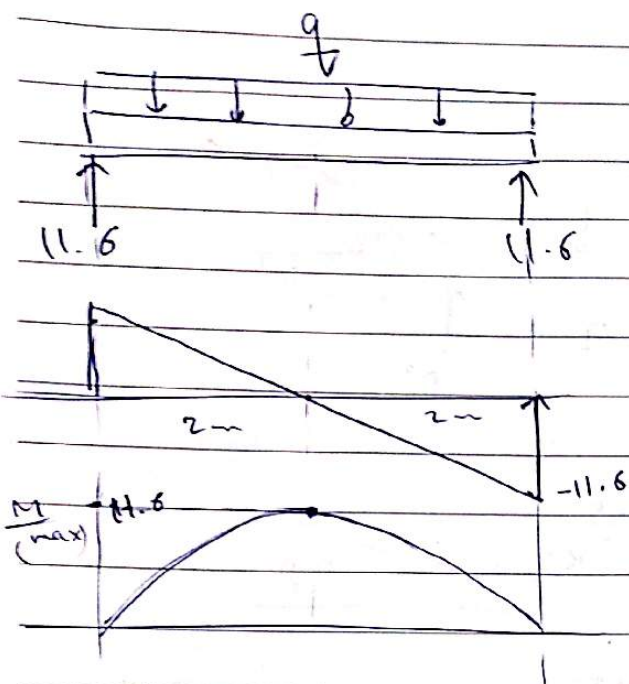
* finding $M_{(max)}$:

$$(5.8)(4) = 23.2 \text{ kN}$$



$d = 1.6 \text{ mm}$ find $E_{(max)}$?

$$R = 500 \text{ mm}$$

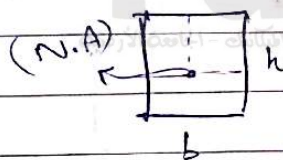


$$M(\max) = \frac{(11.6)(2)}{2}$$

(center)

$$\Rightarrow 11.6 \text{ kN.m}$$

* Finding I



$$I = \frac{bh^3}{12}$$

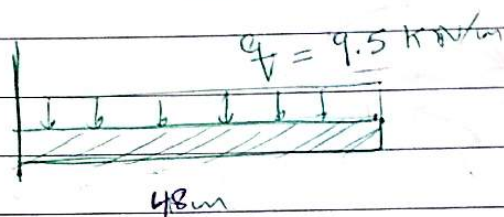
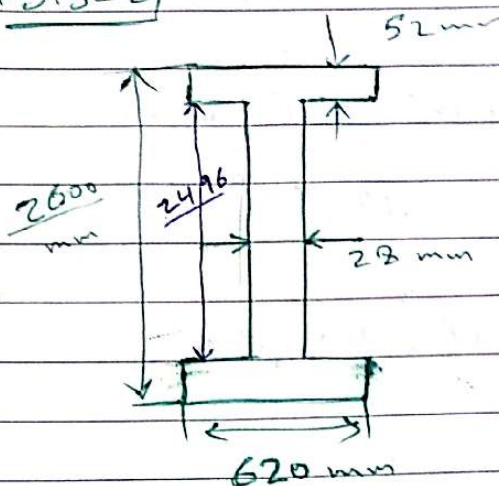
$$I = 161.2 \text{ mm}^4$$

Now

$$V(\max) = \frac{(11.6 \times 10^3)(120 \times 10^{-3})}{161.2 \times 10^{-12}}$$

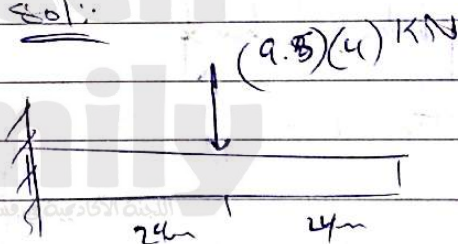
$$\sigma_{\max} = 8.63 \text{ MPa}$$

* 5.5-8



Find: $\sigma(\max)$?

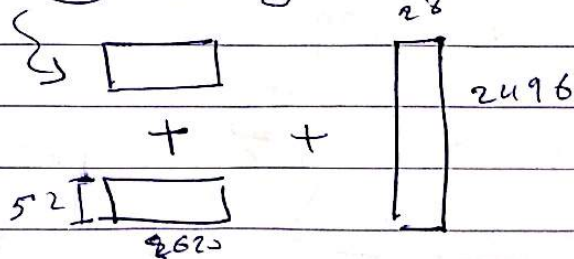
Sol:



یہ منہ کا ہے
(c) مرکزہ
لا دے گی لایا دے

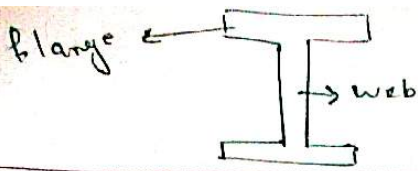
$$M(\max) = (9.5)(48)(24) = 10944 \text{ kN.m}$$

* I : (using Parallel Thm.)



$$I = \frac{1}{12} (620)(52)^3 + \frac{1}{12} (28)(2496)^3 + (620)(52)(1224)^2$$

$$I = 1.15 \times 10^{11} \text{ mm}^4$$



(Illumin formula)

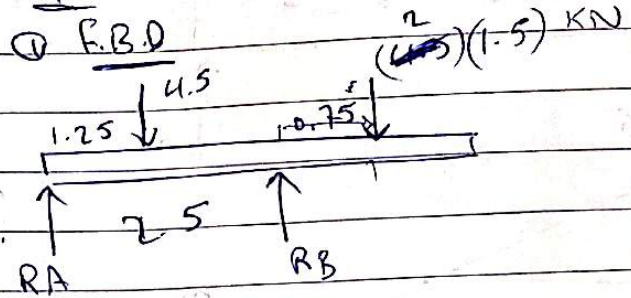
11

Now

Sol:

① F.B.D

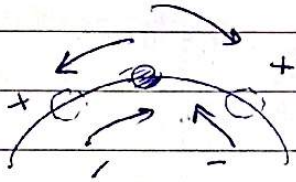
$$\sigma_{max} = \frac{(10944) \times 10^3 \cdot (1300 \times 10^{-3})}{1.15 \times 10^4 \text{ mm}^4}$$



$$\sigma_{max} = 123.7 \text{ MPa}$$

MA=0

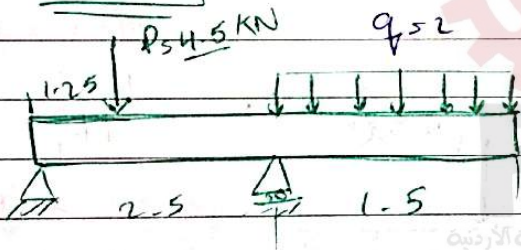
$$\Rightarrow R_B = 6.15 \text{ kN}$$



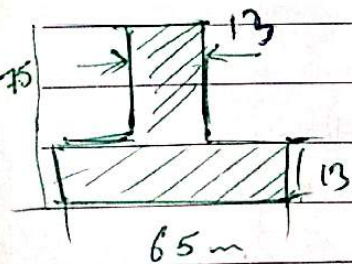
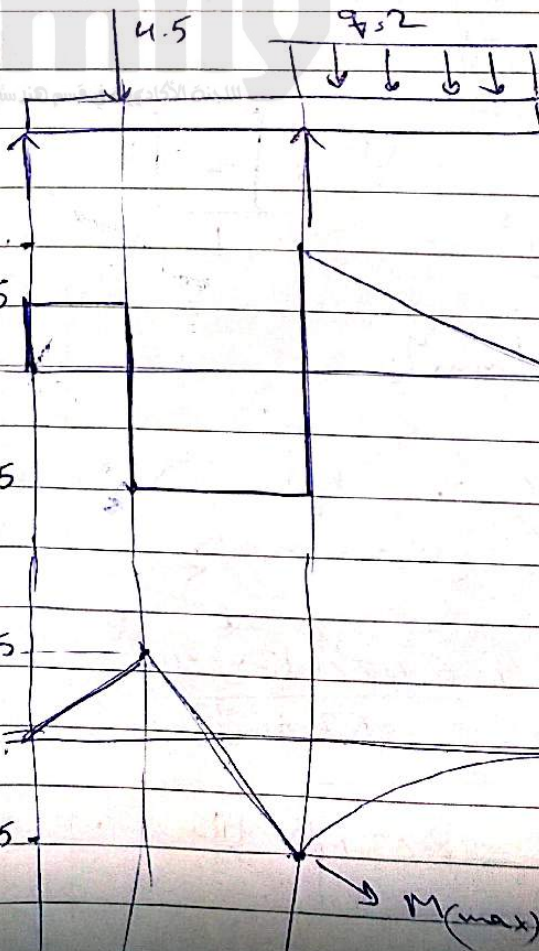
$\sum F_{y=0}$

$$\Rightarrow R_A = 1.35 \text{ kN}$$

* 5.5 - 32



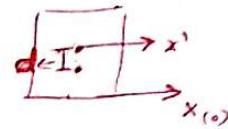
* Moment diag.



* Find:

σ_{max} , σ_{min} ??

مركز الثقل (C) الجيد = 12.5 (مقدم)
(التي فرضناه بالبرهان)



$$\therefore M(\max) = -2.25 \text{ kN.m}$$

$$d_1 = 24.8 - 6.5 = 18.3 \text{ mm}$$

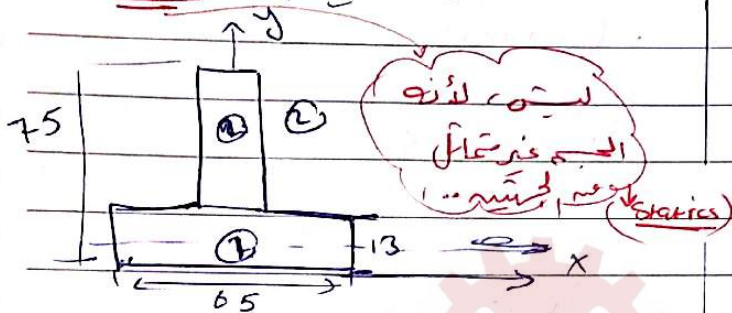
$$\Rightarrow \sigma(\max) = \frac{-M y}{I}$$

Now (I)

* (أولاً احس بطول centroid)

$$I_x = \sum (\bar{I}_i + A_i d_i^2)$$

$$= \bar{I}_1 + A_1 d_1^2 + \bar{I}_2 + A_2 d_2^2$$



$$= \frac{1}{12} (65)(13)^3 + (65)(13)(18.3)^2$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

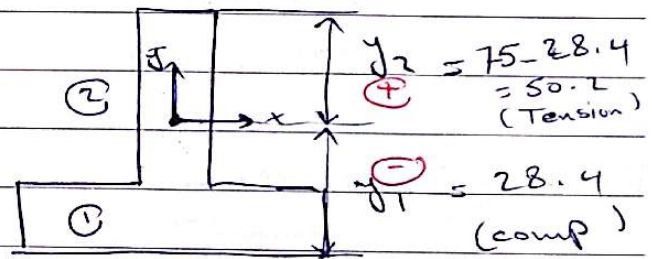
$$+ \left(\frac{1}{12} (13)(62)^3 + (62)(13)(19.3)^2 \right)$$

$$= \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

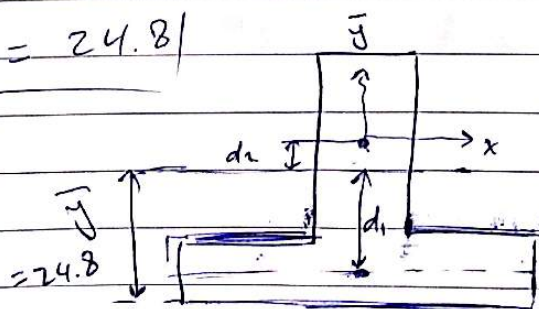
$$\therefore I = 850193 \text{ mm}^4$$

$$= \frac{(13)(65)(6.5) + ((62)(13)(44))}{(13)(65) + (13)(62)}$$

$\Rightarrow$ (To find y_1, y_2)



$$\bar{y} = 24.8$$

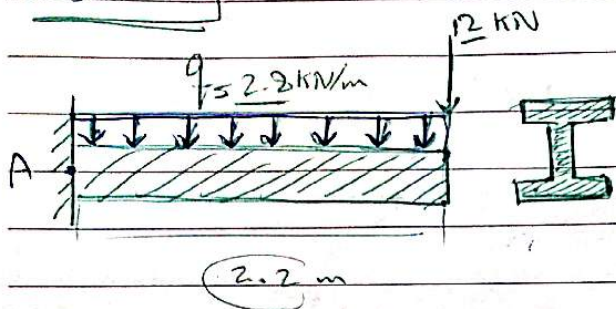


$$\sigma_t = \frac{(2.25) \times 10^3 (50.2)}{0.850 \times 10^6 \text{ mm}^4}$$

$$[\sigma_t = 133 \text{ MPa}]$$

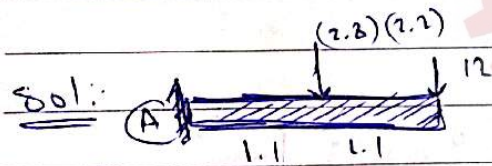
$$\sigma_c = \frac{(2.25) \times 10^3 (28.4)}{0.85 \times 10^6 \text{ mm}^4} = -65 \text{ MPa}$$

* 5.6-3



$$\sigma_{all} = 110 \text{ MPa}$$

⇒ find section modulus??



(بدون ما أرسى ... واضح)

✓ A is M_{max}

$$M_{(max)} = -12(2.2) - (2.2)(2.2)(1.1)$$

$$M = -33.18 \text{ kN}\cdot\text{m}$$

$$\sigma_{all} = \frac{-M}{S} \quad \left(\frac{-M}{I} \cdot \frac{1}{3} \right)$$

$$110 \times 10^6 = \frac{33.18 \times 10^3}{S}$$

$$S = 301.6 \times 10^3 \text{ mm}^3$$

⇒ (E) ~~المادة~~ ~~المادة~~

Choose $S = 311.5 \times 10^3 \text{ mm}^3$

(S=301.6) الأقرب لـ
(مأخوذ الأقرب لأكبر)

(Sahay)

{ HE 160 B } ✓
(Beam ~~المادة~~)

{ G = 42.6 Kg/m }
(Beam) ~~المادة~~
م. ك. ب. م

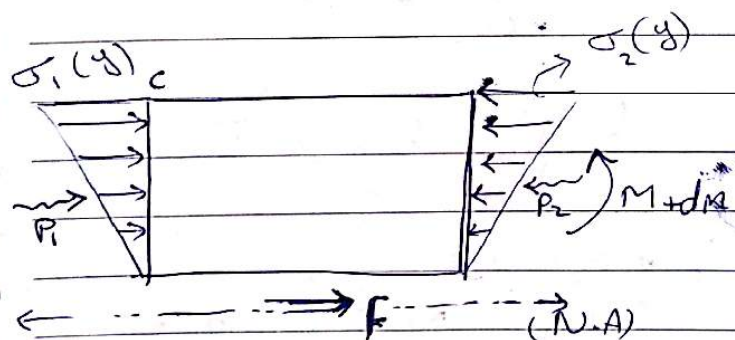
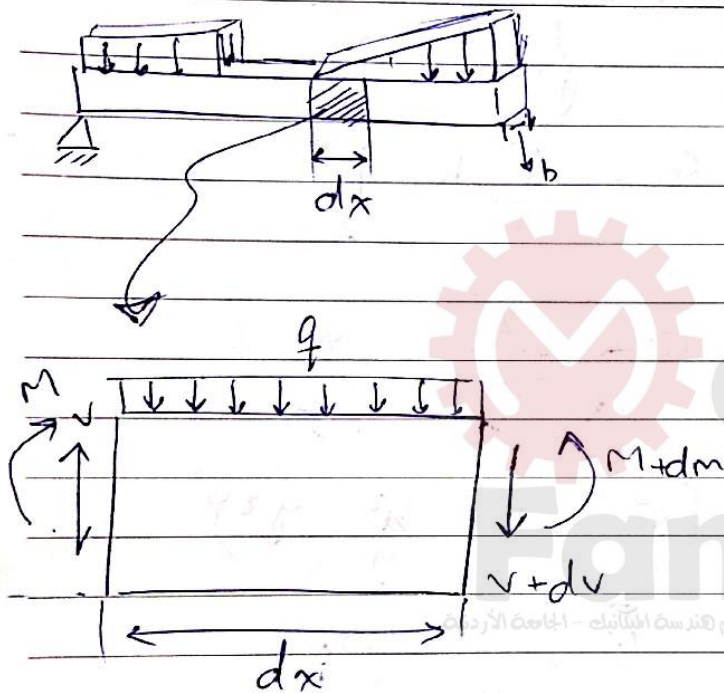
$$\text{New load} = (42.6 \times 10) \text{ N/m} = 426 \text{ N/m}$$

$$q = q_0 + 426$$

$$q_2 = 3.2 \text{ kN/m}$$

** Shear stress in beams

→ if a shear load exist on the beam, then shear stress between the layers the beam exist.



$$P_2 = P_1 + F$$

$$P_1 = \int_y^c \frac{M y}{I} dA$$

(body)

$$P_1 = \int_y^c \frac{M y}{I} dA$$

$$P_2 = \int_y^c \frac{(M + dM) y}{I} dA$$

$$F = P_2 - P_1$$

$$= \int_y^c \frac{(M + dM) y}{I} dA - \int_y^c \frac{M y}{I} dA$$

$$F = \int_y^c \frac{dM y}{I} dA$$

$$\tau = \frac{F}{A} = \frac{dM}{I b(y) dx} \int y dA$$

$$= \frac{V}{I b(y)} \int y dA$$

(first moment of area)

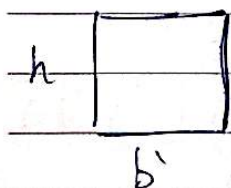
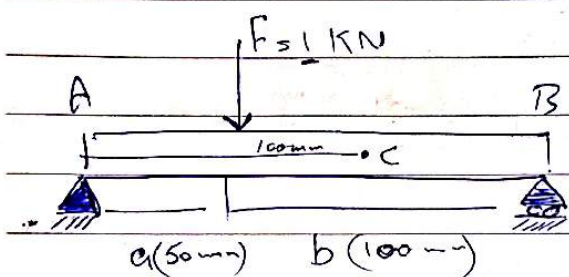
$$\tau_y = \frac{V Q(y)}{I b(y)}$$

##

• Shear stress.

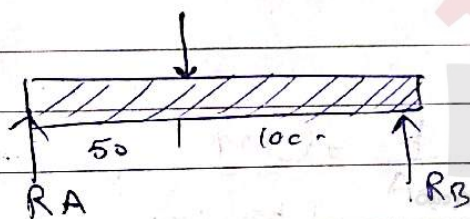
(at $y = c$ $\tau(y) = \text{Zero}$)

Exp: (تفسير)



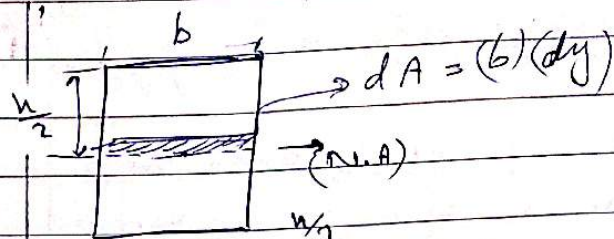
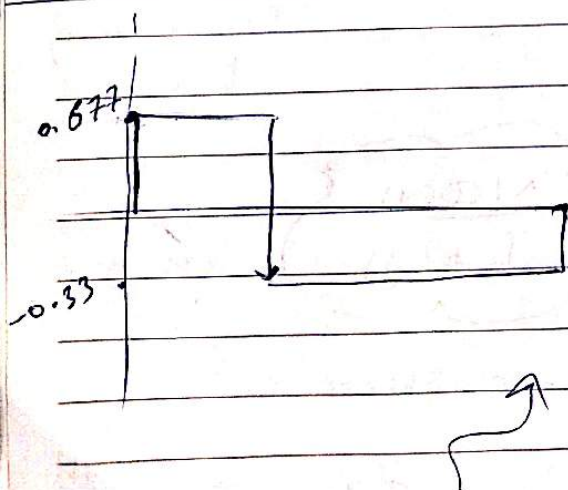
⇒ Find (τ) distributed @ C

Sol:-



$$R_B = 0.33$$

$$R_A = 0.677$$



$$Q = \int y dA$$

$$Q = \int_{-h/2}^{h/2} y b dy$$

$$Q = \frac{y^2}{2} b \Big|_{-h/2}^{h/2}$$

$$Q \Rightarrow \frac{h^2 b}{8} - \frac{y^2 b}{2}$$

$$Q = \frac{b}{2} \left(\frac{h^2}{4} - y^2 \right)$$

بالتعويض

$$\Rightarrow y = \frac{h}{2}$$

$$Q = 0$$

$$b(y) = b \quad / \quad I = \frac{1}{12} b h^3$$

Now

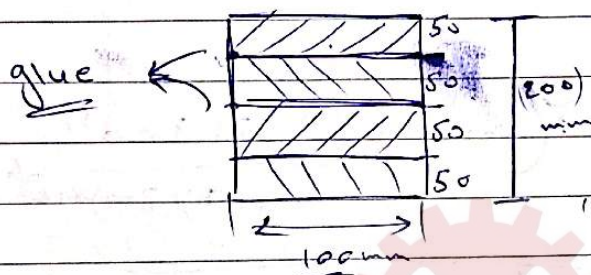
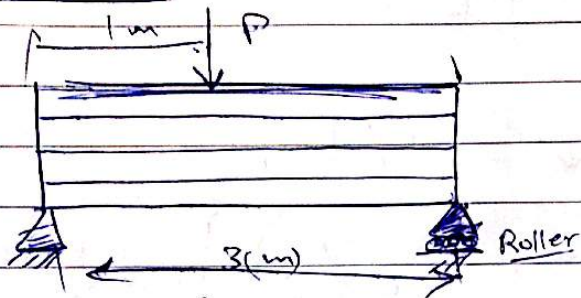
$$\tau(y) = \frac{V Q}{b I}$$

$$\tau = \frac{6 V \left(\frac{h^2}{4} - y^2 \right)}{b h^3}$$

$$\tau_{max} = \frac{6 V}{4 b h}$$

$$\frac{KN}{m^3} \times A \times L$$

*5.8-7 :- (Wood beam)



$$\tau_{(all)}_{glue} = (0.35 \text{ MPa})$$

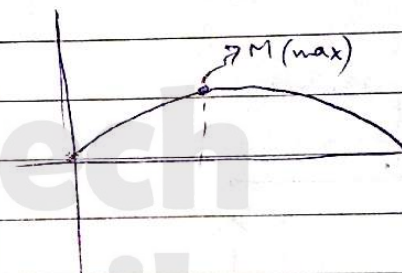
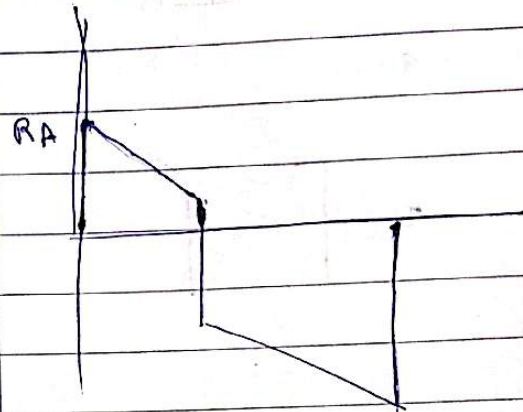
$$\sigma_{(all)}_{wood} = (11 \text{ MPa})$$

~~Find~~ Beam weight = 5.5 kN/m³

Find: P_{max}

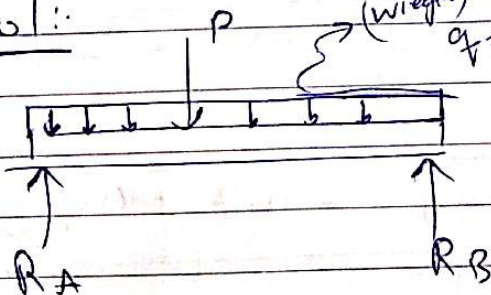
$$R_B = \frac{P}{3} + 0.165$$

$$R_A = \frac{2P}{3} - 0.055$$



$V_{(max)} @ R_B$

Sol: $(weight) = (5.5)(0.1)(0.2^2) = 0.11 \text{ kN/m}$



$$\tau_{(max)} = 0.35 \text{ MPa} = \frac{3}{2} \frac{V}{bh}$$

$$0.35 \times 10^6 = \frac{3}{2} \left(\frac{P}{3} + 0.165 \right) \frac{1}{(100)(200)}$$

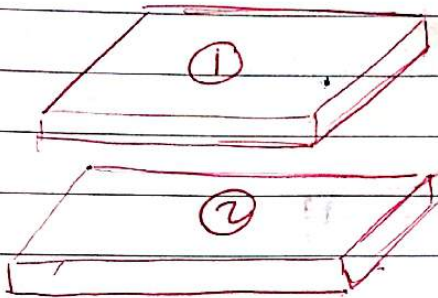
$$P = \quad \#$$

$$\sum M_A = 0$$

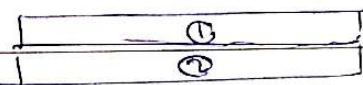
$$-P(1) - (0.11)(3)\left(\frac{3}{2}\right) + R_B(3) = 0$$

5-10-7

**Built up section:-

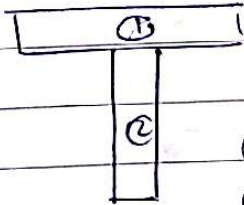


①



(خونه بوض)

②

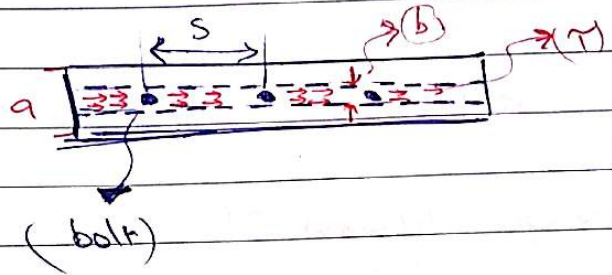
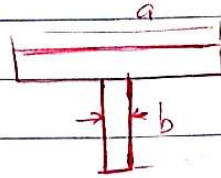


($I_B > I_A$)
(P) $\leftarrow$
(A) $\rightarrow$

- These section are composed from different parts bonded together using:

- glue
- bolts
- welds

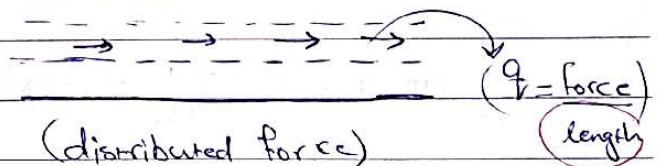
- The built up section has advantages over the as larger (I, s)



$$\tau = \frac{VQ}{Ib}$$

$$\tau \times s \Rightarrow q \text{ (shear flow)}$$

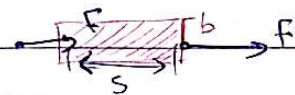
$$\Rightarrow \frac{VQ}{I}$$



(distributed force)

$$\Rightarrow (b \times (\tau) \text{ equal})$$

(concentrated force)



$$q = \frac{F}{s} \Rightarrow F = q s$$

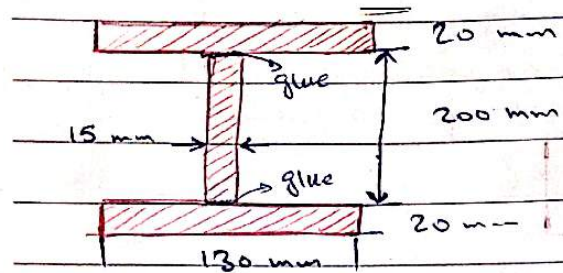
$$F = \tau b s$$

$$= \frac{VQ}{Ib} (b) (s)$$

shear flow

$$F = q s$$

* 5-11.11



$q_{\text{glue}} = 12 \text{ kN/m}$

- determine $V(\max)$??

Sol:

$$V = \frac{V \phi}{I}$$

↓
(Shear flow عرفية) ←
(Built up) ←

$$\downarrow$$
$$I = \frac{1}{12} (15)(200)^2 + 2 \left[\frac{1}{12} (30)(20)^2 \right]$$

$$(20)(130)(230-120)]$$

$$I = 24.7 \times 10^6 \text{ mm}^4$$

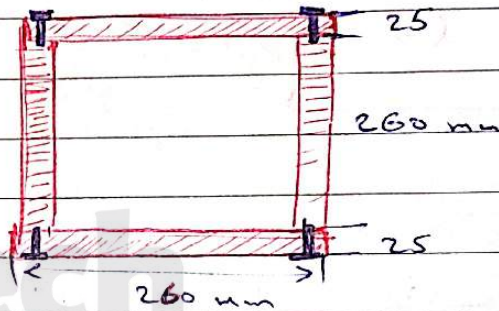
$$Q = \underbrace{(20)(130)}_A \underbrace{(110)}_B$$

$$Q = 286 \times 10^3 \text{ mm}^3$$

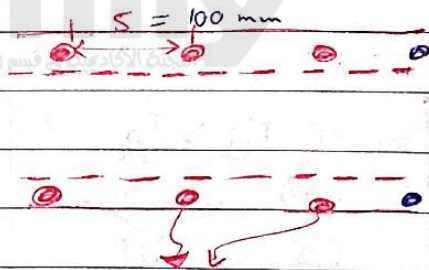
$$\frac{12 \text{ kN}}{\text{m}} = \frac{(V)(286 \times 10^3 \text{ mm}^3)}{24.7 \times 10^6 \text{ mm}^4}$$

$$\boxed{N = 1.16 \text{ kN}} \quad \#$$

$\times 15 = 11.4$ (Box beam)

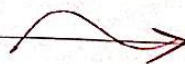


↓
(Top View)

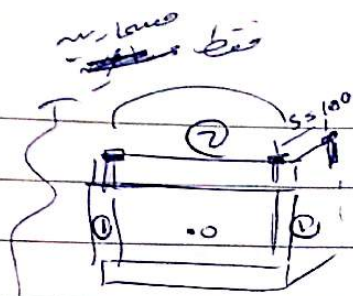


$$F_{(all)} = 1200 \text{ N}$$

Find $(v)_{\max}$??



$$q = \frac{VQ}{I}$$



$$q = \frac{F}{S} = \frac{(2)(1200)}{100 \text{ mm}}$$

$$q =$$

$$Q = \underbrace{(260)(25)}_A \underbrace{(12.5)}_y = 926.25 \times 10^{-6}$$

(بعد من 0)
 مركز 2
 $\Rightarrow \frac{260}{2} + \frac{25}{2}$

$$I = \frac{1}{12} \left[(260)(260+50)^3 - (260-50)(260)^3 \right]$$

$$I = 338 \times 10^{-8}$$

Now

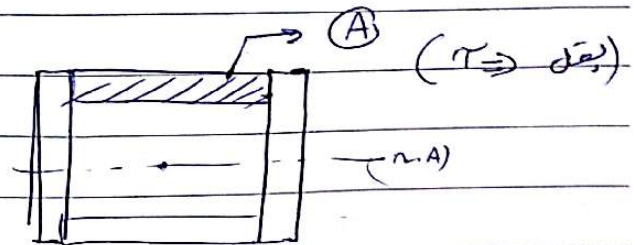
$$q = \frac{VQ}{I} = \frac{F}{S} = \frac{(2)(1200)}{100 \text{ mm}}$$

$$\frac{2400}{0.1} = \frac{V(926.25 \times 10^{-6})}{338 \times 10^{-8}}$$

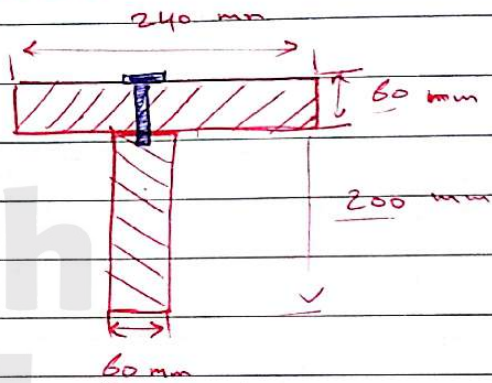
$$V = 8.76 \text{ kN}$$

* 524.44

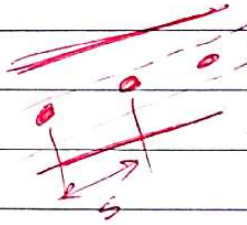
(لو كان نفس الارتفاع لكان
 بس لموقعه لوكنت مختلفه)



* 5-11.8



$V = 1500 \text{ N}$
 $F = 760$
 $\Rightarrow$ find s ??



$$\text{Sol: } q = \frac{F}{S} = \frac{VQ}{I}$$



(بعد من الطول)

$$I = 155 \times 10^{-6} \quad \bar{y} = 171$$

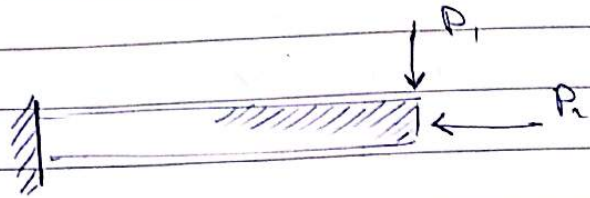
$$Q = \underbrace{(240)(60)}_A \underbrace{(59)}_y$$

$$Q = 850 \times 10^{-3} \quad q = \frac{F}{S}$$

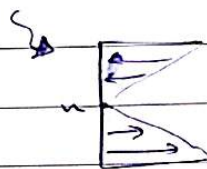
$$S = 185 \text{ mm}$$

(15.12)

xx



$$\sigma = \frac{Mc}{I} + \frac{F}{A}$$

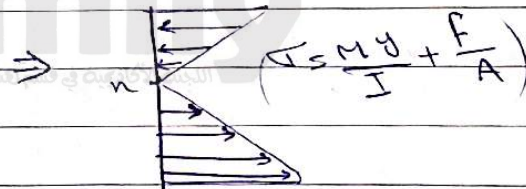


(bending)

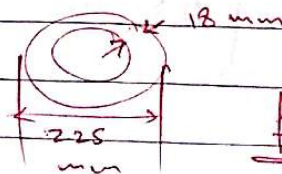
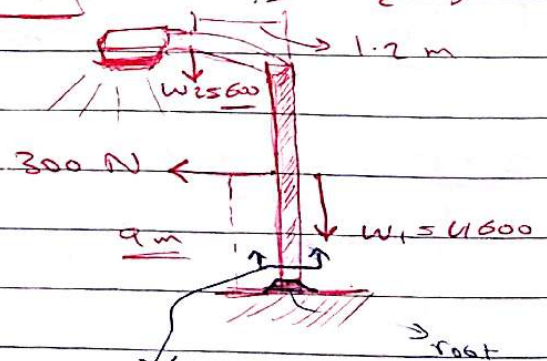


(axial)

$$\sigma_b = -\frac{My}{I} + \sigma_a = \frac{F}{A}$$



* 5-12.2 (sl. 22E)



find: σ_b
 σ_c

Sol:

Compressive force = $w_1 + w_2$

$$= 5260 \text{ N}$$

Bending moment @ the root

$$\Rightarrow (660)(1.2) + 300(9)$$

$$= 3492 \text{ N.m}$$

$$\Rightarrow \sigma_b = \frac{M y}{I}$$

$$I = \frac{\pi}{64} ((225)^4 - (189)^4)$$

$$I = 63.17 \times 10^6 \text{ mm}^4$$

$$y = 225/2 = 112.5 \text{ mm}$$

$$\sigma_b = \frac{(3492)(112.5)}{63.17 \times 10^6}$$

$$\sigma_b = 6.2 \text{ MPa}$$

#

Now for (σ_c)

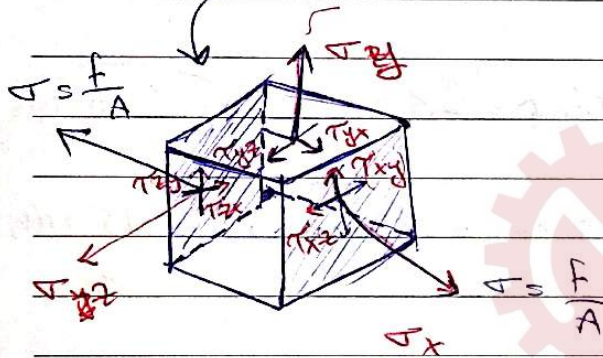
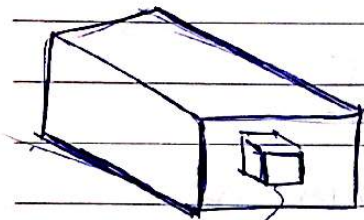
$$\begin{aligned} \sigma_c &= \frac{\Sigma F}{A} \quad (\downarrow) \\ &= \frac{-(5260)}{\frac{\pi}{4} (225^2 - 189^2)} \end{aligned}$$

$$\sigma_c = -0.45 \text{ MPa} \quad \#$$

$$\Rightarrow \sigma_{\text{comp}} = -6.2 - 0.45 = -6.65 \text{ MPa}$$

$$\Rightarrow \sigma_{\text{Tension}} = 6.2 - 0.45 = 5.75 \text{ MPa} \quad \#$$

** CH. 7 $\Rightarrow$ stress analysis

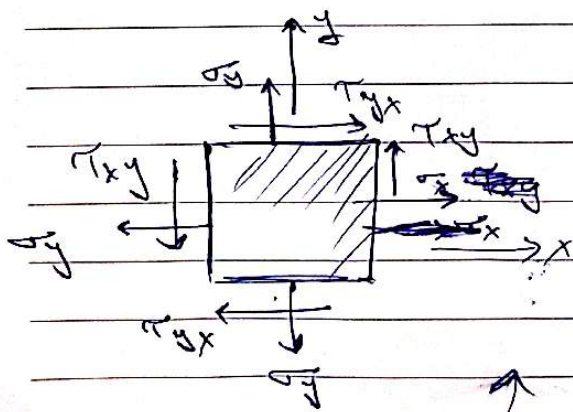


$$\tau_{zx} = -\tau_{xz}$$

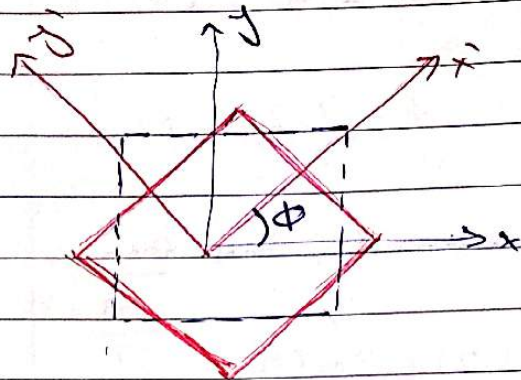
$$\tau_{zy} = -\tau_{yz}$$

$$\tau_{yx} = -\tau_{xy}$$

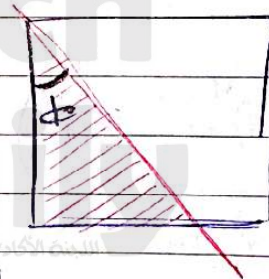
$\Rightarrow$ Two dimensional stress:



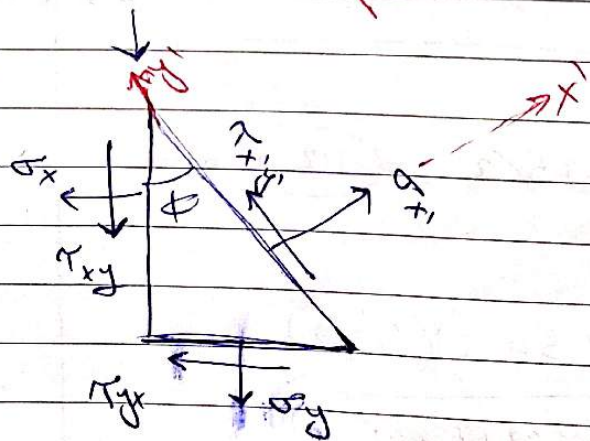
(x', y' is principal)



($\sigma_{x'}, \sigma_{y'}, \tau_{y'x'}, \tau_{x'y'}$) ??



(equilibrium)

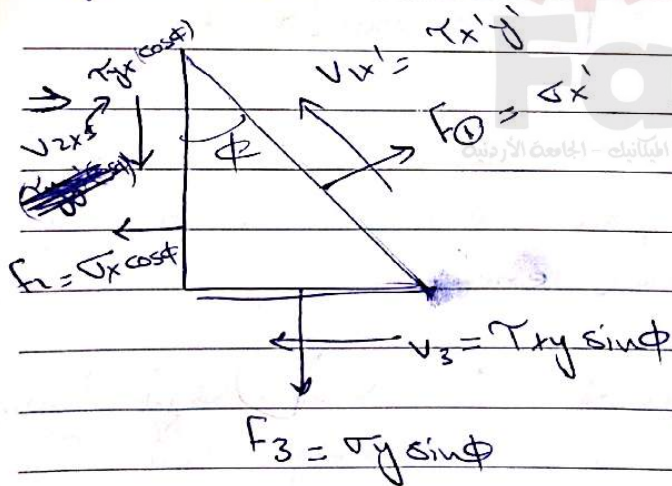
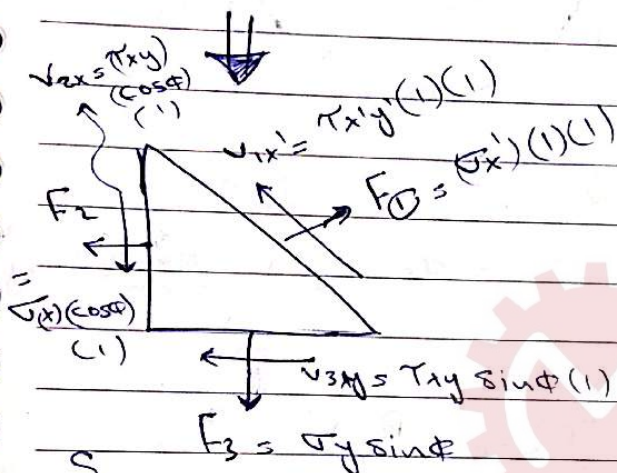
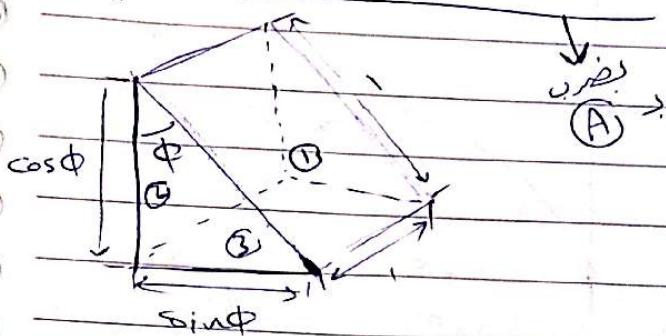


(we will finding $\sigma_{x'}, \tau_{x'y'}$ by applying equilibrium eqn.)

$$\sum F_{x'} = 0$$

$$\sum F_{y'} = 0$$

① Convert stresses to forces:



Now, I can apply (equ.)

(x y) are old axes

direction

$$\sum F_{x'} = 0$$

$$\sigma_{x'} = \sigma_x \cos^2 \phi + 2\tau_{xy} \cos \phi \sin \phi - \sigma_y \sin^2 \phi = 0$$

$$\sigma_{x'} = \sigma_x \cos^2 \phi + \sigma_y \sin^2 \phi + \tau_{xy} \sin 2\phi$$

$$\sum F_{y'} = 0$$

$$\tau_{x'y'} + \sigma_x \cos \phi \sin \phi - \tau_{xy} \cos^2 \phi + \tau_{xy} \sin^2 \phi - \sigma_y \sin \phi \cos \phi = 0$$

$$\tau_{x'y'} = -\frac{(\sigma_x - \sigma_y) \sin 2\phi}{2} + \tau_{xy} \cos 2\phi$$

$$\cos^2 \phi = \frac{1 + \cos 2\phi}{2}$$

$$\sin^2 \phi = \frac{1 - \cos 2\phi}{2}$$

$$\sigma_{x'} =$$

$$@ \phi = 0 \Rightarrow \sigma_{x'} = \sigma_x$$

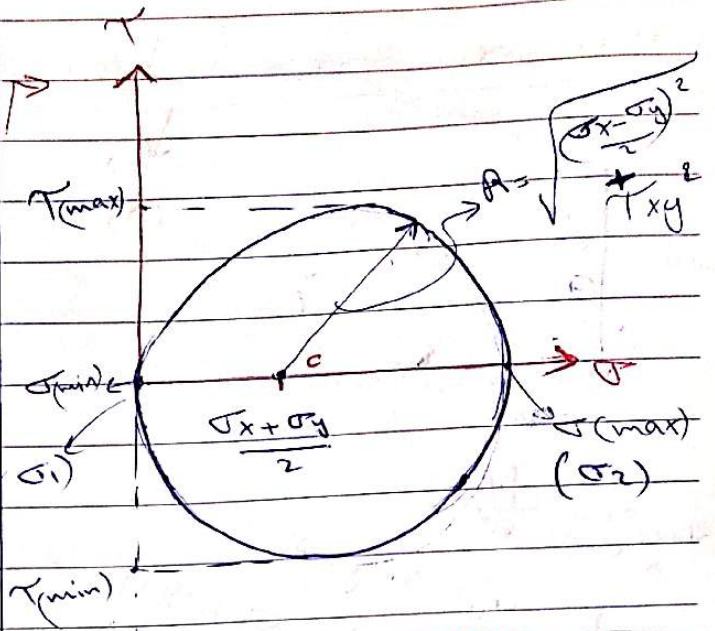
$$@ \phi = 90 \Rightarrow \sigma_{x'} = \sigma_y$$

$$@ \phi = 180 \Rightarrow \sigma_{x'}$$

For τ_{xy}

@ $\phi = 0 \Rightarrow \tau_{x'y'} = \tau_{xy}$

@ $\phi = 90 \Rightarrow \tau_{x'y'} = -\tau_{xy}$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The $+$ sign corresponds to σ_1 (max) and the $-$ sign corresponds to σ_2 (min).

$$\tau_{\max} = R$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right) = \frac{\sigma_x - \sigma_y}{2} \cos 2\phi + \tau_{xy} \sin 2\phi$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\phi + \tau_{xy} \cos 2\phi$$

Square both eqs and add

Now find ϕ @ which $[\sigma_{\max}, \sigma_{\min}, \tau_{\max}, \tau_{\min}]$ are exists!

$$\left(\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

@ $\phi = 0$ $\left. \begin{array}{l} \sigma_{x'} = \sigma_x \\ \tau_{x'y'} = \tau_{xy} \end{array} \right\} \textcircled{1}$

$$\left(\tau_{x'y'}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

Radius

@ $\phi = 90^\circ$ $\left. \begin{array}{l} \sigma_{x'} = \sigma_y \\ \tau_{x'y'} = \tau_{yx} = -\tau_{xy} \end{array} \right\} \textcircled{2}$

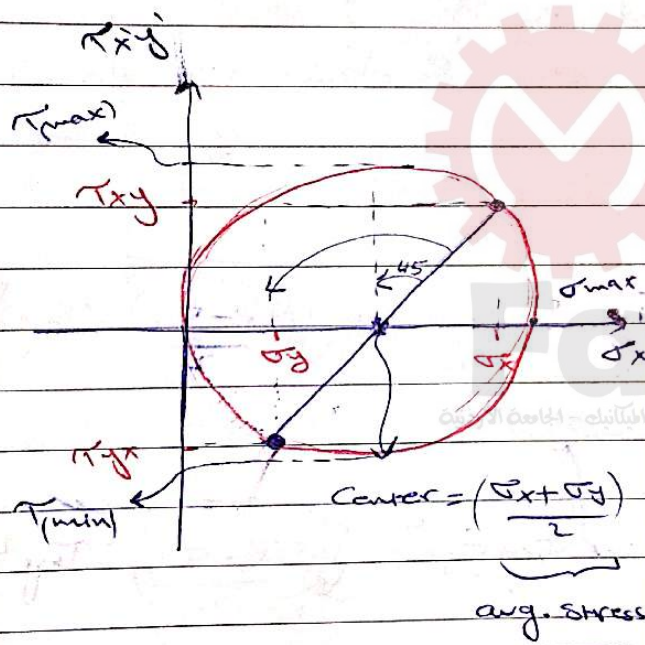
Circle eqn.

(1, 2) $\Rightarrow$ coordinates of the point on Mohr's circle.

→ The angle the radial lines through these two on the circle is $(2 \times 90^\circ) = 180^\circ$

- Therefore the line passing through the points represents the diameter on the circle.

- Therefore, given $\sigma_x, \sigma_y, \tau_{xy}$ Mohr's circle could be constructed.



Note that

@ $\sigma_{max}, \sigma_{min}$
 $\Rightarrow \tau = (\text{Zero})$

@ τ_{max}, τ_{min}
 $\Rightarrow \sigma = \text{avg. stress}$

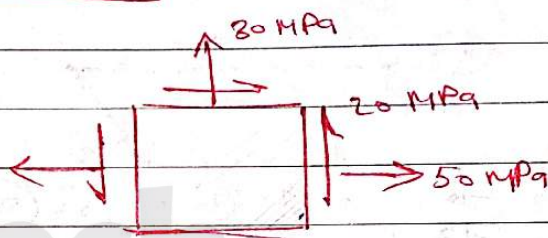
$$\sigma = \left(\frac{\sigma_x + \sigma_y}{2} \right)$$

~~the~~ (ϕ) between σ_{max}, τ_{min}
 $= 90^\circ$

→ To match the equations assign $\tau_{xy}(\text{c.w.})$ as positive on the Mohr's circle coordinate.

The plane angles are measured (c.c.w) from the reference point @ $\phi = 0 (\sigma_x, \tau_{xy})$

* Ex. 7 :-



Find: $\sigma_{x'}, \tau_{x'y'} @ \phi = 50^\circ$

- Sketch the element @ $\phi = 50^\circ$

Sol:-

* Principal Stress
 $\sigma_{x'}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\phi + \tau_{xy} \sin 2\phi$$

$$= \frac{(50 + 30)}{2} + \frac{(50 - 30)}{2} \cos 100 + (20) \sin 100$$

$$\sigma_{x'} = 61.4 \text{ MPa}$$

$\sigma_{max}, \sigma_{min} \Rightarrow$ Principle stresses

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\phi + \tau_{xy} \cos 2\phi$$

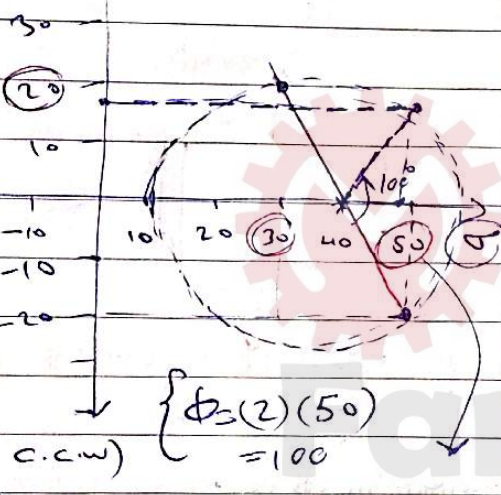
$$= -10 \sin 100 + 20 \cos 100$$

$$= -13.3$$

$$\tau = -13.3 \text{ MPa}$$

Mohr's circle
المعنى بالاسماء

(C.W)

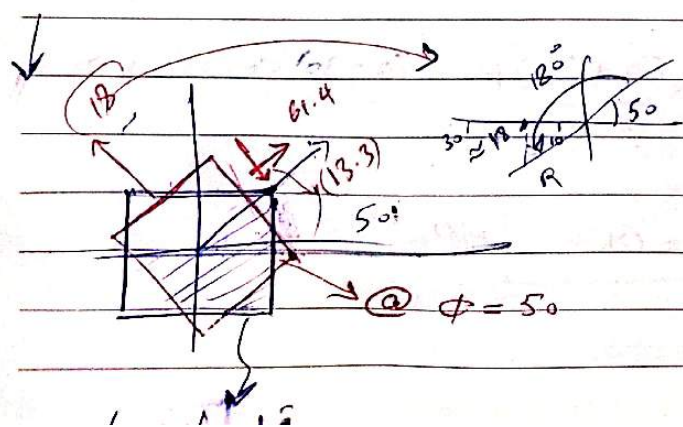


$$\left\{ \begin{aligned} \phi &= (2)(50) \\ &= 100 \end{aligned} \right.$$

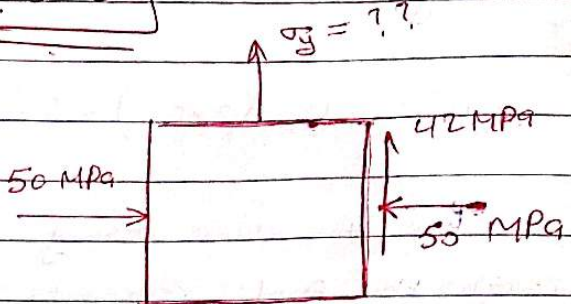
50 (C.C.W) 10 MPa

المعنى بالاسماء
المعنى بالاسماء

(Sketch element @ $\phi = 50^\circ$)



$$7.3-20$$



$\sigma_{max}, \sigma_{min}$

σ_1, σ_2

$$\sigma_1 = 33 \text{ MPa}$$

find σ_y ?

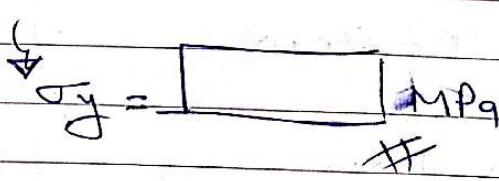
σ_2 , & draw the element @ σ_1, σ_2

Sol.

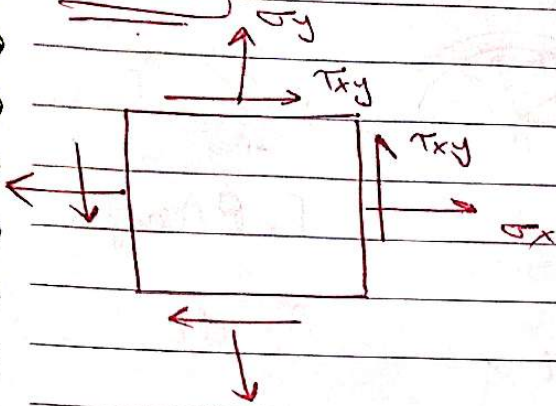
$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\left(\sigma_1 - \frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$(33 \text{ MPa} - (-50 + \sigma_y))^2 = \left(\frac{-50 - \sigma_y}{2}\right)^2 + 42^2$$



* 7.4-111



Where:

$$\sigma_x = 31 \text{ MPa}$$

$$\sigma_y = 97 \text{ MPa}$$

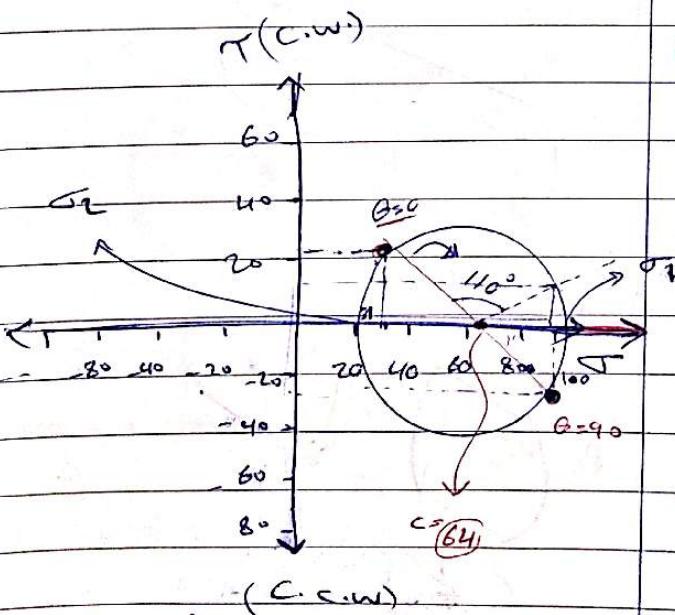
$$\tau_{xy} = -21 \text{ MPa}$$

Find:

① $\sigma_{x'}$, $\tau_{x'y'}$ @ $\theta = -55^\circ$

② σ_1, σ_2

Sol: (Using Mohr's circle)

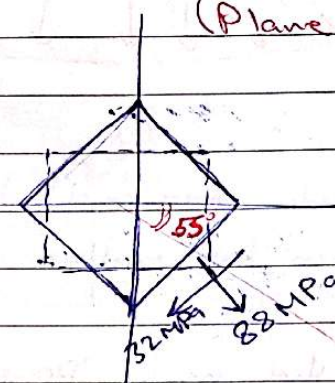


بالنسبة للزاوية المطلوبة
 $\theta = -55^\circ$
 $\sigma_{x'} = 88 \text{ MPa}$
 $\tau_{x'y'} = 32 \text{ MPa}$
 $\theta = 110^\circ \leftarrow \text{at } \sigma = 0$

ملاحظة تقريبية $\leftarrow$

$$\left(\begin{array}{l} \sigma_{x'} = 88 \text{ MPa} \\ \tau_{x'y'} = 32 \text{ MPa} \end{array} \right)$$

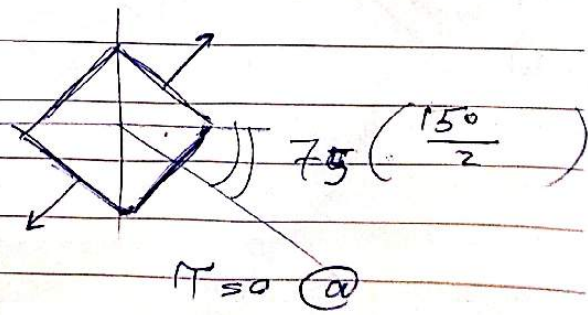
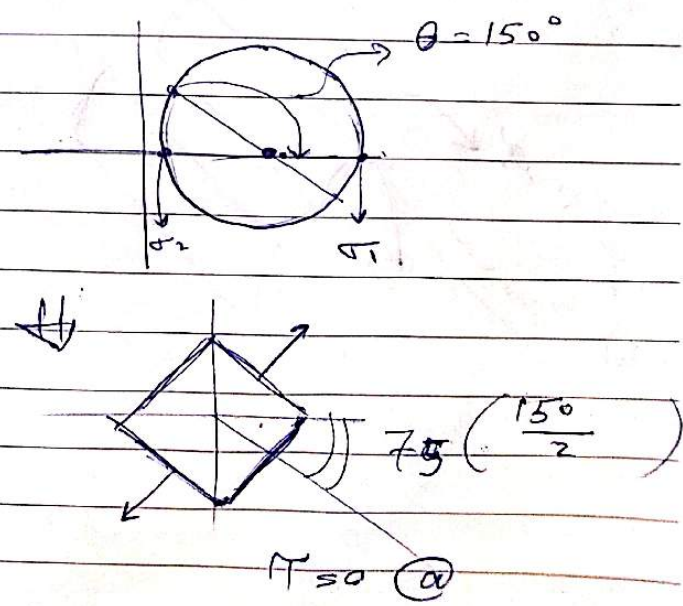
(Plane for $\sigma_{x'}$)
 $\tau_{x'y'}$



② $\sigma_1 = 103 \text{ MPa}$ (تقريباً)
 $\sigma_2 = 25 \text{ MPa}$

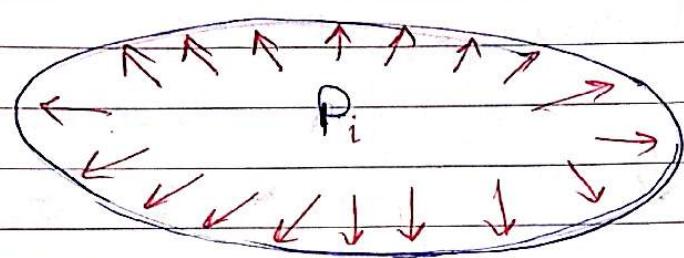
ملاحظة تقريبية

(Plane for σ_1, σ_2)



--- / sk / apl ←

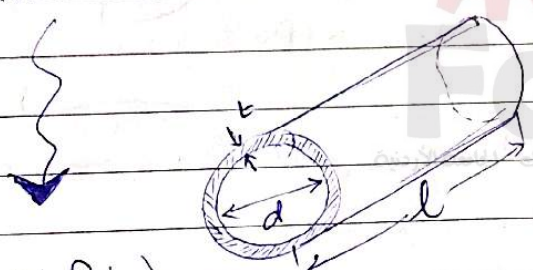
CH-8 → Pressure Vessels



(Tank)

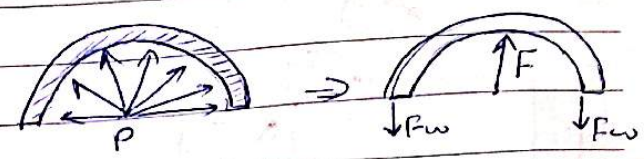
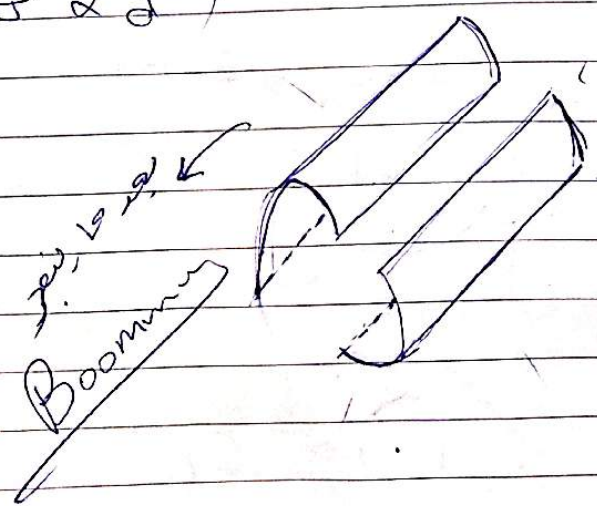
→ The inside pressure (P_i) produces stresses in the walls.

→ Relate the inside pressure to the value of stresses in the wall.



$\sigma \propto P_i$
 $\sigma \propto \frac{1}{t}$
 $\sigma \propto d$

① (في حالة كانه مفتوح)

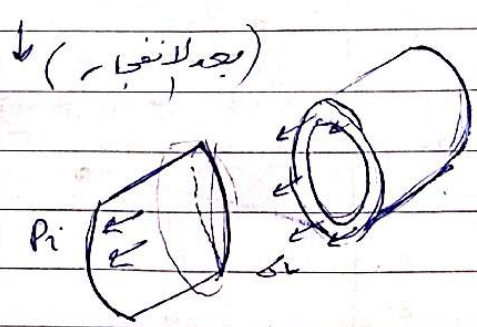
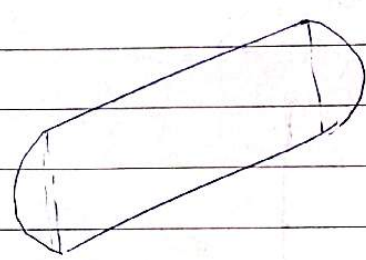


$F = P A(\text{proj})$
 $A_p = dL$
 $F_w = \sigma t L$
 $\sum F_y = PdL - 2\sigma t L = 0$

$\sigma_t = \frac{P_i d}{2t}$

→ tangential stress

② (في حالة كانه مغلق)

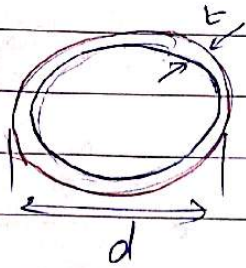


$F = P_i A$
 $F = (P) \left(\frac{\pi d^2}{4} \right)$

Now

the stress in the wall

$$\sigma_L = \frac{F}{A}$$



→ (for thin wall)

$$A(\text{wall}) = \pi d t$$

$$\sigma_L = \frac{\pi d^2 p}{\pi d t}$$

$$\sigma_L = \frac{p d}{4 t}$$

* 8.2-6 (spherical pressure vessel)

$$d = 480 \text{ mm}$$

$$t = 8 \text{ mm}$$

$$E = 180 \times 10^6$$

$$E = 205 \text{ GPa}$$

$$\nu = 0.3$$

~~if~~

عوض

Find P_i ?

Sol:

$$\sigma_r = \frac{p d}{4 t}$$



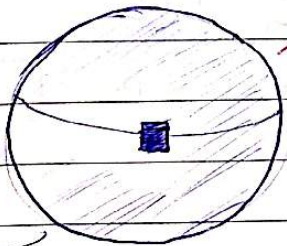
$$\sigma_r = \frac{p d}{4 t}$$

$$E = \frac{\sigma_1}{\epsilon} = \frac{\nu \sigma_2}{\epsilon}$$

$$180 \times 10^6 = \frac{p d}{4 t E} - (0.3) \left(\frac{p d}{4 t E} \right)$$

عوض

* for spherical



هذا هو الشكل
للجدار
الذي يتحمل
الضغط الداخلي

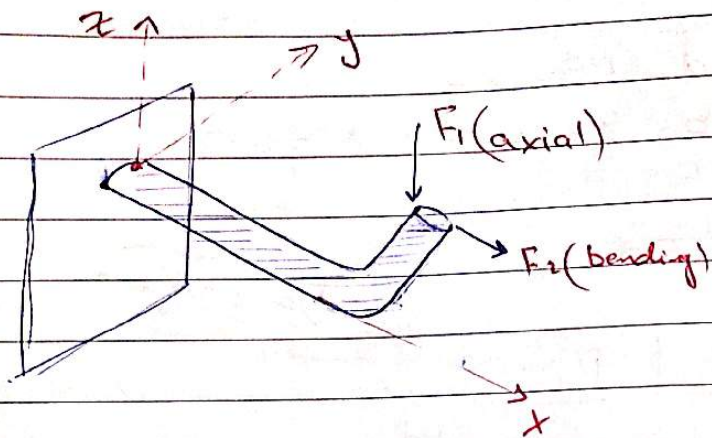
(closed vessel) ν

$$\sigma_r = \frac{p d}{4 t}$$

$P =$

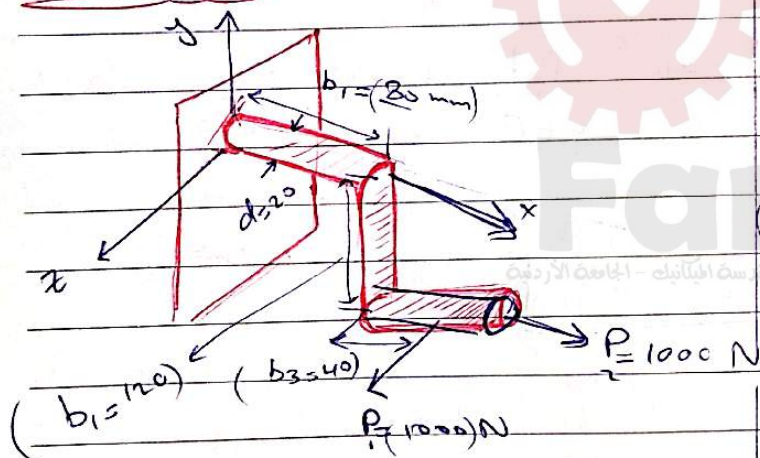
#

** Combined loading:



(Combined loading)
(axial, Torsional, bending)

* 8.5-20) (Crank shaft)



Find:

$\sigma_t, \sigma_c @ (A, B)$

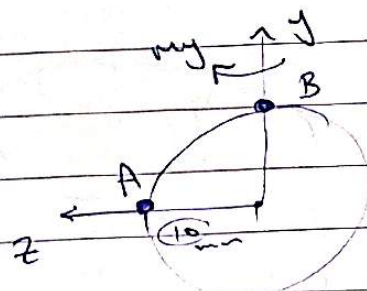
τ ??

Sol:

@ (A)

Force (P_1) $\rightarrow$ bending about (y)
 $\rightarrow$ Torsion (x)
 $\rightarrow$ shear stress in (y-z plane)

Force (P_1) $\rightarrow$ comp (y-z) plane
 $\rightarrow$ bending about (z)



$$* M_y = P_1 (b_2 + b_1)$$

(moment about y-axis)

$$\Rightarrow 1000 (0.08 + 0.04)$$

$$\Rightarrow 120 \text{ N.m}$$

$$* \sigma_b(A) = - \frac{M_y y}{I}$$

(bending)

$$= \frac{(120)(10 \text{ mm})}{\frac{\pi}{64} (20)^4 \text{ mm}^4}$$

$$\sigma_b(A) = -152 \text{ MPa [comp]}$$

Stress

$$* (P_2 \text{ no } (\sigma_c) \sim 152 \text{ MPa})$$

$$\sigma_A = \frac{P_2}{A} = \frac{1000}{\frac{\pi}{4} (20)^2 \text{ mm}^2}$$

$$\sigma_A = 3.18 \text{ MPa}$$

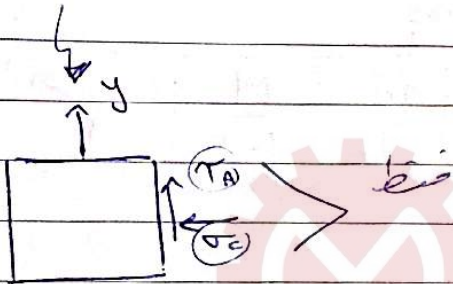
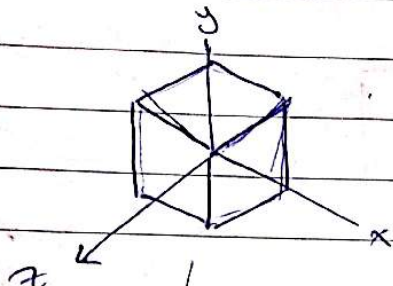
$$\Rightarrow \sigma_c = 152 + 3.18 = 155.18 \text{ MPa}$$

H

$$T_{\text{(for circular)}} = \frac{4}{3} \frac{V}{A}$$

$$\frac{\tau_A(P_1)}{\pi d^3} \Rightarrow \frac{16 T}{\pi d^3} = \frac{16 (P_1 b_2)}{\pi (20 \text{ mm})^3}$$

$$\tilde{T}(A) = 76.6 \text{ MPa}$$



* * * لا إله إلا الله !!
 (section) لا إله إلا الله
 لا إله إلا الله
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$$\tau = 0 \text{ (Shear stress)}$$

Now @ B

$$\sigma_B = \frac{(120)(10)}{\frac{\pi}{64}(20)^4} = 152 \text{ MPa}$$

(Tension)

$$x \nabla(B)(c) = \boxed{-3.18} \text{ comp.}$$

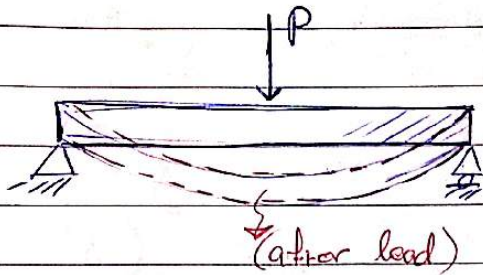
↙

(axial)

$$\Rightarrow \sigma_T = 152 - 3.18$$

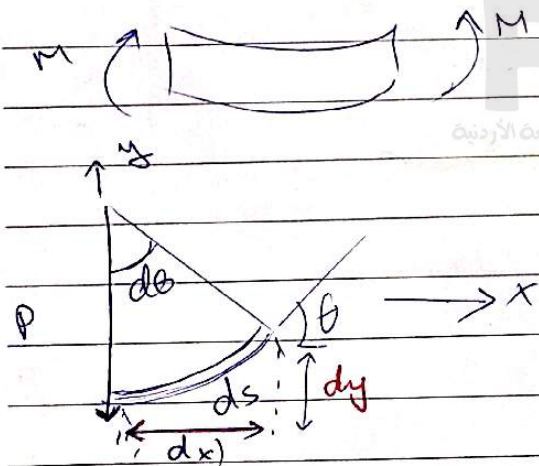
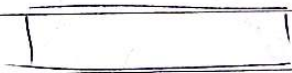
= 148 ... MPa Tension

** CH.9 ⇒ Deflection



- change in shape due to the loading on the member.

→ What is the deformed shape??



$$P d\theta = ds$$

$$\frac{d\theta}{ds} = \frac{1}{P} = \frac{M}{EI}$$

[for small deflection]

$$ds \approx dx$$

$$\Rightarrow \frac{d\theta}{ds} \approx \frac{M}{EI}$$

$$\frac{dy}{dx} = \tan \theta \approx \theta$$

إذا كانت الزاوية صغيرة
تقريباً $\tan \theta \Rightarrow \theta$

$$\textcircled{1} \frac{dy}{dx} = \theta$$

$$\textcircled{2} \frac{d\theta}{dx} = \frac{M}{EI}$$

- given the (load, material, cross section)

$$\text{find } \frac{M}{EI}$$

$$\textcircled{1} \int \frac{M}{EI} dx = \theta(x)$$

$$\textcircled{2} \int \theta(x) dx = y(x)$$

(deflected shape)

~~Q.2~~

مسألة
التي مطلوب

$$V = \frac{dM}{dx}, \quad q = -\frac{dV}{dx}$$

$$\theta(x) = \frac{dy}{dx} \Rightarrow y' = \int \frac{M}{EI} dx$$

$$y'' = \frac{M}{EI}$$

$$y''' = \frac{V}{EI}$$

$$y^{(4)} = -\frac{q}{EI}$$

$$q = -EI y^{(4)}$$

$$\times \frac{q \cdot 2 - 1}{(q \cdot 2) - 1}$$

$$y = \frac{-q_0 x}{360(EI)} (7L^4 - 10L^2 x^2 + 3x^4)$$

(deflected shape) vib!

Find: $q(x)$??



Sol:

$$q = -EI y^{(4)}$$

$$y' = \frac{-q_0}{(360)(EI)} (7L^4 - 10L^2 x^2 + 3x^4)$$

$$y'' = \frac{-q_0}{(360)(EI)} (-60L^2 x + 120x^3)$$

$$y''' = \frac{-q_0}{(360)(EI)} (-60L^2 + 360x)$$

$$y^{(4)} = \frac{-q_0}{(360)(EI)} (360x)$$

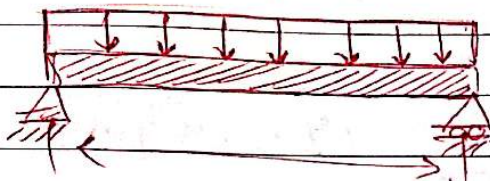
$$q = -x q_0 (-EI)$$

$$q = \frac{-x q_0}{L}$$

$$q = \frac{q_0 x}{L} \quad \#$$

* 9.3-4

$$q = 2 \text{ kN/m}$$



2m

$$E = 70 \text{ GPa}$$

Find: σ_{max}

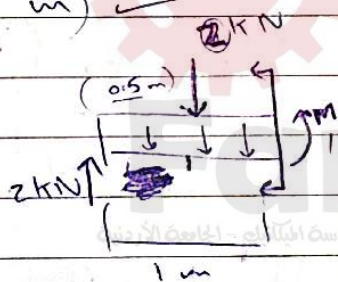
$$\tau_{\text{max}} = 60 \text{ MPa}$$

Sol:

$$M_{\text{max}} @ x = (1 \text{ m})$$

$$M_{\text{max}} = (2)(1)$$

$$(2)\left(\frac{1}{2}\right)$$



$$M_{\text{max}} = 1 \text{ kN}\cdot\text{m}$$

$$\tau_{\text{max}} = M_{\text{max}} \left(\frac{q}{2} \right)$$

$$\frac{1}{12} (a)^4$$

$$(a=b)$$

$$(\text{But } \tau_{\text{max}} = 60 \times 10^6)$$

$$\Rightarrow a = 46.9 \text{ mm}$$

$$(b a^3) = (a^4)$$

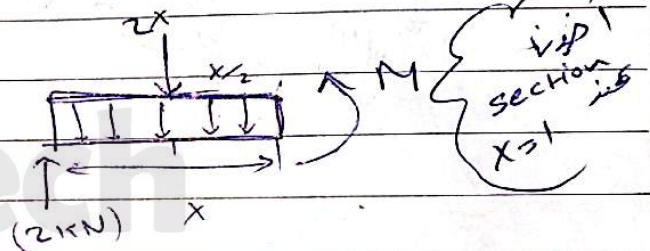
$$\Rightarrow I = \frac{1}{12} a^4$$

$$I = 0.386 \times 10^6 \text{ mm}^4$$

$$EI = (70 \times 10^9)(0.386 \times 10^6 \text{ mm}^4)$$

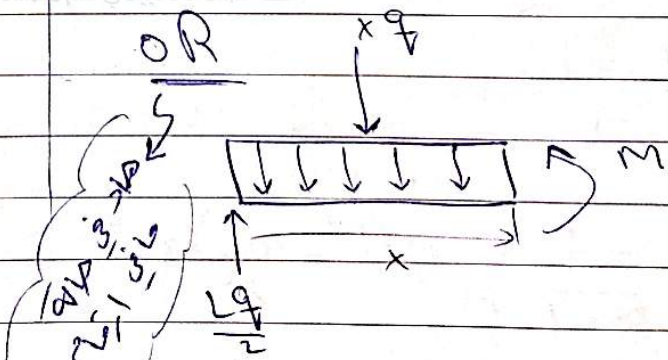
$$\Rightarrow 27 \text{ N}\cdot\text{m}$$

Now: (find M as a func of x)



$$M(x) = 2x - 2x\left(\frac{x}{2}\right)$$

OR



$$M(x) = \frac{Lq}{2}x - xq\left(\frac{x}{2}\right)$$

$$M = \frac{q}{2} (Lx - x^2)$$

قسم (3) مسائل

$$q = -EI y^{(4)} \rightarrow \text{معطى}$$

$$V = EI y'''$$

$$M = EI y''$$

$$M = EI y''$$

$$y'' = \frac{M}{EI} \Rightarrow y' = \int y'' dx$$

$$y' = \int \frac{M}{EI} dx$$

$$\Rightarrow \frac{1}{EI} \int_0^x \frac{q}{2} (Lx - x^2) dx$$

$$= \frac{q}{2EI} \left[L \frac{x^2}{2} - \frac{x^3}{3} \right]_0^x \quad \text{(constant)}$$

$$y' = \frac{q}{2EI} \left(L \frac{x^2}{2} - \frac{x^3}{3} \right)$$

↓

$$y = \int y' dx$$

$$y = \frac{q}{2EI} \int_0^x \left(L \frac{x^3}{2} - \frac{x^4}{3} \right) dx$$

$$y(x) = \frac{q}{2EI} \left[\frac{Lx^4}{8} - \frac{x^5}{12} \right] + C_1 x + C_2$$

(لايجاد قيمتي ثابتي C_1, C_2)

$$\hookrightarrow (y(0) = 0) \Rightarrow C_2 = 0$$

$$\downarrow C_1 = -\frac{q}{2EI} \left(\frac{L^3}{8} - \frac{L^3}{12} \right)$$

$$= \left(-\frac{qL^3}{24EI} \right)$$

الآن

$$y(x) = \frac{q}{2EI} \left(\frac{Lx^3}{8} - \frac{x^4}{12} \right) - \frac{qL^3}{24EI}$$

↓

(القيمة القصوى δ_{max})

والتي هي عند

$$x = (L/2)$$

$$y(L/2) = \frac{q}{2EI} \left(\frac{L^4}{48} - \frac{L^4}{96} \right) - \frac{qL^3}{24EI}$$

$$y(L/2) = \dots \quad \text{(بعد تبسيط)}$$

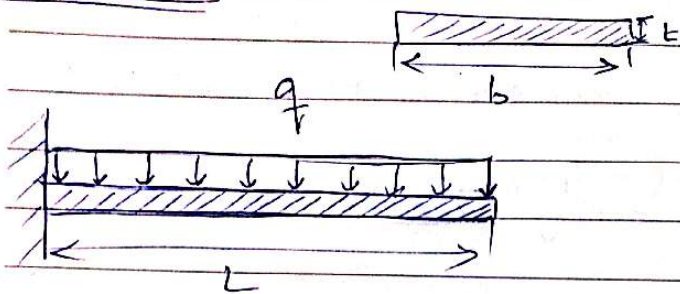
↓

$$y(L/2) = \delta_{max} = \left[-\frac{qL^4}{192EI} \right]$$

الآن

$y'(0) \Rightarrow$ to find max. deflection,
@ fixed point

Q.3-61. (سؤال فائيل)



$$L = 27.5 \text{ mm} \quad \delta_{\text{max}} = 2.46$$

$$b = 4 \text{ mm}$$

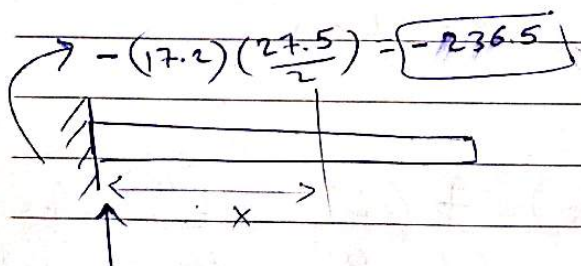
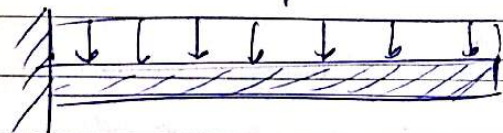
$$t = 0.28 \text{ mm}$$

$$\Rightarrow \text{Total load} = (17.2 \text{ MN})$$

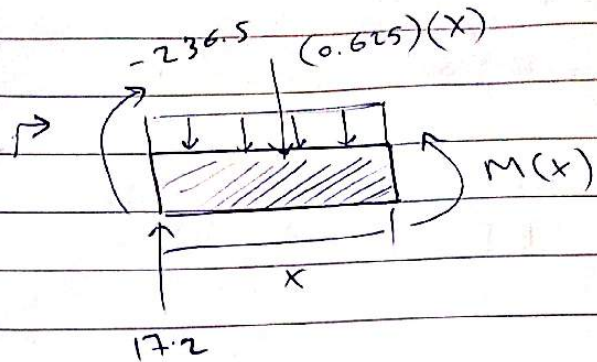
* Find ~~max~~ ?
(Eg)

Sol:

$$q = \frac{17.2 \text{ MN}}{27.5 \text{ mm}} \Rightarrow \boxed{0.625}$$



$$(17.2 \text{ MN})$$



$$\uparrow M = M(x) + 236.5 - 17.2(x) + (0.625)\left(\frac{x^2}{2}\right) = 0$$



$$M(x) = -236.5 + 17.2(x) - 0.625\left(\frac{x^2}{2}\right)$$

$$y'' = \frac{M(x)}{EI}$$

$$y'(x) = \int \frac{M(x)}{EI} dx$$

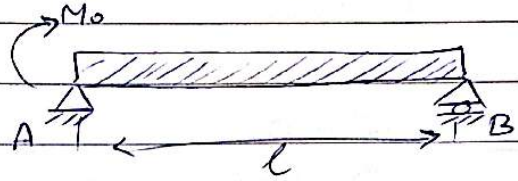
$$\Rightarrow -236.5x + \frac{17.2(x^2)}{2}$$

$$-0.625\left(\frac{x^3}{6}\right) + C_1$$

$$y'(0) = 0 \Rightarrow C_1 = 0$$

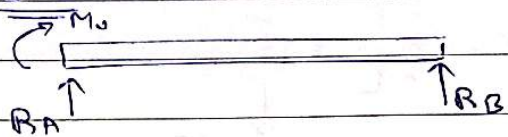
$$y'(x) =$$

Ex 9.3-9 :-



Find: $y(x)$, δ_{max}

Sol:

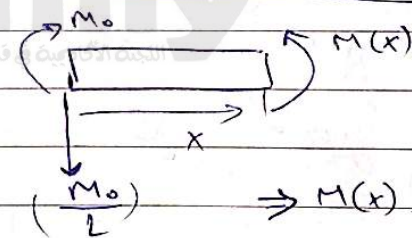


$$+\circlearrowleft M_A = 0$$

$$-M_o + R_B(l)$$

$$\Rightarrow R_B = \frac{M_o}{l} \quad R_A = -\frac{M_o}{l}$$

(To find $M(x)$)



$$\Rightarrow M(x) = M_o - \left(\frac{M_o}{l}\right)x$$

$$M(x) = M_o \left(1 - \frac{x}{l}\right)$$

$$y'' = \frac{M(x)}{EI} \Rightarrow y' = \int y''(x) dx$$

$$y' = \frac{M_o}{EI} \left(1 - \frac{x}{l}\right) + C$$

$$y = \frac{M_o}{EI} \left(\frac{x}{l} - \frac{x^2}{2l}\right) + C_1 x + C_2$$

$$[C_2 = 0 \text{ (} y(0) = 0 \text{)}]$$

$$\& y(l) = 0 \Rightarrow C_1 = \frac{M_o}{EI} \left(\frac{1}{2} - \frac{l}{2l}\right)$$

ويعبر عن δ_{max}

Now (to find the location of δ_{max})

$\Rightarrow$ Set $y' = 0$

$$y'(x) = \frac{M_0}{EI} \left(x - \frac{x^2}{2L} \right) + c_1$$

$$0 = x - \frac{x^2}{2L} + c_1$$

$$c_1 = \left[\frac{-M_0 L}{3EI} \right]$$

$$0 = x - \frac{x^2}{2L} - \frac{M_0 L}{3EI}$$

$$x =$$

$\Rightarrow$ δ_{max} $\leq$ $y(x)$ $\Rightarrow$ δ_{max} $\leq$ $y(x)$ $\Rightarrow$ δ_{max} $\leq$ $y(x)$

Back to (Pressure vessels)

8-3.11

(Or) Find



$$P_{(in)} = 12 \text{ MPa}$$

$n = 2$ (Factor of safety)

$$\sigma_y = 200 \text{ MPa}$$

$$\tau_y = 140 \text{ MPa}$$

$$d = 150 \text{ mm}$$

$\Rightarrow$ Find (L)

Sol.