The University of Jordan
Department of Mathematics

Eng. Math.(II)

Second Exam
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Lecture time: 8 - 9

Q1.(6 points) Use the Divergence Theorem to evaluate $\iint_S \vec{F}\cdot\hat{n}\,dA$, where

$$\vec{F}(x,y,z)=\left[\frac{x^3}{3},\,x^2\cos z+\frac{y^3}{3},\,\frac{z^3}{3}\right]$$

and S is the surface of the solid bounded by the sphere $x^2+y^2+z^2=4$ and the planes $x=0$, $y=0$ and $z=0$, that lies in the first octant. Assume that S is oriented inward.

5.5

$$\iint_S \vec{F}\cdot n\,dA = -\iiint_T \operatorname{div}\vec{F}\cdot dV$$

$$\operatorname{div}\vec{F} = x^2+y^2+z^2$$

$$\iiint_T x^2+y^2+z^2 \cdot dV$$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{2} \rho^2\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$\int_0^{\pi/2}\int_0^{\pi/2} \frac{\rho^5}{5}\Big|_0^2 \sin\phi\,d\phi\,d\theta$$

$$\cancel{\frac{32}{5}\int^{\pi/2} \cos\phi \Big|^{\pi/2}_0 d\phi\,d\theta}$$

$$\frac{32}{5}\int_0^{\pi/2}\int_0^{\pi/2} \sin\phi\,d\phi\,d\theta$$

$$-\frac{32}{5}\int_0^{\pi/2} \cos\phi\Big|_0^{\pi/2} d\theta$$

$$\Rightarrow \frac{32}{5}\times\theta\Big|_0^{\pi/2} = \cancel{\frac{16}{5}\pi}$$

$$\boxed{-\frac{16}{5}\pi}$$

sherical cordinat

$x=\rho\cos\theta\sin\phi$

$y=\rho\sin\theta\sin\phi$

$z=\rho\cos\phi$

$dz=\rho^2\sin\phi\,d\phi\,d\rho\,d\theta$

$0\geq\theta\geq\pi/2$

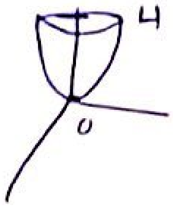
$0\geq\phi\geq\frac{\pi}{2}$

$0\geq\rho\geq2$

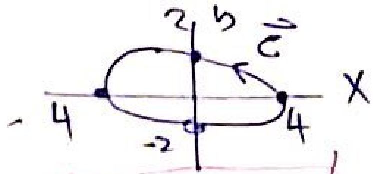
Q2.(6 points) Use the Stokes's Theorem to evaluate $\iint_S (curl\vec{F})\cdot\hat{n}\,dA$, where

$$\vec{F}(x,y,z)=\left[6yz,\ 5x,\ yze^{x^2}\right]$$

and S is the surface of the paraboloid $z=\frac{1}{4}x^2+y^2$ for $0\le z\le 4$. Assume that S is oriented upward.



$$\iint_S (\text{curl}\,\vec{F})\cdot\hat{n}\,dA = \oint_C \vec{F}\cdot d\vec{r}$$



$$\vec{r}(t) = [\cancel{2}\cos t, \cancel{2}\sin t, \cancel{4}\,4]$$

$$\vec{r}'(t) = [-2\sin t, \cos t, 0]$$

3.5

$$C = \boxed{\frac{1}{4}x^2+y^2=4}$$

$$2\pi \ge t \ge 0$$

$$\Rightarrow \int \vec{F}(\vec{r}(t))\cdot\vec{r}'(t)\,dt \Rightarrow \int [24\sin t, 10\cos t, --]\,[-2\sin t, \cos t, 0]$$

$$= \int_0^{2\pi} -48\sin^2 t + 10\cos^2 t\; dt = \alpha$$

$$C = \frac{1}{16}x^2 + \frac{y^2}{4} = 1$$

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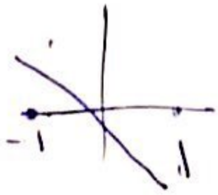
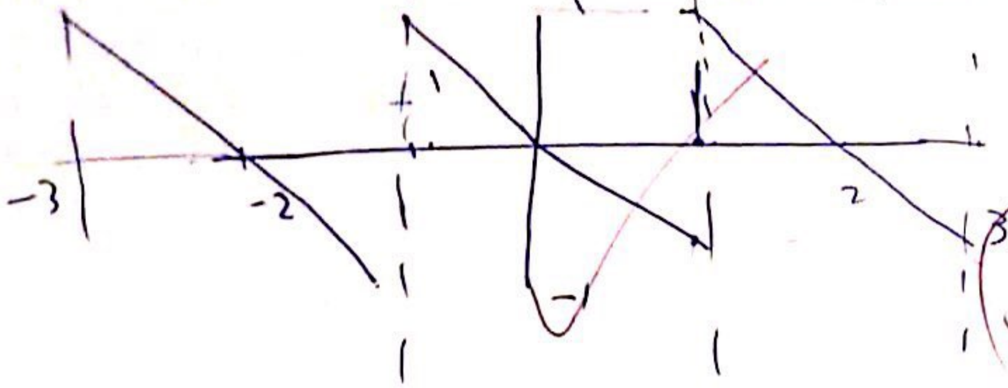
Q3. (10 points) Consider $f(x) = -x,\ -1 < x < 1,\ f(x+2) = f(x)$.



(a) Sketch the graph of the function $f(x)$ in three periods.



9.5

(b) Write down the Fourier series for the function $f(x)$.

$F(x)$ is odd Func. $\Rightarrow a_0\ \&\ a_n = 0$ / $L = 1$

$F(x) \sim \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$.

$b_n = \frac{2}{L}\int_0^1 -x \sin(n\pi x)\cdot dx$

$-x$ $\sin n\pi x$

-1 $+$ $-\frac{\cos(n\pi x)}{n\pi}$

0 $-$ $-\frac{\sin(n\pi x)}{n^2\pi^2}$

$= 2\left[x\frac{\cos n\pi x}{n\pi} - \frac{\sin(n\pi x)}{n^2\pi^2}\right]_0^1$

$2\left[\frac{\cos n\pi}{n\pi} - \frac{\sin n\pi}{n^2\pi^2}\right]$

$= 2\left[\frac{(-1)^n}{n\pi} - \boxed{\frac{(-1)^{n+1}}{n^2\pi^2}}\right]$

$F(x) \sim \sum_{n=1}^{\infty} \ \sin(n\pi x)$

(c) How should $f(x)$ be defined at $x = -1$ and $x = 1$ in order that the Fourier series will converge to $f(x)$, $-1 \le x \le 1$.

$F(1) = \frac{F(1)^+ + F(1)^-}{2} = \frac{1 + -1}{2} = 0$

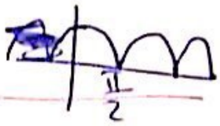
$F(-1) = \frac{F(-1)^+ + F(-1)^-}{2} = \frac{1 + -1}{2} = 0$

$F(x) = \begin{cases} -x, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

$\frac{\pi}{4} - \frac{2}{\pi} \quad \frac{\pi-2}{\pi}$ $\qquad \cos 2n\frac{\pi}{2} = \pi \quad 2\pi \quad 3\pi$

Q4. **(3 points)** **The Fourier cosine series for the function** $f(x) = |\cos x|$, $0 < x < \pi$ **is**

$f(x) \sim \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n \cos 2nx}{4n^2-1}$. **Find the sum of** $\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$.



(a) $x = 0$, $f(0) = 1$

$$1 = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$$

$$1 - \frac{2}{\pi} = -\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$$

$$\cancel{\frac{\pi-2}{\pi} * \frac{-\pi}{4} = \sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}}$$

$$1 - \frac{2}{\pi} * \frac{-\pi}{4} \sim \sum_{n=1}^{\infty}\frac{(-1)^n}{4n^2-1}$$

2.5

Q5. **(5 points)** **Find the Fourier transform for function** $f(x) = (1-x^2)e^{-x^2}$,

using the facts: $\mathcal{F}\{e^{-x^2}\} = \frac{1}{\sqrt{2}}e^{\frac{-1}{4}w^2}$, $\mathcal{F}\{f'(x)\} = iw\mathcal{F}\{f(x)\}$, **and**

$\mathcal{F}\{f''(x)\} = -w^2\mathcal{F}\{f(x)\}$.

$$F(f(x)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(x) \cdot e^{-iwx} \, dx$$

zero

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(1-x^2)\, e^{-x^2+iwx} \, dx$$

$(-2x+2x^3)$

$$\tilde{F}'(x) = (1-2x)e^{-x^2} + (1-x^2) \cdot -2x \, e^{-x^2}$$

$$F''(x) = (1-2)e^{-x^2} + (-2+6x^2)e^{-x^2} + (+4x^2+2x^4)e^{-x^2}$$