$\frac{20}{25}$

The University of Jordan
Mathematics Department

Eng. Math.II (0331302)

Second Exam
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Set Number: ~~53~~ 43

$-(x-4) = y^2+z^2$

Q1. Find the area of the portion of the paraboloid $x + y^2 + z^2 = 4$ that lies above the ring $1 \le y^2 + z^2 \le 4$ in the yz plane. (5 points)

4

$$\boxed{x = 4 - y^2 - z^2}$$

$$\cancel{A = \iint \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + 1}\, dx\, dy}$$

$$A = \iint \sqrt{\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2 + 1}\, dx\, dy$$

$$= \iint \sqrt{(-2y)^2 + (-2z)^2 + 1}\, dx\, dy$$

$$= \iint \sqrt{4y^2 + 4z^2 + 1}\, dx\, dy$$

$$= \iint \sqrt{4[y^2 + z^2] + 1}\, dx\, dy$$

$$= \int_0^{2\pi} \int_1^{2} \cancel{\sqrt{(4r^2 + 1)}}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_1^{2} 4r^3 + r\, dr\, d\theta$$

$\cancel{r\cos = [(x-4)] = 4\cos}$

$x = 0$

$$\int_0^{2\pi} \left[r^4 + \frac{r^2}{2}\right]_1^2 d\theta$$

$$= \int_0^{2\pi} (16 + 2) - \left(1 - \frac{1}{2}\right)$$

$$= \int_0^{2\pi} \left(18 - \frac{1}{2}\right) d\theta$$

$$= \int_0^{2\pi} (17.5)\, d\theta$$

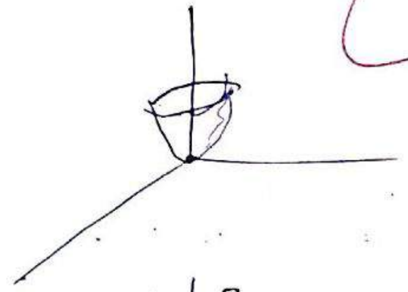
$$\boxed{A = 35\pi}$$

$\iiint_T \text{div } F$ $\quad\quad$ $\iint \text{curl } f$

Q2. Let $F = z\ \tan^{-1}(y^2)\, i + z^3 \ln(x^2+1)\, j + z\, k$, find the $\iint_S\ F.n\, dS$ where S is the part of the paraboloid $x^2+y^2+z=2$, $z \geq 1$ and is oriented upward. $x^2+y^2 = -(z-2)$ (6 points)

5.5

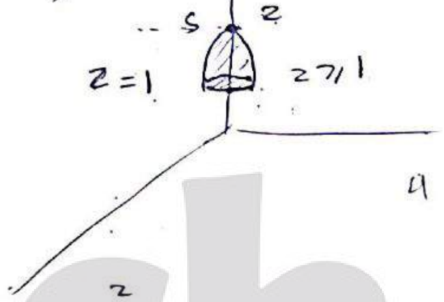
$$\iint_A F \cdot N\, ds = \iiint_T \text{div } f$$



$\text{div} f = 1$

$dz\, r\, dr\, d\theta$

$$\iiint 1\ dx\, dy\, dz$$



$$= \int_0^{2\pi}\int_0^{\sqrt2}\int_2^{2-x^2-y^2} r\, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt2} rz \Big|_2^{2-x^2-y^2} dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt2} r\left[\cancel{2}-x^2-y^2-\cancel{2}\right] dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt2} -rx^2 - ry^2\, dr\, d\theta$$

$$= \int_0^{2\pi}\left[-\frac{r^2 x}{2} - \frac{r^2 y}{2}\right]_0^{\sqrt2} d\theta$$

$$= \int_0^{2\pi} -\frac{\cancel{2}x^2}{\cancel{2}} - \frac{\cancel{2}y^2}{\cancel{2}}\ d\theta$$

$$\int_0^{2\pi}\int_0^{\sqrt2}\int_2^{2-r^2} r\, dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt2} rz \Big|_2^{2-r^2} dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt2} r\left(\cancel{2}-r^2-\cancel{2}\right) dr\, d\theta$$

$$\int_0^{2\pi} \frac{-r^4}{4}\Big|_0^{\sqrt2} d\theta$$

$$= \int_0^{2\pi} -1\ d\theta = \boxed{-2\pi}$$

$z=1$ $\quad x^2+y^2=2$ / $N = [0, 0, -1]$

$r(x,y) = [x, y, 1]$

$$\iint F.N\, dA$$

$$= \iint 1\ dx\, dy$$

$= \text{Area of circle}$

$= \pi(\sqrt2)^2 = 2\pi \quad -\pi$

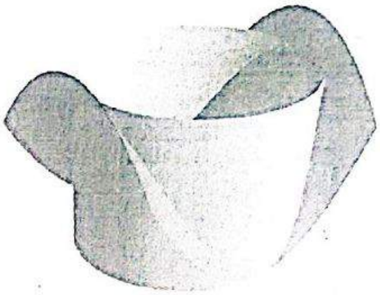
$A = -2\pi - 2\pi = -4\pi$

Q3. Find $\oint_C F.dr$ where $F = x^2yi + \frac{x^3}{3}j + xy\,k$ and C is the curve of intersection of the hyperbolic paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$ oriented counterclockwise. (6 points)

$\frac{1}{2} - \frac{u}{2}$

4

Stokes theorm



$$\text{curl} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & \frac{x^3}{3} & xy \end{vmatrix}$$

$$x\,i - [y] + [x^2 - x^2]k$$

$$= [x, -y, 0]$$

$$\oint_C F \cdot dr = \iint_S \text{curl} F \cdot N$$

$$= \iint [x, -y, 0] \cdot [\tfrac{1}{2}, 0, 2u]$$

$$= \int^{1} \int^{1-y^2} \frac{1}{2} x \, dx\, dy$$

$$\int_0^{1-y^3} \int \frac{x^2}{4} \Big|_0^{1-y^2}$$

$$= \int_0^{2\pi} \int_0^{1} \frac{r\cos\theta}{2} r\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^{1} \frac{r^2\cos\theta}{2} dr\,d\theta$$

$\Longleftarrow$ يتبع

$$x^2 + y^2 + 1 = y^2 - x^2 - z$$

$$z + 2x^2 - 1 = 0$$

$$x^2 = \frac{(1-z)}{2}$$

$$r(u,v) = [u^2, v, \tfrac{1-u}{2}]$$

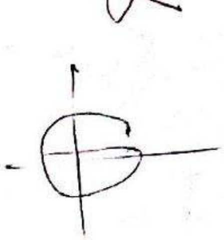
$$r_u = [2u, 0, -\tfrac{1}{2}]$$

$$r_v = [0, 1, 0]$$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 2u & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \end{vmatrix}$$

$$= +\frac{1}{2} i + 2u\,k$$

$$= [\tfrac{1}{2}, 0, 2u]$$



$$= \int_0^{2\pi} \int_0^1 \frac{r^2 \cos\theta}{2} \, dr \, d\theta$$

$$= \int_0^{2\pi} \frac{r^3}{6} \cos\theta \Big|_0^1 \, d\theta$$

$$= \int_0^{2\pi} \left\{ \frac{1}{6} \cos\theta - \frac{1}{6} \right\} d\theta$$

$$= \left[\frac{\sin\theta}{6} - \frac{1}{6}\theta \right] \Big|_0^{2\pi}$$

$$= \boxed{-\frac{\pi}{3}}$$

Q4. Let $f(x) = \begin{cases} 3\pi + 2x, & -\pi < x < 0 \\ \pi + 2x & 0 < x < \pi \end{cases}$ and $f(x+2\pi) = f(x)$ (8 points)

a. Find Fourier series of $f(x)$,

b. Find the sum $\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$.

6.5

$\boxed{L = \pi}$

$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{\pi x n}{L} + B_n \sin \frac{\pi x n}{L}$

$a_0 = \frac{1}{2L} \int_{-\pi}^{0} 3\pi + 2x \, dx + \frac{1}{2L} \int_{0}^{\pi} \pi + 2x \, dx$

$= \frac{1}{2\pi} \left[3\pi x + x^2 \Big|_{-\pi}^{0} + \pi x + x^2 \Big|_{0}^{\pi} \right]$

$= \frac{1}{2\pi} \left[+3\pi^2 - \pi^2 + \pi^2 + \pi^2 \right]$

$= 2\pi$

$a_n = \frac{1}{L} \int_{-\pi}^{0} [3\pi + 2x] \cos \frac{n\pi x}{L} \, dx + \frac{1}{L} \int_{0}^{\pi} [\pi + 2x] \cos \frac{n\pi x}{L} \, dx$

$= \frac{1}{\pi} \left[\underbrace{\int_{-\pi}^{0} [3\pi + 2x] \cos nx}_{a_1} + \underbrace{\int_{0}^{\pi} [\pi + 2x] [\cos nx] \, dx}_{a_2} \right]$

$a_1 = \frac{1}{\pi} \left\{ -2x \frac{1}{n} \sin nx + \frac{2}{n^2} \cos nx \right\} \Big|_{-\pi}^{0}$

$+ \frac{3\pi \sin nx}{n} \Big|_{-\pi}^{0} \Big]$

$2x \quad \cos nx$

$2 \quad \frac{-1}{n} \sin nx$

$0 \quad \frac{-1}{n^2} \cos nx$

$= \frac{1}{\pi} \left[\frac{2}{n^2} \right.$

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Q41

$a_1 = \frac{1}{\pi}\left[\frac{-2x}{n} \cancel{\sin nx} + \frac{2}{n^2}\cos nx + \cancel{\frac{3\pi \cos nx}{n}} \right]_{-\pi}^{0}$ zero $\sin nx$

$= \frac{1}{\pi}\left[\frac{2}{n^2} - \left[\frac{2}{n^2}\cos -\pi x \right]\right]$

$= \frac{1}{\pi}\left[\frac{2}{n^2} - \frac{2}{n^2}\cos \pi x \right]$

$= \frac{2}{n^2\pi}\left[1-(-1)^n\right]$

$a_2 = \frac{1}{\pi}\left[\cancel{\frac{-2x}{n}\sin nx} + \frac{2}{n^2}\cos nx + \cancel{\frac{\pi \sin nx}{n}} \right]_0^{\pi}$

$= \frac{1}{\pi}\left[\frac{2}{n^2}\cos \pi n - \frac{2}{n^2} \right]$

$= \frac{2}{\pi n^2}\left[\cos \pi n - 1\right] = \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$

$a_n = a_1 + a_2 = 0$

$b_n = \frac{1}{L}\int_{-\pi}^{\pi 0} (3\pi + 2x) \frac{\sin n\pi x}{L}\, dx + \int_0^{\pi} (\pi + 2x) \frac{\sin n\pi x}{L}\, dx$

$= \frac{1}{\pi}\int_{-\pi}^{0} (3\pi + 2x)(\sin nx) + \int_0^{\pi} (\pi + 2x)\sin nx\, dx$

$b_1 = \frac{1}{\pi}\left[\frac{2x}{n}\cos nx + \frac{2}{n^2}\sin xn - \frac{3\pi\cos nx}{n} \right]_{-\pi}^{0}$

$2x \xrightarrow{+} \sin nx$

$2 \xrightarrow{-} \frac{1}{n}\cos nx$

$0 \rightarrow \frac{-1}{n^2}\sin nx$

$= \frac{1}{\pi}\left[\cancel{\frac{-n\pi}{n}} - \frac{2\pi}{n}\cos n\pi - \frac{3\pi \cos n\pi}{n} \right]$

$+ \left[\frac{3\pi}{n}\right] = \frac{1}{\pi}\left[\frac{-5\pi \cos n\pi}{n} + \frac{3\pi}{n} \right]$

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Q4S

$F_0 = a_0 + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$

$= \cancel{2\pi + \sum_{n=1}^{\infty} 2 [1 - 2\cos (-1)^n]}$

$$\boxed{f = 2\pi + \frac{4}{\pi} \sum_{n=1}^{\infty} (-1)^n \frac{\sin nx}{n}}$$

$f(1) = 2\pi + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\sin nx}{n}$

$0 = 2\pi + 4 \sum_{n=1}^{\infty} \frac{\sin n}{n}$

$\sum \frac{\sin}{n} = -\frac{2\pi}{4}$

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b_2

$= \frac{1}{\pi} \Big[\frac{2x}{n} \cos nx + \frac{2}{n^2} \sin xn - \frac{\pi \sin nx}{n} \Big]_0^{\pi}$

$\frac{1}{\pi} \Big[\frac{2\pi}{n} \cos n\pi - \frac{\pi \cos n\pi}{n} + \left(\frac{\pi}{n} \right) \Big]$

$= \frac{2}{n} \cos n\pi - \frac{\cos n\pi}{n} + \frac{1}{n} \Big]$

$b_2 = \frac{\cos n\pi + 1}{n}$

$b_1 = \frac{-5 \cos n\pi + 3}{n}$

$b = b_1 + b_2$

$= \frac{-5 \cos n\pi + 3 + \cos n\pi + 1}{n}$

$= \frac{-4 \cos n\pi + 4}{n}$

$= \frac{2[1 - \cos n\pi]}{n}$

$$\boxed{b = \frac{4}{n} [(-1)^n]}$$