Control Theory Applications
Balancing Scooters

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Inverted Pendulum

- Hoover boards, balancing scooters, wiziwig scooters…etc. are forms of Balancing Inverted Pendulums

Short Video, Balancing Scooter

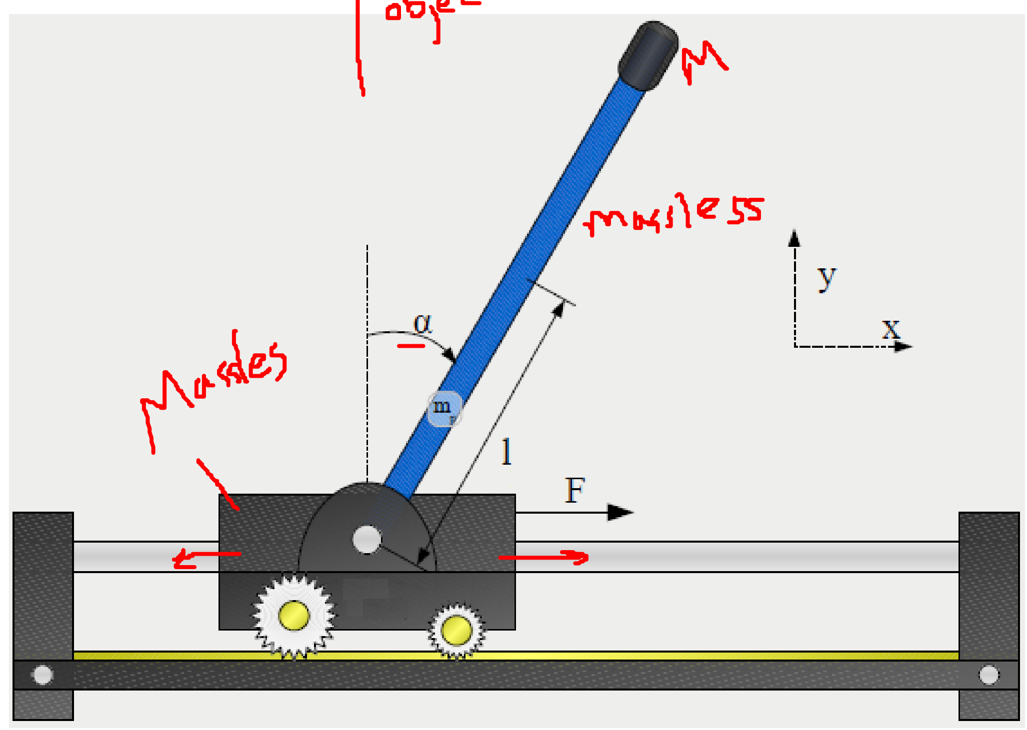
Inverted Pendulum

HW

- Consider the balancing of an inverted pendulum problem as shown by the diagram
- Suppose that the cart is massless and it moves on a frictionless rails
- Assuming a first order motor that generates a traction force

objective : to stabilize M in the vertical

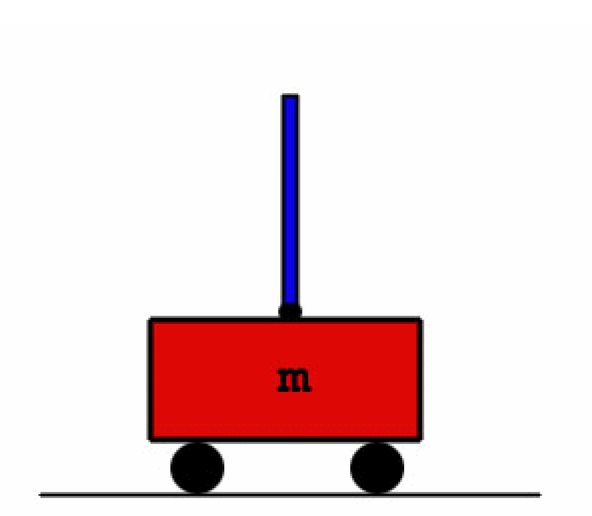
$$\frac{F(S)}{V(S)} = \frac{C}{\tau S + 1}$$

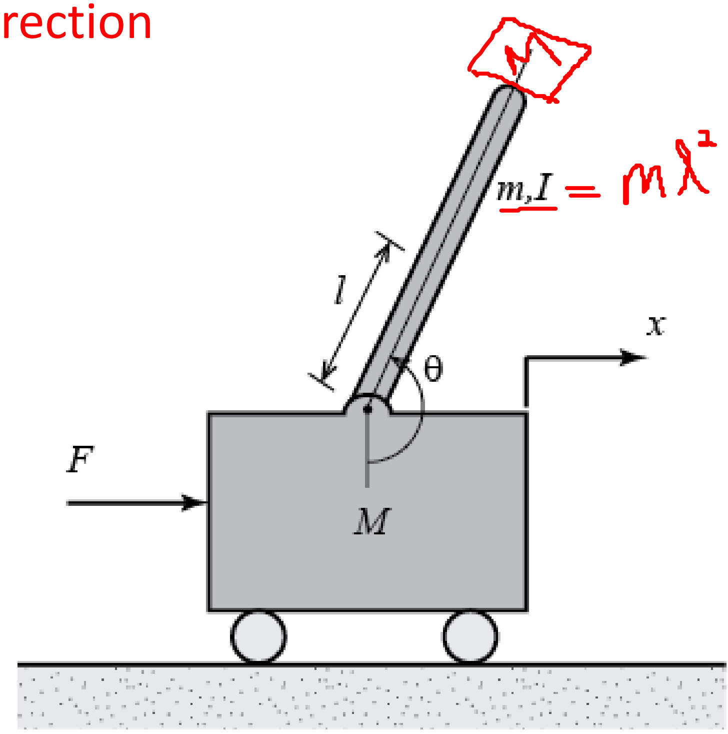


- Where
  - C is a motor constant
  - τ is a motor time constant
  - V is a motor input voltage

Inverted Pendulum

- First let us draw a sketch of the physical model as shown in the diagram, with the following assumptions:
  - The person will be modeled as a rod with mass *m* and mass moment of inertia *I*
  - The rod's center of gravity is located in the middle
  - The rod is pivoted to a massless cart as shown
  - The cart can move back and forth in the x-direction
  - The motor generates a varying force F





Inverted Pendulum

- From the free body diagram of the cart
  - Summation of forces in the x-direction

$$M\ddot{x} + b\dot{x} + N = F$$

  - Assuming massless cart and frictionless ground surface

$$N = F$$

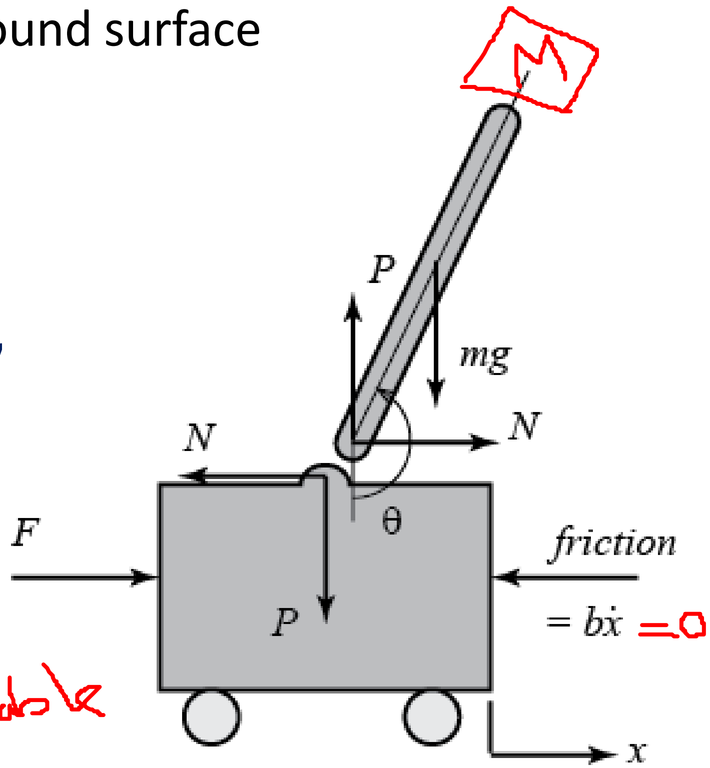
- From the free body diagram of the rod
  - Taking summation of torques about the piv

$$T = Fl = I\ddot{\alpha} - mgl\sin(\alpha)$$

Sin(∝)≈∝ *for a small vertical angle*

$T = Fl$ *Torque*

Show the system is unstable



Inverted Pendulum

- The system transfer function

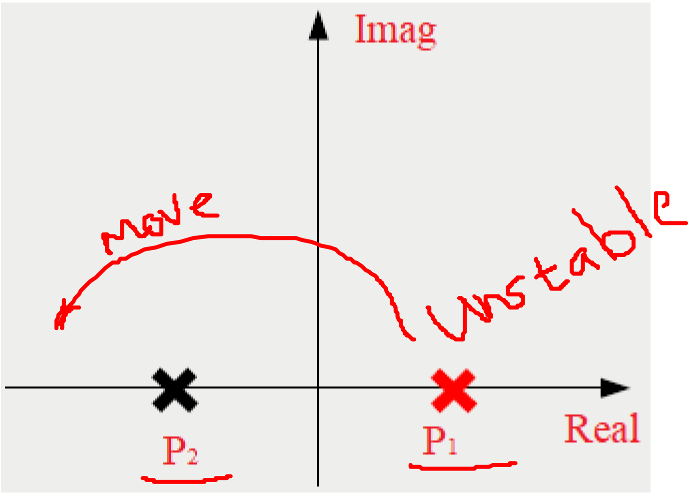
$$G(S) = \frac{\propto(S)}{T(S)} = \frac{1}{IS^2 - mgl}$$

- The characteristic Equation

$$D(S) = IS^2 - mgl = 0$$

$$P_{1,2} = \pm\sqrt{\frac{mgl}{I}}$$

- The inverted pendulum is unstable



Inverted Pendulum

- Stabilization using a P-Controller
  - The closed loop transfer function

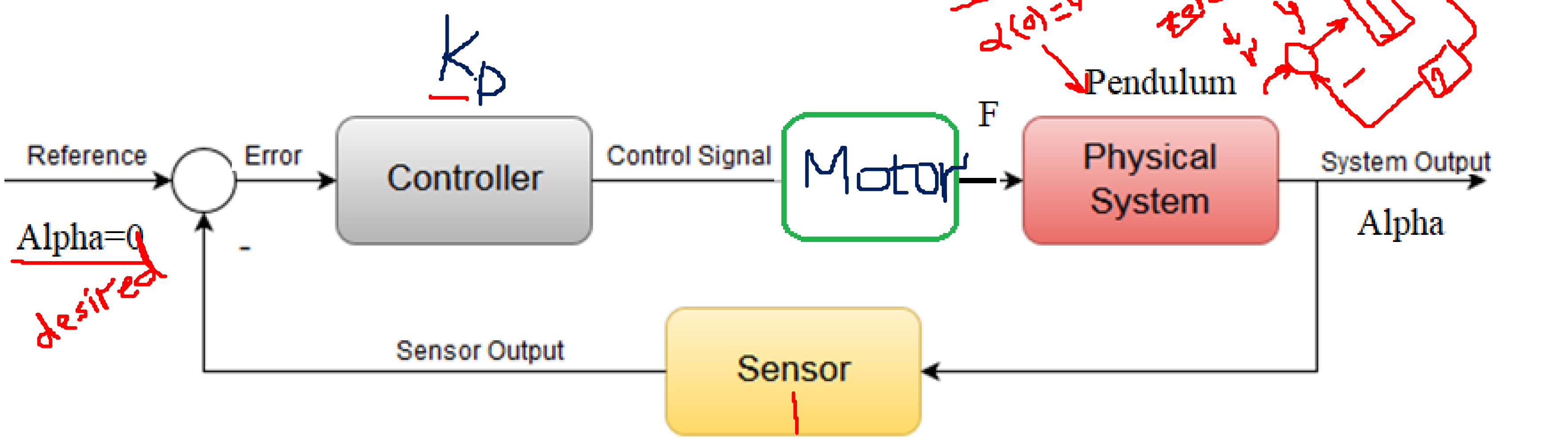
$$\frac{\propto(S)}{\propto_d(S)} = \frac{K_p \dfrac{C}{\tau S+1}\dfrac{1}{IS^2-mgl}}{1+K_p \dfrac{C}{\tau S+1}\dfrac{1}{IS^2-mgl}}$$

DE
Excitation
Non homog.
ICs

Ex. $\frac{dy}{dt} + 2y = 0 = r$

ICs

$\dot{y} = -2y + r$



$$\frac{\propto(S)}{\propto_d(S)} = \frac{K_p \frac{C}{\tau S+1}\frac{1}{IS^2-mgl}}{1+K_p\frac{C}{\tau S+1}\frac{1}{IS^2-mgl}}$$

✓ $$\frac{\propto(S)}{\propto_d(S)} = \frac{K_pC}{(\tau S+1)(IS^2-mgl)+K_pC}$$ Type equation here.

$$D(S) = (\tau S+1)(IS^2-mgl)+K_pC = 0$$

$$= \tau I\, S^3 + IS^2 - mgl\,\tau S + \boxed{K_pC - mgl} = 0 \rightarrow \; < 0$$

S^3	τI	$-mgl\tau$
S^2	I	K_p
S	h_1	
S^0	h_2	

$$h_1 = \frac{-1}{I}\left(K_p\tau I + I(\cdot)\right)$$

$$h_2 = K_p > 0$$

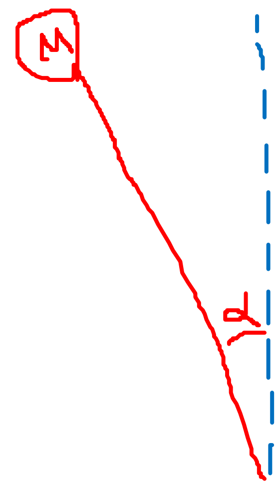
?

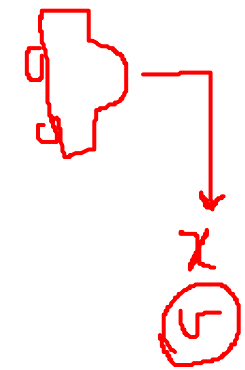
✓ $K_p < 0$

Why K_p alone cannot stablize the pendulum?

$u = K_p e = -K_p \alpha$

$e = S_p - \alpha = -\alpha$





Idea

$$u = K_p e + K_D \frac{de}{dt}$$

Proportional + Derivative   PD

CO: Reaction based α +
"   " $\dot{\alpha}$