

Metrology:-

- Objective of Metrology

1. Inspect the capability of device.

working Error.
Env.

2. Effective maintenance $\left\{ \begin{array}{l} \text{less time.} \\ \text{low cost.} \end{array} \right.$

3. correctness & accuracy of process.

4. Standardization. for measurement [code البناء كود]

Statistical Treatment of Data.

$\Rightarrow$ Random Error :- contro. جميع Error causes :- Error.

1. Instrument.

2. observer.

3. Process.

4. Calibration.

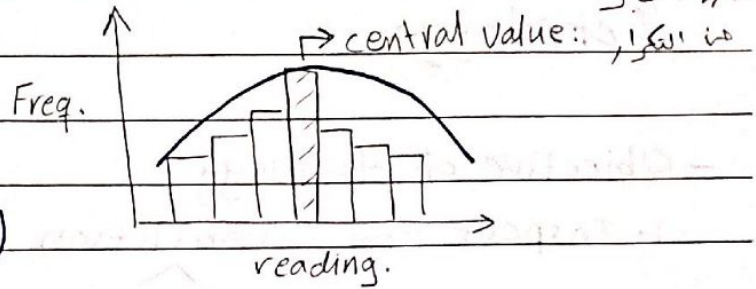
Random Error $\left\{ \begin{array}{l} \text{single sample: دون تغيير اي دة:} \\ \text{multisample: نقوم بتغيير اقل شيء اثنين منهم} \end{array} \right.$
(هو الذي يتولد سبب Error)

single sample : change the time

multisample : change all. (with time).

* Multi sample :-

1. step (1) : draw Heistogram.



2. step (2) : Find True Value. ($\bar{x}$)

$$\bar{x} = \frac{\sum (X * F)}{\sum F}$$

(normal distribution)

3. step (3) : Find Deviation $d_i = X - \bar{x}$

4. step (4) : Find the average deviation ($\bar{D}$)

avg. Error. متوسط خطأ

$$\bar{D} = \frac{\sum |F * d|}{\sum F}$$

i	X	F	X*F	Dev.
1	5	1		$5 - \bar{x}$
2	5.5	10		$5.5 - \bar{x}$
3	6	30		
4	6.5	12		
5	7	2		
		$\Sigma =$	$\Sigma =$	

5. step (5) : Calculate stand. deviation (σ) : if range $\uparrow$, Peak $\downarrow$

اعلى قيمة ل Error مقبولة في التجربة

$\Rightarrow$ So Error $\uparrow$

$$\sigma = \sqrt{\frac{\sum (F d^2)}{\sum F}}$$

check (1) : distribution.
check (2) :-
 $\bar{D} < \sigma$

لازم يكون σ اكبر $\bar{D}$ اذا كان

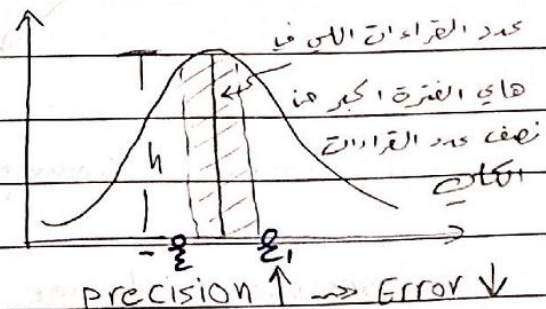
اقل لازم اعيد التجربة اذا متساويان

لا فكل نفيد التجربة.

6. step (6) : Find precision index "h"

نسبة القراءات حول True value

$$h = \frac{1}{\sigma \sqrt{2}} > 50\%$$



check (3) : "h" should be greater than 50%.

7. step (7): Probable Error of one reading $\bar{z}_1$.

$[\bar{x} - \bar{z}_1, \bar{x} + \bar{z}_1] > 50\%$; check (4): should be more than 50%.

$$\bar{z}_1 = \frac{0.4769}{h}$$

$$= 0.6745 \times \text{stand. deviation.}$$

Example 9 :-

<u>T (x)</u>	<u>F</u>	<u>T * F</u>	<u>d_i</u>	<u> F * d_i </u>	<u>F * d_i²</u>	<u>d_i / 6</u>
397	1	397	-3.78	3.78	14.2884	
398	3	1194	-2.78	8.34	23.1852	
399	12	4788	-1.78	21.36		
400	23					
401	37					
402	16					
403	4					
404	2					
405	2					
$\Sigma = 100$		40078		102.84	191.16	

I) The data follow nearly normal dist.

II) calculate the true value " $\bar{T}$ "

$$\bar{T} = \frac{\Sigma T * F}{\Sigma F} = \frac{40078}{100} = 400.78^\circ \text{C}$$

III) Find the deviation " d_i " for each Group

$$d = T - \bar{T} = 397 - 400.78 = -3.78^\circ \text{C}$$

بما أن "معدل" في الامتحان

IV) Find the avg. deviation " $\bar{D}$ "

$$= \frac{\Sigma |F * d|}{\Sigma F} = \frac{102.84}{100} = 1.0284 \checkmark$$

V) Find the stand. deviation (σ):

$$\sigma = \sqrt{\frac{\sum F d^2}{F}} = \sqrt{\frac{191.16}{100}} = 1.382^\circ \text{C}$$

since $\sigma > \bar{D}$ the test was carried out correctly.

VI) Find the value of precision index. 'h'

$$h = \frac{1}{\sigma \sqrt{2}} = 0.5115^\circ \text{C}$$

since $h > 50\%$ the test was carried out correctly.

VII) check the validity of assumption that $\bar{T}$ represents the true values by calculating the value of z_1

$$z_1 = 0.6745 \sigma = 0.9323^\circ \text{C}$$

since the number reading between [] $\geq 50\%$

The value is True.

* Chauvenet's Criteria.

of reading d_i/σ (should be less than this number)

50

2.57

100

2.81

إذا كانت عدد القراءات غير موجودة ما بين عدد القراءات العقلية Interpolation

* How to specify the data.

$$X_{tru} = \bar{X}_{err} \pm r_i$$

± 0 more loss

± 2.5 more loss

± 3.5 more loss (high Error than prev.)

* Limiting Errors & Uncertainty :-

max. Error

Exact Error

Limiting Errors : $\pm \delta A$

Uncertainty : $\pm W$

$$A_{tru} = A_{err} \pm \delta A$$

; err: erroneous (لَبَّاءُ لَبَّاءُ)

$$\text{Relative Limiting Error } (\pm E_R) = \pm \frac{\delta A}{A}; A \rightarrow A_1 (A_{err})$$

→ Full scale

Division (FSD)

$$E_R = \pm \frac{\delta A}{A_1}$$

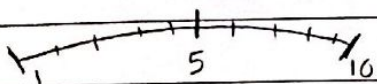
$$E_{RFS} = \pm \frac{\delta A}{FSD}$$

$$E_R A_1 = E_{RFS} FSD$$

$$E_R = \frac{E_{RFS} * FSD}{A_1}$$

= 2%

لَبَّاءُ لَبَّاءُ 1%



لَبَّاءُ لَبَّاءُ 1% 15 0 -

لَبَّاءُ لَبَّاءُ 1% 15 0 -

$$X = f(x_1, x_2, x_3, \dots, x_n)$$

$$X = X_1 + X_2 + \dots$$

$$X = X_1 - X_2 - \dots$$

$$X = X_1 * X_2 * X_3 \dots$$

$$X = X_1 + X_2 + X_3$$

$$\text{obj} = \frac{\delta X}{X} \quad (\text{max. Error})$$

For small change of X

means For small Error of X

$$dx = d(\quad)$$

$$dx = \delta X$$

$$\frac{dx}{X} = \frac{dX_1}{X} + \frac{dX_2}{X} + \frac{dX_3}{X}$$

$$\frac{\delta X}{X} = \frac{\delta X_1}{X} + \frac{\delta X_2}{X} + \frac{\delta X_3}{X}$$

$$= \frac{X_1}{X} \frac{\delta X_1}{X_1} + \frac{X_2}{X} \frac{\delta X_2}{X_2} + \frac{X_3}{X} \frac{\delta X_3}{X_3}$$

$$\frac{\delta X}{X} = \pm \left[\sum_{i=1}^n \left| \frac{X_i}{X} \frac{\delta X_i}{X_i} \right| \right]$$

$$\delta X = \sum \delta x_i$$

$$= \pm \left[\left| \frac{X_1}{X} \epsilon_{X_1} \right| + \left| \frac{X_2}{X} \epsilon_{X_2} \right| + \dots \right]$$

$$X = X_1 - X_2$$

$$dX = dX_1 - dX_2$$

$$\delta X = \delta X_1 - \delta X_2$$

$$\frac{\delta X}{X} = \frac{\delta X_1}{X} - \frac{\delta X_2}{X}$$

$$\frac{\delta X}{X} = \frac{X_1}{X} \frac{\delta X_1}{X_1} - \frac{X_2}{X} \frac{\delta X_2}{X_2}$$

$$= \pm \left[\left| \frac{X_1}{X} \frac{\delta X_1}{X_1} \right| + \left| \frac{X_2}{X} \frac{\delta X_2}{X_2} \right| \right]$$

$$X = X_1^a X_2^b X_3^c$$

Where : a, b, c + or - or $\pm$ or $\pm$ or $\pm$

$$\ln X = a \ln X_1 + b \ln X_2 + c \ln X_3$$

$$\frac{1}{X} = \frac{a}{X_1} \frac{dX_1}{dX} + \frac{b}{X_2} \frac{dX_2}{dX} + \frac{c}{X_3} \frac{dX_3}{dX}$$

$$\frac{dX}{X} = a \frac{dX_1}{X_1} + b \frac{dX_2}{X_2} + c \frac{dX_3}{X_3}$$

e.g:- $\delta X_1 = \pm 1$

$$\frac{\delta X}{X} = a \frac{\delta X_1}{X_1} + b \frac{\delta X_2}{X_2} + c \frac{\delta X_3}{X_3}$$

$\delta X_2 = \pm 2$

$\delta X_3 = \pm 3$

$$= \pm \left[\right]$$

e.g:-

$$Re = \rho v d \mu^{-1}$$

$$\frac{\delta Re}{Re} = \pm \left[\frac{\delta \rho}{\rho} + \frac{\delta v}{v} + \frac{\delta d}{d} + \frac{\delta \mu}{\mu} \right]$$

Note

$$\underbrace{A_1 \pm \delta A}$$

Should Be the Same Unit.

$$U = C \sqrt{2gh}$$

$$= 2^{\frac{1}{2}} C' g^{\frac{1}{2}} h^{\frac{1}{2}}$$

$$\frac{\delta U}{U} = \pm \left[\frac{\delta C}{C} + \frac{1}{2} \frac{\delta g}{g} + \frac{1}{2} \frac{\delta h}{h} \right]$$

$$\frac{\delta X}{X} = \pm \left[\sum \alpha_i E_{ri} \right]$$

$\rightsquigarrow$

في حالة الفرق او الف

Example 1 :-

$$R = (500 \pm 10\%) \Omega \quad \leftarrow \text{Rel. Limiting.}$$

$$10\% \rightarrow E_{R1} = \frac{\delta R}{500}$$

$$\delta R = \pm \frac{10}{100} * 500 = \pm 50 \Omega$$

$$R = [450 - 550] \Omega$$

Example 2 :-

$$E_{RFS} = \pm 5\% \text{ (FS)}$$

$$FS = 5 \times 10^{-6} \text{ m}^3/\text{s}$$

* إذا كانت نسبة الخطأ ← Full scale Error
scale

$$\dot{Q}_1 = 2.5 \times 10^{-6} \text{ m}^3/\text{s}$$

$$E_{R1} = \frac{E_{RFS} * FS}{\dot{Q}_1} = \frac{\pm 5\% * 5 \times 10^{-6}}{2.5 \times 10^{-6}} = \pm 10\%$$

$$E_{RFS} = \frac{\delta \dot{Q}_1}{FS} \Rightarrow \delta \dot{Q} = FS * E_{RFS}$$

$$= 0.25 \times 10^{-6}$$

الخطأ المسموح به

$$E_{RFS} = \frac{\delta A}{FS}$$

$$E_{R1} = \frac{\delta \dot{Q}_1}{\dot{Q}_1} = \frac{\pm 0.25 \times 10^{-6}}{2.5 \times 10^{-6}} = \pm 10\%$$

$$E_{R1} = \frac{\delta A}{A}$$

Example 3 :-

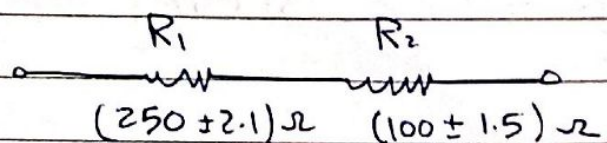
$$E_{RFS} = \pm 1\% \text{ FS}$$

$$FS = 1000 \text{ kPa}$$

$$P_1 = 100 \text{ kPa}$$

$$E_{R1} = \frac{E_{RFS} * FS}{P_1} = \frac{\pm 1\% * 1000}{100} = \pm 10\%$$

Ex Example 5 :-



$$R_{eq} = R_1 + R_2 = 350 \Omega$$

$$\frac{\Delta R_{eq}}{R_{eq}} = \pm \left[\frac{R_1}{R_{eq}} \frac{\Delta R_1}{R_1} + \frac{R_2}{R_{eq}} \frac{\Delta R_2}{R_2} \right] = \pm 0.010285$$

$$\Delta R_{eq} = \pm 3.6 \Omega$$

$$\text{if } R_1 = (250 \pm 3\%) \Omega$$

$$R_2 = (100 \pm 2\%) \Omega$$

$$\frac{\Delta R_{eq}}{R_{eq}} = \pm \left[\frac{R_1}{R_{eq}} 3\% + \frac{R_2}{R_{eq}} 2\% \right] = \pm 2.175\%$$

$$\frac{2.175}{100} \times 350$$

Example. 6 :-

$$E_{r,p} = \pm 1.5\%$$

$$E_{r,I} = \pm 1\%$$

$$R = I^2 P^{-1}$$

$$\frac{\Delta R}{R} = \pm \left[2 \left(\frac{\Delta I}{I} \right) + \left(\frac{\Delta P}{P} \right) \right] = \pm 3.5\%$$

Pr

$$X(R_1) = \frac{R_2 R_3}{R_4} + \frac{R_5}{R_6}$$

$\delta x_1, x_1$ $x_2, \delta x_2$

$$A = A_1 \pm \delta A$$

$$= A_1 (1 \pm \% E_r)$$

* Uncertainty propagation (W_A) :-

$$A = A_1 \pm W_A$$

$$= A_1 (1 \pm \% E_{WA})$$

$$X = f(x_1, x_2, x_3, \dots)$$

$$W_x = \pm \sqrt{\left(\frac{dx}{dx_1} W_{x_1}\right)^2 + \left(\frac{\partial x}{\partial x_2} W_{x_2}\right)^2 + \dots}$$

← General Case.

$x_1, x_2, \dots$ ← $\frac{dx}{dx_1}, \frac{\partial x}{\partial x_2}, \dots$

Ex:-

$$PV = mRT$$

$$T = PV m^{-1} R^{-1}$$

$\frac{-1}{x^2} \therefore \left(\frac{1}{x}\right)$ ←

$$\frac{dT}{dp} = \frac{V}{Rm} = \frac{T}{P}$$

$$\frac{dT}{dm} = -\frac{PV}{m^2 R} = -\frac{T}{m}$$

$$\frac{dT}{dV} = \frac{P}{Rm} = \frac{T}{V}$$

$$\frac{dT}{dR} = -\frac{PV}{mR^2} = -\frac{T}{R}$$

$$\frac{W_T}{T} = \pm \sqrt{\left(\frac{V}{Rm} W_P\right)^2 + \left(\frac{P}{Rm} W_V\right)^2 + \left(\frac{PV}{mR^2} W_m\right)^2 + \left(-\frac{PV}{mR^2} W_R\right)^2}$$

$\frac{T}{P}$ $\frac{T}{V}$ $\frac{T}{m}$ $\frac{T}{R}$

$$E_{wx} = \pm \sqrt{\sum_{i=1}^n (\alpha E_{wxi})^2}$$

Ex:-

$$I = \frac{bh^3}{12}$$

$$\frac{dI}{db} = \frac{h^3}{12} = \frac{I}{b}$$

$$\frac{dI}{dh} = \frac{3bh^2}{12} = \frac{3I}{h}$$

$$\frac{WI}{I} = \pm \sqrt{\left(\frac{W_b}{b}\right)^2 + \left(\frac{3W_h}{h}\right)^2}$$

نستخدم هذه الطريقة في حالة الضرب
نقط، فإذا وجد إشارة $\pm$ استخدمها.

Ex:-

$$\dot{m}_w = \frac{\rho A}{\sqrt{1-\beta^4}} \sqrt{2g\Delta h}$$

$$= \phi \rho 2^{\frac{1}{2}} g^{\frac{1}{2}} h^{\frac{1}{2}} = \psi \rho g^{\frac{1}{2}} h^{\frac{1}{2}}$$

أي

$$\frac{Wm}{m} = \pm \sqrt{\left(\frac{W_p}{\rho}\right)^2 + \left(\frac{1}{2} \frac{W_g}{g}\right)^2 + \left(\frac{1}{2} \frac{W_h}{h}\right)^2}$$

اللجنة الأكاديمية في قسم هندسة الميكانيك - الجامعة الأردنية

$$\frac{dm_w}{d\rho} = \psi h^{\frac{1}{2}} g^{\frac{1}{2}} = \frac{m}{\rho}$$

$$\frac{dm_w}{dg} = \frac{1}{2} \psi \rho h^{\frac{1}{2}} g^{-\frac{1}{2}} = \frac{1}{2} \frac{m}{g}$$

$$\frac{dm_w}{dh} = \frac{1}{2} \frac{m_w}{h}$$

Ex :-

$$S = \frac{W I^4 g^{3/4}}{z^2 h^5}$$

$$\frac{W_S}{S} = \pm \sqrt{\left(\frac{W_N}{N}\right)^2 + \left(\frac{4W_I}{I}\right)^2 + \left(\frac{3}{4} \frac{W_g}{g}\right)^2 + \left(2 \frac{W_z}{z}\right)^2 + \left(5 \frac{W_h}{h}\right)^2}$$

Example 10 :-

$$V = 110 \pm 0.2 \text{ V} \quad W_V$$

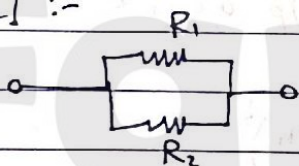
$$I = 5.3 \pm 0.06 \text{ A} \quad W_A$$

$$P = V * I = 584 \text{ watt}$$

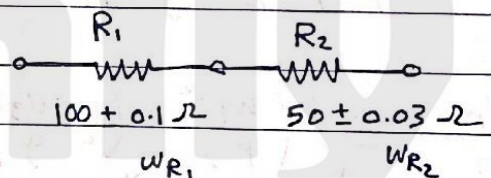
$$\frac{W_P}{P} = \pm \sqrt{\left(\frac{W_V}{V}\right)^2 + \left(\frac{W_I}{I}\right)^2} = \pm 1.15 \%$$

$$\therefore W_P = \pm \frac{1.15}{100} * 584 = \pm 6.7 \text{ watt}$$

Example 11 :-



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = 33.33 \Omega$$



$$R_{eq} = R_1 + R_2 = 150 \Omega$$

$$R_{eq} = R_1 R_2 (R_1 + R_2)^{-1}$$

$$\frac{dR_{eq}}{dR_1} = R_2 \left(\right)^{-1} - R_1 R_2 \left(\right)^{-2} \quad W_{Req} = \pm \sqrt{\left(\frac{dR_{eq}}{dR_1} W_{R_1}\right)^2 + \left(\frac{dR_{eq}}{dR_2} W_{R_2}\right)^2}$$

1 * 0.3 1 * 0.3

$$\frac{dR_{eq}}{dR_2} = R_1 \left(\right)^{-1} - R_1 R_2 \left(\right)^{-2} = \pm 0.1044 \Omega$$

0.444

$$R_{eq} = 150 \pm 0.1044 \Omega$$

$$W_{Req} = \pm 0.01734 \Omega$$

$$R_{eq} = 33.33 \pm 0.01734 \Omega$$

* Known Error :-

$$A = A_1 + \Delta A$$

$$A = A_1 - \Delta A$$

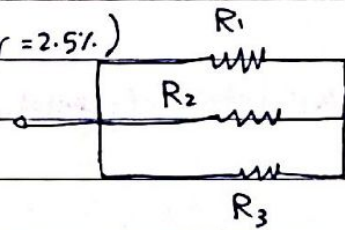
Known Error $\times$ Error
controlable ان نوع

Example 12 :-

$$R_1 = 250 + 2.5\% \quad (0.025 \text{ Fractional Error} = 2.5\%)$$

$$R_2 = 500 - 3.6\%$$

$$R_3 = 375 + \underbrace{1.4\%}_{\text{C.F}}$$



$$A_{\text{true}} = A_{\text{erroneous}} + \Delta A$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

1) $R_{eq} \text{ (erroneous)} = \frac{R_{1e} R_{2e} R_{3e}}{R_{1e} + R_{2e} + R_{3e}} = \frac{250 \times 500 \times 375}{250 + 500 + 375} = 115.4 \, \Omega$ check!

2) $R_{\text{true}} = 250 + \frac{2.5}{100} \times 250 = 256.25 \, \Omega$

$$R_{2,\text{true}} = 500 - \frac{3.6}{100} \times 500 = 482 \, \Omega$$

$$R_{3,\text{true}} = 380.2502$$

3) $R_{eq,\text{true}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = 116.3 \, \Omega$

$$E_{R_{eq}} = \frac{R_{eq,\text{erron}} - R_{eq,\text{tru}}}{R_{eq,\text{true}}} = \frac{115.4 - 116.3}{116.3} \times 100\%$$

$$= -0.776\%$$

↪ erronous ان النوع

$$R_{\text{true}} = 115.4 + 0.776\%$$

Drift :-

المؤشر ما يكون على الصفر .

~~Value~~ ~~القيمة~~ ~~مقابلة~~ ~~output~~

resolution :-

امكانية قراءة اي تغير في pointer .

repeatability :-

تغير الوقت

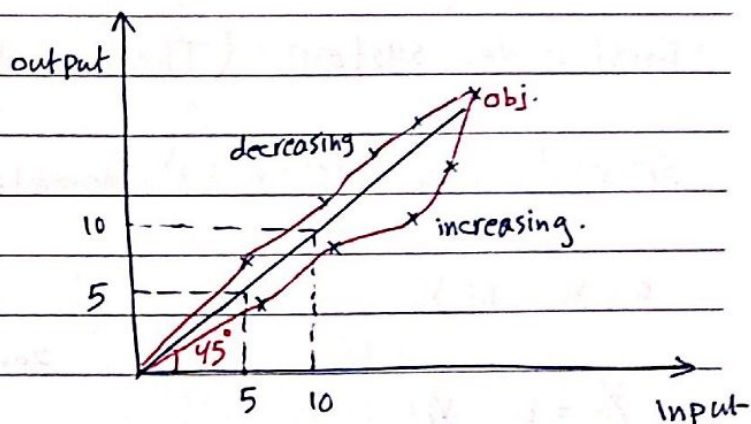
reproducibility : (دقة التكرار) parameter

تغير جميع ال

اللجنة الأكاديمية في قسم هندسة الميكانيك - الجامعة الأردنية

* Calibration Procces :-

- Random data $\uparrow, \downarrow$.
- max. scale.
- take as small as possible step
- standard.



* correction factor = $-d_i$

الملاحظة بين Input , output في أي حال
 يجب أن تكون Linear أو Linear
 إذا لم تكن Error

input	output		Diff	($\bar{x}_i$) avg. output	$d_i = x_i - \bar{x}$
#	inc	Dec.			
5	4.5	7	2.5	5.75	-0.75
10	9.6	11.4		10.5	-0.5
15	14.2	16.2		15.2	-0.2
20	18	21		19.8	+0.5
25	22.5	26	3.5	24.25	0.75
30	29	29		29	+1

1) Hysteresis = $\frac{2.5}{1}$

القيم التي
 High
 Stresses.

Zero Order system (Potentiometer)

First order system (Thermometer)

Second Order system (Manometer)

Step Input

$$a_0 Y_0 = b_i Y_i$$

Zero order:- Error free system.

$$Y_0 = \left(\frac{b_i}{a_0} \right) Y_i$$

K_s (static sensitivity)

$$\left(\frac{a_2}{a_0} \right) \ddot{Y}_0 + \left(\frac{a_1}{a_0} \right) \dot{Y}_0 + \frac{a_0}{a_0} Y_0 = \left(\frac{b_i}{a_0} \right) Y_i$$

$$\dot{Y} = \frac{dy}{dt}$$

τ : time constant

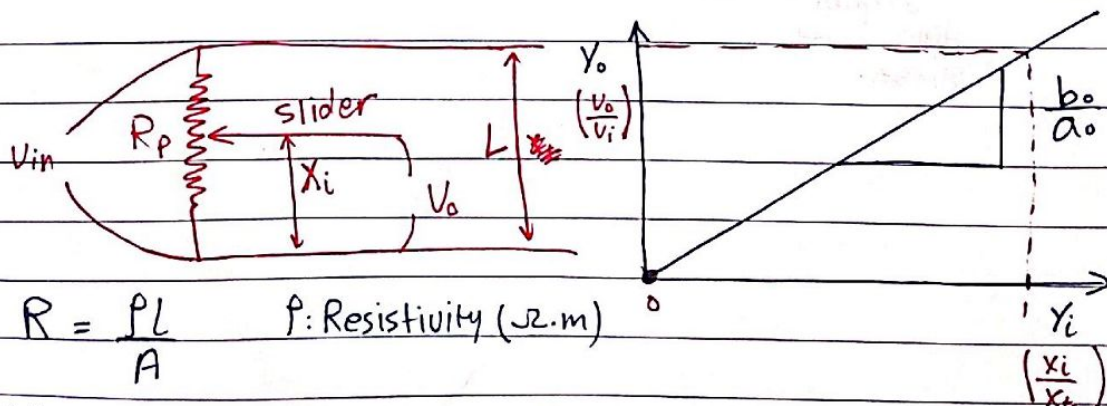
K_s

$$\left(\frac{a_2}{a_0} \right) \ddot{Y}_0 + \left(\frac{a_1}{a_0} \right) \dot{Y}_0 + \frac{a_0}{a_0} Y_0 = b_i Y_i$$

$$\frac{1}{\omega_n^2}$$

$f(\xi)$

* Zero Order system:-



$$R = \frac{\rho L}{A}$$

ρ : Resistivity ($\Omega \cdot m$)

$$V_o \cdot L = x_i V_i$$

$$V_o = \left(\frac{x_i}{L} V_i \right) K_s$$

* First Order System :-

$$a_1 \dot{Y}_o + a_0 Y_o = b_1 \dot{Y}_i$$

$$\tau \dot{Y}_o + Y_o = K_s Y_i$$

→ For step input

$$Y_i = A, \quad A: \text{any const.}$$

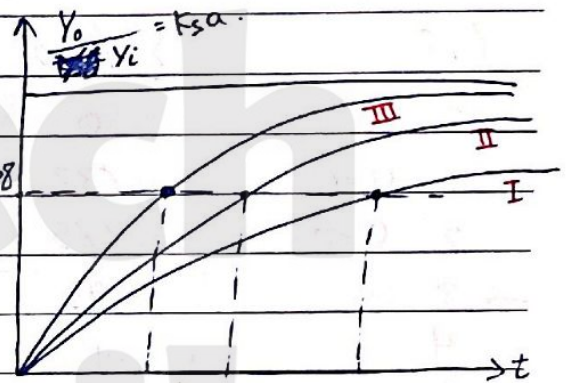
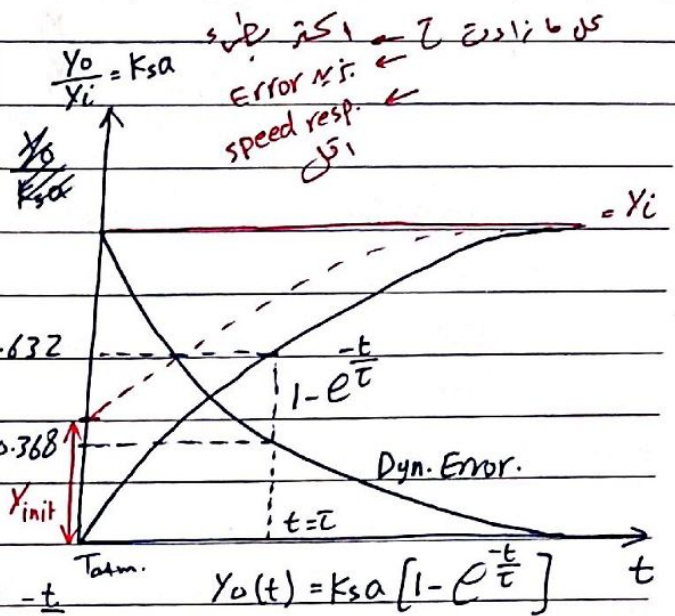
$$Y_o(t=0) = Y_{init.}$$

$$Y(t) = K_s A \left[1 - e^{-t/\tau} \right] + Y_{init.} e^{-t/\tau}$$

$$Y_o(t) = \underbrace{K_s A}_{\text{steady state output}} + \underbrace{(Y_{init.} - K_s A) e^{-t/\tau}}_{\text{Transient output}}$$

As $t \rightarrow \infty$, $Y_o(t) \rightarrow K_s A$ (Steady state output)

$$Y_{init.} e^{-t/\tau}$$



As $\tau \uparrow$: Error $\uparrow$

speed of resp. $\downarrow$

time $\uparrow$

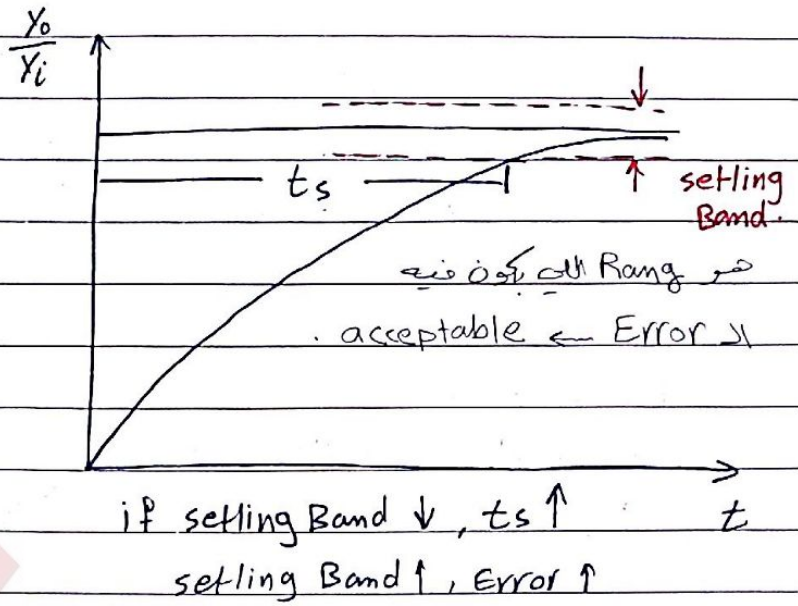
$$\begin{aligned} 1) DE &= Y_i - Y_o \\ &= K_s A - K_s A (1 - e^{-t/\tau}) \\ &= K_s A e^{-t/\tau} \quad \text{max at } t=0 \end{aligned}$$

$$2) S.S.E = \lim_{t \rightarrow \infty} DE = 0$$

$$3) S.S.O = \lim_{t \rightarrow \infty} Y_o(t) = K_s A$$

$$4) \text{ at } t = \tau$$

$$\frac{Y_o}{Y_i} = 0.632$$



t/τ	Y_o / K_{sa}	DE	
0	0	1	
1 ($t = \tau$)	0.632	0.368	
2 ($t = 2\tau$)	0.865	0.135	
3	0.95	0.05	عادة ينتظر في $(3\tau - 4\tau)$
4	0.982	0.018	
5	0.993	0.007	
6			
$\vdots$			
∞	1	0	

5) $\tau \uparrow$, speed of response system becomes more slow :-

Modelling of Thermometer

$$Q_{\text{stored}} = Q_{\text{in}} - Q_{\text{out}}$$

$$Q_{\text{in}} = H A_b (\theta_i - \theta)$$

$$\theta_{\text{out}} = 0$$

$$\theta_{\text{in}} = \theta$$

if $\theta_i > \theta$

$$Q_{\text{out}} = H A_b (\theta - \theta_i)$$

if $\theta > \theta_i$

$$Q_{\text{stored}} = \rho V C \frac{d\theta}{dt}$$

$$\rho V_b C \frac{d\theta}{dt} = H A_b (\theta_i - \theta)$$

$$\rho V_b C \frac{d\theta}{dt} + H A_b \theta = H A_b \theta_i \quad \text{----- (*)}$$

$$\Rightarrow \text{Area.} \times \text{linear Exp.} = \Delta T_b$$

$$\overset{\text{m}^2}{A_c} \times \overset{\text{m}}{dX_o} = \alpha \overset{\frac{\text{m}^3}{\text{m}^2 \times \text{C}}}{T_b} \overset{\text{m}^3 \times \text{C}}{d\theta}$$

$$A_c dX_o = \alpha T_b d\theta$$

α : vol. exp. coeff.

$$d\theta = \frac{A_c}{\alpha T_b} dX_o \quad \left. \vphantom{\frac{A_c}{\alpha T_b}} \right\} \text{sub in (*)}$$

$$\theta = \frac{A_c}{\alpha T_b} X_o$$

$$\underbrace{\rho C A_c}_{a_1} \frac{dX_o}{dt} + \underbrace{A_b H A_c}_{a_0} X_o = \underbrace{H A_b \theta_i}_{b_0}$$

$$\frac{\rho C T_b}{H A_b} \dot{X}_o + X_o = \underbrace{\frac{T_b}{A_c}}_{\text{sensitivity}} \theta_i$$

القدرة على قراءة القراء

$$\tau = \frac{\rho C T_b}{H A_b} \Rightarrow \text{so, } \tau \text{ is known.}$$

- τ should be minimize.

Ex:-

$$k_s \leftarrow d_c = 0.25 \text{ mm}$$

Find:-

$$\tau \left[\begin{array}{l} \rho = 13.5 \times 10^3 \\ C = 139 \text{ KJ/kg} \cdot ^\circ\text{C} \end{array} \right.$$

$$t_b ??$$

$$\tau ??$$

$$k_s \leftarrow \alpha = 0.55 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$$

$$H = 60 \text{ J/m}^2 \cdot \text{s} \cdot ^\circ\text{C}$$

$$k_s = 1.5 \text{ mm/} ^\circ\text{C}$$

هذا τ هو مقدار الطاقة في m^2

$$k_s = 1.5 = \alpha \frac{t_b}{A_c}$$

$$1) \quad t_b = \frac{k_s A_c}{\alpha} = \frac{1.5 \times (\frac{\pi}{4} \times 0.25^2)}{0.55 \times 10^{-3}} = 133.875 \text{ mm}^3$$

$$2) \quad \tau = \frac{\rho C t_b}{H A_b} \quad ; \quad t_b = \frac{4}{3} \pi R_b^3 \rightarrow R_b = 3.173 \text{ mm}$$

$$D_b = 6.346 \text{ mm}$$

$$\tau = \frac{13500 \times 139 \times 133.875 \times 10^{-9}}{60 \times 26.5 \times 10^{-6}}$$

$$A_b = \pi D_b^2 = 126.51 \text{ mm}^2$$

$$\tau = 33.18 \text{ sec.}$$

$$3) \quad \tau \dot{x}_0 + x_0 = k_s T_i$$

$$x_0 = k_s T_i \left[1 - e^{-t/\tau} \right] + \frac{k_s}{\tau} T_{init} e^{-t/\tau}$$

المعادلة العامة لدرجة الحرارة في output كالتالي

$$\frac{x_0}{k_s} = T_i \left[1 - e^{-t/\tau} \right] + T_{init} e^{-t/\tau}$$

أو تكتبها هكذا $T = T_i (1 - e^{-t/\tau}) + T_{init} e^{-t/\tau}$

$$= 125 \left[1 - e^{-10/33.1} \right] + 25 e^{-10/33.1}$$

$$= 51.07 ^\circ\text{C}$$

~~assume put in cold water (20°C) for 15 sec~~

$$\frac{X_0}{K_s} = T_i [1 - e^{-t/\tau}] + T_{\text{init}} e^{-t/\tau} = 20 [1 - e^{-\frac{15}{2.1}}] + 51 e^{-\frac{15}{2.1}} = 39.77^\circ\text{C}$$

if this thermo meter dept in cold ~~hot~~ water after 10 sec of the hot water in cold water of 20°C, How much be the temp. indicate after 15 sec ???

EX. 1 :-

$$T_i = 200^\circ\text{C}, T_{\text{initial}} = 0^\circ\text{C}, t = 1.5 \text{ s}, \tau = 2.5 \text{ s}$$

المجموعه اقل
 $0.63 \times 200 \approx 126$
 من 126

case (I): $T_{\text{initial}} = 0^\circ\text{C}$ $T_0 = T_i [1 - e^{-t/\tau}] = 90.24^\circ\text{C}$

case (II): $T_{\text{initial}} = 25^\circ\text{C}$ $T_0 = T_i [1 - e^{-t/\tau}] + T_{\text{initial}} e^{-t/\tau} = 103.96^\circ\text{C}$

OR

$$DE = 200 e^{-\frac{1.5}{2.5}} = 109.75 \rightarrow \therefore T_0 = 90.25$$

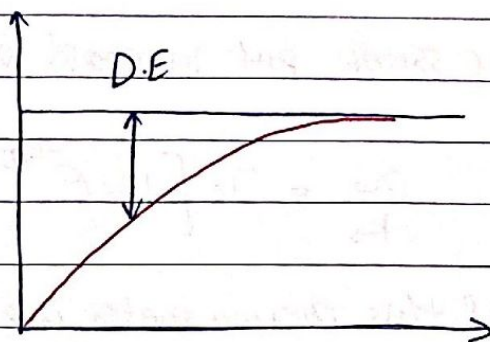
$$DE = 200 e^{-\frac{1.5}{2.5}} + T_{\text{initial}} e^{-t/\tau} + T_i$$

EX. 2 :-

$$T_0 = T_i [1 - e^{-t/\tau}] = 34.85^\circ\text{C}$$

$$DE = 65.144^\circ\text{C}$$

$$\begin{aligned}
 D.E &= T_i - T_o \\
 &= T_i - [T_i (1 - e^{-t/\tau}) + T_{init} e^{-t/\tau}] \\
 &= e^{-t/\tau} (T_i - T_{init})
 \end{aligned}$$



Example 4 :-

$$t_s = 10s$$

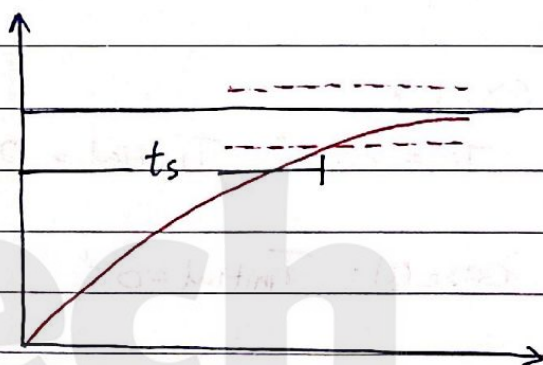
$$\frac{t_s}{\tau} = 5$$

$$\therefore \tau = 2 \text{ sec.}$$

$$T_o = T_i (1 - e^{-t/\tau})$$

$$\frac{T_o}{T_i} = (1 - e^{-t/\tau}) = \frac{1}{2}$$

$$\Rightarrow t = 1.38 \text{ sec.}$$



إذا لم يتغير T_{init} لا تغيرها أبداً
تكونت صفر.

* Second Order System :-

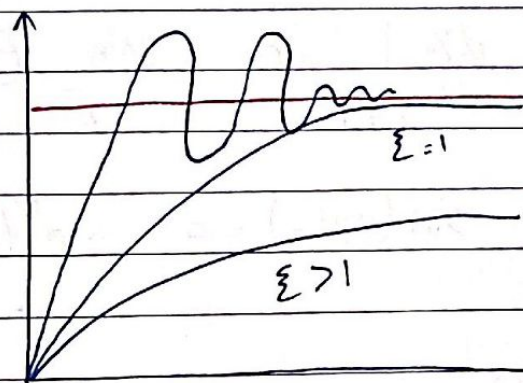
$$\begin{aligned}
 \left(\frac{a_2}{a_0} \right) \frac{d^2 y_o}{dt^2} + \left(\frac{a_1}{a_0} \right) \frac{dy_o}{dt} + \frac{a_0}{a_0} y_o &= \left(\frac{b}{a_0} \right) y_i \\
 \frac{1}{\omega_n^2} \frac{d^2 y_o}{dt^2} + \frac{2\xi}{\omega_n} \frac{dy_o}{dt} + y_o &= \frac{K_s}{\omega_n^2} y_i
 \end{aligned}$$

$$\frac{1}{\omega_n^2} s^2 + \frac{2\xi}{\omega_n} s + 1 = 0$$

$$s_{1,2} = -\xi \omega_n \pm \omega_n \sqrt{\xi^2 - 1} \quad ; \quad \begin{aligned} \xi &> 1 \text{ over damped} \\ \xi &< 1 \text{ under damped.} \\ \xi &= 1 \end{aligned}$$

For $0 < \xi < 1$:-

Step input = a



$$y_o(t) = 1 - e^{-\xi \omega_n t} \left[\cos(\omega_d t) + \frac{\xi}{\sqrt{1-\xi^2}} \sin(\omega_d t) \right]$$

$$\frac{y_o(t)}{K_s a} = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cos(\omega_d t - \phi)$$

, where $\omega_d = \omega_n \sqrt{1-\xi^2}$

$\phi = \cos^{-1}(\xi)$

$= \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$

1) Steady state output

$$\lim_{t \rightarrow \infty} y_o = K_s a$$

$$\begin{aligned} 2) DE &= K_s a - K_s a * \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cos(\omega_d t - \phi) \right] \\ &= K_s a \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cos(\omega_d t - \phi) \end{aligned}$$

t_r : the time needed to reach input @ first time :-

$$t_r = \frac{y_o(t=t_r)}{K_s a} = 1$$

$$t_r = \frac{\pi - \phi}{\omega_d}$$

π : rad

ϕ : Degree ($* \frac{\pi}{180}$)

proof
is at p.

t_p : Peak Time (t_p)

$$\left. \frac{dy_0}{dt} \right|_{t=t_p} = 0 = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t_p} \sin(\omega_d t_p)$$

$$\sin(\omega_d t_p) = 0 \Rightarrow \boxed{t_p = \frac{n\pi}{\omega_d}}, n=1$$

$$\text{Peak Overshoot (M}_p) = \frac{y_0(t=t_p) - y_0(t=\infty)}{y_0(t=\infty)} \times 100\%$$

$$M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$$\Rightarrow \xi = \sqrt{\frac{(\ln(M_p))^2}{(\ln(M_p))^2 + \pi^2}}$$

$$\Rightarrow t_s = -\frac{\ln\left(\frac{E_s}{100}\right)}{\xi \omega_n} \quad \text{Ex } E_s = 3\% \xrightarrow{\text{sub}} \left(\frac{3}{100}\right)$$

Example 7:-

$M_p = 10\%$, $t_r = 0.2 \text{ s}$, find:- ξ , ω_n , ω_d

$$\frac{10}{100} = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \Rightarrow \xi = 0.591$$

$$t_r = \frac{\pi - \phi}{\omega_d} \quad \text{where } \phi = \cos^{-1}(\xi) = 53.77^\circ = 0.9385 \text{ rad}$$

$$0.2 = \frac{\pi - 0.9385}{\omega_d} \Rightarrow \omega_d = 11.05 \text{ rad/s}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = 13.65 \frac{\text{rad}}{\text{s}}$$

Extra: $E_s = \pm 3\%$

$$t_p = \frac{T}{\omega_d} = 0.284 \text{ sec.}$$

$$t_s = \frac{-\ln(\epsilon)}{\xi \omega_n} = \frac{-\ln\left(\frac{3}{100}\right)}{0.591(13.65)} = 0.434$$

Mass-spring system :-

Manometer :-

B: coeff of viscous friction $\frac{N}{s}$

$$B = \frac{\mu}{A}$$

$$F_i = m \ddot{x} = P \theta \frac{dx}{dx^2}$$

$$F_\mu = B \frac{dx}{dt}$$

$$F_{tot} = \rho g a (H - h) = 2 \rho g a x$$

$$\underbrace{\theta \ddot{x}}_{a_2} + \underbrace{B \dot{x}}_{a_1} + \underbrace{2 \rho g a x}_{a_0} = \underbrace{P \theta}_{b_0}$$

cross section Area

$$\left[\frac{P \theta}{a} s^2 + \frac{B}{a} s + 2 \rho g \right] X(s) = P(s)$$

$$\frac{X(s)}{P(s)} = \frac{1}{\frac{P \theta}{a} s^2 + \frac{B}{a} s + 2 \rho g}$$

$$\frac{X(s)}{P(s)} = \frac{Pa/P\theta}{s^2 + \frac{B}{P\theta}s + \frac{2ag}{\theta}}, \quad G(s) = 2Pa g \frac{X(s)}{P(s)}$$

$$G(s) = \frac{2ag/\theta}{s^2 + \frac{B}{P\theta}s + \frac{2ag}{\theta}}$$

$$\omega_n = \sqrt{\frac{2ag}{\theta}}; \quad \zeta = \frac{B}{2P\sqrt{2ag\theta}}$$

OR

$$\frac{P\theta}{2Pa g} \ddot{X} + \frac{B}{2Pa g} \dot{X} + \frac{2Pa g}{2Pa g} X = \frac{Pa}{2Pa g}$$

$$\frac{1}{\omega_n^2} = \frac{P\theta}{2Pa g}, \quad \frac{2\zeta}{\omega_n} = \frac{B}{2Pa g}, \quad K_s = \frac{1}{2Pa g}$$

$$\Rightarrow \omega_n = \sqrt{\frac{2ag}{\theta}} \Rightarrow \zeta = \frac{B}{2P\sqrt{2ag\theta}}$$

$$\omega_n \propto \frac{1}{d}$$

الجامعة الأردنية - الجامعة الأردنية

Basic Electrical/Mechanical Detecting Devices:-

→ Def.

→ physical phenomenon.

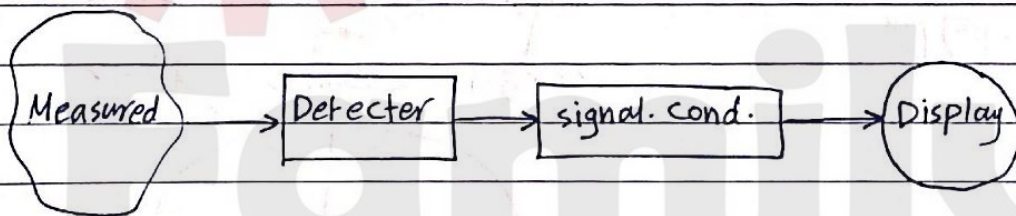
→ Mech. → Diaphragm.

→ Bellow.

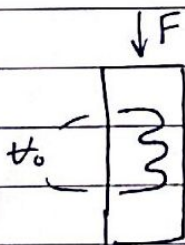
→ Electrical :-

1. potentiometer
2. Piezoelectric.
3. RTD
4. Thermistor.
5. Capacitor.
6. Strain Gauge.

Transducers:-



← transducer & Detector هما بكونوا في نفس الجزء Thermometer



← هما بكونوا في نفس الجزء :-

Detector: coulombn $\{F \rightarrow \Delta L\}$

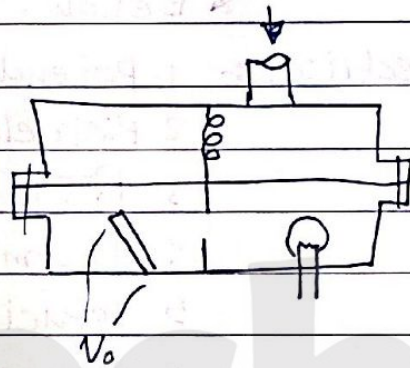
Transducer: Strain Gauge. $[\Delta L \rightsquigarrow \Delta V]$

In General :-
 Detector mech. $\rightarrow$ mech.
 Transducer mech. $\rightarrow$ elec.

Bellow }
 spring } Elastic Members. $F, P \rightsquigarrow \Delta x$
 Diaphragm }

mass }
 seismic mass } mass $F \rightsquigarrow \Delta h$
 Manometer }

Venturi }
 orifice } $v \rightarrow P$
 Pitot }



* Diaphragm :-

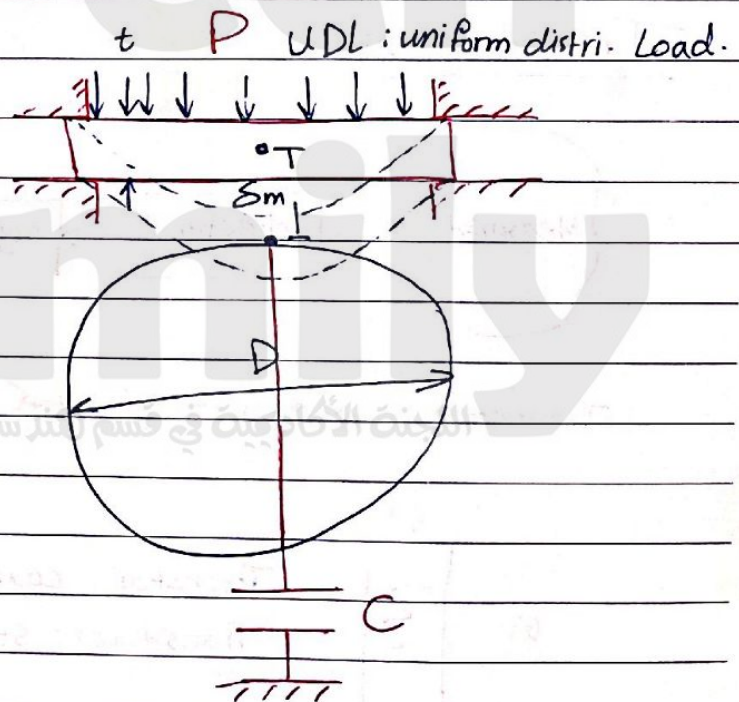
$$P = \frac{256 E t^3 \delta_m}{3(1-\nu^2) D^4}$$

$$K_s = \frac{\delta_m}{P}$$

To use this method :-

$$\delta_m < \frac{1}{2} t$$

$$\sigma_{max} = \frac{3 D^2 P}{16 t^2}$$



Basic Electrical/Mechanical Detecting Devices:-

→ Def.

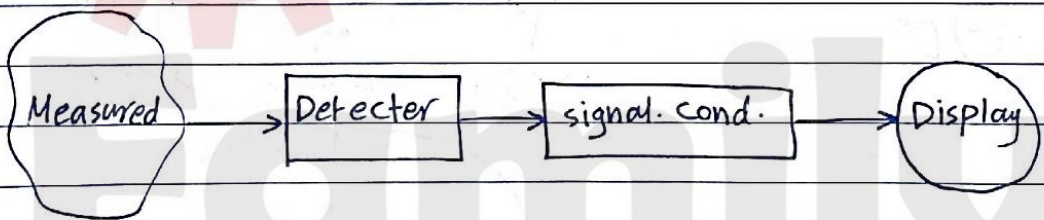
→ physical phenomenon.

→ Mech. → Diaphragm.
→ Bellow.

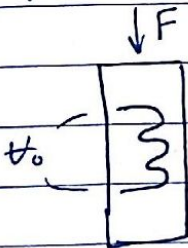
→ Electrical :-

1. Potentiometer
2. Piezoelectric.
3. RTD
4. Thermistor.
5. Capacitor.
6. Strain Gauge.

Transducers:-



Thermometer & Detector ← transducer
Thermo couple.



Detector: coulomb $\{F \rightarrow \Delta L\}$

Transducer: Strain Gauge. $[\Delta L \rightsquigarrow \Delta V]$

In General :-
 Detector mech. $\rightarrow$ mech.
 Transducer mech. $\rightarrow$ elec.

Bellow
 spring
 Diaphragm

Elastic Members. $F, P \rightsquigarrow \Delta x$

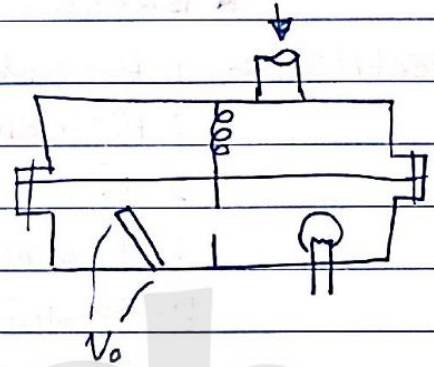
mass
 seismic mass
 Manometer

mass

$F \rightsquigarrow \Delta h$

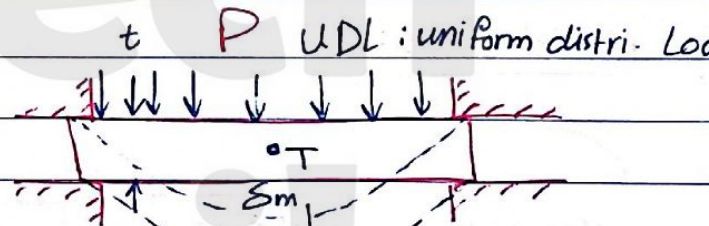
Venturi
 orifice
 Pitot

$v \rightarrow P$



* Diaphragm :-

$$P = \frac{256 E t^3 \delta_m}{3(1-\nu^2) D^4}$$

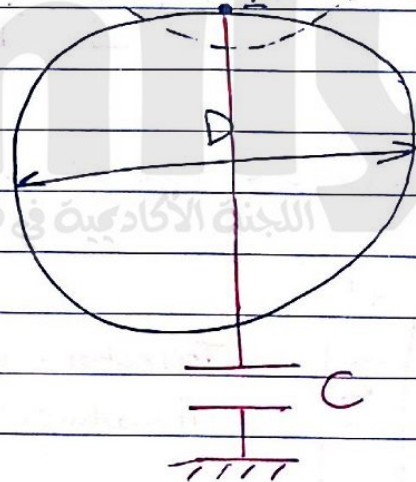


$$K_s = \frac{\delta_m}{P}$$

To use this method :-

$$\delta_m < \frac{1}{2} t$$

$$\sigma_{max} = \frac{3 D^2 P}{16 t^2}$$

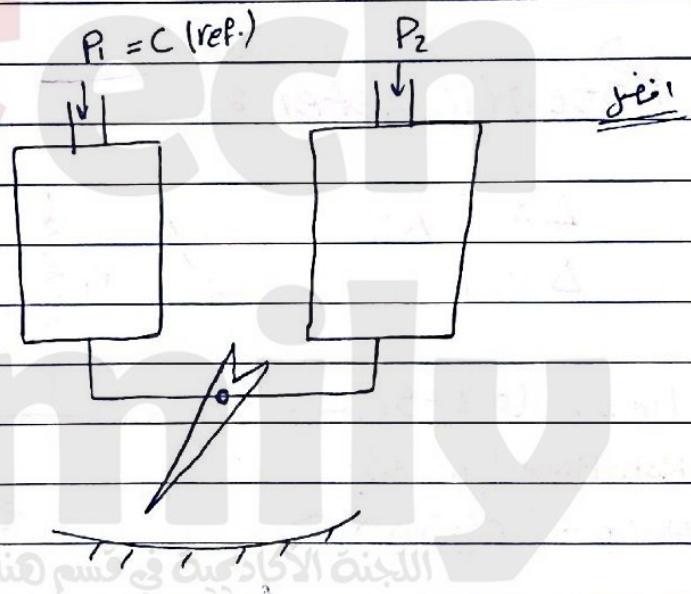
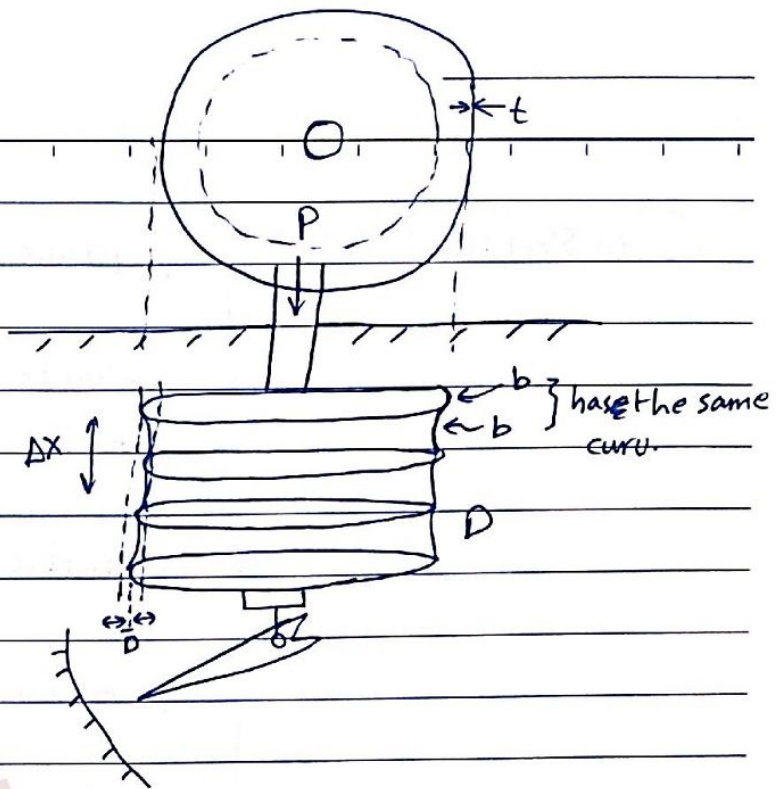


$$\omega_n = \frac{20 t}{D^2} \sqrt{\frac{E}{3 P (1-\nu^2)}}$$

Bellow:-

$$\Delta x = \frac{0.452 * P * b * d^2 \sqrt{1-\nu^2}}{E t^3}$$

$$K_s = \frac{\Delta x}{P}$$



* Phenomena :-

- ΔV (Voltage) Pot. $\Delta x \rightarrow \Delta V$
- ΔR (Resist.) RTD, thermomet. $\Delta T \rightarrow \Delta R$
- ΔC Capacitor $\left. \begin{matrix} \Delta x \\ \Delta A \\ \Delta E \end{matrix} \right\} \Delta C$

Classification :-

Primary (Detector)

secondary (Transducer) primary $\rightarrow$ output ipk

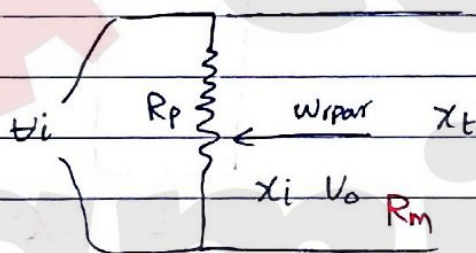
Passive: External source.

Active : consume / Generates its own Power from Input

Analogue.

Digital.

Potentiometer :-

$$\left. \begin{array}{l} \Delta x \\ \Delta \theta \end{array} \right\} \Delta v$$


For Ideal system

$$R_{\text{in}} = \infty$$

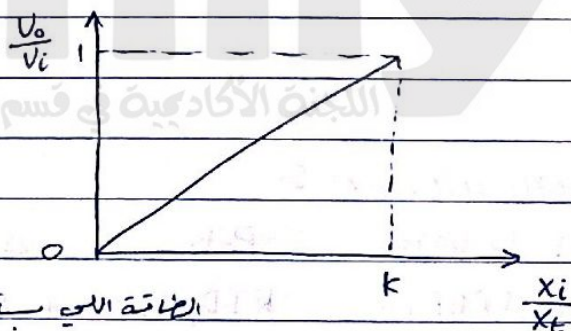
$R_m \downarrow \rightarrow \text{Error} \uparrow$

-linear (0.2-50)cm

Rotational (10-60°)

- Helipot 

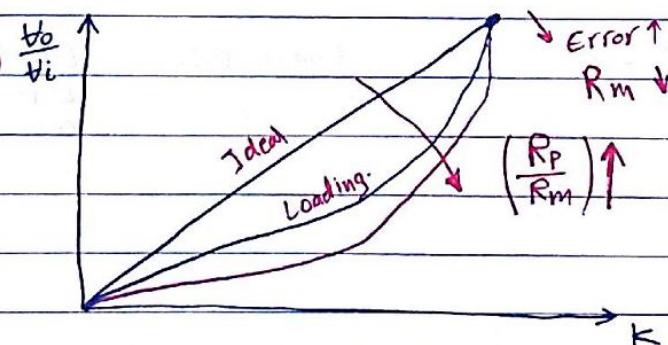
Ideal :- $V_{o,i} = \left(\frac{U_i}{x_{t,i}} \right) x_i$ - ①



with loading error:- $\frac{\text{المقاومة الفعلية}}{\text{المقاومة الاسمية}}$

$$\frac{V_{OL}}{V_i} = \frac{K}{K(1+K) \left(\frac{R_p}{R_m} \right) + 1}$$

High internal resistance
OR
lower impedance



$$E = V_{OL} - V_{OI}$$

$$= -V_i \left[\frac{K^2 (K-1)}{K(1-K) + (R_m/R_p)} \right] \quad - (3)$$

$$V_{i,max} = \sqrt{P R_p}$$

• P max Power rat. $\frac{P}{R_p}$ $\frac{V_i^2}{R_p}$ $\frac{I_i^2 R_p}{R_p}$

→ Power Rating :- max. heat dissipation

For good t_o :- $\star$ increase t_i

For $E \rightarrow 0$:- use $R_p \downarrow$, $R_m \uparrow$

$$P = \frac{V_{in}^2}{R_p}$$

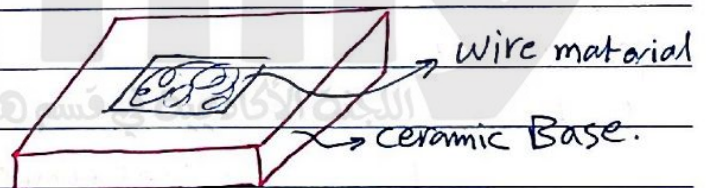
material selection Based on : P , R_p

$$\Sigma_{max} = 15 \times \frac{R_p}{R_m} \%$$

Wire

Continuous.

1. Cermet



2. Thin Metal

3. Hot carbon Film.

Resistance Temp. Detector (RTD) $\rightarrow$ material $R_T \uparrow$ as $T \uparrow$

Thermister $\rightarrow$ semicond. chip $R_T \uparrow$ as $T \downarrow$

$\rightarrow$ Used for Temp. measurement

$\rightarrow$ measure "T" by observing change in "R"
قياس

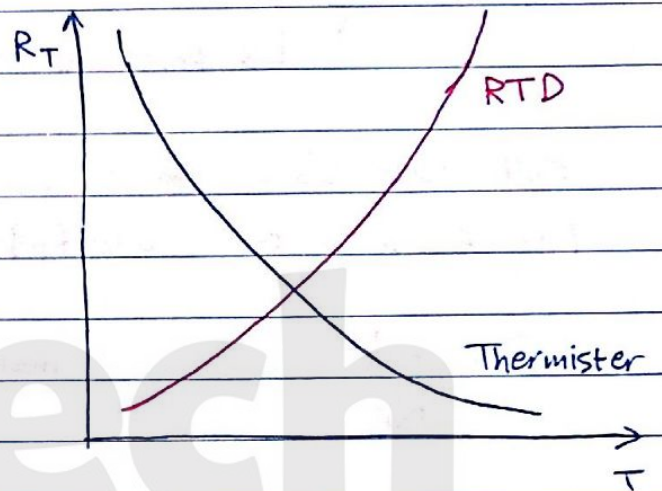
Thermister used @ T low

RTD:-

$$R_T = R_{T_0} [1 + \alpha_1 \Delta T + \alpha_2 \Delta T^2]$$

Thermister:- $R \left[\frac{1}{T_1} - \frac{1}{T_2} \right]$

$$R_{T_1} = R_{T_2} e^{B \left[\frac{1}{T_1} - \frac{1}{T_2} \right]}$$

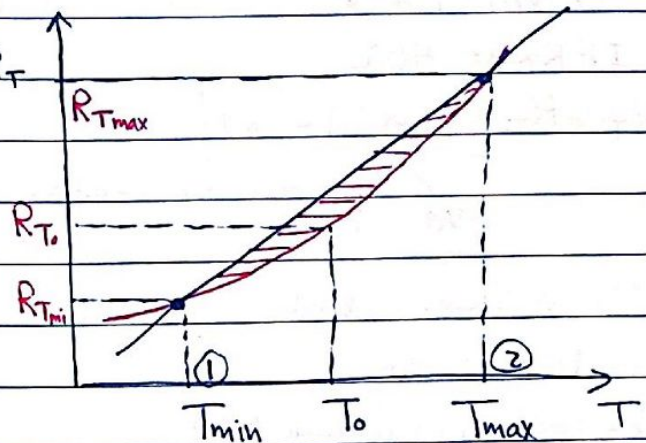


Material Properties:-

- cheap
- Available
- melting point, $T_{m.p} > T_{max, working}$
- stable
- pure metal
- Uniform $\left\{ \begin{array}{l} \rightarrow \text{Geometry} \\ \rightarrow \text{Chemical composition.} \end{array} \right.$
- corrosive Resistance
- Non-Toxic
- linear R-T relation $\rightarrow$ [أهم ميزة لها السؤاال بغيره]:

RTD :-

Resistance at this Temp. $\leftarrow R_T$



- Linear Approximation :-

$$R(T) = R(T_0) [1 + \alpha (T - T_0)]$$

- Quadratic :-

$$R(T) = R(T_0) [1 + \alpha_1 (T - T_0) + \alpha_2 (T - T_0)^2]$$

In measurement :-

$$\alpha = \frac{1}{R_{T_0}} \left[\frac{R_{T_2} - R_{T_1}}{T_2 - T_1} \right]_{\text{slope.}} \quad \{ \text{for linear} \}$$

$$T_0 = \frac{T_{\max} + T_{\min}}{2}$$

In Quadratic Divide Interval to 2 segment

segment (1) $[T_1 - T_0]$ $\frac{R_{T_1}}{R_{T_0}} = R_{T_0} [1 + \alpha_1 \overset{-ve}{(T_1 - T_0)} + \alpha_2 (T_1 - T_0)^2]$

segment (2) $[T_2 - T_0]$ $\frac{R_{T_2}}{R_{T_0}} = R_{T_0} [1 + \alpha_1 \overset{+ve}{(T_2 - T_0)} + \alpha_2 (T_2 - T_0)^2]$

Add. above eq.n's :-

$$\frac{R_{T_1} + R_{T_2}}{R_{T_0}} = 2 + 2\alpha_2 (T_2 - T_0)^2$$

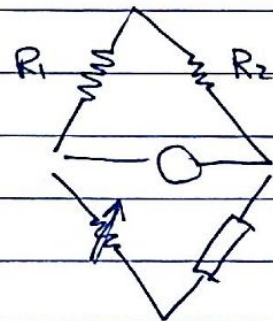
الجهاز يقيس كم R ويرجع
كل المعادلة و يعطينا T .

$$R_T = R_{T_0} [T - T_0]$$

α_1 يكون سالبة .

α_2 : ممكن يكون موجبة , سالبة

اذا كانت سالبة (يكون قيمها صغيرة) .



$$\frac{R_1}{R_2} = \frac{R_3}{R_T}$$

$$\text{if } R_1 = R_2 \rightarrow R_3 = R_T$$

Thermistor :-

* IF Range 50°C

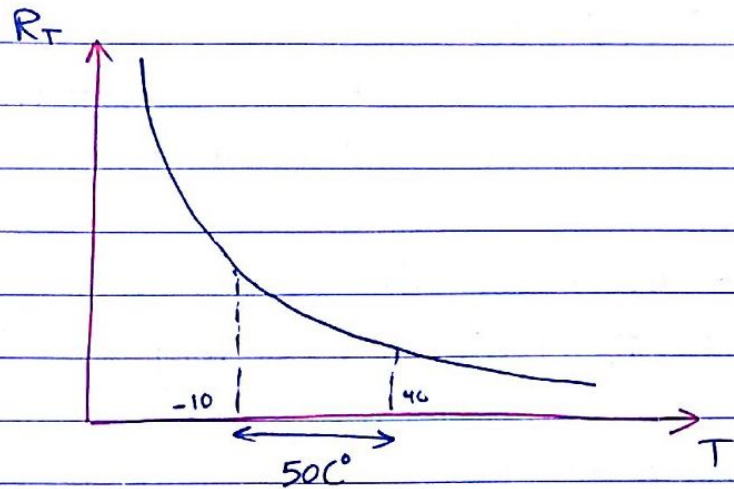
$$R_T = R_{T_0} [1 + \alpha (T - T_0)]$$

-ve % ex: $-5\% = -0.05$.

$T_0 \neq T_{avg}$, But T_0 can be:

✓ 1- Ice point.

✓ 2- Least in the range (-10)



* IF Range $> 50^{\circ}\text{C}$ [logarithmic]

$$R_T = \alpha R_{T_0} e^{\frac{B}{T}}$$

$\frac{B}{T}$ is in Kelvin.

$$R_{T_1} = R_{T_2} e^{B \left[\frac{1}{T_1} - \frac{1}{T_2} \right]}$$

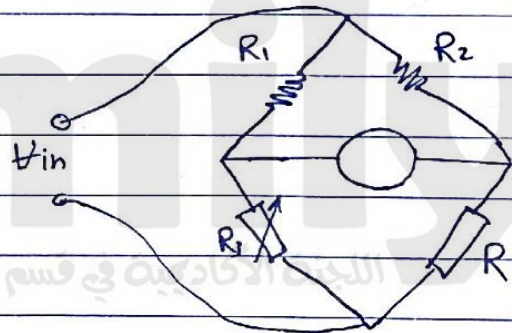
$\frac{B}{T}$ is in Kelvin.

اذا وجد (0) في Range فهو T_0
 حتى لو كان في قيم سالبة.
 لكن اذا لم توجد ال (0) بنقطة
 قاعد في range.

$$\frac{R_1}{R_2} = \frac{R_3}{R}$$

$$\text{IF } R_1 = R_2$$

$$R = R_3$$



Null current $\{I = 0\}$

Q1) $D = 15 \times 10^{-3} \text{ m}$, $E = 200 \times 10^9 \text{ N/m}^2$, $\nu = 0.28$, $t = ??$

if $\sigma_{\max} = 300 \times 10^6 \text{ N/m}^2$

$P_{\max} = 300 \text{ kPa}$

Extra: $\delta_m = ??$ if $P_{\text{act}} = 150 \text{ kPa}$

max shear stress $\sigma_{\max} = \frac{3D^2 P}{16t^2}$

$\therefore t = \sqrt{\frac{3D^2 P_{\text{act}}}{16\sigma_{\max}}} = \sqrt{\frac{(15 \times 10^{-3})^2 \times 3 \times 300 \times 10^3}{16 \times 300 \times 10^6}} = 2.05 \times 10^{-4} \text{ m}$

$t = 0.205 \text{ mm}$

max deflection $\delta_m = \frac{3(1-\nu^2) D^4 P_{\text{act}}}{256 E t^3} = \frac{3(1-0.28^2) (15 \times 10^{-3})^4 \times 150 \times 10^3}{256 (200 \times 10^9) \times (2.05 \times 10^{-4})^3}$
 $= 47.5 \times 10^{-6} \text{ m}$

since $\delta_m < \frac{1}{2}t$, this design is valid.

if $\delta_m = 2.1 \times 10^{-4} \rightarrow$ not valid $\rightarrow$ so, ① decrease diameter.

② increase thickness.

Q2) $R_p = 2 \times 10^3 \Omega$, $V_i = 10 \text{ volt}$,

$R_m = 5 \times 10^3 \Omega$, find: $E = ??$ @ $K = 0, 0.25, 0.5, 0.75, 1.0$

$|E| = +V_i \left[\frac{K^2 (K-1)}{K(1-K) + R_m/R_p} \right] =$ @ $K=0 \rightarrow E=0$

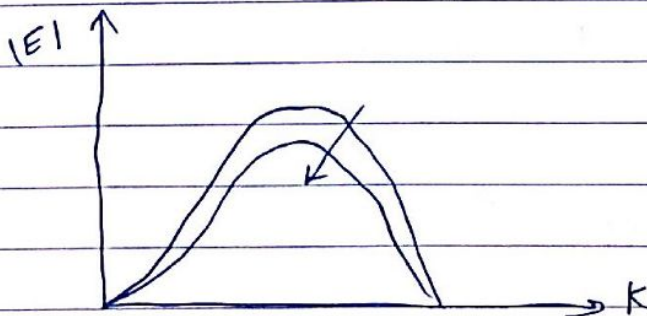
$= 0.25 \rightarrow 0.174$

$|E|_{K=0.25} = 10 \left[\frac{0.25^2 (1-0.25)}{0.25(1-0.25) + (\frac{5}{2})} \right] = 0.174$

$0.5 \rightarrow 0.454 \text{ Volt}$

$0.75 \rightarrow 0.524$

$1.0 \rightarrow 0$



Q10)

Linear Approx.

$$R_T = R_{T_0} [1 + \alpha (T - T_0)]$$

$$\alpha = \frac{1}{R_{T_0}} \left[\frac{R_{T_2} - R_{T_1}}{T_2 - T_1} \right]$$

$$= \frac{1}{589.5} \left[\frac{605.5 - 573.4}{130 - 100} \right] = 0.00182 \text{ } ^\circ\text{C}^{-1}$$

$$\therefore R_T = 589.5 [1 + 0.00182 (T - 115)]$$

Quadratic Approx. :-

$$R_T = R_{T_0} [1 + \alpha_1 (T - T_0) + \alpha_2 (T - T_0)^2]$$

$$[100 - 115] \quad R_{T_1} = R_{T_0} [1 + \alpha_1 \dots]$$

$$573.4 = 589.5 [1 + \alpha_1 (100 - 115) + \alpha_2 (\dots)^2] \dots (1)$$

[130 - 115]:

$$605.5 = 589.5 [1 + \alpha_1 (130 - 115) + \alpha_2 (130 - 115)^2] \dots (2)$$

$$\alpha_2 = -15.098 \times 10^{-6} \text{ } \Omega / \Omega \text{ } ^\circ\text{C}^{-2} = -3.7696 \times 10^{-7}$$

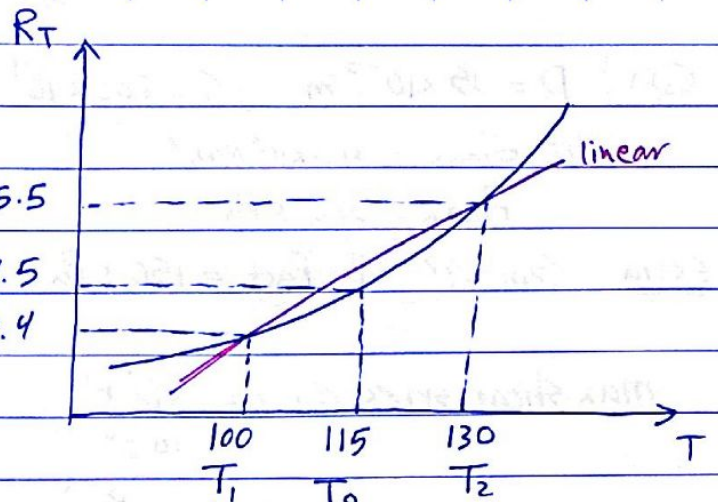
$$\alpha_1 = 0.00159 \text{ } \Omega / \Omega \text{ } ^\circ\text{C}^{-1} = 0.001815$$

Q11) $\alpha = 0.005392 \text{ } ^\circ\text{C}^{-1}$

$$R_{T_0} = 90 \Omega \text{ @ } T_0 = 25^\circ\text{C}, \quad R_T = 115 \Omega$$

$$R_T = R_{T_0} [1 + \alpha (T - T_0)] = 76.51^\circ\text{C}$$

$$115 \quad 90 \quad 0.005392$$

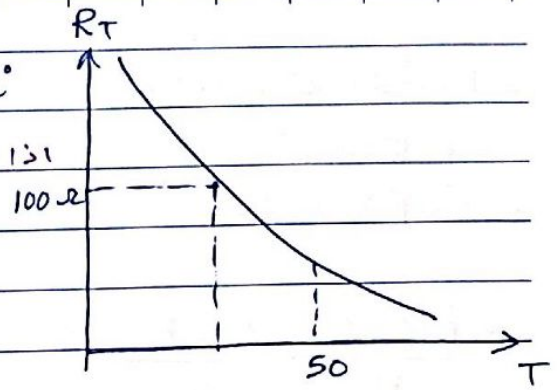


Q12) $\alpha = -0.05$, $R_{T_0} = 100 \Omega$ @ $T_0 = 25^\circ C$

إذا كان α استخدم linear لأنه الوحد في α

$$R_T = 35^\circ C = 100 [1 - 0.05(30 - 25)]$$

$$= 50 \Omega$$



Q13)

$R_{T_0} = 3980 \Omega$ @ $T_0 = 0^\circ C$

$$R_T = A R_{T_0} e^{B/T}$$

I) at $T = 0$

$$R_{T @ 0^\circ C} = R_{T_0} A e^{B/273}$$

$$1 = A e^{B/273} \quad \text{----- (1)}$$

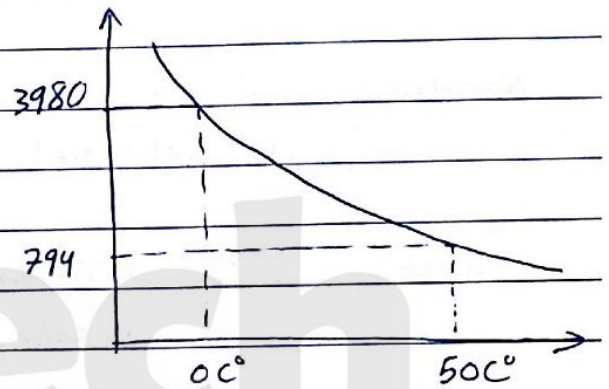
at $T = 50$

$$794 = 3980 A e^{B/323} \quad \text{----- (2)}$$

$$\frac{794}{3980} = \frac{e^{B/323}}{e^{B/273}} \Rightarrow \frac{794}{3980} = e^{B[\frac{1}{323} - \frac{1}{273}]}$$

$\rightarrow B = 2843$ (دائماً كبير بالألف)

$\rightarrow A = 30 \times 10^{-6}$ (دائماً صغيرة صغرة)



- Piezo electric

- LVDT

- Capacitor

* Piezo electric:-

P

t_0

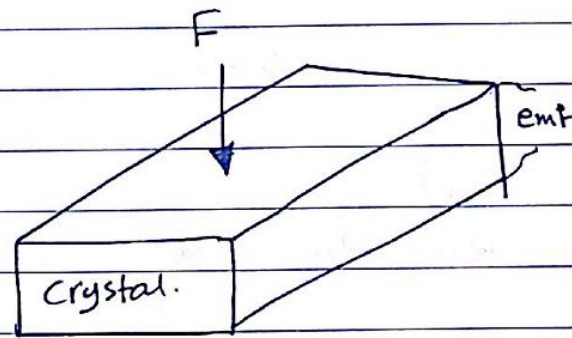
convert pressure to Voltage.

Natural

Quartz

Rochelle salts.

used for calibration.



Synthetic

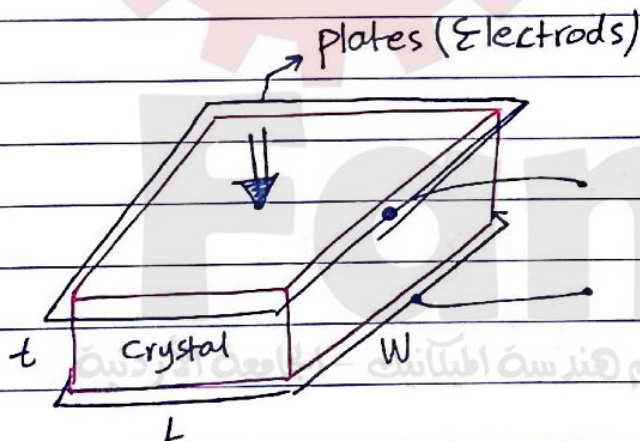
Barium Titanate (B.T)

Lithium sulphate (L.S)

for normal operation.

can Apply

1. compressive load
2. tensile load
3. longitudinal
4. Torsion.
5. shear force.



$$t_p = t_i - \Delta t$$

$$Q = F * K_c \quad \text{--- [1]}$$

charge (C) force (N) crystal sensitivity (C/N)

if Pressure

(bar) ~~(C/N)~~ (C/bar)

Hockeslow:-

$$F = \frac{E A_c \Delta t}{t} \quad \text{--- [2]}$$

$$t_0 = \frac{Q}{C} \quad \text{--- [3]}$$

capacitance (F)

$$C = \frac{\epsilon_0 \epsilon_r A_{\text{overlap}}}{t} \quad \text{--- [4]}$$

From eq [3] & [4] :-

$$V_0 = \frac{Q t}{E_0 \epsilon A} \quad \text{----- [5] } E_0 = 8.85 \times 10^{-12} \text{ F/m}$$

From eq [1] & [5]

$$V_0 = \frac{F}{K_c} t$$

$(E_0 \epsilon_r) A \rightarrow$ crystal voltage sensitivity $\cdot (K_{vc}) [\text{V}\cdot\text{m}/\text{N}]$

$$\boxed{V_0 = K_{vc} \frac{F}{A} t} \quad \# \quad K_{vc} = \frac{V_0}{P t} = \frac{V_0}{E \frac{\Delta t}{t}} = \frac{V_0}{E \epsilon t}$$

$\epsilon t \rightarrow$ strain.

Quartz sensitivity ϵ و E و ϵt $\therefore$ Quartz $\leftarrow$

Q3 $Q = P \cdot K_c$

$$Q = 1.4 \times 80 \times 10^{-12} = 112 \text{ pC.}$$

$$V_0 = \frac{Q}{C} = \frac{112 \times 10^{-12}}{1 \times 10^{-9}} = 112 \text{ mV}$$

Q4 $t = 2 \text{ mm}$, $K_{vc} = 0.055 \frac{\text{V}\cdot\text{m}}{\text{N}}$, $P = 1.5 \text{ MPa}$, Find V_0 , K_c
 $E_0 \epsilon_r = 40.6 \times 10^{-12} \text{ F/m}$

$$V_0 = K_{vc} P t$$

$$= 0.055 \times 1.5 \times 10^6 \times \frac{2}{1000} = 165 \text{ V}$$

$$K_c = \epsilon_r E_0 \cdot K_{vc} = 2.233 \text{ pC/N.}$$

Q5] B.T, $5 \times 5 \times 1.5$, $F = 5N$, $K_c = 150 \text{ Pc/N}$

$E_0 E_r = 12.5 \times 10^{-9} \text{ F/m}$, $E = 12 \text{ MPa}$

Find $\therefore Q, E, C$

1) $F = EAE$

$\therefore E = \frac{F}{EA} = 0.0167$; $A = 25 \times 10^{-6} \text{ m}^2$

2) $Q = F \times K_c = 5 \times 150 \text{ Pc/N} = 750 \text{ Pc}$

3) $t_0 = \frac{K_c}{E_r E_0} \cdot \frac{F}{A} \cdot t = 3.6 \text{ t}$

Q9] $K_{vc} = 0.055 \text{ V.m/N}$, $5 \times 5 \times 1.5$, $t_0 = 100 \text{ V}$

$t_0 = K_{vc} t \times \frac{F}{A}$

$\therefore F = 30.3 \text{ N}$

General Req. $\therefore$ 1. high Sensitivity

2. high purity

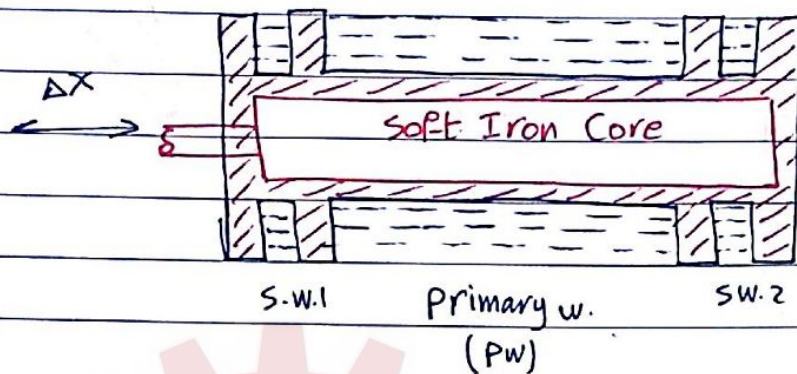
3. uniform $\rightarrow$ Geometry.
 $\rightarrow$ Composition.

4. it can be formed in multiple shapes.

5. Insensitive to change in T, humidity.

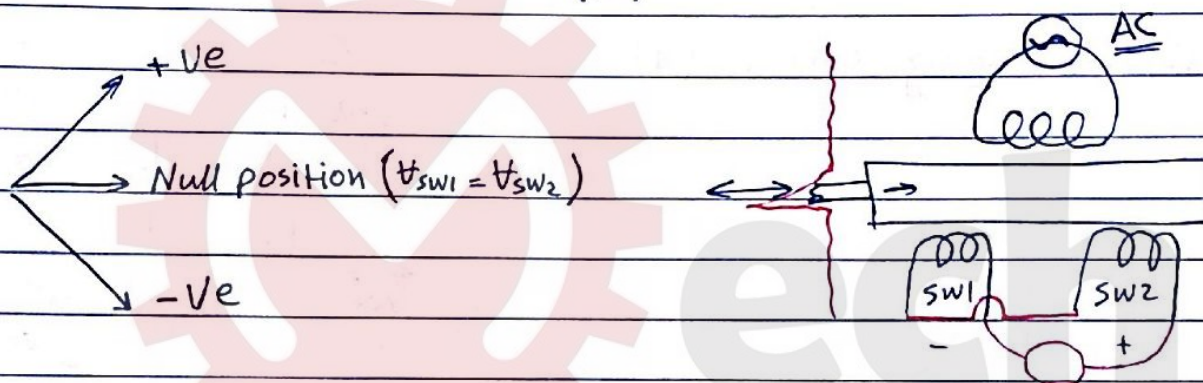
[Quartez fails at 45°C].

* LVDT (Linear Variable Diff. Transformer) :-



why soft??

إذا كان السطح حديدياً فسيكون
على تسمية نقطة المجال المغناطيسي
Error في 2' 13



ADV. :-

Power consum. $< 1W$.

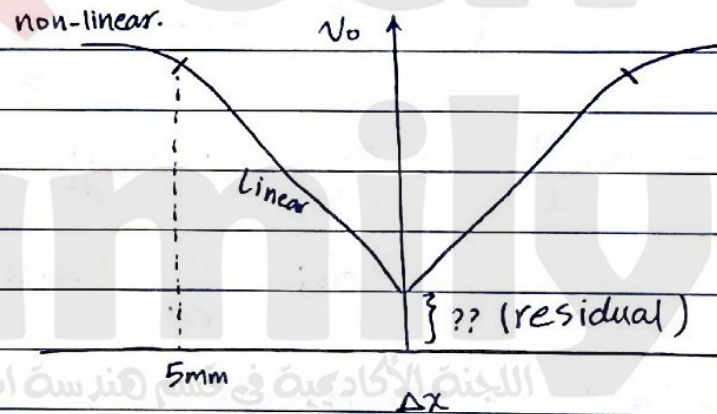
Friction less.

sensitivity $40V/mm$.

Dis. adv. :-

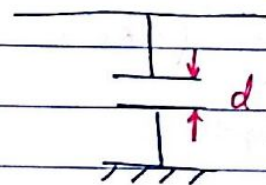
1. تأخر المجال المغناطيسي الكاربي.

2.

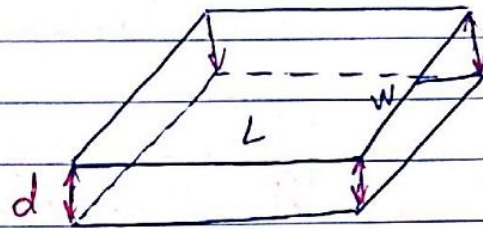
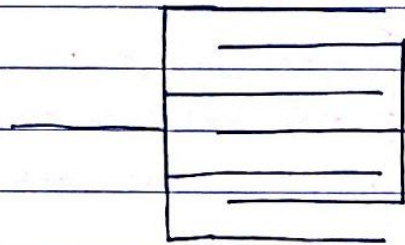


* Capacitor :-

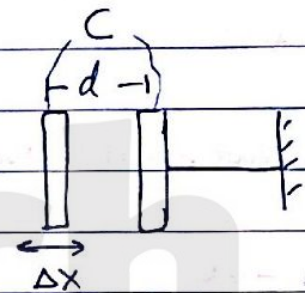
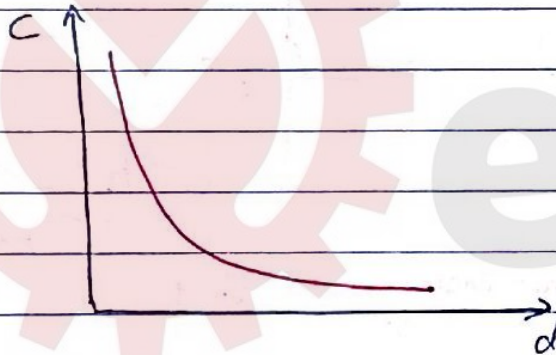
$$\left. \begin{matrix} P \\ F \\ T \\ \sigma \\ E \end{matrix} \right\} \rightarrow \Delta C = \frac{\epsilon_0 \epsilon_r A}{d}$$



$$\Delta C = \frac{\epsilon_0 \epsilon_r L W}{d} \times n \rightarrow \text{multiple}$$



Case (I) : changing distance (d)



$$C = \frac{\epsilon_0 \epsilon_r A}{d} \Rightarrow \frac{dC}{dd} = -\epsilon_0 \epsilon_r \frac{A}{d^2}$$

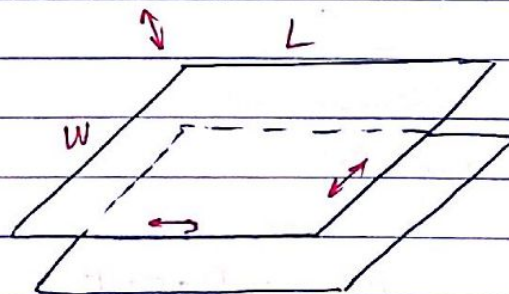
$$K_{s\downarrow} = -\frac{C}{d\uparrow} \quad \{ \text{sensitivity} \}$$

Case (II) : Changing Area :-

$$C = \epsilon_0 \epsilon_r \frac{L * W}{d}$$

$$\Theta = \left[\frac{\epsilon_0 \epsilon_r}{d} \right] (L * W)$$

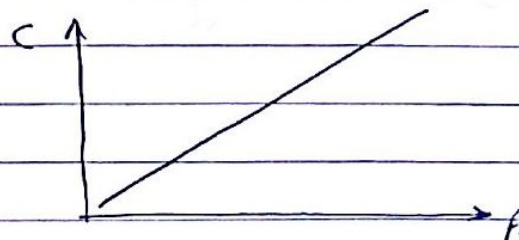
slope.



$$\frac{dC}{dL} = K_s W$$

$$\frac{dC}{dA} = K_s$$

$$\frac{dC}{dW} = K_s L$$

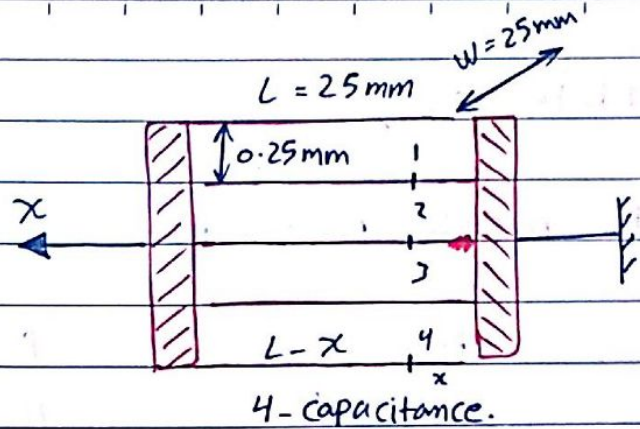


Q14 :-

$$\epsilon_0 \epsilon_r = 8.55 \times 10^{-12} \text{ F/m}$$

Before $\Rightarrow C = \frac{n \epsilon_0 \epsilon_r L W}{d}$

After $\Rightarrow C = \frac{n \epsilon_0 \epsilon_r (L-x) W}{d}$



$$\frac{dC}{dx} = \frac{n \epsilon_0 \epsilon_r W}{d} = 3.54 \text{ PF/mm}$$

Q15 :-

$$A_{\text{int}} = 500 \text{ mm}^2$$

$$d_i = 0.2 \text{ mm}, d_f = 0.18 \text{ mm}$$

$$\epsilon_0 \epsilon_r = 8.85 \times 10^{-12} \text{ F/m}$$

$$C_i = \frac{8.85 \times 10^{-12} \times 500 \times 10^{-6}}{0.2 \times 10^{-3}} = 22.125 \text{ PF}$$

$$C_f = \frac{8.85 \times 10^{-12} \times 500 \times 10^{-6}}{0.2 \times 10^{-3}} = 24.582 \text{ PF}$$

$$\Delta C = C_f - C_i = 2.458$$

$$\text{Cap. factor} = \frac{\Delta C/C}{\Delta d/d} \text{ OR } \frac{\Delta C/C}{\Delta W/W} \text{ OR } \frac{\Delta C/C}{\Delta L/L} \text{ OR } \frac{\Delta C/C}{\Delta LW/LW}$$

For this question :-

$$\text{Cap. factor} = \frac{\Delta C/C}{\Delta d/d} = 1.111$$

For $d = 1.9$ instead of 1.8

$$\text{Cap. factor} = 1.049$$

اختار الجواب الذي هو cap. fact. اعلى

* Strain Gauge :-

Gauge Factor $\uparrow$, دقة

accuracy $\uparrow$

wire, metal, $\Delta L \rightsquigarrow \Delta R$

sensitivity $\uparrow$

$$R = \frac{\rho L}{A}$$

ρ : Resistivity ($\Omega \cdot m$)

A : Area (m^2)

L : Length. (m)

Piezo Resistivity

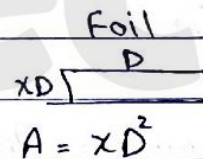
$$\uparrow R = \uparrow \rho L \uparrow$$

$$A \downarrow$$



$$\ln R = \ln \rho + \ln L - \ln A$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$$



$$A = \pi D^2$$

$$x = \frac{\pi}{4}$$

$$x < 1$$

$\rightsquigarrow$ so, $x = \text{const.}$

$$\left(\frac{dR}{R} \right) / \left(\frac{dL}{L} \right) = \frac{d\rho/\rho}{dL/L} + 1 + 2 \frac{dD/D}{dL/L}$$

$$\frac{dA}{A} = 2 \frac{dD}{D}$$

$\rightarrow$ Gauge Factor (G_F).

$$G_F = \frac{\Delta \rho / \rho}{\Delta L / L} + 1 + 2 \nu$$

$\rightarrow$ Piezo Resistivity (مقاومة تغير مع تغير)

$$G_F = 1 + 2 \nu$$

; max value: $\Delta R = 2 \epsilon R$

$\rightarrow$ max. value = 2.

المواصفات المطلوبة :-

4. Linear Relation between ϵ & R

1. higher gauge factor ~~value~~

5. Resistance temp. coeff. must be very low.

2. higher initial resistance

3. Hysteresis low as possible.

1) unbonded $\rightarrow$ wire.

2) Bonded $\rightarrow$ wire

3) $\rightarrow$ Foil



4) x

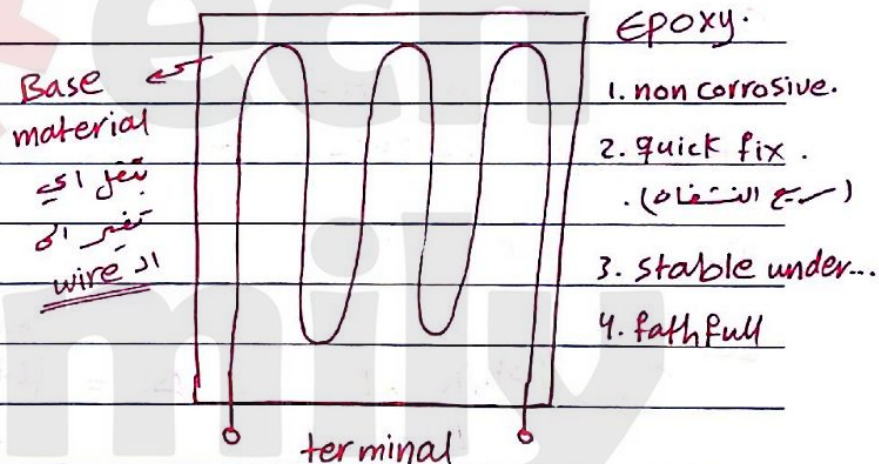
5) x

6) Bonded semiconductor $\{ L \uparrow \rightarrow R \downarrow \}$

7)

* Strain Rosette :-

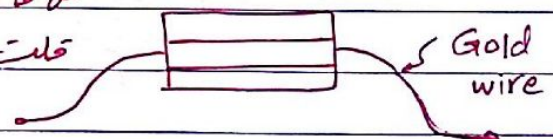
multiple strain ~~gauge~~ to capture all the mode of the deflection.



$\rightarrow$ Germanium $\Rightarrow G_p = (-)133 \cdot = 2ER$.

كل ما زدت الطول

قلت R



To calculate :-

$\Rightarrow$ Force, Flow, pressure, Torque, Strain.

Q1 :-

$$G_p = 2, \sigma_T = 100 \text{ MN/m}^2, E = 200 \text{ GN/m}^2$$

* اذا لم يحدد نوع

ال stress اعتبره

Tensile

$$E = \frac{\sigma}{\epsilon}$$

$$G_p = \frac{\Delta R/R}{\epsilon}$$

$$\therefore \frac{\Delta R}{R} = G_p \times \frac{\sigma}{E} \times 100\% = 0.1\%$$

- change in R is very small to be detected, so we need to amplify
- OR wire with high initial R.

Q2 :-

$$\text{Beam: } L = 0.1 \text{ m}, A = 3 \text{ m}^2, E = 207 \text{ GN/m}^2,$$

$$\text{Strain Gauge: } R = 240 \Omega, G_p = 2.2, \Delta R = 0.013 \Omega$$

Find: $\Delta L, F$

Sol.

$$G_p = \frac{(\Delta R/R) \times L}{\Delta L} \Rightarrow \Delta L = \frac{(\Delta R/R) \times L}{G_p} = 2.46 \times 10^{-6} \text{ m}$$

$$F = \sigma A = F \left(\frac{\Delta L}{L} \right) A = 2.037 \times 10^3 \text{ N}$$

Q3 :-

$$G_p = 1 + 2\nu \Rightarrow \nu = 1.6$$

Q4 :-

$$G_F = \frac{\Delta R}{R_i \in}$$

$$\Delta R = G_p \in R_i$$

$$R_f = G_f \in R_i + R_i$$

$$= (-12.1 \times 5) \times 10^{-6} + 1 \times 120$$

$$= 120.0072 \Omega$$

$$\begin{aligned} R_F &= G_F \in R_i + R_i \\ &= (2 * (-5 \times 10^{-6}) + 1) * 120 \\ &= 119. \dots \Omega \end{aligned}$$

1) $G_F = -ve \rightarrow$ Design for compressive load.

in compressive load $\rightarrow R_f \uparrow$

• Tensile load $\rightarrow R_f \downarrow$

Gauge factor :- یہ وزن کما ہو یا بڑھاتے

compressive $\rightarrow \therefore \frac{\Delta L}{L} = \frac{L_f - L_i}{L}$, $L_f < L_i$

$$\therefore \frac{\Delta L}{L} = -Ve$$

2] $G_f = +ve \rightarrow$ Design for Tensile load

$\therefore$ comp. load $\rightarrow R_f \downarrow$

$\therefore$ Tensile load $\rightarrow R_f \uparrow$

* Flow & Velocity Measurement :-

velocity $\rightarrow$ Pitot tube.

→ Hot wire Anemometer.

Flow

Orifice

Venturi

Flow Nozzle

Rotometer.

- liquids / Gases

- laminar / turbulent

- Continuity, Conservation of mass

$$\rho = \rho(P)$$

ρ : if change compressible (Gas)

ρ : if not change non-comp.

Continuity: إذا عرضنا الفلويـد Force فكل جزيـة، في كل الـ Force الـ في

وهذا واضح وموجود non comp. fluid

- For comp. : $\rho_1 A_1 U_1 = \rho_2 A_2 U_2$ mass flow rate

For incomp. : $A_1 U_1 = A_2 U_2$ Volume flow rate

→ @ measurement.

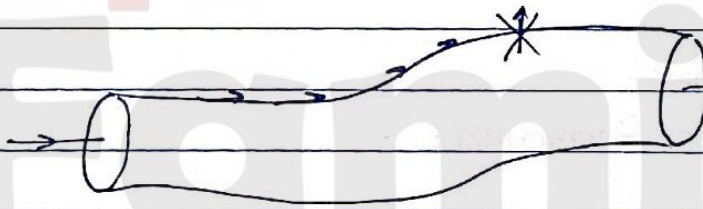
$$Re_d = \frac{\rho U_{act} d}{\mu}$$

U : actual velocity

- $Re \leq 2000$ Laminar flow . Re اكل: حركته اكل من مقاومته للحركة .

$Re > 2000$ Turbulent flow .

- Continuity, Conservation of mass :-



- Stream Tube

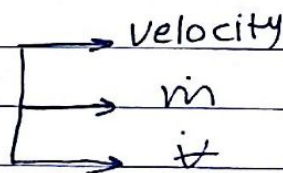
- No leakage

- Stream line no v

is normal always

tangent.

Flow Techniques :-



Methods :-

1. Const. Head Type (Flow Tech) $\rho(\Delta P)$ ✓

الـ variable الـ في

Flow rate في

2. Turbine (") $\rho(\omega)$

→ ang. velocity

3. Thermal EM (") $\rho(\Delta T)$ ✓

- mag. ~~flux~~ flux.
4. Magnetic flow Measurement - $f(B)$
 5. Variable head - $f(Y)$ ✓
 6. positive Disp. pump - (ψ)

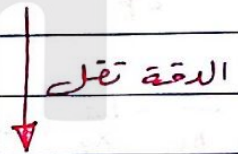
* Differential Pressure (Meters)

- Obstruction Flow Tech. (القياس بالعتبة)

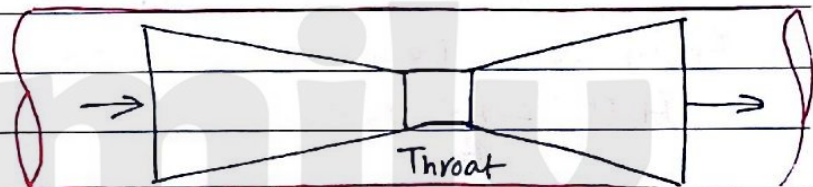
work on: Bernoulli Theory

$$Q (m^3/s) \propto (P_1 - P_2)^n \begin{cases} 1/2 & (\text{turb. flow}) \\ 1 & (\text{laminar flow}) \end{cases}$$

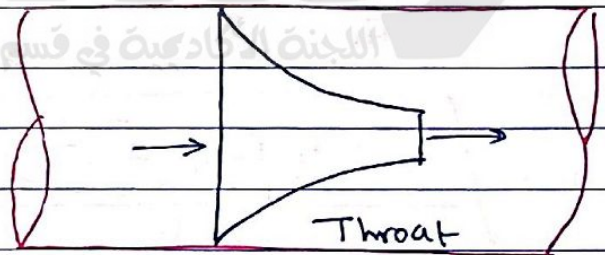
Obstruction Flow Tech.
 ↳ Venturi meter
 ↳ Flow Nozzle
 ↳ Orifice plate



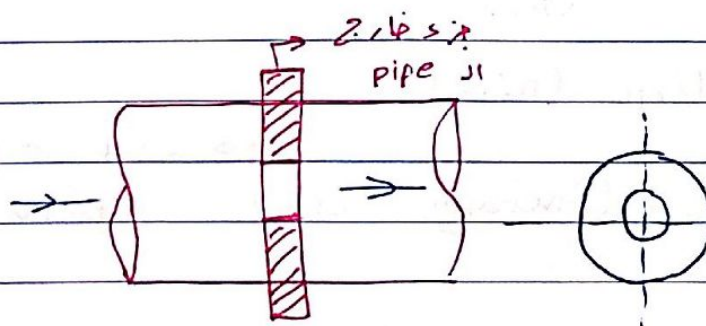
1. Venturi

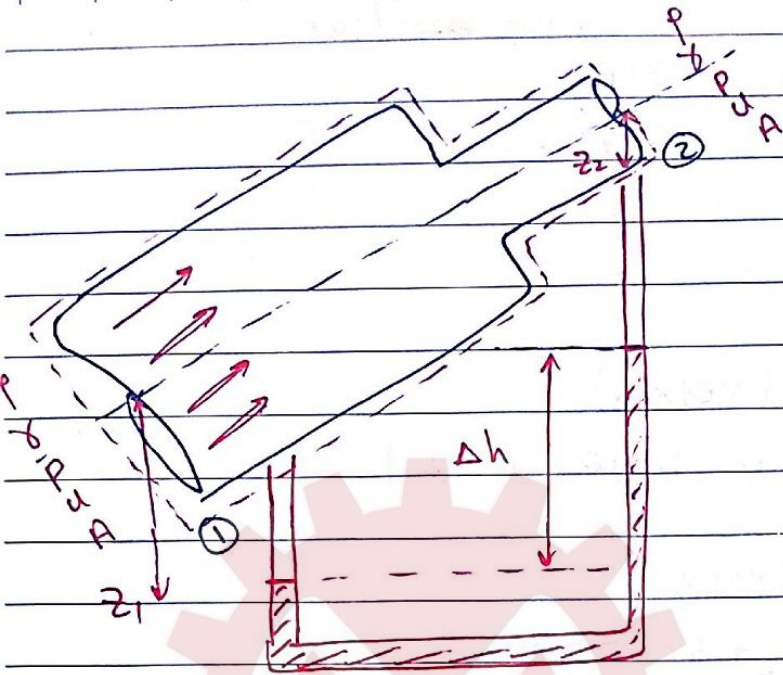


2. Orifice



3. orifice.





$$\frac{P_1}{\gamma_1} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\gamma_2} + \frac{u_2^2}{2g} + z_2$$

$$\left(\frac{P_1}{\gamma_1} - \frac{P_2}{\gamma_2} \right) + (z_1 - z_2) = \frac{u_2^2 - u_1^2}{2g} \quad \text{----- (1)}$$

From cont. Eq.:-

$$A_1 u_1 = A_2 u_2$$

$$\therefore u_1 = \frac{A_2 u_2}{A_1}$$

إذا كان مخرج من مضخة :-

should be negative.

$$u_{z,th} = \sqrt{\frac{2g \left[\frac{P_1 - P_2}{\gamma_f} + (z_1 - z_2) \right]}{1 - \left(\frac{A_2}{A_1} \right)^2}} \quad \text{Flow} \rightarrow$$

$$Q_{th} = A_2 u_{z,th} \quad (m^3/s)$$

الظاهرة هي تضايق C_D :-

$$Q_{act} = C_D \text{ (Discharge coeff.)}$$

Vena Contracta.

$$Q_{th}$$

"Reducing Area"

$$\begin{aligned}\dot{Q}_{act} &= C_D \dot{Q}_{th} \\ &= C_D A_2 U_{2th} \\ &= A_2 U_{act} \quad [U_{act} = C_D U_{2th}]\end{aligned}$$

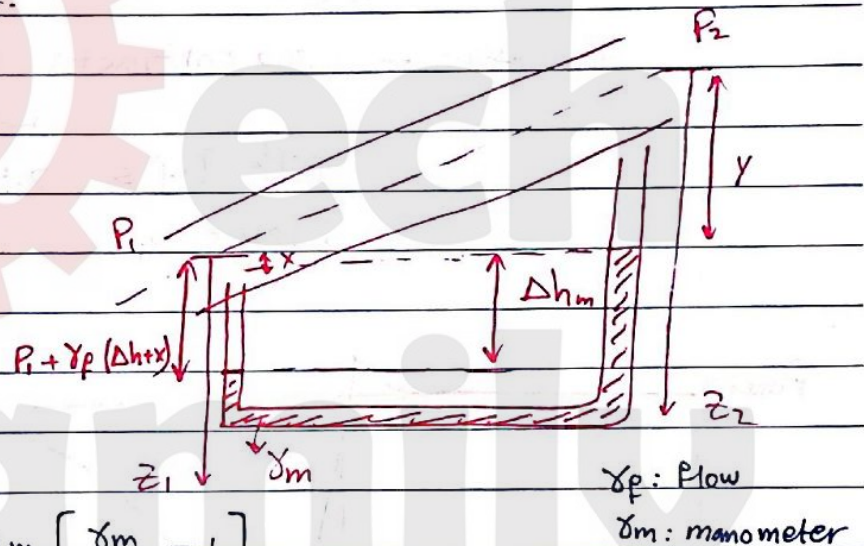
$$\dot{Q}_{act} = Y * C_D * A_2 * \sqrt{\dots}$$

↳ compressibility factor.

$Y = 1$ Incomp.

$Y < 1$ comp.

$$\dot{m}_{act} = \rho_2 \dot{Q}_{act}$$



$$\frac{P_1 - P_2}{\gamma_f} + (z_1 - z_2) = \Delta h_m \left[\frac{\gamma_m}{\gamma_f} - 1 \right]$$

zero for Horizontal.

for liquid (water) $\left[\frac{\gamma_m}{\gamma_f} - 1 \right] \rightarrow 1$

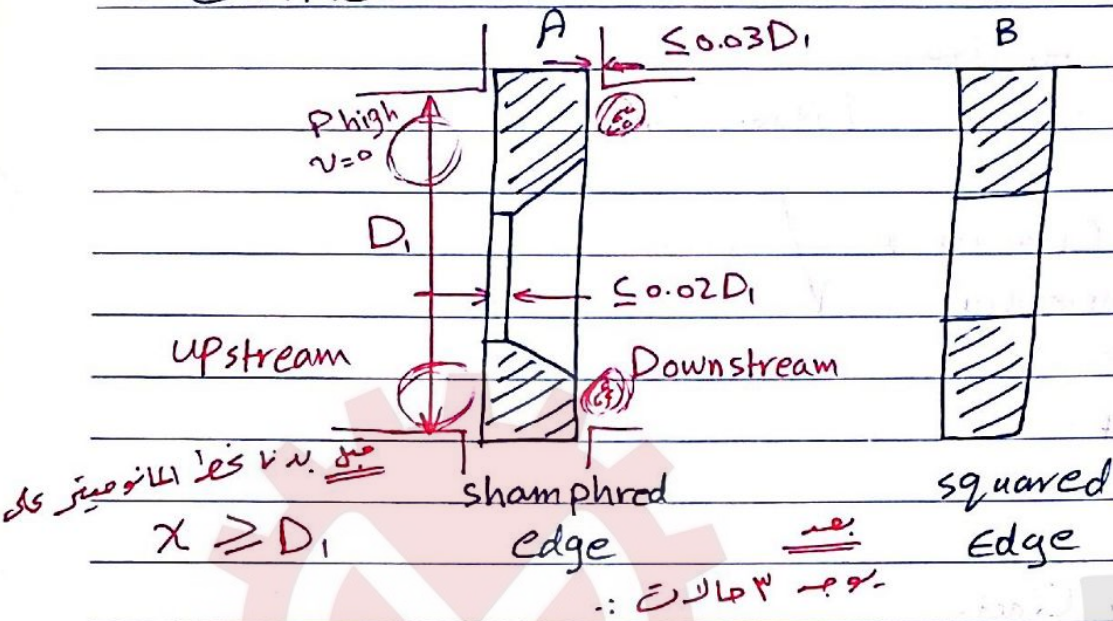
$$\dot{Q}_{act} = \frac{C_D Y A_2}{\sqrt{1 - \left(\frac{A_2}{A_1} \right)^2}} \sqrt{2g \Delta h_m \left(\frac{\gamma_m}{\gamma_f} - 1 \right)}$$

$$\frac{P_1 - P_2}{\gamma_f} + (z_1 - z_2) = \Delta h_m$$

$$\left(\frac{D_2}{D_1} \right)^4 = \beta^4 \quad \text{OR} \quad 2g \left[\frac{\Delta P_{1-2}}{\gamma_f} + (z_1 - z_2) \right]$$

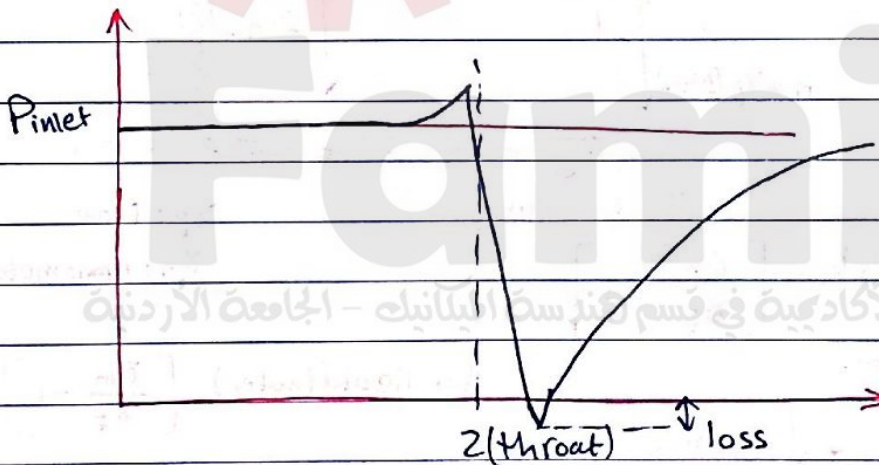
$$\dot{m}_{act} = \rho_2 \dot{Q}_{act}$$

* Orifice :



Vena Contracta Taps $\approx \frac{1}{2} D_1$

	before	After
Pipe Taps	$2.5 D_1$	$8 D_1$
Flange Taps	$1 D_1$	$1 D_1$

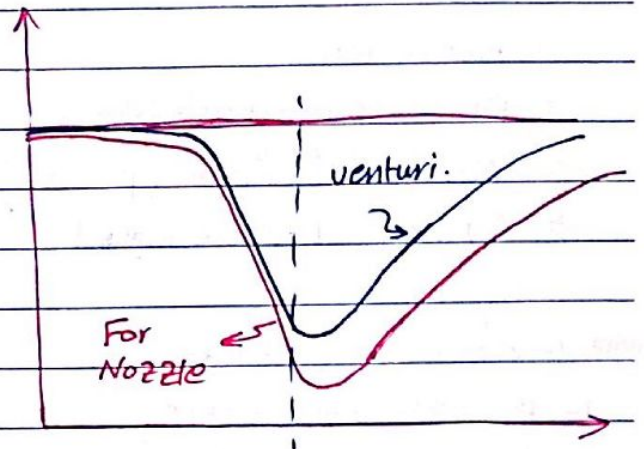
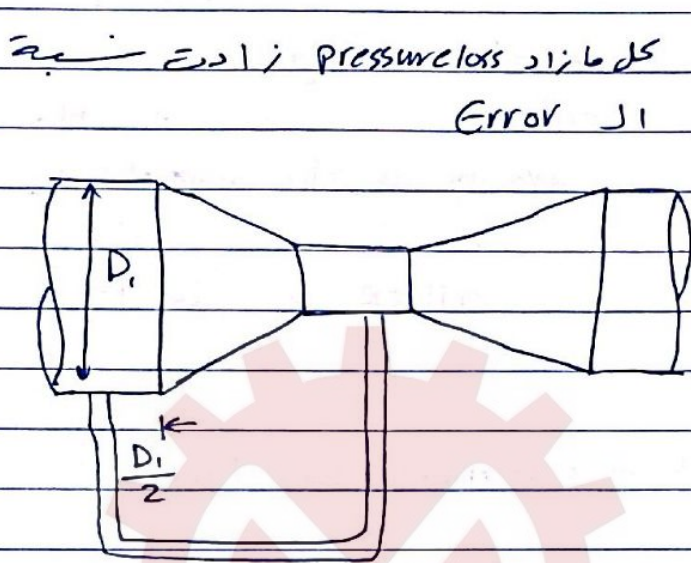


∴ Orifice

(ΔP_{21}) pressure drop is a pressure loss

- pressure losses. (65%)
- cavetation
- مانع الزئبق ينتقل من mana.
- Flow

Venturimeter :-



C_D For Venturi : 0.98 - 0.99

C_D For Nozzle : 0.95 - 0.97.

* compensating pump :-

* اذا مضى orifice راح يغير P_{loss} على

$$W_{sh} = \Delta P$$

اذن ان P_{loss} راح يغير راح يغير W_{sh} ، كمان

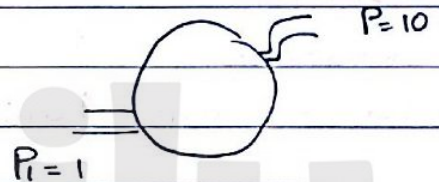
على cost عشان هيرى ما يستعمل

orifice.

* او بنزل (compensating pump) راح يغير

ان P_{loss} فيكون $(P_m = 1.5)$ فلا يغير loss

بغير $P = 1$



Meter selection :-

1. cost
2. Accuracy
3. Placement available.
4. Nature of flow (phase)
5. ΔP_{loss} . (orifice higher)

بناثر اکثر شیبہ عاکس $\rightarrow$ impiority

venturi.

← اکثر واحد محتاج مسافہ ← venturi

← اقل ← ← ← ← orifice

← velocity بناثر عاکس Nature of flow ←

← اقل تقنا ← venturi

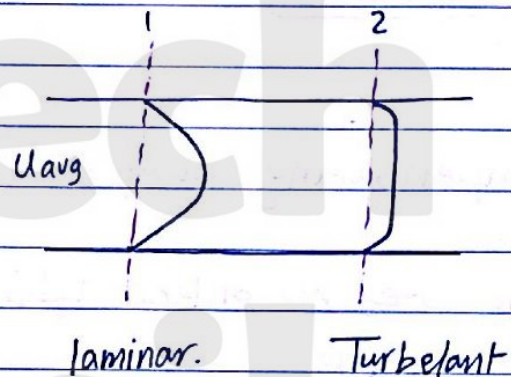
← اقل تقنا ← orifice

العوامل التي بناثر عاکس accuracy :-

- 1- β (diameter ratio)
- 2- ρ (density of flow) $\rightarrow$ (Change by Ambient condition "T")
- 3- C_d (discharge coeff.)

COST :-

	venturi	orific
Initial cost.	↑	↓
calibration	↑	↓
maintinamce	↑	↓
instalation cost	↑ critical	↓



$$\Delta P_{loss} = \% \Delta P_{12}$$

$$W_p (\text{compensating pump}) = \rho g h_L \frac{Q}{\eta_{PM} g_c}$$

للغرفة التي يديها
الركبها .

$$\boxed{W_p = \frac{Q_{act.} * \Delta P_{loss}}{\eta_{PM}}} \rightarrow (Pa)$$

$\frac{N-m}{s}$

$$\text{time} * \text{tariff rate} * \text{Power} = \text{cost}$$

$$\frac{hr}{Yr} \quad \frac{JD}{Kw \cdot hr} \quad \frac{KW}{\equiv} \quad \frac{JD}{Yr}$$

Example 1 :-

(f) water : $\gamma_f = 9810 \text{ N/m}^3$, $\rho_f = 1000 \text{ kg/m}^3$, $V = 1$ compressibil.

Pipe : $D_1 = 400 \text{ mm}$, horizontal $\Rightarrow z_1 = z_2$ (venturimeter)

venturi :- $h_m = 250 \text{ mm}$, $C_D = 0.98$, $SG_m = 13.6$

Sol

$$Q_{act} = 0.98 * \frac{\pi}{4} \left(\frac{150}{1000} \right)^2 * 1000 * 1 * \sqrt{\frac{2 * 9.81 * 250}{1000} \left[\frac{13.6}{1} - 1 \right]}$$

$$= 0.1375$$

Example 2

$\gamma_f =$, $\rho_f =$

$C_D = 0.97$, $P_1 - P_2 = 30.6 \text{ kPa}$

$\Delta z_{2-1} = 112 \text{ mm}$

$$v = \frac{M}{\rho} = 1 \times 10^{-6} \frac{\text{m}^2}{\text{s}^2}$$

Sol

$$U_{2,th} = \sqrt{\frac{2g \left[\frac{\Delta P_{1-2}}{\gamma_f} + (z_1 - z_2) \right]}{1 - \beta^4}}$$

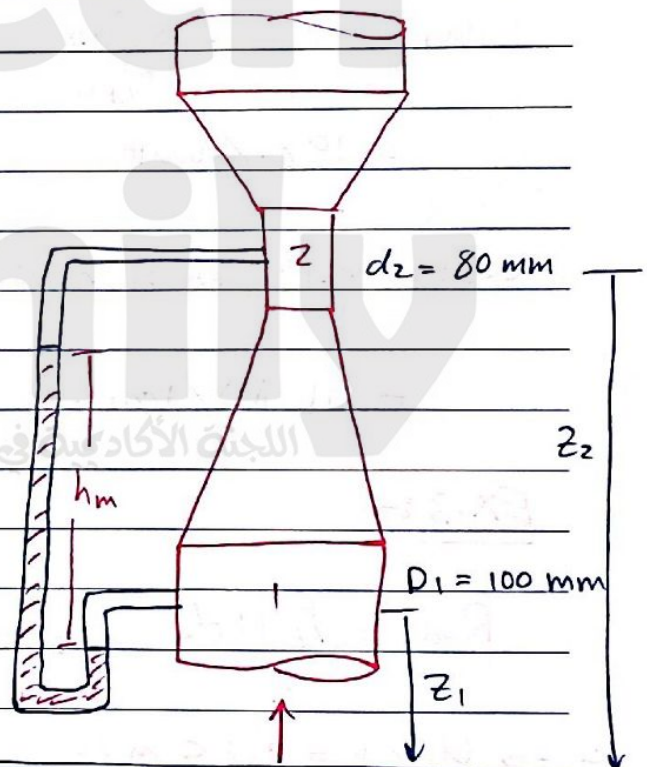
$$\frac{\Delta P_{1-2}}{\gamma} + (z_1 - z_2) = h_m \left[\frac{SG_m}{SG_f} - 1 \right]$$

$$\frac{30.6 \times 10^3}{9810} + \frac{-112}{1000} = h_m \left[\frac{13.6}{1} - 1 \right]$$

$$h_m = 0.238 \text{ m}$$

$$M_f = 1.08 \times 10^{-3}$$

$$U_{2,th} = \sqrt{2 * 9.81 \left[\frac{30.6 \times 10^3}{9810} - \frac{112}{1000} \right]} = 9.996 \text{ m/s}$$



$$U_{2,act} = C_D U_{2,th} = 9.69$$

$$2) Red_2 = \frac{\rho_f U_{2,act} dz}{\mu} = \frac{U_{2,act} dz}{\nu} = 775,753$$

extra 3) calculate the cost of using compensating pump having $\eta_{PM} = 75\%$, to overcome losses of 10%, running for 7000 hr/year with tariff rate 0.08 JD/kW-hr

$$\Delta P_{loss} = \frac{10}{100} * 30.6 = 3.06 \text{ kPa}$$

$$\begin{aligned} Q_{act} &= C_D A_2 U_{2,th} \\ &= 0.97 * \frac{\pi}{4} \left(\frac{80}{1000}\right)^2 * 9.996 = 0.048 \frac{m^3}{s} \end{aligned}$$

$$\begin{aligned} W_p &= \frac{Q_{act} * \Delta P_{loss}}{\eta_{PM}} = \frac{0.048 * 3.06 * 10^3}{0.75} \\ &= 198.8 \text{ watt} \end{aligned}$$

$$\begin{aligned} \text{Cost} &= 0.198 \text{ kW} * 7000 \frac{\text{hrs}}{\text{Year}} * 0.08 \frac{\text{JD}}{\text{kW} \cdot \text{hr}} \\ &= 111 \text{ JD/Year} \end{aligned}$$

EX. 3:-

$$Red_2 = \frac{\rho_f U dz}{\mu} = \frac{U_{act} dz}{\nu} \Rightarrow 100,000 = \frac{U_{act} * 30 * 10^{-3}}{1 * 10^{-6}}$$

$$\Rightarrow U_{act} = 3.35 \text{ m/s}$$

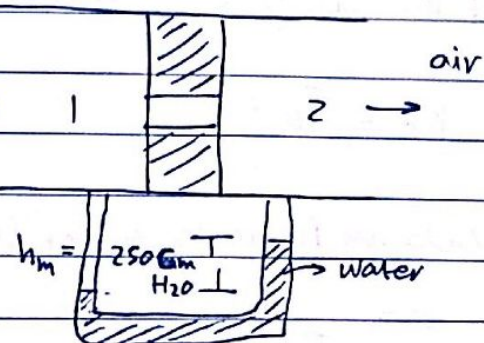
$$U_{2,act} = C_D \sqrt{\frac{2 * 9.81 [\Delta P / \gamma_f]}{1 - \beta^4}} \Rightarrow \Delta P = 130 \text{ kPa}$$

$\beta = \left(\frac{0.3}{0.5}\right)^4$

EX. 4:-

20°C
 $P_1 = 93.7 \text{ kPa}$
 $\gamma = 0.92$
 $\beta = 0.4$

$C_D = 0.625$



Sol:-

$$Q = \frac{C_D \gamma A_2}{\sqrt{1 - \beta^4}} \sqrt{2g \left(\frac{\Delta P_{1-2}}{\gamma_f} \right)}$$

هذا هو ما

$$Q = \dots \sqrt{2gh_m \left(\frac{SG_m}{SG_f} - 1 \right)}$$

$$\Delta P_{1-2} (\text{air}) = \rho_m g h_m$$

very high

$$= \frac{9810 \times 250}{100} = 24.525$$

لذلك لا نستطيع أن نحس interaction بين الماء والهواء

في الحالة، إذا كان

$$Q = \frac{C_D \gamma A_2}{\sqrt{1 - \beta^4}} \sqrt{2g \left(\frac{\Delta P_{1-2}}{\gamma_f} \right)} = 0.0588$$

في 2 Fluid كادى هو interaction

$$\rightarrow \frac{P}{RT} \times 9.81$$

استخدم المعادلة التالية

if we should find m

$$Q = \dots \sqrt{2gh_m \left(\frac{\rho_m}{\rho_f} - 1 \right)}$$

$$[1] \dot{m} = \frac{C_D \gamma A_2 \rho_{a1}}{\sqrt{1 - \beta^4}} \sqrt{2g \left(\frac{\Delta P_{1-2}}{\gamma_f} \right)}$$

OR

$$[2] \dot{m} = \frac{C_D A_2 \rho_{a2}}{\sqrt{1 - \beta^4}} \sqrt{2g \left(\frac{\Delta P_{1-2}}{\gamma_f} \right)} \quad \& \rho_{a2} = \rho_{a1} \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}}$$

Ex:-

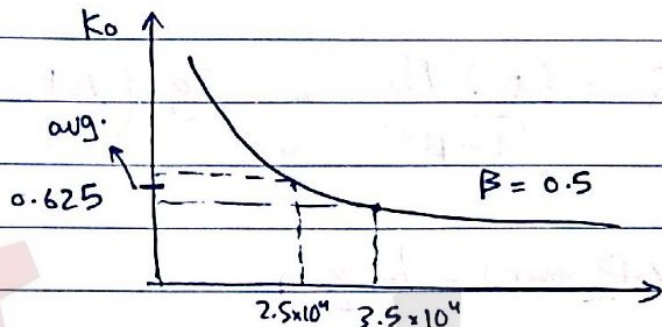
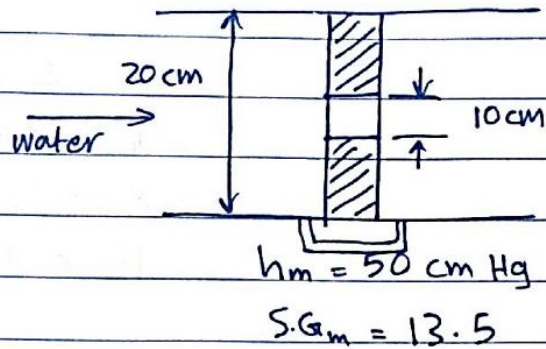
Flow coefficient (K_o)

$$K_o = C_D E$$

Velocity Approach factor (E)

$$E = \frac{1}{\sqrt{1 - \beta^4}}$$

$$\therefore K_o = \frac{C_D}{\sqrt{1 - \beta^4}}$$



$$Re_d = \frac{\rho_1 u_1 d_1}{\mu}$$

$$Q = A_1 u_1$$

$$Re_d = \frac{4Q}{\pi d_1 v}$$

$$\dot{Q} = \frac{C_D A_2 Y}{\frac{\sqrt{1 - \beta^4}}{K_o}} \sqrt{2gh_m [\dots - 1]} = 0.055 \text{ m}^3/\text{s}$$

Re_d from calculation

$$\text{if } Re_d > 3.5 \times 10^4 \quad \checkmark$$

$$Re_d = 3.5 \times 10^4 \quad \checkmark$$

if $Re_d < 3.5 \times 10^4$ (Take $\overline{Re_d}$ and recalculate).

* Rotometer :-

- variable head.

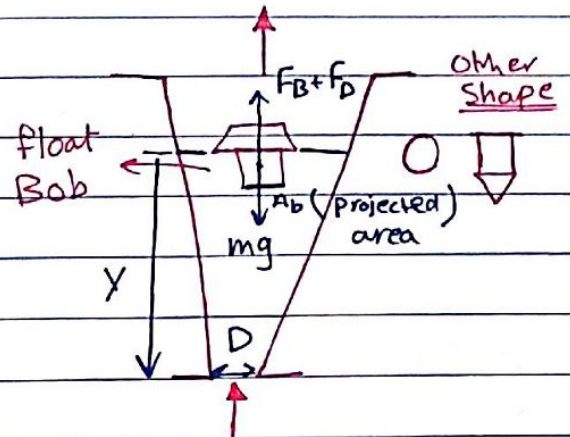
@ Equilibrium:-

$$F_w = F_B + F_D$$

$$F_D = C_D A_b \rho_f U_m^2$$

$$F_B = \rho_b \theta_b g$$

$$F_w = \rho_b \theta_b g$$



F_B : Bouency Force

F_D : Drag Force

$$U_m = \sqrt{\frac{2(\rho_b - \rho_f) \theta_b g}{C_D A_b \rho_f}}$$

1] → losses due to Bob Rotation.

2] → losses due to Derts increase velocity due to decrease Area
∴ Drag Force increase.

$$U_{man} = \sqrt{\frac{2g}{C_D} \left(\frac{\rho_b}{\rho_f} - 1 \right) \left(\frac{\theta_b}{A_b} \right)}$$

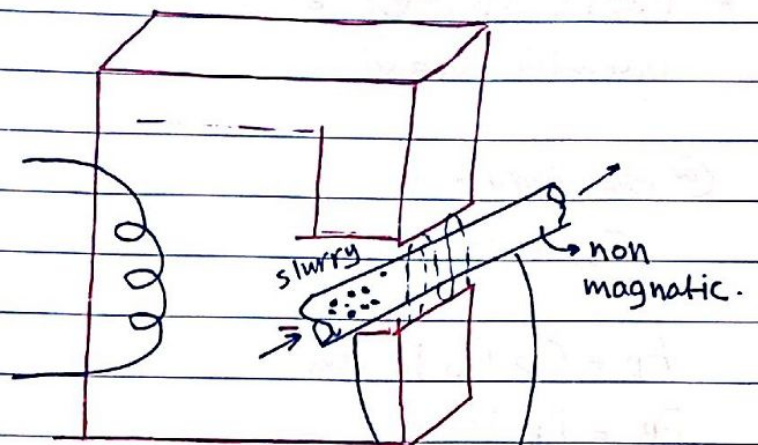
$$Q = A * U_m \quad ; \quad A = \frac{\pi}{4} \left[(D + \overset{\text{slope}}{\alpha} Y)^2 - d_b^2 \right]$$

$$\dot{m}_f = C Y \sqrt{\rho_f (\rho_b - \rho_f)}$$

$$\rho_b = 2 \rho_f \therefore$$

To prevent Bob to go up and $m=0$ (failure)

$$\dot{m}_f = \frac{C_1 Y \rho_b}{2}$$



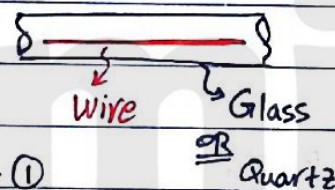
$$V = B \times L \times U_p$$

* Hot Wire Anemometer (thermal Measurement) :- "like RTD"

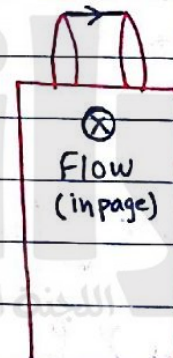
→ To measure velocity.

cooling effect $\propto U_p$

protected Mode.



R_w, I_w $d = 4-10 \mu m$
 $L \approx 1 mm$



$$I^2 R_w = h A_w (T_w - T_f) \quad - (1)$$

$$R_w = R_{ref.} [1 + \alpha (T_w - T_{ref.})] \quad - (2)$$

Kings law for convection H.T :-

$$h = a + b U_p^c \quad - (3)$$

sub. (3) & (2) in (1)

$$U_p = \left\{ \left(\frac{I^2 R_{ref.} (1 + \alpha (T_w - T_{ref.})) - a}{A_w (T_w - T_f)} \right) / b \right\}$$

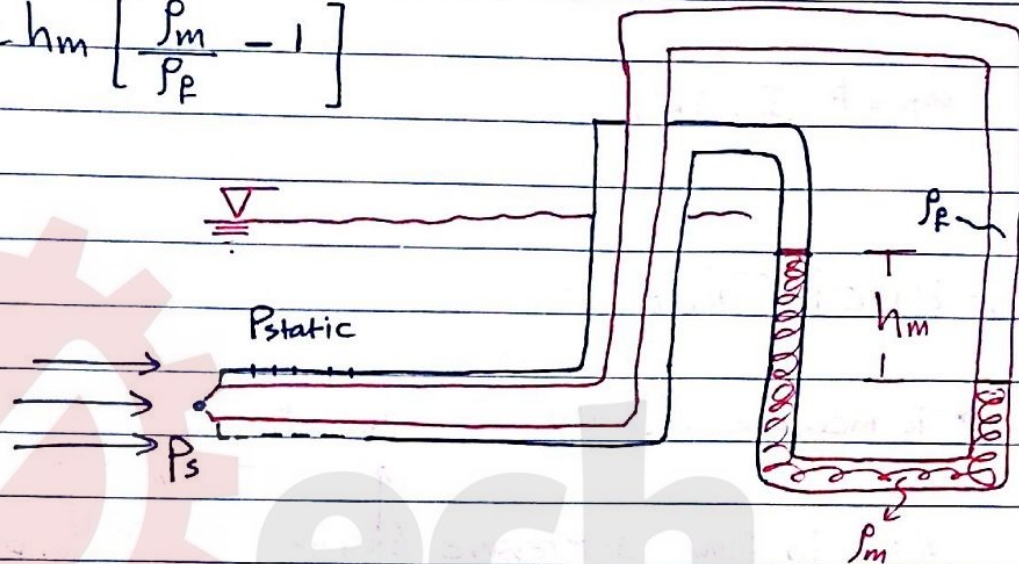
controlling
parameter

I, T_f, T_w

$$U_{p,act} = C \sqrt{2 * g * H}$$

→ For compressible Flow, they should be use manometer :-

$$U_{p,act} = C \sqrt{2g h_m \left[\frac{\rho_m}{\rho_f} - 1 \right]}$$



disadv.

→ Flow should Be clean, straight

→ sensitive for Flow type (lam., Turb.)

→ not work @ low speed.

Example 5 :-

$$h_m = 8 \text{ cm}$$

$$SG_{p,L} = 1.6, \therefore \rho_f = 1600 \frac{\text{Kg}}{\text{m}^3}$$

$$C = 1$$

$$U_p = ??$$



$$U_p = C \sqrt{2g h_m \left(\frac{\rho_m}{\rho_f} - 1 \right)}$$

$$\gamma_m = Hg (13.6)$$

$$\therefore \rho_m = 13600 \frac{\text{Kg}}{\text{m}^3}$$

$$= 1 \sqrt{2 \times 9.81 \times \frac{8}{100} \left(\frac{13600}{1600} - 1 \right)} = 3.43 \frac{\text{m}}{\text{s}}$$

Example 6 :-

Sol:-

$$U_f = C \sqrt{2gH}$$

$$5 = 1 \sqrt{2 \times 9.81 \times H}$$

$$H = 1.275 \text{ m}$$

$$U_1 = 18 \text{ km/hr}$$

$$= 5 \text{ m/s}$$

$$\rho_{\text{seawater}} = 1025 \text{ kg/m}^3$$



$$P_i = 8 \times 20 \text{ m}$$

$$\frac{P_s - P_i}{\gamma} = H \Rightarrow P_s = \gamma h + P_i$$
$$= 213.9 \text{ kPa}$$

* Flow Devices calibration:-

$$X_{\text{act}} = \text{Const.} \times X_{\text{th}}$$

L → from calibration. (device: Flow pover)

Device: Flow pover

قياس الوزن: catch & weight.

اللجنة الأكاديمية في قسم هندسة الميكانيك - الجامعة الأردنية