12 → 1:30
1 → 2:30
2 → 3:30

Hydrometer : measures viscosity

omega.com
engineersedge.com
efunda.com

ch5) Basics of electrical / mechanical Transducers

potentiometer ($x_i \rightarrow$ voltage)

strain gauge ($\epsilon \Rightarrow \Delta R$)

piezoelectric ($\frac{P}{F} \Rightarrow C$)
C: capacitance

Bellow ($\frac{F}{P} \Rightarrow \delta$)
δ: deflection

Diaphragm ($\frac{F}{P} \Rightarrow \delta$)

RTD
thermistor } Tem → ΔR

capacitor $C = \epsilon \frac{A}{d}$ A → overlap Area

---

strain Gauge ex
Beam is detector and primary transducer (الجزء يلي بيتلقى ال force)
strain gauge secondary transducer

detectors only (thermometer)

Bourdon tube : pressure Gauge
UDL : uniformly distributed load

$$K_s = \frac{d_m}{P} = \frac{3(1-\nu^2)D^4}{256\,E t^3}$$

Ex 528 ص

maximum shear stress at certain pressure

$\delta_m < 0.5t$

$t \uparrow, K_s \downarrow$

ثم نحسب σ ونقارنها ب τ_{max}

if $\delta_m > 0.5t$ → نقلل thickness
→ نزيد thickness

www.uspto.gov

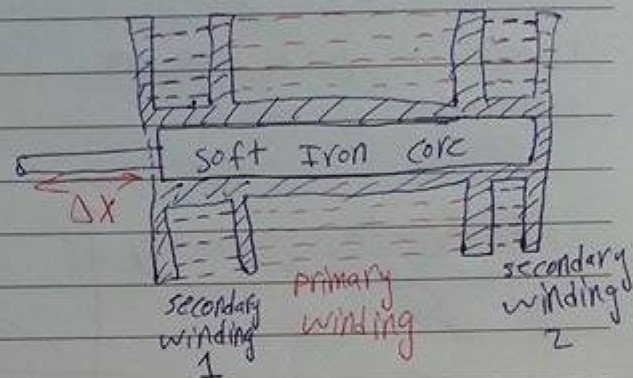
Electrical Transducer

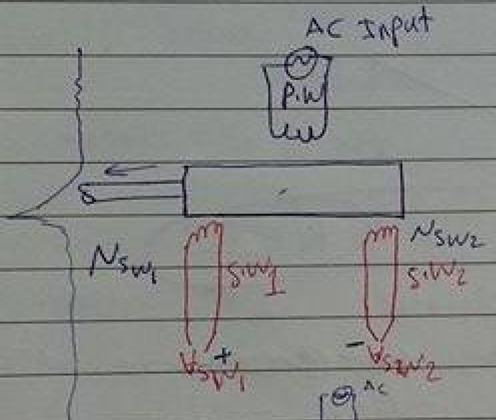
1) Inductance (LVDT)

LVDT: Linear variable differential Transformer

small distance (crack) used for crack displacement measurement

linear $\longrightarrow$ Voltage

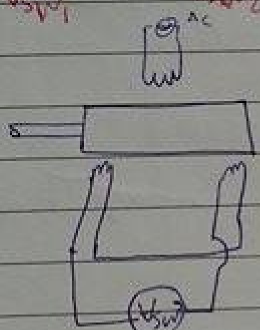




N: number of winding

$N_{sw_1} = N_{sw_2}$

$|V_{sw_1}| = |V_{sw_2}|$



1- very sensitive for vibration

2- Human factor

~~3- sensitive for~~

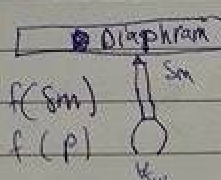
$\boxed{K_s = 40\ V/mm\ ,\ W_p \approx 1\ W}$

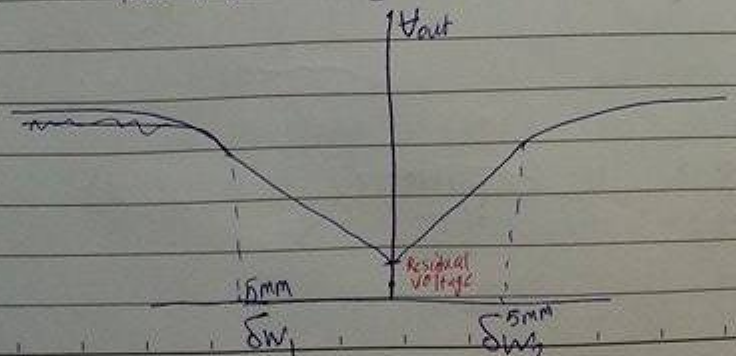
Iron: to concentrate the magnatic field

soft: لتقليل ال volt الناتج عن تقاطع خطوط المجال المغناطيسي من wire خفة

$V_{sw} = f(\delta m)$

$= f(P)$

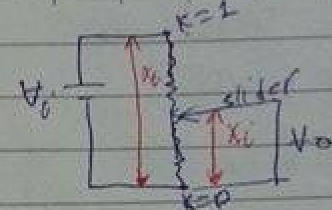




Residual: ~~not~~ known error

زي ال Roughness كأنه

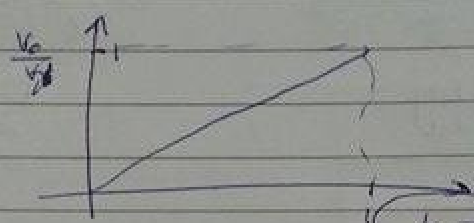
2) potentiometer



linear (translation)

linear : 2 to 500 mm
Rotational : 10° to 60°

$$V_{o_I} = \underbrace{\frac{V_i}{X_t}}_{K_s} \cdot X_i$$



$$K = \frac{X_i}{X_t}$$

source of errors :-
1- distance between wires cause quasistatic
2- loading error : as a result of voltage device ~~(internal resistance)~~ internal resistance

$V_{o_{Ideal}} > V_{o_{\text{Loading error}}}$

$$V_{o_L} = V_i \left[\frac{K}{K(1-K)\left(\frac{R_p}{R_m}\right)+1} \right]$$

R_m : voltage Resistance

$$\lim_{R_m \to \infty} V_{o_L} = V_{o_I}$$

for good linearity :- $\varepsilon \to 0$
1- $R_p \downarrow$
OR 2- $R_m \uparrow$

for $K \uparrow$:- $V_i \uparrow$ problem of power rating (تم احتراق الجهاز)

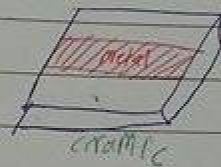
Five Apple

$P = \frac{V_c^2}{R_p}$

solution for k↑, nonlinearity↓

CerMet

thin metal Film

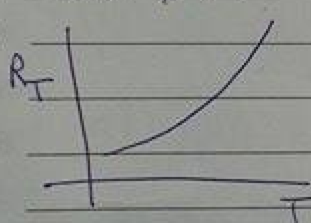
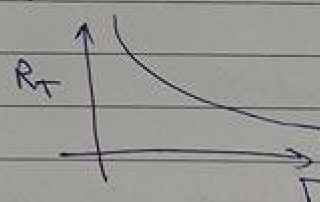


Moulded carbon or Moulded metal

Temperature transducers

- RTD
- Thermistor

} $\Delta T \longrightarrow \Delta R$

RTD	thermistor
- metal	semi conductor
R↑ as T↑	R↓ as T↑
	
	- Low temperature
High sensitivity at high temp so used for High temp measure	

خصائص اختيار الـ material

١- ~~High~~ ~~melting~~ ~~point~~ melting temp > temp working

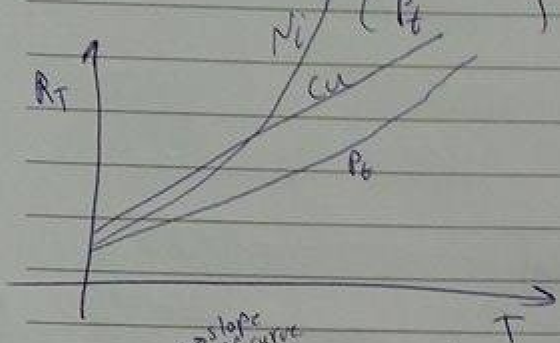
٢- العلاقة بين R_T and T ← linear على مدى Range معين

٣- Homogenous material → chemical composition, geometrical (uniform geometri)

ع- corrosion resistance

ه- non-toxic (غير سام)

افضل معدن ل RTD هو platine (P_t)



	α (slope of curve)	Tmin	Tmax	Tmelting	
P_t	0.39	−260°c	1100°c	1773	
Cu	0.39	0	180	1083	
Ni	0.62	−220	300	1455	
Tingistone	0.45	−200	1000	3390	Highly Britel

RTD equation

$$R_T = R_{T_0}\left[1 + \alpha_1 \Delta T + \alpha_2 \Delta T^2 + \cdots\right]$$

T_0 : calibrated (Reference) temperature
R_{T_0} : ~~exist~~ ance resistence at T_0
$\Delta T = T - T_0$

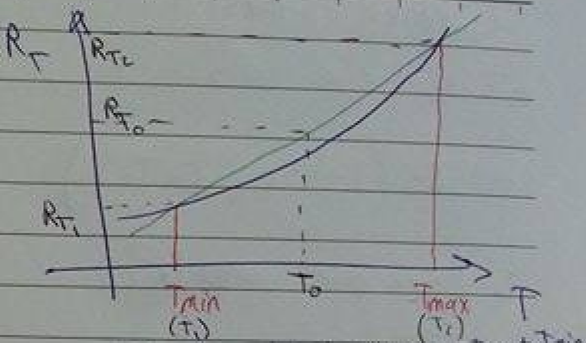
Thermistor

$$\cancel{R_{T_1} = R_{T_2}} \qquad R_{T_1} = R_{T_2}\left(e^{\beta\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}\right)$$

β : coeff. of variation between resistance and temp

~~RTD approximation~~

RTD → linear
→ quadratic

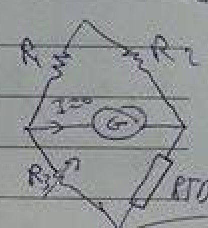


$$R_T = R_{T_0}\left[1 + \alpha_{T_0}(T - T_0)\right]$$

$$\alpha_{T_0} = \frac{1}{R_{T_0}}\left[\frac{\overbrace{\Delta R_T}^{R_{T_2}-R_{T_1}}}{\underbrace{\Delta T}_{T_2-T_1}}\right]$$

T_0 : is the averag $\frac{T_{max}+T_{min}}{2}$

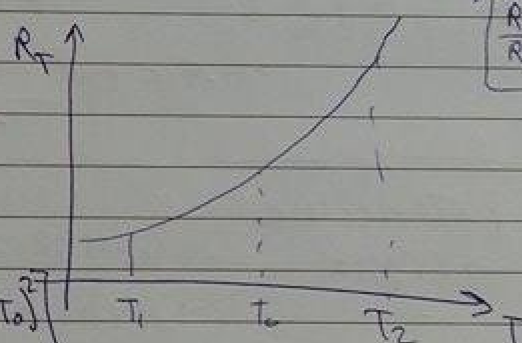
R_T : winston Bridge نعوضها عن طريقة وضعها في



$$\boxed{\frac{R_2}{R_1} = \frac{RTD}{R_3}}$$

quadratic

(I) $(T_1 \rightarrow T_0)$
(II) $(T_0 \rightarrow T_2)$



$\boxed{I}$ $R_{T_1} = R_{T_0}\left[1 + \alpha_1(T_1 - T_0) + \alpha_2(T_1 - T_0)^2\right]$

$\boxed{II}$ $R_{T_2} = R_{T_0}\left[1 + \alpha_1(T_2 - T_0) + \alpha_2(T_2 - T_0)^2\right]$

$$\frac{R_{T_1}}{R_{T_0}} + \frac{R_{T_2}}{R_{T_0}} = 2 + 2\alpha_2(T_2 - T_0)^2 \quad \text{(take five digits)}$$

اذا جاب جدول take average

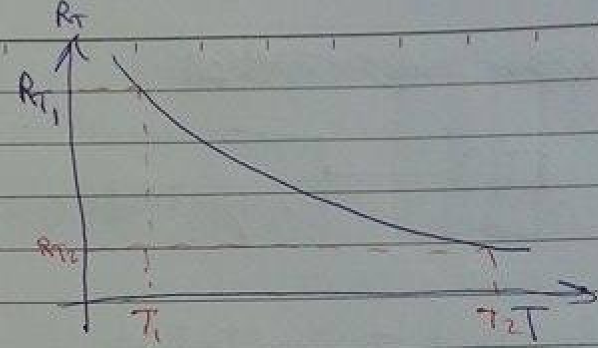
اذا سؤال الجدول على $^{\circ}C$ ← ~~R~~ R_{T_0}

اذا كان الصفر داخل الـ Range take ← R_{T_0} at $0^{\circ}C$

" " " خارج الـ Range ← take ← R_{T_0} ← at average temp

اذا بالسؤال ما أعطى R_{T_0} نأخذ افتراضية

~~thermost~~ thermistor



$\Delta T \leq 50^\circ c \rightarrow$ linear

$R_T = R_{T_0}\left(1 - \alpha_{T_0}(T - T_0)\right)$

T_0 : Ice point ~~temp~~ ($0^\circ c$)

if $\Delta T > 50^\circ c$

$\frac{R_{T_1}}{R_{T_2}} = e^{\beta\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$

in K

$R_T = A\, R_{T_0}\, e^{B/T}$ → in K ; A, B : are constants

→ at Ice point only

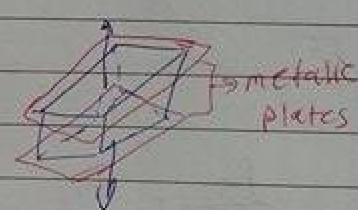
$B = \beta$ (K)

---

piezoelectric

→ crystal → Quartz, → Rochelle salts } natural

→ P, F → Voltage

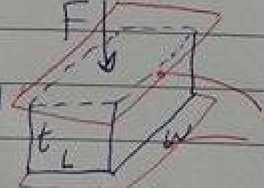


synthatic crystal (صناعي

BT : Barium Titanate

LS : Lithium sulphate

Dimension for plate = Dimension for crystal



(L w t)

$t_f = t_i - \Delta t$

charge ← $Q = K_c * F$ —(1)

K_c → charge sensitivity

$(C) = \left(\frac{C}{N}\right)(N)$

$$F = \frac{EA}{t} \Delta t \quad \text{—(2)}$$

$$V_0 = \frac{Q}{C} \quad \text{——(3)}$$

$$C = \epsilon_0 \epsilon_r \frac{A}{t} \quad \text{—(4)}$$

$$V_0 = \left(\frac{K_c}{\epsilon_0 \epsilon_r}\right)\left(P t\right)$$

pressure

Crystal voltage sensitivity (K_{vc})

$$V_0 = K_{vc}\, P\, t$$
$$= K_{vc} \frac{F}{A} t$$

$$V_0 = K_{vc} \frac{E}{t} \Delta t \; t$$

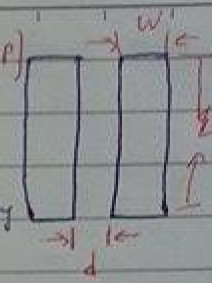
$$= K_{vc}\, E\, \Delta t$$

$$K_s = \frac{V_0}{\Delta t} = K_{vc} E$$

	K_{vc}	$\epsilon_0 \epsilon_r$	K_c pico/N (10^{-12})
BT	12×10^{-3}	12.5×10^{-9}	150
quartz	50×10^{-3}	40.6×10^{-12}	2

capacitor

$$C = \epsilon\epsilon \frac{A}{d}$$ → wL (overlap) Area



Modes of operations → W → L → A

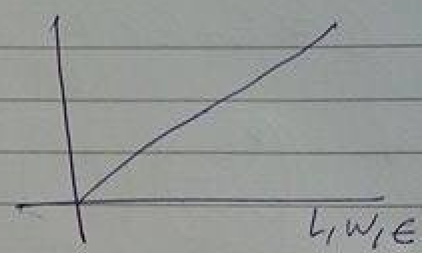
$\frac{\partial C}{\partial ?}$ sensitivity

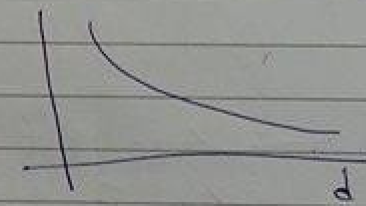
→ W → $\frac{\partial C}{\partial W} = \frac{\epsilon\epsilon L}{d}$

→ $\frac{\partial C}{\partial L} = \frac{\epsilon\epsilon W}{d}$

→ d → $\frac{\partial C}{\partial d} = -\frac{\epsilon A}{d^2}$

→ ϵ → $\frac{\partial C}{\partial \epsilon} = \frac{A}{d}$



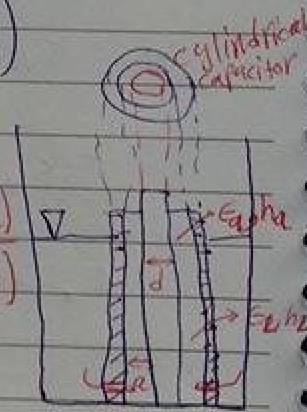


$$\frac{\frac{\Delta C}{C}}{\frac{\Delta X}{X}} = K_c$$

%out

%input

(capacitance factor
capacitor sensitivity)



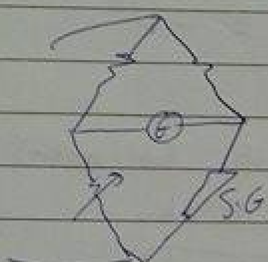
$$C = \frac{2\pi(h_1\epsilon_1 + h_2\epsilon_2)}{2\ln(1-\frac{a}{r})}$$

strain gauge :-

it measures $\left.\begin{matrix}\text{stress}\\ \text{strain}\end{matrix}\right\} \rightarrow \Delta R$

$R = \frac{\rho L}{A}$, ρ : Resistivity $\Omega - m$

strain $= \frac{\Delta L}{L}$



wheatstone Bridge

strain gauges

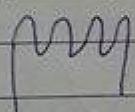
$\boxed{\text{metals}}$ → wire, foil

→ semiconductor
tension , R↓ (wire case)
compression , R↑

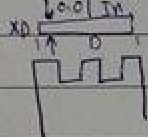
wire

$A = \pi D^2$

$D = 10 - 25\ \mu m$



foil



$A = \pi D^2$
is const

max. for metals $\frac{\frac{\Delta R}{R}}{\epsilon} = 2$

max. for semiconductor $\frac{\frac{\Delta R}{R}}{\epsilon} = -133$

[illegible]
tensile

$R = \frac{\rho L}{A}$

$\ln R = \ln \rho + \ln L - \ln A$

$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$

$\frac{dA}{A} = \frac{d(\pi D^2)}{A} = 2\frac{dD}{D}$

$$\frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta L}{L} - 2\frac{\Delta D}{D}$$

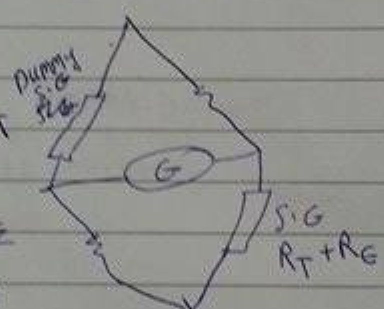
(G_f) strain Gauge factor $\dfrac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = G_f$

0, very small

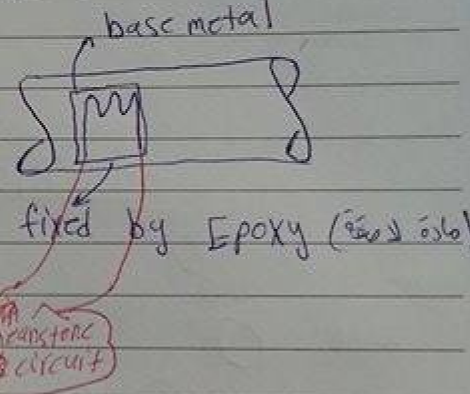
$$\frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = \frac{\Delta\rho/\rho}{\frac{\Delta L}{L}} + 1 + 2\upsilon \quad , \frac{\Delta D}{D} \Big/ \frac{\Delta L}{L} = -\upsilon$$

$G_f = 1 + 2\upsilon$ max $G_f = 2$, max $\upsilon = 0.5$ for metal

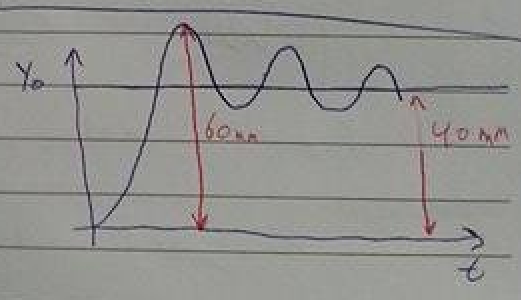
to eliminat the effect of temperature



$$\frac{R_1}{R_2} = \frac{R_{S.G}}{R_3}$$



Exam question 2



$$M_p = \frac{60-40}{40} = 0.5$$

$$= e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\zeta = 0.2154$$

$$\omega_d = \frac{\pi}{t_p} \Rightarrow \omega_d = 7.618 \text{ rad/s}$$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 7.681 \text{ rad/s}$$

$$\phi = \cos^{-1}(\zeta) = 77.55^\circ$$

$$= 0.4308\pi$$

$$t_r = \frac{\pi - \phi}{\omega_d} = 0.683$$

$$\eta = \frac{B}{2S\sqrt{2ag\forall}} \quad , \omega_n = \sqrt{\frac{2ag}{\forall}}$$

$$\frac{\eta_1}{\eta_2} = \frac{\sqrt{\forall_2}}{\sqrt{\forall_1}} = \sqrt{\frac{1}{2}}$$

$\eta_2 = 0.3047$

$\omega_{n_2} = 3.8$ rad/s

$\omega_{d_2} = 3.61$ rad/s

$t_{r_2} = 0.52$ sec

Q2) First order because is controlling parameter is τ

$K_s = 3$ mm/°C $= \frac{\alpha \forall_b}{A_c}$

$\forall_b = 123.75$ mm^3

$\tau = \frac{\delta C \forall_b}{H A_b} = 34.135$ sec

for cylinderical bulb

$d = 4.75$ mm

$\tau_{cyl} = 39.3$ sec

$$X_o = K_s T_i \left(1 - e^{-t/\tau}\right) + K_s T_{init}\, e^{-t/\tau}$$

$X_o = 151.183$ mm

$T_o = 50.4$°C

Q3) $\bar{\bar{T}} = 12.06$°C

$\bar{D} = 0.752$°C

$\sigma = 1.0084$

$r_1 = 0.6801$

$V = \sigma^2 = 1.0169$°C

$h =$

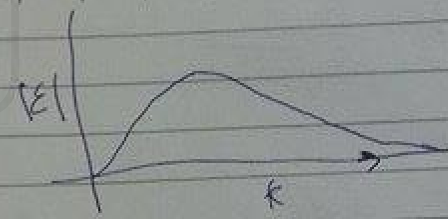
chapter (5) : problems

Q2) $R_p = 2\,k\Omega$

$v_{in} = 10\,V$

$R_m = 5\,k\Omega$

$\frac{x_i}{x_t} = k = 0, 0.25, 0.5, 0.75, 1$



$E = \cancel{m} v_i \left[\frac{k^2(k-1)}{k(1-k) + \frac{R_m}{R_p}} \right]$ $\quad E = 0$ at $k = 0, 1$

$E(k=0.5) = \cancel{m} 10 \left[\frac{0.5^2(0.5-1)}{0.5(1-0.5) + \frac{5}{2}} \right] = -0.454$

Q3) $\underset{\frac{C}{N}}{Q = K_c}\ \underset{N}{F}$

$Q = \underset{80 \times 10^{-12} \frac{C}{bar}}{K_c}\ \underset{bar}{P}$

$C = 1 \times 10^{-9}\,F$

$Q = 80 \times 10^{-12}(1.4) = 112\,pC$

$v_0 = \frac{Q}{C} = \frac{112 \times 10^{-12}}{1 \times 10^{-9}} = 112\,mV$

Q4) $t = 2\,mm$

$K_{vc} = 0.055\ V\text{-}m/N$

$P = 1.5\,MPa$

$\epsilon = 40.6 \times 10^{-12}\,F/m$

find v_0, K_c ?

$v_0 = K_{vc}\, t\, P = 165\,V$

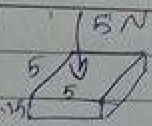
$K_{vc} = \frac{K_c}{\epsilon} \Rightarrow K_c = 2.23\,pC/N$

Q5)

$K_C = 150\ pC/N$

$\epsilon = 12.5 \times 10^{-9}\ F/M$

$E = 12 \times 10^{6}\ N/m^2$



find strain, Q, C ?

$$\text{strain} = \frac{\text{stress}(F/A)}{E} = \frac{5/5\times5\times10^{-6}}{12\times18} = 0.0167$$

$Q = K_c F$

$= 150\times10^{-12}\times5 = 750\ pC$

$$C = \frac{Q}{V_0} = \frac{Q}{K_c t\,P} = \frac{750}{3} = 250\ pF$$

$\cancel{Q/V_0}$ $\quad K_c \to \frac{K_c}{\epsilon}$ $\quad P \to F/A$

Q10) $T_1 = 100$, $T_2 = 130$, $T_0 = \frac{T_1+T_2}{2} = 115$

$R_{T_1} = 573.4$, $R_{T_2} = 606.5$, $R_{T_0} = 589.5$

$$R_T = R_{T_0}\left(1+\alpha_{T_0}(T-T_0)\right)$$

$$\alpha_{T_0} = \frac{1}{R_{T_0}}\left(\frac{R_{T_2}-R_{T_1}}{T_2-T_1}\right) = 0.00182 \quad \text{(دائماً موجبة)}$$

linear $\qquad$ معادلة الخط الواصل بين T_1 و T_2

$$R_T = 589.5\left(1+0.00182(T-115)\right)$$

quadratic

Range(I) $\quad T_1 \to T_0$

$$R_{T_1} = R_{T_0}\left(1+\alpha_1(T_1-T_0)+\alpha_2(T_1-T_0)^2\right)$$

Range (II)

$$R_{T_2} = R_{T_0}\left(1+\alpha_2(T_2-T_0)+\alpha_2(T_2-T_0)^2\right)$$

$\alpha_1 = 0.00159$ (موجبة دائماً)

$\alpha_2 = -15.098\times10^{-6}$ (معظم تكون سالبة)

RTD $\alpha = +ve$

thermistor $\alpha = -ve$

Q11) $\alpha_{T_0} = 0.005392 \ \frac{\Omega}{\Omega \cdot C}$

$T_0 = 25^\circ C \rightarrow R_0 = 90\ \Omega$ (lowest)

$R_T = 115\ \Omega \rightarrow T\ ??$

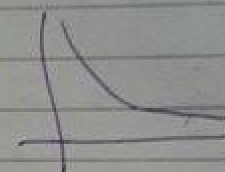
$$R_T = R_{T_0}(1 + \alpha_{T_0}(T - T_0))$$

$$115 = 90(1 + 0.005392(T - 25)) \Rightarrow T = 76.51^\circ C$$

Q12) thermistor

$\alpha_{T_0} = -0.05$

25 – 50 assume linear



$R_{T_0} = 100\ \Omega \rightarrow T_0 = 25^\circ C$

$T = 35^\circ C \rightarrow R_T\ ??$

$$R_T = R_{T_0}\left[1 + \alpha(T - T_0)\right]$$

$$= 100(1 - 0.05(35 - 25))$$

$$R_T = 50\ \Omega$$

Q13) $R_{T_0} = 3980\ \Omega$; $T_0 = 0^\circ C$

$R_T = 794\ \Omega$; $T = 50^\circ C$

→ Border case so logerithmic

$$R_T = A R_{T_0} e^{B/T}$$

logerithmic valid (Range لها حدود)

At $T = 0 = 273\ K$

$R_T = R_{T_0} A e^{B/T}$

$3980 = A\, 3980\, e^{B/273} \Rightarrow 1 = A e^{B/273}$ — (1)

at $T = 50 = 323\ K$

$794 = A\, 3980\, e^{B/323} \Rightarrow \frac{794}{3980} = A e^{B/273}$

$A = 30 \times 10^{-6}$

$B = 2845\ K$

OR $T_1 = 0\,^\circ C$, $R_{T_1} = 3980\,\Omega$

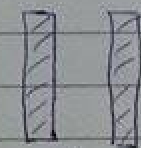
$T_2 = 50\,^\circ C$, $R_{T_2} = 794\,\Omega$

$$R_{T_2} = R_{T_1}\, e^{B\left(\frac{1}{T_2} - \frac{1}{T_1}\right)}$$

$B = 2845\ K$

then use any Temp to find A

Q13)



$$C_1 = \epsilon_0 \epsilon_r \frac{A}{d_1}$$
$$= 22.125\ pF$$

$A = 500\ mm^2$

$d_1 = 0.2\ mm$

$\epsilon_0 = 8.85 * 10^{-12}\ F$

$\epsilon_r = 1$ (air)

$d_2 = 0.18\ mm$

$$C_2 = \epsilon_0 \epsilon_r \frac{A}{d_2}$$
$$= 24.583\ pF$$

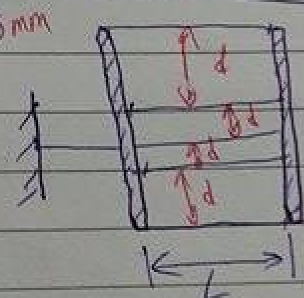
ratio per unit change of capacitance to per unit change in displacement $= K_c$

$$K_c = \frac{\frac{\Delta C}{C_1}}{\frac{\Delta d}{d_1}} = 1.11$$

Q14)

$$C = \frac{\overset{4}{N}\ \overset{8.85\times10^{-12}}{\epsilon_0}\ \epsilon_r\ (\overset{25mm}{L} * \overset{25mm}{W})}{d}$$

$d = 0.25mm$



$$C = \frac{N\, \epsilon_0 \epsilon_r (L - x)\, W}{d}$$

$$S_x = \frac{\partial C}{\partial x} = -\frac{N\, \epsilon_0 \epsilon_r W}{d} = -3540\ pF/M$$

for one plate divide on 4

strain Gauge examples

EX1: wire S.G, $G_f = 2$

$\sigma = 100$ Mpa

$E = 200$ Gpa

$$\frac{\Delta R}{R} = G_f \, \epsilon$$

$$\frac{\Delta R}{R} = 2 \; \frac{100 \times 10^{6}}{200 \times 10^{9}} = 0.1\%$$

Rosette: multi strain Gauge to measure one Input at series

EX2:- beam → $L = 0.1$ m

→ $A_c = 3 \, m^2$

→ $E = 207$ Gpa

unstrained Resistance $= 240 \, \Omega$

$G_f = 2.2$

$\Delta R = 0.013$ (tensile force)

$$G_f = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} \Rightarrow \Delta L = 2.46 \times 10^{-6} \, m$$

$\cancel{\frac{\Delta R}{R} = \epsilon \, G_f}$

$$\epsilon = \frac{\Delta L}{L} = \frac{\sigma}{E}$$

$$\sigma = 5.092 \times 10^{6} \; N/m^2$$

$\sigma = F A$

$F = 2.037 \times 10^{3}$ N ~~α~~

$$F = \frac{5.092 \times 10^{6}}{3}$$

$= 1697333.333$

Five Apple

EX3:- $G_f = 4.2$
not semiconductor (composite material)
tensile force

$G_f = 1 + 2\nu + \frac{\Delta \rho \nearrow 0}{\cancel{\varepsilon_L}}$

$\frac{4.2 - 1}{2} = \nu \Rightarrow \nu = 1.6$

EX4) $\epsilon = 5\mu$, S.G$_1$ = Ni, $G_{f_1} = -12.1$ (designed for compressive loading)

Nichrome 2 (designed for tensile loading)

$R = 120\,\Omega$

$R_f = \epsilon G_f * R_i + R_i$

compressive force ϵ negative

for Ni

$R_f = -5 \times 10^{-6} * -12.1 * 120 + 120$
$= 120 + 7.26 \times 10^{-3}$

$\Delta R = 7.26\, m\Omega$

for Nichrome

$R_f = -5 \times 10^{-6} * 2 * 120 + 120$
$R_f = 120 - 1.2 \times 10^{-3}$

$\Delta R = -1.2 \times 10^{-3}\, \Omega$

Five Apple

Flow, pressure and velocity measurements :-

3 methods to calculate ~~Fo~~ Flow
- → velocity ~~(co...)~~
- → mass
- → volume flow rate

Methods
- * → Head type $\dot{V} = f(\Delta P)$ (orifice, venturi)
- * → variable head $\dot{V} = f(y)$ (Rotameter)
- → turbine meter (عداد المية)
- → magnetic flow meters $\dot{V} = f(\phi)$ → flux
- * → Thermal flow meters (Hot wire Anemometer) $\dot{V} = f(\Delta T)$
- → positive displacement (reciprocated pumps) $\dot{V} = f(\Delta P)$
- → mass method $\dot{m} = f(\rho)$

Flow → laminar, Turbulent

$Re \leq 2000$ laminar

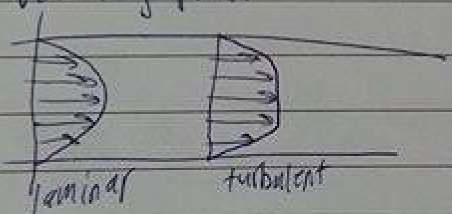
Flow → compressible, In compressible $\Delta V = f(P)$

compressibility factor y

$y = 1$ liquid (In compressible)

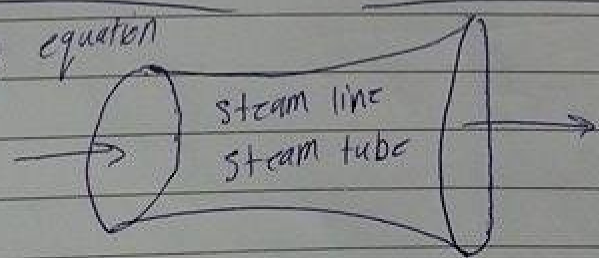
$y < 1$ compressible

velocity profile



continuetiy equation

$\sum_{in} \dot{m} = \sum_{out} \dot{m}$



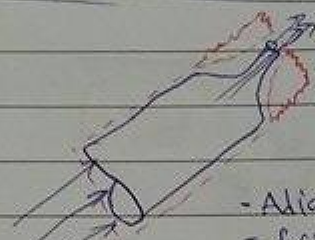
steam tube : tube with no leakage.
(no cross flow)

Bernoulli theorems:-

$$\dot{Q} \propto (P_1 - P_2)^{n \,\to\, 1 \text{ laminar},\ 0.5 \text{ turbulent}}$$

$\dot{Q}$: volumetric flow rate

Obstruction flow measurements devices (Head type)



$z_2, u_2, A_2, \rho_2, \gamma_2$

$z_1, u_1, A_1, \rho_1, \gamma_1$

- Adiabatic
- frictionless
- 1-D
- Irrotational
- Isothermal
- stream Tube
- Incompressible

$$\cancel{\rho_1} A_1 u_1 = \cancel{\rho_2} A_2 u_2$$

$$A_1 u_1 = A_2 u_2 \quad \text{—— (1)}$$

$$\frac{P_1}{\gamma_f} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\gamma_f} + \frac{u_2^2}{2g} + z_2$$

$$\left[\frac{P_1 - P_2}{\gamma_f} + z_1 - z_2 + \frac{u_1^2}{2g}\right] * 2g = u_2^2$$

$$u_{2th} = \sqrt{2g\left[\frac{(P_1 - P_2)}{\gamma_f} + \underbrace{z_1 - z_2}_{\text{negative}}\right] \Big/ \left(1 - \left(\frac{A_2}{A_1}\right)^2\right)} \quad m/s$$

$$\left(\frac{A_2}{A_1}\right)^2 = \left(\frac{d_2}{d_1}\right)^4 = \beta^4$$

$$\dot{Q}_{th} = A_2 u_{2th} \quad m^3/s$$

$$\dot{m}_{th} = \rho_2 A_2 u_{2th}$$

Five Ap

$C_D = \frac{\dot{Q}_{act}}{\dot{Q}_{th}}$ (vena contracta)

$\dot{Q}_{act} = C_D A_2 u_{2th}$

ΔP_{1-2} : in ~~pascal~~ pa

$$\dot{Q}_{act} = Y C_D A_2 \sqrt{\frac{2g\left(\frac{\Delta P_{1-2}}{\gamma_f} + z_1 - z_2\right)}{1-\beta^4}}$$

$Y \to 1$ incomp

$Y \to < 1$ comp

$\dot{m}_{act} = \rho_2 \dot{Q}_{act} \to \rho_2 = \rho_1$ liquid

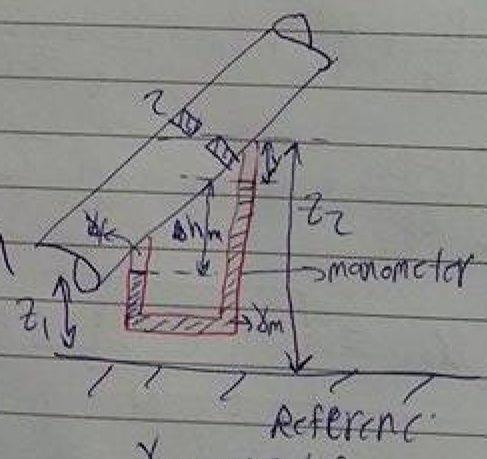
$\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)^k$ (gas)

$$\boxed{\rho_2 = \left(\frac{P_2}{P_1}\right)^{1/k} \rho_1}$$

$P_1 + \gamma_f x = P_2 + \gamma_f y + \gamma_m h$

$P_1 - P_2 = \cancel{x}\gamma_f = (y - x) + \gamma_m h$

$\frac{P_1 - P_2}{\gamma_f} + (z_1 - z_2) = h_m\left(\frac{\gamma_m}{\gamma_f} - 1\right)$



$$\dot{Q}_{act} = \frac{Y C_D A_2}{\sqrt{1-\beta^4}} \sqrt{2g h_m\left[\frac{\gamma_m}{\gamma_f} - 1\right]}$$

γ_m : manometer

γ_f : fluid

$$\frac{\gamma_m}{\gamma_f} = \frac{\rho_m}{\rho_f} = \frac{S.G_m}{S.G_f}$$

$$Re_{d_2} = \frac{\rho_2 u_{2_{act}} d_2}{\mu_f}$$

$$Q_{act} = C_D A_2 U_{2_{th}}$$

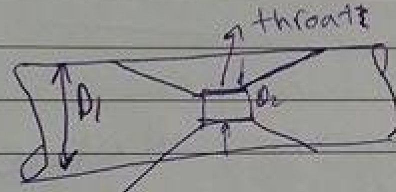
$$u_{2_{act}} = C_D u_{2_{th}}$$

compressible → mass flow rate
incompressible → volume flow rate

Orifice plate

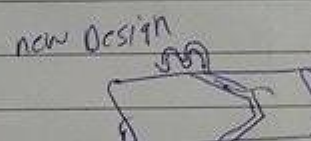
sudden change in Area

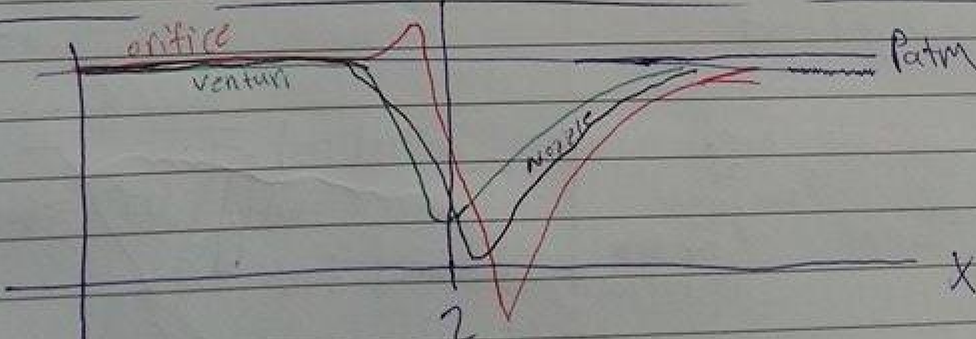
Venturimeter



venturi is better because Area changes gradually.

flow Nozzle





$\Delta P_{loss\,max}$ for orifice

$\Delta P_{loss\,min}$ for venturi

ΔP has an effect if there is a steady flow device after the obstruction device.

to eliminate that effect :

1- put the stead flow device (pump) far away from the obstruction device

2- use compensating pump to increase the inlet pressu

$$\dot{W}_p = \frac{Q_{act} * \Delta P_{loss} \rightarrow pa}{\eta_{pump}} \quad (w)$$

$$cost = \dot{W}_p * tariff\ rate * time$$

$$kw * \frac{JD}{kwh} * \frac{hrs}{year}$$

---

selection criteria :-

1) space 2) cost 3) Accuracy 4) ΔP_{loss} tolerated

5) Nature of flow

accuracy dependes on

1) β : حساب الـ $\frac{Area}{ratio}$ بدقة عالية

2) C_D حساب الـ C_D بدقة عالية

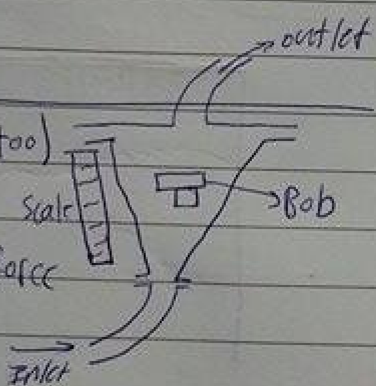
3) $\rho(T)$ حساب الـ density على الظروف الجديدة

4) Temperature effect

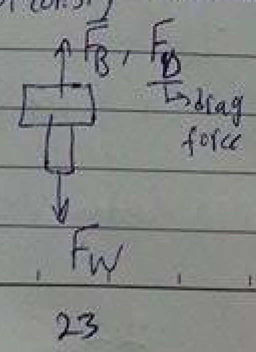
Rotameter : variable head (act as control too)

variable → Area not constant

head → the height of the Bob is not const → Boiancy force

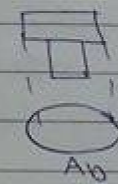


$$F_D + F_B = F_W$$



frontal area of the Bob (projected Area)

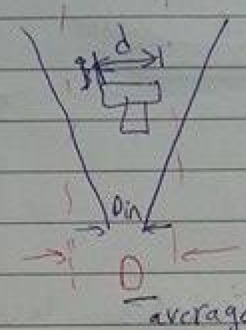
$$\frac{C_D A_b \rho_f U_m^2}{2} + \rho_f \forall_b g = \rho_b \forall_b g$$



controlling parameter

$$U_m = \sqrt{\frac{2(\rho_b - \rho_f) \forall_b g}{C_D A_b \rho_f}}$$

$$= \sqrt{\frac{2g}{C_D}\left(\frac{\rho_b}{\rho_f} - 1\right)\left(\frac{\forall_b}{A_b}\right)}$$



Din قطر طرف کون

$$\dot{Q} = U_m * A$$

$$A = \frac{\pi}{4}\left[(D_{in} + ay)^2 - d^2\right]$$

$$= \frac{\pi}{4}\left[(D_{out} - ay)^2 - d^2\right]$$

y: hight of the float

optimum $\dot{m}_f$
when $\rho_b = 2\rho_f$

disaduantges

1- glass
2- ~~...~~ dirt ~~...~~ existanc in water

---

velocity → pitot Tube
→ Hot wire Anemometer
metal

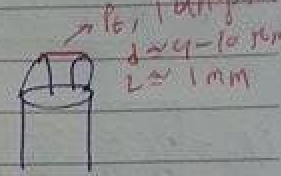
$$R = \frac{\rho L}{A}, \quad R = f(T)$$

based on: forced convection

كل ما بصغر ال Length التغير بالقطاعات ~~ع~~ ال length بقل

// // // // // ل loose بقل، والقطاع الو بيتقطع بقل

لن يأخذ من ال cross sectional area of the flow



1- to protect it from dirt

2- to protect it from thermal shock ~~[illegible]~~ Insulation

OR

$$I^2 R_w = h A_w (T_w - T_f)$$

$$R_w = R_{ref}\left[1 + \alpha (T_w - T_{ref})\right]$$

King's law

$$h = a + b U_f^c$$; a, b, c : properties for flow

$$U_f = \left\{ \frac{\frac{\left[I^2 R_{ref}\left[1+\alpha(T_w - T_{ref})\right]\right]}{A_w (T_w - T_f)} - a}{b} \right\}^{1/C}$$

process

1- constant Temp (T)
variable current (I)
R_w : constant
امسك لا وحافظ على فرق درجات ثابت
موجود convection.

$$U_f = f(I, T_f)$$

T_w : wire temp

2- constant current (I)
variable Temp T
R_w : variable

$$U_f = f(T_w, T_f)$$

Examples :-

$\mu_{water} = 1.08 \times 10^{-3}$ (تقريباً)

$\rho_w = 1000$

$D_1 = 400$ mm

$d_2 = 150$ mm

$h_m = 250$ mm of mercury

$S.G\ Hg = 13.6$

$C_D = 0.98$

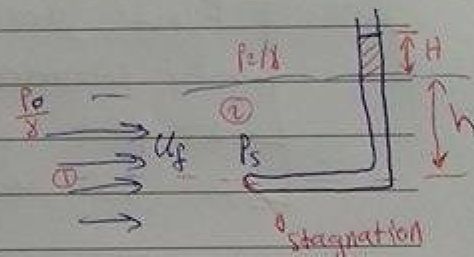
$\cancel{\times}\ \Delta P_{1-2} = \rho g h$ (single homogenous fluid) خطأ فرق لا تستخدمه

$\cancel{\Delta P_{1-2}}\ Q = \dfrac{C_D Y A_2}{\sqrt{1-\beta^4}} \sqrt{2g\, h_m \left[\dfrac{SG_m}{SG_f} - 1\right]}$

---

pitot tube $\longrightarrow$ velocity

$\Delta KE_{flow} \rightarrow$ pressure E



$\dfrac{P_o}{\gamma} + \dfrac{u_o^2}{2g} + \cancel{Z_1} = \dfrac{P_s}{\gamma} + \cancel{\dfrac{u_s^2}{2g}}^{\,0} + \cancel{Z_s}$

stagnation point

$u_s = 0$

$\dfrac{u_o^2}{2g} = \dfrac{P_s - P_o}{\gamma}$

$u_o = \sqrt{\dfrac{2}{\rho}\left(P_s - P_o\right)}$

$P_o = \gamma h = P_2$

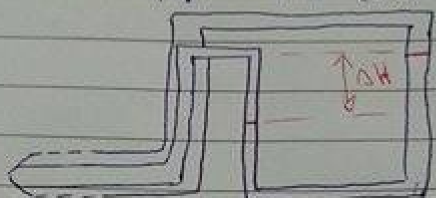
$P_s = \gamma = h + H$

$P_s - P_o = \gamma H$

$u_{o_{th}} = \sqrt{\dfrac{2}{\rho}\gamma H} = \sqrt{2gH}$

$U_{o_{act}} = C \; U_{o_{th}}$

↳ flow coeff.
piptot tube coeff



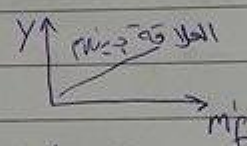
اهم مشكلة
اذا dust علق ب فتحة pitot

used when
1- Gas flow
2 - High speed

$$U_{o_{act}} = C_f \sqrt{2g\, H_m \left[\frac{S.G_m}{S.G_f} - 1\right]}$$

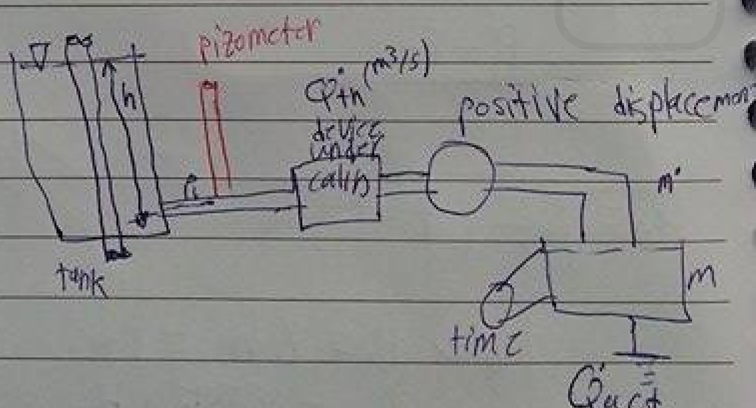
---

calibration process
- → to find discharge coeff. for obstruction Flow measurement devices
- → for Rotameter, find slope a,

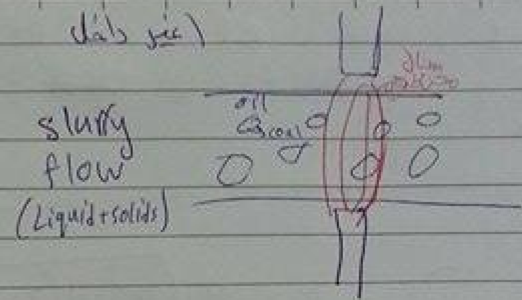


- → for pitot to find C
- → Hot wire Anemometer, to find a, b, C

*flow prover
(theory) principle : catch and weigh
اهم مصدر لل error هو density
P_1 must be const
$P_1 = \gamma h$



magnatic flow meter (غير قابل)



Ex:- 1

$\gamma_{water} = 9810 \ \frac{N}{m^3}$

$\beta = \frac{150}{400} = 0.375$

$y = 1$ (water)

$z_1 - z_2 = 0$

$h_m = 250$ mm Hg

$$\dot{Q} = \frac{C_D \, Y \, A_2}{\sqrt{1-\beta^4}} \sqrt{2g \, \underbrace{h_m}_{\frac{250}{1000}} \left[\frac{S.G_m}{S.G_f} - 1\right]}$$

$$= \frac{(0.98)(1)\left(\frac{\pi}{4}\left(\frac{150}{1000}\right)^2\right)}{\sqrt{1-(0.375)^4}} \sqrt{2(9.81)\left(\frac{250}{1000}\right)\left[\frac{13.6}{1} - 1\right]}$$

$= 0.1375 \ m^3/s$

OR

$$\dot{Q} = \frac{C_D \, Y \, A_2}{\sqrt{1-\beta^4}} \sqrt{2g\left(\frac{\Delta P_{1-2}}{\gamma_f} + \cancel{z_1 - z_2}^{\,0}\right)}$$

$(\cancel{z_1 - z_2})^0 + \frac{\Delta P_{1-2}}{\gamma_f} = h_m \left[\frac{13.6}{1} - 1\right] = 3.15 \ m$

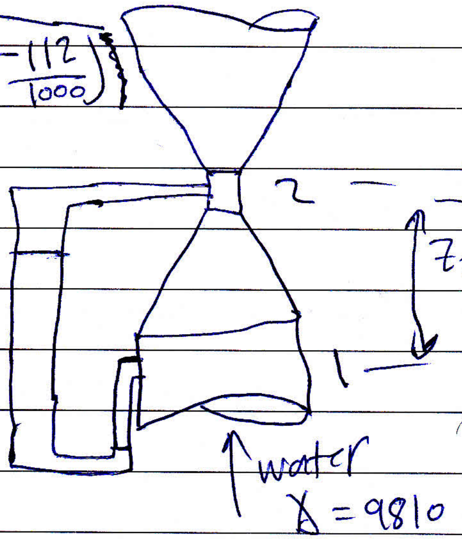
if said $\Delta P_{1-2} = 280$ kPa

$\underline{\Delta P_{1-2}}$ عوض بـ ~~[illegible]~~ المعادلة الثانية

in pa

EX:2

$$Q^{\cdot} = \frac{1 * 0.97 * \frac{\pi}{4}\left(\frac{80}{1000}\right)^2}{\sqrt{1-\left(\frac{80}{100}\right)^4}} \sqrt{2*9.81\left(\frac{30.6*10^3}{98100} - \frac{112}{1000}\right)}$$

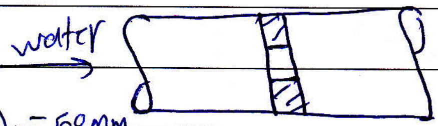


$$U_{2\,act} = \frac{Q^{\cdot}}{\frac{\pi}{4}\left(\frac{80}{1000}\right)^2} = 9.69 \text{ m/s}$$

$$Re_{d_2} = \frac{\rho\, U_{2\,act}\, d_2}{\mu}$$

$$\left(\frac{30.6*10^3}{9810} - \frac{112}{1000}\right) = h_m\left[13.6 - 1\right]$$

EX3:-



$C_D = 0.61$, $\rho_f = 1000$

find U_{2act}, ΔP_{1-2}

ν_f : kinematic viscosity $= 1*10^{-6}$

assume: Horizontal

$D_1 = 50$ mm

$d_2 = 30$ mm

$\beta = 0.6$

$Re_{d_2} = 100000$

$$Re_{d_2} = \frac{U_{2act}\, d_2}{\nu_f}$$

$$100000 = \frac{U_{2act}(0.03)}{1*10^{-3}} \Rightarrow U_{2act} = 3.33 \text{ m/s}$$

$$U_{2actual} = C_D \sqrt{\frac{2(9.81)\left(\frac{\Delta P_{1-2}}{1000*9.81}\right)}{1-(0.61)^4}} \Rightarrow \Delta P_{1-2} = 12992.71 \text{ pa}$$

ex 4 Air , $D_1 = 60$ mm

$\beta = 0.4$

$h_m = 250$ cm

$P_1 = 93.7$ kPa (absolute) , $T_1 = 20°C$

$C_D = 0.625$, $Y = 0.92$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{93.7}{(0.287)(293)} = 1.1135 \text{ kg/m}^3$$

$$\dot{Q} = \frac{C_D Y A_2}{\sqrt{1-\beta^4}} \sqrt{2g h_m \left[\frac{S.G_m}{S.G_f} - 1\right]} = 0.0528 \text{ m}^3/\text{s}$$

$S.G_m \rightarrow 1$, $S.G_f \rightarrow \frac{1.1135}{1000}$

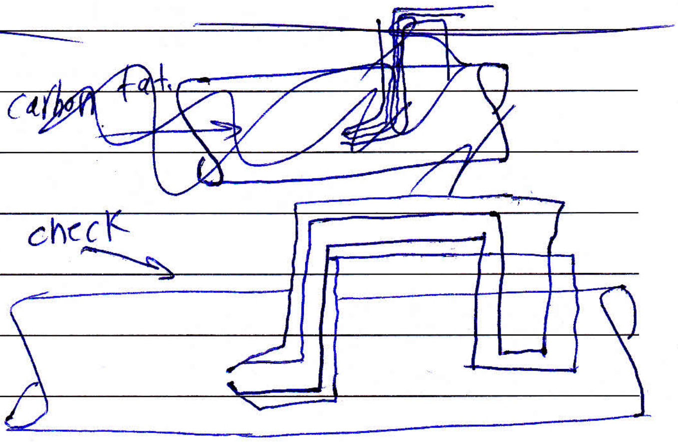
$$\dot{m} = (1.1135)(0.0528) = 0.0588$$

EX5 $S.G_f = 1.6$

$C = 1$

$h_m = 8$ cm Hg

$S.G_m = 13.6$



$$U_{f_{act}} = C \sqrt{2g h_m \left(\frac{SG_m}{SG_f} - 1\right)}$$

$SG_m \rightarrow 13.6$, $h_m \rightarrow \frac{8}{100}$, $SG_f \rightarrow 1.6$

$= 3.43$ m/s

EX6 pitot tube application

بما أنه لم يحدد
نعوض اول واحد

$$U_{f_{act}} = C\sqrt{2gh}$$

$$5 \text{ m/s} = 1\sqrt{2g * h} \Rightarrow h = 1.274 \text{ m}$$

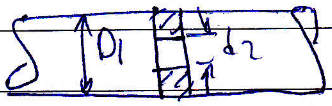
$$h = \frac{P_s - P_o}{\gamma_f} = \frac{P_s - \gamma_f z}{\gamma_f} \Rightarrow P_s = 213.91 \text{ kPa}$$

EX: $D_1 = 20$ cm

$d_2 = 10$ cm

$h_m = 50$ cm Hg

$S.G_m = 13.5$



?? ??

$$\dot{Q} = \frac{Y\, C_D\, A_2}{\sqrt{1-\beta^4}} \sqrt{2 g h_m \left[\frac{S.G_m}{S.G_f} - 1\right]}$$

flow coeff (K_o)

$\frac{1}{\sqrt{1-\beta^4}}$ velocity approach factor (E)

using Figure 10.6

$\beta = 0.5$

$$Re_{d_1} = \frac{\rho_f\, U_{1act}\, d_1}{\mu_f}$$

$$\dot{Q}_{act} = A_1 U_{1act} \Rightarrow U_{1act} = \frac{\dot{Q}_{act}}{A_1}$$

$$Re_{d_1} = \frac{4\, \dot{Q}_{act}}{\pi\, d_1\, \nu_f}$$

find ~~Re~~ K_o when the curve becomes flat

$K_o = 0.625$

$$K_o = \frac{C_D\, Y}{\sqrt{1-\beta^4}} \Rightarrow C_D =$$

$\dot{Q} = 0.054\ m^3/s$ now to ~~ve~~ vertify the answer

find $Re_{d_1} = \frac{(4)(0.054)}{\pi\,(0.2)(\nu_f)} = 3.2 \times 10^5$

3.2×10^5 > point at which flat curve start

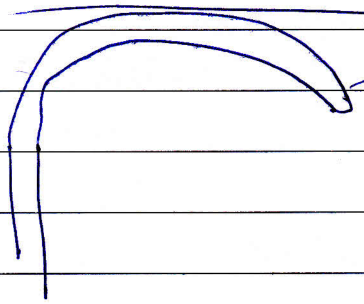
then C_D is true ~~Rf~~ ~~answer~~ < point at which ...

Five Apple

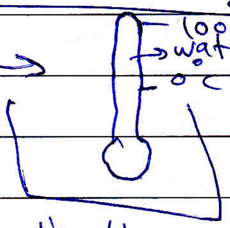
if answer (Re_{d_1}) < point at which flat curve start

then use the value of C_D ~~from~~ at Re_{d_1} calculated and make itteration.

pressure measurement



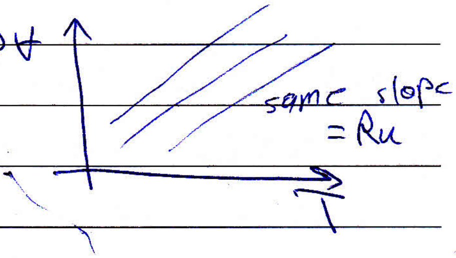
Temperature measurement
not reproducable → using water in thermometer



استخدام ال ~~calibration~~ liquids في calibration غير مجز

we use Gas thermometer for calibration

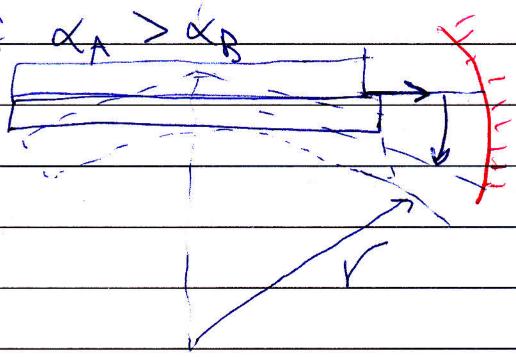
$$\frac{PV}{T} = \overset{=1}{n} R_u \quad , \text{constant slope}$$

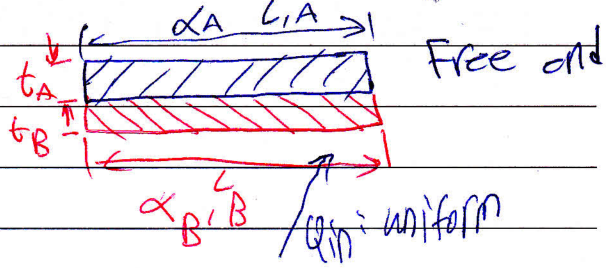


Bimetallic (thermostat) (temperature measurement and control)
Thermocouple
Pyrometer (يستخدم الموجات الكهرومغناطيسية)

Bimetallic ⇒ basic : Expansion of solids with ΔT

if $\alpha_A > \alpha_B$

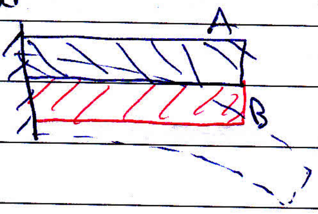




Invar material : very small coeff of expansion
$\alpha = 1.7 * 10^{-6}$ (smallest)

Yellow Brass : $\alpha = 20.2 * 10^{-6}$

Fixed end



for free end

r (radius of curvature) $= \left(\frac{t}{6}\right)\left[\frac{3(1+m)^2 + (1+mn)\left(m^2 + \frac{1}{mn}\right)}{(\alpha_H - \alpha_L)(T_2 - T_1)(1+m)^2}\right]$

$t = t_A + t_B$

$m = \frac{t_{High\,\alpha}}{t_{low\,\alpha}}$, t: thickness

$n = \frac{E_{High\,\alpha}}{E_{low\,\alpha}}$, E: modulus of elastisity

T_2 = temp. to be measured

T_1: Bonding temperature

~~for fixed end~~

if $t_A = t_B \longrightarrow m = 1$

$E_A = E_B \longrightarrow n = 1$

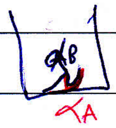
$$r = \frac{2t}{3(\alpha_H - \cancel{3}\alpha_L)(T_2 - T_1)}$$

for fixed end (cantiliver)

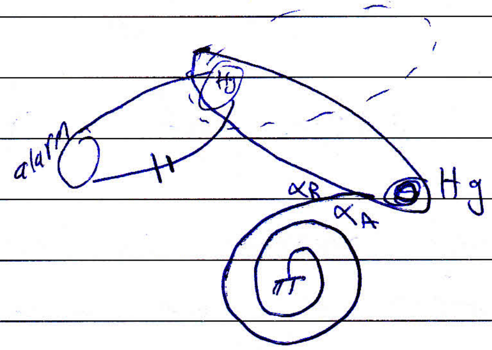
$$r = \frac{t}{2}\left[\frac{1 + \alpha_L(T_2 - T_1)}{(\alpha_H - \alpha_L)(T_2 - T_1)}\right]$$

Application

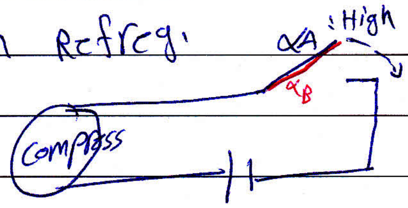
1- vending machine



2- fire alarm



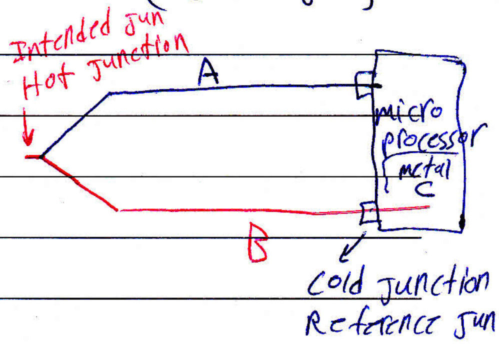
3- thermostat in Refreg.



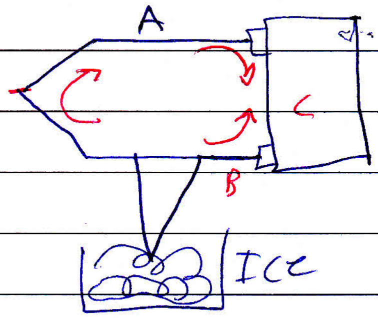
Thermocouple (different metals) (different temp.)

basic: seebeck ~~voltage~~ effect theory: temp can be measured by converting it Electromotive force (voltage)

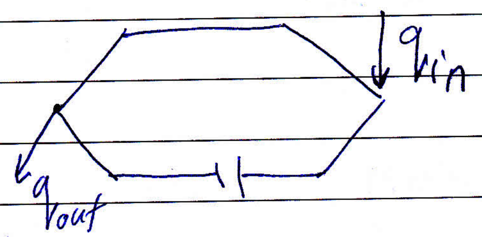
1- calibration: from tables get the voltage of AC and BC and subtract them from the processor reading



2-



Thermoelectric Refreg.



seebeck voltage = peltier emf + Thomson emf

Thomson : voltage produces in case of different temper. of the two ends of the wire

$$emf = \cancel{e}\, c\, T$$

c → voltage const

T → temp