First Exam

(1) رياضيات هندسية

17/20

The University of Jordan

Mathematics Department

Math 202 / First Exam

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(1) Solve the differential equation: $\frac{dy}{dx} = \frac{2y + \sqrt{x^2 - y^2}}{2x}$.

$$\frac{dy}{dx} = \frac{2y}{2x} + \frac{\sqrt{x^2 - y^2}}{2\sqrt{x^2}} \Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{1}{2}\sqrt{\frac{x^2 - y^2}{x^2}}$$

$$\frac{dy}{dx} = \frac{y}{x} + \frac{1}{2}\sqrt{1 - \frac{y^2}{x^2}} \Rightarrow u = \frac{y}{x} \Rightarrow y' = u + xu'$$

3.5

$$y' = u + \frac{1}{2}\sqrt{1 - u^2} \Rightarrow \cancel{u} + xu' = \cancel{u} + \frac{1}{2}\sqrt{1 - u^2}$$

$$x\frac{du}{dx} = \frac{1}{2}\sqrt{1 - u^2} \Rightarrow \int \frac{du}{\sqrt{1 - u^2}} = \int \frac{x}{2} dx \Rightarrow \sin^{-1}(u) = \frac{x^2}{2} + c$$

$$\sin^{-1}\left(\frac{y}{x}\right) = \frac{x^2}{2} + c$$

(2) Solve the (I.V.P): $(\cos 2y - \sin x)dx - 2\tan x \sin 2y \, dy = 0$, $y(0) = 0$.

M N

$$M_y = -2\sin 2y, \quad N_x = -2\sin 2y \sec^2 x \quad \therefore \text{Not exact}$$

$$R = \frac{M_y - N_x}{N} = \frac{-2\sin 2y + 2\sin 2y \sec^2 x}{-2\tan x \sin 2y} = \frac{-\tan^2 x}{\tan x} = -\tan x$$

$$F(x) = e^{\int -\tan x \, dx} = e^{\ln \cos x} = \cos x$$

3.5

$$(\cos 2y \cos x - \sin x \cos x)\, dx - 2\sin x \sin 2y \, dy = 0$$

$$M_y = -2\sin 2y \cos x \qquad N_x = -2\sin 2y \cos x \quad \text{— exact}$$

$$U_x = \cos 2y \cos x - \sin x \cos x \Rightarrow U_x = \cos 2y \cos x - \frac{1}{2}\sin 2x + g'(y)$$

$$U = \cos 2y \sin x + \frac{\cos 2x}{4} + g(y) \qquad U_y = N$$

$$U_y = -2\sin 2y \sin x + g'(y) \qquad -2\sin 2y \sin x + g'(y) = \text{?}$$

variation of paramet.

(3) Find the general solution of: $y'' + 2y' + 2xy = (2x-1)y + e^{-x}\ln x$.

$$y'' + 2y' = (2x-1)y - 2xy + e^{-x}\ln x$$

$$y'' + 2y' + y = 2xy \cancel{- 2xy} + e^{-x}\ln x$$

3

$$r^2 + 2r + 1 = 0$$

$$\cancel{\alpha = -1 \quad \beta = \frac{\sqrt{|D|}}{2a}}$$

$$D = 0$$

$$r = \frac{-b}{2a} = \frac{-2}{2} = -1$$

$$y_h = \underbrace{c_1 e^{-x}}_{y_1} + \underbrace{c_2 x e^{-x}}_{y_2}$$

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(4) Let $y'' - y = 4\sinh x + \frac{x}{\sec^2 x}$. Determine a suitable form for the particular solution y_p if the method of undetermined coefficients is to be used. $\underline{\underline{y_p}}$

$$r^2 - 1 = 0 \Rightarrow r^2 = 1 \Rightarrow r = \pm 1$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

$$y = y_h + y_p \Rightarrow y_h = c_1 e^{x} + c_2 e^{-x}$$

3

$$y_p : y'' - y = 4\sinh x + \frac{x}{\sec^2 x} = 2e^{x} - 2e^{-x} + x\cos^2 x$$

$$\boxed{y_{p_1} = Ae^{x} - be^{-x} \qquad y_{p_2} = \frac{1}{2}x\cos 2x + \frac{x}{2}}$$

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(5) Find the general solution of: $y'' + (y')^2 = 2y'e^{-2y}$.

$$y'' + y(y')^4 = 0$$

$$u = y' \quad , \quad u\frac{du}{dy} = y''$$

$$u\frac{du}{dy} + yu^4 = 0$$

$$\frac{du}{dy} = -yu^3$$

$$\int \frac{du}{u^3} = \int -y\,dy$$

$$\frac{u^{-2}}{-2} = \frac{-y^2}{2} + c$$

4

$$\frac{1}{-2(y')^2} = \frac{-y^2}{2} + K$$

$$2 = 2(y')^2 y^2$$

$$(y')^2 = \frac{1}{y^2}$$