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The University of Jordan — First Exam — 06/11/2013

Mathematics Department — Engineering Math. I (0301202)

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(I) Find the solution of

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{x}{y}+\frac{y}{x}\right), \qquad y(1)=1$$

let $u=\frac{y}{x}$

$y=ux \qquad \frac{dy}{dx}=\frac{du}{dx}x+u$

$$u+x\frac{du}{dx}=\frac{1}{2}\left(\frac{1}{u}+u\right)$$

$\boxed{\text{seperable}}$

$$x\frac{du}{dx}=\frac{1}{2}\left(u+\frac{1}{u}\right)-u$$

$\frac{2}{u}-\frac{u}{2}$

$$\frac{du}{\frac{1}{2}(u+\frac{1}{u})-u}=\frac{dx}{x} \rightarrow \frac{du}{\frac{u}{2}+\frac{2}{u}-u}=\frac{dx}{x}$$

$$\int\frac{2u\,du}{4-u^2}=\int\frac{dx}{x}$$

$$-\ln(4-u^2)=\ln x$$

$$-\ln\left(4-\left(\frac{y}{x}\right)^2\right)=\ln x$$

$4\frac{1}{2}$

$$\left(\frac{\ln 4}{\ln\left(\frac{x}{y}\right)^2}\right)^{-1}=\ln x.$$

$C=?$

$$2(\ln x-\ln y)=\ln x\ln 4$$

$$\ln y=2\ln x-\ln x\ln 4$$

$\ln y=\ln x(2-\ln 4)$

$$\boxed{y=x(2-\ln 4)}$$

(2) **Solve**

$$\frac{dy}{dx} \cancel{=} -\frac{e^x+3y^2}{y^3+2e^x y}$$

$$\underbrace{+(e^x+3y^2)}_{M}\,dx + \underbrace{(y^3+2e^x y)}_{N}\,dy = 0$$

Check exactness

$M_y = 6y$

$N_x = 2y e^x$

$\therefore M_y \neq N_x$

$\Rightarrow$ can be made exact

I.F

$$R(x) = \frac{M_y - N_x}{N} = \frac{6y - 2ye^x}{y^3+2e^x y} = \frac{\cancel{y}(6-2e^x)}{\cancel{y}(y^2+2e^x)}$$

$$\cancel{R(y) = \frac{N_x - M_y}{M} = \frac{2ye^x - 6y}{e^x+3y^2}}$$

$$\frac{(e^x+3y^2)(6y-2e^x y)}{y^3+2e^x y}\,dx + \frac{\cancel{(y^3+2e^x y)}(6y-2e^x y)}{\cancel{y^3+2e^x y}}\,dy = 0$$

$$\frac{(e^x+3y^2)(6-2e^x)}{y^2+2e^x}\,dx + (6y-2e^x y)\,dy = 0$$

$$\frac{d\phi}{dy} = N$$

$$\phi(x,y) = \int N\,dx = \int 6y - 2e^x y\,dx + g(y) = 2ye^x + g(y)$$

$$\frac{d\phi}{dx} = M = 2e^x + g'(y) = \frac{(e^x+3y^2)(6-2e^x)}{y^2+2e^x}$$

(3) **Find the general solution of**

$$y'' + (y')^2 = 2e^{-y}$$

missing $\underline{\underline{x}}$ $\qquad u = y' \qquad y'' = u\frac{du}{dy}$

$$u\frac{du}{dy} + u^2 = 2e^{-y} \qquad \text{divide By } u$$

$$\dot{u} + u = 2e^{-y}u^{-1} \qquad \text{Bernoulli}$$

$P(y) = 1$

$Q(y) = 2e^{-y}$ $\qquad \boxed{n=-1}$

Let $v = u^{1--1} = u^2$

$v' = 2u\,\dot{u}$

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$\rightarrow \quad v' + \frac{v2}{2} = 2e^{-y}$ --- linear

$$v = e^{-\int \frac{2}{2}dy}\left[\int e^{\int \frac{2}{2}dy} \cdot 2e^{-y}\,dy + C\right]$$

$$= e^{-\frac{2y}{2}}\left[\int e^{\frac{2y}{2}} \cdot 2e^{-y}\,dy + C\right] = e^{-\frac{2y}{2}}\left[\int 2e^{\frac{y}{2}}\,dy + C\right]$$

$$v = e^{-\frac{2y}{2}} \cdot \left(2e^{\frac{y}{2}} + C\right) \qquad v = 2e^{-y} + e^{-2y}C = u^2$$

but $v = u^2$

$\cancel{u^2 = e^{-\frac{y}{2}}\left(-4e^{-\frac{y}{2}} + c\right)}$ $\qquad y' = \sqrt{2e^{-y} + ce^{-2y}}$

$\cancel{u = e^{-\frac{y}{4}}\left(-4e^{-\frac{y}{2}} + c\right)^{\frac{1}{2}}}$ $\qquad$ if $c = 0 \rightarrow$

$\cancel{y' = }$ $\qquad y' = \sqrt{2}\,e^{-\frac{y}{2}} \rightarrow y = \frac{\sqrt{2}\,e^{-\frac{y}{2}}}{-\frac{1}{2}}$

if $c \neq 0$

(4) **Given that** $y_1 = e^x$ **is a solution for**

$$xy'' - (x+1)\,y' + y = 0$$

Find the general solution of

$$xy'' - (x+1)\,y' + y = x^2$$

$$y'' - \left(\frac{x+1}{x}\right) y' + \frac{y}{x} = 0$$

$$P(x) = -\frac{x+1}{x}$$

Abel's formulla

$$y_2 = y_1 \int \frac{e^{+\int \frac{x+1}{x}\,dx}}{y_1^2}\,dx$$

$$y_2 = e^x \int \frac{e^{(\ln x + x)}}{e^{2x}}\,dx = e^x \int e^{(+\ln x + x - 2x)}\,dx$$

$$= \frac{e^{\ln x + x - x}}{\frac{1-x}{x}} = \frac{x \cdot x}{1-x} = \frac{x^2}{1-x}$$

$$y_g = c_1 e^x + c_2 \frac{x^2}{1-x} + y_p$$

$$\cancel{y_p = x^2 = (a_0 + a_1 x + a_2 x^2) * x^3}$$

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$$y_g = c_1 e^x + c_2 \frac{x^2}{1-x} + (a_0 + a_1 x + a_2 x^2)\, x^3$$

(5) **Determine a suitable form for the particular solution if the method of undetermined coefficients is to be used for the D.E.**

$y'' + 2y' + 2y = 3e^{-x} + 2x^2e^{-x} \cos x$

roots

$r^2 + 2r + 2 = 0$

$r_1 = \frac{-2 + \sqrt{4 - 4*1*2}}{2}$

$= -1 + \frac{\sqrt{-8}}{2}$

$= -1 + i$

$r_2 = -1 - i$

$a = -1$ $\boxed{b = 1}$

$y_h = c_1 y_1 + c_2 y_2$

$= c_1 e^{-x} \cos x + c_2 e^{-x} \sin x$

y_p :-

y_{p_1} :- $\lambda\, 3e^{-x}$ $\cancel{+ 2x^2 e^{-x}}$ $* \underline{x}$

$= \cancel{(3 + 2x^2) e^{-x}}$ يوجد تشابه

y_{p_2} :- $2x^2 e^{-x} \cos x$

$= (a_0 + a_1 x + a_2 x^2)(e^{-x})(A\cos x + B \sin x)$ $* \underline{x}$

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$y_p = 3\lambda e^{-x} * x + (a_0 + a_1 x + a_2 x^2)(x e^{-x})(A\cos x + b\sin x).$

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