$16 - 4(3) - 4$ $\pm i$

*** Fill in the blank** **(8 points)**

Q1. A second order homogeneous linear differential equation with the general solution $y(x) = c_1 e^{-2x} \sin x + c_2 e^{-2x} \cos x$ is $y'' + 4y' + 5y = 0$

$-2 \pm i$

$\lambda^2 - \lambda(-2+i) - (-4+i^2)$

Q2. The general solution of $y'' = \frac{y'}{x}$ is $y = e^c \frac{x^2}{2} + c_2$

$y' = z$, $y'' = z'$, $\frac{dz}{dx} = \frac{z}{x}$, $\ln|z| = \ln|x| + c$

Q3. An integrating factor that makes the differential equation $\left(2 + \frac{3}{x}\right)\cos y\,dx - \sin y\,dy = 0$ exact is $e^{2x} \cdot x^3$

Q4. Given the I.V.P. $xy^2 \frac{dy}{dx} = y^3 - x^3$, $y(1) = A$. If the general solution of the I.V.P. is $y^3 + 3x^3 \ln|x| = 8x^3$, then the value of A is 2

$y^3 + 0 = 8$

$y(1) = 2$

*** Circle all possible answers. (4 points)**

Q5. Which of the following sets of functions is linearly dependent?

$\frac{3\ln x}{\ln x} =$

a) $e^{3x}, e^{3x} + 3$

b) $\sin 3x, \cos(3x + 2\pi)$

c) $\ln x, \ln x^3$

d) None

Q6. Which of the following differential equations is a Bernoulli's equation?

a) $x^3 y' + 5xy^2 = y^3$

b) $\frac{dy}{dx} = \frac{1 - xy^2}{2x^2 y}$

c) $(y^2 - 3xy^2)dy + (5x^2 + 2y^3)dx =$

d) $y' = \sin x - (\tan x)y + y^2$

$y' - \frac{1}{2x}$

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Q7. (6 points) Use undetermined coefficients method to write a suitable form of a particular solution y_p of the differential equation

$$y^{(4)} + 2y''' + 2y'' = 7 - 3e^{-x}\left(\cos\frac{2x}{2}\right)$$

$\cos^2 x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$\cos^2\frac{x}{x} = \frac{\cos x}{2} + \frac{1}{2}$

$y^{(4)} + 2y''' + 2y'' = 0$

$\lambda^4 + 2\lambda^3 + 2\lambda^2 = 0$

$(\lambda)(\lambda)(\lambda^2 + 2\lambda + 2) = 0$

$\lambda = 0$

$\lambda = 0$

$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-2 \pm \sqrt{4-8}}{2}$

$= -1 \pm i$

$y_p = K + K_1 e^{-x}(A\cos x + B\sin x + K_2)$

this is wrong

$$\boxed{y_p = Kx^2 + K_1 e^{-x} . A\cos x . x + K_1 e^{-x} . B\sin x . x + K_1 K_2 e^{-x}}$$

$y_h = C_1 + C_2 x + C_3 e^{-x}\cos x + C_4 e^{-x}\sin x$

Q8. (5 points) Solve the I.V.P. $(x^2y + xy - y)dx + (x^2y - 2x^2)dy = 0$

$y(1) = 1$

$M_y = x^2 + x - 1$

$N_x = 2xy - 4x$

Not exact, Not reduced to exact

$y(x^2 + x - 1)dx + x^2(y-2)dy = 0$

$y(x^2 + x - 1)dx = (2-y)x^2 dy$

$\frac{(x^2 + x - 1)dx}{x^2} = \left(\frac{2-y}{y}\right)dy$

5

$(1 + x^{-1} - x^{-2})dx = \left(\frac{2}{y} - 1\right)dy$

$x + \ln|x| + \frac{1}{x} = 2\ln|y| - y + C$

$y(1)=1 \quad 1 + 0 + 1 = 2\ln(1) - 1 + C$

$2 = 0 - 1 + C \Rightarrow C = 3$

$\Rightarrow \boxed{x + \ln|x| + \frac{1}{x} = 2\ln|y| - y + 3}$

Q9. (7 points) Solve $yy'' - (y')^2 = y^2 y'$

- X is missing

$$\boxed{\begin{aligned} y' &= z \\ y'' &= z\frac{dz}{dy} \end{aligned}}$$

$$\Rightarrow yzz' - z^2 = y^2 z$$

$$z' - \left(\frac{1}{y}\right)z = y$$

$$z' + P(y)z = r(y) \quad , \ P(y) = \frac{-1}{y}, \ r(y) = y$$

$$z = \frac{1}{e^{\int P(y)dy}}\left(\int e^{\int P(y)dy} \cdot r(y) + c\right)$$

$$e^{\int P(y)dy} = e^{\int \frac{1}{y}dy} = y$$

$$z = \frac{1}{y}\left(\int (y \cdot y)dy + c\right)$$

$$z = \frac{1}{y}\left(\frac{y^3}{3} + c\right)$$

$$z = \frac{1}{3}y^2 + cy^{-1}$$

but $z = y' = \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{3}y^2 + \frac{c}{y}$$

$$\frac{dy}{dx} = \frac{y(y^2)}{y(3)} + \frac{y(c)}{y(y)}$$

$$\frac{dy}{dx} = \frac{y^3}{3y} + \frac{3c}{3y}$$

$$\frac{3y}{y^3+8}$$

$$(y+2)(y^2 - 2y + \ldots$$

$$y^3 \ldots$$

$$3c = d$$

$$\frac{dy}{dx} = \frac{y^3 + d}{3y}$$

$$dx = \frac{3y}{y^3+d}\,dy$$

$$\frac{3y}{y^3+b} = \frac{a}{(y+b)} + k$$

$$\frac{3y}{y^3+d} = \frac{a}{y+d} + \frac{by+k}{(y^2 - dy + d^2)}$$

$$x = \int\left(\frac{a}{y+d} + \frac{(by+k)}{y^2 - dy + d^2}\right)d$$

$$x = a\ln|y+d| + $$

Q9. (7 points) Solve $yy'' - (y')^2 = y^2 y'$

- x is missing

$$\boxed{\begin{aligned} y' &= z \\ y'' &= z\frac{dz}{dy} \end{aligned}}$$

✓

$$\Rightarrow yzz' - z^2 = y^2 z$$