In this example, it would be prudent to limit the current i to approximately 100 μA. Adequate response can be obtained even at these low currents because the sensitivity of a thermistor is so very high. Precise measurements of Δv_o can be made easily with a digital millivoltmeter.

11.4 THERMOCOUPLES

A thermocouple is a simple temperature sensor that consists of two dissimilar materials in thermal contact. The thermal contact, called a junction, may be made by twisting wires together or by welding, soldering, or brazing two materials together. The junctions may also be formed by pressing the two materials together with sufficient pressure. An example of a single thermocouple junction is shown in Fig. 11.12*a*.

The operation of a thermocouple is based on a combination of thermoelectric effects that produce a small open-circuit voltage when two thermocouple junctions are maintained at different temperatures. The classic diagram of the dual-junction thermocouple circuit is shown in Fig. 11.12*b*, where junctions J_1 and J_2 are maintained at temperatures T_1 and T_2, respectively. The thermoelectric voltage v_o is a nonlinear function of temperature that can be represented by an empirical equation having the form

$$v_o = C_1(T_1 - T_2) + C_2(T_1^2 - T_2^2) \tag{11.15}$$

where

C_1 and C_2 are thermoelectric constants that depend on the materials used to form the junctions
T_1 and T_2 are junction temperatures

The generation of the open-circuit voltage indicated by Eq. 11.15 is due to the Seebeck effect (Reference 22), which is produced by diffusion of electrons across the interface between the two materials. The electric potential of the material accepting electrons becomes negative at the interface zone, whereas the potential of the material providing the electrons becomes positive. Thus, an electric field is established by the flow of electrons across the interface. When this electric field becomes sufficient to balance the diffusion forces, a state of equilibrium with respect to electron migration is established. Because the magnitude of the diffusion force is controlled by the temperature of the thermocouple junction, the electric potential developed at the junction provides a measure of the temperature.

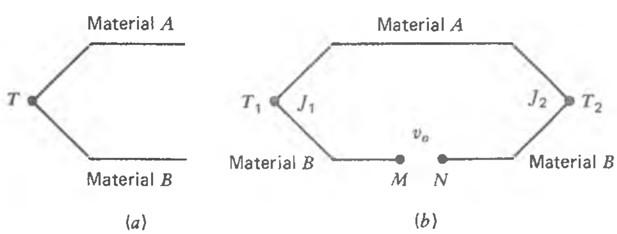

Figure 11.12 Thermocouple sensor and circuit for measuring the temperature difference $T_1 - T_2$. (*a*) Single junction. (*b*) Dual junction.

In addition to the Seebeck effect, two other basic thermoelectric effects occur in a thermocouple circuit. The Peltier effect (Reference 19) and the Thompson effect (Reference 23) are named for the scientists who first observed and explained these thermoelectric phenomena.

The Peltier effect occurs when a current flows in the thermocouple circuit. The presence of the current i in the thermocouple circuit produces the well-known self-heating effect, where the Joule heat transfer is $q = i^2R$. However, the Peltier heat transfer is in addition to the Joule heating effect. The Peltier heat transfer is given by

$$q_P = \pi_{AB}\, i \tag{11.16}$$

where

q_P is the heat transfer in watts (W)
π_{AB} is the Peltier coefficient for the A to B couple

It should be noted that $\pi_{AB} = -\pi_{BA}$ and the Peltier coefficient depends on the direction of current flow through the junction. This fact implies that heat will transfer from the junction to the environment at junction J_1 and from the environment to the junction at junction J_2. This dual-junction heat transfer, illustrated in Fig. 11.13, is the basis of a Peltier refrigerator, which is a cooling device without moving parts.

The Thompson effect is another thermoelectric interaction that affects the behavior of a thermocouple circuit. This effect involves the generation or absorption of heat q_T whenever a temperature gradient and a current exist in a conductor. The Thompson effect, illustrated in Fig. 11.14, results in a quantity of heat q_T being transferred, which is given by

$$q_T = \sigma i(T_1 - T_2) \tag{11.17}$$

where σ is the Thompson coefficient that depends on the conductor material.

Both the Peltier and Thompson effects produce voltages that contribute to the output of a thermocouple circuit and affect the accuracy of the measurement of temperature. Both effects can be minimized by severely limiting the current i that flows through the thermocouple circuit (Fig. 11.12*b*) during the measurement of v_o.

The thermocouple circuit of Fig. 11.12*b* is used to sense an unknown temperature T_1, while junction 2 is maintained at a known reference temperature T_2. Since the reference temperature T_2 is known, it is possible to determine the unknown temperature T_1 by measuring the voltage v_o. It is clear from Eq. 11.15 that the response of a thermocouple is a nonlinear function of the temperature. Also, experience has shown that Eq. 11.15 is not a sufficiently accurate representation of the voltage–temperature rela-

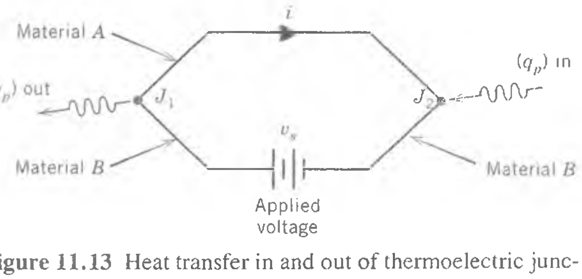

Figure 11.13 Heat transfer in and out of thermoelectric junctions owing to the Peltier effect.

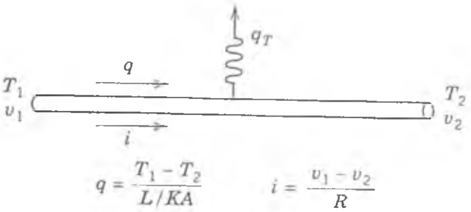

Figure 11.14 Heat transfer from a homogeneous conductor owing to current flow through a temperature gradient.

tionship to be used when precise measurements of temperature are required. Accurate conversion of the output voltage v_o to $(T_1 - T_2)$ is achieved either by using calibration (lookup) tables or by using a higher order polynomial instead of Eq. 11.15. Examples of lookup tables for Chromel-Alumel (Table A.2), Chromel-constantan (Table A.3), copper-constantan (Table A.4), and iron-constantan (Table A.5) thermocouples are presented in Appendix A. It is important to note that the reference temperature is $T_2 = 0°C$ (32°F) in these tables.

The higher order polynomials used for temperature determinations are of the form

$$T_1 - T_2 = a_0 + a_1 v_o + a_2 v_o^2 + \cdots + a_n v_o^n \tag{11.18}$$

where $a_0, a_1, \cdots, a_n$ are coefficients specified for each pair of thermocouple materials, and $T_1 - T_2$ is the difference in junction temperature in °C. The polynomial coefficients for six different types of thermocouples are given in Appendix A (Table A.6).

11.4.1 Principles of Thermocouple Behavior

The practical use of thermocouples is based on the following six operating principles, which are illustrated in Fig. 11.15.

1. A thermocouple circuit must contain at least two dissimilar materials and at least two junctions (Fig. 11.15*a*).
2. The output voltage v_o of a thermocouple circuit depends only on the difference between junction temperatures $(T_1 - T_2)$ and is independent of the temperatures elsewhere in the circuit if no current flows in the circuit (Fig. 11.15*b*).
3. * If a third metal C is inserted into either leg (A or B) of a thermocouple circuit, the output voltage v_o is not affected, provided that the two new junctions (A/C and C/A) are maintained at the same temperature, for example, $T_i = T_j = T_3$ (Fig. 11.15*c*).
4. The insertion of an intermediate metal C into junction 1 does not affect the output voltage v_o, provided that the two junctions formed by insertion of the intermediate metal (A/C and C/B) are maintained at the same temperature T_1 (Fig. 11.15*d*).
5. ** A thermocouple circuit with temperatures T_1 and T_2 produces an output voltage $(v_o)_{1-2} = f(T_1 - T_2)$, and one exposed to temperatures T_2 and T_3 produces an output voltage $(v_o)_{2-3} = f(T_2 - T_3)$. If the same circuit is exposed to temperatures T_1 and T_3, the output voltage $(v_o)_{1-3} = f(T_1 - T_3) = (v_o)_{1-2} + (v_o)_{2-3}$ (Fig. 11.15*e*).

*3) law of Intermediate Metal

**5) Law of Intermediate Temperature

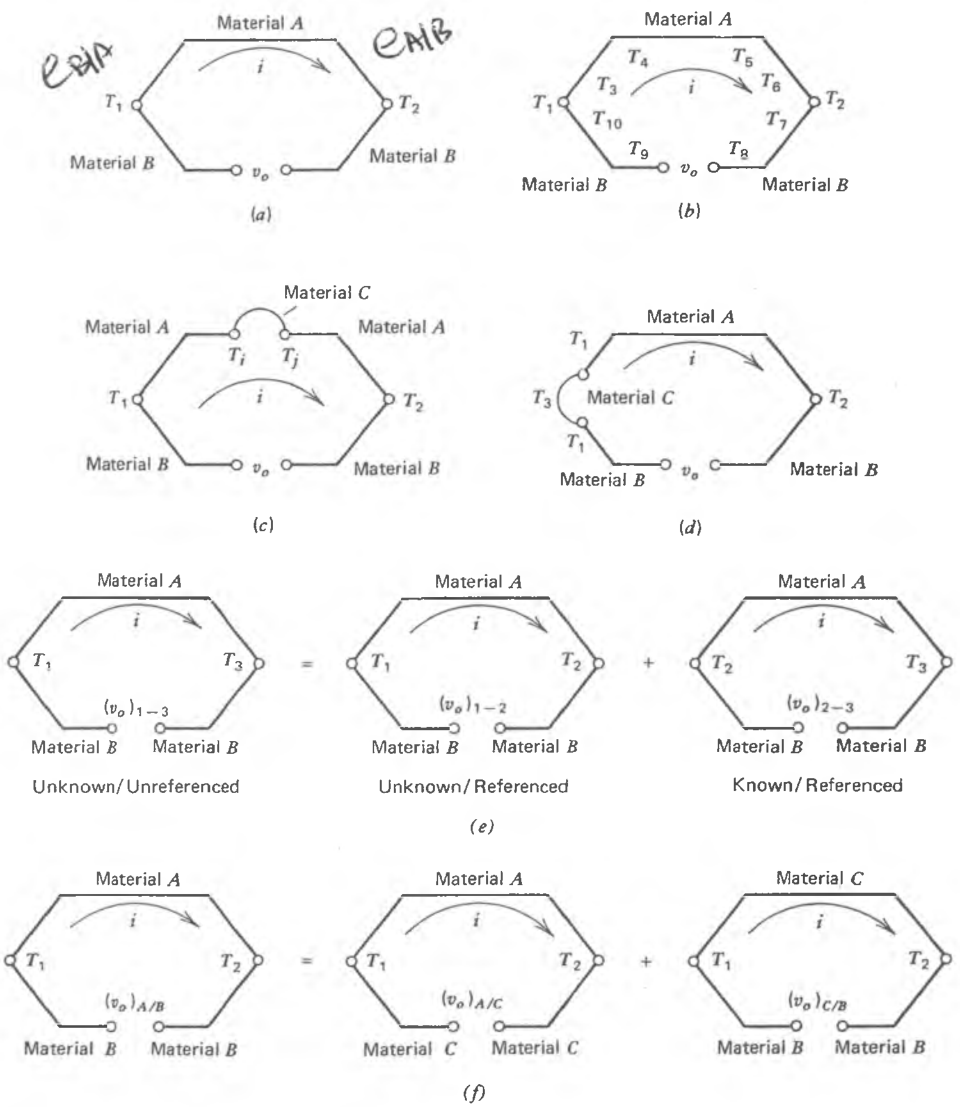

Figure 11.15 Typical situations encountered during use of thermocouples. (*a*) Basic thermocouple circuit. (*b*) Output depends on $(T_1 - T_2)$ only. (*c*) Intermediate metal in circuit. (*d*) Intermediate metal in junction. (*e*) Voltage addition from identical thermocouples at different temperatures. (*f*) Voltage addition from different thermocouples at identical temperatures.

6. A thermocouple circuit fabricated from materials A and C generates an output voltage $(v_o)_{A/C}$ when exposed to temperatures T_1 and T_2, and a similar circuit fabricated from materials C and B generates an output voltage $(v_o)_{C/B}$. Furthermore, a thermocouple fabricated from materials A and B generates an output voltage $(v_o)_{A/B} = (v_o)_{A/C} + (v_o)_{C/B}$ (Fig. 11.15*f*).

The six principles of thermoelectric behavior are important because they provide the basis for the design, circuitry, and application of thermocouples to temperature measurements.

The first principle formalizes the experimental observation that a thermocouple circuit must be fabricated with two different materials so that two junctions are formed. The output voltage v_o has been observed to be a nonlinear function of the difference in temperature $(T_1 - T_2)$ at these two junctions. For clockwise current flow, as illustrated in Fig. 11.15*a*, the output voltage v_o can be expressed as

$$v_o = e_{B/A}T_1 + e_{A/B}T_2 \tag{a}$$

where

- $e_{B/A}$ is the junction potential per unit temperature at a junction as the current flows from material B to material A
- $e_{A/B}$ is the junction potential per unit temperature at a junction as the current flows from material A to material B

Since $e_{B/A} = -e_{A/B}$, Eq. a can be written in its more familiar form as

$$v_o = e_{B/A}(T_1 - T_2) \tag{b}$$

Experiments indicate that the relationship between v_o and the temperature difference $(T_1 - T_2)$, as expressed by Eq. b, depends on the two materials used to fabricate the thermocouple and is nonlinear. Since Eq. b is nonlinear, the junction potential $e_{B/A}$ is not a constant with respect to temperature.

Thermocouple calibration tables, such as Tables A.2, A.3, A.4, and A.5 in Appendix A, are used to relate temperature difference $(T_1 - T_2)$ to a measured output voltage v_o. Since an unknown temperature T_1 is being measured, the reference temperature T_2 must be known. The calibration information presented in Tables A.2, A.3, A.4, and A.5 is based on a reference temperature $T_2 = 0°C$ (32°F). If the reference temperature T_2 is not 0°C, but rather some other known value, such as 100°C, it is still possible to determine T_1, but the procedure involves application of the fifth principle of thermoelectric behavior.

The second principle indicates that the voltage output v_o from a thermocouple circuit is not influenced by the temperature distribution along the conductors except at points where connections are made to form junctions (see Fig. 11.15*b*). This principle provides assurance that the output voltage v_o of the thermocouple circuit is independent of the length of the lead wires and the temperature distribution along their length. The output voltage v_o is determined only by the junction temperatures.

The third principle deals with insertion of an intermediate conductor (such as copper lead wires or a voltage-measuring instrument) into one of the legs of a thermocouple circuit (see Fig. 11.15*c*). The effect of inserting material C into the A-B-type thermocouple can be determined by writing the equation for the output voltage v_o as

$$v_o = e_{B/A}T_1 + e_{A/C}T_i + e_{C/A}T_j + e_{A/B}T_2 \tag{c}$$

Since $e_{B/A} = -e_{A/B}$ and $e_{A/C} = -e_{C/A}$, Eq. c can be written as

$$v_o = e_{B/A}(T_1 - T_2) + e_{A/C}(T_i - T_j) \tag{d}$$

Note, however, that temperature gradients along the length of the lead wires result in heat transfer and affect the junction temperature. Equation d indicates that the effect of the A/C junctions is eliminated if $T_i = T_j$. A similar analysis with the third metal

C inserted in leg B of the thermocouple shows that the effect of B/C junctions is eliminated if $T_i = T_j$. The third principle verifies that insertion of a third material C into the circuit will have no effect on the output voltage v_o, provided that the junctions formed in either leg A or leg B are maintained at the same temperature $T_i = T_j = T_3$.

The fourth principle deals with insertion of an intermediate metal into a junction during fabrication or use of a thermocouple. Such a situation occurs when junctions are formed by twisting the two thermocouple materials A and B together and soldering or brazing the connection with an intermediate metal C (see Fig. 11.15*d*). The influence of the presence of the intermediate metal in the junction can be evaluated by considering the expression for output voltage v_o, which can be written as

$$v_o = e_{B/C}T_1 + e_{C/A}T_1 + e_{A/B}T_2 \tag{e}$$

Since $e_{C/A} = e_{C/B} + e_{B/A}$, Eq. e reduces to

$$v_o = e_{B/A}(T_1 - T_2) \tag{f}$$

Equation f verifies that the output voltage v_o is not affected by the presence of a third material C inserted during fabrication of the thermocouple if the two junctions B/C and C/A are at the same temperature.

The fifth principle deals with the relationship between output voltage v_o and the reference junction temperature. As mentioned previously, Tables A.2, A.3, A.4, and A.5 are based on a reference temperature of 0°C (32°F). In some instances, it may be more convenient to use a different reference temperature (for example, boiling water at 100°C). The effect of this different reference temperature can be accounted for by using the equivalent thermocouple system, illustrated in Fig. 11.15*e*. The output from the equivalent system, which incorporates two thermocouple circuits, is

$$(v_o)_{1-3} = f(T_1 - T_3) = (v_o)_{1-2} + (v_o)_{2-3} \tag{11.19}$$

Use of Eq. 11.19 for the case of an arbitrary reference temperature T_3 can be illustrated by considering the example of a copper-constantan thermocouple exposed to an unknown temperature T_1. Assume that the arbitrary reference temperature T_3 is maintained at 100°C and that an output voltage $(v_o)_{1-3} = 8.388$ mV is recorded under these conditions. The voltage $(v_o)_{2-3}$ of Eq. 11.19 can be determined from Table A.4, because it is known that $T_2 = 0$°C and $T_3 = 100$°C. Thus, $(v_o)_{2-3} = -(v_o)_{3-2} = -4.277$ mV. Solving Eq. 11.19 for $(v_o)_{1-2}$ yields

$$\begin{aligned}(v_o)_{1-2} &= (v_o)_{1-3} - (v_o)_{2-3} \\ &= 8.388 - (-4.277) = 12.665 \text{ mV}\end{aligned}$$

Table A.4 indicates that an output voltage $(v_o)_{1-2} = 12.665$ mV would be produced by a temperature $T_1 = 261.7$°C. The same procedure can be used to correct for any known reference temperature.

The sixth principle illustrates the use of voltage addition (superposition) to analyze thermocouple circuits fabricated from different materials, as shown in Fig. 11.15*f*. The output voltage for the equivalent circuit is

$$(v_o)_{A/B} = (v_o)_{A/C} + (v_o)_{C/B}$$

or

$$(v_o)_{A/B} = (v_o)_{A/C} - (v_o)_{B/C} \tag{11.20}$$

By employing this principle, calibration tables can be developed for any pair of materials if the calibrations for the individual materials are paired with a standard thermocouple material, such as platinum. For example, materials A and B, when paired with the standard material C, would provide $(v_o)_{A/C}$ and $(v_o)_{C/B} = -(v_o)_{B/C}$. The calibration for a junction formed by using materials A and B could then be determined by using Eq. 11.20. Use of this principle eliminates the need to calibrate all possible combinations of materials [for n thermoelectric materials, $n(n-1)$ calibrations are necessary]. Instead, by calibrating all n materials against the standard reference material, platinum, only $(n-1)$ calibrations are required.

11.4.2 Thermoelectric Materials

The thermoelectric effect occurs whenever a thermocouple circuit is fabricated from any two dissimilar metals; therefore, a large number of materials are suitable for use in thermocouples. In most instances, materials are selected to:

1. Provide long-term stability at the upper temperature levels.
2. Ensure compatibility with available instrumentation.
3. Minimize cost.
4. Maximize sensitivity over the range of operation.

The sensitivities of several materials in combination with platinum are presented in Table 11.2. The results from Table 11.2 can be used to determine the sensitivity S at 0°C (32°F) of a thermocouple fabricated from any two materials listed in the table. For instance, the sensitivity of a Chromel-Alumel thermocouple is determined from Eq. 11.20 as

$$S_{\text{Chromel/Alumel}} = 25.8 - (-13.6) = 39.4\ \mu\text{V/°C}$$

Table 11.2 Thermoelectric Sensitivity S of Several Materials in Combination with Platinum at 0°C (32°F)

Material	Sensitivity S µV/°C	Sensitivity S µV/°F	Material	Sensitivity S µV/°C	Sensitivity S µV/°F
Bismuth	−72	−40	Copper	+6.5	+3.6
Constantan	−35	−19.4	Gold	+6.5	+3.6
Nickel	−15	−8.3	Tungsten	+7.5	+4.2
Alumel	−13.6	−7.6	Nicrosil	+15.4	+8.6
Nisil	−10.7	−5.9	Iron	+18.5	+10.3
Platinum	0	0	Chromel	+25.8	+14.3
Mercury	+0.6	+0.3	Germanium	+300	+167
Carbon	+3	+1.7	Silicon	+440	+244
Aluminum	+3.5	+1.9	Tellurium	+500	+278
Lead	+4	+2.2	Selenium	+900	+500
Silver	+6.5	+3.6			

Table 11.3[a] Sensitivity as a Function of Temperature for Seven Types of Thermocouples (mV/°C)

Temperature (°C)	E	J	K	N	R	S	T
−200	25.1	21.9	15.3	9.9	—	—	15.7
−100	45.2	41.1	30.5	20.9	—	—	28.4
0	58.7	50.4	39.5	26.1	5.3	5.4	38.7
100	67.5	54.3	41.4	29.7	7.5	7.3	46.8
200	74.0	55.5	40.0	33.0	8.8	8.5	53.1
300	77.9	55.4	41.4	35.4	9.7	9.1	58.1
400	80.0	55.1	42.2	37.0	10.4	9.6	61.8
500	80.9	56.0	42.6	—	10.9	9.9	—
600	80.7	58.5	42.5	—	11.3	10.2	—
700	79.8	62.2	41.9	—	11.8	10.5	—
800	78.4	—	41.0	—	12.3	10.9	—
900	76.7	—	40.0	—	12.8	11.2	—
1000	74.9	—	38.9	—	13.2	11.5	—

[a]From NBS Monographs 125 (1974) and 161 (1978).

It is important to recall that the sensitivity S of a thermocouple is not constant; the output voltage v_o is a nonlinear function of the difference in junction temperatures $(T_1 - T_2)$. Sensitivity S as a function of temperature for the seven most frequently used material pairs is listed in Table 11.3.

The letters E, J, K, N, R, S, and T are designated by the ANSI standards (Reference 1). The material pairs used in these thermocouple junctions are defined in Table 11.4.

The voltage output v_o as a function of temperature for several popular types of thermocouples is shown in Fig. 11.16. This graphic display shows that E-type (Chromel-constantan) thermocouples generate the largest output voltage at a given temperature; unfortunately, they have an upper temperature limit of only 1000°C (1832°F). The upper limit of the temperature range is increased (but the sensitivity is decreased) to 1260°C (2300°F) with K-type (Chromel-Alumel) thermocouples; to 1538°C (2800°F) with S-type (platinum 10 percent rhodium-platinum) thermocouples; and to 2800°C (5072°F) with G-type (tungsten-tungsten 26 percent rhenium) thermocouples. The operating temperature ranges, together with the span of output voltages, for most of the popular types of thermocouples are listed in Table 11.5.

Long-term thermal stability is an important property of a thermocouple installation if temperature is to be monitored over very long periods of time. A relatively new

Table 11.4 Materials Employed in the Standard Thermocouples

Type	Positive Material	Negative Material
E	Chromel	Constantan
J	Iron	Constantan
K	Chromel	Alumel
N	Nicrosil	Nisil
R	Platinum 13% Rhodium	Platinum
S	Platinum 10% Rhodium	Platinum
T	Copper	Constantan