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The University of Jordan
Department of Mathematics
First Exam

Math 202 14/03/2018

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★ **Fill in the blank** (8 points)

Q1. An integrating factor of the D.E. $\underbrace{(3xy+y^2)}_{M}dx + \underbrace{(x^2+xy)}_{N}dy = 0$ is ....X....

$\frac{\partial M}{\partial y} = 3x+2y$, $\frac{\partial N}{\partial x} = 2x+y$

$R(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{3x+2y-2x-y}{(x^2+xy)} = \frac{(x+y)}{x(x+y)} = \frac{1}{x}$

$F(x) = e^{\int P(x)\,dx} = e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x$

Q2. If $W(f,g)(x) = 2$, then $W(2f,5g)(x) =$ ....20........

$w(f(x), g(x)) = 2$

$w(2f(x), 5g(x)) =$

$w = \begin{vmatrix} f(x) & g(x) \\ f'(x) & g'(x) \end{vmatrix}$ $\boxed{= f(x)g'(x) - f'(x)g(x) = 2}$

$w(2f(x), 5g(x)) = \begin{vmatrix} 2f(x) & 5g(x) \\ 2f'(x) & 5g'(x) \end{vmatrix}$

$= 10f(x)g'(x) - 10f'(x)g(x) =$

$= 10(f(x)g'(x) - f'(x)g(x))$

$= 10(2) = \boxed{20}$

Q3. A third-order linear homogeneous D.E. with solution $y(x) = c_1 + c_2x + c_3e^{4x}$ is ....$y''' - 4y'' = 0$.....

$\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 4$

$(\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) = 0$

$\lambda(\lambda - 0)(\lambda - 4) = 0$

$\lambda^2(\lambda - 4) = 0$

$\lambda^3 - 4\lambda^2 = 0$

$\boxed{y''' - 4y'' = 0}$ O.D.E

Q4. If $y' + \frac{1}{x}y = xy^{2a-1}$ is linear, then the set of all values of the constant a is ....$(\frac{1}{2}, \infty)$

~~$u = y^{2-2a}$~~ ... (crossed-out work)

$2a - 1 > 0$ Bernuli

$2a > 1$

$\boxed{a > \frac{1}{2}}$

1

★ **True or False? Justify your answer !** (12 points)

Q5. The function $y = e^x$ is a solution of the problem $y'' - y = 0, \quad y(0) = 2$

2

$y = e^x$
$y' = e^x \Rightarrow e^x - e^x = 0$
$y'' = e^x$
$0 = 0$ that's true
but $y(0) \neq 2 \Rightarrow y(0) = e^0 \Rightarrow 2 \neq 1$
So it's false because the initial value is incorr

Q6. The equation $y' = 1 + x + y + xy$ is separable

2

$\frac{dy}{dx} = 1 + x + y(1+x) \Rightarrow$ ~~$y' = (1+x)y = 1+x$~~

$dy = ((1+x) + y(1+x))dx \Rightarrow dy = (1+x)(1+y)\,dx \Rightarrow \boxed{\frac{1}{1+y}dy = (1+x)\,dx}$

it's in form $F(x).dx = g(y)\,dy$ so it's seperable (true)

Q7. If y_1 and y_2 are any two linearly independent solutions of $yy'' + (y')^2 = 0$, then $y(x) = c_1y_1 + c_2y_2$ is also a solution

2

~~y missed~~ ~~x missed~~

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Q8. Let f, g be two functions such that $W(f,g)(x) = x\sin^2 x$, then f, g are linearly dependent.

2

it's false because $W(f,g)(x) \neq 0$ so it's independent

Q9. The D.E $\underbrace{\left(x^3 + \frac{y}{x}\right)}_{M} dx + \underbrace{(y^2 - \ln x)}_{N} dy = 0$ is exact

2

$\frac{\partial M}{\partial y} = \frac{1}{x}$, $\frac{\partial N}{\partial x} = \frac{-1}{x} \Rightarrow \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ so it's Not exact

(false)

Q10. The D.E. $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 + x^2}$ is homogeneous !

1

$(y^2+x^2)dy = (x^2+2xy)\,dx$

$(x^2+2xy)dx - (y^2+x^2)dy = 0$

$\Rightarrow r(x) = 0$

So it's homogenous

(True)

Q11. 5 (5 points)

(I) Solve $y'' + 2y' + 2y = 0$

let $y = e^{\lambda x}$

$\Rightarrow \lambda^2 + 2\lambda + 2 = 0$

$\lambda = \frac{-b \mp \sqrt{b^2 - 4(a)(c)}}{2(a)}$

$\lambda = \frac{-2 \mp \sqrt{4 - 4(1)(2)}}{2}$

$\lambda = \frac{-2 \mp \sqrt{-2}}{2}$

$\lambda = -1 \mp i$

$y_1 = e^{-x} \sin x$

$y_2 = e^{-x} \cos x$

$y(x) = e^{-x}(c_1 \sin x + c_2 \cos x)$

(II) Consider $y'' + 2y' + 2y = \pi + xe^{-x} \sin x$. Write a suitable form for the particular solution if the undetermined coefficients is to be used.

Step I: $y_h(x)$

let $\Rightarrow y = e^{\lambda x}$

$\boxed{\begin{array}{l} y_1 = e^{-x} \sin x \\ y_2 = e^{-x} \cos x \end{array}}$

from part (I)

Step II: $y_p(x)$

$y_p(x) = A + x(Bx + C)(D \sin x + F \cos x) e^{-x}$

Q12. Solve $y'' - \frac{2}{x^2}y = 1$ (5 points)

multiply by x^2

$\Rightarrow x^2y'' - 2y = x^2$

let $y = x^m$ Step I : $y_h(x)$

$y' = m x^{m-1}$

$y'' = m(m-1)x^{m-2}$

$\Downarrow$

Substitute

$m(m-1)x^m - 2x^m = 0$

$\cancel{x^m}$

$m(m-1) - 2 = 0$

$m^2 - m - 2 = 0$

$(m-2)(m+1) = 0$

$m = 2$, $m = -1$

$y_1 = x^2$, $y_2 = x^{-1}$

$y(x) = c_1x^2 + c_2x^{-1}$

Step II : $y_p(x)$

~~$x^2y'' - 2y = x^2$~~

~~let $y_p(x) = Ax^2$~~

~~$y_p'(x) = 2Ax$~~

~~$y_p''(x) = 2A$~~

~~Substitute~~

~~$2Ax^2 - 2Ax^2 = x^2$~~

$y_p(x) = -y_1 \int \frac{y_2 \cdot r(x)}{W} \, dx + y_2 \int \frac{y_1 \cdot r(x)}{W}$

$W(x^2, x^{-1}) = \begin{vmatrix} x^2 & x^{-1} \\ 2x & -x^{-2} \end{vmatrix} = -1 - 2 = -3$

$y_p(x) = -x^2 \int \frac{x^{-1} \cdot 1}{-3} \, dx + x^{-1} \int \frac{x^2 \cdot 1}{-3} = \frac{1}{3}x^2(\ln x) - \frac{1}{3}x^{-1} \cdot \frac{x^3}{3}$

$\boxed{y_p(x) = \frac{1}{3}x^2 \ln x - \frac{1}{9}x^2}$

Step III :

$y(x) = y_h(x) + y_p(x)$

$\boxed{y(x) = c_1x^2 + c_2x^{-1} + \frac{1}{3}x^2 \ln x - \frac{1}{9}x^2}$

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Q.7 : $yy'' + (y')^2 = 0$

~~X-missed~~ it's false because it's non linear O.D.E

$\Downarrow$

prove :

X-missed : let $u = y' \Rightarrow u' = \frac{du}{dy} \cdot u$

$u' = \frac{du}{dy} \cdot u \Rightarrow \frac{-u^2}{y} = \frac{du}{dy} \cdot \cancel{u} \Rightarrow +u\, dy = -y\, du$

$\int \frac{1}{u}\, du = \int -\frac{1}{y}\, dy \Rightarrow \ln|u| = -\ln|y| + c$

$\Rightarrow u = e^{-\ln|y| + c} \Rightarrow u = \frac{1}{y} \cdot e^c \Rightarrow u = \frac{k}{y}$

$\Rightarrow y' = \frac{k}{y}$. ~~$\Rightarrow y = k\ln|y| + k_2$~~

~~there is one solution~~

$x' = \frac{y}{k} \Rightarrow$

$\boxed{x = My^2}$

it's one Solution