*The University of Jordan*

**DEPARTMENT OF MATHEMATICS**

27/30

**Eng. Math I**

SECOND EXAM / Summer 2018

Date: 29/07/2018

(11-12 ) :وقـت المحاضـرة (.. ....) :الرقم الجـامعي : الاسم

**Q1 [7 pts].** Find the general solution of:

$$y''' + 8y = -8e^{2x},$$

using the **method of undetermined coefficients.**

$y_h = c_1 e^{\lambda x}$

$\lambda^3 + 8 = 0 \rightarrow \lambda^3 = -8 \rightarrow \boxed{\lambda = -2, -2, -2}$

$y_h = c_1 e^{-2x} + c_2 x e^{-2x} + c_3 x^2 e^{-2x}$

$y_p = Ae^{2x} \rightarrow y_p' = 2Ae^{2x}$, $y_p'' = 4Ae^{2x}$, $y_p''' = 8Ae^{2x}$

$8Ae^{2x} + 8Ae^{2x} = -8e^{2x}$

$16A\cancel{e^{2x}} = -8\cancel{e^{2x}} \rightarrow \boxed{A = \frac{-1}{2}}$

$$\boxed{y = c_1 e^{-2x} + c_2 x e^{-2x} + c_3 x^2 e^{-2x} + \frac{-1}{2} e^{2x}}$$

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**Q2 [6 pts]. (a)** Let $A=\begin{bmatrix}-2&0&0\\-3&2&-2\\-1&5&-3\end{bmatrix}$. Find: **1.** $\det(A)$. **2.** $\det(A^T)$. **3.** $\det(2A)$.

① $\det(A) = \begin{vmatrix}-2&0&0\\-3&2&-2\\-1&5&-3\end{vmatrix} = (-6+10)(-2)-(9-2)(0)+(-15+2)(0)$

$\det(A) = (-8)$

② $\det(A^T) = \det(A) = (-8)$

③ $\det(2A) = 2^3 \det(A) = 8(-8) = -64$ ◡̈

**(b)** Use **Cramer's rule** to find the value of $z$ in the following linear system:

$$\begin{aligned} x-y \quad &= 1 \\ -6x-y-3z &= -3 \\ 2x-y+3z &= 2 \end{aligned}$$

$$\begin{bmatrix}1&-1&0\\-6&-1&-3\\2&-1&+3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\-3\\2\end{bmatrix}$$

$D = \begin{vmatrix}1&-1&0\\-6&-1&-3\\2&-1&3\end{vmatrix} = (-3-3)(1)-(-18+6)(-1)+(6+2)(0)$

$= (-6) - (12) = -18$

$D_3 = \begin{vmatrix}1&-1&1\\-6&-1&-3\\2&-1&2\end{vmatrix} = (-2-3)(1)-(-12+6)(-1)+(6+2)(1)$

$= (-5) - (6) + (8) = (-3)$

$\cancel{z = \frac{D_3}{D} = \frac{+3}{6} = \frac{1}{2}}$

$z = \frac{D_3}{D} = \frac{+3}{+18} = \frac{1}{6}$ ◡̈



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**Q3 [10 pts]. (a)** Find the eigenvalues and the corresponding eigenvectors of:

$$A = \begin{bmatrix} 3 & 2 \\ -2 & 3 \end{bmatrix}.$$

$\det(A - I\lambda) = 0$

$\begin{vmatrix} 3-\lambda & 2 \\ -2 & 3-\lambda \end{vmatrix} = 0 \rightarrow (3-\lambda)(3-\lambda) + 4 = 0$

$9 - 3\lambda - 3\lambda + \lambda^2 + 4 = 0$

$\lambda^2 - 6\lambda + 13 = 0$

$\lambda = \frac{6}{2} \pm \frac{\sqrt{36 - 4*1*13}}{2}$

$\lambda = 3 \pm \frac{4i}{2}$

$\boxed{\lambda = 3 \pm 2i}$

when $\lambda = 3 + 2i$

$\begin{pmatrix} -2i & 2 \\ -2 & -2i \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}$

$-2ix_1 + 2y_1 = 0$

$\boxed{y_1 = ix_1}$

when $\lambda = 3 - 2i$

$\begin{pmatrix} 2i & 2 \\ -2 & 2i \end{pmatrix}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow \vec{v}_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$

$2ix_2 + 2y_2 = 0$

$\boxed{y_2 = -ix_2}$

**(b)** Find the general solution to the following first-order linear system:

$$\frac{dy_1}{dt} = 2y_1 - y_2$$

$$\frac{dy_2}{dt} = 3y_1 - 2y_2$$

$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \longrightarrow \det(A - \lambda I) = 0$

$\begin{vmatrix} 2-\lambda & -1 \\ 3 & -2-\lambda \end{vmatrix} = (2-\lambda)(-2-\lambda) + 3 = -4 \cancel{-2\lambda} + \cancel{2\lambda} + \lambda^2 + 3 = 0$

$\lambda^2 - 1 = 0 \rightarrow \boxed{\lambda = \pm 1}$

when $\boxed{\lambda = 1}$

$\begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow x_1 - y_1 = 0 \longrightarrow \vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$y_1 = x_1$

when $\lambda = -1$

$\begin{pmatrix} 3 & -1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow 3x_2 - y_2 = 0 \longrightarrow \vec{v}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$y_2 = 3x_2$

$y = c_1 e^{t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$y_1 = c_1 e^{t} + c_2 e^{-t}$

$y_2 = c_1 e^{t} + c_2 3 e^{-t}$



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**Q4 [7 pts].** Find a power series solution in powers of $x$ to the given differential equation:

$$y'' + 3xy' - y = 0.$$

$$y = \sum_{n=0}^{\infty} a_n x^n \quad / \quad y' = \sum_{n=1}^{\infty} a_n n x^{(n-1)} \quad / \quad y'' = \sum_{n=2}^{\infty} a_n n(n-1) x^{(n-2)}$$

$$\sum_{n=2}^{\infty} a_n n(n-1) x^{(n-2)} + 3x \sum_{n=1}^{\infty} a_n n x^{(n-1)} - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} a_n n(n-1) x^{(n-2)} + \sum_{n=1}^{\infty} 3a_n n x^n - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\boxed{n = m+2} \qquad \boxed{n = m} \qquad \boxed{n = m}$$

$$\sum_{m=0}^{\infty} a_{m+2}(m+2)(m+1) x^m + \sum_{m=1}^{\infty} 3a_m m x^m - \sum_{m=0}^{\infty} a_m x^m = 0$$

$$2a_2 + \sum_{m=1}^{\infty} a_{m+2}(m+2)(m+1) x^m + \sum_{m=1}^{\infty} 3a_m m x^m - a_0 - \sum_{m=1}^{\infty} a_m x^m = 0$$

$$2a_2 - a_0 + \sum_{m=1}^{\infty} \left(a_{m+2}(m+2)(m+1) + 3a_m m - a_m\right) x^m = 0$$

$$2a_2 - a_0 = 0 \rightarrow \boxed{a_2 = \frac{a_0}{2}} \qquad \boxed{a_{m+2} = \frac{a_m - 3a_m m}{(m+2)(m+1)}} \quad m = 1, 2, 3$$

$$a_3 = \frac{a_1 - 3a_1}{6} \Rightarrow \boxed{a_3 = \frac{-a_1}{3}} \qquad a_4 = \frac{a_2 - 6a_2}{12}$$

$$a_4 = \frac{-5\left(\frac{a_0}{2}\right)}{12} = \boxed{\frac{-5a_0}{24} = a_4}$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

$$y = a_0 + a_1 x + \frac{a_0}{2} x^2 + \frac{-a_1}{3} x^3 - \frac{5a_0}{24} x^4$$

$$y = a_0\left(1 + \frac{x^2}{2} - \frac{5x^4}{24}\right) + a_1\left(x - \frac{x^3}{3} + \dots\right)$$

$$y = a_0 y_1 + a_1 y_2$$



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