Instructor:................ Lecture time:................

4

* **For Questions 1-5, circle the true answer.** (10 points)

Q1. Given $y''' - y' = 2xe^{x} - 5$. If the method of undetermined coefficient is to be used then a suitable form of y_p is

a) $(Ax+B)e^{x}+C$
b) $(Ax+B)e^{x}+Cx$
e) none
c) $(Ax+B)xe^{x}+C$
d) $(Ax+B)xe^{x}+Cx$

2

Q2. If $x=0$ is a regular singular point of $x^2y''+xy'+(2-x)y=0$. Then the indicial equation is

a) $r^2-2=0$
b) $r^2+2=0$
c) $r^2-r-2=0$
d) $r^2+r+2=0$

Q3. The eigenvalues of $A=\begin{bmatrix}-2 & 1\\ -5 & 4\end{bmatrix}$ are

a) 3 and -1
b) 1 and -3
c) 4 and -2
d) $-3\pm i2\sqrt{3}$
e) none

Q4. The eigenvalues of $A=\begin{bmatrix}1 & -2\\ 3 & -4\end{bmatrix}$ are $\lambda_1=-1$ and $\lambda_2=-2$. The eigenvector corresponding to $\lambda_2=-2$ is

a) $\begin{bmatrix}2\\ -3\end{bmatrix}$
b) $\begin{bmatrix}-3\\ 2\end{bmatrix}$
c) $\begin{bmatrix}2\\ 3\end{bmatrix}$
d) $\begin{bmatrix}3\\ 2\end{bmatrix}$
e) none

Q5. Given $y'''-2y''-y'+2y=e^{3x}$. The solution of the corresponding homogeneous equation is $y_h=c_1e^{x}+c_2e^{2x}+c_3e^{-x}$. If the method of variation of parameters is to be used then $W_3(e^{x},e^{2x},e^{-x})$ is

a) $3e^{3x}$
b) e^{3x}
c) $3e^{2x}$
d) $-e^{3x}$
e) none

$a_1 = 0 \qquad a_{n+1} = \dfrac{a_{n-1}}{(n+r+1)(n+r+4)} \qquad 1 \geq n$

$\Rightarrow$

$n=1 \Rightarrow a_2 = \dfrac{a_0}{(r+2)(r+5)}$

$n=2$

$a_3 = \dfrac{a_1}{(r+3)(r+6)} \Rightarrow a_3 = 0$

$n=3$

$a_4 = \dfrac{a_3}{(r+4)(r+6)} = 0 \qquad \therefore r=0$

$y(t) = a_0 + a_1 x^{n+r} + a_2 x^{2+0} + a_3 x^{3+0} + \dots$

for $r=0$

$\Downarrow$

$y(t) = a_0 + \dfrac{a_0 x}{(2)(5)}$

$$\boxed{y(t) = a_0\left(1 + \frac{x}{10}\right)}$$

d

$a_{n+1} = \dfrac{a_{n-1}}{(n+r+1)(n+r+4)} \qquad$ when $r=-3$

$a_{n+1} = \dfrac{a_{n-1}}{(n-2)(n+1)} \qquad n \geq 1$

for $n=1 \Rightarrow a_2 = \dfrac{a_0}{(-1)(2)}$

$a_1 = 0$

for $n=2 \Rightarrow a_3 = \dfrac{a_1}{0}$ does not exist when $n=2$

for $n=3 \quad a_4 = \dfrac{a_3}{(2)(4)}$

$y(t) = a_0 + a_1 X^{n+r} + a_2 x^{n+r} + a_r x^{n+r}$

$y(t) = a_0 - \dfrac{a_0}{2} X^{-2}$

$$\boxed{y(t) = a_0\left(1 - \frac{X^{-2}}{2}\right)}$$

form only

forms $y(t) = a_0 + a_1 X^{n-3} + a_2 X^{n-3} + a_n X^{n-3}$

10

Q7.(12 points) Given $xy'' + 4y' - xy = 0$.

3 a) Show that the given equation has a regular singular point at $x = 0$.

2 b) Determine the indicial equation.

4.5 c) Find the series solution corresponding to the larger root of the indicial equation.

2 1 d) Write ~~the form~~ of the series solution corresponding to the second root of the indicial equation.

ⓐ $a(x) = x \Rightarrow a(x_0) = 0$ So x_0 is singular points

$\lim_{x \to x_0} \frac{p(x)}{a(x)}(x - x_0) = p_0 \Rightarrow \lim_{x \to 0} \frac{4}{x}(x) = 4$ p_0 & q_0 exist

So is regular Singular Points

$\lim_{x \to x_0} \frac{Q(x)}{a(x)}(x - x_0)^2 = q_0 \Rightarrow \lim_{x \to 0} \frac{-x}{x} x^2 = \lim_{x \to 0} -x^2 = 0$

ⓑ $r(r-1) + p_0 r + q_0 = 0$

$\Rightarrow r(r-1) + 4r = 0 \Rightarrow$ ~~$r^2 - r + 4 = 0$~~ ~~indicial equation~~

$r^2 - r + 4r = 0 \Rightarrow \boxed{r^2 + 3r = 0}$ indicial equation

$r(r+3) = 0$

$r = 0$, $r = -3$.

ⓒ when $r = 0$

let $y = \sum_{n=0}^{\infty} a_n x^{n+r} \Rightarrow y' = \sum_{n=1}^{\infty} (n+r) a_n x^{n+r-1} \Rightarrow y'' = \sum_{n=2}^{\infty} (n+r)(n+r-1) a_n x^{n+r-2}$

Substitute

$\sum_{n=2}^{\infty} (n+0)(n+0-1) a_n x^{n+r-1} + \sum_{n=1}^{\infty} 4(n+r) a_n x^{n+r-1} - \sum_{n=0}^{\infty} a_n x^{n+r+1}$

~~$\Rightarrow \sum_{n=1}^{\infty} (n+r+1)(n+r) a_{n+1} x^{n+r} + \sum 4(n+r+1) a_n x^{n+r} - \sum_{n=0}^{\infty} a_n$~~

$\Rightarrow \sum_{n=1}^{\infty} (n+r+1)(n+r) a_{n+1} x^{n+r} + \sum_{n=0}^{\infty} 4(n+r+1) a_{n+1} x^{n+r} - \sum_{n=1}^{\infty} a_{n-1} x^{n+r} = 0$

$\Rightarrow 4(r+1) a_1 x^r + \sum_{n=1}^{\infty} \left((n+r+1)(n+r) a_{n+1} + 4(n+r+1) a_{n+1} - a_{n-1} \right) x^{n+r} = 0$

$4(r+1) a_1 x^r = 0$

$4(r+1) a_1 = 0$

$\boxed{a_1 = 0}$

$\Rightarrow a_{n+1} \left((n+r+1)(n+r) + 4(n+r+1) \right) - a_{n-1} = 0$

$a_{n+1} = \frac{a_{n-1}}{(n+r+1)\cancel{(n+r)}(n+r+1)} \Rightarrow a_{n+1} = \frac{a_{n-1}}{(n+r+1)(n+r+4)}$

$(n+1)(n+4)$

$\Rightarrow$ يتبع خلف الورقة

6

Q6. (8 points) Given $Y' = \begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix} Y + \begin{bmatrix} 4e^{-t} \\ 0 \end{bmatrix}$ and $Y_h = \begin{bmatrix} 3e^{-t} & e^{-5t} \\ e^{-t} & -e^{-5t} \end{bmatrix} C$ is the solution of the corresponding homogeneous system. Use the method of variation of parameters to find the general solution of the system.

$$y(t) = y_h(t)C + y_h(t)\int_0^t y_h^{-1}(\tau)\, g(\tau)\, d\tau$$

$$y_h^{-1}(t) = \frac{1}{\det y_h(t)} \begin{bmatrix} -e^{-5t} & -e^{-5t} \\ -e^{-t} & 3e^{-t} \end{bmatrix} = \frac{-1}{4} e^{6t} \begin{bmatrix} -e^{-5t} & -e^{-5t} \\ -e^{-t} & 3e^{-t} \end{bmatrix}$$

$$\det y(t) = \begin{vmatrix} 3e^{-t} & e^{-5t} \\ e^{-t} & -e^{-5t} \end{vmatrix} = -3e^{-6t} - e^{-6t} = -4e^{-6t}$$

$$y_h^{-1}(t)\, g(t) = \frac{-1}{4} e^{6t} \begin{bmatrix} -e^{-5t} & -e^{-5t} \\ -e^{-t} & 3e^{-t} \end{bmatrix} \begin{bmatrix} 4e^{-t} \\ 0 \end{bmatrix} = \begin{bmatrix} -e^{5t} \\ 0 \end{bmatrix} \begin{bmatrix} -e^{-5t} & -e^{-5t} \\ -e^{-t} & 3e^{-t} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ e^{4t} \end{bmatrix}$$

$$y_p = \int_0^t \begin{bmatrix} 1 \\ e^{4t} \end{bmatrix} d\tau =$$

$$y(t) = Y_h + y_p = \begin{bmatrix} 3e^{-t} & e^{-5t} \\ e^{-t} & -e^{-5t} \end{bmatrix} C + \begin{bmatrix} 1 \\ e^{4t} \end{bmatrix}$$

general solution