19.5/25 | 51 | 51

الأحد ٢٠١٧/٨/٢٠	الامتحان الثاني: رياضيات هندسية ١	الجامعة الأردنية
مدرس المادة: د. بنان		اسم الطالب: ليث مؤيد مصطفى الأحمد
وقت المحاضرة: ٩:٣٠-١١		الرقم الجامعي: 0164511

يتكون الامتحان من ٨ أسئلة في ٤ ورقات:

I- For questions [1] to [5], fill in the blanks to get a correct sentence. (2 marks each)
Show your work

1.5 [1] The general solution of the ODE

$x^3 /$ $xy^{(4)} + 6y^{(3)} = 0$ is $\boxed{y = c_1 x^0 + c_2 x + c_3 x^2 + c_4 x^2 \ln x}$

$x^4 y^4 + x^3 y^3 = 0$

$r(r-1)(r-2)(r-3) + r(r-1)(r-2) = 0$

$r(r-1)(r-2)(r-3+1) = 0$

$(r-2)$

$r = 0, 1, 2, 2$

$(r^2 - r)(r-2)(r-2) = 0$ | $r^4 - 5r^3$

$(r^3 - 2r^2 - r^2 + 2r)(r-2)$

$r^4 - 2r^3 - r^3 + 2r^2 - 2r^3 + 4r^2 + 2r^2 - 4r = 0$

1 [2] The value of α so that the system $x + 3y = 1$

$2x + \alpha y = 0$

has no solution is $\boxed{\alpha = 4}$ 6

$\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 2 & \alpha & 0 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & \alpha-6 & -2 \end{array}\right]$

$\alpha - 6 = -2$

$\boxed{\alpha = 4}$

$\alpha - 6 = \text{zero}$

عشان تكون no solution

$A + \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$

2 [3] If $\left(A + \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}\right)^{-1} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$, then $\boxed{A = \begin{bmatrix} 12 & 7 \\ 7 & 3 \end{bmatrix}}$

$2a - 4 - 2a + 8 = 4$

$(a-2)2 - c = 4$

$(a-3)2 - c = 2$

$c = 2a - 6 - 2$

$2a + -b = 4$

$2a - b = 0$

$b = 2a$

$2by + (d-3)y = 1$

$-(d-2)$

$-dy + 2y + dy - 3y = 1$ $-y = 1$ $\boxed{y = -1}$

1 [4] Find the linear system whose general solution is $\bar{y} = c_1 e^{2t}\begin{bmatrix}4\\0\end{bmatrix} + c_2 e^{3t}\begin{bmatrix}2\\1\end{bmatrix}$

الحل السابق

$\boxed{\begin{array}{l} y_1' = 4y_1 + 2y_2 \\ y_2' = 0 + y_2 \end{array}}$

$y_1 = c_1 4 e^{2t} + c_2 2 e^{3t}$

$y_2 = 0 + c_2 e^{3t}$

$\lambda = 2, 3$

$\begin{bmatrix} a-3 & c \\ b & d-3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$(a-3)x + cy = 2$

$cy = (a-3)x + 2$

$\begin{bmatrix} a-2 & c \\ b & d-2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ $-c =$

$2a - 4 - c = 4$

$(a-2)x + cy = 4$ $2a - c = 8$, $x = 0$

$ax - 2x - ax + 3x = 4$ $c = 2a - 8$

$(a-2)x + cy = 4$

$ax - 2x + 2 - ax + 3x = 4$

$(a-3)x + cy = 2$ $\boxed{x = 2}$

$cy = 2 - (a-3)x$ $2b + (d-2)y = 0$ $2b = -\frac{(d-2)y}{2}$

$-c = 2 - 2a + 6$ $-c = 8 - 2a$

Scanned by CamScanner

Y

X [5] Let $A = \begin{bmatrix} 4 & -3 \\ x & -4 \end{bmatrix}$. The value of x such that $A = A^{-1}$ is $x = \frac{17}{3}$

det $A = -16 + 3x$

$-16 + 3x = 1$

$A^{-1} = \frac{1}{-16+3x}\begin{bmatrix} -4 & 3 \\ -x & 4 \end{bmatrix} = \frac{-16+3x}{-16+3x} = 1$

$x = \frac{-x}{-16+3x} \to -16x + 3x = -1$

$\boxed{x = 5}$

$x = 5$ JWI

II- In questions [6] to [8] give detailed answer. (5 marks each)

5

[6] Solve using undetermined coefficients method

$y''' - y'' + 4y' - 4y = e^x$

$r^3 - r^2 + 4r - 4 = 0$

$r(r^2+4) - (r^2+4) = 0$

$(r^2+4)(r-1) = 0$

$r = \pm 2i$

$r = 1$

$y_h = c_1 e^x + c_2 \cos 2x + c_3 \sin 2x$

$y_p = Axe^x$

$y'_p = A(e^x + xe^x) = Ae^x + Axe^x$

$y''_p = Ae^x + Ae^x + Axe^x = 2Ae^x + Axe^x$

$y'''_p = 2Ae^x + Ae^x + Axe^x$

$= 3Ae^x + Axe^x$

$\Rightarrow 3Ae^x + \cancel{Axe^x} - 2Ae^x - \cancel{Axe^x} + 4Ae^x + 4Axe^x - 4Axe^x = e^x$

$\Rightarrow 5Ae^x = e^x$

$5A = 1$

$\boxed{A = \frac{1}{5}}$

G.S

$y = y_h + y_p$

$= c_1 e^x + c_2 \cos 2x + c_3 \sin 2x + \frac{1}{5} x e^x$

$y_h = c_1 e^x + c_2 \cos 2x + c_3 \sin 2x$

$y_p = \frac{1}{5} x e^x$

r

5

[7] Solve the system: $y_1' = 3y_1 + 2y_2$

$y_2' = -2y_1 - y_2$

$$\vec{y} = \begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 2 \\ -2 & -1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow (3-\lambda)(-1-\lambda) + 4 = 0$$

$$= -3 - 3\lambda + \lambda + \lambda^2 + 4 = 0$$

$$\Rightarrow \lambda^2 - 2\lambda + 1 = 0$$

$$(\lambda - 1)(\lambda - 1) = 0$$

$\lambda = 1, 1 \rightarrow$ eigen value

$\boxed{\lambda = 1}$

$$\begin{bmatrix} 3-1 & 2 \\ -2 & -1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2a + 2b = 0$$

$\boxed{a = -b}$ $\boxed{b = 1}$

$$\lambda_{x_1} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

second $\boxed{\lambda = 1}$

$$\begin{bmatrix} 2 & 2 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\Rightarrow 2a + 2b = -1$$

$$a = \frac{-1 - 2b}{2}$$

$b = 0$

$$\lambda_{x_2} = \begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix}$$

G.S

$$\boxed{\vec{y} = c_1 e^t \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 e^t \left(\begin{bmatrix} -1 \\ 1 \end{bmatrix} t + \begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} \right)}$$

5

[8] Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 5 & -4 & 1 \end{bmatrix}$.

(a) Find A^{-1}.

$\det(A) = 1(1-0) - 0 + 0 = \boxed{1}$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix} \rightarrow \text{Final answer}$$

$a_{11} = \begin{vmatrix} 1 & 0 \\ -4 & 1 \end{vmatrix} = 1$

$a_{12} = \begin{vmatrix} -2 & 0 \\ 5 & 1 \end{vmatrix} = -2$

$a_{13} = \begin{vmatrix} -2 & 1 \\ 5 & -4 \end{vmatrix} = 8 - 5 = 3$

$a_{21} = \begin{vmatrix} 0 & 0 \\ -4 & 1 \end{vmatrix} = 0$

$a_{22} = \begin{vmatrix} 1 & 0 \\ 5 & 1 \end{vmatrix} = 1$

$a_{23} = \begin{vmatrix} 1 & 0 \\ 5 & -4 \end{vmatrix} = -4$

$a_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$a_{32} = \begin{vmatrix} 1 & 0 \\ -2 & 0 \end{vmatrix} = 0$

$a_{33} = \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} = 1$

(b) Find X, if $AX = \begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}_{3\times 2}$

$$A^{-1}AX = A^{-1}\begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}$$

$$IX = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}_{3\times 3}\begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 0 & 2 \end{bmatrix}_{3\times 2}$$

identity matrix

$$IX = \begin{bmatrix} 2+0+0 & 3+0+0 \\ 4-1+0 & 6+1+0 \\ 6-4+0 & 9+4+2 \end{bmatrix} \Rightarrow IX = \begin{bmatrix} 2 & 3 \\ 3 & 7 \\ 2 & 15 \end{bmatrix}$$