Ans Q1

First Test Measurement

A) Thermometer, $\tau = 2.5\,s$, $T_{init} = 15°c$, $T_{in} = 100°c$; $t = 5.5\,s$

$\therefore T_o = T_i[1 - e^{-t/\tau}] + T_{init}\, e^{-t/\tau} = \underline{90.58\,c°}$

If $T_{init} = 0 \Rightarrow T_o = T_i[1 - e^{-t/\tau}] = 88.919\,c°$

$\therefore$ Difference $= \underline{1.66\,c°}$

B) $d = 50 + 0.2\%$, $h = 250 - 0.5\%$. This is case of known error:

$\forall_{err} = \frac{\pi}{4} d^2 h = 490873.8521\ mm^3$

$h_{tre} = 248.75\ mm$, $d_{tre} = 50.1\ mm$

$\therefore \forall_{true} = 490375.1145\ mm^3$, $\frac{\Delta\forall}{\forall} = \frac{\forall_{tre} - \forall_{err}}{\forall_{tre}} * 100\%$

$= \underline{-0.1017\%}$

c) $P_{FSD} = 1000\,kPa$, $P_1 = 100\,kPa$, $\epsilon_{r_{fs}} = \pm 1\%$

$\delta P = \frac{\pm 1}{100} * 1000 = \pm 10\,kPa \Rightarrow$ range $\underline{[90 - 110]}$.

If error was $\epsilon_{r_1} = \pm 1\% \Rightarrow \delta P = \pm \frac{1}{100} * 100 = \pm 1\,kPa$

$\therefore$ range $= \underline{[99 - 101]}$.

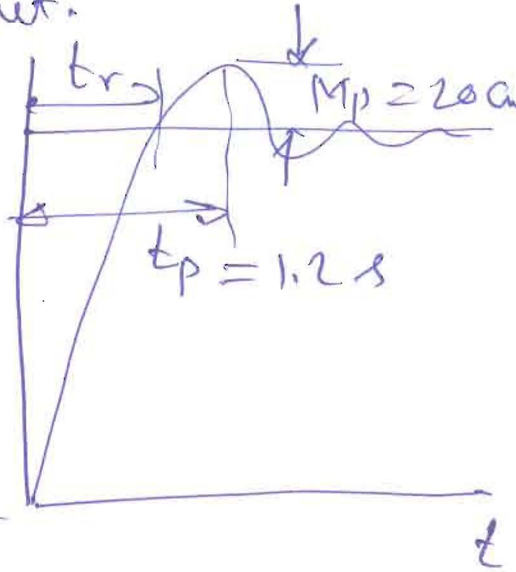
D) Spring, $K_s = \frac{x}{F} = \underline{0.133\ cm/N}$.

E) Range 0-50 kPa, 100 div $\Rightarrow$ Least Count $= \frac{1}{2}\,kPa$

resolution $= \frac{1}{5}$th of $\frac{1}{2}\,kPa = \underline{0.1\,kPa}$

F) $\tau = \frac{4.5}{3.0} = 1.5\,s$, $K_s = \frac{1.5*10^{-3}}{3} = \underline{5*10^{-4}}$

Ans Q2: This is a case of Second order system response to step input.



Given: $Y_o(\infty) = 40\,cm$,

$Y_o(t_p) = 60\,cm$,

$$M_p = \frac{Y_o(t_p) - Y_o(\infty)}{Y_o(\infty)}$$

$$= \frac{60 - 40}{40} = 0.5$$

Now The damping ratio (ζ) is found using

$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} \Rightarrow \zeta = \sqrt{\frac{[\ln(M_p)]^2}{\pi^2 + (\ln M_p)^2}}$$

$\therefore$ $\underline{\zeta = 0.2154}$ — $\frac{1}{2}$

$\therefore$ $\phi = \cos^{-1}(\zeta) = 77.55^\circ = 0.4308\pi$ rad.

The damped natural freq. $(\omega_d) = \frac{\pi}{t_p} = \frac{\pi}{1.2}$ $\frac{1}{2}$

$= \underline{2.618\ rad/s}$

The undamped " " $(\omega_n) = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \underline{2.681\ rad/s}$ $\frac{1}{2}$

The rise time $(t_r) = \frac{\pi - \phi}{\omega_d} = \underline{0.6838}$ $\frac{1}{2}$

Now; If the volume is halved i.e. $\forall_2 = \frac{1}{2}\forall_1$

$$\zeta = \frac{B}{2\rho\sqrt{2ga\forall}} \quad , \quad \omega_n = \sqrt{\frac{2ag}{\forall}}$$

$$\frac{\zeta_1}{\zeta_2} = \frac{\dfrac{\cancel{B}}{\cancel{2\rho}\sqrt{\cancel{2ag}\forall_1}}}{\dfrac{\cancel{B}}{\cancel{2\rho}\sqrt{\cancel{2ag}\forall_2}}} = \sqrt{\frac{\forall_2}{\forall_1}} = \sqrt{\frac{1}{2}}$$

$\frac{1}{2}$

$\therefore$ The damping ratio $(\zeta_2) = \dfrac{\zeta_1}{\sqrt{\frac{1}{2}}} = \underline{0.3047}$

$$\frac{\omega_{n_1}}{\omega_{n_2}} = \frac{\sqrt{\dfrac{\cancel{2ag}}{\forall_1}}}{\sqrt{\dfrac{\cancel{2ag}}{\forall_2}}} = \sqrt{\frac{1}{2}}$$

$\frac{1}{2}$

$\therefore$ The natural freq $(\omega_{n_2}) = \dfrac{\omega_{n_1}}{\sqrt{\frac{1}{2}}} = \underline{3.7915 \text{ rad}}$

$\therefore$ The damped " $(\omega_{d_2}) = \omega_{n_1}\sqrt{1-\zeta^2}$ $\frac{1}{2}$

$= \underline{3.6112 \ \frac{rad}{s}}$

$\therefore$ The rise time $(t_{r_2}) = \dfrac{\pi - \phi_2}{\omega_{d_2}} = \underline{0.52 \text{ sec}}$ $\frac{1}{2}$

$\phi_2 = \cos^{-1}(\zeta_2) = 72.26^\circ = 0.4014\pi$ $\frac{1}{2}$

$\therefore$ The peak time $(t_{p_2}) = \dfrac{\pi}{\omega_{d_2}} = \underline{0.878}$

Ans Q3 Thermometer, $d_c = 0.25\,mm$, Spherical bulb,
$H = 60 \frac{J}{m^2 s c}$,
Alcohol, $\rho = 791.8 \frac{kg}{m^3}$, $C = 2510 \frac{J}{kg c}$,
$\alpha = 0.00119\ c^{-1}$

[1] Thermometer represents a case of First Order System.

[2] This means that the controlling parameter for its response is the Time Constant (τ).

[3] $K_c = 3\ mm/c = \frac{\alpha \forall_b}{A_c}$ } Sensitivity of thermometer

$A_c = \frac{\pi}{4} d_c^2 = \frac{\pi}{4}(0.25)^2$

$\therefore$ we get bulb vol^m ($\forall_b$) $= 123.75\ mm^3$

[4] Time Constant $\tau = \frac{\rho C \forall_b}{H A_b}$

A_b = area of bulb $= \pi d_b^2$

Solving we get $\tau = 34.135\ s$

[5] If the bulb is made cylindrical with height "h" = 7mm, for same $K_c = 3\ mm/c$

$\forall_b = 123.75\ mm^3 = \frac{\pi}{4} d_b^2 h$

we get the bulb diameter $d_b = 4.744\ mm$

[6] $\tau = \frac{\rho C \forall_b}{H A_b}$, but $A_b = \pi d_b h$

hence the new time constant $\tau = 39.29\ s$

[7] $T_{init} = 25c°$, $T_i = 125c°$, $t = 10s$

$$x_o = k_c T_i \left[1 - e^{-t/\tau}\right] + k_c \cancel{\frac{T_{init}}{k_c}}\, e^{-t/\tau}$$

$$= 3 \frac{mm}{c°} * \left[125\left[1 - e^{\frac{-10}{34.135}}\right] + 25 * e^{\frac{-10}{34.135}}\right]$$

$= 151.183$ mm output displacement

but since $k_c = 3$ mm/c

$$\therefore T_o = \frac{x_o}{k_c} = \underline{50.4\ C°} \text{ output temp}^r.$$

[8]
1) low density.
2) high expansion.
3) lower sp. heat
4) lower freezing pt. (below temp^r range)

4 * 1/4

Q4

1) This is a Case of multi-sample test. $\frac{1}{2}$

2) The data represents Random error $\frac{1}{2}$

First (2013/2014) : **Answer Q4**

Section 1

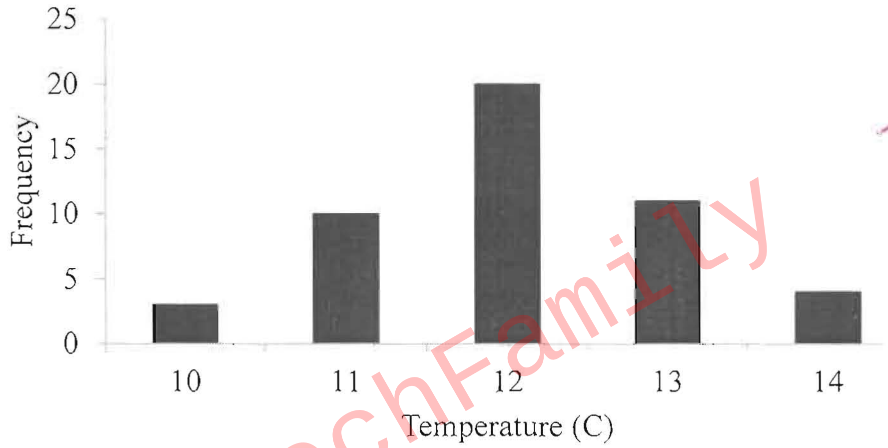
6

Temp	F	T*F	Deviation	D*F	Deviation^2	F*D^2	abs(d)/SD
10	3	30	-2.0625	-6.1875	4.2539	12.7617	2.0453
11	10	110	-1.0625	-10.6250	1.1289	11.2891	1.0536
12	20	240	-0.0625	-1.2500	0.0039	0.0781	0.0620
13	11	143	0.9375	10.3125	0.8789	9.6680	0.9297
14	4	56	1.9375	7.7500	3.7539	15.0156	1.9213
Sum	48	579		36.125		48.8125	

Average Temperature =	12.0625	C
Deviation	shown in table	C
Mean Deviation =	0.752604167	C
Standard Deviation =	1.008428026	C
Variance (v) =	1.016927083	
Probabale Error of one reading =	0.680184703	C
Precision Index =	0.701197074	
i) The following readings are to be rejected	None	

1

1



$\frac{1}{4}$

$\frac{1}{4}$

[1] The data looks to be following normal distribution.

[2] The range is [10 – 14] = 4 °C. $\frac{1}{4}$

[3] The average error in the exp ($\bar{D}$) is less than the maximum allowed (σ). $\frac{1}{4}$

[4] The number of Values (data) that lies between $\bar{T} \pm \gamma_1$ [12.0625 – 0.68018, 12.0625 + 0.68018] is more than 50% of the total No. of readings $\frac{1}{4}$

[5] The precision index h is > 50% of the total readings $\frac{1}{4}$

Q4

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2) The data represents Random error — $\frac{1}{2}$

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First (2013/2014) :

Section 1

Answer Q4

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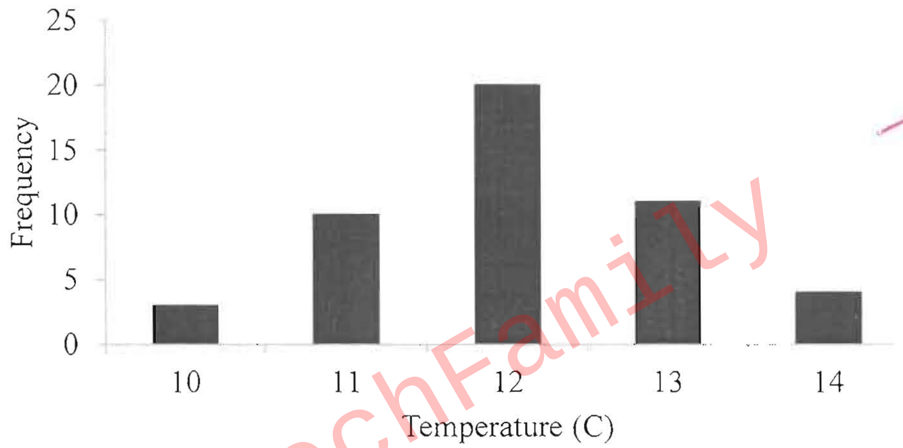

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[5] The precision index h is > 50% of the total readings

$\frac{1}{4}$

* Since we have 48 readings, based on Chauvenet's criteria we take the lowest value i.e. for 25 readings with $\frac{d}{\sigma} = 2.33$, this means that no data will be rejected.

$\frac{1}{2}$

* To specify the data, we need to know the accuracy required, based on that we can write

$\frac{1}{2}$

$$T \pm \sigma$$

OR

$$\pm r_1$$

OR

$$\pm 2\sigma$$

$$\pm 3\sigma$$