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The University of Jordan — Mathematics Department 2-0-2

الأربعاء
Math. Eng 202 / Final Exam
Date: 13/01/2013
1:00-2:00

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(1) Solve the D.E: $dy + dx = \cos^2(x+y)\,dx$.

$x+y=\theta$

$dx+dy=d\theta$

$dy=d\theta-dx$

$d\theta - \cancel{dx} + \cancel{dx} = \cos^2(\theta)dx$

$\frac{d\theta}{\cos^2\theta} - dx = 0$

$\int \sec^2\theta \cdot d\theta - \int dx$

$\tan\theta - x$

$\tan(x+y) - x = C$

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(2) If $W(f,g) = x^5$ and $f(x) = x$. Find $g(x)$.

$\begin{vmatrix} x & g(x) \\ 1 & g'(x) \end{vmatrix} = x^5$

$xg'(x) - g(x) = x^5$

$xg'(x) - x^5 = g(x)$

0

(3) Solve the homogenous system: $\begin{pmatrix} \frac{1}{4} & \frac{-1}{4} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$Y' = AY + F$

$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ 4 & \frac{4}{3} \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$\begin{pmatrix} 4-\lambda & -4 \\ 4 & \frac{4}{3}-\lambda \end{pmatrix}$

$(4-\lambda)(\frac{4}{3}-\lambda)=0$

$\frac{16}{3} - 4\lambda - \frac{4}{3}\lambda + \lambda^2 = 0$

$\lambda^2 - \frac{16}{3}\lambda + \frac{16}{3} = 0$

$(\lambda-4)(\lambda-\frac{4}{3})$

$\lambda_1 = 4 \quad \lambda_2 = \frac{4}{3}$

2

$\lambda_1 = 4$

$\begin{pmatrix} 0 & -4 \\ 4 & - \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-4y = 0$

$\begin{pmatrix} 0 \\ t \end{pmatrix} = t\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\lambda_2 = \frac{4}{3}$

$\begin{pmatrix} \frac{8}{3} & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\frac{8}{3}x - 4y = 0 \qquad \frac{8}{12}x = y$

$\begin{pmatrix} t \\ \frac{8}{12}t \end{pmatrix} = t\begin{pmatrix} 1 \\ \frac{8}{12} \end{pmatrix}$

$Y_h = c_1 e^{4t}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + c_2 e^{\frac{4}{3}t}\begin{pmatrix} 1 \\ \frac{8}{12} \end{pmatrix}$

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(4) Find $\mathcal{L}(f(t))$ if:

3

(a) $f(t) = t^2 \sin t$.

$$(-1)^2 \frac{d^2}{ds^2} \frac{1}{s^2+1} = \frac{d}{ds}\frac{2s}{(s^2+1)^2} = \frac{2(s^2+1)^2 - 2s(2(s^2+1)2s)}{(s^2+1)^4}$$

$$= \frac{2(s^2+1)^2 - 8s^2(s^2+1)}{(s^2+1)^4}$$

(b) $f(t) = \begin{cases} t + \sin t & ,0 \le t < \frac{\pi}{4} \\ \sin t + \cos t & ,\frac{\pi}{4} \le t \end{cases}$

$$t + \sin t\,(1 - u(t - \tfrac{\pi}{4})) + \sin t + \cos t\,(u(t - \tfrac{\pi}{4})$$

$$t + \sin t - t u(t-\tfrac{\pi}{4}) - \sin t\, u(t-\tfrac{\pi}{4}) + \sin t\, u(t-\tfrac{\pi}{4}) + \cos t\, u(t-\tfrac{\pi}{4})$$

$$\frac{1}{s^2} + \frac{1}{s^2+1} - \frac{e^{-\frac{\pi}{4}s}}{s^2} - \cancel{\frac{e^{-\frac{\pi}{4}s}}{s^2+1}} + \cancel{\frac{e^{-\frac{\pi}{4}s}}{s^2+1}} + \frac{s e^{-\frac{\pi}{4}s}}{s^2+1}$$

$1\frac{1}{2}$

$$\frac{1}{s^2} + \frac{1}{s^2+1} - \frac{e^{-\frac{\pi}{4}s}}{s^2} + \frac{s e^{-\frac{\pi}{4}s}}{s^2+1}$$

(5) Find $f(t)$, if :

$t^n \cdot \frac{n!}{s^{n+1}}$

(a) $\mathcal{L}(f(t)) = \frac{e^{-2s}}{s^5}$. $= e^{-2s} \cdot \frac{4!}{s^5\, 4!}$

$$= u(t-2) \cdot \frac{t^4}{4!}$$

1

(b) $\mathcal{L}(f(t)) = \ln(\frac{s}{s-1})$.

$$(\ln s - \ln(s-1))\frac{d}{ds}$$

$$= \frac{1}{s} - \frac{1}{s-1}$$

$$= \int ds - \int e^t\,dt$$

$$= t - e^t$$

(c) $\mathcal{L}(f(t)) = \frac{1}{s^3 - 9s}$.

$$= \frac{1}{s(s^2-9)} = \int_0^t \frac{\sinh 3x}{3} \cdot dx$$

$$= \frac{\cosh 3x}{9}\Big]_0^t = \frac{\cosh 3t}{9} - \frac{\cosh 0}{9}$$

1

(6) Use laplace transform to evaluate the integral: $\int_0^\infty \frac{\sin t}{t}\, dt$.

0 t ∞

$$\mathcal{L}\left(\int_0^t \frac{\sin t}{t} dt + \int_t^\infty \frac{\sin t}{t} dt\right)$$

$$= \frac{1}{s}\mathcal{L}\left(\frac{\sin t}{t}\right) + \frac{K}{s}$$

$u = \frac{1}{t}$ $dv = \sin t$

$du = -\frac{1}{t^2}$ $v = -\cos t$

$-\frac{1}{t}\cos t - \int \frac{\cos t}{t^2}$

$u = \sin t$ $dv = \frac{1}{t}$

$u = \cos t$ $v = \ln t$

$\sin t \ln(t) - \int_t^\infty \ln(t)\cos t$

~~$\sin t \ln t$~~

$u = \ln t$ $dv = \cos t$

$du = \frac{1}{t}$ $v = -\sin t$

$-\ln t \sin t + \int_t^\infty \frac{\sin t}{t} \cdot dt$

$= K$

$\mathcal{L}(y'') + \mathcal{L}(y) = \mathcal{L}(u(t-2))$

(7) Solve the D.E: $y'' + y = u(t-2)$.
given that $y(0) = 0$ and $y'(0) = 1$.

$$s^2 Y(s) - s\cancel{y(0)} - \cancel{y'(0)} + Y(s) = Y(t)$$

0 -1

$$y(s)(s^2+1) = y(t) + 1$$

$$y(s) = \frac{e^{-2s}}{s(s^2+1)} + \frac{1}{(s^2+1)}$$

$$u(t-2)\int_0^t \sin x \cdot dx + \sin t$$

$$u(t-2)\left(-\cos x\Big]_0^t\right) + \sin t$$

$$u(t-2)\left(-\cos t - \cancel{\cos 0}\right) + \sin t$$

$$u(t-2)(-\cos t - 1) + \sin t$$

$\mathcal{L}(u(t-2))$

$$= \frac{e^{-2s}}{s}$$

3

(8) Let $(\sin x)y'' - y' + (\csc x)y = 0$. Find all singular points and classify them.

$$\lim_{x\to 0} (x-0)\left(\frac{-1}{\sin x}\right) = \lim_{x\to 0} \frac{-x}{\sin x} = \lim_{x\to 0} \frac{-1}{\cos x} = -1$$

$$\lim_{x\to 0} (x-0)^2\left(\frac{\csc x}{\sin x}\right) = \lim_{x\to 0} \frac{x^2}{\sin^2 x} = \lim_{x\to 0} \frac{2x}{2\sin x\cos x} = \cancel{\lim_{x\to 0} \frac{2}{2(-\sin^2 x + c}}$$

$$= \lim_{x\to 0} \frac{2}{2(-\sin^2 x + \cos^2 x)} = 1$$

Zero is a singular point

2

(9) Find the recurrence relation of the D.E: $(4-x^2)y'' + 2y = 0$, near $x_0 = 0$.

$y = \sum_0^\infty a_n (x-x_0)^n \quad x_0 = 0 \rightarrow y' = \sum_1^\infty n a_n (x)^{n-1} \rightarrow y'' = \sum_2 n(n-1) a_n x^{n-2}$

$(4-x^2)\sum_2^\infty n(n-1)a_n x^{n-2} + 2\sum_0^\infty a_n x^n = 0$

$\sum_2^\infty 4n(n-1)a_n x^{n-2} - \sum_2^\infty n(n-1)a_n x^n + \sum_0^\infty 2a_n x^n = 0$

$\sum_0^\infty 4(n+2)(n+1)a_{n+2}x^n -$

$8a_2 + 24a_3 x + \sum_2^\infty 4(n+2)(n+1)a_{n+2}x^n - \sum_2^\infty n(n-1)a_n x^n + 2a_0 + 2a_1 x$

$8a_2 + 2a_0 + (24a_2 + 2a_1)x + \sum_2^\infty [4(n+2)(n+1)a_{n+2} - (n(n-1)a_n + 2a_n)] + \sum_2^\infty 2a_n x^n = 0$

$x^n = 0$

$4(n+2)(n+1)a_{n+2} - n(n-1)a_n + 2a_n = 0$

$a_{n+2} = \frac{n(n-1)a_n - 2a_n}{4(n+2)(n+1)} = \frac{a_n(n(n-1)-2)}{4(n+2)(n+1)}$

3

(10) Let $x^2y'' + (\sin x)y' + (x-1)y = 0$. Find the indicial equation. What is the form of the linearly independent solutions y_1 and y_2.

$y'' + \frac{\sin x}{x^2}y' + \frac{(x-1)}{x^2}y = 0$

$(x-0)\frac{\sin x}{x} = \lim \frac{\cos x}{1} = 1 = p_0$

$x^2\frac{(x-1)}{x^2} = -1 = q_0$

$r(r-1) + rp_0 + q_0 = 0$

$r(r-1) + r - 1 = 0$

$r^2 - r + r - 1 = 0$

$r^2 - 1 = 0$

$r_1 = 1 \quad r_2 = -1$

$y_1 = (x-0)^1 \sum_0^\infty a_n (x)^n = \sum_0^\infty a_n x^{n+1}$

$y_2 = (x-0)^{-1} \sum_0^\infty b_n x^n = \sum_0^\infty b_n x^{n-1}$

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Dept. Of Mathematics **Univ. of Jordan** **Eng .Math. 0301202**
Final Exam. **Sat. 29/5/2010** **5 – 7 PM.**

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Name of Student :.......................... **ID.# : ....**
Teacher : .... **Section :.......................**

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#Circle the most correct answer (2 points each)# .

Q1- One of the following is **not true** about the ODE $(x-2)^2 y'' + 5(x-2)y' + y = 1$

(a) This ODE is a non-homogeneous Cauchy – Euler equation .
(b) This ODE has an ordinary point at $x = 0$.
(c) This ODE has a regular singular point at $x = 2$.
(d) All the above are true .
(e) Some of the above is not true .

Q2- If the roots of the characteristic equation of a differential equation are $1,1,-2,-2$ then this differential equation is:

(a) $(D+I)^2(D-2I)^2 y = 0$ (b) $(D-I)^2(D+2I)^2 y = 0$
(c) $(D^2+I)(D^2-2I)y = 0$ (d) $(D^2-I)(D^2+2I)y = 0$ (e) $(D^2-I)^2(D^2+2I)^2 y = 0$

Q3 – Assume that $Y = c_1\begin{bmatrix}1\\-1\end{bmatrix}e^{2t} + c_2\left(\begin{bmatrix}1\\-1\end{bmatrix}t + \begin{bmatrix}1\\-2\end{bmatrix}\right)e^{2t}$ is the general solution of the system $Y' = AY$. About the system $Y' = AY$ we have

(a) the system has repeated eigen values and the eigen space depends on one free variable
(b) the system has repeated eigen values and the eigen space depends on two free variables
(c) the system has real defferent eigen values
(d) the system has complex eigen values
(e) the system has complex eigen values and the eigen space depends on two free variables

Q4- Assume that $Y = c_1\begin{bmatrix}1\\-1\end{bmatrix}e^{-3t} + c_2\begin{bmatrix}1\\1\end{bmatrix}e^{-3t}$ is the general solution of the system $Y' = AY$.

About the system $Y' = AY$ we have

(a) the system has repeated eigen values and the eigen space depends on one free variable
(b) the system has real defferent eigen values
(c) the system has complex eigen values
(d) the system has repeated eigen values and the eigen space depends on two free variables
(e) the system has complex eigen values and the eigen space depends on two free variables

Q5- If the indicial equation of the power series solution of the ode $x^2y'' + (x\sin x)y' - (2\cos x)y = 0$,at a regular singular point $x_0 = 0$ is $r^2 - r - 2 = 0$, then the solutions of this ode are:

(a) $y_1 = x^{-1}\sum_{n=0}^{\infty} a_n x^n$, $y_2 = Ay_1 \ln x + x^2\sum_{n=0}^{\infty} b_n x^n$ (b) $y_1 = x^2\sum_{n=0}^{\infty} a_n x^n$, $y_2 = y_1 \ln x + x^2\sum_{n=0}^{\infty} b_n x^n$

(c) $y_1 = x^2\sum_{n=0}^{\infty} a_n x^n$, $y_2 = Ay_1 \ln x + x^{-1}\sum_{n=0}^{\infty} b_n x^n$

(d) $y_1 = x^{-1}\sum_{n=0}^{\infty} a_n x^n$, $y_2 = y_1 \ln x + x^{-1}\sum_{n=0}^{\infty} b_n x^n$ (e) $y_1 = \cos x$, $y_2 = \sin x$

Q6- About the ODE $y'' + \left(\frac{\cos x}{x^2}\right)y' + \left(\frac{\sin\frac{1}{x}}{x}\right)y = 0$ we have

(a) $x_0 = 0$ is an ordinary point (b) $x_0 = 0$ is an irregular singular point
(c) $x_0 = 0$ is a regular singular point (d) all of the above are true
(e) None of the above is true

Q7- The value of $L\left(e^{\pi t} t\cos \pi t\right)$ is equal to :

(a) $\dfrac{(s-\pi)^2-\pi^2}{((s-\pi)^2+\pi^2)^2}$ (b) $\dfrac{(s+\pi)^2-\pi^2}{((s+\pi)^2+\pi^2)^2}$

(c) $\dfrac{\pi^2-(s-\pi)^2}{((s-\pi)^2+\pi^2)^2}$ (d) $\dfrac{\pi^2-(s+\pi)^2}{((s+\pi)^2+\pi^2)^2}$ (e) $\dfrac{(s-\pi)^2-\pi^2}{((s-\pi)^2+\pi^2)}$

Q8- The value of $L^{-1}\left(\dfrac{(s+3)}{\left[(s+3)^2+\dfrac{\pi^2}{4}\right]^2}\right)$ is equal to

(a) $\dfrac{1}{\pi}te^{-3t}\cos\dfrac{\pi}{2}t$ (b) $\dfrac{1}{\pi}te^{-3t}\cosh\dfrac{\pi}{2}t$

(c) $\dfrac{\pi}{2}te^{-3t}\sinh\dfrac{\pi}{2}t$ (d) $\dfrac{1}{\pi}te^{-3t}\sinh\dfrac{\pi}{2}t$ (e) $\dfrac{1}{\pi}te^{-3t}\sin\dfrac{\pi}{2}t$

Q9- The value of $L^{-1}\left(e^{-\pi s}\dfrac{1}{s^2(s^2+6s+13)}\right)$ is equal to :

(a) $\dfrac{1}{2}u(t-\pi)\int_0^t(\int_0^\tau e^{-3\zeta}\sin 2\zeta\, d\zeta)d\tau$ (b) $\dfrac{1}{2}u(t+\pi)\int_0^{t+\pi}(\int_0^\tau e^{-3\zeta}\sinh 2\zeta\, d\zeta)d\tau$

(c) $\dfrac{1}{2}u(t-\pi)\int_0^{t-\pi}(\int_0^\tau e^{-3\zeta}\sinh 2\zeta\, d\zeta)d\tau$ (d) $\dfrac{1}{2}u(t-\pi)\int_0^{t-\pi}(\int_0^\tau e^{-3\zeta}\sin 2\zeta\, d\zeta)d\tau$

(e) $\dfrac{1}{2}u(t+\pi)\int_0^{t+\pi}(\int_0^\tau e^{-3\zeta}\sin 2\zeta\, d\zeta)d\tau$

II- Write all details (26 points):

Q1-(6 points)Using Laplace transform and Cramer's Rule find the value of y_1 only for the initial value problem of the system of ODE :

$$y_1' = y_1 - 3y_2 \ , \ y_2' = 2y_1 + y_2 \ , \ y_1(0) = 1 \ , \ y_2(0) = -2$$

Q2-(7 points)Find **one** power series solutions of $xy'' + y = 0$ near $x_0 = 0$

Q3-(7 points)Using Laplace transform solve the initial value problem :

$$y'' + 2y' = u\left(t - \frac{\pi}{2}\right)\cos t \quad , \; y(0) = 1 \; , \; y'(0) = 0$$

Q4-(6 points)Using LaPlace transform solve the initial value problem :

$$y'' + 4y = \delta(t-2) \ , \ y(0)=1 \ , \ y'(0)=0$$

Dept. Of Mathematics **Univ. of Jordan** **Eng .Math. 0301202**
Final Exam. **Mon. 16/8/2010** **3 - 5 PM.**
Name of Student :.......................... ID.# : ....
Teacher : .... Section :.......................

I - #Circle the most correct answer #.

Q1- One of the following is **not true** about the ODE $(x+1)^2 y''+3(x+1)y'+2y=e^{-x}$

(a) This ODE is a non-homogeneous Cauchy – Euler equation .
(b) This ODE has an ordinary point at $x=0$.
(c) This ODE has a regular singular point at $x=-1$.
(d) All the above are true . (e) Some of the above is not true .

Q2- If the roots of the characteristic equation of a differential equation are $1,1,1,-2i,2i$ then this differential equation is:

(a) $(D-I)^3(D^2-4I)y=0$ (b) $(D-I)^3(D^2+4I)y=0$
(c) $(D+I)^3(D^2+4I)y=0$ (d) $(D+I)^3(D^2-4I)y=0$ (e) $(D-I)^3(D+4I)^2 y=0$

Q3- An acceptable particular solution y_p of the differential equation $y^{(4)}+4y''=x+\sin 2x$ is :

(a) $y=Ax^3+Bx^2+Cx\cos 2x+Dx\sin 2x$ (b) $y=Ax^2+Bx^1+Cx^2\cos 2x+Dx^2\sin 2x$
(c) $y=Ax^2+Bx+Cx\cos 2x+Dx\sin 2x$
(d) $y=Ax^3+Bx^2+Cx^2\cos 2x+Dx^2\sin 2x$ (e) $y=Ax^3+Bx^2+Cx^2\cos x+Dx^2\sin x$

Q4 – If $Y^{(h)}=c_1\begin{bmatrix}1\\0\end{bmatrix}e^{-t}+c_2\begin{bmatrix}0\\1\end{bmatrix}e^{5t}$ is the general solution of $Y'=\begin{bmatrix}-1&0\\0&5\end{bmatrix}Y$ then a particular solution y_p of the system $Y'=\begin{bmatrix}-1&0\\0&5\end{bmatrix}Y+\begin{bmatrix}2e^t\\-t\end{bmatrix}$ is :

(a) $Y^{(P)}=\begin{bmatrix}e^{-t}&0\\0&e^{5t}\end{bmatrix}\int_0^t\begin{bmatrix}2e^{-2\tau}\\-\tau e^{5\tau}\end{bmatrix}d\tau$ (b) $Y^{(P)}=\begin{bmatrix}e^{t}&0\\0&e^{-5t}\end{bmatrix}\int_0^t\begin{bmatrix}2e^{2\tau}\\-\tau e^{-5\tau}\end{bmatrix}d\tau$

(c) $Y^{(P)}=\begin{bmatrix}e^{-t}&0\\0&e^{5t}\end{bmatrix}\int_0^t\begin{bmatrix}2e^{2\tau}\\-\tau e^{-5\tau}\end{bmatrix}d\tau$

(d) $Y^{(P)}=\begin{bmatrix}e^{-t}&0\\0&e^{5t}\end{bmatrix}\int_0^t\begin{bmatrix}2e^{2\tau}\\-e^{-5\tau}\end{bmatrix}d\tau$ (e) $Y^{(P)}=\begin{bmatrix}e^{5t}&0\\0&e^{-t}\end{bmatrix}\int_0^t\begin{bmatrix}2e^{2\tau}\\-\tau e^{-5\tau}\end{bmatrix}d\tau$

Q5- If the indicial equation of the power series solutionof the ode $y''+P(x)y'+Q(x)y=0$,at a regular singular point $x_0=0$ is $r^2+r-2=0$, then the solutions of this ODE are:

(a) $y_1=x\sum_{n=0}^{\infty}a_nx^n$, $y_2=Ay_1\ln x+x^{-2}\sum_{n=0}^{\infty}b_nx^n$ (b) $y_1=x^{-2}\sum_{n=0}^{\infty}a_nx^n$, $y_2=y_1\ln x+x^{-2}\sum_{n=0}^{\infty}b_nx$

(c) $y_1=x^2\sum_{n=0}^{\infty}a_nx^n$, $y_2=Ay_1\ln x+x^{-1}\sum_{n=0}^{\infty}b_nx^n$

(d) $y_1=x\sum_{n=0}^{\infty}a_nx^n$, $y_2=y_1\ln x+x\sum_{n=0}^{\infty}b_nx^n$ (e) $y_1=x^{-2}\sum_{n=0}^{\infty}a_nx^n$, $y_2=Ay_1\ln x+x\sum_{n=0}^{\infty}b_nx^n$

Q6- About the ODE $x^2y''+(xe^x)y'-(2\cos x)y=0$ we have

(a) $x_0=0$ is an ordinary point (b) $x_0=0$ is a regular singular point

(c) $x_0=0$ is an irregular singular point (d) all of the above are true (e) None of the above is true

Q7-The value of $L^{-1}\left(e^{-3s}\dfrac{1}{s^2(s^2+4)}\right)$ is equal to :

(a) $\frac{1}{2}\int_0^t\int_0^\tau u(\xi+3)\cos 2(\xi+3)d\xi d\tau$ (b) $\frac{1}{2}\int_0^t\int_0^\tau u(\xi-2)\cos 3(\xi-2)d\xi d\tau$

(c) $\frac{1}{2}\int_0^t\int_0^\tau u(\xi-3)\cos 2(\xi-3)d\xi d\tau$ (d) $\frac{1}{2}\int_0^t\int_0^\tau u(\xi-3)\sin 2(\xi-3)d\xi d\tau$ (e) $\frac{1}{3}\int_0^t\int_0^\tau \sin 3(\xi-2)d\xi d\tau$

Q8- The value of $L^{-1}\left(\dfrac{(s+2)}{\left[(s+2)^2+\pi^2\right]^2}\right)$ is equal to

(a) $\frac{1}{2\pi}te^{-2t}\cos\pi t$ (b) $\frac{1}{2\pi}te^{-2t}\sinh\pi t$ (c) $-\frac{1}{2\pi}te^{-2t}\sin\pi t$ (d) $\frac{1}{2\pi}te^{-2t}\cosh\pi t$ (e) $\frac{1}{2\pi}te^{-2t}\sin\pi t$

Q9- If $L(f(t))=\dfrac{s^2}{(s^2+4)^3}$,then value of $L\left(\int_0^t f(\tau)d\tau\right)$ is equal to :

(a) $\dfrac{s}{(s^2+4)^3}$ (b) $\dfrac{s}{(s^2-4)^3}$ (c) $\dfrac{s^2}{(s^2+4)^3}$ (d) $\dfrac{s^2}{(s^2-4)^3}$ (e) $\dfrac{3}{(s^2+4)^2}$

Q10 –Consider the system of linear equations :
$$\begin{aligned} x-y+2z+w&=1\\ 2x+y-3z-w&=2\\ 4x-y+z+w&=4\end{aligned}$$
then

One of the following is **not true**

(a)The system has two linearly independent equations

(b) Echelon form matrix of this system has two non zero rows only

(c) The system has infinite number of solutions .

(d) All of the above statements are true

(e) The system has three linearly independent equations

Q11-If $A=\begin{bmatrix}2&1&2\\0&3&-1\\0&0&2\end{bmatrix}$, $B=\begin{bmatrix}2\\1\\3\end{bmatrix}$ then one of the following is **not true** about the system $AX=B$:

(a)The system $AX=B$ consists of three linearly independent equations

(b)The echelon form matrix for A of this system $AX=B$ consists of three non-zero rows.

(c) The system $AX=B$ has infinitely many solutions

(d) $\det A\neq 0$

(e) A^{-1} exists

II- Write all details :

Q1- Find <u>**one**</u> power series solutions of $xy'' + y' + xy = 0$ near $x_0 = 0$

Q3- Using Laplace transform solve the initial value problem :

$$y'' + 4y = u(t-\pi)t \quad , \; y(0) = 0 \; , \; y'(0) = 0$$

Q4-Using LaPlace transform solve the initial value problem :

$$y'' + 3y' = \delta(t-2) \quad , \quad y(0) = 1 \ , \ y'(0) = 0$$

Final Exam / Eng. I.

Name - - - - - - -   Number - - - - -

1. Find the diff. Equ. whose y_g is:
$$c_1 \sinh x + c_2 \cosh x + 5e^x$$

2. True or False. Justify. $y'' + y'y = 1$ is a linear diff. Equ. with constant cofts. If true what are y_1 and y_2.

3. What is the type of $xy^2dx + (x^2 - y)dy = 0$. If you want to solve it completely, as what type you solve it.

4. If the roots of the associated equation to $L(y) = e^{x}$ are $1+i$, $1+i$, $1+i$, $1-i$, $1-i$, $1-i$, what is the form of y_p?

5. Find Laplace of

(a) $\sin^2 t \cos^2 t$

(b) $\begin{cases} t & t \le 1 \\ e^{t} & t > 1 \end{cases}$

(c) $\dfrac{1-e^{t}}{t}$

6. Find $\mathcal{L}^{-1}$:

(a) $\dfrac{e^{-\pi s}}{s^2+1}$

(b) $\dfrac{s-1}{s^2+2s+10}$

. What are the sing. pts. and classify them.

$$(x\sin x)y'' - (x-1)\sec x\, y' + y = 0$$

8. what is the form of y_1 and y_2 around $x=0$ of the differential Equ.

$$x(1-x)y'' - (\sin x)y' + \frac{x}{\sin^3 x\,(4+x)}\,y = 0$$

9. Use Laplace to solve

$$y' + \int_0^x y(t)\,dt = x^2 \quad , \quad y(0) = 0$$

10. What is the form of Y_p for the system

$$Y' = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{pmatrix} Y + \begin{pmatrix} 3e^{t} \\ 5+e^{t} \end{pmatrix}$$

11. Let $Y' = AY + \begin{pmatrix} \ln t \\ 1 \end{pmatrix}$ where A has eigen values 1 and -1 and eigen vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \end{pmatrix}$. What is the form of Y_p.