Differential

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MechFamily

كُن أنت التغيير..

Notebooks

* Non homogeneous Higher-order DE's

* (undetermined)

* الاولوية الها اذا كانت قابلة للتطبيق.

* لازم يكون المعاملات ثوابت عشان تنطبق.

Ex Solve: $y^{(4)} - y''' = 12x^2 - 18$

① $y^{4} - y''' = 0$

$r^4 - r^3 = 0$

$r^3(r-1) = 0$

$r = 0, 0, 0, 1$

$y_1 = 1, \quad y_2 = x, \quad y_3 = x^2, \quad y_4 = e^x$

The form of Homogeneous:

$$y = c_1(1) + c_2(x) + c_3(x^2) + c_4(e^x)$$

② let $y_{p(x)} = x^3[Ax^2 + Bx + C]$ * بتأكد من التكرار.

عشان التكرار

$$y_{p(x)} = Ax^5 + Bx^4 + Cx^3$$

$y' = 5Ax^4 + 4Bx^3 + 3Cx^2$

$y'' = 20Ax^3 + 12Bx^2 + 6Cx$

$y''' = 60Ax^2 + 24Bx + 6C$

$y'''' = 120Ax + 24B + 0$

$$120Ax + 24B - 60Ax^2 - 24Bx - 6C = 12x^2 - 18$$

Five Apple

$$-60Ax^2 + [120A - 24B]\,x + [24B - 6c] = 12x^2 - 18$$

$-60A = 12$ --- (1) $\quad A = \frac{-1}{5}$ $\quad$ * معامل x^2

$120A - 24B = 0$ --- (2) $\quad B = -1$ $\quad$ * معامل x

$24B - 6c = -18$ --- (3) $\quad c = -1$ $\quad$ معامل الثابت.

$$y_p(x) = \frac{-1}{5}x^5 - x^4 - x^3$$

The general solution is : $\quad y = y_h + y_p$

$$y = c_1 + c_2 x + c_3 x^2 + c_4 e^x - \frac{1}{5}x^5 - x^4 - x^3$$

* لوكان السؤال أكبر من 3^{rd} order بستخدم [illegible]

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12/3/2018

* Nonhomogeneous Higher-order D.E's

(Variation of parameters)

* مراجعة للـ Determinant

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & 4 \end{vmatrix} = (1)\begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} - (1)\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} + (1)\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= (-4-2) - (4-2) + (1+1)$$

$$= -6 - 2 + 2$$

$$= -6$$

* عشان اعرف انه العدد سالب او موجب بحدد رتبته بالاول واللي هي حاصل جمع رقم العامود اللي هو فيه مع رقم السطر فاذا كان العدد فردي (odd) بيكون سالب واذا كان زوجي (even) بيكون موجب.

$$2.\begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix} = \begin{vmatrix} 4 & 8 \\ 6 & 8 \end{vmatrix} = 32-48 = -16$$

* بقدر اضرب العدد اما بالعامود او بالصف فقط احدهما.

$$\Rightarrow \begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix} = -8$$

$$2.\begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix} = -16$$

* بطلع جواب المحصلة وبضربه بالرقم.

Ex

$$W\left[e^{x}, e^{-x}, e^{2x}\right] = \begin{vmatrix} e^{x} & e^{-x} & e^{2x} \\ e^{x} & -e^{-x} & 2e^{2x} \\ e^{x} & e^{-x} & 4e^{2x} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & 4 \end{vmatrix} e^{x} \cdot e^{-x} \cdot e^{2x}$$

$$= e^{2x}(-6) = -6e^{2x}$$

$$W\left\{x, x^2, x^3\right\} = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$$

* يستخدم الصف او العامود الذي يحتوي على اصفار اكثر.

$$= x\begin{vmatrix} 2x & 3x^2 \\ 2 & 6x \end{vmatrix} - (1)\begin{vmatrix} x^2 & x^3 \\ 2 & 6x \end{vmatrix} + 0\left|\;\right|$$

$$= x\left[12x^2 - 6x^2\right] - \left[6x^3 - 2x^3\right]$$

$$= x(6x^2) - 4x^3$$

$$= 2x^3.$$

⑥ $W_1[x, x^2, x^3] = \begin{vmatrix} 0 & x^2 & x^3 \\ 0 & 2x & 3x^2 \\ 1 & 2 & 6x \end{vmatrix}$

$= (1)\begin{vmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{vmatrix} = 3x^4 - 2x^4 = x^4$

⑥ $W_2[x, x^2, x^3] = \begin{vmatrix} x & 0 & x^3 \\ 1 & 0 & 3x^2 \\ 0 & 1 & 6x \end{vmatrix}$

$= (-1)\begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix} = -(3x^3 - x^3) = -2x^3$

⑥ $W_3 = [x, x^2, x^3] = \begin{vmatrix} x & x^2 & 0 \\ 1 & 2x & 0 \\ 0 & 2 & 1 \end{vmatrix}$

$= (1)\begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix} = 2x^2 - x^2 = x^2$

* Variation of parameters

$$y^{(n)} + P_1(x)\, y^{(n-1)} + \text{------} + P_n(x)\, y = r(x)$$

① $y_h = c_1 y_1 + c_2 y_2 + \text{---} + c_n y_n$

② $$\boxed{y_{p(x)} = y_1 \int \frac{W_1}{W} \cdot r + y_2 \int \frac{W_2}{W}\, r + \cdots + y_n \int \frac{W_n}{W} \cdot r}$$

Ex solve : $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 24x^5$

① $x^3 y''' - 3x^2 y'' + 6xy' - 6y = 0$ cauchy

لازم نقسم X
ع المعامل
لما نحل الـ Particular

$y_1 = x$, $y_2 = x^2$, $y_3 = x^3$

$y_h = c_1 x + c_2 x^2 + c_3 x^3$

② $$y_{p(x)} = y_1 \int \frac{W_1}{W} r + y_2 \int \frac{W_2}{W}\, r + y_3 \int \frac{W_3}{W} \cdot r$$

$$= x \int \frac{x^4}{2x^3} \cdot 24x^2\, dx + x^2 \int \frac{-2x^3}{2x^3} \cdot 24x^2\, dx + x^3 \int \frac{x^2}{2x^3} \cdot 24x^2\, dx$$

* كل المعطيات = الي مطلوبة في القانون محلوله في الامثلة السابقة.

$$= 12x \int x^3\, dx - 24x^2 \int x^2\, dx + 12x^3 \int x\, dx$$

$$= 12x \cdot \frac{x^4}{4} - 24x^2 \cdot \frac{x^3}{3} + 12x^3 \cdot \frac{x^2}{2}$$

$$= 3x^5 - 8x^5 + 6x^5$$

$$y_{p(x)} = x^5$$

The general solution is:

$$y = y_h + y_p$$

$$= c_1 x + c_2 x^2 + c_3 x^3 + x^5$$

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* Matries

let $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}_{2\times 2}$ $\quad B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}_{2\times 2}$

$$A+B = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$$

* عملية الضرب يجب ان يكون عدد الاعمدة في الاولى يساوي عدد الصفوف.

$$AB = \begin{bmatrix} a_1a_2+b_1c_2 & a_1b_2+b_1d_2 \\ c_1a_2+d_1c_2 & c_1b_2+d_1d_2 \end{bmatrix}$$

Consider $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Then $\det(A) = |A| = ad - bc$.

① if $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ OR $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

* اذا كان اي شيء خارج القطر الرئيسي يساوي صفر تسمى ب Diagonal matrix

② $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ (Idaintity)

A^{-1} : inverse of A.

$AA^{-1} = A^{-1}A = I$ (Identity.)

Ex $A = \begin{bmatrix} a & b \\ c & D \end{bmatrix}$

Then :

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Ex : $A = \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}$ Find A^{-1}

$$A^{-1} = \frac{1}{-6} \begin{bmatrix} 3 & -3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{-1}{3} \end{bmatrix}$$

Now : $AA^{-1} = \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

*Note :

if $|A| \neq 0$ Then A^{-1} exists.

if $|A| = 0$ Then A^{-1} Does Not exists.

~~equations~~.

consider: $a_{11}x + a_{12}y = b_1$

$a_{21}x + a_{22}y = b_2$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{\underline{b}} \Rightarrow \underbrace{A\underline{x} = \underline{b}}_{\text{Non Homogeneous system } \underline{b} \neq 0}$$

$$\begin{bmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

← رجوعي لنظام المعادلة الاصلية.

① If $\underline{b} = 0$, Then The system is called Homogeneous.

② consider The system:

$a_{11}x + a_{12}y = 0$

$a_{21}x + a_{22}y = 0$

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{0}$$

$A\underline{x} = 0 \Rightarrow$ Homogeneous system.

consider $A\underline{x} = \underline{0}$

* If A^{-1} exists, Then $\Rightarrow A^{-1}A\underline{x} = A^{-1}\underline{0}$

$$I\underline{x} = \underline{0} \Rightarrow \boxed{\underline{x} = 0}$$

* ال Identity في ال vector or matrix بطلعلي ال Vector or matrix نفسه.

* If A^{-1} Does Not Exists. There is (∞) solutions of the equation.

Def: let A be a square matrix.

If There exists $\underline{x} \neq 0$:

$A\underline{x} = \lambda \underline{x}$, Then λ is called an eigen value,

and $\underline{x}$ is called an eigen vector.

① let $\underline{x} \neq 0$: * يا بكون حل واحد او بكون الها (∞) حلول.

$A\underline{x} = \lambda \underline{x}$ ← $I\underline{x}$ بدل

$\lambda I\underline{x} - A\underline{x} = 0 \Rightarrow \underbrace{[\lambda I - A]}_{B}\underline{x} = 0$

$\boxed{\text{Then} \Rightarrow |\lambda I - A| = 0}$ $\Rightarrow$ characteristic equation.

* $|A| = 0 \Rightarrow A^{-1}$ not exists.

$|A| \neq 0 \Rightarrow A^{-1}$ exists.

Ex: Find all eigenvalues and eigenvectors for.

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}$$

$$|\lambda I - A| = \begin{vmatrix} \lambda-2 & -3 \\ -4 & \lambda-3 \end{vmatrix} = 0$$

$$(\lambda-2)(\lambda-3) - 12 = 0$$

$$\lambda^2 - 5\lambda + 6 - 12 = 0$$

$$\lambda^2 - 5\lambda - 6 = 0$$

الحل كدا طول

*Note: $\lambda^2 - (5)\lambda + (-6) = 0$

مجموع عناصر القطر — الـ determine

$$(\lambda-6)(\lambda+1) = 0$$

$\boxed{\lambda = 6, -1}$ ⇒ eigen values.

* To calculate The eigen vectors.

$$[\lambda I - A]\, \underline{x} = \underline{0}$$

$[\lambda I - A]\, X = 0$

$$\begin{bmatrix} \lambda-2 & -3 \\ -4 & \lambda-3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = 6}$

$$\begin{bmatrix} 4 & -3 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

* إذا ال determinant بتساوي صفر الصفوف بتكون dependent

إذا كانت ما بتساوي صفر الصفوف بتكون indep

$4x - 3y = 0$

$x = \dfrac{3y}{4}$

Then The eigen vector $(X^{(1)}) = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \Leftarrow$ where $y = 4$

Now $\boxed{\lambda = -1} \Rightarrow \begin{bmatrix} -3 & -3 \\ -4 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-3x - 3y = 0$

$x + y = 0$

$\boxed{y = -x}$ $\Rightarrow$ Then $(X^{(2)}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

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*

$$\frac{dx}{dt} = ax + by$$

$$\frac{dy}{dt} = cx + dy$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\underline{x}' = A\underline{x}$$

scaler vector

let $\underline{x}(t) = e^{\lambda t}\underline{\xi}$ be a solution.

$$\lambda e^{\lambda t}\underline{\xi} = A e^{\lambda t}\underline{\xi}$$ ← تعويض ال solution بالمعادلة

$$\lambda \xi = A\underline{\xi}$$

$$A\underline{\xi} = \lambda\underline{\xi}$$

* Theorem:

let $\underline{x}'(t) = A\underline{x}(t)$

Then $x(t) = e^{\lambda t}\xi$ is a solution.

Where $\underline{\xi}$: eigen vector

λ : eigen value.

Ex solve:

$$\frac{dx}{dt} = 3x - 2y$$

$$\frac{dy}{dt} = 2x - 2y$$

$$\underline{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

solution:

①
$$\underbrace{\begin{bmatrix} x' \\ y' \end{bmatrix}}_{\underline{X}} = \underbrace{\begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \end{bmatrix}$$

* اكتبها بال Matrix form.

② calculate The eigen value.

$$|\lambda I - A| = 0$$

$$\Rightarrow \begin{vmatrix} \lambda-3 & 2 \\ -2 & \lambda+2 \end{vmatrix} = 0$$

$$(\lambda-3)(\lambda+2) + 4 = 0$$

$$\lambda^2 - \lambda - 6 + 4 = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

$$(\lambda-2)(\lambda+1) = 0$$

$$\lambda = 2, -1$$

③ Calculate The eigen vector.

$\boxed{\lambda = 2}$

$$[\lambda I - A]\, \underline{x} = 0$$

$$\begin{bmatrix} \lambda-3 & 2 \\ -2 & \lambda+2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-x + 2y = 0$$

$$y = \frac{x}{2} \quad \Rightarrow \quad \underline{\xi}^{(1)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

The first solution :

$$\underline{x}^{(1)} = e^{\lambda t}\, \xi = e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\boxed{\lambda = -1}$

$$\begin{bmatrix} -4 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x + y = 0$$

$$y = 2x \quad \Rightarrow \quad \xi^{(2)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

The second solution : $\underline{x}^{(2)} = e^{-t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

The general solution is

$$\underline{X}(t) = c_1 e^{2t}\begin{bmatrix}2\\1\end{bmatrix} + c_2 e^{-t}\begin{bmatrix}1\\2\end{bmatrix}.$$

$$X(t) = c_1 \underline{X}^{(1)} + c_2 \underline{X}^{(2)}$$

(4) The conditions, where $\underline{X}(0) = \begin{bmatrix}1\\-1\end{bmatrix}$

$$\begin{bmatrix}x(0)\\y(0)\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}$$

$$c_1\begin{bmatrix}2\\1\end{bmatrix} + c_2\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}$$

$$\begin{bmatrix}2c_1 + c_2\\c_1 + 2c_2\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}$$

$$2c_1 + c_2 = 1 \quad \text{--(1)}$$

$$c_1 + 2c_2 = -1 \quad \text{...(2)}$$

$$3c_2 = -3 \Rightarrow \boxed{c_2 = -1}$$

$$\boxed{c_1 = 1}$$

Ex solve :

$$x'(t) = \underbrace{\begin{bmatrix} -1 & -6 \\ 3 & 5 \end{bmatrix}}_{A} \underline{x}(t)$$

① $|\lambda I - A| = 0$

$$\begin{vmatrix} \lambda+1 & 6 \\ -3 & \lambda-5 \end{vmatrix} = 0$$

$(\lambda+1)(\lambda-5) + 18 = 0$

$\lambda^2 - 4\lambda - 5 + 18 = 0$

$\lambda^2 - 4\lambda + 13 = 0$ $\qquad a = 1 , b = -4 , c = 13$

$b^2 - 4Ac = 16 - 52 \boxed{= -36}$

complex !

$\lambda = \dfrac{4 \pm \sqrt{-36}}{2}$ $\qquad \lambda = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{4 \pm 6i}{2}$

$= 2 \pm 3i$

$\lambda_1 = 2+3i , \lambda_2 = 2-3i$

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(2) $\boxed{\lambda = 2+3i}$

$$[\lambda I - A]\,\underline{x} = 0$$

$$\begin{bmatrix} \lambda+1 & 6 \\ -3 & \lambda-5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3+3i & 6 \\ -3 & -3+3i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x + (-3+3i)\,y = 0$$

$$-3x = -(-3+3i)\,y$$

$$x = \frac{(-3+3i)\,y}{3}$$

$$x = (-1+i)\,y$$

$$\xi^{(1)} = \begin{bmatrix} -1+i \\ 1 \end{bmatrix}$$

* اذا كانت الـ i بالمقام :
لازم ارفعها للبسط عن طريق
ضربها في المرافق .
لانه بتعادي الحكي عادي
ويكتب الـ x بدلالة الـ y
او بنحل عن الطريقة السريعة

$$\begin{bmatrix} 3-3i \\ -3 \end{bmatrix}$$

The first solution.

$$\underline{x}^{(1)}(t) = e^{\lambda t}\,\underline{\xi} = e^{(2+3i)t}\begin{bmatrix} -1+i \\ 1 \end{bmatrix}$$

$$= e^{2t}\cdot e^{(3t)i}\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix} + i\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$ Very important

$$e^{2t}\left[\cos 3t + i\sin 3t\right]\left(\begin{bmatrix}-1\\1\end{bmatrix} + i\begin{bmatrix}1\\0\end{bmatrix}\right)$$

$$e^{2t}\left(\cos 3t\begin{bmatrix}-1\\1\end{bmatrix} - \sin 3t\begin{bmatrix}1\\0\end{bmatrix}\right) + e^{2t} i\left(\cos 3t\begin{bmatrix}1\\0\end{bmatrix} + \sin 3t\begin{bmatrix}-1\\1\end{bmatrix}\right)$$

* ضربت أل real × real اعطاني real
وال imag × imag اعطاني real

* ضربت ال real × imag اعطاني imag
* وال imag × real اعطاني imag

The first solution:

$$\underline{X}^{(1)}(t) = e^{2t}\left[\cos 3t\begin{bmatrix}-1\\1\end{bmatrix} - \sin 3t\begin{bmatrix}1\\0\end{bmatrix}\right]$$

~~The second solution:~~

$$\underline{X}^{(2)}(t) = e^{2t}\left[\cos 3t\begin{bmatrix}1\\0\end{bmatrix} + \sin 3t\begin{bmatrix}-1\\1\end{bmatrix}\right]$$

The general solution.

$$\underline{X} = c_1\underline{X}^{(1)} + c_2\underline{X}^{(2)}$$

$\underline{\underline{Ex}}$ solve : $\underline{X}'(t) = \underbrace{\begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}}_{A} \underline{X}(t)$

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda - 2 & 5 \\ -1 & \lambda + 2 \end{vmatrix} = 0$$

$$(\lambda - 2)(\lambda + 2) + 5 = 0$$

$$\lambda^2 - 4 + 5 = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$$\lambda = \pm i$$

$$\boxed{\lambda = i}$$

$$[\lambda I - A]\underline{X} = \underline{0}$$

$$\begin{bmatrix} i-2 & 5 \\ -1 & i+2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

real ال مع real ال ~~im~~

$$\underline{\xi}^{(1)} = \begin{bmatrix} -5 \\ i-2 \end{bmatrix}$$

$$\begin{bmatrix} -5 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ i \end{bmatrix}$$

نفصلها

$$\underline{X}^{(1)}_{(t)} = e^{\lambda t} \underline{\xi}^{(1)} \Rightarrow e^{it} \begin{bmatrix} -5 \\ i-2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos t + i \sin t \end{bmatrix} \left(\begin{bmatrix} -5 \\ -2 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$\left(\left(\cos t \begin{bmatrix} -5 \\ -2 \end{bmatrix} - \sin t \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) + i \left(\cos \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \sin \begin{bmatrix} -5 \\ -2 \end{bmatrix} \right) \right)$$

$$\underline{X}^{(1)}(t) = \cos t \begin{bmatrix} -5 \\ -2 \end{bmatrix} - \sin t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\underline{X}^{(2)}(t) = \cos t \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \sin t \begin{bmatrix} -5 \\ -2 \end{bmatrix}$$

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Ex solve :

$$\underline{y}' = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \underline{y}$$

① $|\lambda I - A| = 0$

$$\begin{vmatrix} \lambda - 4 & -1 \\ 1 & \lambda - 2 \end{vmatrix} = 0$$

$$(\lambda - 4)(\lambda - 2) + 1 = 0$$

$$\lambda^2 - 6\lambda + 8 + 1 = 0$$

$$\lambda^2 - 6\lambda + 9 = 0$$

$$(\lambda - 3)(\lambda - 3)$$

$$\lambda = 3, 3.$$

② $[\lambda I - A]\, \underline{y} = 0$

$$\begin{bmatrix} \lambda - 4 & -1 \\ 1 & \lambda - 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\boxed{\lambda = 3}$

عن الطريقة السريعة

$$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow F = \begin{bmatrix} -1 \\ +1 \end{bmatrix}$$

The first solution :

$$(y^{(1)}) = e^{\lambda t}\,\underline{\xi} \Rightarrow (y^{(1)}) = e^{3t}\begin{bmatrix}-1\\1\end{bmatrix}$$

* بنطلع generated igen vector

③ To find The generated igen vector.

$$[\lambda I - A]\,\underline{y} = \cancel{0}\,\underline{\xi} \longleftarrow \text{very important.}$$

$$\begin{bmatrix}-1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}$$

$$x + y = -1 \Rightarrow \text{where } x = 0$$

$$\underline{y} = \begin{bmatrix}0\\-1\end{bmatrix}$$

for The generated igen vector

Now $\Rightarrow$ The second solution is :

$$(y^{(2)}) = t\,e^{\lambda t}\,\underline{\xi} + e^{\lambda t}\,\underline{y}$$

$$= te^{3t}\begin{bmatrix}-1\\1\end{bmatrix} + e^{3t}\begin{bmatrix}0\\-1\end{bmatrix}$$

The general solution is:

$$= c_1 e^{3t}\begin{bmatrix}-1\\1\end{bmatrix} + c_2\left(te^{3t}\begin{bmatrix}-1\\1\end{bmatrix} + e^{3t}\begin{bmatrix}0\\-1\end{bmatrix}\right)$$

The form of the second solution in the repeated is:

$$\boxed{e^{\lambda t}\left[t\underline{\xi} + \underline{\eta}\right]}$$

very important.

$$y^{(2)} = te^{\lambda t}\underline{\xi} + e^{\lambda t}\underline{\eta} = e^{\lambda t}\left[t\underline{\xi} + \underline{\eta}\right]$$

28/3/2018

* Non Homogeneous linear system.

Ex solve :

① $y_1' = -y_1 + y_2$

$y_2' = -y_1 - y_2$

② $\left.\begin{array}{l} y_1' = -y_1 + y_2 + e^{-2t} \\ y_2' = -y_1 - y_2 - e^{-2t} \end{array}\right] \Rightarrow$ solve using undetermined coefficient.

---

①

$$\underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}'}_{\underline{y}'} = \underbrace{\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}_{\underline{y}}$$

This is a homogeneous linear system!

$|\lambda I - A| = 0$

$$\begin{vmatrix} \lambda+1 & -1 \\ 1 & \lambda+1 \end{vmatrix} = 0 \Rightarrow \lambda^2 + 2\lambda + 2 = 0$$

$a=1 \quad b=2 \quad c=2$

$b^2 - 4ac = \boxed{-4}$

$$\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2i}{2} \Rightarrow \lambda = -1 \pm i \longrightarrow \begin{cases} \lambda_1 = -1 + i \\ \lambda_2 = -1 - i \end{cases}$$

$(y^{(1)}) = e^{(-1+i)t} \xi^{(1)}$

OR $(y^{(1)}) = e^{(-1-i)t} \xi^{(2)}$

Take $\lambda_1 = -1+i$

$[\lambda I - A] \underline{y} = \underline{0}$

$$\begin{bmatrix} \lambda+1 & -1 \\ 1 & \lambda+1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore \underline{\xi}^{(1)} = \begin{bmatrix} 1 \\ i \end{bmatrix}$

The first solution is

$i * i = i^2$

$\sqrt{-1} * \sqrt{-1} \quad \sqrt{-1}^2 = -1$

$-1^{\frac{1}{2}} * -1^{\frac{1}{2}} = -1$

$$\underline{y}^{(1)} = e^{\lambda_1 t} \underline{\xi}^{(1)}$$

$$= e^{(-1+i)t} \begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$= e^{-t} e^{it} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= e^{-t} [\cos t + i \sin t] \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

$$= e^{-t} \left(\cos t \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \sin t \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) + i e^{-t} \left(\cos t \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \sin t \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

$$\left(y^{(1)}\right) = e^{-t}\left(\cos t \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \sin t \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = e^{-t}\begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}$$

$$\left(y^{(2)}\right) = e^{-t}\left(\cos t \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \sin t \begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = e^{-t}\begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$$

Then The general solution is:

$$y = c_1 y^{(1)} + c_2 y^{(2)}$$

$$y = c_1 \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$$

---

② $$\underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}'}_{y'} = \underbrace{\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}}_{y} + \underbrace{\begin{pmatrix} e^{-2t} \\ -e^{-2t} \end{pmatrix}}_{g(t)}$$

$$\underline{y}' = A\underline{y} + \underline{g}(t)$$

* ممكن يجي سؤال True or false على ال form

ال Homogeneous ولا لا

$$\underline{y}' = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \underline{y} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$$

① We solve : $\underline{y}' = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \underline{y}$ ⟸ The Homogeneous Term

$$y^{(h)} = c_1 e^{-t}\begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} \sin t \\ \cos t \end{pmatrix}$$

Now : let $y^{(p)} = \underline{a} e^{-2t}$ → Vector.

$$y^{(p)\prime} = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \underline{y}^{(p)} + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-2t}$$

$$y^{(p)} = \underline{a} e^{-2t} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{-2t}$$

$$y^{(p)\prime}_{(t)} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} -2e^{-2t} = \begin{pmatrix} -2a_1 \\ -2a_2 \end{pmatrix} e^{-2t}$$

$$\Rightarrow \begin{pmatrix} -2a_1 e^{-2t} \\ -2a_2 e^{-2t} \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} a_1 e^{-2t} \\ a_2 e^{-2t} \end{pmatrix} + \begin{pmatrix} e^{-2t} \\ -e^{-2t} \end{pmatrix}$$

$$\begin{bmatrix} -2a_1 e^{-2t} \\ -2a_2 e^{-2t} \end{bmatrix} = \begin{bmatrix} -a_1 e^{-2t} + a_2 e^{-2t} \\ -a_1 e^{-2t} - a_2 e^{-2t} \end{bmatrix} + \begin{bmatrix} e^{-2t} \\ -e^{-2t} \end{bmatrix}$$

$$-2a_1 e^{-2t} = -a_1 e^{-2t} + a_2 e^{-2t} + e^{-2t}$$

$-2a_1 = -a_1 + a_2 + 1$ → The first equation

$-a_1 = a_2 + 1$ ---- ①

$a_1 = a_2 - 1$ ----- ②

$$\boxed{\begin{matrix} a_2 = 0 \\ a_1 = 1 \end{matrix}}$$

$$-2a_2 e^{-2t} = -a_1 e^{-2t} - a_2 e^{-2t} - e^{-2t}$$

$-2a_2 = -a_1 - a_2 - 1$ the second equation.

$$\underline{y}^{(p)} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} e^{-2t}$$

$$\underline{y} = \underline{y}^{(h)} + \underline{y}^{(p)}$$

$$= c_1 e^{-t} \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} e^{-2t}$$

---

* consider $\underline{y}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \underline{y} + \begin{pmatrix} 2e^{-t} \\ 3t \end{pmatrix}$

① find $\underline{y}^{(h)}$

$$\underline{y}^{(h)} = c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$$ The solution

② Write a suitable form for $\underline{y}^{(p)}$ if The undetermined cofficiants is to be used!

$$\underline{y}^{(p)} = \underline{a} t e^{-t} + \underline{b} e^{-t} + \underline{c} t + \underline{d}$$

ضربتها عشان التكرار.

$$\boxed{y^{(p)} = t e^{\lambda t} \underline{f} + e^{\lambda t} \underline{g}}$$ في حالة التكرار.

Very important.

Non Homogeneous Term.

$\begin{pmatrix} 2e^{-t} \\ 3t \end{pmatrix} \rightarrow$ نفصلها.

$$\begin{pmatrix} 2e^{-t} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 3t \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 0 \end{pmatrix} e^{-t} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} t$$

2/4/2018

* Nonhomogeneous system. "Variation of parameters"

Ex Solve:

Homo. term

$$\underline{y}' = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \underline{y} + \begin{bmatrix} -6 \\ 2 \end{bmatrix} e^{-2t}$$

Solution:

$$y^{(h)} = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-4t}$$

* The fundamental Matrix $\psi(t)$ is defined by:

$$\psi(t) = \begin{bmatrix} e^{-2t} & e^{-4t} \\ e^{-2t} & -e^{-4t} \end{bmatrix}$$

الـ Fundamental Matrix بطلعها من حل الـ Homogeneous

* consider:

$$\underline{y}' = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \underline{y}$$

Then $\underline{y}^{(h)} = \underline{\psi(t)}\ \underline{c}$

$$= \begin{bmatrix} e^{-2t} & e^{-4t} \\ e^{-2t} & -e^{-4t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

consider : $\underline{y}' = A\underline{y} + \underline{g}(t)$

* Then we can write The General solution as :

$$\underline{y} = \underline{\psi}(t)\,\underline{c} + \underline{\psi}(t)\int \underline{\psi}^{-1}(t)\,\underline{g}(t)\,dt$$

Now :

$$\psi^{-1}(t) = \frac{1}{-2e^{-6t}}\begin{bmatrix} -e^{-4t} & -e^{-4t} \\ -e^{-2t} & e^{-2t} \end{bmatrix} = \frac{e^{6t}}{-2}\begin{bmatrix} -e^{-4t} & -e^{-4t} \\ -e^{-2t} & e^{-2t} \end{bmatrix}$$

|Det| ↗

$$= -\frac{1}{2}\begin{bmatrix} -e^{2t} & -e^{2t} \\ -e^{4t} & e^{4t} \end{bmatrix}$$

$$\psi^{-1}(t)\,\underline{g}(t) = -\frac{1}{2}\begin{bmatrix} -e^{2t} & -e^{2t} \\ -e^{4t} & e^{4t} \end{bmatrix}\begin{bmatrix} -6e^{-2t} \\ 2e^{-2t} \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 4 \\ 8e^{2t} \end{bmatrix} = \begin{bmatrix} -2 \\ -4e^{2t} \end{bmatrix}$$

$$\int \psi^{-1}(t)\,g(t)\,dt = \int \begin{bmatrix} -2 \\ -4e^{2t} \end{bmatrix} dt = \begin{bmatrix} -2t \\ -2e^{2t} \end{bmatrix}$$

$$\psi(t)\int \psi^{-1}(t)\,g(t)\,dt = \begin{bmatrix} e^{-2t} & e^{-4t} \\ e^{-2t} & -e^{-4t} \end{bmatrix}\begin{bmatrix} -2t \\ -2e^{2t} \end{bmatrix}$$

$$= \begin{bmatrix} -2te^{-2t} - 2e^{-2t} \\ -2te^{-2t} + 2e^{-2t} \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix} te^{-2t} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} e^{-2t}$$

The General Solution is:

$$\underline{y}(t) = \begin{bmatrix} e^{-2t} & e^{-4t} \\ e^{-2t} & -e^{-4t} \end{bmatrix} \underline{c} + \begin{bmatrix} -2 \\ -2 \end{bmatrix} t e^{-2t} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} e^{-2t}$$

---

* series Solutions.

consider: $y'' + p(x) y' + q(x) y = 0$

The series solution about x_0 is given by:

$$y_{(x)} = \sum_{n=0} a_n (x - x_0)^n$$

اذا كانت الـ $x = 0$
بصير اسمها ماكلورين سيريز

$$y' = \sum_{n=1} n a_n (x - x_0)^{n-1}$$

$$y'' = \sum_{n=2} n(n-1) a_n (x - x_0)^{n-2}$$

$$y = a_0 + a_1 (x - x_0) + a_2 (x - x_0)^2 + \text{-----}$$

$a(x) \sim y''$ معامل

Ex Find a power series solution of $y'' + y = 0$ about $x_0 = 0$

$a(x) = 1$

$\underbrace{a(x_0) = a(0) = 1 \neq 0}$

Then $x_0 = 0$ is called an ordinary point.

* بعوض النقطة المعطية وبتأكد انها لا تساوي صفر لانه لو تساوي صفر الها طريقة حل اخرى.

let $y = \sum_{n=0} a_n x^n$

$y' = \sum_{n=1} n a_n x^{n-1}$

$y'' = \sum_{n=2} n(n-1) a_n x^{n-2}$

تعويض في المعادلة الاصلية

$$\boxed{\sum_{n=2} n(n-1) a_n x^{n-2} + \sum_{n=0} a_n x^n = 0}$$

* اول خطوة لازم اوحد ال power

* اي زيادة في ال general form بصير فيه نقصان في ال index (العداد)

$$\sum_{n=0} (n+2)(n+1) a_{n+2} x^n + \sum_{n=0} a_n x^n = 0$$

* ما بصير اعمل هاي الخطوة الا اذا كانت ال powers موحدة وال indexes موحدة

$$\sum_{n=0} \left[(n+2)(n+1) a_{n+2} + a_n \right] x^n = 0$$

$$\boxed{(n+2)(n+1)\,a_{n+2} + a_n = 0} \quad n \geq 0 \quad \text{"recursien formula"}$$

↳ مهم

$$a_{n+2} = \frac{-a_n}{(n+2)(n+1)} \qquad n \geq 0$$

Now:

* بنطلع الـ Solution تبع السؤال.

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$\boxed{n=0}$ $\quad a_2 = \dfrac{-a_0}{(2)(1)} = \dfrac{-a_0}{2!}$ ← فكتوريال

$\boxed{n=1}$ $\quad a_3 = \dfrac{-a_1}{(3)(2)} = \dfrac{-a_1}{3!}$

* مطلوبين نحوي نقطتين
مش الكثر

$\boxed{n=2}$ $\quad a_4 = \dfrac{-a_2}{(4)(3)} = \dfrac{a_0}{(4)(3)\,2!} = \dfrac{a_0}{4!}$

$y'' + xy = 0$ شو؟
بتحل فيها على الـ series

$\boxed{n=3}$ $\quad a_5 = \dfrac{-a_3}{(5)(4)} = \dfrac{a_1}{(5)(4)\,3!} = \dfrac{a_1}{5!}$

$$y = a_0 + a_1 x - \frac{a_0}{2!}x^2 - \frac{a_1}{3!}x^3 + \frac{a_0}{4!}x^4 + \frac{a_1}{5!}x^5 + \cdots$$

$$= a_0 \underbrace{\left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right]}_{y_1} + a_1 \underbrace{\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots\right]}_{y_2}$$

4/4/2018

Ex: solve $y'' + x^2 y = 0$

near $x_0 = 0$
about $x_0 = 0$
at $x_0 = 0$

كلام مفاهيم اذا استخدم ال power series.

let $y = \sum_{n=0} a_n x^n$ be a solution.

$$y' = \sum_{n=1} n a_n x^{n-1}$$

$$y'' = \sum_{n=2} n(n-1) a_n x^{n-2}$$

* بنعوض في المعادلة الاصلية *

$$\sum_{n=2} n(n-1) a_n x^{n-2} + x^2 \sum_{n=0} a_n x^n = 0$$

$$\sum_{n=2} n(n-1) a_n x^{n-2} + \sum_{n=0} a_n x^{n+2} = 0$$

* let $n = n+2$ * let $n = n-2$

$$\sum_{n=0} (n+2)(n+1) a_{n+2} x^n + \sum_{n=2} a_{n-2} x^n = 0$$

* بنطلع اول حدين عشان نخلي العبارات تبلش من نفس الرقم.

$$2a_{n_2} + 6a_3 x + \sum_{n=2} (n+2)(n+1) a_{n+2} x^n + \sum_{n=2} a_{n-2} x^n = 0$$

$$2a_{n_2} + 6a_3 x + \sum_{n=2} \left[(n+2)(n+1) a_{n+2} + a_{n-2}\right] x^n = 0$$

$2a_2 = 0$

$6a_3 = 0$ * اذا جواب ال Series بساوي صفر فإنه كل معاملاته =

$$(n+2)(n+1) a_{n+2} + a_{n-2} = 0 \qquad \geq 2$$

recursion formula

Five Apple

$a_2 = 0$, $a_3 = 0$

$$a_{n+2} = \frac{-a_{n-2}}{(n+2)(n+1)} \qquad \geq 2$$

$\boxed{n=2}$ $\quad a_4 = \dfrac{-a_0}{(4)(3)}$

$\boxed{n=3}$ $\quad a_5 = \dfrac{-a_1}{(5)(4)}$

$\boxed{n=4}$ $\quad a_6 = \dfrac{-a_2}{(6)(5)} = 0$ $\quad a_2 = 0$

$\boxed{n=5}$ $\quad a_7 = \dfrac{-a_3}{(7)(6)} = 0$

* ملاحظة مهمة خارج اطار السؤال.

$\boxed{n=6}$ $\quad a_8 = \dfrac{-a_4}{(8)(7)} = \dfrac{a_0}{(8)(7)(4)(3)}$

$a_2 + 4_n = 0$

$\boxed{n=7}$ $\quad a_9 = \dfrac{-a_5}{(9)(8)} = \dfrac{a_1}{(9)(8)\ (5)(4)}$

$a_3 + 4_n = 0$

مهمة

$\boxed{n=8}$ $\quad a_{10} = \dfrac{-a_6}{(10)9} = 0$ $\quad a_6 = 0$

$\boxed{n=9}$ $\quad a_{11} = 0$

نفس

اي قبلها.

Now The solution.

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + \cdots$$

$$= a_0 + a_1 x - \frac{a_0}{12} x^4 - \frac{a_1}{20} x^5 + \cdots$$

$$a_0 \underbrace{\left[1 - \frac{x^4}{12} \cdots\right]}_{y_1} + a_1 \underbrace{\left[x - \frac{x^5}{20} \cdots\right]}_{y_2}$$

Ex solve: $y'' - xy = 0$ about $x_0 = 1$

$a(x) = 1$

$a(x_0) = a(1) = 1 \neq 0$ ordinary point

let $y = \sum_{n=0} a_n (x-1)^n$ $\Longrightarrow$ The general form $y = \sum_{n=0} a_n (x-x_0)^n$

* let $x - 1 = t$

$$y = \sum_{n=0} a_n (t)^n$$

$$y' = \sum_{n=1} n a_n t^{n-1}$$

$$y'' = \sum_{n=2} n(n-1) a_n t^{n-2}$$

$$\sum_{n=2} n(n-1)\, a_n t^{n-2} - (t+1) \sum_{n=0} a_n t^n = 0$$

تعويضها في المعادلة الأصلية.

$$\sum_{n=2} n(n-1)\, a_n t^{n-2} - \sum_{n=0} a_n t^{n+1} - \sum_{n=0} a_n t^n$$

$* \text{ let } n = n+2$ $\qquad\qquad * \text{ let } n = n-1$

$$\sum_{n=0} (n+2)(n+1)\, a_{n+2}\, t^n - \sum_{n=1} a_{n-1}\, t^n - \sum_{n=0} - \sum a_n t^n = 0$$

Now: بنساوي المعادلات ببعض عن طريق انه اطلع الحدود من ال series

$$2a_2 - a_0 + \sum_{n=1} \left[(n+2)(n+1)\, a_{n+2} - a_{n-1} - a_n \right] t^n = 0$$

$$\Rightarrow 2a_2 - a_0 = 0 \quad \Rightarrow \quad a_2 = \frac{1}{2} a_0$$

$$(n+2)(n+1)\, a_{n+2} - a_{n-1} - a_n = 0 \quad \geq 1$$

recursion Formula

$$a_{n+2} = \frac{a_{n-1} + a_n}{(n+2)(n+1)}$$

$\boxed{n=1}$ $\quad a_3 = \dfrac{a_0 + a_1}{(3)(2)}$

$\boxed{n=2}$ $\quad a_4 = \dfrac{a_1 + a_2}{(4)(3)} = \dfrac{a_1 + \frac{1}{2} a_0}{12} = \dfrac{a_1}{12} + \dfrac{a_0}{24}$

$$y = \sum_{n=0} a_n t^n$$

$$y = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots$$

$$a_0 + a_1 t + \frac{1}{2} a_0 t^2 + \left(\frac{a_0}{6} + \frac{a_1}{6}\right) t^3 + \cdots$$

$$= a_0 \left[1 + \frac{1}{2} t^2 + \frac{1}{6} t^3 + \cdots \right] + a_1 \left[t + \frac{1}{6} t^3 + \cdots \right]$$

$$= a_0 \left[1 + \frac{1}{2} (x-1)^2 + \cdots \right] + a_1 \left[(x-1) + \frac{1}{6} (x-1^3) + \cdots \right]$$

26/2/2018

Cauchy - Euler Equation.

$$a\,\overset{r(r-1)}{x^2y''} + b\,\overset{r}{xy'} + c\,\overset{1}{y} = 0 \qquad x>0$$

The equation is called cauchy - Euler.

let $y = x^r$ be a solution.

$y' = r x^{r-1}$

$y'' \; r(r-1) x^{r-2}$

$\Rightarrow \; a x^2 . r(r-1) x^{r-2} + b x . r x^{r-1} + c x^r = 0$

$a r(r-1) x^r + b r x^r + c x^r = 0$

$\boxed{a r(r-1) + b r + c = 0}$ char. equation.

* We have three cases.

① $r_1 \neq r_2$

$y_1 = x^{r_1}, \; y_2 = x^{r_2}$

② $r_1 = r_2 = r$

$y_1 = x^r, \; y_2 = x^r \ln x$

③ $r = \lambda \pm \mu i$

$y_1 = x^\lambda \cos(\mu \ln x), \quad y_2 = x^\lambda \sin(\mu \ln x)$

Five Apple

Ex: solve:

① $2x^2y'' + 3xy' - y = 0$

② $\overset{r(r-1)}{x^2y''} - 3\overset{r}{xy'} + 4\overset{1}{y} = 0$

③ $x^2y'' + 7xy' + 13y = 0$

④ $xy'' - 2y' = 0$

(H.W) $(x+2)^2y'' + (x+2)y' + y = 0$

① $2r(r-1) + 3r - 1 = 0$

$2r^2 + r - 1 = 0$

$(2r-1)(r+1) = 0$

$-r$

$2r$

$2r - 1 = 0$ or $r + 1 = 0$

$r = \frac{1}{2}$ or $r = -1$

$y_1 = x^{\frac{1}{2}}$, $y_2 = x^{-1}$

The general solution is.

$$y = c_1y_1 + c_2y_2 = \boxed{c_1\sqrt{x} + \frac{c_2}{x}}$$

2) $r(r-1) - 3r + 4 = 0$

$r^2 - 4r + 4 = 0$

$(r-2)(r-2) = 0$

$r = 2, 2$

$y_1 = x^2, \quad y_2 = x^2 \ln x$

The general solution.

$y = c_1 x^2 + c_2 x^2 \ln x$

---

3) $r(r-1) + 7r + 13 = 0$

$r^2 + 6r + 13 = 0$

$a = 1 \quad b = 6 \quad c = 13$

$b^2 - 4ac = 36 - 4(1) * 13$

$= 36 - 52$

$= -16$ complex الحل ع القانون العام

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{-16}}{2}$

$= \frac{-6 \pm 4i}{2} = \underset{\lambda}{-3} \pm \underset{\mu}{2}i$

$y_1 = x^{\lambda} \cos(\mu \ln x)$

$y_1 \rightarrow y_1 = x^{-3} \cos(2 \ln x)$

$y_2 \rightarrow y_2 = x^{-3} \sin(2 \ln x)$

The general solution is.

$y = c_1 x^{-3} \cos(2 \ln x) + c_2 x^{-3} \sin(2 \ln x)$

* الاولوية بالحل على ال chara لأنه اسهل

(4) multiply by x

$$\overset{r(r-1)}{x^2 y''} - 2\overset{r}{x y'} = 0$$

$$r(r-1) - 2r = 0$$

$$r^2 - 3r = 0$$

$$r(r-3) = 0$$

$$r = 0, 3$$

$$y_1 = x^0 = 1$$

$$y_2 = x^3$$

The general solution is

$$\boxed{y = c_1 (1) + c_2 x^3}$$

Ex: find a 2^{nd} order linear homogeneous D.E with solution:

$$y(x) = c_1 x + c_2 x^2$$

$$r_1 = 1 \quad , \quad r_2 = 2$$

$$(r-1)(r-2) = 0$$

$$\boxed{r^2 - 3r} + 2 = 0$$

$\updownarrow \longrightarrow$ قارن بينهم

$$r(r-1) - 2r + 2 = 0$$

$$x^2 y'' - 2xy' + 2y = 0$$

$\boxed{H.W}$ سؤال الواجب

let $u = x + 2$ * مهم كثير

$$y'_{(x)} = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 1 \cdot \frac{dy}{dy}$$

$$y''_{(x)} = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{dy}{du}\right) = \frac{dv}{du} \cdot \frac{du}{dx} \quad 1$$

$$\boxed{= \frac{d^2y}{du^2}}$$

* قاعدة عامة
اذا كان معامل x واحد
بقدر اعوض ال u على طول
اما اذا كان معامل x لايساوي 1
بربع الرقم الي عند ال y''
والرقم الي عند y' بطلع زي ماهو

$$u^2 y''_{(u)} + u\, y'_{(u)} + y_{(u)} = 0 \quad \text{cauchy - Euler equation.}$$

27/2/2018

* Nonhomogeneous D.E / Undetermined Coefficient.

consider $y'' + p(x)y' + q(x)y = f(x)$ --- (**)

To solve (**) we find

① y_h: homogeneous solution.

② y_p: particular solution.

The general solution is:

$$y = y_h + y_p$$

Ex Solve: $y'' - 4y = 4x^2 + 10$

ال polynomial يكون على الشكل التالي

① $y'' - 4y = 0$ chara equation

$Ax^2 + \underline{\underline{B}}x + C$

$r^2 - 4 = 0 \qquad r = 2, -2$

معادلة polynomial

لأنه ال function

من الدرجة الثانية

$y_1 = e^{2x}, \quad y_2 = e^{-2x}$

$y_h = c_1 e^{2x} + c_2 e^{-2x}$

② let $y_p(x) = Ax^2 + Bx + C$ be a solution.

$y_p' = 2Ax + B$

$y_p'' = 2A$

$$2A - 4Ax^2 - 4Bx - 4C = 4x^2 + 10$$

$$-4Ax^2 - 4Bx + (2A - 4C) = 4x^2 + 10$$

$$-4A = 4 \quad ----- (1) \qquad A = -1$$

$$-4B = 0 \quad ----- (2) \qquad B = 0$$

$$2A - 4C = 10 \quad --- (3)$$

$$2(-1) - 4C = 10$$
$$-4C = 10 + 2$$
$$C = \frac{12}{-4} = \boxed{-3}$$

$$y_p(x) = -x^2 - 3$$

$\therefore$ The general solution is:

$$y = y_h + y_p = c_1 e^{2x} + c_2 e^{-2x} + (-x^2 - 3)$$

* ملاحظة : ال Particular عشان نحسبه بنوخذ ال general ال(function) form

$\underline{Ex}$ solve: $\quad y'' + 9y = 2e^{3x}$

Solution:

① $y'' + 9y = 0$ chara equation

$$\left.\begin{aligned} r^2 + 9 &= 0 \\ r^2 &= -9 \\ r &= \pm\sqrt{-9} \\ &= \underbrace{0}_{\lambda} \pm \underbrace{3}_{\mu} i \end{aligned}\right\} \Rightarrow \text{complex}$$

$$y_1 = e^{\lambda x} \cos \mu x$$

$$\left.\begin{aligned} y_1 &= \cos 3x \\ y_2 &= \sin 3x \end{aligned}\right] \Rightarrow \boxed{y_h = c_1 \cos 3x + c_2 \sin 3x}$$

② let $y_p(x) = Ae^{3x}$

* كقيمة ال exponential
بنوخذ ال exp كما هو
وال A هي form
ال polynomial

$$y_p' = 3Ae^{3x}$$
$$y_p'' = 9Ae^{3x}$$

$$9Ae^{3x} + 9Ae^{3x} = 2e^{3x}$$

$$18Ae^{3x} = 2e^{3x}$$

$$18A = 2 \Rightarrow A = \frac{1}{9} \Rightarrow y_p(x) = \frac{1}{9}e^{3x}$$

The general solution is.

$$y = y_h + y_p = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{9}e^{3x}$$

Ex solve : $y'' - 2y' = 3\sin 2x$

Solution :

① $y'' - 2y' = 0$

$r^2 - 2r = 0 \Rightarrow r = 0, 2$

$y_1 = 1 \ , \ y_2 = e^{2x}$

$y_h = c_1 + c_2 e^{2x}$

② let $y_p(x) = A\sin 2x + B\cos 2x$

* القاعدة
بنوخذ ال sine كما هو مع الزاوية
داخله و ال A هي ال poly form
بالحل بحط sin و ال cos

$y_p'(x) = 2A\cos 2x - 2B\sin 2x$

$y_p'' = -4A\sin 2x - 4B\cos 2x$

$-4A\sin 2x - 4B\cos 2x - 4A\cos 2x + 4B\sin 2x = 3\sin 2x$

$(-4A + 4B)\sin 2x + (-4B - 4A)\cos 2x = 3\sin 2x$

$-4A + 4B = 3$ --- ①

$4B - 4A = 0$ --- ② $\quad A = -B$

$4B + 4B = 3 \Rightarrow B = \frac{3}{8} \quad A = -\frac{3}{8} \Rightarrow y_p = -\frac{3}{8}\sin 2x + \frac{3}{8}\cos 2x$

The general solution is : $y = y_h + y_p = \frac{-3}{8}\sin 2x + \frac{3}{8}\cos 2x + c_1 + c_2 e^{2x}$

Ex solve ; $y'' - y = 2e^x$

Solution:

① $y'' - y = 0$ chara equation $r^2 - 1 = 0$

$r^2 = 1$ $r = -1, 1$

$y_1 = e^{-x}$, $y_2 = e^x$

$y_h = c_1 e^{-x} + c_2 e^{+x}$

② let $y_{p(x)} = Axe^x$ be a solution

* لازم الفرض للـ particular ما يكون مكرر من الـ Homo اذا كان مكرر بضربه بـ x وبظل اضرب بـ x حتى يبطل مكرر

$y_p' = Axe^x + Ae^x$

$y_p'' = Axe^x + Ae^x + Ae^x$

$y_p' = Axe^x + 2Ae^x$

* تعويضنا الـ condition بكون في الـ general solution.

$y'' - y = 2e^x$

$A\cancel{xe^x} + 2Ae^x - \cancel{Axe^x} + Ae^x = 2e^x$

$2Ae^x = 2e^x$

$2A = 2$

$\boxed{A = 1} \Rightarrow y_{p(x)} = xe^x$

The general solution is.

$y = y_h + y_p \Rightarrow y_{(x)} = c_1 e^x + c_2 e^{-x} + xe^x$

<u>Ex</u> consider: $y'' - 2y' = x^2 + 2e^x + x\sin x$

write a suitable form for $y_p(x)$ if the undetermined coefficients is to be used!

Solution: ① $y'' - 2y' = 0$

$r^2 - 2r = 0 \Rightarrow r = 0, 2$

$y_1 = e^{0x} = 1 \quad , \quad y_2 = e^{2x}$

$y_h = c_1 + c_2 e^{2x}$

ال constant موجود
بال Homo عشان :
هيك ضربت
المعادلة كاملة ب X

② $y_{p(x)} = (A_2 x^2 + A_1 x + A_0)\,x$ ← (term of x^2)

$+ \; Be^x$ ← term of e^x

$+ (Cx + D)\sin x + (Ex + F)\cos x$ ← term of $\sin x$

* بحط كل Term بسطر
عشان ما اتخربط.

5/3/2018

write a suitable form for the particular solution $y_p(x)$ if the undetermined coefficiants is to be used:

1) $y'' - 5y' + 4y = e^{x}\cos x$

2) $y'' + 2y' = 2x^3 + 3x + x^2 e^{-2x} + \cos 2x$

3) $y'' + 2y' + 2y = e^{-x} + xe^{-x}\sin x + e^{-x}\cos x$

---

① $y'' - 5y' + 4y = 0$

$r^2 - 5r + 4 = 0$

$(r-1)(r-4) = 0$

$r = 1, 4$

$y_1 = e^{x}, \quad y_2 = e^{4x}$

$y_h = c_1 e^{x} + c_2 e^{4x}$

$y_p(x) = Ae^{x}\cos x + Be^{x}\sin x$

---

② $y'' + 2y' = 0$

$r^2 + 2r = 0$

$r(r+2) = 0 \Rightarrow r = 0, -2 \Rightarrow y_1 = 1, \; y_2 = e^{-2x}$

$y_h = c_1 + c_2 e^{-2x}$

$$y_{p(x)} = x\left(A_1 x^3 + A_2 x^2 + A_3 x + A_0\right)$$

$$+ x\left(B_2 x^2 + B_1 x + B_0\right)e^{-2x}$$

$$+ A\cos 2x + B\sin 2x$$

---

③ $y'' + 2y' + 2y = 0$

$$r^2 + 2r + 2 = 0$$

$a = 1 \quad b = 2 \quad c = 2$

$$b^2 - 4ac = 4 - 4(1)(2)$$
$$= -4$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2}$$

$$= \underbrace{-1}_{\lambda} \pm \underbrace{1}_{\mu} i$$

$$y_1 = e^{\lambda x}\cos\mu x \quad , \quad y_2 = e^{\lambda x}\sin\mu x$$

$$y_h = c_1 e^{-x}\cos x + c_2 e^{-x}\sin x$$

$$y_{p(x)} = Ae^{-x}$$
$$+ x\left[(Bx + c)e^{-x}\sin x + (Dx + E)e^{-x}\cos x\right]$$
$$+ x\left[\underbrace{Fe^{-x}\cos x + Ge^{-x}\sin x}\right]$$

شو مصيركم الا سعادة
بجبر كل الاذى

عندو ولد مجرورات
بقدر ما اصطدم

والأجمل انه ينشال
اذا زعلنا منكم

Five Apple

* Notes: very important.

1) $\dots = 2\sin x + 4\cos x$

* كيف اتخلص من التكرار:

$$A\sin x + B\cos x$$
$$+ C\cos x + D\sin x$$

$$\underbrace{(B+C)}_{c_1}\cos x + \underbrace{(A+D)}_{c_2}\sin x$$

$$= c_1\cos x + c_2\sin x$$

2) $\dots = \sin x\cos x$

$$= \frac{1}{2}\sin 2x$$

3) $\dots = \sin^2 x$

$$= \frac{1}{2}(1-\cos 2x)$$

$$\frac{1}{2} - \frac{1}{2}\cos 2x$$

* لازم احول لـ Linear
الـ sine والـ cos
والزاوية و التحويل
يكون عن طريق المتطابقات.

$$A + B\cos 2x + C\sin 2x$$

4) $\dots = \cosh x$

$$= \frac{e^{x}+e^{-x}}{2} = \frac{1}{2}e^{x} + \frac{1}{2}e^{-x}$$

* Variation of parameters.

Consider: $y'' + p_{(x)} y' + q(x) y = r(x)$

① $y_h = c_1 y_1 + c_2 y_2$

② $y_p(x) = -y_1 \int \frac{y_2 r}{w} + y_2 \int \frac{y_1 r}{w}$

*لتعويض ال r(x) لازم يكون معامل y" (1) يعني بنقسم عليه.

Ex: solve

$$x^2 y'' - 3x y' + 3y = 24 x^{-3}$$

① $x^2 y'' - 3x y' + 3y = 0$

$r(r-1) - 3r + 3 = 0$ cacye oiler

$r^2 - 4r + 3 = 0 \Rightarrow (r-1)(r-3)$

$r = 1, 3$

$y_1 = x \quad , y_2 = x^3 \quad \Rightarrow y_h = c_1 x + c_2 x^3$

$W[x, x^3] = x(3x^2) - x^3 \Rightarrow 3x^3 - x^3 = 2x^3$

$r(x) = \frac{24 x^{-3}}{x^2} = 24 x^{-5}$

$$y_p(x) = -y_1 \int \frac{y_2 r}{W} + y_2 \int \frac{y_1}{W} r$$

$$= -x \int \frac{x^3 \cdot 24x^{-5}}{2x^3} dx + x^3 \int \frac{x \cdot 24x^{-5}}{2x^3} dx$$

$$= -12x \int x^{-5} dx + 12x^3 \int x^{-7} dx$$

$$= -12x \cdot \frac{x^{-4}}{-4} + 12x^3 \cdot \frac{x^{-6}}{-6}$$

$$= 3x^{-3} - 2x^{-3} = x^{-3}$$

The general solution:

$$y = y_h + y_p$$

$$\boxed{= c_1 x + c_2 x^3 + x^{-5}}$$

H.W) solve: $y'' + y = \sec x$

7/3/2018

ch #13 Higher-order Homogeneous D.E's

Ex Solve:

① $y'''' - y = 0$

② $y'''' + 2y'' + y = 0$

③ $y^{(5)} - 6y'''' + 12y''' - 8y'' = 0$

④ $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$

---

① $r^4 - 1 = 0$ char

$(r^2)^2 - (1)^2 = 0$ فرق بين مربعين

$(r^2-1)(r^2+1) = 0$

$r = 1, 1$, $r^2 = -1$

$r = \pm i$ → $\cos$, $\sin$ على طول

$y_1 = e^x$, $y_2 = e^{-x}$, $y_3 = \cos x$, $y_4 = \sin x$

The general solution:

$y_{(x)} = c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$

Five Apple

② $r^4 + 2r^2 + 1 = 0$

$(r^2)^2 + 2(r^2) + 1 = 0$

$(r^2+1)(r^2+1) = 0$

$r^2+1 = 0$ oR $r^2+1=0$

$r^2 = -1$ $\qquad$ $r^2 = -1$

$r = \pm i$ $\qquad$ $r = \pm i$ $\qquad$ ضربنا بـ X عشان هم مكررين.

$y_1 = \cos x, \quad y_2 = \sin x, \quad \underline{y_3 = X\cos x, \quad y_4 = X\sin x}$

The general Solution :

$y = c_1 \cos x + c_2 \sin x + c_3 X \cos X + c_4 X \sin X$

③ $r^5 - 6r^4 + 12r^3 - 8r^2 = 0$ $\qquad$ * بنحل عن خوارزمية القسمة كمان!!

$r^2(r^3 - 6r^2 + 12r - 8) = 0$

$r^2\left[(r^3-8) - 6r(r-2)\right] = 0$

$r^2\left[(r-2)(r^2+2r+4) - 6r(r-2)\right] = 0$

$r^2(r-2)\left[r^2 + 2r + 4 - 6r\right] = 0$

$r^2(r-2)\left[r^2 - 4r + 4\right] = 0$

$\underline{r*r}$
2 root

$r^2(r-2)(r-2)(r-2) = 0 \Rightarrow r = 0, 0, 2, 2, 2$

$y_1 = 1 \quad , \quad y_2 = x \quad , \quad y_3 = e^{2x} \quad , \quad y_4 = x e^{2x} \quad , \quad y_5 = x^2 e^{2x}$

The general Solution is :

$$y = c_1(1) + c_2(x) + c_3(e^{2x}) + c_4(xe^{2x}) + c_5(x^2 e^{2x})$$

---

(4) * ملاحظة كتابة الـ Euler للـ High order بتكون على الصورة التالي

$$\overset{r(r-1)(r-2)}{x^3 y'''} - 3\overset{r(r-1)}{x^2 y''} + 6\overset{r}{x y'} - 6\overset{1}{y} = 0$$

ولو في حدود اكثر بضل الطرح هالاسي

$$\underbrace{r(r-1)(r-2)} - \underbrace{3r(r-1)} + \underbrace{6r-6}_{6(r-1)} = 0$$

$$(r-1)\left[r(r-2) - 3r + 6\right] = 0$$

$$(r-1)\left[r^2 - 5r + 6\right] = 0$$

$$(r-1)(r-2)(r-3) = 0$$

$r = 1, 2, 3$

$y_1 = x \quad , \quad y_2 = x^2 \quad , \quad y_3 = x^3$

$y = c_1 x + c_2 x^2 + c_3 x^3 \Longleftarrow$ The general Solution.

* مادة امتحان الفيرست.

* Non homogeneous Higher - order DE's

* (undetermined)

* الاولوية الها اذا كانت قابلة للتطبيق.

* لازم يكون المعاملات ثوابت عشان نتطبق.

Ex Solve : $y^{(4)} - y^{''} = 12x^2 - 18$

① $y^4 - y^{''} = 0$

$r^4 - r^3 = 0$

$r^3(r-1) = 0$

$r = 0, 0, 0, 1$

$y_1 = 1 , y_2 = x , y_3 = x^2 , y_4 = e^x$

The ~~gene~~ form of Homogeneous.

$$y = c_1(1) + c_2(x) + c_3(x^2) + c_4(e^x)$$

② let $y_{p(x)} = x^3 \left[Ax^2 + Bx + C \right]$ — بتأكد من التكرار

عشان التكرار

$$y_{p(x)} = Ax^5 + Bx^4 + Cx^3$$

$y' = 5Ax^4 + 4Bx^3 + 3Cx^2$

$y'' = 20Ax^3 + 12Bx^2 + 6Cx$

$y''' = 60Ax^2 + 24Bx + 6C$

$y'''' = 120Ax + 24B + 0$

$$120Ax + 24B - 60Ax^2 - 24Bx - 6C = 12x^2 - 18$$

$$-60Ax^2 + \left[120A - 24B\right]x + \left[24B - 6c\right] = 12x^2 - 18$$

$-60A = 12$ --- ① $\quad A = \frac{-1}{5}$ *معامل x^2

$120A - 24B = 0$ --- ② $\quad B = -1$ *معامل x

$24B - 6c = -18$ --- ③ $\quad c = -1$ معامل الثابت.

$$y_p(x) = \frac{-1}{5}x^5 - x^4 - x^3$$

The general solution is : $y = y_h + y_p$

$$y = c_1 + c_2 x + c_3 x^2 + c_4 e^x - \frac{1}{5}x^5 - x^4 - x^3$$

*لو كان السؤال أكبر من 3^{rd} order بستخدم هاي الطريقة وانا [illegible].

9/4/2018

* consider

$$a(x)\, y'' + p(x)\, y' + q(x)\, y = 0$$

If $a(x_0) = 0$ Then x_0 is called singuler

singuler

regular singuler   irregular singuler

* Def: let x_0 be a singular point

Then x_0 is called regular singuler

If the following two limits exsist:

① $\lim\limits_{x \to x_0} \dfrac{p(x)}{a(x)} (x - x_0) = p_0$

② $\lim\limits_{x \to x_0} \dfrac{q(x)}{a(x)} (x - x_0)^2 = q_0$

Ex Find all reguler singuler points:

① $x(x-1)^2 y'' + 2xy' + (x-1)y = 0$

② $\left(x - \frac{\pi}{2}\right)^2 y'' + (\cos x)y' + (\sin x)y = 0$

③ $x^2(1-x^2)\, y'' + \frac{2}{x} y' + 4y = 0 \Leftarrow$ (H.w)

④ $(\sin x)y'' + xy' + 4y = 0$

① $a(x) = x(x-1)^2$

$a(x) = 0 \Rightarrow x(x-1)^2 = 0$

$\Rightarrow \underbrace{x = 0, 1}_{\text{singuler points.}}$

$\boxed{x=0} \Rightarrow$ ① $\lim_{x \to 0} \frac{2x}{\cancel{x}(x-1)^2}\, \cancel{x} = \lim_{x \to 0} \frac{2x}{(x-1)^2} = \frac{0}{(-1)^2} = 0$ P_0

② $\lim_{x \to 0} \frac{\cancel{(x-1)}}{\cancel{x}(x-1)^{\cancel{2}}}\, x^{\cancel{2}} = \lim_{x \to 0} \frac{x}{x-1} = \frac{0}{-1} = 0$ q_0

$x_0 = 0$ is reguler singuler.

$\boxed{x=1} \Rightarrow$ ① $\lim_{x \to 1} \frac{2x}{x(x-1)^2}(x-1) = \lim_{x \to 1} \frac{2}{x-1} = \frac{2}{0}$ D.N.E

$x_0 = 1$ is irreguler singuler.

Five Apple

* Remark: let x_0 be a reguler singuler point.

Then we define the ~~[illegible]~~ indicial equation as:

$$r(r-1) + p_0 r + q_0 = 0$$

③ in Ex① find the indicial equation at $x_0 = 0$

Solution:
$$r(r-1) + p_0 r + q_0 = 0$$

$r(r-1) = 0$ The equation.

② $a(x) = \left(x - \frac{\pi}{2}\right)^2$

$$a(x) = 0 \Longrightarrow \left(x - \frac{\pi}{2}\right)^2 = 0$$

$x = \frac{\pi}{2}$ singuler point.

① $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\left(x - \frac{\pi}{2}\right)^{2}} \cdot \left(x - \frac{\pi}{2}\right)$

$\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}} \longleftarrow$ اشتق البسط
و اشتق المقام

$\lim_{x \to \frac{\pi}{2}} \frac{-\sin x}{1} = -\sin \frac{\pi}{2} = -1$ p_0

② $\lim_{x \to \frac{\pi}{2}} \frac{\sin x}{\cancel{(x-\frac{\pi}{2})^2}} \cdot \cancel{(x-\frac{\pi}{2})^2}$

$= \sin\frac{\pi}{2} = 1 \quad q_0$

$x_0 = \frac{\pi}{2}$ is regular singular.

---

*Remark: ① $\sin(n\pi) = 0$

② $\cos(n\pi) = (-1)^n$

---

④ $\sin x = 0$

$$\underbrace{x = n\pi, \quad n \in \mathbb{Z} \;(\ldots, -1, 0, 1, \ldots)}_{\text{singular point}}$$

① $\lim_{x \to n\pi} \frac{x}{\sin x}(x - n\pi)$

$\lim_{x \to n\pi} \frac{x^2 - n\pi x}{\sin x}$ ← اشتق

$\lim_{x \to n\pi} \frac{2x - n\pi}{\cos x} = \frac{n\pi}{(-1)^n}$

② $\lim_{x \to n\pi} \frac{4}{\sin x} (x - n\pi)^2$

$$\lim_{x \to n\pi} \frac{8(x-n\pi)}{\cos x} = \frac{0}{(-1)^n} = 0$$

~~$\{n\pi : n \in \mathbb{Z}\}$ are all reguler singuler.~~

(iii) In Ex ④ find the indicial equation at $x = 3\pi$

$$r(r-1) + p_0 r + q_0 = 0$$

$$r(r-1) - 3\pi r = 0$$

$$r^2 - r - 3\pi r = 0$$

$$r^2 - (1+3\pi) r = 0$$

* consider:

$$a(x)\, y'' + p(x)\, y' + q(x)\, y = 0.$$

let x_0 be reguler singuler.

Then we can find a series solution of The

form: $y(x) = \sum_{n=0} a_n (x - x_0)^{n+r}$ حل ال indicial equation. $a_0 \neq 0$

This method is called Frobenius Method.

Ex Solve:

$$2xy'' + y' + xy = 0 \quad \text{about } x_0 = 0$$

$a(x) = 2x$

$a(0) = 0$ singuler. (reguler singuler) ما بيتاج اني اتحبر.

let $y = \sum_{n=0} a_n x^{n+r}$, $a_0 \neq 0$

$y' = \sum_{n=0} (n+r)\, a_n\, x^{n+r-1}$ * العداد بعد ميكش من الصفر عشان عنا ال (r)

$y'' = \sum_{n=0} (n+r)(n+r-1)\, a_n\, x^{n+r-2}$

$$2x\sum_{n=0}(n+r)(n+r-1)a_n x^{n+r-2}+\sum_{n=0}(n+r)a_n x^{n+r-1}$$

$$+x\sum_{n=0}a_n x^{n+r}=0$$

$$\Rightarrow \sum_{n=0}2(n+r)(n+r-1)a_n x^{n+r-1}+\sum_{n=0}(n+r)a_n x^{n+r-1}$$

$$+\underbrace{\sum_{n=0}a_n x^{n+r+1}}=0$$

* دخلنا المعاملات

حة

دولنا

لحاة

$$\sum_{n=2}a_{n-2}x^{n+r-1}$$

$$\Rightarrow 2r(r-1)a_0x^{r-1}+2r(r+1)a_1x^{r}+ra_0x^{r-1}+(r+1)a_1x^{r}$$

$$+\sum_{n=2}\left[2(n+r)(n+r-1)a_n+(n+r)a_n+a_{n-2}\right]x^{n+r-1}=0$$

* طلعنا حدين من اول ثنتين .

$$\Rightarrow \left[2r(r-1)+r\right]a_0x^{r-1}+\left[(r+1)(2r+1)\right]a_1x^{r}$$

$$+\sum_{n=2}\left[2(n+r)(n+r-1)a_n+(n+r)a_n+a_{n-2}\right]x^{n+r-1}=0$$

* طلعنا عوامل مشتركة .

$a_0 \neq 0$

$[2r(r-1)+r]\,a_0 = 0$ ← اول حد بالمعادلة

so

$2r(r-1)+r = 0$

$2r^2 - r = 0$

$r(2r-1) = 0$

$\boxed{r = 0,\ \frac{1}{2}}$

$(r+1)(2r+1)\,a_1 = 0$ ← ثاني حد بالمعادلة

$\boxed{r=0} \Rightarrow a_1 = 0$

$\boxed{r=\frac{1}{2}} \Rightarrow a_1 = 0$

$2(n+r)(n+r-1)\,a_n + (n+r)\,a_n + a_{n-2} = 0 \qquad n \geq 2$ ← ثالث حد

$\boxed{r=0}$ $\quad 2n(n-1)\,a_n + n\,a_n = -a_{n-2}$

$[\underbrace{2n(n-1)+n}_{2n^2-n}]\,a_n = -a_{n-2}$

11/4/2018

$$a_n = -\frac{a_{n-2}}{2n^2-n} \Rightarrow \boxed{a_n = -\frac{a_{n-2}}{n(2n-1)} \quad n \geq 2}$$

$\boxed{n=2}$ $a_2 = \frac{-a_0}{2.3}$

$\boxed{n=3}$ $a_3 = \frac{-a_1}{3.5} = 0$

$\boxed{n=4}$ $a_4 = \frac{-a_2}{4.7} = \frac{a_0}{(2)(4)(3)(7)}$

$\boxed{n=5}$ $a_5 = \frac{-a_3}{5.9} = 0$

$\boxed{n=6}$ $a_6 = \frac{-a_4}{6.11} = \frac{-a_0}{(2)(4)(6)\ (3)(7)(11)}$

$$y_1 = \sum_{n=0} a_n x^n$$

$$= a_0 + a_1 x + a_2 x^2 + \cdots$$

$$= a_0 - \frac{a_0}{6} x^2 + \cdots$$

$$= a_0 \left[1 - \frac{x^2}{6} + \cdots \right]$$

note:

للحدود الفردية. $\boxed{a_{2n-1} = 0 \quad n \geq 1}$

للحدود الزوجية $\boxed{a_{2n} = \frac{(-1)^n a_0}{(2)(4)\cdots(2n)\ (3)(7)\cdots(4n-1)}}$

very important

* كمل حل السؤال.

Find y_2 !!

Five Apple

⓪ Let r_1, r_2 be the roots of the indicial equation.

① if $r_1 - r_2 \notin \mathbb{Z}$, then we have two independent

ليس عدد صحيح.

Solutions. $y_1 = \sum_{n=0} a_n x^{n+r_1}$, $y_2 = \sum_{n=0} a_n x^{n+r_2}$

② if $r_1 - r_2 \in \mathbb{Z}$, $(r_1 - r_2 > 0)$, then

$$y_1 = \sum_{n=0} a_n x^{n+r_1},$$

$$y_2 = \underbrace{k}_{\text{constant}} y_1 \ln x + \sum_{n=0} a_n x^{n+r_2}, \quad a_0 \neq 0$$

* الحل بينبلش بال root الاكبر (r_1)

for the points:

2) may have a ln term.

3) should have a ln term.

③ $r_1 = r_2 = r$ then:

$$y_1 = \sum_{n=0} a_n x^{n+r}$$

$$y_2 = y_1 \ln x + \sum_{n=1} a_n x^{n+r}$$

Ex Solve : $\overset{q(x)}{x} y'' + y = 0$ near $x_0 = 0$

$q(0) = 0$ singular

① $\lim_{x \to 0} \frac{0}{x} \cdot x = 0 \quad p_0$

② $\lim_{x \to 0} \frac{1}{x} x^2 = 0 \quad q_0$

$r(r-1) + \cancel{p_0}^{0} r + \cancel{q_0}^{0} = 0$

$r(r-1) = 0$

$r = 0, 1 \implies 1 - 0 = 1 \in \mathbb{Z}$

let $y = \sum_{n=0} a_n x^{n+1}$

$y' = \sum_{n=0} (n+1) a_n x^n$

$y'' = \sum_{n=1} n(n+1) a_n x^{n-1}$

$x \sum_{n=1} n(n+1) a_n x^{n-1} + \sum_{n=0} a_n x^{n+1} = 0$

$\sum_{n=1} n(n+1) a_n x^n + \sum_{n=0} a_n x^{n+1} = 0$

$\sum_{n=1} n(n+1) a_n x^n + \sum_{n=1} a_{n-1} x^n = 0$

$\sum_{n=1} \left[n(n+1) a_n + a_{n-1} \right] x^n = 0$

Five Apple

$$n(n+1)\,a_n + a_{n-1} = 0 \qquad n \geq 1$$

$$a_n = \frac{-a_{n-1}}{n(n+1)} \qquad n \geq 1$$

$\boxed{n=1}$ $\quad a_1 = \dfrac{-a_0}{(1)(2)}$

$\boxed{n=2}$ $\quad a_2 = \dfrac{-a_1}{(2)(3)} = \dfrac{a_0}{(2)(3)\,(1)(2)}$

$$y_1 = a_0 x + a_1 x^2 + a_2 x^3 + \cdots$$

$$y_1 = a_0 x - \frac{a_0}{2} x^2 + \cdots$$

$$a_0\left[x - \frac{x^2}{2} + \cdots\right]$$

$$\boxed{y_2 = K y_1 \ln x + \sum_{n=0} a_n x^{n+0}}$$

كتابة شكل ال y_2 فقط
تكفي بشرط تعويض
ال root فيه.

16/4/2018

* Laplace Transform.

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt$$

Ex Find $\mathcal{L}\{2\}$

1. $\mathcal{L}\{2\} = \dfrac{2}{s}$, $s > 0$
2. $\mathcal{L}\{a\} = \dfrac{a}{s}$
3. $\mathcal{L}\{e^{at}\} = \dfrac{1}{s-a}$
4. $\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$
5. $\mathcal{L}\{\sin(at)\} = \dfrac{a}{s^2+a^2}$
6. $\mathcal{L}\{\cos at\} = \dfrac{s}{s^2+a^2}$

Five Apple

Ex ① $\mathcal{L}\{t^3\} = \frac{3!}{s^4}$

② $\mathcal{L}\{\sin 2t\} = \frac{2}{s^2+4}$

③ $\mathcal{L}\{e^{-3t}\} = \frac{1}{s+3}$

---

④ $\mathcal{L}\{F(t)\} = F(s)$

$\mathcal{L}\{g(t)\} = Y(s)$

Ex. $\int_0^\infty \cos 2t \, e^{-st} \, dt$

$= \mathcal{L}\{\cos 2t\} = \frac{s}{s^2+4}$

---

⑤ $\mathcal{L}\{F'(t)\} = s\,F(s) - F(0)$

⑥ $\mathcal{L}\{F''(t)\} = s^2 F(s) - s\,F(0) - F'(0)$

Ex Find $\mathcal{L}\{\underbrace{\sin at}_{F(t)}\} = F(s)$

let $F(t) = \sin at$

$f'(t) = a \cos at$

$f''(t) = -a^2 \sin at$

$f''(t) = -a^2 f(t)$

$\mathcal{L}\{f''(t)\} = -a^2 \mathcal{L}\{f(t)\}$

$s^2 F(s) - sF(0) - f'(0) = -a^2 F(s)$

$s^2 F(s) - a = -a^2 F(s)$

$s^2 F(s) + a^2 F(s) = a$

$F(s) = \dfrac{a}{s^2 + a^2}$

---

(3) $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$

Ex Find $\mathcal{L}\{e^{2t} t^2\} = \mathcal{L}\{t^2\}_{s-2}$

$= \dfrac{2}{s^3}\Big|_{s-2} = \dfrac{2}{(s-2)^3}$

Ex $\mathcal{L}\{e^{-2t}\cos 3t\}$

$$= \mathcal{L}\{\cos 3t\}_{s+2}$$

$$= \frac{s}{s^2+9}\Bigg|_{s+2} = \frac{s+2}{(s+2)^2+9}$$

$\mathcal{L}\{1\} = \frac{1}{s}$ * حفظ

$\mathcal{L}\{t\} = \frac{1}{s^2}$ مهم

$\mathcal{L}\{t^2\} = \frac{2}{s^3}$

$\mathcal{L}\{t^3\} = \frac{6}{s^4}$

Ex's

① $\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$

② $\mathcal{L}^{-1}\left\{\frac{4}{s^3}\right\} = 2\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\} = 2t^2$

③ $\mathcal{L}^{-1}\left\{\frac{5}{s^4}\right\} = \frac{5}{6}\mathcal{L}^{-1}\left\{\frac{6}{s^4}\right\} = \frac{5}{6}t^3$ * قسمة على (6)

وضربت بـ (6)

④ $\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} = \cos 3t$

⑤ $\mathcal{L}^{-1}\left\{\frac{2}{s^2+25}\right\} = \frac{2}{5}\mathcal{L}^{-1}\left\{\frac{5}{s^2+25}\right\} = \frac{2}{5}\sin 5t$

⑥ $\mathcal{L}^{-1}\left\{\frac{2}{(s-4)^3}\right\}$

* لازم اشيل ال shift عن طريق ترجيعه لأصله الي هو ال expon.

$= e^{4t}\,\mathcal{L}^{-1}\left\{\frac{2}{s^3}\right\}$

* بنوخذ ال inverse لل function of S

$= e^{4t}\cdot t^2$

⑦ $\mathcal{L}^{-1}\left\{\frac{2}{(s-1)^2+9}\right\}$

$= e^{t}\,\mathcal{L}^{-1}\left\{\frac{2}{s^2+9}\right\}$

$= \frac{2}{3}e^{t}\,\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\} = \frac{2}{3}e^{t}\sin 3t$

⑧ $\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2+1}\right\}$

$= e^{2t}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}$

$= e^{2t}\cos t$

⑨ $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)^2+9}\right\} \Rightarrow \mathcal{L}^{-1}\left\{\frac{(s-2)+2}{(s-2)^2+9}\right\}$ ℓo

$\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2+9}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{(s-2)^2+9}\right\}$

$$e^{2t}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} + \frac{2}{3}\,e^{2t}\,\mathcal{L}^{-1}\left\{\frac{3}{s^2+9}\right\}$$

$$= e^{2t}\cos 3t + \frac{2}{3}\,e^{2t}\sin 3t.$$

(10) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+2s+2}\right\}$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}$$ * اكمال مربع.

$$= e^{-t}\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = e^{-t}\sin t.$$

*Remark ملاحظة للحل بطريقة أخرى.

$$\mathcal{L}^{-1}\left\{\frac{2s-3}{s^2-3s+2}\right\}$$

$$\frac{2s-3}{(s-1)(s-2)} = \frac{1}{s-1} + \frac{1}{s-2}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-1} + \frac{1}{s-2}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} = e^{t} + e^{2t}$$

(11) $\mathcal{L}^{-1}\left\{\dfrac{2s-3}{s^2-2s+2}\right\}$

$$\mathcal{L}^{-1}\left\{\frac{2s-3}{(s-1)^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{(2s-2)-1}{(s-1)^2+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{2(s-1)}{(s-1)^2+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+1}\right\}$$

$$2e^{t}\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} - e^{t}\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$= 2e^{t}\cos t - e^{t}\sin t$$

Note: $\dfrac{s}{s^2-a^2} = \mathcal{L}\{\cosh\}$

$$\frac{a}{s^2-a^2} = \mathcal{L}\{\sinh\}$$

18/4/2018

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{F(s)}{s}$$

*Note $\frac{1}{ab} = \frac{\frac{1}{a}}{b}$

Ex
$$\mathcal{L}\left\{\int_0^t \cos 2u\,du\right\} = \frac{\frac{s}{s^2+4}}{s} = \frac{s}{s(s^2+4)}$$

$$= \frac{1}{s^2+4}$$

$$\mathcal{L}^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(u)\,du$$ ⟸ important

Ex
$$\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\} = \sin 2t.$$

Ex
$$\mathcal{L}^{-1}\left\{\frac{2}{s(s^2+4)}\right\} = \mathcal{L}^{-1}\left\{\frac{\overbrace{\frac{2}{(s^2+4)}}^{F(s)}}{s}\right\}$$

$$= \int_0^t \mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\} = \int_0^t \sin 2u\,du$$

$$= -\frac{\cos 2u}{2}\Big|_0^t$$ ⟶ بشيل ال (-) ويعكس الفترة

$$= \frac{\cos 2u}{2}\Big|_t^0$$

$$= \frac{1}{2}\left[1 - \cos 2t\right]$$

Five Apple

Ex $\mathcal{L}^{-1}\left\{\dfrac{2}{s^2(s^2+4)}\right\}$

$$= \mathcal{L}^{-1}\left\{\dfrac{\overbrace{\dfrac{2}{s(s^2+4)}}^{F(s)}}{s}\right\}$$

$$= \int_0^t \mathcal{L}^{-1}\left\{\frac{2}{s(s^2+4)}\right\} \rightsquigarrow$$

نفس ال حل السؤال السابق

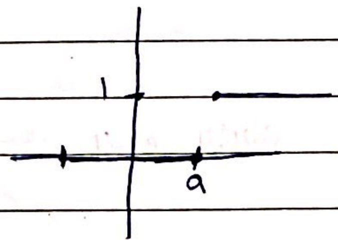
$$= \int_0^t \frac{1}{2}\left[1 - \cos 2u\right] du$$

$$\frac{1}{2}\left[u - \frac{\sin 2u}{2}\right]_0^t = \frac{1}{2}\left[t - \frac{\sin 2t}{2}\right]$$

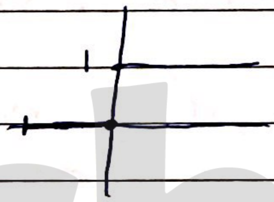
Five Apple

* unit step function. $(u(t-a))$ (Heaviside function)

$$u(t-a) = \begin{cases} 0 & t < a \\ 1 & t \geq a \end{cases}$$



$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$



$u(t) = 1 \quad t \geq 0$ * دايماً ال $u(t)$ في ال $\mathcal{L}$ بتساوي (1)

- $\mathcal{L}\{u(t-a)\} = \frac{e^{-as}}{s}$ ضـ

- $\mathcal{L}\{u(t-a)\ f(t)\} = e^{-as}\ \mathcal{L}\{f(t+a)\}$

Ex $\mathcal{L}\{u(t)\} = \frac{1}{s}$

Ex $\mathcal{L}\{u(t-2)\} = \frac{e^{-2s}}{s}$

Ex $\mathcal{L}\{t^2 u(t-1)\} = e^{-s}\ \mathcal{L}\{f(t+1)\}$

$= e^{-s}\ \mathcal{L}\{(t+1)^2\} = e^{-s}\ \mathcal{L}\{t^2 + 2t + 1\}$

$= e^{-s}\left[\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right]$

* Note :

$$\sin\left(\frac{\pi}{2}+x\right)=\cos x$$

الربع الثاني $\longleftarrow$

$$\cos\left(\frac{\pi}{2}+x\right)=-\sin x$$

$$\sin(\pi+x)=-\sin x$$

الربع الثالث $\longleftarrow$

$$\cos(\pi+x)=-\cos x$$

* ال $\frac{\pi}{2}$ و $\frac{3\pi}{2}$ و ... كلهم بحولوا ال sine ل cos والعكس

* ال π و 2π و ... كلهم بخلوا ال sine زي ما هو وال cos زي ما هو.

Ex

$$\mathcal{L}\{u(t-\pi)\underbrace{\cos t}_{f(t)}\}$$

$$= e^{-\pi s}\,\mathcal{L}\{f(t+\pi)\}$$

$$= e^{-\pi s}\,\mathcal{L}\{\cos(\pi+t)\}$$

$$= e^{-\pi s}\,\mathcal{L}\{-\cos t\}$$

$$= -e^{\pi s}\cdot\frac{s}{s^2+1}$$

Ex $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s}\right\} = u(t-2)$

Ex $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2}\right\}$

* أي سؤال inverse بكون فيه e بربطه ل unit

$= u(t-2)\left[\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}\right]_{t-2}$

$= u(t-2) \cdot [t]_{t-2}$

$= u(t-2)\,(t-2)$

Ex $\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^2+1}\right\}$

$= u(t-3)\left[\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}\right]_{t-3}$

$= u(t-3)\,[\sin t]_{t-3}$

$= u(t-3)\,\sin(t-3)$

Ex $\mathcal{L}^{-1}\left\{\frac{s+e^{-2s}}{s^2+9}\right\}$

$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} + \mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2+9}\right\}$

$= \cos 3t + u(t-2)\left[\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}\right]_{t-2}$

$= \cos 3t + u(t-2) \cdot \left\{ \frac{1}{3} \sin 3t \right\}_{t-2}$

$= \cos 3t + u(t-2) \; \frac{1}{3} \sin 3(t-2)$

---

* pecie wise function

$$f(t) = \begin{cases} f_1(t) & 0 \le t < a \\ f_2(t) & a \le t < b \\ f_3(t) & t \ge b \end{cases}$$

$$f(t) = f_1(t)\left[u(t) - u(t-a)\right] + f_2(t)\left[u(t-a) - u(t-b)\right]$$

$$+ f_3(t) \; u(t-b).$$

Ex consider

$$f(t) = \begin{cases} 2 & 0 \le t < 2 \\ t & t \ge 2 \end{cases}$$

find $F(s)$.

Soln : $f(t) = 2\{u(t) - u(t-2)\} + t\,u(t-2)$

$$f(t) = 2 - 2u(t-2) + t\,u(t-2)$$

$$f(t) = \mathcal{L}\{2\} - 2\,\mathcal{L}\{u(t-2)\} + \mathcal{L}\{t\,u(t-2)\}$$

$$= \frac{2}{s} - \frac{2e^{-2s}}{s} + \mathcal{L}\{t\,u(t-2)\}$$

$$\mathcal{L}\{\overset{f(t)}{t}\,u(t-2)\} = e^{-2s}\,\mathcal{L}\{f(t+2)\}$$

$$= e^{-2s}\,\mathcal{L}\{t+2\}$$

$$= e^{-2s}\left[\frac{1}{s^2} + \frac{2}{s}\right]$$

$$= \frac{2}{s} - \frac{2e^{-2s}}{s} + e^{-2s}\left[\frac{1}{s^2} + \frac{2}{s}\right]$$

Ex $\mathcal{L}\{e^{2t}\,u(t-1)\}$

* الاولوية لقاعدة ال (e)

اذا شفت ال (e) بخليه
وبطلع ال L الي بعده بعدين
بعمل ازاحة.

$$= \mathcal{L}\{u(t-1)\}_{s-2}$$

$$\left.\frac{e^{-s}}{s}\right|_{s-2} = \frac{e^{-(s-2)}}{s-2}$$

Five Apple

23/4/2018

* Dirac – Delta function

$$d(t-a) \begin{cases} 0 & t \neq a \\ \infty & t = a \end{cases}$$

(i) $$\int_0^\infty d(t-a)\, g(t)\, dt = g(a)$$

(ii) $$\int d(t-a) \, dt = 1 \qquad g(t) = 1$$

(iii) $$d(t) = d(t-0) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

Ex $$\int_0^\infty \underbrace{\cos t}_{g(t)}\, d(t-\pi)\, dt = g(\pi)$$

$$= \cos \pi = -1$$

Theorem :

$$\mathcal{L}\{d(t-a)\} =$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$\mathcal{L}\{\delta(t-a)\} = \int_0^\infty \delta(t-a)\, \underbrace{e^{-st}}_{g(t)}\, dt = g(a) = e^{-as}$$

$$\textcircled{\small{!!}} \Rightarrow \mathcal{L}\{\delta(t-a)\} = e^{-as}$$

Ex. Solve $y'' + y = \delta(t-1)$, $y(0) = 0$, $y'(0) = 1$

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2 F(s) - s f(0) - f'(0).$$

Solution:

$$\mathcal{L}\{y''(t)\} + \mathcal{L}\{y(t)\} = \mathcal{L}\{\delta(t-1)\}$$

$$s^2 \underline{Y}(s) - s\, y(0) - y'(0) + \underline{Y}(s) = e^{-s}$$

capital

* لما ناخذ ال inverse لل capital بصير ال small

$$s^2 \underline{Y}(s) - 1 + \underline{Y}(s) = e^{-s}$$

$$(s^2+1)\, \underline{Y}(s) = 1 + e^{-s} \Rightarrow \underline{Y}(s) = \frac{1 + e^{-s}}{s^2+1}$$

Five Apple

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{e^{-s}}{s^2+1}\right\}$$

$$= \sin t + u(t-1)\,\boxed{\sin t}_{\,t-1}$$

* بال Direct بزيد واحد

بال inverse بطرح واحد

$$= \sin t + u(t-1)\,\sin(t-1)$$

Ex: solve: $y'' + 2y' + 2y = \delta(t-3)$ $\quad y(0)=0$, $\quad y'(0)=0$

Solution: $\mathcal{L}\{y''(t)\} + 2\,\mathcal{L}\{y'(t)\} + 2\,\mathcal{L}\{y(t)\} = \mathcal{L}\{\delta(t-3)\}$

$$s^2\,\underline{y}(s) - \cancel{s\,y(0)} - \cancel{y'(0)} + 2\left[s\,\underline{y}(s) - \cancel{y(0)}\right] + 2\,\underline{y}(s) = e^{-3s}$$

$$s^2\,\underline{y} + 2s\,\underline{y} + 2\,\underline{y} = e^{-3s}$$

$$\underline{y}(s) = \frac{e^{-3s}}{s^2+2s+2} \implies y(t) = \mathcal{L}^{-1}\left\{\frac{e^{-3s}}{(s+1)^2+1}\right\}$$

$$y(t) = u(t-3)\,\left[\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+1}\right\}\right]_{t-3} \implies e^{-t}\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = e^{-t}\sin t$$

$$= u(t-3)\, e^{-(t-3)} \sin(t-3)$$

Ex

$$\mathcal{L}\{\cos t\ \delta(t-\pi)\}$$

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)\, e^{-st}\, dt$$

$$\mathcal{L}\{\cos t\ \delta(t-\pi)\} = \int_0^\infty \cos t\ \delta(t-\pi)\, e^{-st}\, dt \quad = g(\pi)$$

$$= \cos \pi\ e^{-\pi s}$$

$$= -e^{-\pi s}$$

① $\mathcal{L}\{t\, f(t)\} = -F'(s) \Rightarrow \dfrac{-d}{ds}\mathcal{L}\{f(t)\}$

② ~~[illegible]~~

Ex $\mathcal{L}\{t \cos t\} = -F'(s)$

$$F(s) = \mathcal{L}\{\cos t\} = \frac{s}{s^2+1}$$

$$-F'(s) = -\frac{s^2+1-s(2s)}{(s^2+1)^2} = -\frac{1-s^2}{(s^2+1)^2}$$

مشتقة بسيطة
كالقاعدة.

$$= \frac{s^2-1}{(s^2+1)^2}$$

$$\mathcal{L}^{-1}\{F'(s)\} = -t\,f(t)$$ "inverse"

$$\mathcal{L}^{-1}\{F(s)\} = f(t)$$ very important Note.

Ex find $\mathcal{L}^{-1}\left\{\underbrace{\ln\left(1+\frac{4}{s^2}\right)}_{F(s)}\right\} = f(t)$

بدور عى هاظا،

$$F(s) = \ln\left(\frac{s^2+4}{s^2}\right)$$

$$F(s) = \ln s^2+4 - \ln s^2$$

$$F(s) = \ln s^2+4 - 2\ln s$$

$$F'(s) = \frac{2s}{s^2+4} - \frac{2}{s}$$

$$\mathcal{L}^{-1}\{F'(s)\} = 2\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} - 2\,\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$-t\,f(t) = 2\cos 2t - 2$$

$$f(t) = \frac{2-2\cos 2t}{t}$$

Ex $\quad \mathcal{L}^{-1}\left\{ \underbrace{\cot^{-1}(s)}_{F(s)} \right\} = f(t)$

$F(s) = \cot^{-1}(s)$

* لو كان في زاوية (θ)
رقم

$F'(s) = -\frac{1}{s^2+1}$

$-\frac{1}{(\theta s)^2+1} \cdot \theta$

$\mathcal{L}^{-1}\{F'(s)\} = -\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$

$= -t\, f(t) = -\sin t$

$f(t) = \frac{\sin t}{t}$

Five Apple

25/4/2018

Ex Solve :

① $x'(t) = 2x(t) + y(t)$ $\quad x(0) = 1$ $\quad$ H.W

$y'(t) = 3x(t) + 4y(t)$ $\quad y(0) = 0$

② $x'(t) + y(t) = 1$ $\quad x(0) = -1$ $\quad$ H.W

$y'(t) - x(t) = 0$ $\quad y(0) = 1$

③ $\frac{dx}{dt} + y = \sin t$ $\quad x(0) = 0$

$\frac{dy}{dt} + x = \cos t$ $\quad y(0) = 2$

* ال d بنستخدمه
لما يكون معطيني
conditions.

* Note :

$$\mathcal{L}^{-1}\left\{\frac{2s-1}{(s^2-1)(s^2+1)}\right\} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{Cs+D}{s^2+1}$$

④ Cramer's rule.

$$\left.\begin{aligned} 2x + 4y &= 2 \\ x - 3y &= 1 \end{aligned}\right\} \qquad x = \frac{D_1}{D}$$

where D is the Det

$$y = \frac{D_2}{D}$$

* يستخدم لحل معادلتين
بمجهولين او 3 معادلات
ب 3 مجاهيل وهكذا !

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$$D = \begin{vmatrix} 2 & 4 \\ 1 & -3 \end{vmatrix} = -6 - 4 = -10$$

Non Homogeneous Term.

$$D_1 = \begin{vmatrix} 2 & 4 \\ 1 & -3 \end{vmatrix} = -10$$

Non Homogeneous Term.

$$D_2 = \begin{vmatrix} 2 & 2 \\ 1 & 1 \end{vmatrix} = 0$$

where $X = \frac{D_1}{D} = \frac{-10}{-10} = 1$

$$y = \frac{D_2}{D_1} = \frac{0}{-10} = 0$$

---

(3)

$$\mathcal{L}\{x'(t)\} + \mathcal{L}\{y(t)\} = \mathcal{L}\{\sin t\} \quad \text{---- (1)}$$

$$\mathcal{L}\{y'(t)\} + \mathcal{L}\{x(t)\} = \mathcal{L}\{\cos t\} \quad \text{---- (2)}$$

For (1) $\Rightarrow$ $sX_{(s)} - x_{(0)} + \underline{y}_{(s)} = \frac{1}{s^2+1}$

$$= sX + \underline{y} = \frac{1}{s^2+1}$$ First equation (*)

For (2) $\Rightarrow$ $s\underline{y}_{(s)} - y_{(0)} + X_{(s)} = \frac{s}{s^2+1}$

$$= X + s\underline{y} = \frac{s}{s^2+1} + 2 = \frac{2s^2+s+2}{s^2+1}$$ second equation. (**)

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Now use cramer's rule.

$$D = \begin{vmatrix} s & 1 \\ 1 & s \end{vmatrix} = s^2 - 1$$

$$D_1 = \begin{vmatrix} \frac{1}{s^2+1} & 1 \\ \frac{2s^2+s+2}{s^2+1} & s \end{vmatrix} = \frac{s}{s^2+1} \quad \frac{-2s^2+s+2}{s^2+1} = \frac{-2s^2-2}{s^2+1}$$

$$\boxed{= -2}$$

$$D_2 = \begin{vmatrix} s & \frac{1}{s^2+1} \\ 1 & \frac{2s^2+s+2}{s^2+1} \end{vmatrix} = \frac{2s^3+s^2+2s}{s^2+1} - \frac{1}{s^2+1} = \frac{2s^3+s^2+2s-1}{s^2+1}$$

$$X_{(s)} = \frac{D_1}{D} = \frac{-2}{s^2-1}$$

$$x_{(t)} = -2 \mathcal{L}^{-1}\left\{\frac{1}{s^2-1}\right\} = -2 \sinh t$$

$$y_{(s)} = \frac{D_2}{D} = \frac{2s^3+s^2+2s-1}{(s^2+1)(s^2-1)} = \underbrace{\frac{As+B}{s^2+1} + \frac{C}{s-1} + \frac{D}{s+1}}$$

$$y_{(t)} = \mathcal{L}^{-1}\left\{\frac{2s^3+s^2+2s-1}{(s^2+1)(s^2-1)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1} + \frac{2}{s^2-1}\right\}$$

* درجة البسط اقل من درجة المقام ← بنحل كسور جزئية

* كمل الحل

Five Apple

Ex Solve :

$$t y'' + (1-t) y' + y = 0$$

$$y_{(0)} = 1 \quad , \quad y'(0) = -1$$

Solution :

$$t y'' + y' - t y' + y = 0$$

$$\mathcal{L}\{t y''\} + \mathcal{L}\{y'\} - \mathcal{L}\{t y'\} + \mathcal{L}\{y\} = 0$$

$$\frac{-d}{ds}\mathcal{L}\{y''\} + s\underline{y}_{(s)} - y_{(0)} + \frac{d}{ds}\mathcal{L}\{y'\} + \underline{y}_{(s)} = 0$$

$$\frac{-d}{ds}\left[s^2 \underline{y}_{(s)} - s\, y_{(0)} - y'_{(0)}\right] + s\underline{y}_{(s)} - y_{(0)} + \frac{d}{ds}\left[s\underline{y}_{(s)} - y_{(0)}\right] + \underline{y}_{(s)} = 0$$

$$\Rightarrow \frac{-d}{ds}\left[s^2 \underline{y}(s) - s + 1\right] + s\underline{y}_{(s)} - 1 + \frac{d}{ds}\left[s\underline{y}(s) - 1\right] + \underline{y}_{(s)} = 0$$

$$-\left[s^2 \underline{y}'(s) + 2s\,\underline{y}(s) - 1\right] + s\underline{y}_{(s)} - 1 + s\underline{y}'(s) + \underline{y}(s) + \underline{y}_{(s)} = 0$$

$$-s^2 \underline{y}'_{(s)} - 2s\, \underline{y}_{(s)} + \cancel{1} + s\, \underline{y}_{(s)} - \cancel{1} + s\underline{y}'(s) + 2\underline{y}_{(s)} = 0$$

$$(-s^2 + s)\, y'(s) + \left[\frac{-2s + s + 2}{2 - s}\right] \underline{y}(s) = 0$$

$$(-s^2+s)\,y'(s) = (s-2)\,y(s)$$

separable first order equation

$$(s-s^2)\frac{dy}{ds} = (s-2)\,y(s)$$

$$\frac{s-2}{s-s^2} = \frac{s-2}{s(1-s)} = \frac{A}{s} + \frac{B}{1-s}$$

$$= \frac{Bs + A(1-s)}{s(1-s)}$$

$$s-2 = Bs + A(1-s)$$

$s=0$ $\boxed{-2=A}$

$s=1$ $\boxed{-1=B}$

$$\int \frac{dy}{y} = \int \frac{s-2}{s-s^2}\,ds$$

$$\ln y = \int\left[\frac{-2}{s} + \frac{-1}{1-s}\right] ds$$

$$\ln y = \ln^{-2} + \ln|1-s| + c$$

$$\ln y = \ln\frac{1-s}{s^2} + c$$

$$y(s) = e^{\ln\frac{1-s}{s^2}} \cdot e^{c}$$

$y(0)=1$

$1=-c$

$\boxed{c=-1}$

$$y(s) = \frac{(1-s)}{s^2}\,c$$

$$y(t) = c\,\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s}\right\}$$

$$y(t) = c\,(t-1)$$

$$y(t) = 1-t$$

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Scanned with CamScanner

30/4/2018

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(u)\, du$$

Ex : find $\mathcal{L}\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{1}{u^2+1}\, du$

$F(s) = \frac{1}{s^2+1}$

$$= \tan^{-1} u \Big|_s^\infty$$

$$= \tan^{-1} \infty - \tan^{-1} s$$

$$= \frac{\pi}{2} - \tan^{-1} s$$

30/4/2018

New chapter.

* let $\underset{a \times b}{A}$, $\underset{c \times d}{B}$ be two matrices.

Then AB is well defined iff $b = c$

Ex : let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}_{2 \times 3}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 0 & -2 \end{bmatrix}_{3 * 2}$

$$AB = \begin{bmatrix} 2 & -3 \\ 10 & 12 \end{bmatrix}_{2 \times 2}$$

Def : $\underset{2 \times 2}{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Identity

$$\underset{3 \times 3}{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Def : A^{-1} is an inverse of A iff

$$AA^{-1} = A^{-1}A = I$$

C: The matrix of cofactors

$adj(A)$: The adjeint matrix   * الصفوف بنظيرها للاعمدة

$$adj(A) = C^T$$ قلب المصفوفة للاعمدة

$$A^{-1} = \frac{1}{det(A)} \, adj(A)$$

Ex: if $A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$ Find A^{-1}

$$det(A) \; (|A|) = 2 \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} - 0 \begin{vmatrix} -2 & 4 \\ -5 & 6 \end{vmatrix} + 1 \begin{vmatrix} -2 & 3 \\ -5 & 5 \end{vmatrix}$$

$$= 2(-2) + (5) = 1$$

* Now solve cofactor matrix $= C$ let $c = \begin{bmatrix} - & - & \\ - & - & - \end{bmatrix}$

$$C = \begin{bmatrix} -2 & -8 & 5 \\ +5 & 17 & -10 \\ -3 & -10 & 6 \end{bmatrix}$$

$$adj(A) = C^T = \begin{bmatrix} -2 & 5 & -3 \\ -8 & 17 & -10 \\ 5 & -10 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

$$= \frac{1}{1} \begin{bmatrix} -2 & 5 & -3 \\ -8 & 17 & -10 \\ 5 & -10 & 6 \end{bmatrix}$$

---

* Properties

① If $\det(A) = 0$, then A^{-1} D.N.E (A is singular)

② $(A^{-1})^T = (A^T)^{-1}$

③ $(AB)^{-1} = B^{-1} A^{-1}$ الترتيب مهم لازم يكون بالاتجاه المعاكس.

④ $(KA)^{-1} = \frac{1}{K} A^{-1}$ حفظ مهم كثير

⑤ $(A+B)^{-1} \neq A^{-1} + B^{-1}$

* Determinent's

④ $\det(AB) = \det(A) \cdot \det(B)$

⑤ $\det(A^{-1}) = \dfrac{1}{\det(A)}$

⑥ $\det(A) = \det(A^T)$

⑦ let A be $n \times n$ matrix ⟸ مهم

Then $\det(KA) = K^n \det(A)$ ⟸ الصفة (القاعدة).

⑧ If the matrix has zero row OR zero column.

then the $\underline{\underline{\det = 0}}$

* تبديل اي صفين او كاعمودين في ال ~~matrix~~ det تضرب ال ~~matrix~~ det بسالب

* اضافة (multiples) لأي row or column مضاعفة لـ row or column أخرى لا يؤثر على ال det.

جمع الصف الاول مثلاً

* اضافة (multiples) لأي row or column بدون ما نجمع لاشي نتجبر نضرب ال det

الاصلية بال (multiples).

⑨ $A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow \det(A) = 0$ تطبيق على ملاحظة ال multiples الاضافة

الضرب ب -1 في صف واجمعه بالصف الي فوق

Five Apple

Ex: let

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 5$$

find.

1) $$\begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = -5$$

2) $$\begin{vmatrix} 2a_1 & 2a_2 & 2a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = -5 * 2 = -10$$

3) $$\begin{array}{c} \\ (2)(3) \leftarrow \\ \\ \end{array} \begin{vmatrix} 2a_1 & a_2 & a_3 \\ 6b_1 & 3b_2 & 3b_3 \\ 2c_1 & c_2 & c_3 \end{vmatrix} = 2 * 3 * 5 = 30$$

4) $$\begin{vmatrix} -a_1 & -a_2 & -a_3 \\ b_1 & b_2 & b_3 \\ c_1 - a_1 & c_2 - a_2 & c_3 - a_3 \end{vmatrix} = -5$$

Ex: find

$$\begin{vmatrix} a & a+1 & a+2 \\ b & b+1 & b+2 \\ c & c+1 & c+2 \end{vmatrix}$$

$$= \begin{vmatrix} a & a+1 & \cancel{a+}2 \\ b & b+1 & \cancel{b+}2 \\ c & c+1 & 2 \end{vmatrix}$$

ضربنا العمود الاول
بسالب.
وجمعناه للعمودين الي قبلوا

$$= 2 \begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix} = 0$$

---

* triangular matrices

$$A = \begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix}$$ lower triangular matrix

$$B = \begin{bmatrix} a & b & c \\ 0 & e & f \\ 0 & 0 & g \end{bmatrix}$$ upper triangular matrix.

Then $\text{Det}(A) = acf$ حاصل ضرب عناصر القطر

$\text{Det}(B) = aeg$

2/5/2018

Ex Solve :

$$x + y - z = 2$$

$$2x + z = 4$$

$$y - 2z = 0$$

امتحان الفاينال
10/5/2018
2-4
مدرج وصفي
التل.

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= (1)\begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} - (2)\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix}$$

$$(1)(-1) - 2(-2+1)$$

$$-1 + 2 = 1$$

Now :

$$x = \frac{D_1}{D}, \quad y = \frac{D_2}{D}, \quad z = \frac{D_3}{D}$$

$$D_1 = \begin{vmatrix} 2 & 1 & -1 \\ 4 & 0 & 1 \\ 0 & 1 & -2 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 0 & 4 \\ 0 & 1 & 0 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & 1 \\ 0 & 0 & -2 \end{vmatrix}$$

* How to solve partial fraction

$$\frac{2s^3+s^2+2s-1}{(s^2+1)(s-1)(s+1)} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{Cs+D}{s^2+1}$$

$$= \frac{A(s+1)(s^2+1) + B(s-1)(s^2+1) + (Cs+D)(s-1)(s+1)}{(s-1)(s+1)(s^2+1)}$$ ← بنوحد المقامات

بما انه المقامات متساوية
معناته البسط متساوي

$$2s^3+s^2+2s-1 = A(s+1)(s^2+1) + B(s-1)(s^2+1) + (Cs+D)(s-1)(s+1)$$

يا بنعوض ال Roots و بنعوض
يا بنعوض كوامل

$\boxed{s=1}$. $4 = 4A \Rightarrow \boxed{A=1}$

$\boxed{s=-1}$ $-4 = -4B \Rightarrow \boxed{B=1}$

$\boxed{s=0}$ $-1 = -D \Rightarrow \boxed{D=1}$

$\boxed{s=2}$ $23 = 20 + 3(2C+1) \Rightarrow \boxed{C=0}$