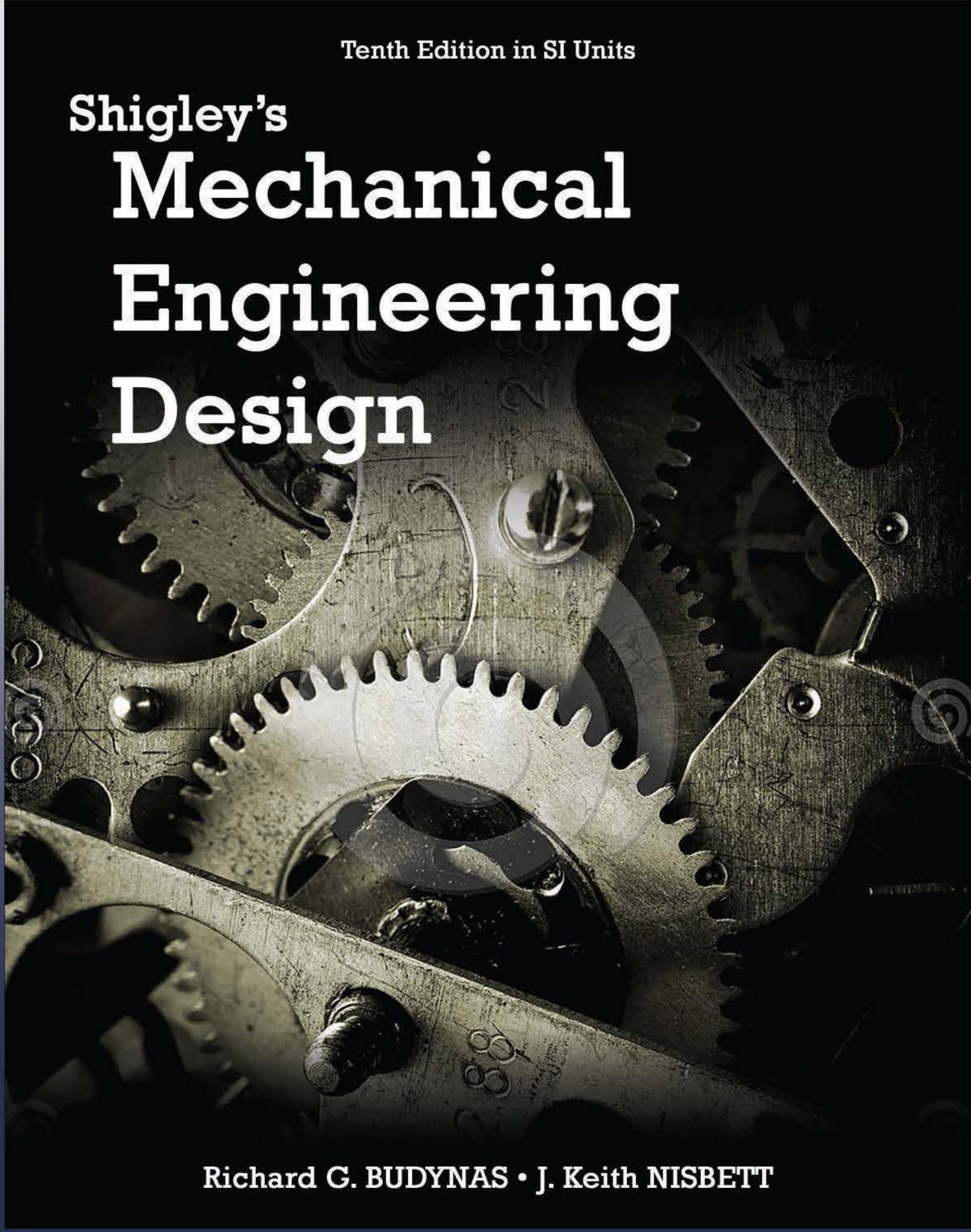
Lecture Slides

Chapter 12

Lubrication and Journal Bearings



Chapter Outline

Types of Lubrication

- Hydrodynamic
- Hydrostatic
- Elastohydrodynamic
- Boundary
- Solid film

Viscosity

- Shear stress in a fluid is proportional to the rate of change of velocity with respect to *y*

$$\tau = \frac{F}{A} = \mu \frac{du}{dy} \qquad (12\text{–}1)$$

- μ is *absolute viscosity*, also called *dynamic viscosity*

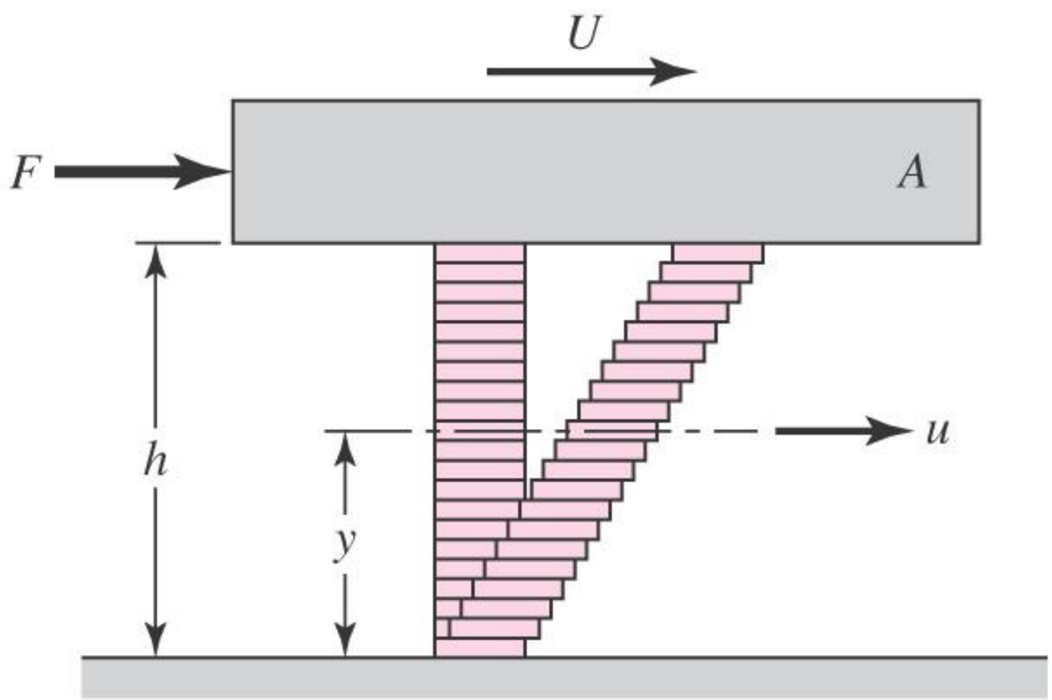


Fig. 12–1

Viscosity

- For most lubricating fluids, the rate of shear is constant, thus

$$du/dy = U/h$$

$$\tau = \frac{F}{A} = \mu \frac{U}{h} \qquad (12\text{–}2)$$

- Fluids exhibiting this characteristic are called *Newtonian fluids*

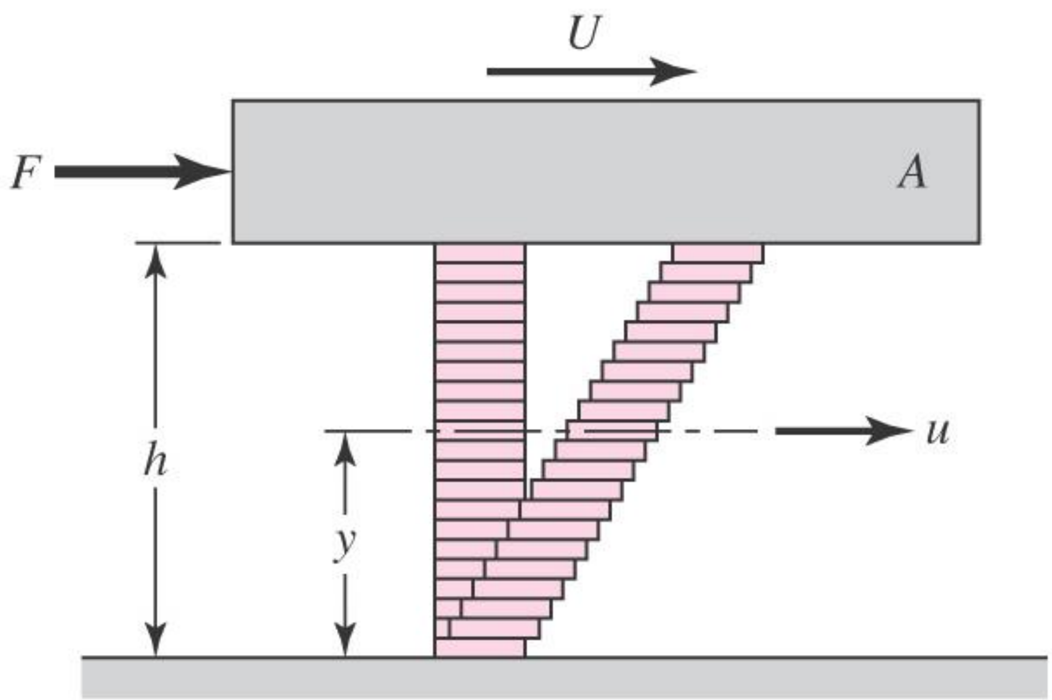


Fig. 12–1

Units of Viscosity

- Units of absolute viscosity
  - ips units: reyn = lbf·s/in^2
  - SI units: Pa·s = N·s/m^2
  - cgs units: Poise =dyn·s/cm^2
- cgs units are discouraged, but common historically in lubrication
- Viscosity in cgs is often expressed in centipoise (cP), designated by *Z*
- Conversion from cgs to SI and ips:

$$\mu(\text{Pa}\cdot\text{s}) = (10)^{-3} Z\ (\text{cP})$$

$$\mu(\text{reyn}) = \frac{Z\ (\text{cP})}{6.89(10)^6}$$

$$\mu(\text{mPa}\cdot\text{s}) = 6.89\ \mu'(\mu\text{reyn})$$

Units of Viscosity

- In ips units, the microreyn (μreyn) is often convenient.
- The symbol μ' is used to designate viscosity in μreyn

$$\mu = \mu'/(10^6)$$

Measurement of Viscosity

- *Saybolt Universal Viscosimeter* used to measure viscosity
- Measures time in seconds for 60 mL of lubricant at specified temperature to run through a tube 17.6 mm in diameter and 12.25 mm long
- Result is *kinematic viscosity*
- Unit is stoke = cm^2/s
- Using *Hagen-Poiseuille law* kinematic viscosity based on seconds Saybolt, also called *Saybolt Universal viscosity* (SUV) in seconds is

$$Z_k = \left(0.22t - \frac{180}{t}\right) \tag{12–3}$$

where Z_k is in centistokes (cSt) and t is the number of seconds Saybolt

Measurement of Viscosity

- In SI, kinematic viscosity n has units of m^2/s
- Conversion is $\nu(\mathrm{m}^2/\mathrm{s}) = 10^{-6} Z_k\ (\mathrm{cSt})$
- Eq. (12–3) in SI units,

$$\nu = \left(0.22t - \frac{180}{t}\right)(10^{-6}) \qquad (12\text{–}4)$$

- To convert to dynamic viscosity, multiply n by density in SI units

$$\mu = \rho\left(0.22t - \frac{180}{t}\right)(10^{-6}) \qquad (12\text{–}5)$$

where ρ is in kg/m^3 and μ is in pascal-seconds

Comparison of Absolute Viscosities of Various Fluids

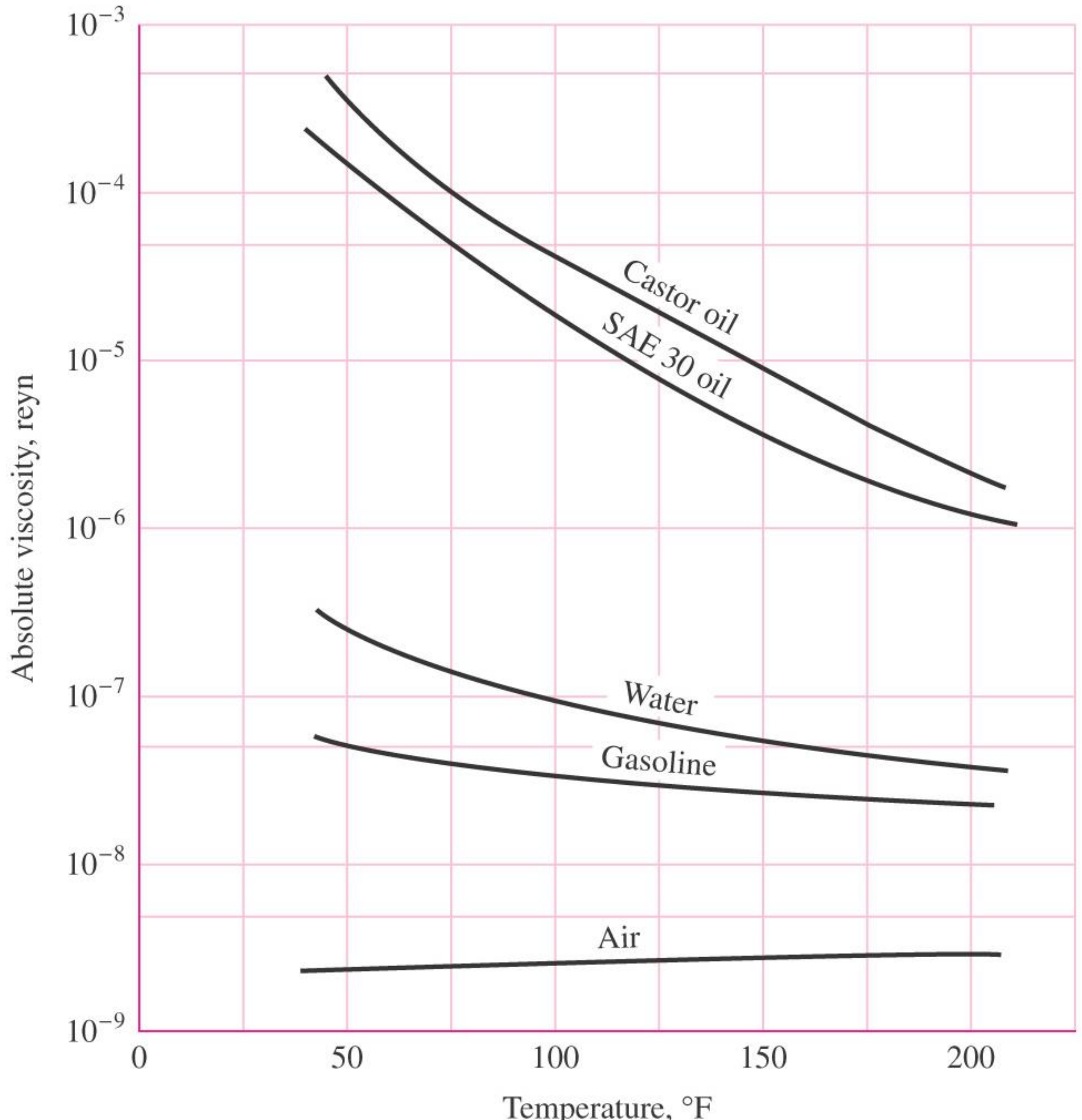


Fig. 12–2

Petroff's Lightly Loaded Journal Bearing

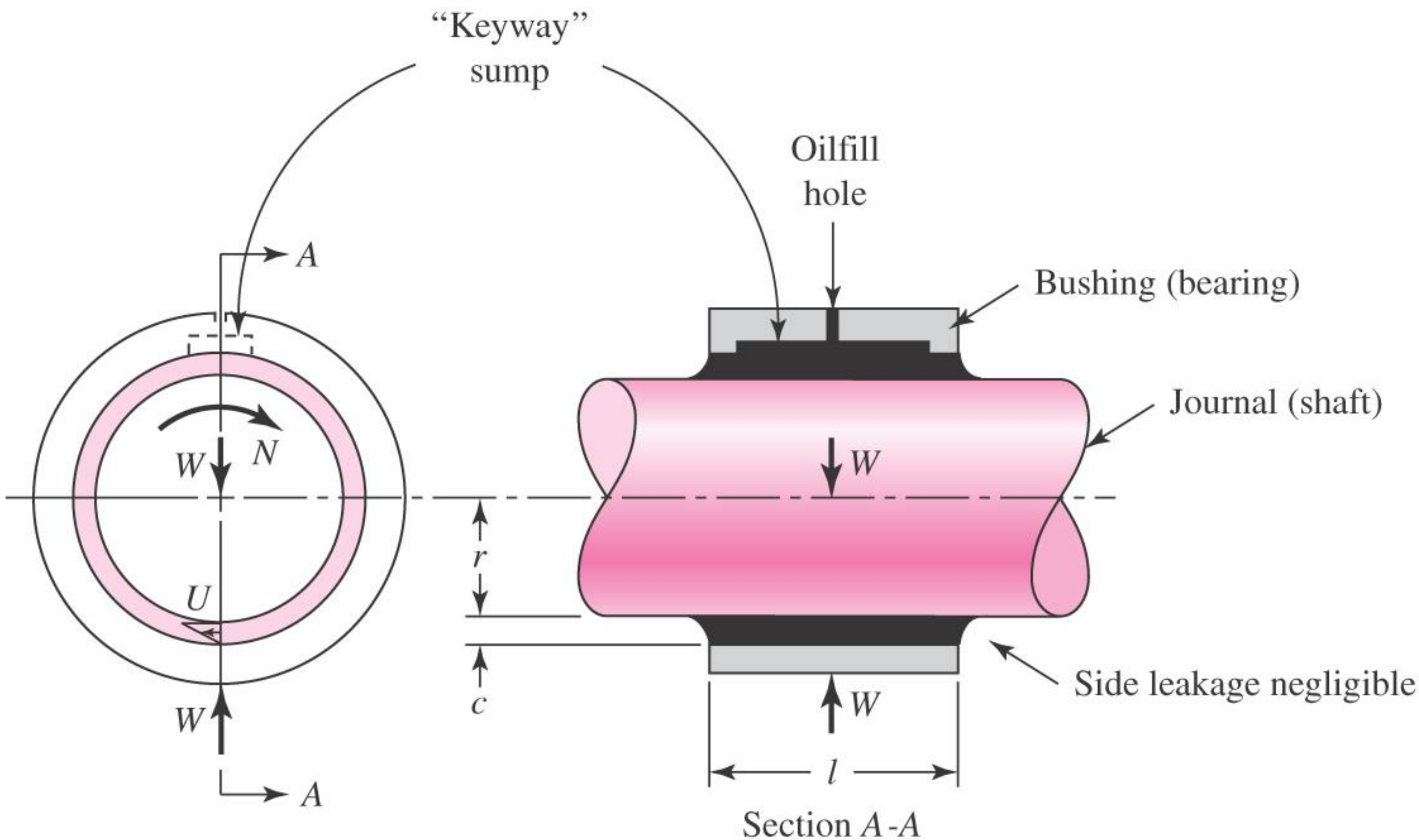


Fig. 12–3

Petroff's Equation

$$\tau = \mu \frac{U}{h} = \frac{2\pi r \mu N}{c} \qquad (a)$$

$$T = (\tau A)(r) = \left(\frac{2\pi r \mu N}{c}\right)(2\pi r l)(r) = \frac{4\pi^2 r^3 l \mu N}{c} \qquad (b)$$

$$T = f W r = (f)(2 r l P)(r) = 2 r^2 f l P \qquad (c)$$

$$f = 2\pi^2 \frac{\mu N}{P} \frac{r}{c} \qquad (12\text{–}6)$$

Important Dimensionless Parameters

- Some important dimensionless parameters used in lubrication
  - *r/c: radial clearance ratio*
  - *$\mu N/P$: Bearing characteristic*
  - *Sommerfeld number* or *bearing characteristic number*

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} \qquad (12\text{–}7)$$

- Interesting relation

$$f\frac{r}{c} = 2\pi^2 \frac{\mu N}{P}\left(\frac{r}{c}\right)^2 = 2\pi^2 S \qquad (12\text{–}8)$$

Stable Lubrication

- To the right of *AB*, changes in conditions are self-correcting and results in stable lubrication
- To the left of *AB*, changes in conditions tend to get worse and results in unstable lubrication
- Point *C* represents the approximate transition between metal-to-metal contact and thick film separation of the parts
- Common design constraint for point *B*, $\frac{\mu N}{P} \geq 1.7(10^{-6})$

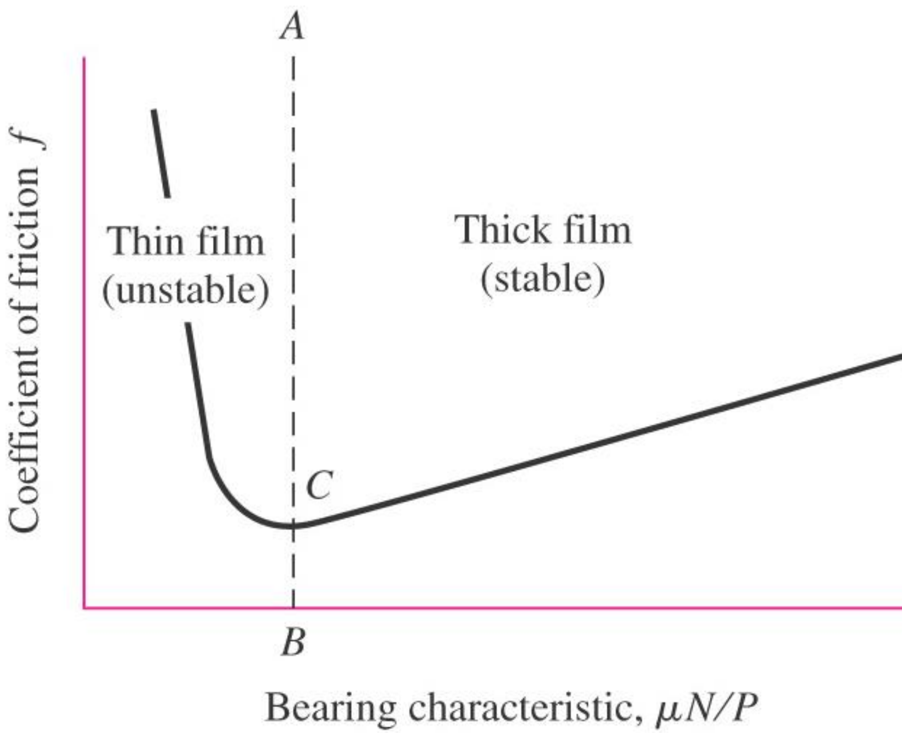


Fig. 12–4

(a)

Thick Film Lubrication

- Formation of a film

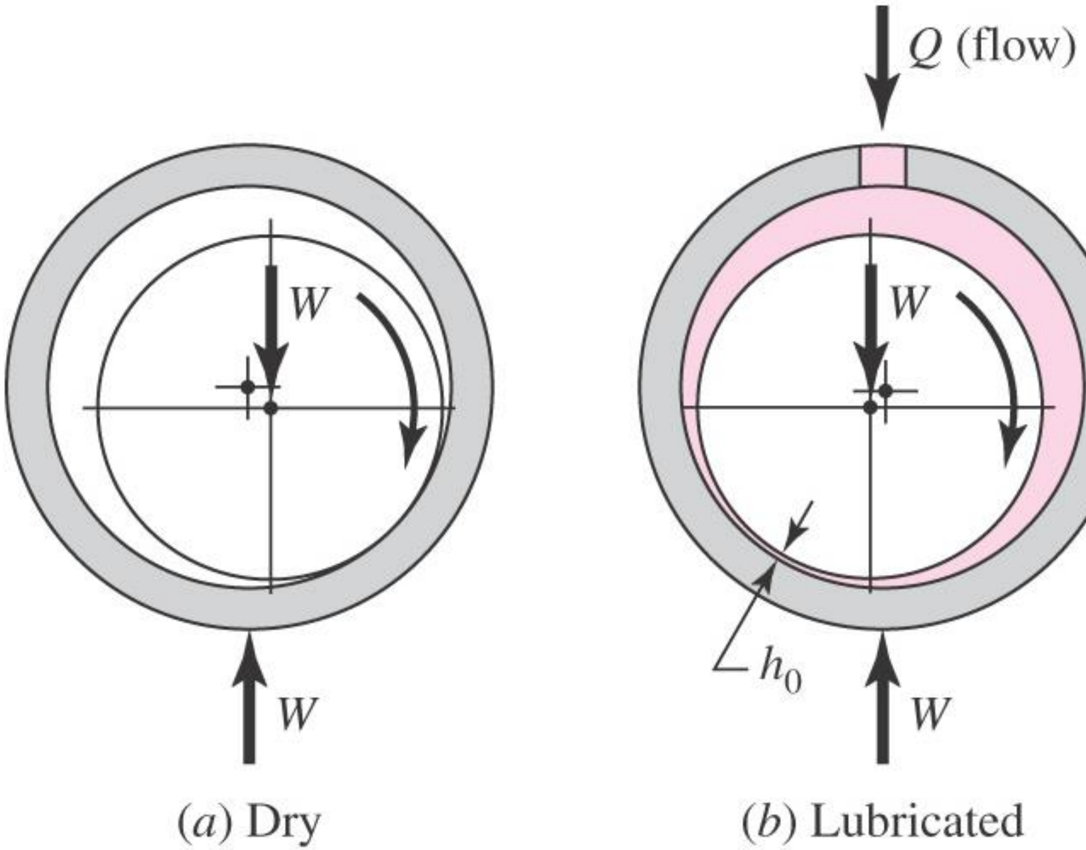


Fig. 12–5

Nomenclature of a Journal Bearing

- Center of journal at O
- Center of bearing at O'
- *Eccentricity* e
- *Minimum film thickness* h_0 occurs at line of centers
- Film thickness anywhere is h
- *Eccentricity ratio*

$$\epsilon = \frac{e}{c}$$

- *Partial bearing* has $\beta < 360$
- *Full bearing* has $\beta = 360$
- *Fitted bearing* has equal radii of bushing and journal

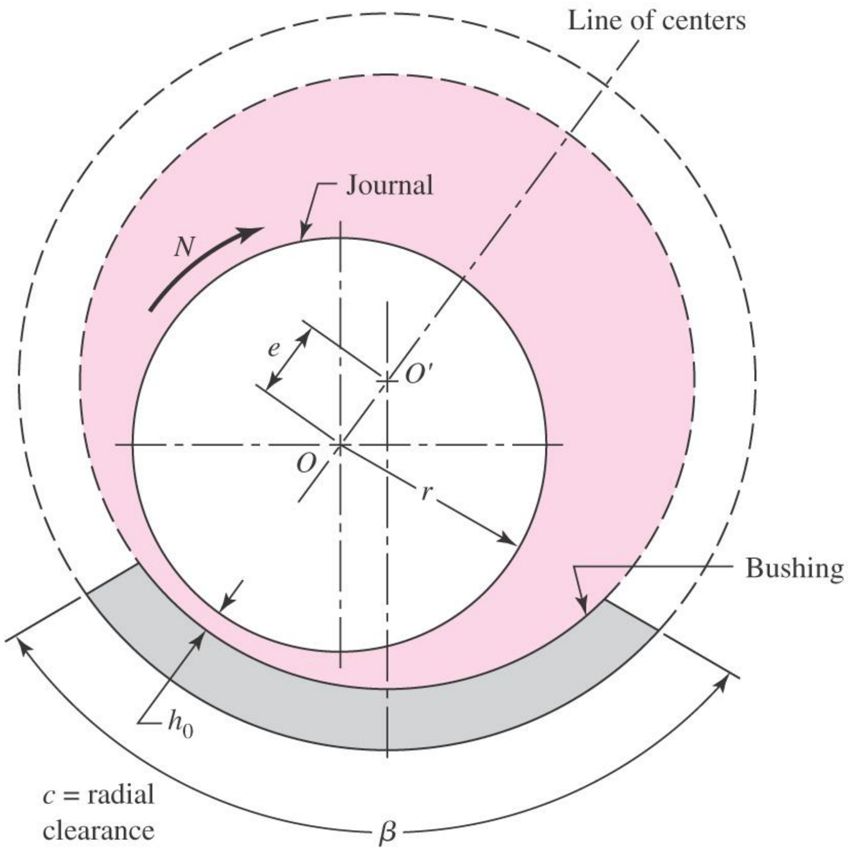


Fig. 12–6

Hydrodynamic Theory

- Present theory originated with experimentation of Beauchamp Tower in early 1880s

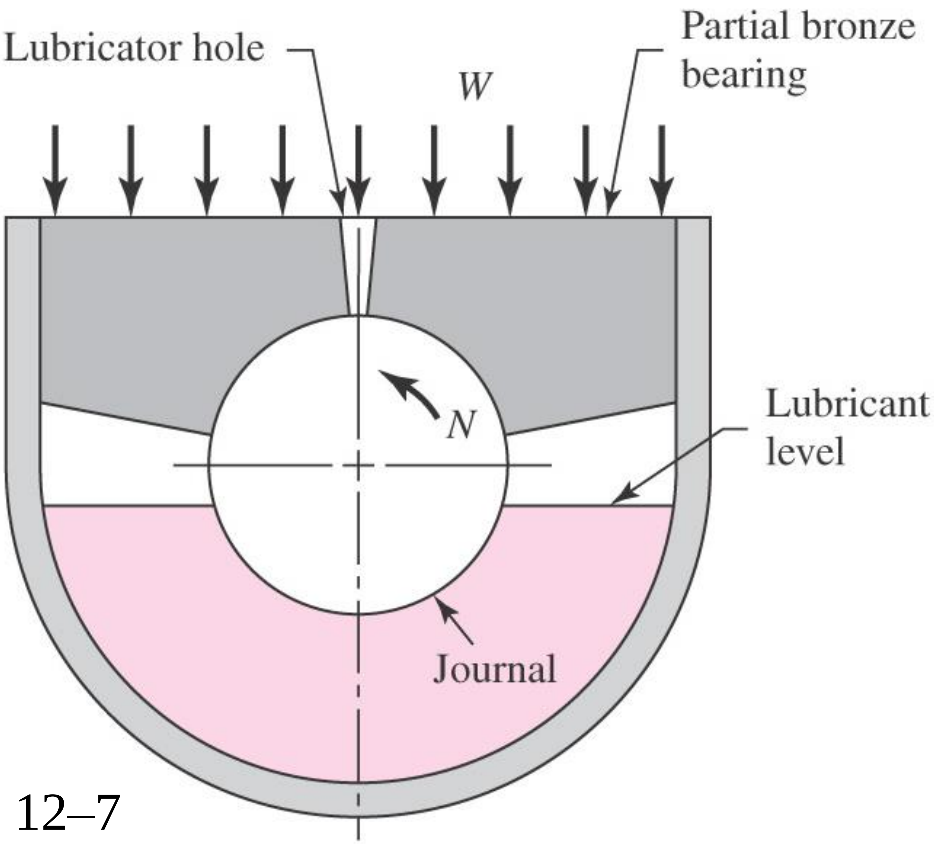


Fig. 12–7

Pressure Distribution Curves of Tower

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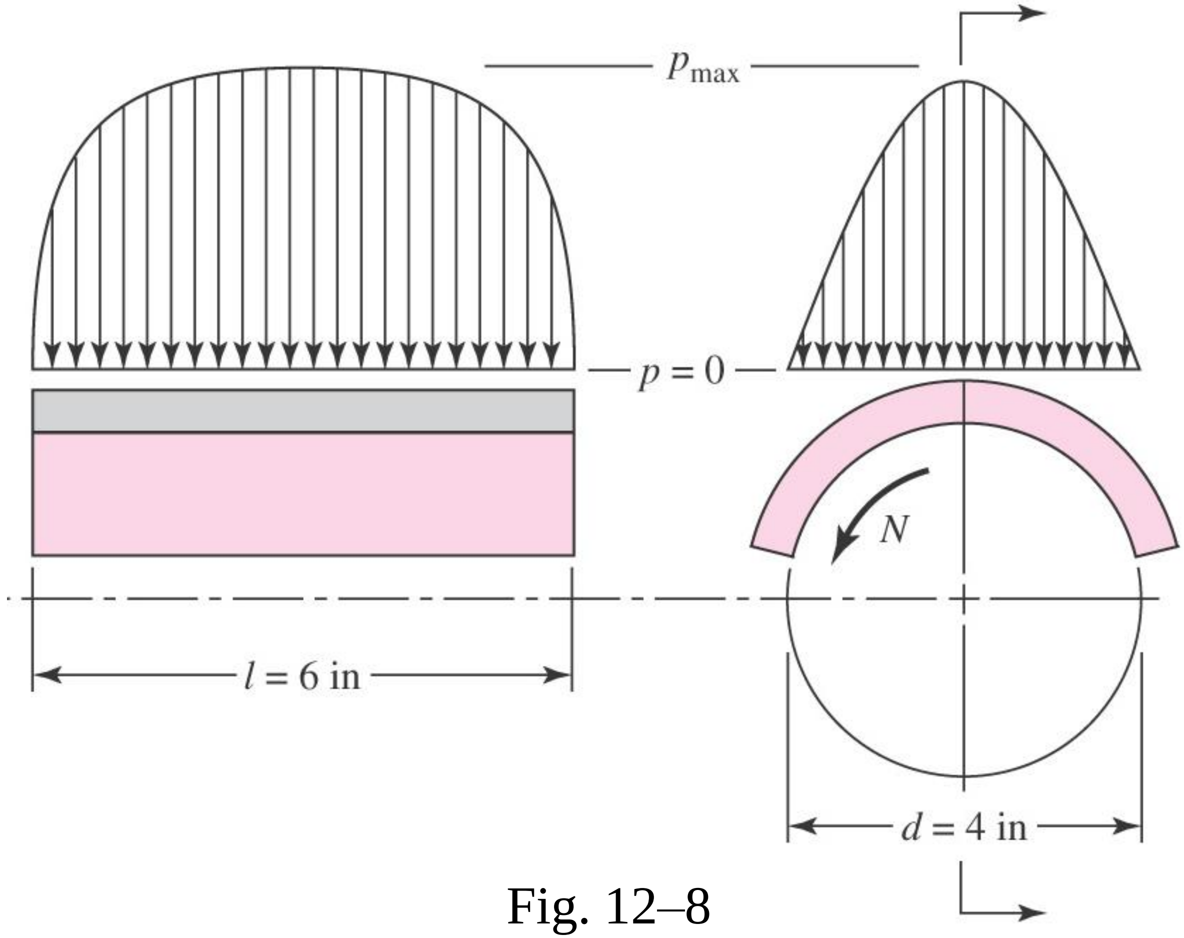


Fig. 12–8

Reynolds Plane Slider Simplification

- Reynolds realized fluid films were so thin in comparison with bearing radius that curvature could be neglected
- Replaced curved bearing with flat bearing
- Called *plane slider bearing*

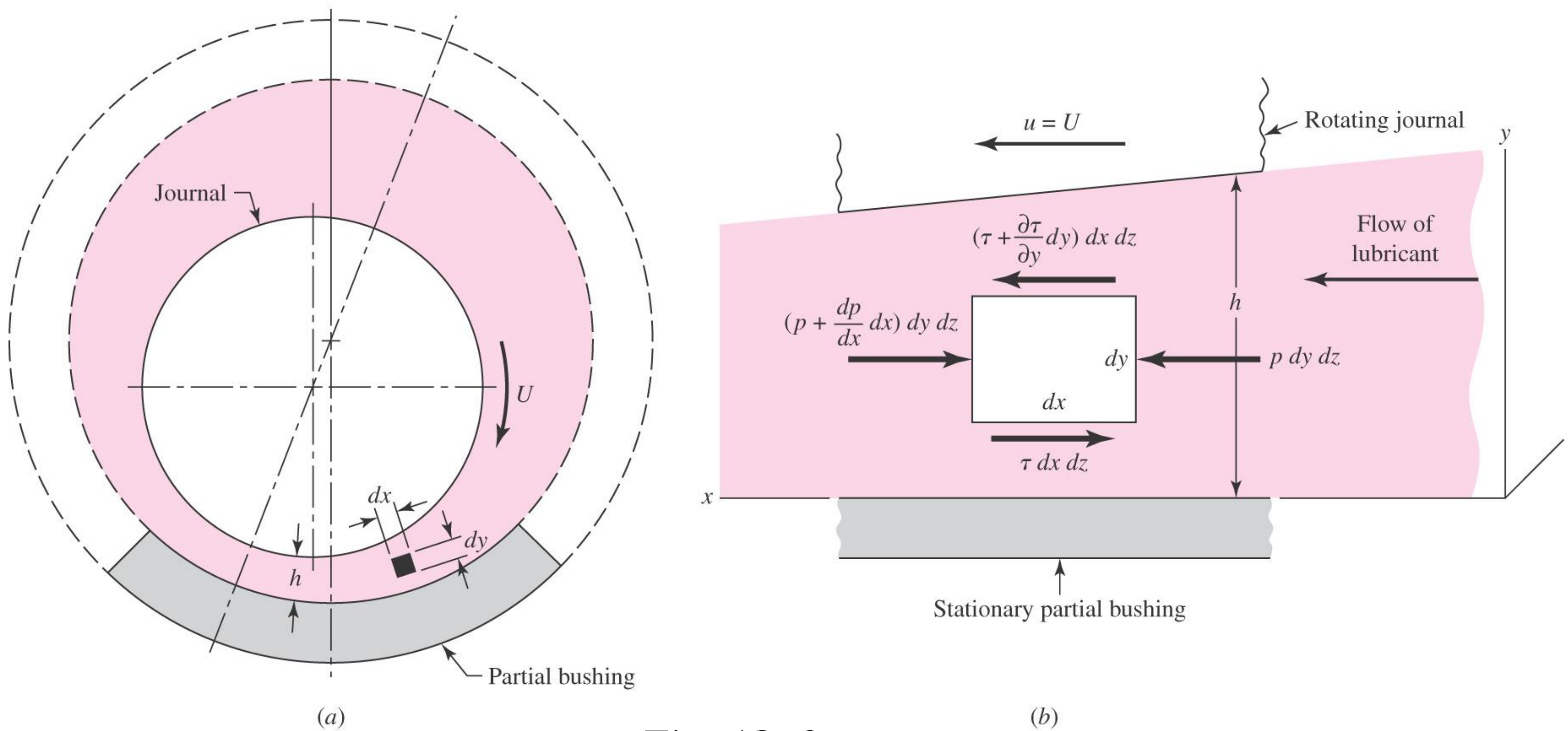


Fig. 12–9

Derivation of Velocity Distribution

$$\sum F_x = p\,dy\,dz - \left(p + \frac{dp}{dx}dx\right) dy\,dz - \tau\,dx\,dz + \left(\tau + \frac{\partial \tau}{\partial y}dy\right) dx\,dz = 0 \quad (a)$$

$$\frac{dp}{dx} = \frac{\partial \tau}{\partial y} \quad (b)$$

$$\tau = \mu \frac{\partial u}{\partial y} \quad (c)$$

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2} \quad (d)$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{dp}{dx}y + C_1$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + C_1 y + C_2 \quad (e)$$

Derivation of Velocity Distribution

$$\text{At} \quad y = 0,\ u = 0$$
$$\text{At} \quad y = h,\ u = U \qquad (f)$$

$$C_1 = \frac{U}{h} - \frac{h}{2\mu}\frac{dp}{dx} \qquad C_2 = 0$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - hy) + \frac{U}{h}y \qquad (12\text{–}9)$$

Velocity Distribution

- Velocity distribution superposes parabolic distribution onto linear distribution
- When pressure is maximum, dp/dx = 0 and $u = \frac{U}{h} y$ (g)

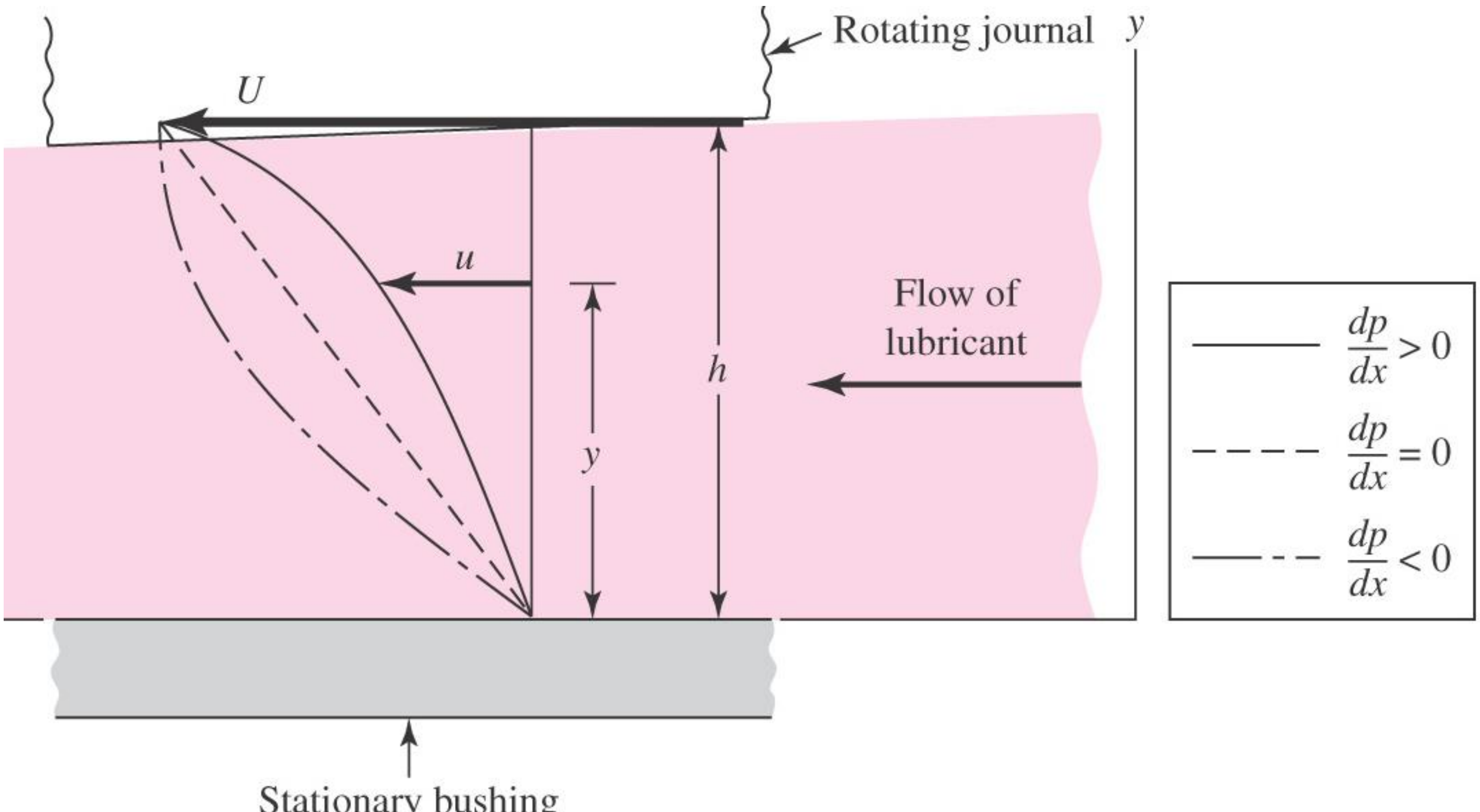


Fig. 12–10

Derivation of Reynolds Equation

$$Q = \int_0^h u \, dy \qquad (h)$$

$$Q = \frac{Uh}{2} - \frac{h^3}{12\mu}\frac{dp}{dx} \qquad (i)$$

$$\frac{dQ}{dx} = 0$$

$$\frac{dQ}{dx} = \frac{U}{2}\frac{dh}{dx} - \frac{d}{dx}\left(\frac{h^3}{12\mu}\frac{dp}{dx}\right) = 0$$

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6U\frac{dh}{dx} \qquad (12\text{–}10)$$

Reynolds Equation

- Classical Reynolds equation for one-dimensional flow, neglecting side leakage,

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6U\frac{dh}{dx} \qquad (12\text{–}10)$$

- With side leakage included,

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 6U\frac{\partial h}{\partial x} \qquad (12\text{–}11)$$

- No general analytical solutions
- One important approximate solution by Sommerfeld,

$$\frac{r}{c}f = \phi\left[\left(\frac{r}{c}\right)^2\frac{\mu N}{P}\right] \qquad (12\text{–}12)$$

Design Considerations

- Variables either given or under control of designer

  1. The viscosity μ
  2. The load per unit of projected bearing area, P
  3. The speed N
  4. The bearing dimensions r, c, β, and l

- Dependent variables, or *performance factors*

  1. The coefficient of friction f
  2. The temperature rise ΔT
  3. The volume flow rate of oil Q
  4. The minimum film thickness h_0

Significant Angular Speed

- Angular speed N that is significant to hydrodynamic film bearing performance is

$$N = |N_j + N_b - 2N_f| \qquad (12\text{–}13)$$

where N_j = journal angular speed, rev/s
N_b = bearing angular speed, rev/s
N_f = load vector angular speed, rev/s

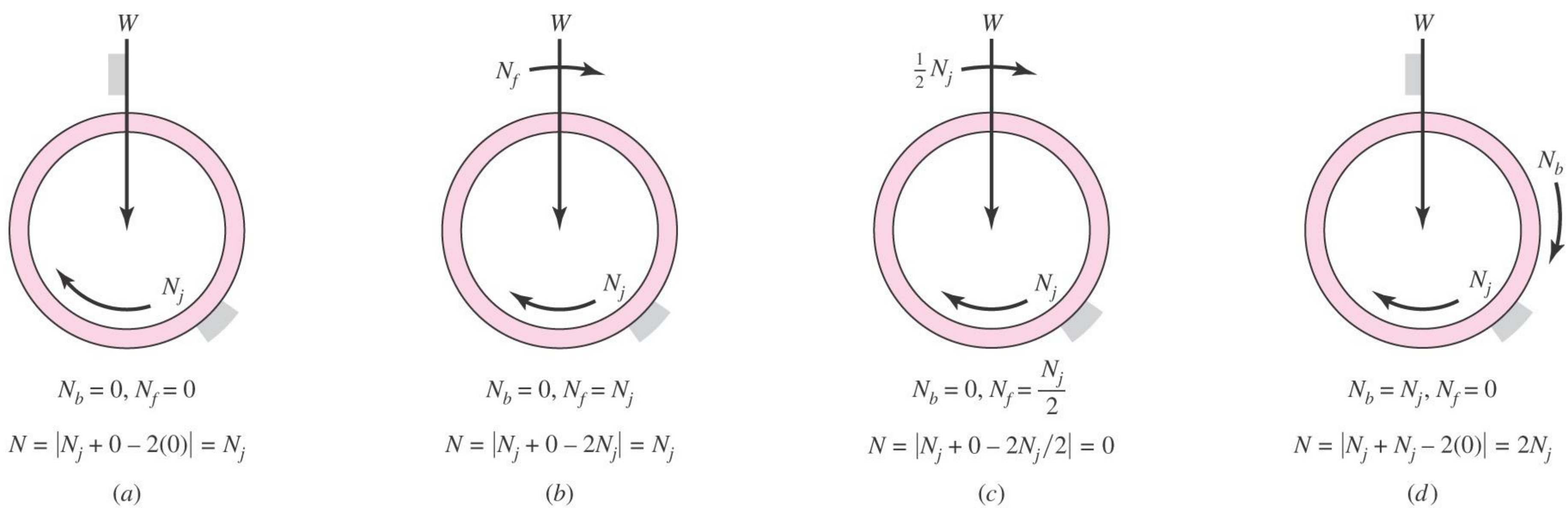


Fig. 12–11

Trumpler's Design Criteria

- Trumpler, a well-known bearing designer, recommended a set of design criteria.
- Minimum film thickness to prevent accumulation of ground off surface particles

$$h_0 \geq 0.0002 + 0.000\,04d \text{ in} \qquad (a)$$

- Maximum temperature to prevent vaporization of lighter lubricant components

$$T_{max} \leq 250^\circ\text{F} \qquad (b)$$

- Maximum starting load to limit wear at startup when there is metal-to-metal contact

$$\frac{W_{st}}{lD} \leq 300 \text{ psi} \qquad (c)$$

- Minimum design factor on running load

$$n_d \geq 2 \qquad (d)$$

The Relations of the Variables

- Albert Raymondi and John Boyd used an iteration technique to solve Reynolds' equation.
- Published 45 charts and 6 tables
- This text includes charts from Part III of Raymondi and Boyd
  - Assumes infinitely long bearings, thus no side leakage
  - Assumes full bearing
  - Assumes oil film is ruptured when film pressure becomes zero

Viscosity Charts

- Viscosity is clearly a function of temperature
- Viscosity charts of common lubricants are given in Figs. 12–12 through 12–14
- Raymondi and Boyd assumed constant viscosity through the loading zone
- Not completely true since temperature rises as work is done on the lubricant passing through the loading zone
- Use average temperature to find a viscosity

$$T_{av} = T_1 + \frac{\Delta T}{2} \qquad (12\text{–}14)$$

Viscosity-Temperature Chart in U.S. Customary Units

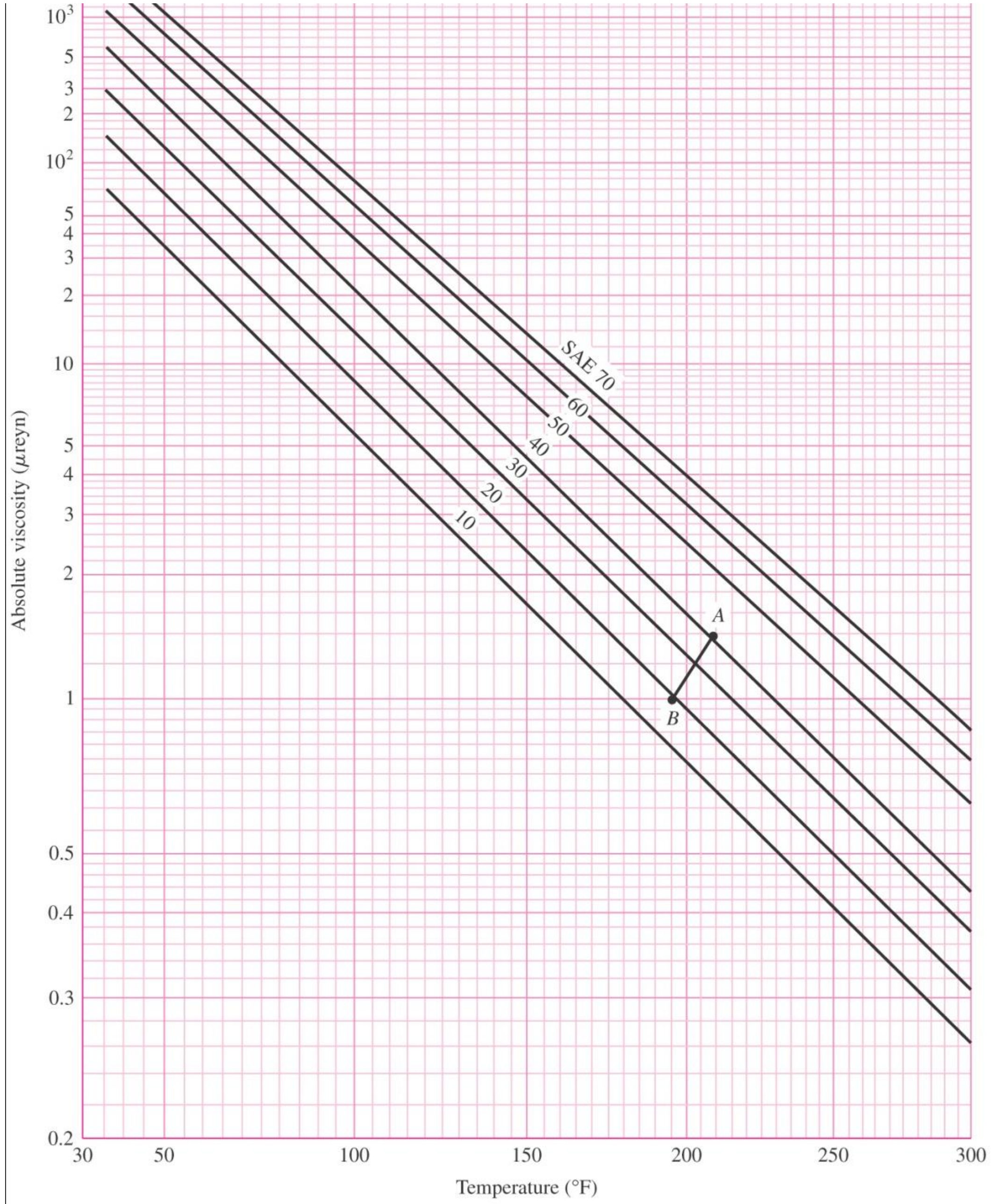


Fig. 12–12

Viscosity-Temperature Chart in Metric Units

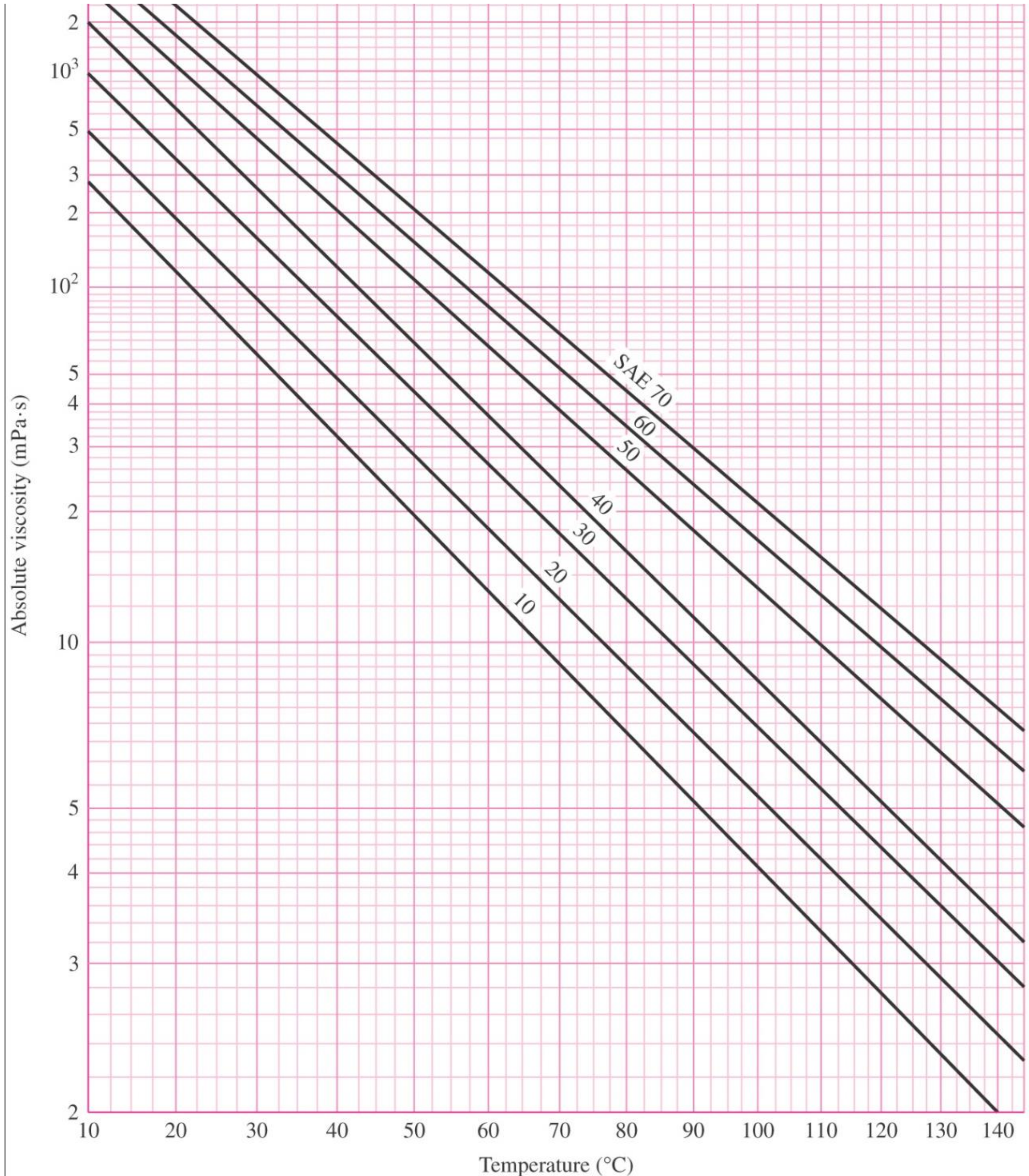


Fig. 12–13

Viscosity-Temperature Chart for Multi-viscosity Lubricants

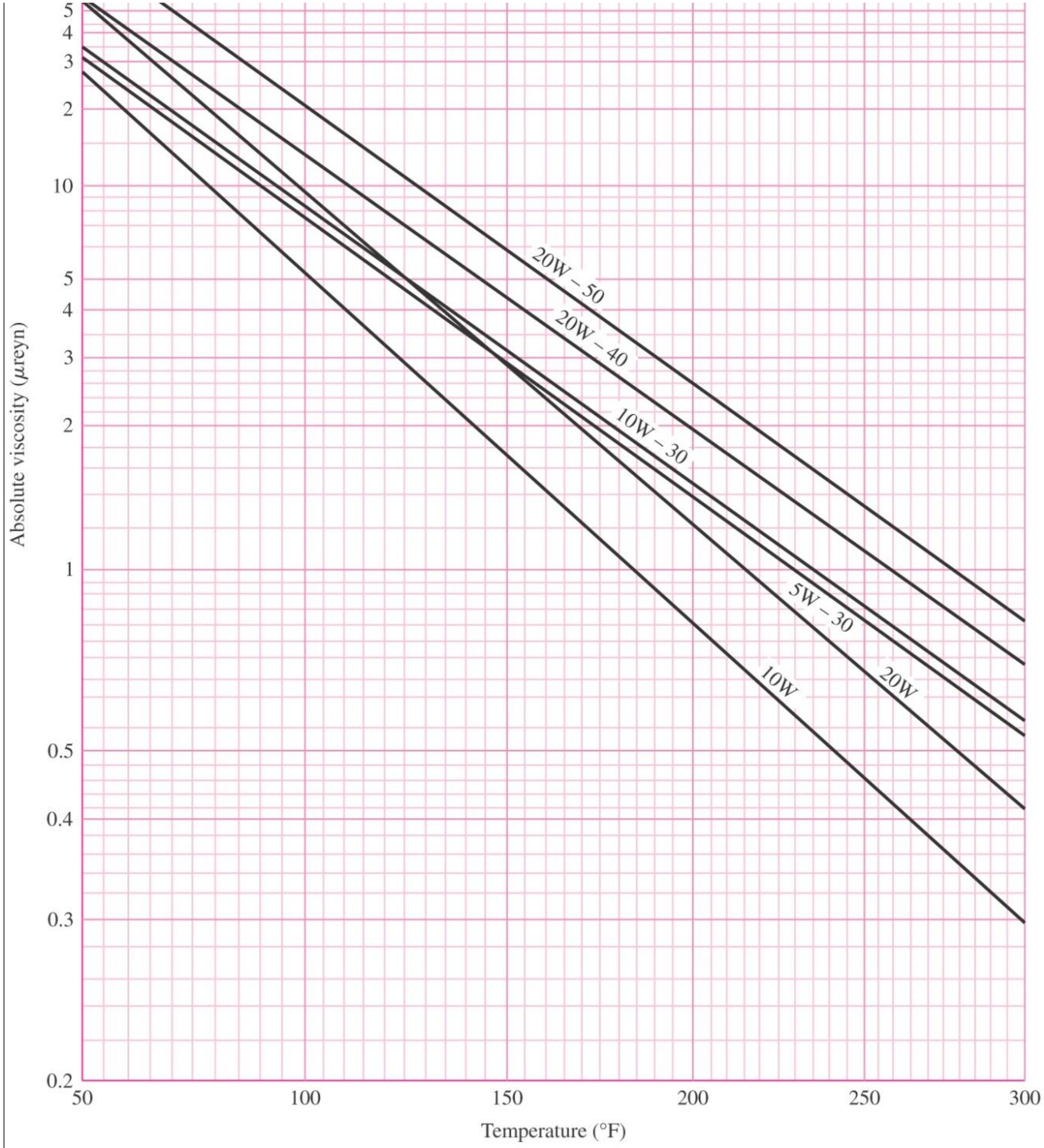


Fig. 12–14

Curve Fits for Viscosity-Temperature Chart

- Approximate curve fit for Fig. 12–12 is given by

$$\mu = \mu_0 \exp\left[b/(T + 95)\right],\ T \text{ in } ^\circ\text{F}.$$

Oil Grade, SAE	Viscosity μ_0, reyn	Constant b, °F
10	$0.0158(10^{-6})$	1157.5
20	$0.0136(10^{-6})$	1271.6
30	$0.0141(10^{-6})$	1360.0
40	$0.0121(10^{-6})$	1474.4
50	$0.0170(10^{-6})$	1509.6
60	$0.0187(10^{-6})$	1564.0

Table 12–1

Notation of Raimondi and Boyd

- Polar diagram of the film pressure distribution showing notation used by Raimondi and Boyd

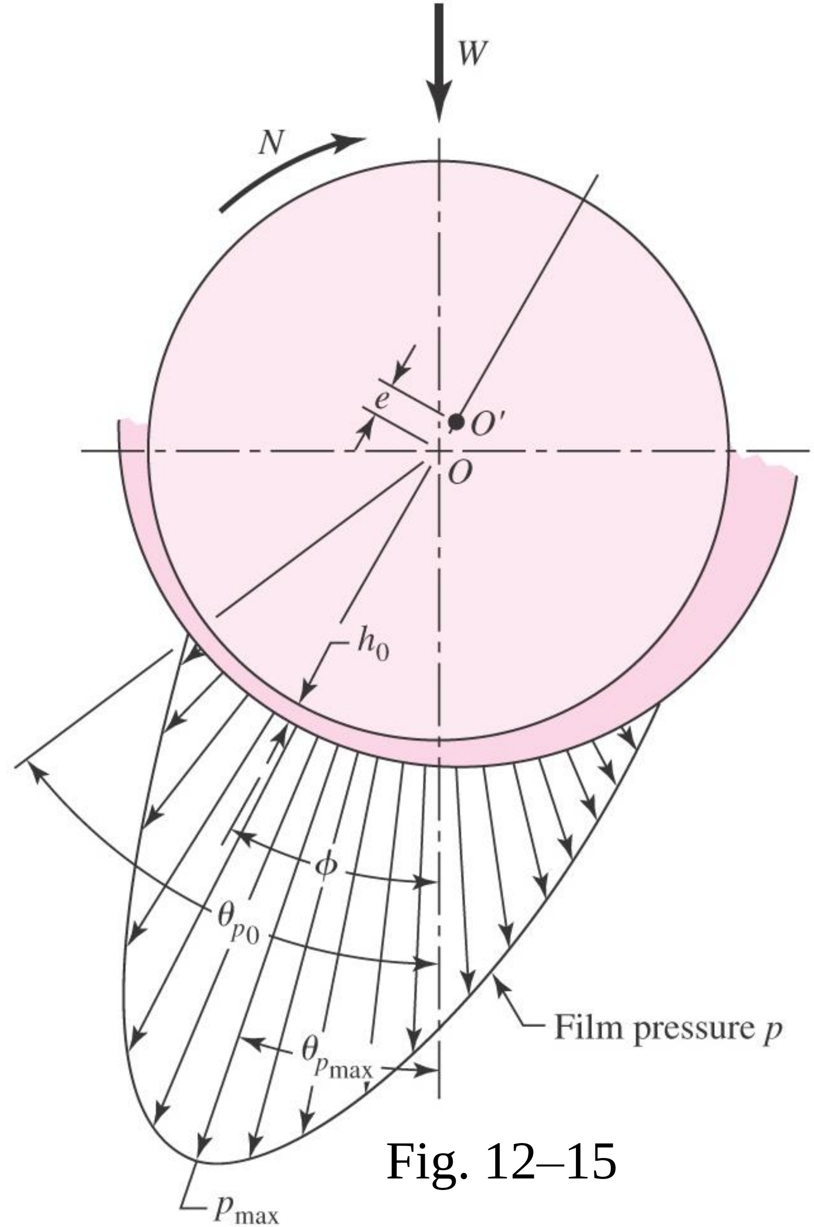


Fig. 12–15

Minimum Film Thickness and Eccentricity Ratio

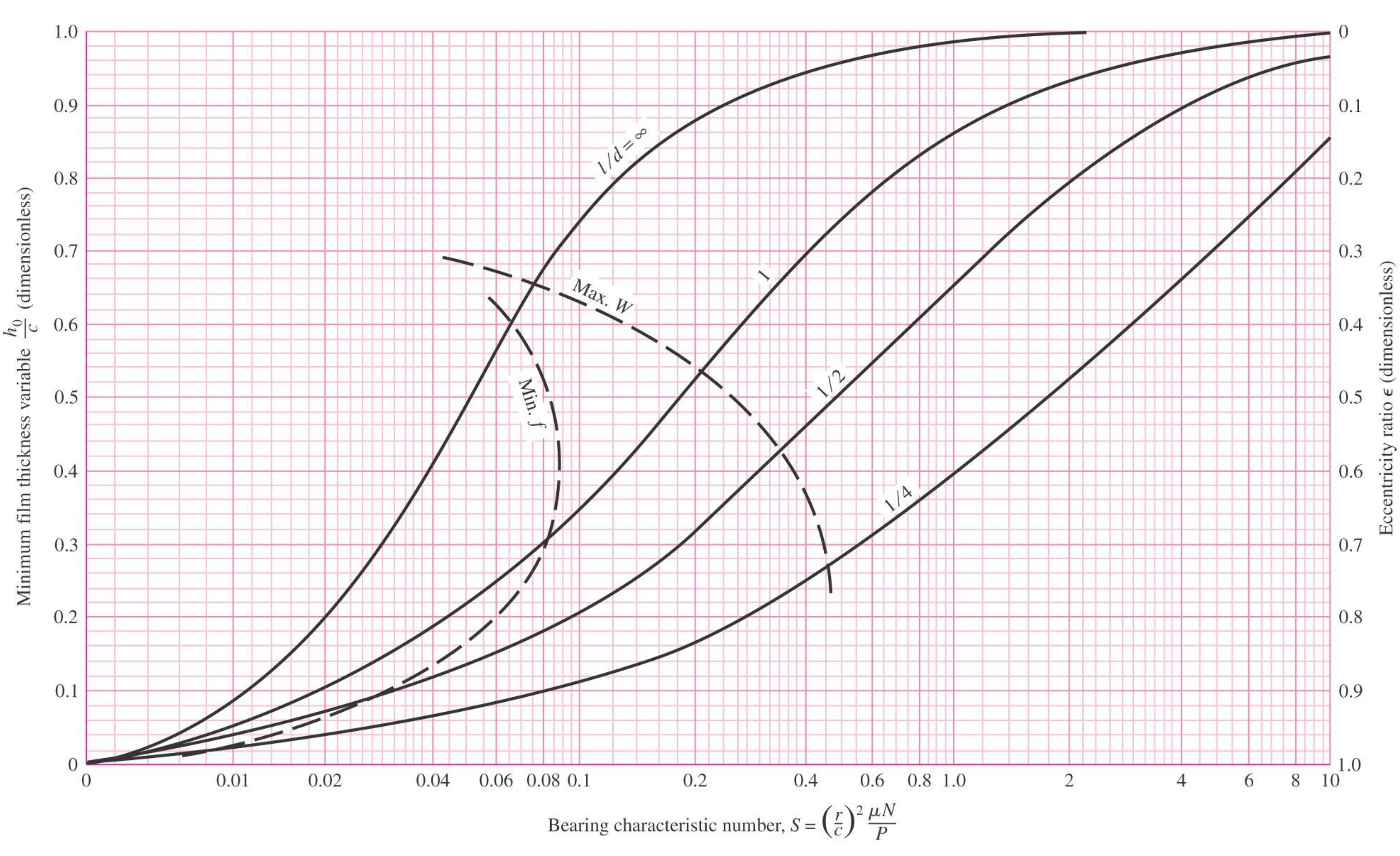


Fig. 12–16

Position of Minimum Film Thickness

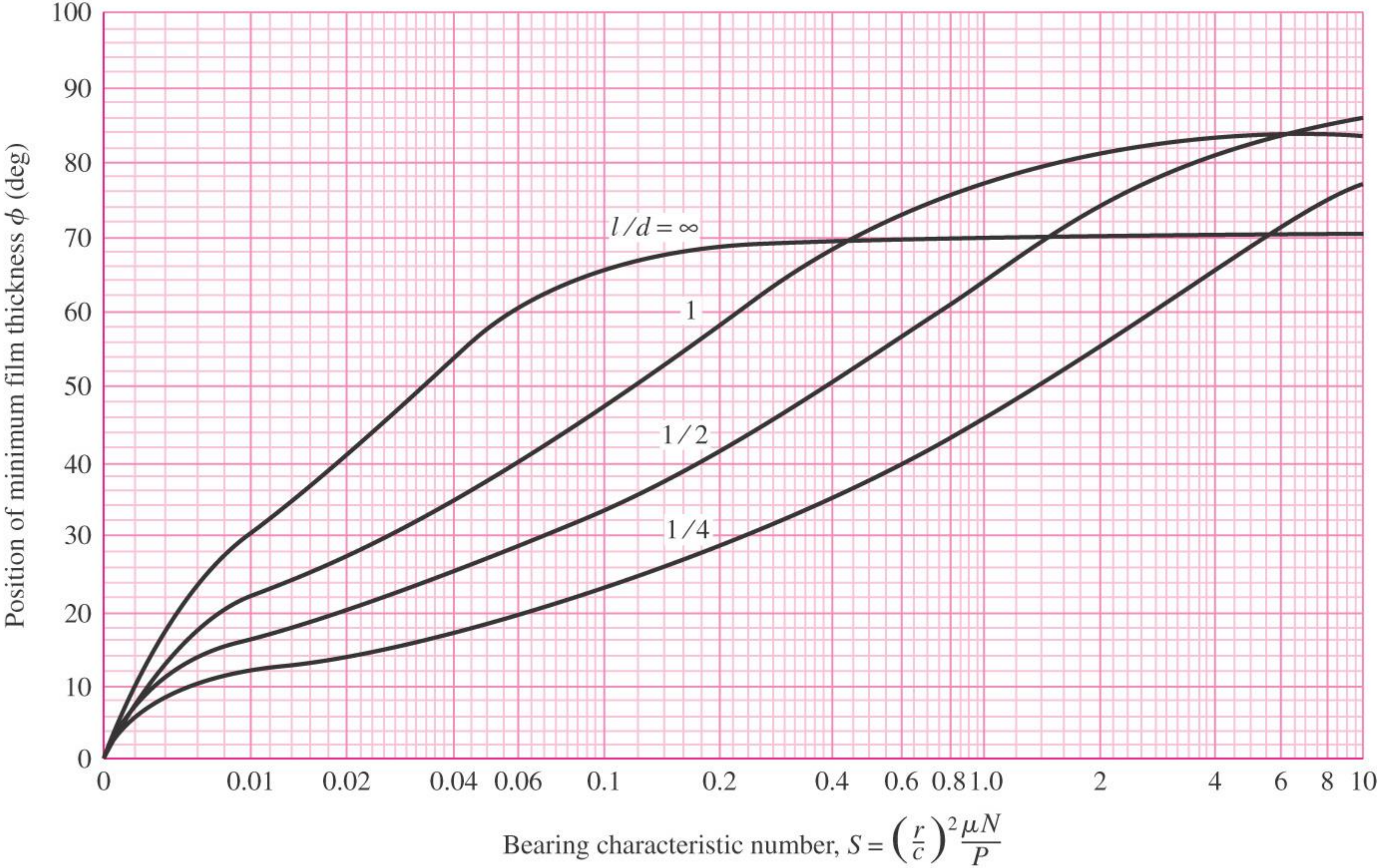


Fig. 12–17

Coefficient of Friction Variable

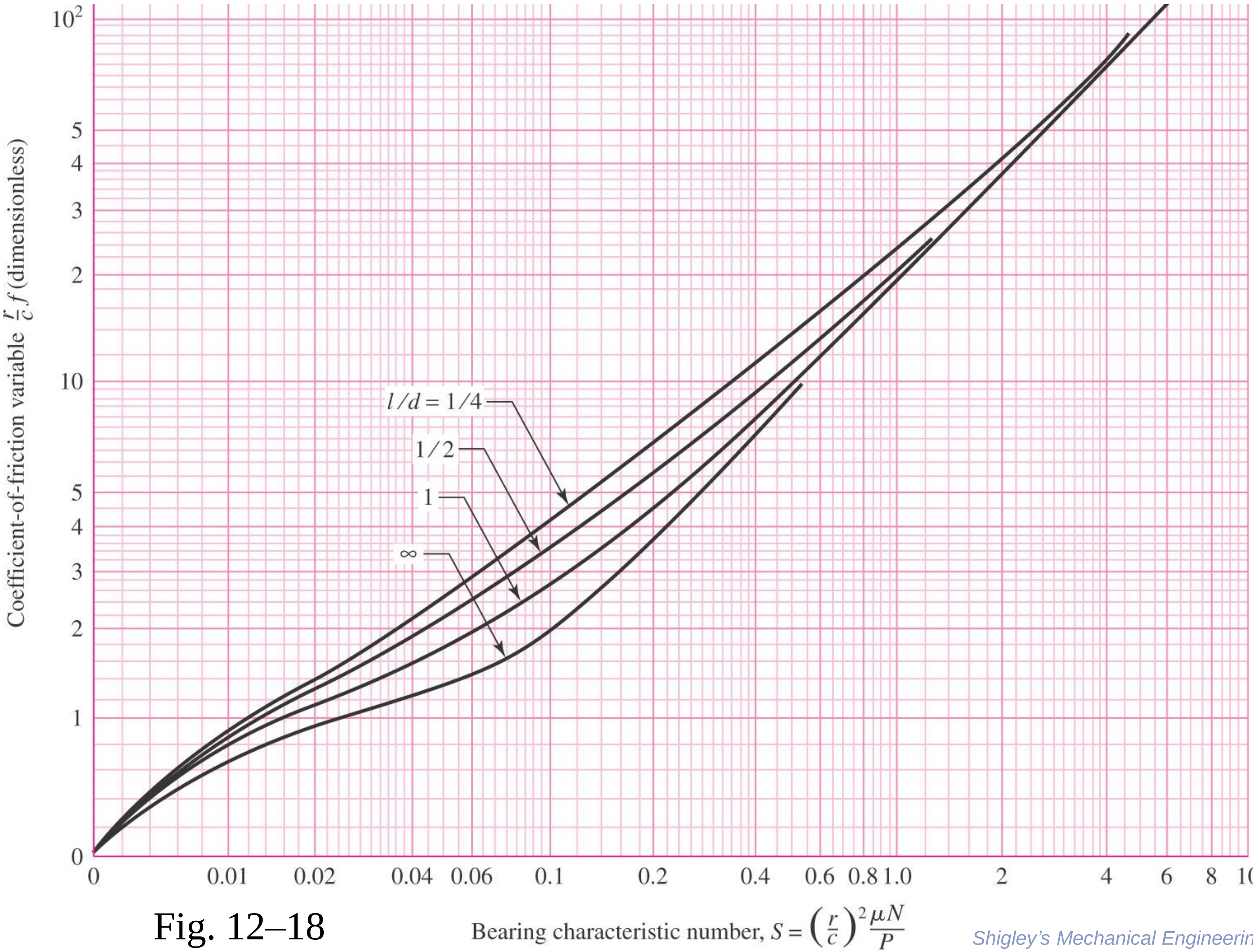


Fig. 12–18

Flow Variable

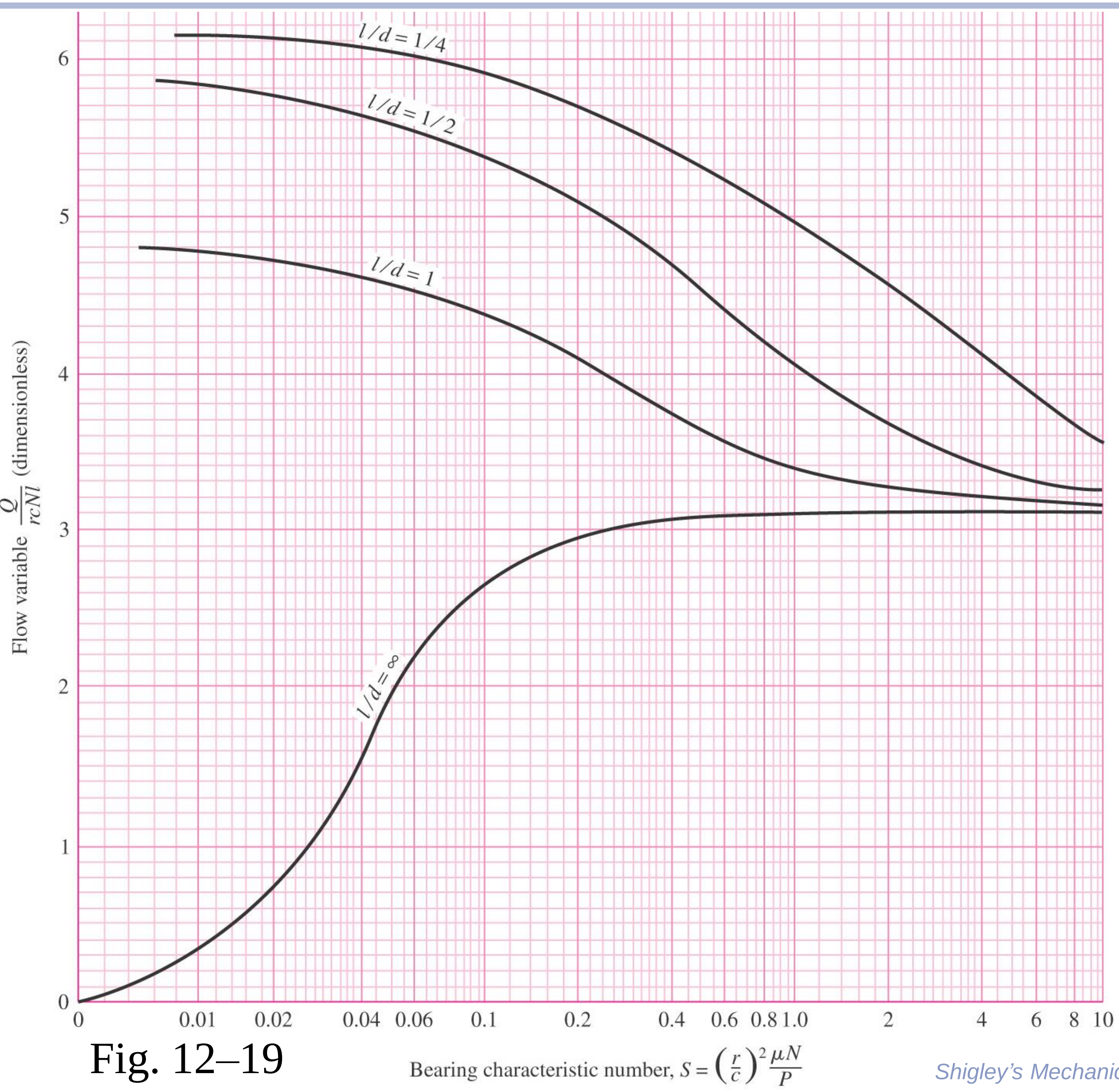


Fig. 12–19

Flow Ratio of Side Flow to Total Flow

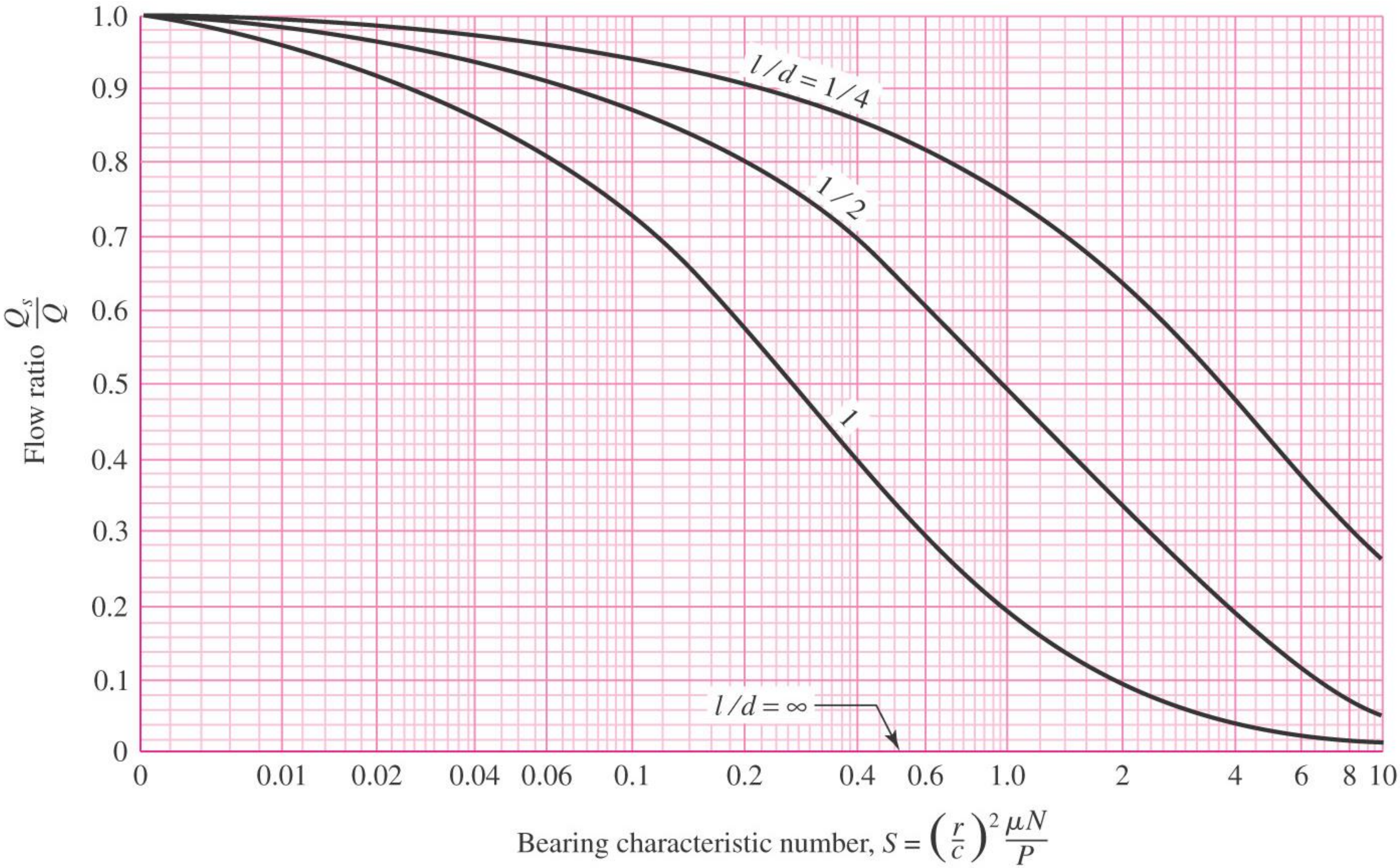


Fig. 12–20

Maximum Film Pressure

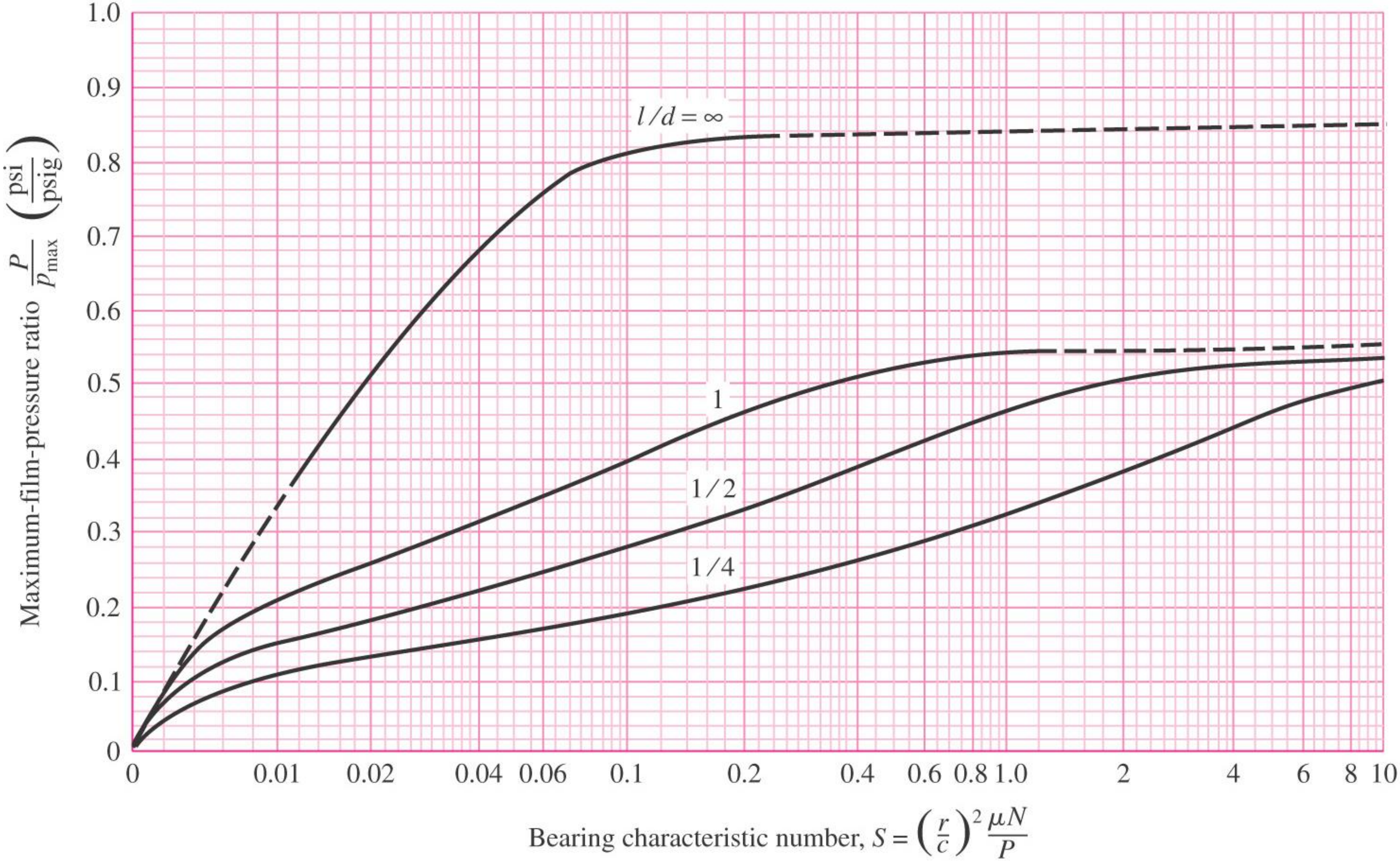


Fig. 12–21

Terminating Position of Film

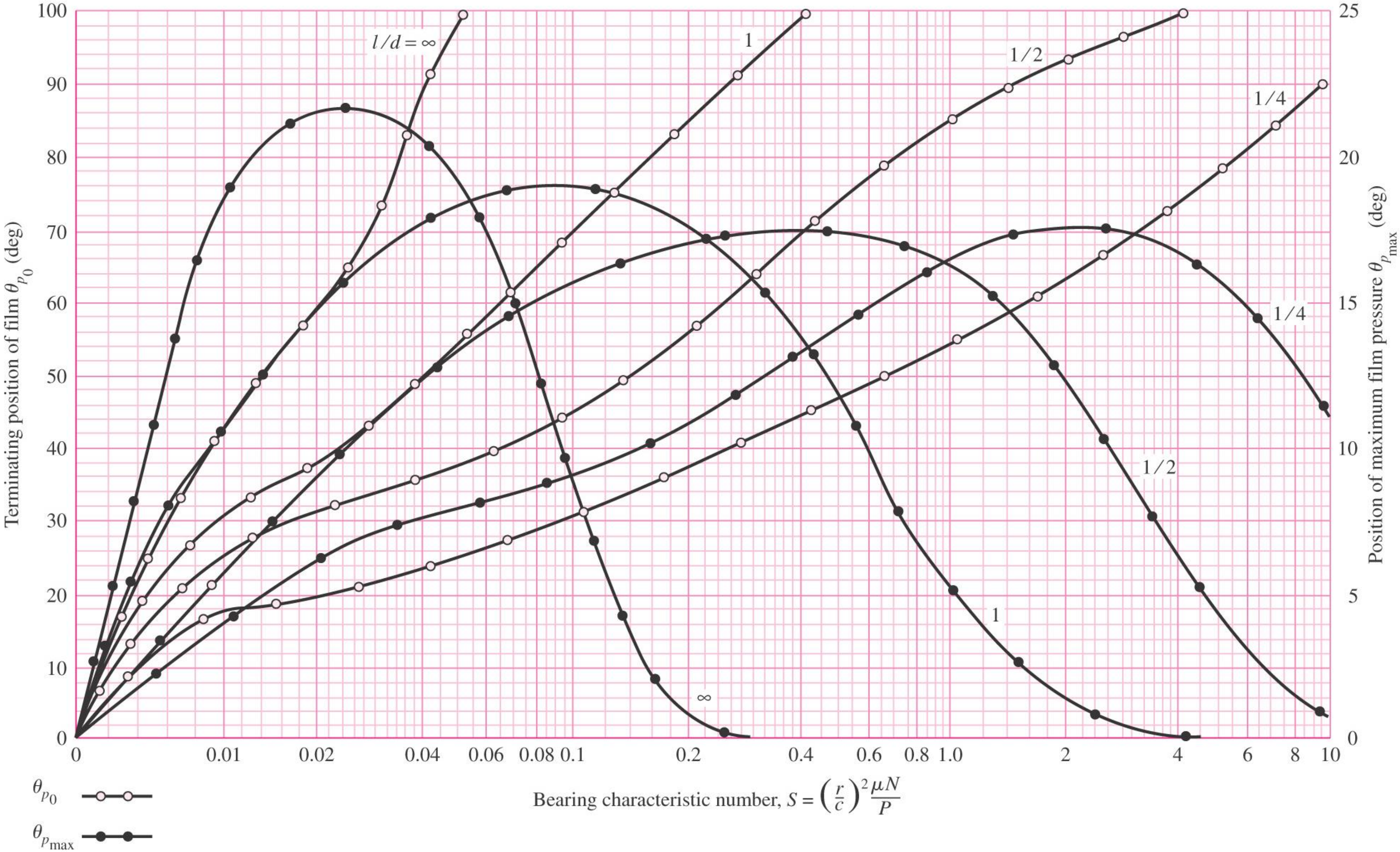


Fig. 12–22

Example 12–1

Determine h_0 and e using the following given parameters: $\mu = 0.027\,56$ Pa · s, $N = 30$ rev/s, $W = 2210$ N (bearing load), $r = 19$ mm, $c = 0.038$ mm, and $l = 38$ mm.

Solution

The nominal bearing pressure (in projected area of the journal) is

$$P = \frac{W}{2rl} = \frac{2210}{2(0.019)0.038} = 1.531 \text{ MPa}$$

The Sommerfeld number is, from Eq. (12–7), where $N = N_j = 30$ rev/s,

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu N}{P}\right) = \left(\frac{19}{0.038}\right)^2 \left[\frac{0.027\,56(30)}{1.531(10^6)}\right] = 0.135$$

Example 12–1 (continued)

Also, $l/d = 38/[2(19)] = 1$. Entering Fig. 12–16 with $S = 0.135$ and $l/d = 1$ gives $h_0/c = 0.42$ and $\epsilon = 0.58$. The quantity h_0/c is called the *minimum film thickness variable*. Since $c = 0.038$ mm, the minimum film thickness h_0 is

$$h_0 = 0.42(0.038) = 0.016 \text{ mm}$$ Answer

We can find the angular location ϕ of the minimum film thickness from the chart of Fig. 12–17. Entering with $S = 0.135$ and $l/d = 1$ gives $\phi = 53°$.

The eccentricity ratio is $\epsilon = e/c = 0.58$. This means the eccentricity e is

$$e = 0.58(0.038) = 0.022 \text{ mm}$$ Answer

Example 12–2

Using the parameters given in Ex. 12–1, determine the coefficient of friction, the torque to overcome friction, and the power loss to friction.

We enter Fig. 12–18 with $S = 0.135$ and $l/d = 1$ and find $(r/c)f = 3.50$. The coefficient of friction f is

$$f = 3.50\,c/r = 3.50(0.038/19) = 0.0070$$ Answer

The friction torque on the journal is

$$T = f\,Wr = 0.007(2210)0.019 = 0.2939\ \text{N}\cdot\text{m}$$ Answer

The power loss is

$$(hp)_{\text{loss}} = \frac{TN}{1050} = \frac{2.62(30)}{1050} = 0.075\ \text{hp}$$ Answer

$$TN\,(2\pi) = 0.2939(30)(2\pi) = 55.4\ \text{W}$$

Example 12–3

Continuing with the parameters of Ex. 12–1, determine the total volumetric flow rate Q and the side flow rate Q_s.

To estimate the lubricant flow, enter Fig. 12–19 with $S = 0.135$ and $l/d = 1$ to obtain $Q/(rcNl) = 4.28$. The total volumetric flow rate is

$$Q = 4.58rcNl = 4.28(19)0.038(30)38 = 3523 \text{ mm}^3/\text{s}$$ Answer

From Fig. 12–20 we find the *flow ratio* $Q_s/Q = 0.655$ and Q_s is

$$Q_s = 0.655Q = 0.655(3523) = 2308 \text{ mm}^3/\text{s}$$ Answer

Example 12–4

Using the parameters given in Ex. 12–1, determine the maximum film pressure and the locations of the maximum and terminating pressures.

Entering Fig. 12–21 with $S = 0.135$ and $l/d = 1$, we find $P/p_{max} = 0.42$. The maximum pressure p_{max} is therefore

$$p_{max} = \frac{P}{0.42} = \frac{1.531}{0.42} = 3.645 \text{ MPa}$$ Answer

With $S = 0.135$ and $l/d = 1$, from Fig. 12–22, $\theta_{p_{max}} = 18.5°$ and the terminating position θ_{p_0} is 75°. Answer

Finding Temperature Rise from Energy Considerations

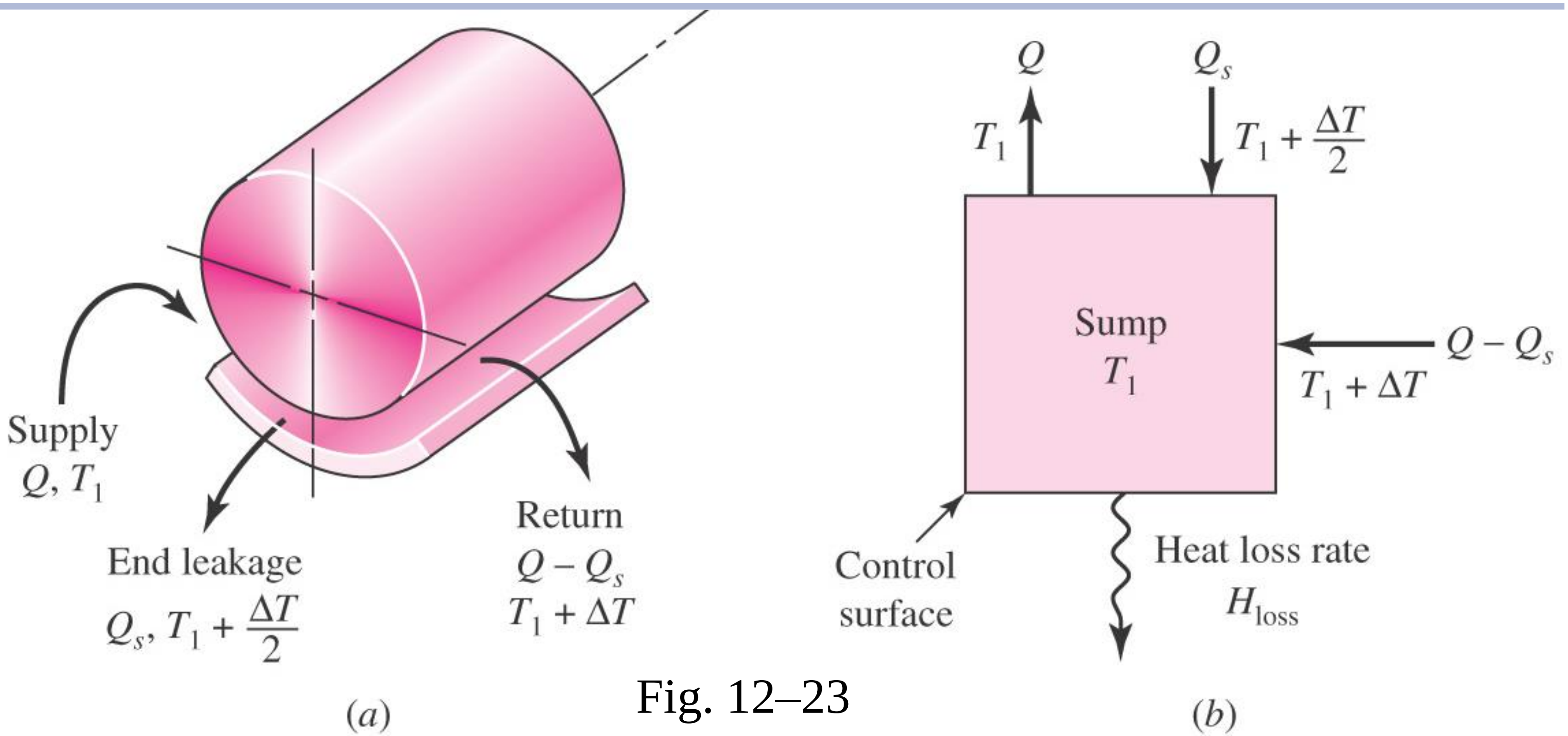


Fig. 12–23

Finding Temperature Rise from Energy Considerations

Q = volumetric oil-flow rate into the bearing, in^3/s

Q_s = volumetric side-flow leakage rate out of the bearing and to the sump, in^3/s

$Q - Q_s$ = volumetric oil-flow discharge from annulus to sump, in^3/s

T_1 = oil inlet temperature (equal to sump temperature T_s), °F

ΔT = temperature rise in oil between inlet and outlet, °F

ρ = lubricant density, lbm/in^3

C_p = specific heat capacity of lubricant, Btu/(lbm · °F)

J = Joulean heat equivalent, in · lbf/Btu

H = heat rate, Btu/s

Finding Temperature Rise from Energy Considerations

$$H_{\text{loss}} = \rho C_p Q_s \Delta T/2 + \rho C_p (Q - Q_s)\Delta T = \rho C_p Q \Delta T \left(1 - 0.5\frac{Q_s}{Q}\right) \qquad (a)$$

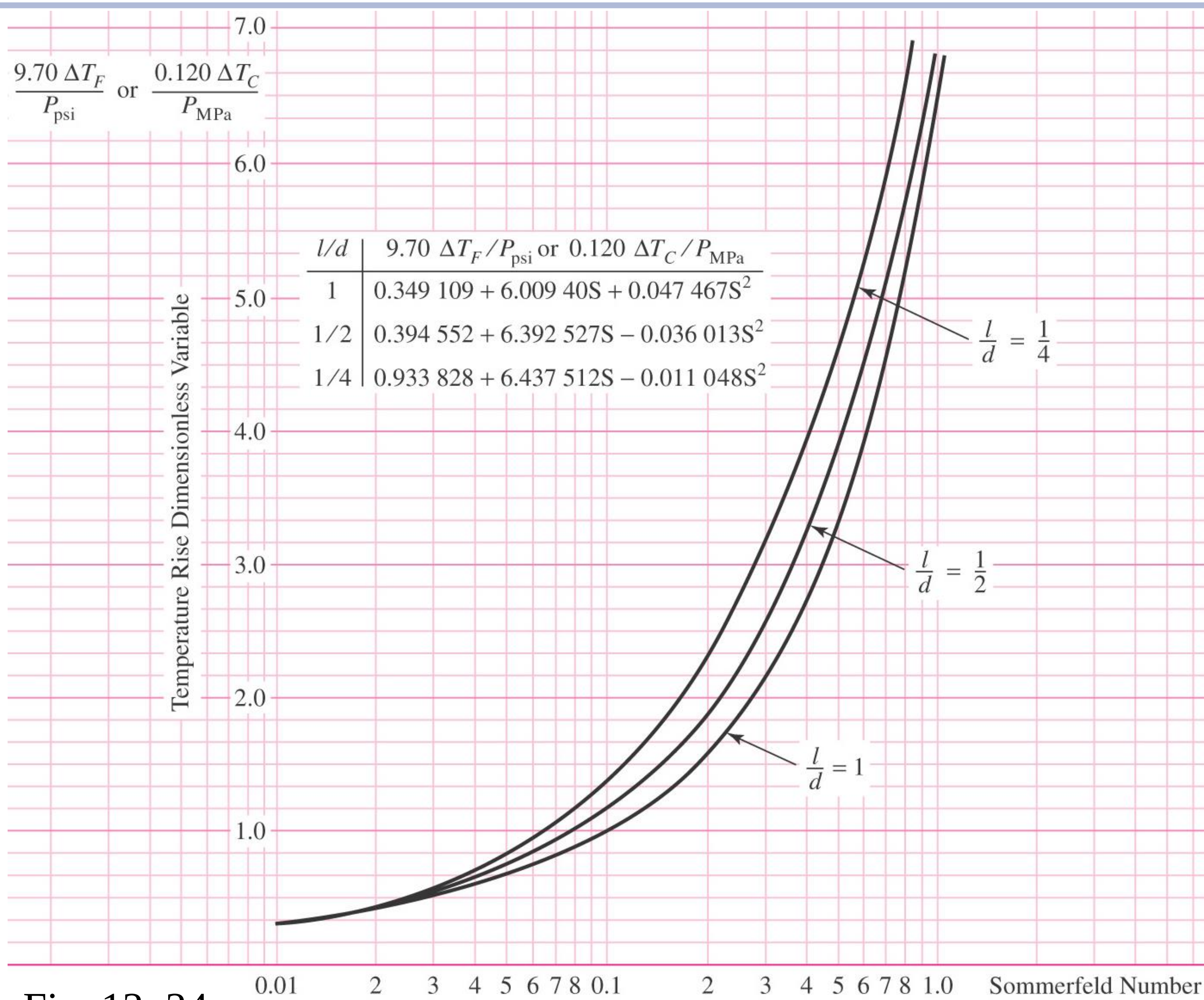
$$H_{\text{loss}} = \frac{4\pi P r l N c}{J}\frac{r f}{c} \qquad (b)$$

$$\frac{J\rho C_p\,\Delta T}{4\pi P} = \frac{rf/c}{(1 - 0.5 Q_s/Q)\,[Q/(rcNl)]} \qquad (c)$$

$$\frac{J\rho C_p\,\Delta T}{4\pi\,P} = \frac{995(10)^6\,862(1.758)\,\Delta T_C}{4\pi\,P_{\text{MPa}}} = 0.12\,\Delta T_C/P_{\text{MPa}}$$

$$\frac{0.12\,\Delta T_C}{P_{\text{MPa}}} = \frac{rf/c}{\left(1 - \frac{1}{2}Q_s/Q\right)[Q/(rcN_j l)]} \qquad (12\text{–}15)$$

Combined Temperature Rise Chart



l/d	$9.70\ \Delta T_F / P_{psi}$ or $0.120\ \Delta T_C / P_{MPa}$
1	$0.349\,109 + 6.009\,40S + 0.047\,467S^2$
1/2	$0.394\,552 + 6.392\,527S - 0.036\,013S^2$
1/4	$0.933\,828 + 6.437\,512S - 0.011\,048S^2$

Fig. 12–24

Interpolation Equation

- Raimondi and Boyd provide interpolation equation for *l/d* ratios other than given in charts

$$y = \frac{1}{(l/d)^3}\left[-\frac{1}{8}\left(1-\frac{l}{d}\right)\left(1-2\frac{l}{d}\right)\left(1-4\frac{l}{d}\right)y_\infty + \frac{1}{3}\left(1-2\frac{l}{d}\right)\left(1-4\frac{l}{d}\right)y_1 - \frac{1}{4}\left(1-\frac{l}{d}\right)\left(1-4\frac{l}{d}\right)y_{1/2} + \frac{1}{24}\left(1-\frac{l}{d}\right)\left(1-2\frac{l}{d}\right)y_{1/4}\right] \quad (12\text{–}16)$$

Steady-State Conditions in Self-Contained Bearings

- Previous analysis assumes lubricant carries away all enthalpy increase
- Bearings in which warm lubricant stays within bearing housing are called *self-contained bearings*
- Heat is dissipated from the housing to the surroundings

Heat Dissipated From Bearing Housing

- Heat given up by bearing housing

$$H_{\text{loss}} = \hbar_{\text{CR}} A (T_b - T_\infty) \qquad (12\text{–}17)$$

H_{loss} = heat dissipated, Btu/h

$\hbar_{\text{CR}}$ = combined overall coefficient of radiation and convection heat transfer, Btu/(h · ft^2 · °F)

A = surface area of bearing housing, ft^2

T_b = surface temperature of the housing, °F

T_∞ = ambient temperature, °F

Overall Coefficient of Heat Transfer

- Overall coefficient of radiation and convection depends on material, surface coating, geometry, roughness, temperature difference between housing and surroundings, and air velocity
- Some representative values

$$\hbar_{CR} = \begin{cases} 11.4\ \text{W}/(\text{m}^2\cdot{}^\circ\text{C}) & \text{for still air} \\ 15.3\ \text{W}/(\text{m}^2\cdot{}^\circ\text{C}) & \text{for shaft-stirred air} \\ 33.5\ \text{W}/(\text{m}^2\cdot{}^\circ\text{C}) & \text{for air moving at 24.5 m/s} \end{cases} \tag{12–18}$$

Difference in Housing and Ambient Temperatures

- The difference between housing and ambient temperatures is given by

$$\bar{T}_f - T_b = \alpha (T_b - T_\infty) \qquad (a)$$

Lubrication System	Conditions	Range of α
Oil ring	Moving air	1–2
	Still air	$\frac{1}{2}$–1
Oil bath	Moving air	$\frac{1}{2}$–1
	Still air	$\frac{1}{5}$–$\frac{2}{5}$

Table 12–2

Housing Temperature

- Bearing heat loss to surroundings

$$H_{\text{loss}} = \frac{\hbar_{\text{CR}} A}{1+\alpha}(\bar{T}_f - T_\infty) \qquad (12\text{–}19a)$$

- Housing surface temperature

$$T_b = \frac{\bar{T}_f + \alpha T_\infty}{1+\alpha} \qquad (12\text{–}19b)$$

Heat Generation Rate

$$T = 4\pi^2 r^3 l\mu/c$$

$$H_{\text{gen}} = \frac{4\pi^2 r^3 l\mu N}{c}(2\pi N) = \frac{248\mu N^2 l r^3}{c} \qquad (b)$$

$$\bar{T}_f = T_\infty + 248(1+\alpha)\frac{\mu N^2 l r^3}{\hbar_{\text{CR}} A c} \qquad (12\text{–}20)$$

Example 12–5

Consider a pillow-block bearing with a keyway sump, whose journal rotates at 900 rev/min in shaft-stirred air at 21°C with $\alpha = 1$. The lateral area of the bearing is 25 800 mm^2. The lubricant is SAE grade 20 oil. The gravity radial load is 450 N and the l/d ratio is unity. The bearing has a journal diameter of 50 mm $+ 0.000/-0.002$ 25 mm, a bushing bore of 50.05 $+ 0.008/-0.000$ mm. For a minimum clearance assembly estimate the steady-state temperatures as well as the minimum film thickness and coefficient of friction.

Example 12–5

The minimum radial clearance, $c_{\min}$, is

$$c_{\min} = \frac{50.05 - 50}{2} = 0.025 \text{ mm}$$

$$P = \frac{W}{ld} = \frac{450}{(50)50} = 0.18 \text{ MPa}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{25}{0.025}\right)^2 \frac{\mu 15}{0.18(10^6)} = 83.3\,\mu$$

The friction loss, F_f, is found as follows:

$$F_f = fWr2\pi N = WNc2\pi\left(\frac{fr}{c}\right) = 450(15)2\pi\left(\frac{fr}{c}\right) = 42\,411\left(\frac{fr}{c}\right)\text{J/s}$$

The heat generation rate H_{gen}, is

$$H_{\text{gen}} = F_f = 42\,411(fr/c) = J/s \text{ or W}$$

Example 12–5

From Eq. (12–19*a*) with $\hbar_{CR} = 15.3\ \text{W}/(\text{m}^2\ °\text{C})$, the rate of heat loss to the environment H_{loss} is

$$H_{\text{loss}} = \frac{\hbar_{CR} A}{\alpha + 1}(\bar{T}_f - 21) = \frac{15.3(0.0258)}{(1 + 1)}(\bar{T}_f - 21) = 1974(\bar{T}_f - 21) J/s \text{ or W}$$

Build a table as follows for trial values of $\bar{T}_f$ of 87.8 and 90.5°C:

Trial $\bar{T}_f$	μ'	S	fr/c	H_{gen}	H_{loss}
87.8	0.008 73	0.73	3.4	145 049	131 863
90.5	0.007 82	0.65	3.06	129 880	137 193

Example 12–5

The temperature at which $H_{gen} = H_{loss} = 135.4$ kJ/s is 89.7°C. Rounding $\bar{T}_f$ to 90°C we find $\mu = 0.0081$ Pa · s and $S = 83.3(0.0081) = 0.67$. From Fig. 12–24, $\dfrac{0.12\Delta T_c}{P_{MPa}} = 4.25$ and thus

$$\Delta T_C = 4.25P/0.12 = 4.25(0.18)/0.12 = 6.4°\text{C}$$

$$T_1 = T_s = \bar{T}_f - \Delta T/2 = 90 - 6.4/2 = 86.8°\text{C}$$

$$T_{max} = T_1 + \Delta T_F = 86.8 + 6.4 = 93.2°\text{C}$$

From Eq. (12–19*b*)

$$T_b = \frac{T_f + \alpha T_\infty}{1 + \alpha} = \frac{90 + (1)21}{1 + 1} = 55.5°\text{C}$$

with $S = 0.67$, the minimum film thickness from Fig. 12–16 is

Example 12–5

$$h_0 = \frac{h_0}{c} c = 0.79(0.025) = 0.0198 \text{ mm}$$

The coefficient of friction from Fig. 12–18 is

$$f = \frac{fr}{c} \frac{c}{r} = 12.8 \frac{0.025}{25} = 0.012\,8$$

The parasitic friction torque T is

$$T = fWr = 0.012\,8(450)(0.025) = 0.144 \text{ N} \cdot \text{m}$$

Effect of Clearance on Example Problems

- Some performance characteristics from Examples 12–1 to 12–4, plotted versus radial clearance

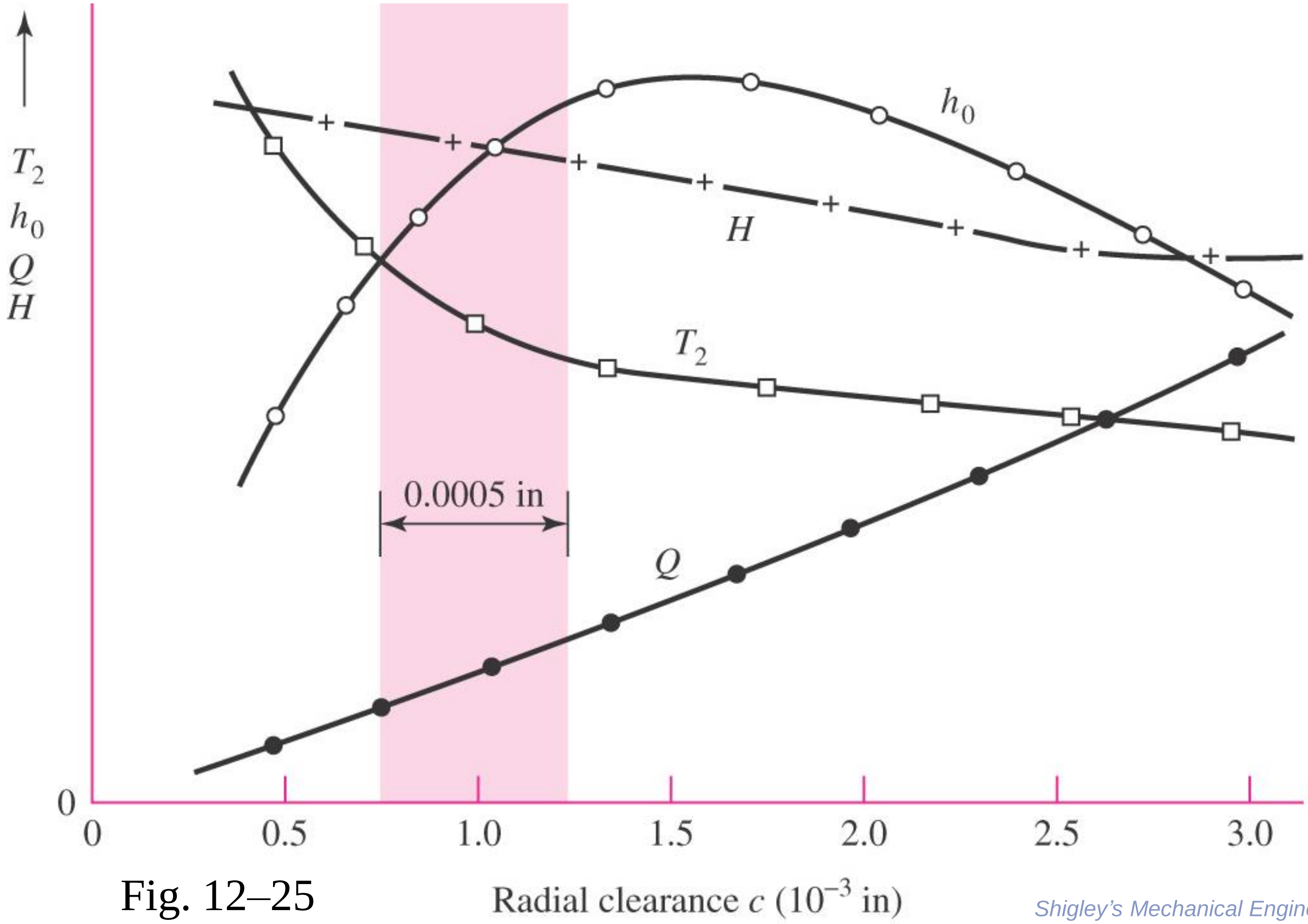


Fig. 12–25

Clearance

Table 12–3

Maximum, Minimum, and Average Clearances for 1.5-in-Diameter Journal Bearings Based on Type of Fit

Type of Fit	Symbol	Clearance c, in Maximum	Average	Minimum
Close-running	H8/f7	0.001 75	0.001 125	0.000 5
Free-running	H9/d9	0.003 95	0.002 75	0.001 55

Table 12–4

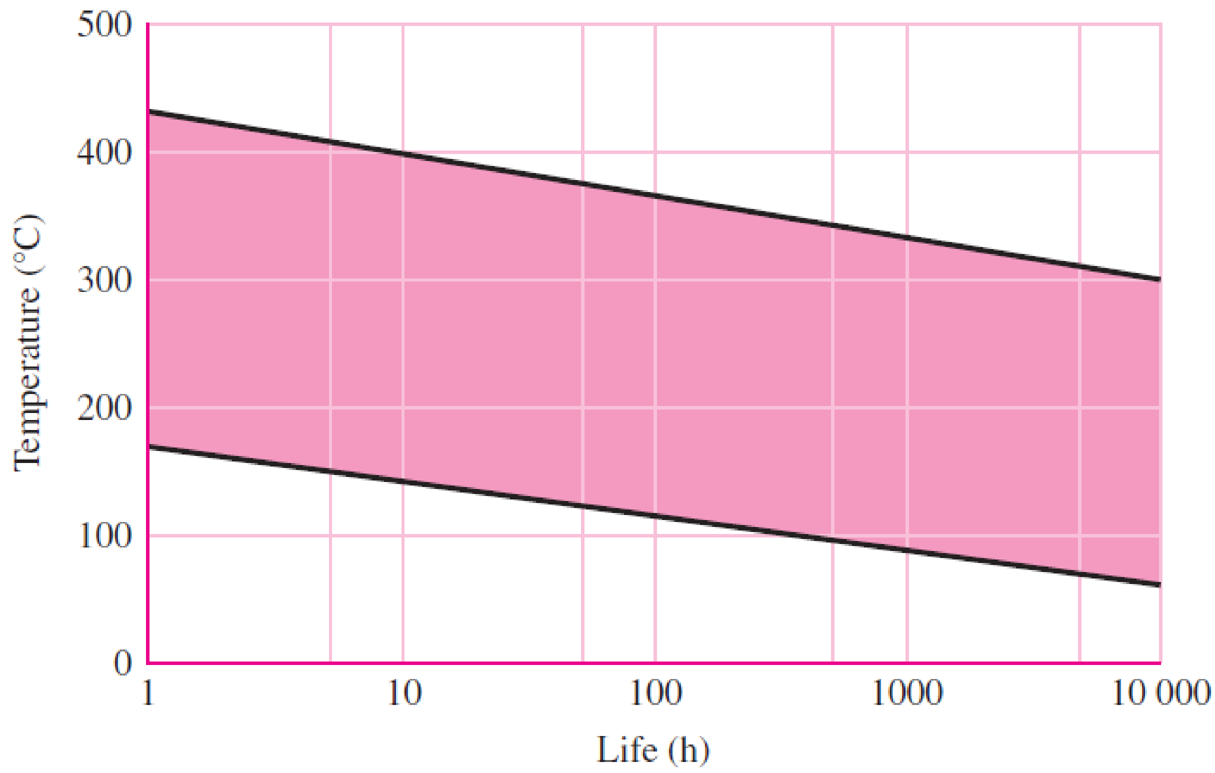
Performance of 1.5-in-Diameter Journal Bearing with Various Clearances. (SAE 20 Lubricant, $T_1 = 100°F$, $N = 30$ r/s, $W = 500$ lbf, $L = 1.5$ in)

c, in	T_2, °F	h_0, in	f	Q, in^3/s	H, Btu/s
0.000 5	226	0.000 38	0.011 3	0.061	0.086
0.001 125	142	0.000 65	0.009 0	0.153	0.068
0.001 55	133	0.000 77	0.008 7	0.218	0.066
0.001 75	128	0.000 76	0.008 4	0.252	0.064
0.002 75	118	0.000 73	0.007 9	0.419	0.060
0.003 95	113	0.000 69	0.007 7	0.617	0.059

Temperature Limits

Figure 12–26

Temperature limits for mineral oils. The lower limit is for oils containing antioxidants and applies when oxygen supply is unlimited. The upper limit applies when insignificant oxygen is present. The life in the shaded zone depends on the amount of oxygen and catalysts present.
(Source: M. J. Neale (ed.), Tribology Handbook, Section B1, Newnes-Butterworth, London, 1975.)



Pressure-Fed Bearings

- Temperature rise can be reduced with increased lubricant flow
- *Pressure-fed bearings* increase the lubricant flow with an external pump
- Common practice is to use circumferential groove at center of bearing
- Effectively creates two half-bearings

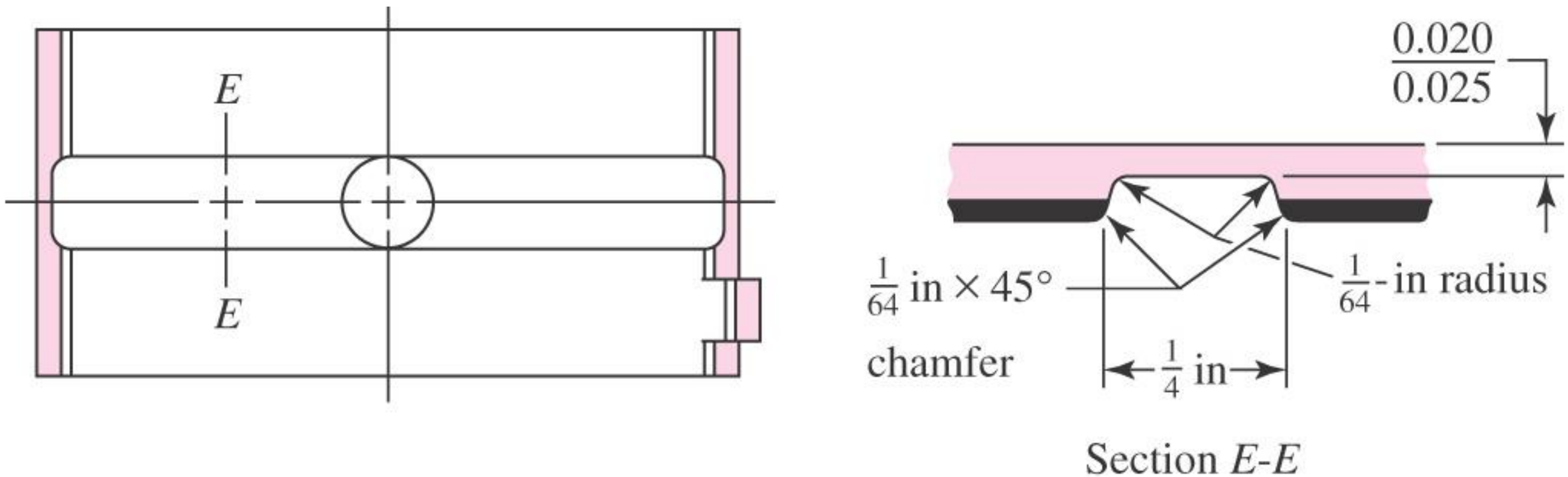


Fig. 12–27

Flow of Lubricant From Central Groove

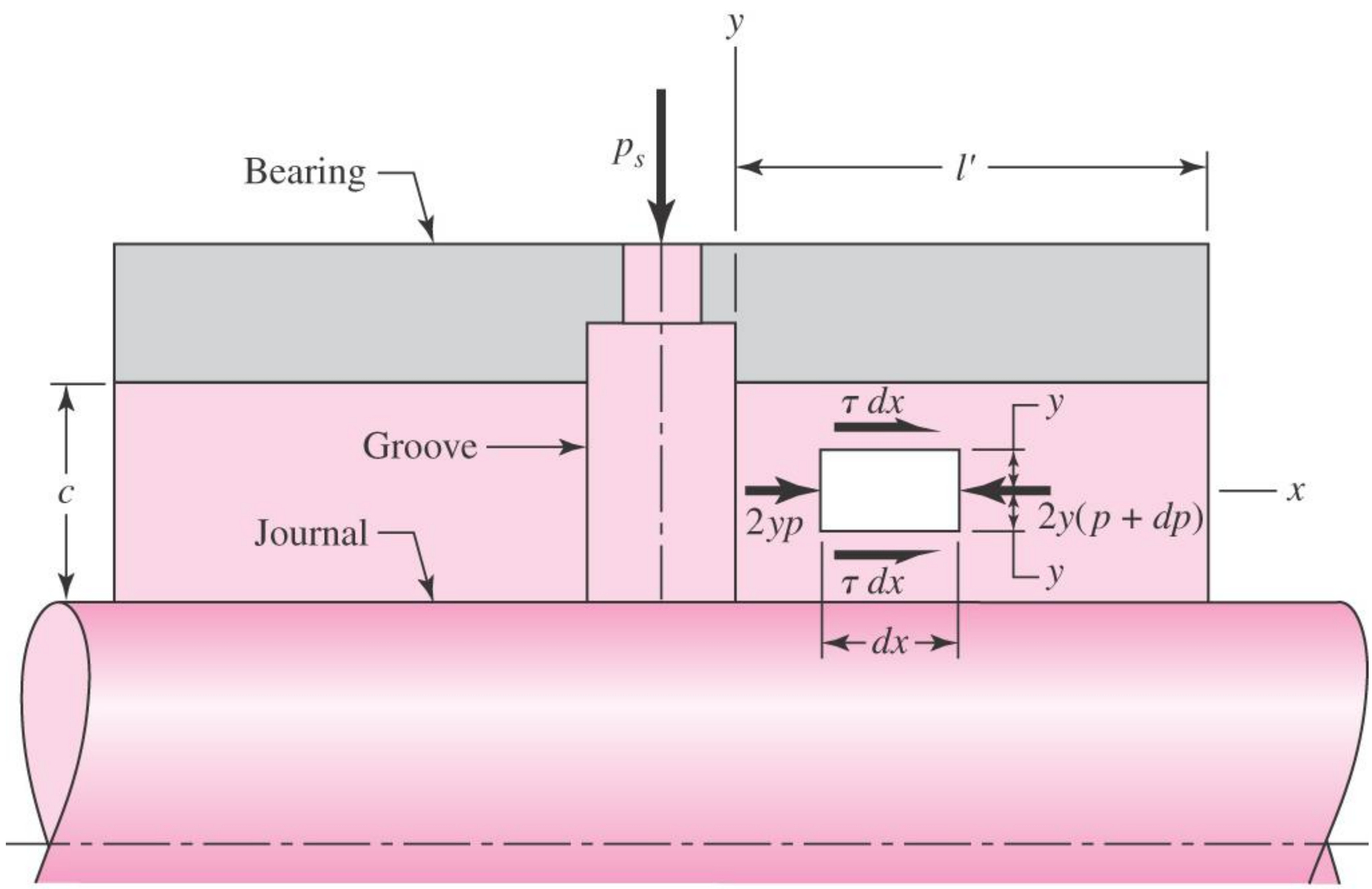


Fig. 12–28

Derivation of Velocity Equation with Pressure-Fed Groove

$$-2y(p + dp) + 2yp + 2\tau\, dx = 0 \qquad (a)$$

$$\tau = y\frac{dp}{dx} \qquad (b)$$

$$\tau = \mu\frac{du}{dy} \qquad (c)$$

$$\frac{du}{dy} = \frac{1}{\mu}\frac{dp}{dx}y \qquad (d)$$

$$u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + C_1 \qquad (e)$$

$$0 = \frac{1}{2\mu}\frac{dp}{dx}\left(\frac{c}{2}\right)^2 + C_1$$

$$C_1 = -\frac{c^2}{8\mu}\frac{dp}{dx}$$

Derivation of Velocity Equation with Pressure-Fed Groove

$$u = \frac{1}{8\mu}\frac{dp}{dx}(4y^2 - c^2) \tag{f}$$

$$p = p_s - \frac{p_s}{l'}x \tag{g}$$

$$\frac{dp}{dx} = -\frac{p_s}{l'} \tag{h}$$

$$u = \frac{p_s}{8\mu l'}(c^2 - 4y^2) \tag{12–21}$$

Distribution of Velocity

$$u = \frac{p_s}{8\mu l'}(c^2 - 4y^2) \qquad (12\text{–}21)$$

$$u_{\max} = \frac{p_s c^2}{8\mu l'} \qquad (i)$$

$$u_{av} = \frac{2}{3}\frac{p_s h^2}{8\mu l'} = \frac{p_s}{12\mu l'}(c - e\cos\theta)^2 \qquad (j)$$

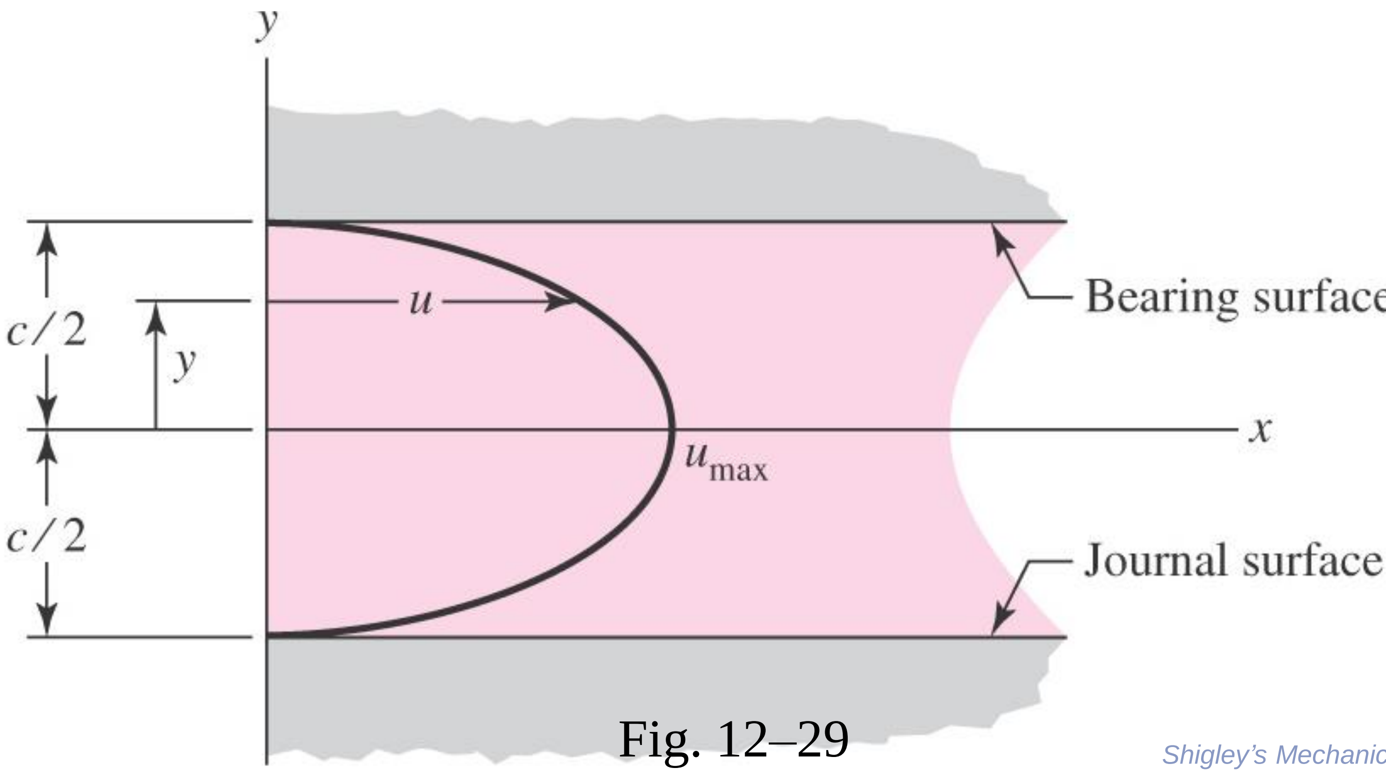


Fig. 12–29

Side Flow Notation

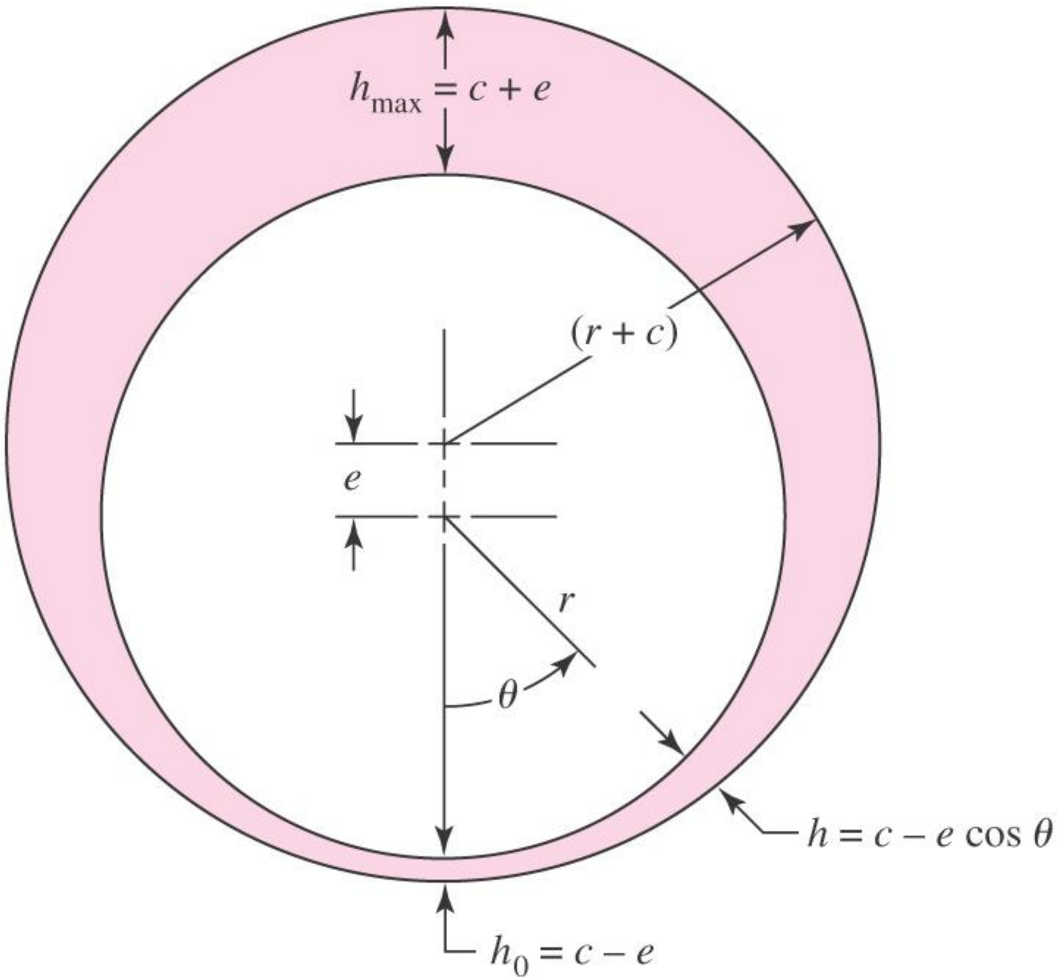


Fig. 12–30

Derivation of Side Flow with Force-fed Groove

$$dQ_s = 2u_{\text{av}}\,dA = 2u_{\text{av}}(r\,h\,d\theta) \qquad (k)$$

$$dQ_s = \frac{p_s r}{6\mu l'}(c - e\cos\theta)^3\,d\theta \qquad (l)$$

$$Q_s = \int dQ_s = \frac{p_s r}{6\mu l'}\int_0^{2\pi}(c - e\cos\theta)^3\,d\theta = \frac{p_s r}{6\mu l'}(2\pi c^3 + 3\pi\,ce^2)$$

$$\epsilon = e/c$$

$$Q_s = \frac{\pi p_s r c^3}{3\mu l'}(1 + 1.5\epsilon^2) \qquad (12\text{–}22)$$

Characteristic Pressure

- The characteristic pressure in each of the two bearings that constitute the pressure-fed bearing assembly is

$$P = \frac{W/2}{2rl'} = \frac{W}{4rl'} \qquad (12\text{–}23)$$

Typical Plumbing with Pressure-fed Groove

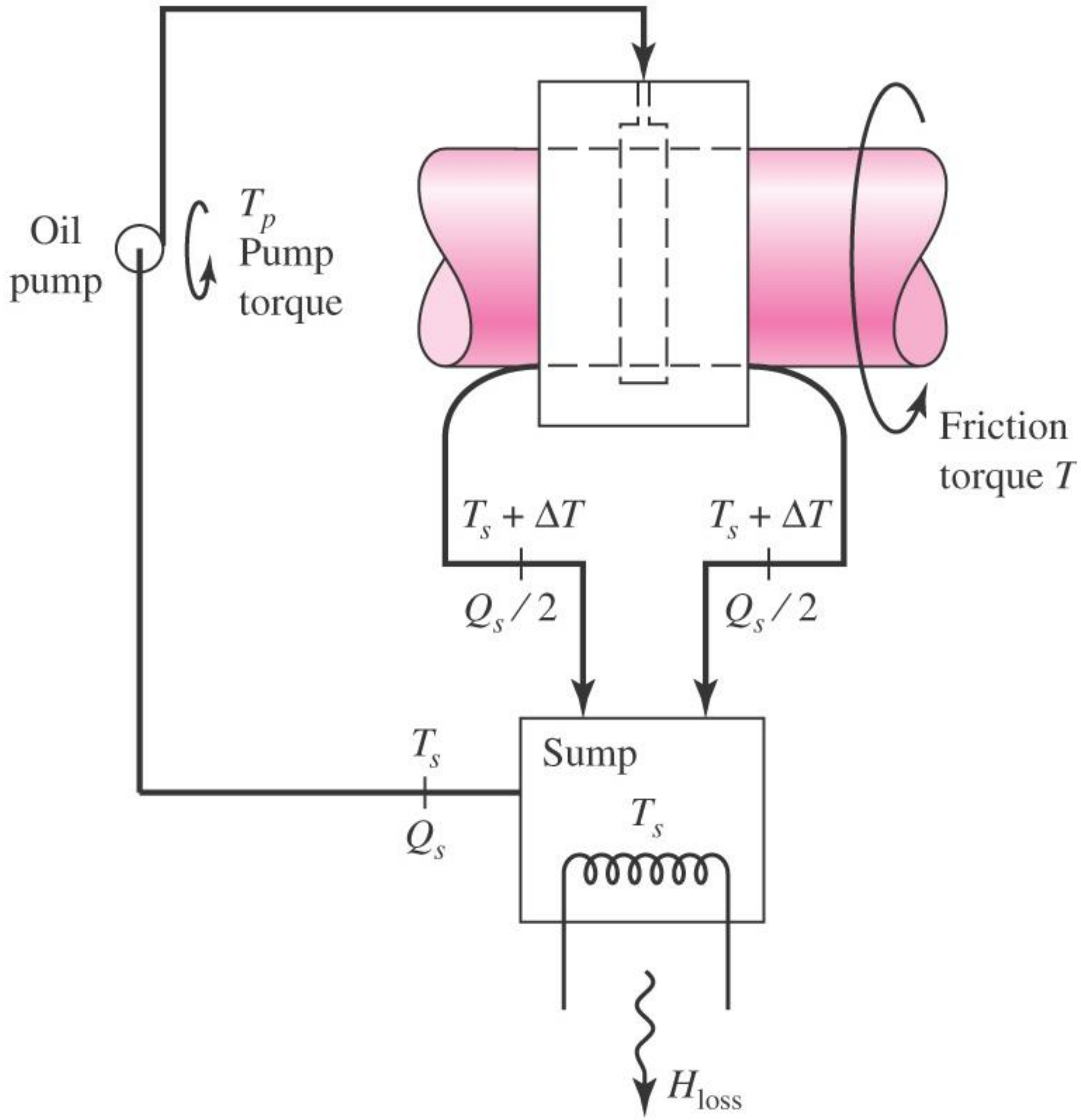


Fig. 12–31

Derivation of Temperature Rise with Pressure-Fed Groove

$$H_{\text{gain}} = 2\,\rho C_p (Q_s/2)\Delta T = \rho C_p Q_s \Delta T \qquad (m)$$

$$H_f = \frac{2\pi T N}{J} = \frac{2\pi f W r N}{J} = \frac{2\pi W N c}{J}\frac{f r}{c} \qquad (n)$$

$$\Delta T = \frac{2\pi W N c}{J \rho C_p Q_s}\frac{f r}{c} \qquad (o)$$

$$\Delta T = \frac{2\pi}{J\rho C_p} W N c \frac{f r}{c}\frac{3\mu l'}{(1 + 1.5\epsilon^2)\pi p_s r c^3}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{r}{c}\right)^2 \frac{4 r l' \mu N}{W}$$

Derivation of Temperature Rise with Pressure-Fed Groove

$$\Delta T_F = \frac{3(fr/c)SW^2}{2J\rho C_p p_s r^4}\,\frac{1}{(1+1.5\epsilon^2)} = \frac{0.0123(fr/c)SW^2}{(1+1.5\epsilon^2)p_s r^4} \qquad (12\text{–}24)$$

$$\Delta T_C = \frac{978(10^6)}{1+1.5\epsilon^2}\,\frac{(fr/c)SW^2}{p_s r^4} \qquad (12\text{–}25)$$

Example 12–6

A circumferential-groove pressure-fed bearing is lubricated with SAE grade 20 oil supplied at a gauge pressure of 207 kPa. The journal diameter d_j is 44.45 mm, with a unilateral tolerance of -0.05 mm. The central circumferential bushing has a diameter d_b of 44.53 mm, with a unilateral tolerance of $+0.1$ mm. The l'/d ratio of the two "half-bearings" that constitute the complete pressure-fed bearing is $1/2$. The journal angular speed is 3000 rev/min, or 50 rev/s, and the radial steady load is 4 kN. The external sump is maintained at 49°C as long as the necessary heat transfer does not exceed 800 Btu/h.

(*a*) Find the steady-state average film temperature.

(*b*) Compare h_0, $T_{\max}$, and P_{st} with the Trumpler criteria.

(*c*) Estimate the volumetric side flow Q_s, the heat loss rate H_{loss}, and the parasitic friction torque.

Solution

(*a*)

$$r = \frac{d_j}{2} = \frac{44.45}{2} = 22.23 \text{ mm}$$

Example 12–6

$$c_{\min} = \frac{(d_b)_{\min} - (d_j)_{\max}}{2} = \frac{44.53 - 44.45}{2} = 0.04 \text{ mm}$$

Since $l'/d = 1/2$, $l' = d/2 = r = 22.23$ mm. Then the pressure due to the load is

$$P = \frac{W}{4rl'} = \frac{4}{4(0.022\,23)^2} = 2024 \text{ kPa}$$

The Sommerfeld number S can be expressed as

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{22.23}{0.04}\right)^2 \frac{\mu'}{2.024} \frac{50}{(10^6)} = 7.63\mu \qquad (1)$$

We will use a tabulation method to find the average film temperature. The first trial average film temperature $\bar{T}_f$ will be 77°C. Using the Seireg curve fit of Table 12–1, we obtain

$$\mu' = 6.89(10^{-3})0.0136 \exp[1271.6/(1.8(77) + 127)] = 0.011\,24 \text{ Pa} \cdot \text{s}$$

From Eq. (1)

Example 12–6

$$7.63\mu = 7.63(0.011\ 24) = 0.0858$$

From Fig. (12–18), $fr/c = 3.3$, and from Fig. (12–16), $\epsilon = 0.80$. From Eq. (12–25),

$$\Delta T_C = \frac{978(10^6)3.3(0.0858)4^2}{[1 + 1.5(0.80)^2]207(22.23)^4} = 44.7°\text{C}$$

$$T_{\text{av}} = T_s + \frac{\Delta T}{2} = 49 + \frac{44.7}{2} = 71.4°\text{C}$$

We form a table, adding a second line with $\bar{T}_f = 71.4°\text{C}$:

Trial $\bar{T}_f$	μ	S	fr/c	ϵ	ΔT_F	T_{av}
77	0.011 24	0.0858	3.3	0.8	44.7	71.4
75.8	0.011 53	0.088	3.39	0.792	53.6	75.8

Example 12–6

If the iteration had not closed, one could plot trial $\bar{T}_f$ against resulting T_{av} and draw a straight line between them, the intersection with a $\bar{T}_f = T_{av}$ line defining the new trial $\bar{T}_f$.

Answer

The result of this tabulation is $\bar{T}_f = 75.8$, $\Delta T_F = 53.6°C$, and $T_{max} = 49 + 53.6 = 102.6°C$

(*b*) Since $h_0 = (1 - \epsilon)c$,

$$h_0 = (1 - 0.792)0.04 = 0.0083 \text{ mm}$$

The required four Trumpler criteria, from "Significant Angular Speed" in Sec. 12–7 are

$$h_0 \geq 0.005\,08 + 0.000\,04(44.45) = 0.006\,86 \text{ mm} \qquad \text{(OK)}$$

Answer

$$T_{max} = T_s + \Delta T = 49 + 53.6 = 102.6°C \qquad \text{(OK)}$$

$$P_{st} = \frac{W_{st}}{4rl'} = \frac{4}{4(0.022\,23)^2} = 2024 \text{ kPa} \qquad \text{(OK)}$$

The factor of safety on the load is approximately unity. (Not OK.)

(*c*) From Eq. (12–22),

Example 12–6

Answer $$Q_s = \frac{\pi(207\,000)22.23(0.04)^3}{3(0.011\,53)22.23}[1 + 1.5(0.792)^2] = 2335 \text{ mm}^3/\text{s}$$

$$H_{\text{loss}} = \rho C_p Q_s \Delta T = 861(1758)(2335 \times 10^{-9})53.6 = 1.89 \text{ J/s}$$

The parasitic friction torque T is

Answer $$T = fWr = \frac{fr}{c}Wr = 3.39(4)(0.04/2) = 0.27 \text{ N} \cdot \text{m}$$

Typical Range of Unit Loads for Sleeve Bearings

Table 12–5

Application	Unit Load psi	Unit Load MPa
Diesel engines:		
Main bearings	900–1700	6–12
Crankpin	1150–2300	8–15
Wristpin	2000–2300	14–15
Electric motors	120–250	0.8–1.5
Steam turbines	120–250	0.8–1.5
Gear reducers	120–250	0.8–1.5
Automotive engines:		
Main bearings	600–750	4–5
Crankpin	1700–2300	10–15
Air compressors:		
Main bearings	140–280	1–2
Crankpin	280–500	2–4
Centrifugal pumps	100–180	0.6–1.2

Some Characteristics of Bearing Alloys

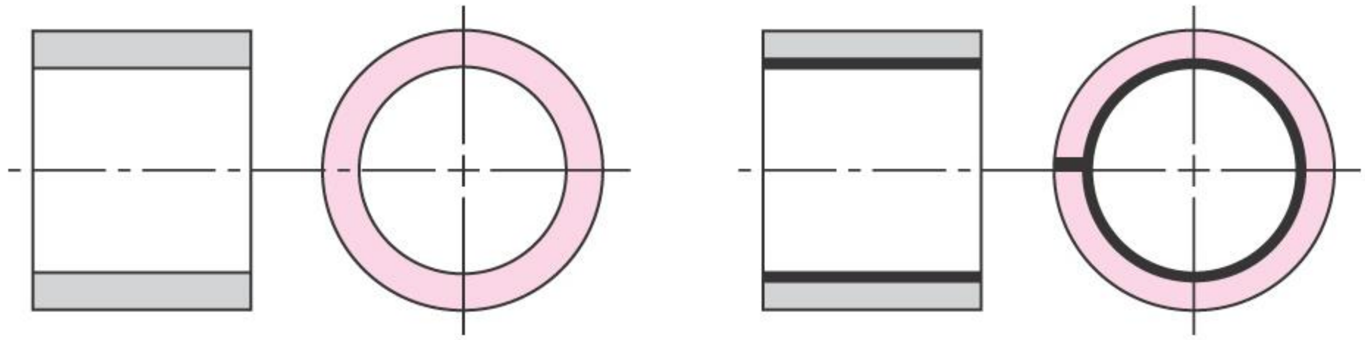
Alloy Name	Thickness, in	SAE Number	Clearance Ratio r/c	Load Capacity	Corrosion Resistance
Tin-base babbitt	0.022	12	600–1000	1.0	Excellent
Lead-base babbitt	0.022	15	600–1000	1.2	Very good
Tin-base babbitt	0.004	12	600–1000	1.5	Excellent
Lead-base babbitt	0.004	15	600–1000	1.5	Very good
Leaded bronze	Solid	792	500–1000	3.3	Very good
Copper-lead	0.022	480	500–1000	1.9	Good
Aluminum alloy	Solid		400–500	3.0	Excellent
Silver plus overlay	0.013	17P	600–1000	4.1	Excellent
Cadmium (1.5% Ni)	0.022	18	400–500	1.3	Good
Trimetal 88*				4.1	Excellent
Trimetal 77†				4.1	Very good

*This is a 0.008-in layer of copper-lead on a steel back plus 0.001 in of tin-base babbitt.

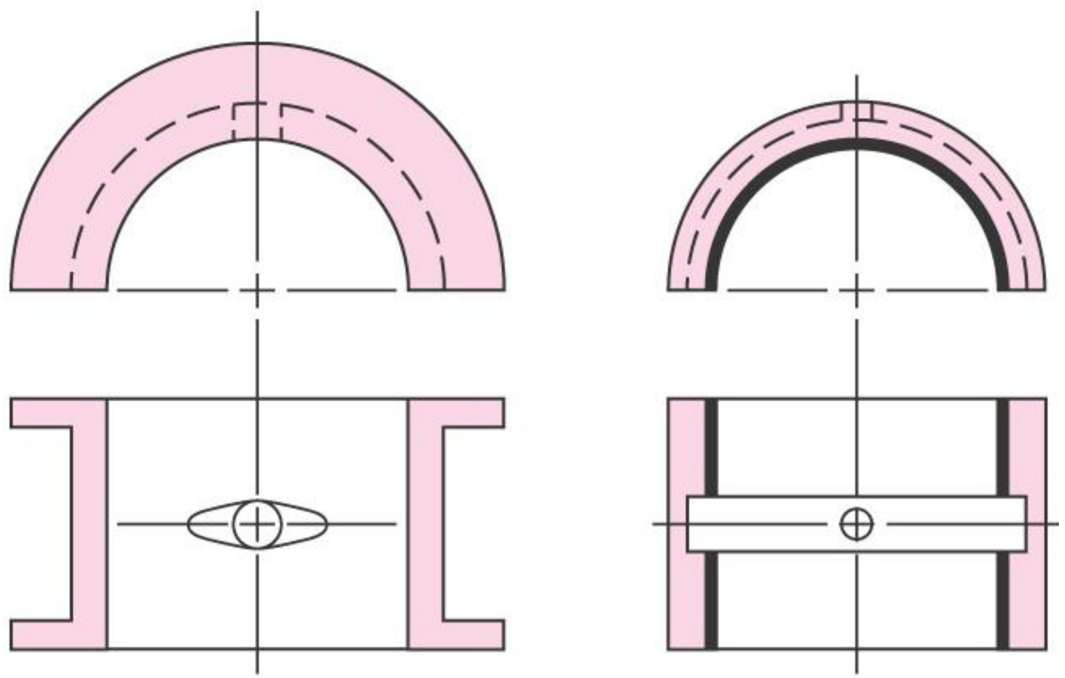
†This is a 0.013-in layer of copper-lead on a steel back plus 0.001 in of lead-base babbitt.

Table 12–6

Bearing Types



(*a*) Solid bushing Fig. 12–32 (*b*) Lined bushing



(*a*) Flanged (*b*) Straight

Fig. 12–33

Typical Groove Patterns

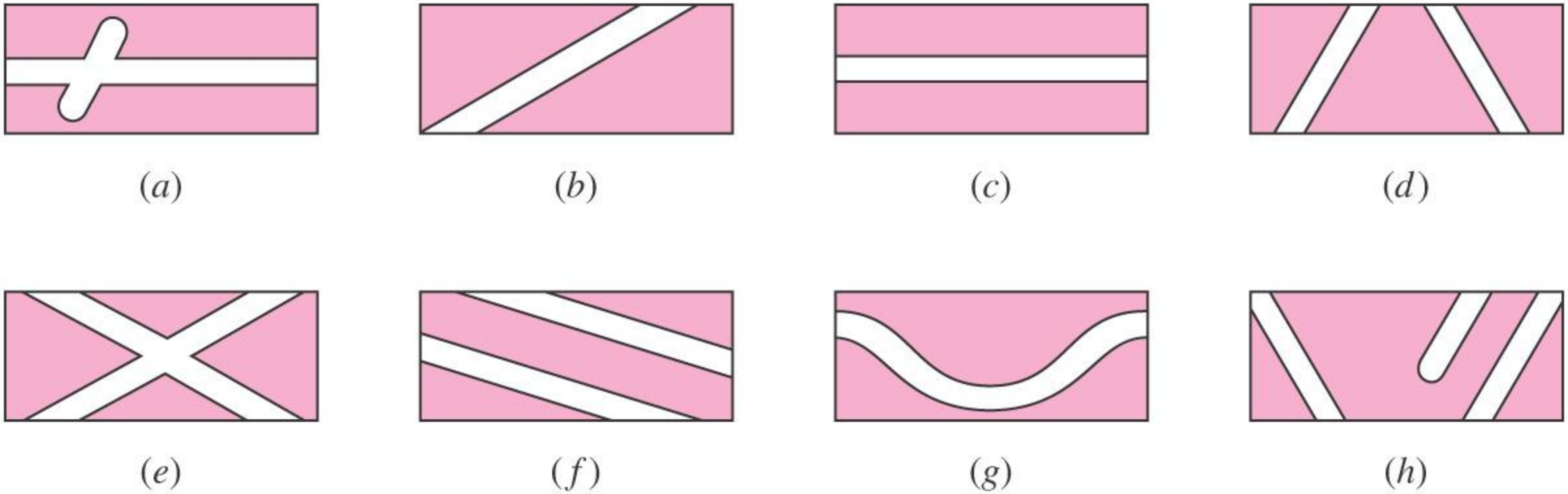


Fig. 12–34

Thrust Bearings

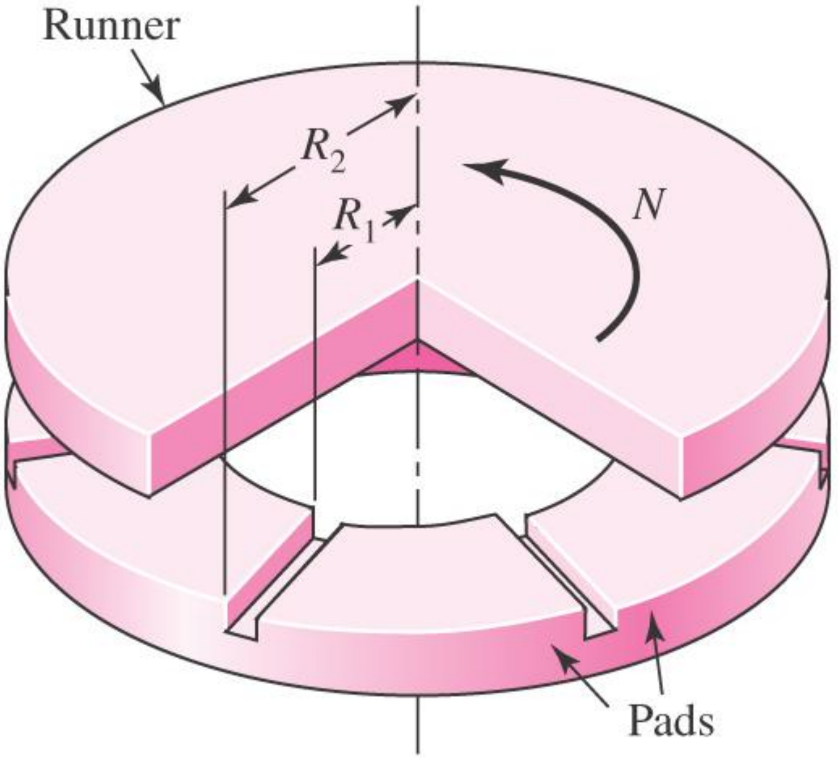


Fig. 12–35

Pressure Distribution in a Thrust Bearing

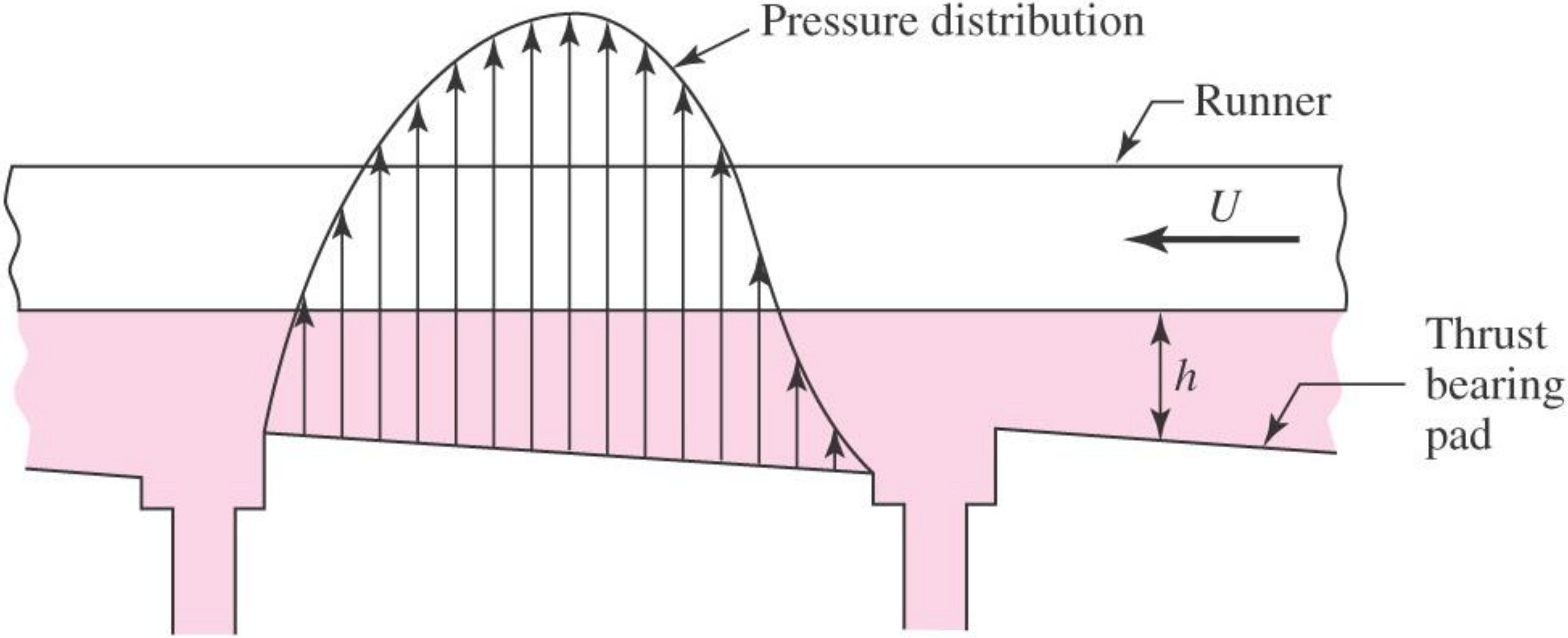


Fig. 12–36

Flanged Sleeve Bearing

- Flanged sleeve bearing can take both radial and thrust loads
- Not hydrodynamically lubricated since clearance space is not wedge-shaped

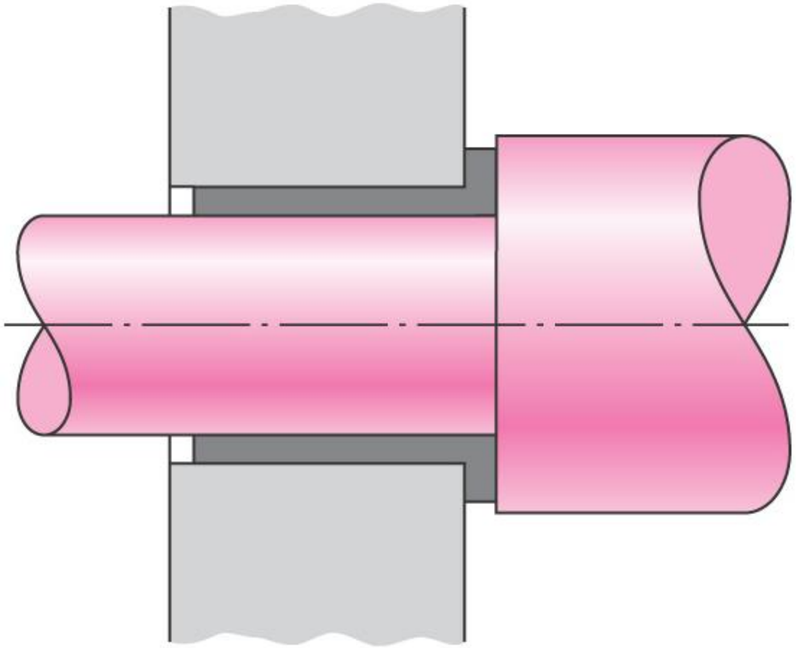


Fig. 12–37

Boundary-Lubricated Bearings

- Relative motion between two surfaces with only a partial lubricant film (not hydrodynamic) is called *boundary lubrication* or *thin-film lubrication.*
- Even hydrodynamic lubrication will have times when it is in thin-film mode, such as at startup.
- Some bearings are boundary lubricated (or dry) at all times.
- Such bearings are much more limited by load, temperature, and speed.

Limits on Some Materials for Boundary-Lubricated Bearings

Material	Maximum Load, MPa	Maximum Temperature, °C	Maximum Speed, m/s	Maximum *PV* Value*
Cast bronze	31.0	163	7.6	1.76
Porous bronze	31.0	66	7.6	1.76
Porous iron	55.2	66	4.1	1.76
Phenolics	41.4	93	12.7	0.53
Nylon	7.0	93	5.1	0.11
Teflon	3.5	260	0.5	0.035
Reinforced Teflon	17.2	260	5.1	0.35
Teflon fabric	413.7	260	0.25	0.88
Delrin	7.0	82	5.1	0.105
Carbon-graphite	4.1	399	12.7	0.53
Rubber	0.3	66	20.3	
Wood	13.8	66	10.2	0.53

*P = load, MPa; V = speed, m/s.

Table 12–7

Linear Sliding Wear

$$wA \propto f_s PAVt$$

$$wA = KPAVt$$

$$w = KPVt \qquad (12\text{–}26)$$

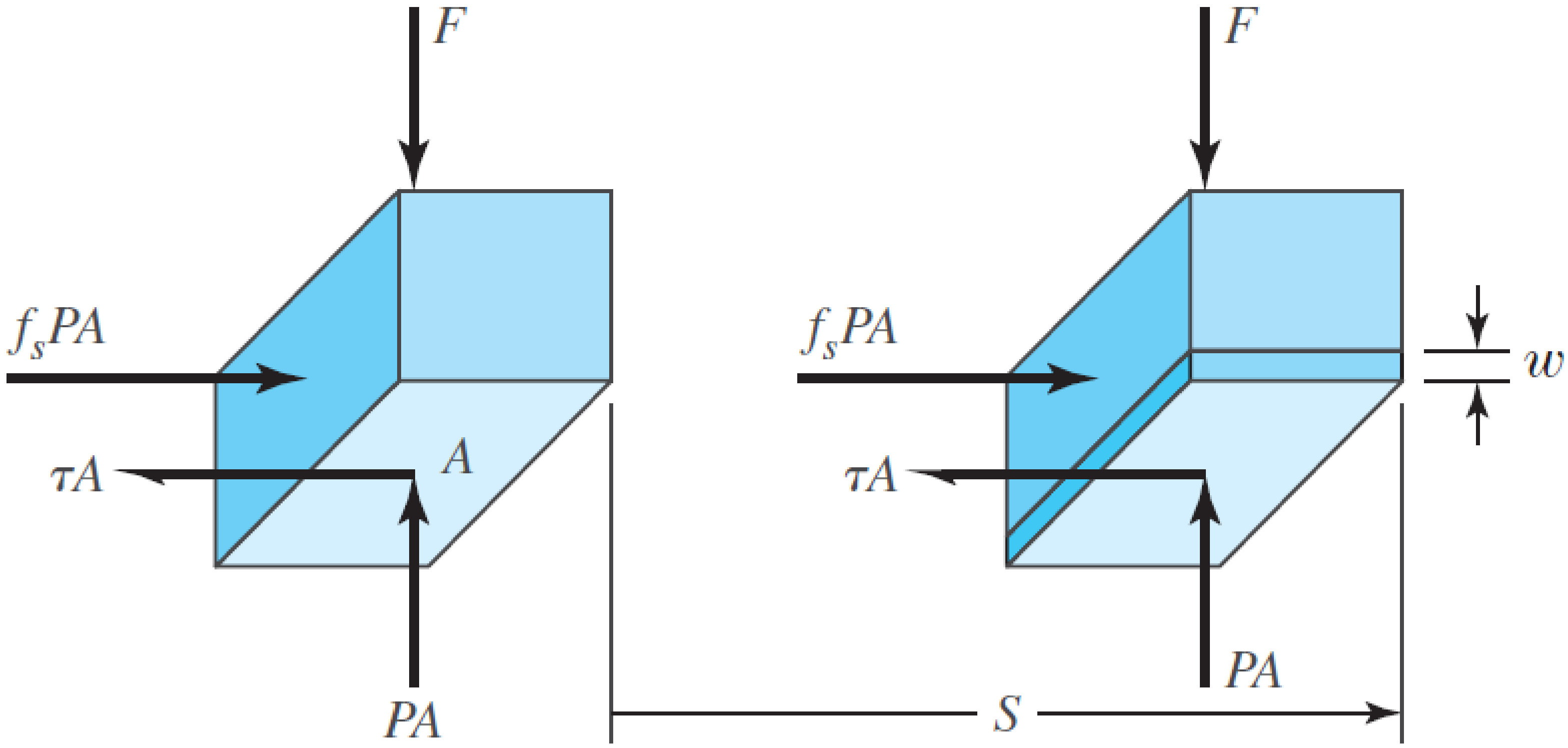


Fig. 12–38

Wear Factors in U.S. Customary Units

Bushing Material	Wear Factor *K*	Limiting *PV*
Oiles 800	$6037(10^{-20})$	0.63
Oiles 500	$1207(10^{-20})$	1.64
Polyactal copolymer	$100\ 615(10^{-20})$	0.18
Polyactal homopolymer	$127\ 730(10^{-20})$	0.11
66 nylon	$402\ 460(10^{-20})$	0.07
66 nylon +15% PTFE	$26\ 160(10^{-20})$	0.25
+15% PTFE +30% glass	$32\ 200(10^{-20})$	0.35
+2.5% MoS_2	$402\ 460(10^{-20})$	0.07
6 nylon	$402\ 460(10^{-20})$	0.07
Polycarbonate +15% PTFE	$150\ 920(10^{-20})$	0.25
Sintered bronze	$205\ 250(10^{-20})$	0.3
Phenol +25% glass fiber	$16\ 100(10^{-20})$	0.4

*dim[K] = $m^3 \cdot s/(N \cdot m \cdot s)$, dim [PV] = MPa · m/s.

Table 12–8

Coefficients of Friction

Type	Bearing	f_s
Placetic	Oiles 80	0.05
Composite	Drymet ST	0.03
	Toughmet	0.05
Met	Cermet M	0.05
	Oiles 2000	0.03
	Oiles 300	0.03
	Oiles 500SP	0.03

Table 12–9

Wear Equation with Practical Modifying Factors

- It is useful to include two modifying factors in the linear wear equation
  - f_1 to account for motion type, load, and speed (Table 12–10)
  - f_2 to account for temperature and cleanliness conditions (Table 12–11)

$$w = f_1 f_2 K P V t \tag{12–27}$$

Motion-Related Factor f_1

Mode of Motion	Characteristic Pressure *P*, MPa		Velocity *V*, mm/s	f_1*
Rotary	5 or less		16.8 or less	1.0
			16.8–168.8	1.0–1.3
			168.8–508	1.3–1.8
	5–25		16.8 or less	1.5
			16.8–168.8	1.5–2.0
			168.8–508	2.0–2.7
Oscillatory	5 or less	>30°	16.8 or less	1.3
			16.8–508	1.3–2.4
		<30°	16.8 or less	2.0
			16.8–508	2.0–3.6
	5–25	>30°	16.8 or less	2.0
			16.8–508	2.0–3.2
		<30°	16.8 or less	3.0
			16.8–508	3.0–4.8
Reciprocating	5 or less		168.8 or less	1.5
			168.8–508	1.5–3.8
	5–25		168.8 or less	2.0
			168.8–508	2.0–7.5

Table 12–10

Environmental Factor f_2

Ambient Temperature, °C	Foreign Matter	f_2
60 or lower	No	1.0
60 or lower	Yes	3.0–6.0
60–99	No	3.0–6.0
60–99	Yes	6.0–12.0

Table 12–11

Pressure Distribution on Boundary-Lubricated Bearing

- Nominal pressure is

$$P = \frac{F}{DL} \qquad (12\text{–}28)$$

- Pressure distribution is given by

$$p = P_{\text{max}} \cos\theta \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

- Vertical component of $p\,dA$ is

$$p\,dA\cos\theta = [pL(D/2)\,d\theta]\cos\theta$$

$$= P_{\text{max}}(DL/2)\cos^2\theta\,d\theta$$

- Integrating gives F,

$$\int_{-\pi/2}^{\pi/2} P_{\text{max}}\left(\frac{DL}{2}\right)\cos^2\theta\,d\theta = \frac{\pi}{4}P_{\text{max}}DL = F$$

$$P_{\text{max}} = \frac{4}{\pi}\frac{F}{DL} \qquad (12\text{–}31)$$

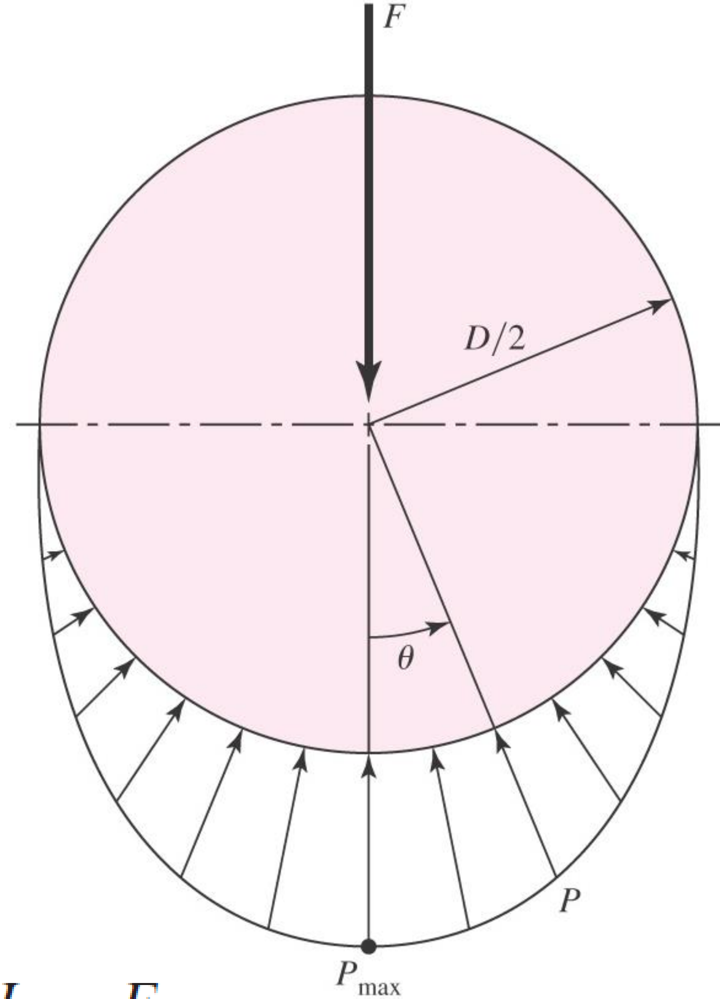


Fig. 12–39

Pressure and Velocity

- Using nominal pressure,

$$P = \frac{F}{DL} \tag{12–28}$$

- Velocity in ft/min,

$$V = \frac{\pi DN}{12} \tag{12–29}$$

- Gives PV in psi·ft/min

$$PV = \frac{F}{DL}\frac{\pi DN}{12} = \frac{\pi}{12}\frac{FN}{L} \tag{12–30}$$

- Note that PV is independent of D

Bushing Wear

- Combining Eqs. (12–29), (12–31), and (12–27), an expression for bushing wear is

$$w = f_1 f_2 K \frac{4}{\pi} \frac{F}{DL} \frac{\pi D N t}{12} = \frac{f_1 f_2 K F N t}{3L} \qquad (12\text{–}32)$$

Length/Diameter Ratio

- Recommended design constraints on length/diameter ratio

$$0.5 \le L/D \le 2 \qquad (12\text{–}33)$$

Example 12–7

An Oiles SP 500 alloy brass bushing is 25 mm long with a 25-mm dia bore and operates in a clean environment at 21°C. The allowable wear without loss of function is 0.125 mm. The radial load is 2250 N. The shaft speed is 200 rev/min. Estimate the number of revolutions for radial wear to be 0.125 mm. See Fig. 12–40 and Table 12–12.

From Table 12–8, $K = 1207(10^{-20})\ \text{m}^3 \cdot \text{s}/(\text{N} \cdot \text{m} \cdot \text{s})$. $P = 2250/[(25)(25)] = 3.6$ MPa, $V = \pi DN = \pi(25)(200)/60 = 262$ mm/s

Tables 12–10 and 12–11:

$$f_1 = 1.8, \quad f_2 = 1$$

Table 12–12:

$$PV_{\max} = 1640\ \text{MPA} \cdot \text{mm/s}, \quad P_{\max} = 24.5\ \text{MPa}, \quad V_{\max} = 510\ \text{mm/s}$$

$$P_{\max} = \frac{4}{\pi}\frac{F}{DL} = \frac{4(2250)}{\pi(25)(25)} = 4.58\ \text{MPa} < 24.5\ \text{MPa} \qquad O.K.$$

$$P = \frac{F}{DL} = 3.6\ MPa \quad V = 262\ \text{mm}/s$$

$$PV = 3.6(262) = 943.2\ \text{MPa} \cdot \text{mm/s} < 1640\ \text{MPa} \cdot \text{mm/s} \qquad O.K.$$

Example 12–7

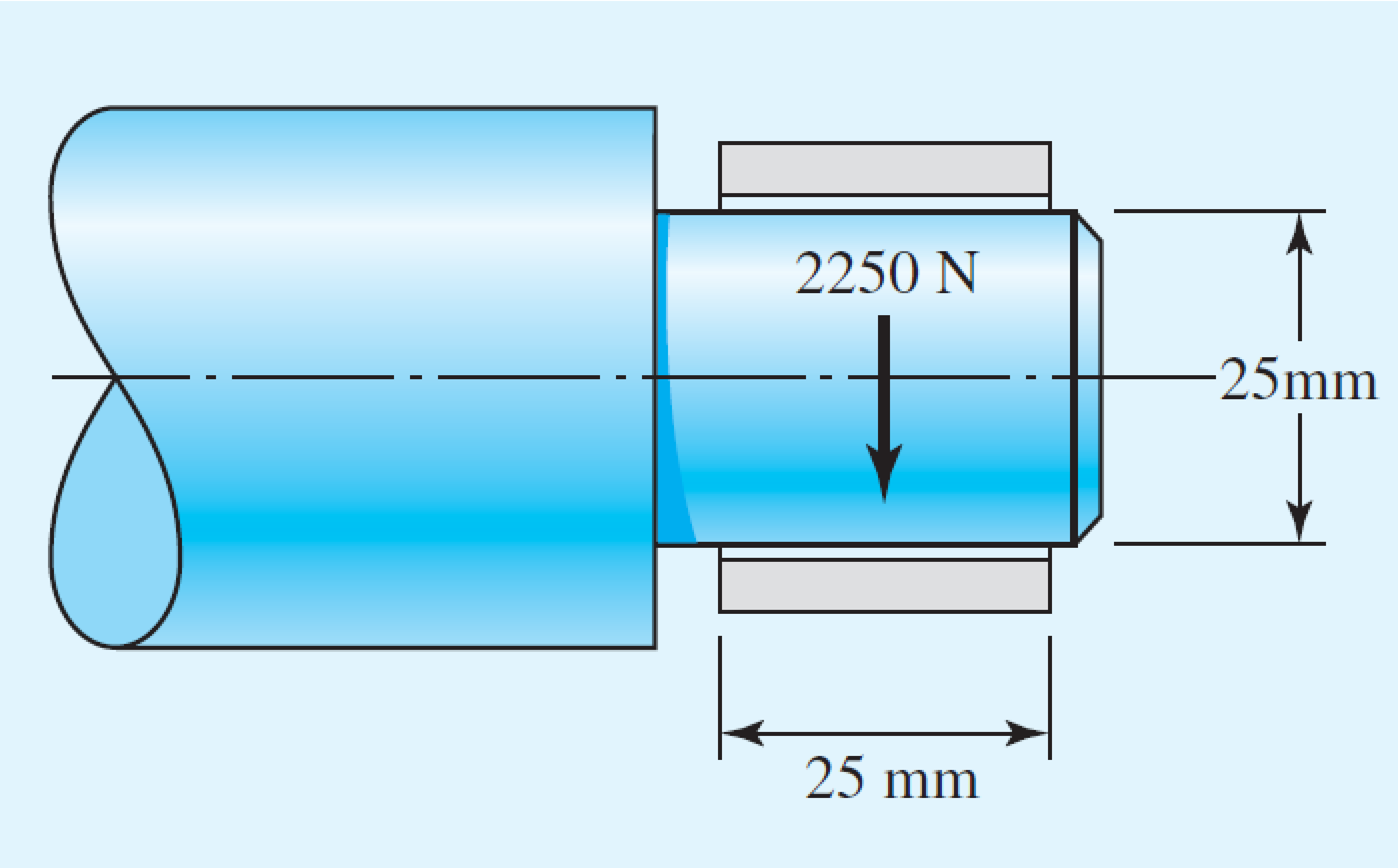


Fig. 12–40

Example 12–7

Service Range	Units	Allowable
Characteristic pressure P_{max}	MPa	<24.5
Velocity V_{max}	m/s	<0.51
PV product	MPa. m/s	<1.64
Temperature T	°C	<149
Properties	**Test Method, Units**	**Value**
Tensile strength	(ASTM E8) psi	>758
Elongation	(ASTM E8) %	>12
Compressive strength	(ASTM E9) psi	67.4
Brinell hardness	(ASTM E10) HB	>210
Coefficient of thermal expansion	(10^{-5}) °C	>1.6
Specific gravity		8.2

Table 12–12

Example 12–7

Solving Eq. (12–32) for t

$$t = \frac{\pi D L w}{4 f_1 f_2 K V F} = \frac{\pi(0.025)(0.025)\,0.125}{4(1.8)(1)(1207)\,(10^{-14})(262)(2250)} = 4\,790\,891 \text{ s} = 1331 \text{ h}$$

$$\text{Cycles} = Nt = 200(79\,848) = 16(10^6) \text{ rev}$$

Answer

Temperature Rise for Boundary-Lubrication

$$H_{\text{gen}} = \frac{f_s F(\pi D)(60N)}{12J} = \frac{5\pi f_s FDN}{J} \qquad (12\text{–}34)$$

$$H_{\text{loss}} = \hbar_{\text{CR}} A \Delta T = \hbar_{\text{CR}} A(T_b - T_\infty) = \frac{\hbar_{\text{CR}} A}{2}(T_f - T_\infty) \qquad (12\text{–}35)$$

where A = housing surface area, ft^2

$\hbar_{\text{CR}}$ = overall combined coefficient of heat transfer, $\text{Btu/(h} \cdot \text{ft}^2 \cdot {}^\circ\text{F)}$

T_b = housing metal temperature, °F

T_f = lubricant temperature, °F

$$T_f = T_\infty + \frac{10\pi f_s FDN}{J \hbar_{\text{CR}} A} \qquad (12\text{–}36)$$

$$A \approx \frac{2\pi DL}{144} \qquad (12\text{–}37)$$

$$T_f \approx T_\infty + \frac{10\pi f_s FDN}{J\hbar_{\text{CR}}(2\pi DL/144)} = T_\infty + \frac{720 f_s FN}{J\hbar_{\text{CR}} L} \qquad (12\text{–}38)$$

Example 12–8

Choose an Oiles 500 bushing to give a maximum wear of 0.025 mm for 800 h of use with a 5 rev/s journal and 220 N radial load. Use $\hbar_{CR} = 13.3\ \mathrm{W/(m^2 \cdot °C)}$, $T_{max} = 149°\mathrm{C}$, $f_s = 0.03$, and a design factor $n_d = 2$. Table 12–13 lists the available bushing sizes from the manufacturer.

Solution

With a design factor n_d, substitute n_dF for F. First estimate the bushing length using Eq. (12-32) with $f_1 = f_2 = 1$, and $K = 1207(10^{-20})$ from Table 12–8:

$$L = \frac{4 f_1 f_2 K n_d F N t}{w} = \frac{4(1)1(1207)10^{-20}(2)220(5)800(3600)}{0.025(10^{-3})} = 0.0122\ \mathrm{m} = 12.2\ \mathrm{mm} \qquad (1)$$

Example 12–8

		L													
ID	OD	12	16	20	22	25	30	38	44	50	62	75	90	100	125
12	20	•	•	•	•	•									
16	22		•	•		•		•							
20	28		•	•		•		•							
22	30			•		•	•	•							
25	35			•		•	•	•	•	•					
25	38			•		•		•		•					
30	40					•	•	•	•	•					
38	50					•	•	•	•	•					
44	58						•	•	•	•	•	•	•	•	
50	60							•		•	•	•			
58	70							•		•	•	•			

Example 12–8

62	75	•	•		•			
70	84	•	•	•	•			
75	90		•	•	•	•		
90	103		•		•		•	
100	120		•		•		•	
115	135				•		•	•
125	150				•		•	•

*In a display such as this a manufacturer is likely to show catolog numbers where the • appears.

Table 12–13

Example 12–8

From Eq. (12–38) with $f_s = 0.03$ from Table 12–9, $\hbar_{CR} = 13.3\ \text{W}/(\text{m}^2 \cdot {}^\circ\text{C})$, and $n_d F$ for F,

$$L \doteq \frac{f_s n_d F N}{J \hbar_{CR}(T_f - T_\infty)} = \frac{0.03(2)220(5)}{13.3(149 - 21)} = 0.0388\ \text{m} = 38.8\ \text{mm}$$

The two results bracket L such that $12.2\ \text{mm} \le L \le 38.8\ \text{mm}$. As a start let $L = 25$ mm. From Table 12–13, we select $D = 25$ mm from the midrange of available bushings.

Trial 1: $D = L = 25$ mm.

Eq. (12–31): $$P_{\max} = \frac{4}{\pi}\frac{n_d F}{DL} = \frac{4}{\pi}\frac{2(220)}{(0.025)(0.025)} = 0.9\ \text{MPa} < 24.5\ \text{MPa} \quad \text{(OK)}$$

$$P = \frac{n_d F}{DL} = \frac{2(220)}{25(25)} = 0.7\ \text{MPa}$$

Eq. (12–29): $$V = \pi DN = \pi(0.025)5 = 0.39\ \text{m/s} < 0.51\ \text{m/s} \qquad \text{(OK)}$$

Example 12–8

$$PV = 0.7(0.39) = 0.27 \text{ MPa} \cdot \text{m/s} < 1.64 \text{ MPa} \cdot \text{m/s} \qquad \text{(OK)}$$

V	f_1	
0.17	1.3	
0.39	f_1	$=> f_1 = 1.64$
0.51	1.8	

Our second estimate is $L \geq 12.2(1.64) = 20$ mm. From Table 12–13, there are many 20-mm bushings to select from. The smallest diameter in Table 12–13 is $D = 12$ mm. This gives an L/D ratio of 1.5, which is acceptable according to Eq. (12–33).

Trial 2: $D = 12.5$ mm, $L = 20$ mm.

$$V = \pi DN = \pi 0.0125(5) = 0.2 \text{ m/s} < 0.51 \text{ m/s} \qquad \text{(OK)}$$

$$P_{\max} = \frac{4}{\pi}\frac{n_d F}{DL} = \frac{4}{\pi}\frac{2(220)}{12.5(20)} = 2.2 \text{ MPa} < 24.5 \text{ MPa} \qquad \text{(OK)}$$

$$P = \frac{n_d F}{DL} = \frac{2(220)}{12.5(20)} = 1.76 \text{ MPa}$$

Example 12–8

$$PV = 1.7(0.2) = 0.34 \text{ MPa} \cdot \text{m/s} < 1.64 \text{ MPa} \cdot \text{m/s} \qquad \text{(OK)}$$

Answer

Select any of the bushings from the trials, where the optimum, from trial 2, is D = 12 mm and L = 20 mm. Other factors may enter in the overall design that make the other bushings more appropriate.