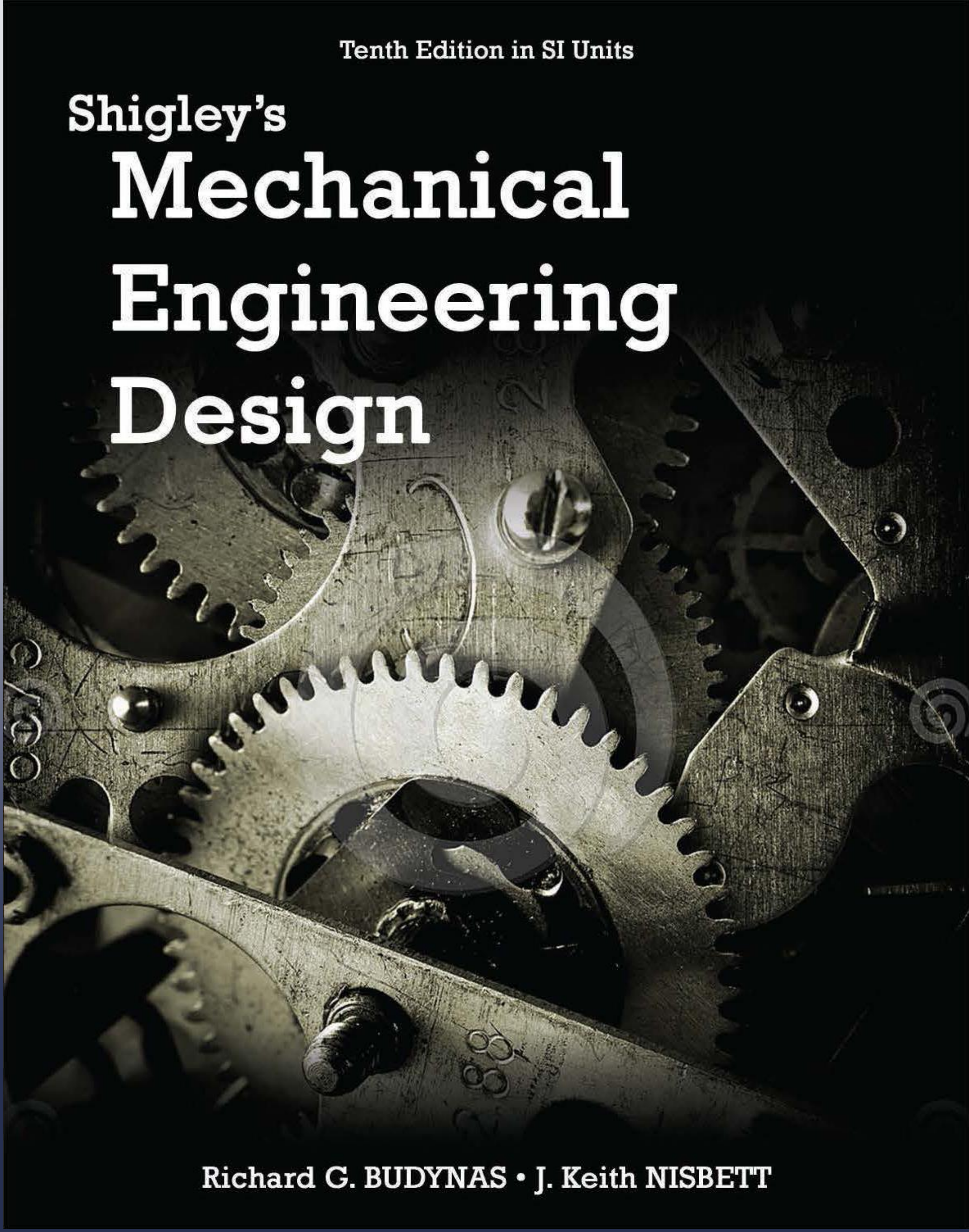
# Lecture Slides




## Chapter 17

## Flexible Mechanical Elements

# Chapter Outline

# Characteristics of Some Common Belt Types

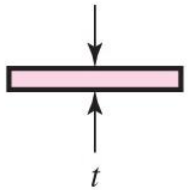
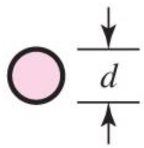
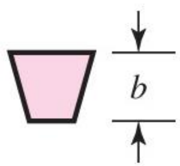
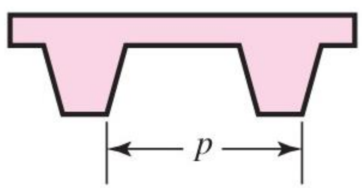
| Belt Type | Figure | Joint | Size Range | Center Distance |
|---|---|---|---|---|
| Flat |  | Yes | $t = \begin{cases} 0.03 \text{ to } 0.20 \text{ in} \\ 0.75 \text{ to } 5 \text{ mm} \end{cases}$ | No upper limit |
| Round |  | Yes | $d = \frac{1}{8}$ to $\frac{3}{4}$ in | No upper limit |
| V |  | None | $b = \begin{cases} 0.31 \text{ to } 0.91 \text{ in} \\ 8 \text{ to } 19 \text{ mm} \end{cases}$ | Limited |
| Timing |  | None | $p = 2$ mm and up | Limited |

Table 17–1

# Flat-Belt Geometry – Open Belt

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\theta_D + d\theta_d)$$

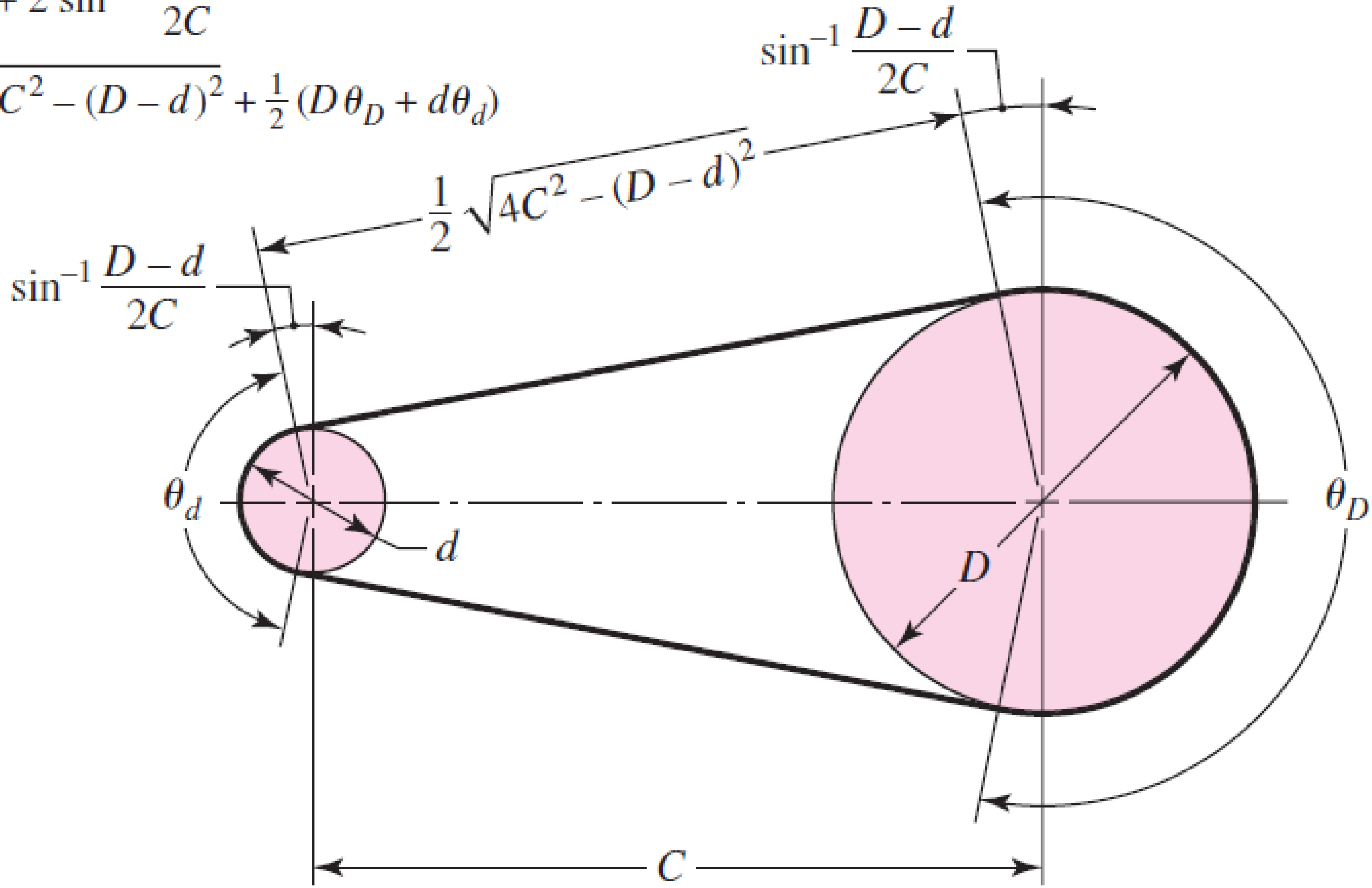



Fig.17–1*a*

# Flat-Belt Geometry – Crossed Belt

$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C}$$

$$L = \sqrt{4C^2 - (D+d)^2} + \frac{1}{2}(D+d)\theta$$

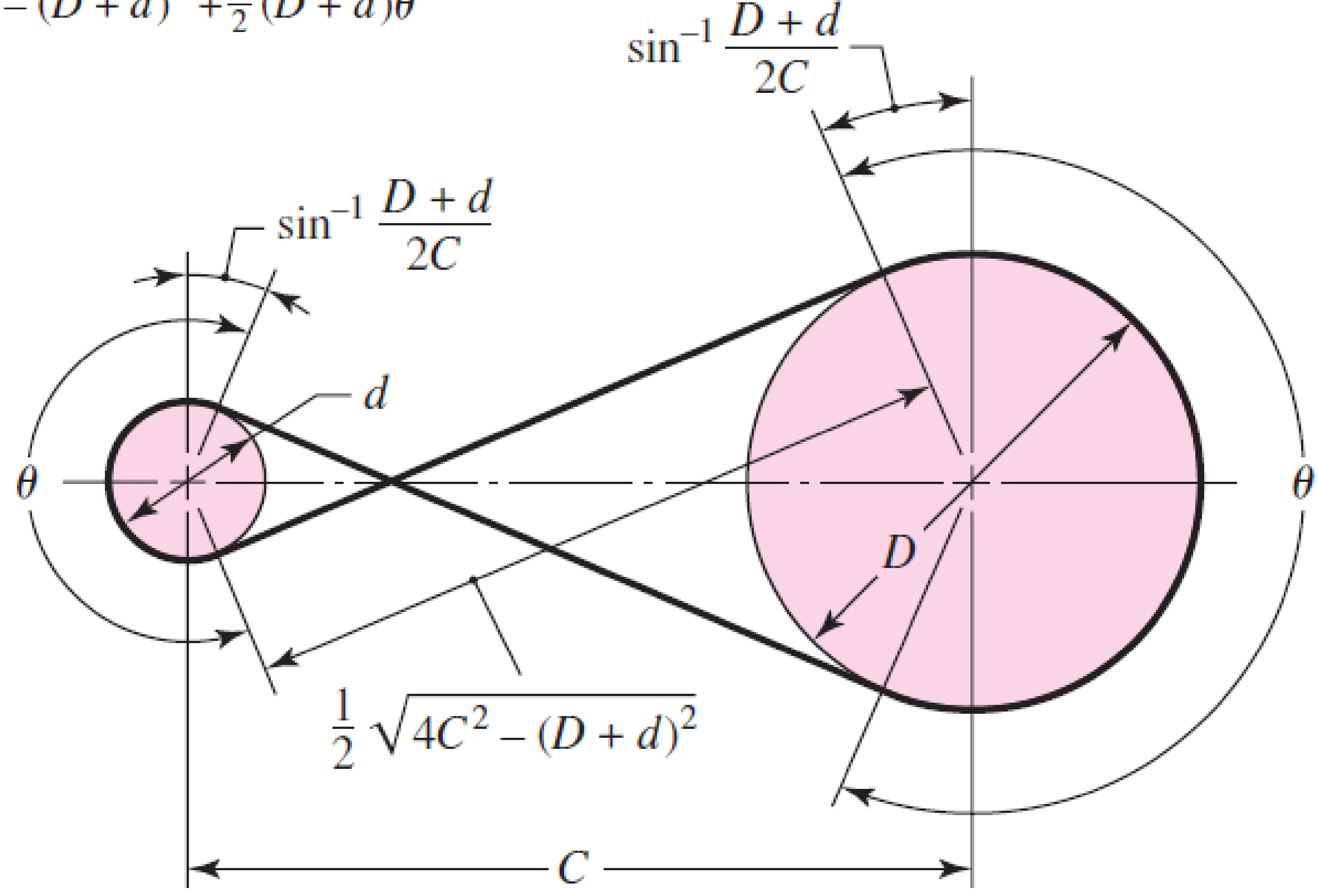



Fig.17–1*b*

# Reversing Belts

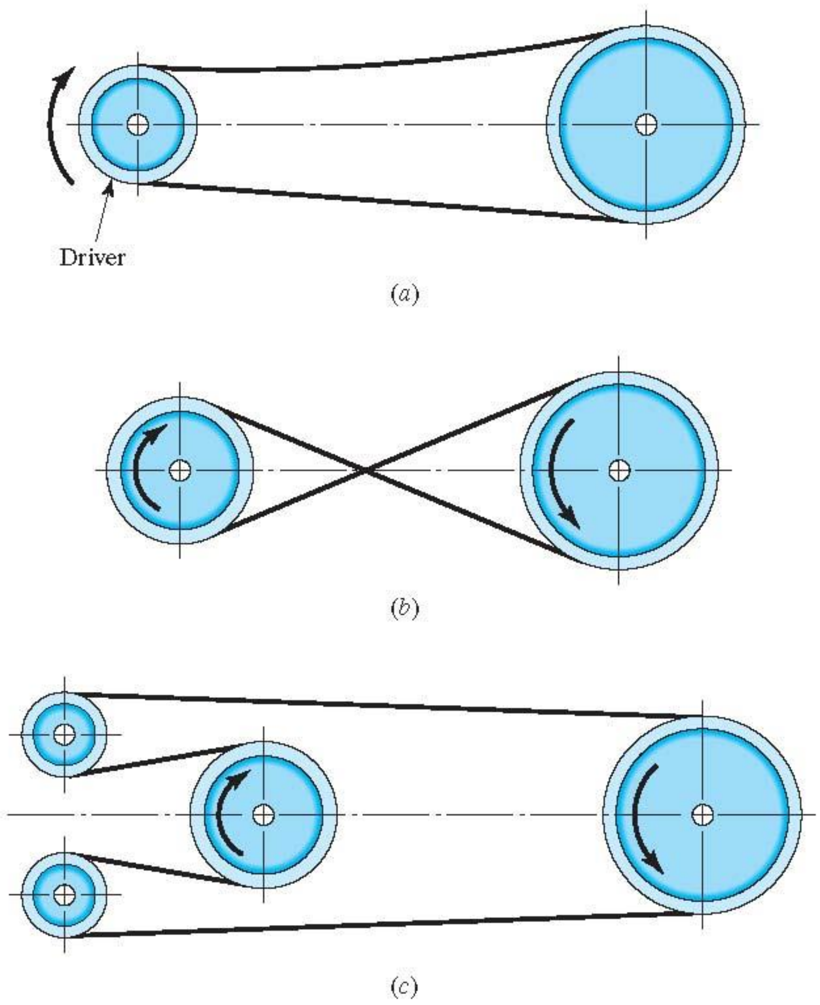



Fig.17–2

# Flat-belt with Out-of-plane Pulleys

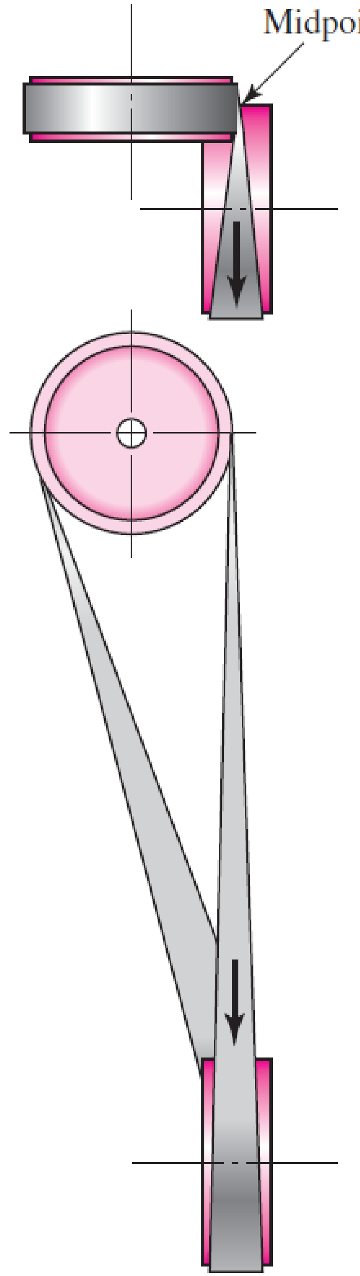



Fig.17–3

# Flat-belt Shifting Without Clutch

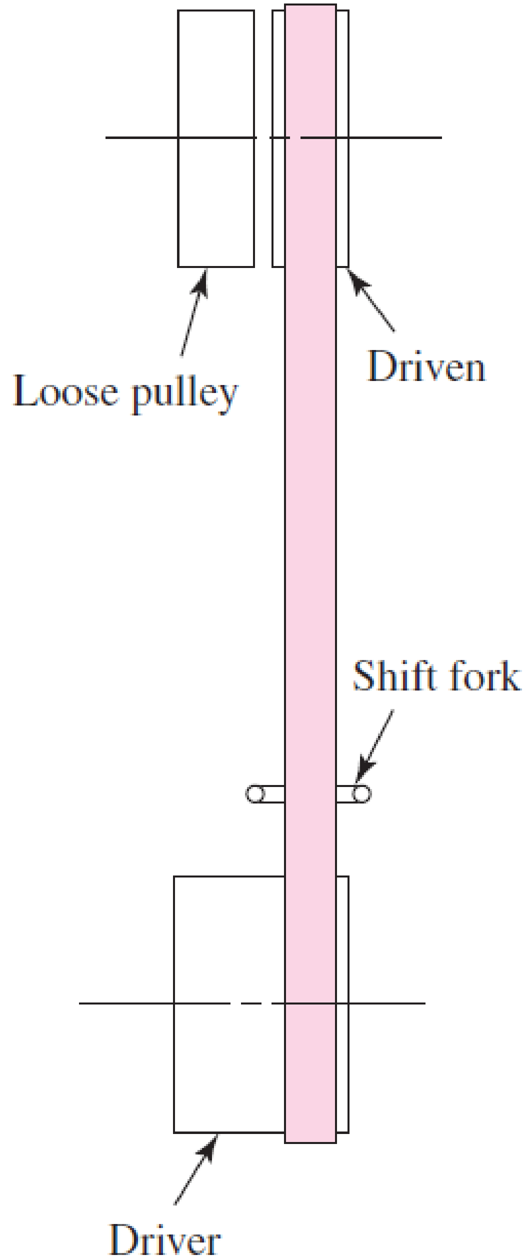
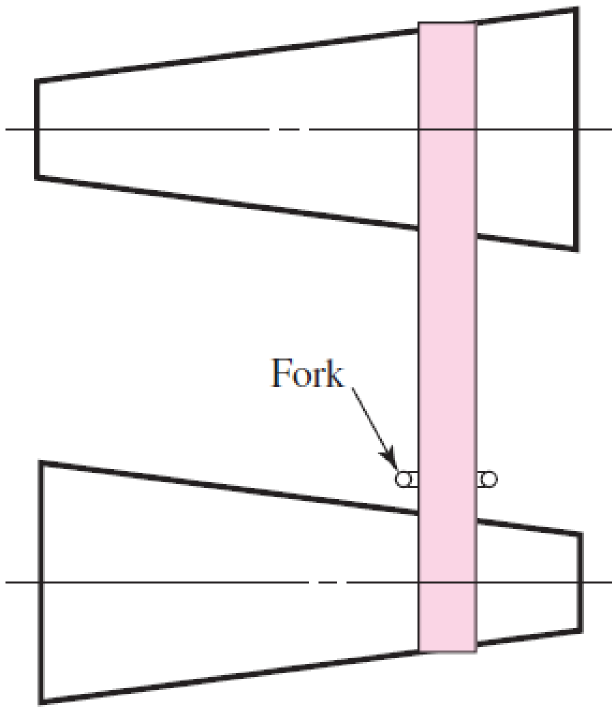



Fig.17–4

# Variable-Speed Belt Drives




(*a*)

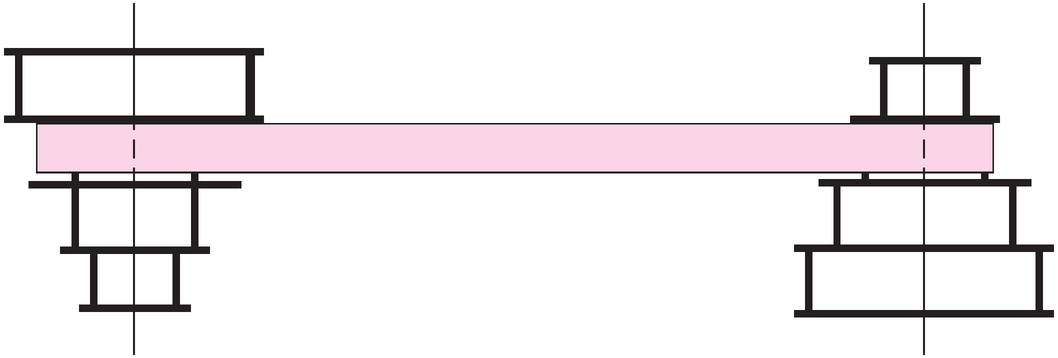


Fig.17–5

(*b*)

# Free Body of Infinitesimal Element of Flat Belt

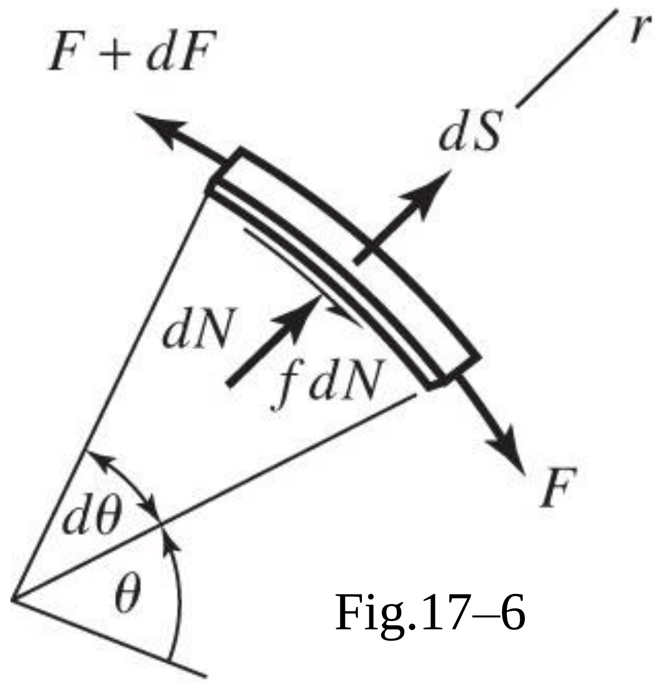



Fig.17–6

$$dS = (mr\,d\theta)r\omega^2 = mr^2\omega^2\,d\theta = mV^2\,d\theta = F_c\,d\theta \qquad (a)$$

$$\sum F_r = -(F + dF)\frac{d\theta}{2} - F\frac{d\theta}{2} + dN + dS = 0$$

$$dN = F\,d\theta - dS \qquad (b)$$

# Free Body of Infinitesimal Element of Flat Belt

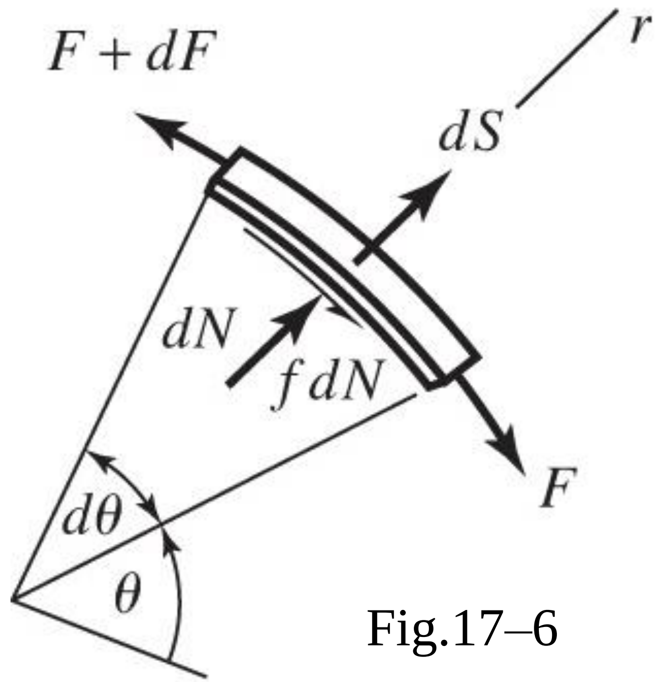



Fig.17–6

$$\sum F_t = -f\,dN - F + (F + dF) = 0$$

$$dF = f\,dN = f F\,d\theta - f\,dS = f F\,d\theta - f m r^2 \omega^2\,d\theta$$

$$\frac{dF}{d\theta} - f F = -f m r^2 \omega^2 \qquad (c)$$

# Analysis of Flat Belt

$$\frac{dF}{d\theta} - fF = -fmr^2\omega^2 \qquad \text{(c)}$$

$$F = A\exp(f\theta) + mr^2\omega^2 \qquad \text{(d)}$$

$$F \text{ at } \theta = 0 \text{ equals } F_2 \text{ gives } A = F_2 - mr^2\omega^2$$

$$F = (F_2 - mr^2\omega^2)\exp(f\theta) + mr^2\omega^2 \qquad \text{(17–5)}$$

$$F|_{\theta=\phi} = F_1 = (F_2 - mr^2\omega^2)\exp(f\phi) + mr^2\omega^2 \qquad \text{(17–6)}$$

$$\frac{F_1 - mr^2\omega^2}{F_2 - mr^2\omega^2} = \frac{F_1 - F_c}{F_2 - F_c} = \exp(f\phi) \qquad \text{(17–7)}$$

$$F_c = mr^2\omega^2$$

$$F_1 - F_2 = (F_1 - F_c)\frac{\exp(f\phi) - 1}{\exp(f\phi)} \qquad \text{(17–8)}$$

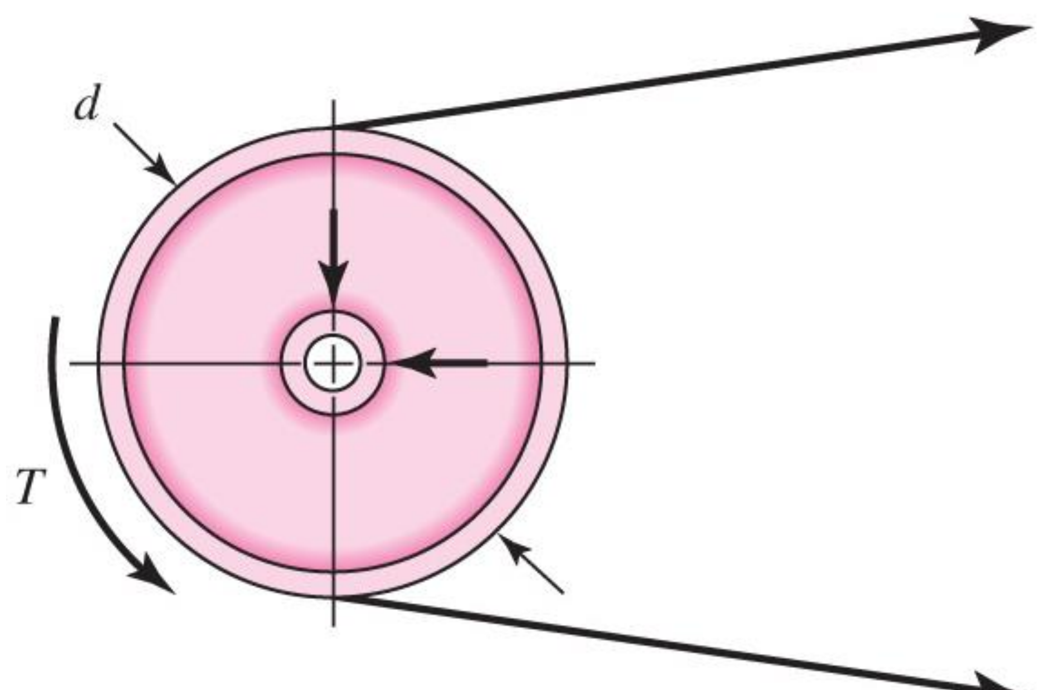
# Hoop Tension Due to Centrifugal Force

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{w}{32.17}\left(\frac{V}{60}\right)^2 \qquad (e)$$

$w = 12\gamma bt$ lbf/ft where $b$ and $t$ are in inches

$V = \pi\, dn/12$ ft/min

# Forces and Torques on a Pulley




$$F_1 = F_i + F_c + \Delta F/2$$
$$= F_i + F_c + \frac{T}{d}$$

Fig.17–7

$$F_2 = F_i + F_c - \Delta F/2$$
$$= F_i + F_c - \frac{T}{d}$$

$F_i$ = initial tension

$F_c$ = hoop tension due to centrifugal force

$\Delta F/2$ = tension due to the transmitted torque $T$

$d$ = diameter of the pulley

# Initial Tension

$$F_1 - F_2 = \frac{2T}{d} \qquad (h)$$

$$F_1 + F_2 = 2F_i + 2F_c$$

$$F_i = \frac{F_1 + F_2}{2} - F_c \qquad (i)$$

$$\frac{F_i}{T/d} = \frac{(F_1 + F_2)/2 - F_c}{(F_1 - F_2)/2} = \frac{F_1 + F_2 - 2F_c}{F_1 - F_2} = \frac{(F_1 - F_c) + (F_2 - F_c)}{(F_1 - F_c) - (F_2 - F_c)}$$

$$= \frac{(F_1 - F_c)/(F_2 - F_c) + 1}{(F_1 - F_c)/(F_2 - F_c) - 1} = \frac{\exp(f\phi) + 1}{\exp(f\phi) - 1}$$

$$F_i = \frac{T}{d}\,\frac{\exp(f\phi) + 1}{\exp(f\phi) - 1} \qquad (17\text{–}9)$$

# Flat Belt Tensions

$$F_1 = F_i + F_c + \frac{T}{d} = F_c + F_i + F_i \frac{\exp(f\phi) - 1}{\exp(f\phi) + 1}$$

$$= F_c + \frac{F_i[\exp(f\phi) + 1] + F_i[\exp(f\phi) - 1]}{\exp(f\phi) + 1}$$

$$F_1 = F_c + F_i \frac{2\exp(f\phi)}{\exp(f\phi) + 1} \qquad (17\text{–}10)$$

$$F_2 = F_i + F_c - \frac{T}{d} = F_c + F_i - F_i \frac{\exp(f\phi) - 1}{\exp(f\phi) + 1}$$

$$= F_c + \frac{F_i[\exp(f\phi) + 1] - F_i[\exp(f\phi) - 1]}{\exp(f\phi) + 1}$$

$$F_2 = F_c + F_i \frac{2}{\exp(f\phi) + 1} \qquad (17\text{–}11)$$

# Plot of Belt Tension vs. Initial Tension

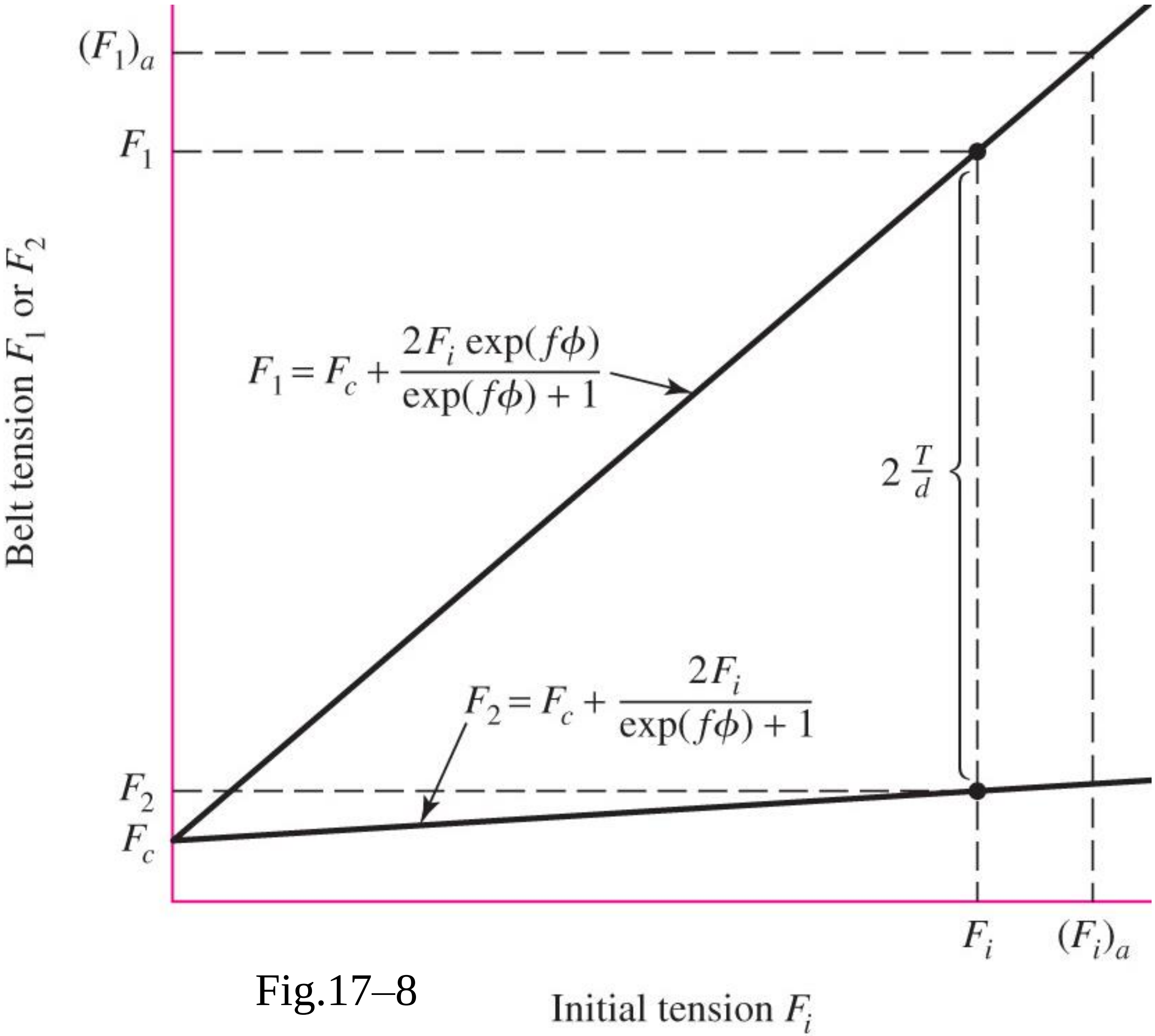



Fig.17–8

# Transmitted Horsepower

$$H = \frac{(F_1 - F_2)V}{33\,000} \qquad (j)$$

# Correction Factors

$$(F_1)_a = bF_aC_pC_v \tag{17–12}$$

where $(F_1)_a$ = allowable largest tension, lbf

$b$ = belt width, in

$F_a$ = manufacturer's allowed tension, lbf/in

$C_p$ = pulley correction factor (Table 17–4)

$C_v$ = velocity correction factor

# Velocity Correction Factor $C_v$ for Leather Belts

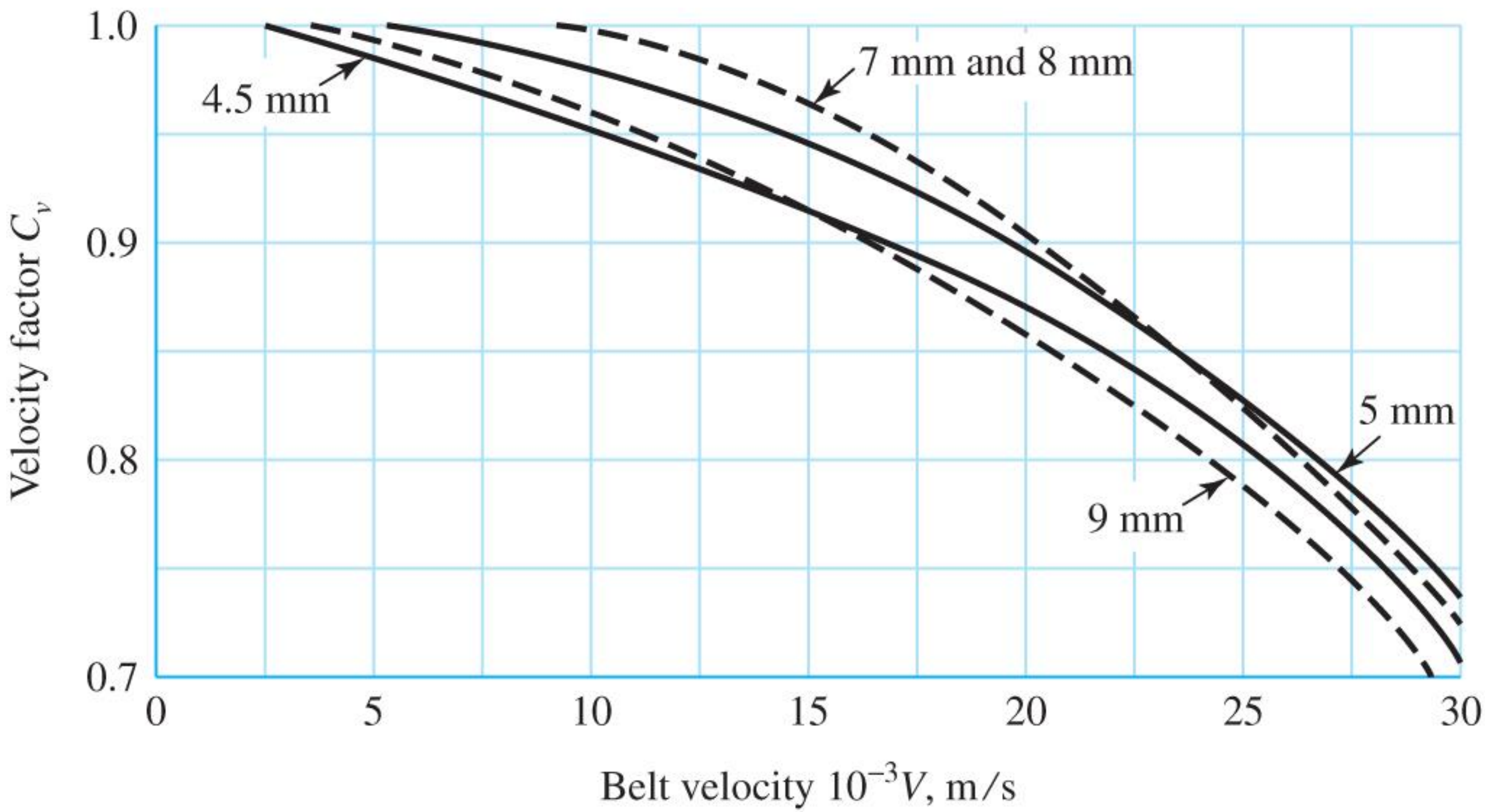



Fig.17–9

# Pulley Correction Factor $C_P$ for Flat Belts

**Table 17–4**

Pulley Correction Factor $C_P$ for Flat Belts*

| | Small-Pulley Diameter, in | | | | | |
|---|---|---|---|---|---|---|
| Material | 1.6 to 4 | 4.5 to 8 | 9 to 12.5 | 14, 16 | 18 to 31.5 | Over 31.5 |
| Leather | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
| Polyamide, F–0 | 0.95 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| F–1 | 0.70 | 0.92 | 0.95 | 1.0 | 1.0 | 1.0 |
| F–2 | 0.73 | 0.86 | 0.96 | 1.0 | 1.0 | 1.0 |
| A–2 | 0.73 | 0.86 | 0.96 | 1.0 | 1.0 | 1.0 |
| A–3 | — | 0.70 | 0.87 | 0.94 | 0.96 | 1.0 |
| A–4 | — | — | 0.71 | 0.80 | 0.85 | 0.92 |
| A–5 | — | — | — | 0.72 | 0.77 | 0.91 |

*Average values of $C_P$ for the given ranges were approximated from curves in the *Habasit Engineering Manual,* Habasit Belting, Inc., Chamblee (Atlanta), Ga.

# Steps for Flat-Belt Analysis

1. Find $\exp(f\phi)$ from belt-drive geometry and friction
2. From belt geometry and speed find $F_c$
3. From $T = 63\,025 H_{\text{nom}} K_s n_d / n$ find necessary torque
4. From torque $T$ find the necessary $(F_1)_a - F_2 = 2T/d$
5. From Tables 17–2 and 17–4, and Eq. (17–12) determine $(F_1)_a$.
6. Find $F_2$ from $(F_1)_a - [(F_1)_a - F_2]$
7. From Eq. ($i$) find the necessary initial tension $F_i$
8. Check the friction development, $f' < f$. Use Eq. (17–7) solved for $f'$:

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c}$$

9. Find the factor of safety from $n_{fs} = H_a / (H_{\text{nom}} K_s)$

# Properties of Some Flat- and Round-Belt Materials

| Material | Specification | Size, mm | Minimum Pulley Diameter, mm | Allowable Tension per Unit Width at 3 m/s, ($10^3$) N/m | Specific Weight, kN/m$^3$ | Coefficient of Friction |
|---|---|---|---|---|---|---|
| Leather | 1 ply | $t = 4.5$ | 75 | 5 | 9.5–12.2 | 0.4 |
| | | $t = 5$ | 90 | 6 | 9.5–12.2 | 0.4 |
| | 2 ply | $t = 7$ | 115 | 7 | 9.5–12.2 | 0.4 |
| | | $t = 8$ | 150 | 9 | 9.5–12.2 | 0.4 |
| | | $t = 9$ | 230 | 10 | 9.5–12.2 | 0.4 |
| Polyamide[b] | F–0[c] | $t = 0.8$ | 15 | 1.8 | 9.5 | 0.5 |
| | F–1[c] | $t = 1.3$ | 25 | 6 | 9.5 | 0.5 |
| | F–2[c] | $t = 1.8$ | 60 | 10 | 13.8 | 0.5 |
| | A–2[c] | $t = 2.8$ | 60 | 10 | 10.0 | 0.8 |
| | A–3[c] | $t = 3.3$ | 110 | 18 | 11.4 | 0.8 |
| | A–4[c] | $t = 5.0$ | 240 | 30 | 10.6 | 0.8 |
| | A–5[c] | $t = 6.4$ | 340 | 48 | 10.6 | 0.8 |
| Urethane[d] | $w = 12.7$ | $t = 1.6$ | See | 1.0[e] | 10.3–12.2 | 0.7 |
| | $w = 19$ | $t = 2.0$ | Table | 1.7[e] | 10.3–12.2 | 0.7 |
| | $w = 32$ | $t = 2.3$ | 17–3 | 3.3[e] | 10.3–12.2 | 0.7 |
| | Round | $d = 6$ | See | 1.4[e] | 10.3–12.2 | 0.7 |
| | | $d = 10$ | Table | 3.3[e] | 10.3–12.2 | 0.7 |
| | | $d = 12$ | 17–3 | 5.8[e] | 10.3–12.2 | 0.7 |
| | | $d = 20$ | | 13[e] | 10.3–12.2 | 0.7 |

# Properties of Some Flat- and Round-Belt Materials

[a]Add 2 in to pulley size for belts 8 in wide or more.

[b]*Source: Habasit Engineering Manual*, Habasit Belting, Inc., Chamblee (Atlanta), Ga.

[c]Friction cover of acrylonitrile-butadiene rubber on both sides.

[d]*Source*: Eagle Belting Co., Des Plaines, Ill.

[e]At 6% elongation; 12% is maximum allowable value.

Table 17–2

# Minimum Pulley Sizes for Flat and Round Urethane Belts

| Belt Style | Belt Size, mm | Ratio of Pulley Speed to Belt Length, rev/(m · s) Up to 14 | 14 to 27 | 28 — 55 |
|---|---|---|---|---|
| Flat | 12.7 × 1.6 | 9.7 | 11.2 | 12.7 |
| | 19 × 2.0 | 12.7 | 16 | 19 |
| | 32 × 2.3 | 12.7 | 16 | 19 |
| Round | 6 | 38.1 | 44.5 | 50.8 |
| | 10 | 57.1 | 66.5 | 76.2 |
| | 12 | 76.2 | 88.9 | 101.6 |
| | 20 | 127 | 152 | 177.8 |

Table 17–3

# Crown Height and ISO Pulley Diameters for Flat Belts

| ISOCrown Pulley Diameter, mm | ISO Height, mm | Pulley Diameter, mm | Crown Height, in $w \leq 250$ mm | $w > 250$ mm |
|---|---|---|---|---|
| 40, 50, 62 | 0.3 | 315, 355 | 0.75 | 0.75 |
| 70, 80 | 0.3 | 315, 355 | 1.0 | 1.0 |
| 90, 100, 115 | 0.3 | 570, 635, 710 | 1.3 | 1.3 |
| 125, 142 | 0.4 | 800, 900 | 1.3 | 1.5 |
| 160, 180 | 0.5 | 1015 | 1.3 | 1.5 |
| 200, 230 | 0.6 | 1140, 1270, 1420 | 1.5 | 2.0 |
| 250, 285 | 0.75 | 1600, 1800, 2030 | 1.8 | 2.5 |

*Crown should be rounded, not angled; maximum roughness is $R_a$ = AA 1500 $\mu$mm.

Table 17–5

# Example 17–1

A polyamide A-3 flat belt 150 mm wide is used to transmit 11 kW under light shock conditions where $K_s = 1.25$, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 2.4 m apart. The 150-mm driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 450 mm in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

(*a*) Estimate the centrifugal tension $F_c$ and the torque $T$.

(*b*) Estimate the allowable $F_1$, $F_2$, $F_i$ and allowable power $H_a$.

(*c*) Estimate the factor of safety. Is it satisfactory?

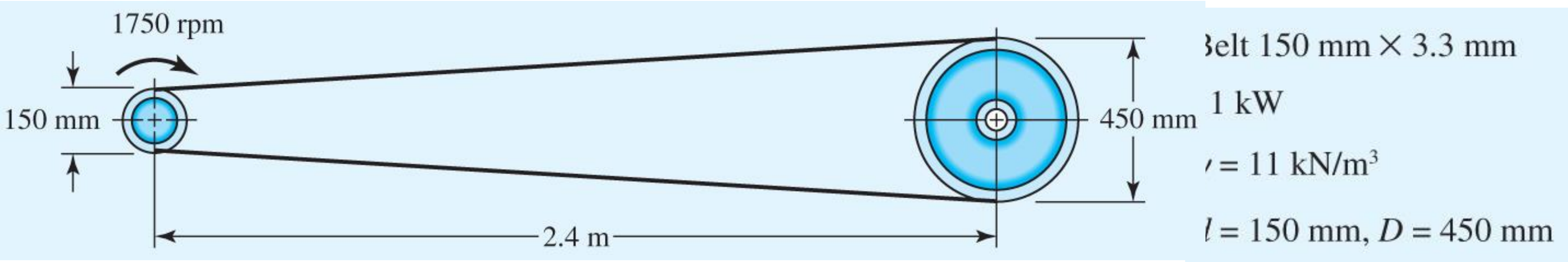



Fig.17–10

# Example 17–1

Solution

(*a*) Eq. (17–1):

$$\phi = \theta_d = \pi - 2\sin^{-1}\left[\frac{450 - 150}{2(2400)}\right] = 3.0165 \text{ rad}$$

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(0.15)1750/60 = 13.7 \text{ m/s}$$

Table 17–2:

$$w = \gamma bt = 11\,000(0.15)0.0033 = 5.4 \text{ N/m}$$

Eq. (*e*):

$$F_c = \frac{w}{g}V^2 = \frac{5.4}{9.81}(13.7)^2 = 103.3 \text{ N}$$

$$T = \frac{H_{\text{nom}} K_s n_d}{2\pi n} = \frac{1.25(1.1)11000}{2\pi 1750/60}$$

$$= 82.5 \text{ N} \cdot \text{m}$$

(*b*) The necessary $(F_1)_a - F_2$ to transmit the torque $T$, from Eq. (*h*), is

# Example 17–1

$$(F_1)_a - F_2 = \frac{2T}{d} = \frac{2(82)}{0.15} = 1093.3 \text{ N}$$

From Table 17–2 $F_a = 18$ kN/m. For polyamide belts $C_v = 1$, and from Table 17–4 $C_p = 0.70$. From Eq. (17–12) the allowable largest belt tension $(F_1)_a$ is

$$(F_1)_a = bF_aC_pC_v = 0.15(18000)0.70(1) = 1890 \text{ N} \qquad \text{Answer}$$

then

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 1890 - 1093 = 796.7 \text{ N}$$

and from Eq. (*i*)

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{1890 + 796.7}{2} - 103 = 1240.4 \text{ N} \qquad \text{Answer}$$

# Example 17–1

The combination $(F_1)_a$, $F_2$, and $F_i$ will transmit the design power of $11(1.25)(1.1) = 15.125$ kW and protect the belt. We check the friction development by solving Eq. (17–7) for $f'$:

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.0165} \ln \frac{1890 - 103}{797 - 103} = 0.314 \qquad \text{Answer}$$

From Table 17–2, $f = 0.8$. Since $f' < f$, that is, $0.314 < 0.80$, there is no danger of slipping.

(*c*)

$$n_{fs} = \frac{H}{H_{\text{nom}} K_s} = \frac{15.125}{11(1.25)} = 1.1 \qquad \text{(as expected)} \qquad \text{Answer}$$

The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 15.125 kW.

# Belt-Tensioning Schemes

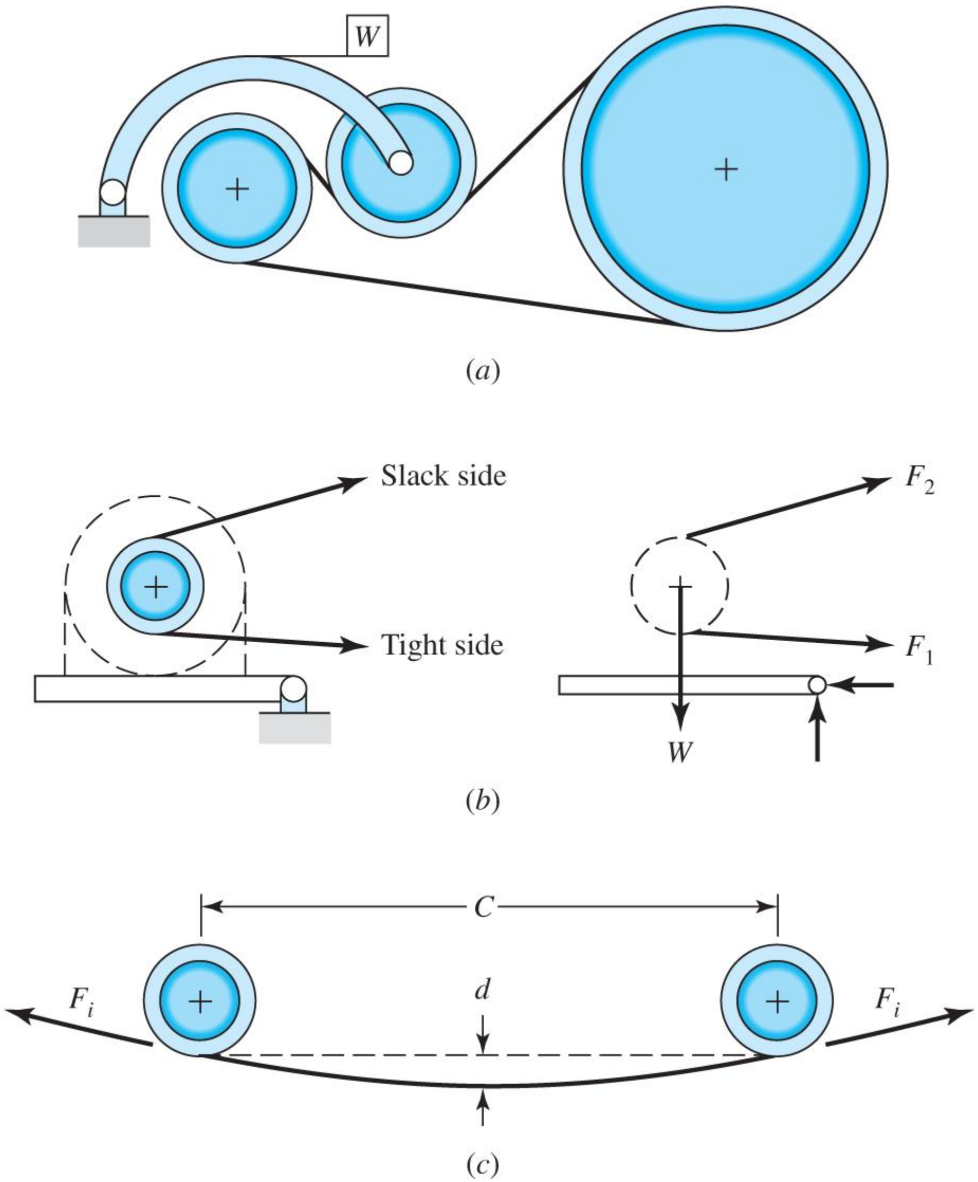



Fig.17–11

# Relation of Dip to Initial Tension

$$d = \frac{L^2 w}{8F_i} \tag{17–13}$$

where $dip$ = dip, in

$C$ = center-to-center distance, in

$w$ = weight per foot of the belt, lbf/ft

$F_i$ = initial tension, lbf

# Example 17–2

Design a flat-belt drive to connect horizontal shafts on 4.8 m centers. The velocity ratio is to be 2.25:1. The angular speed of the small driving pulley is 860 rev/min, and the nominal power transmission is to be 44 760 W under very light shock.

## Solution

- Function: $H_{\text{nom}} = 44\ 760$ W, 860 rev/min, 2.25:1 ratio, $K_s = 1.15$, $C = 4.8$ m
- Design factor: $n_d = 1.05$
- Initial tension maintenance: catenary
- Belt material: polyamide
- Drive geometry, $d$, $D$
- Belt thickness: $t$
- Belt width: $b$

The last four could be design variables. Let's make a few more a priori decisions.

$d = 400$ mm, $D = 2.25d = 900$ mm.

# Example 17–2

Use polyamide A-3 belt; therefore $t = 3.3$ mm and $C_v = 1$.
Now there is one design decision remaining to be made, the belt width $b$.

Table 17–2: $\gamma = 11.4\ \text{kN/m}^3$ $\quad f = 0.8 \quad F_a = 18$ kN/m at 600 rev/min

Table 17–4: $C_p = 0.94$

Eq. (17–12): $F_{1a} = b(18\,000)0.94(1) = 16\,920b$ N (1)

$$H_d = H_{\text{nom}} K_s n_d = 44\,760(1.15)1.05 = 54\,048\ \text{W}$$

$$T = \frac{H_d}{2\pi n} = \frac{54\,048}{2\pi 860/60} = 600\ \text{N}\cdot\text{m}$$

Estimate $\exp(f\phi)$ for full friction development:

Eq. (17–1): $$\phi = \theta_d = \pi - 2\sin^{-1}\frac{900 - 400}{2(4800)} = 3.037\ \text{rad}$$

$$\exp(f\phi) = \exp[0.80(3.037)] = 11.35$$

# Example 17–2

Estimate centrifugal tension $F_c$ in terms of belt width $b$:

$$w = \gamma bt = (11\,400)b(0.0033) = 37.6b \text{ N/m}$$

$$V = \pi dn = \pi(0.4)860/60 = 18 \text{ m/s}$$

Eq. ($e$): $$F_c = \frac{w}{g}V^2 = \frac{(37.6)b(18)^2}{9.81} = 1241.8b \text{ N} \qquad (2)$$

For design conditions, that is, at $H_d$ power level, using Eq. ($h$) gives

$$(F_1)_a - F_2 = 2T/d = 2(600)/0.4 = 3000 \text{ N} \qquad (3)$$

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 16\,920b - 3000 \text{ N} \qquad (4)$$

Using Eq. ($i$) gives

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{16\,920b + 16\,920b - 3000}{2} - 1241.8b = 15\,678.2b - 1500 \text{ N} \qquad (5)$$

# Example 17–2

Place friction development at its highest level, using Eq. (17–7):

$$f\phi = \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \ln \frac{16\,920b - 1241.8b}{16\,920b - 3000 - 1241.8b} = \ln \frac{15\,678.2b}{15\,678.2b - 3000}$$

Solving the preceding equation for belt width $b$ at which friction is fully developed gives

$$b = \frac{3000}{15\,678.2} \frac{\exp(f\phi)}{\exp(f\phi) - 1} = \frac{3000}{15\,678.2} \frac{11.38}{11.38 - 1} = 0.210 \text{ m} = 210 \text{ mm}$$

A belt width greater than 210 mm will develop friction less than $f = 0.80$. The manufacturer's data indicate that the next available larger width is 250 mm.

Use 250 mm-wide belt.
It follows that for a 250-mm-wide belt

Eq. (2): $F_c = 1241.8(0.25) = 310 \text{ N}$

Eq. (1): $(F_1)_a = 16\,920(0.25) = 4230 \text{ N}$

# Example 17–2

Eq. (4): $F_2 = 4230 - 3000 = 1230$ N

Eq. (5): $F_i = 15\,678.2(0.25) - 1500 = 2420$ N

The transmitted power, from Eq. (3), is

$$H_t = [(F_1)_a - F_2]V = 3000(18) = 54\,000 \text{ W}$$

and the level of friction development $f'$, from Eq. (17–7) is

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.037} \ln \frac{4230 - 310}{1230 - 310} = 0.477$$

which is less than $f = 0.8$, and thus is satisfactory. Had a 225-mm belt width been available, the analysis would show $(F_1)_a = 3807$ N, $F_2 = 807$ N, $F_i = 2028$ N, and $f' = 0.63$. With a figure of merit available reflecting cost, thicker belts (A-4 or A-5) could be examined to ascertain which of the satisfactory alternatives is best. From Eq. (17–13) the catenary dip is

$$dip = \frac{L^2 w}{8F_i} = \frac{4.8^2(37.6)0.25}{8(2420)} = 0.011 \text{ m} = 11 \text{ mm}$$

# Variation of Flat-Belt Tensions at Some Cardinal Points

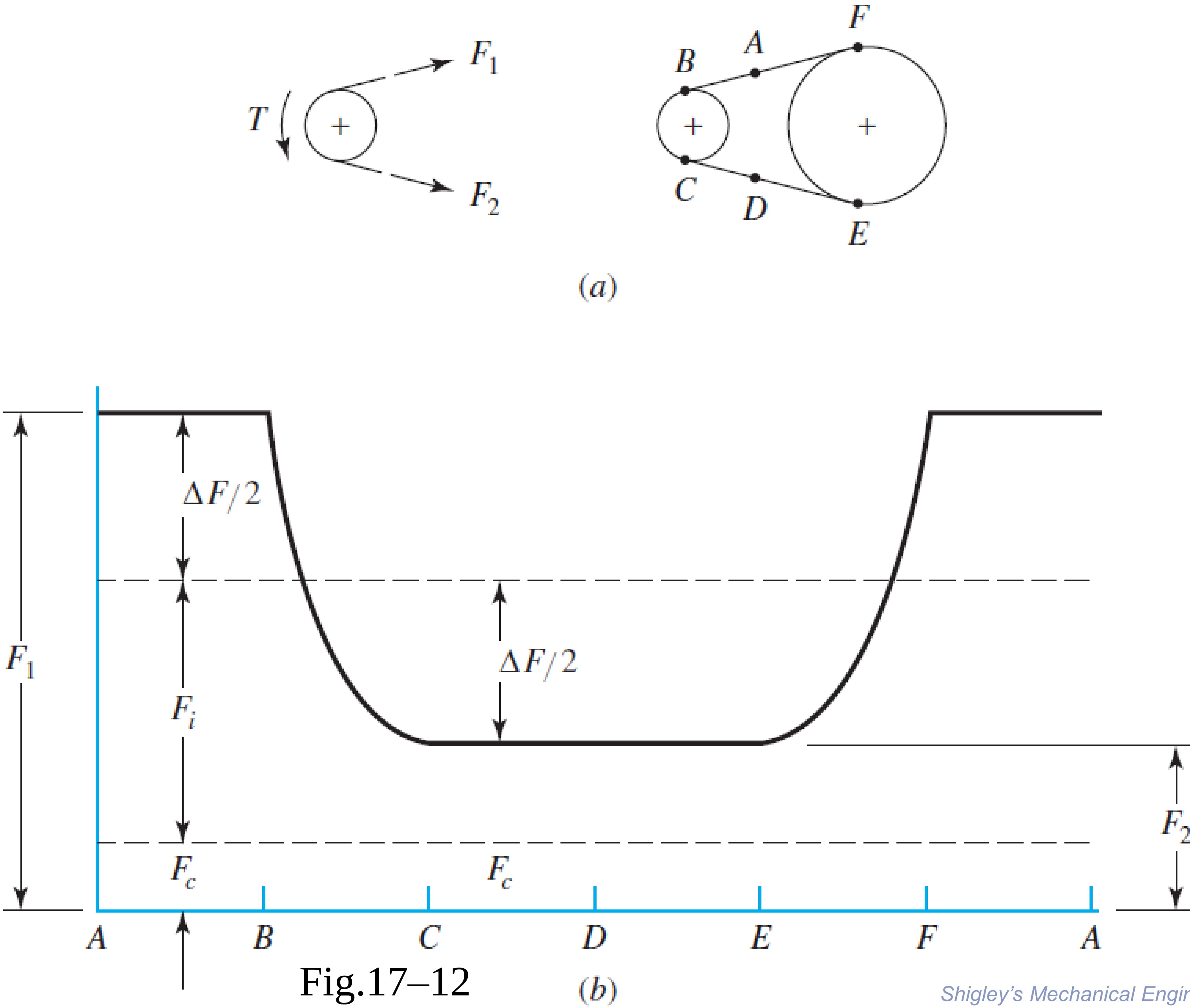



Fig.17–12

# Flat Metal Belts

- Thin metal belts exhibit
  - High strength-to-weight ratio
  - Dimensional stability
  - Accurate timing
  - Usefulness to temperatures up to 700ºF
  - Good electrical and thermal conduction properties

# Tensions and Torques in Thin Flat Metal Belt

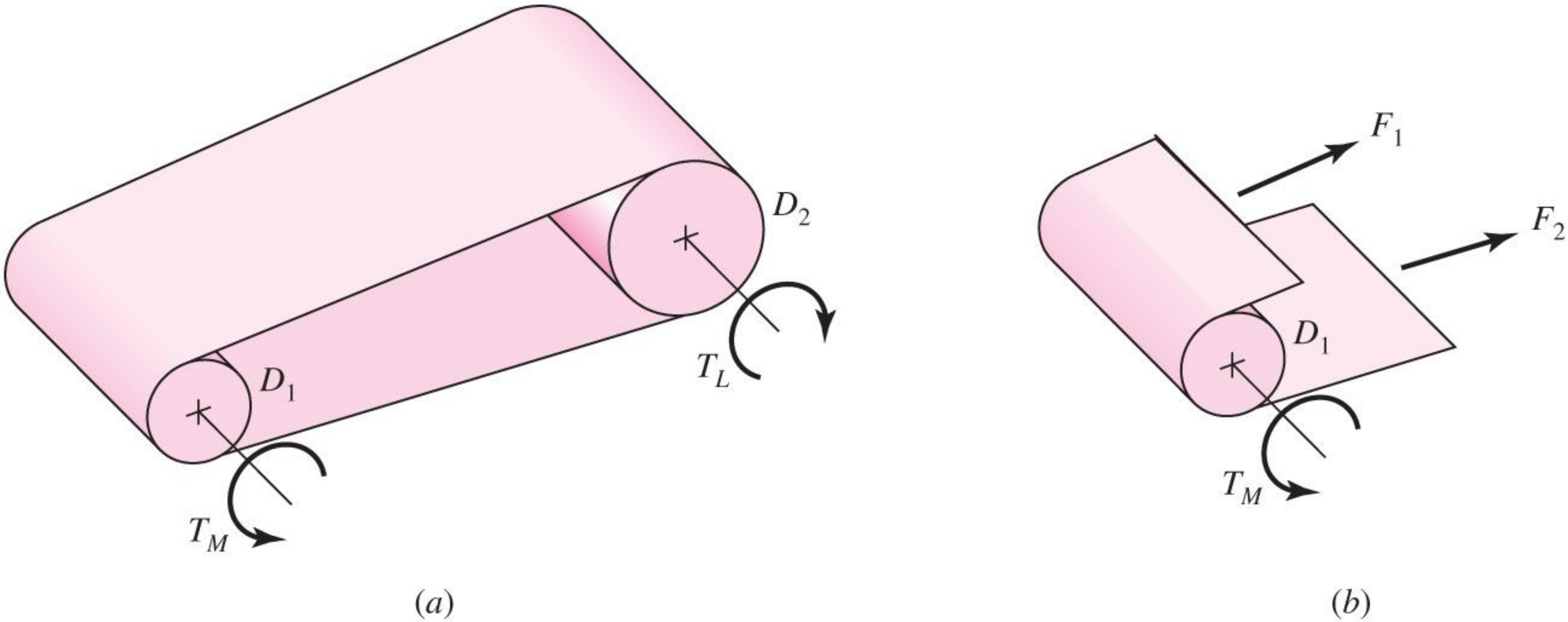



Fig.17–13

# Bending Stress in Flat Metal Belt

$$\sigma_b = \frac{Et}{(1-\nu^2)D} = \frac{E}{(1-\nu^2)(D/t)} \qquad (17\text{–}14)$$

where $E$ = Young's modulus

$t$ = belt thickness

$\nu$ = Poisson's ratio

$D$ = pulley diameter

# Tensile Stresses in Flat Metal Belt

$$(\sigma)_1 = F_1/(bt)$$

$$(\sigma)_2 = F_2/(bt)$$

Largest tensile stress during a belt pass:

$$(\sigma_b)_1 + F_1/(bt)$$

Smallest tensile stress during a belt pass:

$$(\sigma_b)_2 + F_2/(bt)$$

# Belt Life for Stainless Steel Friction Drives

| $\frac{D}{t}$ | Belt Passes |
|---|---|
| 625 | $\geq 10^6$ |
| 400 | $0.500 \cdot 10^6$ |
| 333 | $0.165 \cdot 10^6$ |
| 200 | $0.085 \cdot 10^6$ |

Table 17–6

# Regression Line for Stress and Passes

$$\sigma = 14\,169\,982 N_p^{-0.407} = 14.17(10^6) N_p^{-0.407} \qquad (17\text{–}15)$$

# Minimum Pulley Diameter

| Belt Thickness, mm | Minimum Pulley Diameter, mm |
|---|---|
| 0.05 | 30 |
| 0.08 | 45 |
| 0.13 | 75 |
| 0.20 | 125 |
| 0.25 | 150 |
| 0.38 | 255 |
| 0.50 | 315 |
| 1.00 | 635 |

*Data courtesy of Belt Technologies, Agawam, Mass.

Table 17–7

# Typical Material Properties for Metal Belts

| Alloy | Yield Strength, MPa | Young's Modulus, GPa | Poisson's Ratio |
|---|---|---|---|
| 301 or 302 stainless steel | 1206 | 193 | 0.285 |
| BeCu | 1170 | 117 | 0.220 |
| 1075 or 1095 carbon steel | 1585 | 207 | 0.287 |
| Titanium | 1034 | 103 | — |
| Inconel | 1103 | 207 | 0.284 |

*Data courtesy of Belt Technologies, Agawam, Mass.

Table 17–8

# Steps for Selection of Metal Flat Belt

1. Find $\exp(f\phi)$ from geometry and friction
2. Find endurance strength

$$S_f = 14.17(10^6)N_p^{-0.407} \qquad \text{301, 302 stainless}$$

$$S_f = S_y/3 \qquad \text{others}$$

3. Allowable tension

$$(F_1)_a = \left[S_f - \frac{Et}{(1-\nu^2)D}\right]tb = ab$$

4. $\Delta F = 2T/D$
5. $F_2 = (F_1)_a - \Delta F = ab - \Delta F$
6. $F_i = \dfrac{(F_1)_a + F_2}{2} = \dfrac{ab + ab - \Delta F}{2} = ab - \dfrac{\Delta F}{2}$

# Steps for Selection of Metal Flat Belt

**7** $$b_{\min} = \frac{\Delta F}{a} \frac{\exp(f\phi)}{\exp(f\phi) - 1}$$

**8** Choose $b > b_{\min}$, $(F_1)_a = ab$, $F_2 = ab - \Delta F$,

$$F_i = ab - \Delta F/2, \; T = \Delta F D/2$$

**9** Check frictional development $f'$:

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a}{F_2} \qquad f' < f$$

# Example 17–3

A friction-drive stainless steel metal belt runs over two 100-mm metal pulleys ($f = 0.35$). The belt thickness is to be 0.08 mm. For a life exceeding $10^6$ belt passes with smooth torque ($K_s = 1$), (*a*) select the belt if the torque is to be 3.5 N · m, and (*b*) find the initial tension $F_i$.

## Solution

(*a*) From step 1, $\phi = \theta_d = \pi$, therefore $\exp(0.35\pi) = 3.00$. From step 2,

$$(S_f)_{10^6} = 97\,702(10^6)^{-0.407} = 353 \text{ MPa}$$

From steps 3, 4, 5, and 6,

$$F_{1a} = \left[353(10^6) - \frac{193(10^9)0.08(10^{-3})}{(1-0.285^2)0.1}\right]0.08(10^{-3})b = 14\,796b \text{ N} \tag{1}$$

$$\Delta F = 2T/D = 2(3.5)/0.1 = 70 \text{ N}\cdot\text{m}$$

$$F_2 = F_{1a} - \Delta F = 14\,796b - 70 \text{ N} \tag{2}$$

$$F_i = \frac{F_{1a} + F_2}{2} = \frac{14\,796b + 70}{2} \text{ N} \tag{3}$$

From step 7,

# Example 17–3

$$b_{\min} = \frac{\Delta F}{a}\frac{\exp(f\phi)}{\exp(f\phi)-1} = \frac{70}{14\,796}\frac{3.00}{3.00-1} = 0.0071 \text{ m} = 7.1 \text{ mm}$$

Select an available 19-mm-wide belt 0.08 mm thick.

Eq. (1): $F_{1a} = 14\,796(0.019) = 281 \text{ N}$

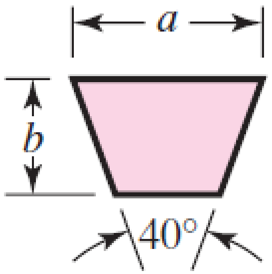
Eq. (2): $F_2 = 281 - 70 = 211 \text{ N}$

Eq. (3): $F_i = (281 + 211)/2 = 246 \text{ N}$

$$f' = \frac{1}{\phi}\ln\frac{F_1}{F_2} = \frac{1}{\pi}\ln\frac{281}{211} = 0.0912$$

Note $f' < f$, that is, $0.0882 < 0.35$.

# Standard V-Belt Sections




| Belt Section | Width *a*, mm | Thickness *b*, mm | Minimum Sheave Diameter, mm | kW Range, One or More Belts |
|---|---|---|---|---|
| A | 12 | 8.5 | 75 | 0.2–7.5 |
| B | 16 | 11 | 135 | 0.7–18.5 |
| C | 22 | 13 | 230 | 11–75 |
| D | 30 | 19 | 325 | 37–186 |
| E | 38 | 25 | 540 | 75 and up |

Table 17–9

# Inside Circumferences of Standard V-Belts

| Section | Circumference, mm |
|---|---|
| A | 650, 775, 825, 875, 950, 1050, 1150, 1200, 1275, 1325, 1375, 1425, 1500, 1550, 1600, 1650, 1700, 1775, 1875, 1950, 2000, 2125, 2250, 2400, 2625, 2800, 3000, 3200 |
| B | 875, 950, 1050, 1150, 1200, 1275, 1325, 1375, 1425, 1500, 1550, 1600, 1650, 1700, 1775, 1875, 1950, 2000, 2125, 2250, 2400, 2625, 2800, 3000, 3200, 3275, 3400, 3450, 3950, 4325, 4500, 4875, 5250, 6000, 6750, 7500 |
| C | 1275, 1500, 1700, 1875, 2025, 2125, 2250, 2400, 2625, 2800, 3000, 3200, 3400, 3600, 3950, 4050, 4350, 4500, 4875, 5250, 6000, 6750, 7500, 8250, 9000, 9750, 10 500 |
| D | 3000, 3200, 3600, 3950, 4050, 4350, 4500, 4875, 5250, 6000, 6750, 7500, 8250, 9000, 9750, 10 500, 12 000, 13 500, 15 000, 16 500 |
| E | 4500, 4875, 5250, 6000, 6750, 7500, 8250, 9000, 9750, 10 500, 12 000, 13 500, 15 000, 16 500 |

Table 17–10

# Length Conversion Dimensions

## Table 17–11

Length Conversion Dimensions (Add the listed quantity to the inside circumference to obtain the pitch length in mm).

| Belt section | A | B | C | D | E |
|---|---|---|---|---|---|
| Quantity to be added | 32 | 45 | 72 | 82 | 112 |

# V-Belt Pitch Length and Center-to-Center Distance

$$L_p = 2C + \pi(D + d)/2 + (D - d)^2/(4C) \qquad (17\text{–}16a)$$

$$C = 0.25\left\{\left[L_p - \frac{\pi}{2}(D + d)\right] + \sqrt{\left[L_p - \frac{\pi}{2}(D + d)\right]^2 - 2(D - d)^2}\right\} \qquad (17\text{–}16b)$$

# Horsepower Ratings of Standard V-Belts

| Belt Section | Sheave Pitch Diameter, mm | Belt Speed, m/s | | | | |
|---|---|---|---|---|---|---|
| | | 5 | 10 | 15 | 20 | 25 |
| A | 65 | 0.35 | 0.46 | 0.40 | 0.11 | |
| | 75 | 0.49 | 0.75 | 0.84 | 0.69 | 0.28 |
| | 85 | 0.60 | 0.98 | 1.17 | 1.64 | 0.84 |
| | 95 | 0.69 | 1.16 | 1.43 | 1.49 | 1.28 |
| | 105 | 0.77 | 1.30 | 1.64 | 1.78 | 1.63 |
| | 115 | 0.83 | 1.41 | 1.82 | 2.01 | 1.93 |
| | 125 and up | 0.87 | 1.51 | 1.97 | 2.21 | 2.16 |
| B | 105 | 0.80 | 1.18 | 1.25 | 0.94 | 0.16 |
| | 115 | 0.95 | 1.48 | 1.71 | 1.55 | 0.92 |
| | 125 | 1.07 | 1.74 | 2.09 | 2.06 | 1.57 |
| | 135 | 1.19 | 1.95 | 2.42 | 2.49 | 2.10 |
| | 145 | 1.28 | 2.14 | 2.69 | 2.87 | 2.57 |
| | 155 | 1.36 | 2.31 | 2.94 | 3.19 | 2.98 |
| | 165 | 1.43 | 2.45 | 3.16 | 3.48 | 3.34 |
| | 175 and up | 1.50 | 2.58 | 3.35 | 3.74 | 3.66 |
| C | 150 | 1.37 | 1.98 | 2.03 | 1.40 | |
| | 175 | 1.85 | 2.94 | 3.46 | 3.31 | 2.33 |
| | 200 | 2.21 | 3.66 | 4.54 | 4.74 | 4.12 |
| | 225 | 2.49 | 4.21 | 5.38 | 5.86 | 5.51 |
| | 250 | 2.72 | 4.66 | 6.05 | 7.16 | 6.63 |
| | 275 | 2.89 | 5.03 | 6.59 | 7.46 | 7.53 |
| | 300 and up | 3.05 | 5.33 | 7.06 | 8.13 | 8.28 |
| D | 250 | 3.09 | 4.57 | 4.89 | 3.80 | 1.01 |
| | 275 | 3.73 | 5.84 | 6.80 | 6.34 | 4.19 |
| | 300 | 4.26 | 6.91 | 8.36 | 8.50 | 6.85 |
| | 325 | 4.71 | 7.83 | 9.70 | 10.30 | 9.10 |
| | 350 | 5.09 | 8.58 | 10.89 | 11.79 | 11.04 |
| | 375 | 5.42 | 9.25 | 11.86 | 13.13 | 12.68 |
| | 400 | 5.71 | 9.85 | 12.76 | 14.32 | 14.17 |
| | 425 and up | 5.98 | 10.37 | 13.50 | 15.37 | 15.44 |
| E | 400 | 6.48 | 10.44 | 13.06 | 13.50 | 11.41 |
| | 450 | 7.40 | 12.46 | 15.82 | 17.16 | 16.04 |
| | 500 | 8.13 | 13.95 | 18.05 | 20.07 | 19.69 |
| | 550 | 8.73 | 15.14 | 19.84 | 22.53 | 22.75 |
| | 600 | 9.25 | 16.11 | 21.34 | 24.54 | 25.22 |
| | 650 | 9.70 | 17.01 | 22.60 | 26.19 | 27.38 |
| | 700 and up | 10.00 | 17.68 | 23.72 | 27.68 | 29.17 |

Table 17–12

# Adjusted Power

$$H_a = K_1 K_2 H_{tab} \qquad (17\text{–}17)$$

where $H_a$ = allowable power, per belt

$K_1$ = angle-of-wrap ($\phi$) correction factor, Table 17–13

$K_2$ = belt length correction factor, Table 17–14

# Angle of Wrap Correction Factor

| $\frac{D-d}{C}$ | $\theta$, deg | $K_1$ VV | $K_1$ V Flat |
|---|---|---|---|
| 0.00 | 180 | 1.00 | 0.75 |
| 0.10 | 174.3 | 0.99 | 0.76 |
| 0.20 | 166.5 | 0.97 | 0.78 |
| 0.30 | 162.7 | 0.96 | 0.79 |
| 0.40 | 156.9 | 0.94 | 0.80 |
| 0.50 | 151.0 | 0.93 | 0.81 |
| 0.60 | 145.1 | 0.91 | 0.83 |
| 0.70 | 139.0 | 0.89 | 0.84 |
| 0.80 | 132.8 | 0.87 | 0.85 |
| 0.90 | 126.5 | 0.85 | 0.85 |
| 1.00 | 120.0 | 0.82 | 0.82 |
| 1.10 | 113.3 | 0.80 | 0.80 |
| 1.20 | 106.3 | 0.77 | 0.77 |
| 1.30 | 98.9 | 0.73 | 0.73 |
| 1.40 | 91.1 | 0.70 | 0.70 |
| 1.50 | 82.8 | 0.65 | 0.65 |

Table 17–13

# Belt-Length Correction Factor

| Length Factor | Nominal Belt Length, m<br>A Belts | B Belts | C Belts | D Belts | E Belts |
|---|---|---|---|---|---|
| 0.85 | Up to 0.88 | Up to 1.15 | Up to 1.88 | Up to 3.2 | |
| 0.90 | 0.95–1.15 | 1.2–1.5 | 2.03–2.4 | 3.6–4.05 | Up to 4.88 |
| 0.95 | 1.2–1.38 | 1.55–1.88 | 2.63–3.0 | 4.33–5.25 | 5.25–6.0 |
| 1.00 | 1.5–1.88 | 1.95–2.43 | 3.2–3.95 | 6.0 | 6.75–7.5 |
| 1.05 | 1.95–2.25 | 2.63–3.0 | 4.05–4.88 | 6.75–8.25 | 8.25–9.75 |
| 1.10 | 2.4–2.8 | 3.2–3.6 | 5.25–6.0 | 9.0–10.5 | 10.5–12.0 |
| 1.15 | 3.0 and up | 3.95–4.5 | 6.75–7.5 | 12.0 | 13.5–15.0 |
| 1.20 | | 4.88 and up | 8.25 and up | 13.5 and up | 16.5 |

*Multiply the rated power per belt by this factor to obtain the corrected power.

Table 17–14

# Belting Equation for V-Belt

$$\frac{F_1 - F_c}{F_2 - F_c} = \exp(0.5123\phi) \qquad (17\text{–}18)$$

# Design Power for V-Belt

$$H_d = H_{\text{nom}} K_s n_d \qquad (17\text{–}19)$$

where $H_{\text{nom}}$ is the nominal power

$K_s$ is the service factor given in Table 17–15

$n_d$ is the design factor

Number of belts:

$$N_b \geq \frac{H_d}{H_a} \qquad N_b = 1, 2, 3, \ldots \qquad (17\text{–}20)$$

# Suggested Service Factors for V-Belt Drives

| Driven Machinery | Source of Power | |
|---|---|---|
| | Normal Torque Characteristic | High or Nonuniform Torque |
| Uniform | 1.0 to 1.2 | 1.1 to 1.3 |
| Light shock | 1.1 to 1.3 | 1.2 to 1.4 |
| Medium shock | 1.2 to 1.4 | 1.4 to 1.6 |
| Heavy shock | 1.3 to 1.5 | 1.5 to 1.8 |

Table 17–15

# V-Belt Tensions

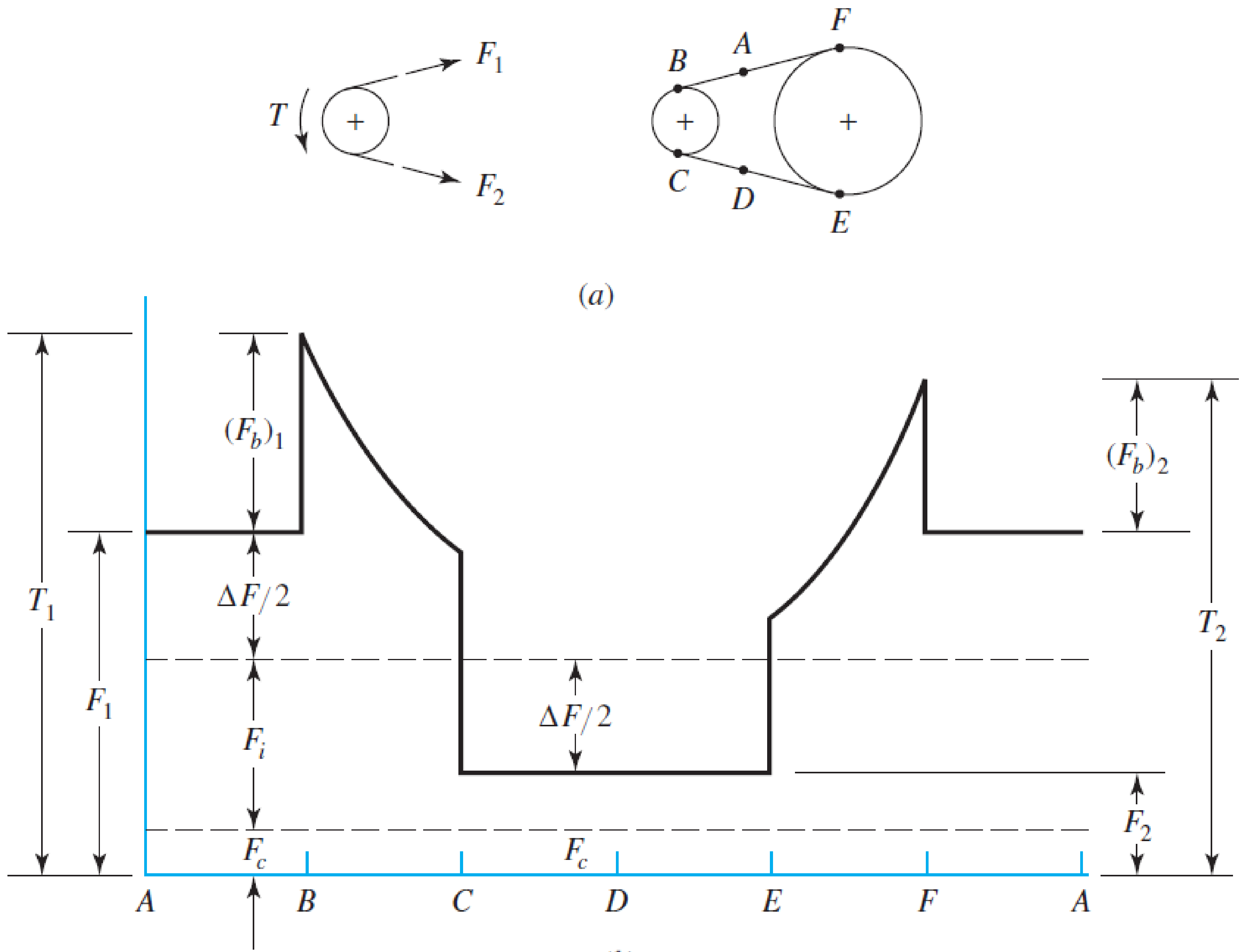



Fig.17–14 (b)

# V-Belt Tensions

$$F_c = K_c \left( \frac{V}{1000} \right)^2 \qquad (17\text{–}21)$$

where $K_c$ is from Table 17–16

$$\Delta F = \frac{63\,025 H_d / N_b}{n(d/2)} \qquad (17\text{–}22)$$

$$F_1 = F_c + \frac{\Delta F \exp(f\phi)}{\exp(f\phi) - 1} \qquad (17\text{–}23)$$

$$F_2 = F_1 - \Delta F \qquad (17\text{–}24)$$

$$F_i = \frac{F_1 + F_2}{2} - F_c \qquad (17\text{–}25)$$

# Some V-Belt Parameters

| Belt Section | $K_b$ | $K_c$ |
|---|---|---|
| A | 220 | 0.561 |
| B | 576 | 0.965 |
| C | 1 600 | 1.716 |
| D | 5 680 | 3.498 |
| E | 10 850 | 5.041 |
| 3V | 230 | 0.425 |
| 5V | 1098 | 1.217 |
| 8V | 4830 | 3.288 |

Table 17–16

# V-Belt Factor of Safety

$$n_{fs} = \frac{H_a N_b}{H_{\text{nom}} K_s} \qquad (17\text{–}26)$$

# V-Belt Tension vs. Passes

$$T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d}$$

$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D}$$

where $K_b$ is given in Table 17–16

$$T^b N_P = K^b$$

$$N_P = \left[ \left( \frac{K}{T_1} \right)^{-b} + \left( \frac{K}{T_2} \right)^{-b} \right]^{-1} \qquad (17\text{–}27)$$

$$t = \frac{N_P L_p}{720V} \qquad (17\text{–}28)$$

# Durability Parameters for Some V-Belt Sections

| Belt Section | $10^8$ to $10^9$ Force Peaks K | $10^8$ to $10^9$ Force Peaks b | $10^9$ to $10^{10}$ Force Peaks K | $10^9$ to $10^{10}$ Force Peaks b | Minimum Sheave Diameter, mm |
|---|---|---|---|---|---|
| A | 2999 | 11.089 | | | 75 |
| B | 5309 | 10.926 | | | 125 |
| C | 9069 | 11.173 | | | 215 |
| D | 18 726 | 11.105 | | | 325 |
| E | 26 791 | 11.100 | | | 540 |
| 3V | 3240 | 12.464 | 4726 | 10.153 | 66 |
| 5V | 7360 | 12.593 | 10 653 | 10.283 | 177 |
| 8V | 16 189 | 12.629 | 23 376 | 10.319 | 312 |

Table 17–17

# Example 17–4

A 7.46-kW split-phase motor running at 1750 rev/min is used to drive a rotary pump, which operates 24 hours per day. An engineer has specified a 188-mm small sheave, a 280-mm large sheave, and three B2800 belts. The service factor of 1.2 was augmented by 0.1 because of the continuous-duty requirement. Analyze the drive and estimate the belt life in passes and hours.

## Solution

The peripheral speed $V$ of the belt is

$$V = \pi\, dn = \pi(0.188)1750/60 = 17 \text{ m/s}$$

Table 17–11: $L_p = L + L_c = 2800 + 45 = 2845$ mm

Eq. (17–16$b$): $$C = 0.25\left\{\left[2845 - \frac{\pi}{2}(280 + 188)\right] + \sqrt{\left[2845 - \frac{\pi}{2}(280 + 188)\right]^2 - 2(280 - 188)^2}\right\}$$

$$= 1054 \text{ mm}$$

# Example 17–4

Eq. (17–1): $\phi = \theta_d = \pi - 2\sin^{-1}(280 - 188)/[2(1054)] = 3.054$ rad

$$\exp[0.5123(3.054)] = 4.781$$

Interpolating in Table 17–12 for $V = 17$ m/s gives $H_{tab} = 3.5$ kW. The wrap angle in degrees is $3.054(180)/\pi = 175°$. From Table 17–13, $K_1 = 0.99$. From Table 17–14, $K_2 = 1.05$. Thus, from Eq. (17–17),

$$H_a = K_1 K_2 H_{tab} = 0.99(1.05)3.5 = 3.64 \text{ kW}$$

Eq. (17–19): $H_d = H_{nom} K_s n_d = 7.46(1.2 + 0.1)(1) = 9.7$ kW

Eq. (17–20): $N_b \geq H_d/H_a = 9.7/3.64 = 2.67 \rightarrow 3$

From Table 17–16, $K_c = 0.965$. Thus, from Eq. (17–21),

$$F_c = 0.965(17/2.4)^2 = 48.4 \text{ N}$$

Eq.(17–22): $$\Delta F = \frac{9700/3}{\pi(1750/60)0.188} = 188 \text{ N}$$

Eq. (17–23): $$F_1 = 48.4 + \frac{188(4.781)}{4.781 - 1} = 286 \text{ N}$$

# Example 17–4

Eq. (17–24): $F_2 = F_1 - \Delta F = 286 - 188 = 98\text{ N}$

Eq. (17–25): $F_i = \dfrac{286 + 98}{2} - 48.4 = 144\text{ N}$

Eq. (17–26): $n_{fs} = \dfrac{H_a N_b}{H_{\text{nom}} K_s} = \dfrac{3.64(3)}{7.46(1.3)} = 1.13$

*Life:* From Table 17–16, $K_b = 576$.

$$F_{b1} = \frac{K_b}{d} = \frac{65}{0.188} = 346\text{ N}$$

$$F_{b2} = 65/0.28 = 232\text{ N}$$

# Example 17–4

$$T_1 = F_1 + F_{b1} = 286 + 346 = 632\ \text{N}$$

$$T_2 = F_1 + F_{b2} = 286 + 232 = 518\ \text{N}$$

From Table 17–17, $K = 5309$ and $b = 10.926$.

Eq. (17–27): $$N_P = \left[\left(\frac{5309}{632}\right)^{-10.926} + \left(\frac{5309}{518}\right)^{-10.926}\right]^{-1} = 11(10^9)\ \text{passes}$$

Answer

Since $N_P$ is out of the validity range of Eq. (17–27), life is reported as greater than $10^9$ passes. Then

Eq. (17–28): $$t > \frac{10^9(2.845)}{3600(17)} = 46\,487\ \text{h}$$ Answer

# Timing Belts

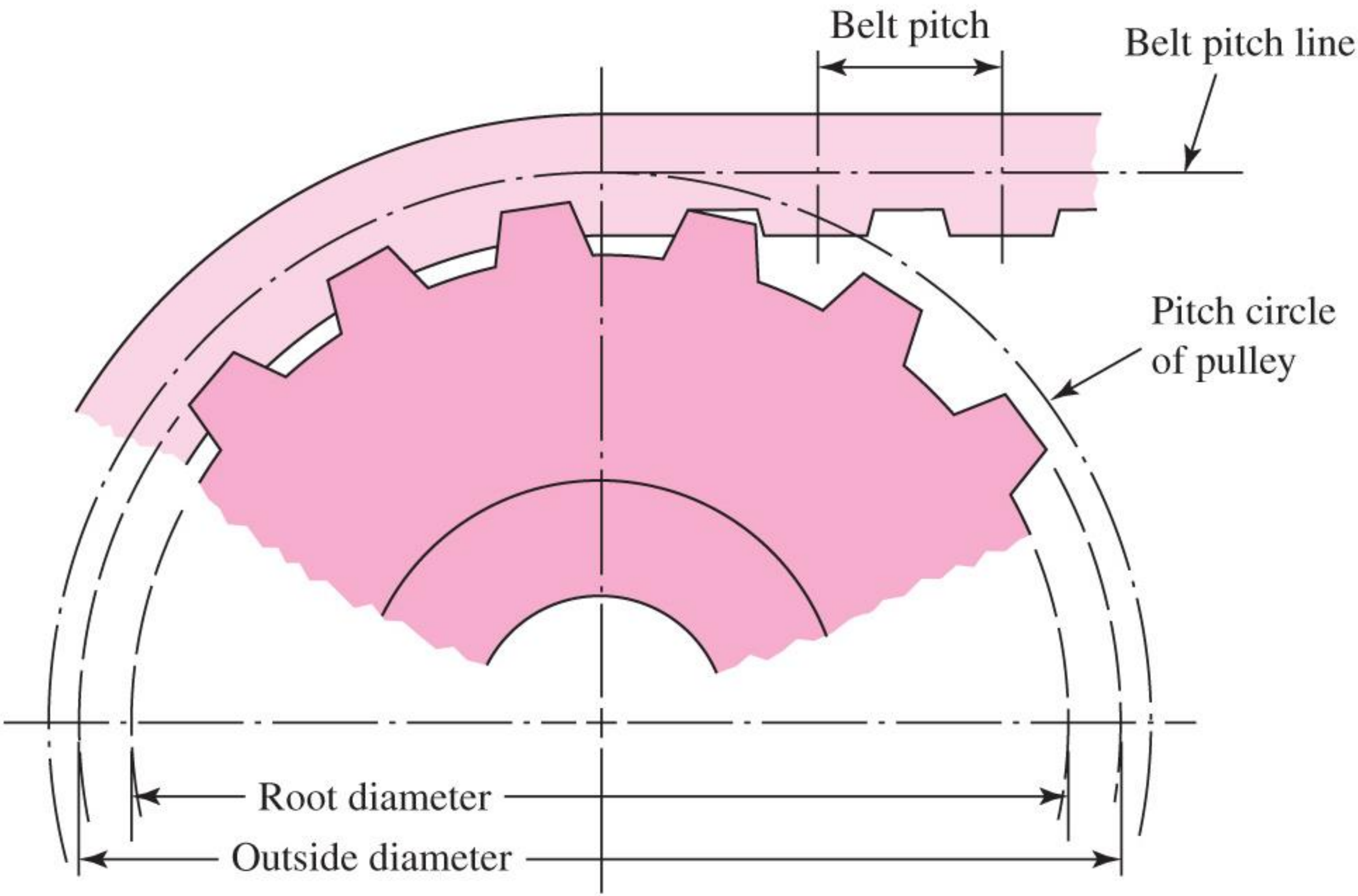



Fig.17–15

# Standard Pitches of Timing Belts

| Service | Designation | Pitch $p$, mm |
|---|---|---|
| Extra light | XL | 5 |
| Light | L | 10 |
| Heavy | H | 12 |
| Extra heavy | XH | 22 |
| Double extra heavy | XXH | 30 |

Table 17–18

# Roller Chain

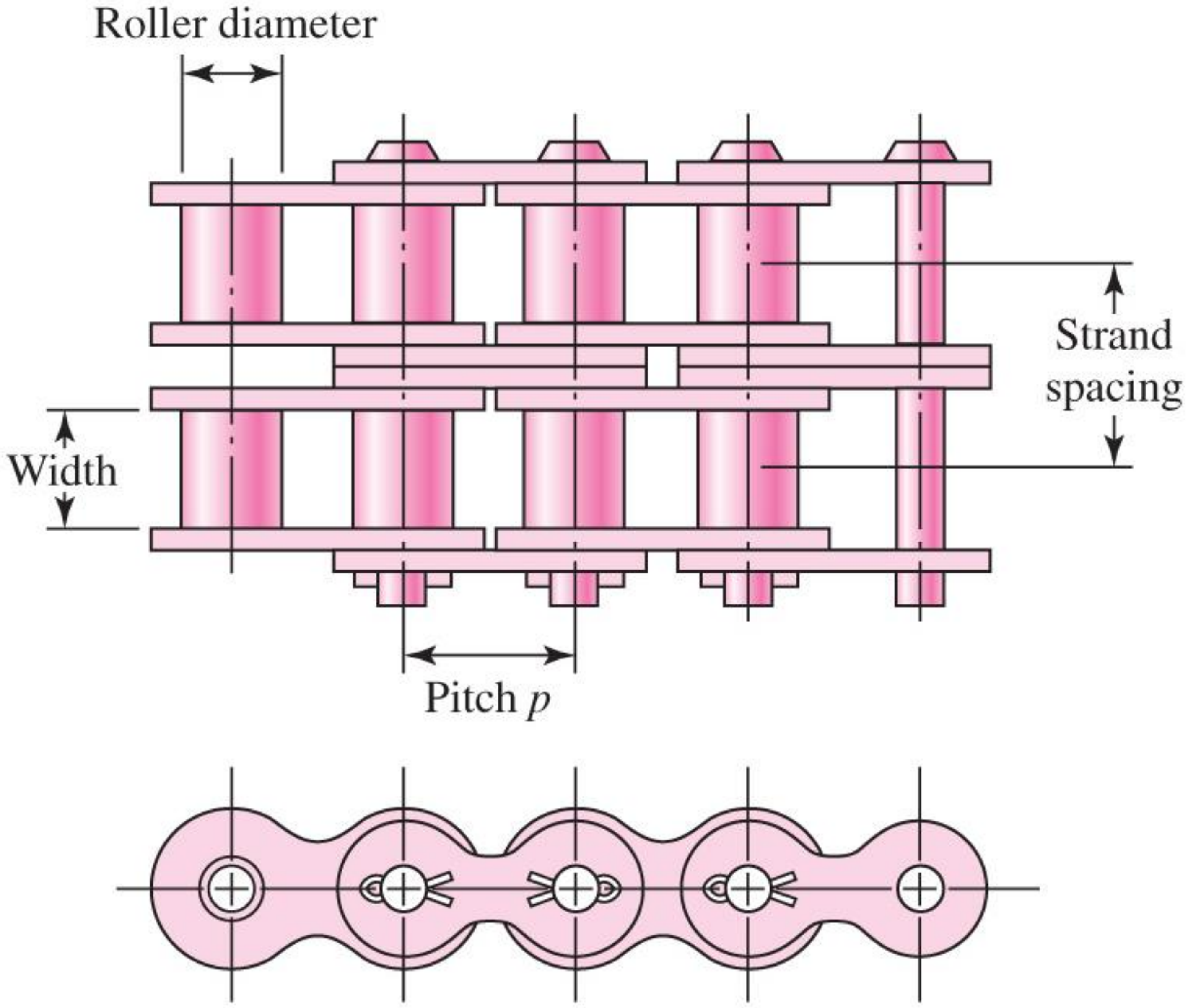



Fig.17–16

# Dimensions of American Standard Roller Chains

| ANSI Chain Number | Pitch, in (mm) | Width, in (mm) | Minimum Tensile Strength, lbf (N) | Average Weight, lbf/ft (N/m) | Roller Diameter, in (mm) | Multiple-Strand Spacing, in (mm) |
|---|---|---|---|---|---|---|
| 25 | 0.250 (6.35) | 0.125 (3.18) | 780 (3 470) | 0.09 (1.31) | 0.130 (3.30) | 0.252 (6.40) |
| 35 | 0.375 (9.52) | 0.188 (4.76) | 1 760 (7 830) | 0.21 (3.06) | 0.200 (5.08) | 0.399 (10.13) |
| 41 | 0.500 (12.70) | 0.25 (6.35) | 1 500 (6 670) | 0.25 (3.65) | 0.306 (7.77) | — — |
| 40 | 0.500 (12.70) | 0.312 (7.94) | 3 130 (13 920) | 0.42 (6.13) | 0.312 (7.92) | 0.566 (14.38) |
| 50 | 0.625 (15.88) | 0.375 (9.52) | 4 880 (21 700) | 0.69 (10.1) | 0.400 (10.16) | 0.713 (18.11) |
| 60 | 0.750 (19.05) | 0.500 (12.7) | 7 030 (31 300) | 1.00 (14.6) | 0.469 (11.91) | 0.897 (22.78) |
| 80 | 1.000 (25.40) | 0.625 (15.88) | 12 500 (55 600) | 1.71 (25.0) | 0.625 (15.87) | 1.153 (29.29) |
| 100 | 1.250 (31.75) | 0.750 (19.05) | 19 500 (86 700) | 2.58 (37.7) | 0.750 (19.05) | 1.409 (35.76) |
| 120 | 1.500 (38.10) | 1.000 (25.40) | 28 000 (124 500) | 3.87 (56.5) | 0.875 (22.22) | 1.789 (45.44) |
| 140 | 1.750 (44.45) | 1.000 (25.40) | 38 000 (169 000) | 4.95 (72.2) | 1.000 (25.40) | 1.924 (48.87) |
| 160 | 2.000 (50.80) | 1.250 (31.75) | 50 000 (222 000) | 6.61 (96.5) | 1.125 (28.57) | 2.305 (58.55) |
| 180 | 2.250 (57.15) | 1.406 (35.71) | 63 000 (280 000) | 9.06 (132.2) | 1.406 (35.71) | 2.592 (65.84) |
| 200 | 2.500 (63.50) | 1.500 (38.10) | 78 000 (347 000) | 10.96 (159.9) | 1.562 (39.67) | 2.817 (71.55) |
| 240 | 3.00 (76.70) | 1.875 (47.63) | 112 000 (498 000) | 16.4 (239) | 1.875 (47.62) | 3.458 (87.83) |

Table 17–19

# Engagement of Chain and Sprocket

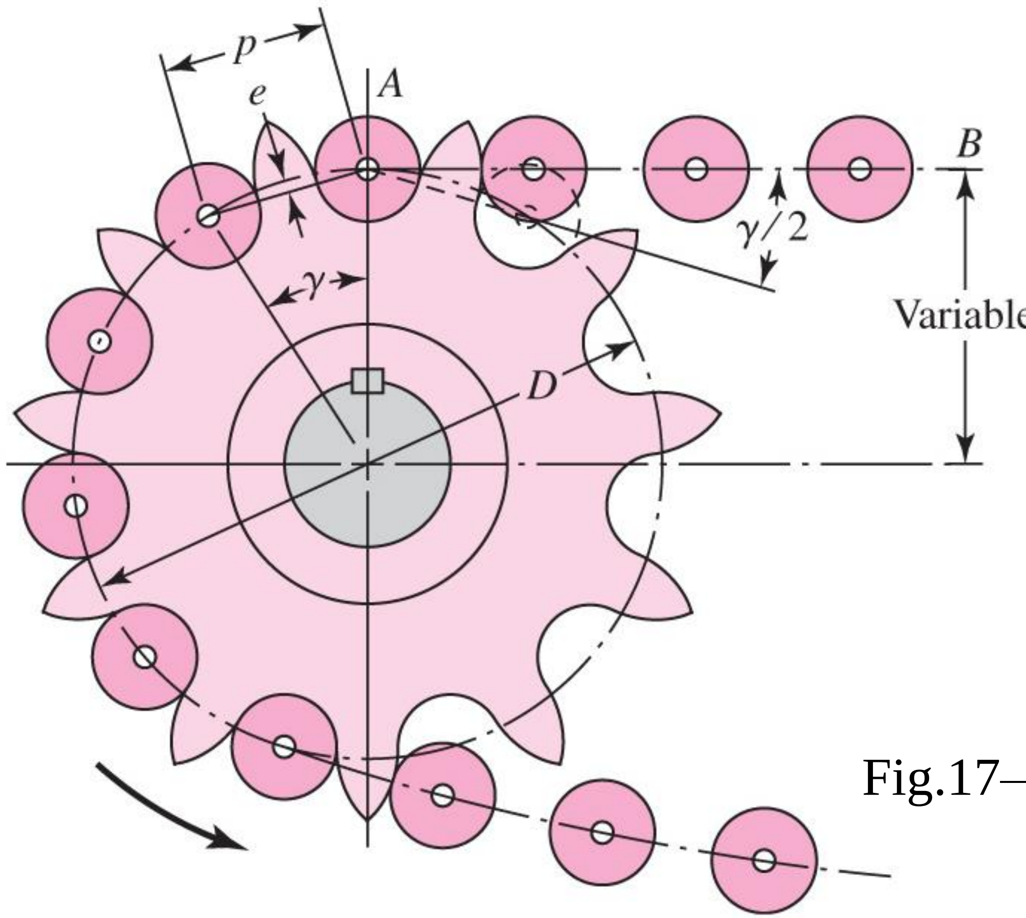



Fig.17–17

$$\sin\frac{\gamma}{2} = \frac{p/2}{D/2} \qquad \text{or} \qquad D = \frac{p}{\sin(\gamma/2)} \qquad (a)$$

$$D = \frac{p}{\sin(180°/N)} \qquad (17\text{–}29)$$

# Chain Velocity

$$V = \frac{Npn}{12} \quad \text{feet per minute} \qquad (17\text{–}30)$$

where $N$ = number of sprocket teeth

$p$ = chain pitch, in

$n$ = sprocket speed, rev/min

$$v_{\max} = \frac{\pi D n}{12} = \frac{\pi n p}{12 \sin(\gamma/2)} \qquad (b)$$

$$d = D \cos\frac{\gamma}{2} \qquad (c)$$

$$v_{\min} = \frac{\pi d n}{12} = \frac{\pi n p}{12} \frac{\cos(\gamma/2)}{\sin(\gamma/2)} \qquad (d)$$

# Chordal Speed Variation

$$\frac{\Delta V}{V} = \frac{v_{max} - v_{min}}{V} = \frac{\pi}{N}\left[\frac{1}{\sin(180^\circ/N)} - \frac{1}{\tan(180^\circ/N)}\right] \qquad (17\text{–}31)$$

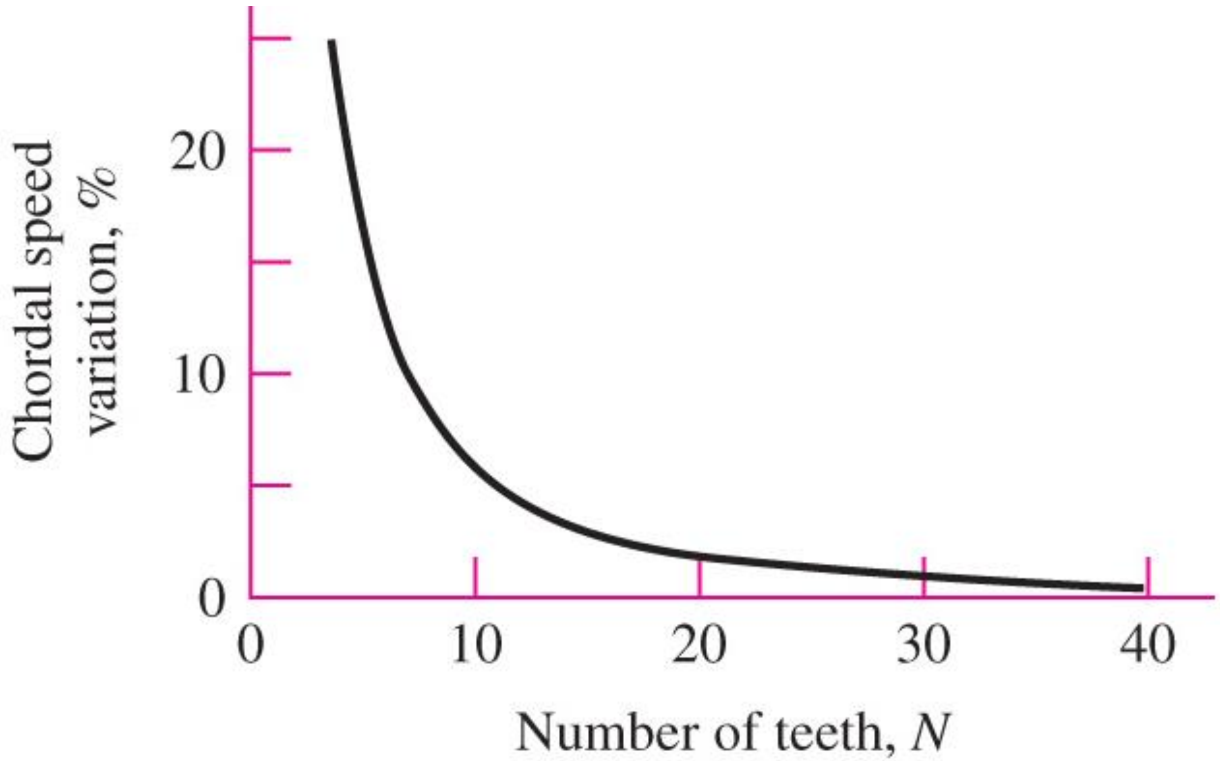



Fig.17–18

# Roller Chain Rated Horsepower Capacity

**Table 17–20**

Rated Horsepower Capacity of Single-Strand Single-Pitch Roller Chain for a 17-Tooth Sprocket

*Source:* Compiled from ANSI B29.1-1975 information only section, and from B29.9-1958.

| Sprocket Speed, rev/min | ANSI Chain Number 25 | 35 | 40 | 41 | 50 | 60 |
|---|---|---|---|---|---|---|
| 50 | 0.05 | 0.16 | 0.37 | 0.20 | 0.72 | 1.24 |
| 100 | 0.09 | 0.29 | 0.69 | 0.38 | 1.34 | 2.31 |
| 150 | 0.13* | 0.41* | 0.99* | 0.55* | 1.92* | 3.32 |
| 200 | 0.16* | 0.54* | 1.29 | 0.71 | 2.50 | 4.30 |
| 300 | 0.23 | 0.78 | 1.85 | 1.02 | 3.61 | 6.20 |
| 400 | 0.30* | 1.01* | 2.40 | 1.32 | 4.67 | 8.03 |
| 500 | 0.37 | 1.24 | 2.93 | 1.61 | 5.71 | 9.81 |
| 600 | 0.44* | 1.46* | 3.45* | 1.90* | 6.72* | 11.6 |
| 700 | 0.50 | 1.68 | 3.97 | 2.18 | 7.73 | 13.3 |
| 800 | 0.56* | 1.89* | 4.48* | 2.46* | 8.71* | 15.0 |
| 900 | 0.62 | 2.10 | 4.98 | 2.74 | 9.69 | 16.7 |
| 1000 | 0.68* | 2.31* | 5.48 | 3.01 | 10.7 | 18.3 |
| 1200 | 0.81 | 2.73 | 6.45 | 3.29 | 12.6 | 21.6 |
| 1400 | 0.93* | 3.13* | 7.41 | 2.61 | 14.4 | 18.1 |
| 1600 | 1.05* | 3.53* | 8.36 | 2.14 | 12.8 | 14.8 |
| 1800 | 1.16 | 3.93 | 8.96 | 1.79 | 10.7 | 12.4 |
| 2000 | 1.27* | 4.32* | 7.72* | 1.52* | 9.23* | 10.6 |
| 2500 | 1.56 | 5.28 | 5.51* | 1.10* | 6.58* | 7.57 |
| 3000 | 1.84 | 5.64 | 4.17 | 0.83 | 4.98 | 5.76 |

Type A    Type B    Type C

# Roller Chain Rated Horsepower Capacity

**Table 17–20**

Rated Horsepower Capacity of Single-Strand Single-Pitch Roller Chain for a 17-Tooth Sprocket (*Continued*)

| Sprocket Speed, rev/min | | ANSI Chain Number 80 | 100 | 120 | 140 | 160 | 180 | 200 | 240 |
|---|---|---|---|---|---|---|---|---|---|
| 50 | Type A | 2.88 | 5.52 | 9.33 | 14.4 | 20.9 | 28.9 | 38.4 | 61.8 |
| 100 | | 5.38 | 10.3 | 17.4 | 26.9 | 39.1 | 54.0 | 71.6 | 115 |
| 150 | | 7.75 | 14.8 | 25.1 | 38.8 | 56.3 | 77.7 | 103 | 166 |
| 200 | | 10.0 | 19.2 | 32.5 | 50.3 | 72.9 | 101 | 134 | 215 |
| 300 | | 14.5 | 27.7 | 46.8 | 72.4 | 105 | 145 | 193 | 310 |
| 400 | | 18.7 | 35.9 | 60.6 | 93.8 | 136 | 188 | 249 | 359 |
| 500 | Type B | 22.9 | 43.9 | 74.1 | 115 | 166 | 204 | 222 | 0 |
| 600 | | 27.0 | 51.7 | 87.3 | 127 | 141 | 155 | 169 | |
| 700 | | 31.0 | 59.4 | 89.0 | 101 | 112 | 123 | 0 | |
| 800 | | 35.0 | 63.0 | 72.8 | 82.4 | 91.7 | 101 | | |
| 900 | | 39.9 | 52.8 | 61.0 | 69.1 | 76.8 | 84.4 | | |
| 1000 | | 37.7 | 45.0 | 52.1 | 59.0 | 65.6 | 72.1 | | |
| 1200 | | 28.7 | 34.3 | 39.6 | 44.9 | 49.9 | 0 | | |
| 1400 | | 22.7 | 27.2 | 31.5 | 35.6 | 0 | | | |
| 1600 | | 18.6 | 22.3 | 25.8 | 0 | | | | |
| 1800 | | 15.6 | 18.7 | 21.6 | | | | | |
| 2000 | | 13.3 | 15.9 | 0 | | | | | |
| 2500 | | 9.56 | 0.40 | | | | | | |
| 3000 | | 7.25 | 0 | | | | | | |

Type C | Type C′

# Available Sprocket Tooth Counts

**Table 17–21**

Single-Strand Sprocket Tooth Counts Available from One Supplier*

| No. | Available Sprocket Tooth Counts |
|---|---|
| 25 | 8-30, 32, 34, 35, 36, 40, 42, 45, 48, 54, 60, 64, 65, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120 |
| 35 | 4-45, 48, 52, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120 |
| 41 | 6-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120 |
| 40 | 8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120 |
| 50 | 8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120 |
| 60 | 8-60, 62, 63, 64, 65, 66, 67, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120 |
| 80 | 8-60, 64, 65, 68, 70, 72, 76, 78, 80, 84, 90, 95, 96, 102, 112, 120 |
| 100 | 8-60, 64, 65, 67, 68, 70, 72, 74, 76, 80, 84, 90, 95, 96, 102, 112, 120 |
| 120 | 9-45, 46, 48, 50, 52, 54, 55, 57, 60, 64, 65, 67, 68, 70, 72, 76, 80, 84, 90, 96, 102, 112, 120 |
| 140 | 9-28, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 42, 43, 45, 48, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 96 |
| 160 | 8-30, 32–36, 38, 40, 45, 46, 50, 52, 53, 54, 56, 57, 60, 62, 63, 64, 65, 66, 68, 70, 72, 73, 80, 84, 96 |
| 180 | 13-25, 28, 35, 39, 40, 45, 54, 60 |
| 200 | 9-30, 32, 33, 35, 36, 39, 40, 42, 44, 45, 48, 50, 51, 54, 56, 58, 59, 60, 63, 64, 65, 68, 70, 72 |
| 240 | 9-30, 32, 35, 36, 40, 44, 45, 48, 52, 54, 60 |

# Tooth Correction Factors $K_1$

| Number of Teeth on Driving Sprocket | $K_1$ Pre-extreme Horsepower | $K_1$ Post-extreme Horsepower |
|---|---|---|
| 11 | 0.62 | 0.52 |
| 12 | 0.69 | 0.59 |
| 13 | 0.75 | 0.67 |
| 14 | 0.81 | 0.75 |
| 15 | 0.87 | 0.83 |
| 16 | 0.94 | 0.91 |
| 17 | 1.00 | 1.00 |
| 18 | 1.06 | 1.09 |
| 19 | 1.13 | 1.18 |
| 20 | 1.19 | 1.28 |
| $N$ | $(N_1/17)^{1.08}$ | $(N_1/17)^{1.5}$ |

Table 17–22

# Multiple-Strand Factors $K_2$

| Number of Strands | $K_2$ |
|---|---|
| 1 | 1.0 |
| 2 | 1.7 |
| 3 | 2.5 |
| 4 | 3.3 |
| 5 | 3.9 |
| 6 | 4.6 |
| 8 | 6.0 |

Table 17–23

# Nominal Power Ratings for Chain

- From American Chain Association publication *Chains for Power Transmission and Materials Handling*
- For single-strand chain
- Nominal power, link-plate limited

$$H_1 = 0.003 N_1^{1.08} n_1^{0.9} p^{(3-0.07p)} \qquad \text{kW} \qquad (17\text{–}32)$$

- Nominal power, roller-limited

$$H_2 = \frac{746 K_r N_1^{1.5} p^{0.8}}{n_1^{1.5}} \qquad \text{kW} \qquad (17\text{–}33)$$

where $N_1$ = number of teeth in the smaller sprocket

$n_1$ = sprocket speed, rev/min

$p$ = pitch of the chain, in

$K_r$ = 29 for chain numbers 25, 35; 3.4 for chain 41; and 17 for chains 40–240

# Chain Dimensions

- Chain length in pitches

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p} \qquad (17\text{–}34)$$

- Center-to-center distance

$$C = \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right] \qquad (17\text{–}35)$$

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p} \qquad (17\text{–}36)$$

# Chain Drive Power

- Allowable power

$$H_a = K_1 K_2 H_{\text{tab}} \qquad (17\text{–}37)$$

where $K_1$ = correction factor for tooth number other than 17 (Table 17–22)

$K_2$ = strand correction (Table 17–23)

- Power that must be transmitted

$$H_d = H_{\text{nom}} K_s n_d \qquad (17\text{–}38)$$

# Variations in Tabulated Power Conditions

- Power ratings in Table 17–20 are for chains of 100 pitch length and 17-tooth sprocket.
- For deviations from this,

$$H_2 = 1000\left[K_r\left(\frac{N_1}{n_1}\right)^{1.5} p^{0.8}\left(\frac{L_p}{100}\right)^{0.4}\left(\frac{15\,000}{h}\right)^{0.4}\right] \tag{17–39}$$

- From a deviation viewpoint,

$$\frac{H_2^{2.5}h}{N_1^{3.75}L_p} = \text{constant} \tag{17–40}$$

# Recommended Maximum Chain Speed

$$n_1 \leq 1000 \left[ \frac{82.5}{7.95^p (1.0278)^{N_1} (1.323)^{F/1000}} \right]^{1/(1.59 \log p + 1.873)} \quad \text{rev/min}$$

where $F$ is the chain tension in pounds

# Example 17–5

Select drive components for a 2:1 reduction, 68 kW input at 300 rev/min, moderate shock, an abnormally long 18-hour day, poor lubrication, cold temperatures, dirty surroundings, short drive $C/p = 25$.

Solution

*Function:* $H_{\text{nom}} = 6$ kW, $n_1 = 300$ rev/min, $C/p = 25$, $K_s = 1.3$
*Design factor:* $n_d = 1.5$

*Sprocket teeth:* $N_1 = 17$ teeth, $N_2 = 34$ teeth, $K_1 = 1$, $K_2 = 1, 1.7, 2.5, 3.3$
*Chain number of strands:*

$$H_{\text{tab}} = \frac{n_d K_s H_{\text{nom}}}{K_1 K_2} = \frac{1.5(1.3)68}{(1)K_2} = \frac{132.6}{K_2}$$

Form a table:

| Number of Strands | 132.6/K2 (Table 17–23) | Chain Number (Table 17–20) | Lubrication Type |
|---|---|---|---|
| 1 | $132.6/1 = 132.6$ | 200 | C′ |
| 2 | $132.6/1.7 = 78$ | 160 | C |
| 3 | $132/2.5 = 53.04$ | 140 | B |
| 4 | $132/3.3 = 40.18$ | 140 | B |

# Example 17–5

3 strands of number 140 chain ($H_{tab}$ is 54 kW).

*Number of pitches in the chain:*

$$\frac{L}{p} = \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p}$$

$$= 2(25) + \frac{17 + 34}{2} + \frac{(34 - 17)^2}{4\pi^2(25)} = 75.79 \text{ pitches}$$

Use 76 pitches. Then $L/p = 76$.

*Identify the center-to-center distance:* From Eqs. (17–35) and (17–36),

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p} = \frac{17 + 34}{2} - 76 = -50.5$$

$$C = \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right]$$

# Example 17–5

$$= \frac{p}{4}\left[50.5 + \sqrt{50.5^2 - 8\left(\frac{34-17}{2\pi}\right)^2}\right] = 25.104p$$

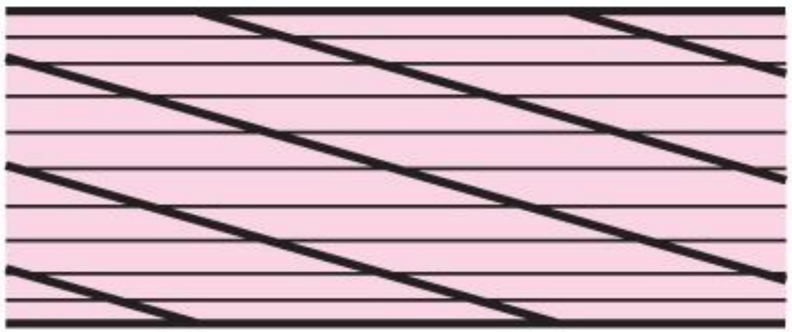
For a 140 chain, $p = 44.45$ mm. Thus,

$$C = 25.104p = 25.104(44.45) = 1115.9 \text{ mm}$$

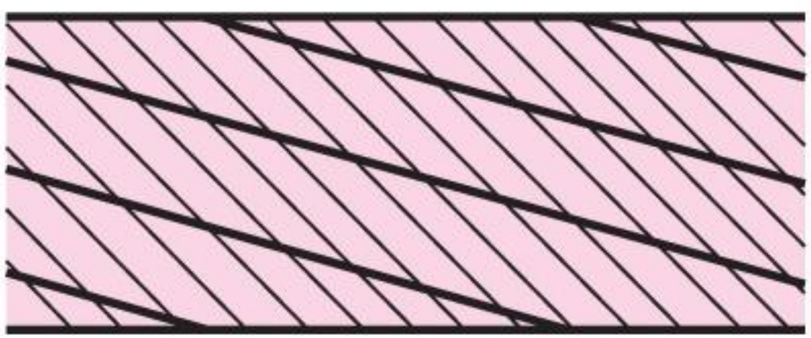
*Lubrication:* Type B

*Comment:* This is operating on the pre-extreme portion of the power, so durability estimates other than 15 000 h are not available. Given the poor operating conditions, life will be much shorter.

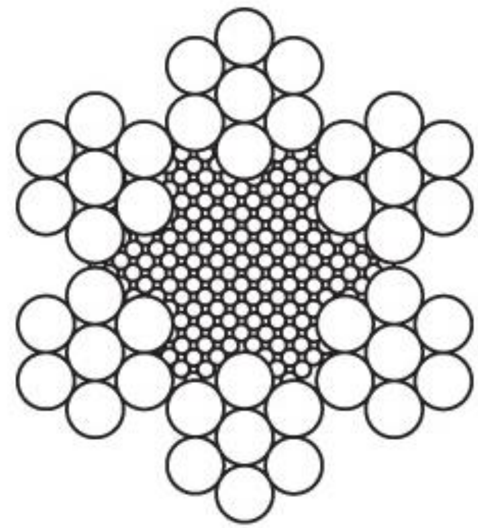
# Types of Wire Rope



(*a*) Regular lay



(*b*) Lang lay



(*c*) Section of $6 \times 7$ rope

Fig.17–19

# Stress in Wire Rope

$$M = \frac{EI}{\rho} \quad \text{and} \quad M = \frac{\sigma I}{c} \qquad (a)$$

$$\sigma = \frac{Ec}{\rho} \qquad (b)$$

$$c = d_w/2$$

where $d_w$ is the wire diameter

$$\sigma = E_r \frac{d_w}{D} \qquad (c)$$

where $E_r$ is the *modulus of elasticity of the rope*, not the wire

# Wire-Rope Data

| Rope | Weight per meter, $(10^{-3})$N | Minimum Sheave Diameter, mm | Standard Sizes $d$, mm | Material | Size of Outer Wires | Modulus of Elasticity,* GPa | Strength,† MPa |
|---|---|---|---|---|---|---|---|
| 6 × 7 haulage | $33.92d^2$ | $42d$ | 6 – 38 | Monitor steel | $d/9$ | 96 | 690 |
| | | | | Plow steel | $d/9$ | 96 | 608 |
| | | | | Mild plow steel | $d/9$ | 96 | 524 |
| 6 × 19 standard hoisting | $36.18d^2$ | $26d$–$34d$ | 6 – 70 | Monitor steel | $d/13$–$d/16$ | 83 | 730 |
| | | | | Plow steel | $d/13$–$d/16$ | 83 | 640 |
| | | | | Mild plow steel | $d/13$–$d/16$ | 83 | 550 |
| 6 × 37 special flexible | $35.05d^2$ | $18d$ | 6 – 90 | Monitor steel | $d/22$ | 76 | 690 |
| | | | | Plow steel | $d/22$ | 76 | 608 |
| 8 × 19 extra flexible | $32.79d^2$ | $21d$–$26d$ | 6 – 38 | Monitor steel | $d/15$–$d/19$ | 69 | 634 |
| | | | | Plow steel | $d/15$–$d/19$ | 69 | 550 |
| 7 × 7 aircraft | $38.45d^2$ | — | 1.6 – 10 | Corrosion-resistant steel | — | — | 850 |
| | | | | Carbon steel | — | — | 850 |
| 7 × 9 aircraft | $39.58d^2$ | — | 3 – 36 | Corrosion-resistant steel | — | — | 930 |
| | | | | Carbon steel | — | — | 986 |
| 19-wire aircraft | $48.62d^2$ | — | 0.8 – 8 | Corrosion-resistant steel | — | — | 1137 |
| | | | | Carbon steel | — | — | 1137 |

*The modulus of elasticity is only approximate; it is affected by the loads on the rope and, in general, increases with the life of the rope.

†The strength is based on the nominal area of the rope. The figures given are only approximate and are based on 25-mm rope sizes and 6-mm aircraft-cable sizes.

Table 17–24

# Equivalent Bending Load

- Wire rope tension giving same tensile stress as sheave bending is called *equivalent bending load* $F_b$

$$F_b = \sigma A_m = \frac{E_r d_w A_m}{D} \qquad (17\text{–}41)$$

# Percent Strength Loss

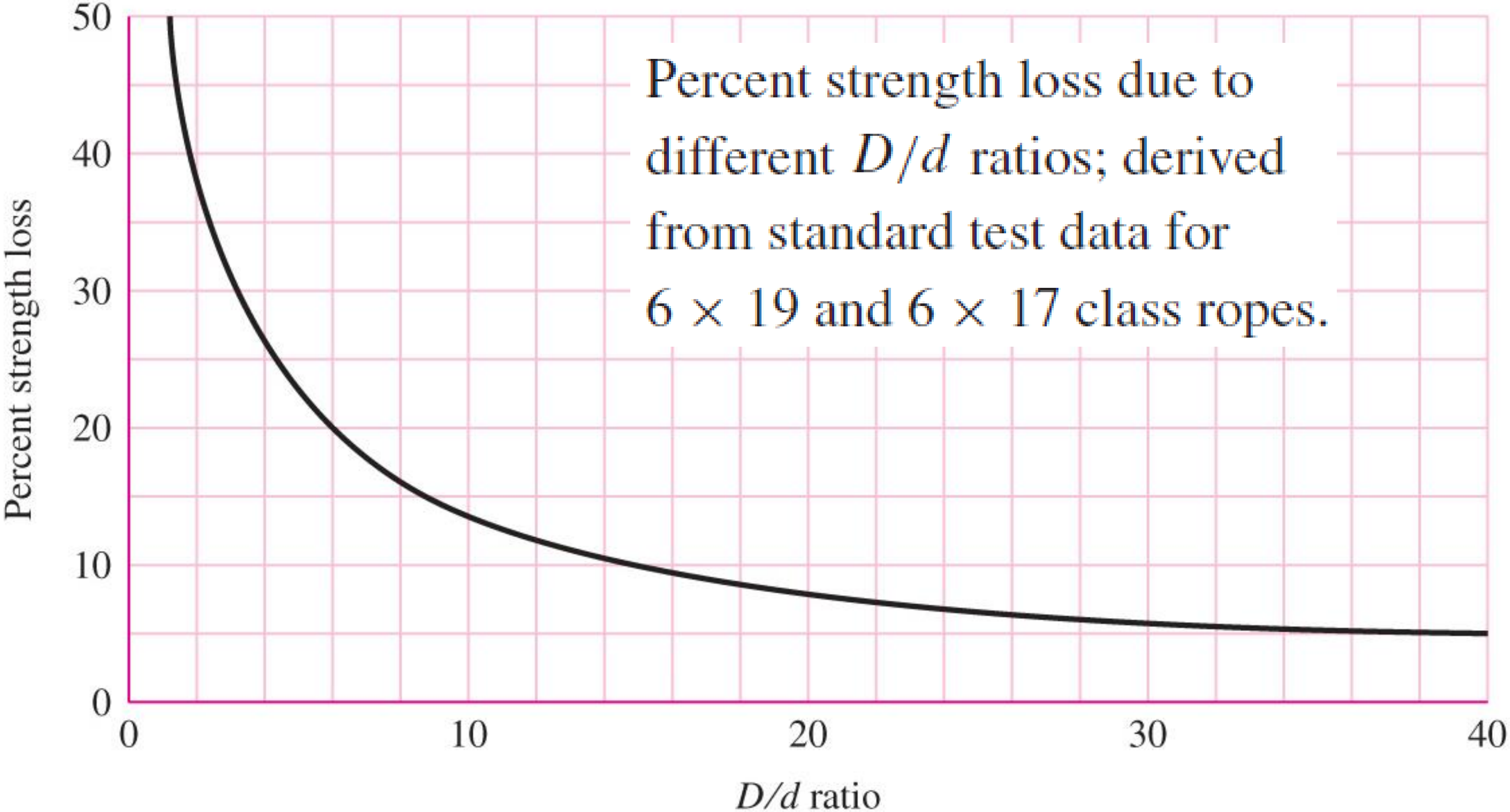



Fig.17–20

# Minimum Factors of Safety for Wire Rope

| | | | |
|---|---|---|---|
| Track cables | 3.2 | Passenger elevators, m/s: | |
| Guys | 3.5 | 0.25 | 7.60 |
| Mine shafts, m: | | 1.52 | 9.20 |
| Up to 152.5 | 8.0 | 4.06 | 11.25 |
| 305–610 | 7.0 | 6.10 | 11.80 |
| 610–915 | 6.0 | 7.62 | 11.90 |
| Over 915 | 5.0 | Freight elevators, m/s: | |
| Hoisting | 5.0 | 0.25 | 6.65 |
| Haulage | 6.0 | 1.52 | 8.20 |
| Cranes and derricks | 6.0 | 4.06 | 10.00 |
| Electric hoists | 7.0 | 6.10 | 10.50 |
| Hand elevators | 5.0 | 7.62 | 10.55 |
| Private elevators | 7.5 | Powered dumbwaiters, m/s: | |
| Hand dumbwaiter | 4.5 | 0.25 | 4.8 |
| Grain elevators | 7.5 | 1.52 | 6.6 |
| | | 4.06 | 8.0 |

*Use of these factors does not preclude a fatigue failure.

Table 17–25

# Bearing Pressure of Wire Rope in Sheave Groove

$$p = \frac{2F}{dD} \tag{17–42}$$

where $F$ = tensile force on rope

$d$ = rope diameter

$D$ = sheave diameter

# Maximum Allowable Bearing Pressures (in psi)

| | Sheave Material | | | | |
|---|---|---|---|---|---|
| Rope | Wood[a] | Cast Iron[b] | Cast Steel[c] | Chilled Cast Irons[d] | Manganese Steel[e] |
| Regular lay: | | | | | |
| 6 × 7 | 1.0 | 2.1 | 3.8 | 4.5 | 10.1 |
| 6 × 19 | 1.7 | 3.3 | 6.2 | 7.6 | 16.6 |
| 6 × 37 | 2.1 | 4.0 | 7.4 | 9.1 | 20.7 |
| 8 × 19 | 2.4 | 4.7 | 8.7 | 10.7 | 24.1 |
| Lang lay: | | | | | |
| 6 × 7 | 1.1 | 2.4 | 4.1 | 4.9 | 11.4 |
| 6 × 19 | 1.9 | 3.8 | 6.9 | 8.3 | 19.0 |
| 6 × 37 | 2.3 | 4.6 | 8.1 | 10.0 | 22.8 |

[a]On end grain of beech, hickory, or gum.

[b]For $H_B$ (min.) = 125.

[c]30–40 carbon; $H_B$ (min.) = 160.

[d]Use only with uniform surface hardness.

[e]For high speeds with balanced sheaves having ground surfaces.

Table 17–26

# Relation Between Fatigue Life of Wire Rope and Sheave Pressure

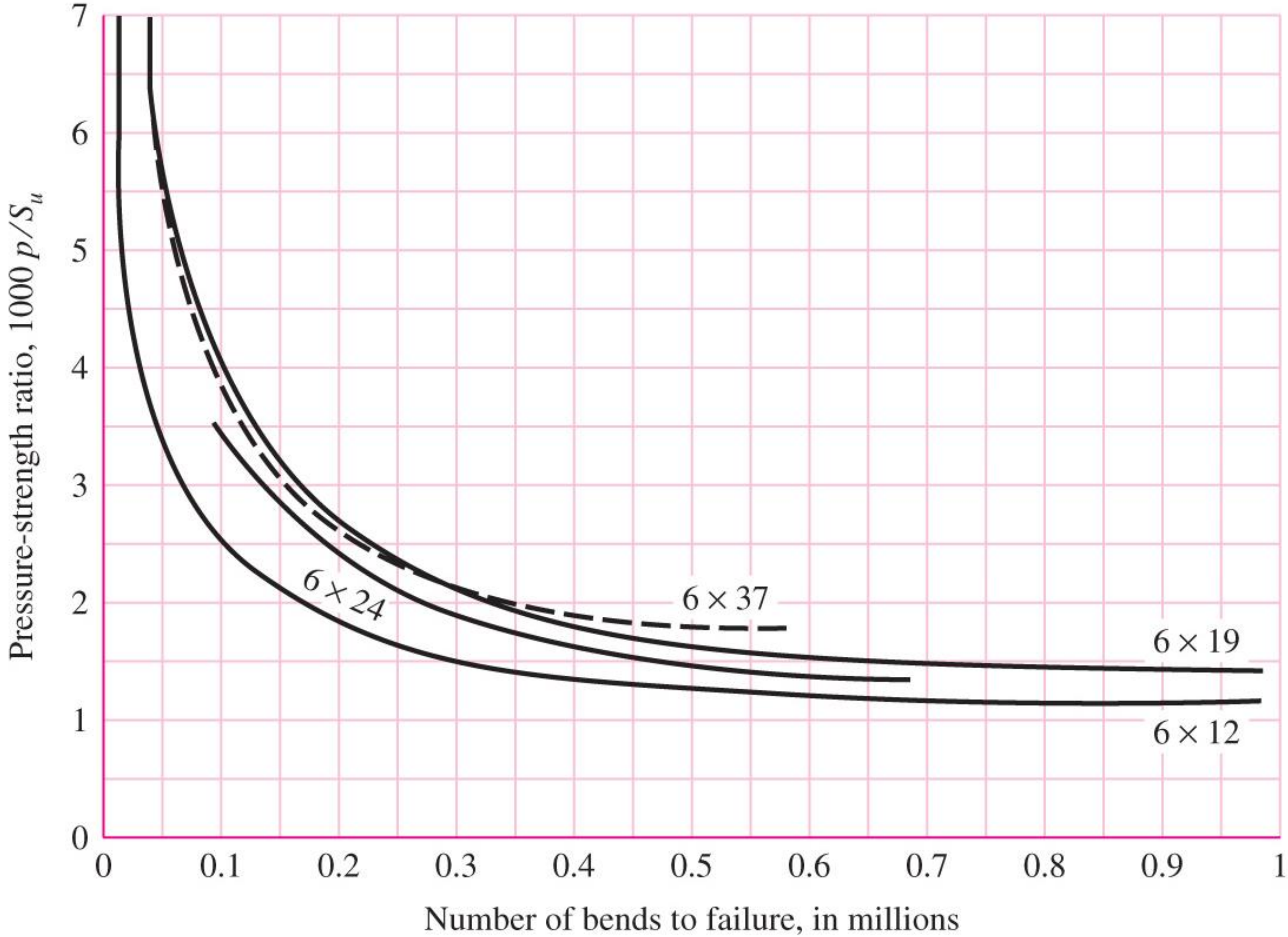



Fig.17–21

# Fatigue of Wire Rope

- Fig. 17–21does not preclude failure by fatigue or wear
- It does show long life if $p/S_u$ is less than 0.001.
- Substituting this ratio in Eq. (17–42),

$$S_u = \frac{2000F}{dD} \tag{17–43}$$

- Dividing both sides of Eq. (17–42) by $S_u$ and solving for $F$, gives allowable fatigue tension,

$$F_f = \frac{(p/S_u)S_u d D}{2} \tag{17–44}$$

- Factor of safety for fatigue is

$$n_f = \frac{F_f - F_b}{F_t} \tag{17–45}$$

# Factor of Safety for Static Loading

- The factor of safety for static loading is

$$n_s = \frac{F_u - F_b}{F_t} \tag{17-46}$$

# Typical Strength of Individual Wires

| | |
|---|---|
| Improved plow steel (monitor) | $240 < S_u < 280$ kpsi |
| Plow steel | $210 < S_u < 240$ kpsi |
| Mild plow steel | $180 < S_u < 210$ kpsi |

# Service-Life Curve Based on Bending and Tensile Stresses

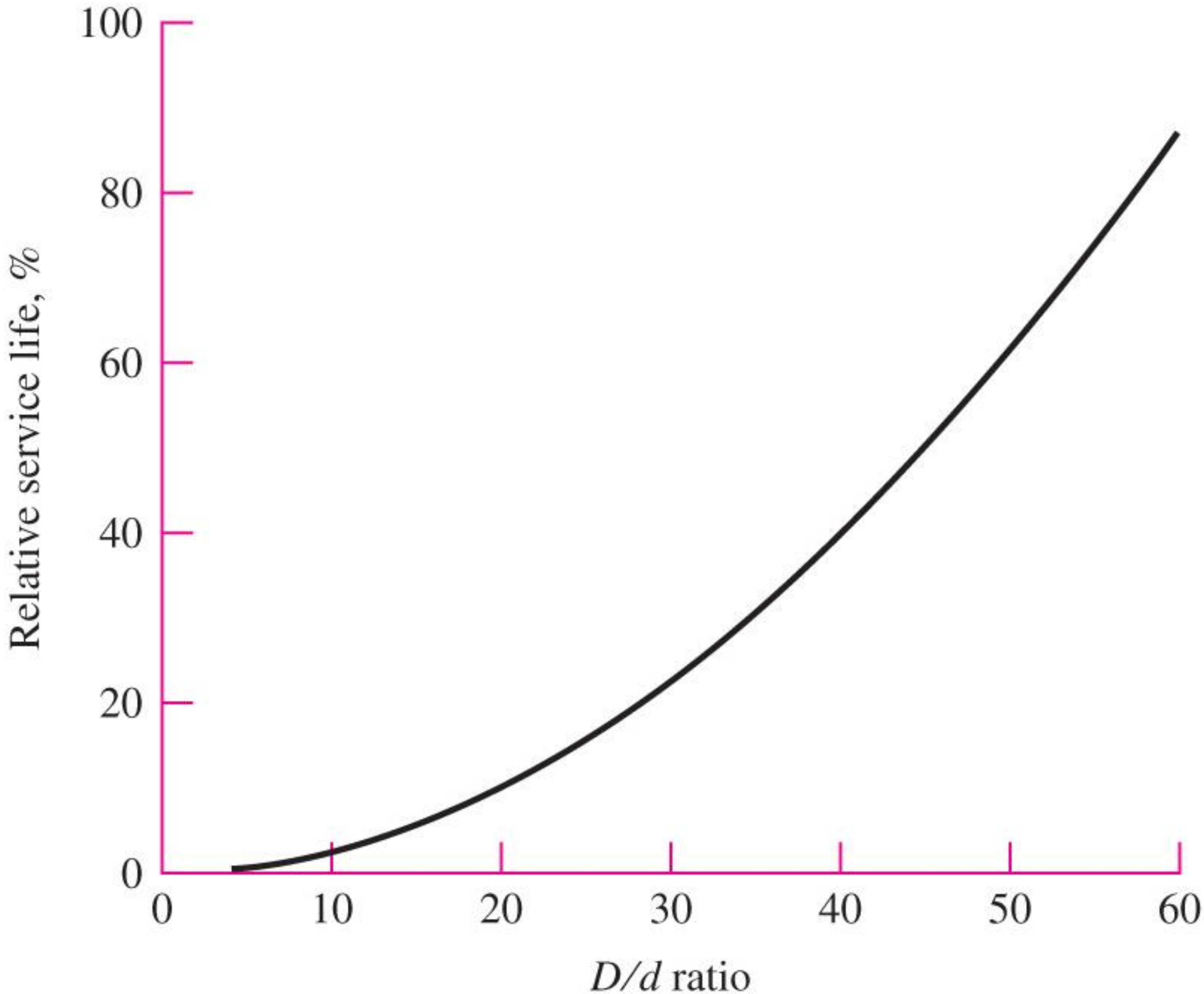



Fig.17–22

# Some Wire-Rope Properties

**Table 17–27**

Some Useful Properties of 6 × 7, 6 × 19, and 6 × 37 Wire Ropes

| Wire Rope | Weight per meter $w$, ($10^{-3}$) N/m | Weight per meter Including Core $w$, ($10^{-3}$) N/m | Minimum Sheave Diameter $D$, mm | Better Sheave Diameter $D$, mm | Diameter of Wires $d_w$, mm | Area of Metal $A_m$, mm$^2$ | Rope Young's Modulus $E_r$, GPa |
|---|---|---|---|---|---|---|---|
| 6 × 7 | $33.92d^2$ | | $42d$ | $72d$ | $0.111d$ | $0.38d^2$ | $13 \times 10^6$ |
| 6 × 19 | $36.18d^2$ | $39.8d^2$ | $30d$ | $45d$ | $0.067d$ | $0.40d^2$ | $12 \times 10^6$ |
| 6 × 37 | $35.05d^2$ | $38.67d^2$ | $18d$ | $27d$ | $0.048d$ | $0.40d^2$ | $12 \times 10^6$ |

# Working Equations for Mine-Hoist Problem

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right) \qquad (17\text{–}47)$$

where $W$ = weight at the end of the rope (cage and load), lbf

$m$ = number of wire ropes supporting the load

$w$ = weight/foot of the wire rope, lbf/ft

$l$ = maximum suspended length of rope, ft

$a$ = maximum acceleration/deceleration experienced, ft/s$^2$

$g$ = acceleration of gravity, ft/s$^2$

# Working Equations for Mine-Hoist Problem

$$F_f = \frac{(p/S_u)S_u D d}{2} \qquad (17\text{–}44)$$

where $(p/S_u)$ = specified life, from Fig. 17–21

$S_u$ = ultimate tensile strength of the wires, psi

$D$ = sheave or winch drum diameter, in

$d$ = nominal wire rope size, in

# Working Equations for Mine-Hoist Problem

$$F_b = \frac{E_r d_w A_m}{D} \qquad (17\text{–}41)$$

where $E_r$ = Young's modulus for the wire rope, Table 17–24 or 17–27, psi

$d_w$ = diameter of the wires, in

$A_m$ = metal cross-sectional area, Table 17–27, $in^2$

$D$ = sheave or winch drum diameter, in

$$n_s = \frac{F_u - F_b}{F_t} \qquad (17\text{–}46)$$

$$n_f = \frac{F_f - F_b}{F_t} \qquad (17\text{–}45)$$

# Example 17–6

A temporary construction elevator is to be designed to carry workers and materials to a height of 27 m. The maximum estimated load to be hoisted is 22 kN at a velocity not to exceed 0.6 m/s. For minimum sheave diameters and acceleration of 1.2 m/s$^2$, specify the number of ropes required if the 25-mm plow-steel $6 \times 19$ hoisting strand is used.

Since this is a design task, a decision set is useful.

*A priori decisions:*

- Function: load, height, acceleration, velocity, life goal
- Design Factor: $n_d$
- Material: IPS, PS, MPS or other
- Rope: Lay, number of strands, number of wires per strand

# Example 17–6

*Decision variables:*

- Nominal wire size: $d$
- Number of load-supporting wires: $m$

From experience with Prob. 17–29, a 25-mm diameter rope is not likely to have much of a life, so approach the problem with the $d$ and $m$ decisions open.

*Function*: 22 kN load, 27 m lift, acceleration $= 1.2\ \mathrm{m/s^2}$, velocity $= 0.6$ m/s, life goal $= 10^5$ cycles

*Design Factor*: $n_d = 2$

*Material*: IPS

*Rope*: Regular lay, 25-mm plow-steel $6 \times 19$ hoisting

# Example 17–6

*Design variables*

Choose 750 mm $D_{min}$. Table 17–27: $w = 0.0362d^2$ N/m

$$wl = 0.0362d^2\,(27) = 0.253d^2\ \text{N, ea.}$$

Eq. (17–46):

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right) = \left(\frac{22\,000}{m} + 0.253d^2\right)\left(1 + \frac{1.2}{9.81}\right)$$

$$= \frac{24\,691}{m} + 0.284d^2\ \text{N},\quad \text{each wire}$$

Eq. (17–47):

$$F_f = \frac{(p/S_u)S_u Dd}{2}$$

From Fig. 17–21 for $10^5$ cycles $p/S_u = 0.004$; $S_u = 1655$ MPa, based on metal area.

# Example 17–6

$$F_f = \frac{0.004(1655)(750)d}{2} = 2482d\ \text{N} \quad \text{each wire}$$

Eq. (17–48) and Table 17–27:

$$F_b = \frac{E_w d_w A_m}{D} = \frac{83\,000(0.067d)(0.4d^2)}{750} = 2.97d^3\ \text{N}, \quad \text{each wire}$$

Eq. (17–45):

$$n_f = \frac{F_f - F_b}{F_t} = \frac{2482d - 2.97d^3}{(24\,691/m) + 0.284d^2}$$

We could use a computer program to build a table similar to that of Ex. 17–6. Alternatively, we could recognize that $0.284d^2$ is small compared to $24\,691/m$, and therefore eliminate the $0.284d^2$ term.

$$n_f \doteq \frac{2482d - 2.97d^3}{24\,691/m} = \frac{m}{24\,691}(2482d - 2.97d^3)$$

Maximize $n_f$,

$$\frac{\partial n_f}{\partial d} = 0 = \frac{m}{24\,691}[2482 - 3(2.97)d^2]$$

# Example 17–6

From which

$$d^* = \sqrt{\frac{2482}{8.91}} = 16.7 \text{ mm}$$

Back-substituting

$$n_f = \frac{m}{24\,691}[2482(16.7) - 2.97(16.7)^3] =$$

Thus $n_f = 1.12, 2.24, 3.35, 4.47$ for $m = 1, 2, 3, 4$ respectively. If we choose $d = 1.25$ mm, then $m = 2$.

$$n_f = \frac{2482(12.5) - 2.97(12.5)^3}{(24\,691/2) + 0.284(12.5)^2} = 2.036$$

This slightly less than $n_d = 2$

*Decisions #1:* $d = 12.5$ mm

# Example 17–6

Answer

*Decisions #2:* $m = 2$ ropes supporting load. Rope should be inspected weekly for any signs of fatigue (broken outer wires).

*Comment*: Table 17–25 gives $n$ for freight elevators in terms of velocity.

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 730\left(\frac{\pi d^2}{4}\right) = 573.3d^2 \text{ N}, \qquad \text{each wire}$$

$$n = \frac{F_u}{F_t} = \frac{573.3(12.5)^2}{(24\,691/2) + 2.97(12.5)^2} = 7.0$$

By comparison, interpolation for 0.6 m/s gives 7.08-close. The category of construction hoists is not addressed in Table 17–25. We should investigate this before proceeding further.

# Flexible Shaft Configurations

Fig.17–24*b*

# Flexible Shaft Construction Details

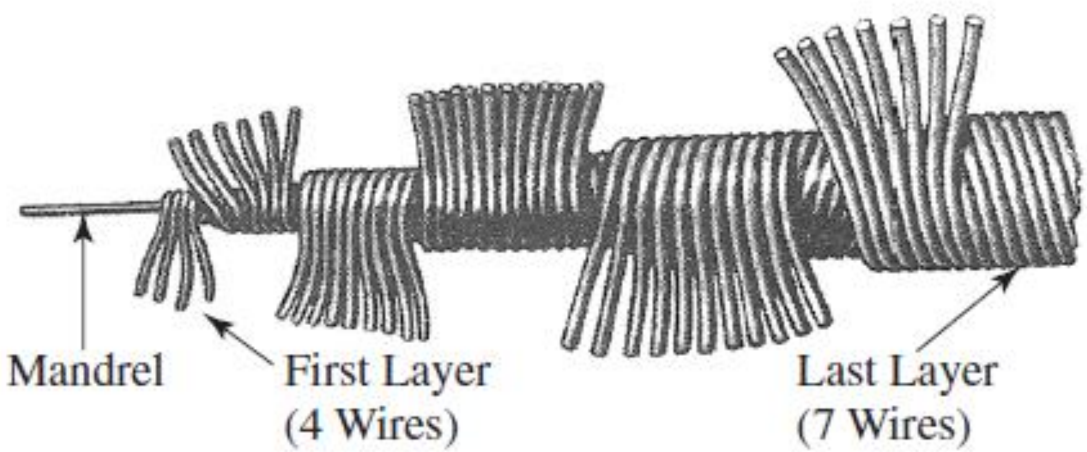



Fig.17–24*a*