Chapter 2

KINEMATICS FUNDAMENTALS

TOPIC/PROBLEM MATRIX

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PROBLEM 2-1

Statement: Find three (or other number as assigned) of the following common devices. Sketch careful kinematic diagrams and find their total degrees of freedom.

a. An automobile hood hinge mechanism
b. An automobile hatchback lift mechanism
c. An electric can opener
d. A folding ironing board
e. A folding card table
f. A folding beach chair
g. A baby swing
h. A folding baby walker
i. A fancy corkscrew as shown in Figure P2-9
j. A windshield wiper mechanism
k. A dump-truck dump mechanism
l. A trash truck dumpster mechanism
m. A pickup tailgate mechanism
n. An automobile jack
o. A collapsible auto radio antenna

Solution: See Mathcad file P0201.

Equation 2.1c is used to calculate the mobility (*DOF*) of each of the models below.

a. An automobile hood hinge mechanism.

The hood (3) is linked to the body (1) through two rocker links (2 and 4).

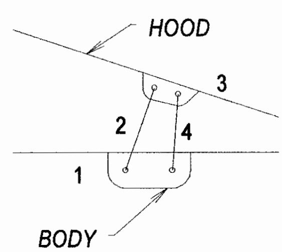
Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

$$M = 1$$



b. An automobile hatchback lift mechanism.

The hatch (2) is pivoted on the body (1) and is linked to the body by the lift arm, which can be modeled as two links (3 and 4) connected through a translating slider joint.

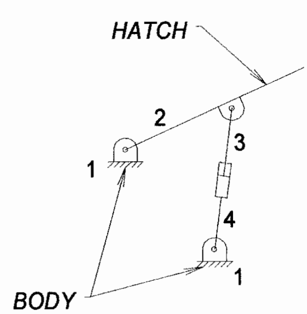
Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

$$M = 1$$



c. An electric can opener has 2 *DOF*.

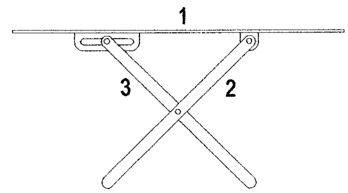
d. A folding ironing board.

The board (1) itself has one pivot (full) joint and one pin-in-slot sliding (half) joint. The two legs (2 and 3) have a common pivot. One leg connects to the pivot joint on the board and the other to the slider joint.

Number of links $L := 3$

Number of full joints $J_1 := 2$

Number of half joints $J_2 := 1$



$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$

e. A folding card table has 7 *DOF*: One for each leg, 2 for location in *xy* space, and one for angular orientation.

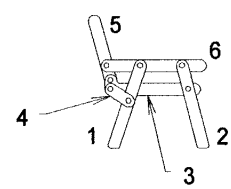
f. A folding beach chair.

The seat (3) and the arms (6) are ternary links. The seat is linked to the front leg(2), the back (5) and a coupling link (4). The arms are linked to the front leg (2), the rear leg (1), and the back (5). Links 1, 2, 4, and 5 are binary links. The analysis below is appropriate when the chair is not fully opened. When fully opened, one or more links are prevented from moving by a stop. Subtract 1 DOF when forced against the stop.

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$



$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$

g. A baby swing has 4 *DOF*: One for the angular orientation of the swing with respect to the frame, and 3 for the location and orientation of the frame with respect to a 2-D frame.

h. A folding baby walker has 4 *DOF*: One for the degree to which it is unfolded, and 3 for the location and orientation of the walker with respect to a 2-D frame.

i. A fancy corkscrew has 2 DOF: The screw can be rotated and the arms rotate to translate the screw.

j. A windshield wiper mechanism has 1 *DOF*: The position of the wiper blades is defined by a single input.

k. A dump-truck dump mechanism has 1 *DOF*: The angle of the dump body is determined by the length of the hydraulic cylinder that links it to the body of the truck.

l. A trash truck dumpster mechanism has 2 *DOF*: These are generally a rotation and a translation.

m. A pickup tailgate mechanism has 1 *DOF*:

n. An automobile jack has 4 *DOF*: One is the height of the jack and the other 3 are the position and orientation of the jack with respect to a 2-D frame.

o. A collapsible auto radio antenna has as many *DOF* as there are sections, less one.

PROBLEM 2-2

Statement: How many DOF do you have in your wrist and hand combined?

Solution: See Mathcad file P0202.

1. Holding the palm of the hand level and facing toward the floor, the hand can be rotated about an axis through the wrist that is parallel to the floor (and perpendicular to the forearm axis) and one perpendicular to the floor (2 *DOF*). The wrist can rotate about the forearm axis (1 *DOF*).
2. Each finger (and thumb) can rotate up and down and side-to-side about the first joint. Additionally, each finger can rotate about each of the two remaining joints for a total of 4 *DOF* for each finger (and thumb).
3. Adding all *DOF*, the total is

Wrist	1
Hand	2
Thumb	4
Fingers 4x4	16
TOTAL	23

✍ PROBLEM 2-3

Statement: How many *DOF* do the following joints have?

a. Your knee
b. Your ankle
c. Your shoulder
d. Your hip
e. Your knuckle

Solution: See Mathcad file P0203.

a. Your knee.
 1 *DOF*: A rotation about an axis parallel to the ground.

b. Your ankle.
 3 *DOF*: Three rotations about mutually perpendicular axes.

c. Your shoulder.
 3 *DOF*: Three rotations about mutually perpendicular axes.

d. Your hip.
 3 *DOF*: Three rotations about mutually perpendicular axes.

e Your knuckle.
 2 *DOF*: Two rotations about mutually perpendicular axes.

PROBLEM 2-4

Statement: How many DOF do the following have in their normal environment?

a. A submerged submarine
b. An earth-orbiting satellite
c. A surface ship
d. A motorcycle
e. The print head on a 9-pin dot matrix computer printer
f. The pen in an x-y plotter

Solution: See Mathcad file P0201.

a. A submerged submarine.
Using a coordinate frame attached to earth, or an inertial coordinate frame, a submarine has 6 *DOF*: 3 linear coordinates and 3 angles.

b. An earth-orbit satellite.
If the satellite was just a particle it would have 3 *DOF*. But, since it probably needs to be oriented with respect to the earth, sun, etc., it has 6 *DOF*.

c. A surface ship.
There is no difference between a submerged submarine and a surface ship, both have 6 *DOF*. One might argue that, for an earth-centered frame, the depth of the ship with respect to mean sea level is constant, however that is not strictly true. A ship's position is generally given by two coordinates (longitude and latitude). For a given position, a ship can also have pitch, yaw, and roll angles. Thus, for all practical purposes, *a surface ship has 5 DOF*.

d. A motorcycle.
At an intersection, the motorcycle's position is given by two coordinates. In addition, it will have some heading angle (turning a corner) and roll angle (if turning). Thus, there are 4 *DOF*.

e. The print head on a 9-pin dot matrix computer printer.
The print head carrier has 1 *DOF* and each pin has 1 *DOF* for a total of 10 *DOF*.

f. The pen in an x-y plotter.
The pen holder has 2 *DOF* (x and y) and the pen may be either up or down, for a total of 3 *DOF*.

PROBLEM 2-5

Statement: Are the joints in Problem 2-3 force closed or form closed?

Solution: See Mathcad file P0205.

They are force closed by ligaments that hold them together. None are geometrically closed.

PROBLEM 2-6

Statement: Describe the motion of the following items as pure rotation, pure translation, or complex planar motion.

a. A windmill
b. A bicycle (in the vertical plane, not turning)
c. A conventional "double-hung" window
d. The keys on a computer keyboard
e. The hand of a clock
f. A hockey puck on the ice
g. The pen in an *XY* plotter
h. The print head in a computer printer
i. A "casement" window

Solution: See Mathcad file P0206.

a. A windmill.
   Pure rotation.

b. A bicycle (in the vertical plane, not turning).
   Pure translation for the frame, complex planar motion for the wheels.

c. A conventional "double-hung" window.
   Pure translation.

d. The keys on a computer keyboard.
   Pure translation.

e. The hand of a clock.
   Pure rotation.

f. A hockey puck on the ice.
   Complex planar motion.

g. The pen in an *XY* plotter.
   Pure translation.

h. The print head in a computer printer.
   Pure translation.

i. A "casement" window.
   Complex planar motion.

PROBLEM 2-7

Statement: Calculate the mobility of the linkages assigned from Figure P2-1 part 1 and part 2.

Solution: See Figure P2-1 and Mathcad file P0207.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

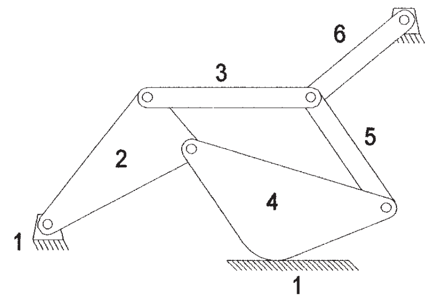
a. Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 1$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

$$M = 0$$



(a)

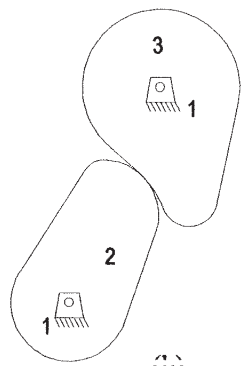
b. Number of links $L := 3$

Number of full joints $J_1 := 2$

Number of half joints $J_2 := 1$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

$$M = 1$$



(b)

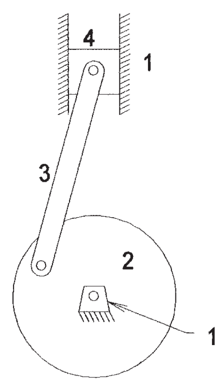
c. Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

$$M = 1$$



(c)

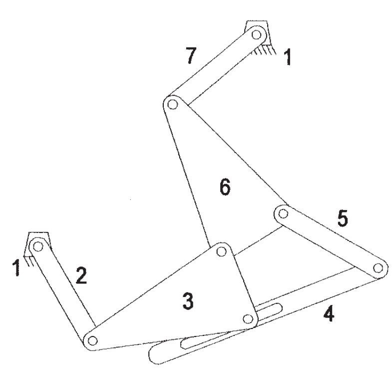
d. Number of links $L := 7$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 1$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

$$M = 3$$



(d)

PROBLEM 2-8

Statement: Identify the items in Figure P2-1 as mechanisms, structures, or preloaded structures.

Solution: See Figure P2-1 and Mathcad file P0208.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility and the definitions in Section 2.5 on page 34 of the text to classify the linkages.

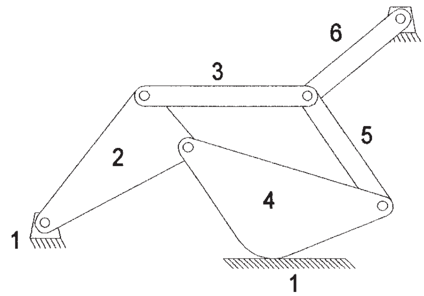
a. Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$M = 0$ Structure



(a)

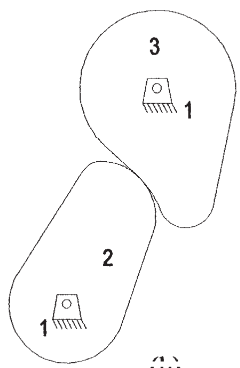
b. Number of links $L := 3$

Number of full joints $J_1 := 2$

Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$M = 1$ Mechanism



(b)

c. Number of links $L := 4$

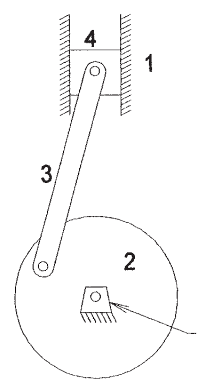
Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$M = 1$ Mechanism

(c)



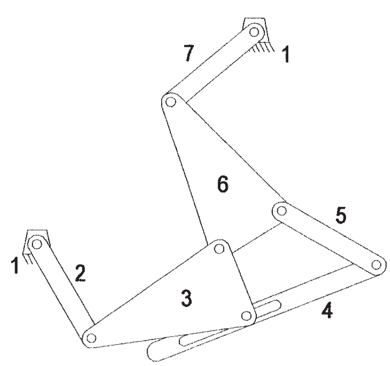
d. Number of links $L := 7$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$M = 3$ Mechanism



(d)

PROBLEM 2-9

Statement: Use linkage transformation on the linkage of Figure P2-1a to make it a 1-*DOF* mechanism.

Solution: See Figure P2-1a and Mathcad file P0209.

1. The mechanism in Figure P2-1a has mobility:

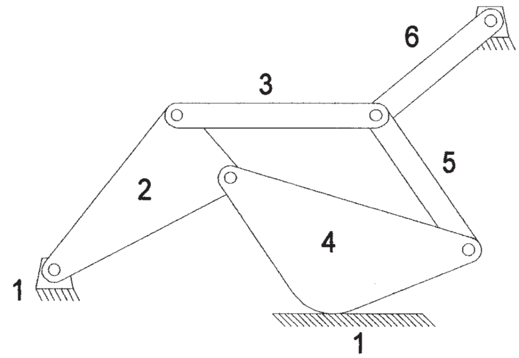
   Number of links $L := 6$

   Number of full joints $J_1 := 7$

   Number of half joints $J_2 := 1$

   $$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

   $$M = 0$$



2. Use rule 2 on page 42, which states: "Any full joint can be replaced by a half joint, but this will increase the *DOF* by one." One way to do this is to replace one of the pin joints with a pin-in-slot joint such as that shown in Figure 2-3c. Choosing the joint between links 2 and 4, we now have mobility:

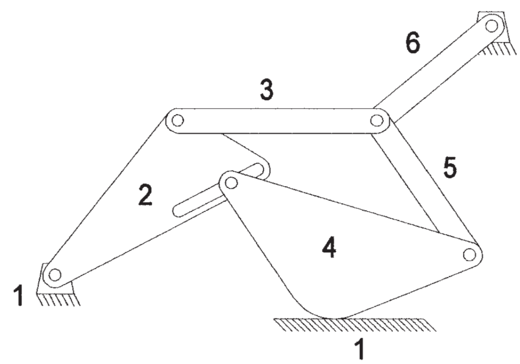
   Number of links $L := 6$

   Number of full joints $J_1 := 6$

   Number of half joints $J_2 := 2$

   $$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

   $$M = 1$$

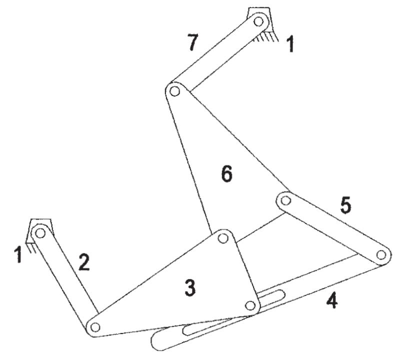


PROBLEM 2-10

Statement: Use linkage transformation on the linkage of Figure P2-1d to make it a 2-*DOF* mechanism.

Solution: See Figure P2-1d and Mathcad file P0210.

1. The mechanism in Figure P2-1d has mobility:



Number of links $L := 7$

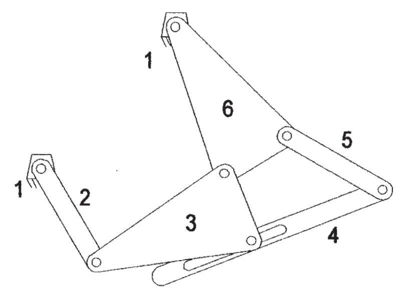
Number of full joints $J_1 := 7$

Number of half joints $J_2 := 1$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

$$M = 3$$

2. Use rule 3 on page 42, which states: "Removal of a link will reduce the *DOF* by one." One way to do this is to remove link 7 such that link 6 pivots on the fixed pin attached to the ground link (1). We now have mobility:



Number of links $L := 6$

Number of full joints $J_1 := 6$

Number of half joints $J_2 := 1$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

$$M = 2$$

PROBLEM 2-11

Statement: Use number synthesis to find all the possible link combinations for 2-*DOF*, up to 9 links, to hexagonal order, using only revolute joints.

Solution: See Mathcad file P0211.

1. Use equations 2.4a and 2.6 with $DOF = 2$ and iterate the solution for valid combinations. Note that the number of links must be odd to have an even *DOF* (see Eq. 2.4). The smallest possible 2-*DOF* mechanism is then 5 links since three will give a structure (the delta triplet, see Figure 2-7).

$$L := B + T + Q + P + H \qquad L - 3 - M := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H \qquad M := 2$$

$$L - 5 := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H$$

2. For $L := 5$

$$0 := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H \qquad 0 = T = Q = P = H \qquad B := 5$$

3. For $L := 7$

$$2 := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H \qquad H := 0 \qquad P := 0$$

Case 1: $Q := 0$ $\quad T := 2 - 2 \cdot Q - 3 \cdot P - 4 \cdot H$ $\quad T = 2$

$B := L - T - Q - P - H$ $\quad B = 5$

Case 2: $Q := 1$ $\quad T := 2 - 2 \cdot Q - 3 \cdot P - 4 \cdot H$ $\quad T = 0$

$B := L - T - Q - P - H$ $\quad B = 6$

4. For $L := 9$

$$4 := T + 2 \cdot Q + 3 \cdot P + 4 \cdot H$$

Case 1: $H := 1$ $\quad T := 0$ $\quad Q := 0$ $\quad P := 0$

$B := L - T - Q - P - H$ $\quad B = 8$

Case 2a: $H := 0$ $\quad 4 := T + 2 \cdot Q + 3 \cdot P$

$9 := B + T + Q + P$

Case 2b: $P := 1$ $\quad 1 := T + 2 \cdot Q$ $\quad Q := 0$ $\quad T := 1$

$B := L - T - Q - P - H$ $\quad B = 7$

Case 2c: $P := 0$ $\quad 4 := T + 2 \cdot Q$

$9 := B + T + Q$

Case 2c1: $Q := 2$ $\quad T := 4 - 2 \cdot Q$ $\quad T = 0$

$B := 9 - T - Q$ $\quad B = 7$

Case 2c2: $Q := 1$ $\quad T := 4 - 2 \cdot Q$ $\quad T = 2$

$B := 9 - T - Q$ $\quad B = 6$

Case 2c3: $Q := 0$ $\quad T := 4 - 2 \cdot Q$ $\quad T = 4$

$B := 9 - T - Q$ $\quad B = 5$

PROBLEM 2-12

Statement: Find all of the valid isomers of the eightbar 1-*DOF* link combinations in Table 2-2 (p. 38) having:

a. Four binary and four ternary links.
b. Five binaries, two ternaries, and one quaternary link.
c. Six binaries and two quaternary links.
d. Six binaries, one ternary, and one pentagonal link.

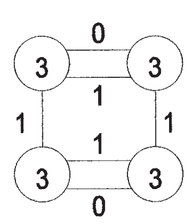
Solution: See Mathcad file P0212.

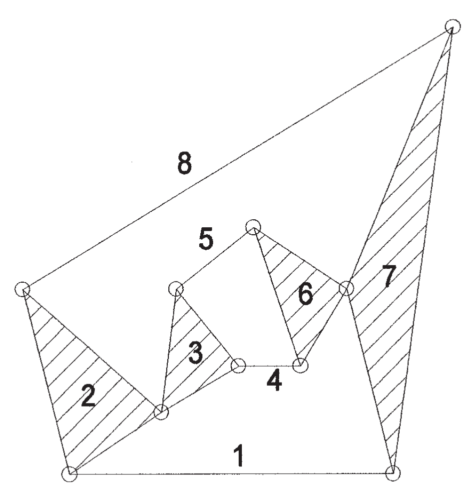
1. Table 2-3 on page 40 lists 16 possible isomers for an eightbar chain. However, Table 2-2 shows that there are five possible link sets, four of which are listed above. Therefore, we expect that the 16 valid isomers are distributed among the five link sets and that there will be fewer than 16 isomers among the four link sets listed above.

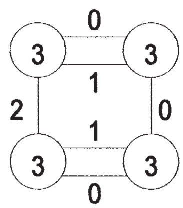
2. One method that is helpful in finding isomers is to represent the linkage in terms of molecules as defined in Franke's Condensed Notations for Structural Synthesis. A summary of the rules for obtaining Franke's molecules follows:
(1) The links of order greater than 2 are represented by circles.
(2) A number is placed within each circle (the "valence" number) to describe the type (ternary, quaternary, etc.) of link.
(3) The circles are connected using straight lines. The number of straight lines emanating from a circle must be equal to its valence number.
(4) Numbers (0, 1, 2, etc.) are placed on the straight lines to correspond to the number of binary links used in connecting the higher order links.
(5) There is one-to-one correspondence between the molecule and the kinematic chain that it represents.

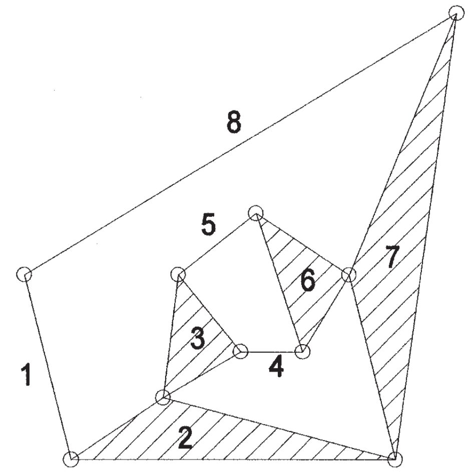
a. Four binary and four ternary links.

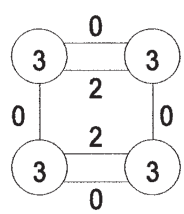
Draw 4 circles with valence numbers of 3 in each. Then find all unique combinations of straight lines that can be drawn that connect the circles such that there are exactly three lines emanating from each circle and the total of the numbers written on the lines is exactly equal to 4. In this case, there are three valid isomers as depicted by Franke's molecules and kinematic chains below.

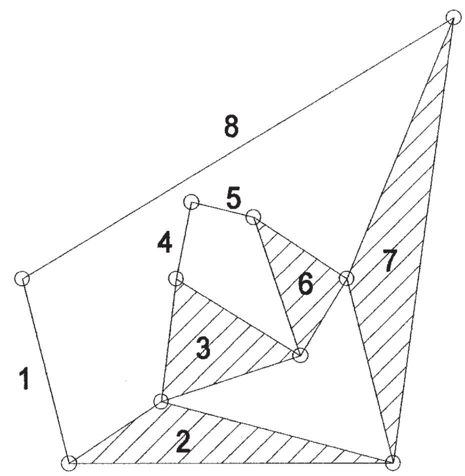








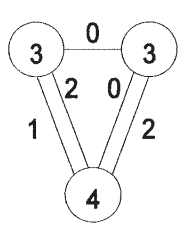


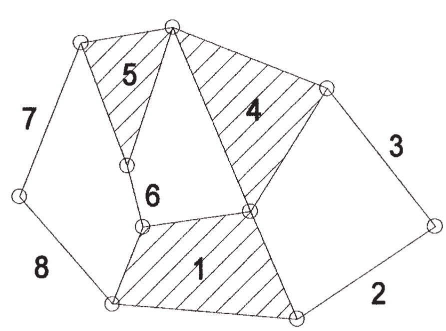


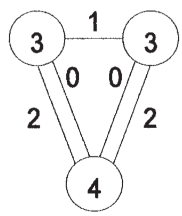
The mechanism shown in Figure P2-5b is the same eightbar isomer as that depicted schematically above.

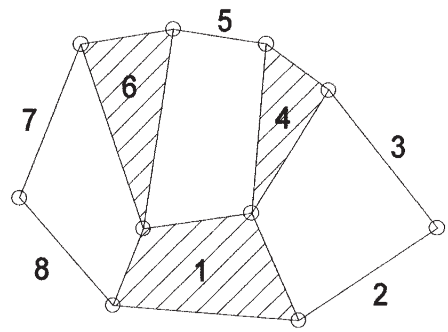
b. Five binaries, two ternaries, and one quaternary link.

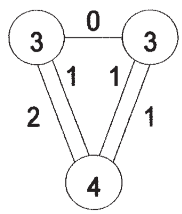
Draw 2 circles with valence numbers of 3 in each and one with a valence number of 4. Then find all unique combinations of straight lines that can be drawn that connect the circles such that there are exactly three lines emanating from each circle with valence of three and four lines from the circle with valence of four; and the total of the numbers written on the lines is exactly equal to 5. In this case, there are five valid isomers as depicted by Franke's molecules and kinematic chains below.

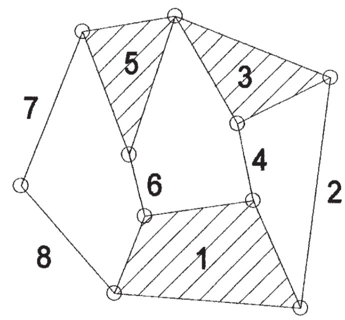


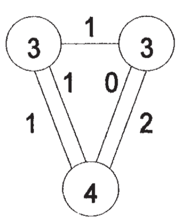


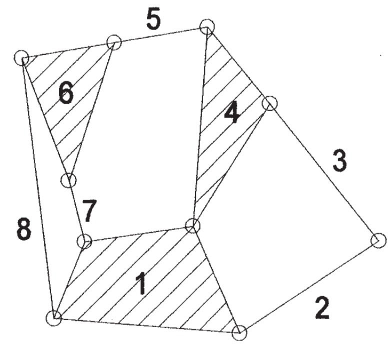


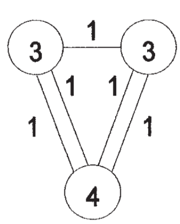


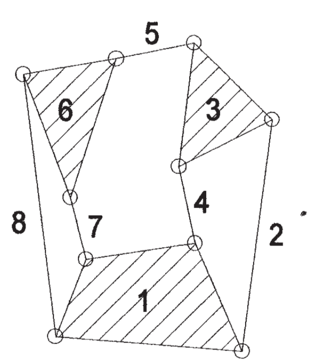






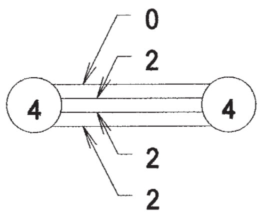


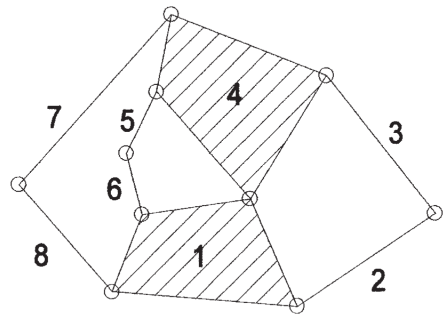


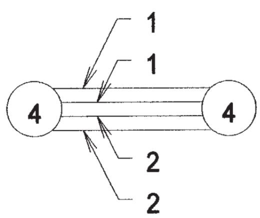


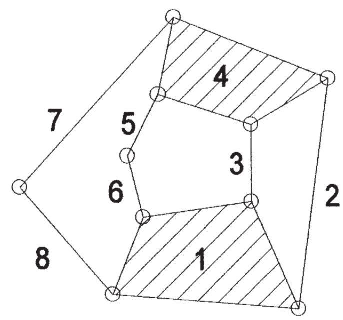
c. Six binaries and two quaternary links.

Draw 2 circles with valence numbers of 4 in each. Then find all unique combinations of straight lines that can be drawn that connect the circles such that there are exactly four lines emanating from each circle and the total of the numbers written on the lines is exactly equal to 6. In this case, there are two valid isomers as depicted by Franke's molecules and kinematic chains below.









d. Six binaries, one ternary, and one pentagonal link.

There are no valid implementations of 6 binary links with 1 pentagonal link.

PROBLEM 2-13

Statement: Use linkage transformation to create a 1-*DOF* mechanism with two sliding full joints from a Stephenson's sixbar linkage as shown in Figure 2-14a (p. 47).

Solution: See Figure 2-14a and Mathcad file P0213.

1. The mechanism in Figure 2-14a has mobility:

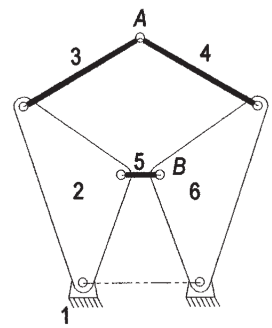
Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

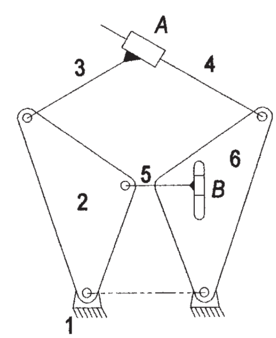
$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2$$

$$M = 1$$



2. Use rule 1 on page 42, which states: "Revolute joints in any loop can be replaced by prismatic joints with no change in *DOF* of the mechanism, provided that at least two revolute joints remain in the loop." One way to do this is to replace pin joints at *A* and *B* with translating full slider joints such as that shown in Figure 2-3b.

Note that the sliders are attached to links 3 and 5 in such a way that they can not rotate relative to the links. The number of links and 1-*DOF* joints remains the same. There are no 2-*DOF* joints in either mechanism.



PROBLEM 2-14

Statement: Use linkage transformation to create a 1-*DOF* mechanism with one sliding full joint a a half joint from a Stephenson's sixbar linkage as shown in Figure 2-14b (p. 48).

Solution: See Figure 2-14a and Mathcad file P0213.

1. The mechanism in Figure 2-14b has mobility:

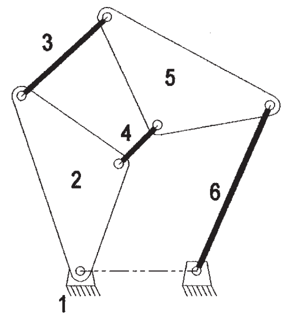
Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

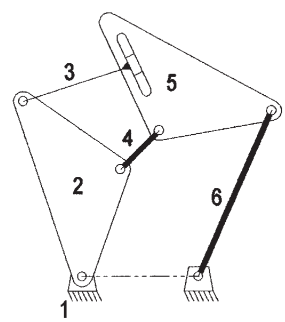
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



2. To get the sliding full joint, use rule 1 on page 42, which states: "Revolute joints in any loop can be replaced by prismatic joints with no change in *DOF* of the mechanism, provided that at least two revolute joints remain in the loop." One way to do this is to replace pin joint links 3 and 5 with a translating full slider joint such as that shown in Figure 2-3b.

Note that the slider is attached to link 3 in such a way that it can not rotate relative to the link. The number of links and 1-*DOF* joints remains the same.



3. To get the half joint, use rule 4 on page 42, which states: "The combination of rules 2 and 3 above will keep the original *DOF* unchanged." One way to do this is to remove link 6 (and its two nodes) and insert a half joint between links 5 and 1.

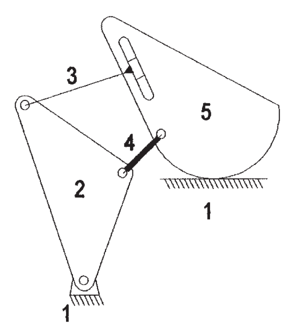
Number of links $L := 5$

Number of full joints $J_1 := 5$

Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



PROBLEM 2-15

Statement: Calculate the Grashof condition of the fourbar mechanisms defined below. Build cardboard models of the linkages and describe the motions of each inversion. Link lengths are in inches (or double given numbers for centimeters).

Part 1.

a.	2	4.5	7	9
b.	2	3.5	7	9
c.	2	4.0	6	8

Part 2.

d.	2	4.5	7	9
e.	2	4.0	7	9
f.	2	3.5	7	9

Solution: See Mathcad file P0215

1. Use inequality 2.8 to determine the Grashof condition.

$$Condition(S,L,P,Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return \text{ "Grashof" } \ if \ SL < PQ \\ return \text{ "Special Grashof" } \ if \ SL = PQ \\ return \text{ "non-Grashof" } \ otherwise \end{array} \right.$$

a. $Condition(2,9,4.5,7) = \text{"Grashof"}$

b. $Condition(2,9,3.5,7) = \text{"non-Grashof"}$

c. $Condition(2,8,4.0,6) = \text{"Special Grashof"}$

This is a special case Grashof since the sum of the shortest and longest is equal to the sum of the other two link lengths.

d. $Condition(2,9,4.5,7) = \text{"Grashof"}$

e. $Condition(2,9,4.0,7) = \text{"Special Grashof"}$

f. $Condition(2,9,3.5,7) = \text{"non-Grashof"}$

PROBLEM 2-16

Statement: Which type(s) of electric motor would you specify

a. To drive a load with large inertia.
b. To minimize variation of speed with load variation.
c. To maintain accurate constant speed regardless of load variations.

Solution: See Mathcad file P0216.

a. Motors with high starting torque are suited to drive large inertia loads. Those with this characteristic include series-wound, compound-wound, and shunt-wound DC motors, and capacitor-start AC motors.

b. Motors with flat torque-speed curves (in the operating range) will minimize variation of speed with load variation. Those with this characteristic include shunt-wound DC motors, and synchronous and capacitor-start AC motors.

b. Speed-controlled DC motors will maintain accurate constant speed regardless of load variations.

PROBLEM 2-17

Statement: Describe the difference between a cam-follower (half) joint and a pin joint.

Solution: See Mathcad file P0217.

1. A pin joint has one rotational *DOF*. A cam-follower joint has 2 *DOF*, rotation and translation. The pin joint also captures its lubricant in the annulus between pin and bushing while the cam-follower joint squeezes its lubricant out of the joint.

PROBLEM 2-18

Statement: Examine an automobile hood hinge mechanism of the type described in Section 2.14. Sketch it carefully. Calculate its *DOF* and Grashof condition. Make a cardboard model. Analyze it with a free-body diagram. Describe how it keeps the hood up.

Solution: Solution of this problem will depend upon the specific mechanism modeled by the student.

PROBLEM 2-19

Statement: Find an adjustable arm desk lamp of the type shown in Figure P2-2. Sketch it carefully. Measure it and sketch it to scale. Calculate its *DOF* and Grashof condition. Make a cardboard model. Analyze it with a free-body diagram. Describe how it keeps itself stable. Are there any positions in which it loses stability? Why?

Solution: Solution of this problem will depend upon the specific mechanism modeled by the student.

PROBLEM 2-20

Statement: Make kinematic sketches, define the types of all the links and joints, and determine the *DOF* of the mechanisms shown in Figure P2-3.

Solution: See Figure P2-3 and Mathcad file P0220.

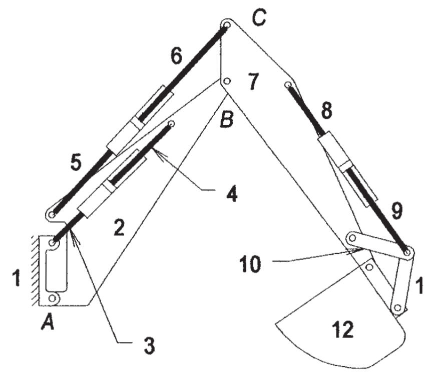
1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

a. Number of links $L := 12$

Number of full joints $J_1 := 15$

Number of half joints $J_2 := 0$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \qquad M = 3$$



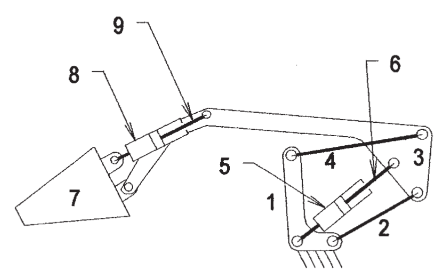
Link 2 is a quaternary, link 7 is pentagonal, and the remainder are binary. All joints are full. There are 3 translating joints and 12 pin joints. The main boom (link 2) is rotated about fixed pivot *A* by hydraulic cylinder 3-4. The secondary boom (link 7) is rotated about the main boom at pivot *B* by hydraulic cylinder 5-6. The bucket is the output link of a fourbar (links 7,10,11, and 12). Its position is controlled by hydraulic cylinder 8-9 acting on the input link (10).

b. Number of links $L := 9$

Number of full joints $J_1 := 11$

Number of half joints $J_2 := 0$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \qquad M = 2$$



Link 1 is ternary, link 7 is pentagonal, and the remainder are binary. All joints are full. There are 2 translating joints and 9 pin joints. The basic linkage (1, 2, 3, 4, 5, and 6) is a Stephenson's sixbar. Hydraulic cylinder 5-6 is the input to the basic linkage and controls the position of the main boom (link 3). The bucket position is determined by hydraulic cylinder 8-9.

PROBLEM 2-21

Statement: Find the mobility of the mechanisms in Figure P2-4.

Solution: See Figure P2-4 and Mathcad file P0221.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

a. This is a basic fourbar linkage. The input is link 2 and the output is link 4. The cross-hatched pivot pins at *O2* and *O4* are attached to the ground link (1).

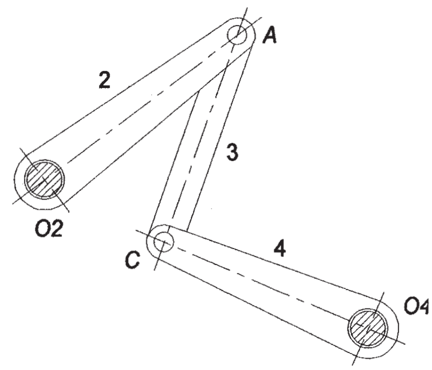
Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



b. This is a fourbar linkage. The input is link 2, which in this case is the wheel 2 with a pin at *A*, and the output is link 4. The cross-hatched pivot pins at *O2* and *O4* are attached to the ground link (1).

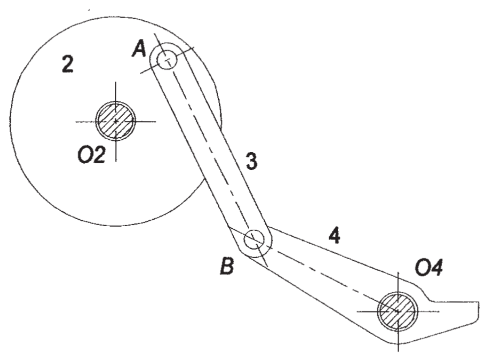
Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



c. This is a 3-cylinder, rotary, internal combustion engine. The pistons (sliders) 6, 7, and 8 drive the output crank (2) through piston rods (couplers 3, 4, and 5). There are 3 full joints at the crank where rods 3, 4and 5 are pinned to crank 2. The cross-hatched crank-shaft at *O2* is supported by the ground link (1) through bearings.

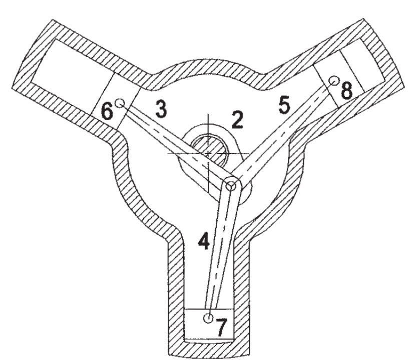
Number of links $L := 8$

Number of full joints $J_1 := 10$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



d. This is a fourbar linkage. The input is link 2, which in this case is a wheel with a pin at A, and the output is the vertical member on the coupler, link 3. Since the lengths of links 2 and 4 (O_2A and O_4B) are the same, the coupler link (3) has curvilinear motion and AB remains parallel to O_2O_4 throughout the cycle. The cross-hatched pivot pins at O_2 and O_4 are attached to the ground link (1).

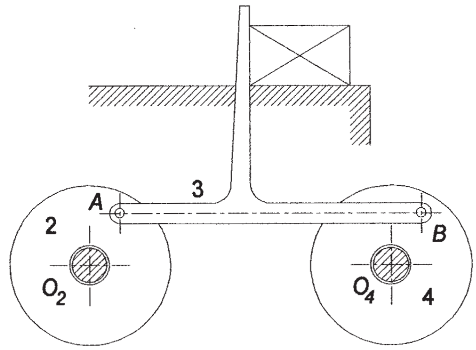
Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



e. This is a fourbar linkage with an output dyad. The input (rocker) is link 2 and the output (rocker) is link 8. Links 5 and 6 are redundant, i.e. the mechanism will have the same motion if they are removed. The input fourbar consists of links 1, 2, 3, and 4. The output dyad consists of links 7 and 8. The cross-hatched pivot pins at $O2$, $O4$ and $O8$ are attached to the ground link (1). In the calculation below, the redundant links and their joints are not counted (subtract 2 links and 4 joints from the totals).

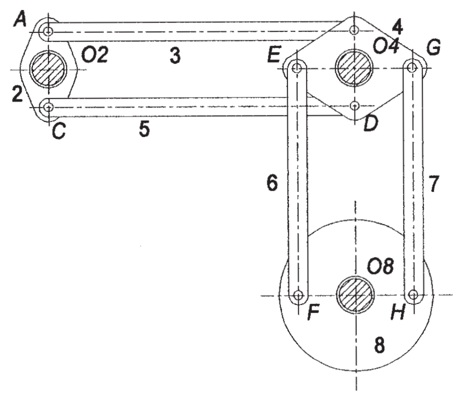
Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



f. This is a fourbar offset slider-crank linkage. The input is link 2 (crank) and the output is link 4 (slider block). The cross-hatched pivot pin at $O2$ is attached to the ground link (1).

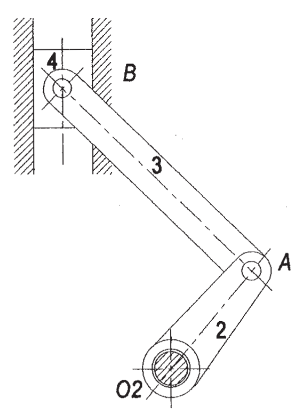
Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



g. This is a fourbar linkage with an alternate output dyad. The input (rocker) is link 2 and the outputs (rockers) are links 4 and 6. The input fourbar consists of links 1, 2, 3, and 4. The alternate output dyad consists of links 5 and 6. The cross-hatched pivot pins at $O2$, $O4$ and $O6$ are attached to the ground link (1).

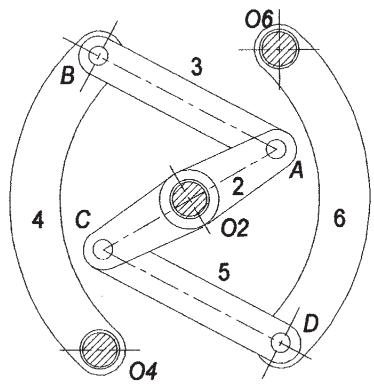
Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



h. This is a ninebar mechanism with three redundant links, which reduces it to a sixbar. Since this mechanism is symmetrical about a vertical centerline, we can split it into two mirrored mechanisms to analyze it. Either links 2, 3 and 5 or links 7, 8 and 9 are redundant. To analyze it, consider 7, 8 and 9 as the redundant links. Analyzing the ninebar, there are two full joints at the pins A, B and C for a total of 12 joints.

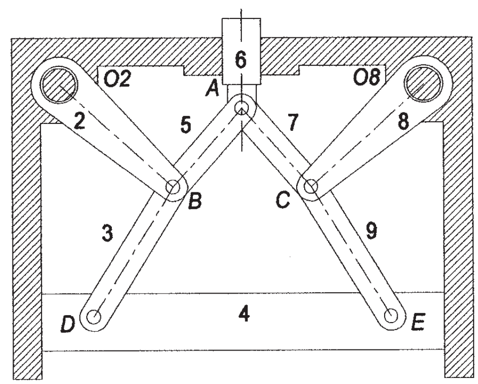
Number of links $L := 9$

Number of full joints $J_1 := 12$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 0$$



The result is that this mechanism seems to be a structure. By splitting it into mirror halves about the vertical centerline the mobility is found to be 1. Subtract the 3 redundant links and their 5 (6 minus the joint at A) associated joints to determine the mobility of the mechanism.

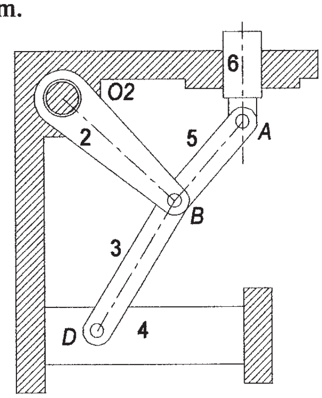
Number of links $L := 9 - 3$

Number of full joints $J_1 := 12 - 5$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



PROBLEM 2-22

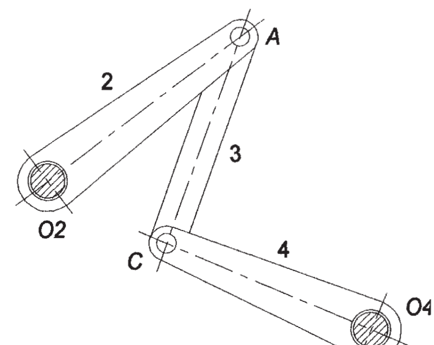
Statement: Find the Grashof condition and Barker classifications of the mechanisms in Figure P2-4a, b, and d.

Solution: See Figure P2-4 and Mathcad file P0222.

1. Use inequality 2.8 to determine the Grashof condition and Table 2-4 to determine the Barker classification.

```
Condition(S,L,P,Q) := | SL ← S + L
                      | PQ ← P + Q
                      | return "Grashof" if SL < PQ
                      | return "Special Grashof" if SL = PQ
                      | return "non-Grashof" otherwise
```

a. This is a basic fourbar linkage. The input is link 2 and the output is link 4. The cross-hatched pivot pins at *O2* and *O4* are attached to the ground link (1).



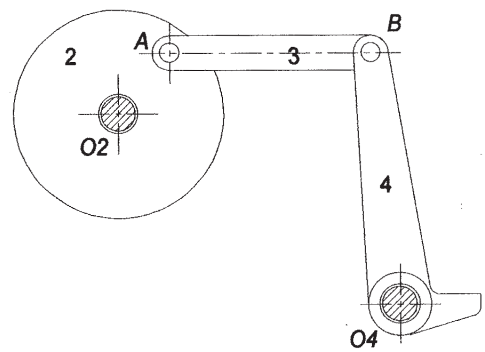
$L_1 := 174$ $L_2 := 116$

$L_3 := 108$ $L_4 := 110$

$Condition(L_3, L_1, L_2, L_4) =$ "non-Grashof"

This is a Barker Type 5 RRR1 (non-Grashof, longest link grounded).

b. This is a fourbar linkage. The input is link 2, which in this case is the wheel with a pin at *A*, and the output is link 4. The cross-hatched pivot pins at *O2* and *O4* are attached to the ground link (1).



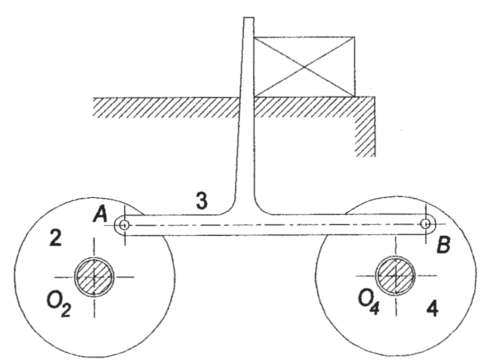
$L_1 := 162$ $L_2 := 40$

$L_3 := 96$ $L_4 := 122$

$Condition(L_2, L_1, L_3, L_4) =$ "Grashof"

This is a Barker Type 2 GCRR (Grashof, shortest link is input).

d. This is a fourbar linkage. The input is link 2, which in this case is a wheel with a pin at *A*, and the output is the vertical member on the coupler, link 3. Since the lengths of links 2 and 4 (O_2A and O_4B) are the same, the coupler link (3) has curvilinear motion and *AB* remains parallel to O_2O_4 throughout the cycle. The cross-hatched pivot pins at O_2 and O_4 are attached to the ground link (1).



$L_1 := 150$ $L_2 := 30$

$L_3 := 150$ $L_4 := 30$

$Condition(L_2, L_1, L_3, L_4) =$ "Special Grashof"

This is a Barker Type 13 S2X (special case Grashof, two equal pairs, parallelogram).

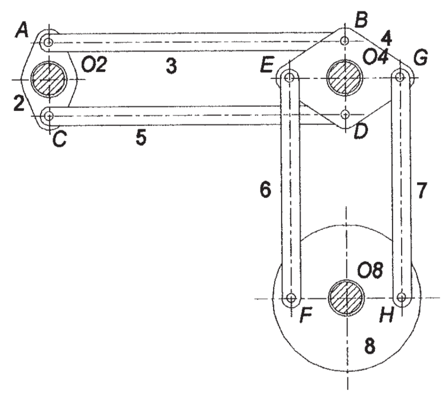
PROBLEM 2-23

Statement: Find the rotability of each loop of the mechanisms in Figure P2-4e, f, and g.

Solution: See Figure P2-4 and Mathcad file P0223.

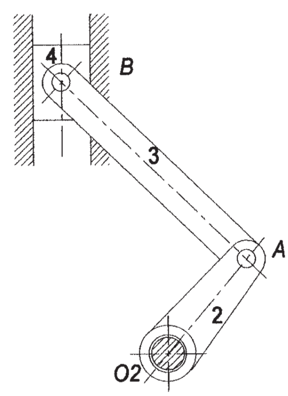
1. Use inequality 2.15 to determine the rotability of each loop in the given mechanisms.

e. This is a fourbar linkage with an output dyad. The input (rocker) is link 2 and the output (rocker) is link 8. Links 5 and 6 are redundant, i.e. the mechanism will have the same motion if they are removed. The input fourbar consists of links 1, 2, 3, and 4. The output dyad consists of links 7 and 8. The cross-hatched pivot pins at *O2, O4* and *O8* are attached to the ground link (1). In the calculation below, the redundant links and their joints are not counted (subtract 2 links and 4 joints from the totals).



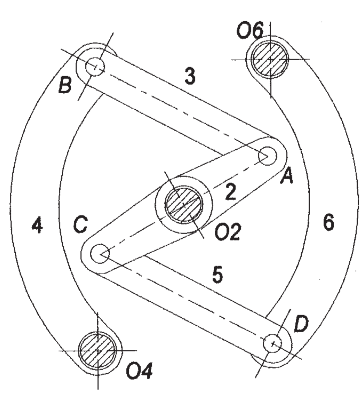
There are two loops in this mechanism. The first loop consists of links 1, 2, 3 (or 5), and 4. The second consists of links 1, 4, 7 (or 6), and 8. By inspection, we see that the sum of the shortest and longest in each loop is equal to the sum of the other two. Thus, both loops are Class III.

f. This is a fourbar offset slider-crank linkage. The input is link 2 (crank) and the output is link 4 (slider block). The cross-hatched pivot pin at *O2* is attached to the ground link (1).



We can analyze this linkage if we replace the slider (4) with an infinitely long binary link that is pinned at B to link 3 and pinned to ground (1). Then links 1 and 4 for are both infinitely long. Since these two links are equal in length and, if we say they are finite in length but very long, the rotability of the mechanism will be determined by the relative lengths of 2 and 3. Thus, this is a Class I linkage since link 2 is shorter than link 3.

g. This is a fourbar linkage with an alternate output dyad. The input (rocker) is link 2 and the outputs (rockers) are links 4 and 6. The input fourbar consists of links 1, 2, 3, and 4. The alternate output dyad consists of links 5 and 6. The cross-hatched pivot pins at *O2*, *O4* and *O6* are attached to the ground link (1).



$r_1 := 87$ $\quad r_2 := 49$

$r_3 := 100$ $\quad r_4 := 153$

Using the notation of inequality 2.15, $N := 4$

$L_N := r_4$ $\quad L_1 := r_2$

$L_2 := r_1$ $\quad L_3 := r_3$

$L_N + L_1 = 202$ $\quad L_2 + L_3 = 187$

Since $L_N + L_1 > L_2 + L_3$, this is a a class II mechanism.

PROBLEM 2-24

Statement: Find the mobility of the mechanisms in Figure P2-5.

Solution: See Figure P2-5 and Mathcad file P0224.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility. In the kinematic representations of the linkages below, binary links are depicted as single lines with nodes at their end points whereas higher order links are depicted as 2-D bars.

a. This is a sixbar linkage with 4 binary (1, 2, 5, and 6) and 2 ternary (3 and 4) links. The inverted U-shaped link at the top of Figure P2-5a is represented here as the binary link 6.

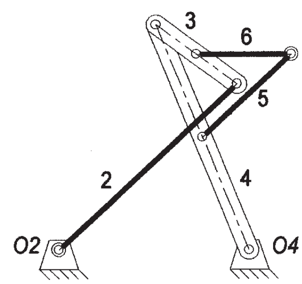
Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



b. This is an eightbar linkage with 4 binary (1, 4, 7, and 8) and 4 ternary (2, 3, 5, and 6) links. The inverted U-shaped link at the top of Figure P2-5b is represented here as the binary link 8.

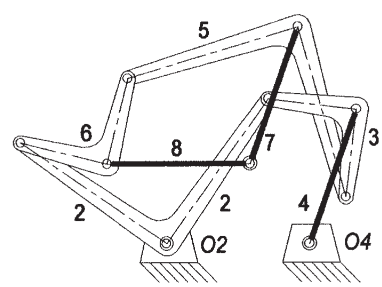
Number of links $L := 8$

Number of full joints $J_1 := 10$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



PROBLEM 2-25

Statement: Find the mobility of the ice tongs in Figure P2-6.
a. When operating them to grab the ice block.
b. When clamped to the ice block but before it is picked up (ice grounded).
c. When the person is carrying the ice block with the tongs.

Solution: See Figure P2-6 and Mathcad file P0225.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

a. In this case there are two links and one full joint and 1 *DOF*.

Number of links $L := 2$

Number of full joints $J_1 := 1$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

b. When the block is clamped in the tongs another link and two more full joints are added reducing the *DOF* to zero (the tongs and ice block form a structure).

Number of links $L := 2 + 1$

Number of full joints $J_1 := 1 + 2$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 0$$

c. When the block is being carried the system has at least 4 *DOF*: *x*, *y*, and *z* position and orientation about a vertical axis.

PROBLEM 2-26

Statement: Find the mobility of the automotive throttle mechanism shown in Figure P2-7.

Solution: See Figure P2-7 and Mathcad file P0226.

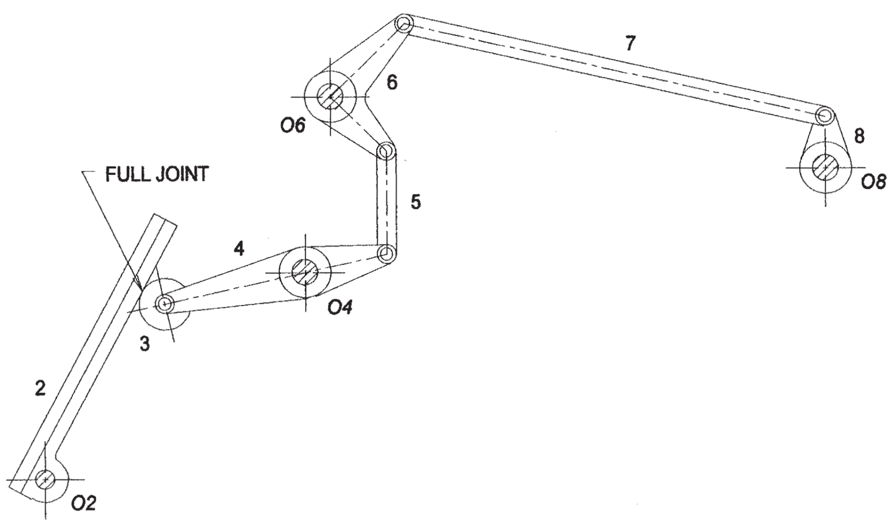
1. This is an eightbar linkage with 8 binary links. It is assumed that the joint between the gas pedal (2) and the roller (3) that pivots on link 4 is a full joint, i.e. the roller rolls without slipping. The pivot pins at *O2*, *O4*, *O6*, and *O8* are attached to the ground link (1). Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

Number of links $L := 8$

Number of full joints $J_1 := 10$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

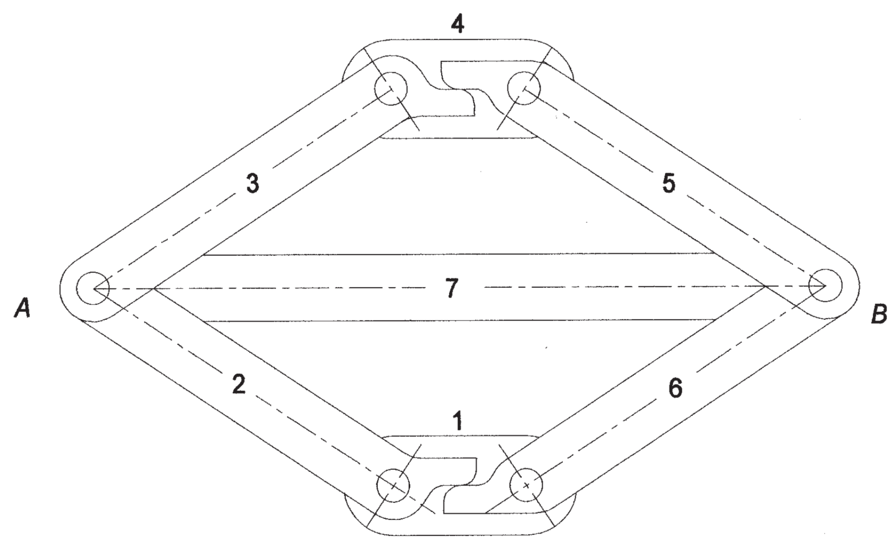


PROBLEM 2-27

Statement: Sketch a kinematic diagram of the scissors jack shown in Figure P2-8 and determine its mobility. Describe how it works.

Solution: See Figure P2-8 and Mathcad file P0227.

1. The scissors jack depicted is a seven link mechanism with eight full and two half joints (see kinematic diagram below). Link 7 is a variable length link. Its length is changed by rotating the screw with the jack handle (not shown). The two blocks at either end of link 7 are an integral part of the link. The block on the left is threaded and acts like a nut. The block on the right is not threaded and acts as a bearing. Both blocks have pins that engage the holes in links 2, 3, 5, and 6. Joints *A* and *B* have 2 full joints apiece. For any given length of link 7 the jack is a structure ($DOF = 0$). When the screw is turned to give the jack a different height the jack has 1 *DOF*.



Number of links $L := 7$

Number of full joints $J_1 := 8$

Number of half joints $J_2 := 2$

$M := 3\cdot(L - 1) - 2\cdot J_1 - J_2$ $M = 0$

PROBLEM 2-28

Statement: Find the mobility of the corkscrew in Figure P2-9.

Solution: See Figure P2-9 and Mathcad file P0228.

1. The corkscrew is made from 4 pieces: the body (1), the screw (2), and two arms with teeth (3), one of which is redundant. The second arm is present to balance the forces on the assembly but is not necessary from a kinematic standpoint. So, kinematically, there are 3 links (body, screw, and arm), 2 full joints (sliding joint between the screw and the body, and pin joint where the arm rotates on the body), and 1 half joint where the arm teeth engage the screw "teeth". Using equation 2.1c, the *DOF* (mobility) is

Number of links $L := 3$

Number of full joints $J_1 := 2$

Number of half joints $J_2 := 1$

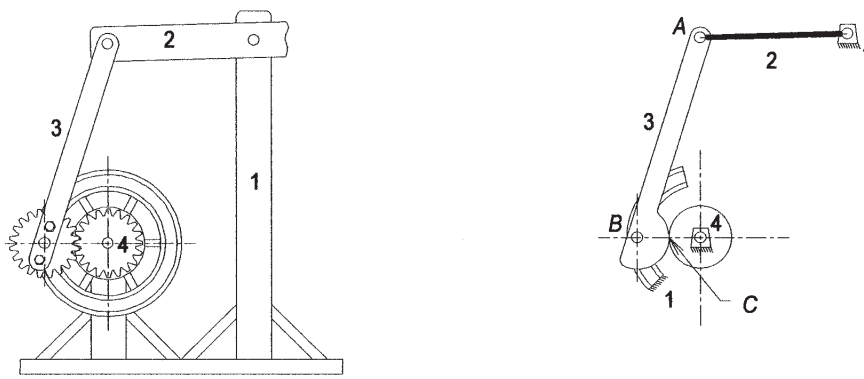
$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-29

Statement: Figure P2-10 shows Watt's sun and planet drive that he used in his steam engine. The beam 2 is driven in oscillation by the piston of the engine. The planet gear is fixed rigidly to link 3 and its center is guided in the fixed track 1. The output rotation is taken from the sun gear 4. Sketch a kinematic diagram of this mechanism and determine its *DOF*. Can it be classified by the Barker scheme? If so, what Barker class and subclass is it?

Solution: See Figure P2-10 and Mathcad file P0229.

1. Sketch a kinematic diagram of the mechanism. The mechanism is shown on the left and a kinematic model of it is sketched on the right. It is a fourbar linkage with 1 *DOF* (see below).



2. Use equation 2.1c to determine the *DOF* (mobility). There are 4 links, 3 full pin joints, 1 half pin-in-slot joint (at *B*), and 1 half joint (at the interface *C* between the two gears, shown above by their pitch circles). Links 1 and 3 are ternary.

Kutzbach's mobility equation (2.1c)

Number of links $L := 4$

Number of full joints $J_1 := 3$

Number of half joints $J_2 := 2$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \qquad M = 1$$

3. The Barker classification scheme requires that we have 4 link lengths. The motion of link 3 can be modeled by a basic fourbar if the half joint at *B* is replaced with a full pin joint and a link is added to connect *B* and the fixed pivot that is coincident with the center of curvature of the slot that guides pin *B*.

$L_1 := 2.15 \qquad L_2 := 1.25$

$L_3 := 1.80 \qquad L_4 := 0.54$

This is a Grashof linkage and the Barker classification is I-4 (type 4) because the shortest link is the output.

PROBLEM 2-30

Statement: Figure P2-11 shows a bicycle hand brake lever assembly. Sketch a kinematic diagram of this device and draw its equivalent linkage. Determine its mobility. Hint: Consider the flexible cable to be a link.

Solution: See Figure P2-11 and Mathcad file P0230.

1. The motion of the flexible cable is along a straight line as it leaves the guide provided by the handle bar so it can be modeled as a translating full slider that is supported by the handlebar (link 1). The brake lever is a binary link that pivots on the ground link. Its other node is attached through a full pin joint to a third link, which drives the slider (link 4).

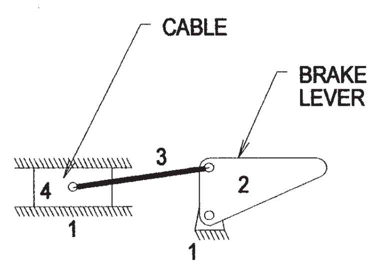
Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



PROBLEM 2-31

Statement: Figure P2-12 shows a bicycle brake caliper assembly. Sketch a kinematic diagram of this device and draw its equivalent linkage. Determine its mobility under two conditions.

a. Brake pads not contacting the wheel rim.
b. Brake pads contacting the wheel rim.

Hint: Consider the flexible cable to be replaced by forces in this case.

Solution: See Figure P2-12 and Mathcad file P0231.

1. The rigging of the cable requires that there be two brake arms. However, kinematically they operate independently and can be analyzed that way. Therefore, we only need to look at one brake arm. When the brake pads are not contacting the wheel rim there is a single lever (link 2) that is pivoted on a full pin joint that is attached to the ground link (1). Thus, there are two links (frame and brake arm) and one full pin joint.

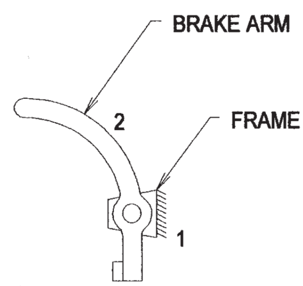
Number of links $L := 2$

Number of full joints $J_1 := 1$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



2. When the brake pad contacts the wheel rim we could consider the joint between the pad, which is rigidly attached to the brake arm and is, therefore, a part of link 2, to be a half joint. The brake arm (with pad), wheel (which is constrained from moving laterally by the frame), and the frame constitute a structure.

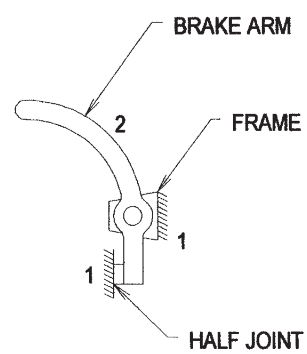
Number of links $L := 2$

Number of full joints $J_1 := 1$

Number of half joints $J_2 := 1$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$$

$$M = 0$$



PROBLEM 2-32

Statement: Find the mobility, the Grashof condition, and the Barker classifications of the mechanism in Figure P2-13.

Solution: See Figure P2-13 and Mathcad file P0232.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

   When there is no cable in the jaw or before the cable is crimped this is a basic fourbar mechanism with with 4 full pin joints:

   Number of links $L := 4$

   Number of full joints $J_1 := 4$

   Number of half joints $J_2 := 0$ $\quad M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \quad M = 1$

   When there is a cable in the jaw this is a threebar mechanism with with 3 full pin joints. While the cable is clamped the jaws are stationary with respect to each other so that link 4 is grounded along with link 1, leaving only three operational links.

   Number of links $L := 3$

   Number of full joints $J_1 := 3$

   Number of half joints $J_2 := 0$ $\quad M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \quad M = 0$

2. Use inequality 2.8 to determine the Grashof condition and Table 2-4 to determine the Barker classification.

$$Condition(S,L,P,Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array} \right.$$

$L_1 := 0.92 \qquad L_2 := 0.27$

$L_3 := 0.50 \qquad L_4 := 0.60$

$Condition(L_2, L_1, L_3, L_4) =$ "non-Grashof"

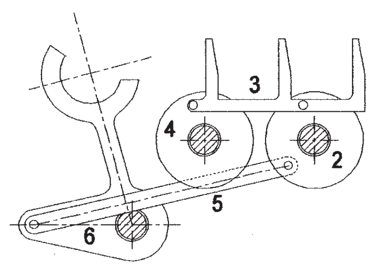
The Barker classification is II-1 (Type 5) RRR1 (non-Grashof, longest link grounded).

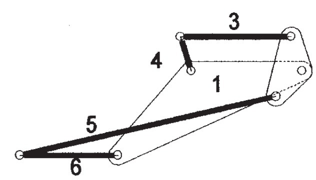
PROBLEM 2-33

Statement: Figure P2-14 shows a "pick-and-place" mechanism in combination with a "walking beam." Sketch its kinematic diagram, determine its mobility and its type (i.e., is it a fourbar, a Watt's sixbar, a Stephenson's sixbar, an eightbar, or what?) Make a cardboard model of all but the gear train portion and examine its motions. Describe what it does.

Solution: See Figure P2-14 and Mathcad file P0233.

1. Sketch a kinematic diagram of the mechanism (without the conveyor). The mechanism is shown on the left and a kinematic model of it is sketched on the right. It is a Watt's sixbar. There are two ternary links (1 and 2) that have a common joint and the remaining joints on the ternary links are connected by two binary links. This is characteristic of a Watt's sixbar. Link 1 is the ground link about which links 2, 4, and 6 are pivoted.





2. Describe what it does: Links 1, 2, 3, and 4 are a fourbar with link 2 as the input that rotates at constant speed. Because links 2 and 4 have the same length, and links 1 and 3 have the same length, link 4 rotates at the same speed as link 2 and link 3 has curvilinear motion, i.e., it is always parallel to link 1, which is the ground link. Link 3 (the coupler) moves to the left while in the position shown, but it is also moving downward. The tips of the comb on link 3 will move below the surface of the conveyor and will then move to the right and, eventually, start moving upward again. When the tips of the comb get back up to the surface of the conveyor they will be displaced to the right a distance of about one diameter of the cylinders on the conveyor. Thus, when they start moving to the left again to complete their cycle, they will have come in contact with the next cylinder down the conveyor to the right and will move it and the others ahead of it to the left. Meanwhile, the dyad composed of links 5 and 6 will be rocking back and forth as link 2 rotates. The rightmost cylinder on the conveyor will be pushed off into the cup that is an extension of link 6. Due to the rocking motion of link 6, the cylinder will roll out of the cup when link 6 is at its extreme counterclockwise position.

3. Use equation 2.1c to calculate the *DOF* (mobility).

Kutzbach's mobility equation (2.1c)

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

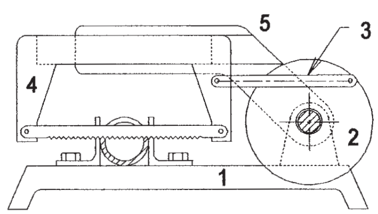
$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \qquad M = 1$$

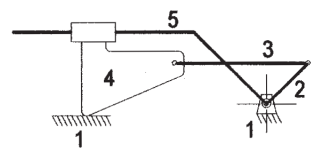
PROBLEM 2-34

Statement: Figure P2-15 shows a power hacksaw, used to cut metal. Link 5 pivots at O5 and its weight forces the sawblade against the workpiece while the linkage moves the blade (link 4) back and forth within link 5 to cut the part. Sketch its kinematic diagram, determine its mobility and its type (i.e., is it a fourbar, a Watt's sixbar, a Stephenson's sixbar, an eightbar, or what?) Use reverse linkage transformation to determine its pure revolute-jointed equivalent linkage.

Solution: See Figure P2-15 and Mathcad file P0234.

1. Sketch a kinematic diagram of the mechanism. The mechanism is shown on the left and a kinematic model of it is sketched on the right. It is a fivebar linkage with 1 *DOF* (see below).





2. Use equation 2.1c to determine the *DOF* (mobility). There are 5 links, 4 full pin joints, 1 full sliding joint, and 1 half joint (at the interface between the hacksaw blade and the pipe being cut).

Kutzbach's mobility equation (2.1c)

Number of links $L := 5$

Number of full joints $J_1 := 5$

Number of half joints $J_2 := 1$

$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$ $M = 1$

3. Use rule 1 (on page 42) to transform the full sliding joint to a full pin joint for no change in *DOF*. Then use rules 2 and 3 by changing the half joint to a full pin joint and adding a link for no change in *DOF*. The resulting kinematically equivalent linkage has 6 links, 7 full pin joints, no half joints, and is shown below.

Kutzbach's mobility equation (2.1c)

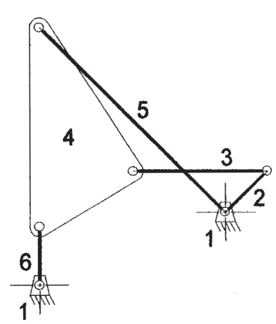
Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$

$M = 1$

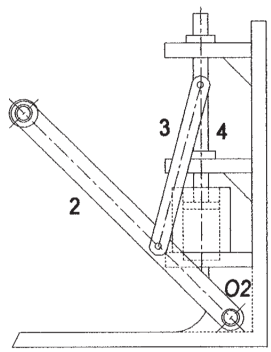


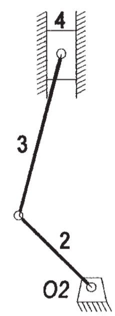
PROBLEM 2-35

Statement: Figure P2-16 shows a manual press used to compact powdered materials. Sketch its kinematic diagram, determine its mobility and its type (i.e., is it a fourbar, a Watt's sixbar, a Stephenson's sixbar, an eightbar, or what?) Use reverse linkage transformation to determine its pure revolute-jointed equivalent linkage.

Solution: See Figure P2-16 and Mathcad file P0235.

1. Sketch a kinematic diagram of the mechanism. The mechanism is shown on the left and a kinematic model of it is sketched on the right. It is a fourbar linkage with 1 *DOF* (see below).





2. Use equation 2.1c to determine the *DOF* (mobility). There are 4 links, 3 full pin joints, 1 full sliding joint, and 0 half joints. This is a fourbar slider-crank.

Kutzbach's mobility equation (2.1c)

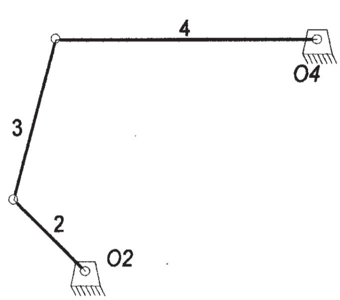
Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2$ $\qquad M = 1$

3. Use rule 1 (on page 42) to transform the full sliding joint to a full pin joint for no change in *DOF*. The resulting kinematically equivalent linkage has 4 links, 4 full pin joints, no half joints, and is shown below.

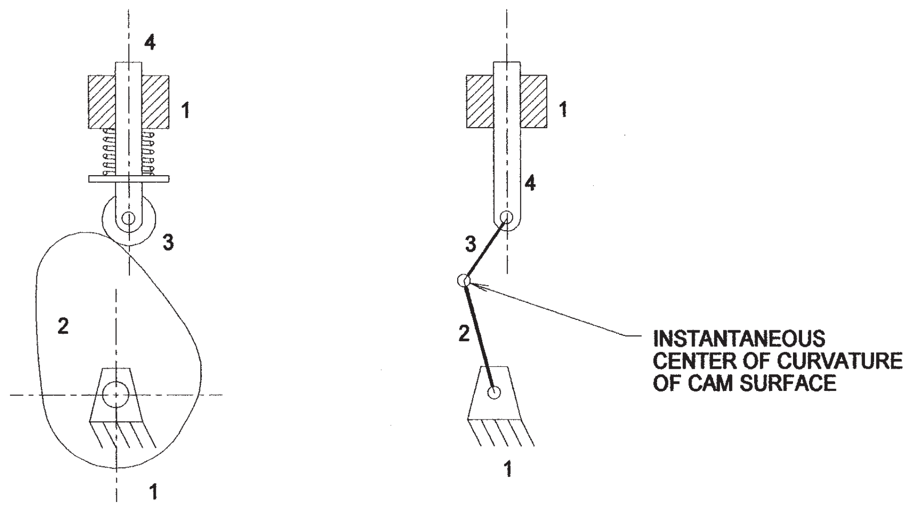


PROBLEM 2-36

Statement: Sketch the equivalent linkage for the cam and follower mechanism in Figure P2-17 in the position shown. Show that it has the same *DOF* as the original mechanism.

Solution: See Figure P2-17 and Mathcad file P0236.

1. The cam follower mechanism is shown on the left and a kinematically equivalent model of it is sketched on the right.



2. Use equation 2.1c to determine the *DOF* (mobility) of the original mechanism. There are 4 links, 2 full pin joints, 1 full sliding joint, 1 pure rolling joint and 0 half joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

3. Use equation 2.1c to determine the *DOF* (mobility) of the equivalent mechanism. There are 4 links, 3 full pin joints, 1 full sliding joint, and 0 half joints. This is a fourbar slider-crank.

Kutzbach's mobility equation (2.1c)

Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-37

Statement: Describe the motion of the following rides, commonly found at an amusement park, as pure rotation, pure translation, or complex planar motion.

a. A Ferris wheel
b. A "bumper" car
c. A drag racer ride
d. A roller coaster whose foundation is laid out in a straight line
e. A boat ride through a maze
f. A pendulum ride
g. A train ride

Solution: See Mathcad file P0211.

a. A Ferris wheel
Pure rotation.

b. A "bumper car"
Complex planar motion.

c. A drag racer ride
Pure translation.

d. A roller coaster whose foundation is laid out in a straight line
Complex planar motion.

e. A boat ride through a maze
Complex planar motion.

f. A pendulum ride
Pure rotation.

g. A train ride
Complex planar motion.

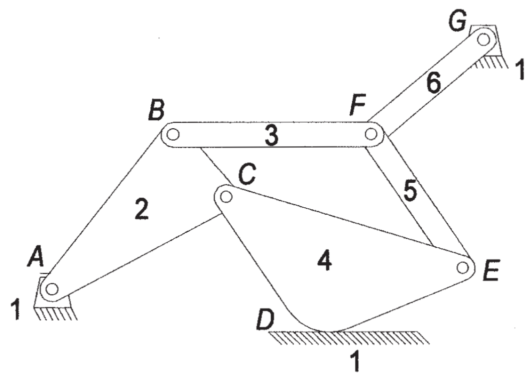
PROBLEM 2-38

Statement: Figure P2-1a is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with *A*.

a. Using the link numbers, describe each link as binary, ternary, etc.
b. Using the joint letters, determine each joint's order.
c. Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1a and Mathcad file P0238.

1. Label the link numbers and joint letters for Figure P2-1a.



a. Using the link numbers, describe each link as binary, ternary, etc.

Link No.	Link Order
1	Ternary
2	Ternary
3	Binary
4	Ternary
5	Binary
6	Binary

b,c. Using the joint letters, determine each joint's order and whether each is a half or full joint.

Joint Letter	Joint Order	Half/Full
A	1	Full
B	1	Full
C	1	Full
D	1	Half
E	1	Full
F	2	Full
G	1	Full

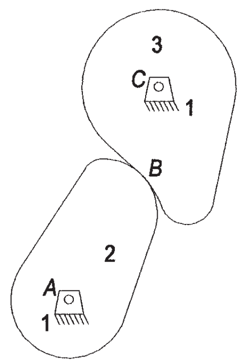
PROBLEM 2-39

Statement: Figure P2-1b is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with *A*.

a. Using the link numbers, describe each link as binary, ternary, etc.
b. Using the joint letters, determine each joint's order.
c. Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1b and Mathcad file P0239.

1. Label the link numbers and joint letters for Figure P2-1b.



a. Using the link numbers, describe each link as binary, ternary, etc.

Link No.	Link Order
1	Binary
2	Binary
3	Binary

b,c. Using the joint letters, determine each joint's order and whether each is a half or full joint.

Joint Letter	Joint Order	Half/Full
A	1	Full
B	1	Half
C	1	Full

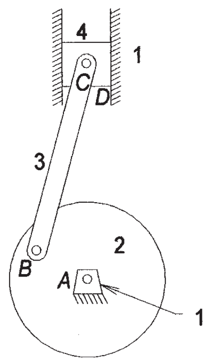
PROBLEM 2-40

Statement: Figure P2-1c is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with *A*.

a. Using the link numbers, describe each link as binary, ternary, etc.
b. Using the joint letters, determine each joint's order.
c. Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1c and Mathcad file P0240.

1. Label the link numbers and joint letters for Figure P2-1c.



a. Using the link numbers, describe each link as binary, ternary, etc.

Link No.	Link Order
1	Binary
2	Binary
3	Binary
4	Binary

b,c. Using the joint letters, determine each joint's order and whether each is a half or full joint.

Joint Letter	Joint Order	Half/Full
A	1	Full
B	1	Full
C	1	Full
D	1	Full

PROBLEM 2-41

Statement: Figure P2-1d is an example of a mechanism. Number the links, starting with 1. (Hint: Don't forget the "ground" link.) Letter the joints alphabetically, starting with *A*.

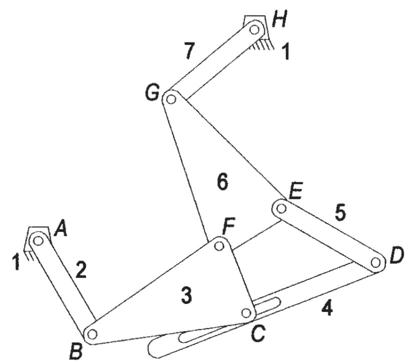
a. Using the link numbers, describe each link as binary, ternary, etc.

b. Using the joint letters, determine each joint's order.

c. Using the joint letters, determine whether each is a half or full joint.

Solution: See Figure P2-1d and Mathcad file P0241.

1. Label the link numbers and joint letters for Figure P2-1d.



a. Using the link numbers, describe each link as binary, ternary, etc.

Link No.	Link Order
1	Binary
2	Binary
3	Ternary
4	Binary
5	Binary
6	Ternary
7	Binary

b,c. Using the joint letters, determine each joint's order and whether each is a half or full joint.

Joint Letter	Joint Order	Half/Full
A	1	Full
B	1	Full
C	1	Half
D	1	Full
E	1	Full
F	1	Full
G	1	Full
H	1	Full

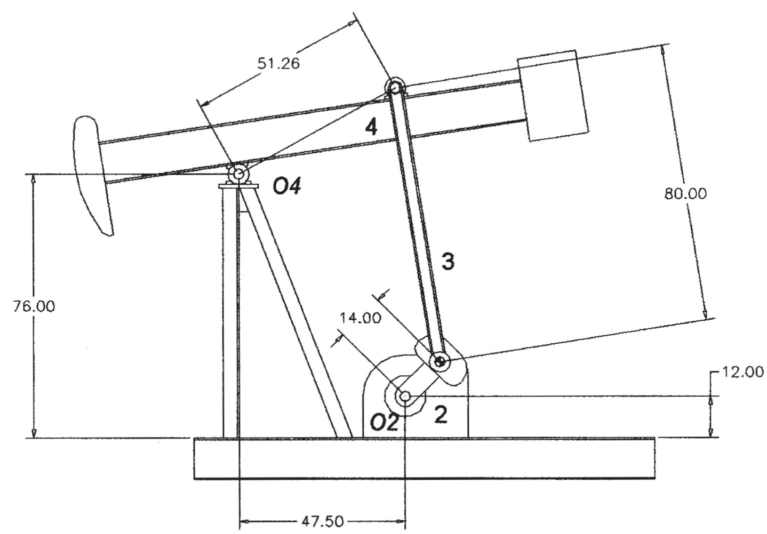
PROBLEM 2-42

Statement: Find the mobility, Grashof condition and Barker classification of the oil field pump shown in Figure P2-18.

Solution: See Figure P2-18 and Mathcad file P0242.

1. Use inequality 2.8 to determine the Grashof condition and Table 2-4 to determine the Barker classification.

$$Condition(S, L, P, Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textit{return } \text{"Grashof"} \textit{ if } SL < PQ \\ \textit{return } \text{"Special Grashof"} \textit{ if } SL = PQ \\ \textit{return } \text{"non-Grashof"} \textit{ otherwise} \end{array} \right.$$



This is a basic fourbar linkage. The input is the 14-in-long crank (link 2) and the output is the top beam (link 4). The mobility (*DOF*) is found using equation 2.1c (Kutzbach's modification):

Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$ $\quad M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \quad M = 1$

The link lengths and Grashof condition are

$$L_1 := \sqrt{(76-12)^2 + 47.5^2} \qquad L_1 = 79.701 \qquad L_2 := 14 \qquad L_3 := 80 \qquad L_4 := 51.26$$

$$Condition(L_2, L_3, L_1, L_4) = \text{"Grashof"}$$

This is a Barker Type 2 GCRR (Grashof, shortest link is input).

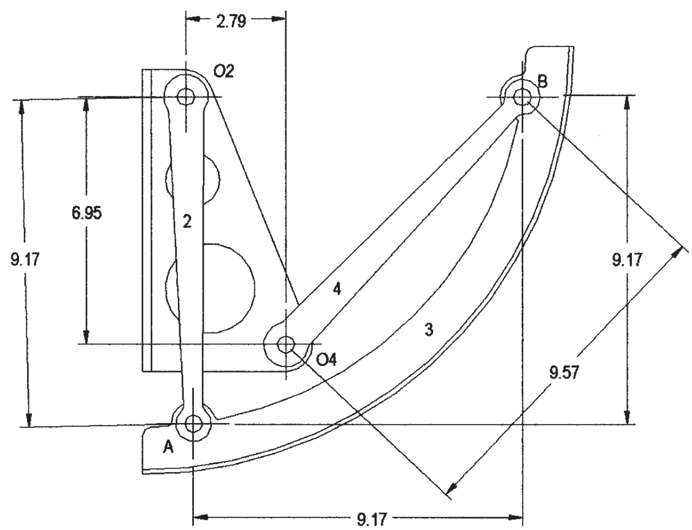
PROBLEM 2-43

Statement: Find the mobility, Grashof condition and Barker classification of the aircraft overhead bin shown in Figure P2-19.

Solution: See Figure P2-19 and Mathcad file P0243.

1. Use inequality 2.8 to determine the Grashof condition and Table 2-4 to determine the Barker classification.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\quad if\ SL < PQ \\ return\ \text{"Special Grashof"}\quad if\ SL = PQ \\ return\ \text{"non-Grashof"}\quad otherwise \end{array}$$



This is a basic fourbar linkage. The input is the link 2 and the output is link 4. The mobility (*DOF*) is found using equation 2.1c (Kutzbach's modification):

Number of links $L := 4$

Number of full joints $J_1 := 4$

Number of half joints $J_2 := 0$ $\quad M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \quad M = 1$

The link lengths and Grashof condition are

$$L_1 := \sqrt{2.79^2 + 6.95^2} \qquad L_1 = 7.489 \qquad L_2 := 9.17$$

$$L_3 := \sqrt{9.17^2 + 9.17^2} \qquad L_3 = 12.968 \qquad L_4 := 9.57$$

$$Condition(L_1, L_3, L_2, L_4) = \text{"non-Grashof"}$$

This is a Barker Type 7 RRR3 (non-Grashof, longest link is coupler).

PROBLEM 2-44

Statement: Figure P2-20 shows a "Rube Goldberg" mechanism that turns a light switch on when a room door is opened and off when the door is closed. The pivot at O_2 goes through the wall. There are two spring-loaded piston-in cylinder devices in the assembly. An arrangement of ropes and pulleys inside the room transfers the door swing into a rotation of link 2. Door opening rotates link 2 CW, pushing the switch up as shown in the figure, and door closing rotates link 2 CCW, pulling the switch down. Find the mobility of the linkage.

Solution: See Figure P2-20 and Mathcad file P0244.

1. Examination of the figure shows 20 links (including the the switch) and 28 full joints. The second piston-in cylinder that actuates the switch is counted as a single binary link of variable length with joints at its ends. The other cylinder consists of two binary links, each link having one pin joint and one slider joint. There are no half joints.
2. Use equation 2.1c to determine the *DOF* (mobility).

Kutzbach's mobility equation (2.1c)

Number of links $L := 20$

Number of full joints $J_1 := 28$

Number of half joints $J_2 := 0$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \qquad M = 1$$

3. An alternative is to ignore the the first piston-in cylinder that acts on the third bellcrank from O_2 since it does not affect the the motion of the linkage (it acts only as a damper.) In that case, subtract two links and three full joints, giving $L = 18$, $J_1 = 25$ and $M = 1$.

PROBLEM 2-45

Statement: Use Working Model to create and animate the mechanism in Figure P2-14.

Solution: See Figure P2-14 and Working Model file P0245.

1. See the Working Model file on the CD for solution.

PROBLEM 2-46

Statement: Use Working Model to create and animate the mechanism in Figure P2-15.

Solution: See Figure P2-15 and Working Model file P0246.

1. See the Working Model file on the CD for solution.

PROBLEM 2-47

Statement: Use Working Model to create and animate the mechanism in Figure P2-18.

Solution: See Figure P2-18 and Working Model file P0247.

1. See the Working Model file on the CD for solution.

PROBLEM 2-48

Statement: Find the mobility of the mechanism shown in Figure 3-33.

Solution: See Figure 3-33 and Mathcad file P0248.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 6 links, 7 full pin joints (two at *B*), and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-49

Statement: Find the mobility of the mechanism shown in Figure 3-34.

Solution: See Figure 3-34 and Mathcad file P0249.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 6 links, 7 full pin joints, and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-50

Statement: Find the mobility of the mechanism shown in Figure 3-35.

Solution: See Figure 3-35 and Mathcad file P0250.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 6 links, 7 full pin joints, and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3 \cdot (L - 1) - 2 \cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-51

Statement: Find the mobility of the mechanism shown in Figure 3-36.

Solution: See Figure 3-36 and Mathcad file P0251.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 8 links, 10 full pin joints (two at O_4), and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 8$

Number of full joints $J_1 := 10$

Number of half joints $J_2 := 0$

$$M := 3\cdot(L - 1) - 2\cdot J_1 - J_2 \qquad M = 1$$

PROBLEM 2-52

Statement: Find the mobility of the mechanism shown in Figure 3-37.

Solution: See Figure 3-37 and Mathcad file P0252.

1. Use equation 2.1c to determine the *DOF* (mobility). There are 6 links, 7 full pin joints (two at O_4), and no half-joints.

Kutzbach's mobility equation (2.1c)

Number of links $L := 6$

Number of full joints $J_1 := 7$

Number of half joints $J_2 := 0$

$$M := 3\cdot(L-1) - 2\cdot J_1 - J_2 \qquad M = 1$$