Chapter 4

POSITION ANALYSIS

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PROBLEM 4-1

Statement: A position vector is defined as having length equal to your height in inches (or centimeters). The tangent of its angle is defined as your weight in lbs (or kg) divided by your age in years. Calculate the data for this vector and:

a. Draw the position vector to scale on Cartesian axes.
b. Write an expression for the position vector using unit vector notation.
c. Write an expression for the position vector using complex number notation, in both polar and Cartesian forms.

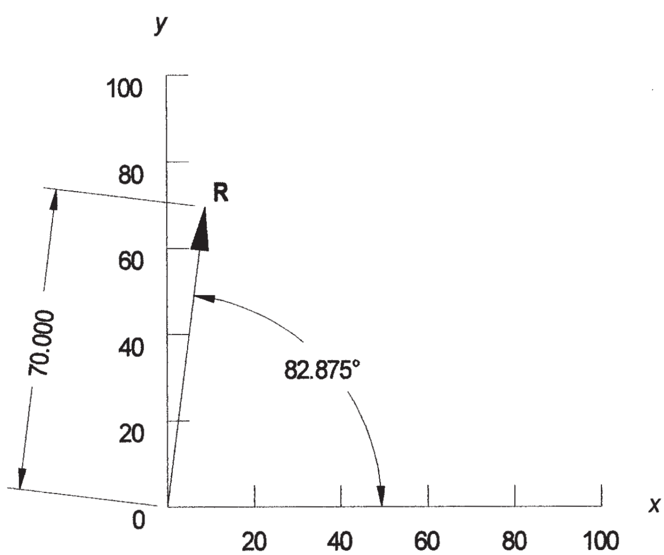
Assumptions: $Height := 70$ $weight := 160$ $age := 20$

Solution: See Mathcad file P0401.

1. The magnitude of the vector is $R := Height$. The angle that the vector makes with the x-axis is

$$\theta := atan\left(\frac{weight}{age}\right) \qquad \theta = 82.875\,deg \qquad \theta = 1.446\,rad$$

a. Draw the position vector to scale on Cartesian axes.



b. Write an expression for the position vector using unit vector notation.

$$\mathbf{R} := R\cdot\begin{pmatrix} cos(\theta) \\ sin(\theta) \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} 8.682 \\ 69.459 \end{pmatrix} \qquad \mathbf{R} = 8.682\,\mathbf{i} + 69.459\,\mathbf{j}$$

c. Write an expression for the position vector using complex number notation, in both polar and Cartesian forms.

Polar form: $\mathbf{R} := 68\cdot\mathbf{e}^{j\cdot 1.446}$

Cartesian form: $\mathbf{R} := 8.682 + j\cdot 69.459$

PROBLEM 4-2

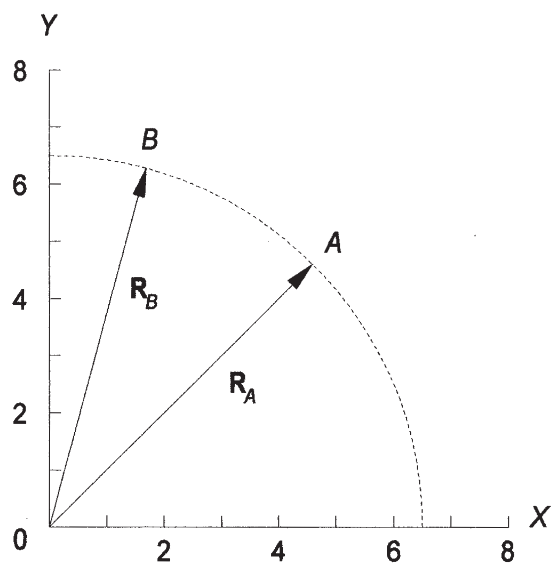
Statement: A particle is traveling along an arc of 6.5 inch radius. The arc center is at the origin of a coordinate system. When the particle is at position *A*, its position vector makes a 45-deg angle with the *X* axis. At position *B*, its vector makes a 75-deg angle with the *X* axis. Draw this system to some convenient scale and:

a. Write an expression for the particle's position vector in position *A* using complex number notation, in both polar and Cartesian forms.
b. Write an expression for the particle's position vector in position *B* using complex number notation, in both polar and Cartesian forms.
c. Write a vector equation for the position difference between points *B* and *A*. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.
d. Check the result of part c with a graphical method.

Given: Circle radius and vector magnitude, $R := 6.5\cdot in$; vector angles: $\theta_A := 45\cdot deg$ $\theta_B := 75\cdot deg$

Solution: See Mathcad file P0402.

1. Establish an *X-Y* coordinate frame and draw a circle with center at the origin and radius *R*.
2. Draw lines from the origin that make angles of 45 and 75 deg with respect to the *X* axis. Label the intersections of the lines with the circles as *A* and *B*, respectively. Make the line segment *OA* a vector by putting an arrowhead at *A*, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment *OB*, labeling it $\mathbf{R}_B$.



a. Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.

Polar form: $\mathbf{R_A} := R\cdot e^{j\cdot\theta_A}$ $\mathbf{R_A} := 6.5\cdot e^{j\cdot\frac{\pi}{4}}$

Cartesian form: $\mathbf{R_A} := R\cdot\left(cos\left(\theta_A\right) + j\cdot sin\left(\theta_A\right)\right)$ $\mathbf{R_A} = 4.596 + 4.596j\ in$

b. Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.

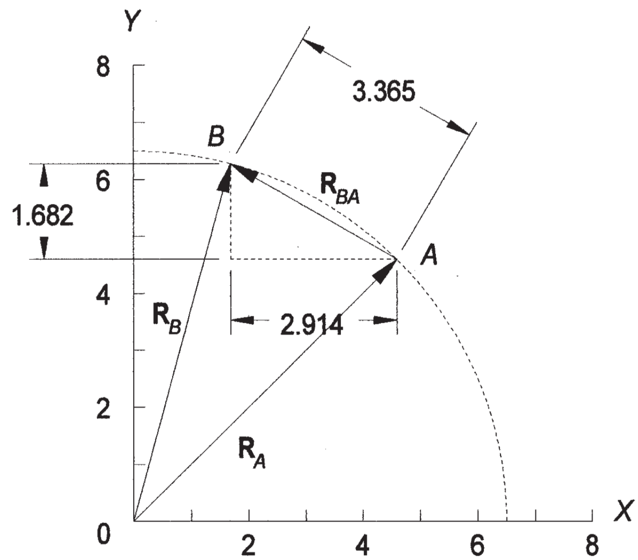
Polar form: $\mathbf{R_B} := R \cdot \mathbf{e}^{j \cdot \theta_B}$ $\mathbf{R_B} := 6.5 \cdot \mathbf{e}^{j \cdot \frac{75 \cdot \pi}{180}}$

Cartesian form: $\mathbf{R_B} := R \cdot \left(\cos\left(\theta_B\right) + j \cdot \sin\left(\theta_B\right)\right)$ $\mathbf{R_B} = 1.682 + 6.279j \; in$

c. *Write a vector equation for the position difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.*

$\mathbf{R_{BA}} := \mathbf{R_B} - \mathbf{R_A}$ $\mathbf{R_{BA}} = -2.914 + 1.682j \; in$

d. *Check the result of part c with a graphical method.*



On the layout above the X and Y components of $\mathbf{R}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 4-3

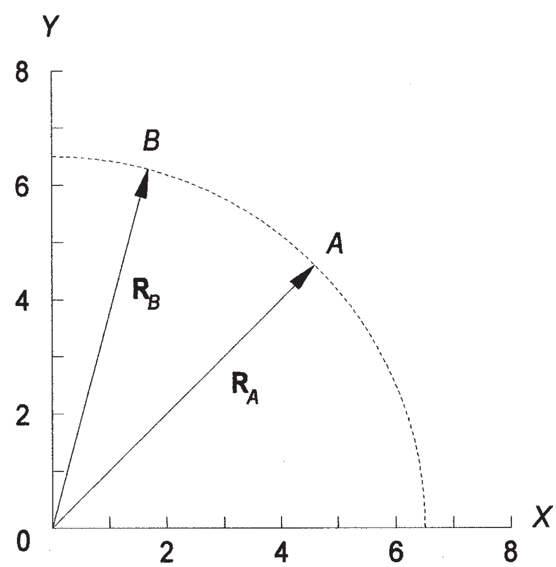
Statement: Two particles are traveling along an arc of 6.5 inch radius. The arc center is at the origin of a coordinate system. When one particle is at position *A*, its position vector makes a 45-deg angle with the *X* axis. Simultaneously, the other particle is at position *B*, where its vector makes a 75-deg angle with the *X* axis. Draw this system to some convenient scale and:

a. Write an expression for the particle's position vector in position *A* using complex number notation, in both polar and Cartesian forms.
b. Write an expression for the particle's position vector in position *B* using complex number notation, in both polar and Cartesian forms.
c. Write a vector equation for the relative position of the particle at *B* with respect to the particle at *A*. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.
d. Check the result of part c with a graphical method.

Given: Circle radius and vector magnitude, $R := 6.5 \cdot in$; vector angles: $\theta_A := 45 \cdot deg$ $\theta_B := 75 \cdot deg$

Solution: See Mathcad file P0403.

1. Establish an *X-Y* coordinate frame and draw a circle with center at the origin and radius *R*.
2. Draw lines from the origin that make angles of 45 and 75 deg with respect to the *X* axis. Label the intersections of the lines with the circles as *A* and *B*, respectively. Make the line segment *OA* a vector by putting an arrowhead at *A*, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment *OB*, labeling it $\mathbf{R}_B$.



a. Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.

Polar form: $\mathbf{R_A} := R \cdot e^{j \cdot \theta_A}$ $\mathbf{R_A} := 6.5 \cdot e^{j \cdot \frac{\pi}{4}}$

Cartesian form: $\mathbf{R_A} := R \cdot \left(cos(\theta_A) + j \cdot sin(\theta_A)\right)$ $\mathbf{R_A} = 4.596 + 4.596j \ in$

b. Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.

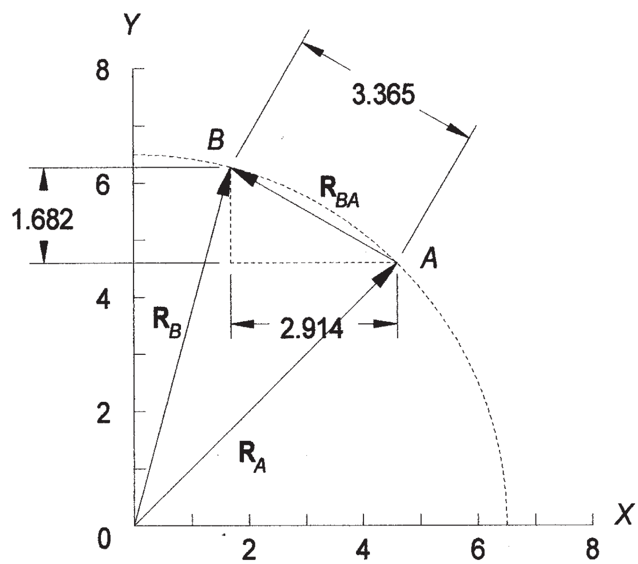
Polar form: $\mathbf{R_B} := R\cdot \mathbf{e}^{j\cdot\theta_B}$ $\mathbf{R_B} := 6.5\cdot \mathbf{e}^{j\cdot\frac{75\cdot\pi}{180}}$

Cartesian form: $\mathbf{R_B} := R\cdot\left(cos\left(\theta_B\right) + j\cdot sin\left(\theta_B\right)\right)$ $\mathbf{R_B} = 1.682 + 6.279j\ in$

c. *Write a vector equation for the relative position of the particle at B with respect to the particle at A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.*

$\mathbf{R_{BA}} := \mathbf{R_B} - \mathbf{R_A}$ $\mathbf{R_{BA}} = -2.914 + 1.682j\ in$

d. *Check the result of part c with a graphical method.*



On the layout above the *X* and *Y* components of $\mathbf{R}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 4-4

Statement: A particle is traveling along the line $y = -2x + 10$. When the particle is at position A, its position vector makes a 45-deg angle with the X axis. At position B, its vector makes a 75-deg angle with the X axis. Draw this system to some convenient scale and:

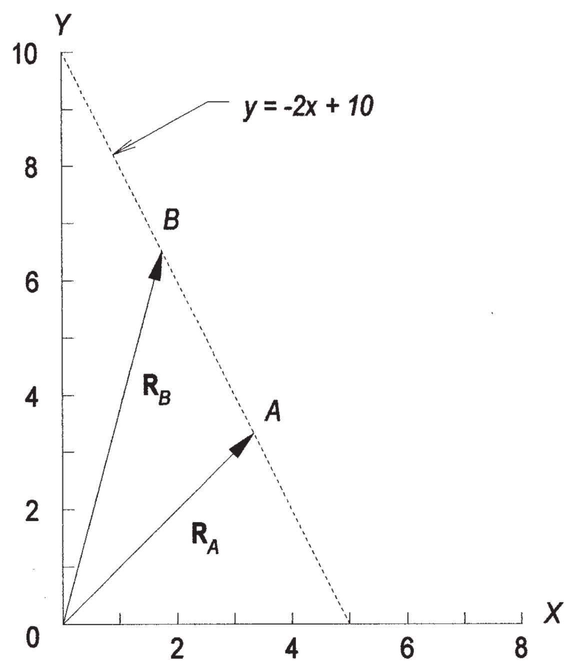
a. Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.
b. Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.
c. Write a vector equation for the position difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.
d. Check the result of part c with a graphical method.

Given: Vector angles: $\theta_A := 45 \cdot deg \quad \theta_B := 75 \cdot deg$

Solution: See Mathcad file P0402.

1. Establish an X-Y coordinate frame and draw the line $y = -2x + 10$.

2. Draw lines from the origin that make angles of 45 and 75 deg with respect to the X axis. Label the intersections of the lines with the line in step 1 as A and B, respectively. Make the line segment OA a vector by putting an arrowhead at A, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment OB, labeling it $\mathbf{R}_B$.



3. Calculate the coordinates of points A and B.

$$x_A \cdot tan(\theta_A) = -2 \cdot x_A + 10 \qquad x_A := \frac{10}{2 + tan(\theta_A)} \qquad x_A = 3.333$$

$$y_A := x_A \cdot tan(\theta_A) \qquad y_A = 3.333$$

$x_B \cdot tan(\theta_B) = -2 \cdot x_B + 10$ $\quad$ $x_B := \dfrac{10}{2 + tan(\theta_B)}$ $\quad$ $x_B = 1.745$

$y_B := x_B \cdot tan(\theta_B)$ $\quad$ $y_B = 6.511$

4. Calculate the distances of points *A* and *B* from the origin.

$R_A := \sqrt{x_A^2 + y_A^2}$ $\quad$ $R_A = 4.714$

$R_B := \sqrt{x_B^2 + y_B^2}$ $\quad$ $R_B = 6.741$

a. *Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.*

Polar form: $\mathbf{R_A} := R_A \cdot \mathbf{e}^{j \cdot \theta_A}$ $\quad$ $\mathbf{R_A} := 4.714 \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}}$

Cartesian form: $\mathbf{R_A} := R_A \cdot (cos(\theta_A) + j \cdot sin(\theta_A))$ $\quad$ $\mathbf{R_A} = 3.333 + 3.333j$

b. *Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.*

Polar form: $\mathbf{R_B} := R_B \cdot \mathbf{e}^{j \cdot \theta_B}$ $\quad$ $\mathbf{R_B} := 6.741 \cdot \mathbf{e}^{j \cdot \frac{75 \cdot \pi}{180}}$

Cartesian form: $\mathbf{R_B} := R_B \cdot (cos(\theta_B) + j \cdot sin(\theta_B))$ $\quad$ $\mathbf{R_B} = 1.745 + 6.511j$

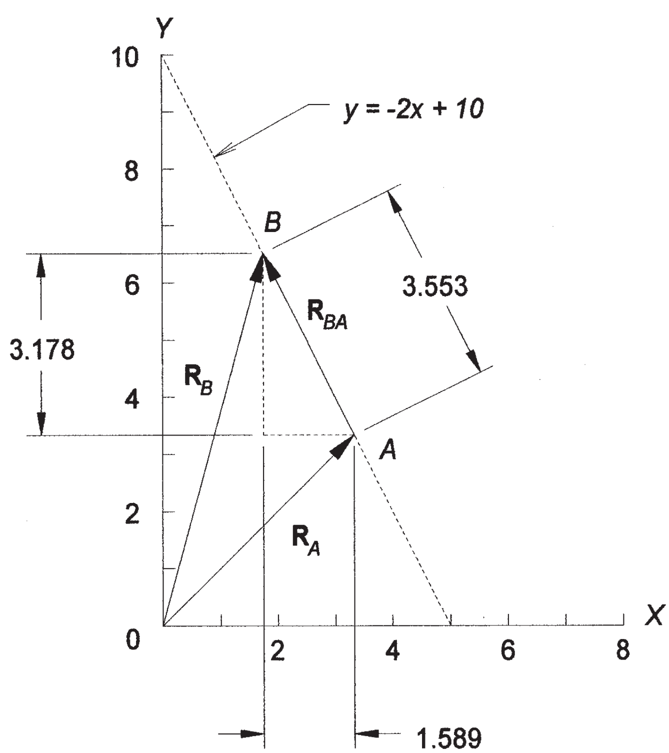
c. *Write a vector equation for the position difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.*

$\mathbf{R_{BA}} := \mathbf{R_B} - \mathbf{R_A}$

$\mathbf{R_{BA}} = -1.589 + 3.178j$

d. *Check the result of part c with a graphical method.*

On the layout above the *X* and *Y* components of $\mathbf{R}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.



PROBLEM 4-5

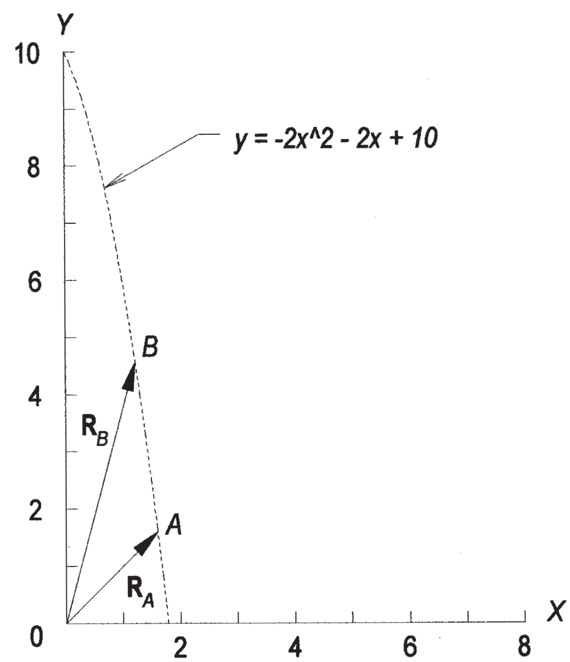
Statement: Two particles are traveling along the line $y = -2x^2 - 2x + 10$. When one particle is at position A, its position vector makes a 45-deg angle with the X axis. Simultaneously, the other particle is at position B, where its vector makes a 75-deg angle with the X axis. Draw this system to some convenient scale and:

a. Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.
b. Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.
c. Write a vector equation for the relative position of the particle at B with respect to the particle at A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.
d. Check the result of part c with a graphical method.

Given: Vector angles: $\theta_A := 45 \cdot deg \quad \theta_B := 75 \cdot deg$

Solution: See Mathcad file P0405.

1. Establish an X-Y coordinate frame and draw the line $y = -2x^2 - 2x + 10$.
2. Draw lines from the origin that make angles of 45 and 75 deg with respect to the X axis. Label the intersections of the lines with the line drawn in step 1 as A and B, respectively. Make the line segment OA a vector by putting an arrowhead at A, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment OB, labeling it $\mathbf{R}_B$.



3. Calculate the coordinates of points A and B.

$$x_A \cdot \tan(\theta_A) = -2 \cdot x_A^2 - 2 \cdot x_A + 10$$

$$x_A := \frac{1}{2} \cdot \left[-\left(1 + \frac{\tan(\theta_A)}{2}\right) + \sqrt{\left(1 + \frac{\tan(\theta_A)}{2}\right)^2 + 20} \right] \qquad x_A = 1.608$$

$$y_A := x_A \cdot tan(\theta_A) \qquad y_A = 1.608$$

$$x_B \cdot tan(\theta_B) = -2 \cdot x_B^2 - 2 \cdot x_B + 10$$

$$x_B := \frac{1}{2} \cdot \left[-\left(1 + \frac{tan(\theta_B)}{2}\right) + \sqrt{\left(1 + \frac{tan(\theta_B)}{2}\right)^2 + 20} \right] \qquad x_B = 1.223$$

$$y_B := x_B \cdot tan(\theta_B) \qquad y_B = 4.564$$

4. Calculate the distances of points A and B from the origin.

$$R_A := \sqrt{x_A^2 + y_A^2} \qquad R_A = 2.275$$

$$R_B := \sqrt{x_B^2 + y_B^2} \qquad R_B = 4.725$$

a. *Write an expression for the particle's position vector in position A using complex number notation, in both polar and Cartesian forms.*

Polar form: $\mathbf{R_A} := R \cdot \mathbf{e}^{j \cdot \theta_A}$ $\qquad \mathbf{R_A} := 2.275 \cdot \mathbf{e}^{j \cdot \frac{\pi}{4}}$

Cartesian form: $\mathbf{R_A} := R_A \cdot (cos(\theta_A) + j \cdot sin(\theta_A))$ $\qquad \mathbf{R_A} = 1.608 + 1.608j$

b. *Write an expression for the particle's position vector in position B using complex number notation, in both polar and Cartesian forms.*

Polar form: $\mathbf{R_B} := R \cdot \mathbf{e}^{j \cdot \theta_B}$ $\qquad \mathbf{R_B} := 4.725 \cdot \mathbf{e}^{j \cdot \frac{75 \cdot \pi}{180}}$

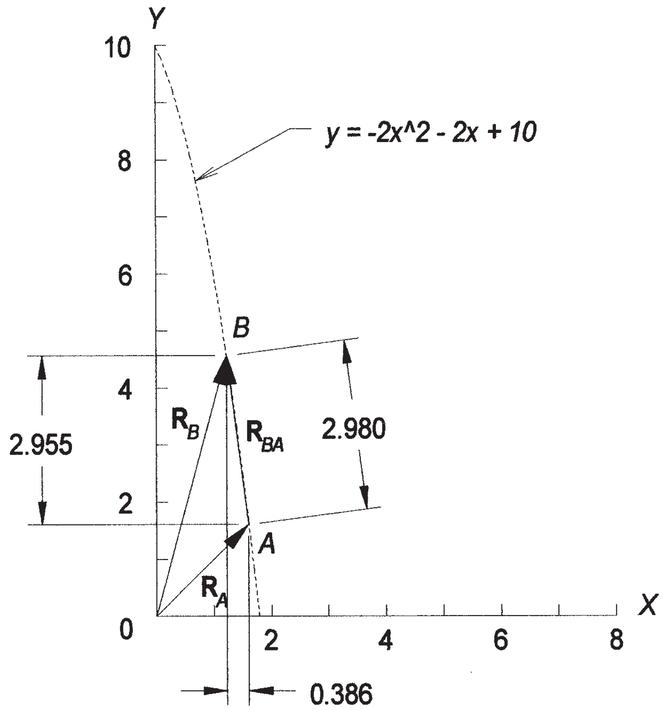
Cartesian form: $\mathbf{R_B} := R_B \cdot (cos(\theta_B) + j \cdot sin(\theta_B))$ $\qquad \mathbf{R_B} = 1.223 + 4.564j$

c. *Write a vector equation for the relative position of the particle at B with respect to the particle at A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.*

$$\mathbf{R_{BA}} := \mathbf{R_B} - \mathbf{R_A} \qquad \mathbf{R_{BA}} = -0.386 + 2.955j$$

d. *Check the result of part c with a graphical method.*

On the layout on the next page the X and Y components of $\mathbf{R}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.



Y
X
y = -2x^2 - 2x + 10
B
A
R_B
R_BA
R_A
2.955
2.980
0.386
10
8
6
4
2
0
2
4
6
8

PROBLEM 4-6a

Statement: The link lengths and value of θ_2 for some fourbar linkages are defined in Table P4-1. The linkage configuration and terminology are shown in Figure P4-1. For row *a*, draw the linkage to scale and graphically find all possible solutions (both open and crossed) for angles θ_3 and θ_4. Determine the Grashoff condition.

Given:

Link 1	$d := 6 \cdot in$	Link 2	$a := 2 \cdot in$	
Link 3	$b := 7 \cdot in$	Link 4	$c := 9 \cdot in$	$\theta_2 := 30 \cdot deg$

Solution: See figure below for one possible solution. Also see Mathcad file P0406a.

1. Lay out an *xy*-axis system. Its origin will be the link 2 pivot, O_2.
2. Draw link 2 to some convenient scale at its given angle.
3. Draw a circle with center at the free end of link 2 and a radius equal to the given length of link 3.
4. Locate pivot O_4 on the x-axis at a distance from the origin equal to the given length of link 1.
5. Draw a circle with center at O_4 and a radius equal to the given length of link 4.
6. The two intersections of the circles (if any) are the two solutions to the position analysis problem, crossed and open. If the circles don't intersect, there is no solution.
7. Draw links 3 and 4 in their two possible positions (shown as solid for open and dashed for crossed in the figure) and measure their angles θ_3 and θ_4 with respect to the x-axis. From the solution below,

OPEN $\theta_{31} := 88.84 \cdot deg$

$\theta_{41} := 117.29 \cdot deg$

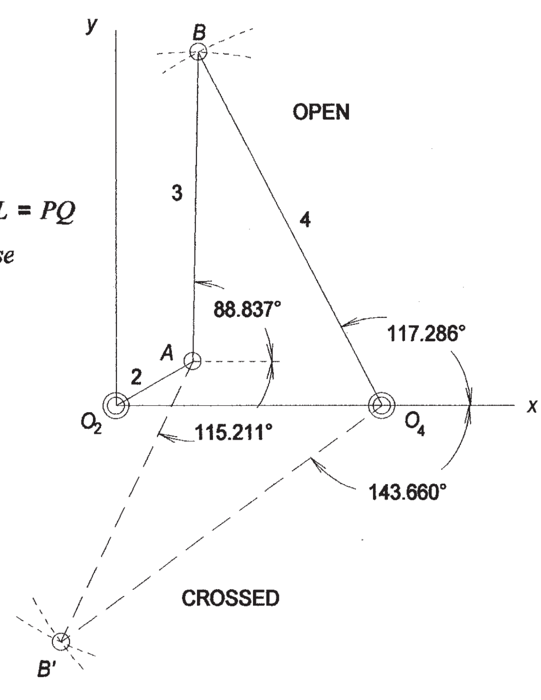
CROSSED $\theta_{32} := 360 \cdot deg - 115.21 \cdot deg$ $\qquad \theta_{32} = 244.790\, deg$

$\theta_{42} := 360 \cdot deg - 143.66 \cdot deg$ $\qquad \theta_{42} = 216.340\, deg$

8. Check the Grashof condition.

$$Condition(S,L,P,Q) := \left| \begin{array}{l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$Condition(a,c,d,b) = \text{"Grashof"}$



PROBLEM 4-7a

Statement: The link lengths and value of θ_2 for some fourbar linkages are defined in Table P4-1. The linkage configuration and terminology are shown in Figure P4-1. For row *a*, find all possible solutions (both open and crossed) for angles θ_3 and θ_4 using the vector loop method. Determine the Grashof condition.

Given:

Link 1 $d := 6 \cdot in$ Link 2 $a := 2 \cdot in$

Link 3 $b := 7 \cdot in$ Link 4 $c := 9 \cdot in$ $\theta_2 := 30 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Mathcad file P0407a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$K_1 = 3.0000$ $K_2 = 0.6667$ $K_3 = 2.0000$

$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$ $A = -0.7113$

$B := -2 \cdot \sin(\theta_2)$ $B = -1.0000$

$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$ $C = 3.5566$

2. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

Open: $\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right)$ $\theta_{41} = 477.286\ deg$

$\theta_{41} := \theta_{41} - 360 \cdot deg$ $\theta_{41} = 117.286\ deg$

Crossed: $\theta_{42} := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right)$ $\theta_{42} = 216.340\ deg$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)}$$

$K_4 = 0.8571$

$K_5 = -0.2857$

$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$ $D = -1.6774$

$E := -2 \cdot \sin(\theta_2)$ $E = -1.0000$

$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$ $F = 2.5906$

4. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

Open: $\theta_{31} := 2\cdot\left(atan2\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right)$ $\theta_{31} = 448.837\ deg$

$\theta_{31} := \theta_{31} - 360\cdot deg$ $\theta_{31} = 88.837\ deg$

Crossed: $\theta_{32} := 2\cdot\left(atan2\left(2\cdot D, -E + \sqrt{E^2 - 4\cdot D\cdot F}\right)\right)$ $\theta_{32} = 244.789\ deg$

5. Check the Grashof condition.

```
Condition(S,L,P,Q) := | SL ← S + L
                      | PQ ← P + Q
                      | return "Grashof" if SL < PQ
                      | return "Special Grashof" if SL = PQ
                      | return "non-Grashof" otherwise
```

Condition(a, c, d, b) = "Grashof"

PROBLEM 4-8

Statement: Expand equation 4.7b and prove that it reduces to equation 4.7c (p. 157).

Solution: See Mathcad file P0408.

1. Write equation 4.7b and expand the two terms that are squared.

$$b^2 := \left(-a\cdot sin(\theta_2) + c\cdot sin(\theta_4)\right)^2 + \left(-a\cdot cos(\theta_2) + c\cdot cos(\theta_4) + d\right)^2 \qquad (6.7b)$$

$$\left(-a\cdot sin(\theta_2) + c\cdot sin(\theta_4)\right)^2 := a^2\cdot sin(\theta_2)^2 - 2\cdot a\cdot c\cdot sin(\theta_2)\cdot sin(\theta_4) + c^2\cdot sin(\theta_4)^2 \qquad (a)$$

$$\left(-a\cdot cos(\theta_2) + c\cdot cos(\theta_4) + d\right)^2 := a^2\cdot cos(\theta_2)^2 - 2\cdot a\cdot c\cdot\left(cos(\theta_2)\cdot cos(\theta_4)\right) - 2\cdot a\cdot d\cdot cos(\theta_2) \ldots$$
$$+ 2\cdot c\cdot d\cdot cos(\theta_4) + c^2\cdot cos(\theta_4)^2 + d^2 \qquad (b)$$

2. Add the two expanded terms, equations *a* and *b*, noting the identity $\sin^2 x + \cos^2 x = 1$.

$$b^2 := a^2 + c^2 + d^2 - 2\cdot a\cdot d\cdot cos(\theta_2) + 2\cdot c\cdot d\cdot cos(\theta_4) - 2\cdot a\cdot c\cdot\left(sin(\theta_2)\cdot sin(\theta_4) + cos(\theta_2)\cdot cos(\theta_4)\right)$$

This is equation 6.7c.

PROBLEM 4-9a

Statement: The link lengths, value of θ_2, and offset for some fourbar slider-crank linkages are defined in Table P4-2. The linkage configuration and terminology are shown in Figure P4-2. For row *a*, draw the linkage to scale and graphically find all possible solutions (both open and crossed) for angles θ_3 and slider position *d*.

Given:

Link 2 $a := 1.4 \cdot in$ Link 3 $b := 4 \cdot in$

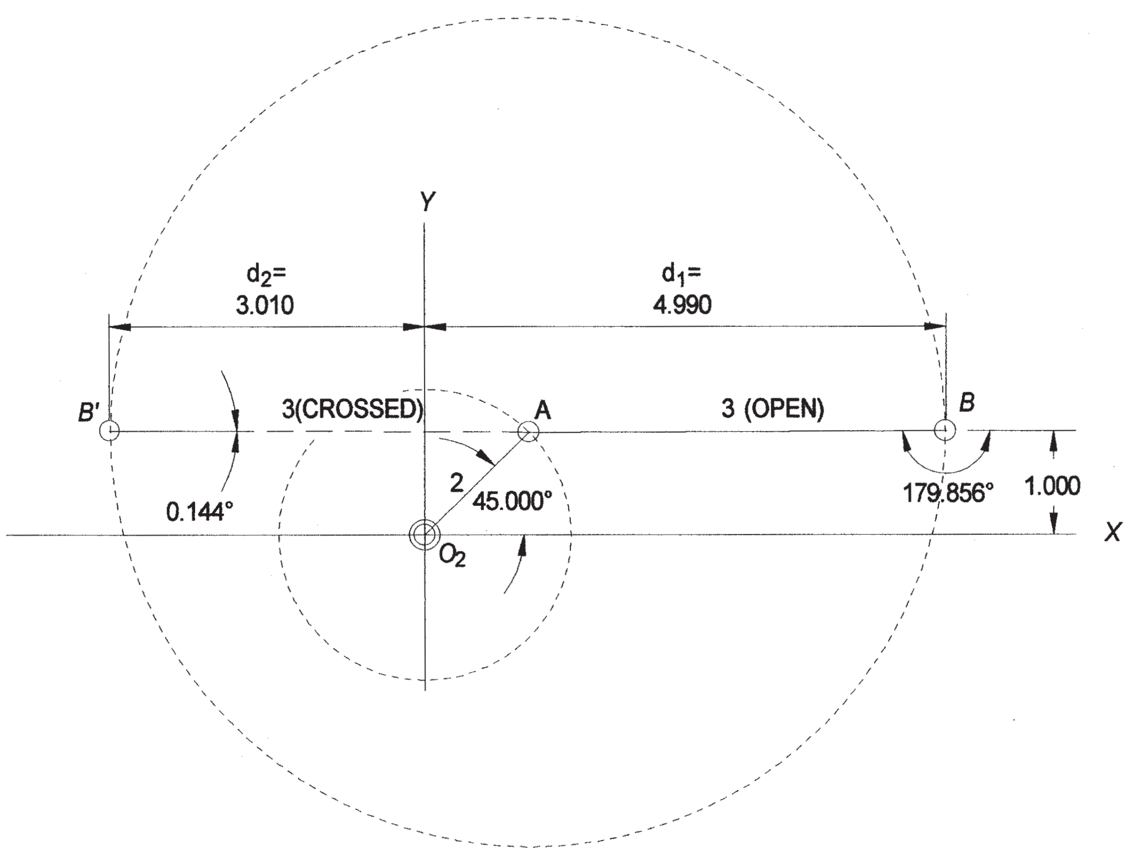
Offset $c := 1 \cdot in$ $\theta_2 := 45 \cdot deg$

Solution: See figure below for one possible solution. Also see Mathcad file P0409a.

1. Lay out an *xy*-axis system. Its origin will be the link 2 pivot, O_2.
2. Draw link 2 to some convenient scale at its given angle.
3. Draw a circle with center at the free end of link 2 and a radius equal to the given length of link 3.
4. Draw a horizontal line through y = c (the offset).
5. The two intersections of the circle with the horizontal line (if any) are the two solutions to the position analysis problem, crossed and open. If the circle and line don't intersect, there is no solution.
6. Draw link 3 and the slider block in their two possible positions (shown as solid for open and dashed for crossed in the figure) and measure the angle θ_3 and length *d* for each circuit. From the solution below,

$\theta_{31} := 360 \cdot deg - 179.856 \cdot deg$ $\theta_{31} = 180.144\, deg$

$\theta_{32} := -0.144 \cdot deg$ $d_1 := 4.990 \cdot in$ $d_2 := -3.010 \cdot in$



PROBLEM 4-10a

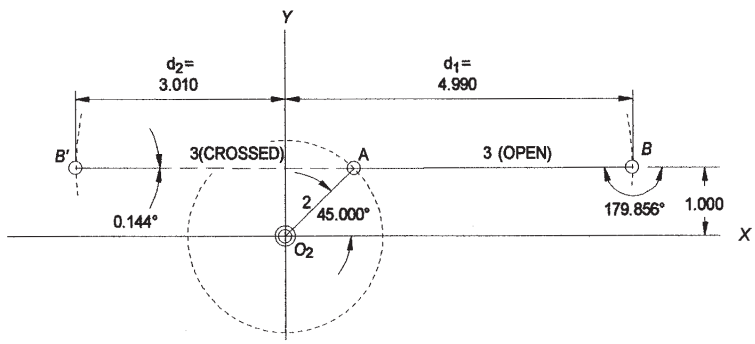
Statement: The link lengths, value of θ_2, and offset for some fourbar slider-crank linkages are defined in Table P4-2. The linkage configuration and terminology are shown in Figure P4-2. For row *a*, using the vector loop method, find all possible solutions (both open and crossed) for angles θ_3 and slider position *d*.

Given:

Link 2 $a := 1.4 \cdot in$ Link 3 $b := 4 \cdot in$

Offset $c := 1 \cdot in$ $\theta_2 := 45 \cdot deg$

Solution: See Figure P4-2 and Mathcad file P0410a.



1. Determine θ_3 and *d* using equations 4.16 and 4.17.

Crossed:

$$\theta_{32} := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_{32} = -0.144\, deg$$

$$d_2 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{32}) \qquad d_2 = -3.010\, in$$

Open:

$$\theta_{31} := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_{31} = 180.144\, deg$$

$$d_1 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{31}) \qquad d_1 = 4.990\, in$$

PROBLEM 4-11a

Statement: The link lengths and the value of θ_2 and γ for some inverted fourbar slider-crank linkages are defined in Table P4-3. The linkage configuration and terminology are shown in Figure P4-3. For row *a*, draw the linkage to scale and graphically find both open and closed solutions for θ_3 and θ_4 and vector $\mathbf{R}_B$.

Given:

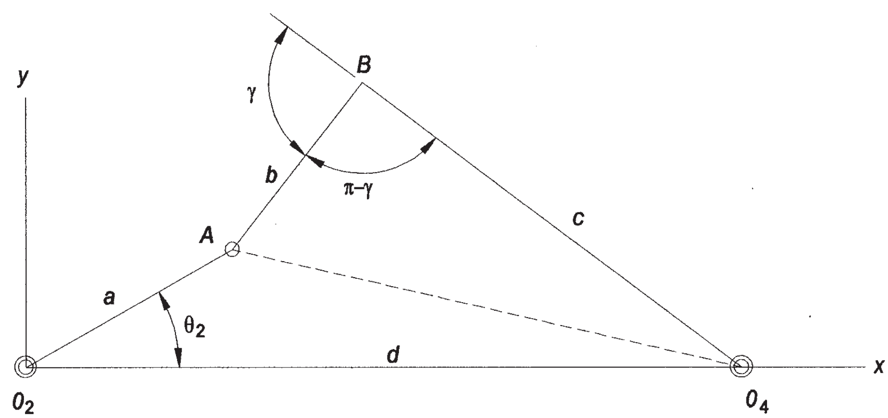
Link 1	$d := 6 \cdot in$	Link 2	$a := 2 \cdot in$		
Link 4	$c := 4 \cdot in$		$\gamma := 90 \cdot deg$		$\theta_2 := 30 \cdot deg$

Solution: See figure below for one possible solution. Also see Mathcad file P04011a.

1. Lay out an *xy*-axis system. Its origin will be the link 2 pivot, O_2.
2. Draw link 2 to some convenient scale at its given angle.

3a. If $\gamma = 90$ deg, locate O_4 on the x-axis at a distance equal the length of link 1 (*d*) from the origin. Draw a circle with center at O_4 and radius equal to the length of link 4 (*c*). From point A, draw two lines that are tangent to the circle. The points of tangency define the location of the points *B* for the open and crossed circuits.

3b. When γ is not 90 deg there are two approaches to a graphical solution for link 3 and the location of point *B*: 1) establish the position of link 4 and the angle γ by trial and error, or 2) calculate the distance from point *A* to point *B* (the instantaneous length of link 3). Using the second approach, from triangle O_2AO_4



$$AO_4^2 = a^2 + d^2 - 2 \cdot a \cdot d \cdot cos(\theta_2)$$

and, from triangle AO_4B (for the open circuit)

$$AO_4^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot cos(\pi - \gamma)$$

where a, b, c, and d are the lengths of links 2, 3, 4, and 1, respectively. Eliminating AO_4 and solving for the unknown distance *b* for the open branch,

$$b_1 := \frac{1}{2} \cdot \left[2 \cdot c \cdot cos(\pi - \gamma) + \sqrt{(2 \cdot c \cdot cos(\pi - \gamma))^2 - 4 \cdot \left(c^2 - a^2 - d^2 + 2 \cdot a \cdot d \cdot cos(\theta_2)\right)} \right]$$

$$b_1 = 1.7932\ in$$

For the closed branch: $AO_4^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot cos(\gamma)$ and

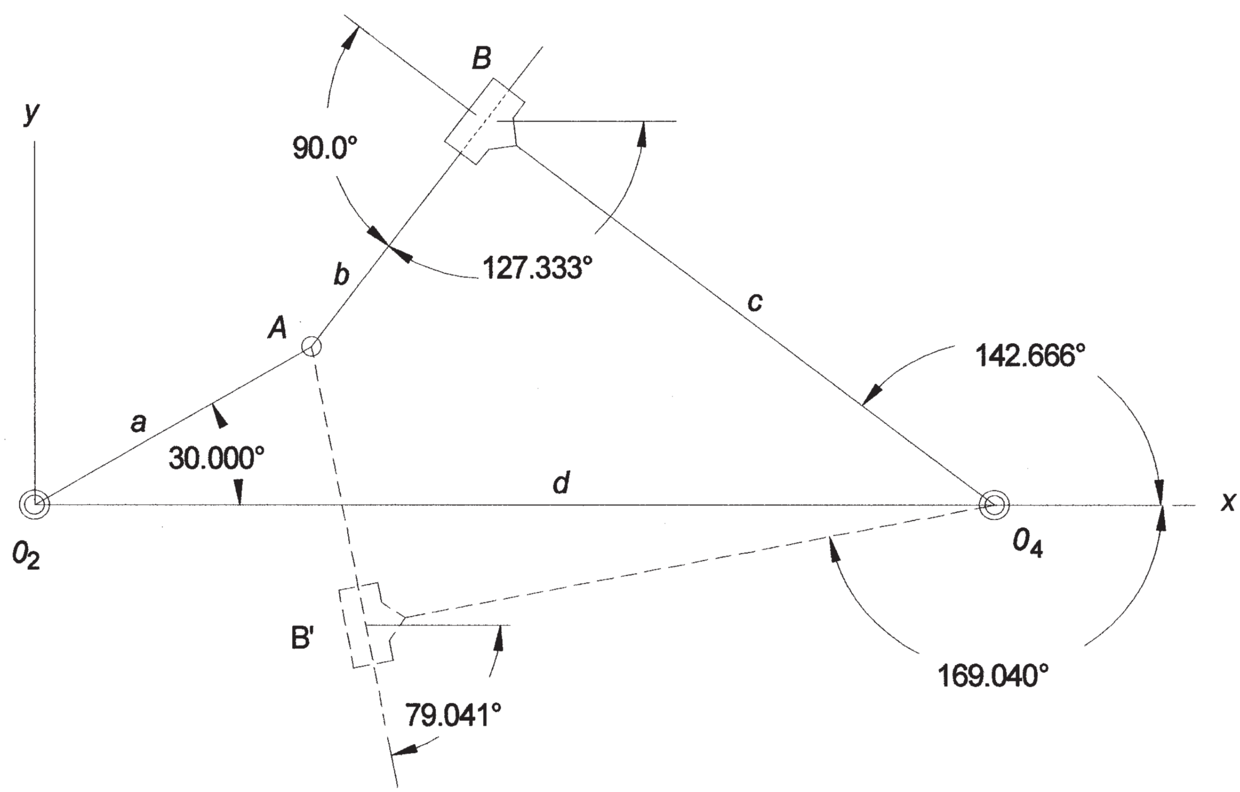
$$b_2 := \frac{1}{2} \cdot \left[2 \cdot c \cdot cos(\gamma) + \sqrt{(2 \cdot c \cdot cos(\gamma))^2 - 4 \cdot \left(c^2 - a^2 - d^2 + 2 \cdot a \cdot d \cdot cos(\theta_2)\right)} \right]$$

$$b_2 = 1.7932\ in$$

Draw a circle with center at point A and radius b_1. Draw a circle with center at O_4 and radius equal to the length of link 4 (c). The intersections of these two circles is the solution for the open and crossed locations of the point B.

4. Draw the complete linkage for the open and crossed circuits, including the slider. The results from the graphical solution below are:

OPEN	$\theta_{31} := -127.333 \cdot deg$	CROSSED	$\theta_{32} := -79.041 \cdot deg$
	$\theta_{41} := 142.666 \cdot deg$		$\theta_{42} := -169.040 \cdot deg$
	$R_{B1} := 3.719$ at 40.708 deg		$R_{B2} := 2.208$ at -20.146 deg



PROBLEM 4-12a

Statement: The link lengths and the value of θ_2 and γ for some inverted fourbar slider-crank linkages are defined in Table P4-3. The linkage configuration and terminology are shown in Figure P4-3. For row *a*, using the vector loop method, find both open and closed solutions for θ_3 and θ_4 and vector $\mathbf{R}_B$.

Given: Link 1 $d := 6 \cdot in$ Link 2 $a := 2 \cdot in$

Link 4 $c := 4 \cdot in$ $\gamma := 90 \cdot deg$ $\theta_2 := 30 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Mathcad file P0412a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$P := a \cdot sin(\theta_2) \cdot sin(\gamma) + (a \cdot cos(\theta_2) - d) \cdot cos(\gamma)$ $P = 1.000\ in$

$Q := -a \cdot sin(\theta_2) \cdot cos(\gamma) + (a \cdot cos(\theta_2) - d) \cdot sin(\gamma)$ $Q = -4.268\ in$

$R := -c \cdot sin(\gamma)$ $R = -4.000\ in$ $T := 2 \cdot P$ $T = 2.000\ in$

$S := R - Q$ $S = 0.268\ in$ $U := Q + R$ $U = -8.268\ in$

2. Use equation 4.22c to find values of θ_4 for the open and crossed circuits.

OPEN $\theta_{41} := 2 \cdot atan2\left(2 \cdot S, -T + \sqrt{T^2 - 4 \cdot S \cdot U}\right)$ $\theta_{41} = 142.667\ deg$

CROSSED $\theta_{42} := 2 \cdot atan2\left(2 \cdot S, -T - \sqrt{T^2 - 4 \cdot S \cdot U}\right)$ $\theta_{42} = -169.041\ deg$

3. Use equation 4.18 to find values of θ_3 for the open and crossed circuits.

OPEN $\theta_{31} := \theta_{41} + \gamma$ $\theta_{31} = 232.667\ deg$

CROSSED $\theta_{32} := \theta_{42} - \gamma$ $\theta_{32} = -259.041\ deg$

4. Determine the magnitude of the instantaneous "length" of link 3 from equation 4.20a.

OPEN $b_1 := \dfrac{a \cdot sin(\theta_2) - c \cdot sin(\theta_{41})}{sin(\theta_{41} + \gamma)}$ $b_1 = 1.793\ in$

CROSSED $b_2 := \left|\dfrac{a \cdot sin(\theta_2) - c \cdot sin(\theta_{42})}{sin(\theta_{42} + \gamma)}\right|$ $b_2 = 1.793\ in$

5. Find the position vector $\mathbf{R}_B$ from the definition given on page 162 of the text.

OPEN

$$\mathbf{R_{B1}} := a\cdot\left(cos\left(\theta_2\right) + j\cdot sin\left(\theta_2\right)\right) - b_1\cdot\left(cos\left(\theta_{31}\right) + j\cdot sin\left(\theta_{31}\right)\right)$$

$R_{B1} := \left|\mathbf{R_{B1}}\right|$ $\qquad R_{B1} = 3.719\ in$

$\theta_{B1} := arg\left(\mathbf{R_{B1}}\right)$ $\qquad \theta_{B1} = 40.707\ deg$

CROSSED

$$\mathbf{R_{B2}} := a\cdot\left(cos\left(\theta_2\right) + j\cdot sin\left(\theta_2\right)\right) - b_2\cdot\left(cos\left(\theta_{32}\right) + j\cdot sin\left(\theta_{32}\right)\right)$$

$R_{B2} := \left|\mathbf{R_{B2}}\right|$ $\qquad R_{B2} = 2.208\ in$

$\theta_{B2} := arg\left(\mathbf{R_{B2}}\right)$ $\qquad \theta_{B2} = -20.145\ deg$

PROBLEM 4-13a

Statement: Find the transmission angles of the linkage in row *a* of Table P4-1.

Given: Link 1 $d := 6 \cdot in$ Link 2 $a := 2 \cdot in$

Link 3 $b := 7 \cdot in$ Link 4 $c := 9 \cdot in$ $\theta_2 := 30 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Mathcad file P0413a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 3.0000 \qquad K_2 = 0.6667 \qquad K_3 = 2.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \qquad A = -0.7113$$

$$B := -2 \cdot \sin(\theta_2) \qquad B = -1.0000$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \qquad C = 3.5566$$

2. Use equation 4.10b to find θ_4 for the open circuit.

$$\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_{41} = 477.286\,deg$$

$$\theta_{41} := \theta_{41} - 360 \cdot deg \qquad \theta_{41} = 117.286\,deg$$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.8571$$

$$K_5 = -0.2857$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -1.6774$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 2.5906$$

4. Use equation 4.13 to find θ_3 for the open circuit.

$$\theta_{31} := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_{31} = 448.837\,deg$$

$$\theta_{31} := \theta_{31} - 360 \cdot deg \qquad \theta_{31} = 88.837\,deg$$

5. Use equations 4.28 to find the transmission angle.

$$\theta_{trans}(\theta_3,\theta_4) := \left| \begin{array}{l} t \leftarrow |\theta_3 - \theta_4| \\ \textit{return } t \textit{ if } t \le 0.5\cdot\pi \\ \pi - t \textit{ otherwise} \end{array} \right.$$

$$\theta_{trans}(\theta_{31},\theta_{41}) = 28.449\, deg$$

6. It can be shown that the triangle ABO_4 in Figure 4-17 is symmetric with respect to the line AO_4 for the crossed branch and, therefore, the transmission angle for the crossed branch is identical to that for the open branch.

PROBLEM 4-14

Statement: Find the minimum and maximum values of the transmission angle for all the Grashof crank-rocker linkages in Table P4-1.

Given: Table P4-1 data: $i := 1, 2 .. 14$

$Row_i :=$	$d_i :=$	$a_i :=$	$b_i :=$	$c_i :=$
"a"	6	2	7	9
"b"	7	9	3	8
"c"	3	10	6	8
"d"	8	5	7	6
"e"	8	5	8	6
"f"	5	8	8	9
"g"	6	8	8	9
"h"	20	10	10	10
"i"	4	5	2	5
"j"	20	10	5	10
"k"	4	6	10	7
"l"	9	7	10	7
"m"	9	7	11	8
"n"	9	7	11	6

Solution: See Table P4-1 and Mathcad file P0414.

1. Determine which of the linkages in Table P4-1 are Grashof.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \textit{return} \text{ "Grashof" } \textit{ if } SL < PQ \\ \textit{return} \text{ "Special Grashof" } \textit{ if } SL = PQ \\ \textit{return} \text{ "non-Grashof" } \textit{ otherwise} \end{array}$$

Row *a* $Condition(2,9,6,7) = \text{"Grashof"}$

Row *b* $Condition(3,9,7,8) = \text{"Grashof"}$

Row *c* $Condition(3,10,6,8) = \text{"Grashof"}$

Row *d* $Condition(5,8,6,7) = \text{"Special Grashof"}$

Row *e* $Condition(5,8,6,8) = \text{"Grashof"}$

Row *f* $Condition(5,9,8,8) = \text{"Grashof"}$

Row *g* $Condition(6,9,8,8) = \text{"Grashof"}$

Row *h* $Condition(10,20,10,10) = \text{"non-Grashof"}$

Row *i* $Condition(2,5,4,5) = \text{"Grashof"}$

Row *j* $Condition(5,20,10,10) = \text{"non-Grashof"}$

Row *k* $Condition(4,10,6,7) = \text{"non-Grashof"}$

Row *l* $Condition(7,10,7,9) = \text{"non-Grashof"}$

Row *m* $Condition(7,11,8,9) = \text{"non-Grashof"}$

Row *n* $Condition(6,11,7,9) = \text{"non-Grashof"}$

2. Determine which of the Grashof linkages are crank-rockers. To be a Grashof crank-rocker, the linkage must be Grashof and the shortest link is either 2 or 4. This is true of rows *a, d,* and *e*.

3. Use equations 4.28 and 4.29 to calculate the maximum and minimum transmission angles.

Row *a* $i := 1$

$$\mu_1 := acos\left[\frac{\left(b_i\right)^2 + \left(c_i\right)^2 - \left(d_i + a_i\right)^2}{2 \cdot b_i \cdot c_i}\right]$$

$$\mu_1 := if\left(\mu_1 > \frac{\pi}{2}, \pi - \mu_1, \mu_1\right) \qquad \mu_1 = 58.412\,deg$$

$$\mu_2 := acos\left[\frac{\left(b_i\right)^2 + \left(c_i\right)^2 - \left(d_i - a_i\right)^2}{2 \cdot b_i \cdot c_i}\right] \qquad \mu_2 = 25.209\,deg$$

Row *d* $i := 4$

$$\mu_1 := acos\left[\frac{\left(b_i\right)^2 + \left(c_i\right)^2 - \left(d_i + a_i\right)^2}{2 \cdot b_i \cdot c_i}\right]$$

$$\mu_1 := if\left(\mu_1 > \frac{\pi}{2}, \pi - \mu_1, \mu_1\right) \qquad \mu_1 = 0.000\,deg$$

$$\mu_2 := acos\left[\frac{\left(b_i\right)^2 + \left(c_i\right)^2 - \left(d_i - a_i\right)^2}{2 \cdot b_i \cdot c_i}\right] \qquad \mu_2 = 25.209\,deg$$

Row *e* $i := 5$

$$\mu_1 := acos\left[\frac{\left(b_i\right)^2 + \left(c_i\right)^2 - \left(d_i + a_i\right)^2}{2 \cdot b_i \cdot c_i}\right]$$

$$\mu_1 := if\left(\mu_1 > \frac{\pi}{2}, \pi - \mu_1, \mu_1\right) \qquad \mu_1 = 44.049\,deg$$

$$\mu_2 := acos\left[\frac{\left(b_i\right)^2 + \left(c_i\right)^2 - \left(d_i - a_i\right)^2}{2 \cdot b_i \cdot c_i}\right] \qquad \mu_2 = 18.573\,deg$$

PROBLEM 4-15

Statement: Find the input angles corresponding to the toggle positions of the non-Grashof linkages in Table P4-1.

Given: Table P4-1 data: $i := 1, 2..14$

$Row_i :=$	$d_i :=$	$a_i :=$	$b_i :=$	$c_i :=$
"a"	6	2	7	9
"b"	7	9	3	8
"c"	3	10	6	8
"d"	8	5	7	6
"e"	8	5	8	6
"f"	5	8	8	9
"g"	6	8	8	9
"h"	20	10	10	10
"i"	4	5	2	5
"j"	20	10	5	10
"k"	4	6	10	7
"l"	9	7	10	7
"m"	9	7	11	8
"n"	9	7	11	6

Solution: See Table P4-1 and Mathcad file P0415.

1. Determine which of the linkages in Table P4-1 are Grashof.

$$Condition(S, L, P, Q) := \begin{vmatrix} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{vmatrix}$$

Row *a* $Condition(2,9,6,7) = \text{"Grashof"}$

Row *b* $Condition(3,9,7,8) = \text{"Grashof"}$

Row *c* $Condition(3,10,6,8) = \text{"Grashof"}$

Row *d* $Condition(5,8,6,7) = \text{"Special Grashof"}$

Row *e* $Condition(5,8,6,8) = \text{"Grashof"}$

Row *f* $Condition(5,9,8,8) = \text{"Grashof"}$

Row *g* $Condition(6,9,8,8) = \text{"Grashof"}$

Row *h* $Condition(10,20,10,10) = \text{"non-Grashof"}$

Row *i* $Condition(2,5,4,5) = \text{"Grashof"}$

Row *j* $Condition(5,20,10,10) = \text{"non-Grashof"}$

Row *k* $Condition(4,10,6,7) = \text{"non-Grashof"}$

Row *l* $Condition(7,10,7,9) = \text{"non-Grashof"}$

Row *m* $Condition(7,11,8,9) = \text{"non-Grashof"}$

Row *n* $Condition(6,11,7,9) = \text{"non-Grashof"}$

2. There are six non-Grashof rows in the Table: Rows *h*, and *j* through *n*. For each row there are two possible arguments to the arccos function given in equation (4.33). They are:

$i := 8 \qquad Row_i = \text{"h"} \qquad j := 1$

$$arg_{(j,1)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{(2 \cdot a_i \cdot d_i)} + \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$$arg_{(j,2)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$i := 10 \qquad Row_i = \text{"j"} \qquad j := 2$

$$arg_{(j,1)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{(2 \cdot a_i \cdot d_i)} + \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$$arg_{(j,2)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$i := 11 \qquad Row_i = \text{"k"} \qquad j := 3$

$$arg_{(j,1)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{(2 \cdot a_i \cdot d_i)} + \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$$arg_{(j,2)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$i := 12 \qquad Row_i = \text{"l"} \qquad j := 4$

$$arg_{(j,1)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{(2 \cdot a_i \cdot d_i)} + \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$$arg_{(j,2)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$i := 13 \qquad Row_i = \text{"m"} \qquad j := 5$

$$arg_{(j,1)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{(2 \cdot a_i \cdot d_i)} + \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$$arg_{(j,2)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$i := 14 \qquad Row_i = \text{"n"} \qquad j := 6$

$$arg_{(j,1)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{(2 \cdot a_i \cdot d_i)} + \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$$arg_{(j,2)} := \frac{(a_i)^2 + (d_i)^2 - (b_i)^2 - (c_i)^2}{2 \cdot a_i \cdot d_i} - \frac{b_i \cdot c_i}{(a_i \cdot d_i)}$$

$$arg = \begin{pmatrix} 1.250 & 0.250 \\ 1.188 & 0.688 \\ 0.896 & -4.938 \\ 0.960 & -1.262 \\ 0.960 & -1.833 \\ 0.833 & -1.262 \end{pmatrix}$$

3. Choose the argument values that lie between plus and minus 1,

$\theta 2_h := acos\left[arg_{(1,2)}\right] \qquad \theta 2_h = 75.5\,deg$

$\theta 2_j := acos\left[arg_{(2,2)}\right] \qquad \theta 2_j = 46.6\,deg$

$\theta 2_k := acos\left[arg_{(3,1)}\right] \qquad \theta 2_k = 26.4\,deg$

$\theta 2_l := acos\left[arg_{(4,1)}\right] \qquad \theta 2_l = 16.2\,deg$

$\theta 2_m := acos\left[arg_{(5,1)}\right] \qquad \theta 2_m = 16.2\,deg$

$\theta 2_n := acos\left[arg_{(6,1)}\right] \qquad \theta 2_n = 33.6\,deg$

PROBLEM 4-16a

Statement: The link lengths, gear ratio, phase angle, and the value of θ_2 for some geared fivebar linkages are defined in Table P4-4. The linkage configuration and terminology are shown in Figure P4-4. For row a, draw the linkage to scale and graphically find all possible solutions for angles θ_3 and θ_4.

Given:

Link 1	$d := 4 \cdot in$	Link 2	$a := 1 \cdot in$		
Link 3	$b := 7 \cdot in$	Link 4	$c := 9 \cdot in$	Link 5	$f := 6 \cdot in$
Gear ratio	$\lambda := 2.0$	Phase angle	$\phi := 30 \cdot deg$	Input angle	$\theta_2 := 60 \cdot deg$

Solution: See Mathcad file P0201.

1. Determine whether or not an idler is required.

$$idler := \begin{vmatrix} \text{"required"} & \text{if } \lambda > 0 \\ \text{"not-required"} & \text{otherwise} \end{vmatrix}$$

$$idler = \text{"required"}$$

2. Choose radii for gears 2 and 5 by making a design choice for their center distance (which must be increased if an idler is required). Let the standard center distance when no idler is required be $C := 0.5 \cdot c$ then

$$C = r_2 + r_5 \qquad \text{and} \qquad |\lambda| = \frac{r_2}{r_5}$$

Solving for r_2 and r_5,

$$r_5 := \frac{C}{|\lambda| + 1} \qquad r_5 = 1.500\,in$$

$$r_2 := r_5 \cdot |\lambda| \qquad r_2 = 3.000\,in$$

If an idler is required, increase the center distance.

$$C := if(idler = \text{"required"}, C + r_5, C) \qquad C = 6.000\,in$$

Note that the amount by which C is increased if an idler is required is a design choice that is made based on the size of the gears and the space available.

3. Using equation 4.23c, determine the angular position of link 5 corresponding to the position of link 2.

$$\theta_5 := \lambda \cdot \theta_2 + \phi \qquad \theta_5 = 150\,deg$$

4. Lay out an xy-axis system. Its origin will be the link 2 pivot, O_2.
5. Draw link 2 to some convenient scale at its given angle.
6. Draw a circle with center at the free end of link 2 and a radius equal to the given length of link 3.
7. Locate pivot O_4 on the x-axis at a distance from the origin equal to the given length of link 1.
8. Draw link 5 to some convenient scale at its calculated angle.
9. Draw a circle with center at the free end of link 5 and a radius equal to the given length of link 4.
10. The two intersections of the circles (if any) are the two solutions to the position analysis problem, crossed and open. If the circles don't intersect, there is no solution.
11. Draw links 3 and 4 in their two possible positions (shown as solid for open and dashed for crossed in the figure) and measure their angles θ_3 and θ_4 with respect to the x-axis. From the solution below,

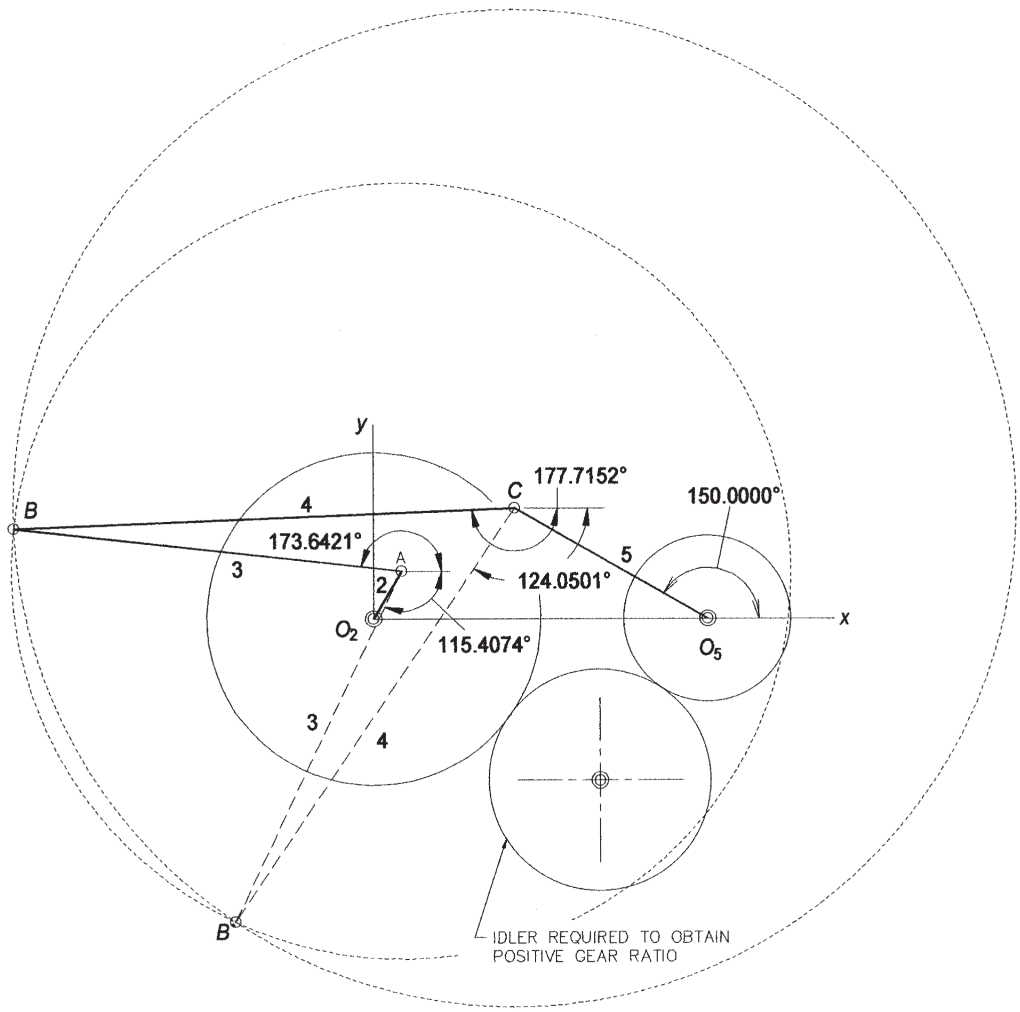
OPEN $\theta_{31} := 173.64 \cdot deg$ $\theta_{41} := 360 \cdot deg - 177.715 \cdot deg$

$\theta_{41} = 182.285\ deg$

CROSSED $\theta_{32} := 360 \cdot deg - 115.407 \cdot deg$ $\theta_{41} := 360 \cdot deg - 124.050 \cdot deg$

$\theta_{32} = 244.593\ deg$ $\theta_{41} = 235.950\ deg$

12. Draw gears 2 and 5 schematically at their calculated radii. If an idler is required, draw it tangent to gears 2 and 5. Its diameter is a design choice that will be made on strength and space requirements. It does not affect the gear ratio.



PROBLEM 4-17a

Statement: The link lengths, gear ratio, phase angle, and the value of θ_2 for some geared fivebar linkages are defined in Table P4-4. The linkage configuration and terminology are shown in Figure P4-4. For row *a*, using the vector loop method, find all possible solutions for angles θ_3 and θ_4.

Given:

Link 1	$d := 4 \cdot in$	Link 2	$a := 1 \cdot in$		
Link 3	$b := 7 \cdot in$	Link 4	$c := 9 \cdot in$	Link 5	$f := 6 \cdot in$
Gear ratio	$\lambda := 2.0$	Phase angle	$\phi := 30 \cdot deg$	Input angle	$\theta_2 := 60 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} \text{return } 0.5 \cdot \pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5 \cdot \pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Mathcad file P0417a.

1. Determine the values of the constants needed for finding θ_3 and θ_4 from equations 4.24h and 4.24i.

$$A := 2 \cdot c \cdot \left(d \cdot cos\left(\lambda \cdot \theta_2 + \phi\right) - a \cdot cos\left(\theta_2\right) + f\right) \qquad A = 36.6462\ in^2$$

$$B := 2 \cdot c \cdot \left(d \cdot sin\left(\lambda \cdot \theta_2 + \phi\right) - a \cdot sin\left(\theta_2\right)\right) \qquad B = 20.412\ in^2$$

$$C := \left(a^2 - b^2 + c^2 + d^2 + f^2\right) - 2 \cdot a \cdot f \cdot cos\left(\theta_2\right) \ldots$$
$$+ -\left[2 \cdot d \cdot \left(a \cdot cos\left(\theta_2\right) - f\right) \cdot cos\left(\lambda \cdot \theta_2 + \phi\right)\right] \ldots$$
$$+ -2 \cdot a \cdot d \cdot sin\left(\theta_2\right) \cdot sin\left(\lambda \cdot \theta_2 + \phi\right) \qquad C = 37.4308\ in^2$$

$$D := C - A \qquad D = 0.78461\ in^2$$

$$E := 2 \cdot B \qquad E = 40.823\ in^2$$

$$F := A + C \qquad F = 74.077\ in^2$$

$$G := 2 \cdot b \cdot \left[-\left(d \cdot cos\left(\lambda \cdot \theta_2 + \phi\right)\right) + a \cdot cos\left(\theta_2\right) - f\right] \qquad G = -28.503\ in^2$$

$$H := 2 \cdot b \cdot \left[-\left(d \cdot sin\left(\lambda \cdot \theta_2 + \phi\right)\right) + a \cdot sin\left(\theta_2\right)\right] \qquad H = -15.876\ in^2$$

$$K := \left(a^2 + b^2 - c^2 + d^2 + f^2\right) - 2 \cdot a \cdot f \cdot cos\left(\theta_2\right) \ldots$$
$$+ -\left[2 \cdot d \cdot \left(a \cdot cos\left(\theta_2\right) - f\right) \cdot cos\left(\lambda \cdot \theta_2 + \phi\right)\right] \ldots$$
$$+ -2 \cdot a \cdot d \cdot sin\left(\theta_2\right) \cdot sin\left(\lambda \cdot \theta_2 + \phi\right) \qquad K = -26.569\ in^2$$

$$L := K - G \qquad L = 1.933\ in^2$$

$$M := 2 \cdot H \qquad M = -31.751\ in^2$$

$$N := G + K \qquad N = -55.072\ in^2$$

2. Use equations 4.24h and 4.24i to find values of θ_3 and θ_4 for the open and crossed circuits.

OPEN $$\theta_{31} := 2 \cdot \left(atan2\left(2 \cdot L, -M + \sqrt{M^2 - 4 \cdot L \cdot N}\right)\right) \qquad \theta_{31} = 173.642\ deg$$

$$\theta_{41} := 2\cdot\left(atan2\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_{41} = -177.715\,deg$$

CROSSED

$$\theta_{32} := 2\cdot\left(atan2\left(2\cdot L, -M - \sqrt{M^2 - 4\cdot L\cdot N}\right)\right) \qquad \theta_{32} = -115.407\,deg$$

$$\theta_{42} := 2\cdot\left(atan2\left(2\cdot D, -E + \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_{42} = -124.050\,deg$$

PROBLEM 4-18a

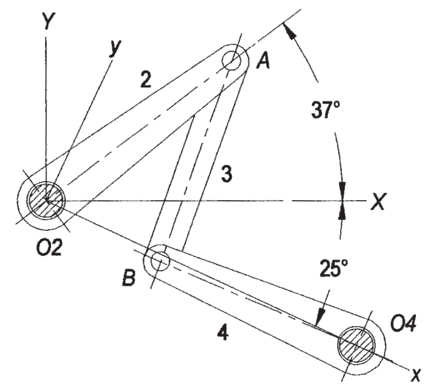
Statement: The angle between the *X* and *x* axes is 25 deg. Find the angular displacement of link 4 when link 2 rotates clockwise from the position shown (+37 deg) to horizontal (0 deg). How does the transmission angle vary and what is its minimum between those two positions? Find the toggle positions of this linkage in terms of the angle of link 2.

Given: Measured link lengths:

Crank	$L_2 := 116$	Coupler	$L_3 := 108$
Rocker	$L_4 := 110$	Ground link	$L_1 := 174$

Crank angle for position shown (relative to AD): $\theta_{21} := 62\cdot deg$

Crank rotation angle from position shown to horizontal: $\Delta\theta_2 := -37\cdot deg$



Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P4-5a and Mathcad file P0418a.

1. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{L_1}{L_2} \qquad K_2 := \frac{L_1}{L_3} \qquad K_3 := \frac{L_2^{\,2} - L_3^{\,2} + L_4^{\,2} + L_1^{\,2}}{(2\cdot L_2\cdot L_4)}$$

$$K_1 = 1.5000 \qquad K_2 = 1.6111 \qquad K_3 = 1.7307$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2\cdot\cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot\sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot\cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

2. Determine θ_4 for the position shown and after the crank has moved to the horizontal position.

$$\theta_{41} := \theta_4(\theta_{21}) \qquad \theta_{41} = 183.5\,deg$$

$$\theta_{42} := \theta_4(\theta_{21} + \Delta\theta_2) \qquad \theta_{42} = 212.8\,deg$$

3. Subtract the two values of θ_4 to find the angular displacement of link 3 when link 2 rotates clockwise from the position shown to the horizontal.

$$\Delta\theta_4 := \theta_{42} - \theta_{41} \qquad \Delta\theta_4 = 29.2\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{L_1}{L_3} \qquad K_5 := \frac{L_4^2 - L_1^2 - L_2^2 - L_3^2}{(2 \cdot L_2 \cdot L_3)} \qquad K_4 = 1.6111 \qquad K_5 = -1.7280$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine θ_3 for the position shown and after the crank has moved to the horizontal position.

$$\theta_{31} := \theta_3(\theta_{21}) \qquad \theta_{31} = 275.1\,deg$$

$$\theta_{32} := \theta_3(\theta_{21} + \Delta\theta_2) \qquad \theta_{32} = 256.1\,deg$$

7. Use equations 4.28 to find the transmission angles.

$$\mu_1 := \pi - |\theta_{31} - \theta_{41}| \qquad \mu_1 = 88.4\,deg$$

$$\mu_2 := |\theta_{32} - \theta_{42}| \qquad \mu_2 = 43.4\,deg$$

The transmission angle is smaller when the crank is in the horizontal position.

8. Check the Grashof condition of the linkage.

```
Condition(S,L,P,Q) := | SL ← S + L
                      | PQ ← P + Q
                      | return "Grashof" if SL < PQ
                      | return "Special Grashof" if SL = PQ
                      | return "non-Grashof" otherwise
```

$Condition(L_3, L_1, L_2, L_4) =$ "non-Grashof"

9. Using equations 4.33, determine the crank angles (relative to the *XY* axes) at which links 3 and 4 are in toggle.

$$arg_1 := \frac{L_2^2 + L_1^2 - L_3^2 - L_4^2}{2 \cdot L_2 \cdot L_1} + \frac{L_3 \cdot L_4}{(L_2 \cdot L_1)} \qquad arg_1 = 1.083$$

$$arg_2 := \frac{L_2^2 + L_1^2 - L_3^2 - L_4^2}{2 \cdot L_2 \cdot L_1} - \frac{L_3 \cdot L_4}{(L_2 \cdot L_1)} \qquad arg_2 = -0.094$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 95.4\,deg$$

The other toggle angle is the negative of this.

PROBLEM 4-18b

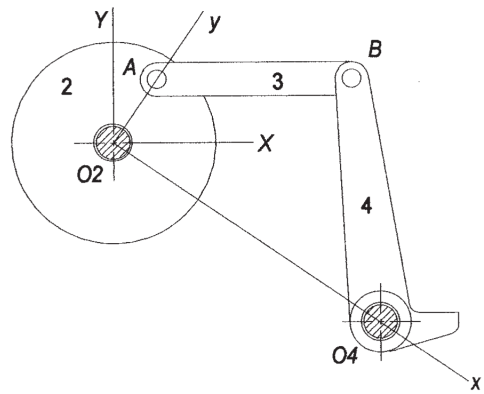
Statement: Find and plot the angular position of links 3 and 4 and the transmission angle as a function of the angle of link 2 as it rotates through one revolution.

Given: Measured link lengths:

Wheel (crank)	$L_2 := 40$	$a := L_2$	Coupler	$L_3 := 96$	$b := L_3$
Rocker	$L_4 := 122$	$c := L_4$	Ground link	$L_1 := 162$	$d := L_1$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5\cdot\pi \;\; \text{if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \;\; \text{if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \;\; \text{if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \;\; \text{otherwise} \end{array}$$



Solution: See Figure P4-5b and Mathcad file P0418b.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array}$$

$$Condition(a,d,b,c) = \text{"Grashof"}$$

2. Define one cycle of the input crank: $\theta_2 := 0\cdot deg, 5\cdot deg .. 360\cdot deg$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 4.0500 \qquad K_2 = 1.3279 \qquad K_3 = 3.4336$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2\cdot\cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot\sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot\cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. If the calculated value of θ_4 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := if\left(\theta_{41}(\theta_2) > 2\cdot\pi, \theta_{41}(\theta_2) - 2\cdot\pi, \theta_{41}(\theta_2)\right)$$

$$\theta_4(\theta_2) := if\left(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2\cdot\pi, \theta_{42}(\theta_2)\right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.6875 \qquad K_5 = -2.8875$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

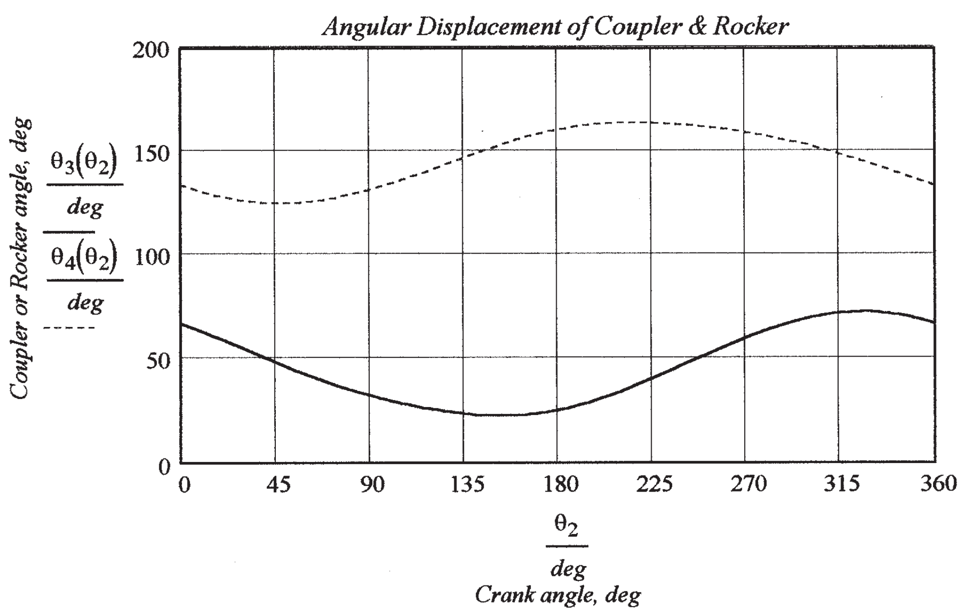
$$\theta_{31}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. If the calculated value of θ_3 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_2) := if(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

$$\theta_3(\theta_2) := if(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) - 2 \cdot \pi, \theta_{32}(\theta_2))$$

8. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).

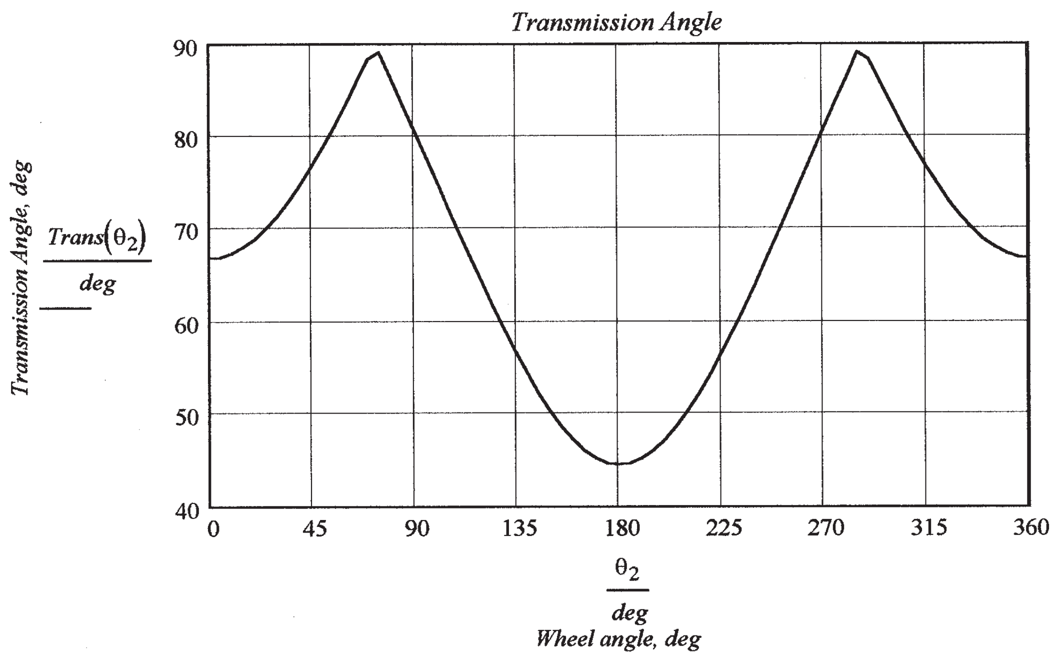


9. Use equations 4.28 to find the transmission angle.

$$Tran(\theta_2) := |\theta_3(\theta_2) - \theta_4(\theta_2)|$$

$$Trans(\theta_2) := if\left(Tran(\theta_2) > \frac{\pi}{2}, |\pi - Tran(\theta_2)|, Tran(\theta_2) \right)$$

10. Plot the transmission angle.

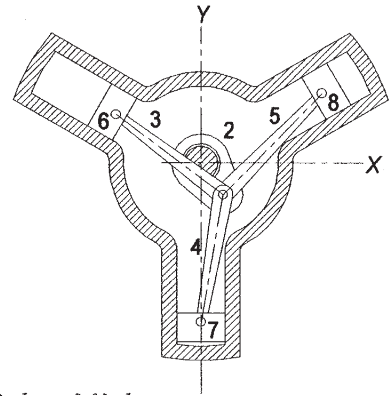


PROBLEM 4-18c

Statement: Find and plot the position of any one piston as a function of the angle of crank 2 as it rotates through one revolution. Once one piston's motion is defined, find the motions of the other two pistons and their phase relationship to the first piston.

Given:

Crank length	$L_2 := 19$	$a := L_2$
Piston-rod length	$L_3 := 70$	$b := L_3$
Offset	$c := 0$	



Solution: See Figure P4-5c and Mathcad file P0418c.

1. Let pistons 1, 2, and 3 be links 7, 6, and 8, respectively.

2. Solve first for piston 6. Establish θ_2 as a range variable: $\theta_{21} := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$

3. Determine θ_3 and d using equations 4.16 and 4.17.

$$\theta_{31}(\theta_{21}) := asin\left(-\frac{a \cdot sin(\theta_{21}) - c}{b}\right) + \pi \qquad d_1(\theta_{21}) := a \cdot cos(\theta_{21}) - b \cdot cos(\theta_{31}(\theta_{21}))$$

4. For each piston (slider) the crank angle is measured counter-clock-wise from the centerline of the piston, which goes through the O_2 in all cases. Thus, when the crank angle for piston 1 is 0 deg, it is 120 deg for piston 2 and 240 deg for piston 3. Thus, the crank angles for pistons 2 and 3 are

$$\theta_{22}(\theta_{21}) := \theta_{21} + 120 \cdot deg \qquad \theta_{23}(\theta_{21}) := \theta_{21} + 240 \cdot deg$$

5. Determine θ_3 and d for pistons 2 and 3.

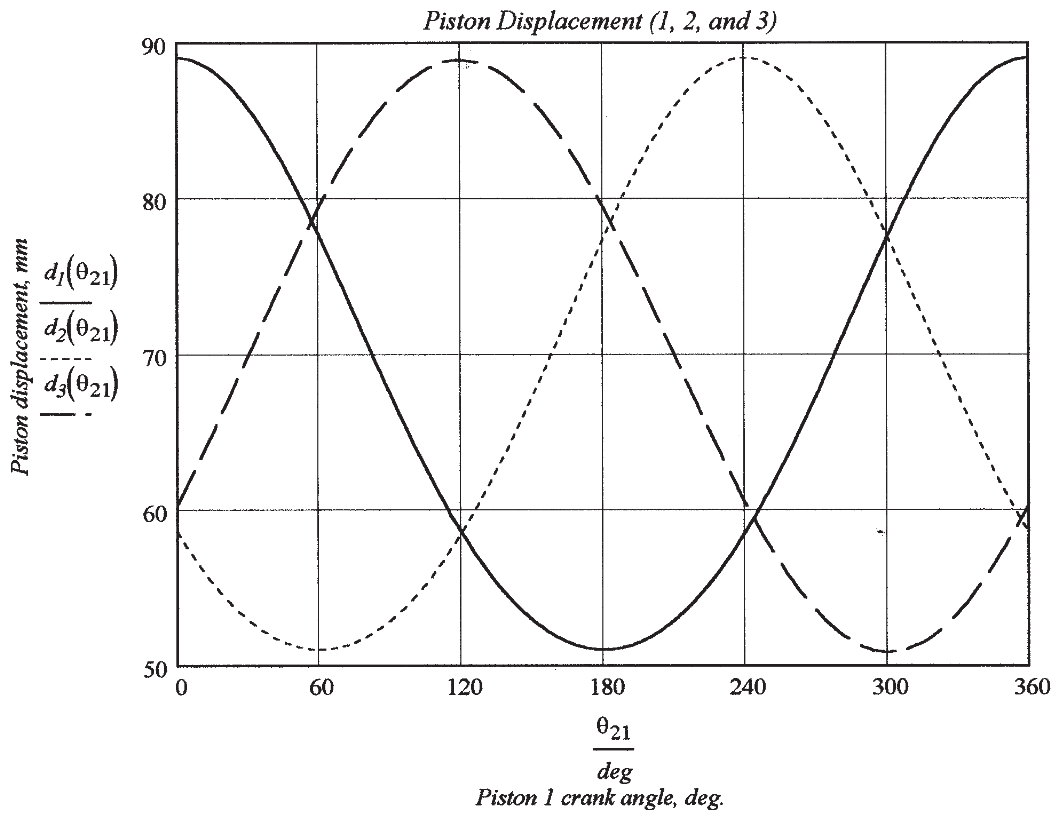
$$\theta_{32}(\theta_{21}) := asin\left(-\frac{a \cdot sin(\theta_{22}(\theta_{21})) - c}{b}\right) + \pi$$

$$d_2(\theta_{21}) := a \cdot cos(\theta_{22}(\theta_{21})) - b \cdot cos(\theta_{32}(\theta_{21}))$$

$$\theta_{33}(\theta_{21}) := asin\left(-\frac{a \cdot sin(\theta_{32}(\theta_{21})) - c}{b}\right) + \pi$$

$$d_3(\theta_{21}) := a \cdot cos(\theta_{23}(\theta_{21})) - b \cdot cos(\theta_{33}(\theta_{21}))$$

6. Plot the piston displacements as a function of crank angle (referenced to line AC (see next page).



The solid line is piston 1, the dotted line is piston 2, and the dashed line is piston 3.

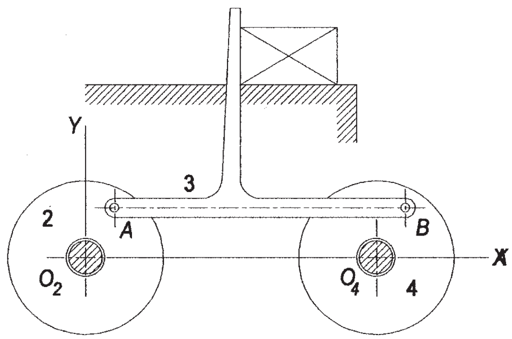
PROBLEM 4-18d

Statement: Find the total angular displacement of link 3 and the total stroke of the box as link 2 makes a complete revolution.

Given:

Ground link	$L_1 := 150$	Input crank	$L_2 := 30$
Coupler link	$L_3 := 150$	Output crank	$L_4 := 30$

Solution: See Figure P4-5d and Mathcad file P0418d.



1. This is a special-case Grashof mechanism in the parallelogram form (see Figure 2-17 in the text). As such, the coupler link 3 executes curvilinear motion and is always parallel to the ground link 1. Thus, the total angular motion of link 3 as crank 2 makes one complete revolution is zero degrees.

2. The stroke of the box will be equal to twice the length of the crank link in one complete revolution of the crank.

$stroke := 2 \cdot L_2 \qquad stroke = 60$

PROBLEM 4-18e

Statement: Determine the ratio of angular displacement between links 8 and 2 as a function of angular displacement of input crank 2. Plot the transmission angle at point *B* for one revolution of crank 2. Comment on the behavior of this linkage. Can it make a full revolution as shown?

Given: Link lengths:

Ground link (O_2O_4) $d_1 := 160$

Crank (O_2A) $a_1 := 20$

Coupler (L_3) $b_1 := 160$

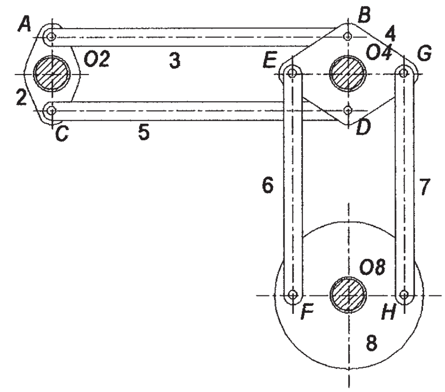
Crank (O_4B) $c_1 := 20$

Ground link (O_4O_8) $d_2 := 120$

Crank (O_4G) $a_2 := 30$

Coupler (L_6) $b_2 := 120$

Crank (O_8F) $c_2 := 30$



Solution: See Figure P4-5e and Mathcad file P0418e.

1. This is an eightbar, 1-DOF linkage with two redundant links (3 and 6 or 5 and 7) making it, effectively, a sixbar. It is composed of a fourbar (1, 2, 3, and 4) with an output dyad (7 and 8). The input fourbar is a special-case Grashof in the parallelogram configuration. Thus, the output angle is equal to the input angle and the couplers execute curvilinear motion with links 3 and 5 always parallel to the horizontal. The output dyad also behaves like a special-case Grashof with parallelogram configuration so that the angular motion of link 8 is equal to that of link 4. Therefore, the ratio of angular displacement between links 8 and 2 is unity. The mechanism is not capable of making a full revolution. The couplers 3 and 5 (also 6 and 7) cannot pass by each other near $\theta_2 = 0$ and 180 deg because of interference with the pins that connect them to their cranks.

2. Define the approximate range of motion of the input crank: $\theta_2 := 0 \cdot deg, 2 \cdot deg .. 180 deg$

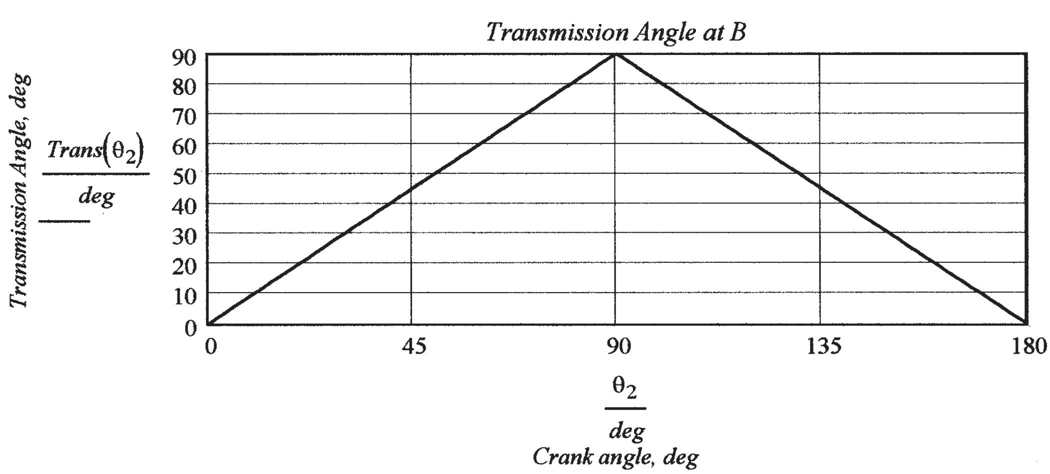
3. Define θ_3 and θ_4.

$$\theta_3 := 0.deg \qquad \theta_4(\theta_2) := \theta_2$$

4. Use equations 4.28 to find and plot the transmission angle.

$$tran(\theta_2) := |\theta_3 - \theta_4(\theta_2)| \qquad Tran(\theta_2) := if(tran(\theta_2) > \pi, tran(\theta_2) - \pi, tran(\theta_2))$$

$$Trans(\theta_2) := if\left(Tran(\theta_2) > \frac{\pi}{2}, |\pi - Tran(\theta_2)|, Tran(\theta_2)\right)$$

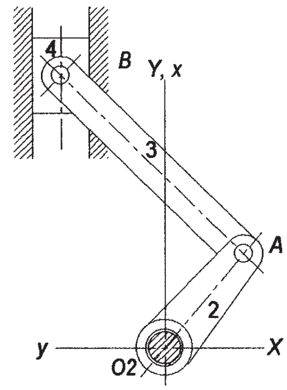


PROBLEM 4-18f

Statement: Find and plot the displacement of piston 4 and the angular displacement of link 3 as a function of the angular displacement of crank 2.

Given: Link lengths:

Crank length, L_2	$a := 63$
Piston-rod length, L_3	$b := 130$
Offset	$c := 52$



Solution: See Figure P4-5f and Mathcad file P0418f.

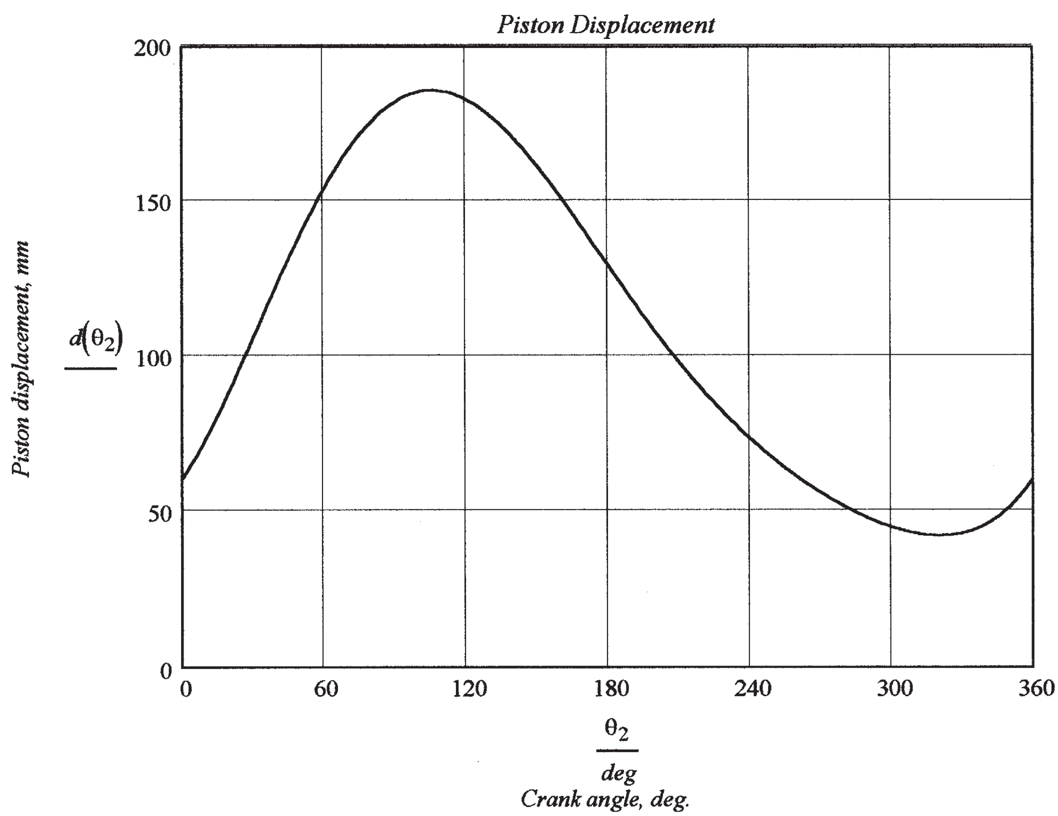
1. Establish θ_2 as a range variable: $\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$

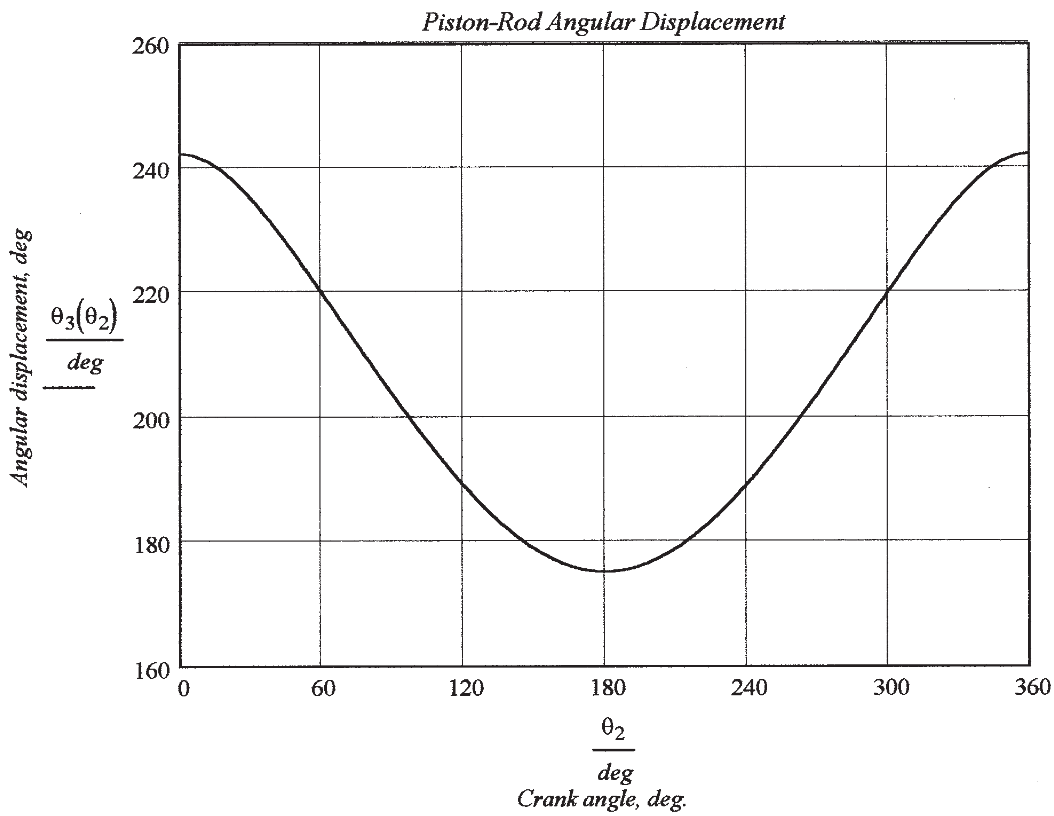
2. Determine θ_3 and d in global XY coord using equations 4.16 and 4.17.

$$\theta_3(\theta_2) := asin\left(\frac{a \cdot sin(\theta_2 - 90 \cdot deg) - c}{b}\right) + \pi$$

$$d(\theta_2) := a \cdot cos(\theta_2 - 90 \cdot deg) - b \cdot cos(\theta_3(\theta_2))$$

3. Plot the piston displacement (directly below) and rod angle (next page) as functions of crank angle in the global XY coordinate frame.




Piston-Rod Angular Displacement
260
240
220
200
180
160
Angular displacement, deg
θ3(θ2)
deg
0
60
120
180
240
300
360
θ2
deg
Crank angle, deg.

PROBLEM 4-18g

Statement: Find and plot the angular displacement of link 6 versus the angle of input link 2 as it is rotated from the position shown (+30 deg) to a vertical position (+90 deg). Find the toggle positions of this linkage in terms of the angle of link 2.

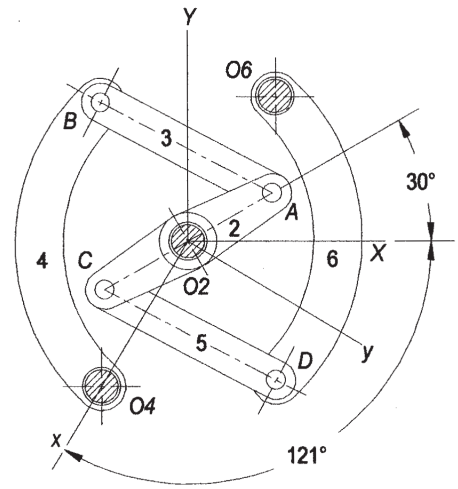
Given: Link lengths:

Input (L_2)	$a := 49$	Coupler (L_3)	$b := 100$
Rocker (L_4)	$c := 153$	Ground link (L_1)	$d := 87$

Angle from x axis to X axis: $\alpha := 121 \cdot deg$

Starting angle: $\theta_{21} := 30 \cdot deg$

Crank rotation angle from position shown to vertical: $\Delta\theta_2 := 60 \cdot deg$



Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P4-5g and Mathcad file P0418g.

1. Define one cycle of the input crank in global coord: $\theta_2 := \theta_{21}, \theta_{21} + 1\cdot deg .. \theta_{21} + \Delta\theta_2$

2. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 1.7755 \qquad K_2 = 0.5686 \qquad K_3 = 1.5592$$

$$A(\theta_2) := cos(\theta_2 + \alpha) - K_1 - K_2\cdot cos(\theta_2 + \alpha) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2 + \alpha) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2 + \alpha) + K_3$$

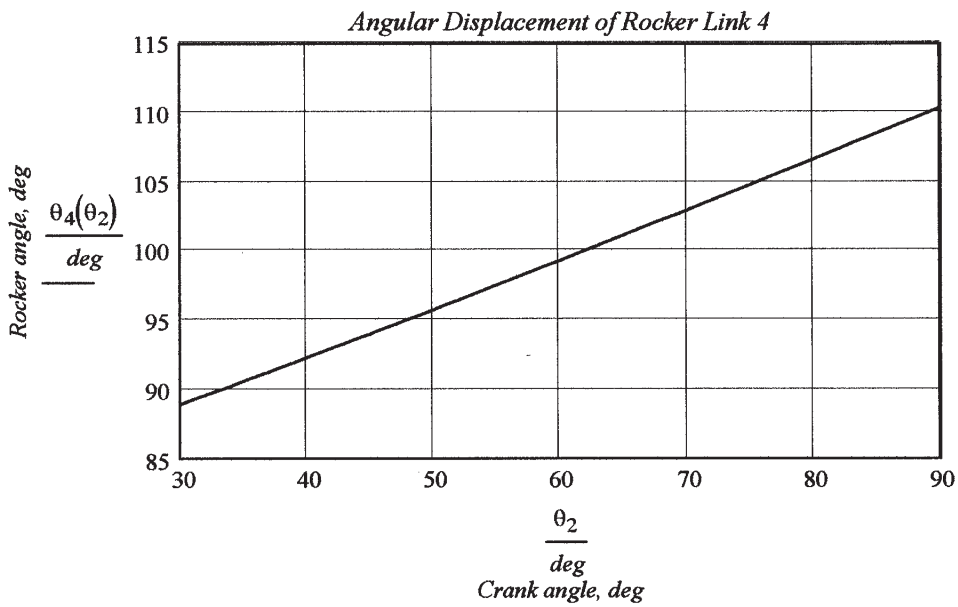
$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right) - \alpha$$

3. If the calculated value of θ_4 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := if(\theta_{41}(\theta_2) > 2\cdot\pi, \theta_{41}(\theta_2) - 2\cdot\pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := if(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2\cdot\pi, \theta_{42}(\theta_2))$$

4. Plot θ_4 as a function of the crank angle θ_2 (measured from the X-axis) as it rotates from the position shown to the vertical position.



4. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array}$$

$Condition(a,c,b,d) = $ "non-Grashof"

5. Using equations 4.33, determine the crank angles (relative to the x-axis) at which links 3 and 4 are in toggle.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} + \frac{b\cdot c}{(a\cdot d)} \qquad arg_1 = 0.840$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} - \frac{b\cdot c}{(a\cdot d)} \qquad arg_2 = -6.338$$

$$\theta_{2toggle} := acos(arg_1) \qquad \theta_{2toggle} = 32.9\,deg$$

The other toggle angle is the negative of this. Thus, in the global XY frame the toggle positions are:

$$\theta_{2XYtoggle} := \theta_{2toggle} - \alpha \qquad \theta_{2XYtoggle} = -88.130\,deg$$

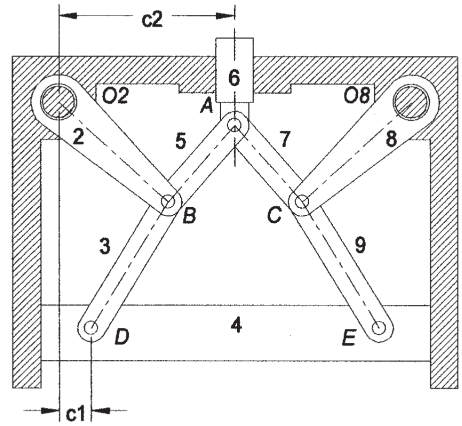
$$\theta_{2XYtoggle} := -\theta_{2toggle} - \alpha \qquad \theta_{2XYtoggle} = -153.870\,deg$$

PROBLEM 4-18h

Statement: Find link 4's maximum displacement vertically downward from the position shown. What will the angle of input link 2 be at that position?

Given: Link lengths:

Crank length, L_2 or L_8	$a := 19.8 \cdot mm$
Coupler length, L_3 or L_5	$b1 := 19.4 \cdot mm$
Offset of 1, 2, 3, 4	$c1 := 4.5 \cdot mm$
Distance from O_2 to O_8	$L_1 := 45.8 \cdot mm$
Coupler length, L_5 or L_7	$b2 := 13.3 \cdot mm$
Offset of 1, 2, 5, 6	$c2 := 22.9 \cdot mm$
Angle of link 2 as shown	$\theta_{20} := 47 \cdot deg$



Solution: See Figure P4-5h and Mathcad file P0418h.

1. Links 1, 2, 3, 4, 5, and 6 make up two offset slider-cranks with a common crank, link 2. Links 7, 8, and 9 are kinematically redundant and contribute only to equalizing the forces in the mirror image links. Slider-crank 1, 2, 3, 4 is in the open circuit, and slider-crank 1, 2, 5, 6 is in the crossed circuit.

2. Calculate the displacement of link 4 with respect to link 2 angle for the position shown in Figure P4-5h using equations 4.17 and 4.16b.

$$\theta_{310} := asin\left[-\left(\frac{a \cdot sin(\theta_{20}) - c1}{b1}\right)\right] + \pi \qquad \theta_{310} = 149.038\, deg$$

$$d_{10} := a \cdot cos(\theta_{20}) - b1 \cdot cos(\theta_{310}) \qquad d_{10} = 30.14\, mm$$

3. Link 4 will reach its maximum downward displacement when links 8 and 9 and links 2 and 3 are in the toggle position. However, it is possible that they may not be able to reach this position because links 5 and 7 may be too short to allow links 2 and 8 to rotate far enough to reach toggle with 3 and 9, respectively.

4. Using equation 4.16a, determine the angle that the crank will make with the x axis (see layout below) when links 5 and 7 are horizontal (θ_5 = -90 deg). This will be the least value of the angle θ_2.

$$\theta_{52} := asin\left(\frac{a \cdot sin(\theta_2) - c2}{b2}\right)$$

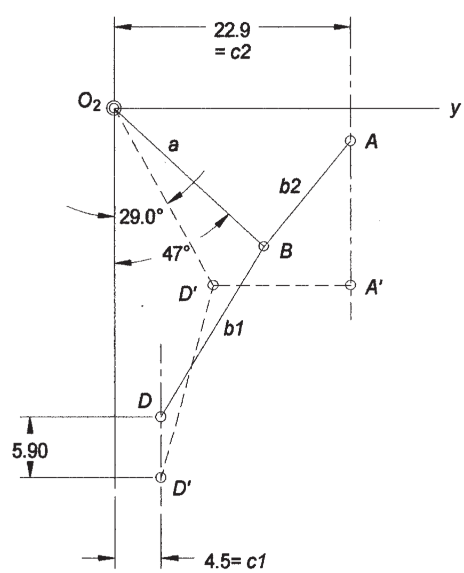
$$\theta_{52} := -90 \cdot deg$$

$$sin(\theta_{52}) = -1.000$$

$$\frac{a \cdot sin(\theta_2) - c2}{b2} := -1.000$$

$$\theta_2 := asin\left(\frac{c2 - b2}{a}\right)$$

$$\theta_2 = 29.00\, deg$$



5. Use equations 4.17 and 4.16b to determine the displacement of D' with respect to O_2.

$$\theta_{31} := asin\left[-\left(\frac{a \cdot sin(\theta_2) - c1}{b1}\right)\right] + \pi \qquad \theta_{31} = 164.759\,deg$$

$$d_1 := a \cdot cos(\theta_2) - b1 \cdot cos(\theta_{31}) \qquad d_1 = 36.03\,mm$$

6. The maximum displacement of link 4 from the position shown in Figure P4-5h is the difference between the displacement found in step 5 and that found in step 2.

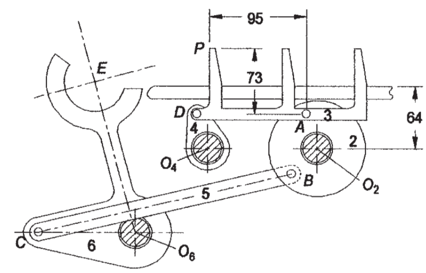
$$\Delta d_{max} := d_1 - d_{10} \qquad \Delta d_{max} = 5.90\,mm$$

PROBLEM 4-19

Statement: For one revolution of the driving link 2 of the walking-beam indexing and pick-and-place mechanism in Figure P4-6, find the horizontal stroke of link 3 for the portion of their motion where their tips are above the platen. Express the stroke as a percentage of the crank length O_2B. What portion of a revolution of link 2 does this stroke correspond to? Also find the total angular displacement of link 6 over one revolution of link 2.

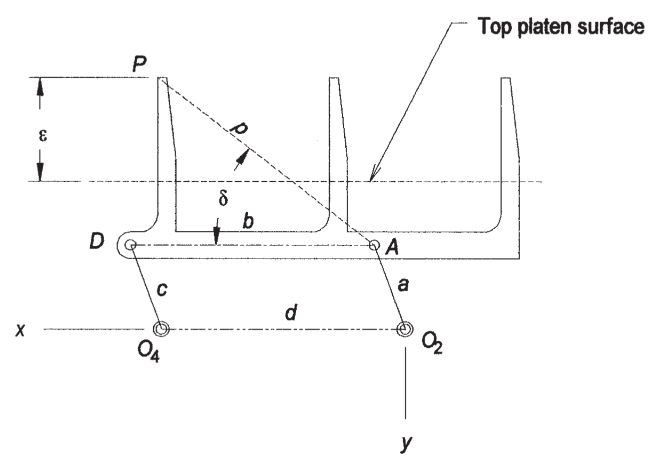
Given: Measured lengths:

Input crank length (O_2A)	$a := 40$
Coupler length (L_3)	$b := 108$
Output crank length (L_4)	$c := 40$
Ground link length (O_2O_4)	$d := 108$
Coupler data (finger closest to D)	$p := 119.81$
	$\delta := -37.54 \cdot deg$
Distance from O_2 to the platen surface	$e := 64$



Solution: See Figure P4-6 and Mathcad file P0419.

1. Links 1, 2, 3 and 4 are a special-case Grashof linkage in the parallelogram form. The tip of the finger closest to point D (left end of the coupler) is used as the coupler point. The distance from the tip to the platen is ε.



2. Define the crank angle as a range variable and define θ_3 ,which is constant because the coupler has curvilinear motion..

$$\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

$$\theta_3 := 0 \cdot deg$$

3. Use equations 4.27 to define the y-component of the vector $\mathbf{R}_P$.

$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a \cdot \left(cos(\theta_2) + j \cdot sin(\theta_2)\right)$$

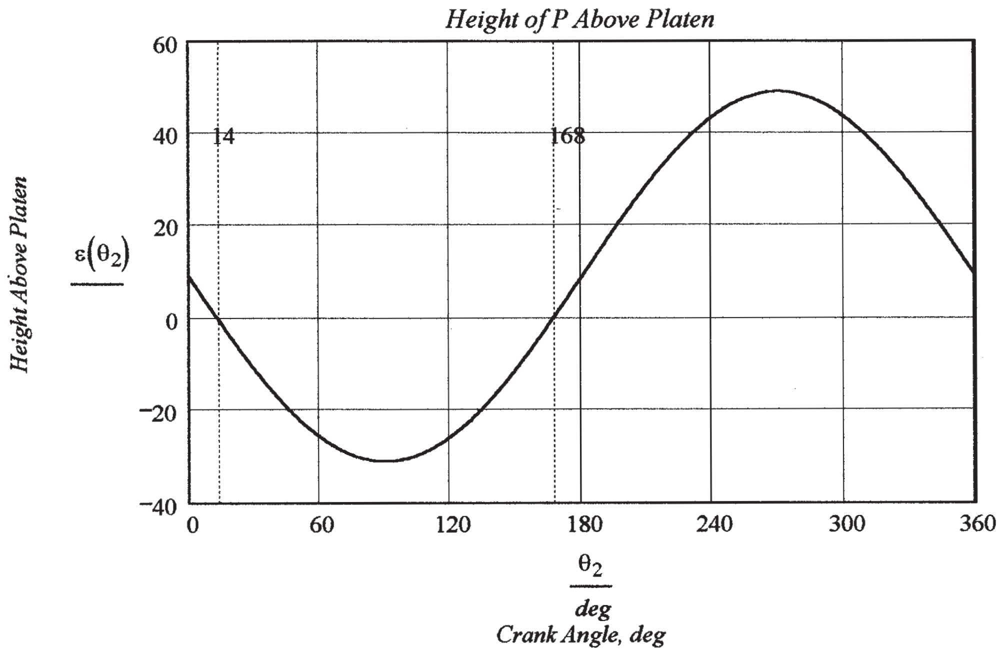
$$\mathbf{R_{PA}} := p \cdot \left(cos(\theta_3 + \delta) + j \cdot sin(\theta_3 + \delta)\right)$$

$$R_{Py}(\theta_2) := a \cdot sin(\theta_2) + p \cdot sin(\theta_3 + \delta)$$

4. Define the distance of point P above the platen (note the direction of the positive y axis in the figure above).

$$\varepsilon(\theta_2) := -e - R_{Py}(\theta_2)$$

5. Plot ε as a function of crank angle θ_2.



6. From the graph we see that the coupler point P is above the platen when the crank angle is greater than 168 deg and less than 14 deg. To find the horizontal stroke during that range of θ_2, calculate the x-components of any point on the coupler, say point A, for those two crank angles and subtract them.

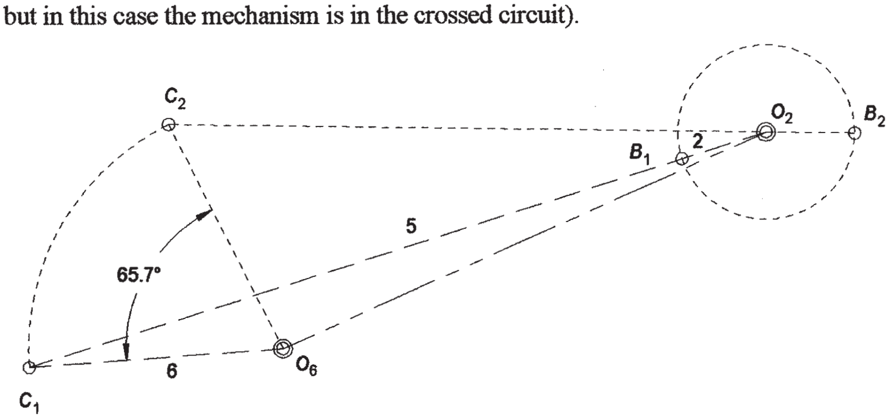
$$A_{x1} := a \cdot cos(14 \cdot deg) \qquad A_{x1} = 38.812$$

$$A_{x2} := a \cdot cos(168 \cdot deg) \qquad A_{x2} = -39.126$$

Horizontal stroke when above the platen normalized by dividing by the crank length

$$Stroke := \frac{A_{x1} - A_{x2}}{a} \qquad Stroke = 1.95 \quad \text{times the crank length}$$

7. Links 1, 2, 5, and 6 constitute a Grashoff crank-rocker-rocker. The extreme positions of the output rocker (link 6) occur when links 2 and 5 are in extended and overlapping toggle positions (see Figure 3-1b in the text for example, but in this case the mechanism is in the crossed circuit).



Given link lengths:

$L_{O2B} := 32$ $L_5 := 260$ $L_{O6C} := 96$ $L_{O2O6} := 200$

In the first position (links 2 and 5 extended), the angle between link 6 and the ground link is:

$$\alpha_1 := acos\left[\frac{L_{O6C}^2 + L_{O2O6}^2 - \left(L_{O2B} + L_5\right)^2}{2 \cdot L_{O6C} \cdot L_{O2O6}}\right] \qquad \alpha_1 = 159.84\,deg$$

In the second position (links 2 and 5 overlapping), the angle between link 6 and the ground link is:

$$\alpha_2 := acos\left[\frac{L_{O6C}^2 + L_{O2O6}^2 - \left(L_5 - L_{O2B}\right)^2}{2 \cdot L_{O6C} \cdot L_{O2O6}}\right] \qquad \alpha_2 = 94.13\,deg$$

The total angular displacement of link 6 is the difference between these two angles.

$\Delta_{DE} := \alpha_1 - \alpha_2$ $\Delta_{DE} = 65.7\,deg$

PROBLEM 4-20

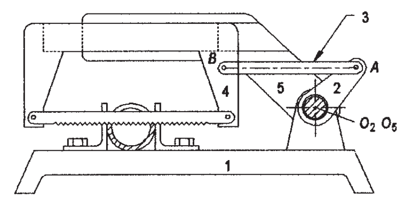
Statement: Figure P4-7 shows a power hacksaw, used to cut metal. Link 5 pivots at O5 and its weight forces the saw blade against the workpiece while the linkage moves the blade (link 4) back and forth on link 5 to cut the part. It is an offset slider-crank mechanism. The dimensions are shown in the figure. For one revolution of the driving link 2 of the hacksaw mechanism on the cutting stroke, find and plot the horizontal stroke of the saw blade as a function of the angle of link 2.

Given: Measured lengths:

Crank length, L_2 $a := 75\cdot mm$

Coupler length, L_3 $b := 170\cdot mm$

Offset $c := 45\cdot mm$



Assumptions: The arm that guides the slider (hacksaw blade carrier) remains horizontal throughout the stroke.

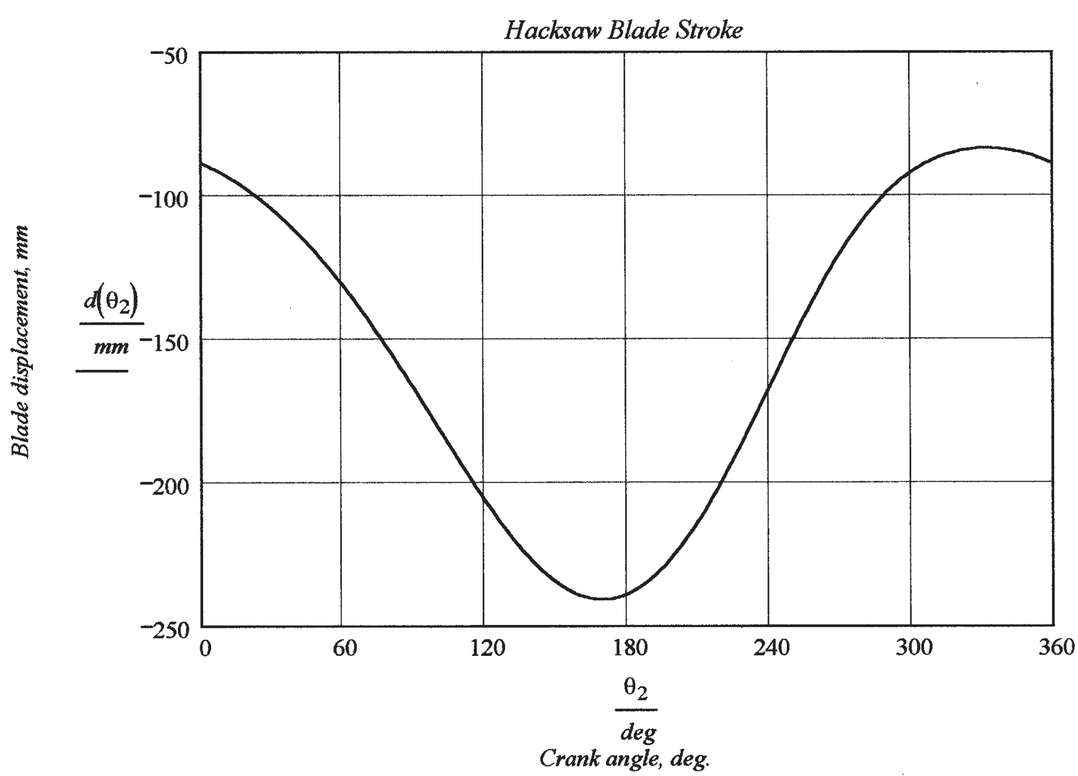
Solution: See Figure P4-7 and Mathcad file P0420.

1. This is a slider-crank mechanism in the crossed circuit. The offset is the vertical distance from the horizontal centerline through O_2 to point B.
2. Establish θ_2 as a range variable: $\theta_2 := 0\cdot deg, 2\cdot deg .. 360 deg$
3. Determine θ_3 and d using equations 4.16a and 4.17.

$$\theta_3(\theta_2) := asin\left(\frac{a\cdot sin(\theta_2) - c}{b}\right)$$

$$d(\theta_2) := a\cdot cos(\theta_2) - b\cdot cos(\theta_3(\theta_2))$$

4. Plot the blade (point B) displacement as a function of crank angle.



PROBLEM 4-21

Statement: For the linkage in Figure P4-8, find its limit (toggle) positions in terms of the angle of link O_2A referenced to the line of centers O_2O_4 when driven from link O_2A. Then calculate and plot the xy coordinates of coupler point P between those limits, referenced to the line of centers O_2O_4.

Given: Link lengths:

Input (O_2A) $a := 5.00 \cdot in$

Coupler (AB) $b := 4.40 \cdot in$

Rocker (O_4B) $c := 5.00 \cdot in$

Ground link $d := 9.50 \cdot in$

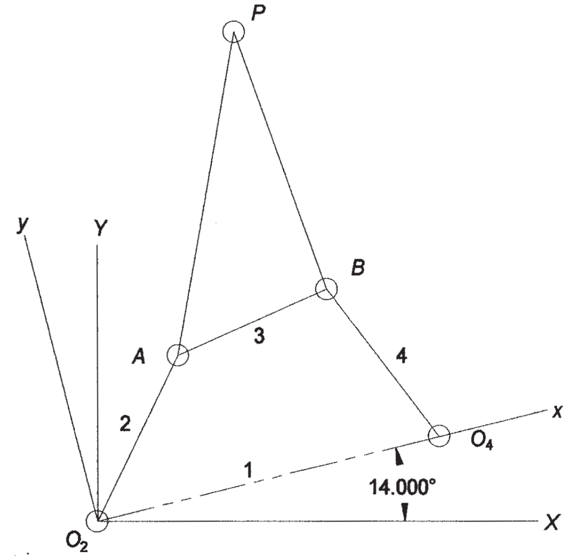
Coupler point data:

$p := 8.90 \cdot in$

$\delta := 56 \cdot deg$

Coordinate transformation angle:

$\alpha := 14 \cdot deg$



Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P4-8 and Mathcad file P0421.

1. Define the coordinate systems. The local frame has origin at O_2 with the positive x axis going through O_4. Let the global frame also have its origin at O_2 with the positive X axis to the right.

2. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{cases} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{cases}$$

$$Condition(b,d,a,c) = \text{"non-Grashof"}$$

3. Using equations 4.33, determine the crank angles (relative to the line AD) at which links 3 and 4 are in toggle.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 1.209$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = 0.283$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 73.6\,deg$$

The other toggle angle is the negative of this.

4. Define one cycle of the input crank between limit positions:

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 2\cdot deg .. \theta_{2toggle}$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \qquad K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)}$$

$$K_1 = 1.9000 \qquad K_4 = 2.1591 \qquad K_5 = -2.4911$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4\cdot\cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot\sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1)\cdot\cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

7. Use equations 4.27 to define the *x*- and *y*-components of the vector $\mathbf{R_P}$.

$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a\cdot(\cos(\theta_2) + j\cdot\sin(\theta_2))$$

$$\mathbf{R_{PA}} := p\cdot(\cos(\theta_3 + \delta) + j\cdot\sin(\theta_3 + \delta))$$

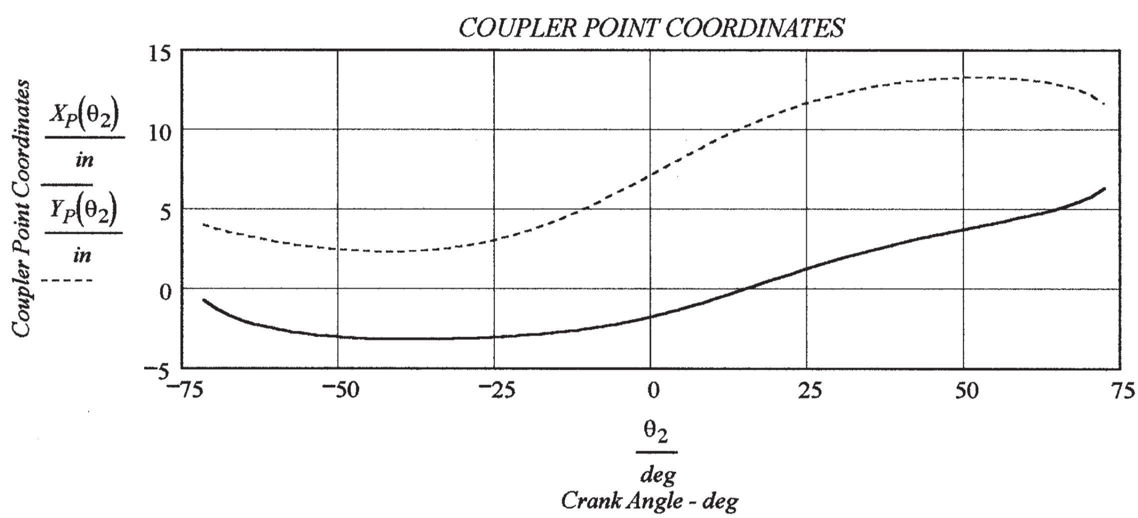
$$R_{Px}(\theta_2) := a\cdot\cos(\theta_2) + p\cdot\cos(\theta_3(\theta_2) + \delta) \qquad R_{Py}(\theta_2) := a\cdot\sin(\theta_2) + p\cdot\sin(\theta_3(\theta_2) + \delta)$$

8. Transform the coupler point coordinates in the local frame to the global frame using coordinate transformation equations.

$$X_P(\theta_2) := R_{Px}(\theta_2)\cdot\cos(\alpha) - R_{Py}(\theta_2)\cdot\sin(\alpha)$$

$$Y_P(\theta_2) := R_{Px}(\theta_2)\cdot\sin(\alpha) + R_{Py}(\theta_2)\cdot\cos(\alpha)$$

9. Plot the coordinates of the coupler point in the global system.



PROBLEM 4-22

Statement: For the walking beam mechanism of Figure P4-9, calculate and plot the x and y components of the position of the coupler point P for one complete revolution of the crank O_2A. Hint: Calculate them first with respect to the ground link O_2O_4 and then transform them into the global XY coordinate system (i.e., horizontal and vertical in the figure).

Given: Link lengths:

Ground link $d := 2.22$ Crank $a := 1$

Coupler $b := 2.06$ Rocker $c := 2.33$

Coupler point data: $p := 3.06$ $\delta := 31.000 \cdot deg$

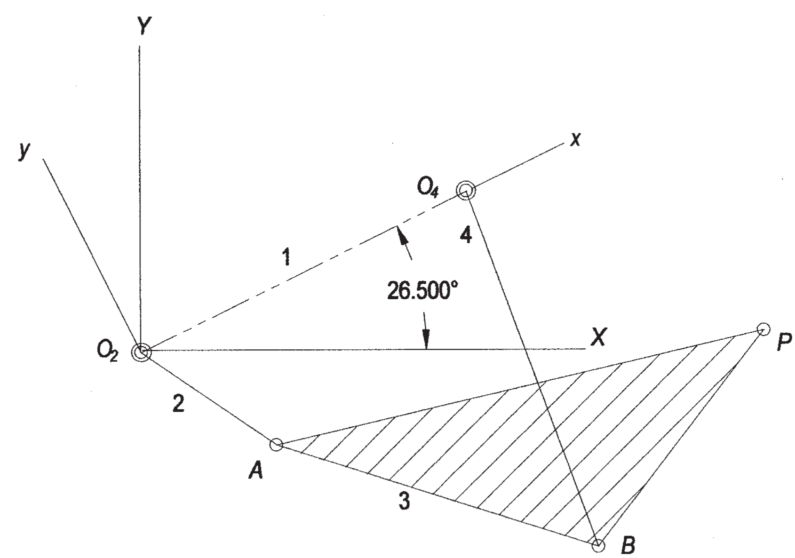
Coordinate transformation angle: $\alpha := 26.5 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P4-9 and Mathcad file P0422.

1. Define the coordinate systems. The local frame has origin at O_2 with the positive x axis going through O_4. Let the global frame also have its origin at O_2 with the positive X axis to the right.



2. Define one revolution of the input crank: $\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \qquad K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)}$$

$$K_1 = 2.2200 \qquad K_4 = 1.0777 \qquad K_5 = -1.1512$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

4. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right)\right)$$

5. Use equations 4.27 to define the *x*- and *y*-components of the vector $\mathbf{R}_P$.

$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{R_{PA}} := p \cdot (\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta))$$

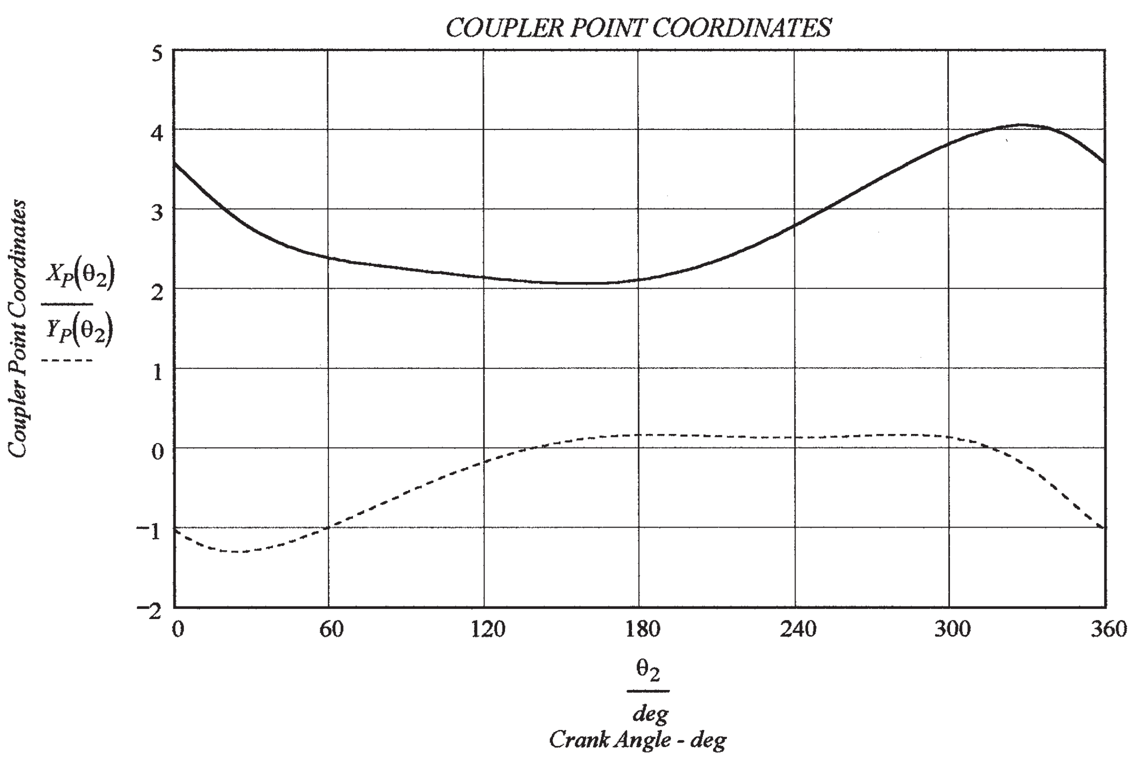
$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta) \qquad R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

6. Transform the coupler point coordinates in the local frame to the global frame using coordinate transformation equations.

$$X_P(\theta_2) := R_{Px}(\theta_2) \cdot \cos(\alpha) - R_{Py}(\theta_2) \cdot \sin(\alpha)$$

$$Y_P(\theta_2) := R_{Px}(\theta_2) \cdot \sin(\alpha) + R_{Py}(\theta_2) \cdot \cos(\alpha)$$

7. Plot the coordinates of the coupler point in the global system. Note that for a crank angle between 180 and 300 deg. the y- coordinate is approximately constant.



PROBLEM 4-23

Statement: For the linkage in Figure P4-10, calculate and plot the angular displacement of links 3 and 4 and the path coordinates of point P with respect to the angle of the input crank O_2A for one revolution.

Given: Link lengths:

Ground link $d := 2.22$ Crank $a := 1.0$

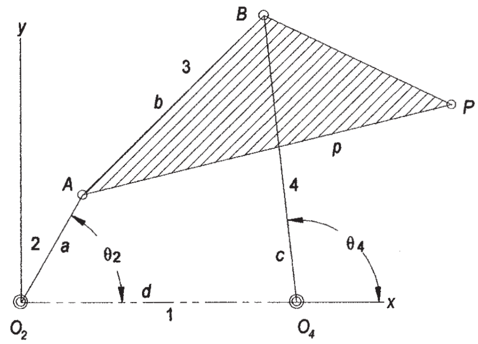
Coupler $b := 2.06$ Rocker $c := 2.33$

Coupler point data:

$p := 3.06$ $\delta := -31.00 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5\cdot\pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{cases}$$



Solution: See Figure P4-10 and Mathcad file P0423.

1. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$

2. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 2.2200 \qquad K_2 = 0.9528 \qquad K_3 = 1.5265$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

3. If the calculated value of θ_4 is greater than 2π, subtract 2π from it.

$$\theta_4(\theta_2) := if(\theta_{41}(\theta_2) > 2\cdot\pi, \theta_{41}(\theta_2) - 2\cdot\pi, \theta_{41}(\theta_2))$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.0777 \qquad K_5 = -1.1512$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

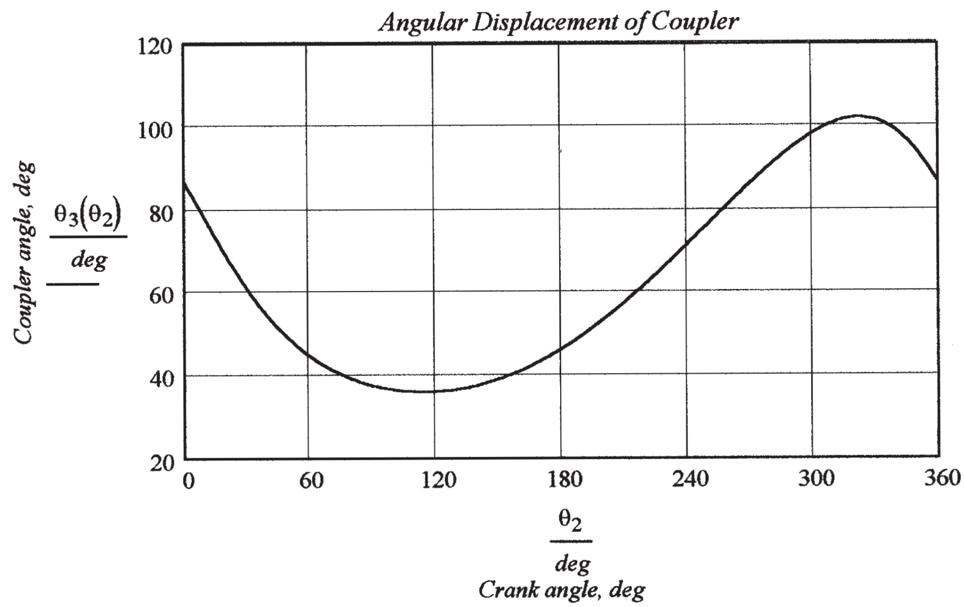
5. Use equation 4.13 to find values of θ_3 for the open circuit.

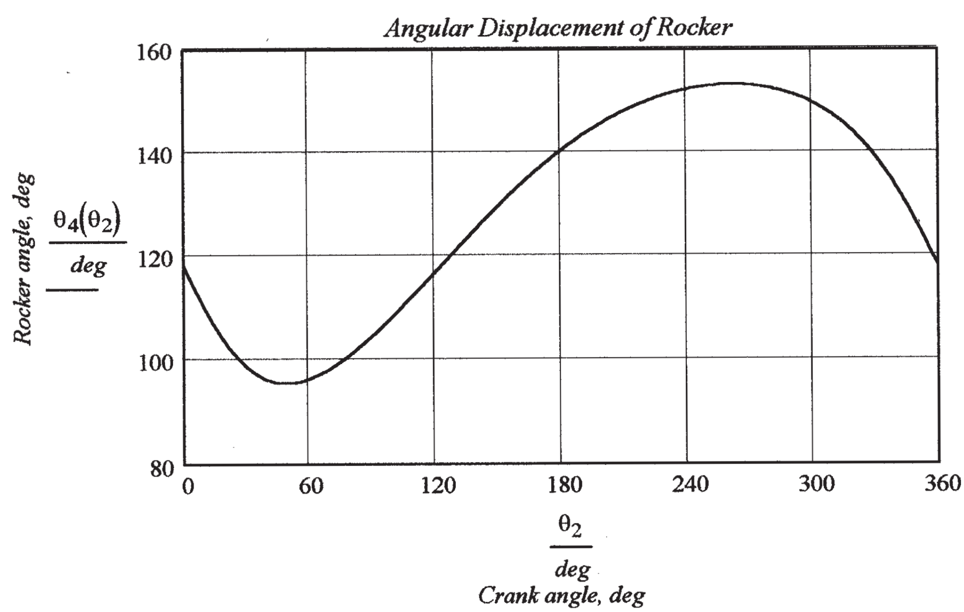
$$\theta_{31}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

6. If the calculated value of θ_3 is greater than 2π, subtract 2π from it.

$$\theta_3(\theta_2) := if\left(\theta_{31}(\theta_2) > 2\cdot\pi, \theta_{31}(\theta_2) - 2\cdot\pi, \theta_{31}(\theta_2)\right)$$

7. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).





8. Use equations 4.27 to define the *x*- and *y*-components of the vector $\mathbf{R}_P$.

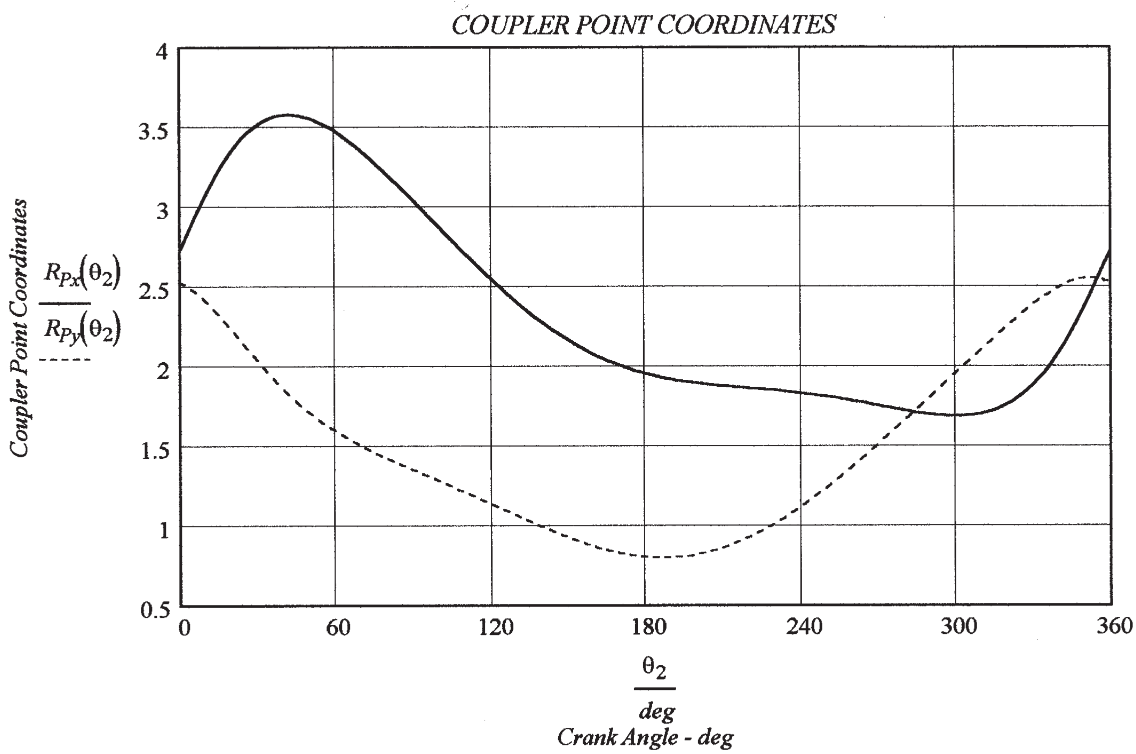
$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a\cdot\left(cos(\theta_2) + j\cdot sin(\theta_2)\right)$$

$$\mathbf{R_{PA}} := p\cdot\left(cos(\theta_3 + \delta) + j\cdot sin(\theta_3 + \delta)\right)$$

$$R_{Px}(\theta_2) := a\cdot cos(\theta_2) + p\cdot cos(\theta_3(\theta_2) + \delta) \qquad R_{Py}(\theta_2) := a\cdot sin(\theta_2) + p\cdot sin(\theta_3(\theta_2) + \delta)$$

9. Plot the coordinates of the coupler point in the local *xy* coordinate system.



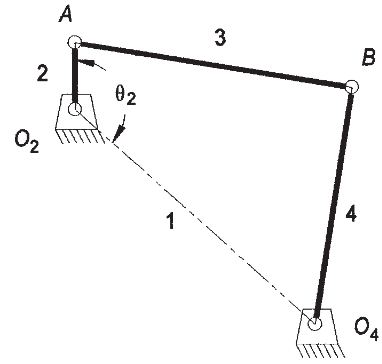
PROBLEM 4-24

Statement: For the linkage in Figure P4-11, calculate and plot the angular displacement of links 3 and 4 with respect to the angle of the input crank O_2A for one revolution.

Given: Link lengths:

Link 2 $a := 2.00 \cdot in$ Link 3 $b := 8.375 \cdot in$

Link 4 $c := 7.187 \cdot in$ Link 1 $d := 9.625 \cdot in$



Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P4-11 and Mathcad file P0424.

1. Define one revolution of the input crank: $\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$
2. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 4.8125 \qquad K_2 = 1.3392 \qquad K_3 = 2.7186$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right)\right)$$

3. If the calculated value of θ_4 is greater than 2π, subtract 2π from it.

$$\theta_4(\theta_2) := if(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.1493 \qquad K_5 = -3.4367$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

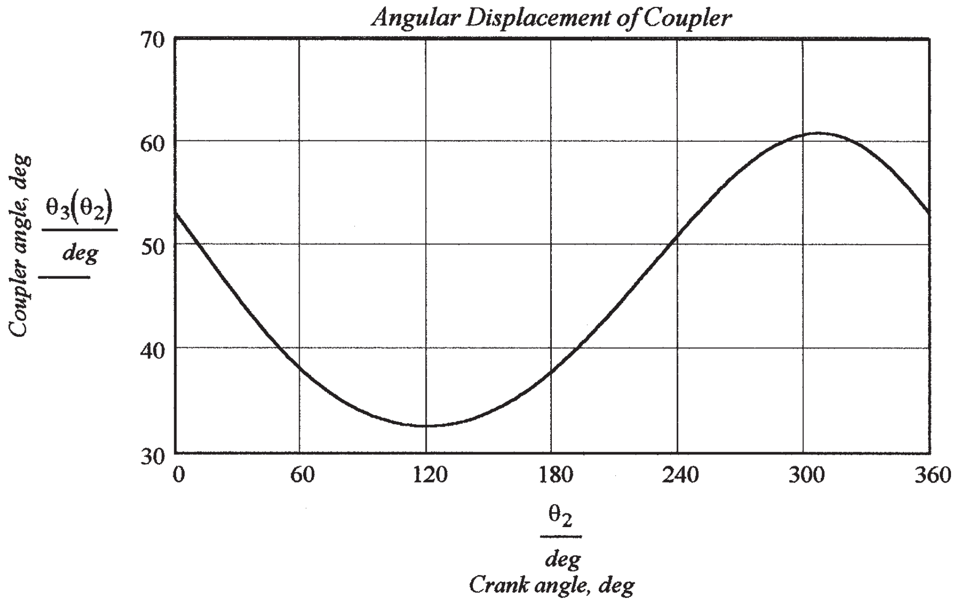
5. Use equation 4.13 to find values of θ_3 for the open circuit.

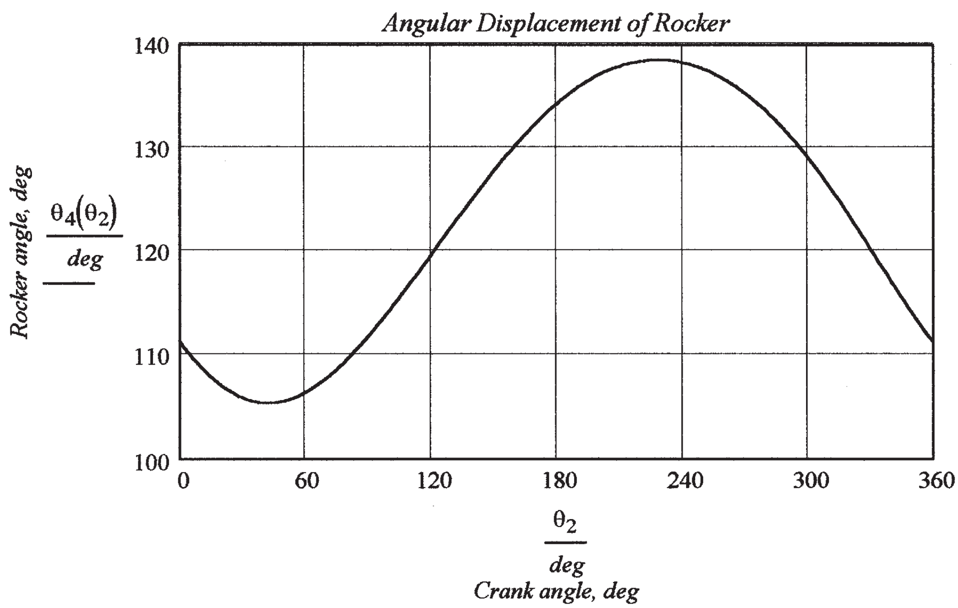
$$\theta_{31}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right)\right)$$

6. If the calculated value of θ_3 is greater than 2π, subtract 2π from it.

$$\theta_3(\theta_2) := if(\theta_{31}(\theta_2) > 2\cdot\pi, \theta_{31}(\theta_2) - 2\cdot\pi, \theta_{31}(\theta_2))$$

7. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).





PROBLEM 4-25

Statement: For the linkage in Figure P4-12, find its limit (toggle) positions in terms of the angle of link O_2A referenced to the line of centers O_2O_4 when driven from link O_2A. Then calculate and plot the angular displacement of links 3 and 4 and the path coordinates of point P with respect to the angle of the input crank O_2A over its possible range of motion referenced to the line of centers O_2O_4.

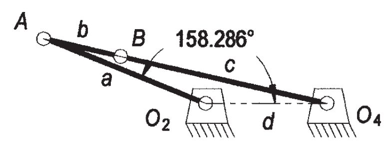
Given: Link lengths:

Input (O_2A) $a := 0.785$

Coupler (AB) $b := 0.356$

Rocker (O_4B) $c := 0.950$

Ground link $d := 0.544$



Coupler point data:

$p := 1.09$

$\delta := 0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P4-12 and Mathcad file P0425.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{cases} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{cases}$$

$Condition(b,c,a,d) = \text{"Grashof"}$ double rocker

2. Using the geometry defined in Figure 3-1a in the text, determine the input crank angles (relative to the line O_2O_4) at which links 2 and 3, and 3 and 4 are in toggle.

$$\theta_{21} := acos\left[\frac{d^2 + (a+b)^2 - c^2}{2\cdot d\cdot(a+b)}\right] \qquad \theta_{21} = 55.937\,deg$$

$$\theta_{22} := acos\left[\frac{a^2 + d^2 - (b+c)^2}{2\cdot a\cdot d}\right] \qquad \theta_{22} = 158.286\,deg$$

3. Define one cycle of the input crank between limit positions:

$$\theta_2 := \theta_{21}, \theta_{21} + 1\cdot deg .. \theta_{22}$$

4. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 0.6930 \qquad K_2 = 0.5726 \qquad K_3 = 1.1317$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. If the calculated value of θ_4 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := if\left(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2)\right)$$

$$\theta_4(\theta_2) := if\left(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2)\right)$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.5281 \qquad K_5 = -0.2440$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the open circuit.

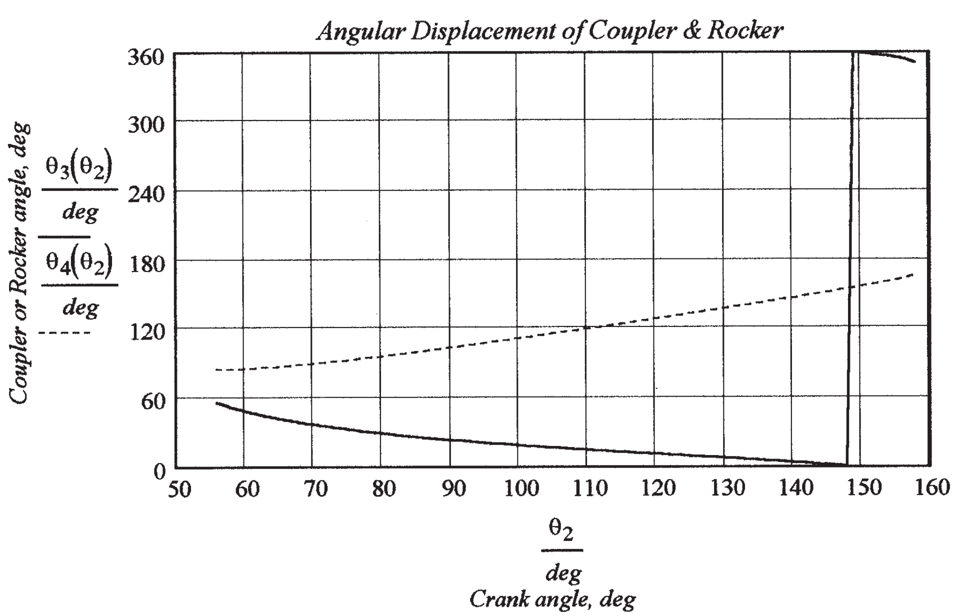
$$\theta_{31}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_2) := if\left(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2)\right)$$

$$\theta_3(\theta_2) := if\left(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) - 2 \cdot \pi, \theta_{32}(\theta_2)\right)$$

9. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).



10. Use equations 4.27 to define the *x*- and *y*-components of the vector $\mathbf{R}_P$.

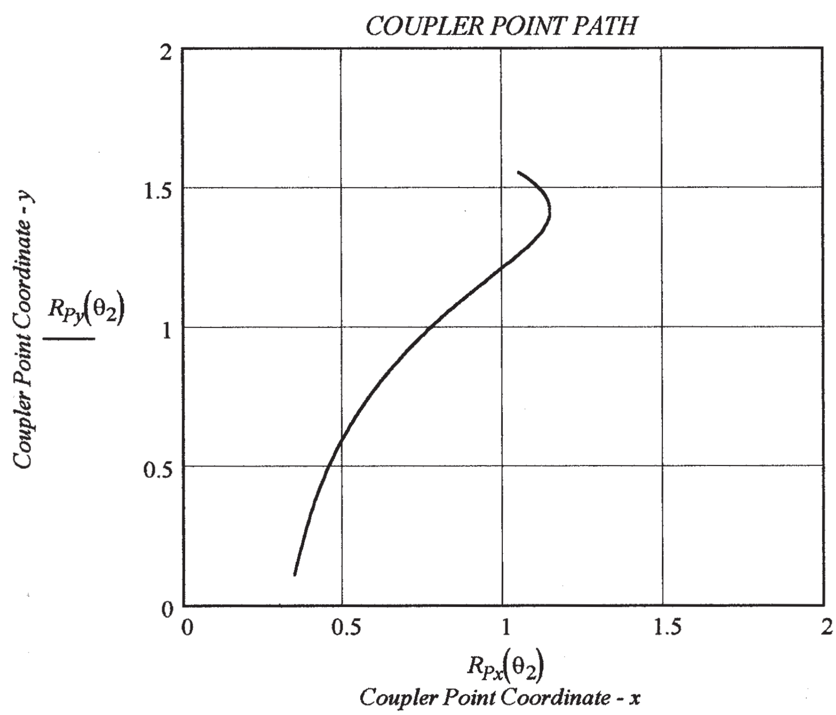
$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a \cdot \left(cos(\theta_2) + j \cdot sin(\theta_2) \right)$$

$$\mathbf{R_{PA}} := p \cdot \left(cos(\theta_3 + \delta) + j \cdot sin(\theta_3 + \delta) \right)$$

$$R_{Px}(\theta_2) := a \cdot cos(\theta_2) + p \cdot cos(\theta_3(\theta_2) + \delta) \qquad R_{Py}(\theta_2) := a \cdot sin(\theta_2) + p \cdot sin(\theta_3(\theta_2) + \delta)$$

11. Plot the coordinates of the coupler point in the local *xy* coordinate system.



PROBLEM 4-26

Statement: For the linkage in Figure P4-13, find its limit (toggle) positions in terms of the angle of link O_2A referenced to the line of centers O_2O_4 when driven from link O_2A. Then calculate and plot the angular displacement of links 3 and 4 and the path coordinates of point P with respect to the angle of the input crank O_2A over its possible range of motion referenced to the line of centers O_2O_4.

Given: Link lengths:

Input (O_2A) $a := 0.86$

Coupler (AB) $b := 1.85$

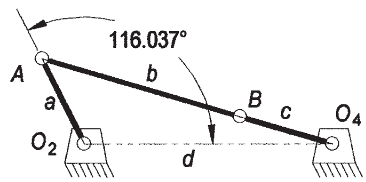
Rocker (O_4B) $c := 0.86$

Ground link $d := 2.22$

Coupler point data:

$p := 1.33$

$\delta := 0 \cdot deg$



Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P4-13 and Mathcad file P0426.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array}$$

$Condition(a,d,b,c) = \text{"non-Grashof"}$

2. Using equations 4.33, determine the crank angles (relative to the line AD) at which links 3 and 4 are in toggle.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} + \frac{b\cdot c}{(a\cdot d)} \qquad arg_1 = 1.228$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} - \frac{b\cdot c}{(a\cdot d)} \qquad arg_2 = -0.439$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 116.037\,deg$$

The other toggle angle is the negative of this.

3. Define one cycle of the input crank between limit positions:

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 2\cdot deg .. \theta_{2toggle}$$

4. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 2.5814 \qquad K_2 = 2.5814 \qquad K_3 = 2.0181$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. If the calculated value of θ_4 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := if(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := if(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.2000 \qquad K_5 = -2.6244$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the open circuit.

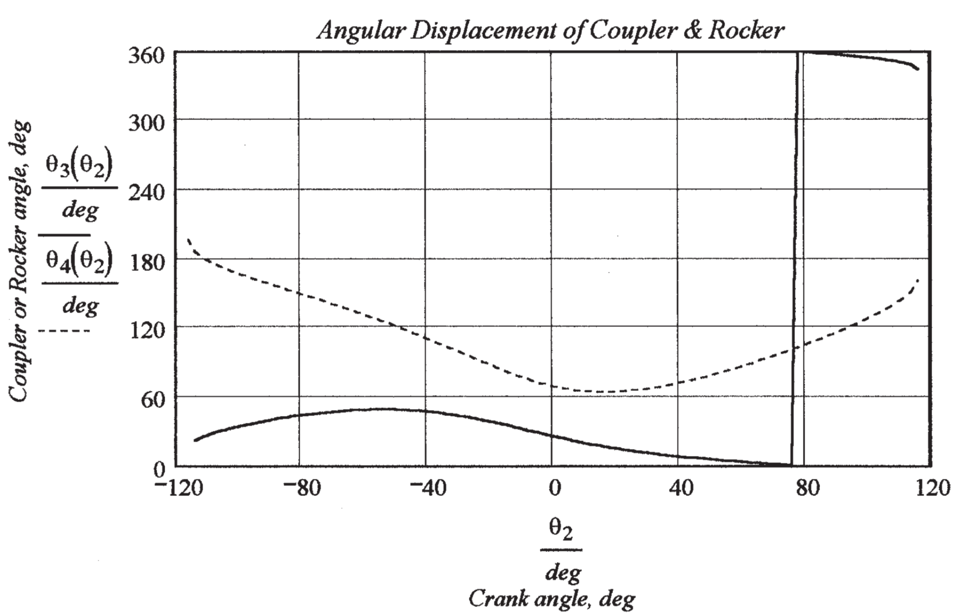
$$\theta_{31}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_2) := if(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2))$$

$$\theta_3(\theta_2) := if(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) - 2 \cdot \pi, \theta_{32}(\theta_2))$$

9. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).



10. Use equations 4.27 to define the *x*- and *y*-components of the vector $\mathbf{R}_P$.

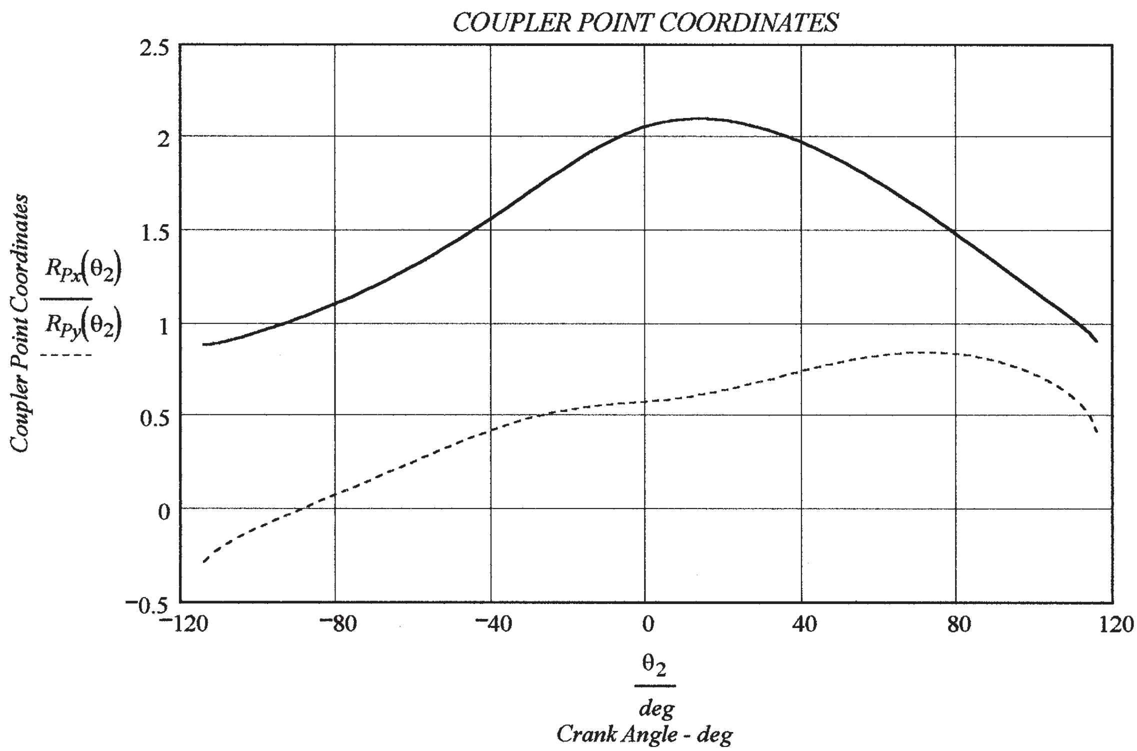
$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a\cdot\left(\cos(\theta_2) + j\cdot\sin(\theta_2)\right)$$

$$\mathbf{R_{PA}} := p\cdot\left(\cos(\theta_3 + \delta) + j\cdot\sin(\theta_3 + \delta)\right)$$

$$R_{Px}(\theta_2) := a\cdot\cos(\theta_2) + p\cdot\cos(\theta_3(\theta_2) + \delta) \qquad R_{Py}(\theta_2) := a\cdot\sin(\theta_2) + p\cdot\sin(\theta_3(\theta_2) + \delta)$$

11. Plot the coordinates of the coupler point in the local *xy* coordinate system.



PROBLEM 4-27

Statement: For the linkage in Figure P4-13, find its limit (toggle) positions in terms of the angle of link O_4B referenced to the line of centers O_4O_2 when driven from link O_4B. Then calculate and plot the angular displacement of links 2 and 3 and the path coordinates of point P with respect to the angle of the input crank O_4B over its possible range of motion referenced to the line of centers O_4O_2.

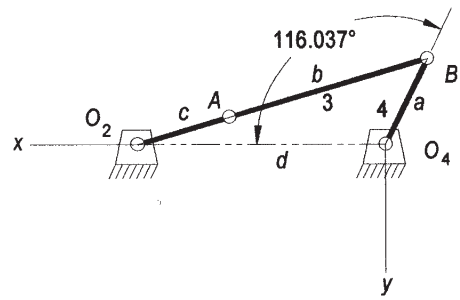
Given: Link lengths:

Input (O_4B) $a := 0.86$

Coupler (AB) $b := 1.85$

Rocker (O_2A) $c := 0.86$

Ground link $d := 2.22$



Coupler point data:

$p := 0.52$

$\delta := 0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P4-13 and Mathcad file P0427.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL \le PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array}$$

$$Condition(a,d,b,c) = \text{"non-Grashof"}$$

2. Using equations 4.33, determine the crank angles (relative to the line O_4O_2) at which links 2 and 3 are in toggle.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} + \frac{b\cdot c}{(a\cdot d)} \qquad arg_1 = 1.228$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} - \frac{b\cdot c}{(a\cdot d)} \qquad arg_2 = -0.439$$

$$\theta_{4toggle} := acos(arg_2) \qquad \theta_{4toggle} = 116.037\,deg$$

The other toggle angle is the negative of this.

3. Define one cycle of the input crank between limit positions:

$$\theta_4 := -\theta_{4toggle}, -\theta_{4toggle} + 2\cdot deg .. \theta_{4toggle}$$

4. Use equations 4.8a and 4.10 to calculate θ_2 as a function of θ_4 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 2.5814 \qquad K_2 = 2.5814 \qquad K_3 = 2.0181$$

$$A(\theta_4) := \cos(\theta_4) - K_1 - K_2 \cdot \cos(\theta_4) + K_3$$

$$B(\theta_4) := -2 \cdot \sin(\theta_4) \qquad C(\theta_4) := K_1 - (K_2 + 1) \cdot \cos(\theta_4) + K_3$$

$$\theta_{21}(\theta_4) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_4), -B(\theta_4) - \sqrt{B(\theta_4)^2 - 4 \cdot A(\theta_4) \cdot C(\theta_4)} \right) \right)$$

5. If the calculated value of θ_2 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{22}(\theta_4) := if\left(\theta_{21}(\theta_4) > 2 \cdot \pi, \theta_{21}(\theta_4) - 2 \cdot \pi, \theta_{21}(\theta_4)\right)$$

$$\theta_2(\theta_4) := if\left(\theta_{22}(\theta_4) < 0, \theta_{22}(\theta_4) + 2 \cdot \pi, \theta_{22}(\theta_4)\right)$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.2000 \qquad K_5 = -2.6244$$

$$D(\theta_4) := \cos(\theta_4) - K_1 + K_4 \cdot \cos(\theta_4) + K_5$$

$$E(\theta_4) := -2 \cdot \sin(\theta_4) \qquad F(\theta_4) := K_1 + (K_4 - 1) \cdot \cos(\theta_4) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the open circuit.

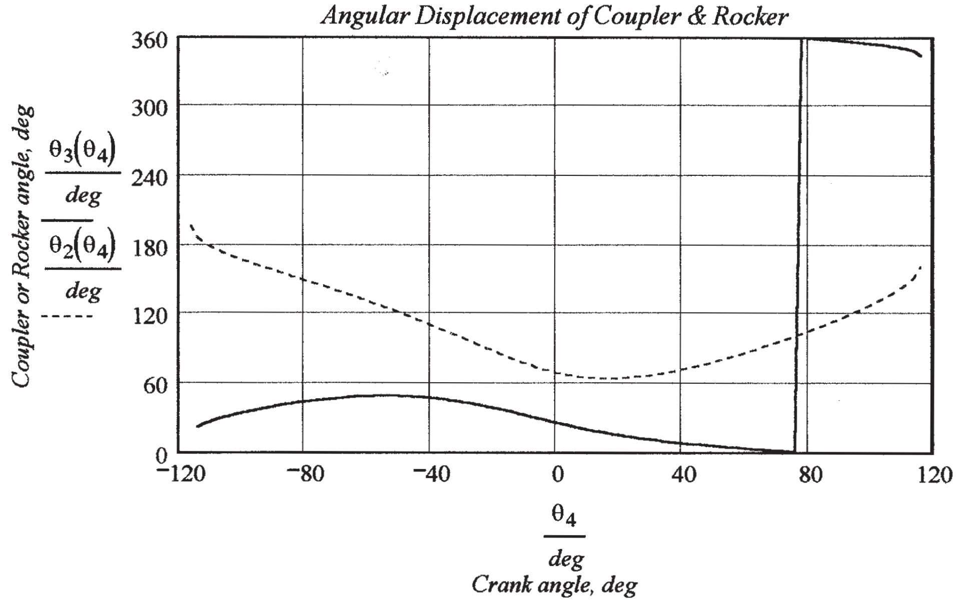
$$\theta_{31}(\theta_4) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_4), -E(\theta_4) - \sqrt{E(\theta_4)^2 - 4 \cdot D(\theta_4) \cdot F(\theta_4)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_4) := if\left(\theta_{31}(\theta_4) > 2 \cdot \pi, \theta_{31}(\theta_4) - 2 \cdot \pi, \theta_{31}(\theta_4)\right)$$

$$\theta_3(\theta_4) := if\left(\theta_{32}(\theta_4) < 0, \theta_{32}(\theta_4) - 2 \cdot \pi, \theta_{32}(\theta_4)\right)$$

9. Plot θ_3 and θ_2 as functions of the crank angle θ_4 (measured from the ground link). See next page.



10. Use equations 4.27 to define the *x*- and *y*-components of the vector $\mathbf{R}_P$.

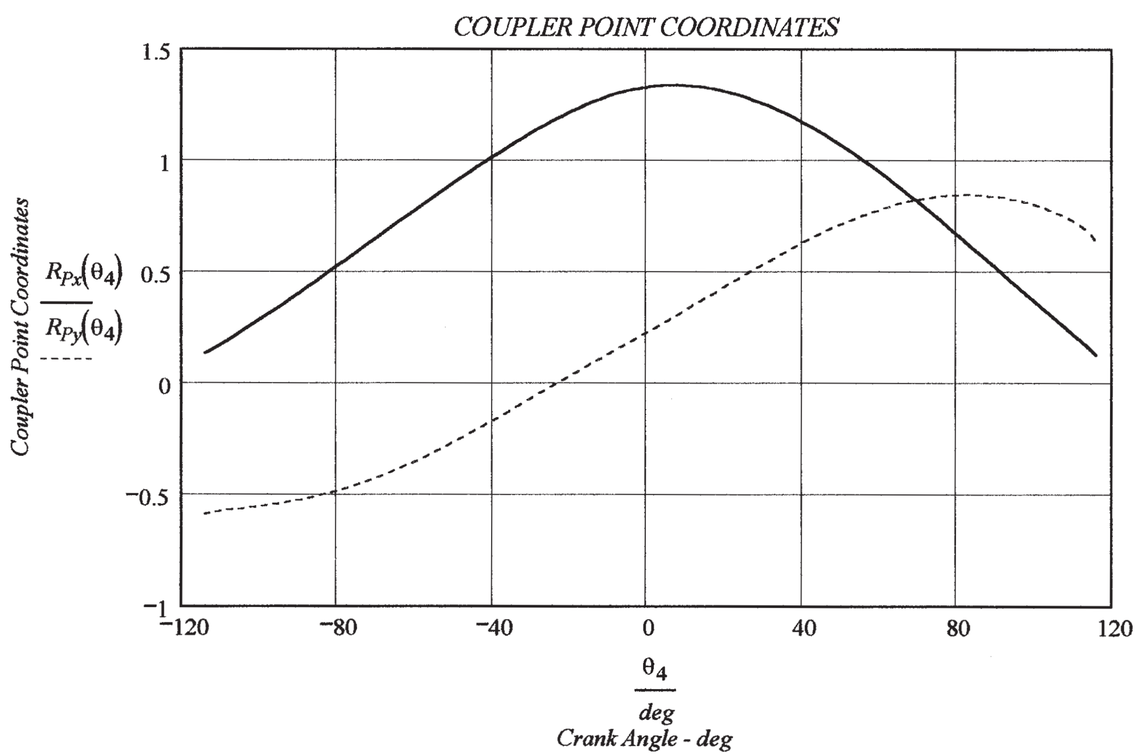
$$\mathbf{R_P} := \mathbf{R_B} + \mathbf{R_{PB}}$$

$$\mathbf{R_B} := a\cdot\left(cos\left(\theta_4\right) + j\cdot sin\left(\theta_4\right)\right)$$

$$\mathbf{R_{PB}} := p\cdot\left(cos\left(\theta_3 + \delta\right) + j\cdot sin\left(\theta_3 + \delta\right)\right)$$

$$R_{Px}\left(\theta_4\right) := a\cdot cos\left(\theta_4\right) + p\cdot cos\left(\theta_3\left(\theta_4\right) + \delta\right) \qquad R_{Py}\left(\theta_4\right) := a\cdot sin\left(\theta_4\right) + p\cdot sin\left(\theta_3\left(\theta_4\right) + \delta\right)$$

11. Plot the coordinates of the coupler point in the local *xy* coordinate system.

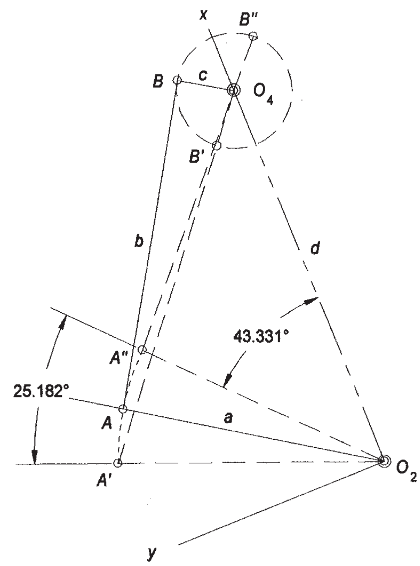


PROBLEM 4-28

Statement: For the rocker-crank linkage in Figure P4-14, find the maximum angular displacement possible for the treadle link (to which force F is applied). Determine the toggle positions. How does this work? Explain why the grinding wheel is able to fully rotate despite the presence of toggle positions when driven from the treadle. How would you get it started if it was in a toggle position?

Given: Link lengths:

Input (O_2A)	$a := 600 \cdot mm$
Coupler (AB)	$b := 750 \cdot mm$
Rocker (O_4B)	$c := 130 \cdot mm$
Ground link	$d := 900 \cdot mm$



Solution: See Figure P4-14 and Mathcad file P0428.

1. Use Figure 3-1(b) in the text to calculate the angles that link O_2A makes with the ground link in the toggle positions.

$$\theta_{21} := acos\left[\frac{a^2 + d^2 - (b-c)^2}{2 \cdot a \cdot d}\right] \qquad \theta_{21} = 43.331\,deg$$

$$\theta_{22} := acos\left[\frac{a^2 + d^2 - (b+c)^2}{2 \cdot a \cdot d}\right] \qquad \theta_{22} = 68.513\,deg$$

2. Subtract these two angles to get the maximum angular displacement of the treadle.

$$\Delta\theta_2 := \theta_{22} - \theta_{21} \qquad \Delta\theta_2 = 25.182\,deg$$

3. Despite having transmission angles of 0 deg twice per revolution, the mechanism will work. That is, one will be able to drive the grinding wheel from the treadle (link 2). The reason is that the grinding wheel will act as a flywheel and will carry the linkage through the periods when the transmission angle is low. Typically, the operator will start the motion by rotating the wheel by hand if it is in or near a toggle position.

PROBLEM 4-29

Statement: For the linkage in Figure P4-15, find its limit (toggle) positions in terms of the angle of link O_2A referenced to the line of centers O_2O_4 when driven from link O_2A. Then calculate and plot the angular displacement of links 3 and 4 and the path coordinates of point P with respect to the angle of the input crank O_2A over its possible range of motion referenced to the line of centers O_2O_4.

Given: Link lengths:

Input (O_2A) $a := 0.72$

Coupler (AB) $b := 0.68$

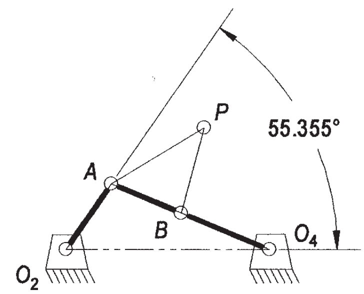
Rocker (O_4B) $c := 0.85$

Ground link $d := 1.82$

Coupler point data:

$p := 0.97$

$\delta := 54 \cdot deg$



Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{cases}$$

Solution: See Figure P4-15 and Mathcad file P0429.

1. Check the Grashof condition of the linkage.

$$Condition(S, L, P, Q) := \begin{cases} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"} \ \ if\ SL < PQ \\ return\ \text{"Special Grashof"} \ \ if\ SL = PQ \\ return\ \text{"non-Grashof"} \ \ otherwise \end{cases}$$

$Condition(b, d, a, c) = \text{"non-Grashof"}$

2. Using equations 4.33, determine the crank angles (relative to the line AD) at which links 3 and 4 are in toggle.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 1.451$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = 0.568$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 55.355\ deg$$

The other toggle angle is the negative of this.

3. Define one cycle of the input crank between limit positions:

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 2 \cdot deg .. \theta_{2toggle}$$

4. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 2.5278 \qquad K_2 = 2.1412 \qquad K_3 = 3.3422$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. If the calculated value of θ_4 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{42}(\theta_2) := if\left(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2)\right)$$

$$\theta_4(\theta_2) := if\left(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2)\right)$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 2.6765 \qquad K_5 = -3.6465$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

7. Use equation 4.13 to find values of q3 for the open circuit.

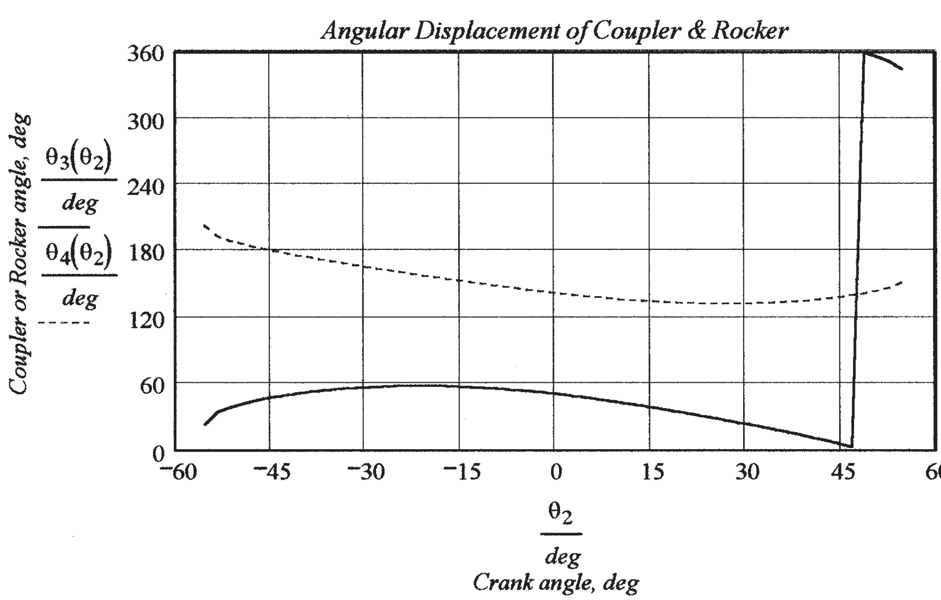
$$\theta_{31}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_2) := if\left(\theta_{31}(\theta_2) > 2 \cdot \pi, \theta_{31}(\theta_2) - 2 \cdot \pi, \theta_{31}(\theta_2)\right)$$

$$\theta_3(\theta_2) := if\left(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) - 2 \cdot \pi, \theta_{32}(\theta_2)\right)$$

9. Plot θ_3 and θ_4 as functions of the crank angle θ_2 (measured from the ground link).



10. Use equations 4.27 to define the *x*- and *y*-components of the vector $\mathbf{R}_P$.

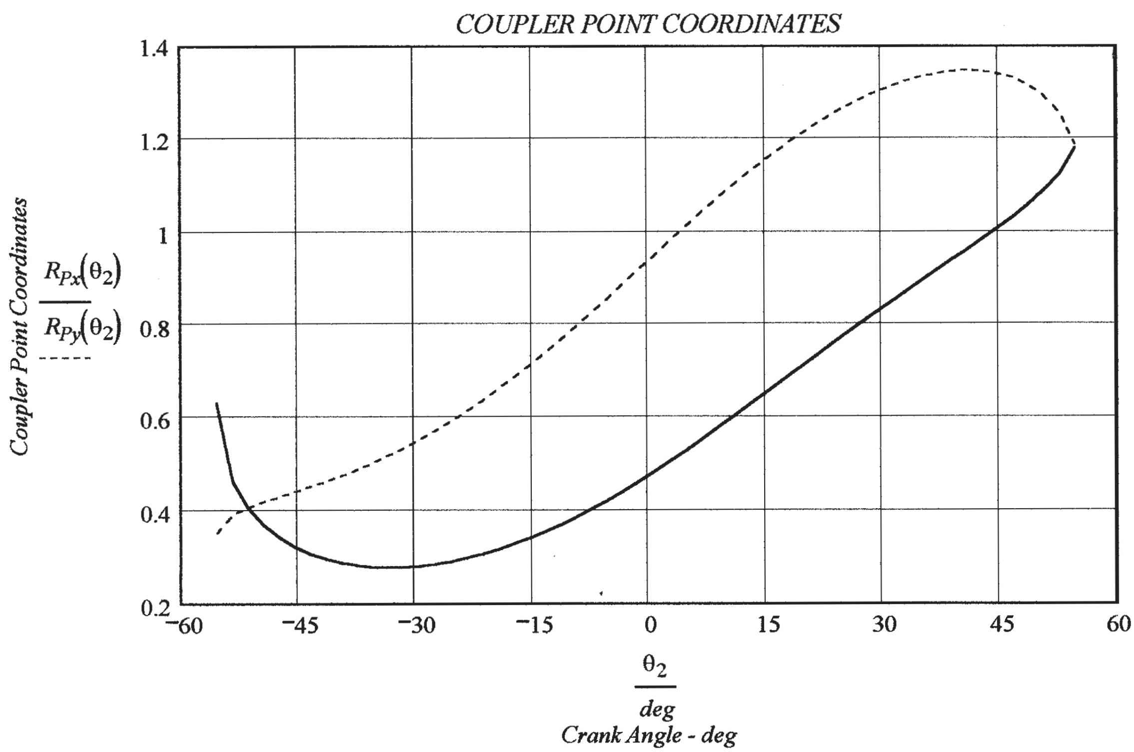
$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a \cdot \left(\cos(\theta_2) + j \cdot \sin(\theta_2)\right)$$

$$\mathbf{R_{PA}} := p \cdot \left(\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta)\right)$$

$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta) \qquad R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

11. Plot the coordinates of the coupler point in the local *xy* coordinate system.



PROBLEM 4-30

Statement: For the linkage in Figure P4-15, find its limit (toggle) positions in terms of the angle of link O_4B referenced to the line of centers O_4O_2 when driven from link O_4B. Then calculate and plot the angular displacement of links 2 and 3 and the path coordinates of point P with respect to the angle of the input crank O_4B over its possible range of motion referenced to the line of centers O_4O_2.

Given: Link lengths:

Input (O_4B) $a := 0.85$

Coupler (AB) $b := 0.68$

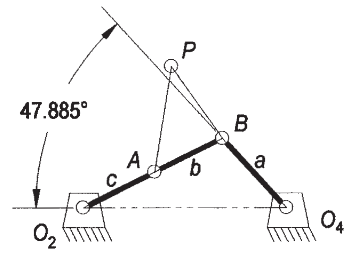
Rocker (O_2A) $c := 0.72$

Ground link $d := 1.82$

Coupler point data:

$p := 0.792$

$\delta := 82.032 \cdot deg$



Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P4-15 and Mathcad file P0430.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"} \ \ if\ SL < PQ \\ return\ \text{"Special Grashof"} \ \ if\ SL = PQ \\ return\ \text{"non-Grashof"} \ \ otherwise \end{array}$$

$$Condition(b,d,a,c) = \text{"non-Grashof"}$$

2. Using equations 4.33, determine the crank angles (relative to the line O_4O_2) at which links 2 and 3 are in toggle.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 1.304$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = 0.671$$

$$\theta_{4toggle} := acos(arg_2) \qquad \theta_{4toggle} = 47.885\ deg$$

The other toggle angle is the negative of this.

3. Define one cycle of the input crank between limit positions:

$$\theta_4 := -\theta_{4toggle}, -\theta_{4toggle} + 2 \cdot deg .. \theta_{4toggle}$$

4. Use equations 4.8a and 4.10 to calculate θ_2 as a function of θ_4 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$K_1 = 2.1412 \qquad K_2 = 2.5278 \qquad K_3 = 3.3422$

$$A(\theta_4) := \cos(\theta_4) - K_1 - K_2 \cdot \cos(\theta_4) + K_3$$

$$B(\theta_4) := -2 \cdot \sin(\theta_4) \qquad C(\theta_4) := K_1 - (K_2 + 1) \cdot \cos(\theta_4) + K_3$$

$$\theta_{21}(\theta_4) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_4), -B(\theta_4) - \sqrt{B(\theta_4)^2 - 4 \cdot A(\theta_4) \cdot C(\theta_4)} \right) \right)$$

5. If the calculated value of θ_2 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{22}(\theta_4) := if\left(\theta_{21}(\theta_4) > 2 \cdot \pi, \theta_{21}(\theta_4) - 2 \cdot \pi, \theta_{21}(\theta_4)\right)$$

$$\theta_2(\theta_4) := if\left(\theta_{22}(\theta_4) < 0, \theta_{22}(\theta_4) + 2 \cdot \pi, \theta_{22}(\theta_4)\right)$$

6. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 2.6765 \qquad K_5 = -3.4420$$

$$D(\theta_4) := \cos(\theta_4) - K_1 + K_4 \cdot \cos(\theta_4) + K_5$$

$$E(\theta_4) := -2 \cdot \sin(\theta_4) \qquad F(\theta_4) := K_1 + (K_4 - 1) \cdot \cos(\theta_4) + K_5$$

7. Use equation 4.13 to find values of θ_3 for the open circuit.

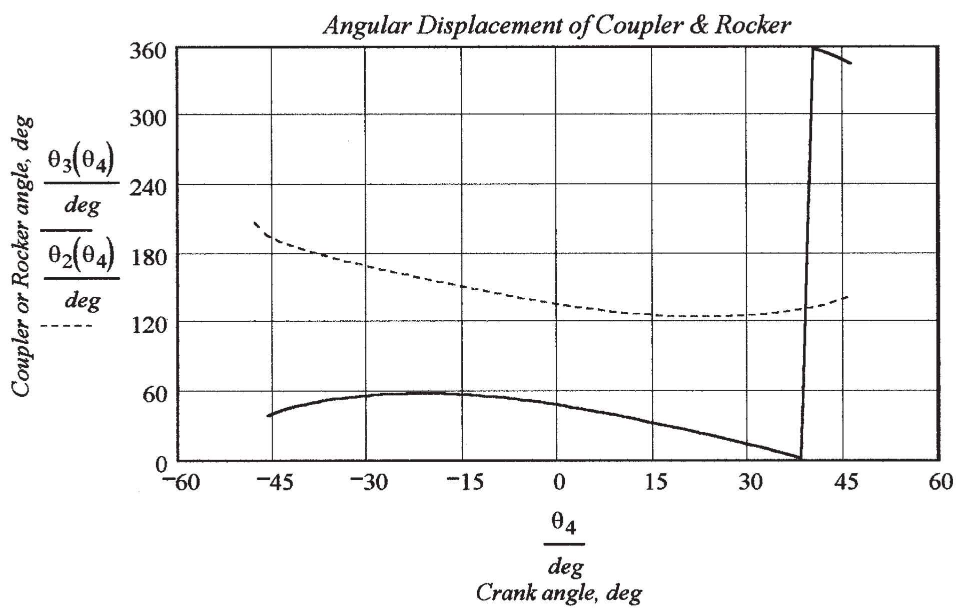
$$\theta_{31}(\theta_4) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_4), -E(\theta_4) - \sqrt{E(\theta_4)^2 - 4 \cdot D(\theta_4) \cdot F(\theta_4)} \right) \right)$$

8. If the calculated value of θ_3 is greater than 2π, subtract 2π from it. If it is negative, make it positive.

$$\theta_{32}(\theta_4) := if\left(\theta_{31}(\theta_4) > 2 \cdot \pi, \theta_{31}(\theta_4) - 2 \cdot \pi, \theta_{31}(\theta_4)\right)$$

$$\theta_3(\theta_4) := if\left(\theta_{32}(\theta_4) < 0, \theta_{32}(\theta_4) - 2 \cdot \pi, \theta_{32}(\theta_4)\right)$$

9. Plot θ_3 and θ_2 as functions of the crank angle θ_4 (measured from the ground link). See next page.



10. Use equations 4.27 to define the *x*- and *y*-components of the vector $\mathbf{R}_P$.

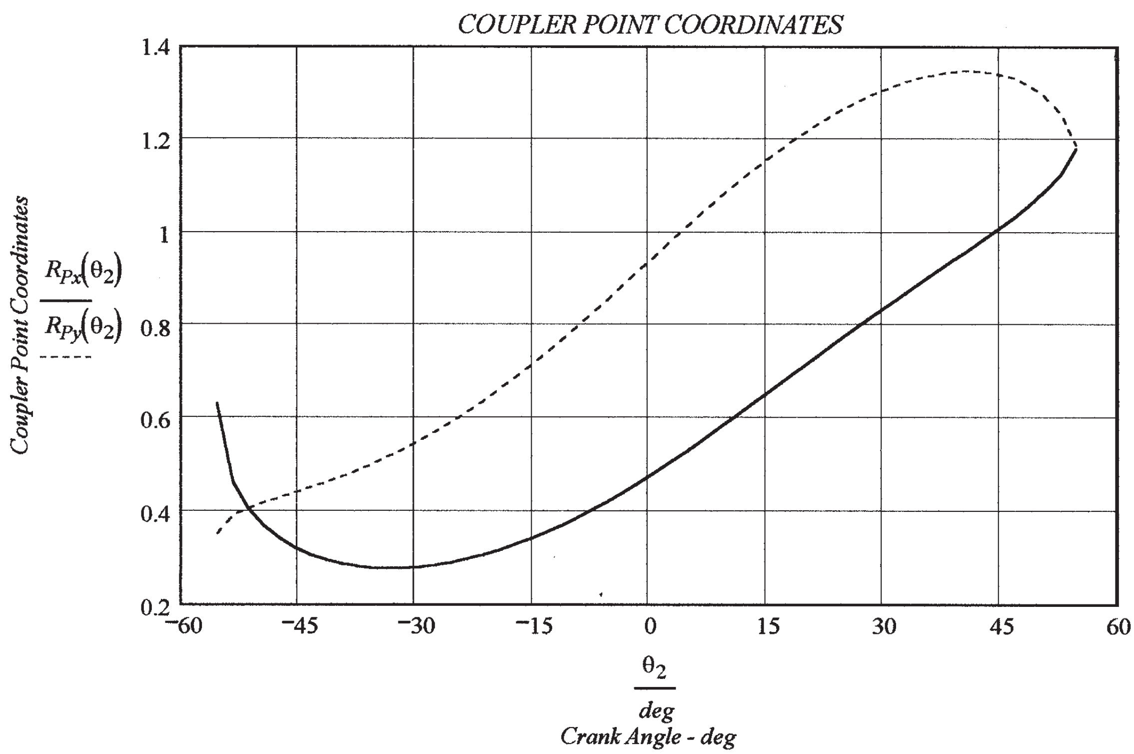
$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a\cdot\left(cos\left(\theta_2\right) + j\cdot sin\left(\theta_2\right)\right)$$

$$\mathbf{R_{PA}} := p\cdot\left(cos\left(\theta_3 + \delta\right) + j\cdot sin\left(\theta_3 + \delta\right)\right)$$

$$R_{Px}\left(\theta_2\right) := a\cdot cos\left(\theta_2\right) + p\cdot cos\left(\theta_3\left(\theta_2\right) + \delta\right) \qquad R_{Py}\left(\theta_2\right) := a\cdot sin\left(\theta_2\right) + p\cdot sin\left(\theta_3\left(\theta_2\right) + \delta\right)$$

11. Plot the coordinates of the coupler point in the local *xy* coordinate system.



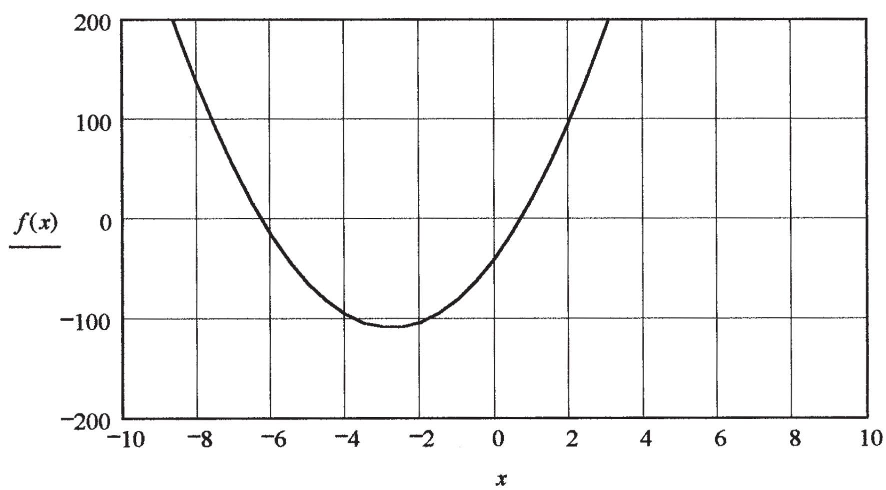
PROBLEM 4-31

Statement: Write a computer program (or use an equation solver such as *Mathcad*, *Matlab*, or *TKSolver*) to find the roots of $y = 9x^2 + 50x - 40$. Hint: Plot the function to determine good guess values.

Solution: See Mathcad file P0431.

1. Plot the function.

$$x := -10, -9.5 .. 10 \qquad f(x) := 9 \cdot x^2 + 50 \cdot x - 40$$



2. From the graph, make guesses of $x_1 := -6$, $x_2 := 0$

3. Define the program using the pseudo code on page 194 of the text.

```
nroot(f, df, x) := | y ← f(x)
                   | return x  if |y| ≤ TOL
                   | while |y| > TOL
                   |   | x ← x − y / df(x)
                   |   | y ← f(x)
                   | x
```

where, $TOL = 1.000 \times 10^{-3}$

4. Define the derivative of the given function. $df(x) := 18 \cdot x + 50$

5. Use the program to find the roots.

$$r_1 := nroot(f, df, x_1) \qquad r_1 = -6.265$$

$$r_2 := nroot(f, df, x_2) \qquad r_2 = 0.709$$

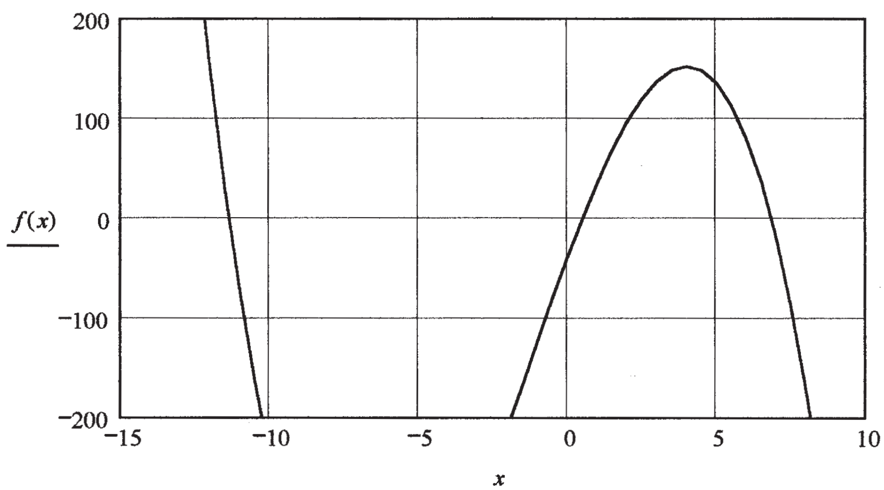
PROBLEM 4-32

Statement: Write a computer program (or use an equation solver such as *Mathcad*, *Matlab*, or *TKSolver*) to find the roots of $y = -x^3 - 4x^2 + 80x - 40$. Hint: Plot the function to determine good guess values.

Solution: See Mathcad file P0432.

1. Plot the function.

$$x := -15, -14.5 .. 10 \qquad f(x) := -x^3 - 4\cdot x^2 + 80\cdot x - 40$$



2. From the graph, make guesses of $x_1 := -11$, $x_2 := 0$, $x_3 := 6$

3. Define the program using the pseudo code on page 194 of the text.

```
nroot(f, df, x) := | y ← f(x)
                   | return x if |y| ≤ TOL
                   | while |y| > TOL
                   |    | x ← x − y/df(x)
                   |    | y ← f(x)
                   | x
```

where, $TOL = 1.000 \times 10^{-3}$

4. Define the derivative of the given function. $df(x) := -3\cdot x^2 - 8\cdot x + 80$

5. Use the program to find the roots.

$r_1 := nroot(f, df, x_1) \qquad r_1 = -11.355$

$r_2 := nroot(f, df, x_2) \qquad r_2 = 0.515$

$r_3 := nroot(f, df, x_3) \qquad r_3 = 6.840$

PROBLEM 4-33

Statement: Figure 4-18 (p. 193) plots the cubic function from equation 4.34. Write a computer program (or use an equation solver such as *Mathcad*, *Matlab*, or *TKSolver*) to investigate the behavior of the Newton-Raphson algorithm as the initial guess value is varied from x = 1.8 to 2.5 in steps of 0.1. Determine the guess value at which the convergence switches roots. Explain this root-switching phenomenon based on your observations from this exercise.

Solution: See Figure 4-18 and Mathcad file P0433.

1. Define the range of the guess value, the function, and the derivative of the function.

$$xguess := 1.8, 1.9 .. 2.5$$

$$f(x) := -x^3 - 2 \cdot x^2 + 50 \cdot x + 60 \qquad df(x) := -3 \cdot x^2 - 4 \cdot x + 50$$

2. Define the root-finding program using the pseudo code on page 175 of the text.

```
nroot(f, df, x) := | y ← f(x)
                   | return x  if |y| ≤ TOL
                   | while |y| > TOL
                   |    | x ← x − y/df(x)
                   |    | y ← f(x)
                   | x
```

3. Find the roots that correspond to the guess values.

$$r(xguess) := nroot(f, df, xguess) \qquad nextx(xguess) := xguess - \frac{f(xguess)}{df(xguess)}$$

xguess =

	1
1	1.800
2	1.900
3	2.000
4	2.100
5	2.200
6	2.300
7	2.400
8	2.500

'(xguess) =

	1
1	33.080
2	31.570
3	30.000
4	28.370
5	26.680
6	24.930
7	23.120
8	21.250

:xtx(xguess) =

	1
1	-2.362
2	-2.564
3	-2.800
4	-3.079
5	-3.410
6	-3.807
7	-4.289
8	-4.882

(xguess) =

	1
1	-1.177
2	-1.177
3	-1.177
4	-1.177
5	-1.177
6	-1.177
7	6.740
8	-7.562

4. Find the roots of the derivative (values of x where the slope is zero).

$$ddf(x) := -6 \cdot x - 4$$

$$xz_1 := nroot(df, ddf, -5) \qquad xz_1 = -4.803$$

$$xz_2 := nroot(df, ddf, 4) \qquad xz_2 = 3.470$$

5. For guess values up to 2.3, the root found is that whose slope is nearly the same as the slope of the function at the guess value. At 2.4, the value of x that is calculated next results in a slope that throws the next x-value to the right of the extreme function value at $x = 3.470$. Subsequent estimates of x then follow down the slope to $x = 6.740$. A a guess value of 2.5, the value of x that is calculated next is to the left of the extreme function value at $x = -4.803$. Subsequent estimates of x follow up the slope to $x = -7.562$.

PROBLEM 4-34

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 4 and the position of slider 6 in Figure 3-33 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2) $a := 2.170$ Fourbar coupler (L_3) $b := 2.067$

Output crank (L_4) $c := 2.310$ Sllider coupler (L_5) $e := 5.40$

Fourbar ground link (L_1) $d := 1.000$

Two argument inverse tangent:

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi & \text{if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi & \text{if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) & \text{if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & \text{otherwise} \end{cases}$$

Solution: See Figure 3-33 and Mathcad file P0434.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a slider-crank mechanism using the output of the fourbar, link 4, as the input to the slider-crank.
2. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 1\cdot deg .. 360\cdot deg$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit) in the global *XY* system.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 0.4608 \qquad K_2 = 0.4329 \qquad K_3 = 0.6755$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2\cdot\cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot\sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot\cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right) - 102\cdot deg$$

4. If the calculated value of θ_4 is greater than 2π, subtract 2π from it and if it is negative, make it positive.

$$\theta_{42}(\theta_2) := if(\theta_{41}(\theta_2) > 2\cdot\pi, \theta_{41}(\theta_2) - 2\cdot\pi, \theta_{41}(\theta_2))$$

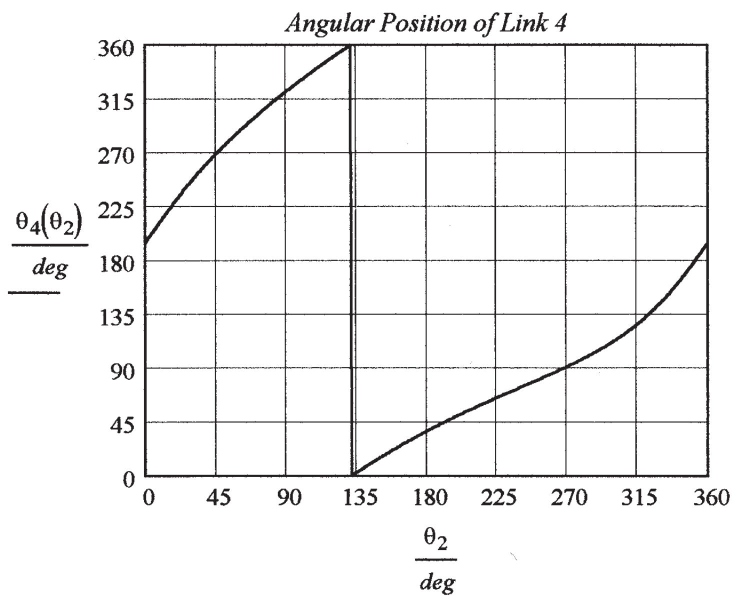
$$\theta_4(\theta_2) := if(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2\cdot\pi, \theta_{42}(\theta_2))$$

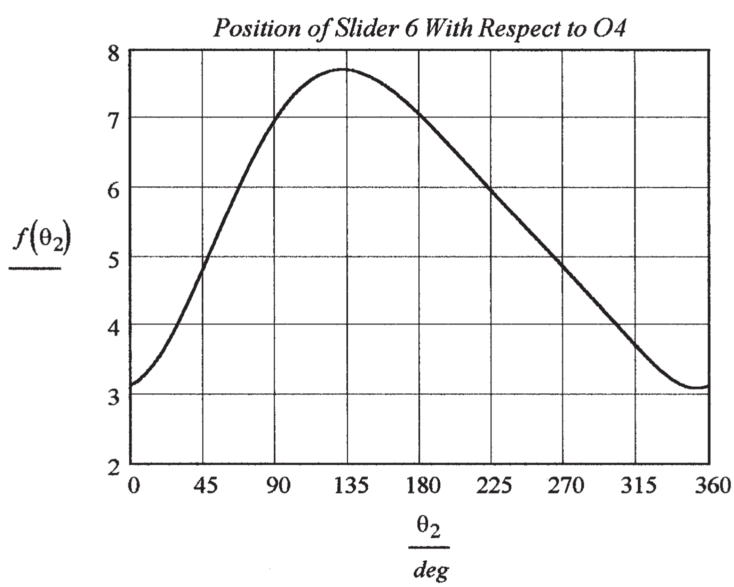
5. Determine the slider-crank motion using equations 4.16 and 4.17 with θ_4 as the input angle.

$$\theta_5(\theta_2) := asin\left(\frac{-c\cdot\sin(\theta_4(\theta_2))}{e}\right) + \pi$$

$$f(\theta_2) := c\cdot\cos(\theta_4(\theta_2)) - e\cdot\cos(\theta_5(\theta_2))$$

6. Plot the angular position of link 4 and the position of link 6 as functions of the angle of input link 2.





PROBLEM 4-35

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B* and *C* of the linkage in Figure 3-33 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2) $a := 2.170$ Fourbar coupler (L_3) $b := 2.067$

Output crank (L_4) $c := 2.310$ Sllider coupler (L_5) $e := 5.40$

Fourbar ground link (L_1) $d := 1.000$

Two argument inverse tangent:

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure 3-33 and Mathcad file P0435.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a slider-crank mechanism using the output of the fourbar, link 4, as the input to the slider-crank.
2. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 1\cdot deg .. 360\cdot deg$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 0.4608 \qquad K_2 = 0.4329 \qquad K_3 = 0.6755$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. If the calculated value of θ_4 is greater than 2π, subtract 2π from it and if it is negative, make it positive.

$$\theta_{42}(\theta_2) := if(\theta_{41}(\theta_2) > 2\cdot\pi, \theta_{41}(\theta_2) - 2\cdot\pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := if(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2\cdot\pi, \theta_{42}(\theta_2))$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.4838$$

$$K_5 = -0.5178$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

7. If the calculated value of θ_3 is greater than 2π, subtract 2π from it and if it is negative, make it positive.

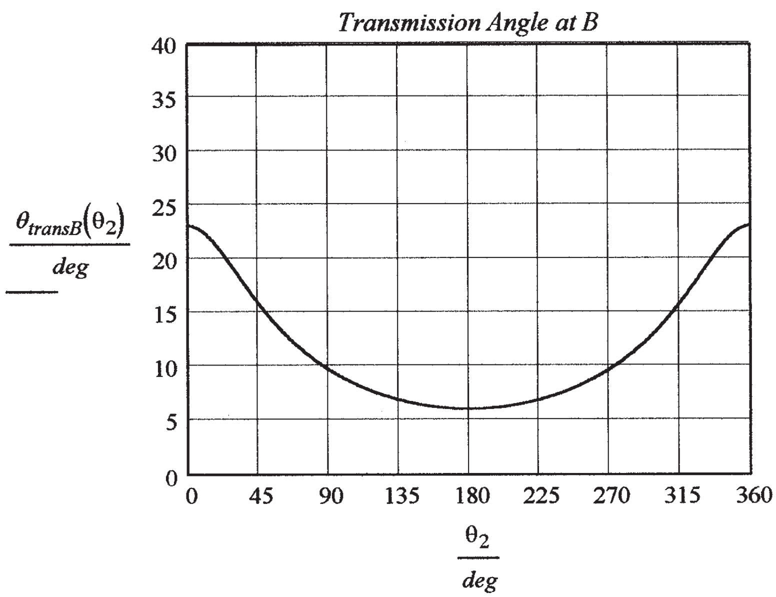
$$\theta_{32}(\theta_2) := if(\theta_{31}(\theta_2) > 2\cdot\pi, \theta_{31}(\theta_2) - 2\cdot\pi, \theta_{31}(\theta_2))$$

$$\theta_3(\theta_2) := if(\theta_{32}(\theta_2) < 0, \theta_{32}(\theta_2) + 2\cdot\pi, \theta_{32}(\theta_2))$$

8. Calculate (using equations 4.28) and plot the transmission angle at *B*.

$$\theta_{transB1}(\theta_2) := |\theta_3(\theta_2) - \theta_4(\theta_2)|$$

$$\theta_{transB}(\theta_2) := \left|if\left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2)\right)\right|$$



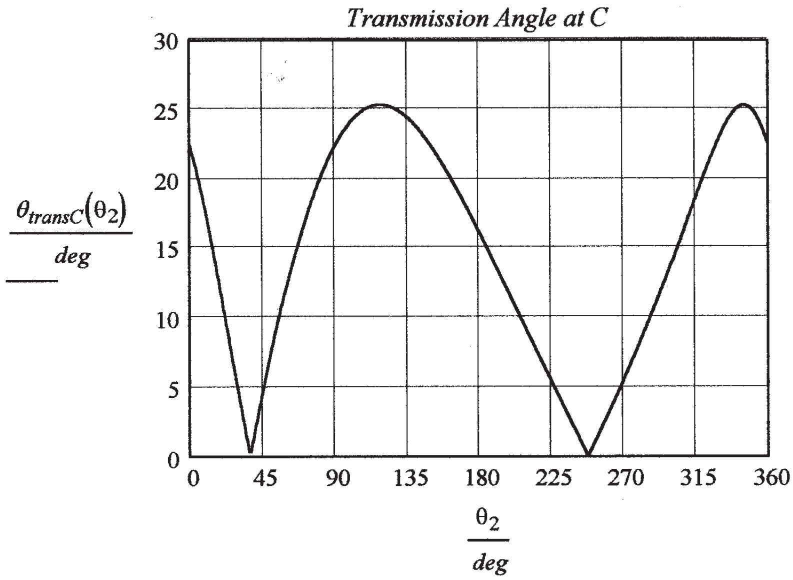
9. Determine the slider-crank motion using equations 4.16 and 4.17 with θ_4 as the input angle.

$$\theta_5(\theta_2) := asin\left(\frac{-c\cdot sin(\theta_4(\theta_2))}{e}\right) + \pi$$

10. Calculate (using equations 4.28) and plot the transmission angle at *C*.

$$\theta_{transC1}(\theta_2) := |\theta_5(\theta_2)|$$

$$\theta_{transC}(\theta_2) := \left|if\left(\theta_{transC1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transC1}(\theta_2), \theta_{transC1}(\theta_2)\right)\right|$$


Transmission Angle at C
30
25
20
15
10
5
0
0
45
90
135
180
225
270
315
360
θtransC(θ2) / deg
θ2 / deg

PROBLEM 4-36

Statement: Create a model of the linkage shown in Figure 3-33 in *Working Model*.

Solution: See Figure 3-33 and Working Model file P0436.

PROBLEM 4-37

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 6 in Figure 3-34 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$g := 1.556$	First coupler (L_3)	$f := 4.248$
First rocker (L_4)	$c := 2.125$	Third coupler (CD)	$b := 2.158$
Output rocker (L_6)	$a := 1.542$	Second ground link (O_4O_6)	$d := 1.000$
Angle CDB	$\delta := 36 \cdot deg$	Distance (BD)	$p := 3.274$
O_2O_4 ground link offsets:	$h_X := 3.259$	$h_Y := 2.905$	

Solution: See Figure 3-34 and Mathcad file P0437.

1. Calculate the length of the O_2O_4 ground link and the angle that it makes with the global XY system.

$$h := \sqrt{h_X^2 + h_Y^2} \qquad h = 4.366 \qquad \gamma := -atan\left(\frac{h_Y}{h_X}\right) \qquad \gamma = -41.713\, deg$$

2. Calculate the distance BC on link 5. This is the length of vector $\mathbf{R}_{51}$. Also, calculate the angle btween vectors $\mathbf{R}_{51}$ and $\mathbf{R}_{52}$

$$e := \sqrt{b^2 + p^2 - 2 \cdot b \cdot p \cdot cos(\delta)} \qquad \text{Second coupler } (BC) \qquad e = 1.986$$

$$\alpha := acos\left(\frac{b^2 + e^2 - p^2}{2 \cdot b \cdot e}\right) \qquad \alpha = 104.305\, deg$$

$$\beta := \pi - \alpha \qquad \beta = 75.695\, deg$$

3. This is a Stephenson's sixbar linkage similar to the one shown in Figure 4-13. Since the output link 6 is known to rotate 180 deg and return for a full revolution of link 2 we can use links 6, 5, and 4 as a first-stage fourbar with known input (link 6) and then solve for vector loop equations to get the corresponding motion of link 2.

4. Define the rotation of the output crank: $\theta_6 := 90 \cdot deg, 91 \cdot deg .. 270 \cdot deg$

5. Use equations 4.8a and 4.10 to calculate θ_4 in the local xy coordinate system as a function of θ_6 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 0.6485 \qquad K_2 = 0.4706 \qquad K_3 = 0.4938$$

$$A(\theta_6) := cos(\theta_6) - K_1 - K_2 \cdot cos(\theta_6) + K_3$$

$$B(\theta_6) := -2 \cdot sin(\theta_6) \qquad C(\theta_6) := K_1 - (K_2 + 1) \cdot cos(\theta_6) + K_3$$

$$\theta_{41}(\theta_6) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_6), -B(\theta_6) + \sqrt{B(\theta_6)^2 - 4 \cdot A(\theta_6) \cdot C(\theta_6)}\right)\right)$$

6. Use equations 4.12 and 4.13 to calculate θ_5 in the local xy coordinate system as a function of θ_6 (for the crossed circuit).

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$K_4 = 0.463 \qquad K_5 = -0.529$

$D(\theta_6) := \cos(\theta_6) - K_1 + K_4 \cdot \cos(\theta_6) + K_5 \qquad E(\theta_6) := -2 \cdot \sin(\theta_6)$

$F(\theta_6) := K_1 + (K_4 - 1) \cdot \cos(\theta_6) + K_5$

$$\theta_{521}(\theta_6) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_6), -E(\theta_6) + \sqrt{E(\theta_6)^2 - 4 \cdot D(\theta_6) \cdot F(\theta_6)} \right) \right)$$

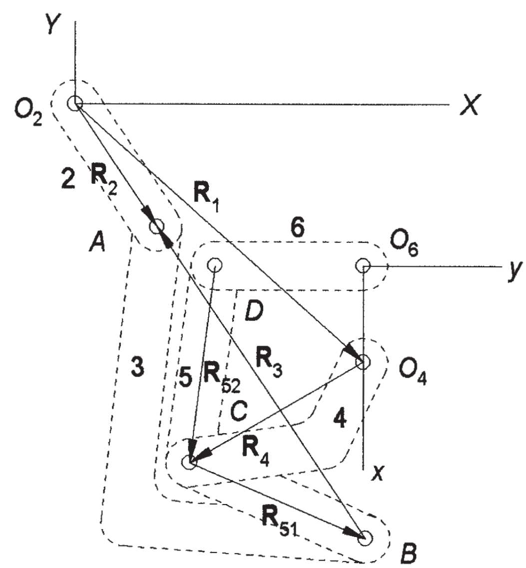
7. Transform the angles for θ_4 and θ_{52} into the global *XY* system and define θ_{51} in the global system.

$$\theta_4(\theta_6) := \theta_{41}(\theta_6) - 90 \cdot deg$$

$$\theta_{52}(\theta_6) := \theta_{521}(\theta_6) - 90 \cdot deg$$

$$\theta_{51}(\theta_6) := \theta_{52}(\theta_6) + \beta$$

8. Define a vector loop for the remaining links and solve the resulting vector equation by separating it into real and imaginary parts using the method of section 4.5 and the identities of equations 4.9.



$\mathbf{R}_1 + \mathbf{R}_4 + \mathbf{R}_{51} + \mathbf{R}_3 - \mathbf{R}_2 = 0$. In this equation the unknowns are θ_3 and θ_2. Following the method of Section 4.5, substitute the complex number notation for each position vector and separate the resulting equations into real and imaginary parts:

$$f \cdot \cos(\theta_3) = g \cdot \cos(\theta_2) - G_1$$

$$f \cdot \sin(\theta_3) = g \cdot \sin(\theta_2) - G_2$$

where $G_1(\theta_6) := h \cdot \cos(\gamma) + c \cdot \cos(\theta_4(\theta_6)) + e \cdot \cos(\theta_{51}(\theta_6))$

$G_2(\theta_6) := h \cdot \sin(\gamma) + (c \cdot \sin(\theta_4(\theta_6)) + e \cdot \sin(\theta_{51}(\theta_6)))$

9. Solve these equations in the manner of equations 4.11 and 4.12 using the identities of equations 4.9 gives:

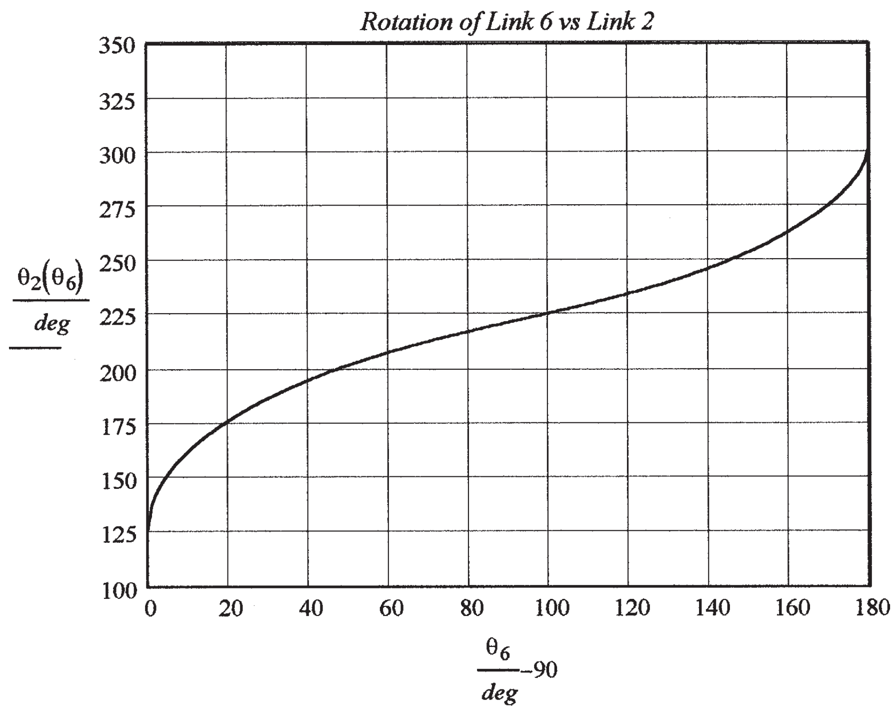
$$G_3(\theta_6) := \frac{g^2 + G_1(\theta_6)^2 + G_2(\theta_6)^2 - f^2}{2 \cdot g}$$

$A'(\theta_6) := -G_1(\theta_6) - G_3(\theta_6) \qquad B'(\theta_6) := 2 \cdot G_2(\theta_6) \qquad C'(\theta_6) := G_1(\theta_6) - G_3(\theta_6)$

$$tan\left(\frac{\theta_2}{2}\right) = \frac{-B' + \sqrt{B'^2 - 4 \cdot A' \cdot C'}}{2 \cdot A'}$$

$$\theta_2(\theta_6) := 2 \cdot \left(atan2\left(2 \cdot A'(\theta_6), -B'(\theta_6) + \sqrt{B'(\theta_6)^2 - 4 \cdot A'(\theta_6) \cdot C'(\theta_6)}\right)\right)$$

10. Plot θ_6 vs θ_2 in global XY coordinates:



PROBLEM 4-38

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B*, *C*, and *D* of the linkage in Figure 3-34 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$g := 1.556$	First coupler (L_3)	$f := 4.248$
First rocker (L_4)	$c := 2.125$	Third coupler (CD)	$b := 2.158$
Output rocker (L_6)	$a := 1.542$	Second ground link (O_4O_6)	$d := 1.000$
Angle *CDB*	$\delta := 36 \cdot deg$	Distance (BD)	$p := 3.274$
O_2O_4 ground link offsets:	$h_X := 3.259$	$h_Y := 2.905$	

Solution: See Figure 3-34 and Mathcad file P0438.

1. Calculate the length of the O_2O_4 ground link and the angle that it makes with the global *XY* system.

$$h := \sqrt{h_X^2 + h_Y^2} \qquad h = 4.366 \qquad \gamma := -atan\left(\frac{h_Y}{h_X}\right) \qquad \gamma = -41.713\,deg$$

2. Calculate the distance *BC* on link 5. This is the length of vector $\mathbf{R}_{51}$. Also, calculate the angle btween vectors $\mathbf{R}_{51}$ and $\mathbf{R}_{52}$

$$e := \sqrt{b^2 + p^2 - 2 \cdot b \cdot p \cdot cos(\delta)} \qquad \text{Second coupler } (BC) \qquad e = 1.986$$

$$\alpha := acos\left(\frac{b^2 + e^2 - p^2}{2 \cdot b \cdot e}\right) \qquad \alpha = 104.305\,deg$$

$$\beta := \pi - \alpha \qquad \beta = 75.695\,deg$$

3. This is a Stephenson's sixbar linkage similar to the one shown in Figure 4-13. Since the output link 6 is known to rotate 180 deg and return for a full revolution of link 2 we can use links 6, 5, and 4 as a first-stage fourbar with known input (link 6) and then solve for vector loop equations to get the corresponding motion of link 2.

4. Define the rotation of the output crank: $\theta_6 := 90 \cdot deg, 91 \cdot deg .. 270 \cdot deg$

5. Use equations 4.8a and 4.10 to calculate θ_4 in the local *xy* coordinate system as a function of θ_6 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 0.6485 \qquad K_2 = 0.4706 \qquad K_3 = 0.4938$$

$$A(\theta_6) := cos(\theta_6) - K_1 - K_2 \cdot cos(\theta_6) + K_3$$

$$B(\theta_6) := -2 \cdot sin(\theta_6) \qquad C(\theta_6) := K_1 - (K_2 + 1) \cdot cos(\theta_6) + K_3$$

$$\theta_{41}(\theta_6) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_6), -B(\theta_6) + \sqrt{B(\theta_6)^2 - 4 \cdot A(\theta_6) \cdot C(\theta_6)}\right)\right)$$

6. Use equations 4.12 and 4.13 to calculate θ_5 in the local *xy* coordinate system as a function of θ_6 (for the crossed circuit).

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$K_4 = 0.463 \qquad K_5 = -0.529$

$$D(\theta_6) := \cos(\theta_6) - K_1 + K_4 \cdot \cos(\theta_6) + K_5 \qquad E(\theta_6) := -2 \cdot \sin(\theta_6)$$

$$F(\theta_6) := K_1 + (K_4 - 1) \cdot \cos(\theta_6) + K_5$$

$$\theta_{521}(\theta_6) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_6), -E(\theta_6) + \sqrt{E(\theta_6)^2 - 4 \cdot D(\theta_6) \cdot F(\theta_6)} \right) \right)$$

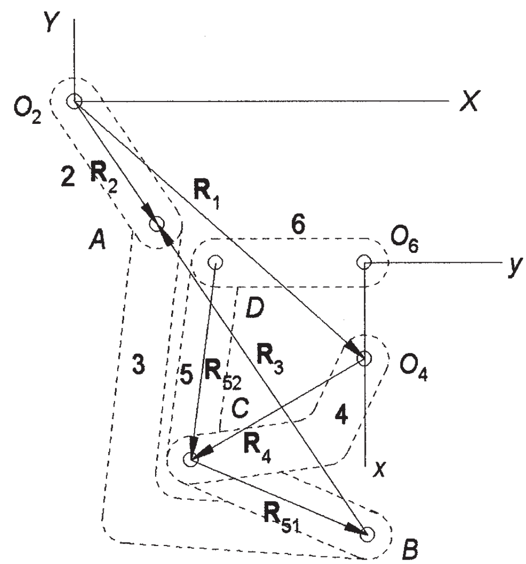
7. Transform the angles for θ_4 and θ_{52} into the global *XY* system and define θ_{51} in the global system.

$$\theta_4(\theta_6) := \theta_{41}(\theta_6) - 90 \cdot deg$$

$$\theta_{52}(\theta_6) := \theta_{521}(\theta_6) - 90 \cdot deg$$

$$\theta_{51}(\theta_6) := \theta_{52}(\theta_6) + \beta$$

8. Define a vector loop for the remaining links and solve the resulting vector equation by separating it into real and imaginary parts using the method of section 4.5 and the identities of equations 4.9.



$\mathbf{R}_1 + \mathbf{R}_4 + \mathbf{R}_{51} + \mathbf{R}_3 - \mathbf{R}_2 = 0$. In this equation the unknowns are θ_3 and θ_2. Following the method of Section 4.5, substitute the complex number notation for each position vector and separate the resulting equations into real and imaginary parts:

$$g \cdot \cos(\theta_2) = f \cdot \cos(\theta_3) + G_1$$

$$g \cdot \sin(\theta_2) = f \cdot \sin(\theta_3) + G_2$$

where $G_1(\theta_6) := h \cdot \cos(\gamma) + c \cdot \cos(\theta_4(\theta_6)) + e \cdot \cos(\theta_{51}(\theta_6))$

$$G_2(\theta_6) := h \cdot \sin(\gamma) + (c \cdot \sin(\theta_4(\theta_6)) + e \cdot \sin(\theta_{51}(\theta_6)))$$

9. Solve these equations for θ_2 in the manner of equations 4.11 and 4.12 using the identities of equations 4.9 gives:

$$G_3(\theta_6) := \frac{g^2 + G_1(\theta_6)^2 + G_2(\theta_6)^2 - f^2}{2 \cdot g}$$

$$A'(\theta_6) := -G_1(\theta_6) - G_3(\theta_6) \qquad B'(\theta_6) := 2 \cdot G_2(\theta_6) \qquad C'(\theta_6) := G_1(\theta_6) - G_3(\theta_6)$$

$$tan\left(\frac{\theta_2}{2}\right) = \frac{-B' + \sqrt{B'^2 - 4 \cdot A' \cdot C'}}{2 \cdot A'}$$

$$\theta_2(\theta_6) := 2 \cdot \left(atan2\left(2 \cdot A'(\theta_6), -B'(\theta_6) + \sqrt{B'(\theta_6)^2 - 4 \cdot A'(\theta_6) \cdot C'(\theta_6)}\right)\right)$$

10. Solve these equations for θ_3 in the manner of equations 4.11 and 4.12 using the identities of equations 4.9 gives:

$$G_4(\theta_6) := \frac{g^2 - G_1(\theta_6)^2 - G_2(\theta_6)^2 - f^2}{2 \cdot f}$$

$$D'(\theta_6) := -G_1(\theta_6) - G_4(\theta_6) \qquad E'(\theta_6) := 2 \cdot G_2(\theta_6) \qquad F'(\theta_6) := G_1(\theta_6) - G_4(\theta_6)$$

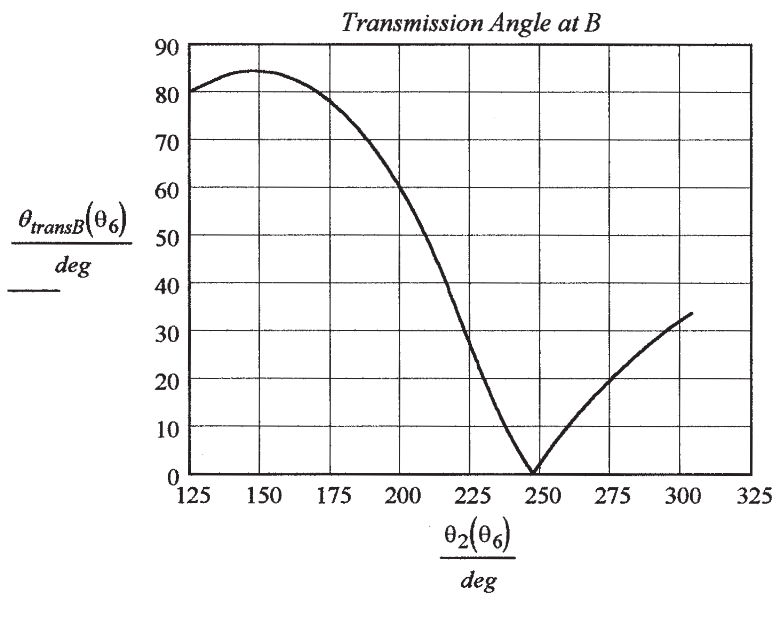
$$tan\left(\frac{\theta_3}{2}\right) = \frac{-E' + \sqrt{E'^2 - 4 \cdot D' \cdot F'}}{2 \cdot D'}$$

$$\theta_3(\theta_6) := 2 \cdot \left(atan2\left(2 \cdot D'(\theta_6), -E'(\theta_6) + \sqrt{E'(\theta_6)^2 - 4 \cdot D'(\theta_6) \cdot F'(\theta_6)}\right)\right)$$

11. Calculate (using equations 4.28) and plot the transmission angle at *B*.

$$\theta_{transB1}(\theta_6) := \left|\theta_3(\theta_6) - \theta_{51}(\theta_6)\right|$$

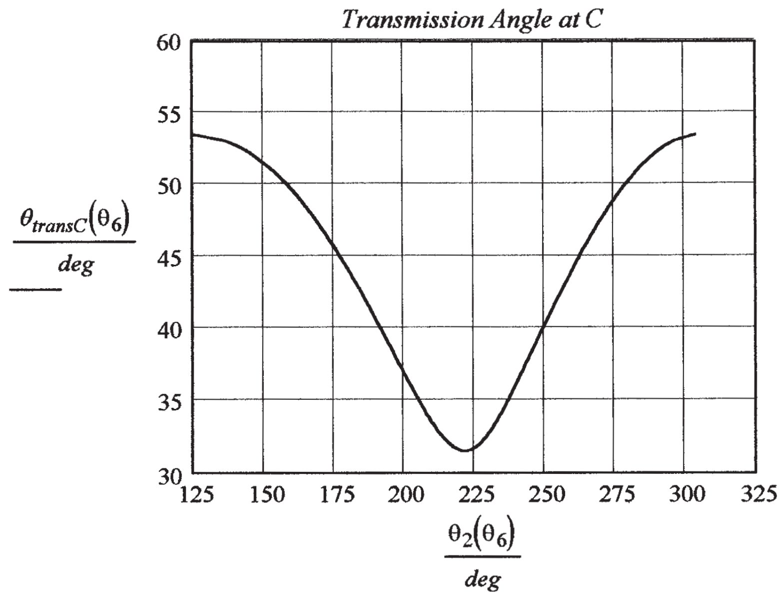
$$\theta_{transB}(\theta_6) := \left| if\left(\theta_{transB1}(\theta_6) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_6), \theta_{transB1}(\theta_6)\right)\right|$$



12. Calculate (using equations 4.28) and plot the transmission angle at *C*.

$$\theta_{transC1}(\theta_6) := \left|\theta_4(\theta_6) - \theta_{51}(\theta_6)\right|$$

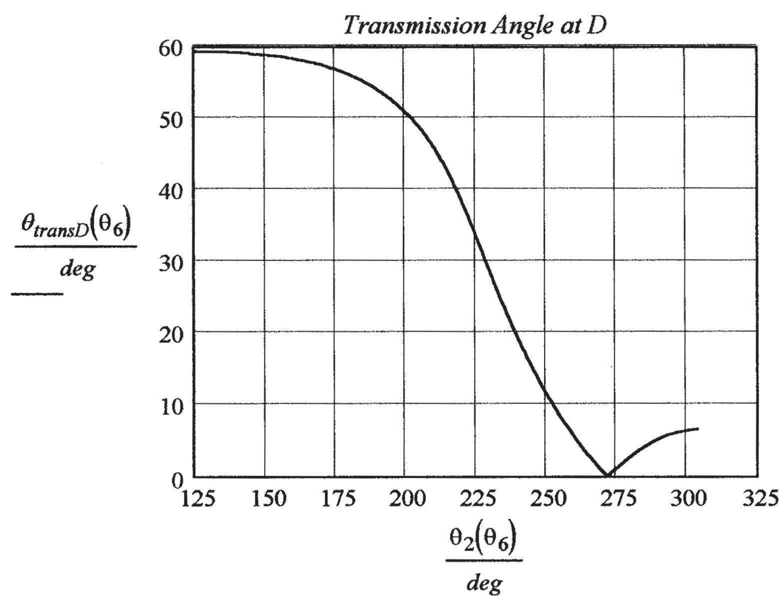
$$\theta_{transC}(\theta_6) := \left| if\left(\theta_{transC1}(\theta_6) > \frac{\pi}{2}, \pi - \theta_{transC1}(\theta_6), \theta_{transC1}(\theta_6)\right)\right|$$



13. Calculate (using equations 4.28) and plot the transmission angle at *D*.

$$\theta_{transD1}(\theta_6) := \left|\theta_6 - \theta_{52}(\theta_6)\right|$$

$$\theta_{transD}(\theta_6) := \left| if\left(\theta_{transD1}(\theta_6) > \frac{\pi}{2}, \pi - \theta_{transD1}(\theta_6), \theta_{transD1}(\theta_6)\right)\right|$$



PROBLEM 4-39

Statement: Create a model of the linkage shown in Figure 3-34 in *Working Model*.

Solution: See Figure 3-34 and Working Model file P0439.

PROBLEM 4-40

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 6 in Figure 3-35 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 1.00$	First coupler (L_3)	$b := 3.80$
Common rocker (O_4B)	$c := 1.29$	Second coupler (L_5)	$b' := 1.29$
First ground link (O_2O_4)	$d := 3.86$	Common rocker (O_4C)	$a' := 1.43$
Output rocker (L_6)	$c' := 0.77$	Second ground link (O_4O_6)	$d' := 0.78$
Angle BO_4C	$\alpha := 157 \cdot deg$		

Two argument inverse tangent:

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P3-35 and Mathcad file P0440.

1. This sixbar drag-link mechanism can be analyzed as two fourbar linkages in series that use the output of the first fourbar, link 4, as the input to the second fourbar.
2. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 1\cdot deg .. 360deg$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 3.8600 \qquad K_2 = 2.9922 \qquad K_3 = 1.2107$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. Use equations 4.8a and 4.10 to calculate θ_6 as a function of θ_2 (for the open circuit).

Input angle to second fourbar: $\theta_{42}(\theta_2) := \theta_{41}(\theta_2) + \alpha$

$$K'_1 := \frac{d'}{a'} \qquad K'_2 := \frac{d'}{c'} \qquad K'_3 := \frac{a'^2 - b'^2 + c'^2 + d'^2}{(2\cdot a'\cdot c')}$$

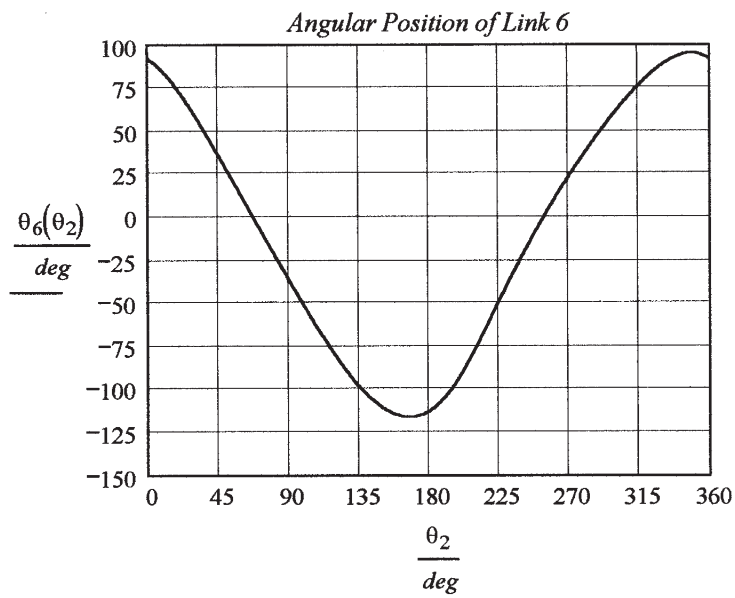
$$K'_1 = 0.5455 \qquad K'_2 = 1.0130 \qquad K'_3 = 0.7184$$

$$A'(\theta_2) := cos(\theta_{42}(\theta_2)) - K'_1 - K'_2\cdot cos(\theta_{42}(\theta_2)) + K'_3$$

$$B'(\theta_2) := -2\cdot sin(\theta_{42}(\theta_2)) \qquad C'(\theta_2) := K'_1 - (K'_2 + 1)\cdot cos(\theta_{42}(\theta_2)) + K'_3$$

$$\theta_6(\theta_2) := 2\cdot\left(atan2\left(2\cdot A'(\theta_2), -B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4\cdot A'(\theta_2)\cdot C'(\theta_2)}\right)\right)$$

5. **Plot the angular position of link 6 as a function of the angle of input link 2.**



PROBLEM 4-41

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B*, *C*, and *D* of the linkage in Figure 3-35 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 1.00$	First coupler (L_3)	$b := 3.80$
Common rocker (O_4B)	$c := 1.29$	Second coupler (L_5)	$b' := 1.29$
First ground link (O_2O_4)	$d := 3.86$	Common rocker (O_4C)	$a' := 1.43$
Output rocker (L_6)	$c' := 0.77$	Second ground link (O_4O_6)	$d' := 0.78$
Angle BO_4C	$\alpha := 157 \cdot deg$		

Two argument inverse tangent:

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure P3-35 and Mathcad file P0441.

1. This sixbar drag-link mechanism can be analyzed as two fourbar linkages in series that use the output of the first fourbar, link 4, as the input to the second fourbar.

2. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 1\cdot deg .. 360\cdot deg$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 3.8600 \qquad K_2 = 2.9922 \qquad K_3 = 1.2107$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. Use equations 4.12 and 4.13 to calculate θ_3 as a function of θ_2 (for the crossed circuit).

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2\cdot a\cdot b}$$

$$K_4 = 1.016 \qquad K_5 = -3.773$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad E(\theta_2) := -2\cdot sin(\theta_2)$$

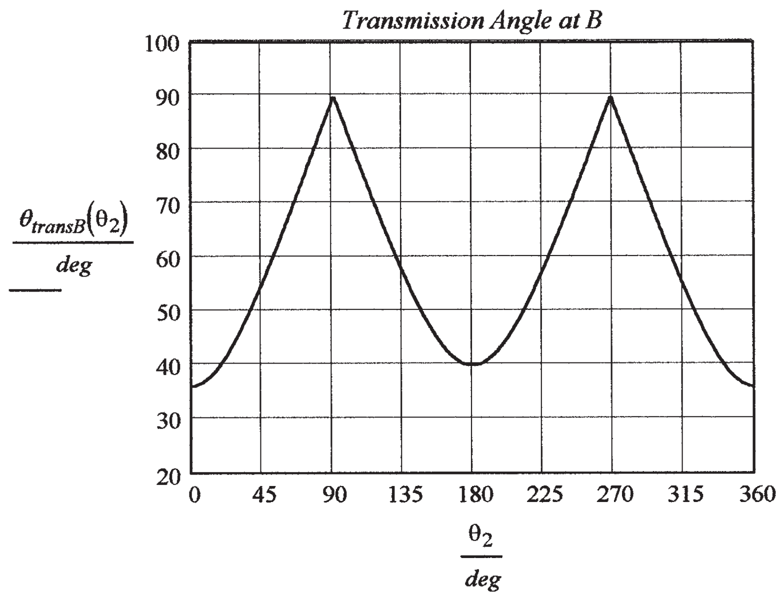
$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

5. Calculate (using equations 4.28) and plot the transmission angle at *B*.

$$\theta_{transB1}(\theta_2) := \left|\theta_3(\theta_2) - \theta_{41}(\theta_2)\right|$$

$$\theta_{transB}(\theta_2) := \left| if\left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2)\right)\right|$$



6. Use equations 4.8a and 4.10 to calculate θ_6 as a function of θ_2 (for the open circuit).

Input angle to second fourbar: $\theta_{42}(\theta_2) := \theta_{41}(\theta_2) + \alpha$

$K'_1 := \frac{d'}{a'}$ $K'_2 := \frac{d'}{c'}$ $K'_3 := \frac{a'^2 - b'^2 + c'^2 + d'^2}{(2 \cdot a' \cdot c')}$

$K'_1 = 0.5455$ $K'_2 = 1.0130$ $K'_3 = 0.7184$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 - K'_2 \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2)) \qquad C'(\theta_2) := K'_1 - (K'_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$\theta_6(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A'(\theta_2), -B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)}\right)\right)$$

7. Use equations 4.12 and 4.13 to calculate θ_5 as a function of θ_2 (for the crossed circuit).

$K'_4 := \frac{d'}{b'}$ $K'_5 := \frac{c'^2 - d'^2 - a'^2 - b'^2}{2 \cdot a' \cdot b'}$

$K'_4 = 0.605$ $K'_5 = -1.010$

$$D'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 + K'_4 \cdot \cos(\theta_{42}(\theta_2)) + K'_5 \qquad E'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

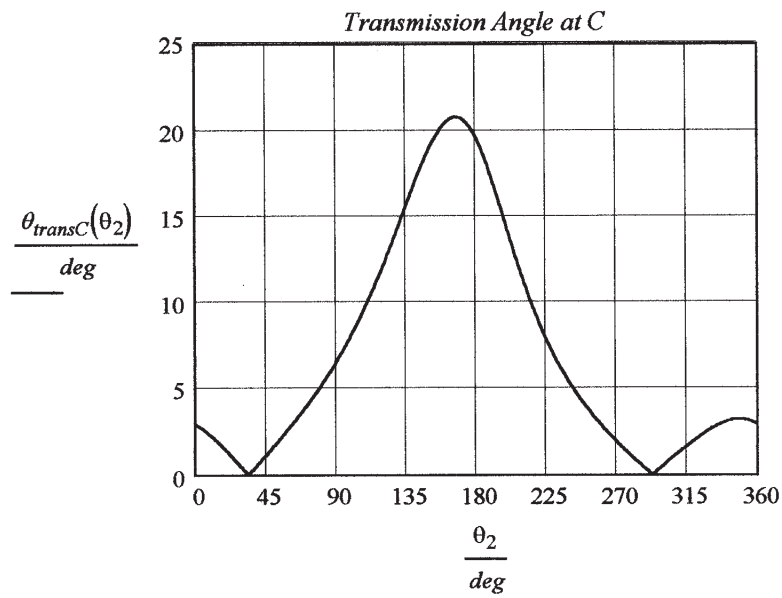
$$F'(\theta_2) := K'_1 + (K'_4 - 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_5$$

$$\theta_5(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D'(\theta_2), -E'(\theta_2) + \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)}\right)\right)$$

8. Calculate (using equations 4.28) and plot the transmission angle at C.

$$\theta_{transC1}(\theta_2) := \left|\theta_5(\theta_2) - \theta_{42}(\theta_2)\right|$$

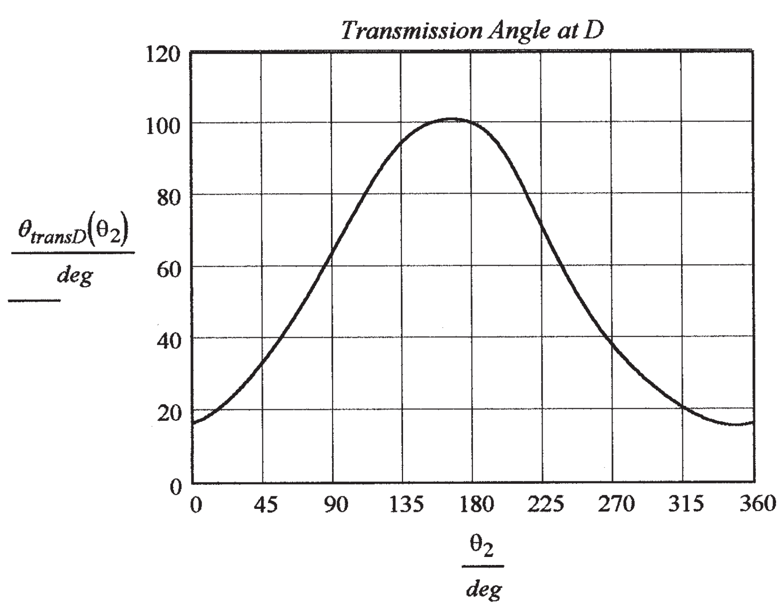
$$\theta_{transC}(\theta_2) := \left|if\left(\theta_{transC1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transC1}(\theta_2), \theta_{transC1}(\theta_2)\right)\right|$$



9. Calculate (using equations 4.28) and plot the transmission angle at D.

$$\theta_{transD1}(\theta_2) := \left|\theta_5(\theta_2) - \theta_6(\theta_2)\right|$$

$$\theta_{transD}(\theta_2) := \left|if\left(\theta_{transD1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transD1}(\theta_2), \theta_{transD1}(\theta_2)\right)\right|$$



PROBLEM 4-42

Statement: Create a model of the linkage shown in Figure 3-35 in *Working Model*.

Solution: See Figure 3-35 and Working Model file P0442.

PROBLEM 4-43

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 8 in Figure 3-36 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 0.450$	First coupler (L_3)	$b := 0.990$
Common rocker (O_4B)	$c := 0.590$	First ground link (O_2O_4)	$d := 1.000$
Common rocker (O_4C)	$a' := 0.590$	Second coupler (CD)	$b' := 0.325$
Output rocker (L_6)	$c' := 0.325$	Second ground link (O_4O_6)	$d' := 0.419$
Link 7 (L_7)	$e := 0.938$	Link 8 (L_8)	$f := 0.572$
Link 5 extension (DE)	$p := 0.823$	Angle DCE	$\delta := 7.0 \cdot deg$
Angle BO_4C	$\alpha := 128.6 \cdot deg$		

Two argument inverse tangent:

$$atan2(x,y) := \begin{vmatrix} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{vmatrix}$$

Solution: See Figure P3-36 and Mathcad file P0443.

1. This eightbar can be analyzed as a fourbar (links 1, 2, 3, and 4) with its output (link 4) as the input to another fourbar (links 1, 4, 5, and 6). Since links 1 and 4 are common to both, we have an eightbar linkage with links 7 & 8 included. Start by analyzing the input fourbar.

2. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 1\cdot deg .. 360\cdot deg$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 2.2222 \qquad K_2 = 1.6949 \qquad K_3 = 1.0744$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2\cdot\cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot\sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot\cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. Use equations 4.8a and 4.10 to calculate θ_6 as a function of θ_2 (for the open circuit).

Input angle to second fourbar: $\theta_{42}(\theta_2) := \theta_{41}(\theta_2) - \alpha$

$$K'_1 := \frac{d'}{a'} \qquad K'_2 := \frac{d'}{c'} \qquad K'_3 := \frac{a'^2 - b'^2 + c'^2 + d'^2}{(2\cdot a'\cdot c')}$$

$$K'_1 = 0.7102 \qquad K'_2 = 1.2892 \qquad K'_3 = 1.3655$$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 - K'_2 \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2)) \qquad C'(\theta_2) := K'_1 - (K'_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_3$$

$$\theta_6(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A'(\theta_2), -B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)} \right) \right)$$

5. Use equations 4.11b, 4.12 and 4.13 to calculate θ_5 as a function of θ_2.

$$K'_4 := \frac{d'}{b'} \qquad K'_5 := \frac{c'^2 - d'^2 - a'^2 - b'^2}{2 \cdot a' \cdot b'}$$

$$K'_4 = 1.289 \qquad K'_5 = -1.365$$

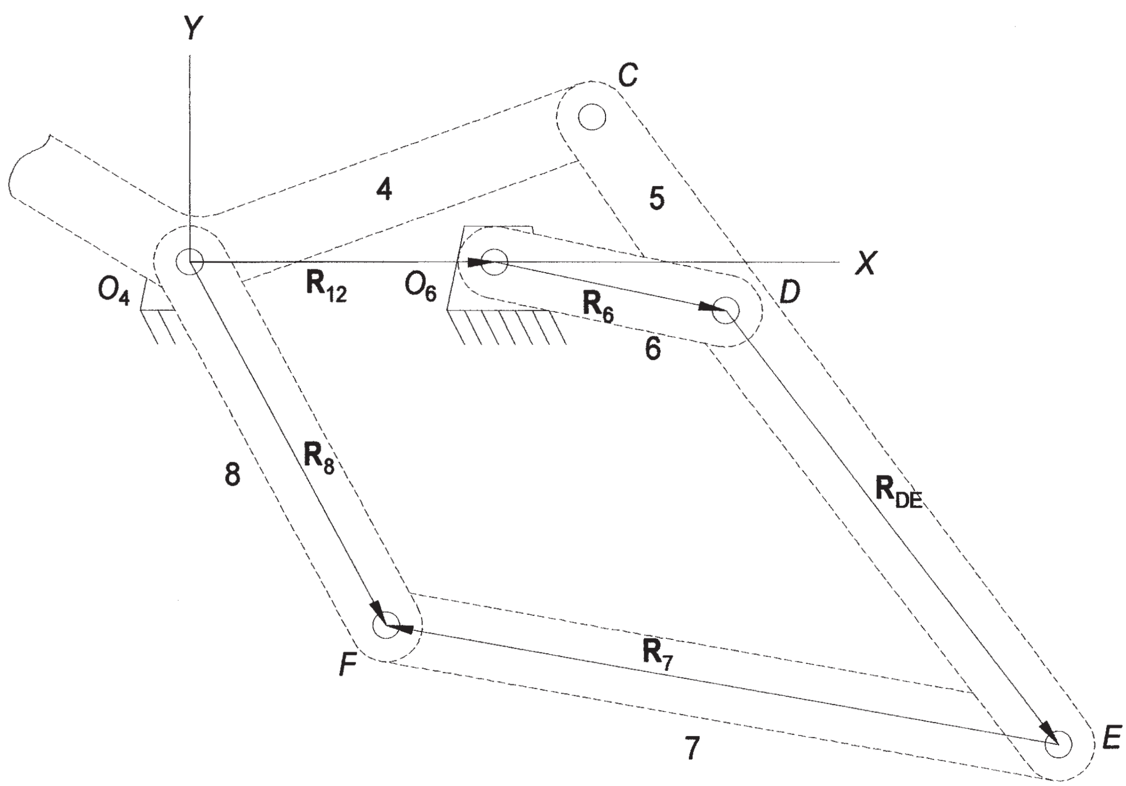
$$D'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K'_1 + K'_4 \cdot \cos(\theta_{42}(\theta_2)) + K'_5$$

$$E'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

$$F'(\theta_2) := K'_1 + (K'_4 - 1) \cdot \cos(\theta_{42}(\theta_2)) + K'_5$$

$$\theta_5(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D'(\theta_2), -E'(\theta_2) - \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)} \right) \right)$$

6. Define a vector loop for links 1, 5, 6, 7, and 8 as shown below and write the vector loop equation.



$\mathbf{R}_{12} + \mathbf{R}_6 + \mathbf{R}_{DE} + \mathbf{R}_7 - \mathbf{R}_8 = 0$. Solving for $\mathbf{R}_7$ gives $\mathbf{R}_7 = \mathbf{R}_8 - \mathbf{R}_{12} - \mathbf{R}_6 - \mathbf{R}_{DE}$. In this equation the only unknown are θ_7 and θ_8. Following the method of Section 4.5, substitute the complex number notation for each position vector and separate the resulting equation into real and imaginary parts:

$$e \cdot cos(\theta_7) = f \cdot cos(\theta_8) - D_1$$

$$e \cdot sin(\theta_7) = f \cdot sin(\theta_8) - D_2$$

where $D_1(\theta_2) := d' + c' \cdot cos(\theta_6(\theta_2)) + p \cdot cos(\theta_5(\theta_2) + \delta)$

$$D_2(\theta_2) := c' \cdot sin(\theta_6(\theta_2)) + p \cdot sin(\theta_5(\theta_2) + \delta)$$

7. Solve these equations in the manner of equations 4.11 and 4.12 using the identities of equations 4.9 gives:

$$D_3(\theta_2) := \frac{f^2 + D_1(\theta_2)^2 + D_2(\theta_2)^2 - e^2}{2 \cdot f}$$

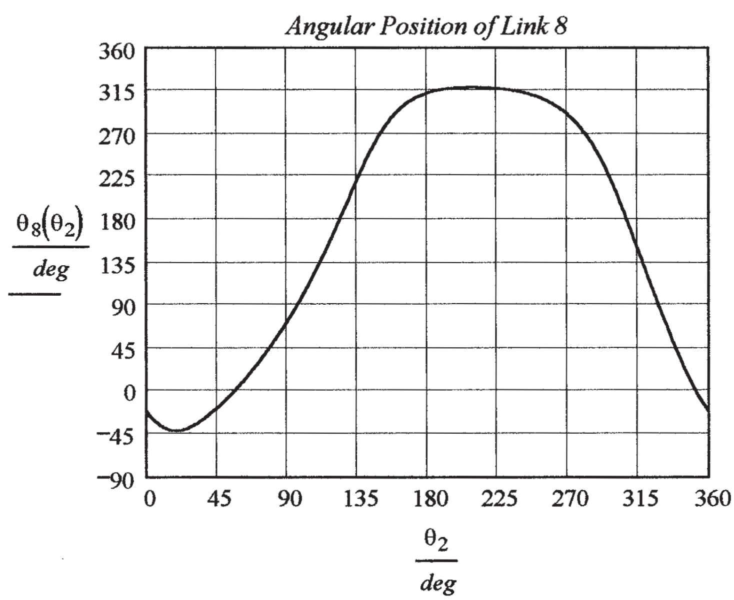
$$A'(\theta_2) := -D_1(\theta_2) - D_3(\theta_2) \qquad B'(\theta_2) := 2 \cdot D_2(\theta_2) \qquad C'(\theta_2) := D_1(\theta_2) - D_3(\theta_2)$$

$$tan\left(\frac{\theta_8}{2}\right) = \frac{-B' + \sqrt{B'^2 - 4 \cdot A' \cdot C'}}{2 \cdot A'}$$

$$\theta_{81}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A'(\theta_2), -B'(\theta_2) + \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)}\right)\right)$$

8. Plot the angular position of link 8 as a function of the angle of input link 2. If θ_{81} is greater than 360 deg, subtract 2π from it.

$$\theta_8(\theta_2) := if\left[\left(\theta_2 < \frac{\pi}{4}\right) \wedge (\theta_{81}(\theta_2) > 0), \theta_{81}(\theta_2) - 2 \cdot \pi, \theta_{81}(\theta_2)\right]$$



The graph shows that link 8 rotates 360 deg between $\theta_2 = 19$ deg and $\theta_2 = 209$ deg.

$$\theta_8(19 \cdot deg) = -42\,deg \qquad \theta_8(209 \cdot deg) - 2 \cdot \pi = -42\,deg$$

$$\theta_8(209 \cdot deg) - \theta_8(19 \cdot deg) = 360.0\,deg$$

PROBLEM 4-44

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B*, *C*, *D*, *E*, and *F* of the linkage in Figure 3-36 as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 0.450$	First coupler (L_3)	$b := 0.990$
Common rocker (O_4B)	$c := 0.590$	First ground link (O_2O_4)	$d := 1.000$
Common rocker (O_4C)	$a' := 0.590$	Second coupler (CD)	$b' := 0.325$
Output rocker (L_6)	$c' := 0.325$	Second ground link (O_4O_6)	$d' := 0.419$
Link 7 (L_7)	$e := 0.938$	Link 8 (L_8)	$f := 0.572$
Link 5 extension (DE)	$p := 0.823$	Angle DCE	$\delta := 7.0 \cdot deg$
Angle BO_4C	$\alpha := 128.6 \cdot deg$		

Two argument inverse tangent:

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P3-36 and Mathcad file P0444.

1. This eightbar can be analyzed as a fourbar (links 1, 2, 3, and 4) with its output (link 4) as the input to another fourbar (links 1, 4, 5, and 6). Since links 1 and 4 are common to both, we have an eightbar linkage with links 7 & 8 included. Start by analyzing the input fourbar.

2. Define one revolution of the input crank: $\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 2.2222 \qquad K_2 = 1.6949 \qquad K_3 = 1.0744$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right)\right)$$

4. Use equations 4.12 and 4.13 to calculate θ_3 as a function of θ_2 (for the open circuit).

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.010 \qquad K_5 = -2.059$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

5. Use equations 4.8a and 4.10 to calculate θ_6 as a function of θ_2 (for the open circuit).

Input angle to second fourbar: $\theta_{42}(\theta_2) := \theta_{41}(\theta_2) - \alpha$

$$K'_1 := \frac{d'}{a'} \qquad K'_2 := \frac{d'}{c'} \qquad K'_3 := \frac{a'^2 - b'^2 + c'^2 + d'^2}{(2\cdot a'\cdot c')}$$

$$K'_1 = 0.7102 \qquad K'_2 = 1.2892 \qquad K'_3 = 1.3655$$

$$A'(\theta_2) := cos(\theta_{42}(\theta_2)) - K'_1 - K'_2\cdot cos(\theta_{42}(\theta_2)) + K'_3$$

$$B'(\theta_2) := -2\cdot sin(\theta_{42}(\theta_2)) \qquad C'(\theta_2) := K'_1 - (K'_2 + 1)\cdot cos(\theta_{42}(\theta_2)) + K'_3$$

$$\theta_6(\theta_2) := 2\cdot\left(atan2\left(2\cdot A'(\theta_2), -B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4\cdot A'(\theta_2)\cdot C'(\theta_2)}\right)\right)$$

6. Use equations 4.11b, 4.12 and 4.13 to calculate θ_5 as a function of θ_2.

$$K'_4 := \frac{d'}{b'} \qquad K'_5 := \frac{c'^2 - d'^2 - a'^2 - b'^2}{2\cdot a'\cdot b'}$$

$$K'_4 = 1.289 \qquad K'_5 = -1.365$$

$$D'(\theta_2) := cos(\theta_{42}(\theta_2)) - K'_1 + K'_4\cdot cos(\theta_{42}(\theta_2)) + K'_5$$

$$E'(\theta_2) := -2\cdot sin(\theta_{42}(\theta_2))$$

$$F'(\theta_2) := K'_1 + (K'_4 - 1)\cdot cos(\theta_{42}(\theta_2)) + K'_5$$

$$\theta_{51}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D'(\theta_2), -E'(\theta_2) - \sqrt{E'(\theta_2)^2 - 4\cdot D'(\theta_2)\cdot F'(\theta_2)}\right)\right)$$

$$\theta_{52}(\theta_2) := if(\theta_{51}(\theta_2) > 2\cdot\pi, \theta_{51}(\theta_2) - 2\cdot\pi, \theta_{51}(\theta_2))$$

$$\theta_5(\theta_2) := if[(105\cdot deg < \theta_2 < 322\cdot deg) \wedge (\theta_{52}(\theta_2) < 0), \theta_{52}(\theta_2) + 2\cdot\pi, \theta_{52}(\theta_2)]$$

7. Define a vector loop for links 1, 5, 6, 7, and 8 as shown on the next page and write the vector loop equation.

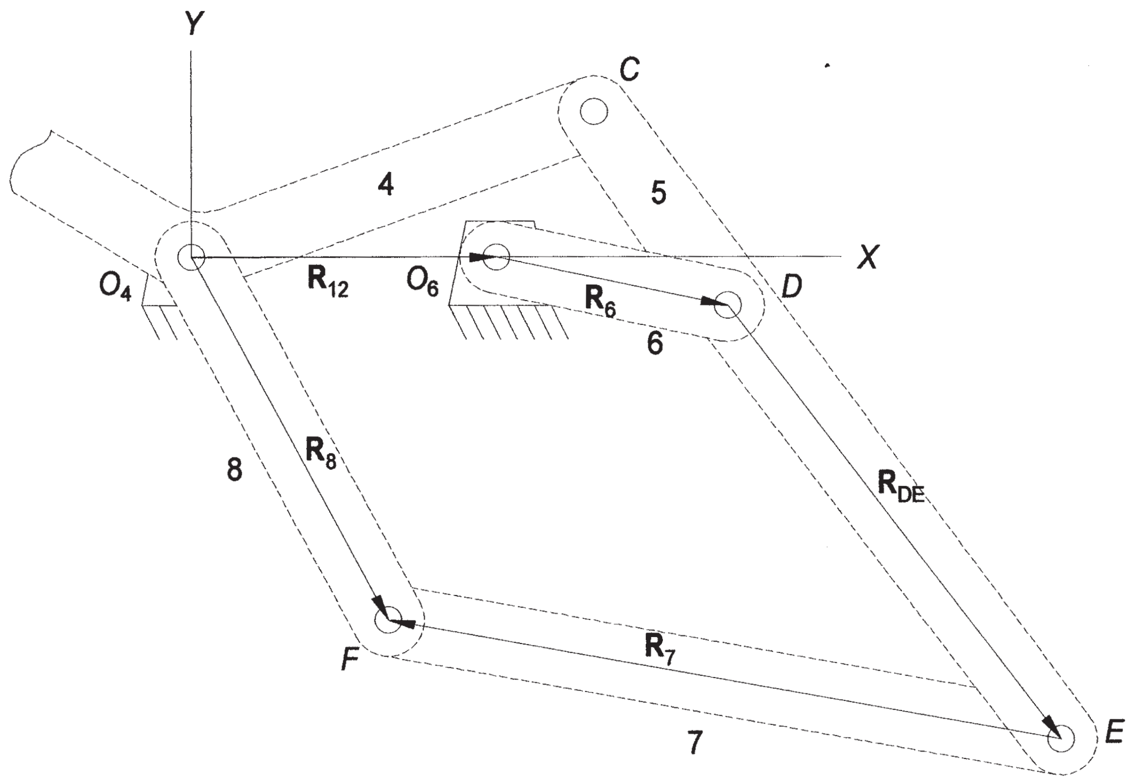
$\mathbf{R}_{12} + \mathbf{R}_6 + \mathbf{R}_{DE} + \mathbf{R}_7 - \mathbf{R}_8 = 0$. Solving for $\mathbf{R}_7$ gives $\mathbf{R}_7 = \mathbf{R}_8 - \mathbf{R}_{12} - \mathbf{R}_6 - \mathbf{R}_{DE}$. In this equation the only unknown are θ_7 and θ_8. Following the method of Section 4.5, substitute the complex number notation for each position vector and separate the resulting equation into real and imaginary parts:

$$e\cdot cos(\theta_7) = f\cdot cos(\theta_8) - D_1$$

$$e\cdot sin(\theta_7) = f\cdot sin(\theta_8) - D_2$$

where $D_1(\theta_2) := d' + c'\cdot cos(\theta_6(\theta_2)) + p\cdot cos(\theta_5(\theta_2) + \delta)$

$$D_2(\theta_2) := c'\cdot sin(\theta_6(\theta_2)) + p\cdot sin(\theta_5(\theta_2) + \delta)$$



8. Solve these equations in the manner of equations 4.10 using the identities of equations 4.9 gives:

$$D_3(\theta_2) := \frac{f^2 + D_1(\theta_2)^2 + D_2(\theta_2)^2 - e^2}{2 \cdot f}$$

$$A'(\theta_2) := -D_1(\theta_2) - D_3(\theta_2) \qquad B'(\theta_2) := 2 \cdot D_2(\theta_2) \qquad C'(\theta_2) := D_1(\theta_2) - D_3(\theta_2)$$

$$\tan\left(\frac{\theta_8}{2}\right) = \frac{-B' + \sqrt{B'^2 - 4 \cdot A' \cdot C'}}{2 \cdot A'}$$

$$\theta_{81}(\theta_2) := 2 \cdot \left(\mathrm{atan2}\left(2 \cdot A'(\theta_2), -B'(\theta_2) + \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)}\right)\right)$$

$$\theta_8(\theta_2) := \mathrm{if}\left[\left(\theta_2 < \frac{\pi}{4}\right) \wedge \left(\theta_{81}(\theta_2) > 0\right), \theta_{81}(\theta_2) - 2 \cdot \pi, \theta_{81}(\theta_2)\right]$$

9. Similarly, solve these equations in the manner of equations 4.11, 4.12 and 4.13 using the identities of equations 4.9 gives:

$$D_4(\theta_2) := \frac{f^2 - D_1(\theta_2)^2 - D_2(\theta_2)^2 - e^2}{2 \cdot e}$$

$$D'(\theta_2) := -D_1(\theta_2) - D_4(\theta_2) \qquad E'(\theta_2) := 2 \cdot D_2(\theta_2) \qquad F'(\theta_2) := D_1(\theta_2) - D_4(\theta_2)$$

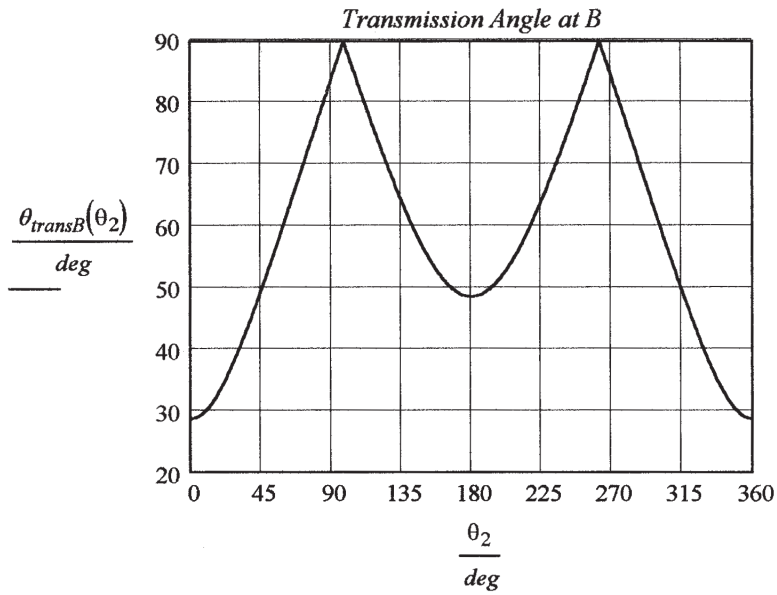
$$\tan\left(\frac{\theta_7}{2}\right) = \frac{-E' + \sqrt{E'^2 - 4 \cdot D' \cdot F'}}{2 \cdot D'}$$

$$\theta_7(\theta_2) := 2 \cdot \left(\mathrm{atan2}\left(2 \cdot D'(\theta_2), -E'(\theta_2) + \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)}\right)\right)$$

10. Calculate (using equations 4.28) and plot the transmission angle at *B*.

$$\theta_{transB1}(\theta_2) := \left|\theta_3(\theta_2) - \theta_{41}(\theta_2)\right|$$

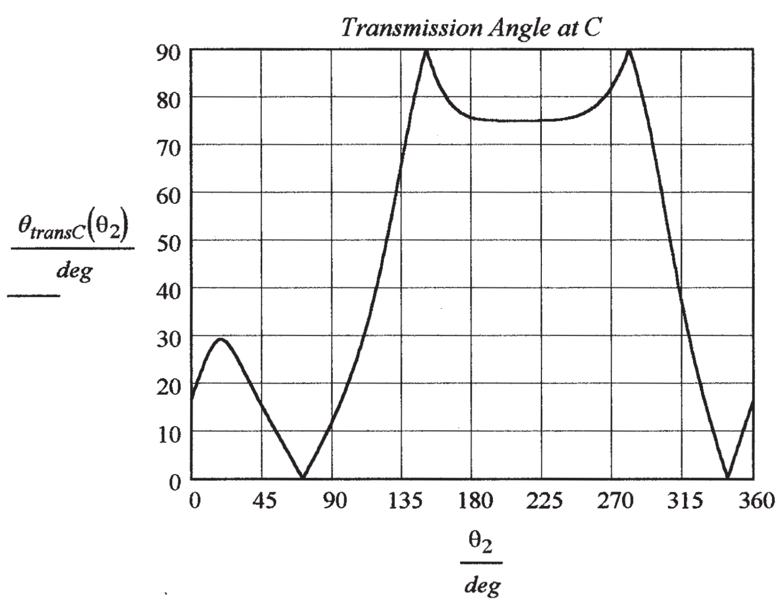
$$\theta_{transB}(\theta_2) := \left| if\left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2)\right)\right|$$



11. Calculate (using equations 4.28) and plot the transmission angle at *C*.

$$\theta_{transC1}(\theta_2) := \left|\theta_5(\theta_2) - \theta_{42}(\theta_2)\right|$$

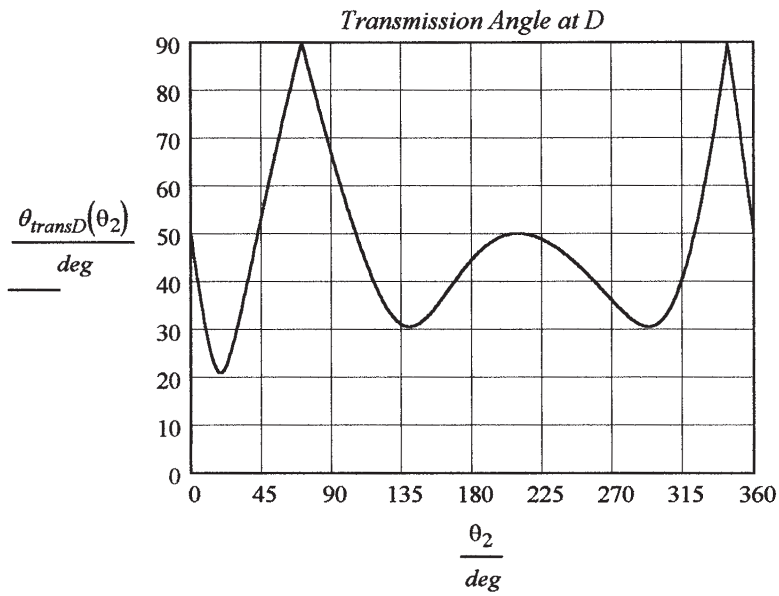
$$\theta_{transC}(\theta_2) := \left| if\left(\theta_{transC1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transC1}(\theta_2), \theta_{transC1}(\theta_2)\right)\right|$$



12. Calculate (using equations 4.28) and plot the transmission angle at D.

$$\theta_{transD1}(\theta_2) := |\theta_5(\theta_2) - \theta_6(\theta_2)| - \pi$$

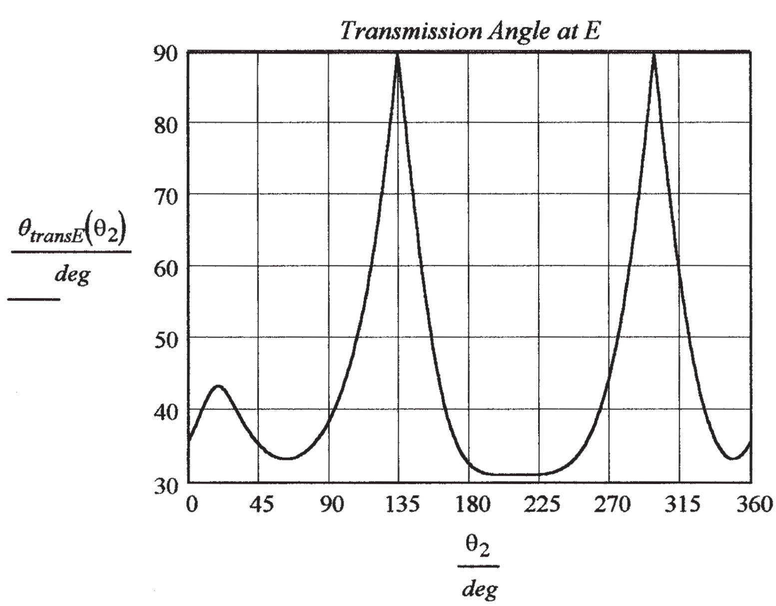
$$\theta_{transD}(\theta_2) := \left| if\left(\theta_{transD1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transD1}(\theta_2), \theta_{transD1}(\theta_2)\right)\right|$$



13. Calculate (using equations 4.28) and plot the transmission angle at E.

$$\theta_{transE1}(\theta_2) := |\theta_7(\theta_2) - \theta_8(\theta_2)|$$

$$\theta_{transE}(\theta_2) := \left| if\left(\theta_{transE1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transE1}(\theta_2), \theta_{transE1}(\theta_2)\right)\right|$$



PROBLEM 4-45

Statement: Create a model of the linkage shown in Figure 3-36 in *Working Model*.

Solution: See Figure 3-36 and Working Model file P0445.

PROBLEM 4-46

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the path of point P in Figure 3-37a as a function of the angle of input link 2. Also plot the variation (error) in the path of point *P* versus that of point *A*.

Given: Link lengths:

Input (O_2A)	$a := 0.136$	Coupler (AB)	$b := 1.000$
Rocker (O_4B)	$c := 1.000$	Ground link	$d := 1.414$

Coupler point data: $p := 2.000$ $\delta := 0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \land y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \land y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{cases}$$

Solution: See Figure 3-37a and Mathcad file P0446.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{cases} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{cases}$$

$Condition(a,b,c,d) = \text{"Grashof"}$ crank rocker

1. Define one cycle of the input crank:

$$\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

2. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \qquad K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)}$$

$$K_1 = 10.3971 \qquad K_4 = 1.4140 \qquad K_5 = -7.4187$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

3. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right)\right)$$

4. Use equations 4.27 to define the *x*- and *y*-components of the vector $\mathbf{R}_P$.

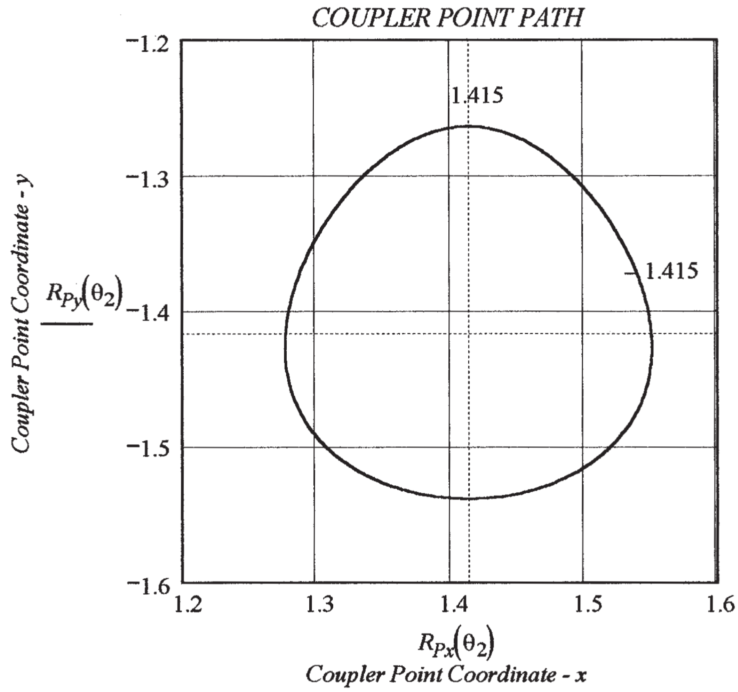
$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a \cdot \left(\cos(\theta_2) + j \cdot \sin(\theta_2)\right)$$

$$\mathbf{R_{PA}} := p \cdot \left(\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta)\right)$$

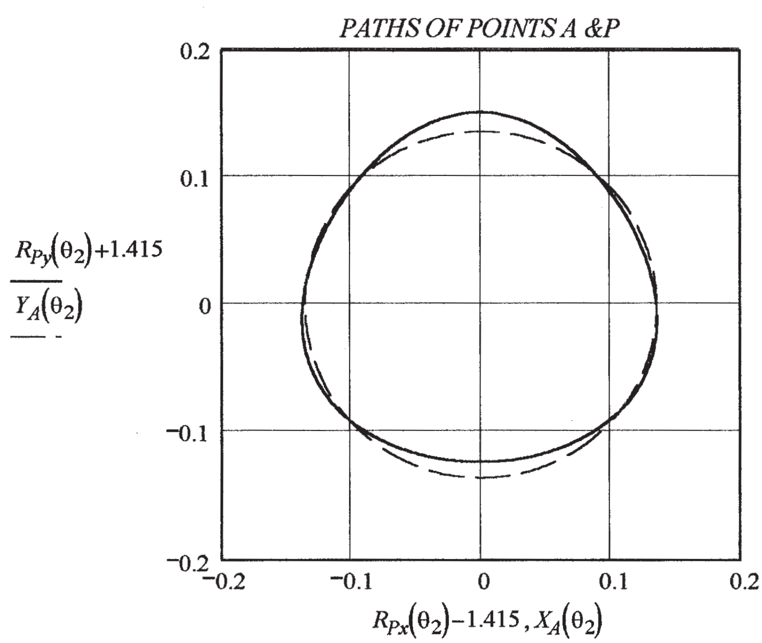
$$R_{Px}(\theta_2) := a \cdot cos(\theta_2) + p \cdot cos(\theta_3(\theta_2) + \delta) \qquad R_{Py}(\theta_2) := a \cdot sin(\theta_2) + p \cdot sin(\theta_3(\theta_2) + \delta)$$

5. Plot the coordinates of the coupler point in the local *xy* coordinate system.



6. Replot, transforming the coupler path to 0,0 and plot the path of point *A*.

$$X_A(\theta_2) := a \cdot cos(\theta_2) \qquad Y_A(\theta_2) := a \cdot sin(\theta_2)$$



PROBLEM 4-47

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angle at point *B* of the linkage in Figure 3-37a as a function of the angle of input link 2.

Given: Link lengths:

Input (O_2A)	$a := 0.136$	Coupler (AB)	$b := 1.000$
Rocker (O_4B)	$c := 1.000$	Ground link	$d := 1.414$

Coupler point data: $p := 2.000$ $\delta := 0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure 3-37a and Mathcad file P0447.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{cases} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{cases}$$

$Condition(a,b,c,d) = \text{"Grashof"}$ crank rocker

1. Define one cycle of the input crank:

$$\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

2. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 10.3971 \qquad K_2 = 1.4140 \qquad K_3 = 7.4187$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right)\right)$$

3. Use equations 4.12 and 4.13 to calculate θ_3 as a function of θ_2 (for the crossed circuit).

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 1.414 \qquad K_5 = -7.419$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

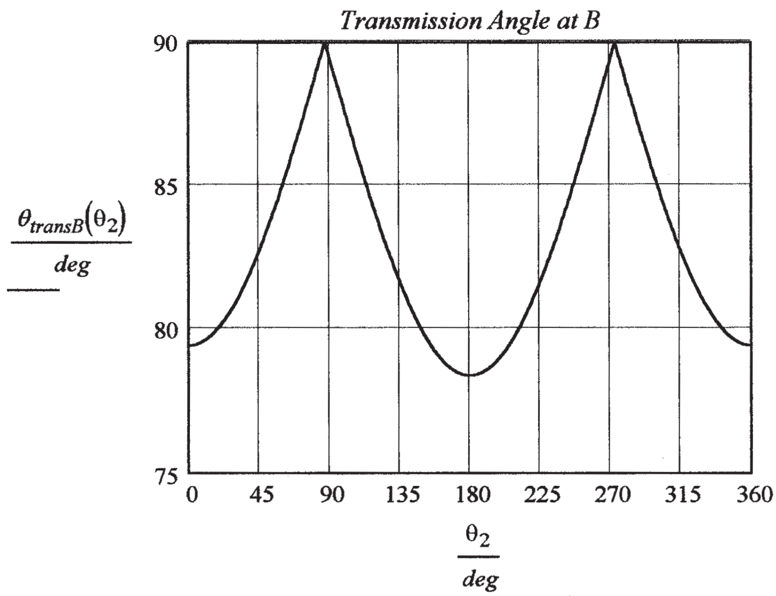
$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

4. Calculate (using equations 4.28) and plot the transmission angle at *B*.

$$\theta_{transB1}(\theta_2) := |\theta_3(\theta_2) - \theta_4(\theta_2)|$$

$$\theta_{transB}(\theta_2) := \left| if\left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2) \right) \right|$$



PROBLEM 4-48

Statement: Create a model of the linkage shown in Figure 3-37a in *Working Model*.

Solution: See Figure 3-37a and Working Model file P0448.

PROBLEM 4-49

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the path of point P in Figure 3-37b as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 0.50$	First coupler (AB)	$b := 1.00$
Rocker 4 (O_4B)	$c := 1.00$	Rocker 5 (L_5)	$c' := 1.00$
Ground link (O_2O_4)	$d := 0.75$	Second coupler 6 (CD)	$b' := 1.00$
Coupler point (DP)	$p := 1.00$	Distance to OP (O_2O_P)	$d' := 1.50$

Two argument inverse tangent:

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{array}$$

Solution: See Figure 3-37b and Mathcad file P0449.

1. Links 4, 5, BC, and CD form a parallelogram whose opposite sides remain parallel throughout the motion of the fourbar 1, 2, AB, 4. Define a position vector whose tail is at point D and whose tip is at point P and another whose tail is at O_4 and whose tip is at point D. Then, since $\mathbf{R}_5 = \mathbf{R}_{AB}$ and $\mathbf{R}_{DP} = -\mathbf{R}_4$, the position vector from O_2 to P is $\mathbf{P} = \mathbf{R}_1 + \mathbf{R}_{AB} - \mathbf{R}_4$. Separating this vector equation into real and imaginary parts gives the equations for the X and Y coordinates of the coupler point P.

$$X_P = d + b\cdot\cos(\theta_3) - c\cdot\cos(\theta_4) \qquad Y_P = b\cdot\sin(\theta_3) - c\cdot\sin(\theta_4)$$

2. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 1\cdot deg .. 360\cdot deg$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 1.5000 \qquad K_2 = 0.7500 \qquad K_3 = 0.8125$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2\cdot\cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot\sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot\cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.7500 \qquad K_5 = -0.8125$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4\cdot\cos(\theta_2) + K_5 \qquad E(\theta_2) := -2\cdot\sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot\cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

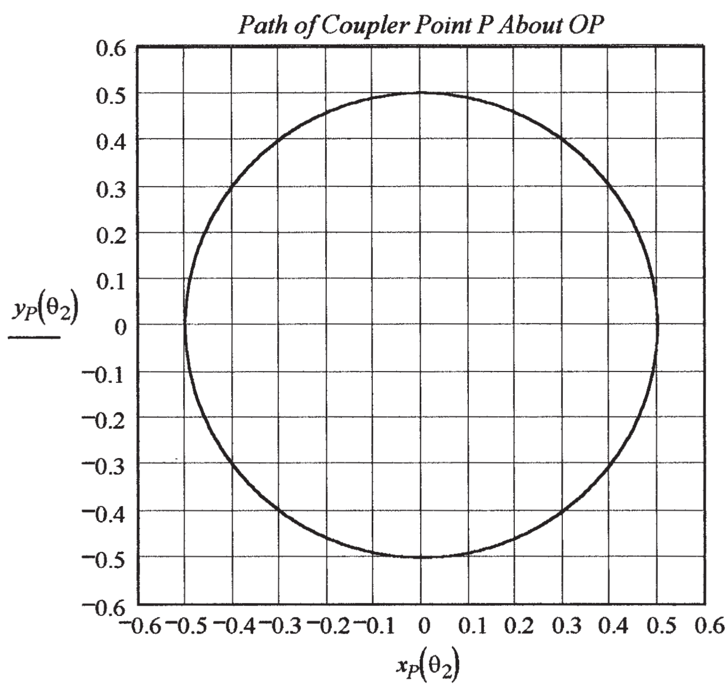
$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Define a local xy coordinate system with origin at O_P and with the positive x axis to the right. The coordinates of P are transformed to $x_P = X_P - d'$, $y_P = Y_P$.

$$x_P(\theta_2) := d + b \cdot \cos(\theta_3(\theta_2)) - c \cdot \cos(\theta_4(\theta_2)) - d'$$

$$y_P(\theta_2) := b \cdot \sin(\theta_3(\theta_2)) - c \cdot \sin(\theta_4(\theta_2))$$

7. Plot the path of P as a function of the angle of link 2.



PROBLEM 4-50

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the transmission angles at points *B*, *C*, and *D* of the linkage in Figure 3-37b as a function of the angle of input link 2.

Given: Link lengths:

Input crank (L_2)	$a := 0.50$	First coupler (*AB*)	$b := 1.00$
Rocker 4 (O_4B)	$c := 1.00$	Rocker 5 (L_5)	$c' := 1.00$
Ground link (O_2O_4)	$d := 0.75$	Second coupler 6 (*CD*)	$b' := 1.00$
Coupler point (*DP*)	$p := 1.00$	Distance to OP (O_2O_P)	$d' := 1.50$

Two argument inverse tangent:

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure 3-37b and Mathcad file P0450.

1. Links 4, 5, *BC*, and *CD* form a parallelogram whose opposite sides remain parallel throughout the motion of the fourbar 1, 2, *AB*, 4. Therefore, the transmission angles at points *B* and *D* will be the same and the transmission angle at point *C* will be the complement of the angle at *B*.
2. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 1\cdot deg .. 360deg$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 1.5000 \qquad K_2 = 0.7500 \qquad K_3 = 0.8125$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.7500 \qquad K_5 = -0.8125$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

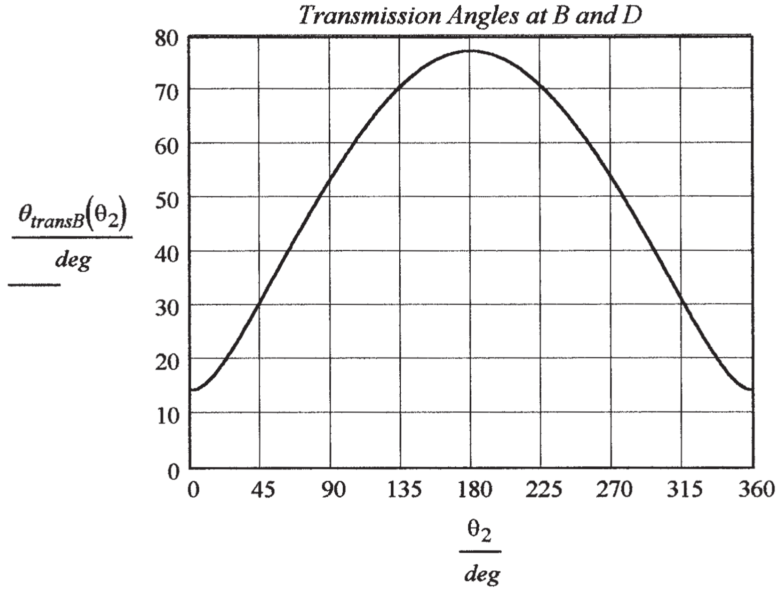
5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

6. Calculate (using equations 4.28) and plot the transmission angles at *B* and *D*.

$$\theta_{transB1}(\theta_2) := |\theta_3(\theta_2) - \theta_4(\theta_2)|$$

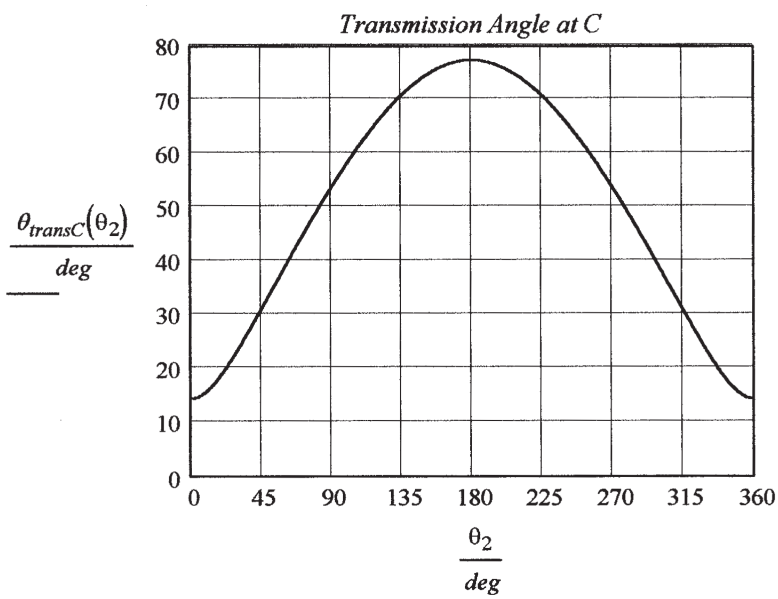
$$\theta_{transB}(\theta_2) := \left| if\left(\theta_{transB1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_2), \theta_{transB1}(\theta_2)\right)\right|$$



6. Calculate and plot the transmission angle at *C*.

$$\theta_{transC1}(\theta_2) := |180 \cdot deg - \theta_{transB}(\theta_2)|$$

$$\theta_{transC}(\theta_2) := \left| if\left(\theta_{transC1}(\theta_2) > \frac{\pi}{2}, \pi - \theta_{transC1}(\theta_2), \theta_{transC1}(\theta_2)\right)\right|$$



PROBLEM 4-51

Statement: Create a model of the linkage shown in Figure 3-37b in *Working Model.*

Solution: See Figure 3-37b and Working Model file P0451.

PROBLEM 4-52

Statement: For the linkage in Figure P4-16, what are the angles that link 2 makes with the positive X-axis when links 2 and 3 are in toggle positions?

Given: Link lengths:

Input (O_2A) $a := 14$ Coupler (AB) $b := 80$

Rocker (O_4B) $c := 51.26$

O_4 offset in XY coordinates: $O_{4X} := -47.5$ $O_{4Y} := 76 - 12$ $O_{4Y} = 64.000$

Ground link: $d := \sqrt{O_{4X}^2 + O_{4Y}^2}$ $d = 79.701$

Solution: See Figure P4-16 and Mathcad file P0452.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{vmatrix} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return \text{ "Grashof" } if\ SL < PQ \\ return \text{ "Special Grashof" } if\ SL = PQ \\ return \text{ "non-Grashof" } otherwise \end{vmatrix}$$

$Condition(a,b,c,d) = \text{"Grashof"}$ crank rocker

2. Define the coordinate frame transformation angle:

$$\delta := \pi + atan\left(\frac{O_{4Y}}{O_{4X}}\right) \qquad \delta = 126.582\, deg$$

3. Calculate the angle of link 2 in the XY system when links 2 and 3 are in the overlapped toggle position.

$$\theta_{21XY} := -acos\left[\frac{(b-a)^2 + d^2 - c^2}{2\cdot(b-a)\cdot d}\right] + \delta \qquad \theta_{21XY} = 86.765\, deg$$

4. Calculate the angle of link 2 in the XY system when links 2 and 3 are in the extended toggle position.

$$\theta_{22XY} := -acos\left[\frac{(b+a)^2 + d^2 - c^2}{2\cdot(b+a)\cdot d}\right] + \delta \qquad \theta_{22XY} = 93.542\, deg$$

PROBLEM 4-53

Statement: The coordinates of the point $P1$ on link 4 in Figure P4-16 are (114.68, 33.19) with respect to the xy coordinate system when link 2 is in the position shown. When link 2 is in another position the coordinates of $P2$ with respect to the xy system are (100.41, 43.78). Calculate the coordinates of $P1$ and $P2$ in the XY system for the two positions of link 2. What is the salient feature of the coordinates of $P1$ and $P2$ in the XY system?

Given: Vertical and horizontal offsets from O_2 to O_4.

$$O2O4_X := 47.5 \cdot in \qquad O2O4_Y := 64 \cdot in$$

Coordinates of P_1 and P_2 in the local system

$$P_{1x} := 114.68 \cdot in \qquad P_{1y} := 33.19 \cdot in$$

$$P_{2x} := 100.41 \cdot in \qquad P_{2y} := 43.78 \cdot in$$

Solution: See Figure P4-16 and Mathcad file P0453.

1. Calculate the angle from the global X axis to the local x axis.

$$\delta := 180 \cdot deg - atan\left(\frac{O2O4_Y}{O2O4_X}\right) \qquad \delta = 126.582\ deg$$

2. Use equations 4.0b to transform the given coordinates from the local to the global system.

$$P_{1X} := P_{1x} \cdot \cos(\delta) - P_{1y} \cdot \sin(\delta) \qquad P_{1X} = -95.00\ in$$

$$P_{1Y} := P_{1x} \cdot \sin(\delta) + P_{1y} \cdot \cos(\delta) \qquad P_{1Y} = 72.31\ in$$

$$P_{2X} := P_{2x} \cdot \cos(\delta) - P_{2y} \cdot \sin(\delta) \qquad P_{2X} = -95.00\ in$$

$$P_{2Y} := P_{2x} \cdot \sin(\delta) + P_{2y} \cdot \cos(\delta) \qquad P_{2Y} = 54.54\ in$$

3. In the global XY system the X-coordinates are the same for each point, which indicates that the head on the end of the rocker beam 4 is designed such that its tangent is always parallel to the Y-axis.

PROBLEM 4-54

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular position of link 4 with respect to the *XY* coordinate frame and the transmission angle at point *B* of the linkage in Figure P4-16 as a function of the angle of input link 2 with respect to the *XY* frame.

Given: Link lengths:

Input (O_2A)	$a := 14$	Coupler (AB)	$b := 80$
Rocker (O_4B)	$c := 51.26$	Ground link	$d := 79.70$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P4-16 and Mathcad file P0454.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array}$$

$Condition(a,b,c,d) = \text{"Grashof"}$ crank rocker

2. Define the coordinate frame transformation angle:

$$\delta := 90\cdot deg + atan\left(\frac{47.5}{64}\right) \qquad \delta = 126.582\,deg$$

3. Define one cycle of the input crank with respect to the *XY* frame:

$$\theta_{2XY} := 0\cdot deg, 1\cdot deg .. 360\cdot deg \qquad \theta_2(\theta_{2XY}) := \theta_{2XY} - \delta$$

4. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 5.6929 \qquad K_2 = 1.5548 \qquad K_3 = 1.9339$$

$$A(\theta_{2XY}) := cos(\theta_2(\theta_{2XY})) - K_1 - K_2\cdot cos(\theta_2(\theta_{2XY})) + K_3$$

$$B(\theta_{2XY}) := -2\cdot sin(\theta_2(\theta_{2XY})) \qquad C(\theta_{2XY}) := K_1 - (K_2 + 1)\cdot cos(\theta_2(\theta_{2XY})) + K_3$$

$$\theta_{41}(\theta_{2XY}) := 2\cdot\left(atan2\left(2\cdot A(\theta_{2XY}), -B(\theta_{2XY}) + \sqrt{B(\theta_{2XY})^2 - 4\cdot A(\theta_{2XY})\cdot C(\theta_{2XY})}\right)\right)$$

$$\theta_4(\theta_{2XY}) := \theta_{41}(\theta_{2XY}) + \delta - 2\cdot\pi$$

5. Use equations 4.12 and 4.13 to calculate θ_3 as a function of θ_2 (for the crossed circuit).

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b}$$

$$K_4 = 0.996 \qquad K_5 = -4.607$$

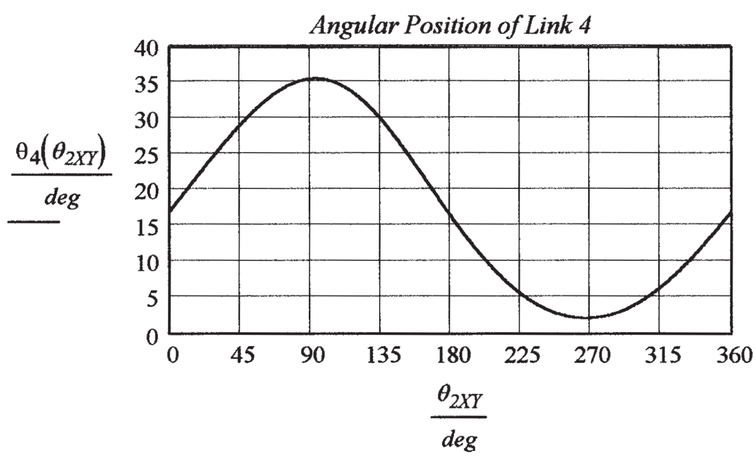
$$D(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 + K_4 \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$E(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$F(\theta_{2XY}) := K_1 + (K_4 - 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$\theta_3(\theta_{2XY}) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_{2XY}), -E(\theta_{2XY}) + \sqrt{E(\theta_{2XY})^2 - 4 \cdot D(\theta_{2XY}) \cdot F(\theta_{2XY})} \right) \right)$$

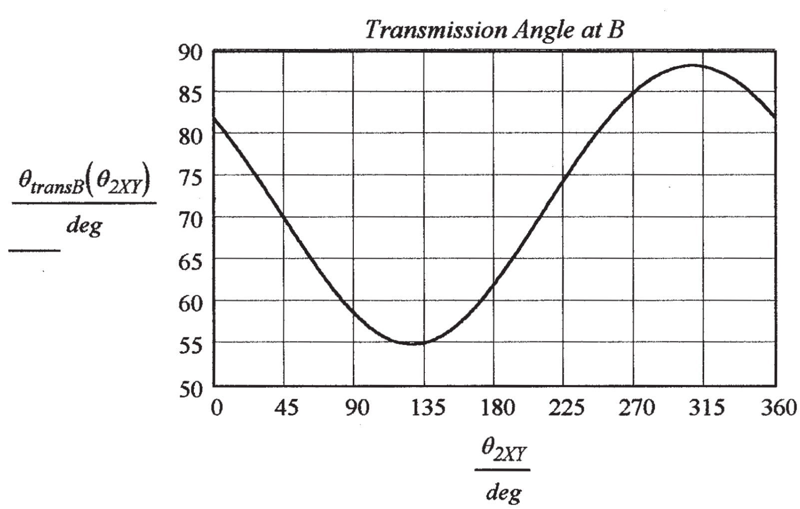
6. Plot the angular position of link 4 as a function of the input angle of link 2 with respect to the *XY* frame.



7. Calculate (using equations 4.28) and plot the transmission angle at *B*.

$$\theta_{transB1}(\theta_{2XY}) := \left| \theta_3(\theta_{2XY}) - \theta_{41}(\theta_{2XY}) \right|$$

$$\theta_{transB}(\theta_{2XY}) := \left| if\left(\theta_{transB1}(\theta_{2XY}) > \frac{\pi}{2}, \pi - \theta_{transB1}(\theta_{2XY}), \theta_{transB1}(\theta_{2XY}) \right) \right|$$



PROBLEM 4-55

Statement: For the linkage in Figure P4-17, calculate the maximum CW rotation of link 2 from the position shown, which is -20.60 deg with respect to the local *xy* system. What angles do link 3 and link 4 rotate through for that excursion of link 2?

Given: Link lengths:

Input (O_2A)	$a := 9.17$	Coupler (AB)	$b := 12.97$
Rocker (O_4B)	$c := 9.57$	Ground link	$d := 7.49$

Initial position of link 2: $\theta_{20} := -26.00 \cdot deg + 2 \cdot \pi$ (with respect to *xy* system)

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P4-17 and Mathcad file P0455.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{cases} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{cases}$$

$$Condition(d,b,a,c) = \text{"non-Grashof"}$$

2. Using equations 4.33, determine the crank angles (relative to the line O_2O_4) at which links 3 and 4 are in toggle.

$$arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} + \frac{b \cdot c}{a \cdot d} \qquad arg_1 = 0.936$$

$$arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2 \cdot a \cdot d} - \frac{b \cdot c}{a \cdot d} \qquad arg_2 = -2.678$$

$$\theta_{21} := acos(arg_1) \qquad \theta_{21} = 20.55\ deg$$

The other toggle angle is the negative of this.

$$\theta_{22} := -\theta_{21} + 2 \cdot \pi \qquad \theta_{22} = 339.45\ deg$$

3. Calculate the CW rotation of link 2 from the initial position to the toggle position.

$$\Delta\theta_2 := \theta_{21} - \theta_{20} \qquad \Delta\theta_2 = -313.45\ deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \qquad K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)}$$

$$K_1 = 0.8168 \qquad K_4 = 0.5775 \qquad K_5 = -0.9115$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

Initial angular position of link 3: $\theta_3(\theta_{20}) - 2\cdot\pi = 72.245\,deg$

Final angular position of link 3: $\theta_3(\theta_{21} + 0.001\cdot deg) = 250.764\,deg$

$$\Delta\theta_3 := \theta_3(\theta_{21} + 0.001\cdot deg) - (\theta_3(\theta_{20}) - 2\cdot\pi) \qquad \Delta\theta_3 = 178.518\,deg$$

7. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 0.8168 \qquad K_2 = 0.7827 \qquad K_3 = 0.3621$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

Initial angular position of link 4: $\theta_4(\theta_{20}) - 2\cdot\pi = 60.538\,deg$

Final angular position of link 4: $\theta_4(\theta_{21} + 0.001\cdot deg) = 250.615\,deg$

$$\Delta\theta_4 := \theta_4(\theta_{21} + 0.001\cdot deg) - (\theta_4(\theta_{20}) - 2\cdot\pi) \qquad \Delta\theta_4 = 190.077\,deg$$

PROBLEM 4-56

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the path of point P in Figure P4-17 with respect to the XY coordinate system as a function of the angle of input link 2 with respect to the XY coordinate system.

Given: Link lengths:

Input (O_2A)	$a := 9.174$	Coupler (AB)	$b := 12.971$
Rocker (O_4B)	$c := 9.573$	Ground link	$d := 7.487$

Coupler point data: $p := 15.00$ $\quad \delta := 0 \cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{array}$$

Solution: See Figure P4-17 and Mathcad file P0456.

1. Check the Grashof condition of the linkage.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ \text{return "Grashof" if } SL < PQ \\ \text{return "Special Grashof" if } SL = PQ \\ \text{return "non-Grashof" otherwise} \end{array}$$

$Condition(d,b,a,c) =$ "non-Grashof"

2. Using equations 4.33, determine the crank angles (relative to the line O_2O_4) at which links 3 and 4 are in toggle.

$$arg_1 := \frac{a^2 + d^2 - b^2 - c^2}{2\cdot a\cdot d} + \frac{b\cdot c}{a\cdot d} \qquad arg_1 = 0.937$$

$$arg_2 := \frac{a^2 + d^2 - b^2 - c^2}{2\cdot a\cdot d} - \frac{b\cdot c}{a\cdot d} \qquad arg_2 = -2.679$$

$$\theta_{2toggle} := acos(arg_1) \qquad \theta_{2toggle} = 20.501\, deg$$

The other toggle angle is the negative of this.

3. Define the coordinate transformation angle.

Transformation angle: $\alpha := -atan\left(\frac{6.95}{2.79}\right)$ $\quad \alpha = -68.128\, deg$

4. Define one cycle of the input crank between limit positions:

$$\theta_2 := \theta_{2toggle}, \theta_{2toggle} + 1\cdot deg .. 2\cdot\pi - \theta_{2toggle}$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_1 := \frac{d}{a} \qquad K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)}$$

$$K_1 = 0.8161 \qquad K_4 = 0.5772 \qquad K_5 = -0.9110$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2) \qquad F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Use equations 4.27 to define the x- and y-components of the vector $\mathbf{R_P}$.

$$\mathbf{R_P} := \mathbf{R_A} + \mathbf{R_{PA}}$$

$$\mathbf{R_A} := a \cdot \left(\cos(\theta_2) + j \cdot \sin(\theta_2) \right)$$

$$\mathbf{R_{PA}} := p \cdot \left(\cos(\theta_3 + \delta) + j \cdot \sin(\theta_3 + \delta) \right)$$

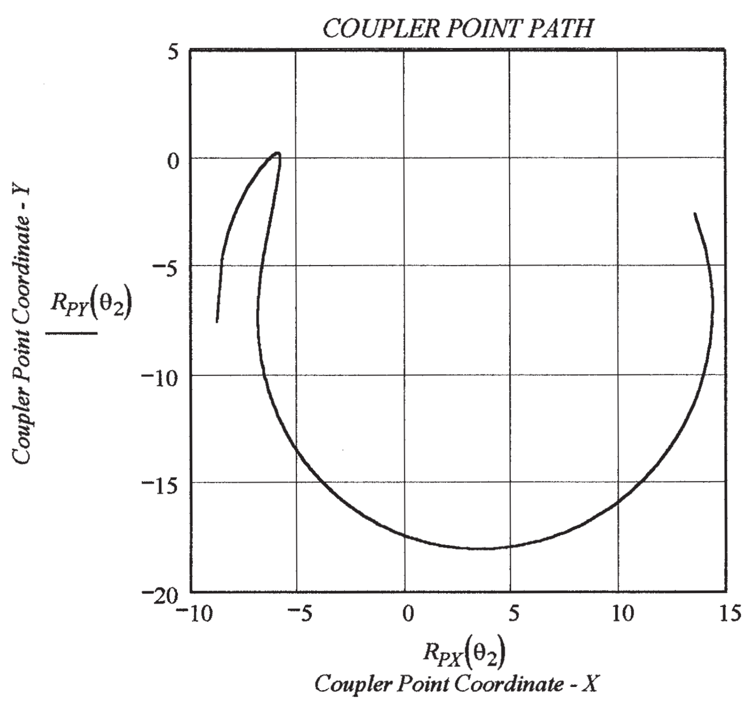
$$R_{Px}(\theta_2) := a \cdot \cos(\theta_2) + p \cdot \cos(\theta_3(\theta_2) + \delta) \qquad R_{Py}(\theta_2) := a \cdot \sin(\theta_2) + p \cdot \sin(\theta_3(\theta_2) + \delta)$$

8. Transform these local xy coordinates to the global XY coordinate system using equations 4.0b.

$$R_{PX}(\theta_2) := R_{Px}(\theta_2) \cdot \cos(\alpha) - R_{Py}(\theta_2) \cdot \sin(\alpha)$$

$$R_{PY}(\theta_2) := R_{Px}(\theta_2) \cdot \sin(\alpha) + R_{Py}(\theta_2) \cdot \cos(\alpha)$$

9. Plot the coordinates of the coupler point in the global XY coordinate system.



PROBLEM 4-57

Statement: For the linkage in Figure P4-17, calculate the coordinates of the point *P* in the *XY* coordinate system if its coordinates in the *xy* system are (2.71, 10.54).

Given: Vertical and horizontal offsets from O_2 to O_4.

$$O2O4_X := 2.790 \cdot in \qquad O2O4_Y := -6.948 \cdot in$$

Coordinates of *P* in the local system

$$P_x := 12.816 \cdot in \qquad P_y := 10.234 \cdot in$$

Solution: See Figure P4-17 and Mathcad file P0457.

1. Calculate the angle from the global *X* axis to the local *x* axis.

$$\delta := \operatorname{atan}\left(\frac{O2O4_Y}{O2O4_X}\right) \qquad \delta = -68.122 \, deg$$

2. Use equations 4.0b to transform the given coordinates from the local to the global system.

$$P_X := P_x \cdot \cos(\delta) - P_y \cdot \sin(\delta) \qquad P_X = 14.273 \, in$$

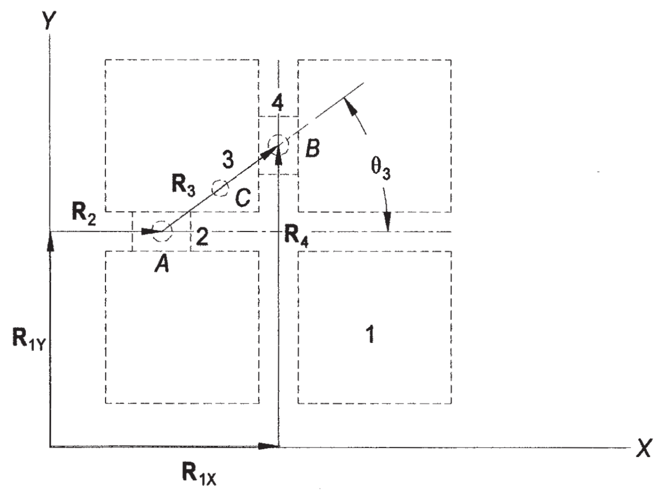
$$P_Y := P_x \cdot \sin(\delta) + P_y \cdot \cos(\delta) \qquad P_Y = -8.079 \, in$$

PROBLEM 4-58

Statement: The elliptical trammel in Figure P4-18 must be driven by rotating link 3 in a full circle. Derive analytical expressions for the positions of points *A*, *B*, and a point *C* on link 3 midway between *A* and *B* as a function of θ_3 and the length *AB* of link 3. Use a vector loop equation. (Hint: Place the global origin off the mechanism, preferably below and to the left and use a total of 5 vectors.) Code your solution in an equation solver such as *Mathcad, Matlab,* or *TKSolver* to calculate and plot the path of point *C* for one revolution of link 3.

Solution: See Figure P4-18 and Mathcad file P0458.

1. Establish the global XY system such that the coordinates of the intersection of the slot centerlines is at (d_X, d_Y). Then, define position vectors $\mathbf{R}_{1X}$, $\mathbf{R}_{1Y}$, $\mathbf{R}_2$, $\mathbf{R}_3$, and $\mathbf{R}_4$ as shown below.



2. Write the vector loop equation: $\mathbf{R}_{1Y} + \mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_{1X} - \mathbf{R}_4 = 0$ then substitute the complex number notation for each position vector. The equation then becomes:

$$d_Y \cdot e^{j\left(\frac{\pi}{2}\right)} + a \cdot e^{j(0)} + c \cdot e^{j(\theta_3)} - d_X \cdot e^{j(0)} - b \cdot e^{j\left(\frac{\pi}{2}\right)} = 0$$

3. Substituting the Euler identity into this equation gives:

$$d_Y \cdot j + a + c \cdot \left(\cos(\theta_3) + j \cdot \sin(\theta_3)\right) - d_X - b \cdot j = 0$$

4. Separate this equation into its real (x component) and imaginary (y component) parts, setting each equal to zero.

$$a + c \cdot \cos(\theta_3) - d_X = 0 \qquad d_Y + c \cdot \sin(\theta_3) - b = 0$$

5. Solve for the two unknowns *a* and *b* in terms of the constants d_X and d_Y and the independent variable θ_3. Where (a, d_Y) and (d_X, b) are the coordinates of points *A* and *B*, respectively, and *c* is the length of link 3. With no loss of generality, let $d_X = d_Y = d$. Then,

$$a = d - c \cdot \cos(\theta_3) \qquad b = d + c \cdot \sin(\theta_3)$$

6. The coordinates of the point C are:

$$C_X = d - 0.5 \cdot \cos(\theta_3) \qquad C_Y = d + 0.5 \cdot c \cdot \sin(\theta_3)$$

7. Using a local coordinate system whose origin is located at the intersection of the centerlines of the two slots and transforming the above functions to the local *xy* system:

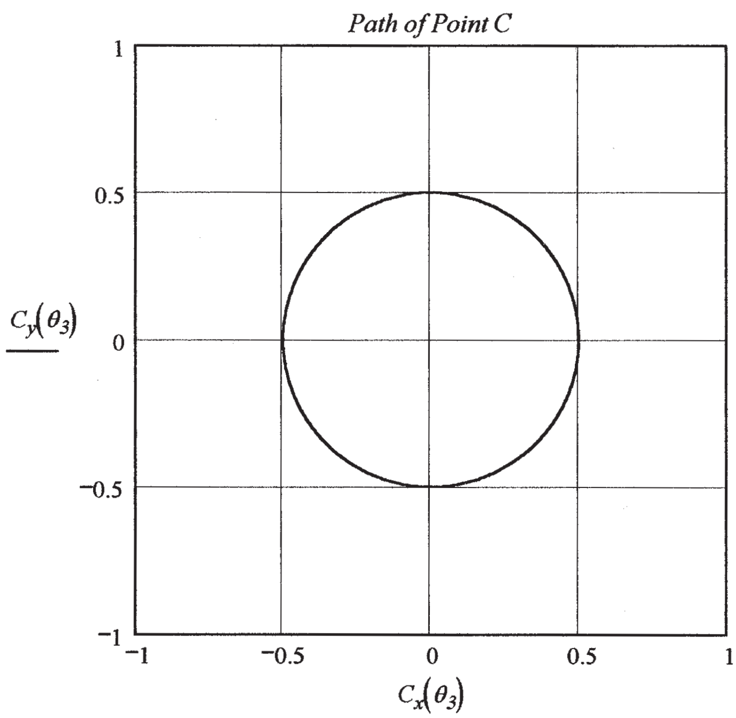
$$a_x = -c \cdot cos(\theta_3) \qquad a_y = 0$$

$$b_x = 0 \qquad b_y = c \cdot sin(\theta_3)$$

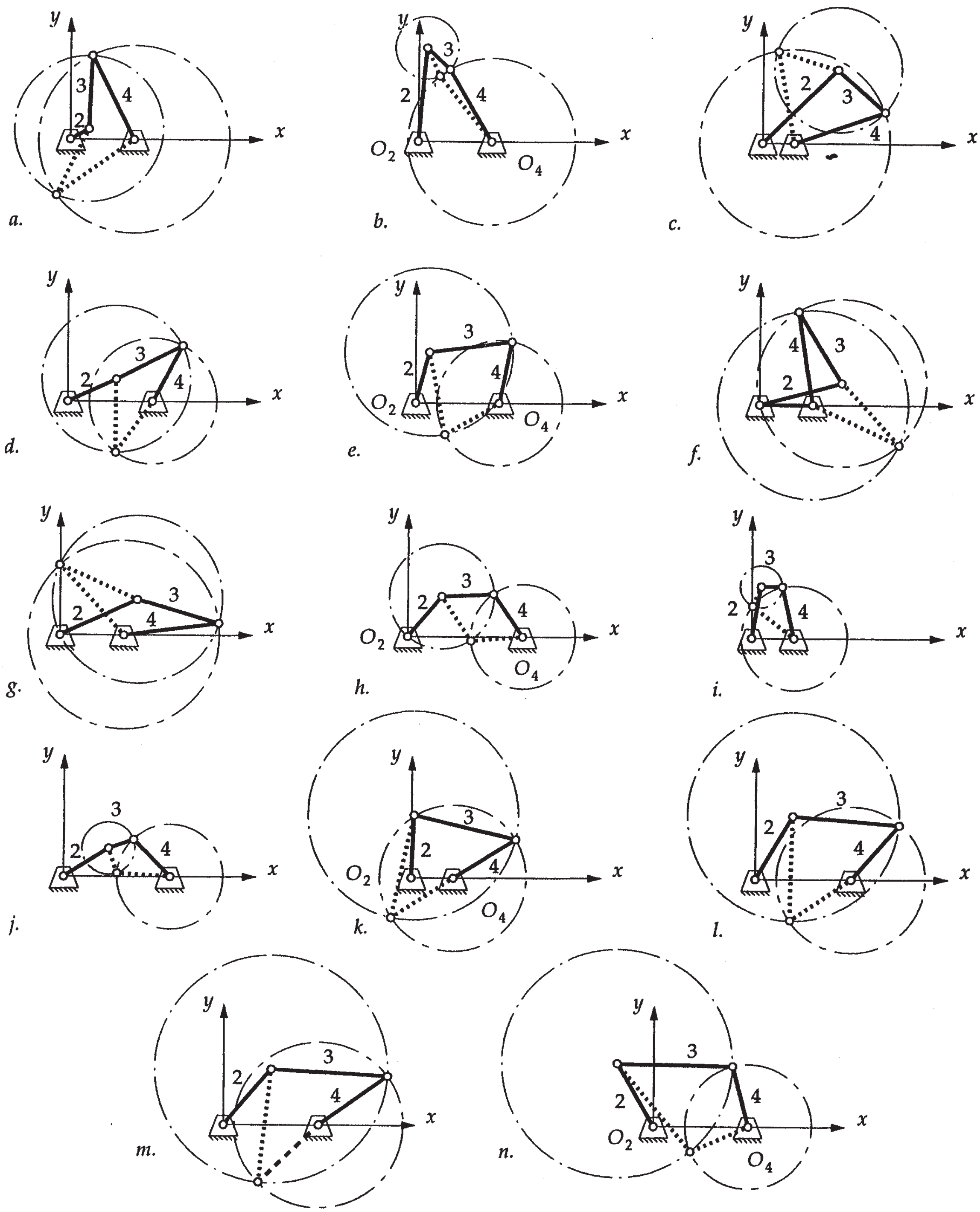
8. To plot the path of point C as a function of θ_3, let $c := 1$ and define a range function for θ_3

$$\theta_3 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

$$C_x(\theta_3) := -0.5 \cdot c \cdot cos(\theta_3) \qquad C_y(\theta_3) := 0.5 \cdot sin(\theta_3)$$



PROBLEM 4–6



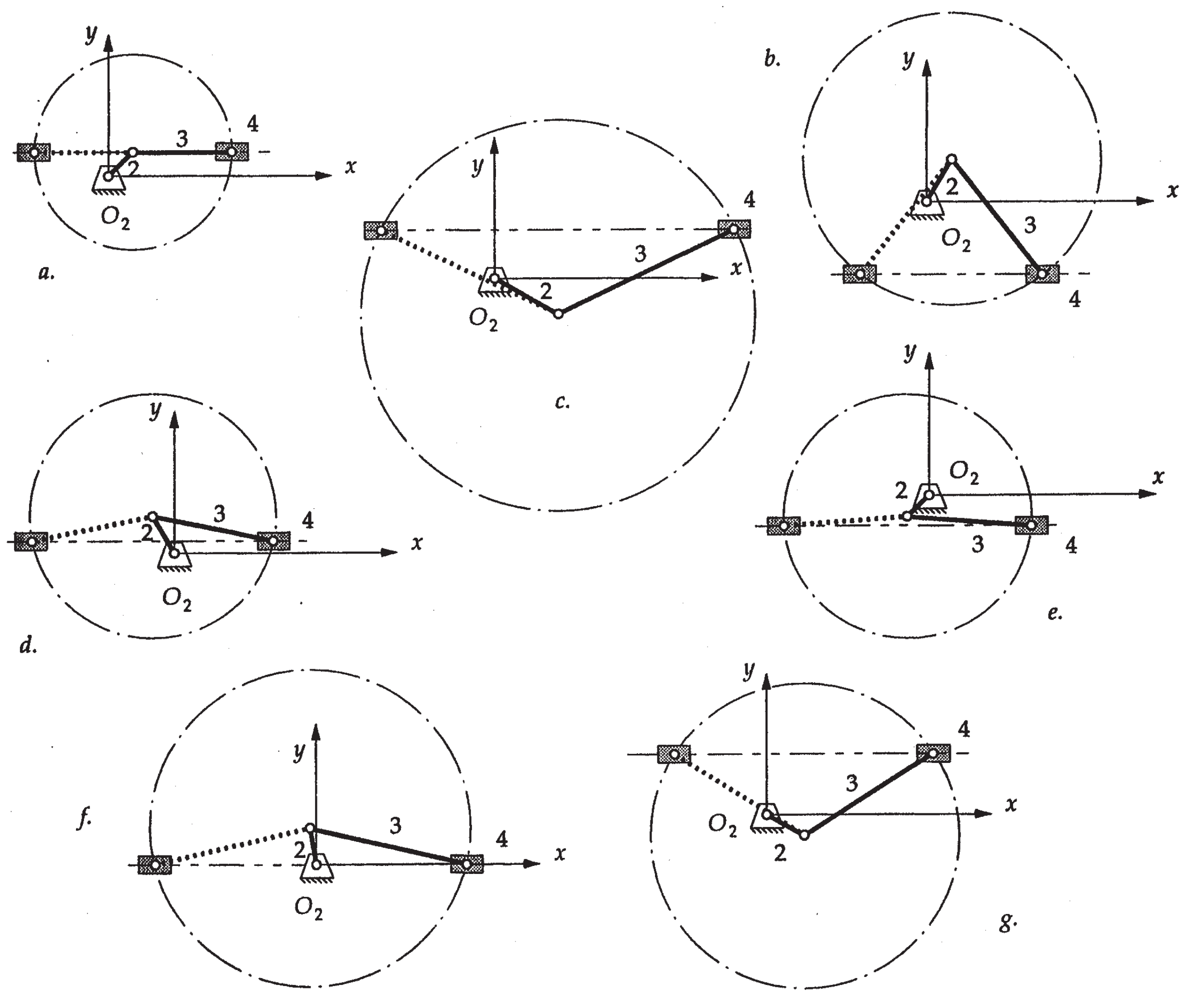
PROBLEMS 4-6 AND 4-7

Row	Open θ_3	Open θ_4	Crossed θ_3	Crossed θ_4
a	88.8	117.3	-115.2	-143.6
b	-43.2	120.2	-67.3	129.2
c	-53.1	16.5	173.3	103.6
d	27.4	62.8	-90.1	-125.5
e	7.5	78.2	-79.0	-149.7
f	-47.3	-25.0	121.6	99.4
g	-16.3	7.2	155.7	132.2
h	9.4	111.7	-68.3	-170.6
i	-1.5	103.1	-113.5	141.8
j	20.6	133.9	-70.9	175.9
k	-13.3	31.9	-102.1	-147.3
l	-3.9	50.2	-91.7	-145.8
m	-3.5	35.9	-96.5	-135.9
n	-1.3	104.5	-50.4	-156.3

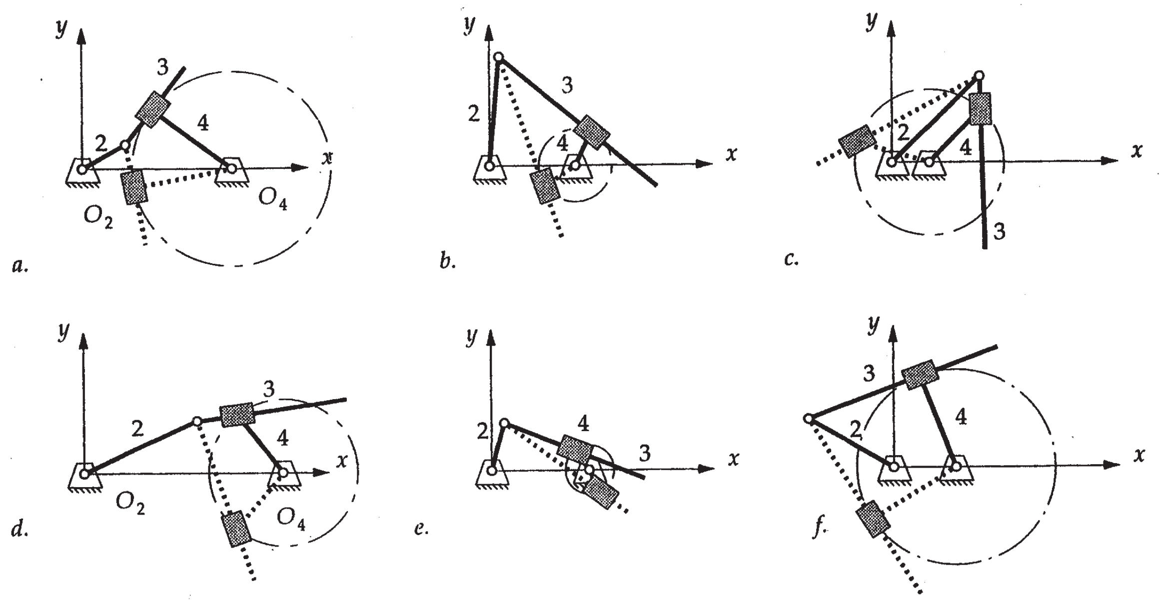
PROBLEMS 4-9 AND 4-10

Row	Open θ_3	Open d	Crossed θ_3	Crossed d
a	180.1	5.0	-0.14	-3.0
b	127.9	4.7	52.1	-2.7
c	205.9	9.8	-25.9	-4.6
d	168.3	8.0	11.7	-11.5
e	175	16.4	4.2	-23.5
f	166.9	12.1	13.1	-13.2
g	212.7	27.1	-32.7	-14.9

PROBLEM 4-9



PROBLEM 4-11



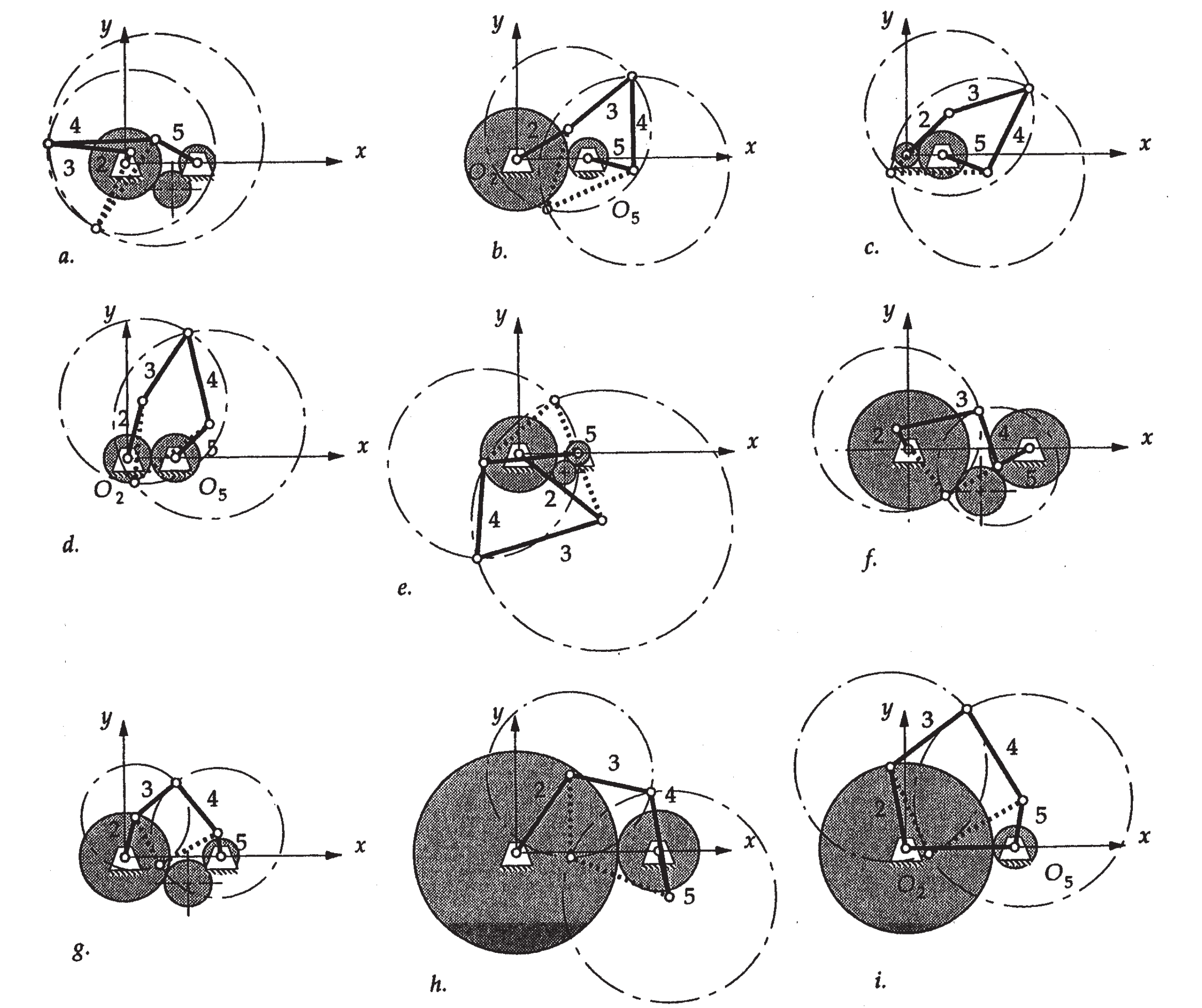
PROBLEMS 4-11 AND 4-12

Row	Open θ_3	Open θ_4	Crossed θ_3	Crossed θ_4
a	232.7	142.7	-79.0	-169.0
b	140.1	65.1	-70.7	-145.7
c	91.4	46.4	208.7	163.7
d	188.4	128.4	-71.1	-131.1
e	158.2	128.2	-36.2	-66.2
f	200.9	110.9	-58.0	-148.0

PROBLEM 4-13

Row	θ_{trans}
a	28.45
b	16.52
c	69.62
d	35.36
e	70.72
f	22.25
g	23.55
h	77.62
i	75.36
j	66.71
k	45.18
l	54.15
m	39.41
n	74.17

PROBLEM 4-16



PROBLEMS 4-16 AND 4-17

Row	Open θ_3	Open θ_4	Crossed θ_3	Crossed θ_4
a	173.6	-177.7	-115.2	-124.0
b	39.6	91.0	-104.7	-156.2
c	17.6	64.0	-133.7	179.9
d	56.3	101.9	-96.1	-141.7
e	-164.0	-94.4	111.2	41.6
f	12.5	108.3	-54.6	-150.4
g	44.2	124.4	-69.1	-149.3
h	-13.0	99.3	-90.3	157.4
i	37.1	120.2	-67.4	-150.5