Chapter 6

VELOCITY ANALYSIS

TOPIC/PROBLEM MATRIX

SECT	TOPIC	PROBLEMS
6.1	Definition of Velocity	6-1, 6-2, 6-3
6.2	Graphical Velocity Analysis	
	Pin-Jointed Fourbar	6-17a, 6-24, 6-28, 6-36, 6-39, 6-84a, 6-87a, 6-94
	Fourbar Slider-Crank	6-16a, 6-32, 6-43
	Other Fourbar	6-18a, 6-98
	Geared Fivebar	6-10
	Sixbar	6-70a, 6-73a, 6-76a, 6-99
	Eightbar	6-103
6.3	Instant Centers of Velocity	6-12, 6-13, 6-14, 6-15, 6-68, 6-72, 6-75, 6-78, 6-83, 6-86, 6-88, 6-97, 6-102
6.4	Velocity Analysis with Instant Centers	6-4, 6-16b, 6-17b, 6-18b, 6-25, 6-29, 6-33, 6-40, 6-70b, 6-73b, 6-76b, 6-84b, 6-87b, 6-92, 6-95, 6-100
	Mechanical Advantage	6-21a, 6-21b, 6-22a, 6-22b, 6-58
6.5	Centrodes	6-23, 6-63, 6-69, 6-89
6.6	Velocity of Slip	6-6, 6-8, 6-19, 6-20, 6-61, 6-64, 6-65, 6-66, 6-91
6.7	Analytical Solutions for Velocity Analysis	6-90
	Pin-Jointed Fourbar	6-26, 6-27, 6-30, 6-31, 6-37, 6-38, 6-41, 6-42, 6-48, 6-62
	Fourbar Slider-Crank	6-7, 6-34, 6-35, 6-44, 6-45, 6-52, 6-60
	Fourbar Inverted Slider-Crank	6-9
	Sixbar	6-70c, 6-71, 6-73c, 6-74, 6-76c, 6-77, 6-93, 6-101
	Eightbar	6-79
	Mechanical Advantage	6-55a, 6-55b, 6-57a, 6-57b, 6-59a, 6-59b, 6-67
6.8	Velocity Analysis of Geared Fivebar	6-11
6.9	Velocity of Any Point on A Linkage	6-5, 6-16c, 6-17c, 6-18c, 6-46, 6-47, 6-49, 6-50,6-51, 6-53, 6-54, 6-56, 6-80, 6-81, 6-82, 6-84c, 6-85, 6-87c, 6-96

PROBLEM 6-1a

Statement: A ship is steaming due north at 20 knots (nautical miles per hour). A submarine is laying in wait 1/2 mile due west of the ship. The sub fires a torpedo on a course of 85 degrees. The torpedo travels at a constant speed of 30 knots. Will it strike the ship? If not, by how many nautical miles will it miss? Hint: Use the relative velocity equation and solve graphically or analytically.

Units: $naut\_mile := 1$ $\qquad knots := \dfrac{naut\_mile}{hr}$

Given: Speed of ship $\quad V_s := 20 \cdot knots \qquad \theta_s := 90 \cdot deg$

Speed of torpedo $\quad V_t := 30 \cdot knots \qquad \theta_t := 15 \cdot deg$

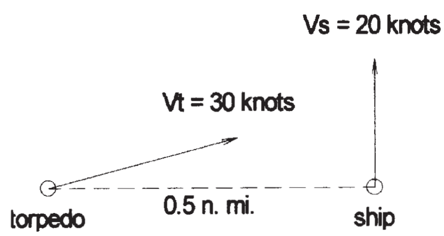
Initial distance between ship and torpedo $\quad d_i := 0.5 \cdot naut\_mile$

Note that, for compass headings, due north is 0 degrees, due east 90 degrees, and the angle increases clockwise. However, in a right-handed Cartesian system, due north is 90 degrees (up) and due east is 0 degrees (to the right). The Cartesian system has been used above to define the ship and torpedo headings.

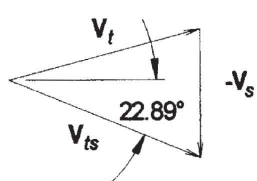
Solution: See Mathcad file P0601a.

1. The key to this solution is to recognize that the only information of interest is the relative velocity of one vessel to the other. The ship captain wants to know the relative velocity of the torpedo versus the ship, $\mathbf{V}_{ts} = \mathbf{V}_t - \mathbf{V}_s$. In effect, we want to resolve the situation with respect to a moving coordinate system attached to the ship.

2. The figure below shows the initial positions of the torpedo and ship and their velocities.



3. The figure below shows the vector diagram that solves the relative velocity equation $\mathbf{V}_{ts} = \mathbf{V}_t - \mathbf{V}_s$. For the torpedo to hit the moving ship, the relative velocity vector has to be perpendicular to the ship's velocity vector (if you were on the ship observing the torpedo, it would appear to be headed directly for you). As the velocity diagram shows, the relative velocity vector is not perpendicular to the ship's velocity vector so it will miss and pass behind the ship.



4. Determine the distance by which the torpedo will miss the ship.

Time required for torpedo to travel 0.5 nautical miles due east

$$t_{t\_east} := \frac{d_i}{V_t \cdot \cos(\theta_t)} \qquad t_{t\_east} = 62.117\ sec$$

Distance traveled by the torpedo due north in that time

$d_{t\_north} := V_t \cdot sin(\theta_t) \cdot t_{t\_east}$ $d_{t\_north} = 0.134\, naut\_mile$

Distance traveled by the ship due north in that time

$d_{s\_north} := V_s \cdot t_{t\_east}$ $d_{s\_north} = 0.345\, naut\_mile$

Distance by which the torpedo will miss the ship

$d_{miss} := d_{s\_north} - d_{t\_north}$ $d_{miss} = 0.211\, naut\_mile$

PROBLEM 6-1b

Statement: A plane is flying due south at 500 mph at 35,000 feet altitude, straight and level. A second plane is initially 40 miles due east of the first plane, also at 35,000 feet altitude, flying straight and level at 550 mph. Determine the compass angle at which the second plane would be on a collision course with the first. How long will it take for the second plane to catch the first? Hint: Use the relative velocity equation and solve graphically or analytically.

Given:

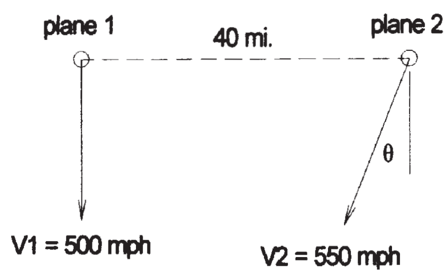
Speed of first plane	$V_1 := 500 \cdot mph$	$\theta_1 := 270 \cdot deg$
Speed of second plane	$V_2 := 550 \cdot mph$	
Initial distance between planes	$d_i := 40 \cdot mi$	

Note that, for compass headings, due north is 0 degrees, due east 90 degrees, and the angle increases clockwise. However, in a right-handed Cartesian system, due north is 90 degrees (up) and due east is 0 degrees (to the right). The Cartesian system has been used above to define the plane headings.

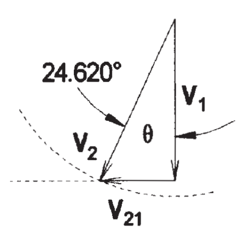
Solution: See Mathcad file P0601b.

1. The key to this solution is to recognize that the only information of interest is the relative velocity of one plane to the other. For a collision to occur, the relative velocity of the second plane with respect to the first must be perpendicular to the velocity vector of the first.

2. The figure below shows the initial positions of the two planes and their velocities.



3. The figure below shows the vector diagram that solves the relative velocity equation $\mathbf{V}_2 = \mathbf{V}_1 + \mathbf{V}_{21}$. To construct this diagram, chose a convenient velocity scale and draw $\mathbf{V}_1$ to its correct length with the arrow head pointing straight down (indicating due south). From the tip of the vector, layoff a horizontal construction line to the left (due west) an undetermined length. From the tail of the $\mathbf{V}_1$ vector, construct a circle whose radius is equal to the scaled length of vector $\mathbf{V}_2$. The intersection of the circle and the horizontal construction line determines the length of $\mathbf{V}_{21}$. Draw the arrowheads for $\mathbf{V}_2$ and $\mathbf{V}_{21}$ pointing toward the intersection of the circle and construction line. Label the horizontal vector $\mathbf{V}_{21}$ and the vector that joins the tail of $\mathbf{V}_1$ with the head of $\mathbf{V}_{21}$ as $\mathbf{V}_2$. The angle θ between $\mathbf{V}_1$ and $\mathbf{V}_2$ is the required direction for $\mathbf{V}_2$ in order that plane 2 collides with plane 1.



4. The angle θ can also be determined analytically from the velocity triangle as follows.

$$\theta := acos\left(\frac{V_1}{V_2}\right) \qquad \theta = 24.620\,deg$$

5. The time it will take for the second plane to catch the first is the time that it will take plane 2 to travel the 40 miles to the west.

$$t := \frac{d_i}{V_2 \cdot sin(\theta)} \qquad t = 628.468\,\text{sec}$$

$$t = 10.474\,min$$

PROBLEM 6-2

Statement: A point is at a 6.5-in radius on a body in pure rotation with ω = 100 rad/sec. The rotation center is at the origin of a coordinate system. When the point is at position *A*, its position vector makes a 45 deg angle with the *X* axis. At position B, its position vector makes a 75 deg angle with the *X* axis. Draw this system to some convenient scale and:

a. Write an expression for the particle's velocity vector in position *A* using complex number notation, in both polar and Cartesian forms.

b. Write an expression for the particle's velocity vector in position *B* using complex number notation, in both polar and Cartesian forms.

c. Write a vector equation for the velocity difference between points *B* and *A*. Substitute the complex number notation for the vectors in this equation and solve for the velocity difference numerically.

d. Check the result of part c with a graphical method.

Given:

Rotation speed	$\omega := 100 \cdot \dfrac{rad}{sec}$	
Vector angles	$\theta_A := 45 \cdot deg$	$\theta_B := 75 \cdot deg$
Vector magnitude	$R := 6.5 \cdot in$	

Solution: See Mathcad file P0602.

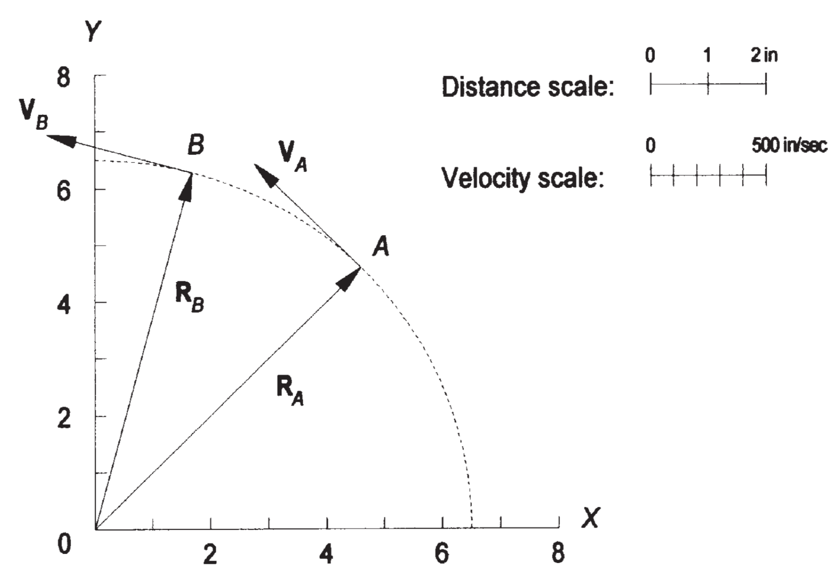
1. Calculate the magnitude of the velocity at points *A* and *B* using equation 6.3.

$$V := R \cdot \omega \qquad V = 650.000 \, \frac{in}{sec}$$

2. Establish an *X-Y* coordinate frame and draw a circle with center at the origin and radius *R*.

3. Draw lines from the origin that make angles of 45 and 75 deg with respect to the *X* axis. Label the intersections of the lines with the circles as *A* and *B*, respectively. Make the line segment *OA* a vector by putting an arrowhead at *A*, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment *OB*, labeling it $\mathbf{R}_B$.

4. Choose a convenient velocity scale and draw the two velocity vectors $\mathbf{V}_A$ and $\mathbf{V}_B$ at the tips of $\mathbf{R}_A$ and $\mathbf{R}_B$, respectively. The velocity vectors will be perpendicular to their respective position vectors.



a. *Write an expression for the particle's velocity vector in position A using complex number notation, in both polar and Cartesian forms.*

Polar form: $\mathbf{R_A} := R \cdot e^{j \cdot \theta_A}$ $\mathbf{R_A} := 6.5 \cdot e^{j \cdot \frac{\pi}{4}}$

$\mathbf{V_A} := R \cdot j \cdot \omega \cdot e^{j \cdot \theta_A}$ $\mathbf{V_A} := 650 \cdot j \cdot e^{j \cdot \frac{\pi}{4}}$

Cartesian form: $\mathbf{V_A} := R \cdot j \cdot \omega \cdot \left(\cos\left(\theta_A\right) + j \cdot \sin\left(\theta_A\right)\right)$

$\mathbf{V_A} = -459.619 + 459.619j \frac{in}{sec}$

b. *Write an expression for the particle's velocity vector in position B using complex number notation, in both polar and Cartesian forms.*

Polar form: $\mathbf{R_B} := R \cdot e^{j \cdot \theta_B}$ $\mathbf{R_A} := 6.5 \cdot e^{j \cdot \frac{\pi}{4}}$

$\mathbf{V_B} := R \cdot j \cdot \omega \cdot e^{j \cdot \theta_B}$ $\mathbf{V_B} := 650 \cdot j \cdot e^{j \cdot \frac{75 \cdot \pi}{180}}$

Cartesian form: $\mathbf{V_B} := R \cdot j \cdot \omega \cdot \left(\cos\left(\theta_B\right) + j \cdot \sin\left(\theta_B\right)\right)$

$\mathbf{V_B} = -627.852 + 168.232j \frac{in}{sec}$

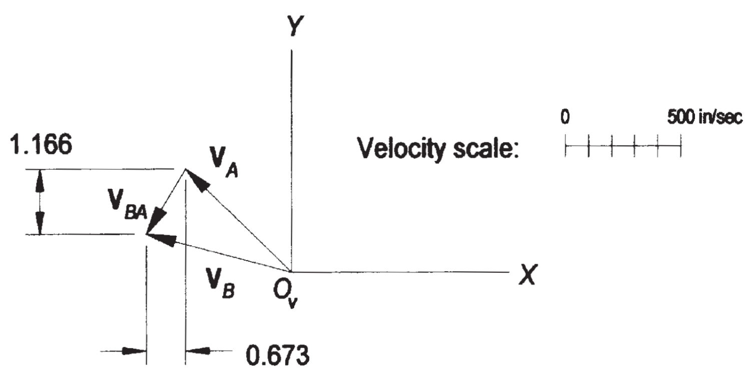
c. *Write a vector equation for the velocity difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.*

$\mathbf{V_{BA}} := \mathbf{V_B} - \mathbf{V_A}$ $\mathbf{V_{BA}} = -168.232 - 291.387j \frac{in}{sec}$

d. *Check the result of part c with a graphical method. Solve the equation* $\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$ using a velocity scale of 250 in/sec per drawing unit.

Velocity scale factor

$k_v := 250 \cdot \frac{in}{sec}$



Horizontal component

$V_{BAx} := -0.673 \cdot k_v$

$V_{BAx} = -168.3 \frac{in}{sec}$

Vertical component

$V_{BAy} := -1.166 \cdot k_v$ $V_{BAy} = -291.5 \frac{in}{sec}$

On the layout above the X and Y components of $\mathbf{V}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 6-3

Statement: A point *A* is at a 6.5-in radius on a body in pure rotation with ω = -50 rad/sec. The rotation center is at the origin of a coordinate system. At the instant considered its position vector makes a 45 deg angle with the *X* axis. A point *B* is at a 6.5-in radius on another body in pure rotation with ω = +75 rad/sec. Its position vector makes a 75 deg angle with the *X* axis. Draw this system to some convenient scale and:

a. Write an expression for the particle's velocity vector in position *A* using complex number notation, in both polar and Cartesian forms.

b. Write an expression for the particle's velocity vector in position *B* using complex number notation, in both polar and Cartesian forms.

c. Write a vector equation for the velocity difference between points *B* and *A*. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.

d. Check the result of part c with a graphical method.

Given:

Rotation speeds $\omega_A := -50 \cdot \dfrac{rad}{sec}$ $\omega_B := 75 \cdot \dfrac{rad}{sec}$

Vector angles $\theta_A := 45 \cdot deg$ $\theta_B := 75 \cdot deg$

Vector magnitude $R := 6.5 \cdot in$

Solution: See Mathcad file P0603.

1. Calculate the magnitude of the velocity at points *A* and *B* using equation 6.3.

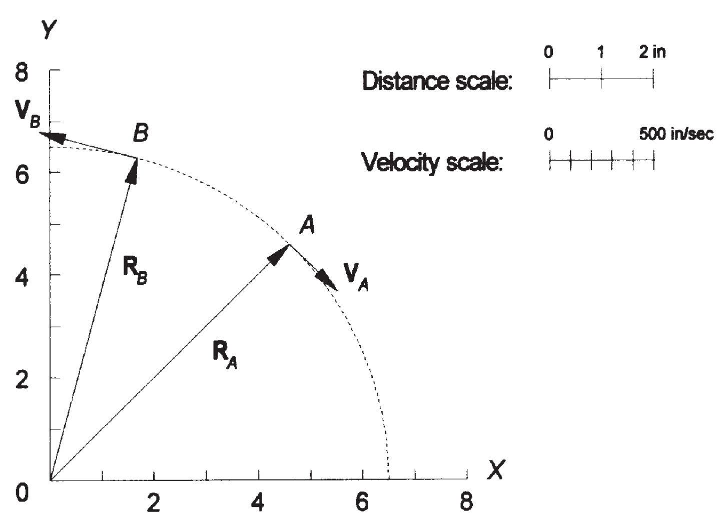
$$V_A := R \cdot \omega_A \qquad V_A = -325.000 \frac{in}{sec}$$

$$V_B := R \cdot \omega_B \qquad V_B = 487.500 \frac{in}{sec}$$

1. Establish an *X-Y* coordinate frame and draw a circle with center at the origin and radius *R*.

2. Draw lines from the origin that make angles of 45 and 75 deg with respect to the *X* axis. Label the intersections of the lines with the circles as *A* and *B*, respectively. Make the line segment *OA* a vector by putting an arrowhead at *A*, pointing away from the origin. Label the vector $\mathbf{R}_A$. Repeat for the line segment *OB*, labeling it $\mathbf{R}_B$.

3. Choose a convenient velocity scale and draw the two velocity vectors $\mathbf{V}_A$ and $\mathbf{V}_B$ at the tips of $\mathbf{R}_A$ and $\mathbf{R}_B$, respectively. The velocity vectors will be perpendicular to their respective position vectors.



a. *Write an expression for the particle's velocity vector on body A using complex number notation, in both polar and Cartesian forms.*

Polar form: $\mathbf{R_A} := R\cdot e^{j\cdot\theta_A}$ $\qquad \mathbf{R_A} := 6.5\cdot e^{j\cdot\frac{\pi}{4}}$

$\mathbf{V_A} := R\cdot j\cdot\omega_A\cdot e^{j\cdot\theta_A}$ $\qquad \mathbf{V_A} := -325\cdot j\cdot e^{j\cdot\frac{\pi}{4}}$

Cartesian form: $\mathbf{V_A} := R\cdot j\cdot\omega_A\cdot\left(\cos\left(\theta_A\right) + j\cdot\sin\left(\theta_A\right)\right)$

$$\mathbf{V_A} = 229.810 - 229.810j\,\frac{in}{sec}$$

b. *Write an expression for the particle's velocity vector on body B using complex number notation, in both polar and Cartesian forms.*

Polar form: $\mathbf{R_B} := R\cdot e^{j\cdot\theta_B}$ $\qquad \mathbf{R_A} := 6.5\cdot e^{j\cdot\frac{\pi}{4}}$

$\mathbf{V_B} := R\cdot j\cdot\omega_B\cdot e^{j\cdot\theta_B}$ $\qquad \mathbf{V_B} := 487.5\cdot j\cdot e^{j\cdot\frac{75\cdot\pi}{180}}$

Cartesian form: $\mathbf{V_B} := R\cdot j\cdot\omega_B\cdot\left(\cos\left(\theta_B\right) + j\cdot\sin\left(\theta_B\right)\right)$

$$\mathbf{V_B} = -470.889 + 126.174j\,\frac{in}{sec}$$

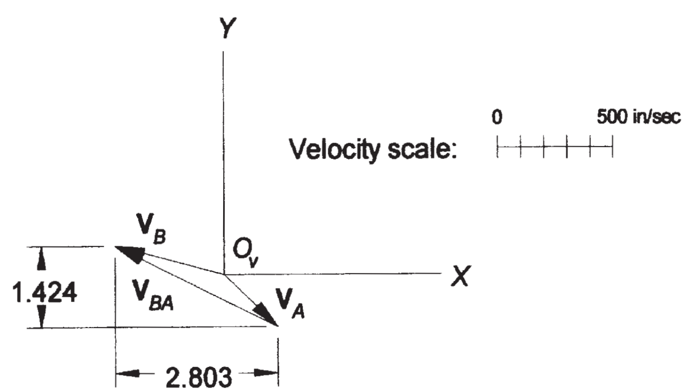
c. *Write a vector equation for the velocity difference between the points on bodies B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.*

$\mathbf{V_{BA}} := \mathbf{V_B} - \mathbf{V_A}$ $\qquad \mathbf{V_{BA}} = -700.699 + 355.984j\,\frac{in}{sec}$

d. *Check the result of part c with a graphical method. Solve the equation* $\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$ using a velocity scale of 250 in/sec per drawing unit.

Velocity scale factor

$$k_v := 250\cdot\frac{in}{sec}$$



Horizontal component

$$V_{BAx} := -2.803\cdot k_v$$

$$V_{BAx} = -700.8\,\frac{in}{sec}$$

Vertical component

$V_{BAy} := 1.424\cdot k_v$ $\qquad V_{BAy} = 356.0\,\frac{in}{sec}$

On the layout above the X and Y components of $\mathbf{V}_{BA}$ are equal to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 6-4a

Statement: For the fourbar defined in Table P6-1, line *a*, find the velocities of the pin joints *A* and *B*, and of the instant centers $I_{1,3}$ and $I_{2,4}$. Then calculate ω_3 and ω_4 and find the velocity of point *P*. Use a graphical method.

Given: Link lengths:

Link 1 $d := 6 \cdot in$ Link 2 $a := 2 \cdot in$

Link 3 $b := 7 \cdot in$ Link 4 $c := 9 \cdot in$

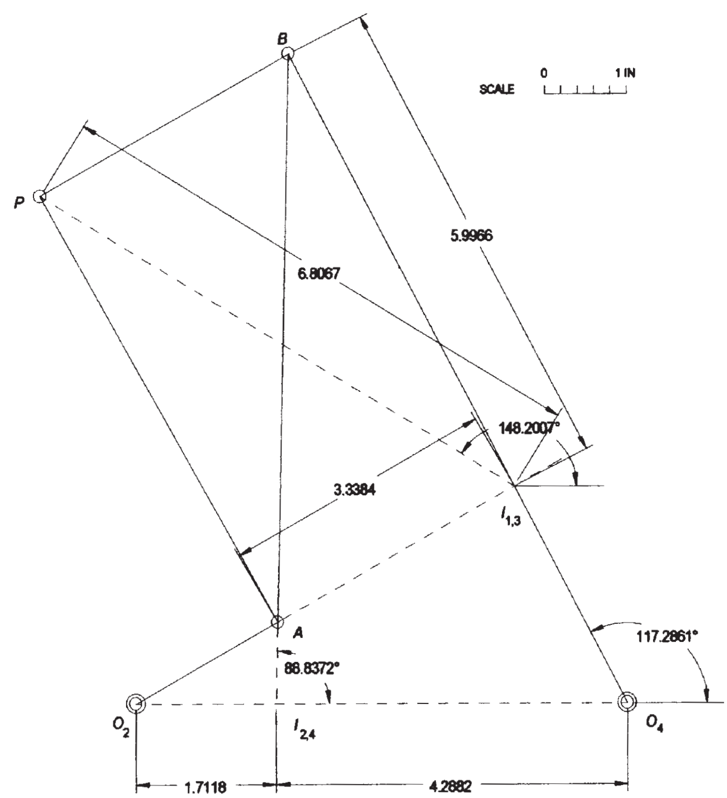
Crank angle: $\theta_2 := 30 \cdot deg$ Crank velocity: $\omega_2 := 10 \cdot rad \cdot sec^{-1}$

Coupler point data:

$R_{pa} := 6 \cdot in$ $\delta_3 := 30 \cdot deg$

Solution: See Figure P6-1 and Mathcad file P0604a.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the *x* axis.



From the layout above:

$O2I24 := 1.7118 \cdot in$ $O4I24 := 4.2882 \cdot in$

$AI13 := 3.3384 \cdot in$ $BI13 := 5.9966 \cdot in$ $PI13 := 6.8067 \cdot in$

$\theta_4 := 117.2861 \cdot deg$ $\theta_3 := 88.8372 \cdot deg$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point *A*.

$V_A := a \cdot \omega_2$ $\qquad V_A = 20.0 \dfrac{in}{sec}$

$\theta_{VA} := \theta_2 + 90 \cdot deg$ $\qquad \theta_{VA} = 120.0\, deg$

3. Determine the angular velocity of link 3 using equation 6.9a.

$\omega_3 := \dfrac{V_A}{AI13}$ $\qquad \omega_3 = 5.991 \dfrac{rad}{sec}$ CW

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. Determine its direction by inspection.

$V_B := BI13 \cdot \omega_3$ $\qquad V_B = 35.925 \dfrac{in}{sec}$

$\theta_{VB} := \theta_4 - 90 \cdot deg$ $\qquad \theta_{VB} = 27.286\, deg$

5. Use equation 6.9c to determine the angular velocity of link 4.

$\omega_4 := \dfrac{V_B}{c}$ $\qquad \omega_4 = 3.99 \dfrac{rad}{sec}$ CW

6. Use equation 6.9d and inspection of the layout to determine the magnitude and direction of the velocity at point *P*.

$V_P := PI13 \cdot \omega_3$ $\qquad V_P = 40.778 \dfrac{in}{sec}$

$\theta_{VP} := 148.2007 \cdot deg - 90 \cdot deg$ $\qquad \theta_{VP} = 58.201\, deg$

PROBLEM 6-5a

Statement: A general fourbar linkage configuration and its notation are shown in Figure P6-1. The link lengths, coupler point location, and the values of θ_2 and ω_2 for the same fourbar linkages as used for position analysis in Chapter 4 are redefined in Table P6-1, which is the same as Table P4-1. For row *a*, find the velocities of the pin joints *A* and *B*, and coupler point P. Calculate ω_3 and ω_4. Draw the linkage to scale and label it before setting up the equations.

Given: Link lengths:

Link 1 $d := 6$ Link 2 $a := 2$ Link 3 $b := 7$ Link 4 $c := 9$

Coupler point: $R_{pa} := 6$ $\delta_3 := 30 \cdot deg$

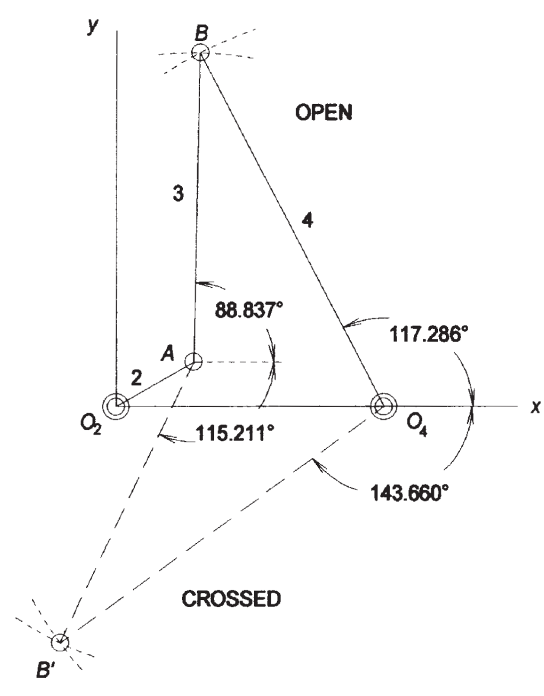
Link 2 position and velocity: $\theta_2 := 30 \cdot deg$ $\omega_2 := 10$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Mathcad file P0605a.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 3.0000 \qquad K_2 = 0.6667$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.7113 \qquad B = -1.0000 \qquad C = 3.5566$$

3. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

Open: $\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right)$ $\qquad \theta_{41} = 477.286\,deg$

$\theta_{41} := \theta_{41} - 360 \cdot deg$ $\qquad \theta_{41} = 117.286\,deg$

Crossed: $\theta_{42} := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right)$ $\qquad \theta_{42} = 216.340\,deg$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.8571$$

$$K_5 = -0.2857$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -1.6774$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 2.5906$$

5. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

Open: $\theta_{31} := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$ $\qquad \theta_{31} = 448.837\,deg$

$\theta_{31} := \theta_{31} - 360 \cdot deg$ $\qquad \theta_{31} = 88.837\,deg$

Crossed: $\theta_{32} := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$ $\qquad \theta_{32} = 244.789\,deg$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31} := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_{31} - \theta_{41})} \qquad \omega_{31} = -5.991$$

$$\omega_{41} := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31})}{\sin(\theta_{41} - \theta_{31})} \qquad \omega_{41} = -3.992$$

7. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$\mathbf{V_A} := a \cdot \omega_2 \cdot \left(-\sin(\theta_2) + j \cdot \cos(\theta_2) \right)$$

$\mathbf{V_A} = -10.000 + 17.321j \qquad |\mathbf{V_A}| = 20 \qquad arg(\mathbf{V_A}) = 120\,deg$

$\mathbf{V_B} := c\cdot\omega_{41}\cdot(-sin(\theta_{41}) + j\cdot cos(\theta_{41}))$

$\mathbf{V_B} = 31.928 + 16.470j \qquad |\mathbf{V_B}| = 35.926 \qquad arg(\mathbf{V_B}) = 27.286\,deg$

8. Determine the velocity of the coupler point P for the open circuit using equations 6.36.

$\mathbf{V_{PA}} := R_{pa}\cdot\omega_{31}\cdot(-sin(\theta_{31} + \delta_3) + j\cdot cos(\theta_{31} + \delta_3))$

$\mathbf{V_{PA}} = 31.488 + 17.337j$

$\mathbf{V_P} := \mathbf{V_A} + \mathbf{V_{PA}}$

$\mathbf{V_P} = 21.488 + 34.658j \qquad |\mathbf{V_P}| = 40.779 \qquad arg(\mathbf{V_P}) = 58.201\,deg$

9. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32} := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{42} - \theta_2)}{sin(\theta_{32} - \theta_{42})} \qquad \omega_{32} = -0.662$$

$$\omega_{42} := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{32})}{sin(\theta_{42} - \theta_{32})} \qquad \omega_{42} = -2.662$$

10. Determine the velocity of point *B* for the crossed circuit using equations 6.19.

$\mathbf{V_B} := c\cdot\omega_{42}\cdot(-sin(\theta_{42}) + j\cdot cos(\theta_{42}))$

$\mathbf{V_B} = -14.195 + 19.295j \qquad |\mathbf{V_B}| = 23.954 \qquad arg(\mathbf{V_B}) = 126.340\,deg$

11. Determine the velocity of the coupler point *P* for the crossed circuit using equations 6.36.

$\mathbf{V_{PA}} := R_{pa}\cdot\omega_{32}\cdot(-sin(\theta_{32} + \delta_3) + j\cdot cos(\theta_{32} + \delta_3))$

$\mathbf{V_{PA}} = -3.960 - 0.332j$

$\mathbf{V_P} := \mathbf{V_A} + \mathbf{V_{PA}}$

$\mathbf{V_P} = -13.960 + 16.989j \qquad |\mathbf{V_P}| = 21.989 \qquad arg(\mathbf{V_P}) = 129.411\,deg$

PROBLEM 6-6a

Statement: The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Fig P6-2. The link lengths and the values of θ_2 and ω_2 are defined in Table P6-2. For row *a*, find the velocities of the pin joints *A* and *B* and the velocity of slip at the sliding joint using a graphical method.

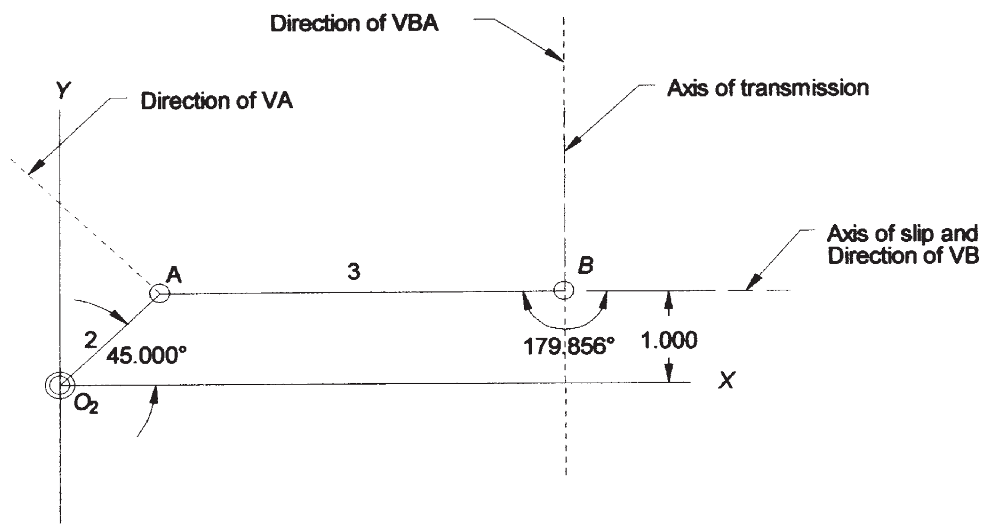
Given: Link lengths:

Link 2 $a := 1.4 \cdot in$ Link 3 $b := 4 \cdot in$

Offset $c := 1 \cdot in$ $\theta_2 := 45 \cdot deg$ $\omega_2 := 10 \cdot rad \cdot sec^{-1}$

Solution: See Figure P6-2 and Mathcad file P0606a.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A*.

$V_A := a \cdot \omega_2$ $V_A = 14.000 \frac{in}{sec}$ $\theta_{VA} := \theta_2 + 90 \cdot deg$ $\theta_{VA} = 135\, deg$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

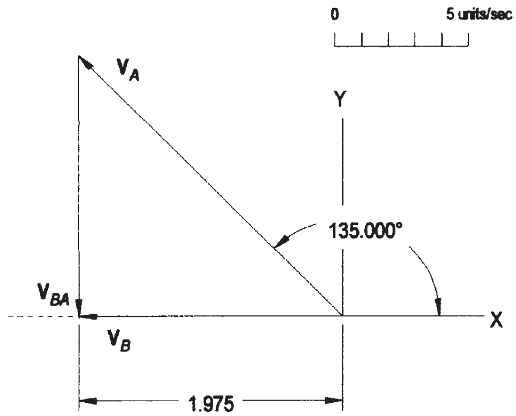
a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

(See next page.)



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{5 \cdot in \cdot sec^{-1}}{in}$

$V_B := 1.975 \cdot in \cdot k_v$ $\qquad V_B = 9.875 \frac{in}{sec}$ $\qquad \theta_{VB} := 180 \cdot deg$

5. Since the slip axis and the direction of the velocity of point B are parallel, $V_{slip} = V_B$.

PROBLEM 6-7a

Statement: The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Fig P6-2. The link lengths and the values of θ_2 and ω_2 are defined in Table P6-2. For row *a*, find the velocities of the pin joints *A* and *B* and the velocity of slip at the sliding joint using the analytic method. Draw the linkage to scale and label it before setting up the equations.

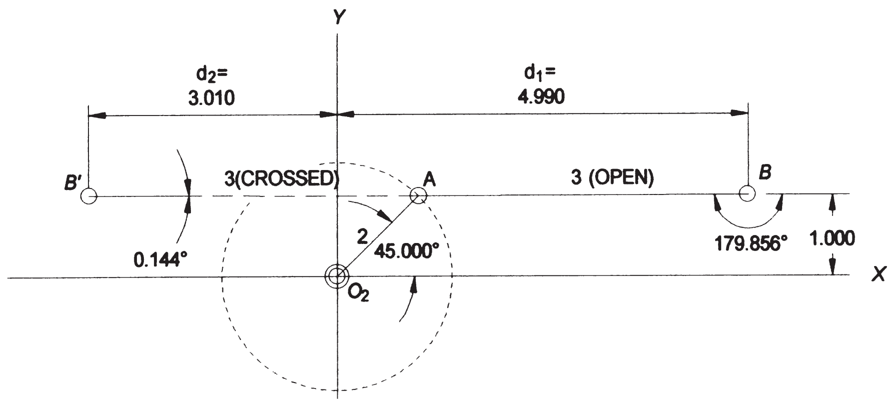
Given: Link lengths:

Link 2 ($O_2 A$) $a := 1.4$ Link 3 (AB) $b := 4$ Offset (y_B) $c := 1$

Crank angle $\theta_2 := 45 \cdot deg$ Crank angular velocity $\omega_2 := 10$

Solution: See Figure P6-2 and Mathcad file P0607a.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and d using equations 4.16 and 4.17.

Crossed:

$$\theta_{32} := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_{32} = -0.144\, deg$$

$$d_2 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{32}) \qquad d_2 = -3.010$$

Open:

$$\theta_{31} := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_{31} = 180.144\, deg$$

$$d_1 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{31}) \qquad d_1 = 4.990$$

3. Determine the angular velocity of link 3 using equation 6.22a:

Open $$\omega_{31} := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_{31})} \cdot \omega_2 \qquad \omega_{31} = -2.475$$

Crossed $$\omega_{32} := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_{32})} \cdot \omega_2 \qquad \omega_{32} = 2.475$$

4. Determine the velocity of pin *A* using equation 6.23a:

$$\mathbf{V_A} := a \cdot \omega_2 \cdot \left(-sin\left(\theta_2\right) + j \cdot cos\left(\theta_2\right)\right)$$

$\mathbf{V_A} = -9.899 + 9.899i$ $\quad\left|\mathbf{V_A}\right| = 14.000$ $\quad arg\left(\mathbf{V_A}\right) = 135.000\,deg$

5. Determine the velocity of pin *B* using equation 6.22b:

Open $\quad V_{B1} := -a \cdot \omega_2 \cdot sin\left(\theta_2\right) + b \cdot \omega_{31} \cdot sin\left(\theta_{31}\right)$ $\quad V_{B1} = -9.875$

Crossed $\quad V_{B2} := -a \cdot \omega_2 \cdot sin\left(\theta_2\right) + b \cdot \omega_{32} \cdot sin\left(\theta_{32}\right)$ $\quad V_{B2} = -9.924$

The angle of $\mathbf{V_B}$ is 0 deg if V_B is positive and 180 deg if V_B negative.

6. The velocity of slip is the same as the velocity of pin *B*.

PROBLEM 6-8a

Statement: The general linkage configuration and terminology for an inverted fourbar slider-crank linkage are shown in Fig P6-3. The link lengths and the values of θ_2 and ω_2 and γ are defined in Table P6-3. For row *a*, using a graphical method, find the velocities of the pin joints *A* and *B* and the velocity of slip at the sliding joint. Draw the linkage to scale.

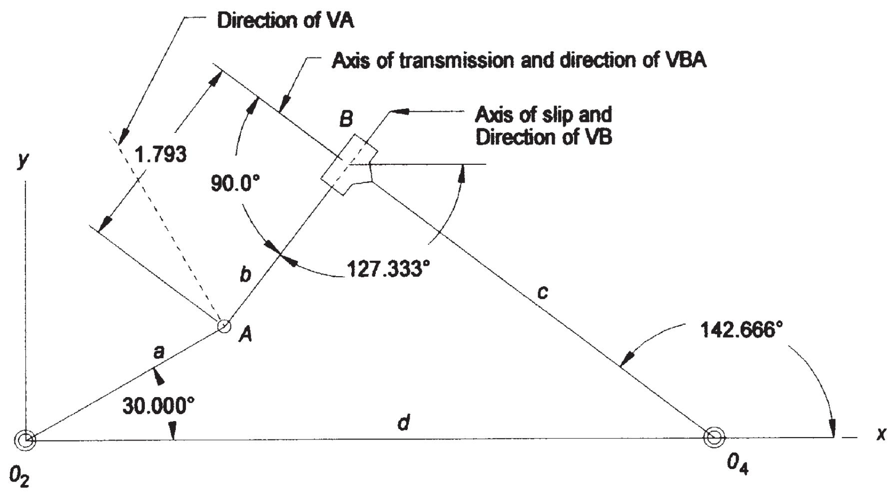
Given: Link lengths:

Link 1 $d := 6 \cdot in$ Link 2 $a := 2 \cdot in$

Link 4 $c := 4 \cdot in$ $\gamma := 90 \cdot deg$ $\theta_2 := 30 \cdot deg$ $\omega_2 := 10 \cdot rad \cdot sec^{-1}$

Solution: See Figure P6-3 and Mathcad file P0608a.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A*.

$V_A := a \cdot \omega_2$ $V_A = 20.000 \dfrac{in}{sec}$ $\theta_{VA} := \theta_2 + 90 \cdot deg$ $\theta_{VA} = 120\,deg$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B* on link 3. The equation to be solved graphically is

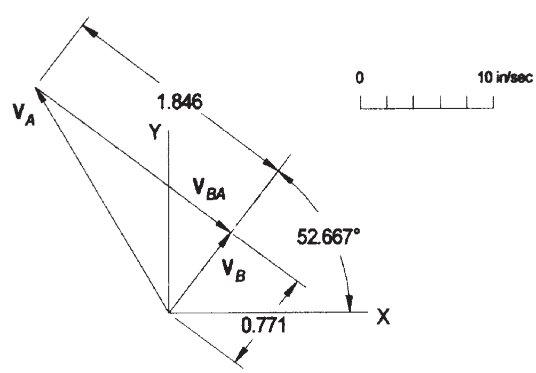
$$\mathbf{V}_{B3} = \mathbf{V}_{A3} + \mathbf{V}_{BA3}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$

$V_{B3} := 0.771 \cdot in \cdot k_v$ $\quad V_{B3} = 7.710 \dfrac{in}{sec}$ $\quad \theta_{VB3} := 52.667 \cdot deg$

$V_{BA3} := 1.846 \cdot in \cdot k_v$ $\quad V_{BA3} = 18.460 \dfrac{in}{sec}$

5. Determine the angular velocity of link 3 using equation 6.7.

From the linkage layout above: $b := 1.793 \cdot in$ and $\theta_4 := 142.666 \cdot deg$

$\omega_3 := \dfrac{V_{BA3}}{b}$ $\quad \omega_3 = 10.296 \dfrac{rad}{sec}$ CW

The way in which link 3 slides in link 4 requires that $\omega_4 := \omega_3$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{B4}$ using equation 6.7.

$V_{B4} := |c \cdot \omega_4|$ $\quad V_{B4} = 41.182 \dfrac{in}{sec}$

$\theta_{VB4} := \theta_4 - 90 \cdot deg$ $\quad \theta_{VB4} = 52.666\, deg$

7. Note that V_{B3} and V_{B4} are in the same direction in this case. The velocity of slip is

$V_{slip} := |V_{B3} - V_{B4}|$ $\quad V_{slip} = 33.472 \dfrac{in}{sec}$

$\theta_{slip} := \theta_{VB4} + 180 \cdot deg$ $\quad \theta_{slip} = 232.666\, deg$

PROBLEM 6-9a

Statement: The general linkage configuration and terminology for an inverted fourbar slider-crank linkage are shown in Fig P6-3. The link lengths and the values of θ_2 and ω_2 and γ are defined in Table P6-3. For row *a*, using an analytic method, find the velocities of the pin joints *A* and *B* and the velocity of slip at the sliding joint. Draw the linkage to scale and label it before setting up the equations.

Given: Link lengths:

Link 1 $d := 6$ Link 2 $a := 2$

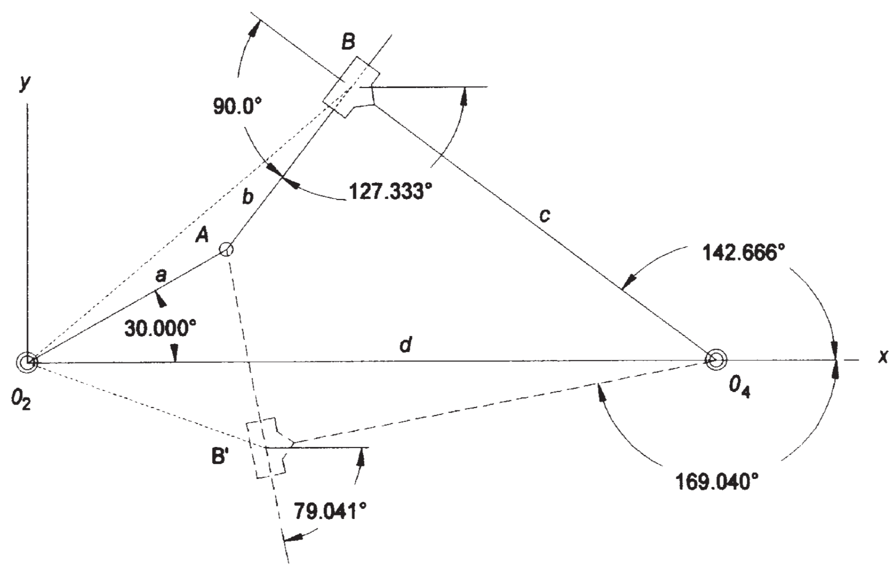
Link 4 $c := 4$ $\gamma := 90 \cdot deg$ $\theta_2 := 30 \cdot deg$ $\omega_2 := 10$

Two argument inverse tangent

$$atan2(x,y) := \begin{vmatrix} return\ 0.5\cdot\pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{vmatrix}$$

Solution: See Mathcad file P0609a.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$P := a \cdot sin(\theta_2) \cdot sin(\gamma) + (a \cdot cos(\theta_2) - d) \cdot cos(\gamma)$ $P = 1.000$

$Q := -a \cdot sin(\theta_2) \cdot cos(\gamma) + (a \cdot cos(\theta_2) - d) \cdot sin(\gamma)$ $Q = -4.268$

$R := -c \cdot sin(\gamma)$ $R = -4.000$ $T := 2 \cdot P$ $T = 2.000$

$S := R - Q$ $S = 0.268$ $U := Q + R$ $U = -8.268$

3. Use equation 4.22c to find values of θ_4 for the open and crossed circuits.

OPEN $\theta_{41} := 2 \cdot atan2\left(2 \cdot S, -T + \sqrt{T^2 - 4 \cdot S \cdot U}\right)$ $\theta_{41} = 142.667\ deg$

CROSSED $\theta_{42} := 2 \cdot atan2\left(2 \cdot S, -T - \sqrt{T^2 - 4 \cdot S \cdot U}\right)$ $\theta_{42} = -169.041\,deg$

4. Use equation 4.18 to find values of θ_3 for the open and crossed circuits.

OPEN $\theta_{31} := \theta_{41} + \gamma$ $\theta_{31} = 232.667\,deg$

CROSSED $\theta_{32} := \theta_{42} - \gamma$ $\theta_{32} = -259.041\,deg$

5. Determine the magnitude of the instantaneous "length" of link 3 from equation 4.20a.

OPEN $$b_1 := \frac{a \cdot sin(\theta_2) - c \cdot sin(\theta_{41})}{sin(\theta_{41} + \gamma)}$$ $b_1 = 1.793$

CROSSED $$b_2 := \left|\frac{a \cdot sin(\theta_2) - c \cdot sin(\theta_{42})}{sin(\theta_{42} + \gamma)}\right|$$ $b_2 = 1.793$

6. Determine the angular velocity of link 4 using equation 6.30c:

OPEN $$\omega_{41} := \frac{a \cdot \omega_2 \cdot cos(\theta_2 - \theta_{31})}{b_1 + c \cdot cos(\gamma)}$$ $\omega_{41} = -10.292$

CROSSED $$\omega_{42} := \frac{a \cdot \omega_2 \cdot cos(\theta_2 - \theta_{32})}{b_2 + c \cdot cos(\gamma)}$$ $\omega_{42} = 3.639$

7. Determine the velocity of pin *A* using equation 6.23a:

$V_A := a \cdot \omega_2 \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right)$

$V_A = -10.000 + 17.321i$ $|V_A| = 20.000$ $arg(V_A) = 120.000\,deg$

8. Determine the velocity of point *B* on link 4 using equation 6.31:

OPEN

$V_{B4x1} := -c \cdot \omega_{41} \cdot sin(\theta_{41})$ $V_{B4x1} = 24.966$

$V_{B4y1} := c \cdot \omega_{41} \cdot cos(\theta_{41})$ $V_{B4y1} = 32.734$

$V_{B41} := \sqrt{V_{B4x1}^2 + V_{B4y1}^2}$ $V_{B41} = 41.168$

$\theta_{VB1} := atan2(V_{B4x1}, V_{B4y1})$ $\theta_{VB1} = 52.667\,deg$

CROSSED

$V_{B4x2} := -c \cdot \omega_{42} \cdot sin(\theta_{42})$ $V_{B4x2} = 2.767$

$V_{B4y2} := c \cdot \omega_{42} \cdot cos(\theta_{42})$ $V_{B4y2} = -14.289$

$V_{B42} := \sqrt{V_{B4x2}^2 + V_{B4y2}^2}$ $V_{B42} = 14.555$

$\theta_{VB2} := atan2(V_{B4x2}, V_{B4y2})$ $\theta_{VB2} = -79.041\,deg$

9. Determine the slip velocity using equation 6.30a:

OPEN

$$V_{slip1} := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_{41} \cdot (b_1 \cdot \sin(\theta_{31}) + c \cdot \sin(\theta_{41}))}{\cos(\theta_{31})}$$

$$V_{slip1} = 33.461$$

CROSSED

$$V_{slip2} := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_{42} \cdot (b_2 \cdot \sin(\theta_{32}) + c \cdot \sin(\theta_{42}))}{\cos(\theta_{32})}$$

$$V_{slip2} = 33.461$$

PROBLEM 6-10a

Statement: The link lengths, gear ratio (λ), phase angle (ϕ), and the values of θ_2 and ω_2 for a geared fivebar from row *a* of Table P6-4 are given below. Draw the linkage to scale and graphically find ω_3 and ω_4, using a graphical method.

Given: Link lengths:

Link 1 $f := 6 \cdot in$ Link 3 $b := 7 \cdot in$ Link 5 $d := 4 \cdot in$

Link 2 $a := 1 \cdot in$ Link 4 $c := 9 \cdot in$

Gear ratio, phase angle, and crank angle:

$\lambda := 2$ $\phi := 30 \cdot deg$ $\theta_2 := 60 \cdot deg$ $\omega_2 := 10 \cdot rad \cdot sec^{-1}$

Solution: See Figure P6-4 and Mathcad file P0610a.

1. Determine the angle of link 5 using the equation in Figure P6-4.

$$\theta_5 := \lambda \cdot \theta_2 + \phi \qquad \theta_5 = 150\,deg$$

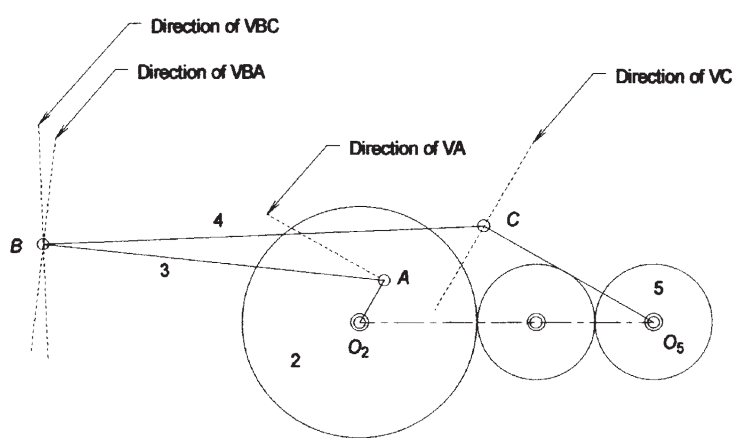
2. Choose the pitch radii of the gears. Since the gear ratio is positive, an idler must be used between gear 2 and gear 5. Let the idler be the same diameter as gear 5, and let all three gears be in line.

$$f := r_2 + 3 \cdot r_5 \qquad \lambda := \frac{r_2}{r_5}$$

$$r_5 := \frac{f}{\lambda + 3} \qquad r_5 = 1.200\,in$$

$$r_2 := \lambda \cdot r_5 \qquad r_2 = 2.400\,in$$

3. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



4. Use equation 6.7 to calculate the magnitude of the velocity at points *A* and *C*.

$$V_A := a \cdot \omega_2 \qquad V_A = 10.00 \frac{in}{sec} \qquad \theta_A := 60 \cdot deg + 90 \cdot deg$$

$$\omega_5 := \lambda \cdot \omega_2 \qquad \omega_5 = 20.000 \frac{rad}{sec}$$

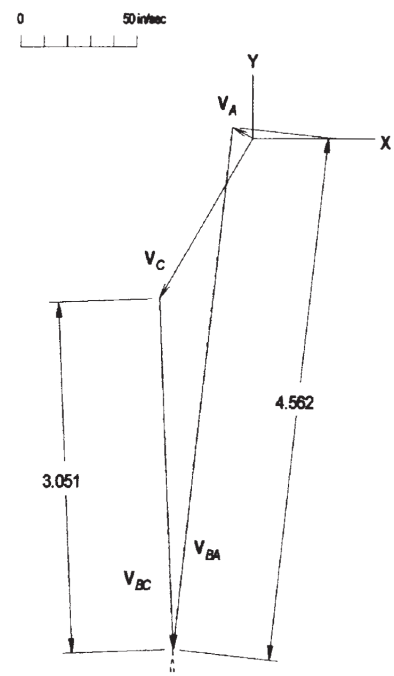
$$V_C := d \cdot \omega_5 \qquad V_C = 80.00 \frac{in}{sec} \qquad \theta_A := 150 \cdot deg + 90 \cdot deg$$

5. Use equation 6.5 to (graphically) determine the magnitudes of the relative velocity vectors $\mathbf{V}_{BA}$ and $\mathbf{V}_{BC}$. The equation to be solved graphically is the last of the following three.

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \qquad \mathbf{V}_B = \mathbf{V}_C + \mathbf{V}_{BC} \qquad \mathbf{V}_A + \mathbf{V}_{BA} = \mathbf{V}_C + \mathbf{V}_{BC}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
c. From the tail of $\mathbf{V}_A$, layout the known vector $\mathbf{V}_C$.
d. From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{BC}$, magnitude unknown.
e. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BC}$ construction line and drawing $\mathbf{V}_{BC}$ from the tip of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

6. From the velocity triangle we have:



Velocity scale factor: $k_v := \dfrac{50 \cdot in \cdot sec^{-1}}{in}$

$V_{BA} := 4.562 \cdot in \cdot k_v \qquad V_{BA} = 228.100 \dfrac{in}{sec}$

$V_{BC} := 3.051 \cdot in \cdot k_v \qquad V_{BC} = 152.550 \dfrac{in}{sec}$

7. Determine the angular velocity of links 3 and 4 using equation 6.7.

$\omega_3 := \dfrac{V_{BA}}{b} \qquad \omega_3 = 32.586 \dfrac{rad}{sec}$

$\omega_3 := \dfrac{V_{BC}}{c} \qquad \omega_3 = 16.950 \dfrac{rad}{sec}$

Both links are rotating CCW.

PROBLEM 6-11a

Statement: The general linkage configuration and terminology for a geared fivebar linkage are shown in Figure P6-4. The link lengths, gear ratio (λ), phase angle (ϕ), and the values of θ_2 and ω_2 are defined in Table P6-4. For row *a*, find ω_3 and ω_4, using an analytic method. Draw the linkage to scale and label it before setting up the equations.

Given: Link lengths:

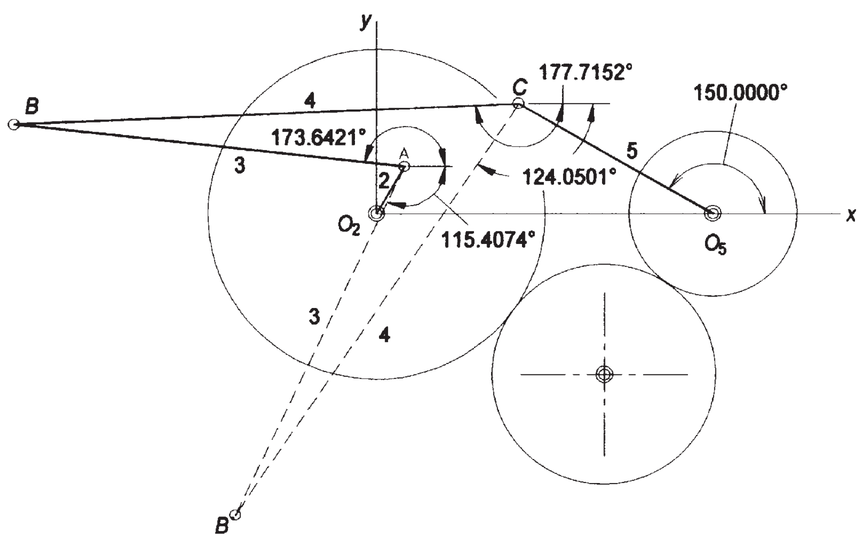
Link 1	$d := 4$	Link 2	$a := 1$	Link 3	$b := 7$
Link 4	$c := 9$	Link 5	$f := 6$	Input angle	$\theta_2 := 60 \cdot deg$
Gear ratio	$\lambda := 2.0$	Phase angle	$\phi := 30 \cdot deg$		$\omega_2 := 10$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{cases}$$

Solution: See Figure P6-4 and Mathcad file P0611a.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_3 and θ_4 from equations 4.24h and 4.24i.

$$A := 2 \cdot c \cdot \left(d \cdot \cos\left(\lambda \cdot \theta_2 + \phi\right) - a \cdot \cos\left(\theta_2\right) + f\right) \qquad A = 36.646$$

$$B := 2 \cdot c \cdot \left(d \cdot \sin\left(\lambda \cdot \theta_2 + \phi\right) - a \cdot \sin\left(\theta_2\right)\right) \qquad B = 20.412$$

$$C := \left(a^2 - b^2 + c^2 + d^2 + f^2\right) - 2 \cdot a \cdot f \cdot \cos\left(\theta_2\right) \ldots$$
$$+ -\left[2 \cdot d \cdot \left(a \cdot \cos\left(\theta_2\right) - f\right) \cdot \cos\left(\lambda \cdot \theta_2 + \phi\right)\right] \ldots$$
$$+ -2 \cdot a \cdot d \cdot \sin\left(\theta_2\right) \cdot \sin\left(\lambda \cdot \theta_2 + \phi\right) \qquad C = 37.431$$

$$D := C - A \qquad D = 0.785$$

$E := 2 \cdot B$ $E = 40.823$

$F := A + C$ $F = 74.077$

$G := 2 \cdot b \cdot \left[-\left(d \cdot \cos\left(\lambda \cdot \theta_2 + \phi \right) \right) + a \cdot \cos\left(\theta_2 \right) - f \right]$ $G = -28.503$

$H := 2 \cdot b \cdot \left[-\left(d \cdot \sin\left(\lambda \cdot \theta_2 + \phi \right) \right) + a \cdot \sin\left(\theta_2 \right) \right]$ $H = -15.876$

$$K := \left(a^2 + b^2 - c^2 + d^2 + f^2 \right) - 2 \cdot a \cdot f \cdot \cos\left(\theta_2 \right) \ldots$$
$$+ -\left[2 \cdot d \cdot \left(a \cdot \cos\left(\theta_2 \right) - f \right) \cdot \cos\left(\lambda \cdot \theta_2 + \phi \right) \right] \ldots$$
$$+ -2 \cdot a \cdot d \cdot \sin\left(\theta_2 \right) \cdot \sin\left(\lambda \cdot \theta_2 + \phi \right)$$
$K = -26.569$

$L := K - G$ $L = 1.933$

$M := 2 \cdot H$ $M = -31.751$

$N := G + K$ $N = -55.072$

3. Use equations 4.24h and 4.24i to find values of θ_3 and θ_4 for the open and crossed circuits.

OPEN $\theta_{31} := 2 \cdot \left(\mathrm{atan2}\left(2 \cdot L, -M + \sqrt{M^2 - 4 \cdot L \cdot N} \right) \right)$ $\theta_{31} = 173.642\, deg$

$\theta_{41} := 2 \cdot \left(\mathrm{atan2}\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$ $\theta_{41} = -177.715\, deg$

CROSSED $\theta_{32} := 2 \cdot \left(\mathrm{atan2}\left(2 \cdot L, -M - \sqrt{M^2 - 4 \cdot L \cdot N} \right) \right)$ $\theta_{32} = -115.407\, deg$

$\theta_{42} := 2 \cdot \left(\mathrm{atan2}\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$ $\theta_{42} = -124.050\, deg$

4. Determine the position and angular velocity of gear 5 from equations 4.23c and 6.32c

$\theta_5 := \lambda \cdot \theta_2 + \phi$ $\theta_5 = 150.000\, deg$

$\omega_5 := \lambda \cdot \omega_2$ $\omega_5 = 20.000$ CCW

Angular velocity of links 3 and 4 from equations 6.33

OPEN
$$\omega_{31} := -\frac{2 \cdot \sin\left(\theta_{41} \right) \cdot \left(a \cdot \omega_2 \cdot \sin\left(\theta_2 - \theta_{41} \right) + d \cdot \omega_5 \cdot \sin\left(\theta_{41} - \theta_5 \right) \right)}{b \cdot \left(\cos\left(\theta_{31} - 2 \cdot \theta_{41} \right) - \cos\left(\theta_{31} \right) \right)}$$

$\omega_{31} = 32.585$ *CCW*

$$\omega_{41} := \frac{a \cdot \omega_2 \cdot \sin\left(\theta_2 \right) + b \cdot \omega_{31} \cdot \sin\left(\theta_{31} \right) - d \cdot \omega_5 \cdot \sin\left(\theta_5 \right)}{c \cdot \sin\left(\theta_{41} \right)}$$

$\omega_{41} = 16.948$ *CCW*

CROSSED
$$\omega_{32} := -\frac{2 \cdot \sin\left(\theta_{42} \right) \cdot \left(a \cdot \omega_2 \cdot \sin\left(\theta_2 - \theta_{42} \right) + d \cdot \omega_5 \cdot \sin\left(\theta_{42} - \theta_5 \right) \right)}{b \cdot \left(\cos\left(\theta_{32} - 2 \cdot \theta_{42} \right) - \cos\left(\theta_{32} \right) \right)}$$

$\omega_{32} = -75.191$ *CW*

$$\omega_{42} := \frac{a \cdot \omega_2 \cdot \sin\left(\theta_2 \right) + b \cdot \omega_{32} \cdot \sin\left(\theta_{32} \right) - d \cdot \omega_5 \cdot \sin\left(\theta_5 \right)}{c \cdot \sin\left(\theta_{42} \right)}$$

$\omega_{42} = -59.554$ *CW*

PROBLEM 6-12

Statement: Find all of the instant centers of the linkages shown in Figure P6-5.

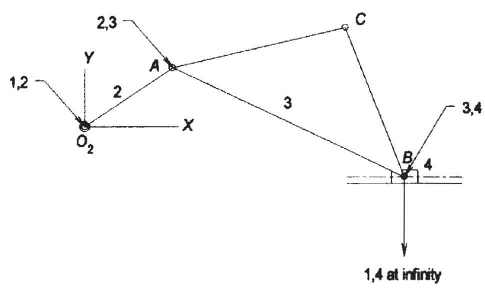
Solution: See Figure P6-5 and Mathcad file P0612.

a. This is a fourbar slider-crank with $n := 4$

1. Determine the number of instant centers for this mechanism using equation 6.8a.

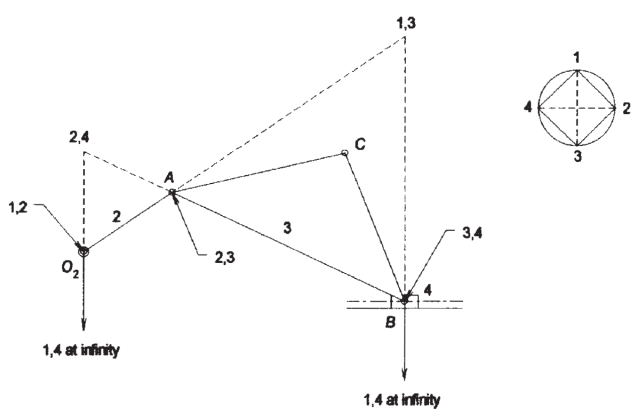
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

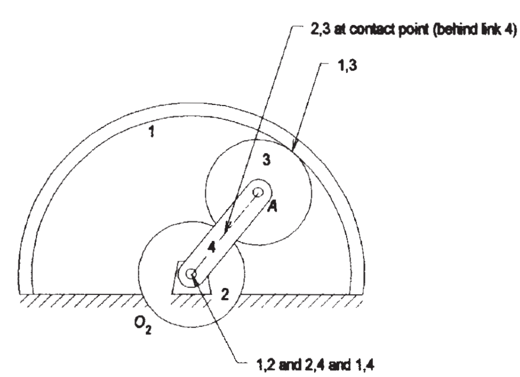


b. This is a fourbar with planetary motion (roller 3), $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection, which in this case, is all of them.

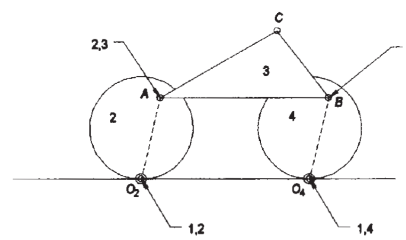


c. This is a fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

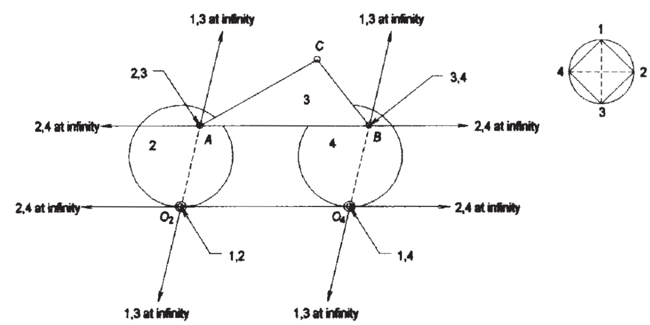
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

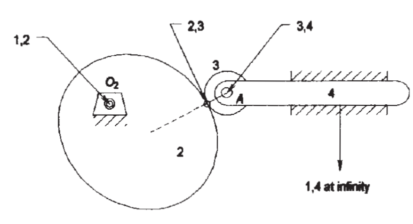


d. This is a fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

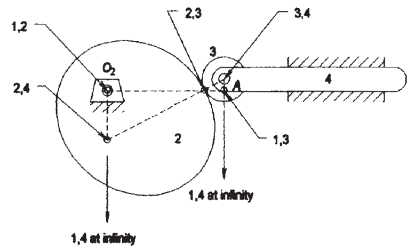
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

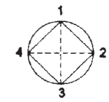
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



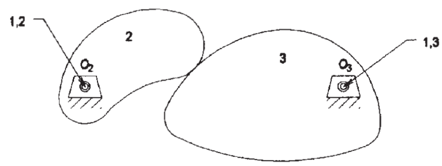


e. This is a threebar with $n := 3$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

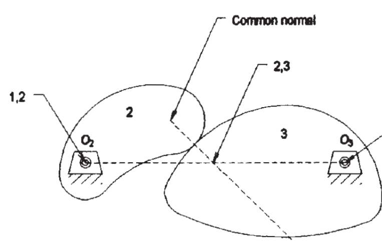
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 3$$

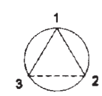
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining IC, $I_{2,3}$

$I_{2,3}$: $I_{1,2}$-$I_{1,3}$



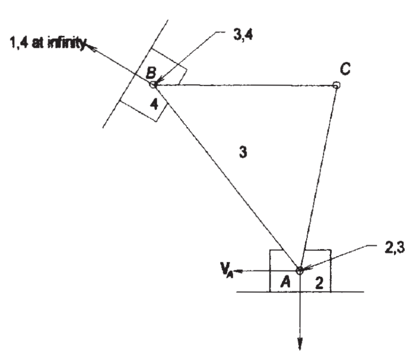


f. This is a fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

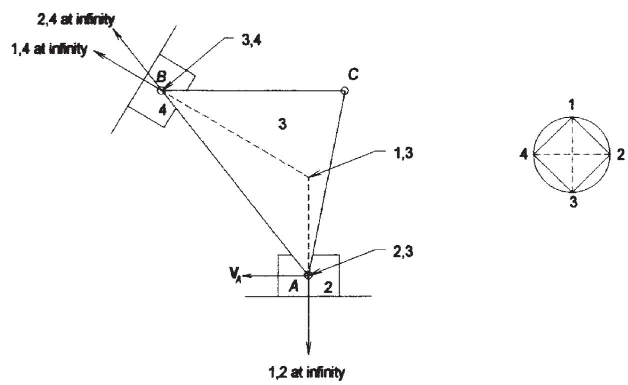
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



PROBLEM 6-13

Statement: Find all of the instant centers of the linkages shown in Figure P6-6.

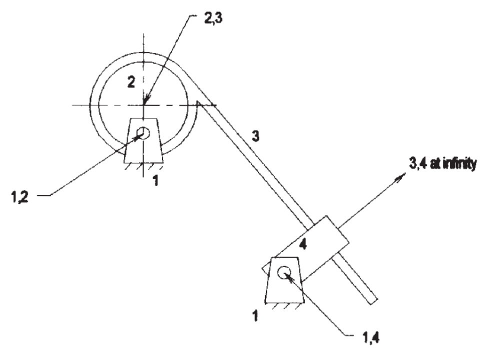
Solution: See Figure P6-6 and Mathcad file P0613.

a. This is a fourbar inverted slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

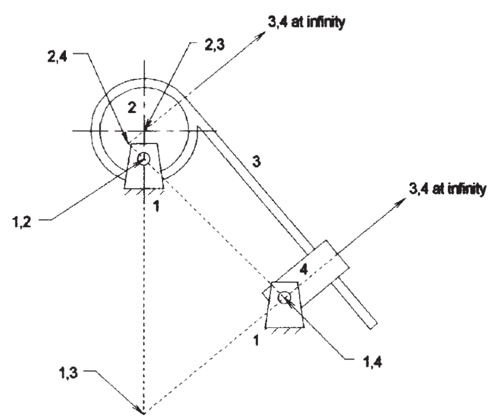
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

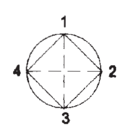
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



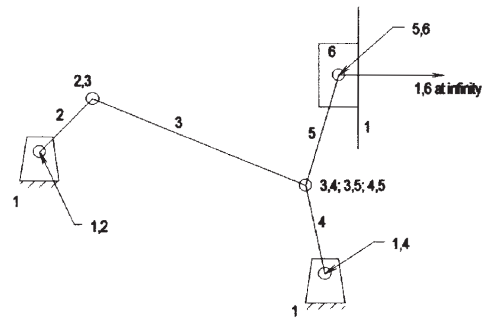


b. This is a sixbar with slider, $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 7 ICs.

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,4}$-$I_{4,5}$

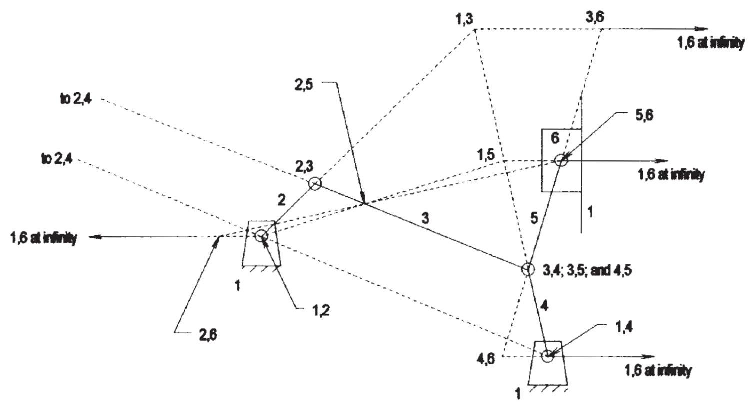
$I_{2,5}$: $I_{1,2}$-$I_{1,5}$ and $I_{2,4}$-$I_{4,5}$

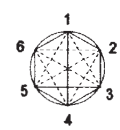
$I_{3,6}$: $I_{1,6}$-$I_{1,3}$ and $I_{3,4}$-$I_{4,6}$

$I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

$I_{4,6}$: $I_{1,6}$-$I_{1,4}$ and $I_{4,5}$-$I_{5,6}$

$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$



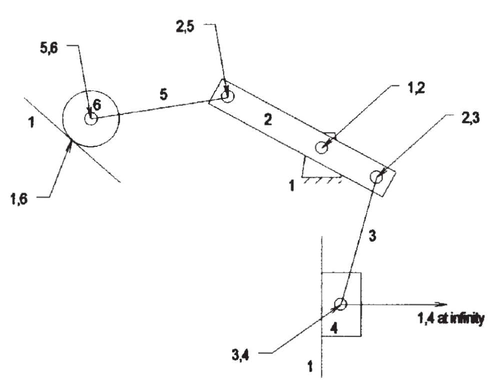


c. This is a sixbar with slider and roller with $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



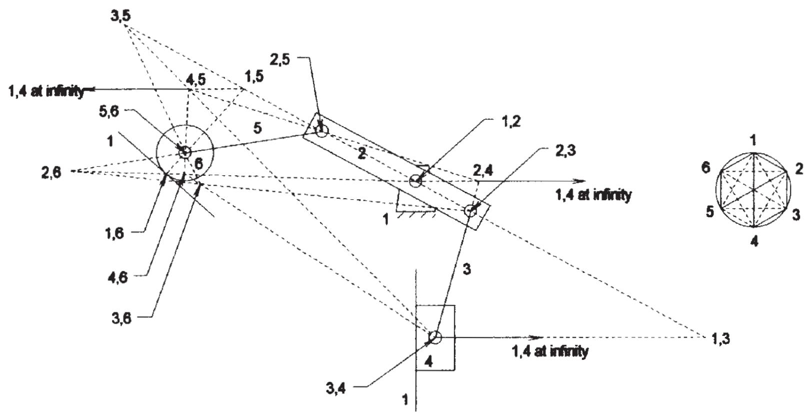
3. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs.

$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$ $\quad$ $I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,2}$-$I_{2,5}$ $\quad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,5}$-$I_{2,4}$

$I_{4,5}$: $I_{1,4}$-$I_{1,5}$ and $I_{2,4}$-$I_{2,5}$ $\quad$ $I_{3,5}$: $I_{3,4}$-$I_{4,5}$ and $I_{2,5}$-$I_{2,3}$

$I_{3,6}$: $I_{3,6}$-$I_{5,6}$ and $I_{2,3}$-$I_{2,6}$ $\quad$ $I_{4,6}$: $I_{4,5}$-$I_{5,6}$ and $I_{3,4}$-$I_{3,6}$

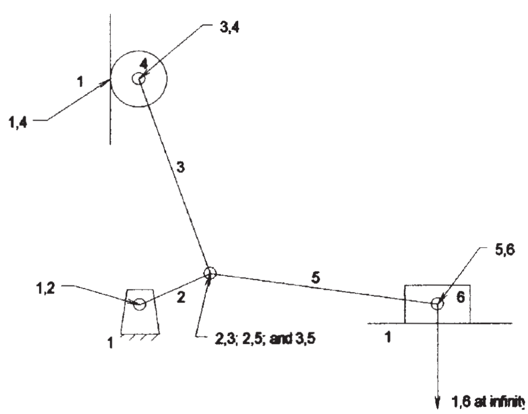


d. This is a sixbar with slider and roller with $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 7 ICs.

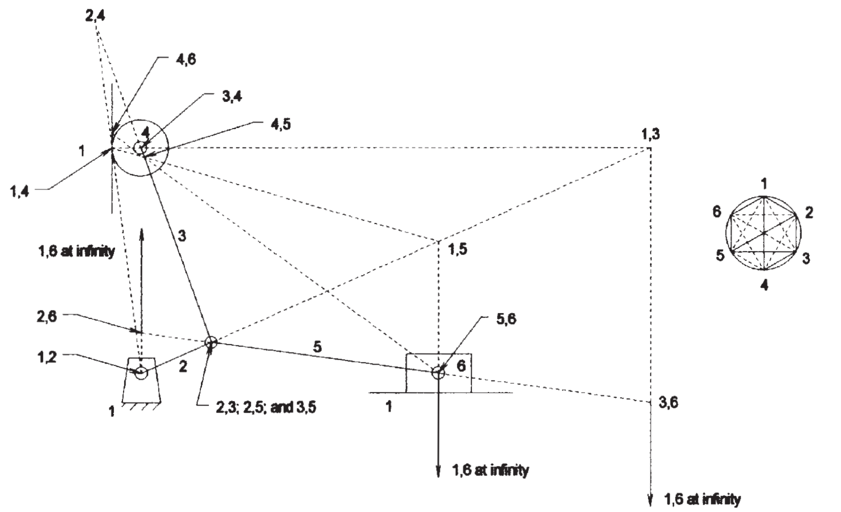
$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\quad$ $I_{3,6}$: $I_{1,6}$-$I_{1,3}$ and $I_{3,5}$-$I_{5,6}$

$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$ $\quad$ $I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,2}$-$I_{2,5}$

14_{5}: $I_{1,4}$-$I_{1,5}$ and $I_{3,5}$-$I_{3,4}$ $\quad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

$I_{4,6}$: $I_{1,6}$-$I_{1,4}$ and $I_{4,5}$-$I_{5,6}$

(See next page)


2,4
4,6
3,4
4,5
1,3
1
4
1,4
3
1,6 at infinity
1,5
2,6
5,6
5
1,2
2
6
1
2,3; 2,5; and 3,5
1
3,6
1,6 at infinity
1,6 at infinity
1
6
2
5
3
4

PROBLEM 6-14

Statement: Find all of the instant centers of the linkages shown in Figure P6-7.

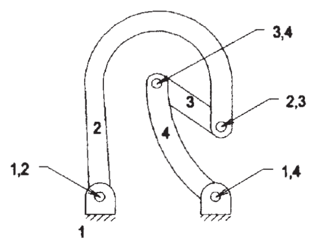
Solution: See Figure P6-7 and Mathcad file P0614.

a. This is a pin-jointed fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

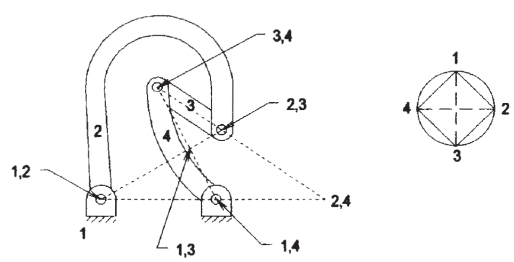
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

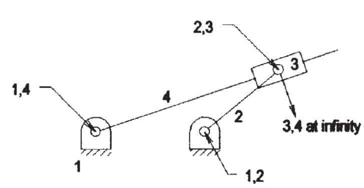


b. This is a fourbar inverted slider-crank, $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

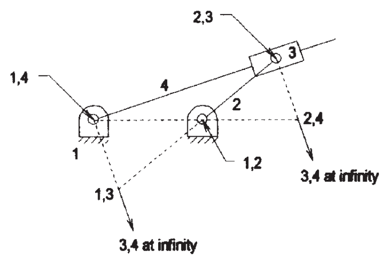
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

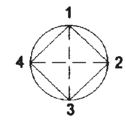
2. Draw the linkage to scale and identify those ICs that can be found by inspection, which in this case, is all of them.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



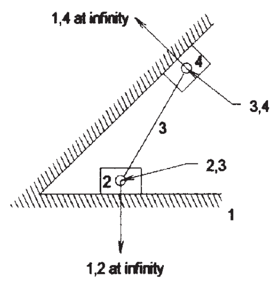


c. This is a fourbar double slider with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

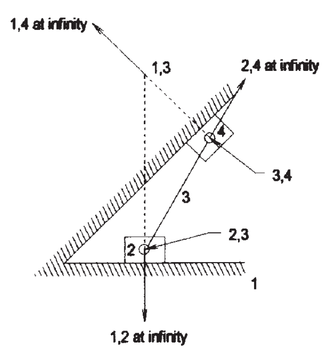
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

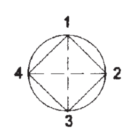
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



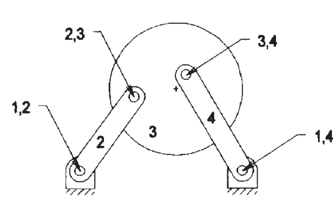


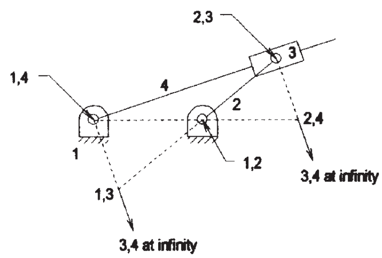
d. This is a pin-jointed fourbar with $n := 4$.

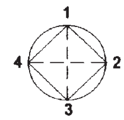
1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.





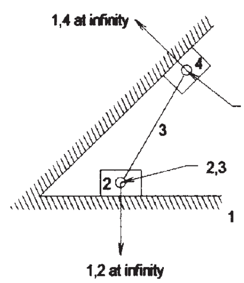


c. This is a fourbar double slider with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

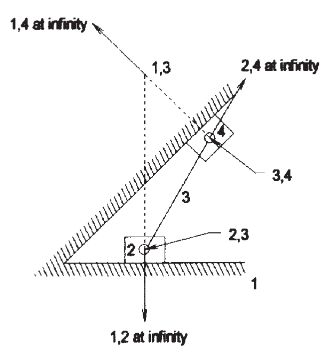
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

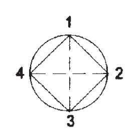
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



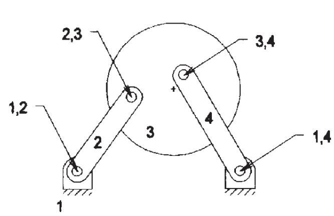


d. This is a pin-jointed fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

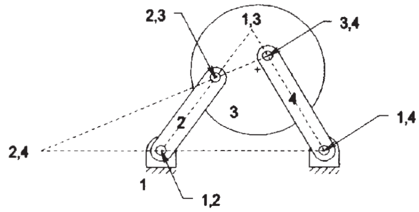
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

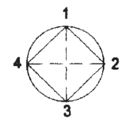
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



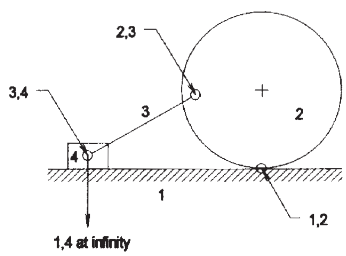


e. This is a fourbar effective slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

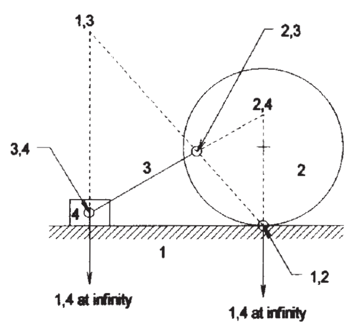
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

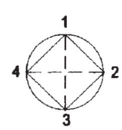
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$





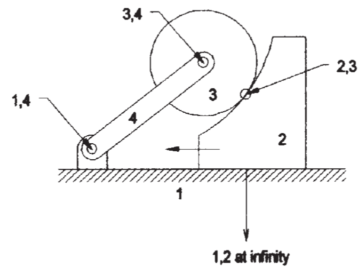
f. This is a fourbar cam-follower with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

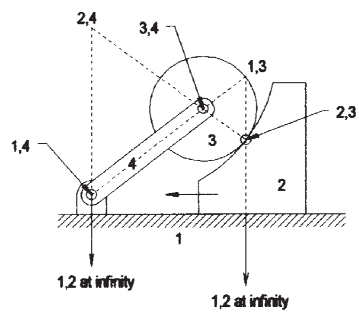
2. Draw the linkage to scale and identify those ICs that can be found by inspection.

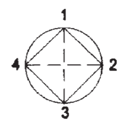
(See next page.)



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



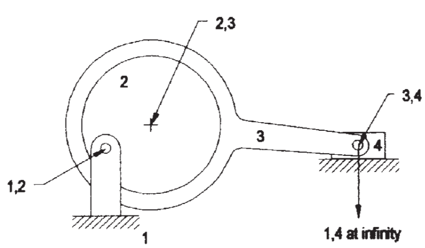


g. This is a fourbar slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

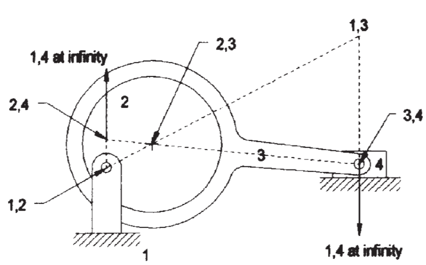
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

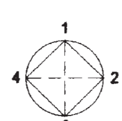
2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



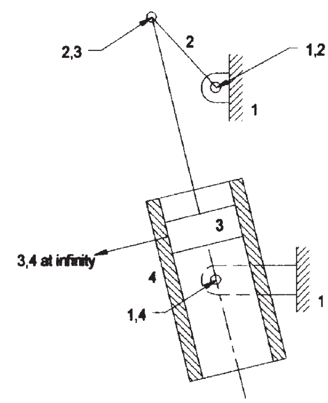


h. This is a fourbar inverted slider-crank with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

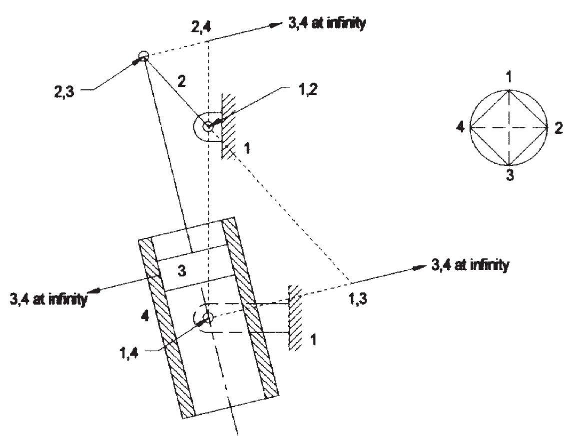
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

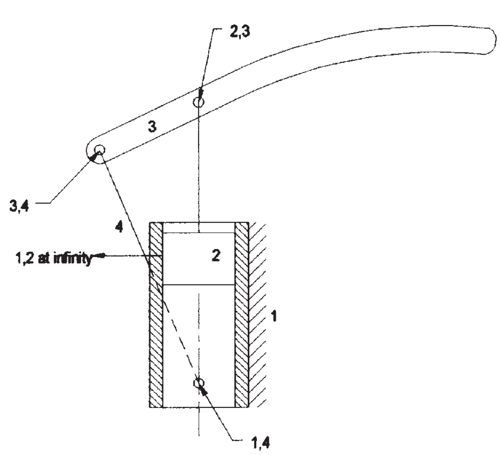


i. This is a fourbar slider-crank (hand pump) with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

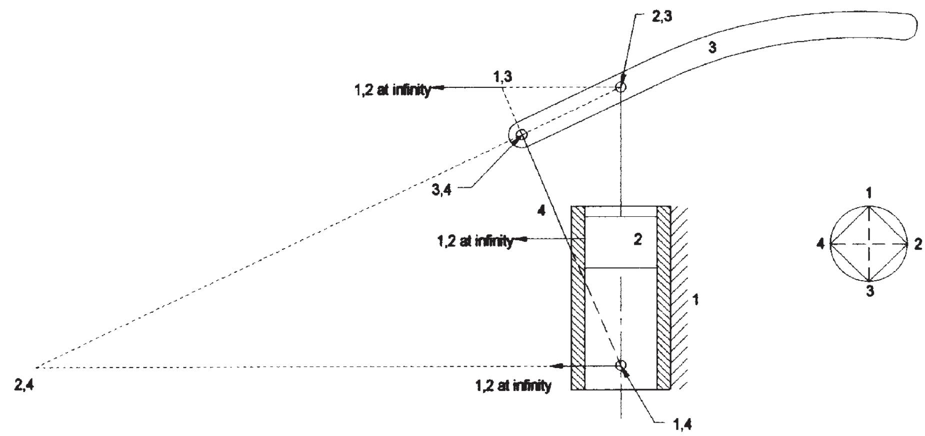
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



PROBLEM 6-15

Statement: Find all of the instant centers of the linkages shown in Figure P6-8.

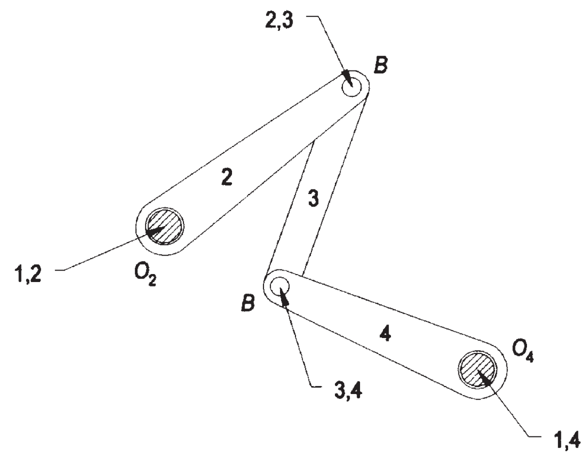
Solution: See Figure P6-8 and Mathcad file P0615.

a. This is a pin-jointed fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

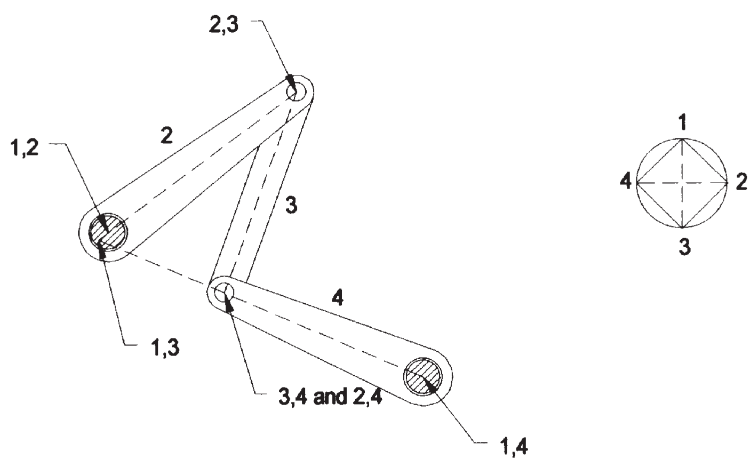
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

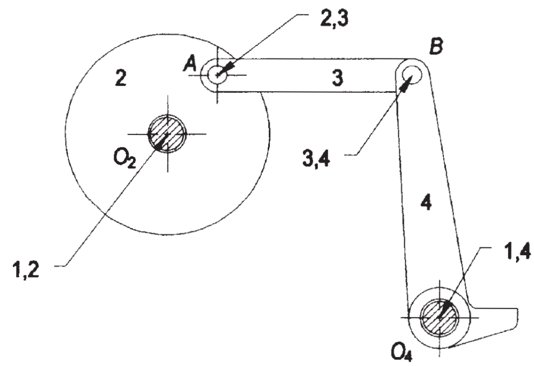


b. This is a pin-jointed fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

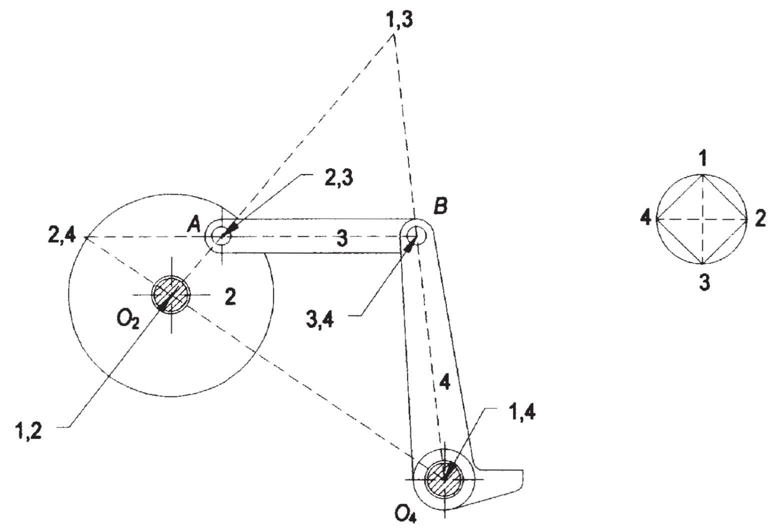
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection, which in this case, is all of them.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

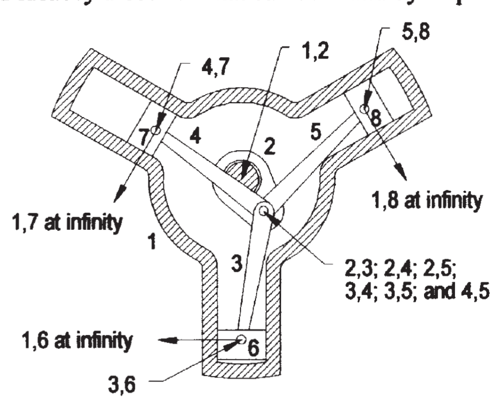


c. This is an eightbar (three slider-cranks with a common crank) with $n := 8$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n\cdot(n-1)}{2} \qquad C = 28$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 15 ICs.

$I_{1,5}$: $I_{1,2}$-$I_{2,5}$ and $I_{1,8}$-$I_{5,8}$ $\qquad$ $I_{2,8}$: $I_{1,2}$-$I_{1,8}$ and $I_{2,5}$-$I_{5,8}$

$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,3}$-$I_{3,6}$

$I_{2,7}$: $I_{1,2}$-$I_{1,7}$ and $I_{2,4}$-$I_{4,7}$

$I_{3,7}$: $I_{1,3}$-$I_{1,7}$ and $I_{3,4}$-$I_{4,7}$

$I_{4,6}$: $I_{1,6}$-$I_{1,4}$ and $I_{3,4}$-$I_{3,6}$

$I_{5,6}$: $I_{2,5}$-$I_{2,6}$ and $I_{3,5}$-$I_{3,6}$

$I_{6,7}$: $I_{2,6}$-$I_{2,7}$ and $I_{3,6}$-$I_{3,7}$

$I_{6,8}$: $I_{2,7}$-$I_{2,8}$ and $I_{3,7}$-$I_{3,8}$

$I_{1,4}$: $I_{1,7}$-$I_{4,7}$ and $I_{1,2}$-$I_{2,4}$

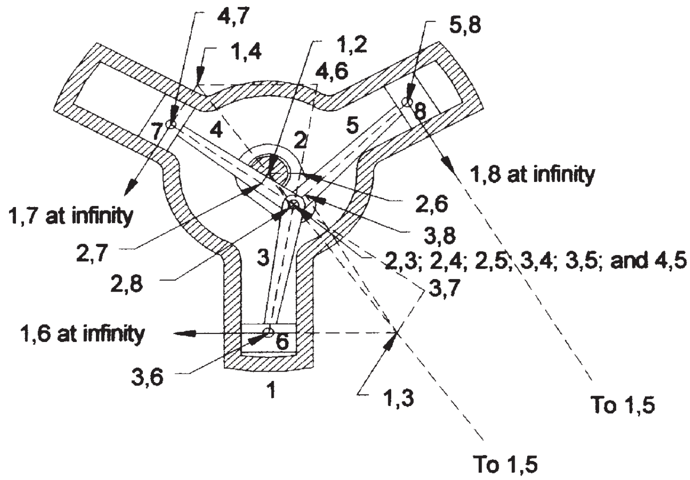
$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,6}$-$I_{3,6}$

$I_{3,8}$: $I_{1,3}$-$I_{3,8}$ and $I_{3,5}$-$I_{5,8}$

$I_{4,8}$: $I_{3,8}$-$I_{3,4}$ and $I_{4,5}$-$I_{5,8}$

$I_{5,7}$: $I_{2,5}$-$I_{2,7}$ and $I_{3,5}$-$I_{3,7}$

$I_{6,8}$: $I_{4,6}$-$I_{4,8}$ and $I_{3,6}$-$I_{3,8}$



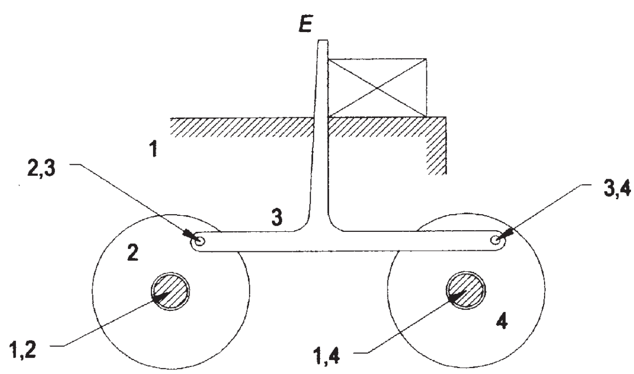
Note that, for clarity, not all ICs are shown.

d. This is a pin-jointed fourbar with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

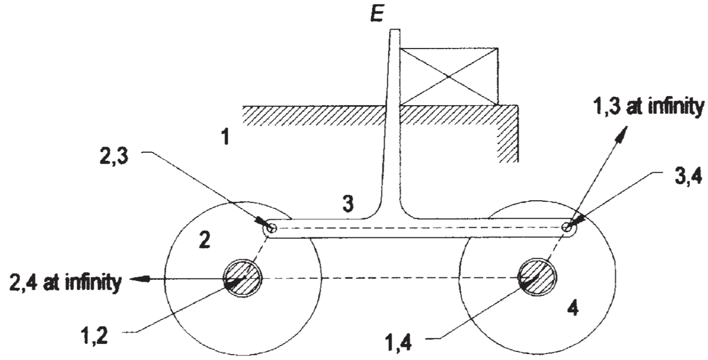
2. Draw the linkage to scale and identify those ICs that can be found by inspection.

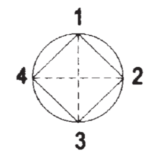


3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$

$I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



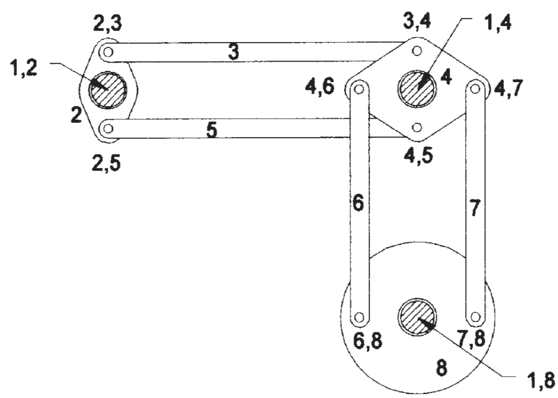


e. This is an eightbar with $n := 8$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 28$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



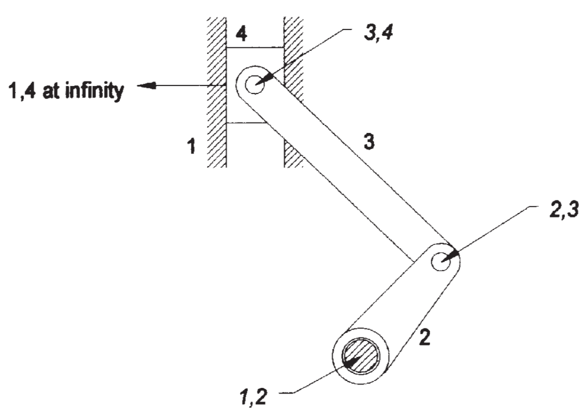
3. The remaining 17 ICs are at infinity.

f. This is an offset crank-slider with $n := 4$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

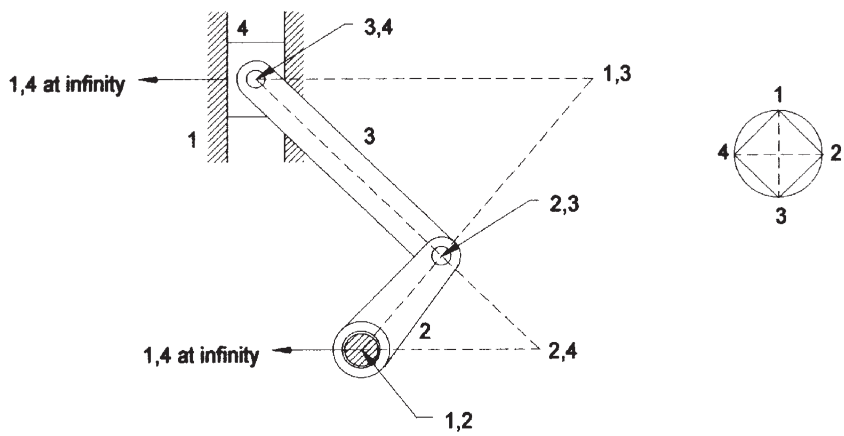
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



3. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs, $I_{1,3}$ and $I_{2,4}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

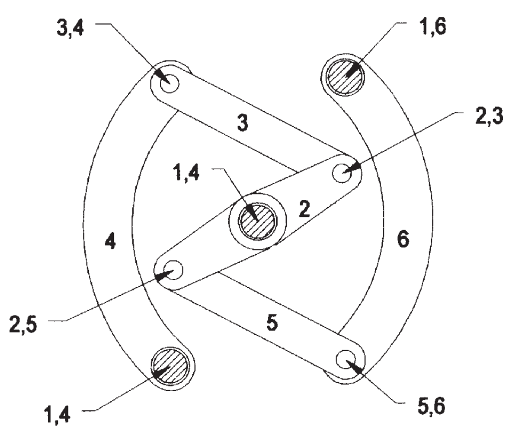


g. *This is a sixbar with* $n := 6$.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection.



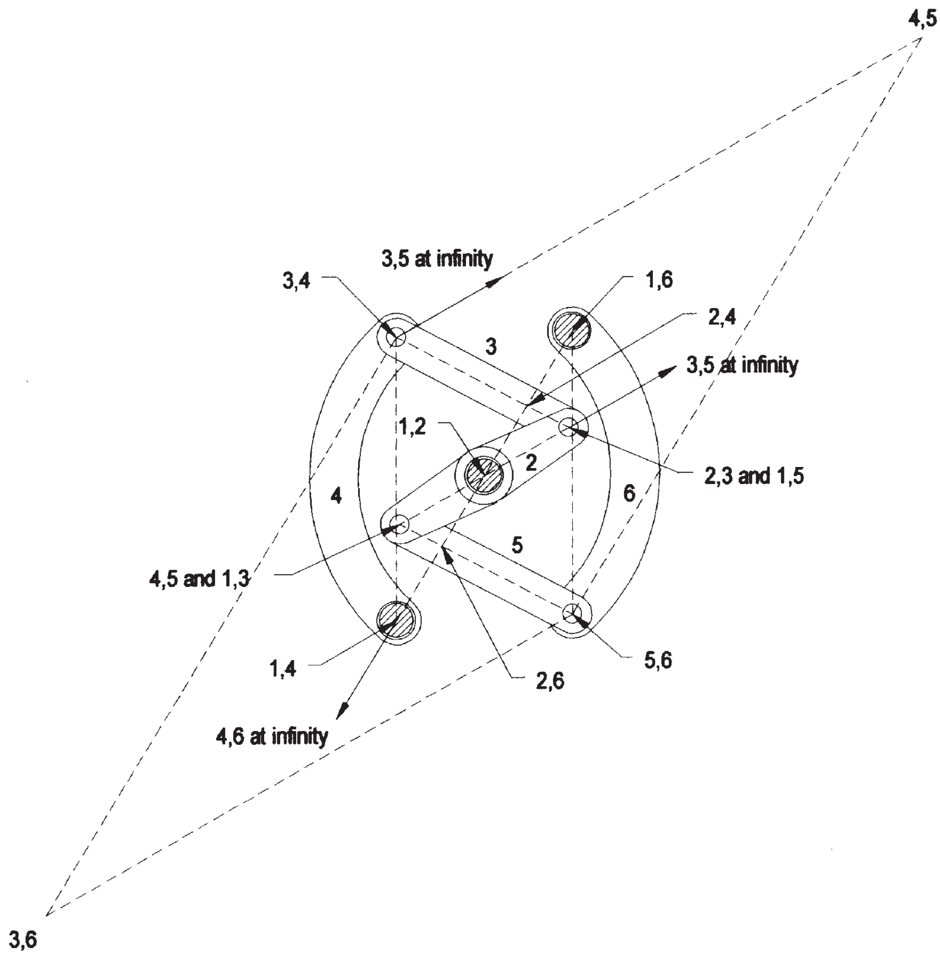
3. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs.

$I_{1,3}$: $I_{1,4}$-$I_{4,3}$ and $I_{1,2}$-$I_{3,2}$ $I_{2,6}$: $I_{1,6}$-$I_{1,2}$ and $I_{2,5}$-$I_{5,6}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,2}$-$I_{2,5}$ $I_{4,6}$: $I_{1,4}$-$I_{1,6}$ and $I_{2,4}$-$I_{2,6}$

$I_{3,5}$: $I_{1,3}$-$I_{1,5}$ and $I_{3,2}$-$I_{2,5}$ $I_{4,5}$: $I_{4,6}$-$I_{5,6}$ and $I_{3,4}$-$I_{3,5}$

$I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{3,4}$-$I_{2,3}$ $I_{3,6}$: $I_{3,4}$-$I_{4,6}$ and $I_{3,5}$-$I_{5,6}$


4,5
3,5 at infinity
3,4
1,6
2,4
3
3,5 at infinity
1,2
2
2,3 and 1,5
4
6
5
4,5 and 1,3
1,4
5,6
2,6
4,6 at infinity
3,6

PROBLEM 6-16a

Statement: The linkage in Figure P6-5a has the dimensions and crank angle given below. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15$ rad/sec in the direction shown. Use the velocity difference graphical method.

Given:

Link lengths:

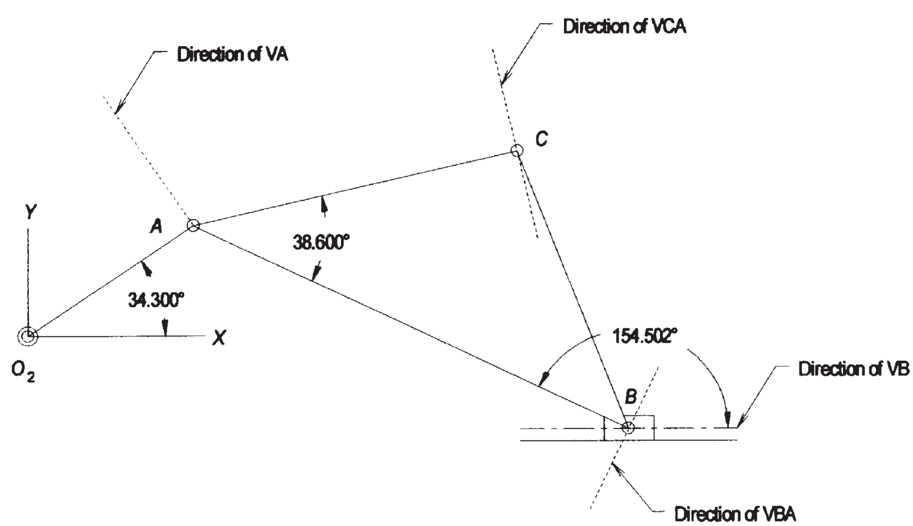
Link 2 (O_2 to A)	$a := 0.80 \cdot in$
Link 3 (A to B)	$b := 1.93 \cdot in$
Offset	$c := -0.38 \cdot in$

Coupler point data:

Distance from A to C	$p := 1.33 \cdot in$
Angle BAC	$\delta := 38.6 \cdot deg$
Crank angle:	$\theta_2 := 34.3 \cdot deg$
Input crank angular velocity	$\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure P6-5a and Mathcad file P0616a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 12.00 \frac{in}{sec} \qquad \theta_A := 34.3 \cdot deg + 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B. The equation to be solved graphically is

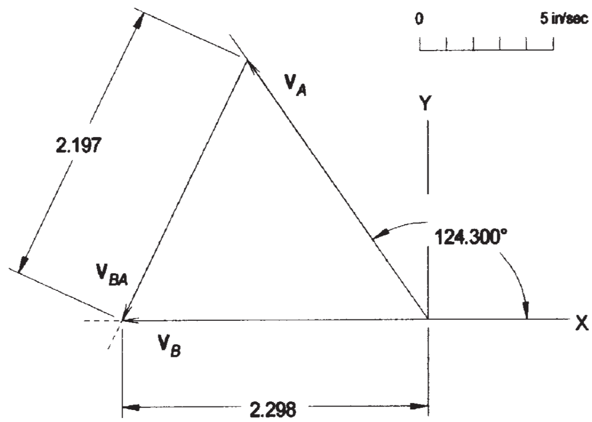
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{5 \cdot in \cdot sec^{-1}}{in}$

$V_B := 2.298 \cdot in \cdot k_v$ $\quad V_B = 11.490 \frac{in}{sec}$ $\quad \theta_B := 180 \cdot deg$

$V_{BA} := 2.197 \cdot in \cdot k_v$ $\quad V_{BA} = 10.985 \frac{in}{sec}$

5. Determine the angular velocity of link 3 using equation 6.7.

$\omega_3 := \frac{-V_{BA}}{b}$ $\quad \omega_3 = -5.692 \frac{rad}{sec}$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{CA}$ using equation 6.7.

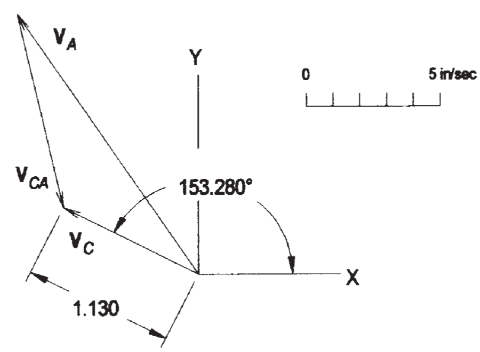
$V_{CA} := |p \cdot \omega_3|$ $\quad V_{CA} = 7.570 \frac{in}{sec}$

$\theta_{CA} := (154.502 - 180 + 38.6 - 90) \cdot deg$ $\quad \theta_{CA} = -76.898\, deg$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{CA}$.
c. Complete the vector triangle by drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{CA}$ vector.



8. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{5 \cdot in \cdot sec^{-1}}{in}$

$V_C := 1.130 \cdot in \cdot k_v$ $V_C = 5.650 \dfrac{in}{sec}$ $\theta_C := 153.28 \cdot deg$

PROBLEM 6-16b

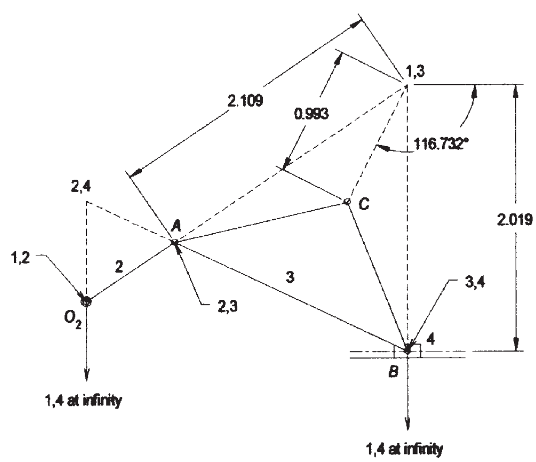
Statement: The linkage in Figure P6-5a has the dimensions and crank angle given below. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for ω_2 = 15 rad/sec in the direction shown. Use the instant center graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 0.80 \cdot in$
Link 3 (A to B)	$b := 1.93 \cdot in$
Offset	$c := -0.38 \cdot in$
Coupler point data: Distance from A to C	$p := 1.33 \cdot in$
Angle BAC	$\delta := 38.6 \cdot deg$
Crank angle:	$\theta_2 := 34.3 \cdot deg$
Input crank angular velocity	$\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure P6-5a and Mathcad file P0616b.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.



From the layout above:

$$AI13 := 2.109 \cdot in \qquad BI13 := 2.019 \cdot in \qquad CI13 := 0.993 \cdot in \qquad \theta_C := (360 - 116.732) \cdot deg$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 12.000 \frac{in}{sec}$$

$$\theta_{VA} := \theta_2 + 90 \cdot deg \qquad \theta_{VA} = 124.3\,deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 5.690 \frac{rad}{sec} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. Determine its direction by inspection.

$V_B := BI13 \cdot \omega_3$ $\qquad V_B = 11.488 \dfrac{in}{sec}$

$\theta_{VC} := 180 \cdot deg$

5. Determine the magnitude of the velocity at point *C* using equation 6.9b. Determine its direction by inspection.

$V_C := CI13 \cdot \omega_3$ $\qquad V_C = 5.650 \dfrac{in}{sec}$

$\theta_{VC} := \theta_C - 90 \cdot deg$ $\qquad \theta_{VC} = 153.268\, deg$

PROBLEM 6-16c

Statement: The linkage in Figure P6-5a has the dimensions and crank angle given below. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15$ rad/sec in the direction shown. Use an analytical method.

Given: Link lengths:

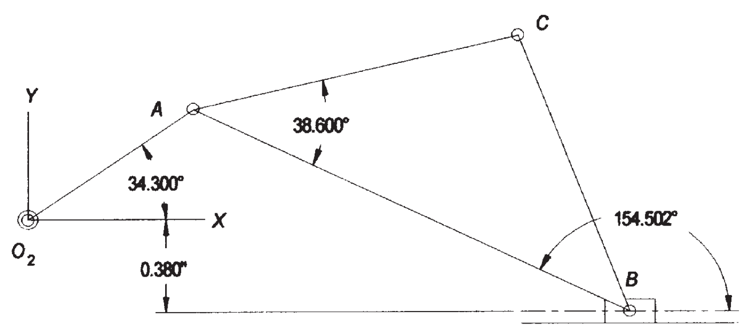
Link 2 (O_2 to A)	$a := 0.80 \cdot in$
Link 3 (A to B)	$b := 1.93 \cdot in$
Offset	$c := -0.38 \cdot in$

Coupler point data:

Distance from A to C	$R_{ca} := 1.33 \cdot in$
Angle BAC	$\delta_3 := 38.6 \cdot deg$
Crank angle:	$\theta_2 := 34.3 \cdot deg$
Input crank angular velocity	$\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure P6-5a and Mathcad file P0616c.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and d using equation 4.17.

$$\theta_{31} := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_{31} = 154.502\, deg$$

$$d_1 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{31}) \qquad d_1 = 2.403\, in$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_{31} := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_{31})} \cdot \omega_2 \qquad \omega_{31} = -5.691 \frac{rad}{sec}$$

4. Determine the velocity of pin A using equation 6.23a:

$$\mathbf{V_A} := a \cdot \omega_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2))$$

$$\mathbf{V_A} = -6.762 + 9.913j \frac{in}{sec} \qquad |\mathbf{V_A}| = 12.000 \frac{in}{sec} \qquad arg(\mathbf{V_A}) = 124.300\, deg$$

5. Determine the velocity of pin B using equation 6.22b:

$$\mathbf{V_B} := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31} \cdot \sin(\theta_{31})$$

$$\mathbf{V_B} = -11.490 \frac{in}{sec} \qquad |\mathbf{V_B}| = 11.490 \frac{in}{sec} \qquad arg(\mathbf{V_B}) = 180.000\ deg$$

6. Determine the velocity of the coupler point C for the open circuit using equations 6.36.

$$\mathbf{V_{CA}} := R_{ca} \cdot \omega_{31} \cdot \left(-\sin(\pi + \theta_{31} + \delta_3) + j \cdot \cos(\pi + \theta_{31} + \delta_3)\right)$$

$$\mathbf{V_{CA}} = 1.716 - 7.371j \frac{in}{sec}$$

$$\mathbf{V_C} := \mathbf{V_A} + \mathbf{V_{CA}}$$

$$\mathbf{V_C} = -5.047 + 2.542j \frac{in}{sec} \qquad |\mathbf{V_C}| = 5.651 \frac{in}{sec} \qquad arg(\mathbf{V_C}) = 153.268\ deg$$

Note that θ_3 is defined at point B for the slider-crank and at point A for the pin-jointed fourbar. Thus, to use equation 6.36a for the slider-crank, 180 deg must be added to the calculated value of θ_3.

PROBLEM 6-17a

Statement: The linkage in Figure P6-5c has the dimensions and effective crank angle given below. Find ω_3, ω_4, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15$ rad/sec in the direction shown. Use the velocity difference graphical method.

Given: Link lengths:

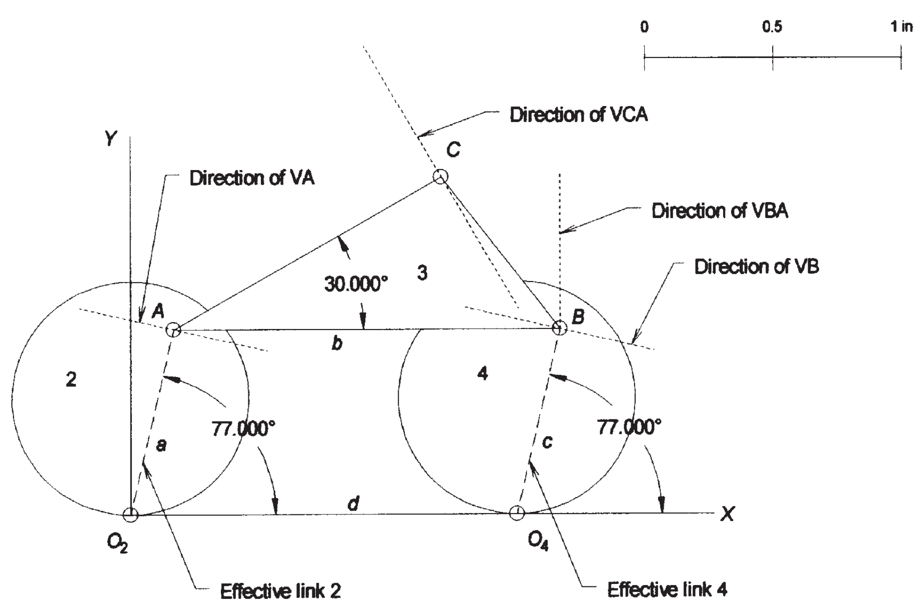
Link 2 (point of contact to *A*)	$a := 0.75 \cdot in$
Link 3 (*A* to *B*)	$b := 1.5 \cdot in$
Link 4 (point of contact to *B*)	$c := 0.75 \cdot in$
Link 1 (between contact points)	$d := 1.5 \cdot in$

Coupler point:

Distance *A* to *C*	$p := 1.2 \cdot in$
Angle *BAC*	$\delta := 30 \cdot deg$
Crank angle:	$\theta_2 := 77 \cdot deg$
Input crank angular velocity	$\omega_2 := 15 \cdot rad \cdot sec^{-1}$

Solution: See Figure P6-5c and Mathcad file P0617a.

1. Although the mechanism shown in Figure P6-5c is not entirely pin-jointed, it can be analyzed for the position shown by its effective pin-jointed fourbar, which is shown below. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A*.

$$V_A := a \cdot \omega_2 \qquad V_A = 11.250 \frac{in}{sec} \qquad \theta_A := 77 \cdot deg + 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

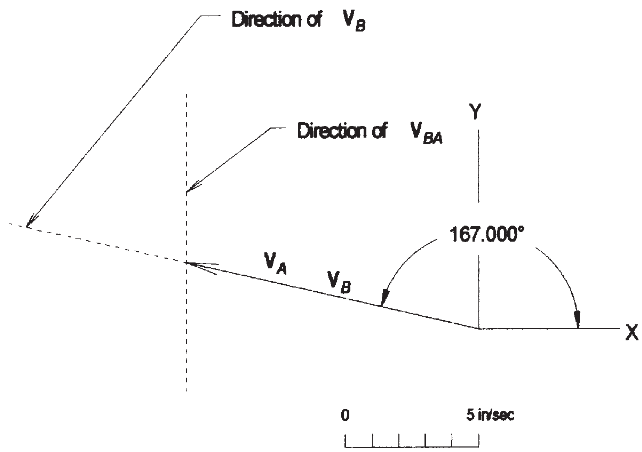
$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

$V_B := V_A \qquad V_B = 11.250 \frac{in}{sec} \qquad \theta_B := 167 \cdot deg$

$V_{BA} := 0 \cdot \frac{in}{sec}$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$\omega_3 := \frac{V_{BA}}{b} \qquad \omega_3 = 0.000 \frac{rad}{sec}$

$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 15.000 \frac{rad}{sec}$

6. Determine the magnitude of the vector $\mathbf{V}_{CA}$ using equation 6.7.

$V_{CA} := p \cdot \omega_3 \qquad V_{CA} = 0.000 \frac{in}{sec}$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$

Normally, we would draw the velocity triangle represented by this equation. However, since $\mathbf{V}_{CA}$ is zero,

$V_C := V_A \qquad V_C = 11.250 \frac{in}{sec}$

$\theta_C := \theta_A \qquad \theta_C = 167.000\, deg$

PROBLEM 6-17b

Statement: The linkage in Figure P6-5c has the dimensions and effective crank angle given below. Find ω_3, ω_4, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for ω_2 = 15 rad/sec in the direction shown. Use the instant center graphical method.

Given: Link lengths:

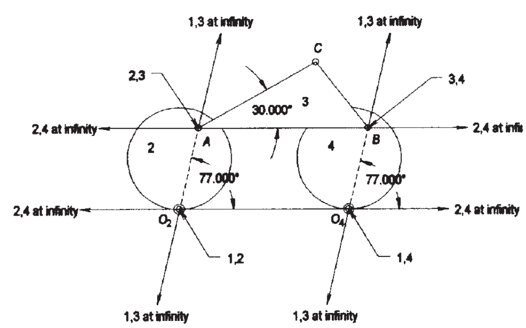
Link 2 (point of contact to A)	$a := 0.75 \cdot in$
Link 3 (A to B)	$b := 1.5 \cdot in$
Link 4 (point of contact to B)	$c := 0.75 \cdot in$
Link 1 (between contact points)	$d := 1.5 \cdot in$

Coupler point:

Distance A to C	$p := 1.2 \cdot in$
Angle BAC	$\delta := 30 \cdot deg$
Crank angle:	$\theta_2 := 77 \cdot deg$
Input crank angular velocity	$\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure P6-5c and Mathcad file P0617b.

1. Although the mechanism shown in Figure P6-5c is not entirely pin-jointed, it can be analyzed for the position shown by its effective pin-jointed fourbar, which is shown below. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.



Since $I_{1,3}$ is at infinity, link 3 is not rotating ($\omega_3 := 0 \cdot rad \cdot sec^{-1}$). Thus, the velocity of every point on link 3 is the same.

From the layout above:

$\theta_4 := 77.000 \cdot deg$ $\theta_3 := 0.000 \cdot deg$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$V_A := a \cdot \omega_2$ $V_A = 11.250 \frac{in}{sec}$

$\theta_{VA} := \theta_2 + 90 \cdot deg$ $\theta_{VA} = 167.0\, deg$

3. Determine the magnitude of the velocity at point B knowing that it is the same as that of point A.

$V_B := V_A$ $\qquad V_B = 11.250 \dfrac{in}{sec}$

$\theta_{VB} := \theta_4 + 90 \cdot deg$ $\qquad \theta_{VB} = 167.000\, deg$

4. Use equation 6.9c to determine the angular velocity of link 4.

$\omega_4 := \dfrac{V_B}{c}$ $\qquad \omega_4 = 15 \dfrac{rad}{sec}$ CCW

5. Determine the magnitude of the velocity at point C knowing that it is the same as that of point A.

$V_C := V_A$ $\qquad V_C = 11.250 \dfrac{in}{sec}$

$\theta_{VC} := \theta_{VA}$ $\qquad \theta_{VC} = 167.000\, deg$

PROBLEM 6-17c

Statement: The linkage in Figure P6-5c has the dimensions and effective crank angle given below. Find ω_3, ω_4, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for ω_2 = 15 rad/sec in the direction shown. Use an analytical method.

Given: Link lengths:

Link 2 (point of contact to *A*) $a := 0.75 \cdot in$

Link 3 (*A* to *B*) $b := 1.5 \cdot in$

Link 4 (point of contact to *B*) $c := 0.75 \cdot in$

Link 1 (between contact points) $d := 1.5 \cdot in$

Coupler point:

Distance *A* to *C* $R_{ca} := 1.2 \cdot in$ Crank angle: $\theta_2 := 77 \cdot deg$

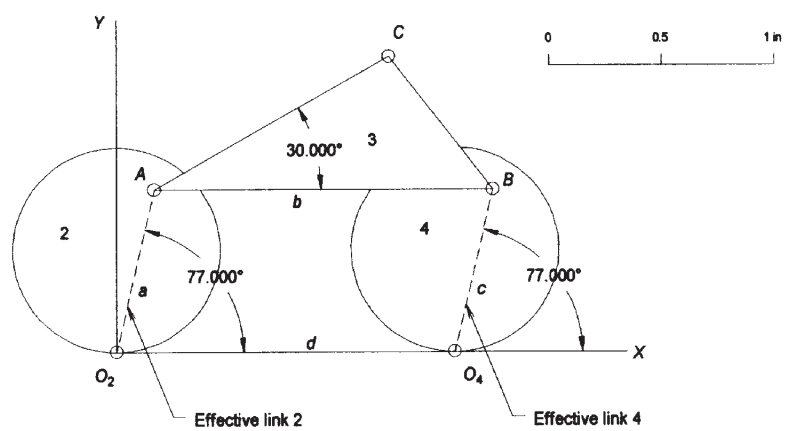
Angle *BAC* $\delta_3 := 30 \cdot deg$ Input crank angular velocity $\omega_2 := 15 \cdot rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P6-5c and Mathcad file P0617c.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.0000 \qquad K_2 = 2.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$A = -1.2250 \quad B = -1.9487 \quad C = 2.3251$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41} := 2\cdot\left(atan2\left(2\cdot A, -B - \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) \qquad \theta_{41} = 437.000\,deg$$

$$\theta_{41} := \theta_{41} - 360\cdot deg \qquad \theta_{41} = 77.000\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.0000$$

$$K_5 = -2.0000$$

$$D := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad D = -3.5501$$

$$E := -2\cdot sin(\theta_2) \qquad E = -1.9487$$

$$F := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5 \qquad F = 0.0000$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31} := 2\cdot\left(atan2\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_{31} = 360.000\,deg$$

$$\theta_{31} := \theta_{31} - 360\cdot deg \qquad \theta_{31} = 0.000\,deg$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31} := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{41} - \theta_2)}{sin(\theta_{31} - \theta_{41})} \qquad \omega_{31} = 0.000\,\frac{rad}{sec}$$

$$\omega_{41} := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{31})}{sin(\theta_{41} - \theta_{31})} \qquad \omega_{41} = 15.000\,\frac{rad}{sec}$$

7. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$\mathbf{V_A} := a\cdot\omega_2\cdot\left(-sin(\theta_2) + j\cdot cos(\theta_2)\right)$$

$$\mathbf{V_A} = -10.962 + 2.531j\,\frac{in}{sec} \qquad \left|\mathbf{V_A}\right| = 11.250\,\frac{in}{sec} \qquad arg(\mathbf{V_A}) = 167.000\,deg$$

$$\mathbf{V_B} := c\cdot\omega_{41}\cdot\left(-sin(\theta_{41}) + j\cdot cos(\theta_{41})\right)$$

$$\mathbf{V_B} = -10.962 + 2.531j\,\frac{in}{sec} \qquad \left|\mathbf{V_B}\right| = 11.250\,\frac{in}{sec} \qquad arg(\mathbf{V_B}) = 167.000\,deg$$

8. Determine the velocity of the coupler point C for the open circuit using equations 6.36.

$$\mathbf{V_{CA}} := R_{ca}\cdot\omega_{31}\cdot\left(-sin(\theta_{31} + \delta_3) + j\cdot cos(\theta_{31} + \delta_3)\right)$$

$$\mathbf{V_{CA}} = 0.000\,\frac{in}{sec}$$

$$\mathbf{V_C} := \mathbf{V_A} + \mathbf{V_{CA}}$$

$$\mathbf{V_C} = -10.962 + 2.531j\,\frac{in}{sec} \qquad \left|\mathbf{V_C}\right| = 11.250\,\frac{in}{sec} \qquad arg(\mathbf{V_C}) = 167.000\,deg$$

PROBLEM 6-18a

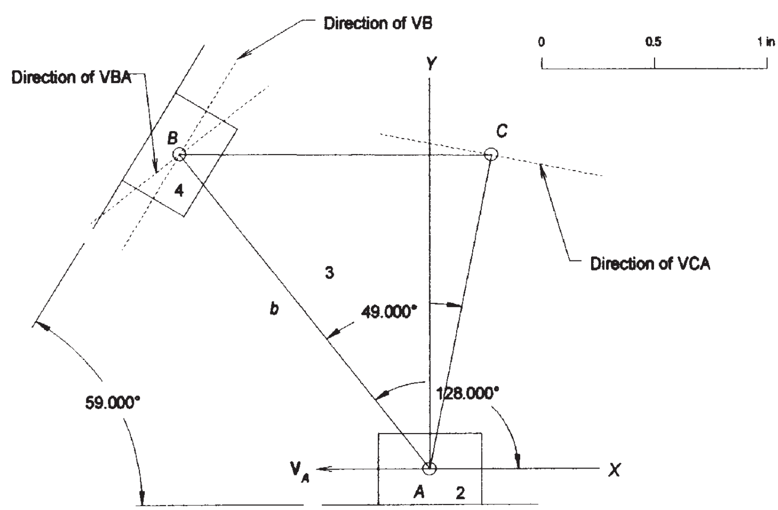
Statement: The linkage in Figure P6-5f has the dimensions and coupler angle given below. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\mathbf{V}_A = 10$ in/sec in the direction shown. Use the velocity difference graphical method.

Given: Link lengths and angles:

Link 3 (*A* to *B*)	$b := 1.8 \cdot in$
Coupler angle	$\theta_3 := 128 \cdot deg$
Slider 4 angle	$\theta_4 := 59 \cdot deg$
Coupler point: Distance *A* to *C*	$p := 1.44 \cdot in$
Angle *BAC*	$\delta := 49 \cdot deg$
Input slider velocity	$V_A := 10 \cdot in \cdot sec^{-1}$

Solution: See Figure P6-5f and Mathcad file P0618a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. The magnitude and sense of the velocity at point *A*.

$$V_A = 10.000 \frac{in}{sec} \qquad \theta_A := 180 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

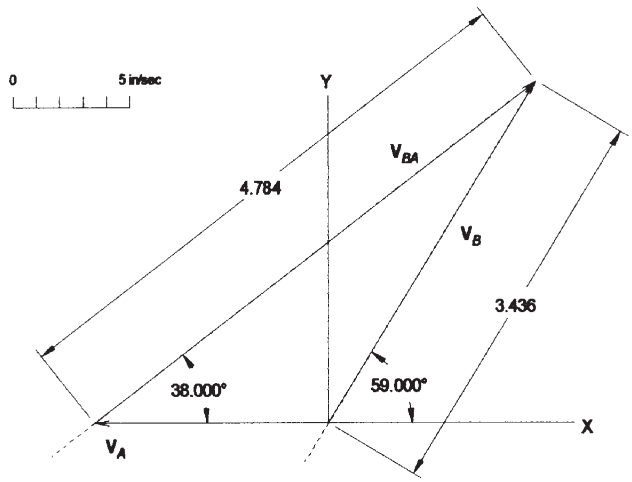
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{5 \cdot in \cdot sec^{-1}}{in}$

$V_B := 3.438 \cdot in \cdot k_v$ $\quad V_B = 17.190 \dfrac{in}{sec}$ $\quad \theta_B := 59 \cdot deg$

$V_{BA} := 4.784 \cdot in \cdot k_v$ $\quad V_{BA} = 23.920 \dfrac{in}{sec}$ $\quad \theta_{BA} := 38 \cdot deg$

5. Determine the angular velocity of link 3 using equation 6.7.

$\omega_3 := \dfrac{-V_{BA}}{b}$ $\quad \omega_3 = -13.289 \dfrac{rad}{sec}$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{CA}$ using equation 6.7.

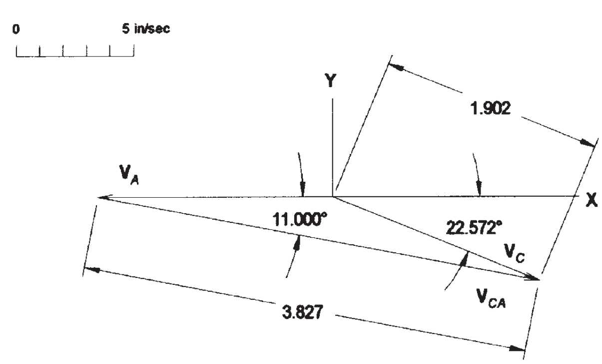
$V_{CA} := \left| p \cdot \omega_3 \right|$ $\quad V_{CA} = 19.136 \dfrac{in}{sec}$

$\theta_{CA} := (128 - 49 - 90) \cdot deg$ $\quad \theta_{CA} = -11.000\, deg$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{CA}$.
c. Complete the vector triangle by drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{CA}$ vector.



8. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{5 \cdot in \cdot sec^{-1}}{in}$

$V_C := 1.902 \cdot in \cdot k_v$ $V_C = 9.510 \frac{in}{sec}$ $\theta_C := -22.572 \cdot deg$

PROBLEM 6-18b

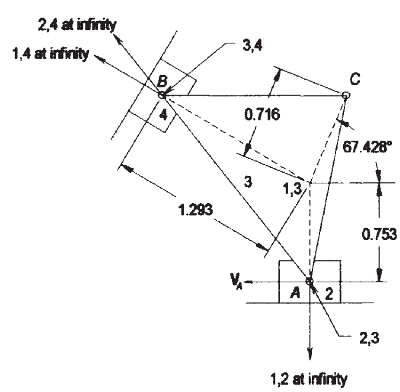
Statement: The linkage in Figure P6-5f has the dimensions and coupler angle given below. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\mathbf{V}_A$ = 10 in/sec in the direction shown. Use the instant center graphical method.

Given: Link lengths and angles:

Link 3 (*A* to *B*)	$b := 1.8 \cdot in$
Coupler angle	$\theta_3 := 128 \cdot deg$
Slider 4 angle	$\theta_4 := 59 \cdot deg$
Coupler point:	
Distance *A* to *C*	$p := 1.44 \cdot in$
Angle *BAC*	$\delta := 49 \cdot deg$
Input slider velocity	$V_A := 10 \cdot in \cdot sec^{-1}$

Solution: See Figure P6-5f and Mathcad file P0618b.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the *x* axis.



From the layout above:

$AI13 := 0.753 \cdot in$ $\quad BI13 := 1.293 \cdot in$ $\quad CI13 := 0.716 \cdot in$ $\quad \theta_C := 67.428 \cdot deg$

2. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 13.280 \frac{rad}{sec} \quad \text{CW}$$

3. Determine the magnitude of the velocity at point *B* using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 17.17 \frac{in}{sec}$$

$$\theta_{VB} := \theta_4 \qquad \theta_{VB} = 59.000\, deg$$

4. Determine the magnitude of the velocity at point *C* using equation 6.9b. Determine its direction by inspection.

$$V_C := CI13 \cdot \omega_3 \qquad V_C = 9.509 \frac{in}{sec}$$

$$\theta_{VC} := \theta_C - 90 \cdot deg \qquad \theta_{VC} = -22.572\, deg$$

PROBLEM 6-18c

Statement: The linkage in Figure P6-5f has the dimensions and coupler angle given below. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\mathbf{V}_A = 10$ in/sec in the direction shown. Use an analytical method.

Given: Link lengths and angles:

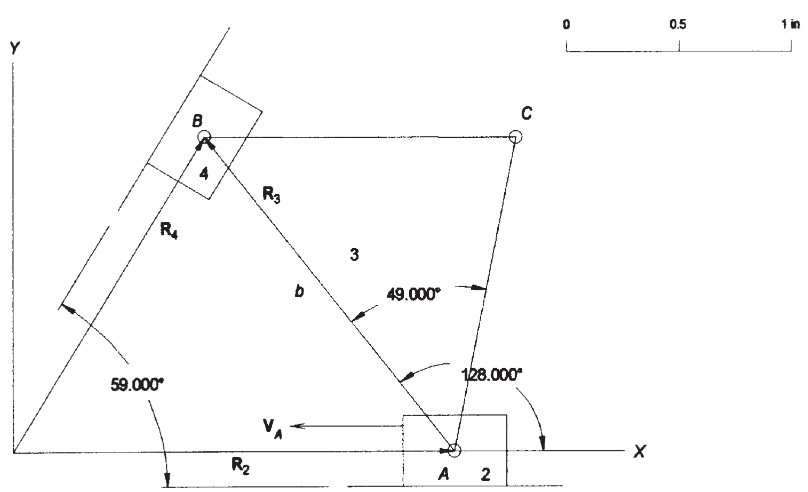
Link 3 (*A* to *B*)	$b := 1.8 \cdot in$
Coupler angle	$\theta_3 := 128 \cdot deg$
Slider 4 angle	$\theta_4 := 59 \cdot deg$

Coupler point:

Distance *A* to *C*	$R_{ca} := 1.44 \cdot in$
Angle *BAC*	$\delta_3 := -49 \cdot deg$
Input slider velocity	$V_A := -10 \cdot in \cdot sec^{-1}$

Solution: See Figure P6-5f and Mathcad file P0618c.

1. Draw the mechanism to scale and define a vector loop using the fourbar slider-crank derivation in Section 6.7 as a model.



2. Write the vector loop equation, differentiate it, expand the result and separate into real and imaginary parts to solve for ω_3 and $\mathbf{V}_B$.

$$\mathbf{R_2} + \mathbf{R_3} := \mathbf{R_4}$$

$$a \cdot e^{j \cdot \theta_2} + b \cdot e^{j \cdot \theta_3} := c \cdot e^{j \cdot \theta_4}$$

where a is the distance from the origin to point *A*, a variable; b is the distance from *A* to *B*, a constant; and c is the distance from the origin to point *B*, a variable. Angle θ_2 is zero, θ_3 is the angle that *AB* makes with the *x* axis, and θ_4 is the constant angle that slider 4 makes with the *x* axis. Differentiating,

$$\frac{d}{dt}a + j \cdot b \cdot \omega_3 \cdot e^{j \cdot \theta_3} := \left(\frac{d}{dt}c\right) \cdot e^{j \cdot \theta_4}$$

Substituting the Euler equivalents,

$$\frac{d}{dt}a + b\cdot\omega_3\cdot\left(-sin(\theta_3) + j\cdot cos(\theta_3)\right) := \left(\frac{d}{dt}c\right)\cdot\left(cos(\theta_4) + j\cdot sin(\theta_4)\right)$$

Separating into real and imaginary components and solving for ω_3 and $\mathbf{V}_B$. Note that $dc/dt = V_B$ and $da/dt = V_A$

$$\omega_3 := \frac{V_A\cdot tan(\theta_4)}{b\cdot\left(sin(\theta_3)\cdot tan(\theta_4) + cos(\theta_3)\right)} \qquad \omega_3 = -13.288\,\frac{rad}{sec}$$

$$V_B := \frac{V_A - b\cdot\omega_3\cdot sin(\theta_3)}{cos(\theta_4)} \qquad V_B = 17.180\,\frac{in}{sec}$$

$$\mathbf{V_B} := V_B\cdot\left(cos(\theta_4) + j\cdot sin(\theta_4)\right) \qquad arg(\mathbf{V_B}) = 59.000\,deg$$

3. Determine the velocity of the coupler point C using equations 6.36.

$$\mathbf{V_{CA}} := R_{ca}\cdot\omega_3\cdot\left(-sin(\theta_3 + \delta_3) + j\cdot cos(\theta_3 + \delta_3)\right)$$

$$\mathbf{V_{CA}} = 18.783 - 3.651j\,\frac{in}{sec}$$

$$\mathbf{V_A} := V_A$$

$$\mathbf{V_C} := \mathbf{V_A} + \mathbf{V_{CA}}$$

$$\mathbf{V_C} = 8.783 - 3.651j\,\frac{in}{sec} \qquad \left|\mathbf{V_C}\right| = 9.512\,\frac{in}{sec} \qquad arg(\mathbf{V_C}) = -22.572\,deg$$

PROBLEM 6-19

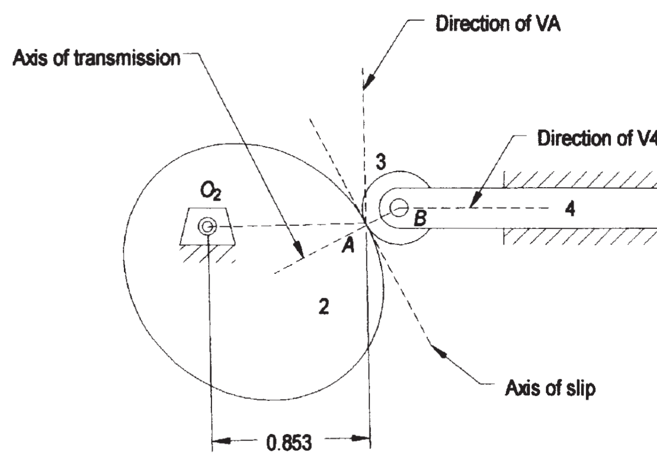
Statement: The cam-follower in Figure P6-5d has O_2A = 0.853 in. Find $\mathbf{V}_4$, $\mathbf{V}_{trans}$, and $\mathbf{V}_{slip}$ for the position shown with ω_2 = 20 rad/sec in the direction (CCW) shown.

Given: $\omega_2 := 20 \cdot rad \cdot sec^{-1}$ $\quad O_2A \quad a := 0.853 in$

Assumptions: Rolling contact (no sliding)

Solution: See Figure P6-5d and Mathcad file P0619.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 17.060 \frac{in}{sec}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A. The equation to be solved graphically is

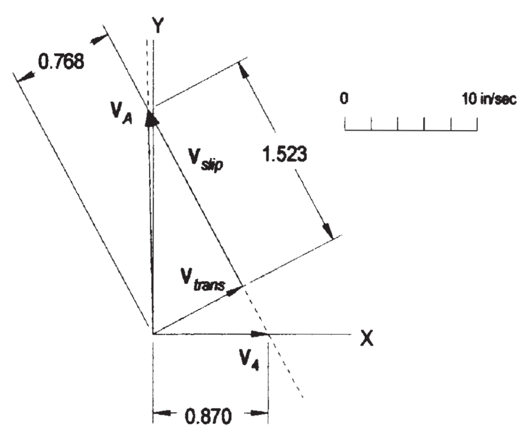
$$\mathbf{V}_A = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_A$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$ $\qquad V_{trans} := 0.768 \cdot in \cdot k_v \qquad V_{trans} = 7.680 \dfrac{in}{sec}$

$$V_{slip} := 1.523 \cdot in \cdot k_v \qquad V_{slip} = 15.230 \frac{in}{sec} \qquad V_4 := 0.870 \cdot in \cdot k_v \qquad V_4 = 8.700 \frac{in}{sec}$$

PROBLEM 6-20

Statement: The cam-follower in Figure P6-5e has O2A = 0.980 in and O3A = 1.344 in. Find ω_3, $\mathbf{V}_{trans}$, and $\mathbf{V}_{slip}$ for the position shown with ω_2 = 10 rad/sec in the direction (CW) shown.

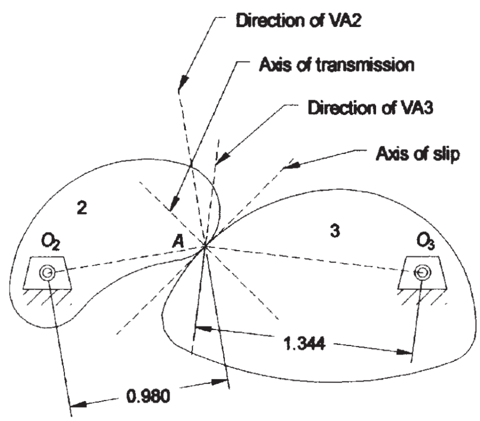
Given: $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ Distance from O_2 to A: $a := 0.980 in$

Distance from O_3 to A: $b := 1.344 in$

Assumptions: Roll-slide contact

Solution: See Figure P6-5e and Mathcad file P0620.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 9.800 \frac{in}{sec}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A. The equation to be solved graphically is

$$\mathbf{V}_{A2} = \mathbf{V}_{trans} + \mathbf{V}_{A2slip}$$

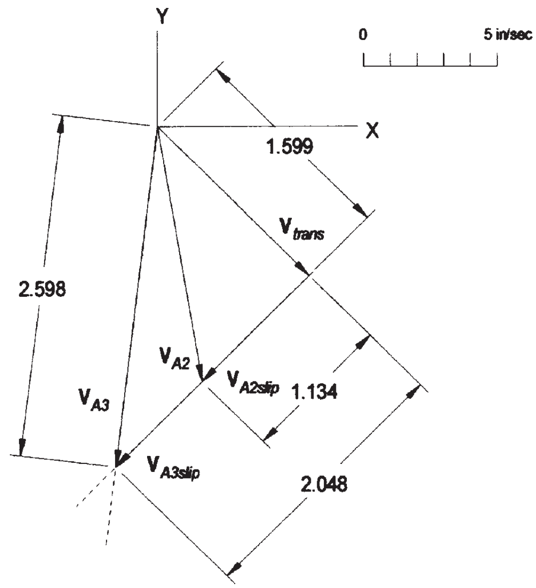
   a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

   b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.

   c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.

   d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_A$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{slip}$ construction line.

(See next page.)



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{5 \cdot in \cdot sec^{-1}}{in}$

$V_{A2slip} := 1.134 \cdot in \cdot k_v \qquad V_{A2slip} = 5.670 \dfrac{in}{sec}$

$V_{trans} := 1.599 \cdot in \cdot k_v \qquad V_{trans} = 7.995 \dfrac{in}{sec}$

$V_{A3slip} := 2.048 \cdot in \cdot k_v \qquad V_{A3slip} = 10.240 \dfrac{in}{sec}$

$V_{A3} := 2.598 \cdot in \cdot k_v \qquad V_{A3} = 12.990 \dfrac{in}{sec}$

5. Determine the angular velocity of link 3 using equation 6.7.

$\omega_3 := \dfrac{V_{A3}}{b} \qquad \omega_3 = 9.665 \dfrac{rad}{sec}$ CCW

6. The relative slip velocity is

$V_{slip} := V_{A3slip} - V_{A2slip} \qquad V_{slip} = 4.570 \dfrac{in}{sec}$

PROBLEM 6-21a

Statement: The linkage in Figure P6-6b has $L1 = 61.9$, $L_2 = 15$, $L_3 = 45.8$, $L_4 = 18.1$, $L_5 = 23.1$ mm. θ_2 is 68.3 d in the *xy* coordinate system, which is at -23.3 deg in the *XY* coordinate system. The *X* component of O_2C is 59.2 mm. Find, for the position shown, the velocity ratio $V_{I5,6}/V_{I2,3}$ and the mechanical advantage from link 2 to link 6. Use the velocity difference graphical method.

Given: Link lengths:

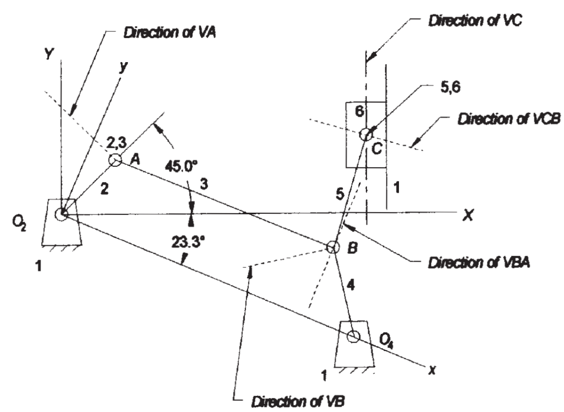
Link 1	$d := 61.9 \cdot mm$	Link 2	$a := 15.0 \cdot mm$
Link 3	$b := 45.8 \cdot mm$	Link 4	$c := 18.1 \cdot mm$
Link 5	$e := 23.1 \cdot mm$	Offset	$f := 59.2 \cdot mm$ from O_2

Crank angle: $\theta_2 := 45 \cdot deg$ Global *XY* system

Coordinate rotation angle $\alpha := -23.3 \cdot deg$ Global *XY* system to local *xy* system

Solution: See Figure P6-6b and Mathcad file P0621a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Choose an arbitrary value for the magnitude of the velocity at $I_{2,3}$ (point *A*). Let link 2 rotate CCW.

$$V_A := 10 \cdot \frac{mm}{sec} \qquad \theta_{VC} := 45 \cdot deg + 90 \cdot deg \qquad \theta_{VC} = 135.000\, deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

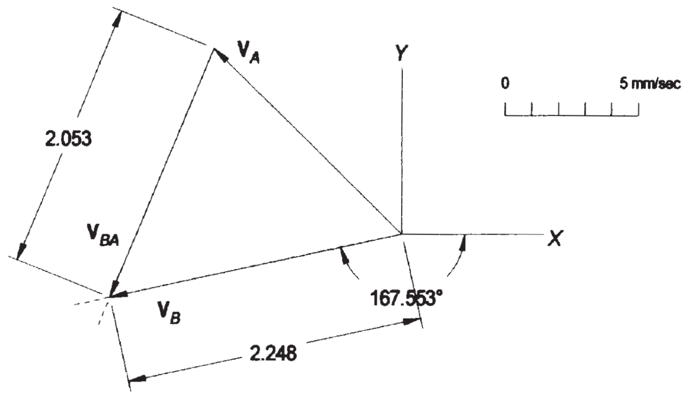
a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

(See next page.)



4. From the velocity triangle we have:

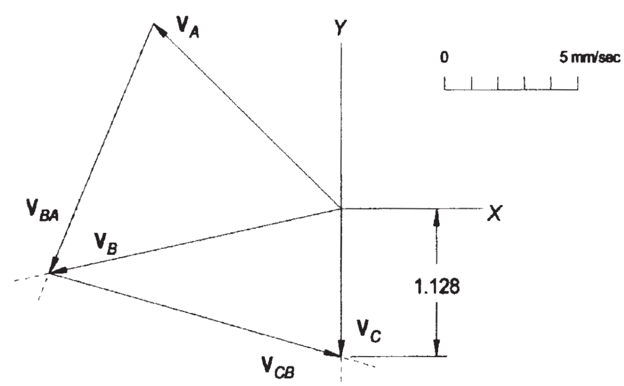
Velocity scale factor: $k_v := \frac{5 \cdot mm \cdot sec^{-1}}{in}$

$V_B := 2.248 \cdot in \cdot k_v$ $\quad V_B = 11.2 \frac{mm}{sec}$ $\quad \theta_{VB} := (360 - 167.558) \cdot deg$

5. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *C*. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

a. Choose a convenient velocity scale and layout the (now) known vector $\mathbf{V}_B$.
b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.



6. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{5 \cdot mm \cdot sec^{-1}}{in}$

$V_C := 1.128 \cdot in \cdot k_v$ $\quad V_C = 5.6 \frac{mm}{sec}$ $\quad \theta_{VC} := 270 \cdot deg$

7. The ratio $V_{I5,6} / V_{I2,3}$ is $\dfrac{V_C}{V_A} = 0.564$

8. **Use equations 6.12 and 6.13 to derive an expression for the mechanical advantage for this linkage where the input is a rotating crank and the output is a slider.**

$$F_{in} = \frac{T_{in}}{r_{in}} = \frac{P_{in}}{r_{in} \cdot \omega_{in}} = \frac{P_{in}}{V_A}$$

$$F_{out} = \frac{P_{out}}{V_{out}} = \frac{P_{out}}{V_C}$$

$$m_A = \frac{F_{out}}{F_{in}} = \frac{P_{out}}{V_C} \cdot \frac{V_A}{P_{in}} \qquad m_A := \frac{V_A}{V_C} \qquad m_A = 1.773$$

PROBLEM 6-21b

Statement: The linkage in Figure P6-6b has $L1 = 61.9$, $L_2 = 15$, $L_3 = 45.8$, $L_4 = 18.1$, $L_5 = 23.1$ mm. θ_2 is 68.3 d in the *xy* coordinate system, which is at -23.3 deg in the *XY* coordinate system. The *X* component of O_2C is 59.2 mm. Find, for the position shown, the velocity ratio $V_{I5,6}/V_{I2,3}$ and the mechanical advantage from link 2 to link 6. Use the instant center graphical method.

Given: Link lengths:

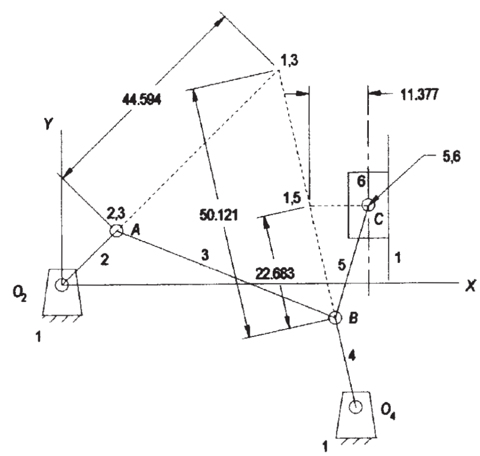
Link 1	$d := 61.9 \cdot mm$	Link 2	$a := 15.0 \cdot mm$
Link 3	$b := 45.8 \cdot mm$	Link 4	$c := 18.1 \cdot mm$
Link 5	$e := 23.1 \cdot mm$	Offset	$f := 59.2 \cdot mm$ from O_2

Crank angle: $\theta_2 := 45 \cdot deg$ Global *XY* system

Coordinate rotation angle $\alpha := -23.3 \cdot deg$ Global *XY* system to local *xy* system

Solution: See Figure P6-6b and Mathcad file P0621b.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the *x* axis.



From the layout:

$AI13 := 44.594 \cdot mm$ $BI13 := 50.121 \cdot mm$ $BI15 := 22.683 \cdot mm$ $CI15 := 11.377 \cdot mm$

2. Choose an arbitrary value for the magnitude of the velocity at $I_{2,3}$ (point *A*). Let link 2 rotate CCW.

$$V_A := 10 \cdot \frac{mm}{sec} \qquad \theta_{VC} := \theta_2 + 90 \cdot deg \qquad \theta_{VC} = 135.000\, deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 0.224 \frac{rad}{sec} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 11.239 \frac{mm}{sec}$$

5. Determine the angular velocity of link 5 using equation 6.9a.

$$\omega_5 := \frac{V_B}{BI15} \qquad \omega_5 = 0.495 \frac{rad}{sec} \quad \text{CW}$$

6. Determine the magnitude of the velocity at point C using equation 6.9b. Determine its direction by inspection.

$$V_C := CI15 \cdot \omega_5 \qquad V_C = 5.637 \frac{mm}{sec} \qquad \text{downward}$$

7. The ratio $V_{I5,6} / V_{I2,3}$ is $\dfrac{V_C}{V_A} = 0.56$

8. Use equations 6.12 and 6.13 to derive an expression for the mechanical advantage for this linkage where the input is a rotating crank and the output is a slider.

$$F_{in} = \frac{T_{in}}{r_{in}} = \frac{P_{in}}{r_{in} \cdot \omega_{in}} = \frac{P_{in}}{V_A}$$

$$F_{out} = \frac{P_{out}}{V_{out}} = \frac{P_{out}}{V_C}$$

$$m_A = \frac{F_{out}}{F_{in}} = \frac{P_{out}}{V_C} \cdot \frac{V_A}{P_{in}} \qquad m_A := \frac{V_A}{V_C} \qquad m_A = 1.77$$

PROBLEM 6-22a

Statement: The linkage in Figure P6-6d has $L_2 = 15$, $L_3 = 40.9$, $L_5 = 44.7$ mm. θ_2 is 24.2 deg in the *XY* coordinat system. Find, for the position shown, the velocity ratio $V_{I5,6}/V_{I2,3}$ and the mechanical advantage from link 2 to link 6. Use the velocity difference graphical method.

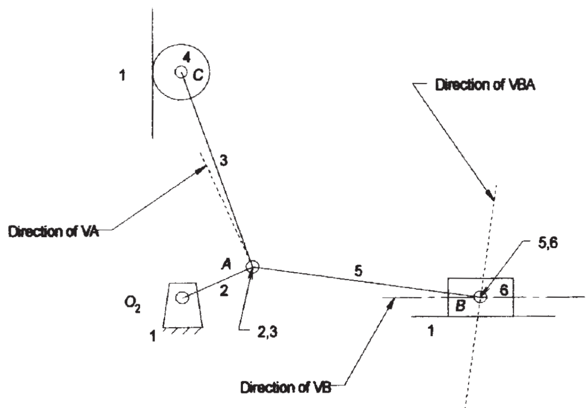
Given: Link lengths:

Link 2	$a := 15.0 \cdot mm$	Link 3	$b := 40.9 \cdot mm$
Link 5	$c := 44.7 \cdot mm$	Offset	$f := 0 \cdot mm$ from O_2

Crank angle: $\theta_2 := 24.2 \cdot deg$

Solution: See Figure P6-6d and Mathcad file P0622a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Choose an arbitrary value for the magnitude of the velocity at $I_{2,3}$ (point *A*). Let link 2 rotate CCW.

$$V_A := 10 \cdot \frac{mm}{sec} \qquad \theta_{VC} := \theta_2 + 90 \cdot deg \qquad \theta_{VC} = 114.200\, deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

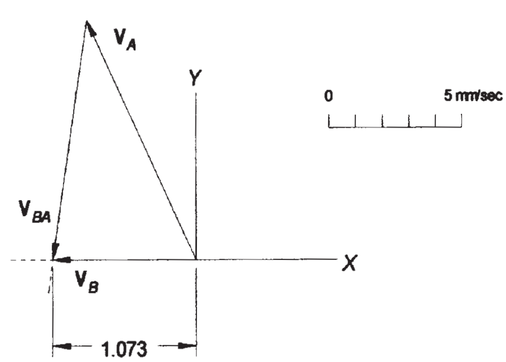
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

   a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

   b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

   c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

   d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{5 \cdot mm \cdot sec^{-1}}{in}$

$V_B := 1.073 \cdot in \cdot k_v$ $\qquad V_B = 5.4 \dfrac{mm}{sec}$ $\qquad \theta_{VB} := 180 \cdot deg$

5. The ratio $V_{I5,6} / V_{I2,3}$ is $\dfrac{V_B}{V_A} = 0.54$

6. Use equations 6.12 and 6.13 to derive an expression for the mechanical advantage for this linkage where the input is a rotating crank and the output is a slider.

$$F_{in} = \frac{T_{in}}{r_{in}} = \frac{P_{in}}{r_{in} \cdot \omega_{in}} = \frac{P_{in}}{V_A}$$

$$F_{out} = \frac{P_{out}}{V_{out}} = \frac{P_{out}}{V_B}$$

$$m_A = \frac{F_{out}}{F_{in}} = \frac{P_{out}}{V_B} \cdot \frac{V_A}{P_{in}} \qquad m_A := \frac{V_A}{V_B} \qquad m_A = 1.86$$

PROBLEM 6-22b

Statement: The linkage in Figure P6-6d has $L_2 = 15$, $L_3 = 40.9$, $L_5 = 44.7$ mm. θ_2 is 24.2 deg in the *XY* coordinat system. Find, for the position shown, the velocity ratio $V_{I5,6}/V_{I2,3}$ and the mechanical advantage from link 2 to link 6. Use the instant center graphical method.

Given: Link lengths:

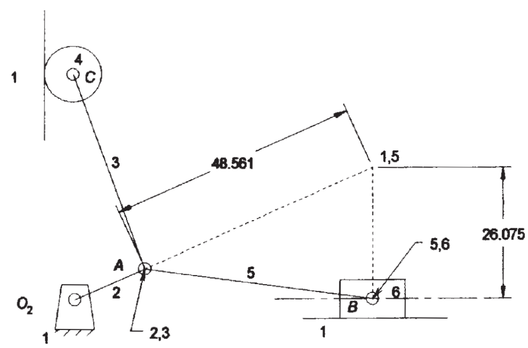
Link 2 $a := 15.0 \cdot mm$ Link 3 $b := 40.9 \cdot mm$

Link 5 $c := 44.7 \cdot mm$ Offset $f := 0 \cdot mm$ from O_2

Crank angle: $\theta_2 := 24.2 \cdot deg$

Solution: See Figure P6-6d and Mathcad file P0622b.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the *x* axis.



From the layout:

$AI15 := 48.561 \cdot mm$ $BI15 := 26.075 \cdot mm$

2. Choose an arbitrary value for the magnitude of the velocity at $I_{2,3}$ (point *A*). Let link 2 rotate CCW.

$V_A := 10 \cdot \frac{mm}{sec}$ $\theta_{VC} := \theta_2 + 90 \cdot deg$ $\theta_{VC} = 114.200\, deg$

3. Determine the angular velocity of link 3 using equation 6.9a.

$\omega_5 := \frac{V_A}{AI15}$ $\omega_5 = 0.206 \frac{rad}{sec}$ CW

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. Determine its direction by inspection.

$V_B := BI15 \cdot \omega_5$ $V_B = 5.370 \frac{mm}{sec}$ to the left

5. The ratio $V_{I5,6} / V_{I2,3}$ is $\frac{V_B}{V_A} = 0.54$

6. Use equations 6.12 and 6.13 to derive an expression for the mechanical advantage for this linkage where the input is a rotating crank and the output is a slider.

$$F_{in} = \frac{T_{in}}{r_{in}} = \frac{P_{in}}{r_{in} \cdot \omega_{in}} = \frac{P_{in}}{V_A} \qquad F_{out} = \frac{P_{out}}{V_{out}} = \frac{P_{out}}{V_C}$$

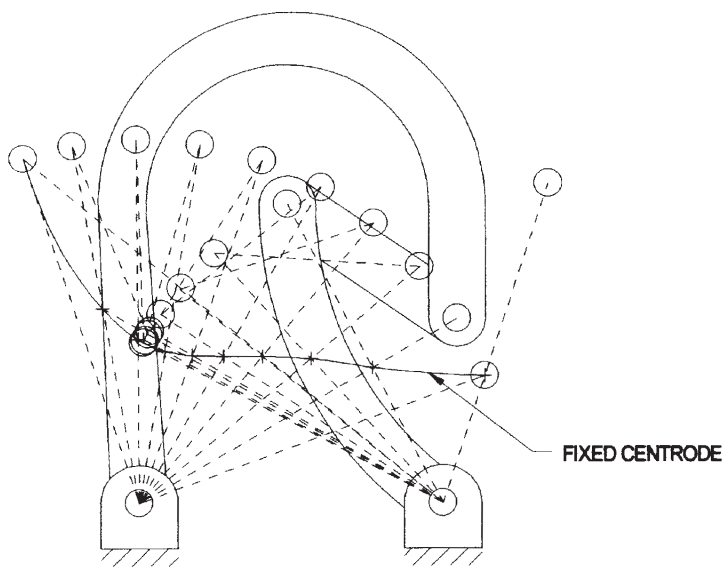
$$m_A = \frac{F_{out}}{F_{in}} = \frac{P_{out}}{V_C} \cdot \frac{V_A}{P_{in}} \qquad m_A := \frac{V_A}{V_B} \qquad m_A = 1.86$$

PROBLEM 6-23

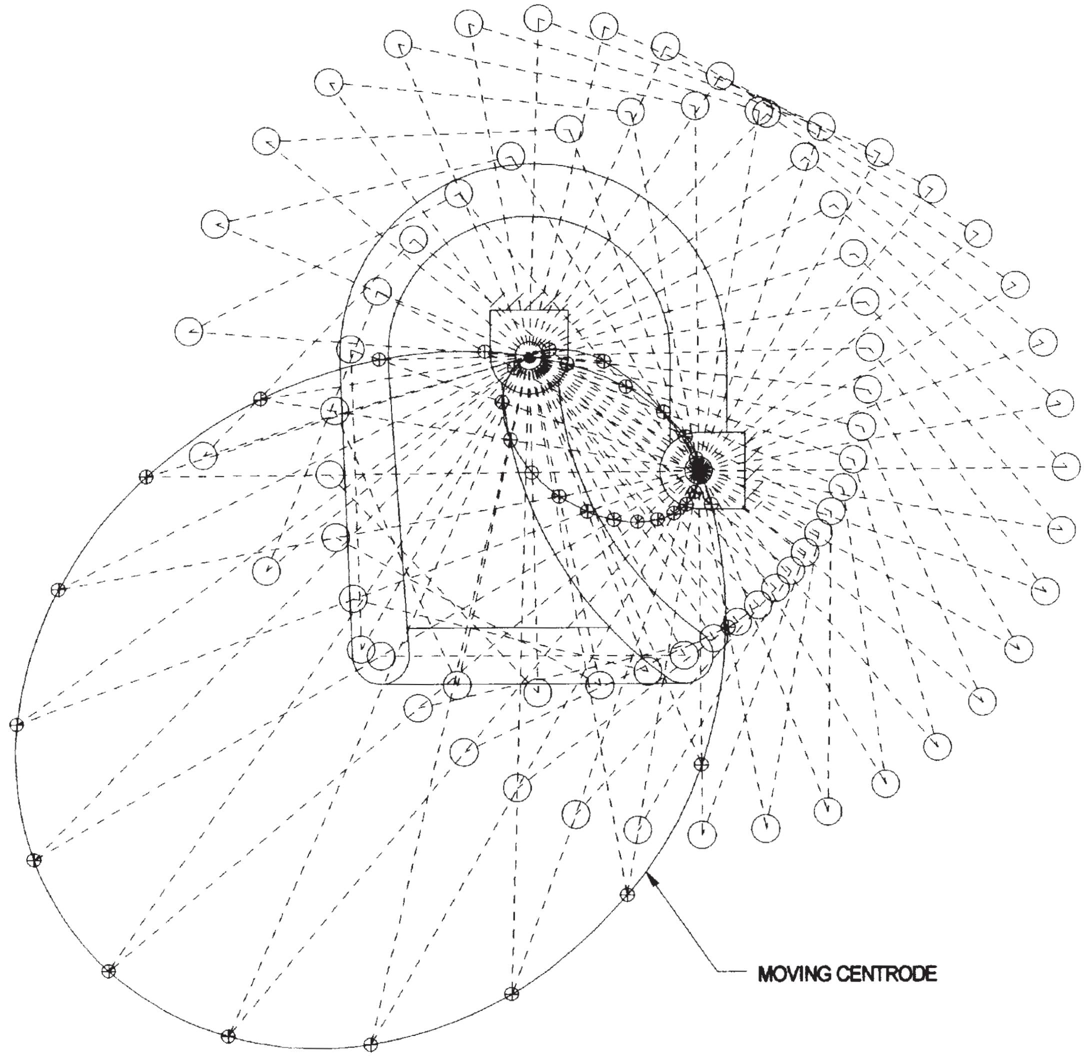
Statement: Generate and draw the fixed and moving centrodes of links 1 and 3 for the linkage in Figure P6-7a.

Solution: See Figure P6-7a and Mathcad file P0623.

1. Draw the linkage to scale, find the instant center $I_{1,3}$, and repeat for several positions of the linkage. The locus of points $I_{1,3}$ is the fixed centrode.



2. Invert the linkage, grounding link 3. Draw the linkage to scale, find the instant center $I_{1,3}$, and repeat for several positions of the linkage. The locus of points $I_{1,3}$ is the moving centrode. (See next page.)


MOVING CENTRODE

PROBLEM 6-24

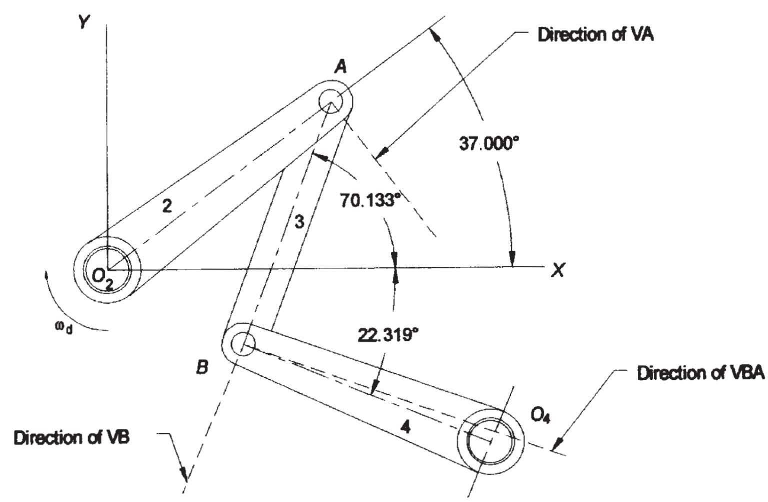
Statement: The linkage in Figure P6-8a has the dimensions and crank angle given below. Find ω_f, $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for ω_2 = 15 rad/sec clockwise (CW). Use the velocity difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 116 \cdot mm$	
Link 3 (A to B)	$b := 108 \cdot mm$	
Link 4 (B to O_4)	$c := 110 \cdot mm$	
Link 1 (O_2 to O_4)	$d := 174 \cdot mm$	
Crank angle:	$\theta_2 := 37 \cdot deg$	Global *XY* system
Input crank angular velocity	$\omega_2 := 15 \cdot rad \cdot sec^{-1}$	
Coordinate rotation angle	$\alpha := -25 \cdot deg$	Global *XY* system to local *xy* system

Solution: See Figure P6-8a and Mathcad file P0624.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 1740.0 \frac{mm}{sec} \qquad \theta_A := 37 \cdot deg - 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

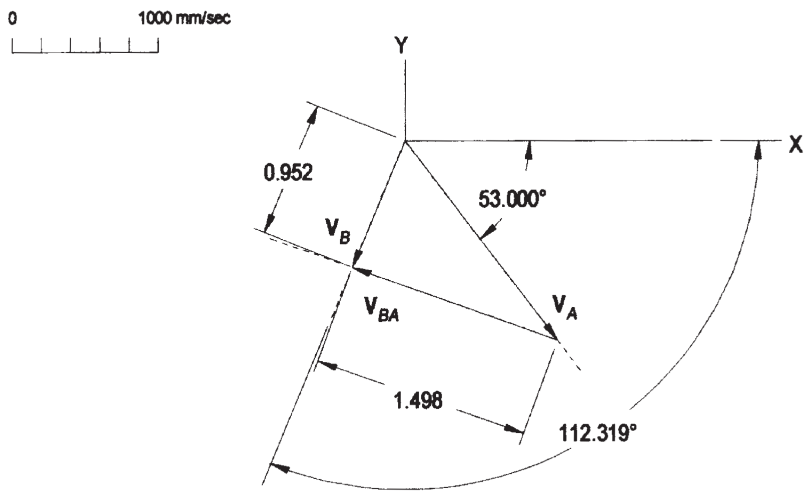
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{1000 \cdot mm \cdot sec^{-1}}{in}$

$V_B := 0.952 \cdot in \cdot k_v$ $\quad V_B = 952.0 \dfrac{mm}{sec}$ $\quad \theta_B := -112.319 \cdot deg$

$V_{BA} := 1.498 \cdot in \cdot k_v$ $\quad V_{BA} = 1498.0 \dfrac{mm}{sec}$

5. Determine the angular velocity of link 3 using equation 6.7.

$\omega_3 := \dfrac{-V_{BA}}{b}$ $\quad \omega_3 = -13.87 \dfrac{rad}{sec}$

6. Determine the angular velocity of link 4 using equation 6.7.

$\omega_4 := \dfrac{V_B}{c}$ $\quad \omega_4 = 8.655 \dfrac{rad}{sec}$

PROBLEM 6-25

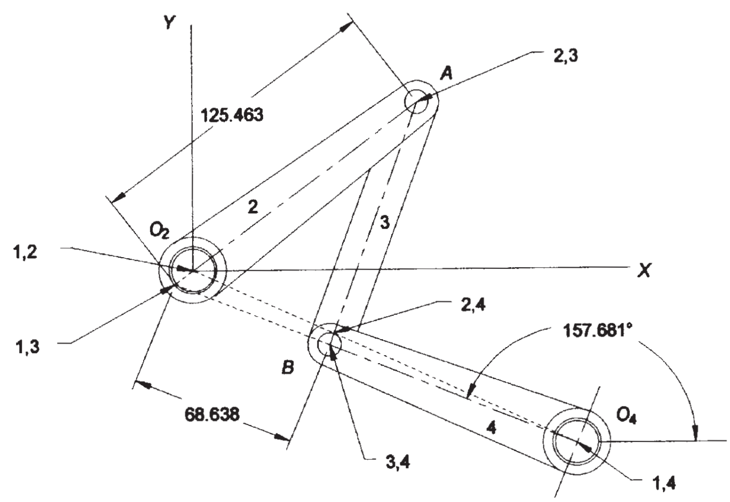
Statement: The linkage in Figure P6-8a has the dimensions and crank angle given below. Find ω_f, $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for ω_2 = 15 rad/sec clockwise (CW). Use the instant center graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 116 \cdot mm$	
Link 3 (A to B)	$b := 108 \cdot mm$	
Link 4 (B to O_4)	$c := 110 \cdot mm$	
Link 1 (O_2 to O_4)	$d := 174 \cdot mm$	
Crank angle:	$\theta_2 := 37 \cdot deg$	Global XY system
Input crank angular velocity	$\omega_2 := 15 \cdot rad \cdot sec^{-1}$	
Coordinate rotation angle	$\alpha := -25 \cdot deg$	Global XY system to local xy system

Solution: See Figure P6-8a and Mathcad file P0625.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.



From the layout above:

$$AI13 := 125.463 \cdot mm \qquad BI13 := 68.638 \cdot mm \qquad \theta_4 := 157.681 \cdot deg$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 1740.0 \frac{mm}{sec}$$

$$\theta_{VA} := \theta_2 - 90 \cdot deg \qquad \theta_{VA} = -53.0 \, deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 13.869 \frac{rad}{sec} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 951.915 \frac{mm}{sec}$$

$$\theta_{VB} := \theta_4 + 90 \cdot deg \qquad \theta_{VB} = 247.681\ deg$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 8.654 \frac{rad}{sec} \quad \text{CCW}$$

PROBLEM 6-26

Statement: The linkage in Figure P6-8a has the dimensions and crank angle given below. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for ω_2 = 15 rad/sec clockwise (CW). Use an analytical method.

Given: Link lengths:

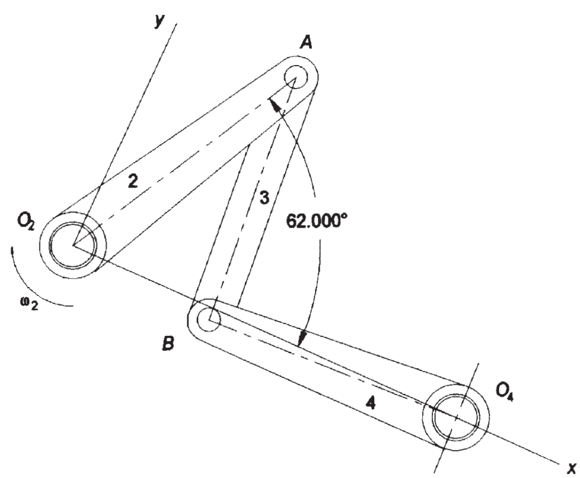
Link 2 (O_2 to A)	$a := 116 \cdot mm$
Link 3 (A to B)	$b := 108 \cdot mm$
Link 4 (B to O_4)	$c := 110 \cdot mm$
Link 1 (O_2 to O_4)	$d := 174 \cdot mm$
Crank angle:	$\theta_2 := 62 \cdot deg$ Global XY system
Input crank angular velocity	$\omega_2 := -15 \cdot rad \cdot sec^{-1}$ CW

Two argument inverse tangent

$$atan2(x,y) := \begin{vmatrix} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{vmatrix}$$

Solution: See Figure P6-8a and Mathcad file P0626.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.5000 \qquad K_2 = 1.5818$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.7307$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$A = -0.0424 \quad B = -1.7659 \quad C = 2.0186$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42} := 2\cdot\left(atan2\left(2\cdot A, -B + \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) \qquad \theta_{42} = 182.681\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.6111$$

$K_5 = -1.7280$

$$D := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad D = -2.0021$$

$$E := -2\cdot sin(\theta_2) \qquad E = -1.7659$$

$$F := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5 \qquad F = 0.0589$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32} := 2\cdot\left(atan2\left(2\cdot D, -E + \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_{32} = 275.133\, deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32} := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{42} - \theta_2)}{sin(\theta_{32} - \theta_{42})} \qquad \omega_{32} = -13.869\,\frac{rad}{sec}$$

$$\omega_{42} := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{32})}{sin(\theta_{42} - \theta_{32})} \qquad \omega_{42} = 8.654\,\frac{rad}{sec}$$

7. Determine the velocity of points A and B for the crossed circuit using equations 6.19.

$$V_A := a\cdot\omega_2\cdot\left(-sin(\theta_2) + j\cdot cos(\theta_2)\right)$$

$$V_A = 1536.329 - 816.881j\,\frac{mm}{sec} \qquad \left|V_A\right| = 1740.000\,\frac{mm}{sec} \qquad arg(V_A) = -28.000\, deg$$

$$V_B := c\cdot\omega_{42}\cdot\left(-sin(\theta_{42}) + j\cdot cos(\theta_{42})\right)$$

$$V_B = 44.524 - 950.875j\,\frac{mm}{sec} \qquad \left|V_B\right| = 951.917\,\frac{mm}{sec} \qquad arg(V_B) = -87.319\, deg$$

PROBLEM 6-27

Statement: The linkage in Figure P6-8a has the dimensions given below. Find and plot ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if ω_2 = 15 rad/sec clockwise (CW).

Given: Link lengths:

Link 2 (O_2 to A)	$a := 116 \cdot mm$	Link 3 (A to B)	$b := 108 \cdot mm$
Link 4 (B to O_4)	$c := 110 \cdot mm$	Link 1 (O_2 to O_4)	$d := 174 \cdot mm$

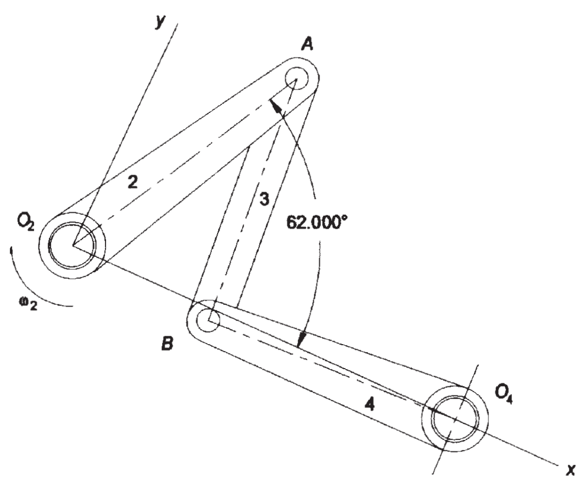
Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$ CW

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P6-8a and Mathcad fil

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 1.083$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = -0.094$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 95.4\,deg$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 1 \cdot deg .. \theta_{2toggle}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.5000 \qquad K_2 = 1.5818$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.7307$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42}(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{(B(\theta_2))^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right] \right]$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.6111$$

$$K_5 = -1.7280$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32}(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{(E(\theta_2))^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right] \right]$$

7. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42}(\theta_2) - \theta_2)}{\sin(\theta_{32}(\theta_2) - \theta_{42}(\theta_2))}$$

$$\omega_{42}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32}(\theta_2))}{\sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))}$$

8. Determine the velocity of points *A* and *B* for the crossed circuit using equations 6.19.

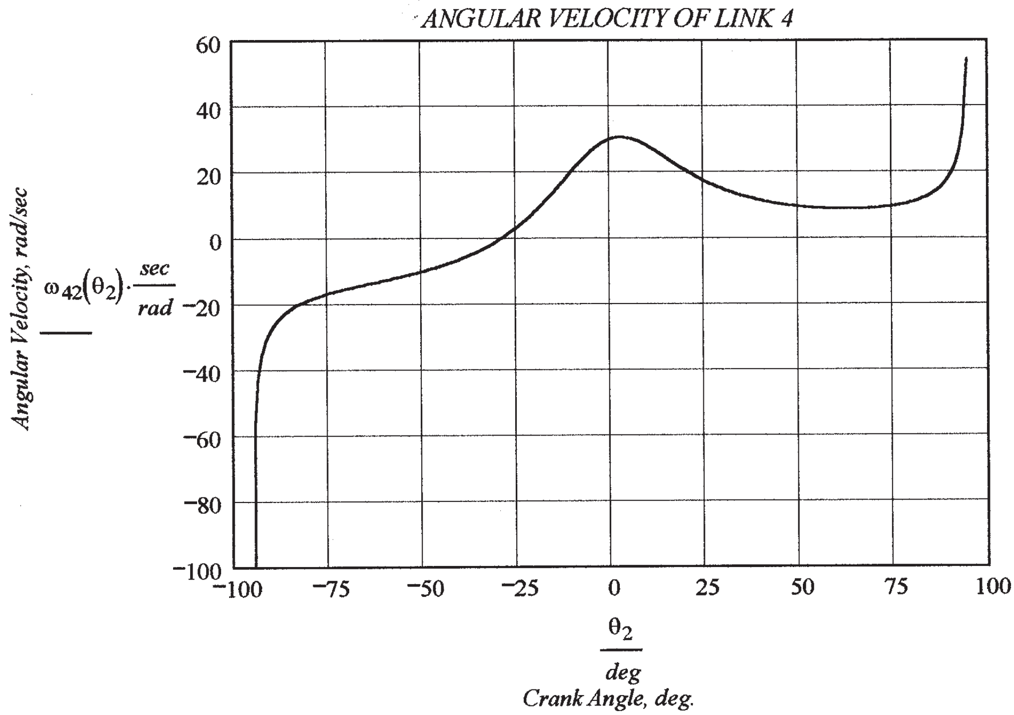
$$V_A(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

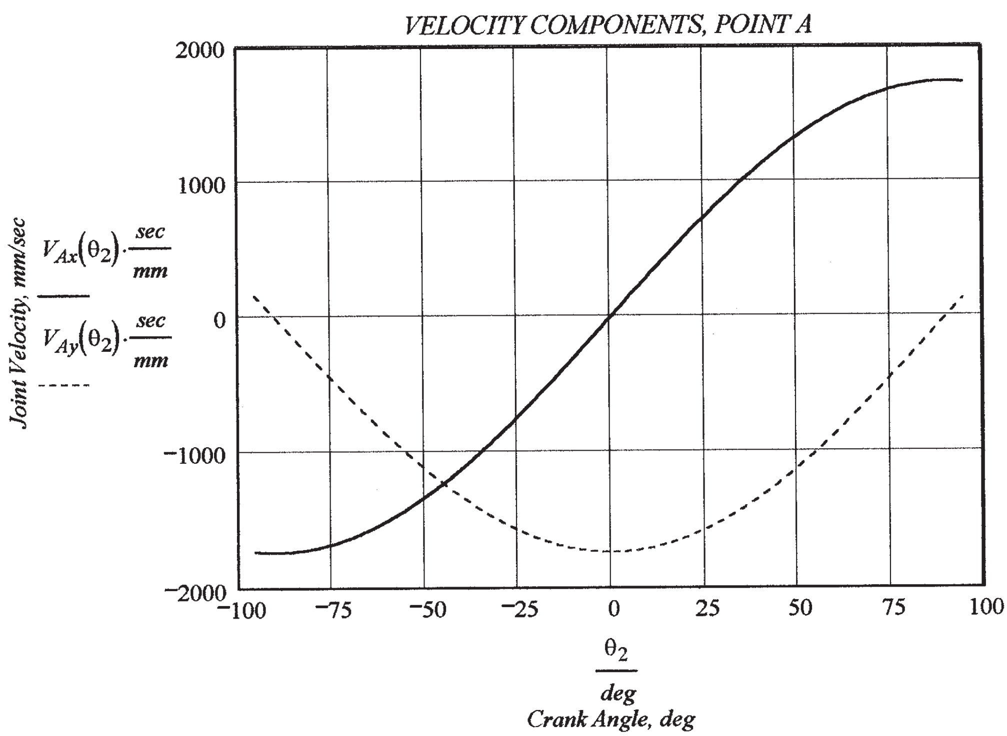
$$V_{Ax}(\theta_2) := Re(V_A(\theta_2)) \qquad V_{Ay}(\theta_2) := Im(V_A(\theta_2))$$

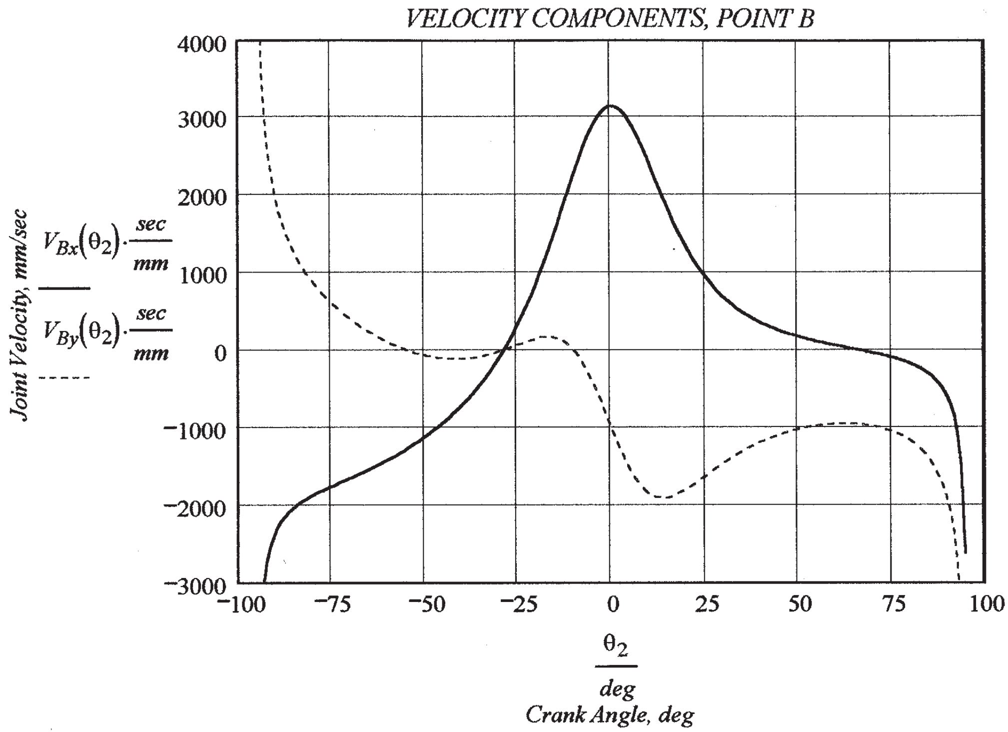
$$V_B(\theta_2) := c \cdot \omega_{42}(\theta_2) \cdot (-\sin(\theta_{42}(\theta_2)) + j \cdot \cos(\theta_{42}(\theta_2)))$$

$$V_{Bx}(\theta_2) := Re(V_B(\theta_2)) \qquad V_{By}(\theta_2) := Im(V_B(\theta_2))$$

9. Plot the angular velocity of the output link, ω_4, and the x and y components of the velocities at points A and B.






VELOCITY COMPONENTS, POINT B
4000
3000
2000
1000
0
−1000
−2000
−3000
−100
−75
−50
−25
0
25
50
75
100
Joint Velocity, mm/sec
$V_{Bx}(\theta_2)\cdot\frac{sec}{mm}$
$V_{By}(\theta_2)\cdot\frac{sec}{mm}$
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 6-28

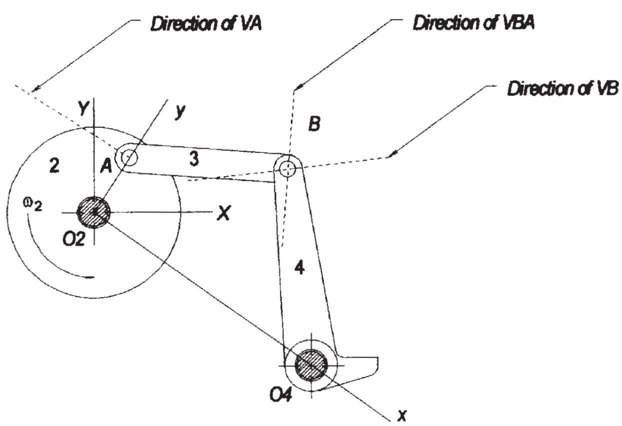
Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for $\omega_2 = 20$ rad/sec counterclockwise (CCW). Use the velocity difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 40 \cdot mm$	
Link 3 (A to B)	$b := 96 \cdot mm$	
Link 4 (B to O_4)	$c := 122 \cdot mm$	
Link 1 (O_2 to O_4)	$d := 162 \cdot mm$	
Crank angle:	$\theta_2 := 57 \cdot deg$	Global XY system
Input crank angular velocity	$\omega_2 := 20 \cdot rad \cdot sec^{-1}$	
Coordinate rotation angle	$\alpha := -36 \cdot deg$	Global XY system to local xy system

Solution: See Figure P6-8b and Mathcad file P0628.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 800.000 \frac{mm}{sec} \qquad \theta_A := \theta_2 + 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

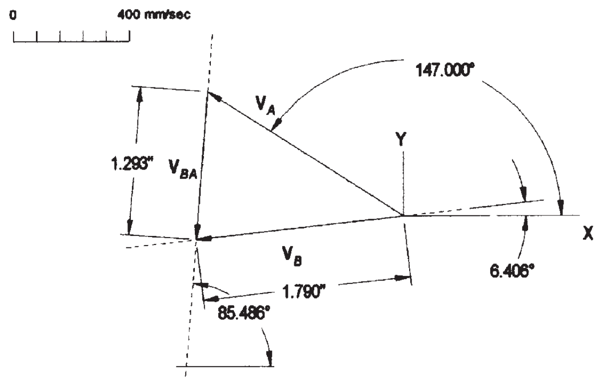
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{400 \cdot mm \cdot sec^{-1}}{in}$

$V_B := 1.790 \cdot in \cdot k_v$ $\qquad V_B = 716.0 \dfrac{mm}{sec}$ $\qquad \theta_B := 186.406 \cdot deg$

$V_{BA} := 1.293 \cdot in \cdot k_v$ $\qquad V_{BA} = 517.2 \dfrac{mm}{sec}$

5. Determine the angular velocity of link 3 using equation 6.7.

$\omega_3 := \dfrac{V_{BA}}{b}$ $\qquad \omega_3 = 5.387 \dfrac{rad}{sec}$

6. Determine the angular velocity of link 4 using equation 6.7.

$\omega_4 := \dfrac{V_B}{c}$ $\qquad \omega_4 = 5.869 \dfrac{rad}{sec}$

PROBLEM 6-29

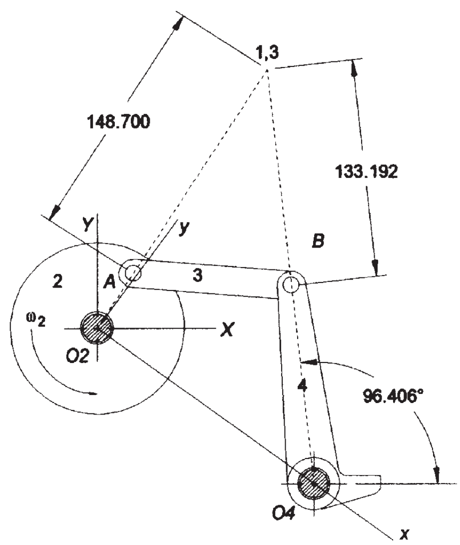
Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for ω_2 = 20 rad/sec counterclockwise (CCW). Use the instant center graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 40 \cdot mm$	
Link 3 (A to B)	$b := 96 \cdot mm$	
Link 4 (B to O_4)	$c := 122 \cdot mm$	
Link 1 (O_2 to O_4)	$d := 162 \cdot mm$	
Crank angle:	$\theta_2 := 57 \cdot deg$	Global *XY* system
Input crank angular velocity	$\omega_2 := 20 \cdot rad \cdot sec^{-1}$	
Coordinate rotation angle	$\alpha := -36 \cdot deg$	Global *XY* system to local *xy* system

Solution: See Figure P6-8b and Mathcad file P0629.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.



From the layout above:

$$AI13 := 148.700 \cdot mm \qquad BI13 := 133.192 \cdot mm \qquad \theta_4 := 96.406 \cdot deg$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 800.0 \frac{mm}{sec}$$

$$\theta_{VA} := \theta_2 + 90 \cdot deg \qquad \theta_{VA} = 147.0\, deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 5.380 \frac{rad}{sec} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b. Determine its direction by inspection.

$V_B := BI13 \cdot \omega_3$ $\qquad V_B = 716.6 \dfrac{mm}{sec}$

$\theta_{VB} := \theta_4 + 90 \cdot deg$ $\qquad \theta_{VB} = 186.406\, deg$

5. Use equation 6.9c to determine the angular velocity of link 4.

$\omega_4 := \dfrac{V_B}{c}$ $\qquad \omega_4 = 5.874 \dfrac{rad}{sec}$ CCW

PROBLEM 6-30

Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for ω_2 = 20 rad/sec counterclockwise (CCW). Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 40 \cdot mm$
Link 3 (A to B)	$b := 96 \cdot mm$
Link 4 (B to O_4)	$c := 122 \cdot mm$
Link 1 (O_2 to O_4)	$d := 162 \cdot mm$

Crank angle: $\theta_{21} := 57 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := 20 \cdot rad \cdot sec^{-1}$

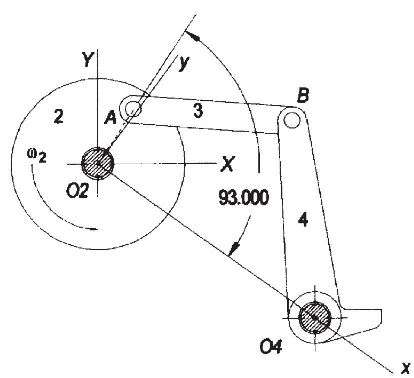
Coordinate rotation angle $\alpha := -36 \cdot deg$ Global XY system to local xy system

Two argument inverse tangent

$$atan2(x,y) := \begin{vmatrix} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{vmatrix}$$

Solution: See Figure P6-8b and Mathcad file P0630.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 4.0500 \qquad K_2 = 1.3279$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 3.4336 \qquad \theta_2 := \theta_{21} - \alpha$$

$$A := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B := -2 \cdot sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

$A = -0.5992 \quad B = -1.9973 \quad C = 7.6054$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2\cdot\left(atan2\left(2\cdot A, -B - \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) - 2\cdot\pi \qquad \theta_4 = 132.386\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.6875$$

$$K_5 = -2.8875$$

$$D := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad D = -7.0782$$

$$E := -2\cdot sin(\theta_2) \qquad E = -1.9973$$

$$F := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5 \qquad F = 1.1265$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2\cdot\left(atan2\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) - 2\cdot\pi \qquad \theta_3 = 31.504\,deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4 - \theta_2)}{sin(\theta_3 - \theta_4)} \qquad \omega_3 = -5.385\,\frac{rad}{sec}$$

$$\omega_4 := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3)}{sin(\theta_4 - \theta_3)} \qquad \omega_4 = 5.868\,\frac{rad}{sec}$$

7. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$V_A := a\cdot\omega_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2))$$

$$V_A = -798.904 - 41.869j\,\frac{mm}{sec} \qquad |V_A| = 800.000\,\frac{mm}{sec} \qquad arg(V_A) = -177.000\,deg$$

$$V_B := c\cdot\omega_4\cdot(-sin(\theta_4) + j\cdot cos(\theta_4))$$

$$V_B = -528.774 - 482.608j\,\frac{mm}{sec} \qquad |V_B| = 715.900\,\frac{mm}{sec} \qquad arg(V_B) = -137.614\,deg$$

PROBLEM 6-31

Statement: The linkage in Figure P6-8b has the dimensions and crank angle given below. Find and plot ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if ω_2 = 20 rad/sec counterclockwise (CCW).

Given: Link lengths:

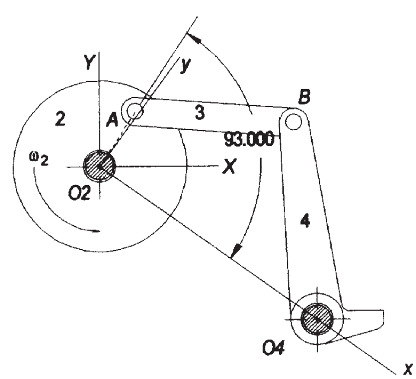
Link 2 (O_2 to A)	$a := 40 \cdot mm$
Link 3 (A to B)	$b := 96 \cdot mm$
Link 4 (B to O_4)	$c := 122 \cdot mm$
Link 1 (O_2 to O_4)	$d := 162 \cdot mm$
Input crank angular velocity	$\omega_2 := 20 \cdot rad \cdot sec^{-1}$ CCW

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P6-8b and Mathcad file P0631.

1. Draw the linkage to scale and label it.



2. Determine Grashof condition.

$$Condition(S,L,P,Q) := \begin{array}{|l} SL \leftarrow S + L \\ PQ \leftarrow P + Q \\ return\ \text{"Grashof"}\ \ if\ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array}$$

$Condition(a,d,b,c) = \text{"Grashof"}$ Crank-rocker

$\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 4.0500 \qquad K_2 = 1.3279$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 3.4336$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left[\operatorname{atan2}\left[2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{(B(\theta_2))^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right] \right]$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.6875$$

$$K_5 = -2.8875$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left[\operatorname{atan2}\left[2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{(E(\theta_2))^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right] \right]$$

7. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Determine the velocity of points B and C for the open circuit using equations 6.19.

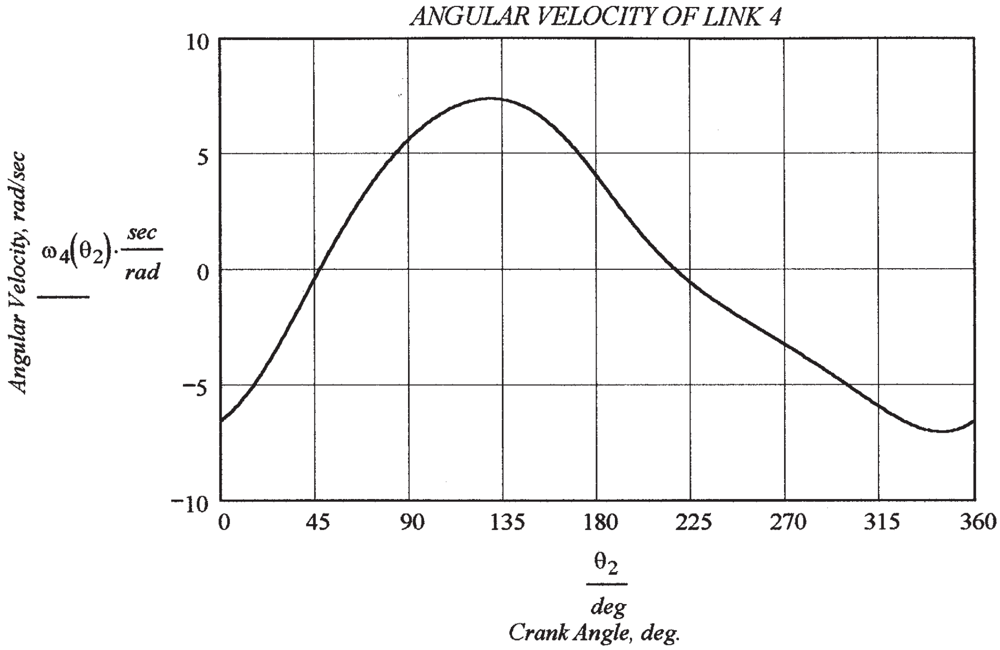
$$\mathbf{V_A}(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

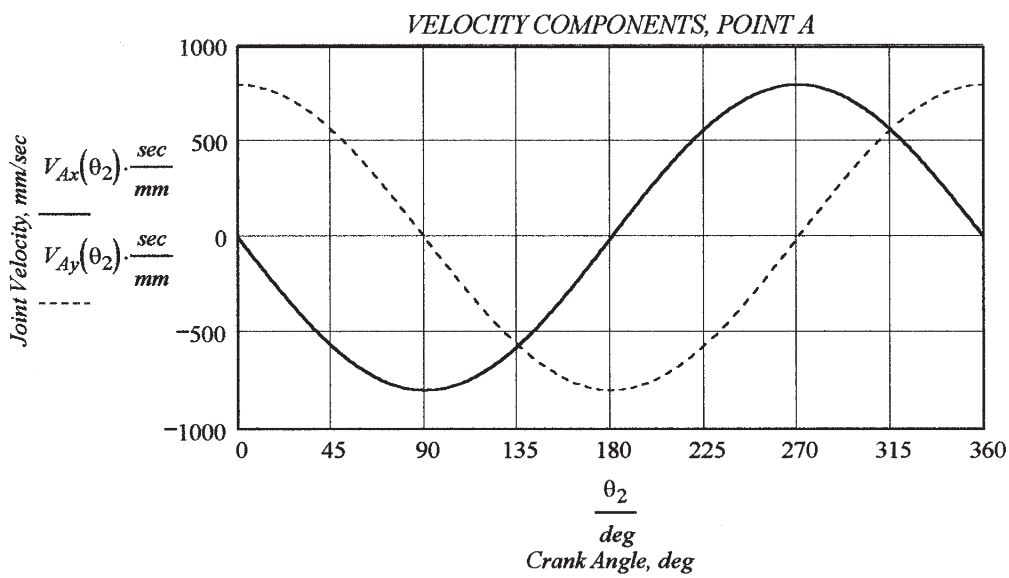
$$V_{Ax}(\theta_2) := Re(\mathbf{V_A}(\theta_2)) \qquad V_{Ay}(\theta_2) := Im(\mathbf{V_A}(\theta_2))$$

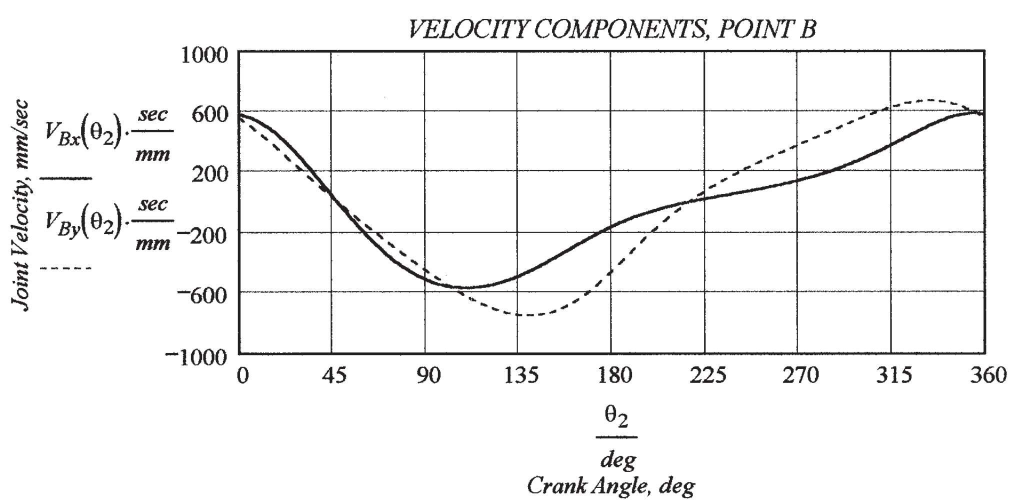
$$\mathbf{V_B}(\theta_2) := c \cdot \omega_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2)))$$

$$V_{Bx}(\theta_2) := Re(\mathbf{V_B}(\theta_2)) \qquad V_{By}(\theta_2) := Im(\mathbf{V_B}(\theta_2))$$

9. Plot the angular velocity of the output link, ω_4, and the magnitudes of the velocities at points B and C.


ANGULAR VELOCITY OF LINK 4
Angular Velocity, rad/sec
$\omega_4(\theta_2)\cdot\frac{sec}{rad}$
10
5
0
−5
−10
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg.


VELOCITY COMPONENTS, POINT A
Joint Velocity, mm/sec
$V_{Ax}(\theta_2)\cdot\frac{sec}{mm}$
$V_{Ay}(\theta_2)\cdot\frac{sec}{mm}$
1000
500
0
−500
−1000
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg


VELOCITY COMPONENTS, POINT B
Joint Velocity, mm/sec
$V_{Bx}(\theta_2)\cdot\frac{sec}{mm}$
$V_{By}(\theta_2)\cdot\frac{sec}{mm}$
1000
600
200
−200
−600
−1000
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 6-32

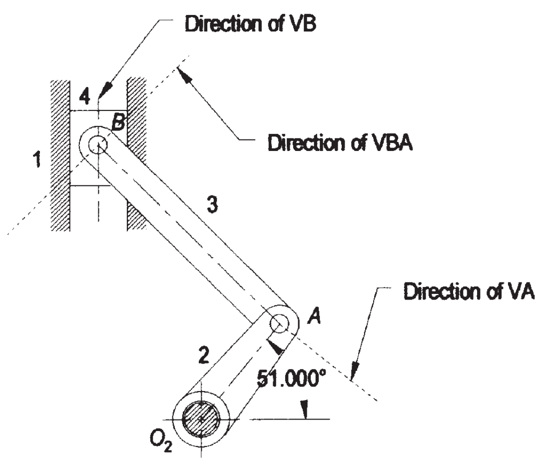
Statement: The offset slider-crank linkage in Figure P6-8f has the dimensions and crank angle given below. Find $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown if $\omega_2 = 25$ rad/sec CW. Use the velocity difference graphical method.

Given: Link lengths:

Link 2	$a := 63 \cdot mm$
Link 3	$b := 130 \cdot mm$
Offset	$c := -52 \cdot mm$
Crank angle:	$\theta_2 := 51 \cdot deg$
Input crank angular velocity	$\omega_2 := 25 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-8f and Mathcad file P0632.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A*.

$$V_A := a \cdot \omega_2 \qquad V_A = 1575.0 \frac{mm}{sec} \qquad \theta_A := 51 \cdot deg - 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*. The equation to be solved graphically is

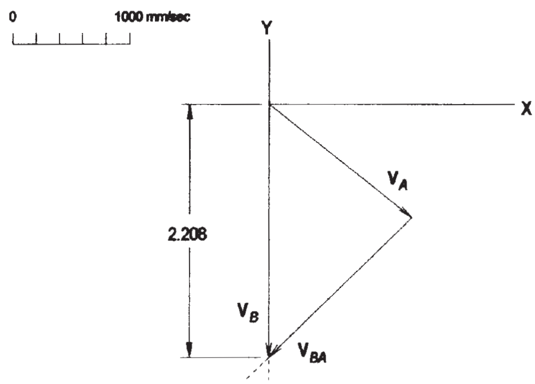
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{1000 \cdot mm \cdot sec^{-1}}{in}$

$V_B := 2.208 \cdot in \cdot k_v$ $\qquad V_B = 2208 \dfrac{mm}{sec}$ $\qquad \theta_B := 270 \cdot deg$

PROBLEM 6-33

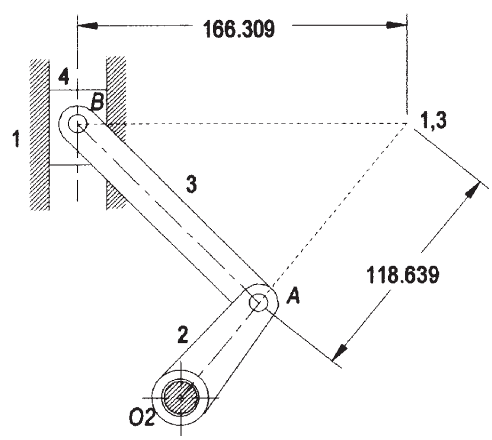
Statement: The offset slider-crank linkage in Figure P6-8f has the dimensions and crank angle given below. Find $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for $\omega_2 = 25$ rad/sec CW. Use the instant center graphical method.

Given: Link lengths:

Link 2	$a := 63 \cdot mm$
Link 3	$b := 130 \cdot mm$
Offset	$c := -52 \cdot mm$
Crank angle:	$\theta_2 := 51 \cdot deg$
Input crank angular velocity	$\omega_2 := 25 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-8f and Mathcad file P0633.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the *x* axis.



From the layout above:

$AI13 := 118.639 \cdot mm$ $\qquad BI13 := 166.309 \cdot mm$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point *A*.

$V_A := a \cdot \omega_2 \qquad V_A = 1575.0 \dfrac{mm}{sec}$

$\theta_{VA} := \theta_2 - 90 \cdot deg \qquad \theta_{VA} = -39.0\,deg$

3. Determine the angular velocity of link 3 using equation 6.9a.

$\omega_3 := \dfrac{V_A}{AI13} \qquad \omega_3 = 13.276 \dfrac{rad}{sec}$ CCW

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. Determine its direction by inspection.

$V_B := BI13 \cdot \omega_3 \qquad V_B = 2207.8 \dfrac{mm}{sec}$

$\theta_{VB} := 270 \cdot deg$

PROBLEM 6-34

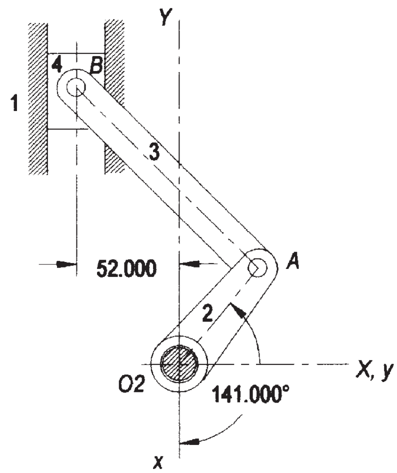
Statement: The offset slider-crank linkage in Figure P6-8f has the dimensions and crank angle given below. Find $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for $\omega_2 = 25$ rad/sec CW. Use an analytical method.

Given: Link lengths:

Link 2	$a := 63 \cdot mm$
Link 3	$b := 130 \cdot mm$
Offset	$c := -52 \cdot mm$
Crank angle:	$\theta_2 := 141 \cdot deg$ Local *xy* coordinate system
Input crank angular velocity	$\omega_2 := -25 \cdot rad \cdot sec^{-1}$

Solution: See Figure P6-8f and Mathcad file P0634.

1. Draw the linkage to a convenient scale.



2. Determine θ_3 and d using equations 4.16 for the crossed circuit.

$$\theta_3 := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_3 = 44.828\,deg$$

$$d := a \cdot cos(\theta_2) - b \cdot cos(\theta_3) \qquad d = -141.160\,mm$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3)} \cdot \omega_2 \qquad \omega_3 = 13.276\,\frac{rad}{sec}$$

4. Determine the velocity of pin A using equation 6.23a:

$$V_A := a \cdot \omega_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2))$$

$$V_A = 991.180 + 1224.005i\,\frac{mm}{sec} \qquad |V_A| = 1575.000\,\frac{mm}{sec} \qquad arg(V_A) = 51.000\,deg$$

In the global coordinate system, $\theta_{VA} := arg(V_A) - 90 \cdot deg$ $\qquad \theta_{VA} = -39.000\,deg$

5. Determine the velocity of pin *B* using equation 6.22b:

$V_B := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_3 \cdot \sin(\theta_3)$ $\qquad V_B = 2207.849 \frac{mm}{sec}$

$\mathbf{V_B} := V_B$

In the global coordinate system, $\theta_{VB} := arg(\mathbf{V_B}) - 90 \cdot deg$ $\qquad \theta_{VB} = -90.000\, deg$

PROBLEM 6-35

Statement: The offset slider-crank linkage in Figure P6-8f has the dimensions and crank angle given below. Find and plot $\mathbf{V}_A$, and $\mathbf{V}_B$ in the global coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 25$ rad/sec CW.

Given: Link lengths:

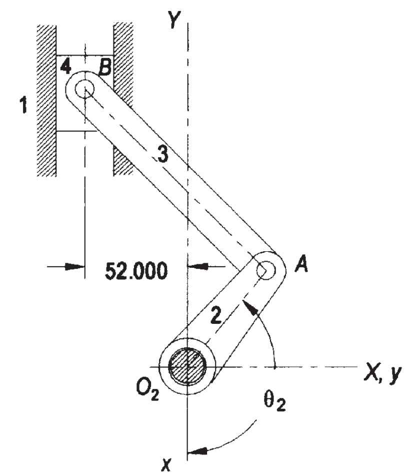
Link 2 $a := 63 \cdot mm$ Offset $c := -52 \cdot mm$

Link 3 $b := 130 \cdot mm$

Input crank angular velocity $\omega_2 := -25 \cdot rad \cdot sec^{-1}$

Solution: See Figure P6-8f and Mathcad file P0635.

1. Draw the linkage to a convenient scale. The coordinate rotation angle is $\alpha := -90 \cdot deg$



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine θ_3 using equations 4.16 for the crossed circuit.

$$\theta_3(\theta_2) := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right)$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Determine the *x* and *y* components of the velocity of pin *A* using equation 6.23a:

$$V_A(\theta_2) := a \cdot \omega_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2))$$

$$V_{Ax}(\theta_2) := Re(V_A(\theta_2)) \qquad V_{Ay}(\theta_2) := Im(V_A(\theta_2))$$

In the global coordinate system,

$$V_{AX}(\theta_2) := V_{Ay}(\theta_2) \qquad V_{AY}(\theta_2) := -V_{Ax}(\theta_2)$$

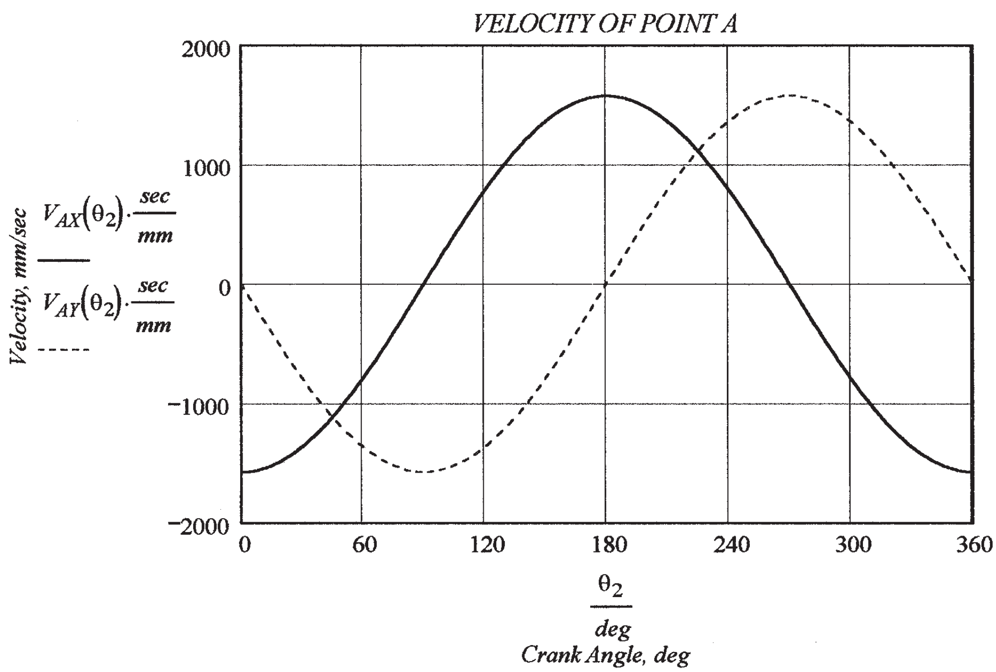
6. Determine the velocity of pin *B* using equation 6.22b:

$$V_{Bx}(\theta_2) := -a\cdot\omega_2\cdot sin(\theta_2) + b\cdot\omega_3(\theta_2)\cdot sin(\theta_3(\theta_2))$$

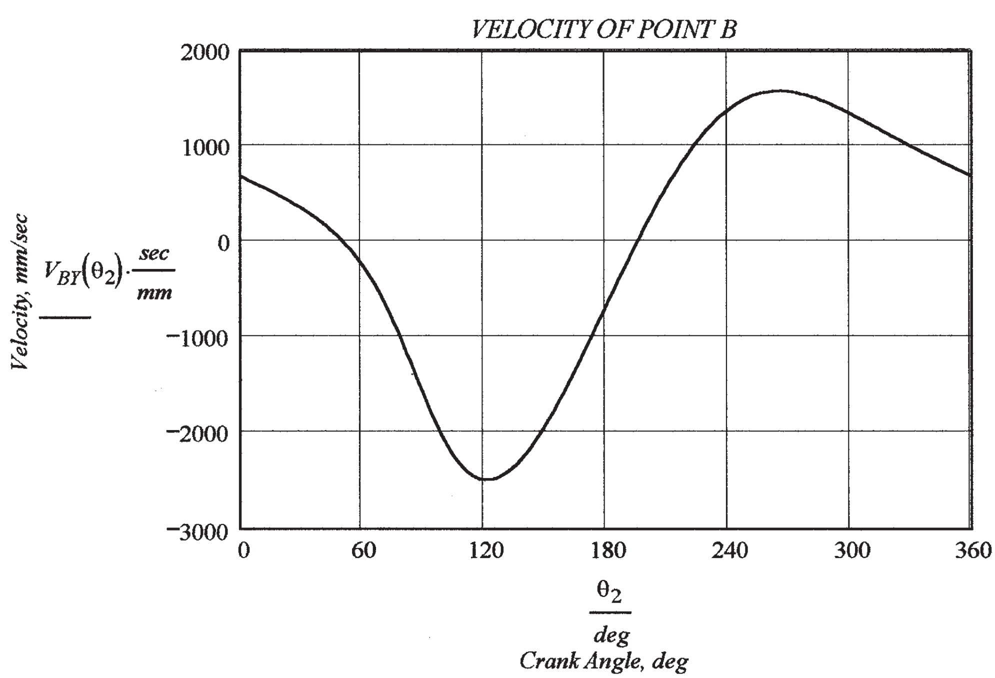
In the global coordinate system,

$$V_{BY}(\theta_2) := -V_{Bx}(\theta_2)$$

7. Plot the *x* and *y* components of the velocity of *A*.



7. Plot the velocity of point *B*.



PROBLEM 6-36

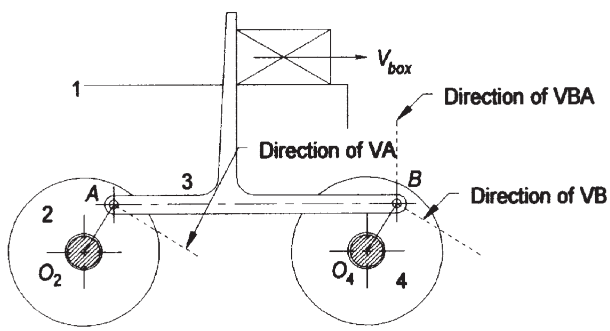
Statement: The linkage in Figure P6-8d has the dimensions and crank angle given below. Find $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ for the position shown for $\omega_2 = 30$ rad/sec clockwise (CW). Use the velocity difference graphical method.

Given: Link lengths:

Link 2	$a := 30 \cdot mm$	
Link 3	$b := 150 \cdot mm$	
Link 4	$c := 30 \cdot mm$	
Link	$d := 150 \cdot mm$	
Crank angle:	$\theta_2 := 58 \cdot deg$	Global *XY* system
Input crank angular velocity	$\omega_2 := 30 \cdot rad \cdot sec^{-1}$	CW

Solution: See Figure P6-8d and Mathcad file P0636.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A*.

$$V_A := a \cdot \omega_2 \qquad V_A = 900.000 \frac{mm}{sec} \qquad \theta_A := 58 \cdot deg - 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

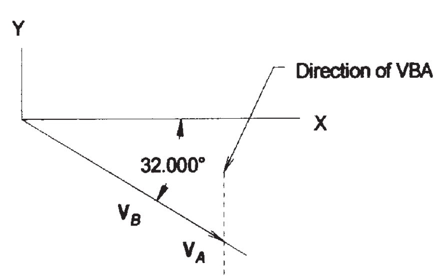
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

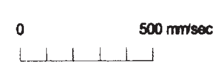
a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.





4. From the velocity triangle we have:

$V_B := V_A$ $\qquad V_B = 900.000\,\frac{mm}{sec}$ $\qquad \theta_B := -32 \cdot deg$

$V_{BA} := 0 \cdot \frac{in}{sec}$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$\omega_3 := \frac{V_{BA}}{b}$ $\qquad \omega_3 = 0.000\,\frac{rad}{sec}$

$\omega_4 := \frac{-V_B}{c}$ $\qquad \omega_4 = -30.000\,\frac{rad}{sec}$

6. Determine the magnitude of the vector $\mathbf{V}_{box}$. This is a special case Grashof mechanism in the parallelogram configuration. Link 3 does not rotate, therefore all points on link 3 have the same velocity. The velocity $\mathbf{V}_{box}$ is the horizontal component of $\mathbf{V}_A$.

$V_{box} := V_A \cdot cos(\theta_A)$ $\qquad V_{box} = 763.243\,\frac{mm}{sec}$

PROBLEM 6-37

Statement: The linkage in Figure P6-8d has the dimensions and effective crank angle given below. Find $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ in the global coordinate system for the position shown for $\omega_2 = 30$ rad/sec CW. Use an analytical method.

Given: Link lengths:

Link 2	$a := 30 \cdot mm$	Link 3	$b := 150 \cdot mm$
Link 4	$c := 30 \cdot mm$	Link 1	$d := 150 \cdot mm$

Crank angle: $\theta_2 := 58 \cdot deg$

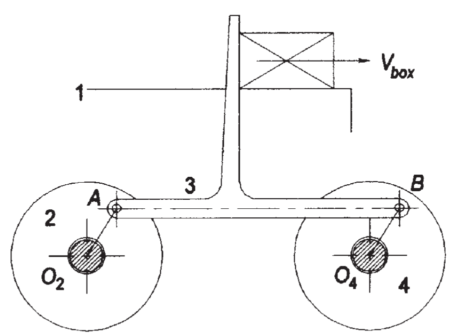
Input crank angular velocity $\omega_2 := -30 \cdot rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P6-8d and Mathcad file P0637.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 5.0000 \qquad K_2 = 5.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -6.1197 \qquad B = -1.6961 \qquad C = 2.8205$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41} := 2\cdot\left(atan2\left(2\cdot A, -B - \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) \qquad \theta_{41} = 418.000\,deg$$

$$\theta_{41} := \theta_{41} - 360\cdot deg \qquad \theta_{41} = 58.000\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.0000$$

$$K_5 = -5.0000$$

$$D := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad D = -8.9402$$

$$E := -2\cdot sin(\theta_2) \qquad E = -1.6961$$

$$F := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5 \qquad F = 0.0000$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31} := 2\cdot\left(atan2\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_{31} = 360.000\,deg$$

$$\theta_{31} := \theta_{31} - 360\cdot deg \qquad \theta_{31} = 0.000\,deg$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31} := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{41} - \theta_2)}{sin(\theta_{31} - \theta_{41})} \qquad \omega_{31} = 0.000\,\frac{rad}{sec}$$

$$\omega_{41} := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{31})}{sin(\theta_{41} - \theta_{31})} \qquad \omega_{41} = -30.000\,\frac{rad}{sec}$$

7. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$\mathbf{V_A} := a\cdot\omega_2\cdot\left(-sin(\theta_2) + j\cdot cos(\theta_2)\right)$$

$$\mathbf{V_A} = 763.243 - 476.927j\,\frac{mm}{sec} \qquad \left|\mathbf{V_A}\right| = 900.000\,\frac{mm}{sec} \qquad arg(\mathbf{V_A}) = -32.000\,deg$$

$$\mathbf{V_B} := c\cdot\omega_{41}\cdot\left(-sin(\theta_{41}) + j\cdot cos(\theta_{41})\right)$$

$$\mathbf{V_B} = 763.243 - 476.927j\,\frac{mm}{sec} \qquad \left|\mathbf{V_B}\right| = 900.000\,\frac{mm}{sec} \qquad arg(\mathbf{V_B}) = -32.000\,deg$$

8. Determine the velocity V_{box}. Since link 3 does not rotate (this is a special case Grashof linkage in the parallelogram mode), all points on it have the same velocity. Therefore,

$$V_{box} := \left|\mathbf{V_A}\right| \qquad V_{box} = 900.000\,\frac{mm}{sec}$$

PROBLEM 6-38

Statement: The linkage in Figure P6-8d has the dimensions and effective crank angle given below. Find and plot $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ in the global coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 30$ rad/sec CW.

Given: Link lengths:

Link 2	$a := 30 \cdot mm$	Link 3	$b := 150 \cdot mm$
Link 4	$c := 30 \cdot mm$	Link 1	$d := 150 \cdot mm$

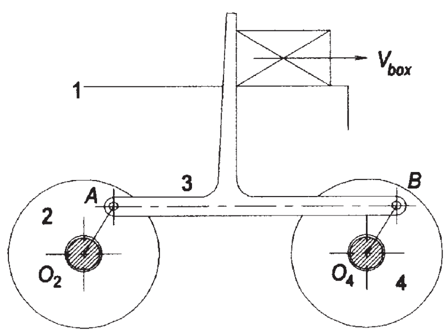
Input crank angular velocity $\omega_2 := -30 \cdot rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \land y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \land y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{cases}$$

Solution: See Figure P6-8d and Mathcad file

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this special-case Grashof double crank.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 5.0000 \qquad K_2 = 5.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.0000$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.0000$$

$$K_5 = -5.0000$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{41}(\theta_2) - \theta_2)}{sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{31}(\theta_2))}{sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of points *A* and *B* for the open circuit using equations 6.19.

$$\mathbf{V_A}(\theta_2) := a\cdot\omega_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2))$$

$$V_A(\theta_2) := |\mathbf{V_A}(\theta_2)|$$

$$\mathbf{V_B}(\theta_2) := c\cdot\omega_{41}(\theta_2)\cdot(-sin(\theta_{41}(\theta_2)) + j\cdot cos(\theta_{41}(\theta_2)))$$

$$V_B(\theta_2) := |\mathbf{V_B}(\theta_2)|$$

9. Plot the angular velocity of the output link, ω_4, and the magnitudes of the velocities at points *A* and *B*.

Since this is a special-case Grashof linkage in the parallelogram configuration, $\omega_3 = 0$ and $\omega_4 = \omega_2$ for all values of θ_2. Similarly, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ all have the same constant magnitude through all values of θ_2.

$$\omega_{41}(5\cdot deg) = -30.000\,\frac{rad}{sec} \qquad \omega_{41}(135\cdot deg) = -30.000\,\frac{rad}{sec}$$

$$V_A(5\cdot deg) = 900.000\,\frac{mm}{sec} \qquad V_A(135\cdot deg) = 900.000\,\frac{mm}{sec}$$

$$V_B(5\cdot deg) = 900.000\,\frac{mm}{sec} \qquad V_B(135\cdot deg) = 900.000\,\frac{mm}{sec}$$

PROBLEM 6-39

Statement: The linkage in Figure P6-8g has the dimensions and crank angle given below. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for ω_2 = 15 rad/sec clockwise (CW). Use the velocity difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot mm$	Link 2 (O_2 to C)	$a' := 49 \cdot mm$
Link 3 (A to B)	$b := 100 \cdot mm$	Link 5 (C to D)	$b' := 100 \cdot mm$
Link 4 (B to O_4)	$c := 153 \cdot mm$	Link 6 (D to O_6)	$c' := 153 \cdot mm$
Link 1 (O_2 to O_4)	$d := 87 \cdot mm$	Link 1 (O_2 to O_6)	$d' := 87 \cdot mm$

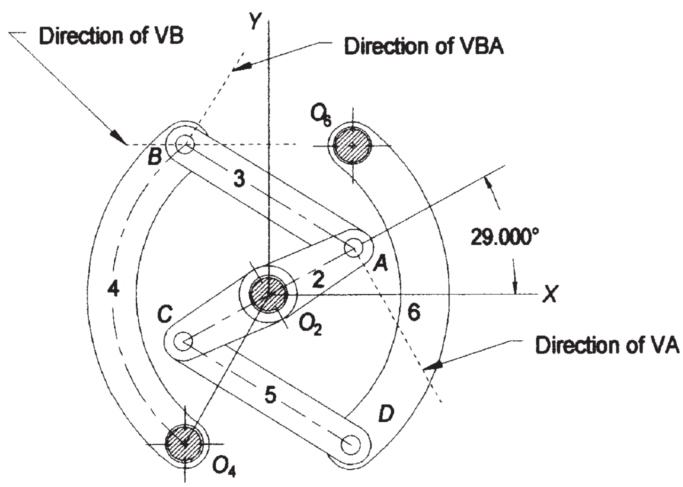
Crank angle: $\theta_2 := 29 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CW

Coordinate rotation angle $\alpha := 119 \cdot deg$ Global XY system to local xy system

Solution: See Figure P6-8g and Mathcad file P0639.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point B.

$$V_B := a \cdot \omega_2 \qquad V_B = 735.0 \frac{mm}{sec} \qquad \theta_B := 29 \cdot deg - 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point A, the magnitude of the relative velocity $\mathbf{V}_{AB}$, and the angular velocity of link 3. The equation to be solved graphically is

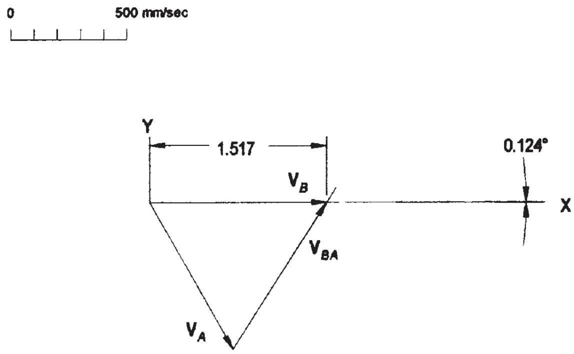
$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{AB}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.

b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{AB}$, magnitude unknown.

c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_A$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{AB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_A$ construction line and drawing $\mathbf{V}_A$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{AB}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{500 \cdot mm \cdot sec^{-1}}{in}$

$V_A := 1.517 \cdot in \cdot k_v$ $\qquad V_A = 758.5 \dfrac{mm}{sec}$ $\qquad \theta_A := -0.124 \cdot deg$

5. Determine the angular velocity of link 4 using equation 6.7.

$\omega_4 := \dfrac{-V_A}{c}$ $\qquad \omega_4 = -4.958 \dfrac{rad}{sec}$

PROBLEM 6-40

Statement: The linkage in Figure P6-8g has the dimensions and crank angle given below. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for ω_2 = 15 rad/sec clockwise (CW). Use the instant center graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot mm$	Link 2 (O_2 to C)	$a' := 49 \cdot mm$
Link 3 (A to B)	$b := 100 \cdot mm$	Link 5 (C to D)	$b' := 100 \cdot mm$
Link 4 (B to O_4)	$c := 153 \cdot mm$	Link 6 (D to O_6)	$c' := 153 \cdot mm$
Link 1 (O_2 to O_4)	$d := 87 \cdot mm$	Link 1 (O_2 to O_6)	$d' := 87 \cdot mm$

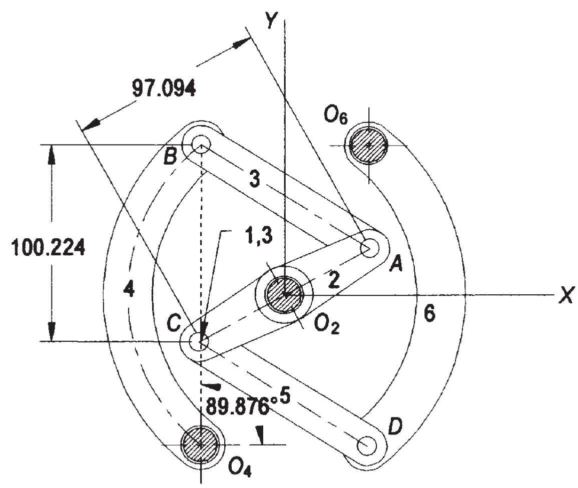
Crank angle: $\theta_2 := 29 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CW

Coordinate rotation angle $\alpha := 119 \cdot deg$ Global XY system to local xy system

Solution: See Figure P6-8g and Mathcad file P0640.

1. Draw the linkage to scale in the position given, find the instant centers, distances from the pin joints to the instant centers and the angles that links 3 and 4 make with the x axis.



From the layout above:

$AI13 := 97.094 \cdot mm$ $BI13 := 100.224 \cdot mm$ $\theta_4 := 89.876 \cdot deg$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$V_A := a \cdot \omega_2$ $V_A = 735.0 \frac{mm}{sec}$

$\theta_{VA} := \theta_2 - 90 \cdot deg$ $\theta_{VA} = -61.0 \, deg$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 7.570\,\frac{rad}{sec} \qquad \text{CW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. Determine its direction by inspection.

$$V_B := BI13\cdot\omega_3 \qquad V_B = 758.694\,\frac{mm}{sec}$$

$$\theta_{VB} := \theta_4 - 90\cdot deg \qquad \theta_{VB} = -0.124\,deg$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 4.959\,\frac{rad}{sec} \qquad \text{CW}$$

PROBLEM 6-41

Statement: The linkage in Figure P6-8g has the dimensions and crank angle given below. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ for the position shown for ω_2 = 15 rad/sec clockwise (CW). Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot mm$	Link 2 (O_2 to C)	$a' := 49 \cdot mm$
Link 3 (A to B)	$b := 100 \cdot mm$	Link 5 (C to D)	$b' := 100 \cdot mm$
Link 4 (B to O_4)	$c := 153 \cdot mm$	Link 6 (D to O_6)	$c' := 153 \cdot mm$
Link 1 (O_2 to O_4)	$d := 87 \cdot mm$	Link 1 (O_2 to O_6)	$d' := 87 \cdot mm$

Crank angle: $\theta_2 := 148 \cdot deg$ Local *xy* system

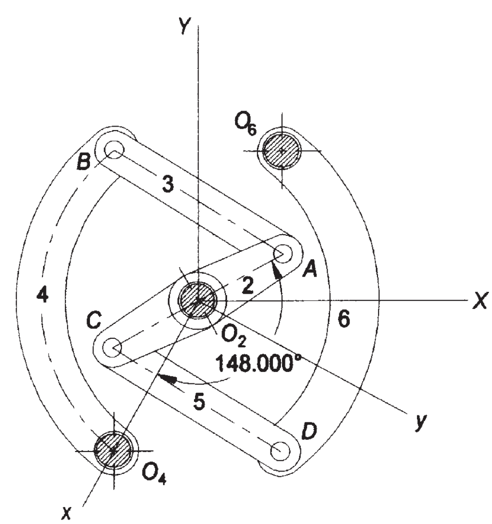
Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi & if\ (x = 0 \land y > 0) \\ return\ 1.5 \cdot \pi & if\ (x = 0 \land y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) & if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & otherwise \end{cases}$$

Solution: See Figure P6-8g and Mathcad file P0641.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.7755 \qquad K_2 = 0.5686$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.5592$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$A = -0.5821 \quad B = -1.0598 \quad C = 4.6650$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42} := 2\cdot\left(atan2\left(2\cdot A, -B + \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) \qquad \theta_{42} = 208.876\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.8700$$

$K_5 = 0.3509$

$$D := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad D = -3.0104$$

$$E := -2\cdot sin(\theta_2) \qquad E = -1.0598$$

$$F := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5 \qquad F = 2.2367$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32} := 2\cdot\left(atan2\left(2\cdot D, -E + \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_{32} = 266.892\,deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32} := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{42} - \theta_2)}{sin(\theta_{32} - \theta_{42})} \qquad \omega_{32} = -7.570\,\frac{rad}{sec}$$

$$\omega_{42} := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{32})}{sin(\theta_{42} - \theta_{32})} \qquad \omega_{42} = -4.959\,\frac{rad}{sec}$$

7. Determine the velocity of points *B* and *A* for the crossed circuit using equations 6.19.

$$\mathbf{V_A} := a\cdot\omega_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2))$$

$$\mathbf{V_A} = 389.491 + 623.315j\,\frac{mm}{sec} \qquad |\mathbf{V_A}| = 735.000\,\frac{mm}{sec} \qquad arg(\mathbf{V_A}) = 58.000\,deg$$

$$\mathbf{V_B} := c\cdot\omega_{42}\cdot(-sin(\theta_{42}) + j\cdot cos(\theta_{42}))$$

$$\mathbf{V_B} = -366.389 + 664.362j\,\frac{mm}{sec} \qquad |\mathbf{V_B}| = 758.694\,\frac{mm}{sec} \qquad arg(\mathbf{V_B}) = 118.876\,deg$$

PROBLEM 6-42

Statement: The linkage in Figure P6-8g has the dimensions and crank angle given below. Find and plot ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if ω_2 = 15 rad/sec clockwise (CW).

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot mm$	Link 2 (O_2 to C)	$a' := 49 \cdot mm$
Link 3 (A to B)	$b := 100 \cdot mm$	Link 5 (C to D)	$b' := 100 \cdot mm$
Link 4 (B to O_4)	$c := 153 \cdot mm$	Link 6 (D to O_6)	$c' := 153 \cdot mm$
Link 1 (O_2 to O_4)	$d := 87 \cdot mm$	Link 1 (O_2 to O_6)	$d' := 87 \cdot mm$

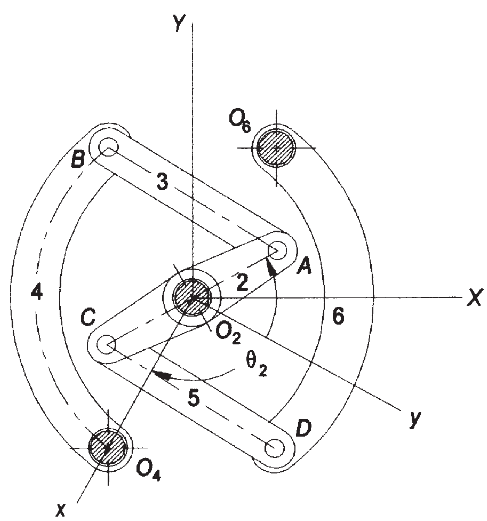
Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P6-8g and Mathcad file P0642.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 0.840$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = -6.338$$

$$\theta_{2toggle} := acos(arg_1) \qquad \theta_{2toggle} = 32.9\, deg$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := \theta_{2toggle}, \theta_{2toggle} + 1 \cdot deg .. 360 \cdot deg - \theta_{2toggle}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.7755 \qquad K_2 = 0.5686$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.5592$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{42}(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{(B(\theta_2))^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right] \right]$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.8700$$

$$K_5 = 0.3509$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_{32}(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{(E(\theta_2))^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right] \right]$$

7. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_{32}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{42}(\theta_2) - \theta_2)}{\sin(\theta_{32}(\theta_2) - \theta_{42}(\theta_2))}$$

$$\omega_{42}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{32}(\theta_2))}{\sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))}$$

8. Determine the velocity of points *A* and *B* for the crossed circuit using equations 6.19.

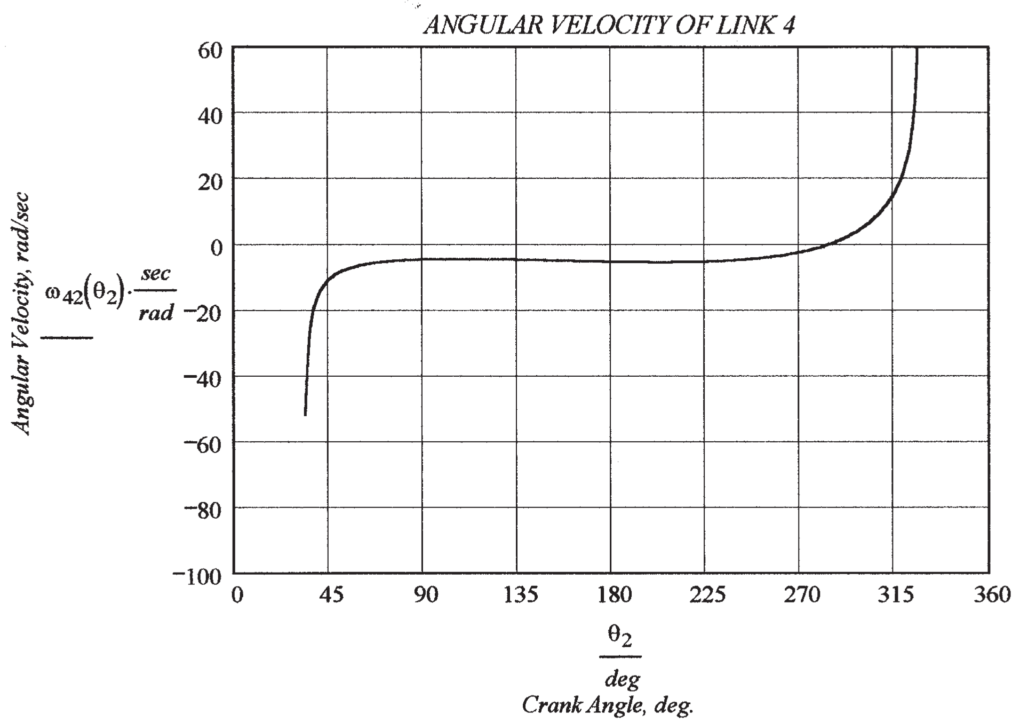
$$\mathbf{V_B}(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

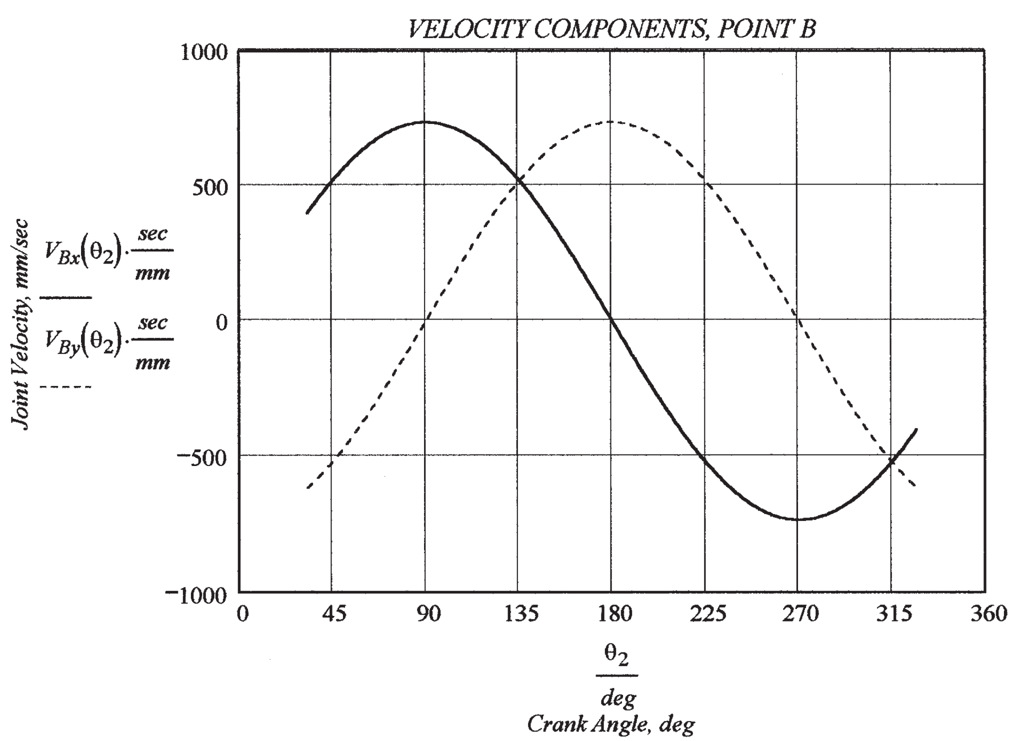
$$V_{Bx}(\theta_2) := Re(\mathbf{V_B}(\theta_2)) \qquad V_{By}(\theta_2) := Im(\mathbf{V_B}(\theta_2))$$

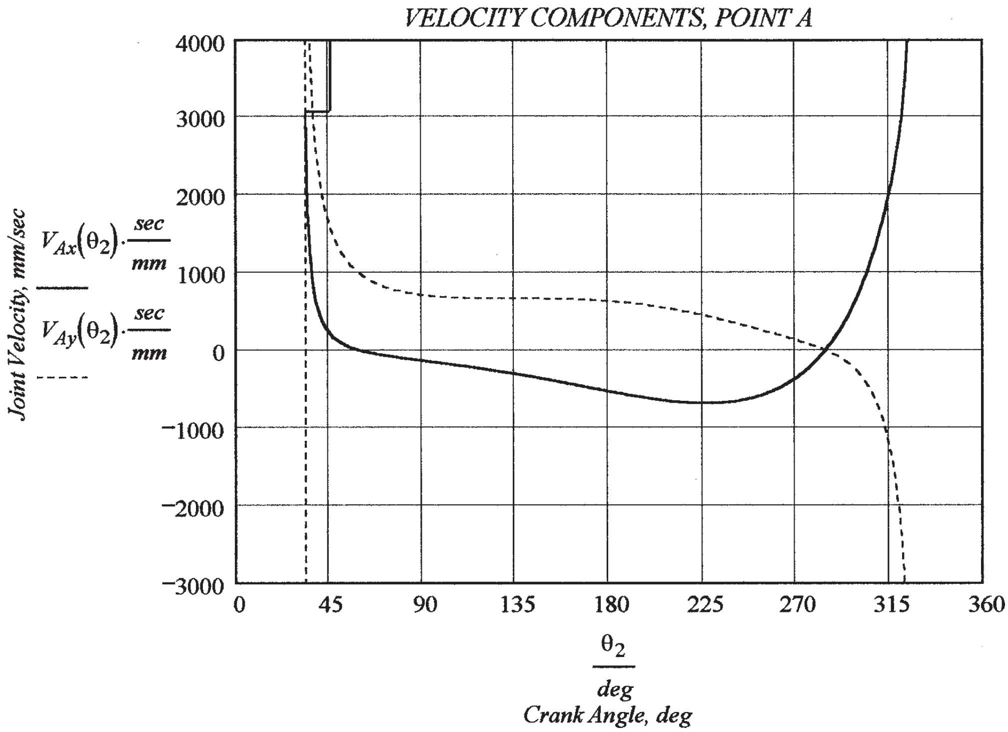
$$\mathbf{V_A}(\theta_2) := c \cdot \omega_{42}(\theta_2) \cdot (-\sin(\theta_{42}(\theta_2)) + j \cdot \cos(\theta_{42}(\theta_2)))$$

$$V_{Ax}(\theta_2) := Re(\mathbf{V_A}(\theta_2)) \qquad V_{Ay}(\theta_2) := Im(\mathbf{V_A}(\theta_2))$$

9. Plot the angular velocity of the output link, ω_4, and the *x* and *y* components of the velocities at points *B* and *A*.






VELOCITY COMPONENTS, POINT A
4000
3000
2000
1000
0
−1000
−2000
−3000
0
45
90
135
180
225
270
315
360
Joint Velocity, mm/sec
$V_{Ax}(\theta_2)\cdot\frac{sec}{mm}$
$V_{Ay}(\theta_2)\cdot\frac{sec}{mm}$
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 6-43

Statement: The 3-cylinder radial compressor in Figure P6-8c has the dimensions and crank angle given below. Find piston velocities $\mathbf{V}_6$, $\mathbf{V}_7$, and $\mathbf{V}_8$ for the position shown for $\omega_2 = 15$ rad/sec clockwise (CW). Use the velocity difference graphical method.

Given: Link lengths:

Link 2	$a := 19 \cdot mm$	Offset	$c := 0 \cdot mm$
Links 3, 4, and 5	$b := 70 \cdot mm$		

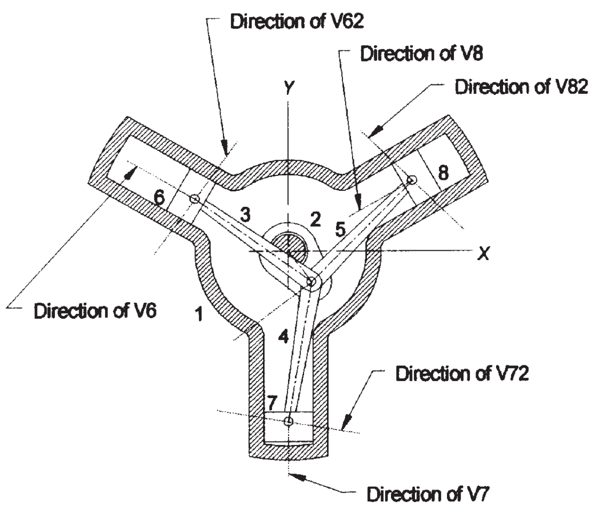
Crank angle: $\theta_2 := -53 \cdot deg$ Global *XY* system

Input crank angular velocity $\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CW

Cylinder angular spacing $\alpha := 120 \cdot deg$

Solution: See Figure P6-8c and Mathcad file P0643.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at rod pin on link *2*.

$$V_2 := a \cdot \omega_2 \qquad V_2 = 285.0 \frac{mm}{sec} \qquad \theta_2 := -53 \cdot deg - 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at pistons 6, 7, and 8.. The equations to be solved graphically are

$$\mathbf{V}_7 = \mathbf{V}_2 + \mathbf{V}_{72} \qquad \mathbf{V}_6 = \mathbf{V}_2 + \mathbf{V}_{62} \qquad \mathbf{V}_8 = \mathbf{V}_2 + \mathbf{V}_{82}$$

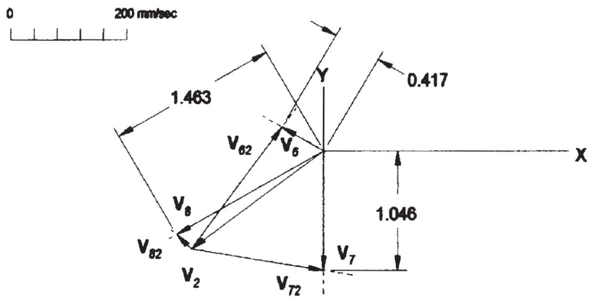
a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_2$.

b. From the tip of $\mathbf{V}_2$, draw a construction line with the direction of $\mathbf{V}_{72}$, magnitude unknown.

c. From the tail of $\mathbf{V}_2$, draw a construction line with the direction of $\mathbf{V}_7$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{72}$ from the tip of $\mathbf{V}_2$ to the intersection of the $\mathbf{V}_7$ construction line and drawing $\mathbf{V}_7$ from the tail of $\mathbf{V}_2$ to the intersection of the $\mathbf{V}_{72}$ construction line.

e. Repeat for $\mathbf{V}_6$ and $\mathbf{V}_8$.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{200 \cdot mm \cdot sec^{-1}}{in}$

$V_7 := 1.046 \cdot in \cdot k_v$ $\quad V_7 = 209.2 \dfrac{mm}{sec}$ $\quad \theta_{V7} := 270 \cdot deg$

$V_6 := 0.417 \cdot in \cdot k_v$ $\quad V_6 = 83.4 \dfrac{mm}{sec}$ $\quad \theta_{V6} := 150 \cdot deg$

$V_8 := 1.463 \cdot in \cdot k_v$ $\quad V_8 = 292.6 \dfrac{mm}{sec}$ $\quad \theta_{V8} := 210 \cdot deg$

PROBLEM 6-44

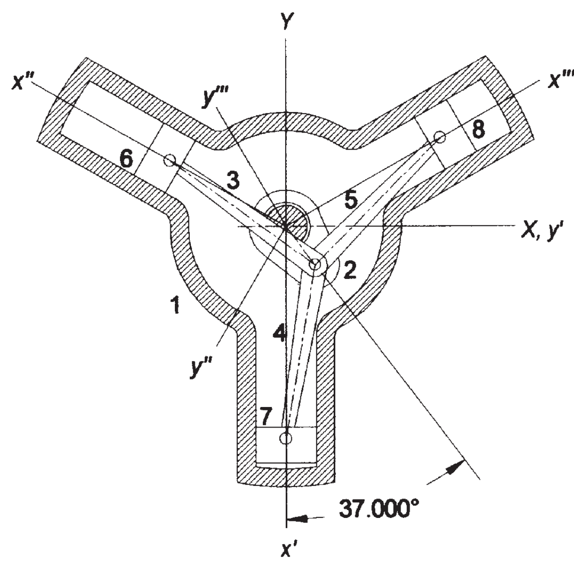
Statement: The 3-cylinder radial compressor in Figure P6-8c has the dimensions and crank angle given below. Find V_6, V_7, and V_8 for the position shown for $\omega_2 = 15$ rad/sec clockwise (CW). Use an analytical method.

Given: Link lengths:

Link 2	$a := 19 \cdot mm$	
Links 3, 4, and 5	$b := 70 \cdot mm$	
Offset	$c := 0 \cdot mm$	
Crank angle:	$\theta_2 := 37 \cdot deg$	Local $x'y'$ system
Input crank angular velocity	$\omega_2 := -15 \cdot rad \cdot sec^{-1}$	CW
Cylinder angular spacing	$\alpha := 120 \cdot deg$	

Solution: See Figure P6-8c and Mathcad file P0644.

1. Draw the linkage to scale and label it.



2. Determine θ_4 and d' using equation 4.17.

$$\theta_4 := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_4 = 170.599\, deg \qquad (x'y' \text{ system})$$

$$d' := a \cdot cos(\theta_2) - b \cdot cos(\theta_4) \qquad d' = 3.316\, in$$

3. Determine the angular velocity of link 4 using equation 6.22a:

$$\omega_4 := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_4)} \cdot \omega_2 \qquad \omega_4 = 3.296 \frac{rad}{sec}$$

4. Determine the velocity of the rod pin on link 2 using equation 6.23a:

$$\mathbf{V_2} := a \cdot \omega_2 \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right)$$

$$\mathbf{V_2} = 171.517 - 227.611i \frac{mm}{sec} \qquad \left|\mathbf{V_2}\right| = 285.000 \frac{mm}{sec} \qquad arg(\mathbf{V_2}) = -53.000\, deg$$

In the global coordinate system, $\theta_{V2} := arg(\mathbf{V_2}) - 90 \cdot deg$ $\qquad \theta_{V2} = -143.000\,deg$

5. Determine the velocity of piston 7 using equation 6.22b:

$$\mathbf{V_7} := -a \cdot \omega_2 \cdot sin(\theta_2) + b \cdot \omega_4 \cdot sin(\theta_4)$$

$$\mathbf{V_7} = 209.204\,\frac{mm}{sec} \qquad |\mathbf{V_7}| = 209.204\,\frac{mm}{sec} \qquad arg(\mathbf{V_7}) = 0.000\,deg$$

In the global coordinate system, $\theta_{V7} := arg(\mathbf{V_7}) - 90 \cdot deg$ $\qquad \theta_{V7} = -90.000\,deg$

6. Determine θ_3 and d'' using equation 4.17.

$$\theta_2 := \theta_2 + 120 \cdot deg \qquad \theta_2 = 157.000\,deg \qquad (x''y''\text{ system})$$

$$\theta_3 := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_3 = 173.912\,deg$$

$$d'' := a \cdot cos(\theta_2) - b \cdot cos(\theta_3) \qquad d'' = 2.052\,in$$

7. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3)} \cdot \omega_2 \qquad \omega_3 = -3.769\,\frac{rad}{sec}$$

8. Determine the velocity of the rod pin on link 2 using equation 6.23a:

$$\mathbf{V_2} := a \cdot \omega_2 \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right)$$

$$\mathbf{V_2} = 111.358 + 262.344i\,\frac{mm}{sec} \qquad |\mathbf{V_2}| = 285.000\,\frac{mm}{sec} \qquad arg(\mathbf{V_2}) = 67.000\,deg$$

In the global coordinate system, $\theta_{V2} := arg(\mathbf{V_2}) + 150 \cdot deg$ $\qquad \theta_{V2} = 217.000\,deg$

9. Determine the velocity of piston 6 using equation 6.22b:

$$\mathbf{V_6} := -a \cdot \omega_2 \cdot sin(\theta_2) + b \cdot \omega_3 \cdot sin(\theta_3)$$

$$\mathbf{V_6} = 83.378\,\frac{mm}{sec} \qquad |\mathbf{V_6}| = 83.378\,\frac{mm}{sec} \qquad arg(\mathbf{V_6}) = 0.000\,deg$$

In the global coordinate system, $\theta_{V6} := arg(\mathbf{V_6}) + 150 \cdot deg$ $\qquad \theta_{V6} = 150.000\,deg$

10. Determine θ_5 and d''' using equation 4.17.

$$\theta_2 := \theta_2 + 120 \cdot deg \qquad \theta_2 = 277.000\,deg \qquad (x'''y'''\text{ system})$$

$$\theta_5 := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_5 = 195.629\,deg$$

$$d''' := a \cdot cos(\theta_2) - b \cdot cos(\theta_5) \qquad d''' = 2.745\,in$$

11. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_5)} \cdot \omega_2 \qquad \omega_5 = 0.515 \frac{rad}{sec}$$

12. Determine the velocity of the rod pin on link 2 using equation 6.23a:

$$\mathbf{V_2} := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V_2} = -282.876 - 34.733i \frac{mm}{sec} \qquad |\mathbf{V_2}| = 285.000 \frac{mm}{sec} \qquad arg(\mathbf{V_2}) = -173.000\, deg$$

In the global coordinate system, $\theta_{V2} := arg(\mathbf{V_2}) + 30 \cdot deg$ $\qquad \theta_{V2} = -143.000\, deg$

13. Determine the velocity of piston 8 using equation 6.22b:

$$\mathbf{V_8} := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_5 \cdot \sin(\theta_5)$$

$$\mathbf{V_8} = -292.592 \frac{mm}{sec} \qquad |\mathbf{V_8}| = 292.592 \frac{mm}{sec} \qquad arg(\mathbf{V_8}) = 180.000\, deg$$

In the global coordinate system, $\theta_{V8} := arg(\mathbf{V_8}) + 30 \cdot deg$ $\qquad \theta_{V8} = 210.000\, deg$

PROBLEM 6-45

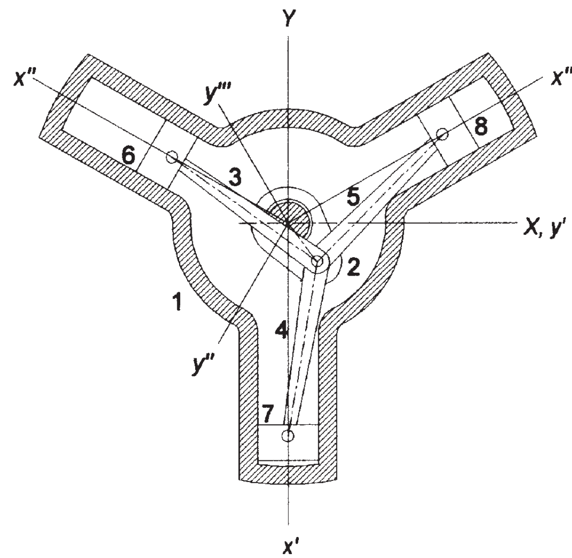
Statement: The 3-cylinder radial compressor in Figure P6-8c has the dimensions and crank angle given below. Find and plot $\mathbf{V}_6$, $\mathbf{V}_7$, and $\mathbf{V}_8$ for one revolution of the crank if $\omega_2 = 15$ rad/sec clockwise (CW).

Given: Link lengths:

Link 2	$a := 19 \cdot mm$	Offset	$c := 0 \cdot mm$
Links 3, 4, and 5	$b := 70 \cdot mm$		
Input crank angular velocity	$\omega_2 := -15 \cdot rad \cdot sec^{-1}$ CW		
Cylinder angular spacing	$\alpha := 120 \cdot deg$		

Solution: See Figure P6-8c and Mathcad file P0645.

1. Draw the linkage to scale and label it. Note that there are three local coordinate systems.



2. Determine the range of motion for this slider-crank linkage. This will be the same in each coordinate frame.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine θ_4 using equation 4.17.

$$\theta_4(\theta_2) := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad (x'y' \text{ system})$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_4(\theta_2) := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_4(\theta_2))} \cdot \omega_2$$

5. Determine the velocity of piston 7 using equation 6.22b:

$$V_7(\theta_2) := -a \cdot \omega_2 \cdot sin(\theta_2) + b \cdot \omega_4(\theta_2) \cdot sin(\theta_4(\theta_2))$$

6. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := asin\left(-\frac{a \cdot sin(\theta_2 + \alpha) - c}{b}\right) + \pi \qquad (x''y'' \text{ system})$$

7. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2 + \alpha)}{\cos(\theta_3(\theta_2))} \cdot \omega_2$$

8. Determine the velocity of piston 6 using equation 6.22b:

$$V_6(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2 + \alpha) + b \cdot \omega_3(\theta_2) \cdot \sin(\theta_3(\theta_2))$$

9. Determine θ_5 and d''' using equation 4.17.

$$\theta_5(\theta_2) := \operatorname{asin}\left(\frac{a \cdot \sin(\theta_2 + 2 \cdot \alpha) - c}{b}\right) + \pi \qquad (x'''y''' \text{ system})$$

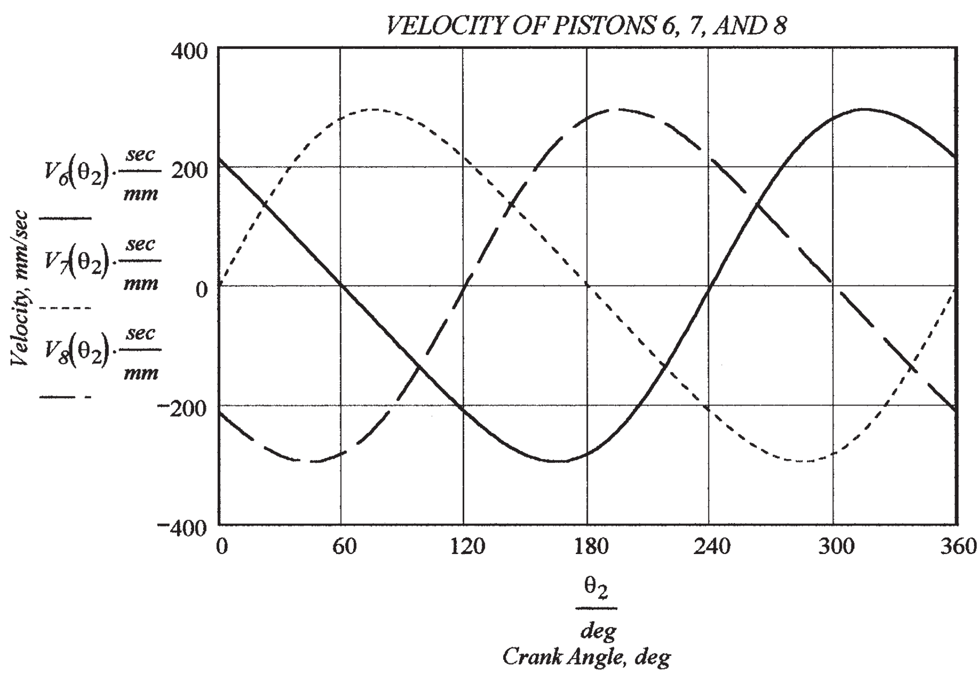
10. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2 + 2 \cdot \alpha)}{\cos(\theta_5(\theta_2))} \cdot \omega_2$$

11. Determine the velocity of piston 8 using equation 6.22b:

$$V_8(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2 + 2 \cdot \alpha) + b \cdot \omega_5(\theta_2) \cdot \sin(\theta_5(\theta_2))$$

12. Plot the velocities of pistons 6, 7, and 8.



PROBLEM 6-46

Statement: Figure P6-9 shows a linkage in one position. Find the instantaneous velocities of points A, B, and P if link O_2A is rotating CW at 40 rad/sec.

Given: Link lengths:

Link 2	$a := 5.00 \cdot in$	Link 3	$b := 4.40 \cdot in$
Link 4	$c := 5.00 \cdot in$	Link 1	$d := 9.50 \cdot in$

Coupler point: $R_{pa} := 8.90 \cdot in$ $\quad \delta_3 := 56 \cdot deg$

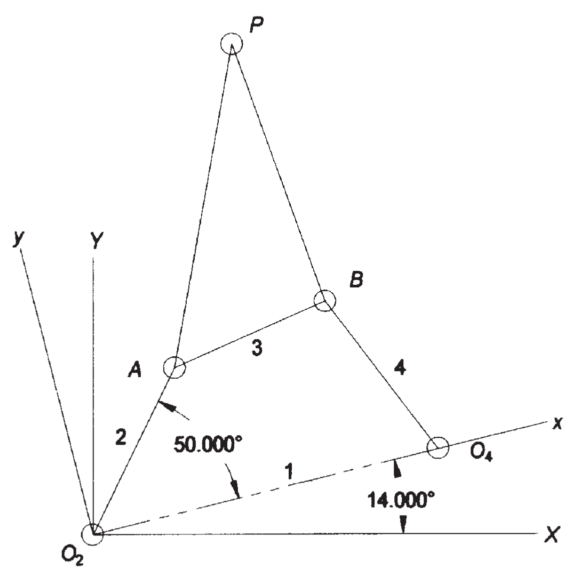
Crank angle and speed: $\theta_2 := 50 \cdot deg$ $\quad \omega_2 := -40 \cdot rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi & if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi & if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) & if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & otherwise \end{cases}$$

Solution: See Figure P6-9 and Mathcad file P0646.

1. Draw the linkage to scale and label it. All calculated angles are in the local xy system.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.9000 \qquad K_2 = 1.9000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.4178$$

$$A := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B := -2 \cdot sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

$A = -0.0607 \quad B = -1.5321 \quad C = 2.4537$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41} := 2\cdot\left(atan2\left(2\cdot A, -B - \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) \qquad \theta_{41} = 473.008\,deg$$

$$\theta_{41} := \theta_{41} - 360\cdot deg \qquad \theta_{41} = 113.008\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 2.1591$$

$$K_5 = -2.4911$$

$$D := \cos(\theta_2) - K_1 + K_4\cdot\cos(\theta_2) + K_5 \qquad D = -2.3605$$

$$E := -2\cdot\sin(\theta_2) \qquad E = -1.5321$$

$$F := K_1 + (K_4 - 1)\cdot\cos(\theta_2) + K_5 \qquad F = 0.1539$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31} := 2\cdot\left(atan2\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_{31} = 370.105\,deg$$

$$\theta_{31} := \theta_{31} - 360\cdot deg \qquad \theta_{31} = 10.105\,deg$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31} := \frac{a\cdot\omega_2}{b}\cdot\frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_{31} - \theta_{41})} \qquad \omega_{31} = 41.552\,\frac{rad}{sec}$$

$$\omega_{41} := \frac{a\cdot\omega_2}{c}\cdot\frac{\sin(\theta_2 - \theta_{31})}{\sin(\theta_{41} - \theta_{31})} \qquad \omega_{41} = -26.320\,\frac{rad}{sec}$$

7. Determine the velocity of points A and B for the open circuit using equations 6.19.

$$\mathbf{V_A} := a\cdot\omega_2\cdot\left(-\sin(\theta_2) + j\cdot\cos(\theta_2)\right)$$

$$\mathbf{V_A} = 153.209 - 128.558j\,\frac{in}{sec} \qquad \left|\mathbf{V_A}\right| = 200.000\,\frac{in}{sec} \qquad arg(\mathbf{V_A}) = -40.000\,deg$$

$$\mathbf{V_B} := c\cdot\omega_{41}\cdot\left(-\sin(\theta_{41}) + j\cdot\cos(\theta_{41})\right)$$

$$\mathbf{V_B} = 121.130 + 51.437j\,\frac{in}{sec} \qquad \left|\mathbf{V_B}\right| = 131.598\,\frac{in}{sec} \qquad arg(\mathbf{V_B}) = 23.008\,deg$$

8. Determine the velocity of the coupler point P for the open circuit using equations 6.36.

$$\mathbf{V_{PA}} := R_{pa}\cdot\omega_{31}\cdot\left(-\sin(\theta_{31} + \delta_3) + j\cdot\cos(\theta_{31} + \delta_3)\right)$$

$$\mathbf{V_{PA}} = -338.121 + 149.797j\,\frac{in}{sec}$$

$$\mathbf{V_P} := \mathbf{V_A} + \mathbf{V_{PA}}$$

$$\mathbf{V_P} = -184.912 + 21.239j\,\frac{in}{sec} \qquad \left|\mathbf{V_P}\right| = 186.128\,\frac{in}{sec} \qquad arg(\mathbf{V_P}) = 173.448\,deg$$

PROBLEM 6-47

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the magnitude and direction of the velocity of point *P* in Figure 3-37a as a function of θ_2 for a constant ω_2 = 1 rad/sec CCW. Also calculate and plot the velocity of point *P* versus point *A*.

Given: Link lengths:

Link 2 (O_2 to *A*) $a := 1.00$ Link 3 (*A* to *B*) $b := 2.06$

Link 4 (*B* to O_4) $c := 2.33$ Link 1 (O_2 to O_4) $d := 2.22$

Coupler point: $R_{pa} := 3.06$ $\delta_3 := -31 \cdot deg$

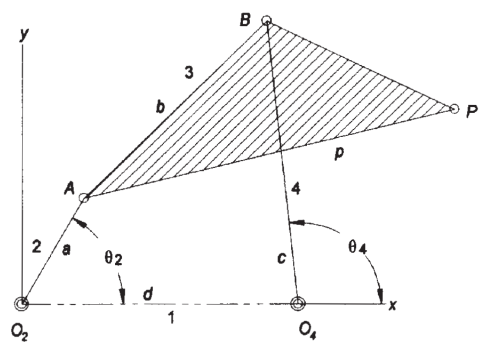
Crank speed: $\omega_2 := 1 \cdot rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure P6-10 and Mathcad file P0647.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.2200 \qquad K_2 = 0.9528$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.5265$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.0777 \qquad K_5 = -1.1512$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{41}(\theta_2) - \theta_2)}{sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{31}(\theta_2))}{sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point A using equations 6.19.

$$\mathbf{V_A}(\theta_2) := a\cdot\omega_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) \qquad V_A(\theta_2) := |\mathbf{V_A}(\theta_2)|$$

9. Determine the velocity of the coupler point P using equations 6.36.

$$\mathbf{V_{PA}}(\theta_2) := R_{pa}\cdot\omega_{31}(\theta_2)\cdot(-sin(\theta_{31}(\theta_2) + \delta_3) + j\cdot cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V_P}(\theta_2) := \mathbf{V_A}(\theta_2) + \mathbf{V_{PA}}(\theta_2)$$

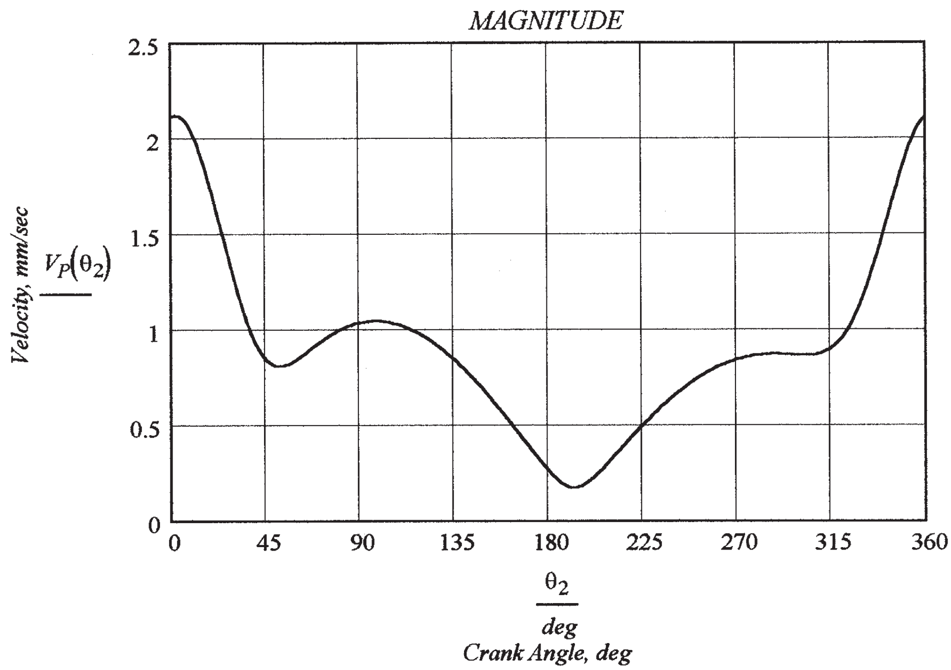
10. Plot the magnitude and direction of the velocity at coupler point P.

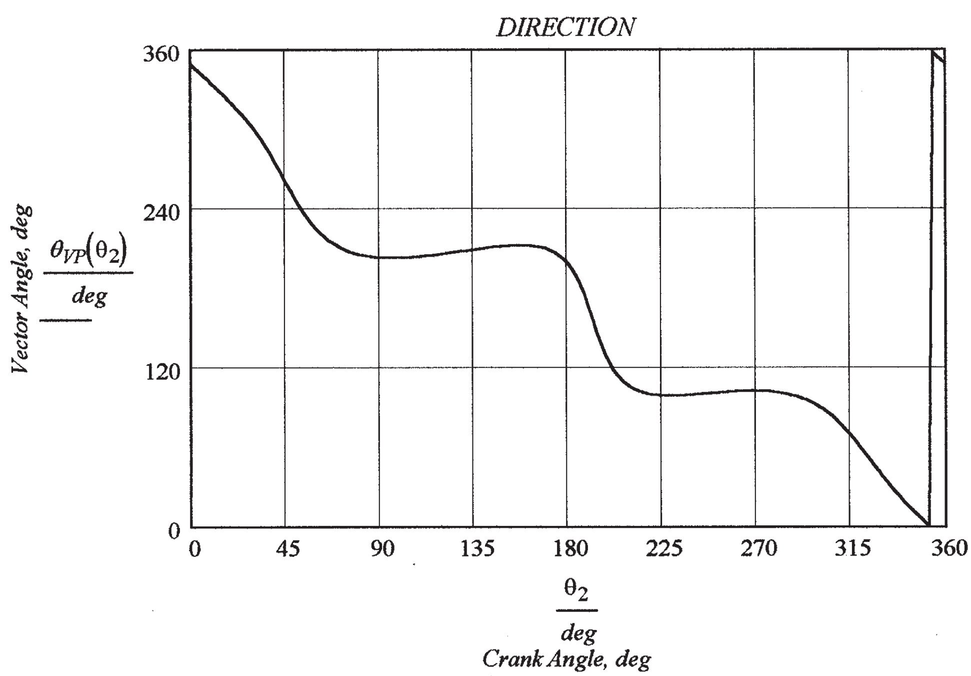
Magnitude: $V_P(\theta_2) := |\mathbf{V_P}(\theta_2)|$

Direction: $\theta_{VP1}(\theta_2) := arg(\mathbf{V_P}(\theta_2))$

$$\theta_{VP}(\theta_2) := if(\theta_{VP1}(\theta_2) < 0, \theta_{VP1}(\theta_2) + 2\cdot\pi, \theta_{VP1}(\theta_2))$$

(See next page)


MAGNITUDE
2.5
2
1.5
1
0.5
0
Velocity, mm/sec
$V_P(\theta_2)$
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg


DIRECTION
360
240
120
0
Vector Angle, deg
$\frac{\theta_{VP}(\theta_2)}{deg}$
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 6-48

Statement: Figure P6-11 shows a linkage that operates at 500 crank rpm. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of point B at 2-deg increments of crank angle. Check your result with program FOURBAR.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.000 \cdot in$	Link 3 (A to B)	$b := 8.375 \cdot in$
Link 4 (B to O_4)	$c := 7.187 \cdot in$	Link 1 (O_2 to O_4)	$d := 9.625 \cdot in$

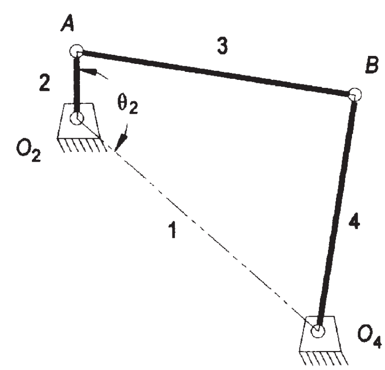
Input crank angular velocity $\omega_2 := 500 \cdot rpm$ $\qquad \omega_2 = 52.360\, rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P6-11 and Mathcad file P06

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 4.8125 \qquad K_2 = 1.3392$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.7186$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.1493$$

$$K_5 = -3.4367$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{41}(\theta_2) - \theta_2)}{sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

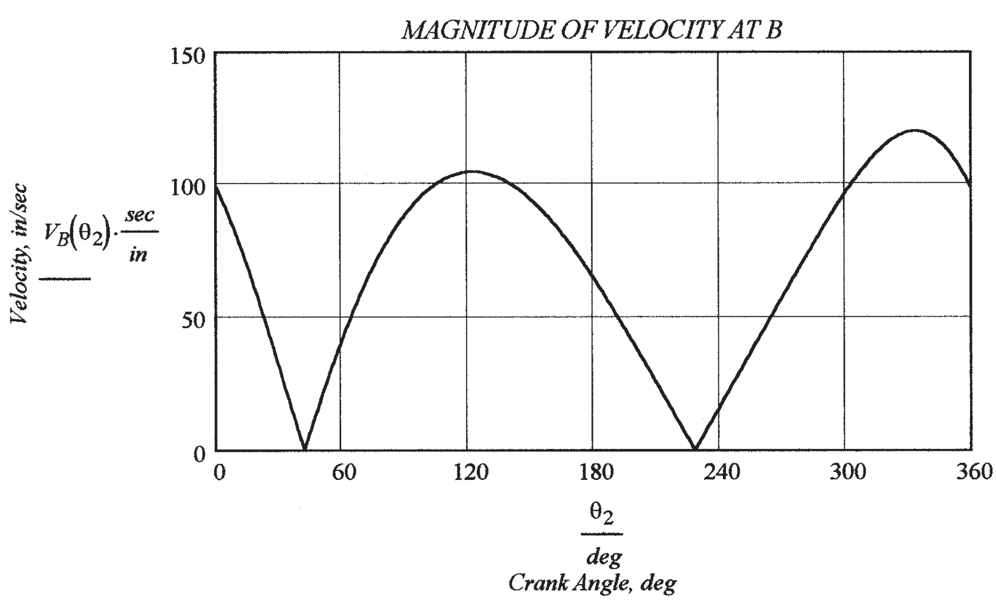
$$\omega_{41}(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{31}(\theta_2))}{sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point B using equations 6.19.

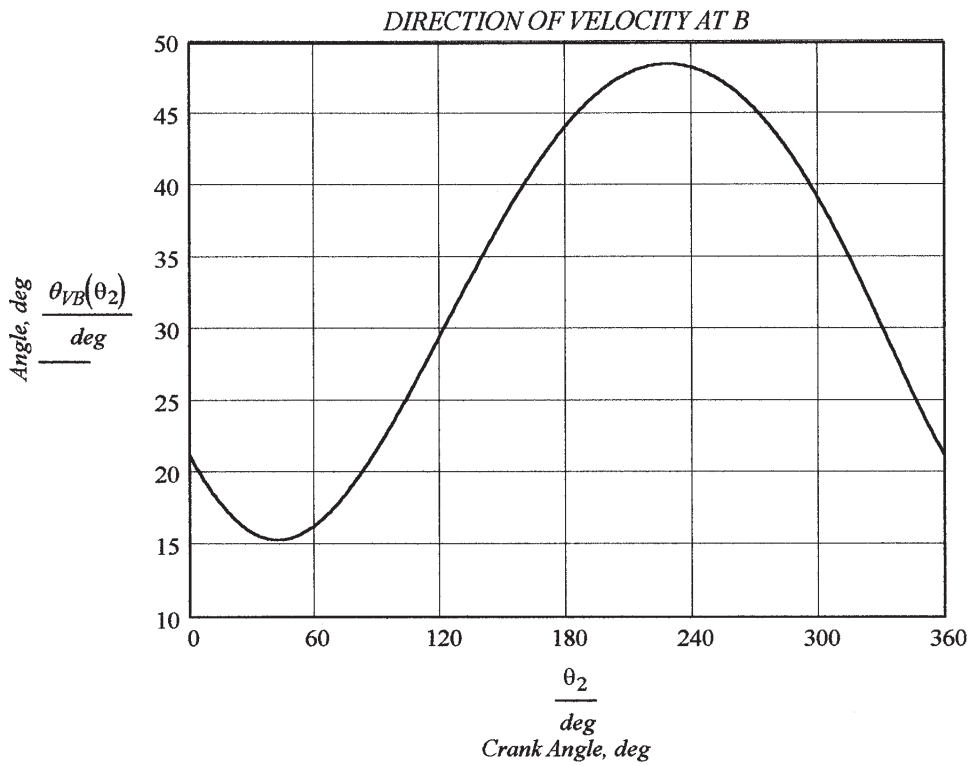
$$\mathbf{V_B}(\theta_2) := c\cdot\omega_{41}(\theta_2)\cdot\left(-sin(\theta_{41}(\theta_2)) + j\cdot cos(\theta_{41}(\theta_2))\right)$$

$$V_B(\theta_2) := \left|\mathbf{V_B}(\theta_2)\right| \qquad \theta_{VB1}(\theta_2) := arg(\mathbf{V_B}(\theta_2))$$

9. Plot the magnitude and angle of the velocity at point B.



$$\theta_{VB}(\theta_2) := if(\theta_{VB1}(\theta_2) < 0, \theta_{VB1}(\theta_2) + \pi, \theta_{VB1}(\theta_2))$$



PROBLEM 6-49

Statement: Figure P6-12 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point *P* at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to *A*) $a := 0.785 \cdot in$ Link 3 (*A* to *B*) $b := 0.356 \cdot in$

Link 4 (*B* to O_4) $c := 0.950 \cdot in$ Link 1 (O_2 to O_4) $d := 0.544 \cdot in$

Coupler point: $R_{pa} := 1.09 \cdot in$ $\delta_3 := 0 \cdot deg$

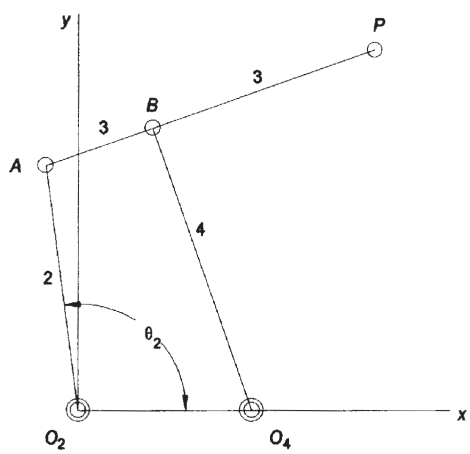
Crank speed: $\omega_2 := 20 \cdot rpm$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} \text{return } 0.5 \cdot \pi & \text{if } (x = 0 \wedge y > 0) \\ \text{return } 1.5 \cdot \pi & \text{if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) & \text{if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & \text{otherwise} \end{cases}$$

Solution: See Figure P6-12 and Mathcad file P0649.

1. Draw the linkage to scale and label it.



2. Using the geometry defined in Figure 3-1a in the text, determine the input crank angles (relative to the line O_2O_4) at which links 2 and 3, and 3 and 4 are in toggle.

$$\theta_{20} := acos\left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d}\right] \qquad \theta_{20} = 158.286\,deg$$

$$\theta_2 := -\theta_{20}, -\theta_{20} + 2 \cdot deg .. \theta_{20}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.6930 \qquad K_2 = 0.5726$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.1317$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.5281$$

$$K_5 = -0.2440$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point A using equations 6.19.

$$\mathbf{V_A}(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

9. Determine the velocity of the coupler point P using equations 6.36.

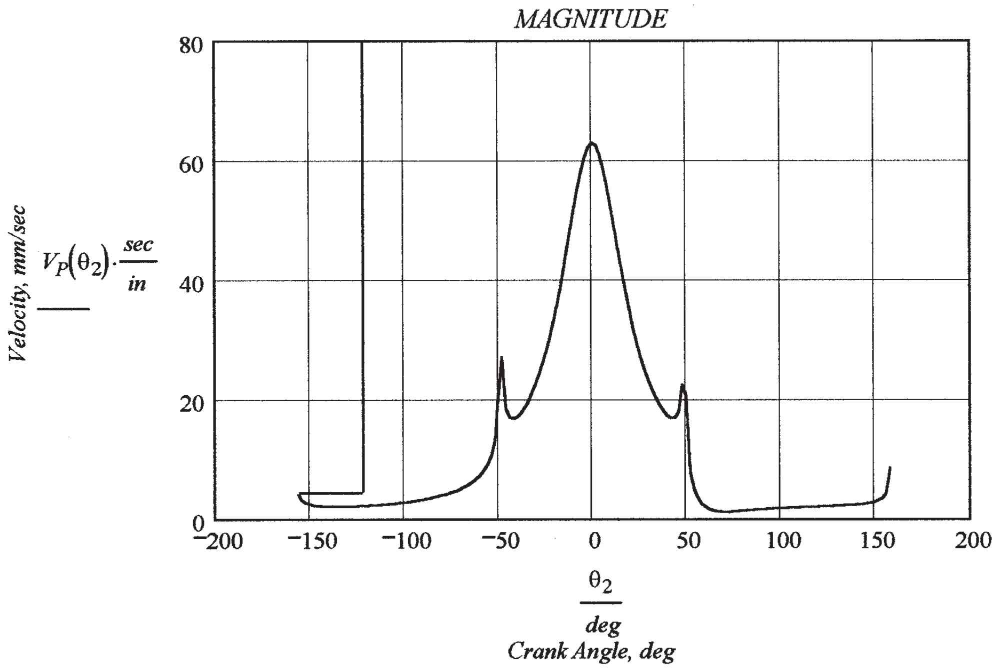
$$\mathbf{V_{PA}}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

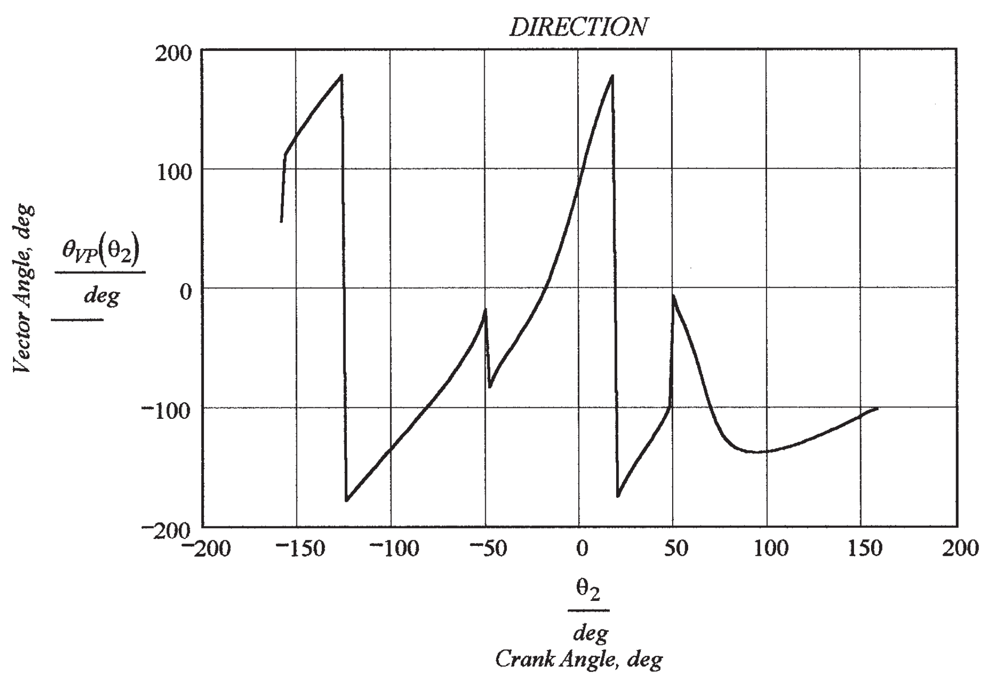
$$\mathbf{V_P}(\theta_2) := \mathbf{V_A}(\theta_2) + \mathbf{V_{PA}}(\theta_2)$$

10. Plot the magnitude and direction of the coupler point P.

Magnitude: $V_P(\theta_2) := |\mathbf{V_P}(\theta_2)|$

Direction: $\theta_{VP}(\theta_2) := arg(\mathbf{V_P}(\theta_2))$





PROBLEM 6-50

Statement: Figure P6-13 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$a := 0.86\cdot in$	Link 3 (A to B)	$b := 1.85\cdot in$
Link 4 (B to O_4)	$c := 0.86\cdot in$	Link 1 (O_2 to O_4)	$d := 2.22\cdot in$

Coupler point: $R_{pa} := 1.33\cdot in$ $\quad \delta_3 := 0\cdot deg$

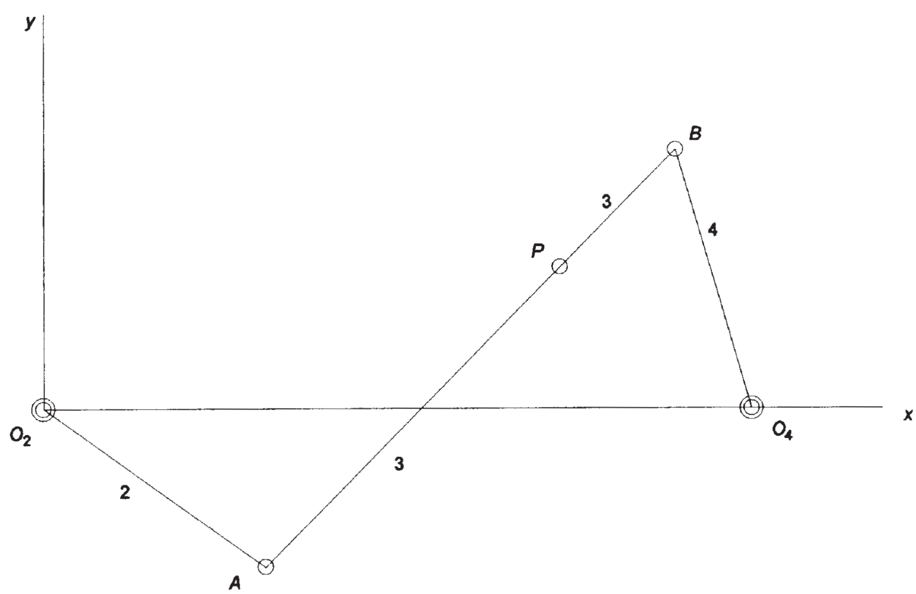
Crank speed: $\omega_2 := 80\cdot rpm$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P6-13 and Mathcad file P0650.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} + \frac{b\cdot c}{(a\cdot d)} \qquad arg_1 = 1.228$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} - \frac{b\cdot c}{(a\cdot d)} \qquad arg_2 = -0.439$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 116.0\,deg$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 1\cdot deg .. \theta_{2toggle}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.5814 \qquad K_2 = 2.5814$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)} \qquad K_3 = 2.0181$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.2000$$

$$K_5 = -2.6244$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{41}(\theta_2) - \theta_2)}{sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{31}(\theta_2))}{sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point *A* using equations 6.19.

$$\mathbf{V_A}(\theta_2) := a\cdot\omega_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2))$$

9. Determine the velocity of the coupler point *P* using equations 6.36.

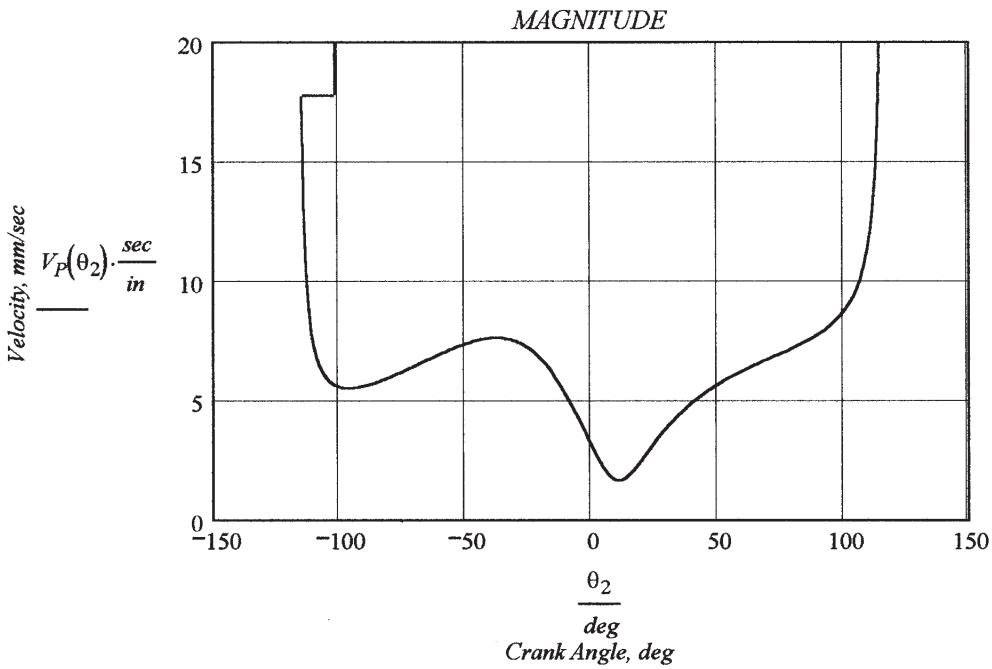
$$\mathbf{V_{PA}}(\theta_2) := R_{pa}\cdot\omega_{31}(\theta_2)\cdot(-sin(\theta_{31}(\theta_2) + \delta_3) + j\cdot cos(\theta_{31}(\theta_2) + \delta_3))$$

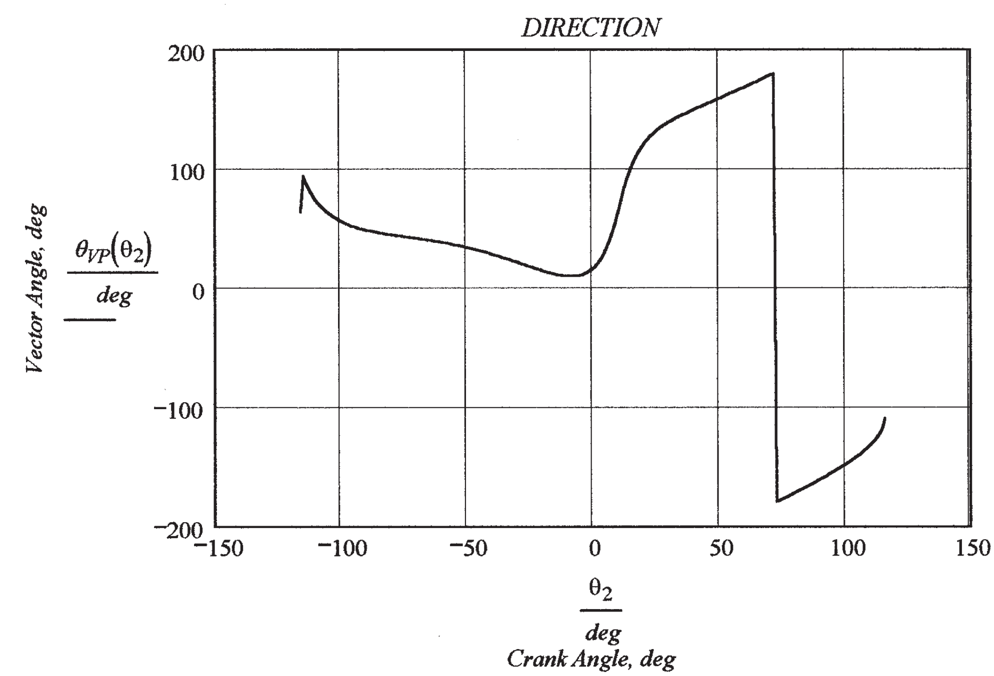
$$\mathbf{V_P}(\theta_2) := \mathbf{V_A}(\theta_2) + \mathbf{V_{PA}}(\theta_2)$$

10. Plot the magnitude and direction of the coupler point *P*.

Magnitude: $V_P(\theta_2) := \left|\mathbf{V_P}(\theta_2)\right|$

Direction: $\theta_{VP}(\theta_2) := arg\left(\mathbf{V_P}(\theta_2)\right)$





PROBLEM 6-51

Statement: Figure P6-14 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point *P* at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to *A*) $a := 0.72 \cdot in$ Link 3 (*A* to *B*) $b := 0.68 \cdot in$

Link 4 (*B* to O_4) $c := 0.85 \cdot in$ Link 1 (O_2 to O_4) $d := 1.82 \cdot in$

Coupler point: $R_{pa} := 0.97 \cdot in$ $\delta_3 := 54 \cdot deg$

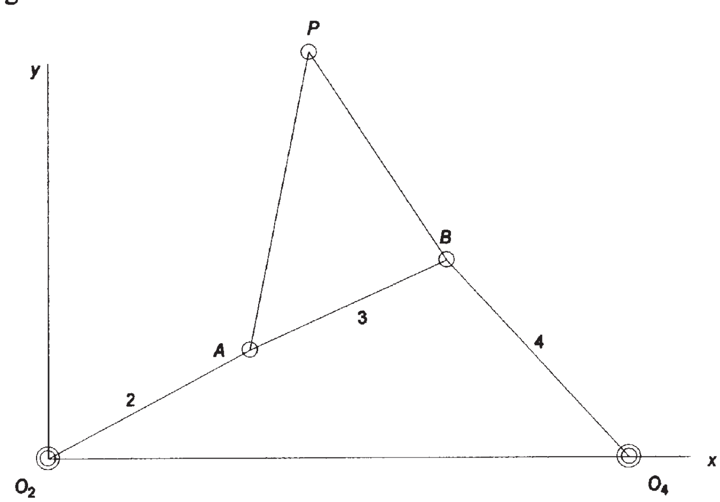
Crank speed: $\omega_2 := 80 \cdot rpm$

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P6-14 and Mathcad file P0651.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 1.451$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = 0.568$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 55.4\,deg$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 1 \cdot deg .. \theta_{2toggle}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.5278 \qquad K_2 = 2.1412$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 3.3422$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(\mathrm{atan2}\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 2.6765$$

$$K_5 = -3.6465$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(\mathrm{atan2}\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point *A* using equations 6.19.

$$\mathbf{V_A}(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

9. Determine the velocity of the coupler point *P* using equations 6.36.

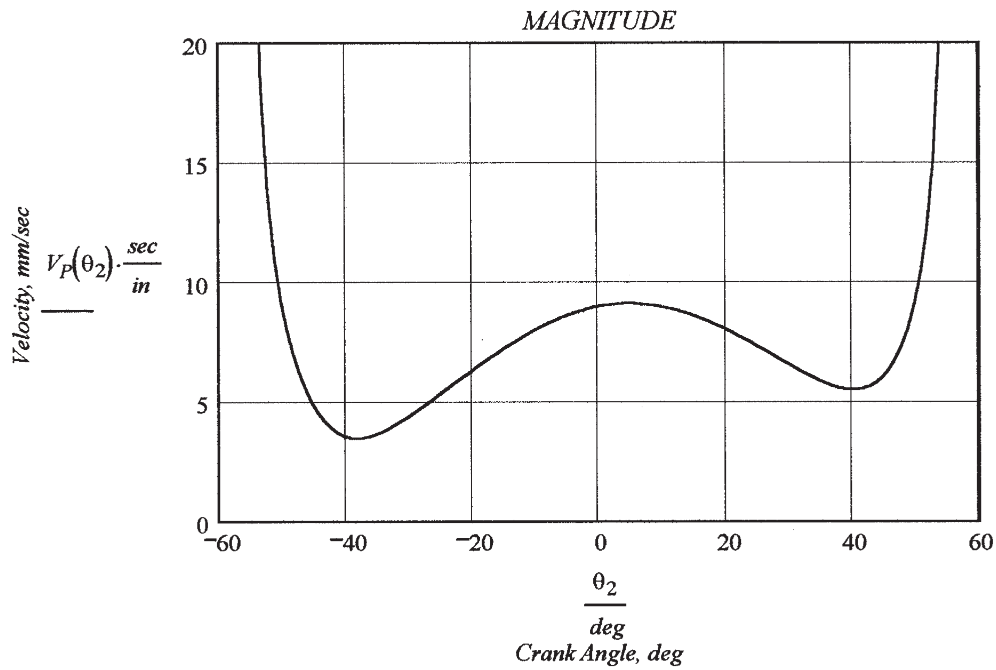
$$\mathbf{V_{PA}}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-\sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot \cos(\theta_{31}(\theta_2) + \delta_3))$$

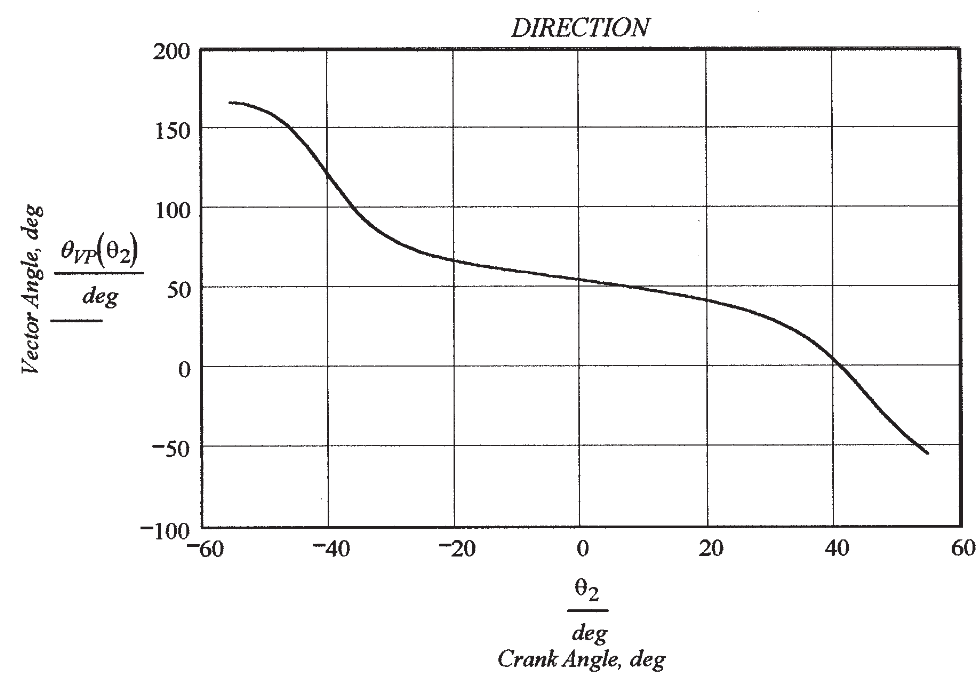
$$\mathbf{V_P}(\theta_2) := \mathbf{V_A}(\theta_2) + \mathbf{V_{PA}}(\theta_2)$$

10. Plot the magnitude and direction of the coupler point P.

Magnitude: $V_P(\theta_2) := |\mathbf{V_P}(\theta_2)|$

Direction: $\theta_{VP}(\theta_2) := arg(\mathbf{V_P}(\theta_2))$





PROBLEM 6-52

Statement: Figure P6-15 shows a power hacksaw that is an offset slider-crank mechanism that has the dimensions given below. Draw an equivalent linkage diagram; then calculate and plot the velocity of the saw blade with respect to the piece being cut over one revolution of the crank, which rotates at 50 rpm.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

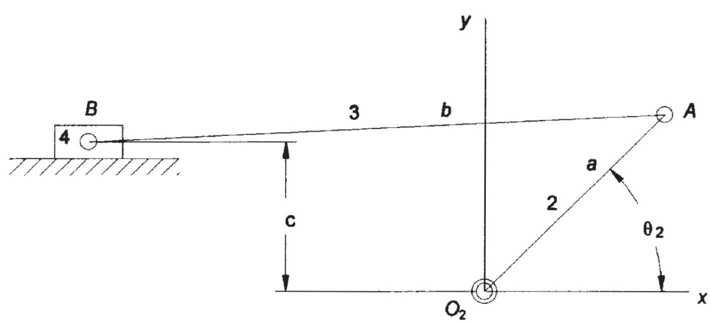
Link 2 $a := 75 \cdot mm$ Offset $c := 45 \cdot mm$

Link 3 $b := 170 \cdot mm$

Input crank angular velocity $\omega_2 := 50 \cdot rpm$

Solution: See Figure P6-15 and Mathcad file P0652.

1. Draw the equivalent linkage to a convenient scale and label it.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine θ_3 using equations 4.16 for the crossed circuit.

$$\theta_3(\theta_2) := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right)$$

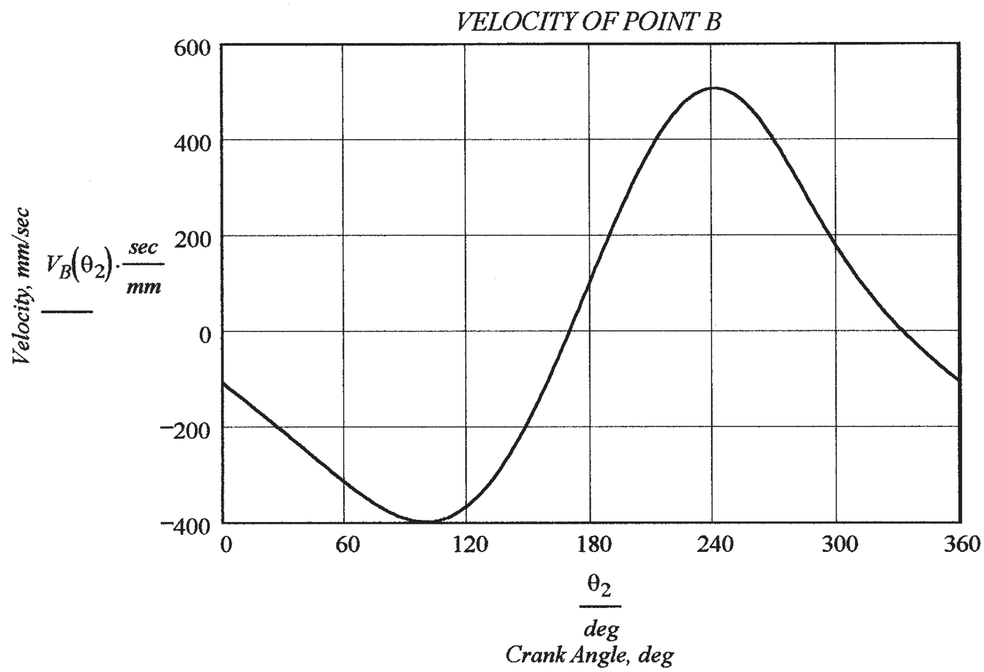
4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Determine the velocity of pin *B* using equation 6.22b:

$$V_B(\theta_2) := -a \cdot \omega_2 \cdot sin(\theta_2) + b \cdot \omega_3(\theta_2) \cdot sin(\theta_3(\theta_2))$$

7. Plot the magnitude of the velocity of *B*. (See next page.)


VELOCITY OF POINT B
600
400
200
0
−200
−400
0
60
120
180
240
300
360
Velocity, mm/sec
$V_B(\theta_2)\cdot\frac{sec}{mm}$
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 6-53

Statement: Figure P6-16 shows a walking-beam indexing and pick-and-place mechanism that can be analyzed as two fourbar linkages driven by a common crank. Calculate and plot the absolute velocities of points E and P and the relative velocity between points *E* and *P* for one revolution of gear 2.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Link lengths (walking-beam linkage):

Link 2 (O_2 to A)	$a := 40\cdot mm$	Link 3 (A to D)	$b' := 108\cdot mm$
Link 4 (O_4 to D)	$c' := 40\cdot mm$	Link 1 (O_2 to O_4)	$d' := 108\cdot mm$

Link lengths (pick and place linkage):

Link 2 (O_2 to B)	$a := 32\cdot mm$	Link 5 (B to C)	$b := 260\cdot mm$
Link 4 (C to O_6)	$c := 96\cdot mm$	Link 1 (O_2 to O_6)	$d := 200\cdot mm$

Rocker point E: $u := 160\cdot mm$ $\delta_4 := -75\cdot deg$

Crank speed: $\omega_2 := 100\cdot rpm$

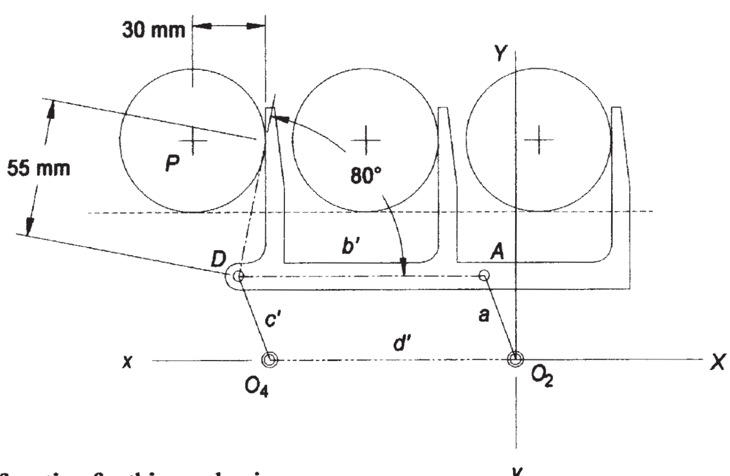
Crank-pin phase angle $\phi := 120\cdot deg$

Two argument inverse tangent

$$atan2(x,y) := \begin{vmatrix} return\ 0.5\cdot\pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5\cdot\pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{vmatrix}$$

Solution: See Figure P6-16 and Mathcad file P0653.

1. Draw the walking-beam linkage to scale and label it.



2. Determine the range of motion for this mechanism.

$\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$ (local $x'y'$ coordinate system)

3. This part of the mechanism is a special-case Grashof in the parallelogram configuration. As such, the coupler does not rotate, but has curvilinear motion with ever point on it having the same velocity. Therefore, it is only necessary to calculate the X-component of the velocity at point A in order to determine the velocity of the cylinder center, P.

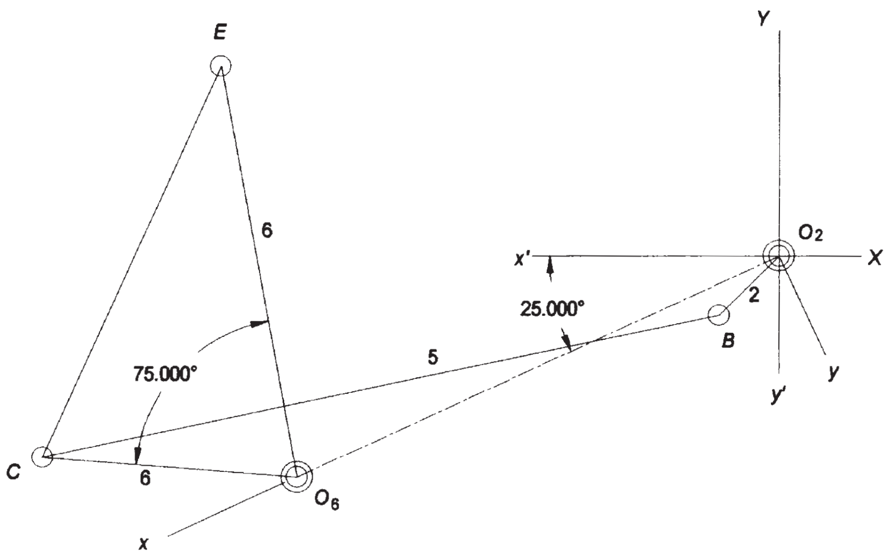
$$\mathbf{V_A}(\theta_2) := a\cdot\omega_2\cdot(-\sin(\theta_2) + j\cdot\cos(\theta_2))$$

$$V_{Ax}(\theta_2) := Re(\mathbf{V_A}(\theta_2))$$

In the global coordinate frame,

$$\mathbf{V_P}(\theta_2) := -V_{Ax}(\theta_2)$$

4. Draw the pick and place linkage to scale and label it.



Coordinate rotation angle: $\alpha := 25\cdot deg$

5. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 6.2500 \qquad K_2 = 2.0833$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)} \qquad K_3 = -2.8255$$

$$A(\theta_2) := \cos(\theta_2 + \phi - \alpha) - K_1 - K_2\cdot\cos(\theta_2 + \phi - \alpha) + K_3$$

$$B(\theta_2) := -2\cdot\sin(\theta_2 + \phi - \alpha)$$

$$C(\theta_2) := K_1 - (K_2 + 1)\cdot\cos(\theta_2 + \phi - \alpha) + K_3$$

6. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_6(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

7. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.7692 \qquad K_5 = -5.9740$$

$$D(\theta_2) := \cos(\theta_2 + \phi - \alpha) - K_1 + K_4\cdot\cos(\theta_2 + \phi - \alpha) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2 + \phi - \alpha)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2 + \phi - \alpha) + K_5$$

8. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_5(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

9. Determine the angular velocity of links 5 and 6 using equations 6.18.

$$\omega_5(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin[\theta_6(\theta_2) - (\theta_2 + \phi - \alpha)]}{sin(\theta_5(\theta_2) - \theta_6(\theta_2))}$$

$$\omega_6(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 + \phi - \alpha - \theta_5(\theta_2))}{sin(\theta_6(\theta_2) - \theta_5(\theta_2))}$$

10. Determine the velocity of the rocker point E using equations 6.35.

$$\mathbf{V_E}(\theta_2) := u\cdot\omega_6(\theta_2)\cdot(-sin(\theta_6(\theta_2) + \delta_4) + j\cdot cos(\theta_6(\theta_2) + \delta_4))$$

11. Transform this into the global XY system.

$$V_{Ex}(\theta_2) := Re(\mathbf{V_E}(\theta_2)) \qquad V_{Ey}(\theta_2) := Im(\mathbf{V_E}(\theta_2))$$

$$V_{EX}(\theta_2) := V_{Ex}(\theta_2)\cdot cos(\alpha) - V_{Ey}(\theta_2)\cdot sin(\alpha)$$

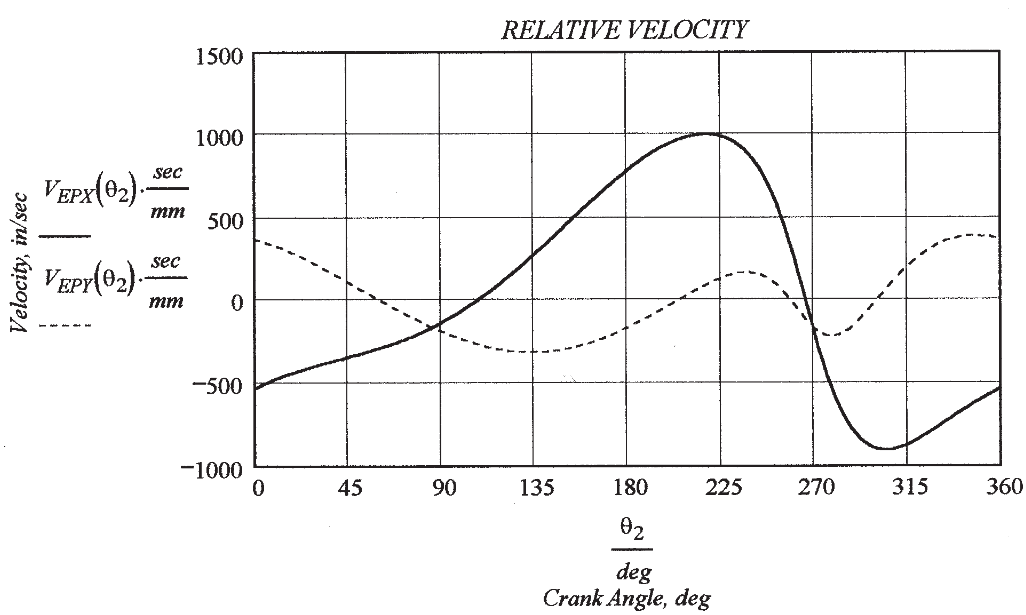
$$V_{EY}(\theta_2) := V_{Ex}(\theta_2)\cdot sin(\alpha) + V_{Ey}(\theta_2)\cdot cos(\alpha)$$

$$\mathbf{V_{EXY}}(\theta_2) := V_{EX}(\theta_2) + j\cdot V_{EY}(\theta_2)$$

12. Calculate and plot the velocity of E relative to P.

$$\mathbf{V_{EP}}(\theta_2) := \mathbf{V_{EXY}}(\theta_2) - \mathbf{V_P}(\theta_2)$$

$$V_{EPX}(\theta_2) := Re(\mathbf{V_{EP}}(\theta_2)) \qquad V_{EPY}(\theta_2) := Im(\mathbf{V_{EP}}(\theta_2))$$



PROBLEM 6-54

Statement: Figure P6-17 shows a paper roll off-loading mechanism driven by an air cylinder. In the position shown, it has the dimensions given below. The V-links are rigidly attached to O_4A. The air cylinder is retracted at a constant velocity of 0.2 m/sec. Draw a kinematic diagram of the mechanism, write the necessary equations, and calculate and plot the angular velocity of the paper roll and the linear velocity of its center as it rotates through 90 deg CCW from the position shown.

Given: Link lengths and angles:

Link 4 (O_4 to A)	$c := 300 \cdot mm$	
Link 1 (O_2 to O_4)	$d := 930 \cdot mm$	
Link 4 initial angle	$\theta_{40} := 62.8 \cdot deg$	

Paper roll location from O_4:

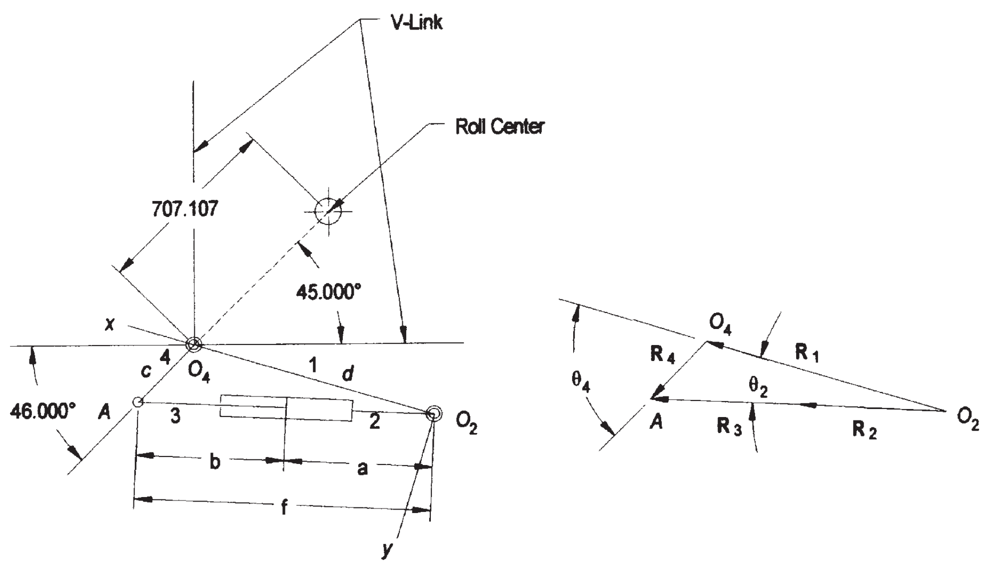
$u := 707.1 \cdot mm$

$\delta_4 := -181 \cdot deg$ with respect to local x axis

Input cylinder velocity $adot := -200 \cdot mm \cdot sec^{-1}$

Solution: See Figure P6-17 and Mathcad file P0654.

1. Draw the mechanism to scale and define a vector loop using the fourbar derivation in Section 6.7 as a model.



2. Write the vector loop equation, differentiate it, expand the result and separate into real and imaginary parts to solve for f, θ_2, and ω_4.

$$\mathbf{R_1} + \mathbf{R_4} := \mathbf{R_2} + \mathbf{R_3} \tag{a}$$

$$d \cdot e^{j \cdot \theta_1} + c \cdot e^{j \cdot \theta_4} := a \cdot e^{j \cdot \theta_2} + b \cdot e^{j \cdot \theta_2} \tag{b}$$

where a is the distance from the origin to the cylinder piston, a variable; b is the distance from the cylinder piston to A, a constant; and c is the distance from θ_4 to point A, a constant. Angle θ_1 is zero, $\theta_3 = \theta_2$, and θ_4 is the variable angle that the rocker arm makes with the x axis. Solving the position equations:

Let $f := a + b$ then, making this substitution and substituting the Euler equivalents,

$$d + c \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) := f \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \tag{c}$$

Separating into real and imaginary components and solving for θ_2 and f,

$$\theta_2(\theta_4) := atan2(d + c \cdot \cos(\theta_4), c \cdot \sin(\theta_4)) \tag{d}$$

$$f(\theta_4) := \frac{c \cdot sin(\theta_4)}{sin(\theta_2(\theta_4))} \qquad (e)$$

Differentiate equation *b*.

$$j \cdot c \cdot \omega_4 \cdot e^{j \cdot \theta_4} := \left(\frac{d}{dt} f\right) \cdot e^{j \cdot \theta_2} + j \cdot f \cdot \omega_2 \cdot e^{j \cdot \theta_2} \qquad (f)$$

Substituting the Euler equivalents,

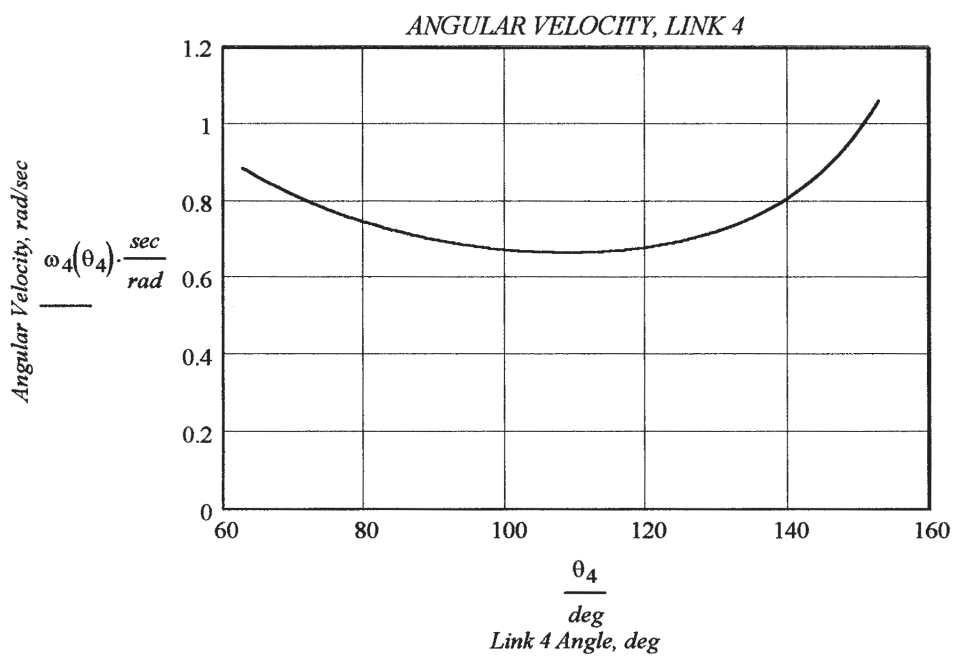
$$c \cdot \omega_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) := \left(\frac{d}{dt} f\right) \cdot (cos(\theta_2) + j \cdot sin(\theta_2)) + f \cdot \omega_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) \qquad (g)$$

Separating into real and imaginary components and solving for ω_4. Note that *df/dt* = *adot*.

$$\omega_4(\theta_4) := \frac{adot}{c \cdot sin(\theta_2(\theta_4) - \theta_4)} \qquad (h)$$

3. Plot ω_4 over a range of θ_4 of

$$\theta_4 := \theta_{40}, \theta_{40} + 1 \cdot deg .. \theta_{40} + 90 \cdot deg$$

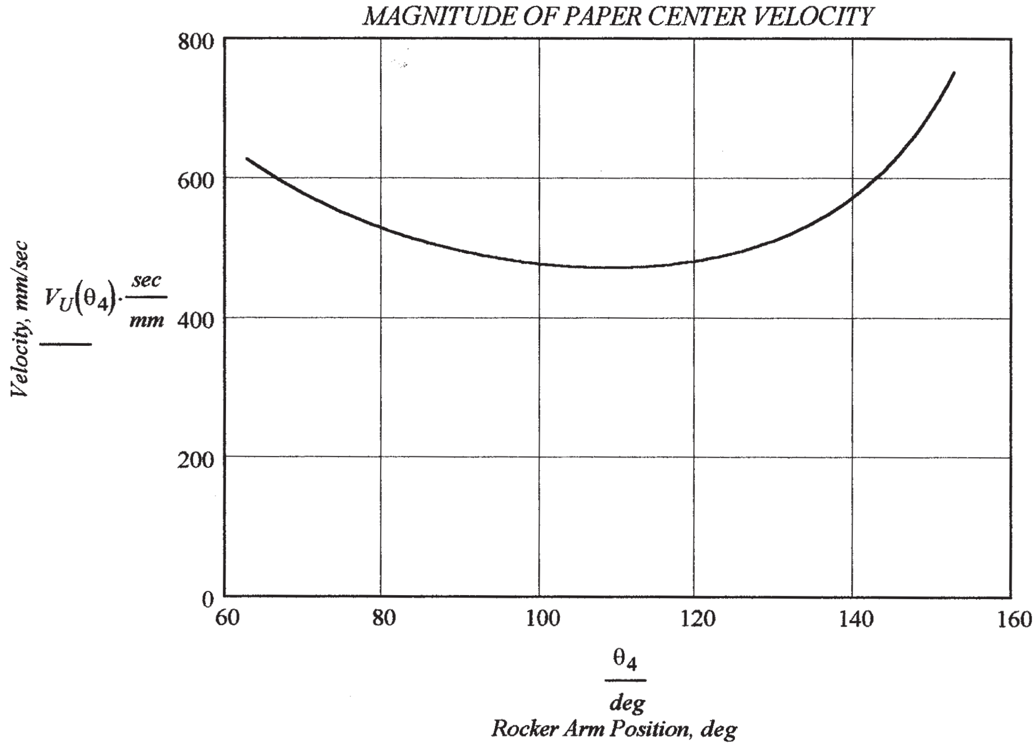


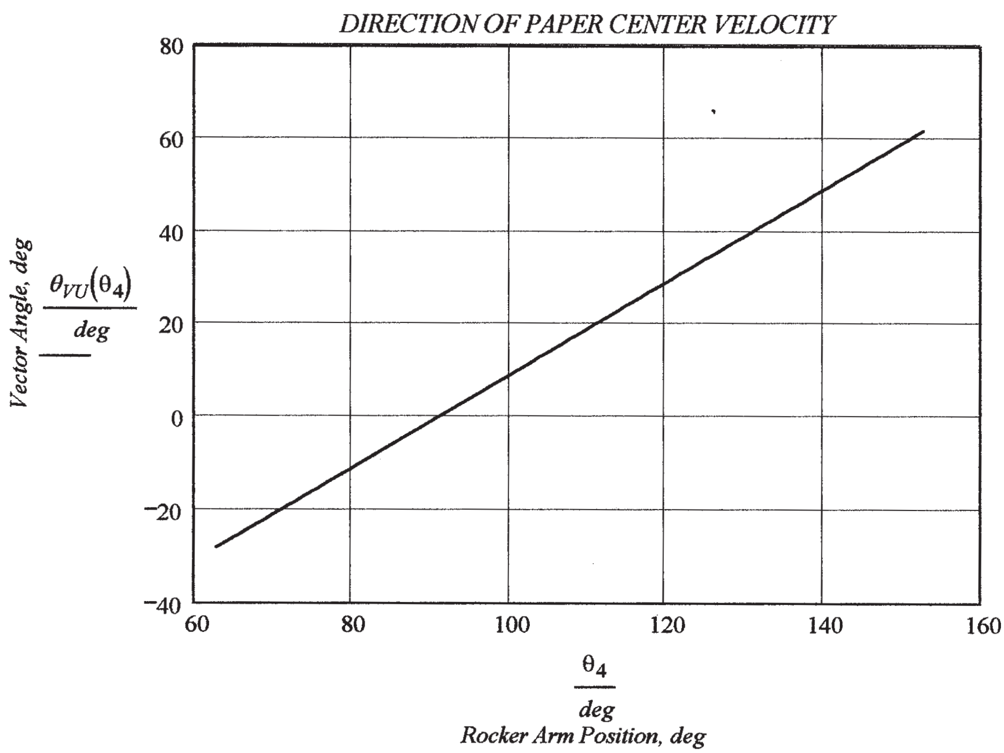
4. Determine the velocity of the center of the paper roll using equation 6.35. The direction is in the local *xy* coordinate system.

$$\mathbf{V_U}(\theta_4) := u \cdot \omega_4(\theta_4) \cdot (-sin(\theta_4 + \delta_4) + j \cdot cos(\theta_4 + \delta_4))$$

$$V_U(\theta_4) := |\mathbf{V_U}(\theta_4)| \qquad \theta_{VU}(\theta_4) := arg(\mathbf{V_U}(\theta_4))$$

5. Plot the magnitude and direction of the velocity of the paper roll center. (See next page.)


MAGNITUDE OF PAPER CENTER VELOCITY
800
600
400
200
0
60
80
100
120
140
160
Velocity, mm/sec
$V_U(\theta_4)\cdot\frac{sec}{mm}$
$\frac{\theta_4}{deg}$
Rocker Arm Position, deg


DIRECTION OF PAPER CENTER VELOCITY
80
60
40
20
0
−20
−40
60
80
100
120
140
160
Vector Angle, deg
$\frac{\theta_{VU}(\theta_4)}{deg}$
$\frac{\theta_4}{deg}$
Rocker Arm Position, deg

PROBLEM 6-55a

Statement: Figure P6-18 shows a powder compaction mechanism. Calculate its mechanical advantage for the position shown.

Given: Link lengths:

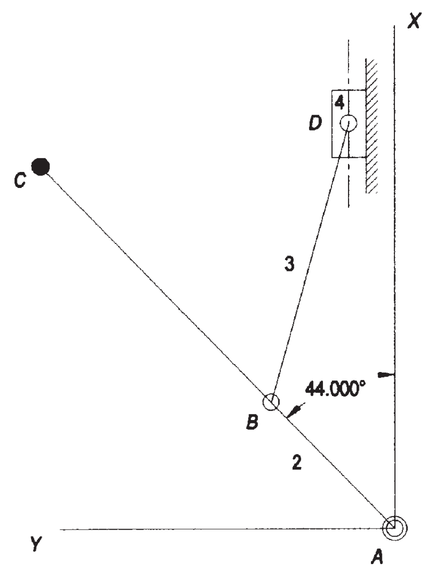
Link 2 (*A* to *B*)	$a := 105 \cdot mm$	Offset	$c := 27 \cdot mm$
Link 3 (*B* to *D*)	$b := 172 \cdot mm$		

Distance to force application:

Link 2 (*AC*)	$r_{in} := 301 \cdot mm$		
Position of link 2:	$\theta_2 := 44 \cdot deg$	Let	$\omega_2 := 1 \cdot rad \cdot sec^{-1}$

Solution: See Figure P6-18 and Mathcad file P0655a.

1. Draw the linkage to scale and label it.



2. Determine θ_3 using equation 4.17.

$$\theta_3 := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_3 = 164.509\, deg$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3)} \cdot \omega_2$$

4. Determine the velocity of pin *D* using equation 6.22b:

$$V_D := -a \cdot \omega_2 \cdot sin(\theta_2) + b \cdot \omega_3 \cdot sin(\theta_3) \qquad \text{Positive upward}$$

5. Calculate the velocity of point *C* using equation 6.23a:

$$\mathbf{V_C} := r_{in} \cdot \omega_2 \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right)$$

$$v_C := |\mathbf{V_C}|$$

6. **Calculate the mechanical advantage using equation 6.13.**

$$m_A := \frac{v_C}{|v_D|} \qquad m_A = 3.206$$

PROBLEM 6-55b

Statement: Figure P6-18 shows a powder compaction mechanism. Calculate and plot its mechanical advantage as a function of the angle of link AC as it rotates from 15 to 60 deg.

Given: Link lengths:

Link 2 (*A* to *B*) $a := 105 \cdot mm$ Offset $c := 27 \cdot mm$

Link 3 (*B* to *D*) $b := 172 \cdot mm$

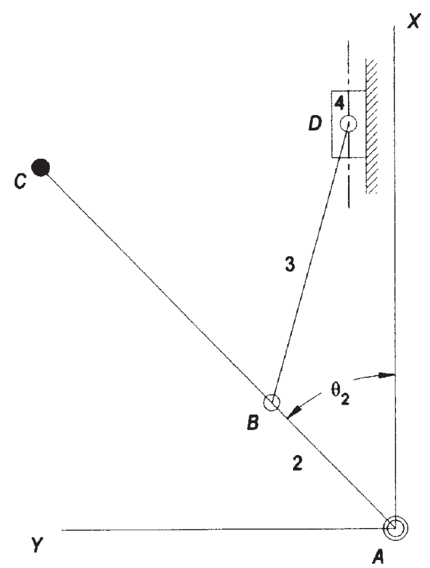
Distance to force application:

Link 2 (*AC*) $r_{in} := 301 \cdot mm$

Initial and final positions of link 2: $\theta_{20} := 15 \cdot deg$ $\theta_{21} := 60 \cdot deg$ Let $\omega_2 := 1 \cdot rad \cdot sec^{-1}$

Solution: See Figure P6-18 and Mathcad file P0655b.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := \theta_{20}, \theta_{20} + 1 \cdot deg .. \theta_{21}$$

3. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Determine the velocity of pin *D* using equation 6.22b:

$$V_D(\theta_2) := -a \cdot \omega_2 \cdot sin(\theta_2) + b \cdot \omega_3(\theta_2) \cdot sin(\theta_3(\theta_2))$$ Positive upward

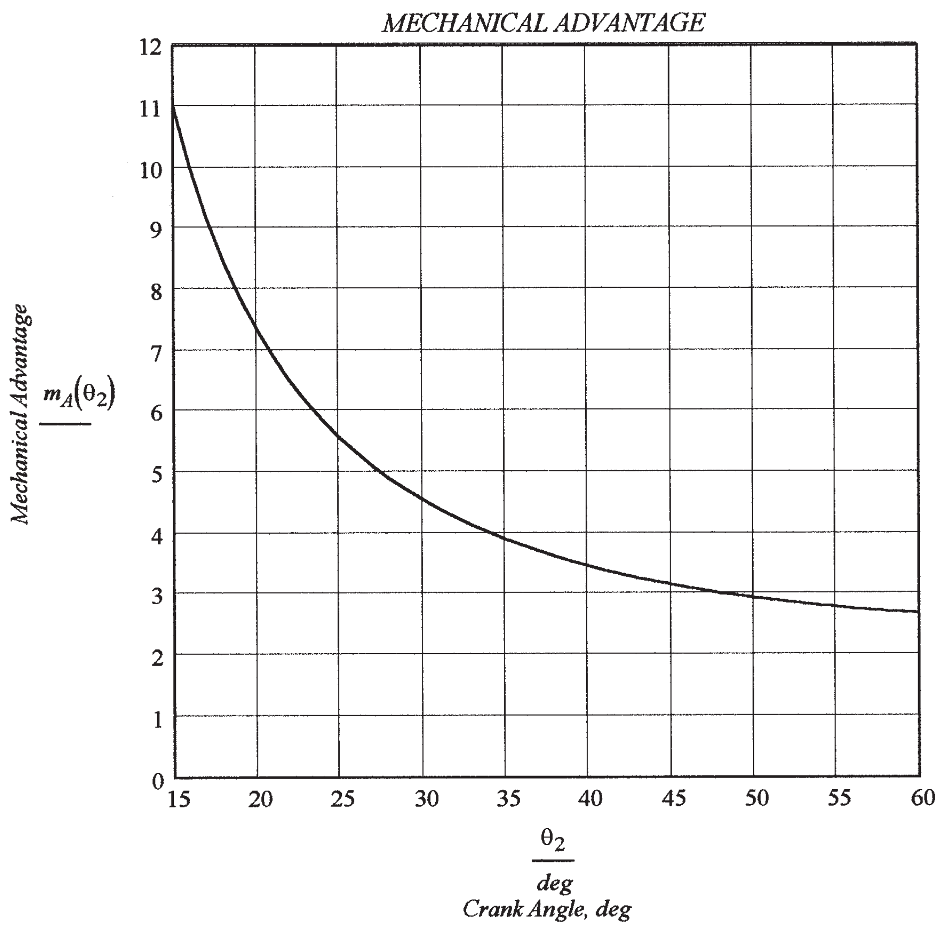
6. Calculate the velocity of point C using equation 6.23a:

$$\mathbf{V_C}(\theta_2) := r_{in} \cdot \omega_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2))$$

$$V_C(\theta_2) := |\mathbf{V_C}(\theta_2)|$$

7. Calculate the mechanical advantage using equation 6.13.

$$m_A(\theta_2) := \frac{V_C(\theta_2)}{|V_D(\theta_2)|}$$



PROBLEM 6-56

Statement: Figure P6-19 shows a walking beam mechanism. Calculate and plot the velocity V_{out} for one revolution of the input crank 2 rotating at 100 rpm.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.00 \cdot in$	Link 3 (A to B)	$b := 2.06 \cdot in$
Link 4 (B to O_4)	$c := 2.33 \cdot in$	Link 1 (O_2 to O_4)	$d := 2.22 \cdot in$

Coupler point: $R_{pa} := 3.06$ $\quad \delta_3 := 31 \cdot deg$

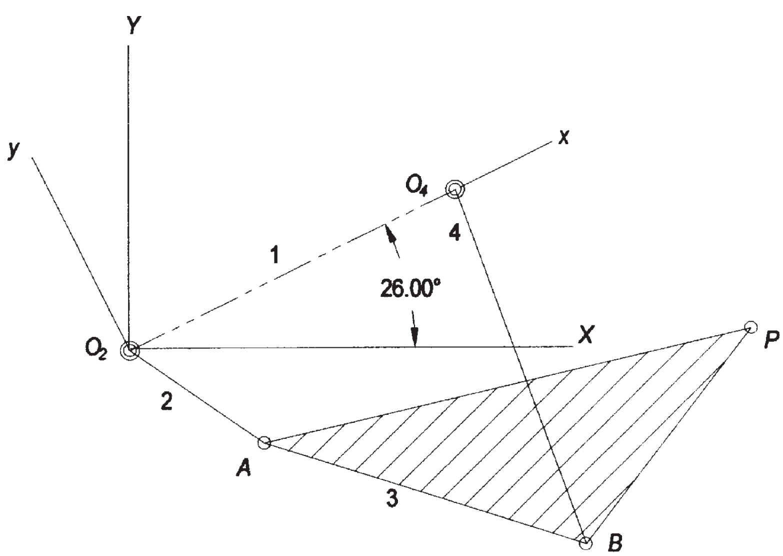
Crank speed: $\omega_2 := 100 \cdot rpm$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} \text{return } 0.5 \cdot \pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5 \cdot \pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{array}$$

Solution: See Figure P6-19 and Mathcad file P0656.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.2200 \qquad K_2 = 0.9528$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.5265$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right)\right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.0777$$

$$K_5 = -1.1512$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right)\right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{sin(\theta_{41}(\theta_2) - \theta_2)}{sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{sin(\theta_2 - \theta_{31}(\theta_2))}{sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

8. Determine the velocity of point A using equations 6.19.

$$\mathbf{V_A}(\theta_2) := a \cdot \omega_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2))$$

9. Determine the velocity of the coupler point P using equations 6.36.

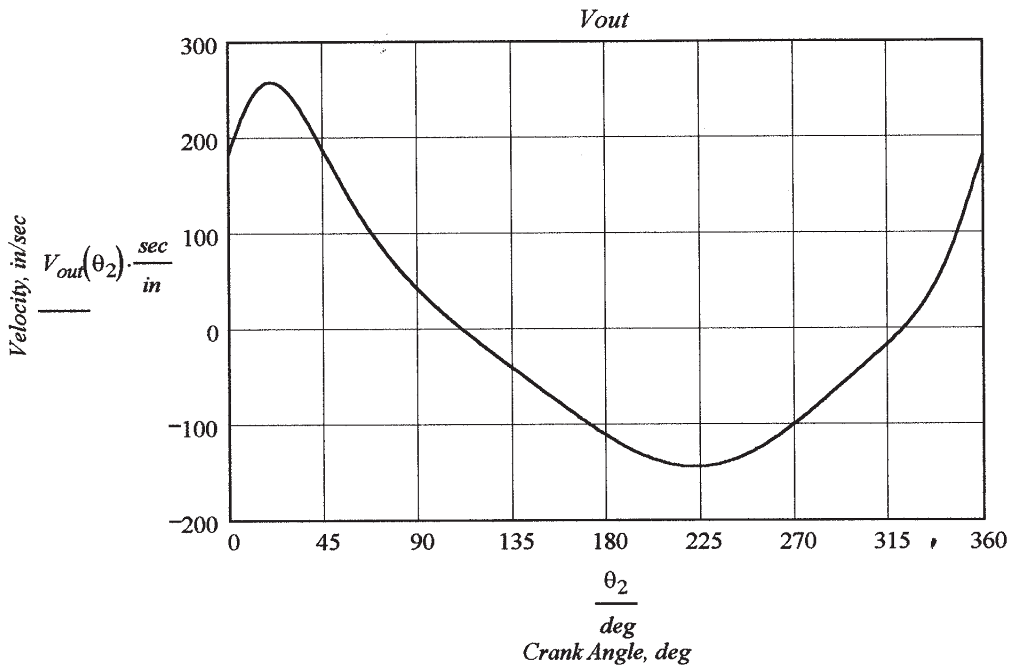
$$\mathbf{V_{PA}}(\theta_2) := R_{pa} \cdot \omega_{31}(\theta_2) \cdot (-sin(\theta_{31}(\theta_2) + \delta_3) + j \cdot cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V_P}(\theta_2) := \mathbf{V_A}(\theta_2) + \mathbf{V_{PA}}(\theta_2)$$

10. Plot the X-component (global coordinate system) of the velocity of the coupler point P.

Coordinate rotation angle: $\alpha := 26 \cdot deg$

$$V_{out}(\theta_2) := Re(\mathbf{V_P}(\theta_2)) \cdot cos(\alpha) - Im(\mathbf{V_P}(\theta_2)) \cdot sin(\alpha)$$


Vout
300
200
100
0
−100
−200
Velocity, in/sec
$V_{out}(\theta_2)\cdot\frac{sec}{in}$
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 6-57a

Statement: Figure P6-20 shows a crimping tool. For the dimensions given below, calculate its mechanical advantage for the position shown.

Given: Link lengths:

Link 2 (*AB*) $a := 0.80 \cdot in$ Link 3 (*BC*) $b := 1.23 \cdot in$

Link 4 (*CD*) $c := 1.55 \cdot in$ Link 1 (*AD*) $d := 2.40 \cdot in$

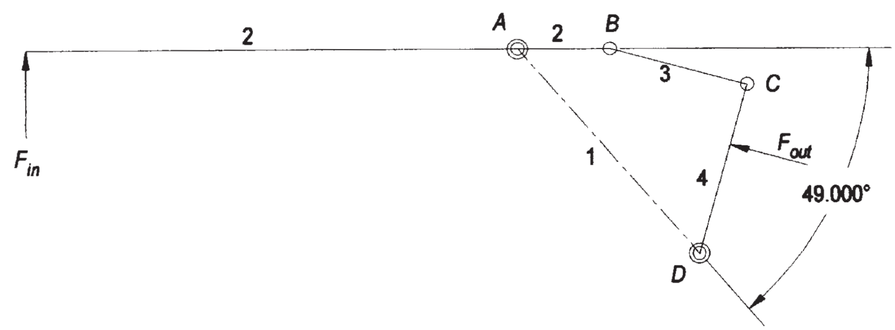
Distance to force application:

Link 2 (*AB*) $r_{in} := 4.26 \cdot in$ Link 4 (*CD*) $r_{out} := 1.00 \cdot in$

Initial position of link 2: $\theta_2 := 49 \cdot deg$

Solution: See Figure P6-20 and Mathcad file P0657a.

1. Draw the mechanism to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 3.0000 \qquad K_2 = 1.5484$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.9394$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.4204 \qquad B = -1.5094 \qquad C = 4.2675$$

3. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_4 = -236.482\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)}$$

$K_4 = 1.9512$

$K_5 = -2.8000$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$ $D = -3.8638$

$$E := -2 \cdot \sin(\theta_2)$$ $E = -1.5094$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$ $F = 0.8241$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$$ $\theta_3 = -325.961\ deg$

6. Referring to Figure 6-10, calculate the values of the angles ν and μ.

$$\nu := \theta_2 - \theta_3$$ $\nu = 374.961\ deg$

If $\nu > 360$ deg, subtract 360 deg from it.

$$\nu := if(\nu > 360 \cdot deg, \nu - 360 \cdot deg, \nu)$$ $\nu = 14.961\ deg$

$$\mu := \theta_4 - \theta_3$$ $\mu = 89.479\ deg$

If $\mu > 90$ deg, subtract it from 180 deg.

$$\mu := if(\mu > 90 \cdot deg, 180 \cdot deg - \mu, \mu)$$ $\mu = 89.479\ deg$

7. Using equation 6.13e, calculate the mechanical advantage of the linkage in the position shown.

$$m_A := \frac{c \cdot \sin(\mu)}{a \cdot \sin(\nu)} \cdot \frac{r_{in}}{r_{out}}$$ $m_A = 31.969$

PROBLEM 6-57b

Statement: Figure P6-20 shows a crimping tool. For the dimensions given below, calculate and plot its mechanical advantage as a function of the angle of link AB as it rotates from 60 to 45 deg.

Given: Link lengths:

Link 2 (AB)	$a := 0.80 \cdot in$	Link 3 (BC)	$b := 1.23 \cdot in$
Link 4 (CD)	$c := 1.55 \cdot in$	Link 1 (AD)	$d := 2.40 \cdot in$

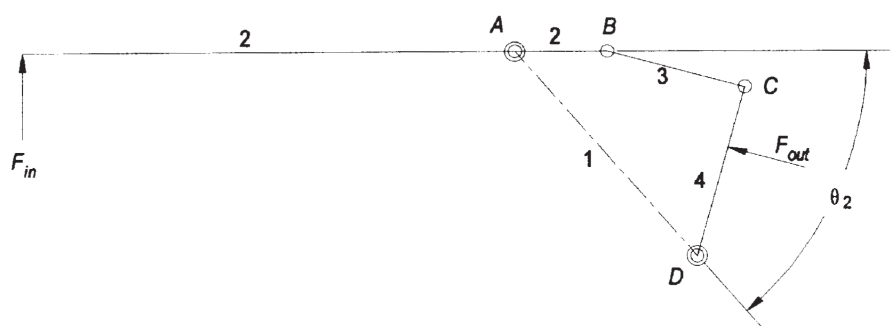
Distance to force application:

Link 2 (AB)	$r_{in} := 4.26 \cdot in$	Link 4 (CD)	$r_{out} := 1.00 \cdot in$

Range of positions of link 2: $\theta_{20} := 60 \cdot deg$ $\theta_{21} := 45 \cdot deg$

Solution: See Figure P6-20 and Mathcad file P0657b.

1. Draw the mechanism to scale and label it.



2. Calculate the range of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

$$\theta_2 := \theta_{20}, \theta_{20} - 1 \cdot deg .. \theta_{21}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 3.0000 \qquad K_2 = 1.5484$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.9394$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{(B(\theta_2))^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right] \right]$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.9512$$

$$K_5 = -2.8000$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{\left(E(\theta_2)\right)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right] \right]$$

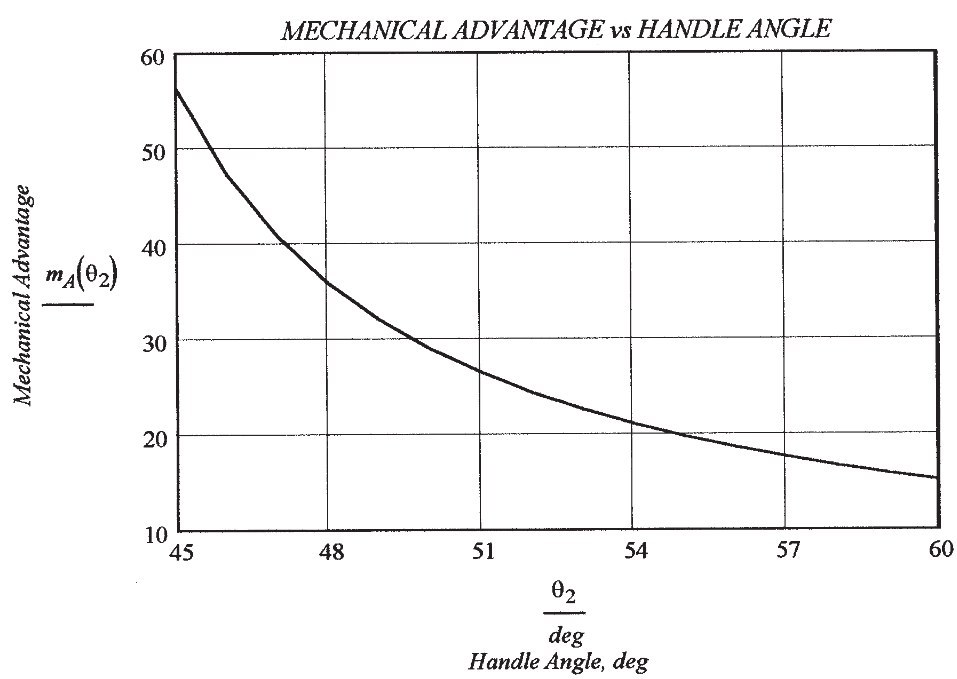
7. Referring to Figure 6-10, calculate the values of the angles ν and μ.

$$\nu(\theta_2) := \theta_2 - \theta_3(\theta_2)$$

$$\mu(\theta_2) := \theta_4(\theta_2) - \theta_3(\theta_2)$$

8. Using equation 6.13e, calculate and plot the mechanical advantage of the linkage over the given range.

$$m_A(\theta_2) := \frac{c \cdot sin(\mu(\theta_2))}{a \cdot sin(\nu(\theta_2))} \cdot \frac{r_{in}}{r_{out}}$$

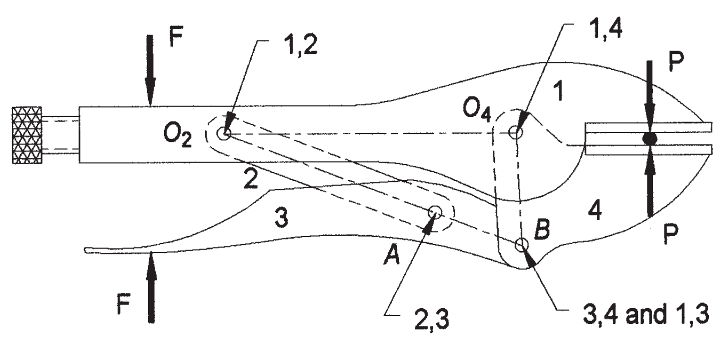


PROBLEM 6-58

Statement: Figure P6-21 shows a locking pliers. Calculate its mechanical advantage for the position shown. Scale any dimensions needed from the diagram.

Solution: See Figure P6-21 and Mathcad file P0658.

1. Draw the linkage to scale in the position given and find the instant centers.



2. Note that the linkage is in a toggle position (links 2 and 3 are in line) and the angle between links 2 and 3 is 0 deg. From the discussion below equation 6.13e on page 285 in the text, we see that the mechanical advantage for this linkage in this position is theoretically infinite.

PROBLEM 6-59a

Statement: Figure P6-22 shows a fourbar toggle clamp used to hold a workpiece in place by clamping it at D. The linkage will toggle when link 2 reaches 90 deg. For the dimensions given below, calculate its mechanical advantage for the position shown.

Given: Link lengths:

Link 2 (O_2A) $a := 70 \cdot mm$ Link 3 (AB) $b := 35 \cdot mm$

Link 4 (O_4B) $c := 34 \cdot mm$ Link 1 (O_2O_4) $d := 48 \cdot mm$

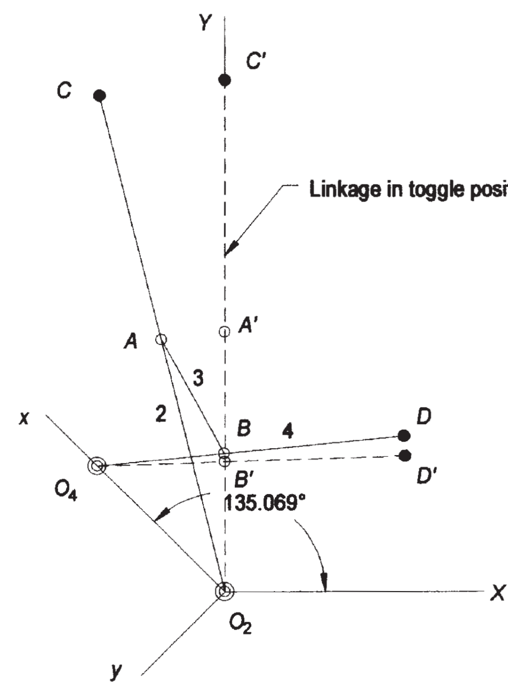
Distance to force application:

Link 2 (O_2C) $r_{in} := 138 \cdot mm$ Link 4 (O_4D) $r_{out} := 82 \cdot mm$

Initial position of link 2: $\theta_{20} := 104 \cdot deg$ Global *XY* system

Solution: See Figure P6-22 and Mathcad file P0659a.

1. Draw the mechanism to scale and label it. To establish the position of O_4 with respect to O_2 (in the global coordinate frame), draw the linkage in the toggle position with $\theta_2 = 90$ deg. The fixed pivot O_4 is then 48 mm from O_2 and 34 mm from B' (see layout).



2. Calculate the value of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

Rotation angle of local xy system to global XY system: $\alpha := 135.069 \cdot deg$

$\theta_2 := \theta_{20} - \alpha$ $\theta_2 = -31.069\,deg$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$K_1 := \frac{d}{a}$ $K_2 := \frac{d}{c}$

$K_1 = 0.6857$ $K_2 = 1.4118$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.4989$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = 0.4605 \qquad B = 1.0321 \qquad C = 0.1189$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_4 = -129.480\, deg$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.3714$$

$$K_5 = -1.4843$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -0.1388$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = 1.0321$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = -0.4804$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_3 = -196.400\, deg$$

7. Referring to Figure 6-10, calculate the values of the angles ν and μ.

$$\nu := \theta_2 - \theta_3 \qquad \nu = 165.331\, deg$$

If $\nu > 90$ deg, subtract it from 180 deg.

$$\nu := if(\nu > 90 \cdot deg, 180 \cdot deg - \nu, \nu) \qquad \nu = 14.669\, deg$$

$$\mu := \theta_4 - \theta_3 \qquad \mu = 66.920\, deg$$

If $\mu > 90$ deg, subtract it from 180 deg.

$$\mu := if(\mu > 90 \cdot deg, 180 \cdot deg - \mu, \mu) \qquad \mu = 66.920\, deg$$

8. Using equation 6.13e, calculate the mechanical advantage of the linkage in the position shown.

$$m_A := \frac{c \cdot \sin(\mu)}{a \cdot \sin(\nu)} \cdot \frac{r_{in}}{r_{out}} \qquad m_A = 2.970$$

PROBLEM 6-59b

Statement: Figure P6-22 shows a fourbar toggle clamp used to hold a workpiece in place by clamping it at D. The linkage will toggle when link 2 reaches 90 deg. For the dimensions given below, calculate and plot its mechanical advantage as a function of the angle of link *AB* as link 2 rotates from 120 to 90 deg (in the global coordinate system).

Given: Link lengths:

Link 2 (O_2A) $a := 70 \cdot mm$ Link 3 (AB) $b := 35 \cdot mm$

Link 4 (O_4B) $c := 34 \cdot mm$ Link 1 (O_2O_4) $d := 48 \cdot mm$

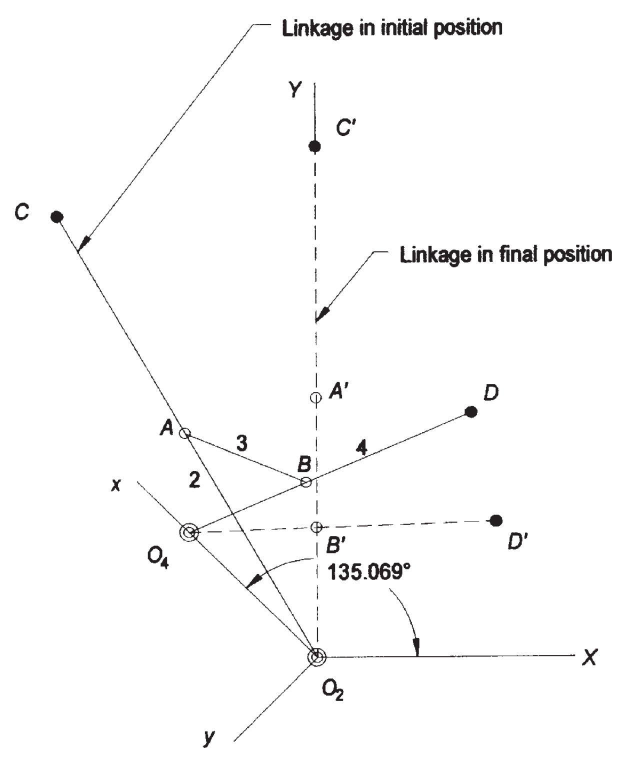
Distance to force application:

Link 2 (O_2C) $r_{in} := 138 \cdot mm$ Link 4 (O_4D) $r_{out} := 82 \cdot mm$

Range of positions of link 2: $\theta_{20} := 120 \cdot deg$ $\theta_{21} := 90.1 \cdot deg$ Global *XY* system

Solution: See Figure P6-22 and Mathcad file P0659b.

1. Draw the mechanism to scale and label it. To establish the position of O_4 with respect to O_2 (in the global coordinate frame), draw the linkage in the toggle position with $\theta_2 = 90$ deg. The fixed pivot O_4 is then 48 mm from O_2 and 34 mm from B' (see layout).



2. Calculate the range of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

Rotation angle of local xy system to global XY system: $\alpha := 135.069 \cdot deg$

$$\theta_2 := \theta_{20} - \alpha, \theta_{20} - \alpha - 1 \cdot deg .. \theta_{21} - \alpha$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.6857 \qquad K_2 = 1.4118$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.4989$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{\left(B(\theta_2)\right)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right] \right]$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.3714$$

$$K_5 = -1.4843$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{\left(E(\theta_2)\right)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right] \right]$$

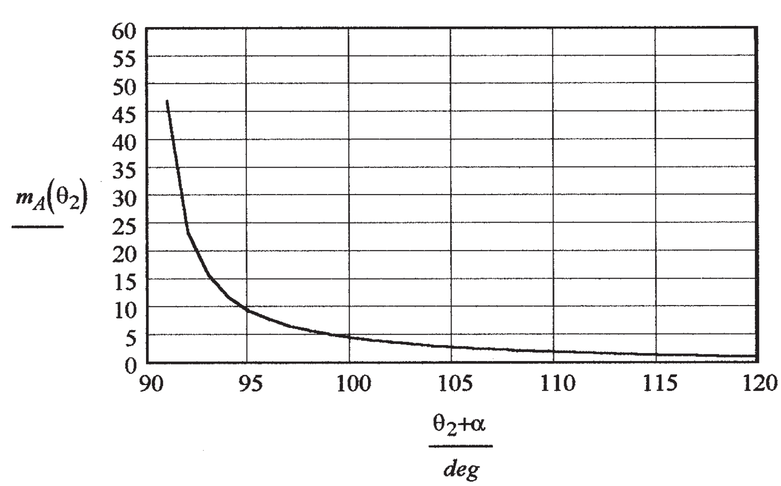
7. Referring to Figure 6-10, calculate the values of the angles ν and μ.

$$\nu(\theta_2) := \theta_2 - \theta_3(\theta_2)$$

$$\mu(\theta_2) := \theta_4(\theta_2) - \theta_3(\theta_2)$$

8. Using equation 6.13e, calculate and plot the mechanical advantage of the linkage over the given range.

$$m_A(\theta_2) := \frac{c \cdot \sin(\mu(\theta_2))}{a \cdot \sin(\nu(\theta_2))} \cdot \frac{r_{in}}{r_{out}}$$



PROBLEM 6-60

Statement: Figure P6-23 shows a surface grinder. The workpiece is oscillated under the spinning grinding wheel by the slider-crank linkage that has the dimensions given below. Calculate and plot the velocity of the grinding wheel contact point relative to the workpiece over one revolution of the crank.

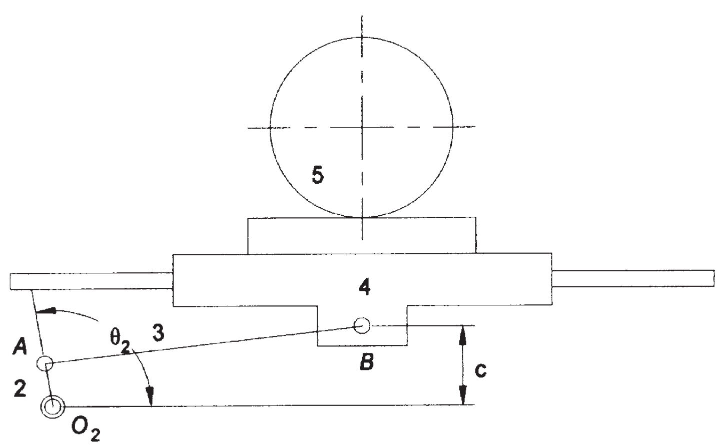
Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$a := 22\cdot mm$	Offset	$c := 40\cdot mm$
Link 3 (A to B)	$b := 157\cdot mm$		
Grinding wheel diameter	$d := 90\cdot mm$		
Input crank angular velocity	$\omega_2 := 120\cdot rpm$	CCW	
Grinding wheel angular velocity	$\omega_5 := 3450\cdot rpm$	CCW	

Solution: See Figure P6-23 and Mathcad file P0660.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$$

3. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := asin\left(-\frac{a\cdot sin(\theta_2) - c}{b}\right) + \pi$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b}\cdot\frac{cos(\theta_2)}{cos(\theta_3(\theta_2))}\cdot\omega_2$$

5. Determine the velocity of pin B using equation 6.22b:

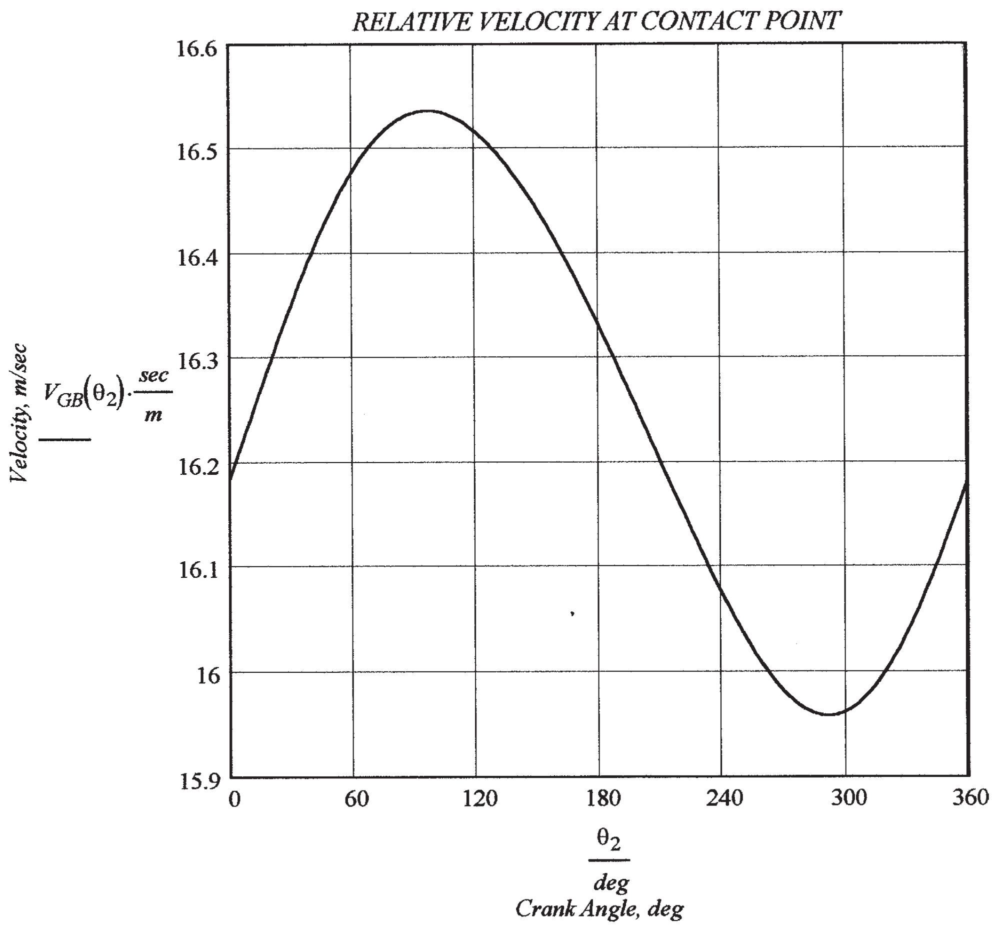
$$V_B(\theta_2) := -a\cdot\omega_2\cdot sin(\theta_2) + b\cdot\omega_3(\theta_2)\cdot sin(\theta_3(\theta_2))$$ Positive to the right

6. Calculate the velocity of the grinding wheel contact point using equation 6.7:

$V_G := \frac{d}{2} \cdot \omega_5$ $V_G = 16.258 \frac{m}{sec}$ Directed to the right

7. The velocity of the grinding wheel contact point relative to the workpiece, which has velocity V_B, is

$$V_{GB}(\theta_2) := V_G - V_B(\theta_2)$$



PROBLEM 6-61

Statement: Figure P6-24 shows an inverted slider-crank mechanism. Given the dimensions below, find ω_2, ω_3, ω_4, V_{A4}, V_{trans}, and V_{slip} for the position shown with V_{A2} = 20 in/sec in the direction shown.

Given: Link lengths:

Link 2 (O_2A) $a := 2.5 \cdot in$ Link 4 (O_4A) $c := 4.1 \cdot in$ Link 1 (O_2O_4) $d := 3.9 \cdot in$

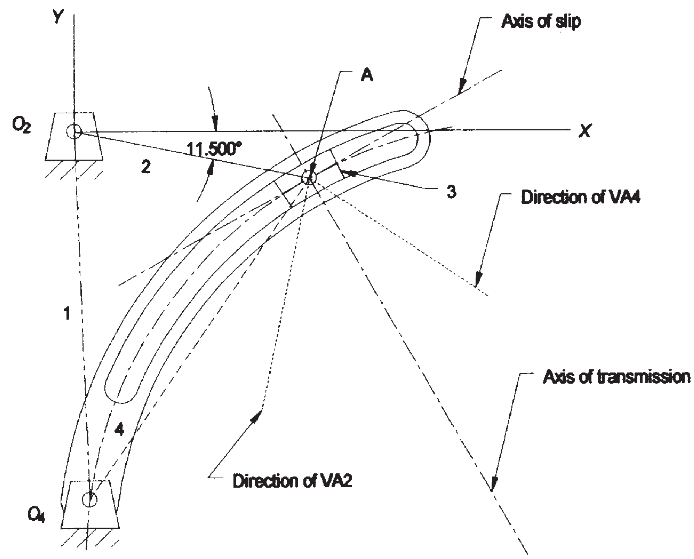
Measured angles:

$\theta_2 := 75.5 \cdot deg$ $\theta_{trans} := 26.5 \cdot deg$ $\theta_{slip} := 116.5 \cdot deg$

Velocity of point A on links 2 and 3: $V_{A2} := 20 \cdot in \cdot sec^{-1}$ $V_{A3} := V_{A2}$

Solution: See Figure P6-24 and Mathcad file P0661.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the angular velocity of link 2.

$$\omega_2 := \frac{V_{A2}}{a} \qquad \omega_2 = 8.000 \frac{rad}{sec} \quad \text{CW}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on links 2 and 3.. The equation to be solved graphically is

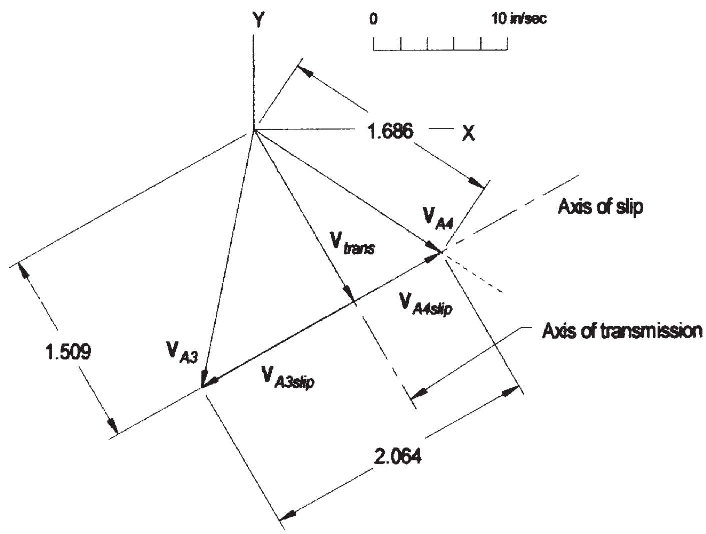
$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{A3slip}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.

b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{A3slip}$, magnitude unknown.

c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{A3slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{A3slip}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$

$V_{A4} := 1.686 \cdot in \cdot k_v$ $\qquad V_{A4} = 16.9 \dfrac{in}{sec}$

$V_{slip} := 2.064 \cdot in \cdot k_v$ $\qquad V_{slip} = 20.6 \dfrac{in}{sec}$

$V_{trans} := 1.509 \cdot in \cdot k_v$ $\qquad V_{trans} = 15.1 \dfrac{in}{sec}$

5. Determine the angular velocity of link 4 using equation 6.7.

$\omega_4 := \dfrac{V_{A4}}{c}$ $\qquad \omega_4 = 4.1 \dfrac{rad}{sec}$ $\qquad$ CW

Because link 3 slides within link 4, $\omega_3 = \omega_4$.

PROBLEM 6-62

Statement: Figure P6-25 shows a drag-link mechanism with dimensions. Write the necessary equations and solve them to calculate and plot the angular velocity of link 4 for an input of $\omega_2 = 1$ rad/sec. Comment on the uses for this mechanism.

Given: Link lengths:

Link 2 (L_2)	$a := 1.38 \cdot in$	Link 3 (L_3)	$b := 1.22 \cdot in$
Link 4 (L_4)	$c := 1.62 \cdot in$	Link 1 (L_1)	$d := 0.68 \cdot in$

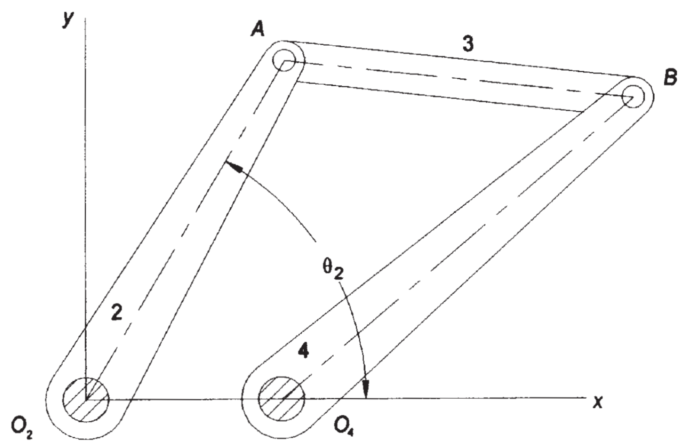
Input crank angular velocity $\quad \omega_2 := -1 \cdot rad \cdot sec^{-1}$ CW

Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi & if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi & if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) & if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & otherwise \end{cases}$$

Solution: See Figure P6-25 and Mathcad file P0662.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof double crank.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.4928 \qquad K_2 = 0.4198$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.7834$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.5574$$

$$K_5 = -0.3655$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4\cdot\cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot\sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot\cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

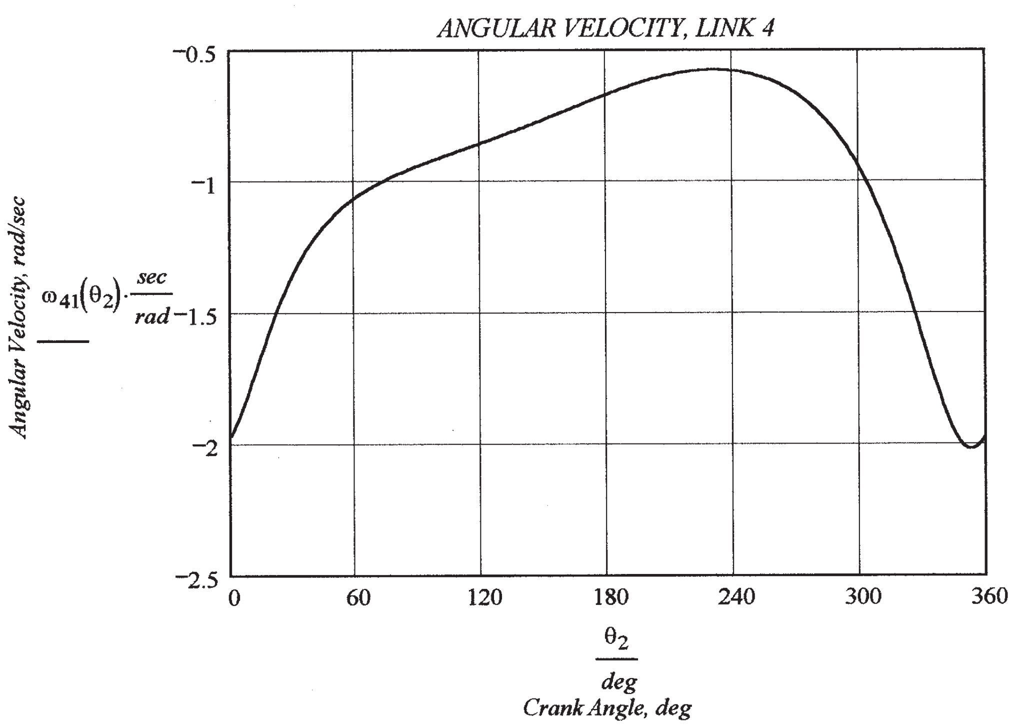
$$\theta_{31}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{\sin(\theta_2 - \theta_{31}(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

9. Plot the angular velocity of link 4.



PROBLEM 6-63

Statement: Figure P6-25 shows a drag-link mechanism with dimensions. Write the necessary equations and solve them to calculate and plot the centrodes of instant center $I_{2,4}$.

Given: Measured link lengths:

Link 2 (L_2)	$L_2 := 1.38 \cdot in$	Link 3 (L_3)	$L_3 := 1.22 \cdot in$
Link 4 (L_4)	$L_4 := 1.62 \cdot in$	Link 1 (L_1)	$L_1 := 0.68 \cdot in$

Input crank angular velocity $\omega_2 := -1 \cdot rad \cdot sec^{-1}$

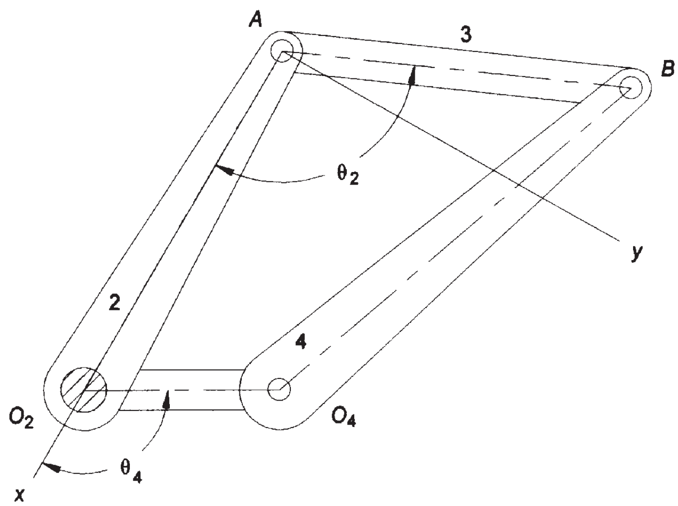
Two argument inverse tangent

$$atan2(x,y) := \begin{cases} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{cases}$$

Solution: See Figure P6-25 and Mathcad file P0663.

1. Draw the linkage to scale and label it. Instant center $I_{2,4}$ is at the intersection of line AB with line O_2O_4. To get the first centrode, ground link 2 and let link 3 be the input. Then we have

$a := L_3$ $\quad b := L_4$ $\quad c := L_1$ $\quad d := L_2$



2. Determine the range of motion for this Grashof double rocker. From Figure 3-1a on page 80, one toggle angle is

$$\theta_{20} := acos\left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d}\right] \qquad \theta_{20} = 124.294\,deg$$

The other toggle angle is the negative of this. The range of motion is

$$\theta_2 := -\theta_{20}, -\theta_{20} + 1 \cdot deg .. \theta_{20}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.1311 \qquad K_2 = 2.0294$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.7418$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Calculate the coordinates of the intersection of line *BC* with line *AD*. This will be the instant center $I_{2,4}$.

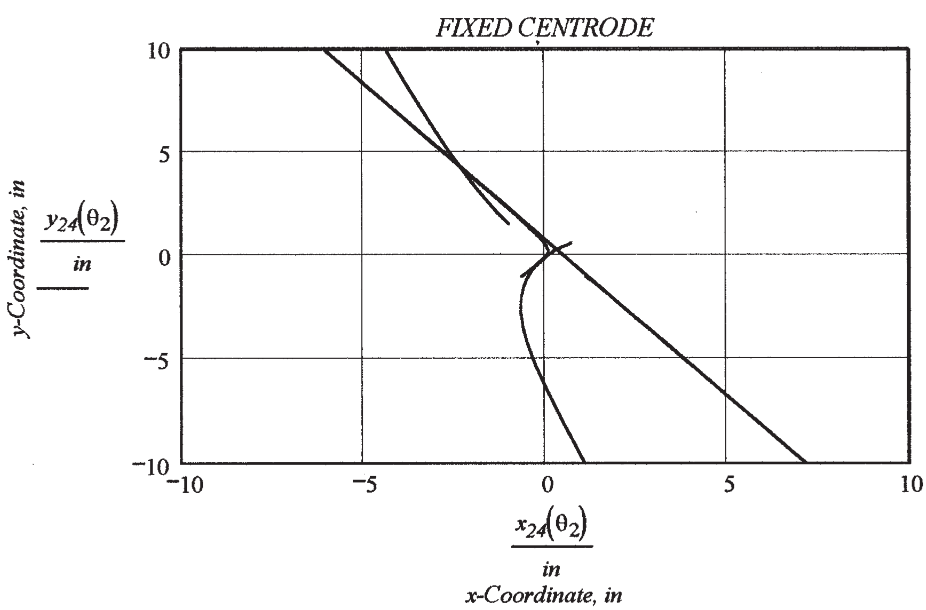
Line *BC*: $y := x \cdot \tan(\theta_2)$

Line *AD*: $y - 0 := (x - d) \cdot \tan(\theta_4)$

Eliminating y and solving for the x- and y-coordinates of the intersection,

$$x_{24}(\theta_2) := \frac{d \cdot \tan(\theta_{41}(\theta_2))}{\tan(\theta_{41}(\theta_2)) - \tan(\theta_2)} \qquad y_{24}(\theta_2) := x_{24}(\theta_2) \cdot \tan(\theta_2)$$

6. Plot the fixed centrode.

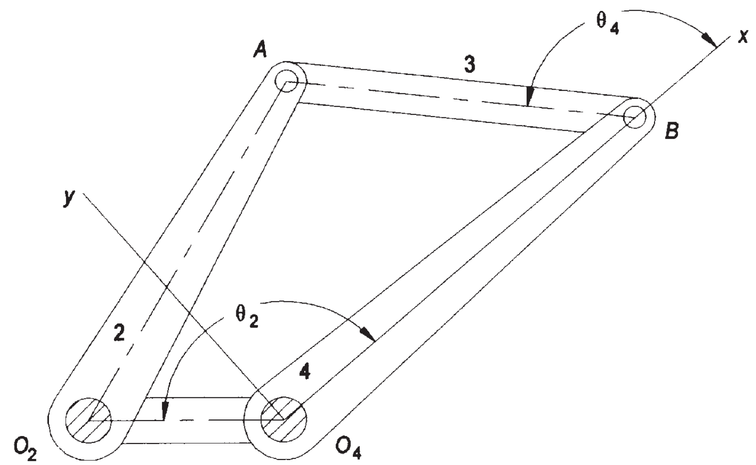


7. Invert the linkage, making C and D the fixed pivots. Then,

$a := L_1 \qquad b := L_2 \qquad c := L_3 \qquad d := L_4$

8. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 0.5 \cdot deg .. 360 \cdot deg$$



9. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.3824 \qquad K_2 = 1.3279$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.6097$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

10. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{42}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

11. Calculate the coordinates of the intersection of line BC with line AD. This will be the instant center $I_{2,4}$.

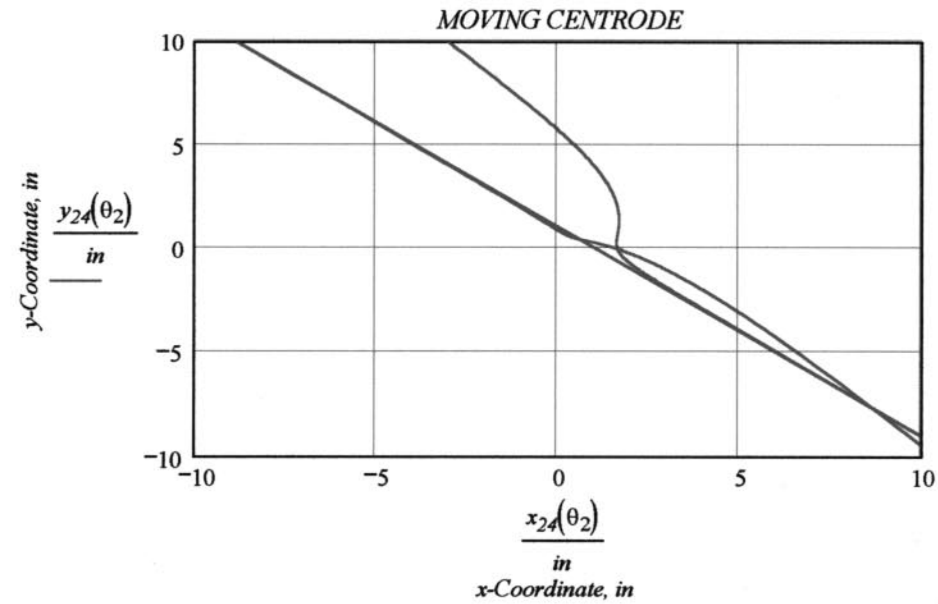
Line AD: $y := x \cdot tan(\theta_2)$

Line BC: $y - 0 := (x - d) \cdot tan(\theta_4)$

Eliminating y and solving for the x- and y-coordinates of the intersection,

$$x_{24}(\theta_2) := \frac{d \cdot tan(\theta_{42}(\theta_2))}{tan(\theta_{42}(\theta_2)) - tan(\theta_2)} \qquad y_{24}(\theta_2) := x_{24}(\theta_2) \cdot tan(\theta_2)$$

6. Plot the moving centrode. (See next page.)


MOVING CENTRODE
10
5
0
−5
−10
−10
−5
0
5
10
$\frac{y_{24}(\theta_2)}{in}$
y-Coordinate, in
$\frac{x_{24}(\theta_2)}{in}$
x-Coordinate, in

PROBLEM 6-64

Statement: Figure P6-26 shows a mechanism with dimensions. Use a graphical method to calculate the velocities of points *A*, *B*, and *C* and the velocity of slip for the position shown.

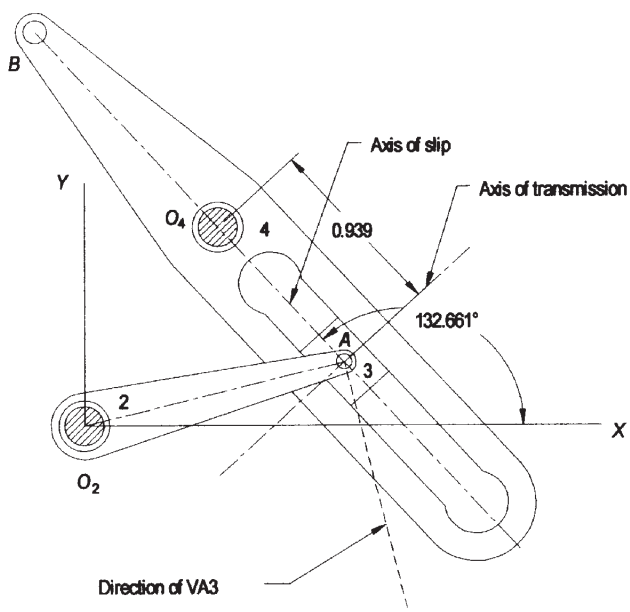
Given: Link lengths and angles:

Link 1 (O_2O_4)	$d := 1.22 \cdot in$	Angle O_2O_4 makes with *X* axis	$\theta_1 := 56.5 \cdot deg$
Link 2 (O_2A)	$a := 1.35 \cdot in$	Angle θ_2 makes with *X* axis	$\theta_2 := 14 \cdot deg$
Link 4 (O_4B)	$e := 1.36 \cdot in$		
Link 5 (BC)	$f := 2.69 \cdot in$		
Link 6 (O_6C)	$g := 1.80 \cdot in$	Angle O_6C makes with *X* axis	$\theta_6 := 88 \cdot deg$

Angular velocity of link 2 $\qquad \omega_2 := 20 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-26 and Mathcad file P0664.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A* on links 2 and 3.

$$V_{A3} := a \cdot \omega_2 \qquad V_{A3} = 27.000 \frac{in}{sec} \qquad \theta_{VA3} := \theta_2 - 90 \cdot deg \qquad \theta_{VA3} = -76.0 \, deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point *A* on link 3. The equation to be solved graphically is

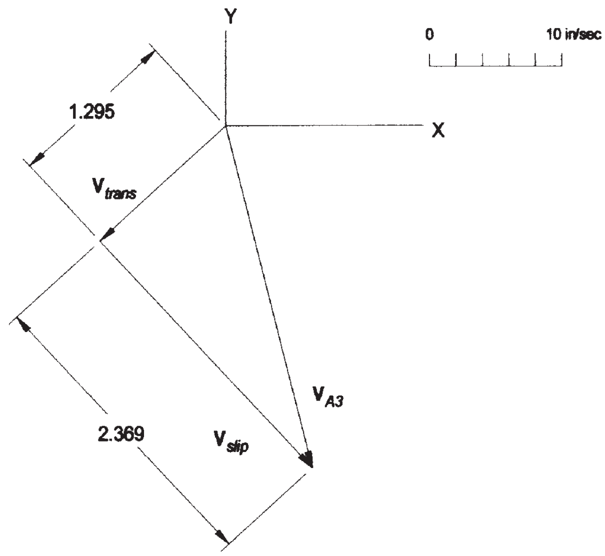
$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.

b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.

c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$

$V_{slip} := 2.369 \cdot in \cdot k_v$ $\qquad V_{slip} = 23.690 \dfrac{in}{sec}$

$V_{trans} := 1.295 \cdot in \cdot k_v$ $\qquad V_{trans} = 12.950 \dfrac{in}{sec}$

5. The true velocity of point A on link 4 is V_{trans},

$V_{A4} := V_{trans}$ $\qquad V_{A4} = 12.95 \dfrac{in}{sec}$

6. Determine the angular velocity of link 4 using equation 6.7.

From the linkage layout above: $c := 0.939 \cdot in$ and $\theta_4 := 132.661 \cdot deg$

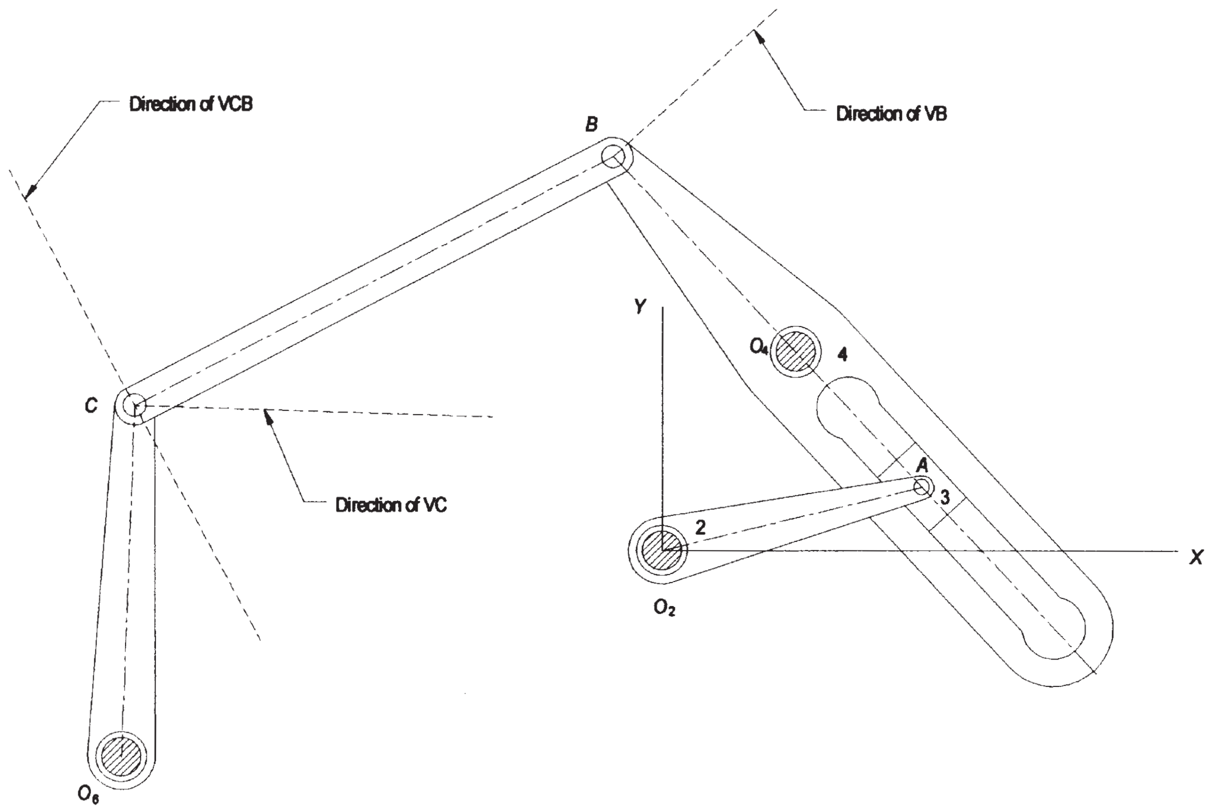
$\omega_4 := \dfrac{V_{A4}}{c}$ $\qquad \omega_4 = 13.791 \dfrac{rad}{sec}$ CW

7. Determine the magnitude and sense of the vector $\mathbf{V}_B$ using equation 6.7.

$V_B := e \cdot \omega_4$ $\qquad V_B = 18.756 \dfrac{in}{sec}$

$\theta_{VA4} := \theta_4 - 90 \cdot deg$ $\qquad \theta_{VA4} = 42.661\ deg$

8. Draw links 1, 4, 5, and 6 to a convenient scale. Indicate the directions of the velocity vectors of interest. (See next page.)



9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

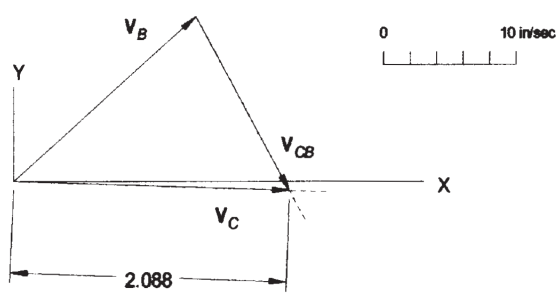
$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.

b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.

c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.



10. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$ $\qquad \theta_{VC} := \theta_6 - 90 \cdot deg$

$V_C := 2.088 \cdot in \cdot k_v$ $\qquad V_C = 20.9 \dfrac{in}{sec}$ $\qquad \theta_{VC} = -2.0\,deg$

PROBLEM 6-65

Statement: Figure P6-27 shows a cam and follower. Distances are given below. Find the velocities of points *A* and *B*, the velocity of transmission, velocity of slip, and ω_3 if $\omega_2 = 50$ rad/sec (CW). Use a graphical method.

Given: $\omega_2 := 50 \cdot rad \cdot sec^{-1}$

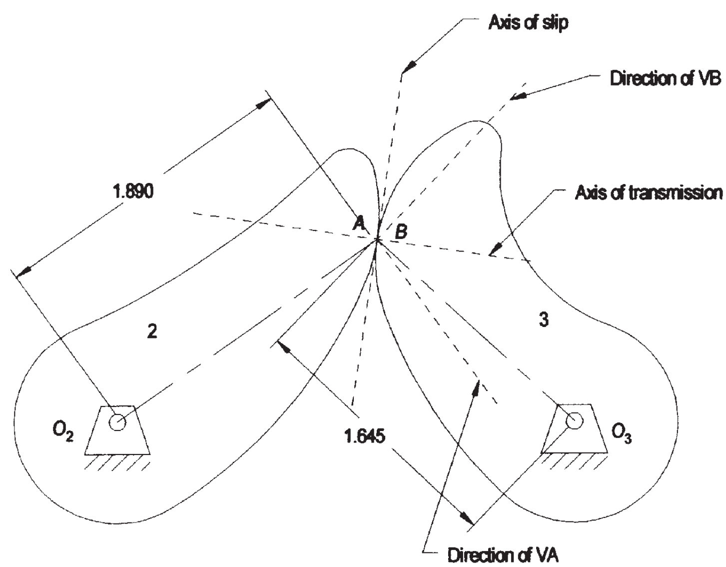
Distance from O_2 to *A*: $a := 1.890 \cdot in$

Distance from O_3 to *B*: $b := 1.645 \cdot in$

Assumptions: Roll-slide contact

Solution: See Figure P6-27 and Mathcad file P0665.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A*.

$$V_A := a \cdot \omega_2 \qquad V_A = 94.500 \frac{in}{sec}$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point *A*. The equation to be solved graphically is

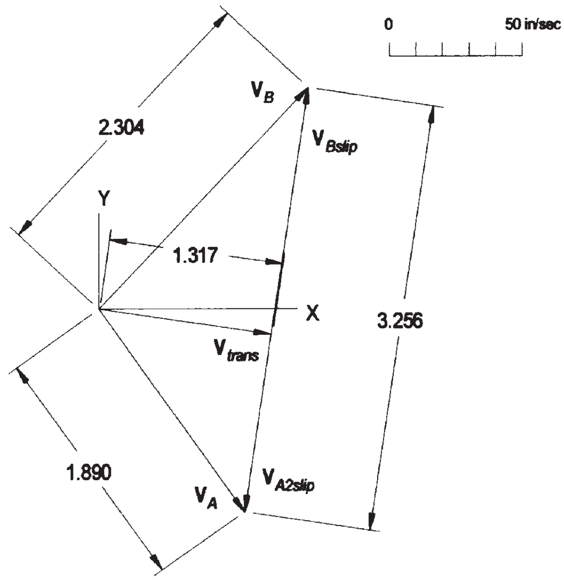
$$\mathbf{V}_A = \mathbf{V}_{trans} + \mathbf{V}_{Aslip}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_A$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{50 \cdot in \cdot sec^{-1}}{in}$

$V_A := 1.890 \cdot in \cdot k_v$ $\qquad V_A = 94.5 \frac{in}{sec}$

$V_B := 2.304 \cdot in \cdot k_v$ $\qquad V_B = 115.2 \frac{in}{sec}$

$V_{slip} := 3.256 \cdot in \cdot k_v$ $\qquad V_{slip} = 162.8 \frac{in}{sec}$

$V_{trans} := 1.317 \cdot in \cdot k_v$ $\qquad V_{trans} = 65.9 \frac{in}{sec}$

5. Determine the angular velocity of link 3 using equation 6.7.

$\omega_3 := \frac{V_B}{b}$ $\qquad \omega_3 = 70.0 \frac{rad}{sec}$ CW

PROBLEM 6-66

Statement: Figure P6-28 shows a quick-return mechanism with dimensions. Use a graphical method to calculate the velocities of points *A*, *B*, and *C* and the velocity of slip for the position shown.

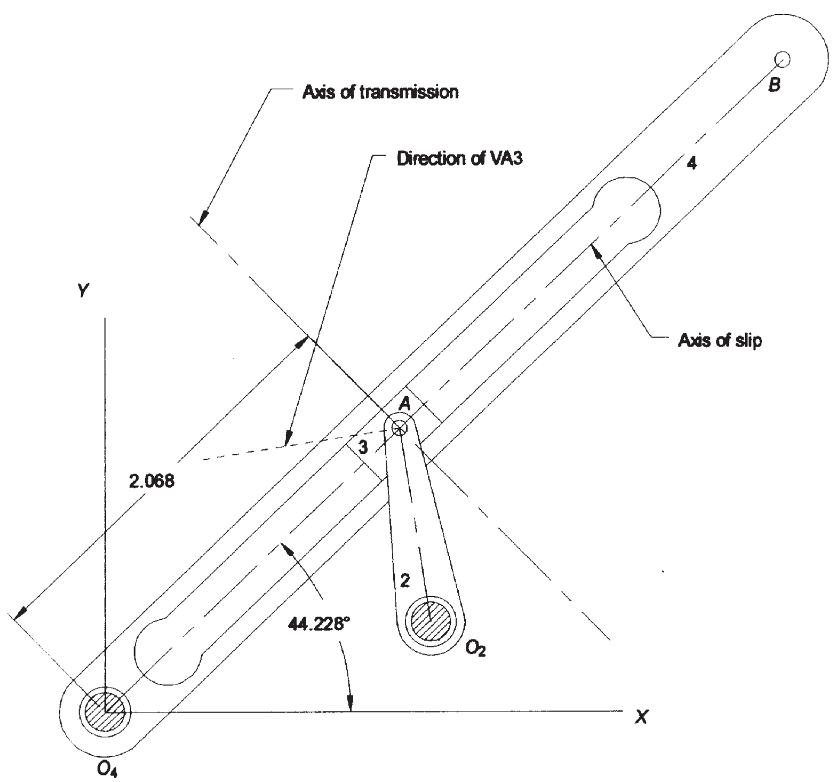
Given: Link lengths and angles:

Link 1 (O_2O_4)	$d := 1.69 \cdot in$	Angle O_2O_4 makes with X axis	$\theta_1 := 15.5 \cdot deg$
Link 2 (L_2)	$a := 1.00 \cdot in$	Angle link 2 makes with X axis	$\theta_2 := 99 \cdot deg$
Link 4 (L_4)	$e := 4.76 \cdot in$		
Link 5 (L_5)	$f := 4.55 \cdot in$		
Offset (O_2C)	$g := 2.86 \cdot in$		

Angular velocity of link 2 $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure P6-28 and Mathcad file P0666.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A* on links 2 and 3.

$$V_{A2} := a \cdot \omega_2 \qquad V_{A2} = 10.000 \frac{in}{sec} \qquad \theta_{VA2} := \theta_2 + 90 \cdot deg \qquad \theta_{VA2} = 189.0\,deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point *A* on link 3. The equation to be solved graphically is

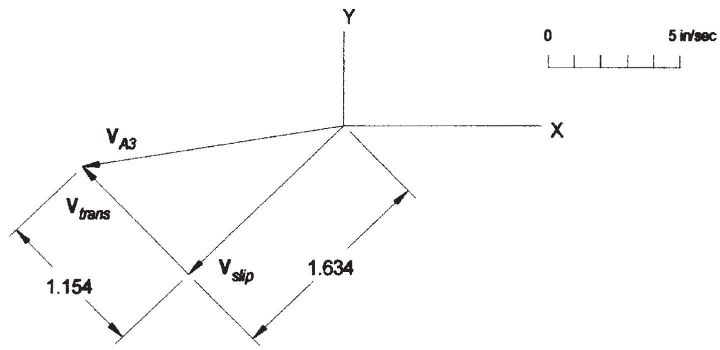
$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.

b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.

c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{5 \cdot in \cdot sec^{-1}}{in}$

$V_{slip} := 1.634 \cdot in \cdot k_v$ $\qquad V_{slip} = 8.170 \dfrac{in}{sec}$

$V_{trans} := 1.154 \cdot in \cdot k_v$ $\qquad V_{trans} = 5.770 \dfrac{in}{sec}$

5. The true velocity of point A on link 4 is V_{trans},

$V_{A4} := V_{trans}$ $\qquad V_{A4} = 5.77 \dfrac{in}{sec}$

6. Determine the angular velocity of link 4 using equation 6.7.

From the linkage layout above: $c := 2.068 \cdot in$ and $\theta_4 := 44.228 \cdot deg$

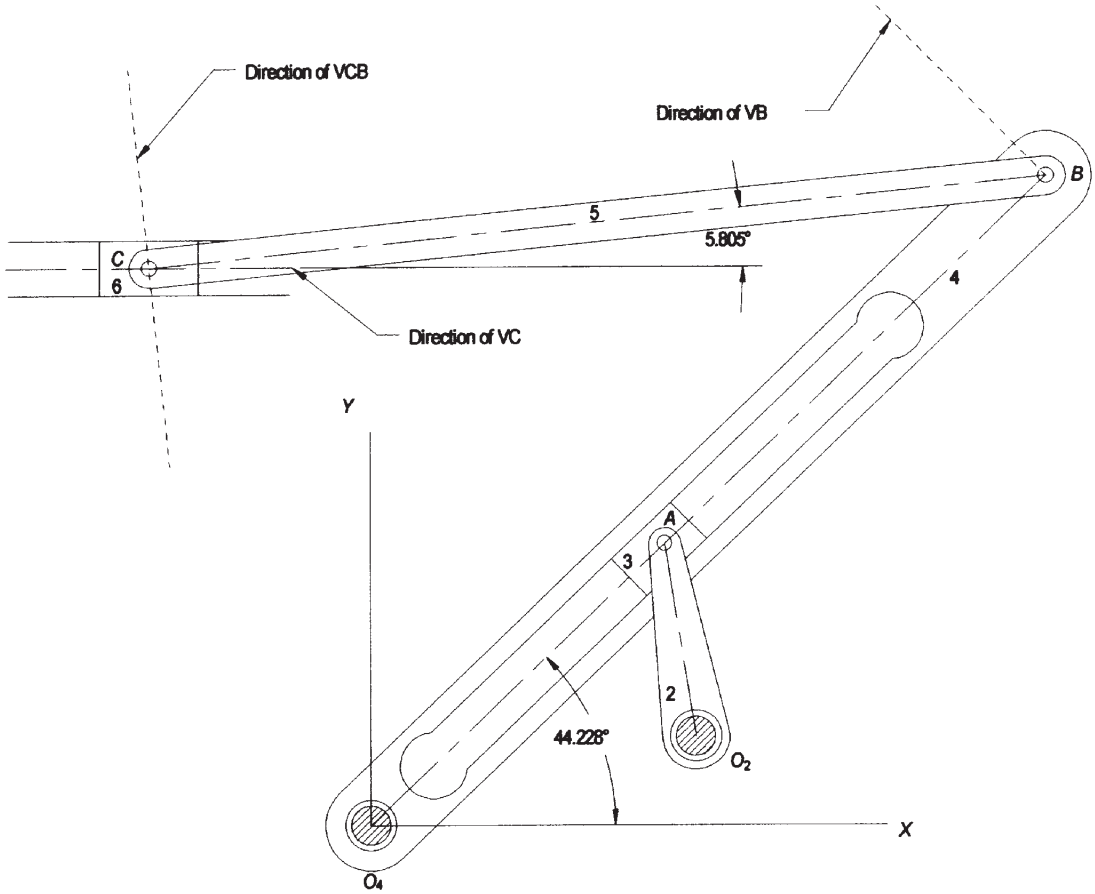
$\omega_4 := \dfrac{V_{A4}}{c}$ $\qquad \omega_4 = 2.790 \dfrac{rad}{sec}$ CCW

7. Determine the magnitude and sense of the vector $\mathbf{V}_B$ using equation 6.7.

$V_B := e \cdot \omega_4$ $\qquad V_B = 13.281 \dfrac{in}{sec}$

$\theta_{VB} := \theta_4 + 90 \cdot deg$ $\qquad \theta_{VB} = 134.228\ deg$

8. Draw links 1, 4, 5, and 6 to a convenient scale. Indicate the directions of the velocity vectors of interest. (See next page.)



9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *C*. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.

b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.

c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.

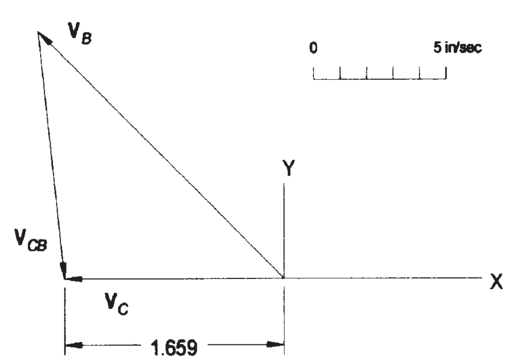
d. Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.

10. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{5 \cdot in \cdot sec^{-1}}{in}$

$V_C := 1.659 \cdot in \cdot k_v$ $\qquad V_C = 8.30 \dfrac{in}{sec}$

$\theta_{VC} := 180 \cdot deg$



PROBLEM 6-67

Statement: Figure P6-29 shows a drum pedal mechanism . For the dimensions given below, find and plot the mechanical advantage and the velocity ratio of the linkage over its range of motion. If the input velocity V_{in} is a constant and F_{in} is constant, find the output velocity, output force, and power in over the range of motion.

Given: Link lengths:

Link 2 (O_2A)	$a := 100 \cdot mm$	Link 3 (AB)	$b := 28 \cdot mm$
Link 4 (O_4B)	$c := 64 \cdot mm$	Link 1 (O_2O_4)	$d := 56 \cdot mm$

Distance to force application:

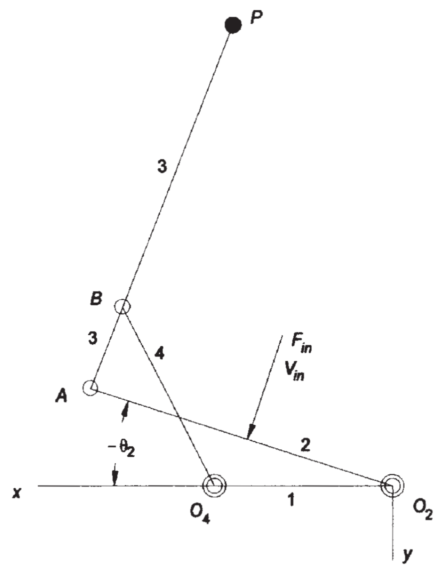
Link 2	$r_{in} := 48 \cdot mm$	Link 3 (AP)	$r_{out} := 124 \cdot mm$

Input force and velocity: $F_{in} := 50 \cdot N$ $V_{in} := 3 \cdot m \cdot sec^{-1}$

Range of positions of link 2: $\theta_{20} := 162 \cdot deg$ $\theta_{21} := 171 \cdot deg$

Solution: See Figure P6-29 and Mathcad file P0667.

1. Draw the mechanism to scale and label it.



2. Calculate the range of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

Rotation angle of local xy system to global XY system: $\alpha := 180 \cdot deg$

$$\theta_2 := \theta_{20} - \alpha, (\theta_{20} - \alpha) + 1 \cdot deg .. \theta_{21} - \alpha$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.5600 \qquad K_2 = 0.8750$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.2850$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{(B(\theta_2))^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right] \right]$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 2.0000$$

$$K_5 = -1.7543$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

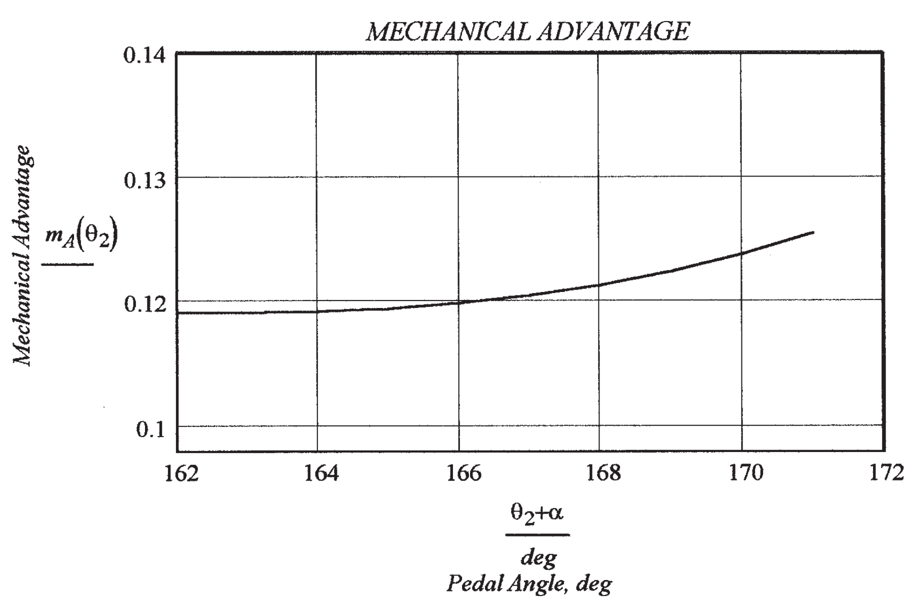
$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{(E(\theta_2))^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right] \right]$$

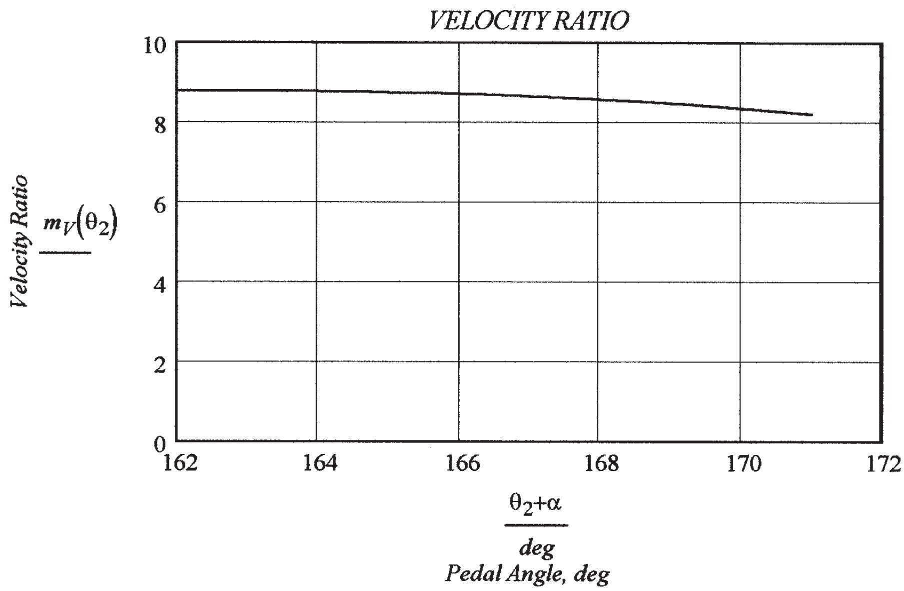
7. Using equations 6.13d and 6.18a, where ω_{out} is ω_3, calculate and plot the mechanical advantage of the linkage over the given range.

$$m_A(\theta_2) := \frac{b \cdot sin(\theta_3(\theta_2) - \theta_4(\theta_2))}{a \cdot sin(\theta_4(\theta_2) - \theta_2)} \cdot \frac{r_{in}}{r_{out}}$$



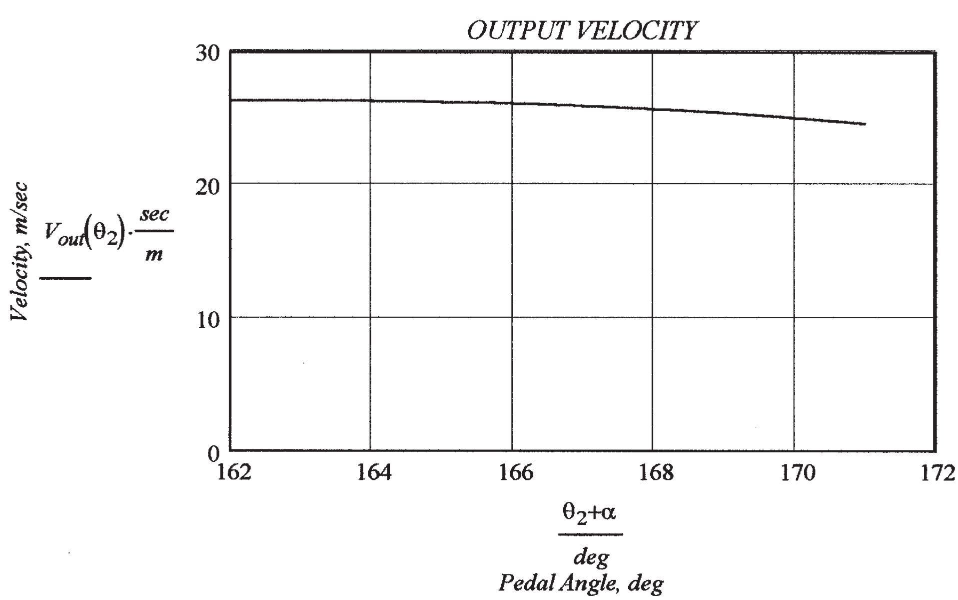
8. Calculate and plot the velocity ratio using equation 6.13d,

$$m_V(\theta_2) := \frac{1}{m_A(\theta_2)}$$



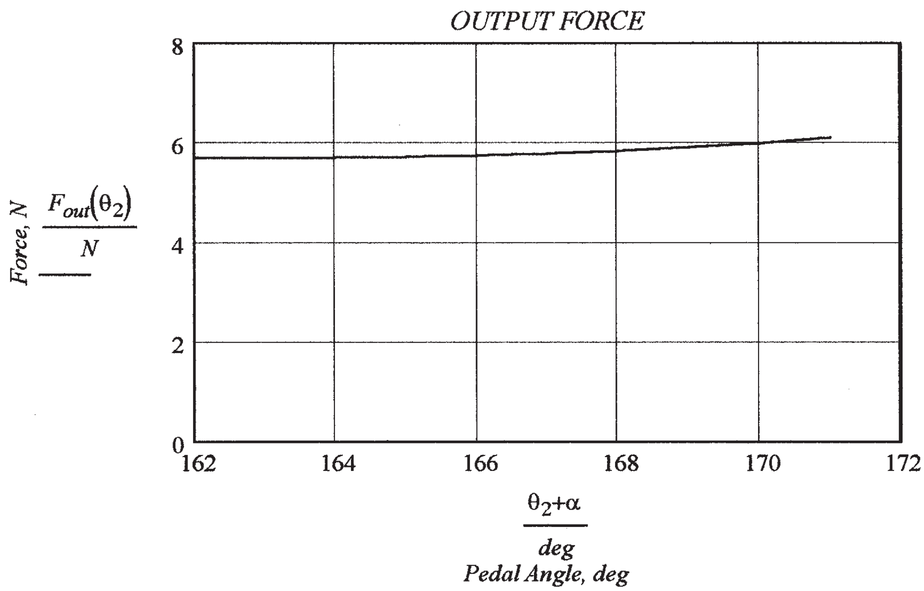
9. Calculate and plot the output velocity using equation 6.13a.

$$V_{out}(\theta_2) := V_{in} \cdot m_V(\theta_2)$$



10. Calculate and plot the output force using equation 6.13a.

$$F_{out}(\theta_2) := F_{in} \cdot m_A(\theta_2)$$



PROBLEM 6-68

Statement: Figure 3-33 shows a sixbar slider crank linkage. Find all of its instant centers in the position shown:

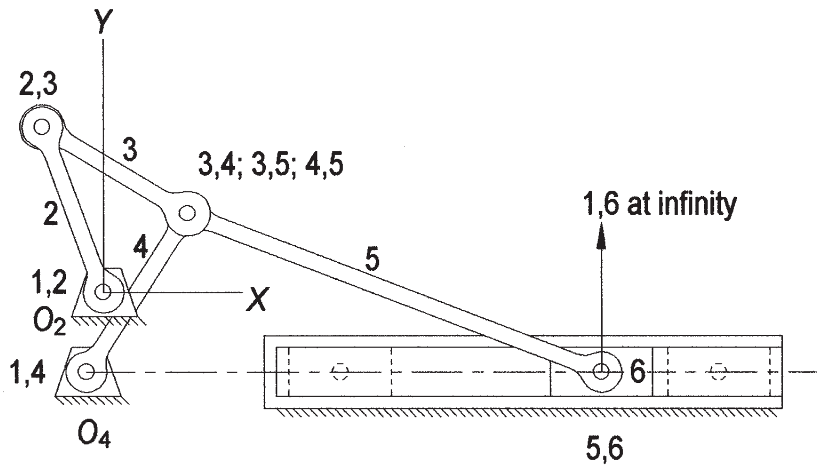
Given: Number of links $n := 6$

Solution: See Figure 3-33 and Mathcad file P0668.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

2. Draw the linkage to scale and identify those ICs that can be found by inspection (8).



2. Use Kennedy's Rule and a linear graph to find the remaining 7 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,6}$; and $I_{4,6}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,4}$-$I_{4,5}$

$I_{2,5}$: $I_{1,2}$-$I_{1,5}$ and $I_{2,3}$-$I_{3,5}$

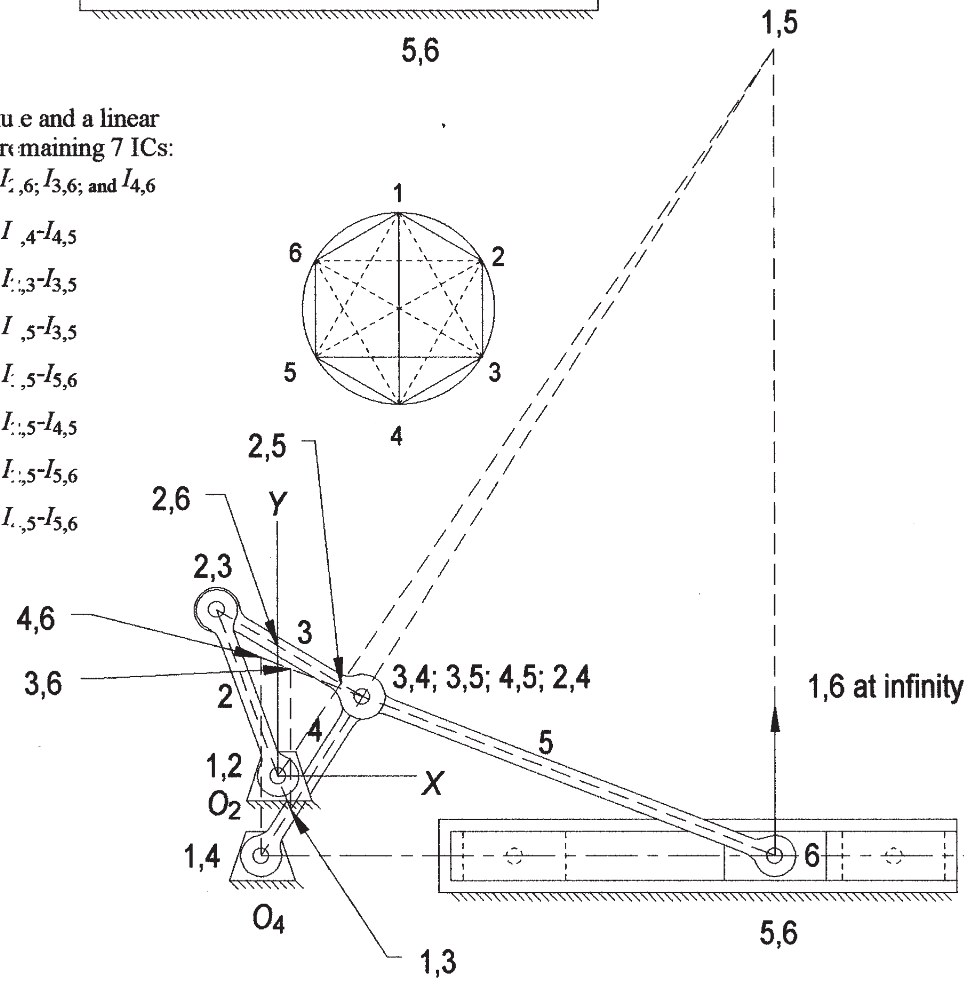
$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,5}$-$I_{3,5}$

$I_{3,6}$: $I_{1,6}$-$I_{1,3}$ and $I_{3,5}$-$I_{5,6}$

$I_{2,4}$: $I_{2,3}$-$I_{3,4}$ and $I_{2,5}$-$I_{4,5}$

$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$

$I_{4,6}$: $I_{1,4}$-$I_{1,6}$ and $I_{4,5}$-$I_{5,6}$



PROBLEM 6-69

Statement: Calculate and plot the centrodes of instant center I_{24} of the linkage in Figure 3-33 so that a pair of noncircular gears can be made to replace the driver dyad 23.

Given: Link lengths:

Input crank (L_2)	$L_2 := 2.170$	Fourbar coupler (L_3)	$L_3 := 2.067$
Output crank (L_4)	$L_4 := 2.310$	Fourbar ground link (L_1)	$L_1 := 1.000$

Two argument inverse tangent:

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure 3-33 and Mathcad file P0669.

1. Invert the linkage, grounding link 2 such that the input link is 3, the coupler is 4, and the output link is 1.

$$a := L_3 \qquad b := L_4 \qquad c := L_1 \qquad d := L_2$$

2. Define the input crank motion for this inversion: $\theta_2 := 47\cdot deg, 47.5 deg .. 102.5 deg$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 1.0498 \qquad K_2 = 2.1700 \qquad K_3 = 1.1237$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

$$\theta_4(\theta_2) := if(\theta_{41}(\theta_2) > 2\cdot\pi, \theta_{41}(\theta_2) - 2\cdot\pi, \theta_{41}(\theta_2))$$

5. Calculate the coordinates of the intersection of links 1 and 3 in the *xy* coordinate system.

$$x_{242}(\theta_2) := -\frac{d\cdot tan(\theta_4(\theta_2))}{tan(\theta_2) - tan(\theta_4(\theta_2))} \qquad y_{242}(\theta_2) := x_{242}(\theta_2)\cdot tan(\theta_2)$$

6. Invert the linkage, grounding link 4 such that the input link is 1, the coupler is 2, and the output link is 3.

$$a := L_1 \qquad b := L_2 \qquad c := L_3 \qquad d := L_4$$

8. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 2.3100 \qquad K_2 = 1.1176 \qquad K_3 = 1.4271$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

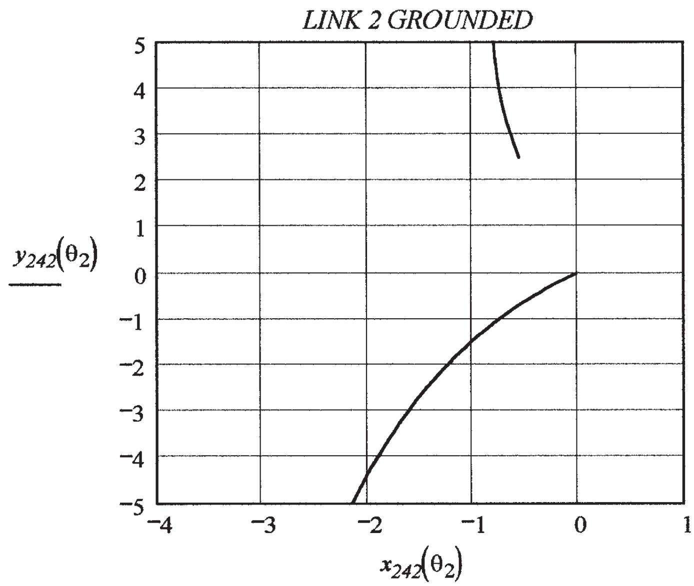
$$B(\theta_2) := -2\cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

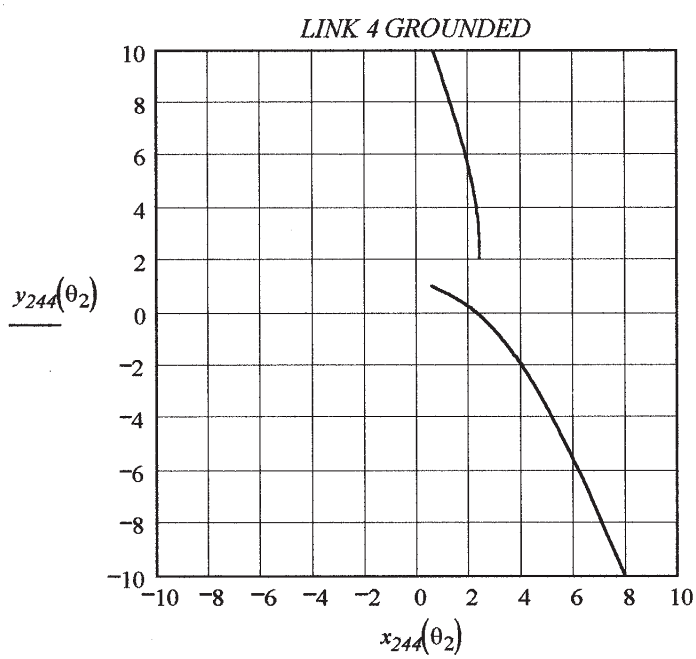
$$\theta_4(\theta_2) := if(\theta_{41}(\theta_2) > 2\cdot\pi, \theta_{41}(\theta_2) - 2\cdot\pi, \theta_{41}(\theta_2))$$

5. Calculate the coordinates of the intersection of links 1 and 3 in the *xy* coordinate system.

$$x_{244}(\theta_2) := -\frac{d\cdot tan(\theta_4(\theta_2))}{tan(\theta_2) - tan(\theta_4(\theta_2))} \qquad y_{244}(\theta_2) := x_{244}(\theta_2)\cdot tan(\theta_2)$$



7. Define the input crank motion for this inversion: $\theta_2 := 41\cdot deg, 42\cdot deg..241\cdot deg$



PROBLEM 6-70a

Statement: Find the velocity of the slider in Figure 3-33 for $\theta_2 = 110$ deg with respect to the global *X* axis assuming $\omega_2 = 1$ rad/sec CW. Use a graphical method.

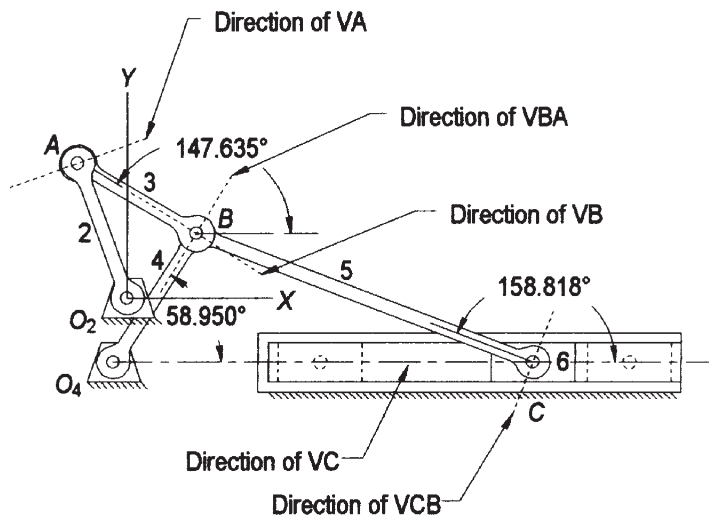
Given: Link lengths:

Link 2 (O_2 to *A*)	$a := 2.170 \cdot in$	Link 3 (*A* to *B*)	$b := 2.067 \cdot in$
Link 4 (O_4 to *B*)	$c := 2.310 \cdot in$	Link 1 (O_2 to O_4)	$d := 1.000 \cdot in$
Link 5 (*B* to *C*)	$e := 5.400$		
Coordinate angle	$\delta := -102 \cdot deg$	Crank angle:	$\theta_2 := 110 \cdot deg$

Input crank angular velocity $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-33 and Mathcad file P0670a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A*.

$$V_A := a \cdot \omega_2 \qquad V_A = 2.170 \frac{in}{sec}$$

$$\theta_{VA} := \theta_2 - 90 \cdot deg \qquad \theta_{VA} = 20.000\,deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

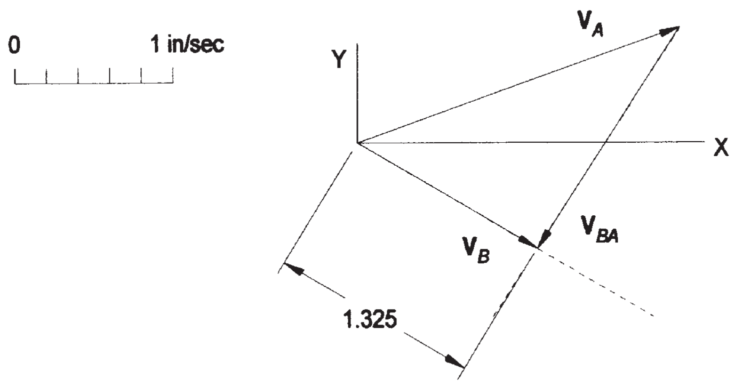
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

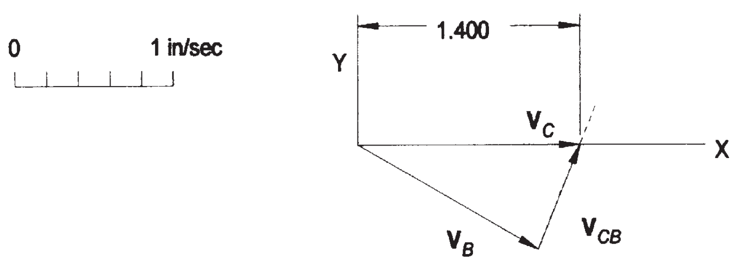
Velocity scale factor: $k_v := \frac{1 \cdot in \cdot sec^{-1}}{in}$

$V_B := 1.325 \cdot in \cdot k_v$ $\quad V_B = 1.325 \frac{in}{sec}$ $\quad \theta_{VB} := -31.050 \cdot deg$

5. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *C*, the magnitude of the relative velocity $\mathbf{V}_{CB}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.
c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{1 \cdot in \cdot sec^{-1}}{in}$

$V_C := 1.400 \cdot in \cdot k_v$ $\quad V_C = 1.400 \frac{in}{sec}$ $\quad \theta_{VC} := 0.0 \cdot deg$

PROBLEM 6-70b

Statement: Find the velocity of the slider in Figure 3-33 for $\theta_2 = 110$ deg with respect to the global X axis assuming $\omega_2 = 1$ rad/sec CW. Use the method of instant centers.

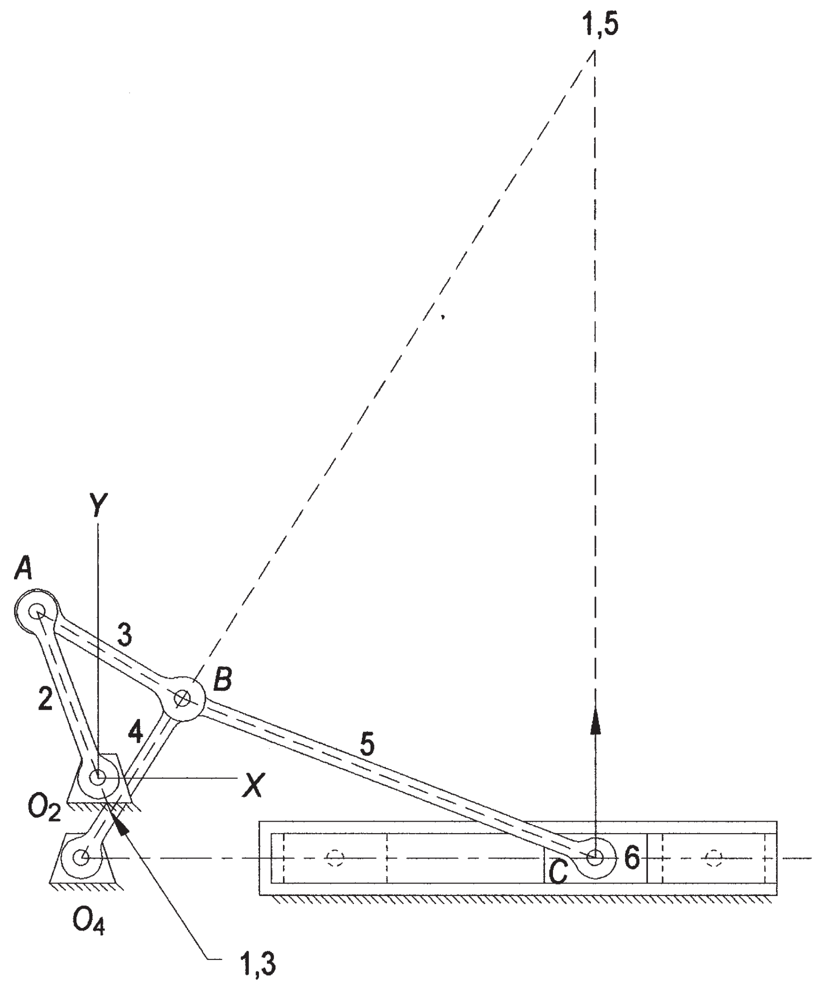
Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.170 \cdot in$	Link 3 (A to B)	$b := 2.067 \cdot in$
Link 4 (O_4 to B)	$c := 2.310 \cdot in$	Link 1 (O_2 to O_4)	$d := 1.000 \cdot in$
Link 5 (B to C)	$e := 5.400$		
Coordinate angle	$\delta := -102 \cdot deg$	Crank angle:	$\theta_2 := 110 \cdot deg$

Input crank angular velocity $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure 3-33 and Mathcad file P0670b.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers. See Problem 6-68 for the determination of IC locations.



From the layout above:

$AI13 := 2.609 \cdot in$ $BI13 := 1.641 \cdot in$ $BI15 := 9.406 \cdot in$ $CI15 := 9.896 \cdot in$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point *A*.

$$V_A := a \cdot \omega_2 \qquad V_A = 2.170 \frac{in}{sec}$$

$$\theta_{VA} := \theta_2 - 90 \cdot deg \qquad \theta_{VA} = 20.0\,deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 0.832 \frac{rad}{sec} \qquad \text{CW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b. The direction of $\mathbf{V}_B$ is down and to the right

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 1.365 \frac{in}{sec}$$

5. Use equation 6.9c to determine the angular velocity of link 5.

$$\omega_5 := \frac{V_B}{BI15} \qquad \omega_5 = 0.145 \frac{rad}{sec} \qquad \text{CCW}$$

6. Determine the magnitude of the velocity at point *C* using equation 6.9b.

$$V_C := CI15 \cdot \omega_5 \qquad V_C = 1.436 \frac{in}{sec} \qquad \text{to the right}$$

PROBLEM 6-70c

Statement: Find the velocity of the slider in Figure 3-33 for θ_2 = 110 deg with respect to the global X axis assuming ω_2 = 1 rad/sec CW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.170 \cdot in$	Link 3 (A to B)	$b := 2.067 \cdot in$
Link 4 (O_4 to B)	$c := 2.310 \cdot in$	Link 1 (O_2 to O_4)	$d := 1.000 \cdot in$
Link 5 (B to C)	$e := 5.400 \cdot in$		
Coordinate angle	$\delta := -102 \cdot deg$	Crank angle:	$\theta_{2XY} := 110 \cdot deg$

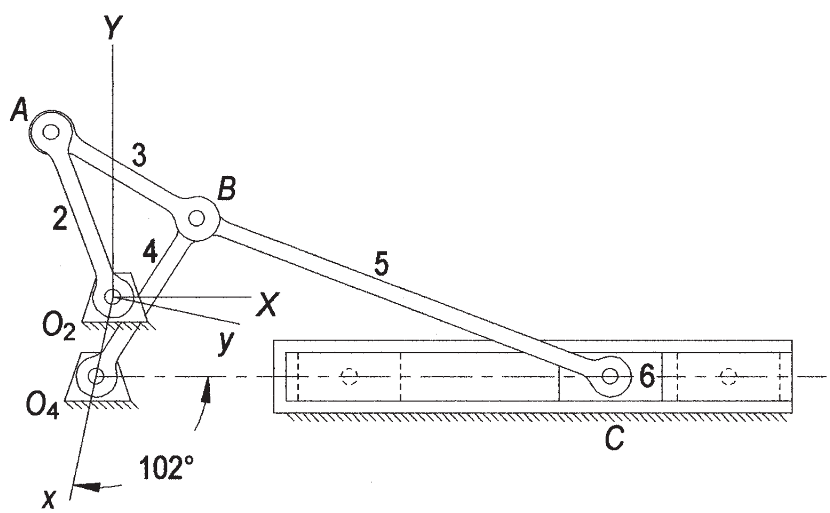
Input crank angular velocity $\omega_2 := -1 \cdot rad \cdot sec^{-1}$ CW

Two argument inverse tangent

$$atan2(x, y) := \begin{cases} return\ 0.5 \cdot \pi & if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi & if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) & if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & otherwise \end{cases}$$

Solution: See Figure P6-33 and Mathcad file P0670c.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

Transform crank angle to the local xy coordinate system:

$\theta_2 := \theta_{2XY} - \delta$ $\qquad\qquad \theta_2 = 212.000\ deg$

$K_1 := \frac{d}{a}$ $\qquad K_2 := \frac{d}{c}$

$K_1 = 0.4608$ $\qquad K_2 = 0.4329$

$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$ $\qquad K_3 = 0.6755$

$$A := \cos(\theta_2) - K_1 - K_2\cdot\cos(\theta_2) + K_3$$

$$B := -2\cdot\sin(\theta_2)$$

$$C := K_1 - (K_2 + 1)\cdot\cos(\theta_2) + K_3$$

$A = -0.2662 \quad B = 1.0598 \quad C = 2.3515$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{4xy} := 2\cdot\left(\operatorname{atan2}\left(2\cdot A, -B - \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) - 2\cdot\pi \qquad \theta_{4xy} = 159.635\ deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.4838$$

$K_5 = -0.5178$

$$D := \cos(\theta_2) - K_1 + K_4\cdot\cos(\theta_2) + K_5 \qquad D = -2.2370$$

$$E := -2\cdot\sin(\theta_2) \qquad E = 1.0598$$

$$F := K_1 + (K_4 - 1)\cdot\cos(\theta_2) + K_5 \qquad F = 0.3808$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{3xy} := 2\cdot\left(\operatorname{atan2}\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) - 2\cdot\pi \qquad \theta_{3xy} = 70.950\ deg$$

6. Determine the angular velocity of link 4 for the open circuit using equations 6.18.

$$\omega_4 := \frac{a\cdot\omega_2}{c}\cdot\frac{\sin(\theta_2 - \theta_{3xy})}{\sin(\theta_{4xy} - \theta_{3xy})} \qquad \omega_4 = -0.591\,\frac{rad}{sec}$$

7. Transform θ_4 back to the global *XY* system.

$$\theta_4 := \theta_{4xy} + \delta \qquad \theta_4 = 57.635\ deg$$

5. Determine θ_5 and d, with respect to O_4, using equation 4.17.

Offset: $cc := 0\cdot in$

$$\theta_5 := \operatorname{asin}\left(\frac{c\cdot\sin(\theta_4) - cc}{e}\right) + \pi \qquad \theta_5 = 158.818\ deg$$

$$dd := c\cdot\cos(\theta_4) - e\cdot\cos(\theta_5) \qquad dd = 6.272\ in$$

6. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5 := \frac{c}{e}\cdot\frac{\cos(\theta_4)}{\cos(\theta_5)}\cdot\omega_4 \qquad \omega_5 = 0.145\,\frac{rad}{sec}$$

7. Determine the velocity of pin C using equation 6.22b:

$$\mathbf{V_C} := -c\cdot\omega_4\cdot\sin(\theta_4) + e\cdot\omega_5\cdot\sin(\theta_5)$$

$$\mathbf{V_C} = 1.436\,\frac{in}{sec} \qquad |\mathbf{V_C}| = 1.436\,\frac{in}{sec} \qquad \arg(\mathbf{V_C}) = 0.000\ deg$$

PROBLEM 6-71

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular velocity of link 4 and the linear velocity of slider 6 in the sixbar slider-crank linkage of Figure 3-33 as a function of the angle of input link 2 for a constant $\omega_2 = 1$ rad/sec CW. Plot V_c both as a function of θ_2 and separately as a function of slider position as shown in the figure. What is the percent deviation from constant velocity over 240 deg < θ_2 < 270 deg and over 190 < θ_2 < 315 deg?

Given: Link lengths:

Input crank (L_2)	$a := 2.170$	Fourbar coupler (L_3)	$b := 2.067$
Output crank (L_4)	$c := 2.310$	Sllider coupler (L_5)	$e := 5.40$
Fourbar ground link (L_1)	$d := 1.000$		

Two argument inverse tangent:

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Crank velocity: $\omega_2 := -1\cdot\frac{rad}{sec}$

Solution: See Figure 3-33 and Mathcad file P0671.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a slider-crank mechanism using the output of the fourbar, link 4, as the input to the slider-crank.

2. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 1\cdot deg .. 360\cdot deg$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit) in the global XY coordinate system.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 0.4608 \qquad K_2 = 0.4329 \qquad K_3 = 0.6755$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2\cdot\cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot\sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot\cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right) - 102\cdot deg$$

4. If the calculated value of θ_4 is greater than 2π, subtract 2π from it and if it is negative, make it positive.

$$\theta_{42}(\theta_2) := if(\theta_{41}(\theta_2) > 2\cdot\pi, \theta_{41}(\theta_2) - 2\cdot\pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := if(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2\cdot\pi, \theta_{42}(\theta_2))$$

5. Determine the slider-crank motion using equations 4.16 and 4.17 with θ_4 as the input angle.

$$\theta_5(\theta_2) := asin\left(\frac{-c\cdot\sin(\theta_4(\theta_2))}{e}\right) + \pi$$

$$f(\theta_2) := c \cdot cos(\theta_4(\theta_2)) - e \cdot cos(\theta_5(\theta_2))$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.4838$$

$$K_5 = -0.5178$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

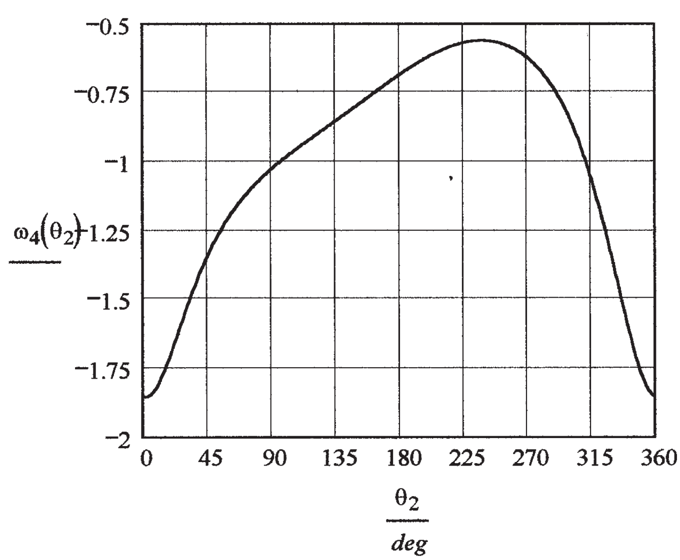
$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{(E(\theta_2))^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right] \right]$$

7. Determine the angular velocity of link 4 for the open circuit using equations 6.18.

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) + 102 \cdot deg - \theta_3(\theta_2))}$$

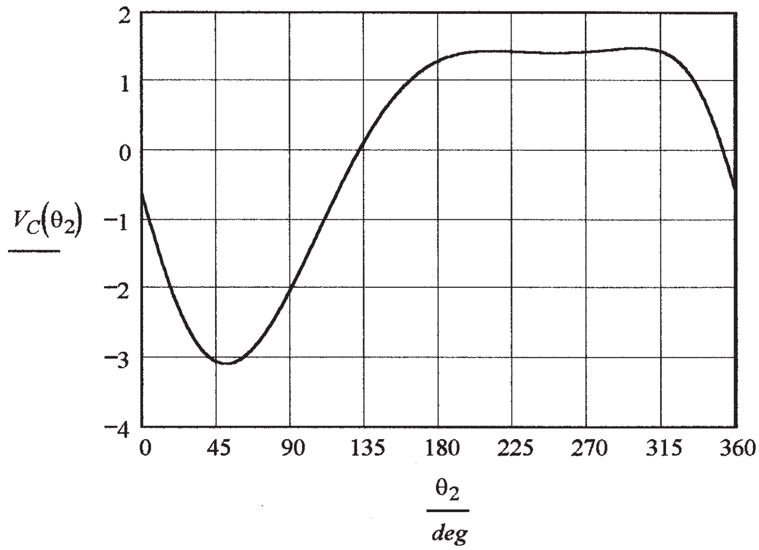


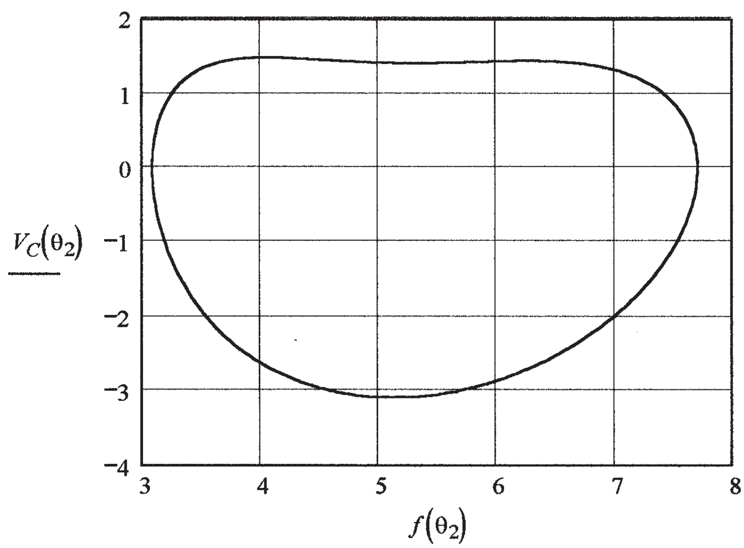
8. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5(\theta_2) := \frac{c}{e} \cdot \frac{cos(\theta_4(\theta_2))}{cos(\theta_5(\theta_2))} \cdot \omega_4(\theta_2)$$

9. Determine the velocity of pin C using equation 6.22b:

$$V_C(\theta_2) := -c \cdot \omega_4(\theta_2) \cdot sin(\theta_4(\theta_2)) + e \cdot \omega_5(\theta_2) \cdot sin(\theta_5(\theta_2))$$


2
1
0
$V_C(\theta_2)$
−1
−2
−3
−4
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$


2
1
0
$V_C(\theta_2)$
−1
−2
−3
−4
3
4
5
6
7
8
$f(\theta_2)$

PROBLEM 6-72

Statement: Figure 3-34 shows a Stephenson's sixbar mechanism. Find all of its instant centers in the position shown:

Given: Number of links $n := 6$

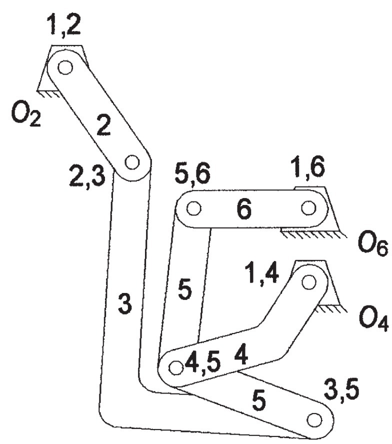
Solution: See Figure 3-34 and Mathcad file P0672.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

a. In part (a) of the figure.

1. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,4}$; $I_{3,6}$; and $I_{4,6}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,4}$-$I_{4,5}$

$I_{2,5}$: $I_{1,2}$-$I_{1,5}$ and $I_{2,3}$-$I_{3,5}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,5}$-$I_{3,5}$

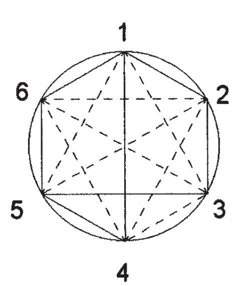
$I_{3,4}$: $I_{1,4}$-$I_{1,3}$ and $I_{4,5}$-$I_{3,5}$

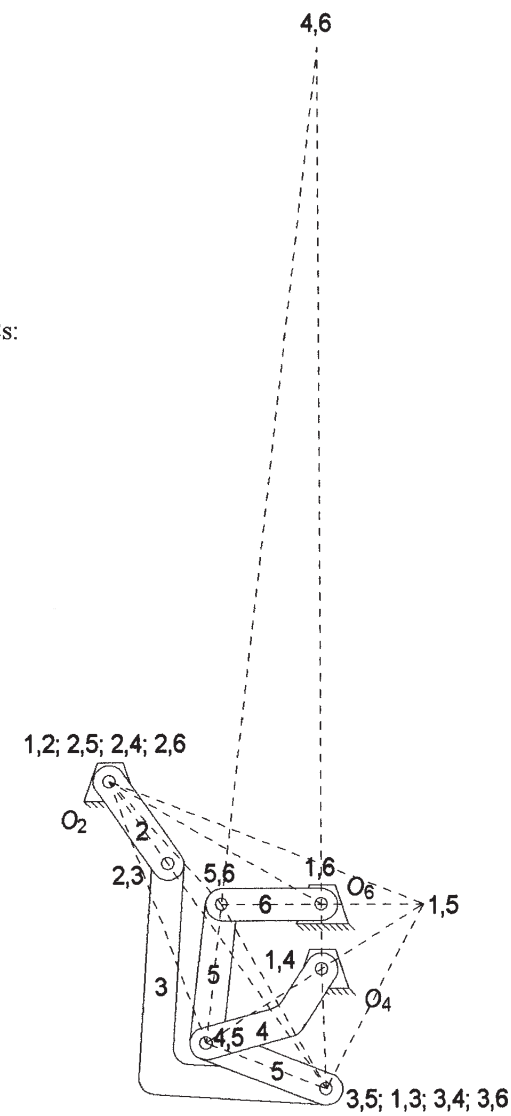
$I_{2,8}$: $I_{2,3}$-$I_{3,4}$ and $I_{2,5}$-$I_{4,5}$

$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$

$I_{3,6}$: $I_{1,3}$-$I_{1,6}$ and $I_{3,5}$-$I_{5,6}$

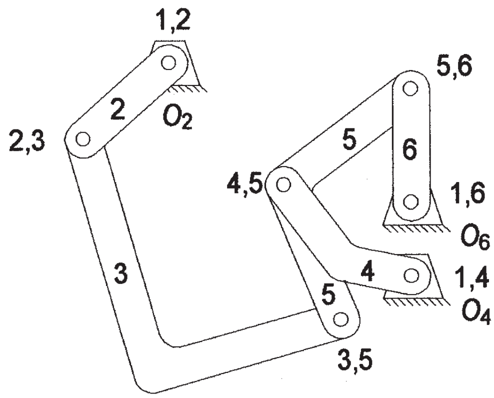
$I_{4,6}$: $I_{1,4}$-$I_{1,6}$ and $I_{4,5}$-$I_{5,6}$





b. In part (b) of the figure.

1. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,4}$; $I_{3,6}$; and $I_{4,6}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,4}$-$I_{4,5}$

$I_{2,5}$: $I_{1,2}$-$I_{1,5}$ and $I_{2,3}$-$I_{3,5}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,5}$-$I_{3,5}$

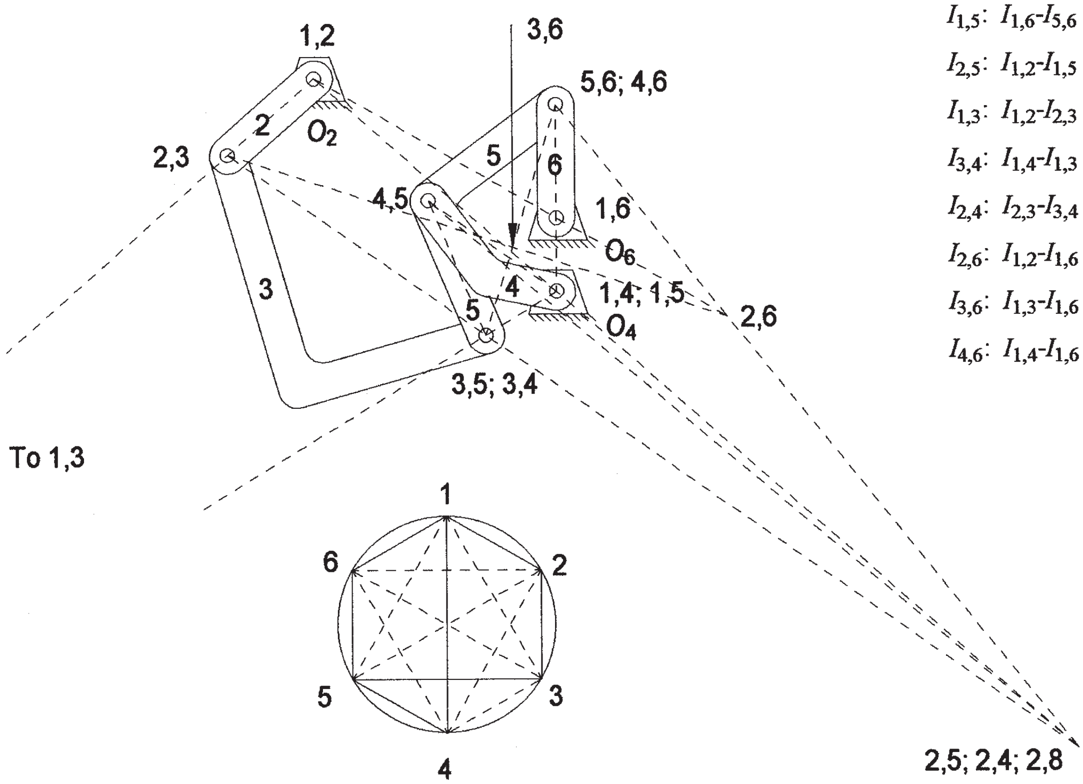
$I_{3,4}$: $I_{1,4}$-$I_{1,3}$ and $I_{4,5}$-$I_{3,5}$

$I_{2,4}$: $I_{2,3}$-$I_{3,4}$ and $I_{2,5}$-$I_{4,5}$

$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$

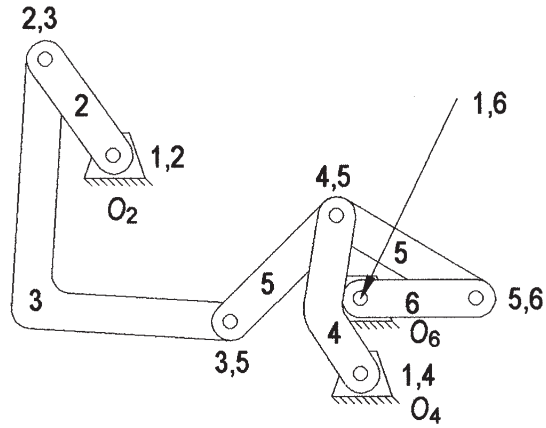
$I_{3,6}$: $I_{1,3}$-$I_{1,6}$ and $I_{3,5}$-$I_{5,6}$

$I_{4,6}$: $I_{1,4}$-$I_{1,6}$ and $I_{4,5}$-$I_{5,6}$



c. *In part (c) of the figure.*

1. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,4}$; $I_{3,6}$; and $I_{4,6}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,4}$-$I_{4,5}$

$I_{2,5}$: $I_{1,2}$-$I_{1,5}$ and $I_{2,3}$-$I_{3,5}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,5}$-$I_{3,5}$

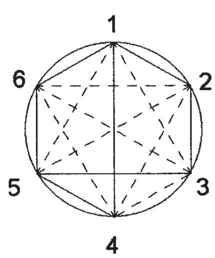
$I_{3,4}$: $I_{1,4}$-$I_{1,3}$ and $I_{4,5}$-$I_{3,5}$

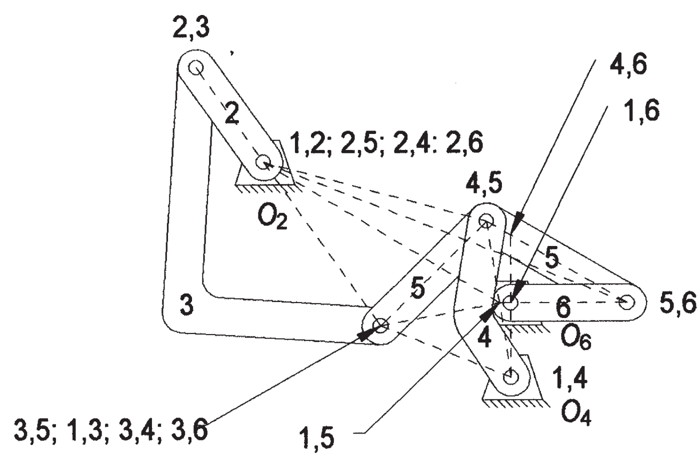
$I_{2,4}$: $I_{2,3}$-$I_{3,4}$ and $I_{2,5}$-$I_{4,5}$

$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$

$I_{3,6}$: $I_{1,3}$-$I_{1,6}$ and $I_{3,5}$-$I_{5,6}$

$I_{4,6}$: $I_{1,4}$-$I_{1,6}$ and $I_{4,5}$-$I_{5,6}$





PROBLEM 6-73a

Statement: Find the angular velocity of link 6 in Figure 3-34b for $\theta_6 = 90$ deg with respect to the global X axis assuming $\omega_2 = 10$ rad/sec CW. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot in$	Link 3 (A to B)	$f := 4.248 \cdot in$
Link 4 (O_4 to C)	$c := 2.125 \cdot in$	Link 5 (C to D)	$b := 2.158 \cdot in$
Link 6 (O_6 to D)	$a := 1.542 \cdot in$	Link 5 (B to D)	$p := 3.274 \cdot in$
Link 1 X-offset	$d_X := 3.259 \cdot in$	Link 1 Y-offset	$d_Y := -2.905 \cdot in$

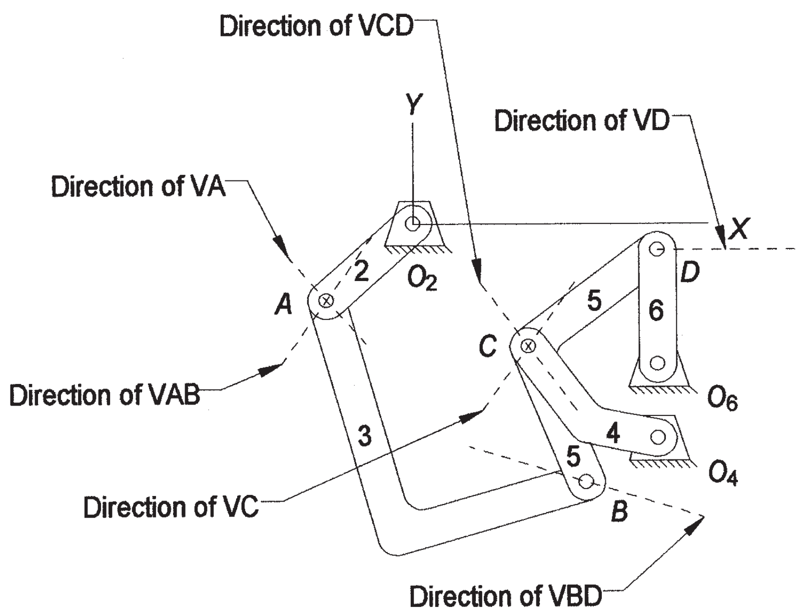
Angle CDB $\delta_5 := 36.0 \cdot deg$

Output rocker angle: $\theta_6 := 90 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure 3-34b and Mathcad file P0673a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Since this linkage is a Stephenson's II sixbar, we will have to start at link 6 and work back to link 2. We will assume a value for ω_6 and eventually find a value for ω_2. We will then multiply the magnitudes of all velocities by the ratio of the actual ω_2 to the found ω_2. Use equation 6.7 to calculate the magnitude of the velocity at point D.

Assume: $\omega_6 := 1 \cdot rad \cdot sec^{-1}$ CW

$V_D := a \cdot \omega_6$ $\qquad V_D = 1.542 \frac{in}{sec}$ $\qquad \theta_{VD} := 0 \cdot deg$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C, the magnitude of the relative velocity $\mathbf{V}_{CD}$, and the angular velocity of link 5. The equation to be solved graphically is

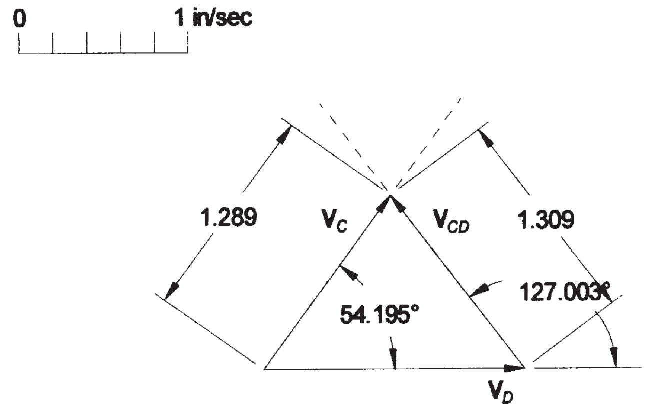
$$\mathbf{V}_C = \mathbf{V}_D + \mathbf{V}_{CD}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.

b. From the tip of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_{CD}$, magnitude unknown.

c. From the tail of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{CD}$ from the tip of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_{CD}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{1 \cdot in \cdot sec^{-1}}{in}$

$V_C := 1.289 \cdot in \cdot k_v$ $\quad V_C = 1.289\dfrac{in}{sec}$ $\quad \theta_{VC} := 54.195 \cdot deg$

$V_{CD} := 1.309 \cdot in \cdot k_v$ $\quad V_{CD} = 1.309\dfrac{in}{sec}$ $\quad \theta_{VCD} := 127.003 \cdot deg$

5. Determine the angular velocity of links 5 and 4 using equation 6.7.

$\omega_5 := \dfrac{-V_{CD}}{b}$ $\quad \omega_5 = -0.607\dfrac{rad}{sec}$

$\omega_4 := \dfrac{-V_C}{c}$ $\quad \omega_4 = -0.607\dfrac{rad}{sec}$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{BD}$ using equation 6.7.

$V_{BD} := \left|p \cdot \omega_5\right|$ $\quad V_{BD} = 1.986\dfrac{in}{sec}$

$\theta_{VBD} := \theta_{VCD} + \delta_5$ $\quad \theta_{VBD} = 163.003\,deg$

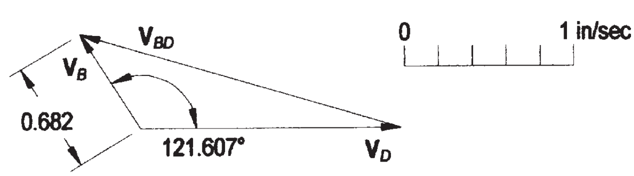
7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_D + \mathbf{V}_{BD}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.

b. From the tip of $\mathbf{V}_D$, layout the (now) known vector $\mathbf{V}_{BD}$.

c. Complete the vector triangle by drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_D$ to the tip of the $\mathbf{V}_{BD}$ vector.



8. From the velocity triangle we have:

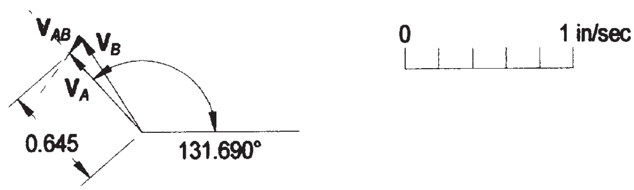
Velocity scale factor: $k_v := \dfrac{1 \cdot in \cdot sec^{-1}}{in}$

$V_B := 0.682 \cdot in \cdot k_v$ $\quad V_B = 0.682 \dfrac{in}{sec}$ $\quad \theta_{VB} := 121.607 \cdot deg$

9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *A*, the magnitude of the relative velocity $\mathbf{V}_{AB}$, and the angular velocity of link 2. The equation to be solved graphically is

$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{AB}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{AB}$, magnitude unknown.
c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_A$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{AB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_A$ construction line and drawing $\mathbf{V}_A$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{AB}$ construction line.



10. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{1 \cdot in \cdot sec^{-1}}{in}$

$V_A := 0.645 \cdot in \cdot k_v$ $\quad V_A = 0.645 \dfrac{in}{sec}$ $\quad \theta_{VA} := 131.690 \cdot deg$

11. Determine the angular velocity of link 2 with respect to the assumed value of ω_6 using equation 6.7.

$\omega_{26} := \dfrac{V_A}{g}$ $\quad \omega_{26} = 0.415 \dfrac{rad}{sec}$ $\quad$ CW

12. Calculate the actual value of the angular velocity of link 6.

$\omega_6 := \dfrac{\omega_2}{\omega_{26}} \cdot \dfrac{rad}{sec}$ $\quad \omega_6 = 24.124 \dfrac{rad}{sec}$

PROBLEM 6-73b

Statement: Find the angular velocity of link 6 in Figure 3-34b for $\theta_6 = 90$ deg with respect to the global X axis assuming $\omega_2 = 10$ rad/sec CW. Use the method of instant centers.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot in$	Link 3 (A to B)	$f := 4.248 \cdot in$
Link 4 (O_4 to C)	$c := 2.125 \cdot in$	Link 5 (C to D)	$b := 2.158 \cdot in$
Link 6 (O_6 to D)	$a := 1.542 \cdot in$	Link 5 (B to D)	$p := 3.274 \cdot in$
Link 1 X-offset	$d_X := 3.259 \cdot in$	Link 1 Y-offset	$d_Y := -2.905 \cdot in$

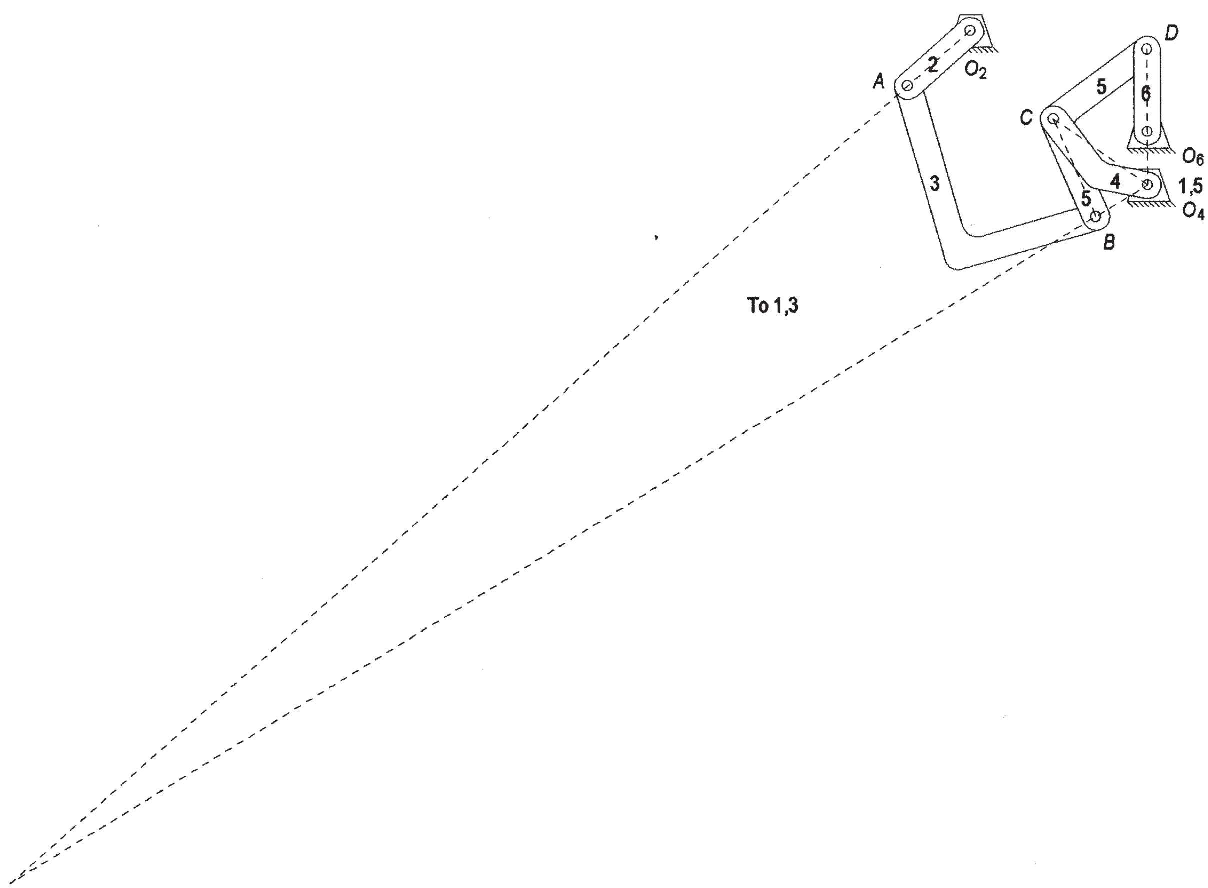
Angle CDB $\delta_5 := 36.0 \cdot deg$

Output rocker angle: $\theta_6 := 90 \cdot deg$ Global XY system

Input crank angular velocity $\omega_{21} := 10 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure 3-34b and Mathcad file P0673b.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers. See Problem 6-72b for determination of IC locations.



From the layout above:

$AI13 := 22.334 \cdot in$ $BI13 := 23.650 \cdot in$ $BI15 := 1.124 \cdot in$ $DI15 := 2.542 \cdot in$

2. Start from point *D* with an assumed value for ω_6 and work to find ω_2. Then, use the ratio of the actual vale of ω_2 to the found value to calculate the actual value of ω_6. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point *D*.

$$\omega_6 := 1 \cdot \frac{rad}{sec}$$

$$V_D := a \cdot \omega_6 \qquad V_D = 1.542 \frac{in}{sec}$$

$$\theta_{VD} := \theta_6 - 90 \cdot deg \qquad \theta_{VD} = 0.0\,deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_5 := \frac{V_D}{DI15} \qquad \omega_5 = 0.607 \frac{rad}{sec} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b.

$$V_B := BI15 \cdot \omega_5 \qquad V_B = 0.682 \frac{in}{sec}$$

5. Use equation 6.9c to determine the angular velocity of link 3.

$$\omega_3 := \frac{V_B}{BI13} \qquad \omega_3 = 0.029 \frac{rad}{sec} \quad \text{CCW}$$

6. Determine the magnitude of the velocity at point *A* using equation 6.9b.

$$V_A := AI13 \cdot \omega_3 \qquad V_A = 0.644 \frac{in}{sec} \quad \text{to the left}$$

7. Use equation 6.9c to determine the angular velocity of link 2 based on the assumed value of ω_6.

$$\omega_{22} := \frac{V_A}{g} \qquad \omega_{22} = 0.414 \frac{rad}{sec} \quad \text{CW}$$

8. Multiply the assumed vaue of ω_6 by the ratio of ω_{21} over ω_{22} to get the value of ω_6 for ω_2 = 10 rad/sec.

$$\omega_6 := \frac{\omega_{21}}{\omega_{22}} \cdot \frac{rad}{sec} \qquad \omega_6 = 24.166 \frac{rad}{sec} \quad \text{CW}$$

PROBLEM 6-73c

Statement: Find the angular velocity of link 6 in Figure 3-34b for $\theta_6 = 90$ deg with respect to the global X axis assuming $\omega_2 = 10$ rad/sec CW. Use an analytic method.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot in$	Link 3 (A to B)	$f := 4.248 \cdot in$
Link 4 (O_4 to C)	$c := 2.125 \cdot in$	Link 5 (C to D)	$b := 2.158 \cdot in$
Link 6 (O_6 to D)	$a := 1.542 \cdot in$	Link 5 (B to D)	$p := 3.274 \cdot in$
Link 1 X-offset	$d_X := 3.259 \cdot in$	Link 1 Y-offset	$d_Y := -2.905 \cdot in$
Angle CDB	$\delta_5 := 36.0 \cdot deg$	Link 1 (O_4 to O_6)	$d := 1.000 \cdot in$
Output rocker angle:	$\theta_{6XY} := 90 \cdot deg$	Global XY system	

Input crank angular velocity $\omega_2 := -10 \cdot rad \cdot sec^{-1}$ CW

Coordinate rotationm angle $\delta := -90 \cdot deg$

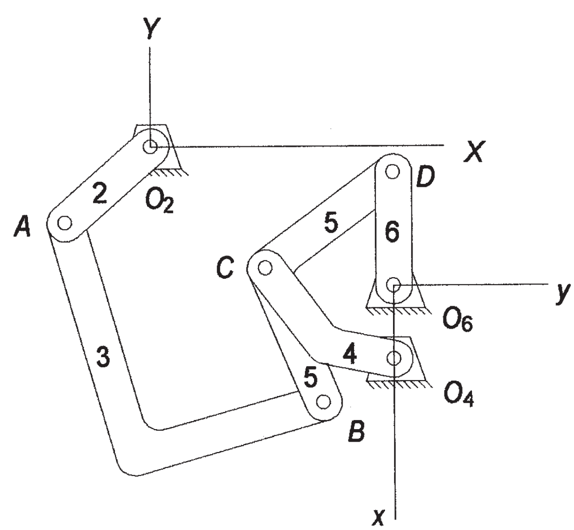
Two argument inverse tangent

$$atan2(x,y) := \begin{cases} \text{return } 0.5 \cdot \pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5 \cdot \pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Solution: See Figure 3-34b and Mathcad file P0673c.

1. Transform the crank angle to the local coordinate system. Draw the linkage to scale and label it.

$\theta_6 := \theta_{6XY} - \delta$ $\quad\quad$ $\theta_6 = 180.000\ deg$



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \quad\quad K_2 := \frac{d}{c} \quad\quad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$K_1 = 0.6485$ $\quad\quad$ $K_2 = 0.4706$ $\quad\quad$ $K_3 = 0.4938$

$A := \cos(\theta_6) - K_1 - K_2 \cdot \cos(\theta_6) + K_3$ $\qquad A = -0.6841$

$B := -2 \cdot \sin(\theta_6)$ $\qquad B = 1.5314 \times 10^{-15}$

$C := K_1 - (K_2 + 1) \cdot \cos(\theta_6) + K_3$ $\qquad C = 2.6129$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_4 = 234.195\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.4634$$

$K_5 = -0.5288$

$D := \cos(\theta_6) - K_1 + K_4 \cdot \cos(\theta_6) + K_5$ $\qquad D = -2.6407$

$E := -2 \cdot \sin(\theta_6)$ $\qquad E = 1.5314 \times 10^{-15}$

$F := K_1 + (K_4 - 1) \cdot \cos(\theta_6) + K_5$ $\qquad F = 0.6563$

5. Use equation 4.13 to find values of θ_5 for the crossed circuit.

$$\theta_{51} := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_{51} = 307.003\, deg$$

6. Determine the angular velocity of links 4 and 5 for the open circuit using equations 6.18. Initially assume $\omega_6 = 1$ rad/sec. then by trial and error, change it to make $\omega_2 = 10$ rad/ sec CW.

$$\omega_6 := 1 \cdot rad \cdot sec^{-1}$$

$$\omega_5 := \frac{a \cdot \omega_6}{b} \cdot \frac{\sin(\theta_4 - \theta_6)}{\sin(\theta_{51} - \theta_4)} \qquad \omega_5 = 0.607 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_6}{c} \cdot \frac{\sin(\theta_6 - \theta_{51})}{\sin(\theta_4 - \theta_{51})} \qquad \omega_4 = 0.607 \frac{rad}{sec}$$

PROBLEM 6-74

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular velocity of link 6 in the sixbar linkage of Figure 3-34 as a function of θ_2 for a constant ω_2 = 1 rad/sec CW.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot in$	Link 3 (A to B)	$f := 4.248 \cdot in$
Link 4 (O_4 to C)	$c := 2.125 \cdot in$	Link 5 (C to D)	$b := 2.158 \cdot in$
Link 6 (O_6 to D)	$a := 1.542 \cdot in$	Link 5 (B to D)	$p := 3.274 \cdot in$
Link 1 X-offset	$d_X := 3.259 \cdot in$	Link 1 Y-offset	$d_Y := -2.905 \cdot in$
Angle CDB	$\delta_5 := 36.0 \cdot deg$	Link 1 (O_4 to O_6)	$d := 1.000 \cdot in$
Output rocker angle:	$\theta_{6XY} := 90 \cdot deg$	Global XY system	

Input crank angular velocity $\omega_2 := -10 \cdot rad \cdot sec^{-1}$ CW

Coordinate rotationm angle $\delta := -90 \cdot deg$

Solution: See Figure P6-34 and Mathcad file P0674.

1. This problem is long and may be more appropriate for a project assignment. The solution involves defining vector loops and solving the resulting equations use a method such as Newton-Raphson.

PROBLEM 6-75

Statement: Figure 3-35 shows a Stephenson's sixbar mechanism. Find all of its instant centers in the position shown:

Given: Number of links $n := 6$

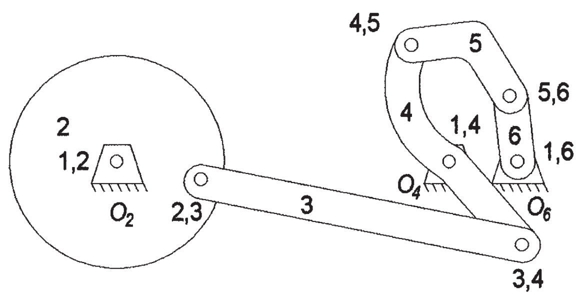
Solution: See Figure 3-35 and Mathcad file P0675.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

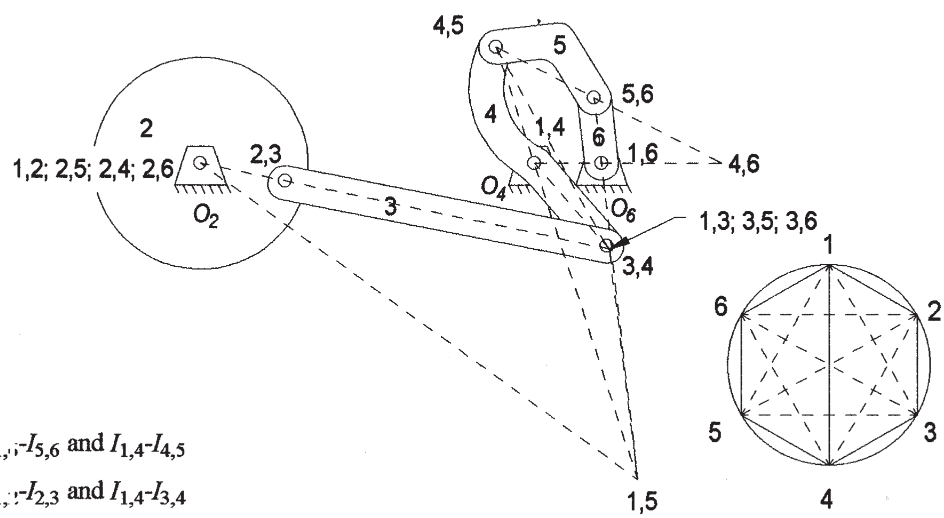
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 15$$

a. In part (a) of the figure.

1. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,5}$; $I_{3,6}$; and $I_{4,6}$



$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,4}$-$I_{4,5}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$

$I_{3,5}$: $I_{1,5}$-$I_{1,3}$ and $I_{3,4}$-$I_{4,5}$

$I_{2,5}$: $I_{1,2}$-$I_{1,5}$ and $I_{2,3}$-$I_{3,5}$

$I_{2,4}$: $I_{2,3}$-$I_{3,4}$ and $I_{2,5}$-$I_{4,5}$

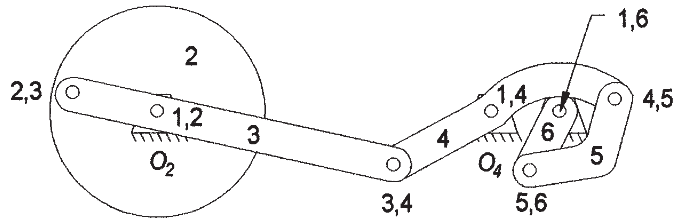
$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$

$I_{3,6}$: $I_{1,3}$-$I_{1,6}$ and $I_{3,5}$-$I_{5,6}$

$I_{4,6}$: $I_{1,4}$-$I_{1,6}$ and $I_{4,5}$-$I_{5,6}$

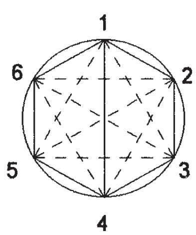
b. In part (b) of the figure.

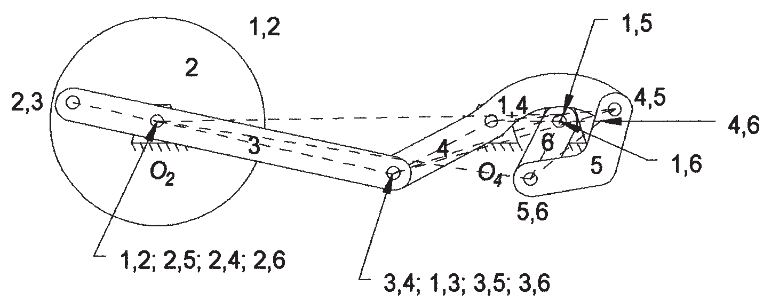
1. Draw the linkage to scale and identify those ICs that can be found by inspection (7).



2. Use Kennedy's Rule and a linear graph to find the remaining 8 ICs: $I_{1,3}$; $I_{1,5}$; $I_{2,4}$; $I_{2,5}$; $I_{2,6}$; $I_{3,5}$; $I_{3,6}$; and $I_{4,6}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,4}$-$I_{4,5}$	$I_{2,4}$: $I_{2,3}$-$I_{3,4}$ and $I_{2,5}$-$I_{4,5}$
$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$	$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$
$I_{3,5}$: $I_{1,5}$-$I_{1,3}$ and $I_{3,4}$-$I_{4,5}$	$I_{3,6}$: $I_{1,3}$-$I_{1,6}$ and $I_{3,5}$-$I_{5,6}$
$I_{2,5}$: $I_{1,2}$-$I_{1,5}$ and $I_{2,3}$-$I_{3,5}$	$I_{4,6}$: $I_{1,4}$-$I_{1,6}$ and $I_{4,5}$-$I_{5,6}$





PROBLEM 6-76a

Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular velocity of link 6 assuming $\omega_2 = 10$ rad/sec CCW. Use a graphical method.

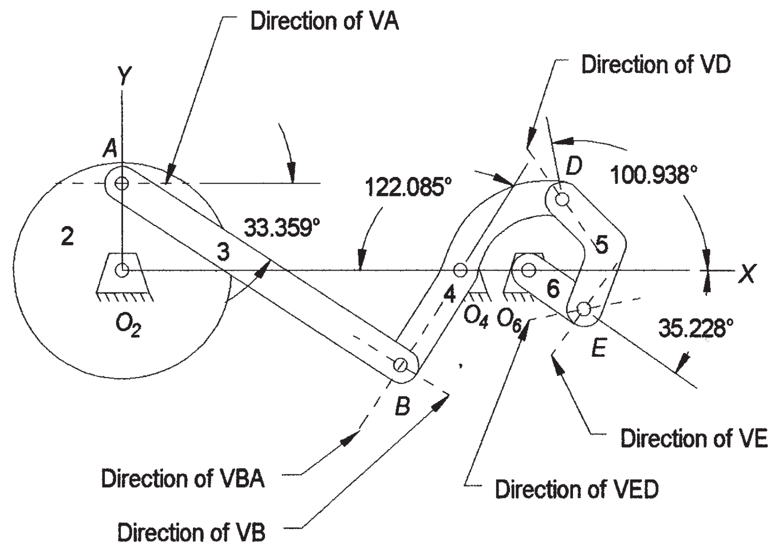
Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot in$	Link 3 (A to B)	$b := 3.800 \cdot in$
Link 4 (O_4 to B)	$c := 1.286 \cdot in$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot in$
Link 4 (O_4 to D)	$e := 1.429 \cdot in$	Link 5 (D to E)	$f := 1.286$
Link 6 (O_6 to E)	$g := 0.771 \cdot in$	Crank angle:	$\theta_2 := 90 \cdot deg$

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure 3-35 and Mathcad file P0676a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

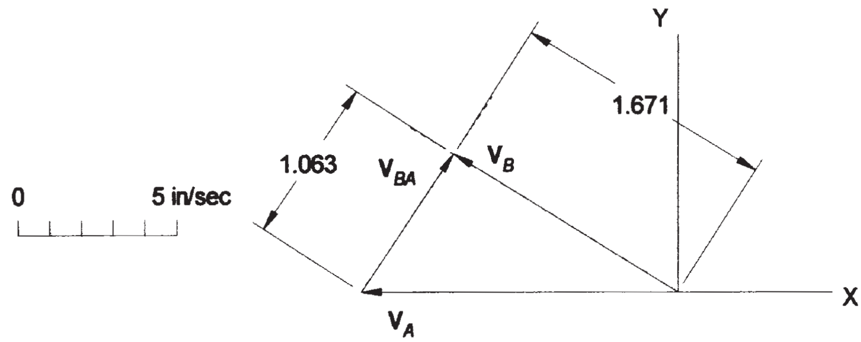
$$V_A := a \cdot \omega_2 \qquad V_A = 10.000 \frac{in}{sec}$$

$$\theta_{VA} := \theta_2 + 90 \cdot deg \qquad \theta_{VA} = 180.000\, deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{5 \cdot in \cdot sec^{-1}}{in}$

$V_B := 1.671 \cdot in \cdot k_v$ $\quad V_B = 8.355 \frac{in}{sec}$ $\quad \theta_{VB} := 147.915 \cdot deg$

$V_{BA} := 1.063 \cdot in \cdot k_v$ $\quad V_{BA} = 5.315 \frac{in}{sec}$ $\quad \theta_{VBA} := 56.641 \cdot deg$

$\omega 4 := \frac{V_B}{c}$ $\quad \omega 4 = 6.497 \frac{rad}{sec}$ CW

5. Calculate the magnitude and direction of $\mathbf{V}_D$.

$V_D := e \cdot \omega 4$ $\quad V_D = 9.284 \frac{in}{sec}$ $\quad \theta_{VD} := -55.085 \cdot deg$

6. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *E*, the magnitude of the relative velocity $\mathbf{V}_{ED}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_E = \mathbf{V}_D + \mathbf{V}_{ED}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.
b. From the tip of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_{ED}$, magnitude unknown.
c. From the tail of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_E$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{ED}$ from the tip of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_E$ construction line and drawing $\mathbf{V}_E$ from the tail of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_{ED}$ construction line.

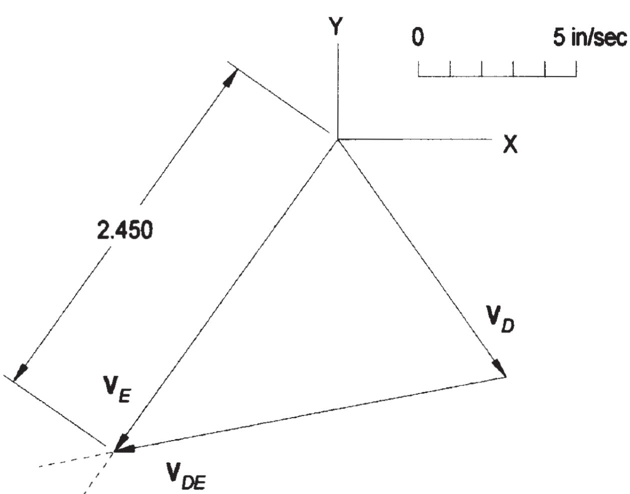
4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{5 \cdot in \cdot sec^{-1}}{in}$

$V_E := 2.450 \cdot in \cdot k_v$ $\quad V_E = 12.250 \frac{in}{sec}$

$\omega 6 := \frac{V_E}{g}$

$\omega 6 = 15.888 \frac{rad}{sec}$ CW



PROBLEM 6-76b

Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular velocity of link 6 assuming $\omega_2 = 10$ rad/sec CCW. Use the method of instant centers.

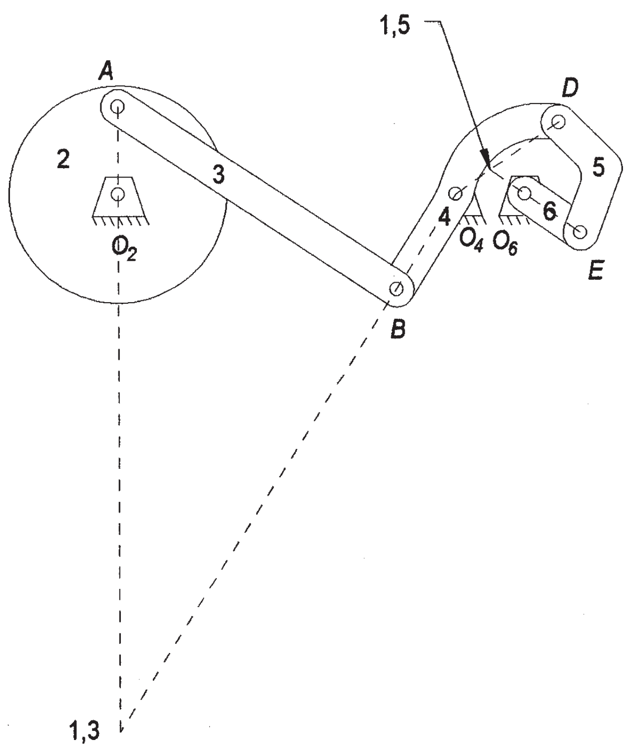
Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot in$	Link 3 (A to B)	$b := 3.800 \cdot in$
Link 4 (O_4 to B)	$c := 1.286 \cdot in$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot in$
Link 4 (O_4 to D)	$e := 1.429 \cdot in$	Link 5 (D to E)	$f := 1.286$
Link 6 (O_6 to E)	$g := 0.771 \cdot in$	Crank angle:	$\theta_2 := 90 \cdot deg$

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure 3-35 and Mathcad file P0676a.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers. See Problem 6-68 for the determination of IC locations.



From the layout above:

$AI13 := 7.152 \cdot in$ $BI13 := 5.975 \cdot in$ $BI15 := 1.740 \cdot in$ $DI15 := 0.947 \cdot in$ $EI15 := 1.249 \cdot in$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 10.000 \frac{in}{sec}$$

$$\theta_{VA} := \theta_2 + 90 \cdot deg \qquad \theta_{VA} = 180.0\, deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 1.398 \frac{rad}{sec} \quad \text{CCW}$$

4. Determine the magnitude of the velocity at point *B* using equation 6.9b.

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 8.354 \frac{in}{sec}$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 6.496 \frac{rad}{sec} \quad \text{CW}$$

6. Determine the magnitude of the velocity at point *D* using equation 6.9b.

$$V_D := e \cdot \omega_4 \qquad V_D = 9.283 \frac{in}{sec}$$

7. Use equation 6.9c to determine the angular velocity of link 5.

$$\omega_5 := \frac{V_D}{DI15} \qquad \omega_5 = 9.803 \frac{rad}{sec} \quad \text{CW}$$

8. Determine the magnitude of the velocity at point *E* using equation 6.9b.

$$V_E := EI15 \cdot \omega_5 \qquad V_E = 12.244 \frac{in}{sec}$$

9. Use equation 6.9c to determine the angular velocity of link 6.

$$\omega_6 := \frac{V_E}{g} \qquad \omega_6 = 15.88 \frac{rad}{sec} \quad \text{CW}$$

PROBLEM 6-76c

Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular velocity of link 6 assuming $\omega_2 = 10$ rad/sec CCW. Use an analytical method.

Given: Link lengths:

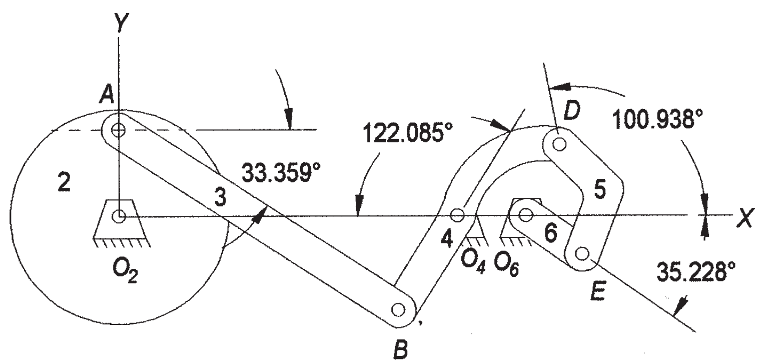
Link 2 (O_2 to A)	$a := 1.000 \cdot in$	Link 3 (A to B)	$b := 3.800 \cdot in$
Link 4 (O_4 to B)	$c := 1.286 \cdot in$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot in$
Link 4 (O_4 to D)	$e := 1.429 \cdot in$	Link 5 (D to E)	$f := 1.286 \cdot in$
Link 6 (O_6 to E)	$g := 0.771 \cdot in$	Link 1 (O_4 to O_6)	$h := 0.786 \cdot in$

Crank angle: $\theta_2 := 90 \cdot deg$

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure 3-35 and Mathcad file P0676c.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 3.8570 \qquad K_2 = 2.9992$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.2015$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -2.6555 \qquad B = -2.0000 \qquad C = 5.0585$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \qquad \theta_{41} = 237.915\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.0150 \qquad K_5 = -3.7714$$

$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$ $\qquad D = -7.6284$

$E := -2 \cdot \sin(\theta_2)$ $\qquad E = -2.0000$

$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$ $\qquad F = 0.0856$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \qquad \theta_3 = 326.641\ deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_3 - \theta_{41})} \qquad \omega_3 = 1.398 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_{41} - \theta_3)} \qquad \omega_4 = -6.496 \frac{rad}{sec}$$

7. Link 4 will be the input to the second fourbar (links 4, 5, 6, and 1). Let the angle that link 4 makes in the second fourbar be

$$\theta_{42} := \theta_{41} + 157 \cdot deg - 360 \cdot deg \qquad \theta_{42} = 34.915\ deg$$

8. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$K_1 := \frac{h}{e}$ $\qquad K_2 := \frac{h}{g}$

$K_1 = 0.5500$ $\qquad K_2 = 1.0195$

$K_3 := \frac{e^2 - f^2 + g^2 + h^2}{(2 \cdot e \cdot g)}$ $\qquad K_3 = 0.7263$

$A := \cos(\theta_{42}) - K_1 - K_2 \cdot \cos(\theta_{42}) + K_3$

$B := -2 \cdot \sin(\theta_{42})$

$C := K_1 - (K_2 + 1) \cdot \cos(\theta_{42}) + K_3$

$A = 0.1603 \qquad B = -1.1447 \qquad C = -0.3796$

9. Use equation 4.10b to find values of θ_6 for the open circuit.

$$\theta_6 := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \qquad \theta_6 = -35.228\ deg$$

10. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$K_4 := \frac{h}{f}$ $\qquad K_5 := \frac{g^2 - h^2 - e^2 - f^2}{(2 \cdot e \cdot f)}$ $\qquad K_4 = 0.6112$

$K_5 = -1.0119$

$D := \cos(\theta_{42}) - K_1 + K_4 \cdot \cos(\theta_{42}) + K_5$ $\qquad D = -0.2408$

$E := -2 \cdot \sin(\theta_{42})$ $\qquad E = -1.1447$

$F := K_1 + (K_4 - 1) \cdot \cos(\theta_{42}) + K_5$ $\qquad F = -0.7807$

11. Use equation 4.13 to find values of θ_5 for the open circuit.

$$\theta_5 := 2\cdot\left(atan2\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_5 = 280.938\ deg$$

12. Determine the angular velocity of link6 for the open circuit using equations 6.18.

$$\omega_6 := \left(\frac{e\cdot\omega_4}{g}\right)\cdot\frac{\sin\left(\theta_{42} - \theta_5\right)}{\sin\left(\theta_6 - \theta_5\right)} \qquad \omega_6 = -15.885\ \frac{rad}{sec}$$

PROBLEM 6-77

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular velocity of link 6 in the sixbar linkage of Figure 3-35 as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CCW.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot in$	Link 3 (A to B)	$b := 3.800 \cdot in$
Link 4 (O_4 to B)	$c := 1.286 \cdot in$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot in$
Link 4 (O_4 to D)	$e := 1.429 \cdot in$	Link 5 (D to E)	$f := 1.286 \cdot in$
Link 6 (O_6 to E)	$g := 0.771 \cdot in$	Link 1 (O_4 to O_6)	$h := 0.786 \cdot in$

Input crank angular velocity $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure 3-35 and Mathcad file P0677.

1. Define the range of the input angle: $\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 deg$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 3.8570 \qquad K_2 = 2.9992$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.2015$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.0150 \qquad K_5 = -3.7714$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Link 4 will be the input to the second fourbar (links 4, 5, 6, and 1). Let the angle that link 4 makes in the second fourbar be

$$\theta_{42}(\theta_2) := \theta_4(\theta_2) + 157 \cdot deg - 360 \cdot deg$$

8. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{h}{e} \qquad K_2 := \frac{h}{g}$$

$$K_1 = 0.5500 \qquad K_2 = 1.0195$$

$$K_3 := \frac{e^2 - f^2 + g^2 + h^2}{(2 \cdot e \cdot g)} \qquad K_3 = 0.7263$$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K_1 - K_2 \cdot \cos(\theta_{42}(\theta_2)) + K_3$$

$$B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

$$C'(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K_3$$

9. Use equation 4.10b to find values of θ_6 for the open circuit.

$$\theta_6(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A'(\theta_2), B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)} \right) \right)$$

10. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$$K_4 := \frac{h}{f} \qquad K_5 := \frac{g^2 - h^2 - e^2 - f^2}{(2 \cdot e \cdot f)} \qquad K_4 = 0.6112 \qquad K_5 = -1.0119$$

$$D'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K_1 + K_4 \cdot \cos(\theta_{42}(\theta_2)) + K_5$$

$$E'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

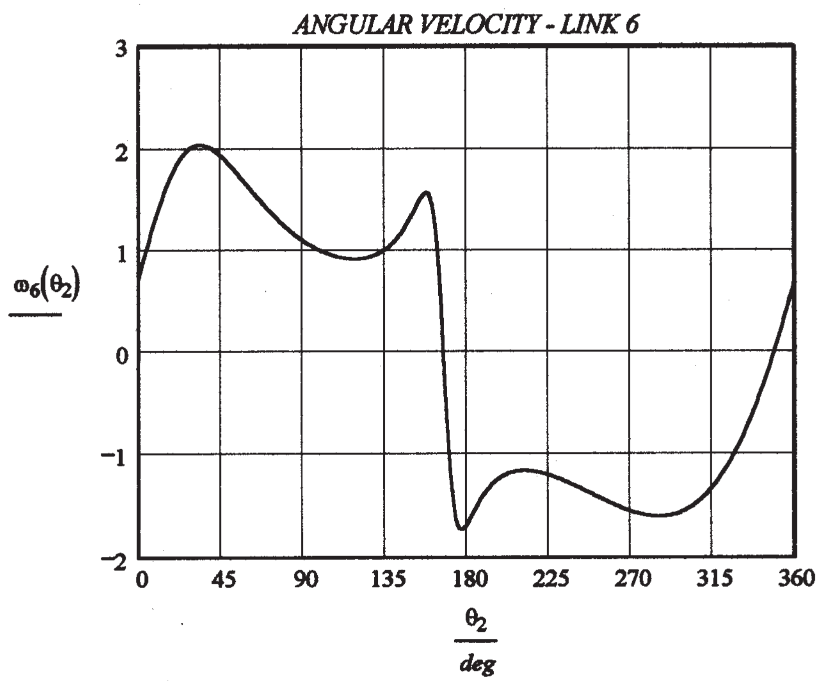
$$F'(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_{42}(\theta_2)) + K_5$$

11. Use equation 4.13 to find values of θ_5 for the open circuit.

$$\theta_5(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D'(\theta_2), -E'(\theta_2) - \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)} \right) \right)$$

12. Determine the angular velocity of link6 for the open circuit using equations 6.18.

$$\omega_6(\theta_2) := \left(\frac{e \cdot \omega_4(\theta_2)}{g} \right) \cdot \frac{\sin(\theta_{42}(\theta_2) - \theta_5(\theta_2))}{\sin(\theta_6(\theta_2) - \theta_5(\theta_2))}$$


ANGULAR VELOCITY - LINK 6
3
2
1
0
−1
−2
ω6(θ2)
0
45
90
135
180
225
270
315
360
θ2
deg

PROBLEM 6-78

Statement: Figure 3-36 shows an eightbar mechanism. Find all of its instant centers in the position shown in part (a) of the figure:

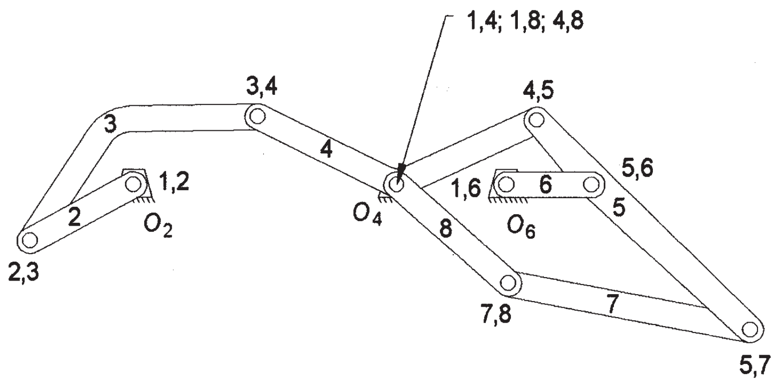
Given: Number of links $n := 8$

Solution: See Figure 3-36a and Mathcad file P0678.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 28$$

1. Draw the linkage to scale and identify those ICs that can be found by inspection (11).



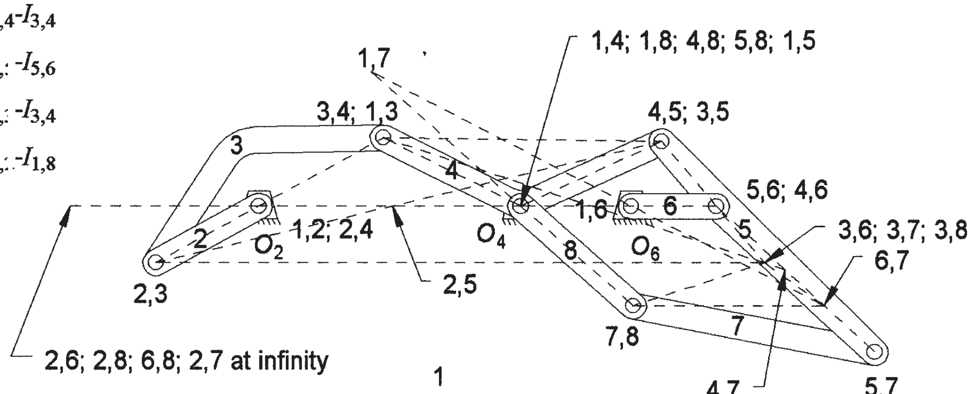
2. Use Kennedy's Rule and a linear graph to find the remaining 17 ICs.

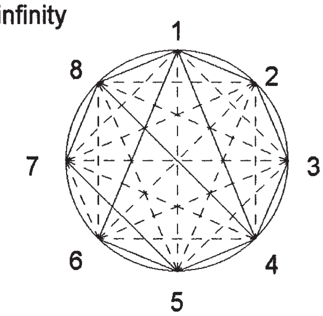
$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$

$I_{4,6}$: $I_{1,4}$-$I_{1,6}$ and $I_{4,5}$-$I_{5,6}$

$I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

$I_{2,8}$: $I_{2,4}$-$I_{4,8}$ and $I_{1,2}$-$I_{1,8}$





$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,4}$-$I_{4,6}$

$I_{6,8}$: $I_{2,8}$-$I_{2,6}$ and $I_{1,8}$-$I_{1,6}$

$I_{6,7}$: $I_{5,6}$-$I_{5,7}$ and $I_{6,8}$-$I_{7,8}$

$I_{2,5}$: $I_{2,6}$-$I_{5,6}$ and $I_{2,4}$-$I_{4,5}$

$I_{2,7}$: $I_{2,8}$-$I_{7,8}$ and $I_{2,6}$-$I_{6,7}$

$I_{3,5}$: $I_{3,4}$-$I_{4,5}$ and $I_{2,3}$-$I_{2,5}$

$I_{3,6}$: $I_{2,3}$-$I_{2,6}$ and $I_{3,5}$-$I_{5,6}$

$I_{3,7}$: $I_{3,6}$-$I_{6,7}$ and $I_{2,3}$-$I_{2,7}$

$I_{3,8}$: $I_{3,7}$-$I_{7,8}$ and $I_{2,3}$-$I_{2,8}$

$I_{4,7}$: $I_{4,6}$-$I_{6,7}$ and $I_{3,4}$-$I_{3,7}$

$I_{5,8}$: $I_{5,6}$-$I_{6,8}$ and $I_{4,5}$-$I_{4,8}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,4}$-$I_{4,5}$

$I_{1,7}$: $I_{1,8}$-$I_{7,8}$ and $I_{1,6}$-$I_{6,7}$

PROBLEM 6-79

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular velocity of link 8 in the linkage of Figure 3-36 as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CCW.

Given: Link lengths:

Input crank (L_2)	$a := 0.450$	First coupler (L_3)	$b := 0.990$
Common rocker (O_4B)	$c := 0.590$	First ground link (O_2O_4)	$d := 1.000$
Common rocker (O_4C)	$a' := 0.590$	Second coupler (CD)	$b' := 0.325$
Output rocker (L_6)	$c' := 0.325$	Second ground link (O_4O_6)	$d' := 0.419$
Link 7 (L_7)	$e := 0.938$	Link 8 (L_8)	$f := 0.572$
Link 5 extension (DE)	$p := 0.823$	Angle DCE	$\delta := 7.0 \cdot deg$
Angle BO_4C	$\alpha := 128.6 \cdot deg$		

Input crank angular velocity $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure 3-36 and Mathcad file P0679.

1. See problem 4-43 for the position solution. The velocity solution will use the same vector loop equations for links 5, 6, 7, and 8, differentiated with respect to time. This problem is suitable for a project assignment and is probably too long for an overnight assignment.

PROBLEM 6-80

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the magnitude and direction of the velocity of point *P* in Figure 3-37a as a function of θ_2 for a constant ω_2 = 1 rad/sec CCW. Also calculate and plot the velocity of point *P* versus point *A*.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 0.136$	Link 3 (A to B)	$b := 1.000$
Link 4 (B to O_4)	$c := 1.000$	Link 1 (O_2 to O_4)	$d := 1.414$

Coupler point: $R_{pa} := 2.000$ $\quad \delta_3 := 0 \cdot deg$

Crank speed: $\omega_2 := 1 \cdot rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{vmatrix} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{vmatrix}$$

Solution: See Figure 3-37a and Mathcad file P0680.

1. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 10.3971 \qquad K_2 = 1.4140$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 7.4187$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right)\right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.4140 \qquad K_5 = -7.4187$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{41}(\theta_2) - \theta_2)}{sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{31}(\theta_2))}{sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

7. Determine the velocity of point A using equations 6.19.

$$\mathbf{V_A}(\theta_2) := a\cdot\omega_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) \qquad V_A(\theta_2) := |\mathbf{V_A}(\theta_2)|$$

8. Determine the velocity of the coupler point P using equations 6.36.

$$\mathbf{V_{PA}}(\theta_2) := R_{pa}\cdot\omega_{31}(\theta_2)\cdot(-sin(\theta_{31}(\theta_2) + \delta_3) + j\cdot cos(\theta_{31}(\theta_2) + \delta_3))$$

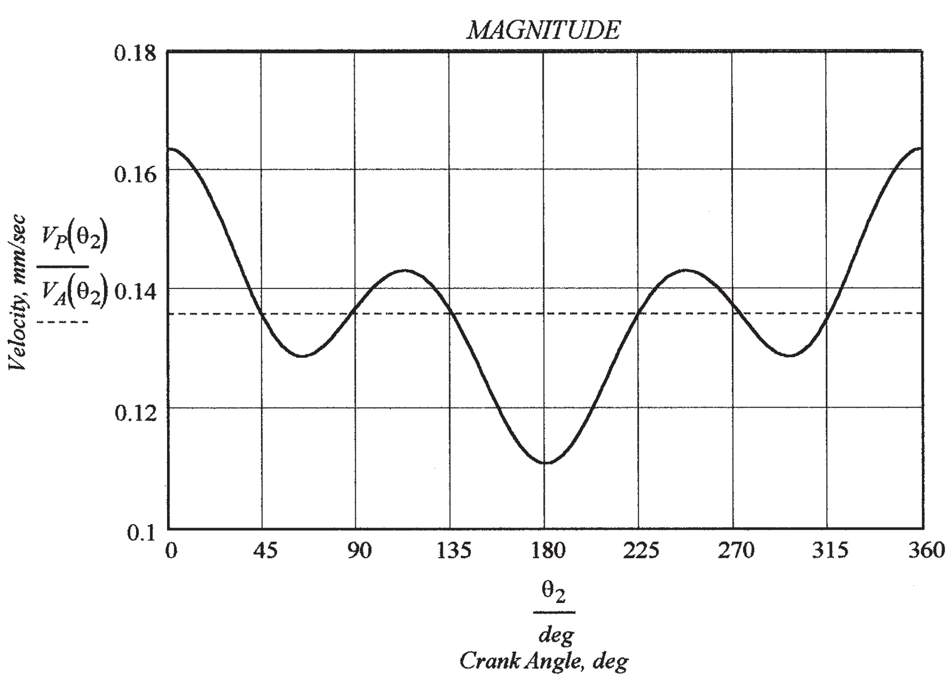
$$\mathbf{V_P}(\theta_2) := \mathbf{V_A}(\theta_2) + \mathbf{V_{PA}}(\theta_2)$$

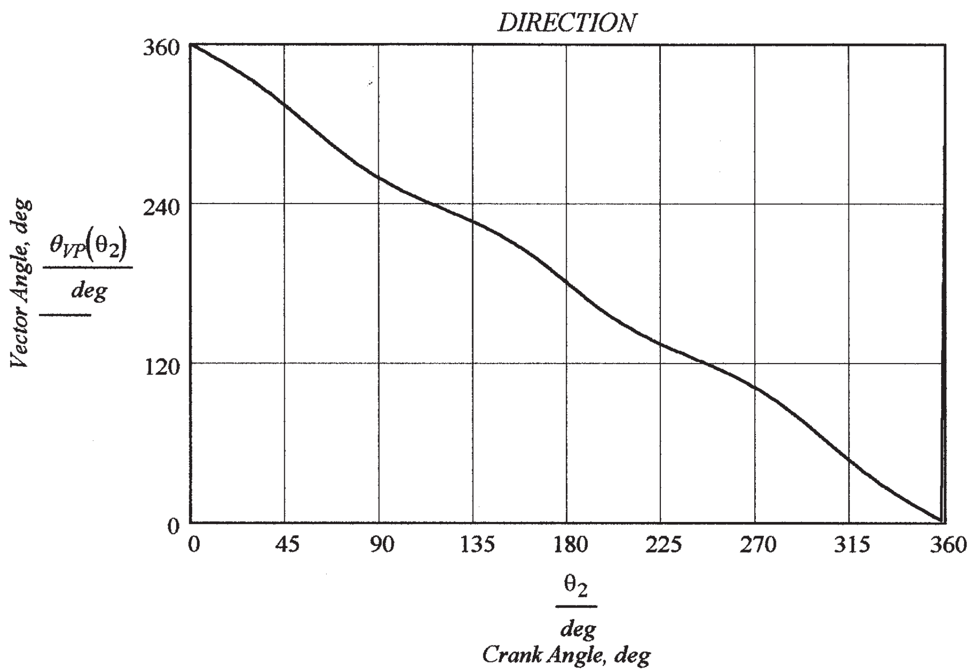
9. Plot the magnitude and direction of the velocity at coupler point P.

Magnitude: $V_P(\theta_2) := |\mathbf{V_P}(\theta_2)|$

Direction: $\theta_{VP1}(\theta_2) := arg(\mathbf{V_P}(\theta_2))$

$$\theta_{VP}(\theta_2) := if(\theta_{VP1}(\theta_2) < 0, \theta_{VP1}(\theta_2) + 2\cdot\pi, \theta_{VP1}(\theta_2))$$




DIRECTION
360
240
120
0
0
45
90
135
180
225
270
315
360
Vector Angle, deg
θVP(θ2)
deg
θ2
deg
Crank Angle, deg

PROBLEM 6-81

Statement: Calculate the percent error of the deviation from constant velocity magnitude of point *P* in Figure 3-37a.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 0.136$	Link 3 (A to B)	$b := 1.000$
Link 4 (B to O_4)	$c := 1.000$	Link 1 (O_2 to O_4)	$d := 1.414$

Coupler point: $R_{pa} := 2.000$ $\quad \delta_3 := 0 \cdot deg$

Crank speed: $\omega_2 := 1 \cdot rad \cdot sec^{-1}$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure 3-37a and Mathcad file P0681.

1. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 10.3971 \qquad K_2 = 1.4140$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 7.4187$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.4140 \qquad K_5 = -7.4187$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{31}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_{31}(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_{41}(\theta_2) - \theta_2)}{sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_{41}(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_{31}(\theta_2))}{sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

7. Determine the velocity of point A using equations 6.19.

$$\mathbf{V_A}(\theta_2) := a\cdot\omega_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) \qquad V_A(\theta_2) := |\mathbf{V_A}(\theta_2)|$$

8. Determine the velocity of the coupler point P using equations 6.36.

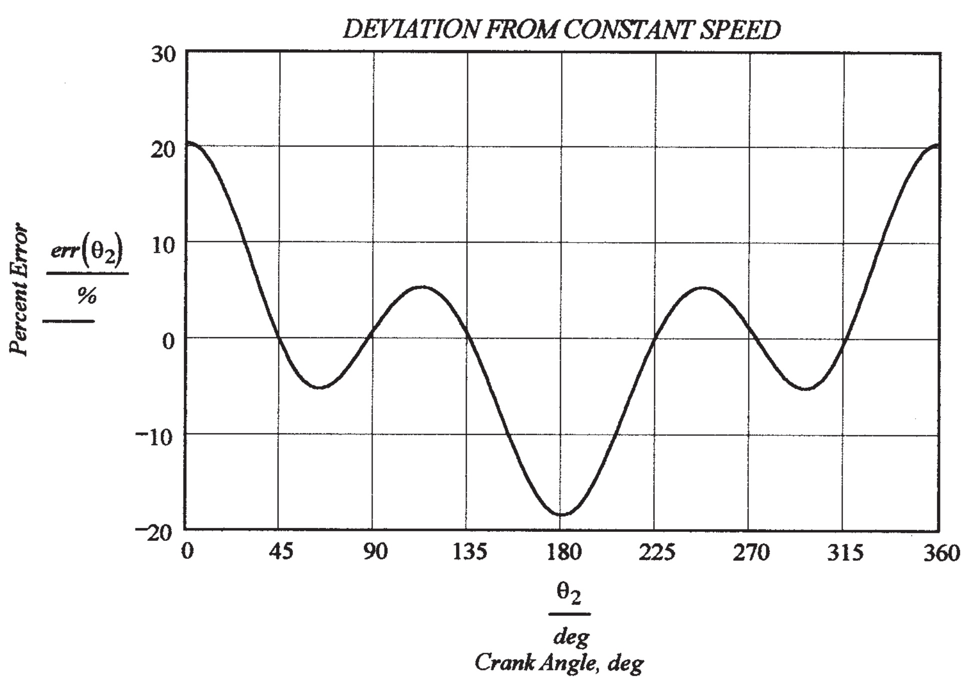
$$\mathbf{V_{PA}}(\theta_2) := R_{pa}\cdot\omega_{31}(\theta_2)\cdot(-sin(\theta_{31}(\theta_2) + \delta_3) + j\cdot cos(\theta_{31}(\theta_2) + \delta_3))$$

$$\mathbf{V_P}(\theta_2) := \mathbf{V_A}(\theta_2) + \mathbf{V_{PA}}(\theta_2)$$

9. Calculate the magnitude of the velocity at coupler point P.

Magnitude: $V_P(\theta_2) := |\mathbf{V_P}(\theta_2)|$

Deviation from constant speed: $err(\theta_2) := \dfrac{V_P(\theta_2) - V_A(\theta_2)}{V_A(\theta_2)}$



PROBLEM 6-82

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the magnitude and the direction of the velocity of point *P* in Figure 3-37b as a function of θ_2. Also calculate and plot the velocity of point *P* versus point *A*.

Given: Link lengths:

Input crank (L_2)	$a := 0.50$	First coupler (AB)	$b := 1.00$
Rocker 4 (O_4B)	$c := 1.00$	Rocker 5 (L_5)	$c' := 1.00$
Ground link (O_2O_4)	$d := 0.75$	Second coupler 6 (CD)	$b' := 1.00$
Coupler point (DP)	$p := 1.00$	Distance to O_P (O_2O_P)	$d' := 1.50$

Two argument inverse tangent:

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi \text{ if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi \text{ if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) \text{ if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \text{ otherwise} \end{cases}$$

Crank speed: $\omega_2 := 1\cdot\frac{rad}{sec}$

Solution: See Figure 3-37b and Mathcad file P0682.

1. Links 4, 5, *BC*, and *CD* form a parallelogram whose opposite sides remain parallel throughout the motion of the fourbar 1, 2, *AB*, 4. Define a position vector whose tail is at point *D* and whose tip is at point *P* and another whose tail is at O_4 and whose tip is at point *D*. Then, since $\mathbf{R}_5 = \mathbf{R}_{AB}$ and $\mathbf{R}_{DP} = -\mathbf{R}_4$, the position vector from O_2 to *P* is $\mathbf{P} = \mathbf{R}_1 + \mathbf{R}_{AB} - \mathbf{R}_4$. Separating this vector equation into real and imaginary parts gives the equations for the *X* and *Y* coordinates of the coupler point *P*.

$$X_P = d + b\cdot cos(\theta_3) - c\cdot cos(\theta_4) \qquad Y_P = b\cdot sin(\theta_3) - c\cdot sin(\theta_4)$$

2. Define one revolution of the input crank: $\theta_2 := 0\cdot deg, 0.5\cdot deg .. 360\cdot deg$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)}$$

$$K_1 = 1.5000 \qquad K_2 = 0.7500 \qquad K_3 = 0.8125$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right) - 2\cdot\pi$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.7500 \qquad K_5 = -0.8125$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right) - 2\cdot\pi$$

6. Define a local xy coordinate system with origin at O_P and with the positive x axis to the right. The coordinates of P are transformed to $x_P = X_P - d'$, $y_P = Y_P$.

$$x_P(\theta_2) := d + b\cdot cos(\theta_3(\theta_2)) - c\cdot cos(\theta_4(\theta_2)) - d'$$

$$y_P(\theta_2) := b\cdot sin(\theta_3(\theta_2)) - c\cdot sin(\theta_4(\theta_2))$$

7. Use equations 6.18 to calculate ω_3 and ω_4.

$$\omega_3(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4(\theta_2) - \theta_2)}{sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

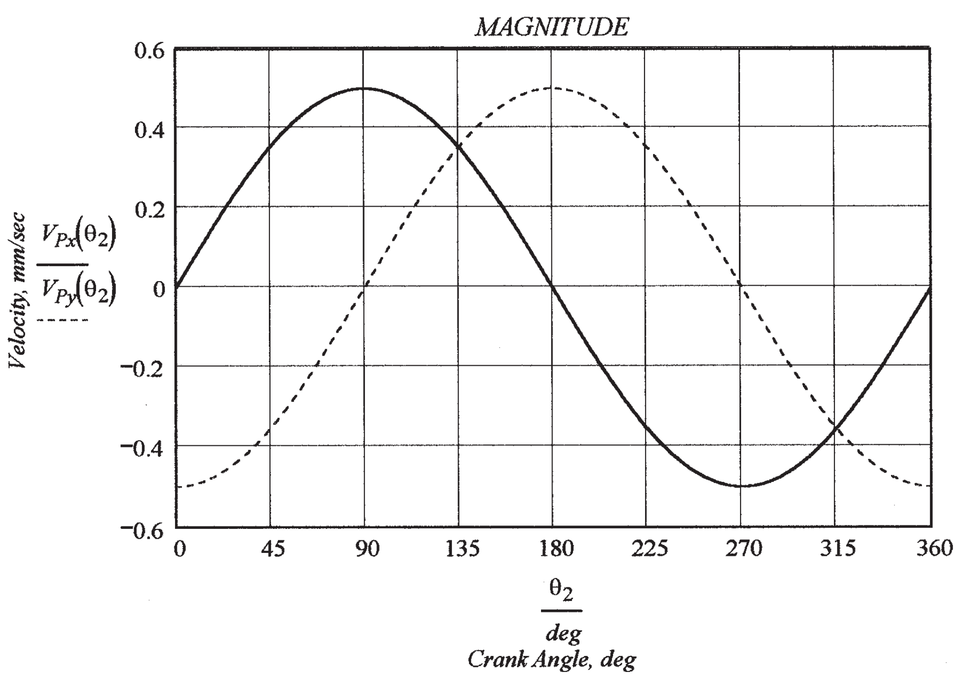
$$\omega_4(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Differentiate the position equations with respect to time to get the velocity components.

$$V_{Px}(\theta_2) := -b\cdot\omega_3(\theta_2)\, sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)\cdot sin(\theta_4(\theta_2))$$

$$V_{Py}(\theta_2) := b\cdot\omega_3(\theta_2)\cdot cos(\theta_3(\theta_2)) - c\cdot\omega_4(\theta_2)\cdot cos(\theta_4(\theta_2))$$

$$V_P(\theta_2) := \sqrt{(V_{Px}(\theta_2))^2 + (V_{Py}(\theta_2))^2}$$



PROBLEM 6-83

Statement: Find all instant centers of the linkage in Figure P6-30 in the position shown.

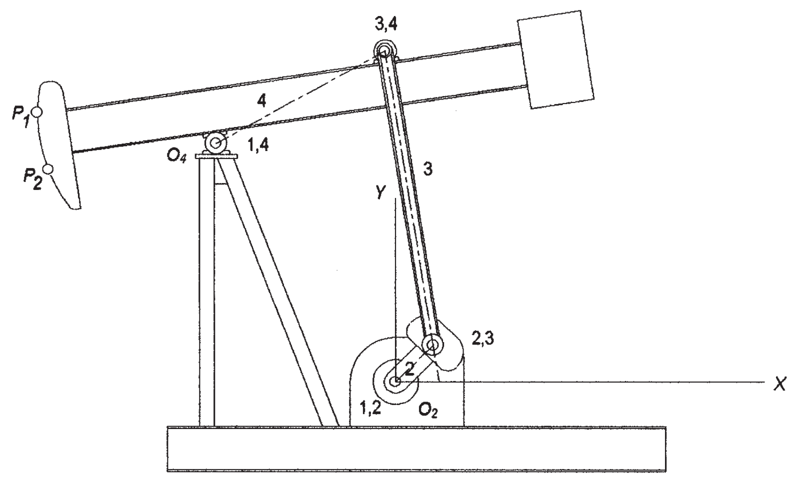
Given: Number of links $n := 4$

Solution: See Figure P6-30 and Mathcad file P0683.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

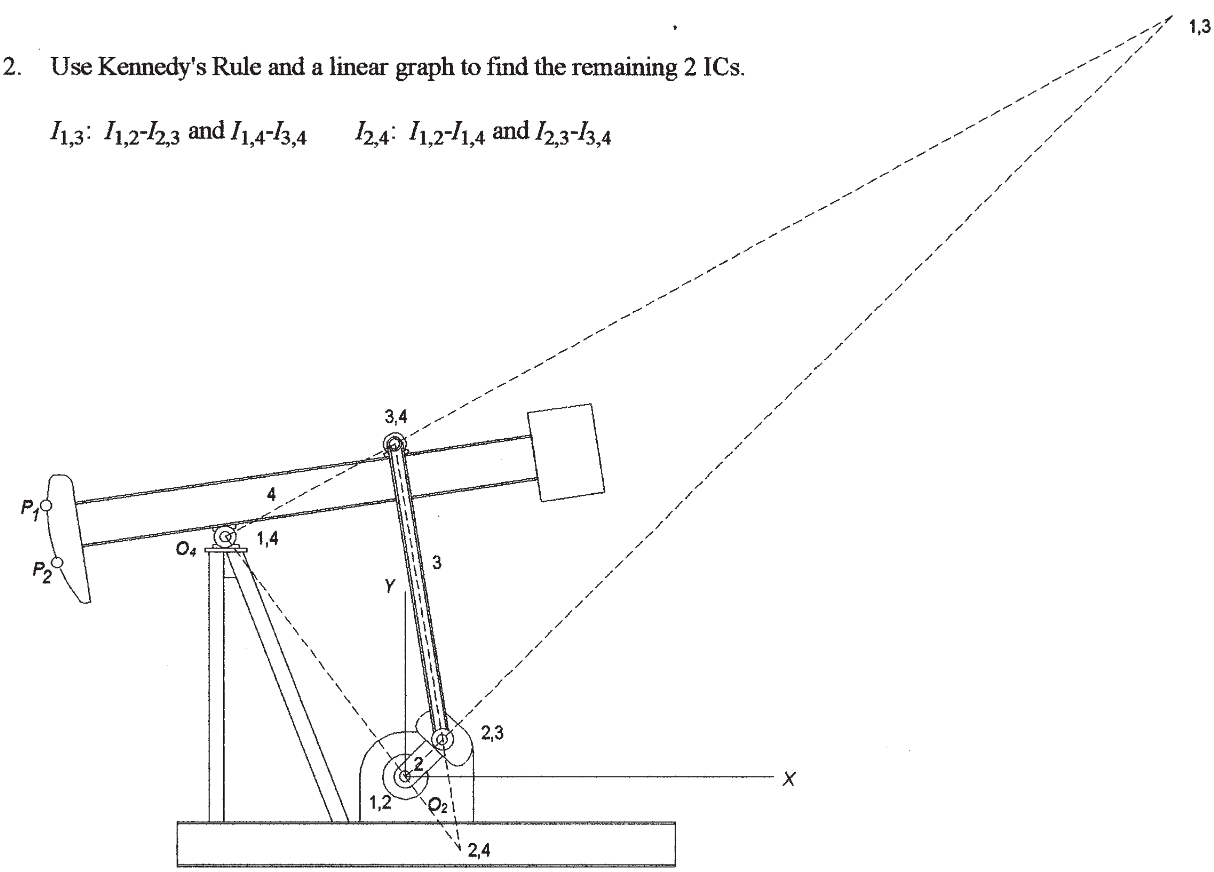
$$C := \frac{n\cdot(n-1)}{2} \qquad C = 6$$

1. Draw the linkage to scale and identify those ICs that can be found by inspection (4).



2. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs.

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



PROBLEM 6-84a

Statement: Find the angular velocities of links 3 and 4 and the linear velocity of points A, B and P_1 in the XY coordinate system for the linkage in Figure P6-30 in the position shown. Assume that $\theta_2 = 45$ deg in the XY coordinate system and $\omega_2 = 10$ rad/sec. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use a graphical method.

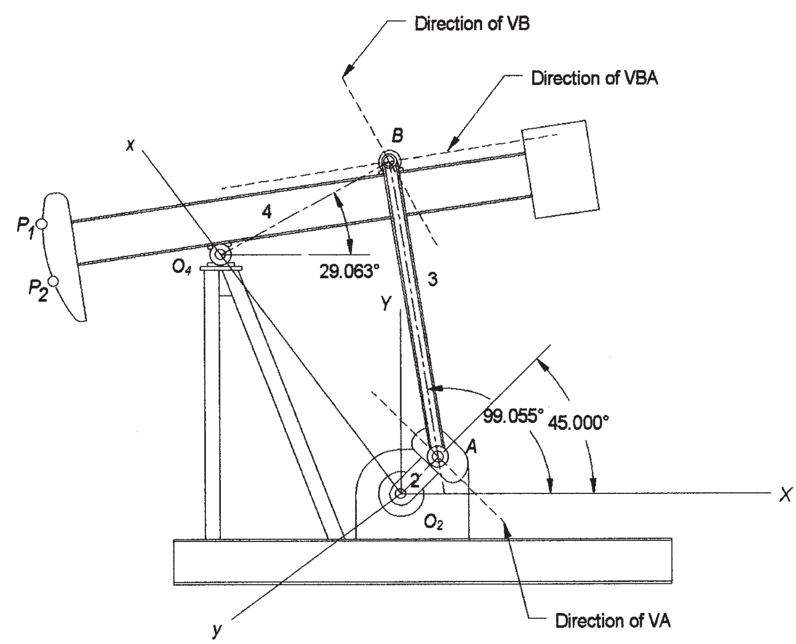
Given: Link lengths:

Link 2 (O_2 to A)	$a := 14.00 \cdot in$	Link 3 (A to B)	$b := 80.00 \cdot in$
Link 4 (O_4 to B)	$c := 51.26 \cdot in$		
Link 1 X-offset	$d_X := 47.5 \cdot in$	Link 1 Y-offset	$d_Y := 76.00 \cdot in - 12.00 \cdot in$
Coupler point x-offset	$p_x := 114.68 \cdot in$	Coupler point y-offset	$p_y := 33.19 \cdot in$
Crank angle:	$\theta_2 := 45 \cdot deg$		

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure P6-30 and Mathcad file P0684a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 140.000 \frac{in}{sec} \qquad \theta_{VA} := 45 \cdot deg + 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

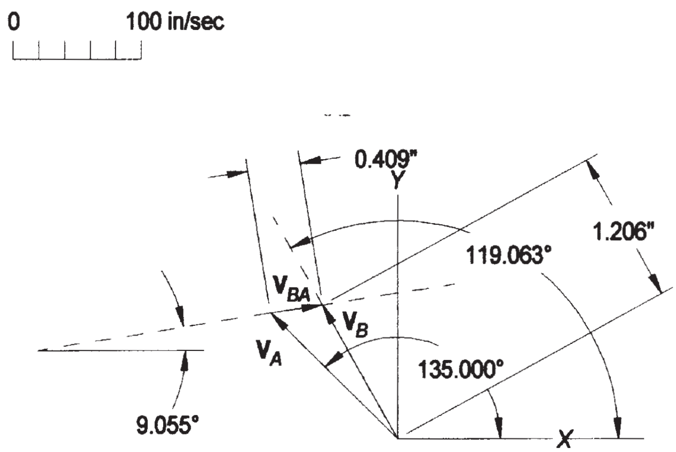
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{100 \cdot in \cdot sec^{-1}}{in}$

$V_B := 1.206 \cdot in \cdot k_v$ $\qquad V_B = 120.600 \dfrac{in}{sec}$ $\qquad \theta_{VB} := 119.063 \cdot deg$

$V_{BA} := 0.409 \cdot in \cdot k_v$ $\qquad V_{BA} = 40.900 \dfrac{in}{sec}$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$\omega_3 := \dfrac{-V_{BA}}{b}$ $\qquad \omega_3 = -0.511 \dfrac{rad}{sec}$

$\omega_4 := \dfrac{V_B}{c}$ $\qquad \omega_4 = 2.353 \dfrac{rad}{sec}$

6. Transform the *xy* coordinates of point P1 into the *XY* system using equations 4.0b.

Coordinate transformation angle $\delta := atan2(-d_X, d_Y)$ $\qquad \delta = 126.582\ deg$

$P_X := p_x \cdot cos(\delta) - p_y \cdot sin(\delta)$ $\qquad P_X = -94.998\ in$

$P_Y := p_x \cdot sin(\delta) + p_y \cdot cos(\delta)$ $\qquad P_Y = 72.308\ in$

7. Calculate the distance from O_4 to P_1 and use equation 6.7 to calculate the velocity at P_1.

Distance from O_4 to P_1: $e := \sqrt{(P_X + d_X)^2 + (P_Y - d_Y)^2}$ $\qquad e = 48.219\ in$

$V_{P1} := e \cdot \omega_4$ $\qquad V_{P1} = 113.446 \dfrac{in}{sec}$

PROBLEM 6-84b

Statement: Find the angular velocities of links 3 and 4 and the linear velocity of points A, B and P_1 in the XY coordinate system for the linkage in Figure P6-30 in the position shown. Assume that $\theta_2 = 45$ deg in the XY coordinate system and $\omega_2 = 10$ rad/sec. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use the method of instant centers.

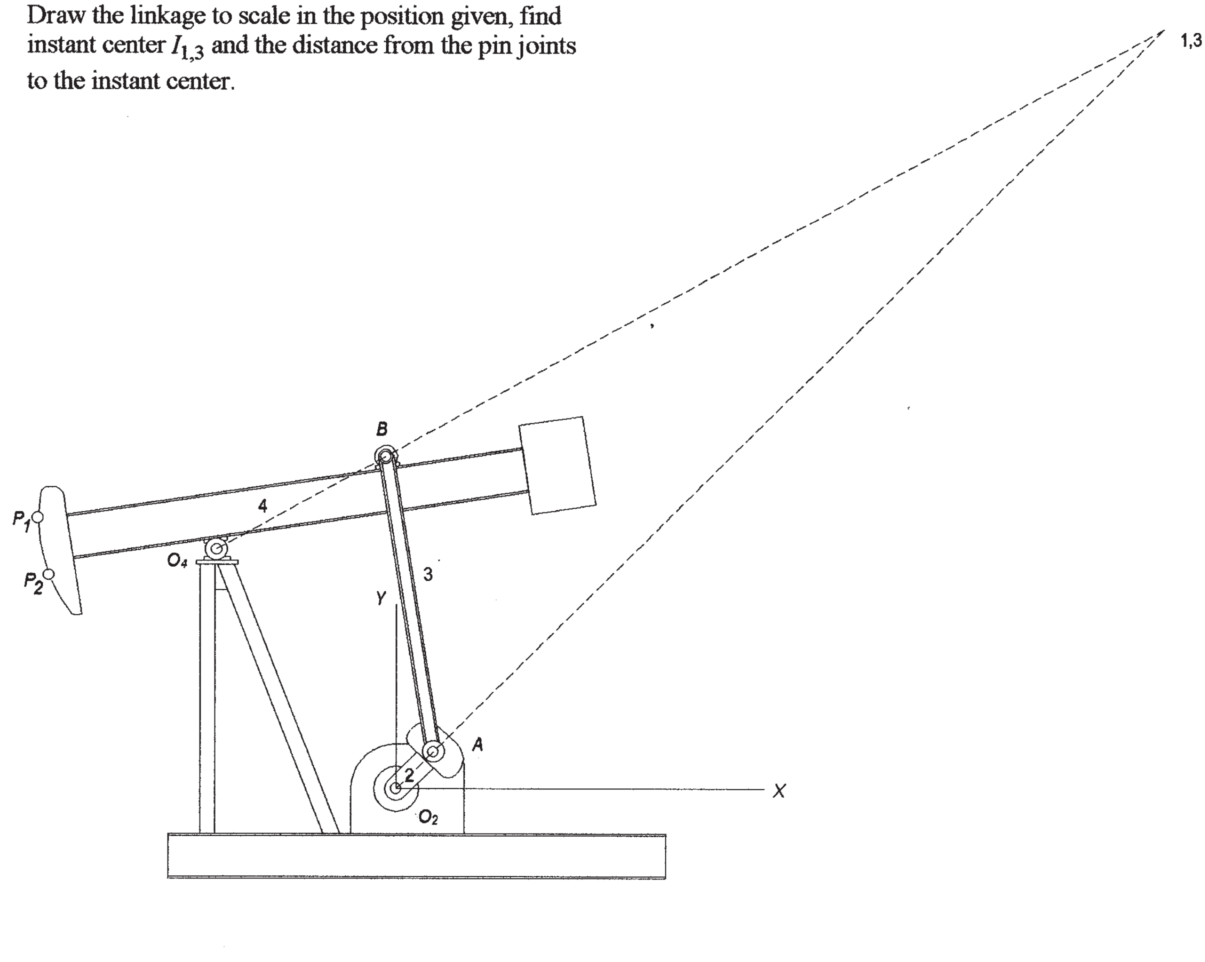
Given: Link lengths:

Link 2 (O_2 to A)	$a := 14.00 \cdot in$	Link 3 (A to B)	$b := 80.00 \cdot in$
Link 4 (O_4 to B)	$c := 51.26 \cdot in$		
Link 1 X-offset	$d_X := 47.5 \cdot in$	Link 1 Y-offset	$d_Y := 76.00 \cdot in - 12.00 \cdot in$
Coupler point x-offset	$p_x := 114.68 \cdot in$	Coupler point y-offset	$p_y := 33.19 \cdot in$
Crank angle:	$\theta_2 := 45 \cdot deg$		

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure P6-30 and Mathcad file P0684b.

1. Draw the linkage to scale in the position given, find instant center $I_{1,3}$ and the distance from the pin joints to the instant center.



From the layout above:

$$AI13 := 273.768 \cdot in \qquad BI13 := 235.874 \cdot in$$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 140.000 \frac{in}{sec}$$

sec

$$\theta_{VA} := \theta_2 + 90 \cdot deg \qquad \theta_{VA} = 135.0\,deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 0.511\,\frac{rad}{sec} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b.

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 120.622\,\frac{in}{sec}$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 2.353\,\frac{rad}{sec} \quad \text{CCW}$$

6. Transform the xy coordinates of point P1 into the XY system using equations 4.0b.

Coordinate transformation angle $\delta := atan2(-d_X, d_Y)$ $\qquad \delta = 126.582\,deg$

$$P_X := p_x \cdot \cos(\delta) - p_y \cdot \sin(\delta) \qquad P_X = -94.998\,in$$

$$P_Y := p_x \cdot \sin(\delta) + p_y \cdot \cos(\delta) \qquad P_Y = 72.308\,in$$

7. Calculate the distance from O_4 to P_1 and use equation 6.7 to calculate the velocity at P_1.

Distance from O_4 to P_1: $\quad e := \sqrt{(P_X + d_X)^2 + (P_Y - d_Y)^2} \qquad e = 48.219\,in$

$$V_{P1} := e \cdot \omega_4 \qquad V_{P1} = 113.467\,\frac{in}{sec}$$

PROBLEM 6-84c

Statement: Find the angular velocities of links 3 and 4 and the linear velocity of points *A*, *B* and P_1 in the *XY* coordinate system for the linkage in Figure P6-30 in the position shown. Assume that $\theta_2 = 45$ deg in the *XY* coordinate system and $\omega_2 = 10$ rad/sec. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the *xy* coordinate system. Use an analytical method.

Given: Link lengths:

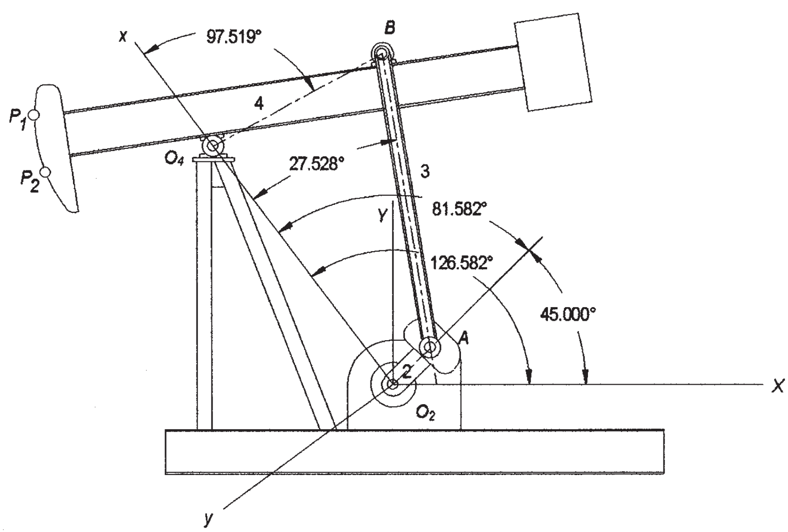
Link 2 (O_2 to *A*)	$a := 14.00 \cdot in$	Link 3 (*A* to *B*)	$b := 80.00 \cdot in$
Link 4 (O_4 to *B*)	$c := 51.26 \cdot in$		
Link 1 *X*-offset	$d_X := 47.5 \cdot in$	Link 1 *Y*-offset	$d_Y := 76.00 \cdot in - 12.00 \cdot in$
Coupler point *x*-offset	$p_x := 114.68 \cdot in$	Coupler point *y*-offset	$p_y := 33.19 \cdot in$
Crank angle:	$\theta_{2XY} := 45 \cdot deg$	*XY* coord system	

Coordinate transformation angle: $\delta := 126.582 \cdot deg$

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure P6-30 and Mathcad file P0684c.

1. Draw the linkage to scale and label it.



Transform the crank angle from global to local coordinate system:

$$\theta_2 := \theta_{2XY} - \delta \qquad \theta_2 = -81.582\,deg$$

Distance O_2O_4: $$d := \sqrt{d_X^{\,2} + d_Y^{\,2}} \qquad d = 79.701\,in$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 5.6929 \qquad K_2 = 1.5548$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.9340$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$A = -3.8401 \qquad B = 1.9785 \qquad C = 7.2529$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \qquad \theta_4 = 262.482\ deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.9963$$

$K_5 = -4.6074$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -10.0081$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = 1.9785$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 1.0849$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \qquad \theta_3 = 332.475\ deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_3 = -0.511 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \qquad \omega_4 = 2.353 \frac{rad}{sec}$$

7. Determine the velocity of points A and B for the crossed circuit using equations 6.19.

$$\mathbf{V_A} := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$\mathbf{V_A} = 138.492 + 20.495j \frac{in}{sec} \qquad |\mathbf{V_A}| = 140.000 \frac{in}{sec}$$

$$\theta_{VAXY} := arg(\mathbf{V_A}) + \delta \qquad \theta_{VAXY} = 135.000\ deg$$

$$\mathbf{V_B} := c \cdot \omega_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4))$$

$$\mathbf{V_B} = 119.588 - 15.781j \frac{in}{sec} \qquad |\mathbf{V_B}| = 120.624 \frac{in}{sec}$$

$$\theta_{VBXY} := arg(\mathbf{V_B}) + \delta \qquad \theta_{VBXY} = 119.064\ deg$$

8. Calculate the distance from O_4 to P_1 and the angle BO_4P_1.

Distance from O_4 to P_1: $$e := \sqrt{(p_x - d)^2 + (p_y)^2} \qquad e = 48.219\ in$$

$$\delta_4 := 180 \cdot deg - atan\left(\frac{p_y}{p_x - d} \right) \qquad \delta_4 = 136.503\ deg$$

9. Determine the velocity of point P_1 using equations 6.35.

$$\mathbf{V_{P1}} := e \cdot \omega_4 \cdot \left(-\sin\left(\theta_4 + \delta\right) + j \cdot \cos\left(\theta_4 + \delta\right)\right)$$

$$\mathbf{V_{P1}} = -55.122 + 99.181j \, \frac{in}{sec} \qquad \left|\mathbf{V_{P1}}\right| = 113.469 \, \frac{in}{sec}$$

$$\theta_{VPXY} := \arg\left(\mathbf{V_{P1}}\right) + \delta \qquad \theta_{VPXY} = 245.646 \, deg$$

PROBLEM 6-85

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the absolute velocity of point P_1 in Figure P6-30 as a function of θ_2 for $\omega_2 = 10$ rad/sec. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the *xy* coordinate system.

Given: Link lengths:

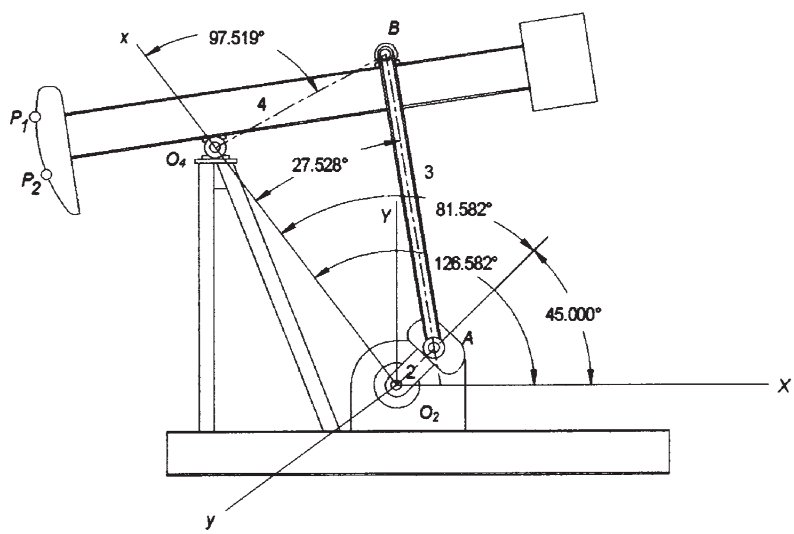
Link 2 (O_2 to A)	$a := 14.00 \cdot in$	Link 3 (A to B)	$b := 80.00 \cdot in$
Link 4 (O_4 to B)	$c := 51.26 \cdot in$		
Link 1 X-offset	$d_X := 47.5 \cdot in$	Link 1 Y-offset	$d_Y := 76.00 \cdot in - 12.00 \cdot in$
Coupler point x-offset	$p_x := 114.68 \cdot in$	Coupler point y-offset	$p_y := 33.19 \cdot in$
Crank angle:	$\theta_{2XY} := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$		XY coord system

Coordinate transformation angle: $\delta := 126.582 \cdot deg$

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure P6-30 and Mathcad file P0685.

1. Draw the linkage to scale and label it.



Transform the crank angle from global to local coordinate system:

$$\theta_2(\theta_{2XY}) := \theta_{2XY} - \delta$$

Distance O_2O_4: $\quad d := \sqrt{d_X^2 + d_Y^2} \qquad d = 79.701\ in$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 5.6929 \qquad K_2 = 1.5548$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.9340$$

$$A(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 - K_2 \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

$$B(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$C(\theta_{2XY}) := K_1 - (K_2 + 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_{2XY}) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_{2XY}), -B(\theta_{2XY}) + \sqrt{B(\theta_{2XY})^2 - 4 \cdot A(\theta_{2XY}) \cdot C(\theta_{2XY})}\right)\right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.9963 \qquad K_5 = -4.6074$$

$$D(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 + K_4 \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$E(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$F(\theta_{2XY}) := K_1 + (K_4 - 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_{2XY}) := 2 \cdot \left[atan2\left[2 \cdot D(\theta_{2XY}), -E(\theta_{2XY}) + \sqrt{\left(E(\theta_{2XY})^2 - 4 \cdot D(\theta_{2XY}) \cdot F(\theta_{2XY})\right)}\right]\right]$$

6. Determine the angular velocity of link 4 for the crossed circuit using equations 6.18.

$$\omega_4(\theta_{2XY}) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2(\theta_{2XY}) - \theta_3(\theta_{2XY}))}{\sin(\theta_4(\theta_{2XY}) - \theta_3(\theta_{2XY}))}$$

7. Calculate the distance from O_4 to P_1 and the angle BO_4P_1.

Distance from O_4 to P_1: $\quad e := \sqrt{(p_x - d)^2 + (p_y)^2} \qquad e = 48.219\ in$

$$\delta_4 := 180 \cdot deg - atan\left(\frac{p_y}{p_x - d}\right) \qquad \delta_4 = 136.503\ deg$$

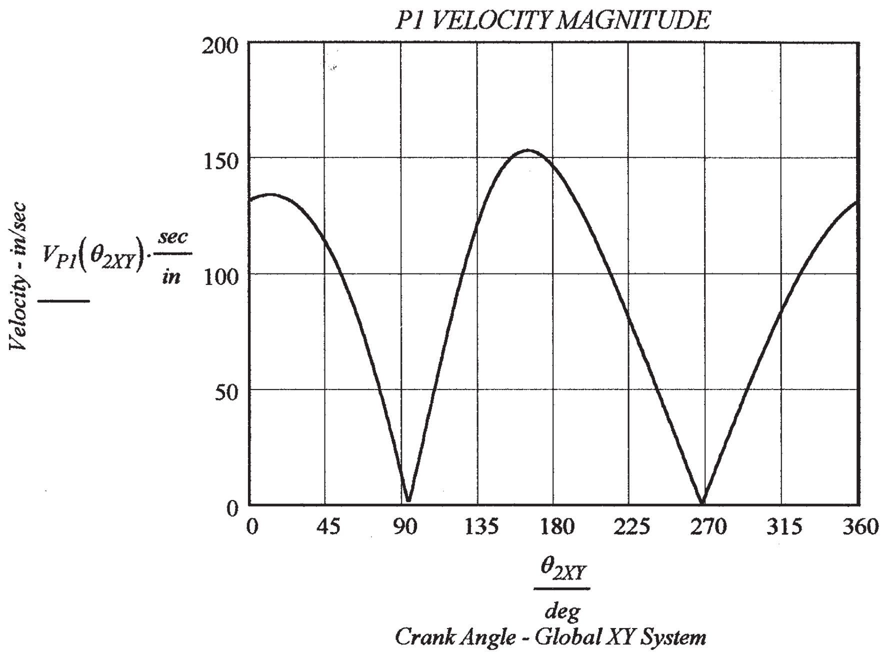
8. Determine and plot the velocity of point P_1 using equations 6.35.

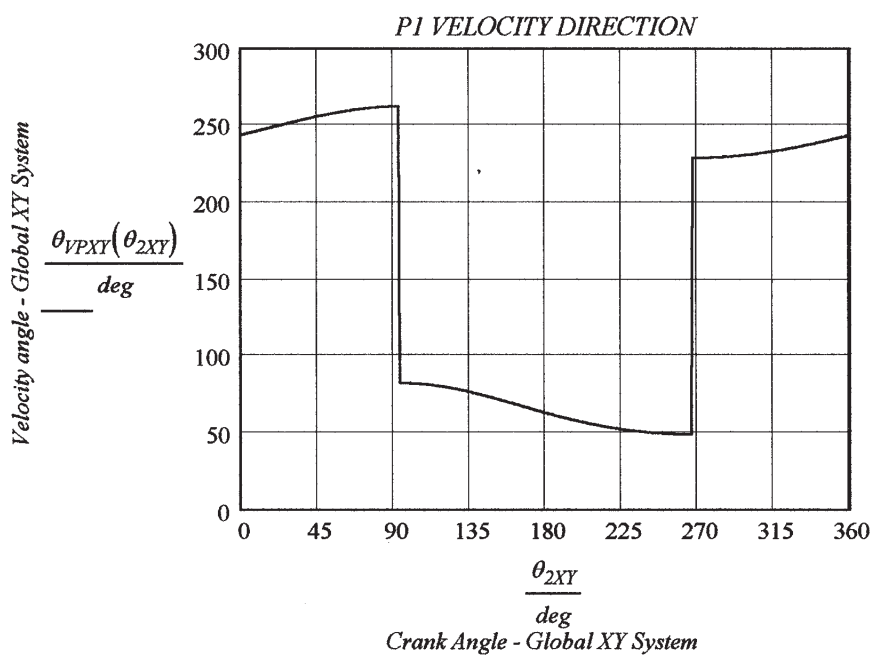
$$\mathbf{V_{P1}}(\theta_{2XY}) := e \cdot \omega_4(\theta_{2XY}) \cdot \left(-\sin(\theta_4(\theta_{2XY}) + \delta_4) + j \cdot \cos(\theta_4(\theta_{2XY}) + \delta_4)\right)$$

$$V_{P1}(\theta_{2XY}) := \left|\mathbf{V_{P1}}(\theta_{2XY})\right|$$

$$\theta_{VPXY}(\theta_{2XY}) := arg(\mathbf{V_{P1}}(\theta_{2XY})) + \delta$$

(See next page)


P1 VELOCITY MAGNITUDE
Velocity - in/sec
$V_{P1}(\theta_{2XY}) \cdot \frac{sec}{in}$
200
150
100
50
0
0
45
90
135
180
225
270
315
360
$\frac{\theta_{2XY}}{deg}$
Crank Angle - Global XY System


P1 VELOCITY DIRECTION
Velocity angle - Global XY System
$\frac{\theta_{VPXY}(\theta_{2XY})}{deg}$
300
250
200
150
100
50
0
0
45
90
135
180
225
270
315
360
$\frac{\theta_{2XY}}{deg}$
Crank Angle - Global XY System

PROBLEM 6-86

Statement: Find all instant centers of the linkage in Figure P6-31 in the position shown.

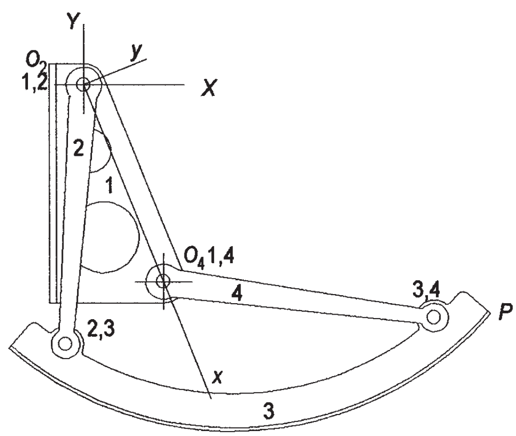
Given: Number of links $n := 4$

Solution: See Figure P6-31 and Mathcad file P0686.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

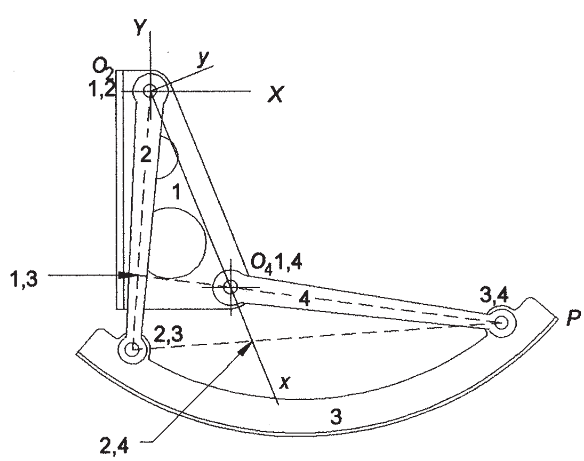
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

1. Draw the linkage to scale and identify those ICs that can be found by inspection (4).



2. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs.

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$



PROBLEM 6-87a

Statement: Find the angular velocities of links 3 and 4 and the linear velocity of point P in the XY coordinate system for the linkage in Figure P6-31 in the position shown. Assume that $\theta_2 = -94.121$ deg in the XY coordinate system and $\omega_2 = 1$ rad/sec. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0$ deg. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 9.17 \cdot in$	Link 3 (A to B)	$b := 12.97 \cdot in$
Link 4 (O_4 to B)	$c := 9.57 \cdot in$		
Link 1 X-offset	$d_X := 2.79 \cdot in$	Link 1 Y-offset	$d_Y := -6.95 \cdot in$

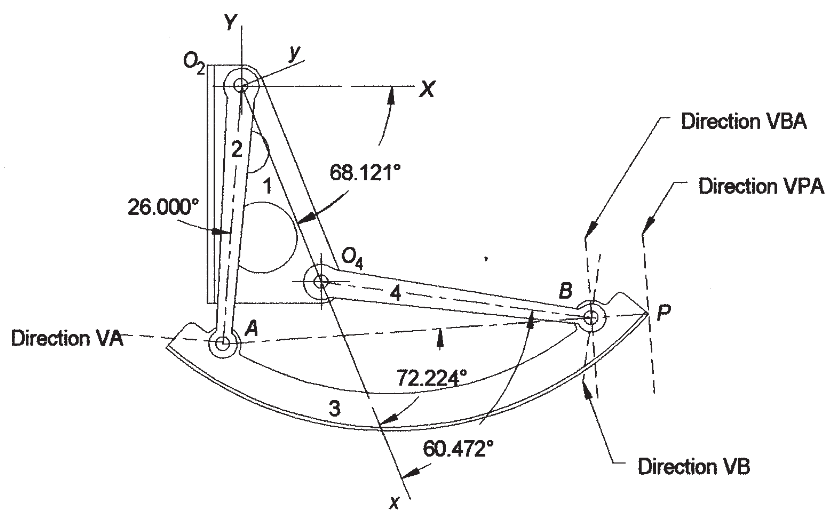
Coupler point data: $p := 15.00 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

Crank angle: $\theta_2 := -94.121 \cdot deg$

Input crank angular velocity $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-31 and Mathcad file P0687a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

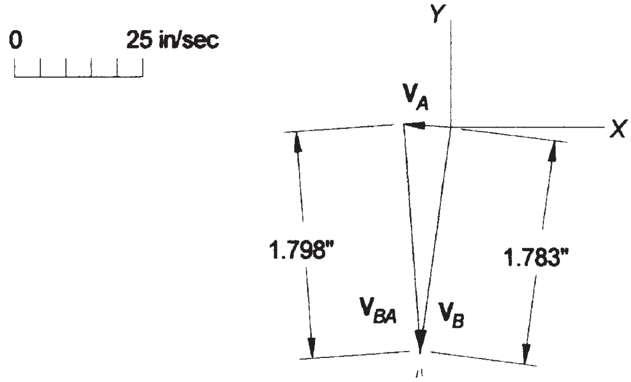
$$V_A := a \cdot \omega_2 \qquad V_A = 9.170 \frac{in}{sec}$$

$$\theta_{VA} := \theta_2 - 90 \cdot deg \qquad \theta_{VA} = -184.121\ deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{25 \cdot in \cdot sec^{-1}}{in}$

$V_B := 1.783 \cdot in \cdot k_v$ $\qquad V_B = 44.575 \dfrac{in}{sec}$ $\qquad \theta_{VB} := 262.352 \cdot deg$

$V_{BA} := 1.798 \cdot in \cdot k_v$ $\qquad V_{BA} = 44.950 \dfrac{in}{sec}$ $\qquad \theta_{VBA} := 274.103 \cdot deg$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$\omega_3 := \dfrac{-V_{BA}}{b}$ $\qquad \omega_3 = -3.466 \dfrac{rad}{sec}$

$\omega_4 := \dfrac{-V_B}{c}$ $\qquad \omega_4 = -4.658 \dfrac{rad}{sec}$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{PA}$ using equation 6.7.

$V_{PA} := \left| p \cdot \omega_3 \right|$ $\qquad V_{PA} = 51.985 \dfrac{in}{sec}$

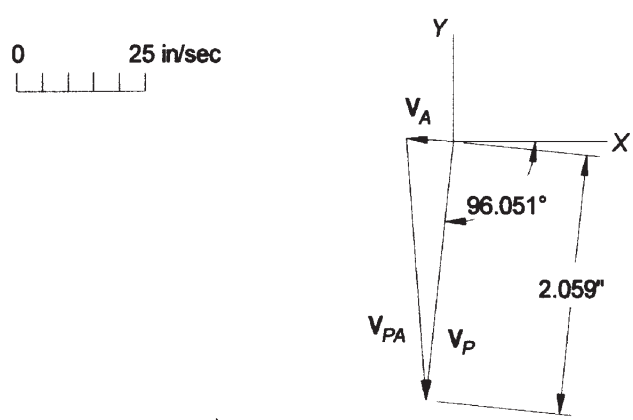
$\theta_{VPA} := \theta_{VBA}$ $\qquad \theta_{VPA} = 274.103\, deg$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *P*. The equation to be solved graphically is

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{PA}$.
c. Complete the vector triangle by drawing $\mathbf{V}_P$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{PA}$ vector.

8. From the velocity triangle we have:



Velocity scale factor: $k_v := \dfrac{25 \cdot in \cdot sec^{-1}}{in}$

$V_P := 2.059 \cdot in \cdot k_v$ $\qquad V_P = 51.475 \dfrac{in}{sec}$

$\theta_P := -96.051 \cdot deg$

PROBLEM 6-87b

Statement: Find the angular velocities of links 3 and 4 and the linear velocity of point P in the XY coordinate system for the linkage in Figure P6-31 in the position shown. Assume that θ_2 = -94.121 deg in the XY coordinate system and ω_2 = 1 rad/sec. The position of the coupler point P on link 3 with respect to point A is: p = 15.00, δ_3 = 0 deg. Use the method of instant centers.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 9.174 \cdot in$	Link 3 (A to B)	$b := 12.971 \cdot in$
Link 4 (O_4 to B)	$c := 9.573 \cdot in$		
Link 1 X-offset	$d_X := 2.790 \cdot in$	Link 1 Y-offset	$d_Y := -6.948 \cdot in$

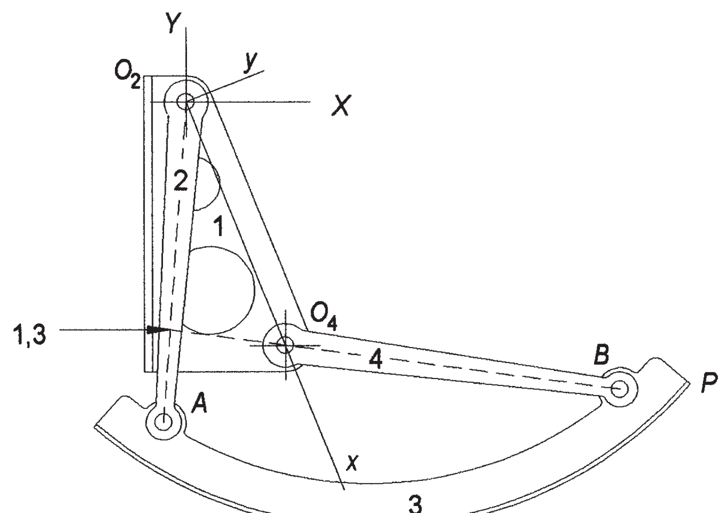
Coupler point data: $p := 15.00 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

Crank angle: $\theta_2 := -94.121 \cdot deg$

Input crank angular velocity $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-31 and Mathcad file P0687b.

1. Draw the linkage to scale in the position given, find instant center $I_{1,3}$ and the distance from the pin joints to the instant center.



From the layout above:

$AI13 := 2.647 \cdot in \quad BI13 := 12.862 \cdot in \quad PI13 := 14.825 \cdot in$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 9.174 \frac{in}{sec}$$

$$\theta_{VA} := \theta_2 - 90 \cdot deg \qquad \theta_{VA} = -184.121 \, deg$$

3. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_A}{AI13} \qquad \omega_3 = 3.466 \frac{rad}{sec} \quad \text{CW}$$

4. Determine the magnitude of the velocity at point B using equation 6.9b.

$$V_B := BI13 \cdot \omega_3 \qquad V_B = 44.577 \frac{in}{sec}$$

5. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 4.657 \frac{rad}{sec} \quad \text{CW}$$

6. Determine the magnitude of the velocity at point *P* using equation 6.9b.

$$V_P := PI13 \cdot \omega_3 \qquad V_P = 51.381 \frac{in}{sec}$$

PROBLEM 6-87c

Statement: Find the angular velocities of links 3 and 4 and the linear velocity of point P in the XY coordinate system for the linkage in Figure P6-31 in the position shown. Assume that $\theta_2 = -94.121$ deg in the XY coordinate system and $\omega_2 = 1$ rad/sec. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0$ deg. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 9.174 \cdot in$	Link 3 (A to B)	$b := 12.971 \cdot in$
Link 4 (O_4 to B)	$c := 9.573 \cdot in$		
Link 1 X-offset	$d_X := 2.790 \cdot in$	Link 1 Y-offset	$d_Y := -6.948 \cdot in$

Coupler point data: $p := 15.00 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

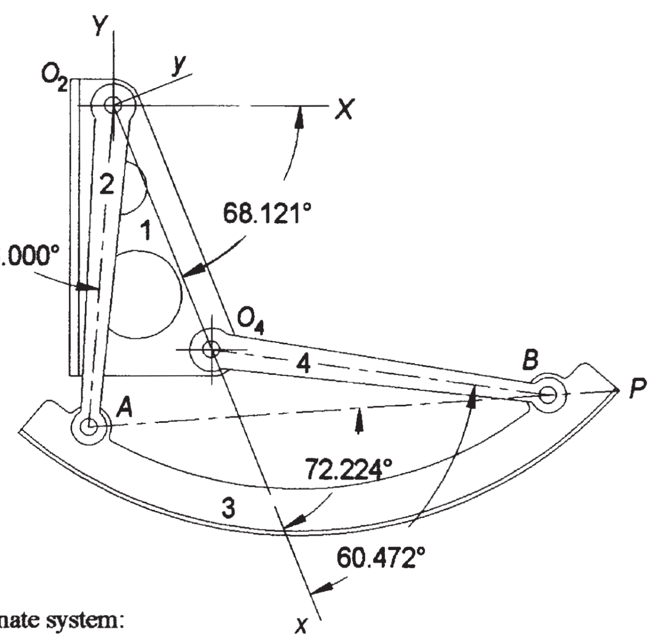
Crank angle: $\theta_{2XY} := -94.121 \cdot deg$ Global XY coord system

Coordinate transformation angle: $\delta := -68.121 \cdot deg$

Input crank angular velocity $\omega_2 := -1 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-31 and Mathcad file P0687c.

1. Draw the linkage to scale and label it.



Transform crank angle into local xy coordinate system:

$$\theta_2 := \theta_{2XY} - \delta \qquad \theta_2 = -26.000\,deg$$

Calculate distance O_2O_4: $\quad d := \sqrt{d_X^2 + d_Y^2} \qquad d = 7.487\,in$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.8161 \qquad K_2 = 0.7821$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.3622$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.2581 \qquad B = 0.8767 \qquad C = -0.4234$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) + 2 \cdot \pi \qquad \theta_4 = 60.488\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.5772 \qquad K_5 = -0.9111$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -0.3096$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = 0.8767$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = -0.4749$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) + 2 \cdot \pi \qquad \theta_3 = 72.236\, deg$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_3 = -3.467 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \qquad \omega_4 = -4.658 \frac{rad}{sec}$$

7. Determine the velocity of points *A* and *B* using equations 6.19.

$$\mathbf{V_A} := a \cdot \omega_2 \cdot \left(-\sin(\theta_2) + j \cdot \cos(\theta_2) \right)$$

$$\mathbf{V_A} = -4.022 - 8.246i \frac{in}{sec} \qquad \left| \mathbf{V_A} \right| = 9.174 \frac{in}{sec}$$

$$\theta_{VAXY} := \arg(\mathbf{V_A}) + \delta \qquad \theta_{VAXY} = -184.121\, deg$$

$$\mathbf{V_B} := c \cdot \omega_4 \cdot \left(-\sin(\theta_4) + j \cdot \cos(\theta_4) \right)$$

$$\mathbf{V_B} = 38.807 - 21.967i \frac{in}{sec} \qquad \left| \mathbf{V_B} \right| = 44.593 \frac{in}{sec}$$

$$\theta_{VBXY} := \arg(\mathbf{V_B}) + \delta \qquad \theta_{VBXY} = -97.633\, deg$$

8. Determine the velocity of the coupler point *P* for the crossed circuit using equations 6.36.

$$\mathbf{V_{PA}} := p \cdot \omega_3 \cdot \left(-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3) \right)$$

$$\mathbf{V_{PA}} = 4.127 - 1.322i \frac{ft}{sec}$$

$$\mathbf{V_P} := \mathbf{V_A} + \mathbf{V_{PA}} \qquad \mathbf{V_P} = 3.792 - 2.009i \frac{ft}{sec} \qquad \left| \mathbf{V_P} \right| = 51.501 \frac{in}{sec}$$

$$\theta_{VPXY} := \arg(\mathbf{V_P}) + \delta \qquad \theta_{VPXY} = -96.039\, deg$$

PROBLEM 6-88

Statement: Figure P6-32 shows a fourbar double slider known as an elliptical trammel. F ind all its instant centers in the position shown.

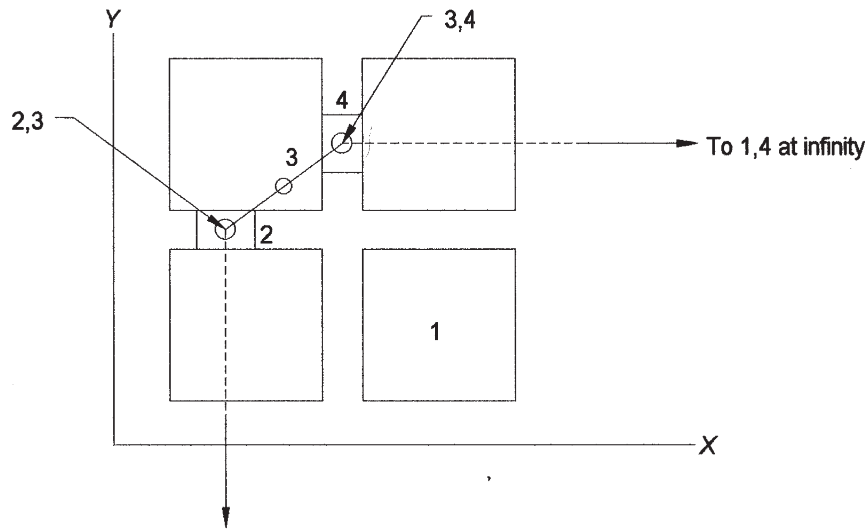
Given: Number of links $n := 4$

Solution: See Figure P6-32 and Mathcad file P0688.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

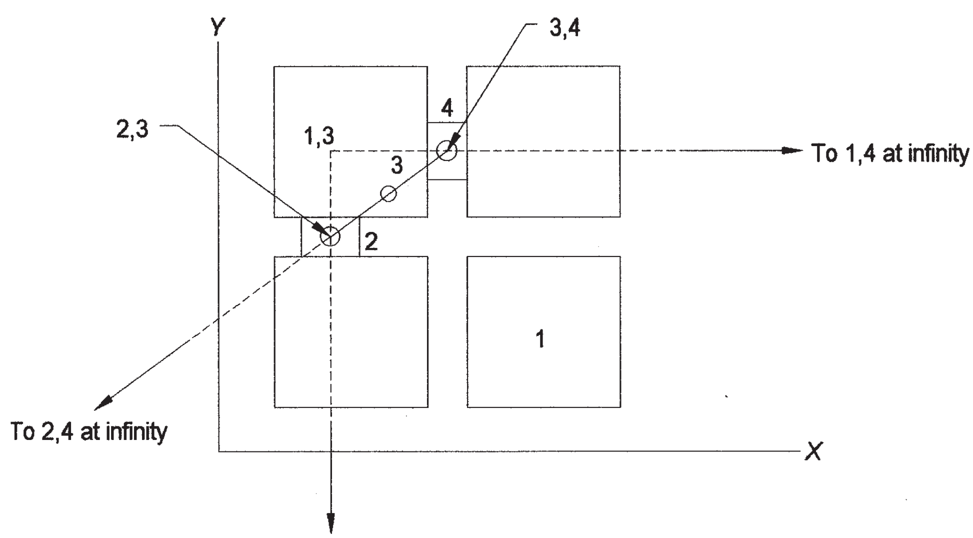
$$C := \frac{n \cdot (n-1)}{2} \qquad C = 6$$

1. Draw the linkage to scale and identify those ICs that can be found by inspection (4).



2. Use Kennedy's Rule and a linear graph to find the remaining 2 ICs.

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$ $\qquad$ $I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

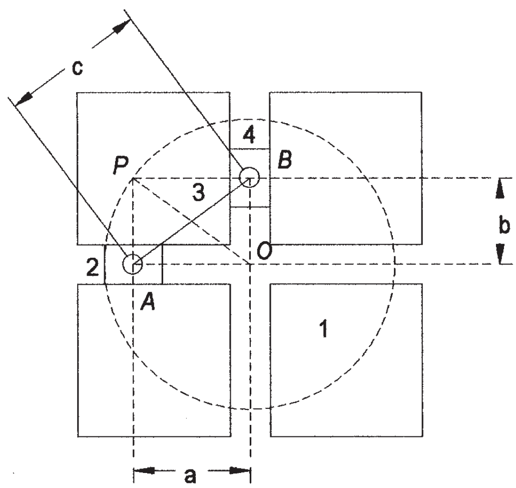


PROBLEM 6-89

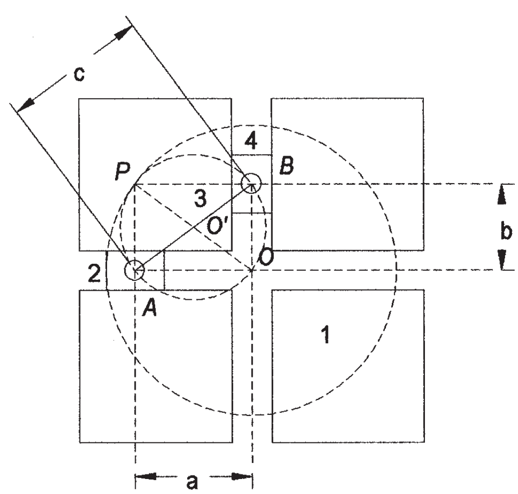
Statement: The elliptical trammel in Figure P6-32 must be driven by rotating link 3 in a full circle. Points on line AB describe ellipses. Find and draw (mannually or with a computer) the fixed and moving centrodes of instant center I_{13}. (Hint: These are called the Cardan circles.)

Solution: See Figure P6-32 and Mathcad file P0689.

1. The instant center I_{13} is shown as the point P in the diagram below. The length of link 3, c, is constant and it is a diagonal of the rectangle $OBPA$. Therefore, the other diagonal, OP, has a constant length, c, regardless of the current lengths a and b. Thus, the point P (I_{13}) travels on a circle of radius c. This circle is the fixed centrode.



2. Now, invert the mechanism by holding link 3 fixed and allowing the slots to move, which can only be in a circle about O. The locus of points P will then be a circle of radius $0.5c$ with center at O'. This is the moving centrode.

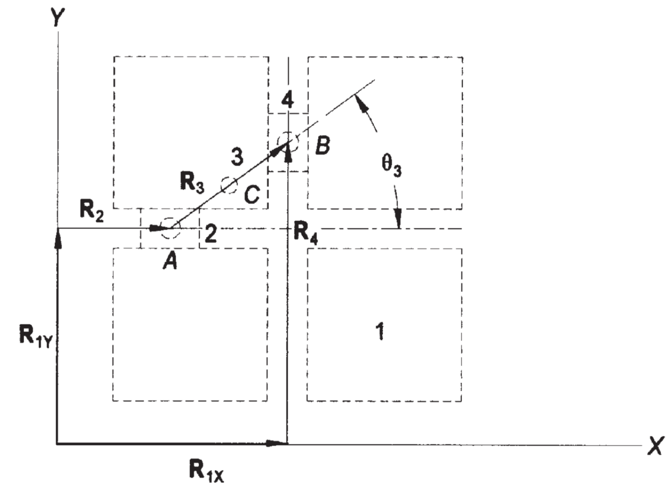


PROBLEM 6-90

Statement: Derive analytical expressions for the velocities of points A and B in Figure P6-32 as a function of θ3, ω3 and the length AB of link 3. Use a vector loop equation.

Solution: See Figure P6-32 and Mathcad file P0690.

1. Establish the global XY system such that the coordinates of the intersection of the slot centerlines is at (d_X, d_Y). Then, define position vectors $\mathbf{R}_{1X}$, $\mathbf{R}_{1Y}$, $\mathbf{R}_2$, $\mathbf{R}_3$, and $\mathbf{R}_4$ as shown below.



2. Write the vector loop equation: $\mathbf{R}_{1Y} + \mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_{1X} - \mathbf{R}_4 = 0$ then substitute the complex number notation for each position vector. The equation then becomes:

$$d_Y \cdot e^{j\left(\frac{\pi}{2}\right)} + a \cdot e^{j(0)} + c \cdot e^{j(\theta_3)} - d_X \cdot e^{j(0)} - b \cdot e^{j\left(\frac{\pi}{2}\right)} = 0$$

3. Differentiate this equation with respect to time.

$$\frac{d}{dt}a + j \cdot c \cdot \omega_3 \cdot e^{j(\theta_3)} - \frac{d}{dt}b \cdot e^{j\left(\frac{\pi}{2}\right)} = 0$$

3. Substituting the Euler identity into this equation gives:

$$V_a + jc \cdot \omega_3 \cdot \left(\cos(\theta_3) + j \cdot \sin(\theta_3)\right) - V_b \cdot j = 0$$

4. Separate this equation into its real (x component) and imaginary (y component) parts, setting each equal to zero.

$$V_a - c \cdot \omega_3 \cdot \sin(\theta_3) = 0 \qquad c \cdot \omega_3 \cdot \cos(\theta_3) - V_b = 0$$

5. Solve for the two unknowns V_a and V_b in terms of the independent variables θ_3 and ω_3

$$V_a = c \cdot \omega_3 \cdot \sin(\theta_3) \qquad V_b = c \cdot \omega_3 \cdot \cos(\theta_3)$$

PROBLEM 6-91

Statement: The linkage in Figure P6-33a has link 2 at 120 deg in the global *XY* coordinate system. Find ω_6 and $\mathbf{V}_D$ in the global *XY* coordinate system for the position shown if ω_2 = 10 rad/sec CCW. Use the velocity difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot in$	Link 3 (A to B)	$b := 4.50 \cdot in$
Link 4 (O_4 to B)	$c := 3.00 \cdot in$	Link 5 (C to D)	$e := 5.60 \cdot in$
Link 3 (A to C)	$p := 2.25 \cdot in$	Link 4 (O_4 to D)	$f := 3.382 \cdot in$
Link 1 X-offset	$d_X := -7.80 \cdot in$	Link 1 Y-offset	$d_Y := 0.62 \cdot in$

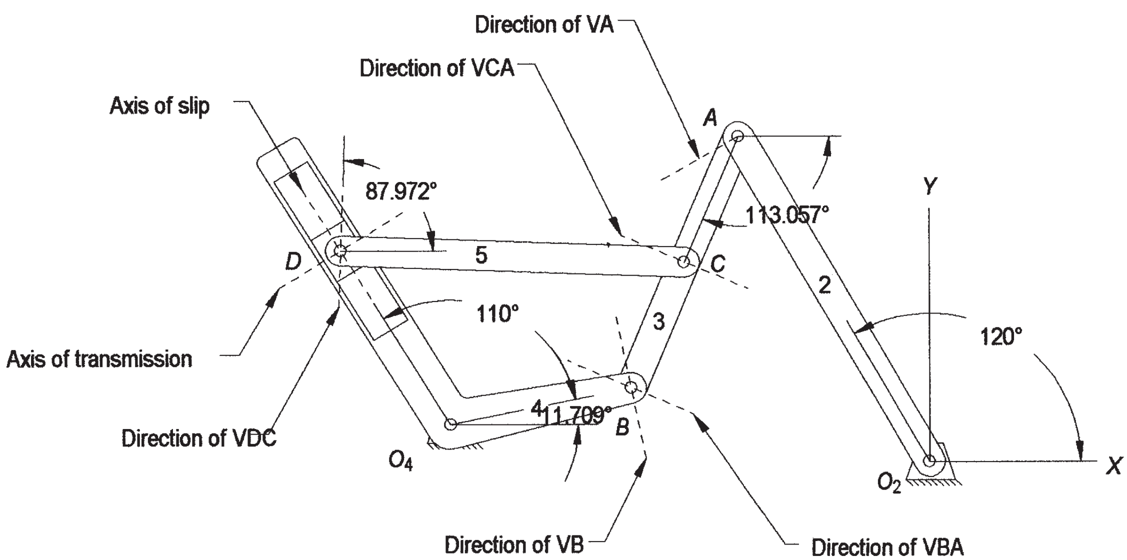
Angle ACB $\quad \delta_3 := 0.0 \cdot deg$ $\quad$ Angle BO_4D $\quad \delta_4 := 110.0 \cdot deg$

Input rocker angle: $\quad \theta_2 := 120 \cdot deg$ $\quad$ Global XY system

Input crank angular velocity $\quad \omega_2 := 10 \cdot rad \cdot sec^{-1}$ $\quad$ CW

Solution: See Figure P6-33a and Mathcad file P0691.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



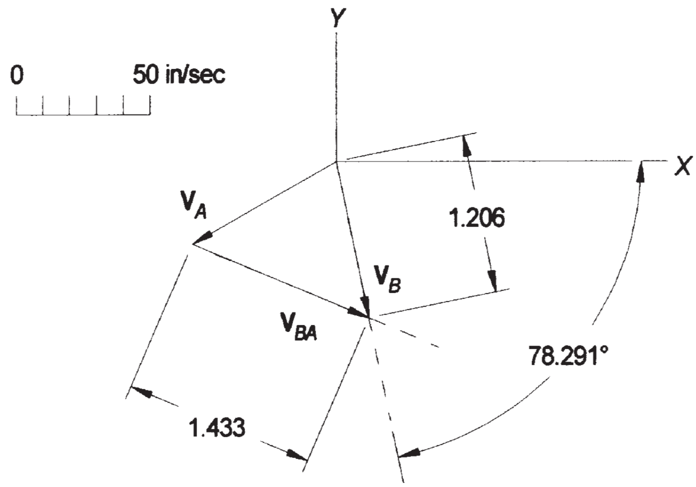
2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 62.000 \frac{in}{sec} \qquad \theta_{VA} := 120 \cdot deg + 90 \cdot deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 5. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{50 \cdot in \cdot sec^{-1}}{in}$

$V_B := 1.206 \cdot in \cdot k_v$ $\quad V_B = 60.300 \frac{in}{sec}$ $\quad \theta_{VB} := -78.291 \cdot deg$

$V_{BA} := 1.433 \cdot in \cdot k_v$ $\quad V_{BA} = 71.650 \frac{in}{sec}$ $\quad \theta_{VBA} := -23.057 \cdot deg$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$\omega_3 := \frac{V_{BA}}{b}$ $\quad \omega_3 = 15.922 \frac{rad}{sec}$ $\quad$ CCW

$\omega_4 := \frac{-V_B}{c}$ $\quad \omega_4 = -20.100 \frac{rad}{sec}$ $\quad$ CW

6. Determine the magnitude and sense of the vector $\mathbf{V}_{CA}$ using equation 6.7.

$V_{CA} := |p \cdot \omega_3|$ $\quad V_{CA} = 35.825 \frac{in}{sec}$

$\theta_{VCA} := \theta_{VBA} + \delta_3$ $\quad \theta_{VCA} = -23.057 \, deg$

7. Determine the magnitude and sense of the vector $\mathbf{V}_{trans}$ using equation 6.7.

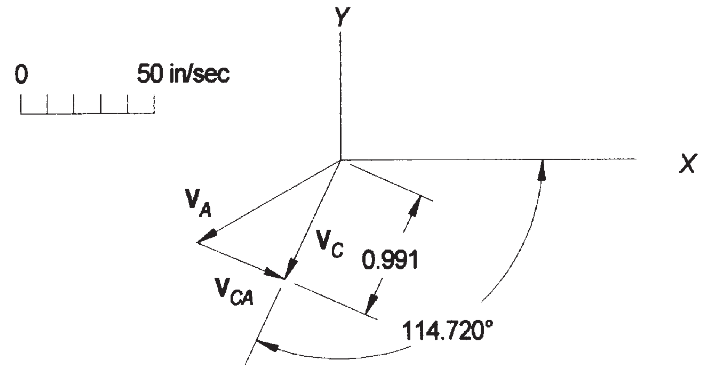
$V_{trans} := |f \cdot \omega_4|$ $\quad V_{trans} = 67.978 \frac{in}{sec}$

$\theta_{trans} := \theta_{VB} + \delta_4$ $\quad \theta_{trans} = 31.709 \, deg$

8. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{CA}$.
c. Complete the vector triangle by drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{CA}$ vector.



9. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{50 \cdot in \cdot sec^{-1}}{in}$

$V_C := 0.991 \cdot in \cdot k_v$ $\qquad V_C = 49.550 \dfrac{in}{sec}$ $\qquad \theta_{VC} := 114.720 \cdot deg$

9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *D*, the magnitude of the relative velocity $\mathbf{V}_{DC}$. The equations to be solved (simultaneously) graphically are

$$\mathbf{V}_D = \mathbf{V}_C + \mathbf{V}_{DC} \qquad \text{and} \qquad \mathbf{V}_D = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_C$.
b. From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{DC}$, magnitude unknown.
c. Repeat steps a and b with $\mathbf{V}_{trans}$ and $\mathbf{V}_{slip}$.
c. From the tail of $\mathbf{V}_C$, draw a construction line to the intersection of the two construction lines.
d. Complete the vector polygon by drawing $\mathbf{V}_{DC}$ from the tip of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_D$ construction line and drawing $\mathbf{V}_D$ from the tail of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_{DC}$ construction line.

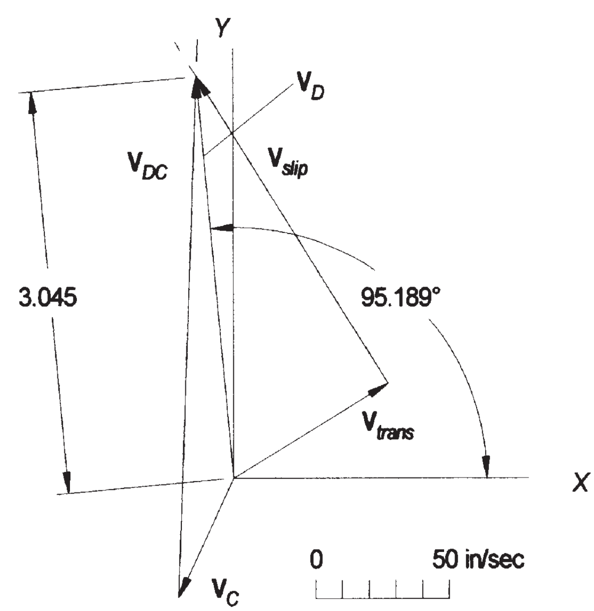
10. From the velocity polygon we have:

Velocity scale factor: $k_v := \dfrac{50 \cdot in \cdot sec^{-1}}{in}$

$V_D := 3.045 \cdot in \cdot k_v$ $\qquad V_D = 152.250 \dfrac{in}{sec}$

$\theta_{VD} := 95.189 \cdot deg$

$\omega_6 := \omega_4$ $\qquad \omega_6 = -20.100 \dfrac{rad}{sec}$



PROBLEM 6-92

Statement: The linkage in Figure P6-33a has link 2 at 120 deg in the global *XY* coordinate system. Find ω_6 and $\mathbf{V}_D$ in the global *XY* coordinate system for the position shown if ω_2 = 10 rad/sec CCW. Use the instant center graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot in$	Link 3 (A to B)	$b := 4.50 \cdot in$
Link 4 (O_4 to B)	$c := 3.00 \cdot in$	Link 5 (C to D)	$e := 5.60 \cdot in$
Link 3 (A to C)	$p := 2.25 \cdot in$	Link 4 (O_4 to D)	$f := 3.382 \cdot in$
Link 1 X-offset	$d_X := -7.80 \cdot in$	Link 1 Y-offset	$d_Y := 0.62 \cdot in$

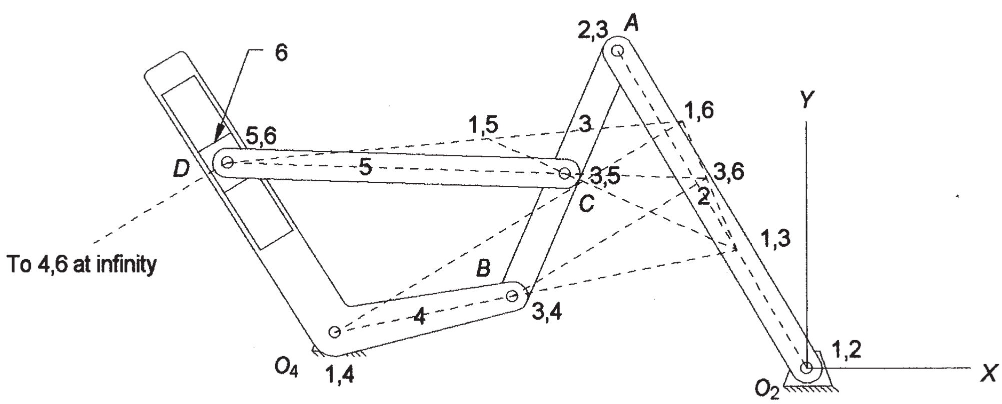
Angle *ACB* $\delta_3 := 0.0 \cdot deg$ Angle BO_4D $\delta_4 := 110.0 \cdot deg$

Input rocker angle: $\theta_2 := 120 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-33a and Mathcad file P0692.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers.



From the layout above:

$AI13 := 3.893 \cdot in$ $BI13 := 3.788 \cdot in$ $CI13 := 3.113 \cdot in$ $CI15 := 1.411 \cdot in$ $DI15 := 4.333 \cdot in$ $DI16 := 7.573 \cdot in$

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at point *A*.

$V_A := a \cdot \omega_2$ $V_A = 62.000 \dfrac{in}{sec}$

$\theta_{VA} := \theta_2 + 90 \cdot deg$ $\theta_{VA} = 210.0\,deg$

3. Determine the angular velocity of link 3 using equation 6.9a.

$\omega_3 := \dfrac{V_A}{AI13}$ $\omega_3 = 15.926 \dfrac{rad}{sec}$ CCW

4. Determine the magnitude of the velocity at point *B* using equation 6.9b.

$V_B := BI13 \cdot \omega_3$ $V_B = 60.328 \dfrac{in}{sec}$

5. Use equation 6.9c to determine the angular velocity of links 4 and 6.

$$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 20.109 \frac{rad}{sec} \qquad \text{CW}$$

$$w_6 := \omega_4 \qquad w_6 = 20.109 \frac{rad}{sec} \qquad \text{CW}$$

6. Determine the magnitude of the velocity at point C using equation 6.9b.

$$V_C := CI13 \cdot \omega_3 \qquad V_C = 49.578 \frac{in}{sec}$$

7. Determine the angular velocity of link 5 using equation 6.9a.

$$\omega_5 := \frac{V_C}{CI15} \qquad \omega_5 = 35.137 \frac{rad}{sec} \qquad \text{CW}$$

8. Determine the magnitude of the velocity at point D using equation 6.9b.

$$V_D := DI15 \cdot \omega_5 \qquad V_D = 152.247 \frac{in}{sec} \quad @\, 95.185 \text{ deg}$$

PROBLEM 6-93

Statement: The linkage in Figure P6-33a has link 2 at 120 deg in the global *XY* coordinate system. Find ω_6 and $\mathbf{V}_D$ in the global *XY* coordinate system for the position shown if ω_2 = 10 rad/sec CCW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot in$	Link 3 (A to B)	$b := 4.50 \cdot in$
Link 4 (O_4 to B)	$c := 3.00 \cdot in$	Link 5 (C to D)	$e := 5.60 \cdot in$
Link 3 (A to C)	$p := 2.25 \cdot in$	Link 4 (O_4 to D)	$f := 3.382 \cdot in$
Link 1 X-offset	$d_X := -7.80 \cdot in$	Link 1 Y-offset	$d_Y := 0.62 \cdot in$

Angle ACB $\qquad \delta_3 := 0.0 \cdot deg$ $\qquad$ Angle BO_4D $\qquad \delta_4 := 110.0 \cdot deg$

Input rocker angle: $\qquad \theta_{2XY} := 120 \cdot deg$ $\qquad$ Global XY system

Coordinate transformation angle: $\qquad \delta := 175.455 \cdot deg$

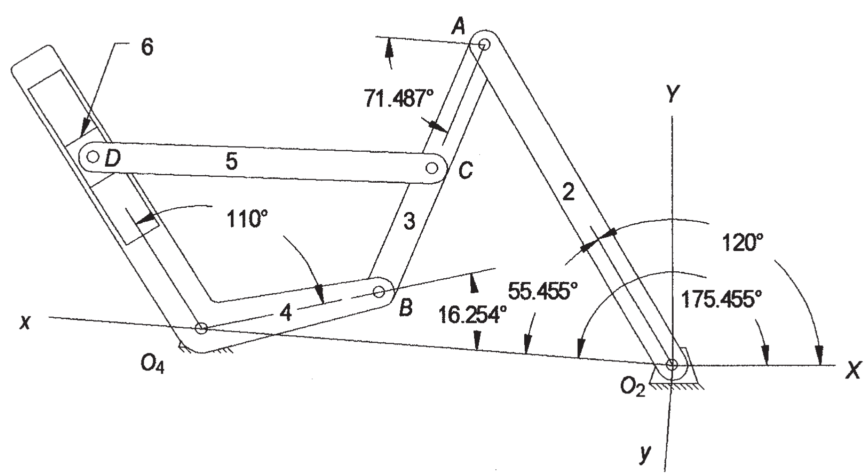
Input crank angular velocity $\qquad \omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi\ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi\ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right)\ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi\ \ otherwise \end{array}$$

Solution: See Figure P6-33a and Mathcad file P0691.

1. Draw the linkage to scale and label it.



Calculate the distance O_2O_4: $\qquad d := \sqrt{d_X^{\,2} + d_Y^{\,2}}$ $\qquad d = 7.825\ in$

Transform θ_{2XY} into the local coordinate system: $\qquad \theta_2 := \theta_{2XY} - \delta$ $\qquad \theta_2 = -55.455\ deg$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.2620 \qquad K_2 = 2.6082$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.3767$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = 0.2028 \qquad B = 1.6474 \qquad C = 1.5927$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_{41} = -163.746\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.7388$$

$$K_5 = -1.9877$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -1.6967$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = 1.6474$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = -0.3067$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) - 2 \cdot \pi \qquad \theta_3 = 71.488\, deg$$

6. At this point write vector loop equations for links 3, 4, 5, and 6 to solve for the position and velocty of link 6.

PROBLEM 6-94

Statement: The linkage in Figure P6-33b has link 3 perpendicular to the X-axis and links 2 and 4 are parallel to each other. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_P$ if ω_2 = 15 rad/sec CW. Use the velocity difference graphical method.

Given: Link lengths:

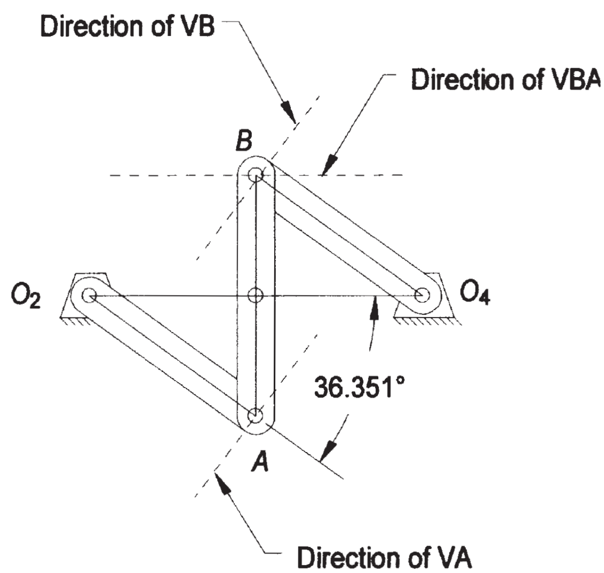
Link 2 (O_2 to A)	$a := 2.75 \cdot in$	Link 3 (A to B)	$b := 3.26 \cdot in$
Link 4 (O_4 to B)	$c := 2.75 \cdot in$	Link 1 (O_4 to B)	$d := 4.43 \cdot in$

Coupler point data: $p := 1.63 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

Input crank angular velocity $\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-33b and Mathcad file P0694.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



$\theta_2 := -36.352 \cdot deg$

2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

$$V_A := a \cdot \omega_2 \qquad V_A = 41.250 \frac{in}{sec}$$

$$\theta_{VA} := \theta_2 - 90 \cdot deg \qquad \theta_{VA} = -126.352\, deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point B, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equation to be solved graphically is

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.

b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.

4. Without actually drawing the vector polygon, we see that $\mathbf{V}_A$ and $\mathbf{V}_B$ have the same angle with respect to the X axis. Therefore,

$$V_B := V_A \qquad \text{and} \qquad V_{BA} := 0 \cdot \frac{in}{sec} \qquad \theta_{VB} := \theta_{VA} \qquad \theta_{VBA} := 0 \cdot deg$$

5. Determine the angular velocity of links 3 and 4 using equation 6.7.

$$\omega_3 := \frac{-V_{BA}}{b} \qquad \omega_3 = 0.000 \frac{rad}{sec}$$

$$\omega_4 := \frac{-V_B}{c} \qquad \omega_4 = -15.000 \frac{rad}{sec}$$

6. Determine the magnitude and sense of the vector $\mathbf{V}_{PA}$ using equation 6.7.

$$V_{PA} := |p \cdot \omega_3| \qquad V_{PA} = 0.000 \frac{in}{sec}$$

$$\theta_{VPA} := \theta_{VBA} \qquad \theta_{VPA} = 0.000\,deg$$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point P. The equation to be solved graphically is

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA}$$

   a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
   b. From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{PA}$.
   c. Complete the vector triangle by drawing $\mathbf{V}_P$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{PA}$ vector.

8. Again, without drawing the vector polygon we see that,

$$V_P := V_A$$

At this instant, link 3 is in pure translation with every point on it having the same velocity.

PROBLEM 6-95

Statement: The linkage in Figure P6-33b has link 3 perpendicular to the X-axis and links 2 and 4 are parallel to each other. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_P$ if ω_2 = 15 rad/sec CW. Use the instant center graphical method.

Given: Link lengths:

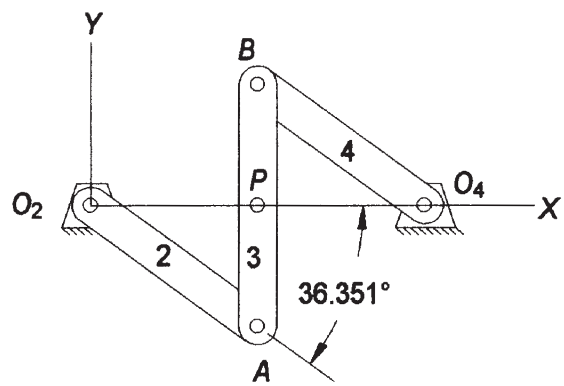
Link 2 (O_2 to A)	$a := 2.75 \cdot in$	Link 3 (A to B)	$b := 3.26 \cdot in$
Link 4 (O_4 to B)	$c := 2.75 \cdot in$	Link 1 (O_4 to B)	$d := 4.43 \cdot in$

Coupler point data: $p := 1.63 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

Input crank angular velocity $\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-33b and Mathcad file P0695.

1. Draw the linkage to scale in the position given, find instant center $I_{1,3}$ and the distance from the pin joints to the instant center.



Since O_2A and O_4B are parallel, $I_{1,3}$ is at infinity and link 3 does not rotate for this instantaneous position of the linkage. Thus, link 3 is in pure translation and all points on it will have the same velocity.

2. Use equation 6.7 and inspection of the layout to determine the magnitude and direction of the velocity at points A, B, and P, which will be the same.

$$V_A := a \cdot \omega_2 \qquad V_A = 41.250 \frac{in}{sec}$$

$$\theta_2 := -36.351 \cdot deg$$

$$\theta_{VA} := \theta_2 - 90 \cdot deg \qquad \theta_{VA} = -126.351\ deg$$

The velocity vectors for points B and P are identical to $\mathbf{V}_A$.

PROBLEM 6-96

Statement: The linkage in Figure P6-33b has link 3 perpendicular to the X-axis and links 2 and 4 are parallel to each other. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_P$ if ω_2 = 15 rad/sec CW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A) $a := 2.75 \cdot in$ Link 3 (A to B) $b := 3.26 \cdot in$

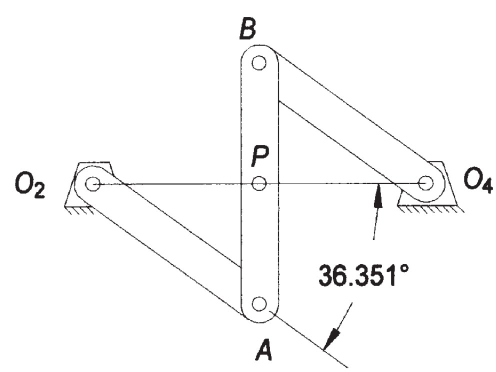
Link 4 (O_4 to B) $c := 2.75 \cdot in$ Link 1 (O_4 to B) $d := 4.43 \cdot in$

Coupler point data: $p := 1.63 \cdot in$ $\delta_3 := 0 \cdot deg$

Input crank angular velocity $w_2 := -15 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-33b and Mathcad file P0696.

1. Draw the linkage to a convenient scale and label it.



$\theta_2 := -36.351 \cdot deg$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.6109 \qquad K_2 = 1.6109$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.5949$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \qquad A = -0.5081$$

$$B := -2 \cdot \sin(\theta_2) \qquad B = 1.1855$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \qquad C = 1.1029$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) + 2 \cdot \pi \qquad \theta_4 = 143.649\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.3589 \qquad K_5 = -1.6873$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -1.3983$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = 1.1855$$

$$F := K_1 + (K_4 - 1)\cdot\cos(\theta_2) + K_5 \qquad F = 0.2127$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2\cdot\left(atan2\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) + 2\cdot\pi \qquad \theta_3 = 89.995\,deg$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a\cdot w_2}{b}\cdot\frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_3 = -0.000\,\frac{rad}{sec}$$

$$\omega_4 := \frac{a\cdot w_2}{c}\cdot\frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \qquad \omega_4 = 15.000\,\frac{rad}{sec}$$

7. Determine the velocity of points *A* and *B* using equations 6.19.

$$\mathbf{V_A} := a\cdot w_2\cdot(-\sin(\theta_2) + j\cdot\cos(\theta_2))$$

$$\mathbf{V_A} = -24.450 - 33.223i\,\frac{in}{sec} \qquad |\mathbf{V_A}| = 41.250\,\frac{in}{sec}$$

$$\theta_{VAXY} := arg(\mathbf{V_A}) \qquad \theta_{VAXY} = -126.351\,deg$$

$$\mathbf{V_B} := c\cdot\omega_4\cdot(-\sin(\theta_4) + j\cdot\cos(\theta_4))$$

$$\mathbf{V_B} = -24.450 - 33.223i\,\frac{in}{sec} \qquad |\mathbf{V_B}| = 41.250\,\frac{in}{sec}$$

$$\theta_{VBXY} := arg(\mathbf{V_B}) \qquad \theta_{VBXY} = -126.351\,deg$$

8. Determine the velocity of the coupler point *P* using equations 6.36.

$$\mathbf{V_{PA}} := p\cdot\omega_3\cdot(-\sin(\theta_3 + \delta_3) + j\cdot\cos(\theta_3 + \delta_3))$$

$$\mathbf{V_{PA}} = 1.936 \times 10^{-4} - 1.676i \times 10^{-8}\,\frac{in}{sec}$$

$$\mathbf{V_P} := \mathbf{V_A} + \mathbf{V_{PA}} \qquad |\mathbf{V_P}| = 41.250\,\frac{in}{sec}$$

$$\theta_{VPXY} := arg(\mathbf{V_P}) \qquad \theta_{VPXY} = -126.351\,deg$$

PROBLEM 6-97

Statement: The crosshead linkage shown in Figure P6-33c has 2 *DOF* with inputs at crossheads 2 and 5. Find instant centers $I_{1,3}$ and $I_{1,4}$.

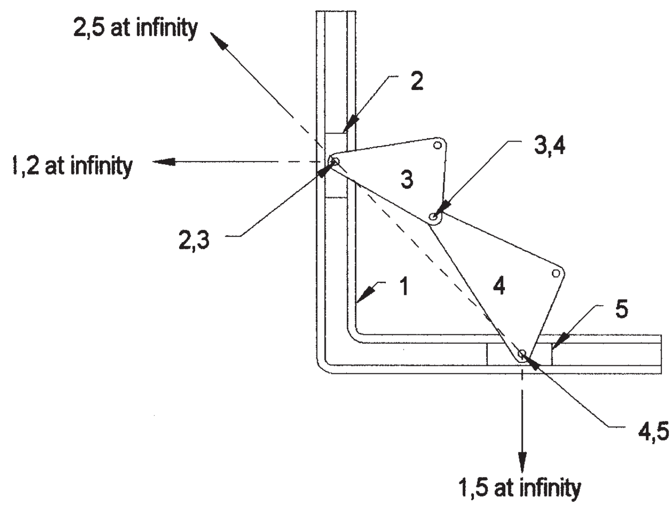
Given: Number of links $n := 5$

Solution: See Figure P6-33c and Mathcad file P06976.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 10$$

1. Draw the linkage to scale and identify those ICs that can be found by inspection (4).



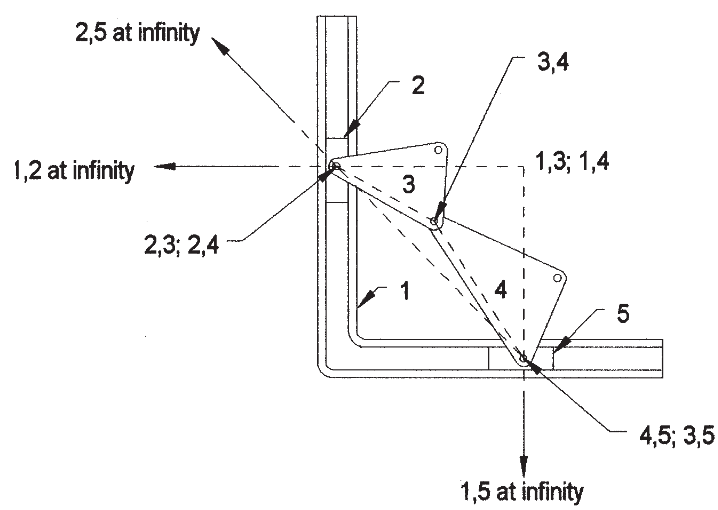
2. Use Kennedy's Rule and a linear graph to find the remaining ICs required to find $I_{1,3}$ and $I_{1,4}$.

$I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

$I_{3,5}$: $I_{2,3}$-$I_{2,5}$ and $I_{4,5}$-$I_{3,4}$

$I_{1,4}$: $I_{1,2}$-$I_{2,4}$ and $I_{1,5}$-$I_{4,5}$

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$



PROBLEM 6-98

Statement: The crosshead linkage shown in Figure P6-33c has 2 *DOF* with inputs at cross heads 2 and 5. Find $\mathbf{V}_B$, $\mathbf{V}_{P3}$, and $\mathbf{V}_{P4}$ if the crossheads are each moving toward the origin of the *XY* coordinate system with a speed of 20 in/sec. Use a graphical method.

Given: Link lengths:

Link 3 (*A* to *B*)	$b := 34.32 \cdot in$	Link 4 (*B* to *C*)	$c := 50.4 \cdot in$
Link 2 *Y*-offset	$a := 59.5 \cdot in$	Link 5 *X*-offset	$d := 57 \cdot in$

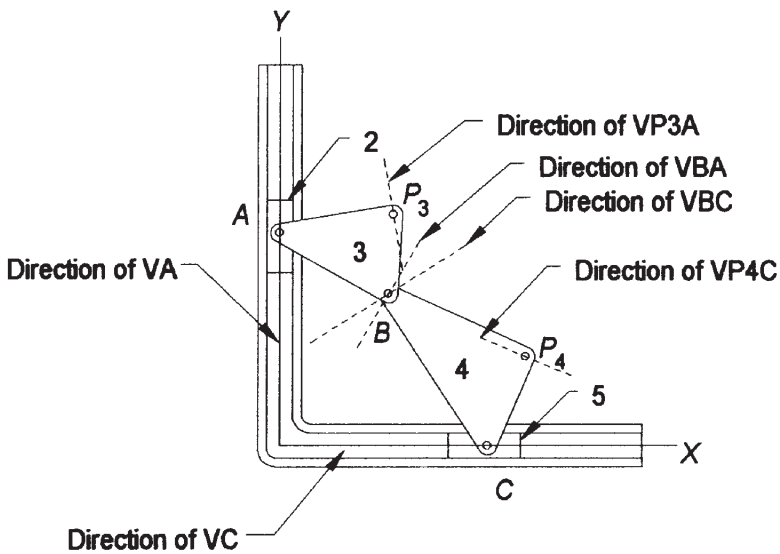
Coupler point data: $AP_3 := 31.5 \cdot in$ $\quad BP_3 := 22.2 \cdot in$

$BP_4 := 41.52 \cdot in$ $\quad CP_4 := 27.0 \cdot in$

Input velocities: $V_{2Y} := -20 \cdot in \cdot sec^{-1}$ $\quad V_{5X} := -20 \cdot in \cdot sec^{-1}$

Solution: See Figure P6-33c and Mathcad file P0698.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocities $\mathbf{V}_{BA}$, $\mathbf{V}_{BC}$ and the angular velocities of links 3 and 4. The equations to be solved (simultaneously) graphically are

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \qquad \mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

a. Choose a convenient velocity scale and layout the known vectors $\mathbf{V}_A$ and $\mathbf{V}_C$

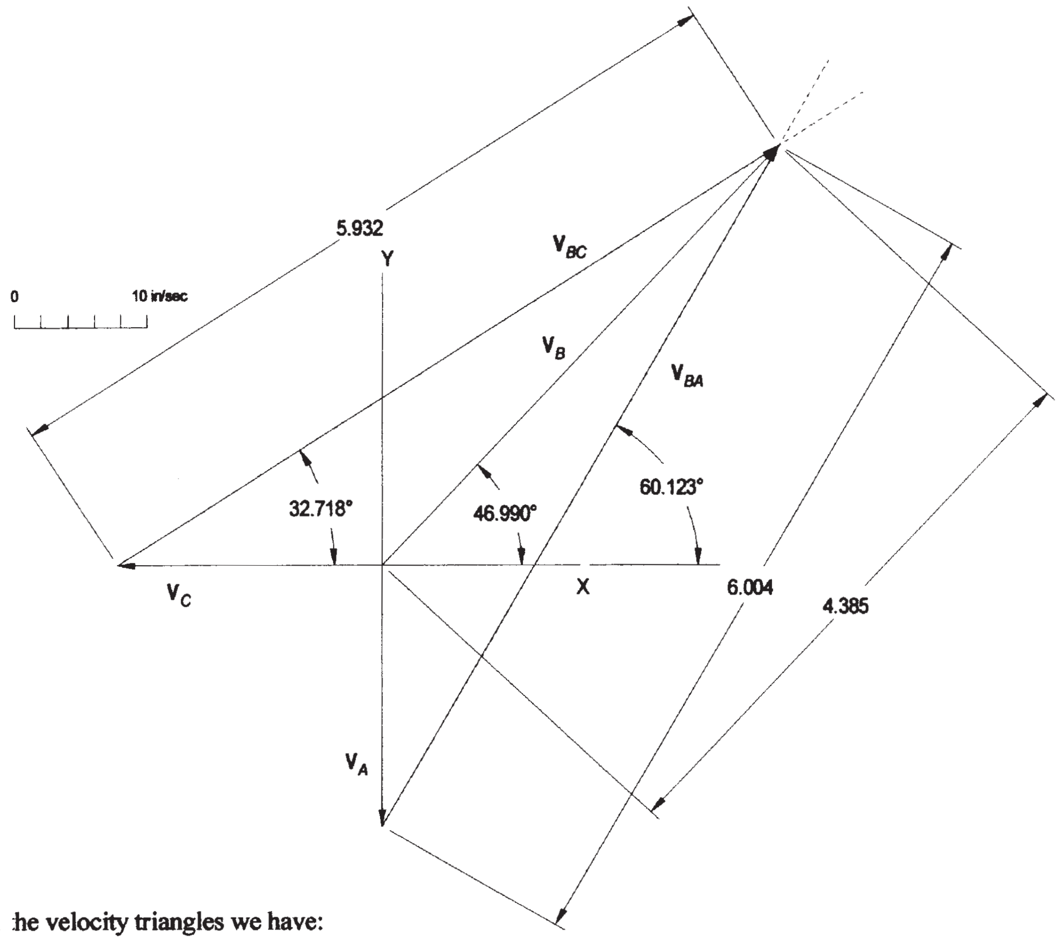
b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.

c. From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.

d. From the origin, draw a construction line to the intersection of the two construction lines in steps b and c.

d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line. And then do the same with the $\mathbf{V}_{CB}$ vector

(See next page)



3. From the velocity triangles we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$

$V_B := 4.385 \cdot in \cdot k_v$	$V_B = 43.850 \dfrac{in}{sec}$	$\theta_{VB} := 46.990 \cdot deg$
$V_{BA} := 6.004 \cdot in \cdot k_v$	$V_{BA} = 60.040 \dfrac{in}{sec}$	$\theta_{VBA} := 60.123 \cdot deg$
$V_{BC} := 5.932 \cdot in \cdot k_v$	$V_{BC} = 59.320 \dfrac{in}{sec}$	$\theta_{VBC} := 32.718 \cdot deg$

4. Determine the angular velocity of links 3 and 4 using equation 6.7.

$\omega_3 := \dfrac{V_{BA}}{b}$ $\qquad \omega_3 = 1.749 \dfrac{rad}{sec}$ CCW

$\omega_4 := \dfrac{-V_{BC}}{c}$ $\qquad \omega_4 = -1.177 \dfrac{rad}{sec}$ CW

6. Determine the magnitudes of the vectors $\mathbf{V}_{P3A}$ and $\mathbf{V}_{P4C}$ using equation 6.7.

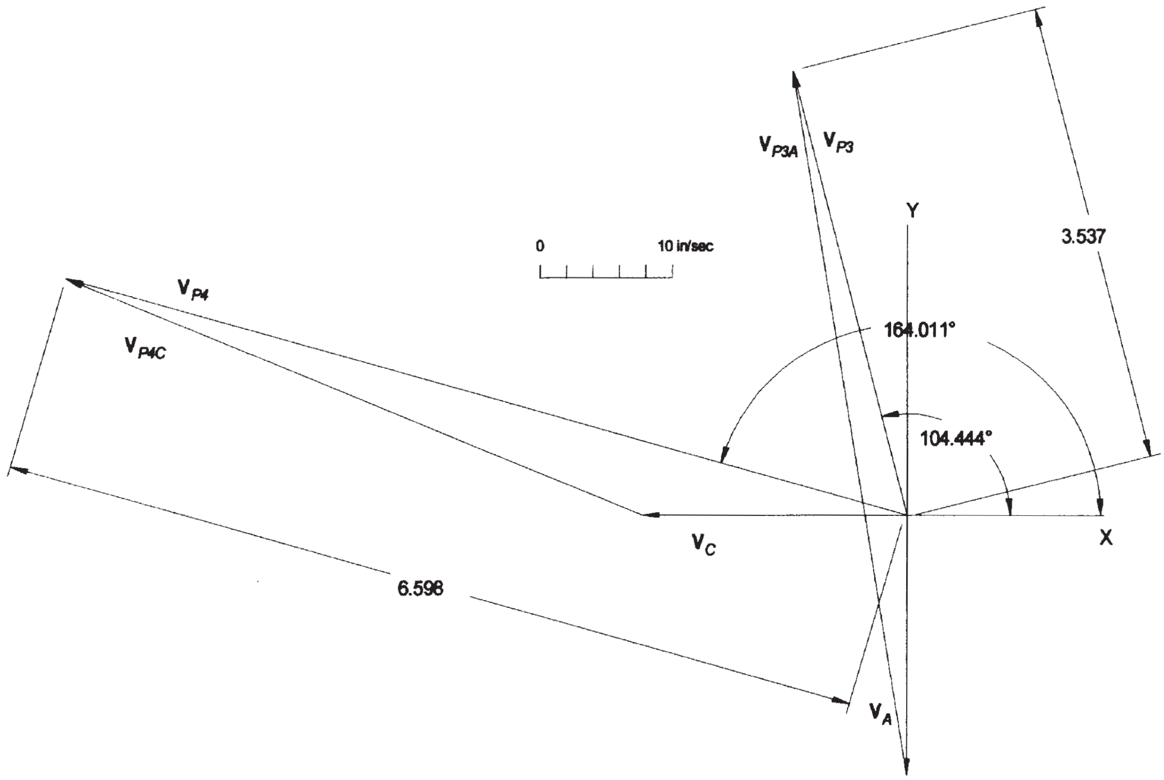
$V_{P3A} := \left| AP_3 \cdot \omega_3 \right|$ $\qquad V_{P3A} = 55.107 \dfrac{in}{sec}$

$V_{P4C} := \left| CP_4 \cdot \omega_3 \right|$ $\qquad V_{P4C} = 47.234 \dfrac{in}{sec}$

7. Use equation 6.5 to (graphically) determine the magnitude of the velocities at points P_3 and P_4. The equations to be solved graphically are

$$\mathbf{V}_{P3} = \mathbf{V}_A + \mathbf{V}_{P3A} \qquad \mathbf{V}_{P4} = \mathbf{V}_C + \mathbf{V}_{P4C}$$

   a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
   b. From the tip of $\mathbf{V}_A$, layout the (now) known vector $\mathbf{V}_{P3A}$.
   c. Complete the vector triangle by drawing $\mathbf{V}_{P3}$ from the tail of $\mathbf{V}_A$ to the tip of the $\mathbf{V}_{P3A}$ vector.
   d. Repeat for $\mathbf{V}_{P4}$.



8. From the velocity triangles we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$

$V_{P3} := 3.537 \cdot in \cdot k_v$ $\quad V_{P3} = 35.370 \dfrac{in}{sec}$ $\quad \theta_{P3} := 104.444 \cdot deg$

$V_{P4} := 6.598 \cdot in \cdot k_v$ $\quad V_{P4} = 65.980 \dfrac{in}{sec}$ $\quad \theta_{P4} := 164.011 \cdot deg$

PROBLEM 6-99

Statement: The linkage in Figure P6-33d has the path of slider 6 perpendicular to the global X-axis and link 2 aligned with the global X-axis. Find $\mathbf{V}_A$ in the position shown if the velocity of the slider is 20 in/sec downward. Use the velocity difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 12 \cdot in$	Link 3 (A to B)	$b := 24 \cdot in$
Link 4 (O_4 to B)	$c := 18 \cdot in$	Link 5 (C to D)	$f := 24 \cdot in$
Link 4 (O_4 to C)	$e := 18 \cdot in$		
Link 1 X-offset	$d_X := 19 \cdot in$	Link 1 Y-offset	$d_Y := 28 \cdot in$

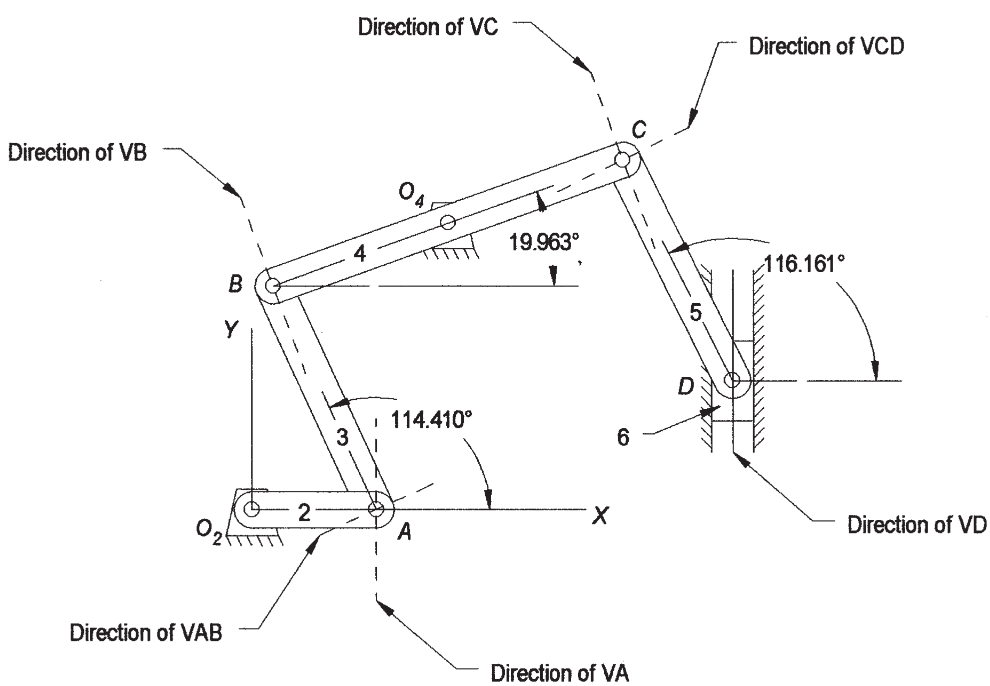
Angle BO_4C $\quad \delta_4 := 0.0 \cdot deg$

Output crank angle: $\quad \theta_2 := 0 \cdot deg \quad$ Global XY system

Input slider velocity $\quad V_D := 20 \cdot in \cdot sec^{-1} \quad \theta_{VD} := -90.0 \cdot deg$

Solution: See Figure P6-33d and Mathcad file P0699.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



2. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

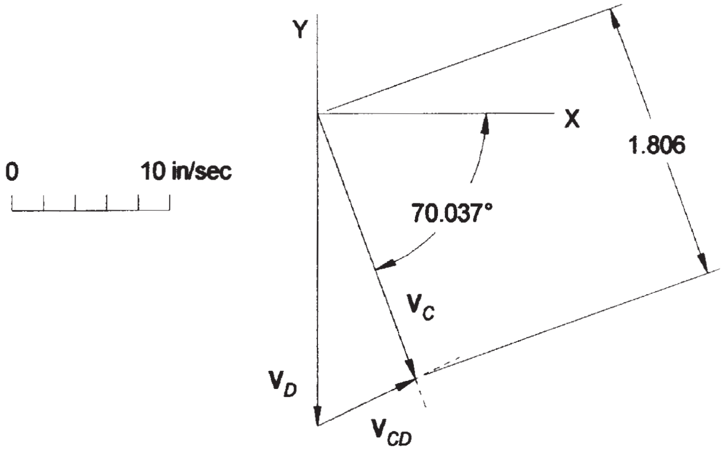
$$\mathbf{V}_C = \mathbf{V}_D + \mathbf{V}_{CD}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.

b. From the tip of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_{CD}$, magnitude unknown.

c. From the tail of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{CD}$ from the tip of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_{CD}$ construction line.



3. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$

$V_C := 1.806 \cdot in \cdot k_v$ $\quad V_C = 18.060 \dfrac{in}{sec}$ $\quad \theta_{VC} := -70.037 \cdot deg$

4. Determine the magnitude and sense of the vector $\mathbf{V}_B$.

$V_B := V_C$ $\quad V_B = 18.060 \dfrac{in}{sec}$

$\theta_{VB} := \theta_{VC} + 180 \cdot deg$ $\quad \theta_{VB} = 109.963\, deg$

5. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *A*. The equation to be solved graphically is

$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{AB}$$

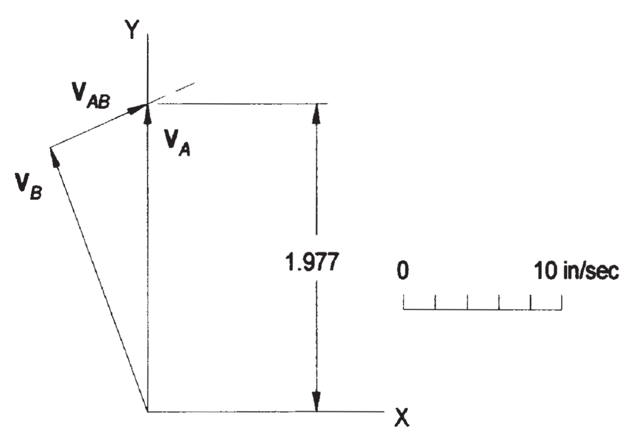
a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.
b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{AB}$, magnitude unknown.
c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_A$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{AB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_A$ construction line and drawing $\mathbf{V}_A$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{AB}$ construction line.

6. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$

$V_A := 1.977 \cdot in \cdot k_v$ $\quad V_A = 19.770 \dfrac{in}{sec}$

$\theta_{VA} := 90.0 \cdot deg$



PROBLEM 6-100

Statement: The linkage in Figure P6-33a has the path of slider 6 perpendicular to the global *X*-axis and link 2 aligned with the global *X*-axis. Find $\mathbf{V}_A$ in the position shown if the velocity of the slider is 20 in/sec downward. Use the instant center graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 12 \cdot in$	Link 3 (A to B)	$b := 24 \cdot in$
Link 4 (O_4 to B)	$c := 18 \cdot in$	Link 5 (C to D)	$f := 24 \cdot in$
Link 3 (A to C)	$p := 2.25 \cdot in$	Link 4 (O_4 to C)	$e := 18 \cdot in$
Link 1 *X*-offset	$d_X := 19 \cdot in$	Link 1 *Y*-offset	$d_Y := 28 \cdot in$

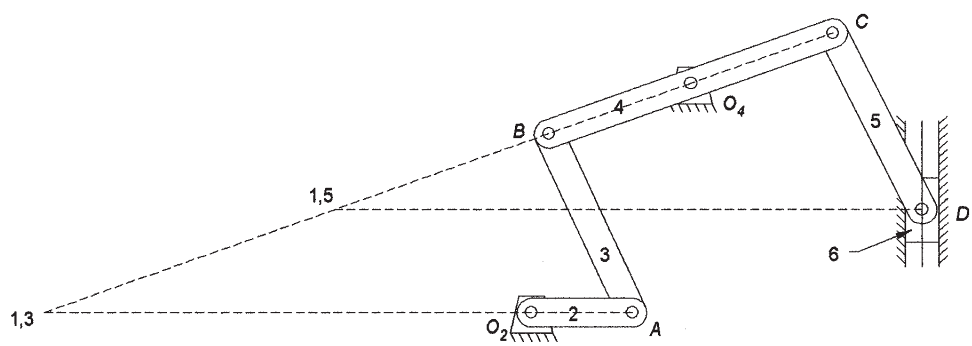
Angle BO_4C $\quad \delta_4 := 0.0 \cdot deg$

Output crank angle: $\quad \theta_2 := 0 \cdot deg$ $\quad$ Global XY system

Input slider velocity $\quad V_D := 20 \cdot in \cdot sec^{-1}$ $\quad \theta_{VD} := -90.0 \cdot deg$

Solution: See Figure P6-33d and Mathcad file P06100.

1. Draw the linkage to scale in the position given, find instant centers $I_{1,3}$ and $I_{1,5}$, and the distances from the pin joints to the instant centers.



From the layout above:

$AI13 := 70.085 \cdot in \quad BI13 := 64.013 \cdot in \quad CI15 := 63.095 \cdot in \quad DI15 := 69.886 \cdot in$

2. Use equation 6.9c to determine the angular velocity of link 5.

$$\omega_5 := \frac{V_D}{DI15} \qquad \omega_5 = 0.286 \frac{rad}{sec} \quad \text{CW}$$

3. Determine the magnitude of the velocity at point C using equation 6.9b.

$$V_C := CI15 \cdot \omega_5 \qquad V_C = 18.057 \frac{in}{sec}$$

4. Use equation 6.9c to determine the angular velocity of link 4.

$$\omega_4 := \frac{V_C}{e} \qquad \omega_4 = 1.003 \frac{rad}{sec} \quad \text{CW}$$

5. Determine the magnitude of the velocity at point B using equation 6.9b.

$$V_B := c \cdot \omega_4 \qquad V_B = 18.057 \frac{in}{sec}$$

6. Determine the angular velocity of link 3 using equation 6.9a.

$$\omega_3 := \frac{V_B}{BI13} \qquad \omega_3 = 0.282 \frac{rad}{sec} \qquad \text{CCW}$$

7. Determine the magnitude of the velocity at point *A* using equation 6.9b.

$$V_A := AI13 \cdot \omega_3 \qquad V_A = 19.769 \frac{in}{sec} \qquad \text{Upward}$$

PROBLEM 6-101

Statement: For the linkage of Figure P6-33e, write a computer program or use an equation solver to find and plot $\mathbf{V}_D$ in the global coordinate system for one revolution of link 2 if $\omega_2 = 10$ rad/sec CW.

Given: Link lengths:

Input crank (L_2)	$a := 5.00$	Fourbar coupler (L_3)	$b := 5.00$
Output crank (O_4B)	$c := 6.00$	Slider coupler (L_5)	$e := 15.00$
Fourbar ground link (L_1)	$d := 2.500$	Angle BO_4C	$\delta := 83.621 \cdot deg$

Two argument inverse tangent:

$$atan2(x,y) := \begin{cases} \text{return } 0.5\cdot\pi & \text{if } (x = 0 \wedge y > 0) \\ \text{return } 1.5\cdot\pi & \text{if } (x = 0 \wedge y < 0) \\ \text{return } atan\left(\left(\frac{y}{x}\right)\right) & \text{if } x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & \text{otherwise} \end{cases}$$

Crank velocity: $\omega_2 := -10 \cdot \frac{rad}{sec}$

Solution: See Figure P6-33e and Mathcad file P06101.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a slider-crank mechanism using the output of the fourbar, link 4, as the input to the slider-crank.
2. Define one revolution of the input crank: $\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 deg$
3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the open circuit) in the global XY coordinate system.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 0.5000 \qquad K_2 = 0.4167 \qquad K_3 = 0.7042$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. If the calculated value of θ_4 is greater than 2π, subtract 2π from it and if it is negative, make it positive.

$$\theta_{42}(\theta_2) := if(\theta_{41}(\theta_2) > 2 \cdot \pi, \theta_{41}(\theta_2) - 2 \cdot \pi, \theta_{41}(\theta_2))$$

$$\theta_4(\theta_2) := if(\theta_{42}(\theta_2) < 0, \theta_{42}(\theta_2) + 2 \cdot \pi, \theta_{42}(\theta_2))$$

5. Determine the slider-crank motion using equations 4.16 and 4.17 with θ_4 as the input angle.

$$\theta_5(\theta_2) := asin\left(\frac{-c \cdot \sin(\theta_4(\theta_2) - \delta)}{e} \right) + \pi$$

$$f(\theta_2) := c \cdot \cos(\theta_4(\theta_2) - \delta) - e \cdot \cos(\theta_5(\theta_2))$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.5000$$

$$K_5 = -0.4050$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left[atan2\left[2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{(E(\theta_2))^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right] \right]$$

7. Determine the angular velocity of link 4 for the open circuit using equations 6.18.

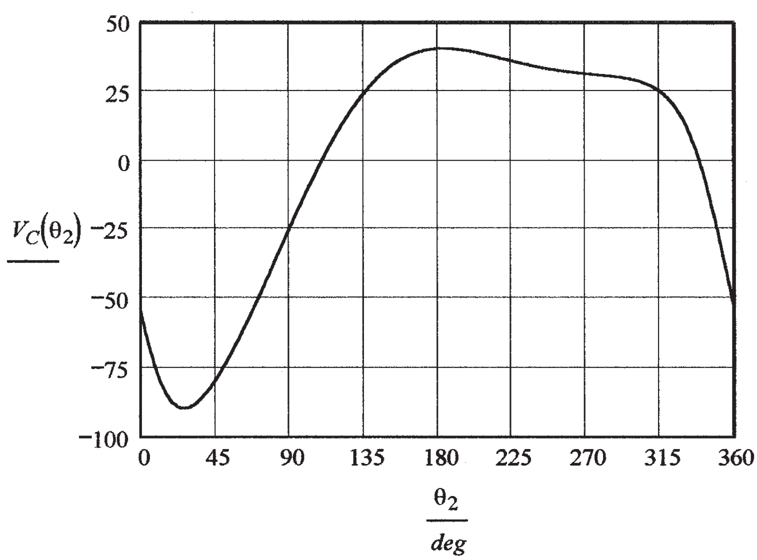
$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5(\theta_2) := \frac{c}{e} \cdot \frac{cos(\theta_4(\theta_2) - \delta)}{cos(\theta_5(\theta_2))} \cdot \omega_4(\theta_2)$$

9. Determine the velocity of pin *D* using equation 6.22b:

$$V_C(\theta_2) := -c \cdot \omega_4(\theta_2) \cdot sin(\theta_4(\theta_2) - \delta) + e \cdot \omega_5(\theta_2) \cdot sin(\theta_5(\theta_2))$$



PROBLEM 6-102

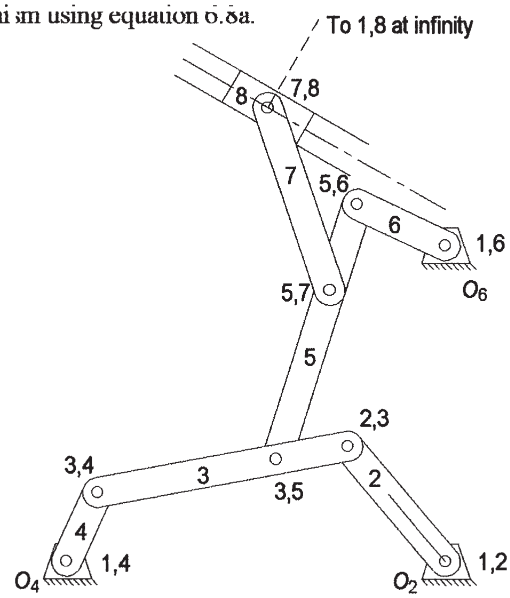
Statement: For the linkage of Figure P6-33f, locate and identify all instant centers.

Given: Number of links $n := 8$

Solution: See Figure P6-33f and Mathcad file P06102.

1. Determine the number of instant centers for this mechanism using equation 6.8a.

$$C := \frac{n \cdot (n-1)}{2} \qquad C = 28$$



1. Draw the linkage to scale and identify those ICs that can be found by inspection (10).

2. Use Kennedy's Rule and a linear graph to find the remaining 18 ICs.

$I_{1,3}$: $I_{1,2}$-$I_{2,3}$ and $I_{1,4}$-$I_{3,4}$

$I_{1,5}$: $I_{1,6}$-$I_{5,6}$ and $I_{1,3}$-$I_{3,5}$

$I_{2,4}$: $I_{1,2}$-$I_{1,4}$ and $I_{2,3}$-$I_{3,4}$

$I_{4,5}$: $I_{1,4}$-$I_{1,5}$ and $I_{3,5}$-$I_{3,4}$

$I_{2,5}$: $I_{1,2}$-$I_{1,5}$ and $I_{2,4}$-$I_{4,5}$

$I_{2,6}$: $I_{1,2}$-$I_{1,6}$ and $I_{2,5}$-$I_{5,6}$

$I_{3,6}$: $I_{2,3}$-$I_{2,6}$ and $I_{1,2}$-$I_{1,6}$

$I_{5,8}$: $I_{5,7}$-$I_{7,8}$ and $I_{1,5}$-$I_{1,8}$

$I_{4,8}$: $I_{4,5}$-$I_{5,8}$ and $I_{1,8}$-$I_{1,4}$

$I_{2,8}$: $I_{2,4}$-$I_{4,8}$ and $I_{1,2}$-$I_{1,8}$

$I_{3,8}$: $I_{3,4}$-$I_{4,8}$ and $I_{1,3}$-$I_{1,8}$

$I_{6,8}$: $I_{5,8}$-$I_{5,6}$ and $I_{2,8}$-$I_{2,6}$

$I_{2,7}$: $I_{2,8}$-$I_{7,8}$ and $I_{2,5}$-$I_{5,7}$

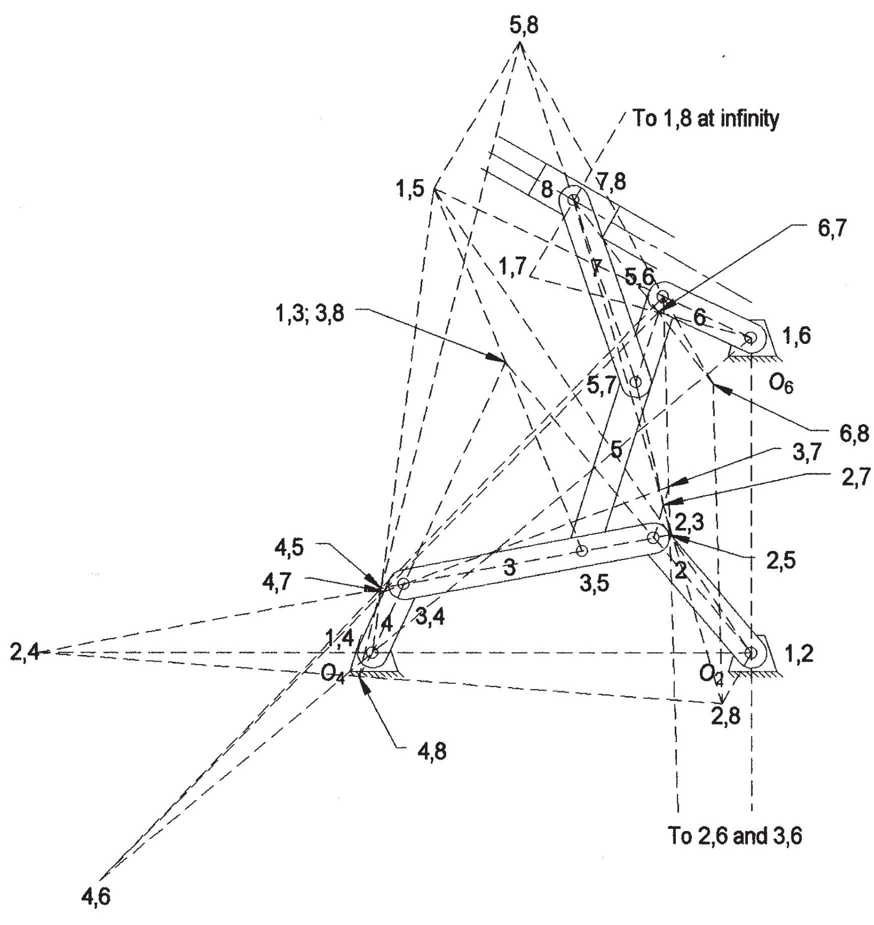
$I_{6,7}$: $I_{5,6}$-$I_{5,7}$ and $I_{6,8}$-$I_{7,8}$

$I_{3,7}$: $I_{3,6}$-$I_{6,7}$ and $I_{2,3}$-$I_{2,7}$

$I_{4,6}$: $I_{1,4}$-$I_{1,6}$ and $I_{4,5}$-$I_{5,6}$

$I_{4,7}$: $I_{4,6}$-$I_{6,7}$ and $I_{3,4}$-$I_{3,7}$

$I_{1,7}$: $I_{1,8}$-$I_{7,8}$ and $I_{1,6}$-$I_{6,7}$



PROBLEM 6-103

Statement: The linkage of Figure P6-33f has link 2 at 130 deg in the global XY coordinate system. Find VD in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW. Use any graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 5.0 \cdot in$	Link 3 (A to B)	$b := 8.4 \cdot in$
Link 4 (O_4 to B)	$c := 2.5 \cdot in$	Link 1 (O_2 to O_4)	$d_1 := 12.5 \cdot in$
Link 5 (C to E)	$e := 8.9 \cdot in$	Link 5 (C to D)	$h := 5.9 \cdot in$
Link 6 (O_6 to E)	$f := 3.2 \cdot in$	Link 7 (D to F)	$k := 6.4 \cdot in$
Link 1 (O_2 to O_6)	$d_2 := 10.5 \cdot in$	Link 3 (A to C)	$g := 2.4 \cdot in$

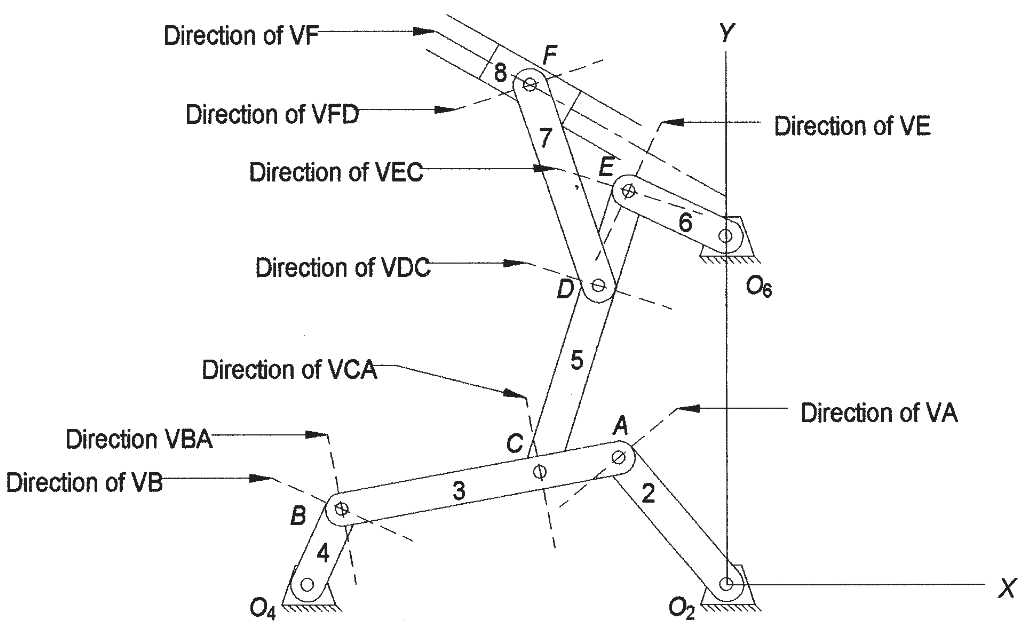
Crank angle: $\theta_2 := 130 \cdot deg$

Slider axis offset and angle: $s := 11.7 \cdot in$ $\delta_7 := 150 \cdot deg$

Input crank angular velocity $\omega_2 := 15 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure P6-33f and Mathcad file P06103.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



Angles measured from layout:

$\theta_{VB} := -24.351 \cdot deg$ $\theta_{VBA} := -79.348 \cdot deg$ $\theta_{VCA} := \theta_{VBA}$

$\theta_{VE} := 64.594 \cdot deg$ $\theta_{VEC} := 162.461 \cdot deg$ $\theta_{VDC} := \theta_{VEC}$

$\theta_{VF} := 150.00 \cdot deg$ $\theta_{VFD} := 198.690 \cdot deg$

2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

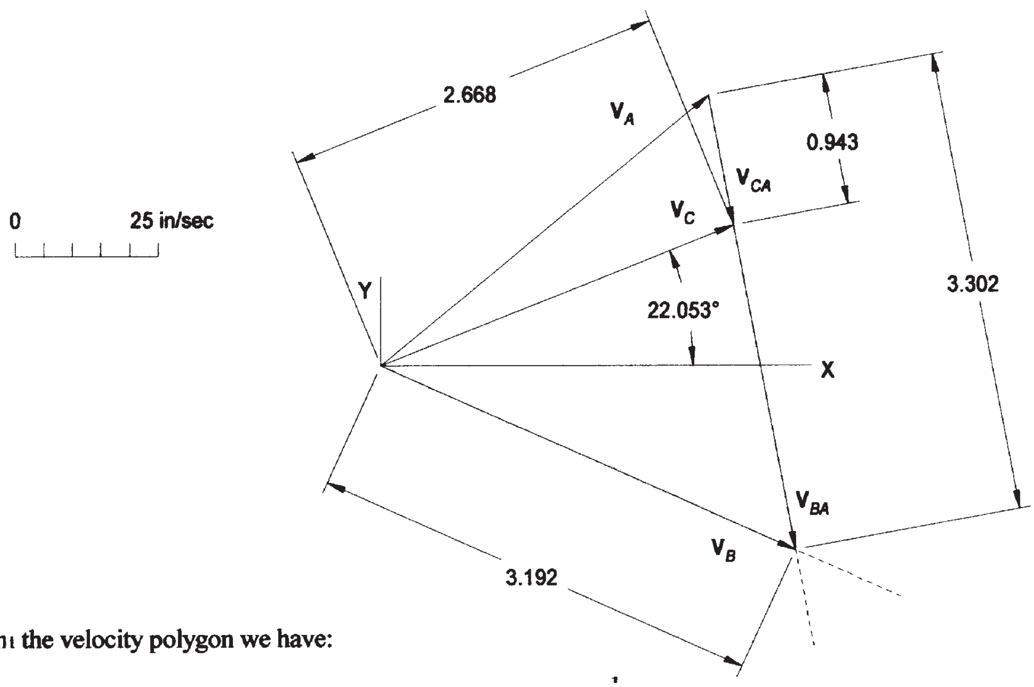
$V_A := a \cdot \omega_2$ $V_A = 75.000 \frac{in}{sec}$

$\theta_{VA} := \theta_2 - 90 \cdot deg$ $\theta_{VA} = 40.000\, deg$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equations to be solved graphically are

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \quad \text{and} \quad \mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity polygon we have:

Velocity scale factor: $k_v := \dfrac{25 \cdot in \cdot sec^{-1}}{in}$

$V_B := 3.192 \cdot in \cdot k_v$ $\qquad V_B = 79.800 \dfrac{in}{sec}$ $\qquad \theta_{VB} = -24.351\, deg$

$V_{BA} := 3.302 \cdot in \cdot k_v$ $\qquad V_{BA} = 82.550 \dfrac{in}{sec}$ $\qquad \theta_{VBA} = -79.348\, deg$

$\omega 3 := \dfrac{V_{BA}}{b}$ $\qquad \omega 3 = 9.827 \dfrac{rad}{sec}$ CCW

5. Calculate the magnitude and direction of $\mathbf{V}_{CA}$ and determine the magnitude and velocity of $\mathbf{V}_C$ from the velocity polygon above.

$V_{CA} := g \cdot \omega 3$ $\qquad V_{CA} = 23.586 \dfrac{in}{sec}$ $\qquad \theta_{VCA} = -79.348\, deg$

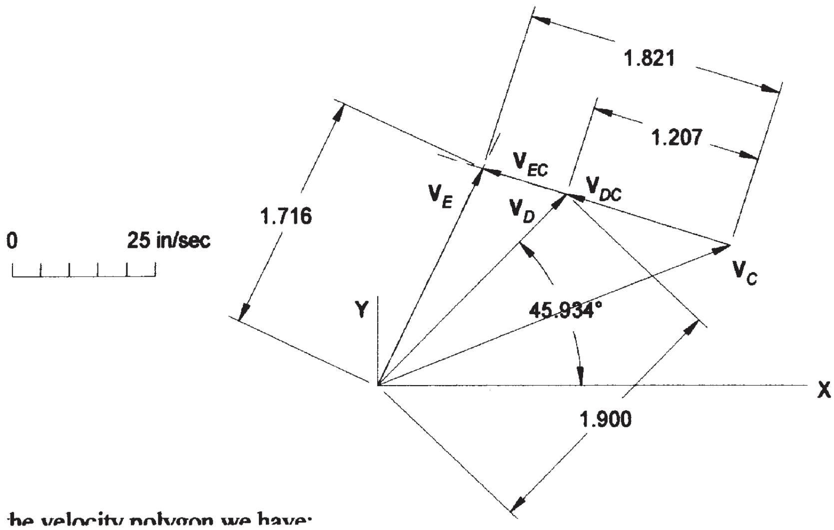
Length of $\mathbf{V}_{CA}$ on velocity polygon: $\quad v_{CA} := \dfrac{V_{CA}}{k_v}$ $\qquad v_{CA} = 0.943\, in$

$V_C := 2.668 \cdot in \cdot k_v$ $\qquad V_C = 66.700 \dfrac{in}{sec}$ $\qquad \theta_{VC} := 22.053 deg$

6. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *E*, the magnitude of the relative velocity $\mathbf{V}_{ED}$, and the angular velocity of link 5. The equations to be solved graphically are

$$\mathbf{V}_E = \mathbf{V}_C + \mathbf{V}_{EC} \quad \text{and} \quad \mathbf{V}_D = \mathbf{V}_C + \mathbf{V}_{DC}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_C$.
b. From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{EC}$, magnitude unknown.
c. From the tail of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_E$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{EC}$ from the tip of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_E$ construction line and drawing $\mathbf{V}_E$ from the tail of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_{EC}$ construction line.



7. From the velocity polygon we have:

Velocity scale factor: $k_v := \dfrac{25 \cdot in \cdot sec^{-1}}{in}$

$V_E := 1.716 \cdot in \cdot k_v$ $\quad V_E = 42.900 \dfrac{in}{sec}$ $\quad \theta_{VE} = 64.594\,deg$

$V_{EC} := 1.821 \cdot in \cdot k_v$ $\quad V_{EC} = 45.525 \dfrac{in}{sec}$ $\quad \theta_{VEC} = 162.461\,deg$

$\omega 5 := \dfrac{V_{EC}}{e}$ $\quad \omega 5 = 5.115 \dfrac{rad}{sec}$ CCW

8. Calculate the magnitude and direction of $\mathbf{V}_{DE}$ and determine the magnitude and velocity of $\mathbf{V}_D$ from the velocity polygon above.

$V_{DC} := h \cdot \omega 5$ $\quad V_{DC} = 30.179 \dfrac{in}{sec}$ $\quad \theta_{VDC} = 162.461\,deg$

Length of $\mathbf{V}_{DC}$ on velocity polygon: $\quad v_{DC} := \dfrac{V_{DC}}{k_v}$ $\quad v_{DC} = 1.207\,in$

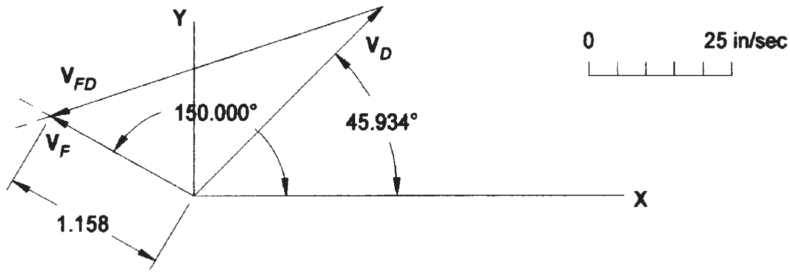
$V_D := 1.900 \cdot in \cdot k_v$ $\quad V_D = 47.500 \dfrac{in}{sec}$ $\quad \theta_{VD} := 45.934 deg$

9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *F*, the magnitude of the relative velocity $\mathbf{V}_{FD}$, and the angular velocity of link 5. The equation to be solved graphically is

$$\mathbf{V}_F = \mathbf{V}_D + \mathbf{V}_{FD}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.
b. From the tip of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_{FD}$, magnitude unknown.
c. From the tail of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_F$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{FD}$ from the tip of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_F$ construction line and drawing $\mathbf{V}_F$ from the tail of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_{FD}$ construction line.



10. From the velocity polygon we have:

Velocity scale factor: $k_v := \dfrac{25 \cdot in \cdot sec^{-1}}{in}$

$V_F := 1.158 \cdot in \cdot k_v$ $\qquad V_F = 28.950 \dfrac{in}{sec}$ $\qquad \theta_{VF} = 150.000\, deg$

PROBLEMS 6-4 AND 6-5

Row	Open ω_3	Open ω_4	V_P Mag	V_P Ang	Crossed ω_3	Crossed ω_4	V_P Mag	V_P Ang
a	-6.0	-4.0	40.8	58.2	-0.7	-2.7	22.0	129.4
b	73.1	-37.3	690.3	63.0	-88.3	22.1	734.2	234.4
c	-12.7	-19.8	273.8	-53.3	-22.7	-15.7	119.1	199.9
d	-18.1	-1.4	88.8	66.3	-14.6	-31.3	177.3	150.8
e	1.9	-40.8	260.5	-12.1	-23.3	19.3	139.9	42.0
f	-76.4	-93.5	1003.6	-34.9	-118.3	-101.2	971.6	167.3
g	76.4	146.8	798.4	92.9	239.0	168.6	1435.3	153.9
h	58.6	-43.3	343.9	-18.9	-43.3	58.6	664.5	-62.8
i	-25.3	25.6	103.1	-13.4	56.9	6.0	476.5	70.4
j	-53.4	5.8	198.1	126.3	32.9	-26.4	244.2	115.5
k	-56.2	-94.8	436.0	-77.4	-55.6	-16.9	362.7	79.3
l	-13.2	-99.7	603.9	-24.4	-33.8	52.7	783.1	-24.1
m	18.3	83.0	680.8	149.2	7.7	-57.0	571.3	133.5
n	2.6	15.5	128.9	194.3	9.6	-3.0	148.6	141.1

PROBLEMS 6-6 AND 6-7

Row	V_A Mag	V_A Ang	Open ω_3	Open V_B	Crossed ω_3	Crossed V_B
a	14	135	-2.47	-9.87	2.47	-9.92
b	24	-30	3.25	36.18	-3.25	5.39
c	45	-120	5.42	-41.5	-5.42	-3.54
d	84	-150	4.29	-64.03	-4.29	-81.46
e	250	135	-8.86	-189.7	8.86	-163.8
f	135	10	-1.85	127.48	1.85	138.42
g	700	60	-28.8	738.9	28.8	-38.9

PROBLEMS 6-8 AND 6-9

Row	Open ω_3	Open ω_4	Open V_{slip}	Crossed ω_3	Crossed ω_4	Crossed V_{slip}
a	-10.3	-10.3	33.46	3.64	3.64	-33.46
b	-7.34	-7.34	-89.51	-11.7	-11.7	89.51
c	23.75	23.75	73	33	33	-73
d	76.75	76.75	-270.76	8.54	8.54	270.76
e	-2.7	-2.7	-176	-8.24	-8.24	176
f	51.91	51.91	205.98	72.73	72.73	-205.98

PROBLEMS 6-10 AND 6-11

Row	Open ω_3	Open ω_4	Crossed ω_3	Crossed ω_4
a	32.6	16.9	-75.2	-59.6
b	39.62	17.25	12.58	26.01
c	10.7	-2.6	-8.2	5.1
d	-26.95	3.42	16.57	-13.8
e	-158.3	-81.3	-116.8	-193.9
f	25.84	-29.49	-13.10	42.23
g	-8.9	-40.9	-48.5	-16.5
h	61.68	12.54	3.7	52.84
i	-40.1	47.9	59.6	-28.4