Chapter 7

ACCELERATION ANALYSIS

TOPIC/PROBLEM MATRIX

SECT	TOPIC	PROBLEMS
7.1	Definition of Acceleration	7-1, 7-2, 7-10, 7-56
7.2	Graphical Acceleration Analysis	
	Pin-Jointed Fourbar	7-3, 7-14a, 7-21, 7-24, 7-30, 7-33, 7-70a, 7-72a, 7-77
	Fourbar Slider-Crank	7-5, 7-13a, 7-27, 7-36
	Other Fourbar	7-15a
	Fivebar	7-79
	Sixbar	7-52, 7-53, 7-61a, 7-63a, 7-65a, 7-75, 7-82
	Eightbar	7-86
7.3	Analytical Solutions for Acceleration Analysis	
	Pin-Jointed Fourbar	7-22, 7-23, 7-25, 7-26, 7-34, 7-35, 7-41, 7-46, 7-51,7-74
	Fourbar Slider-Crank	7-6, 7-28, 7-29, 7-37, 7-38, 7-45, 7-50, 7-58
	Coriolis Acceleration	7-12, 7-20
	Fourbar Inverted Slider-Crank	7-7, 7-8, 7-16, 7-59
	Other Fourbar	7-15b, 7-74
	Fivebar	7-80, 7-81
	Sixbar	7-17, 7-18, 7-19, 7-48, 7-54, 7-61b, 7-62, 7-63b, 7-64, 7-65b, 7-66, 7-76, 7-83, 7-84, 7-85
	Eightbar	7-67
7.5	Acceleration of Any Point on A Linkage	
	Pin-Jointed Fourbar	7-4, 7-13b, 7-14b, 7-31, 7-32, 7-39, 7-40, 7-42, 7-43, 7-44, 7-49, 7-55, 7-68, 7-70b, 7-71, 7-72b, 7-73, 7-78
	Other Fourbar	7-15b, 7-47
	Geared Fivebar	7-9, 7-60
	Sixbar	7-69
7.7	Jerk	7-11, 7-57

PROBLEM 7-1

Statement: A point at a 6.5-in radius is on a body that is in pure rotation with ω = 100 rad/sec and a constant α = -500 rad/sec^2 at point *A*. The rotation center is at the origin of a coordinate system. When the point is at position *A*, its position vector makes a 45 deg angle with the *X* axis. It takes 0.01 sec to reach point *B*. Draw this system to some convenient scale, calculate the θ and ω of position *B*, and:

a. Write an expression for the particle's acceleration vector in position *A* using complex number notation, in both polar and Cartesian forms.

b. Write an expression for the particle's acceleration vector in position *B* using complex number notation, in both polar and Cartesian forms.

c. Write a vector equation for the acceleration difference between points *B* and *A*. Substitute the complex number notation for the vectors in this equation and solve for the acceleration difference numerically.

d. Check the result of part c with a graphical method.

Given:

Initial rotation speed	$\omega_A := 100 \cdot \frac{rad}{sec}$
Rotation acceleration	$\alpha := -500 \cdot \frac{rad}{sec^2}$
Time to reach point *B*	$t := 0.01 \cdot sec$
Vector angle	$\theta_A := 45 \cdot deg$
Vector magnitude	$R := 6.5 \cdot in$

Solution: See Mathcad file P0701.

1. Calculate the position and angular velocity at point *B* using equations 6.1 and 7.1.

$$\omega_B := \alpha \cdot t + \omega_A \qquad \omega_B = 95.000 \frac{rad}{sec}$$

$$\theta_B := \frac{\alpha \cdot t^2}{2} + \omega_A \cdot t + \theta_A \qquad \theta_B = 100.863\, deg$$

2. Calculate the magnitudes and directions of the normal and tangential components of acceleration at points *A* and *B* using equations 7.2.

$$a_{An} := R \cdot \omega_A^{\,2} \qquad a_{An} = 65000 \frac{in}{sec^2} \qquad \theta_{An} := \theta_A + 180 \cdot deg \qquad \theta_{An} = 225.000\, deg$$

$$a_{At} := |R \cdot \alpha| \qquad a_{At} = 3250 \frac{in}{sec^2} \qquad \theta_{At} := \theta_A + sign(\alpha) \cdot 90 \cdot deg$$

$$\theta_{At} = -45.000\, deg$$

$$a_{Bn} := R \cdot \omega_B^{\,2} \qquad a_{Bn} = 58662 \frac{in}{sec^2} \qquad \theta_{Bn} := \theta_B + 180 \cdot deg \qquad \theta_{Bn} = 280.863\, deg$$

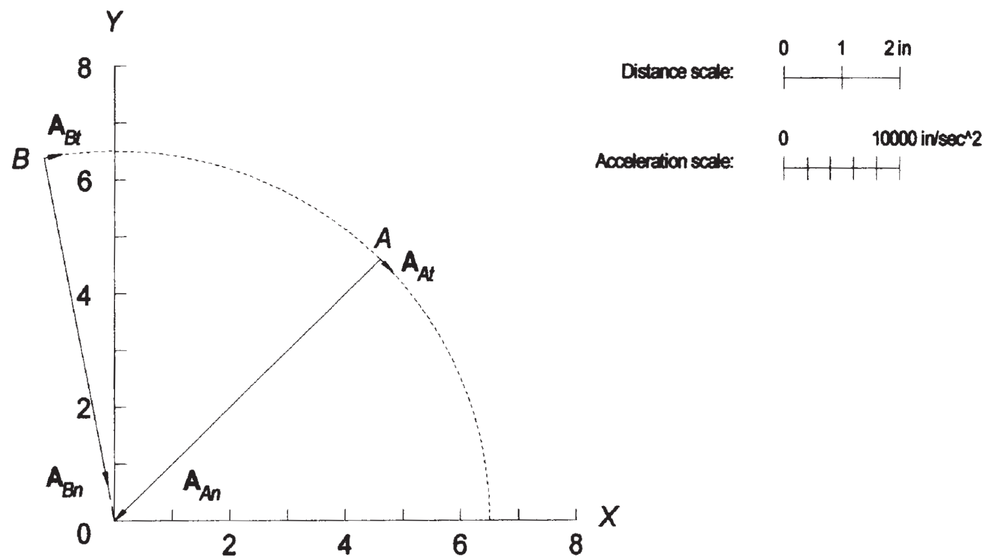
$$a_{Bt} := |R \cdot \alpha| \qquad a_{Bt} = 3250 \frac{in}{sec^2} \qquad \theta_{Bt} := \theta_B + sign(\alpha) \cdot 90 \cdot deg$$

$$\theta_{Bt} = 10.863\, deg$$

3. Establish an *X-Y* coordinate frame and draw a circle with center at the origin and radius *R*.

4. Draw lines from the origin that make angles of 45 and 100.863 deg with respect to the X axis. Label the intersections of the lines with the circles as A and B, respectively.

5. Choose a convenient acceleration scale and draw the two acceleration component vectors at points A and B.



a. Write an expression for the particle's acceleration vector in position A using complex number notation, in both polar and Cartesian forms.

Polar form:

$$\mathbf{R_A} := R\cdot e^{j\cdot\theta_A} \qquad \mathbf{R_A} := 6.5\cdot e^{j\cdot\frac{\pi}{4}}$$

$$\mathbf{V_A} := R\cdot j\cdot\omega_A\cdot e^{j\cdot\theta_A} \qquad \mathbf{V_A} := 650\cdot j\cdot e^{j\cdot\frac{\pi}{4}}$$

$$\mathbf{A_A} := R\cdot j\cdot\alpha\cdot e^{j\cdot\theta_A} - R\cdot j\cdot\omega_A^{\,2}\cdot e^{j\cdot\theta_A}$$

$$\mathbf{A_A} := 6.50\cdot j\cdot\alpha\cdot e^{j\cdot\frac{\pi}{4}} - 6.50\cdot\omega_A^{\,2}\cdot e^{j\cdot\frac{\pi}{4}}$$

Cartesian form:

$$\mathbf{A_A} := R\cdot\alpha\cdot\left(-sin\left(\theta_A\right) + j\cdot cos\left(\theta_A\right)\right) - R\cdot\omega_A^{\,2}\cdot\left(cos\left(\theta_A\right) + j\cdot sin\left(\theta_A\right)\right)$$

$$\mathbf{A_A} = -43664 - 48260j\frac{in}{sec^2}$$

b. Write an expression for the particle's velocity vector in position B using complex number notation, in both polar and Cartesian forms.

Polar form:

$$\mathbf{R_B} := R\cdot e^{j\cdot\theta_B} \qquad \mathbf{R_B} := 6.5\cdot e^{j\cdot\frac{\pi}{180}\cdot 100.863}$$

$$\mathbf{V_B} := R\cdot j\cdot\omega_B\cdot e^{j\cdot\theta_B} \qquad \mathbf{V_A} := 650\cdot j\cdot e^{j\cdot\frac{\pi}{180}\cdot 100.863}$$

$$\mathbf{A_B} := R \cdot j \cdot \alpha \cdot e^{j \cdot \theta_B} - R \cdot j \cdot \omega_B^{\,2} \cdot e^{j \cdot \theta_B}$$

$$\mathbf{A_B} := 6.50 \cdot j \cdot \alpha \cdot e^{j \cdot \frac{\pi}{180} \cdot 100.863} - 6.50 \cdot \omega_B^{\,2} \cdot e^{j \cdot \frac{\pi}{180} \cdot 100.863}$$

Cartesian form: $$\mathbf{A_B} := R \cdot \alpha \cdot \left(-\sin(\theta_B) + j \cdot \cos(\theta_B)\right) - R \cdot \omega_B^{\,2} \cdot \left(\cos(\theta_B) + j \cdot \sin(\theta_B)\right)$$

$$\mathbf{A_B} = 14248 - 56999j \frac{in}{sec^2}$$

c. *Write a vector equation for the velocity difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.*

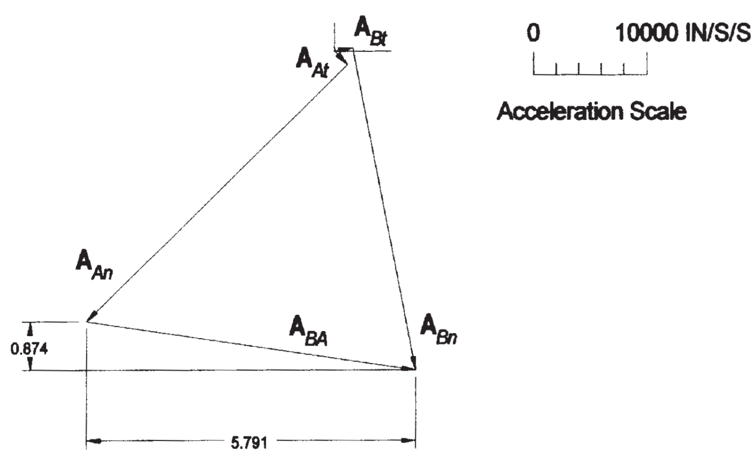
$$\mathbf{A_{BA}} := \mathbf{A_B} - \mathbf{A_A} \qquad \mathbf{A_{BA}} = 57912 - 8739j \frac{in}{sec^2}$$

d. *Check the result of part c with a graphical method. Solve the equation* $\mathbf{A}_{Bt} + \mathbf{A}_{Bn} = \mathbf{A}_{At} + \mathbf{A}_{An} + \mathbf{A}_{BA}$ using an acceleration scale of 10000 in/sec^2 per drawing unit.

Acceleration scale factor $$k_a := 10000 \cdot \frac{in}{sec^2}$$

Horizontal component $$A_{BAx} := 5.791 \cdot k_a \qquad A_{BAx} = 57910 \frac{in}{sec^2}$$

Vertical component $$A_{BAy} := -0.874 \cdot k_a \qquad A_{BAy} = -8740 \frac{in}{sec^2}$$



On the layout above the X and Y components of $\mathbf{A}_{BA}$ are equal (to three significant figures) to the real and imaginary components calculated, confirming that the calculation is correct.

PROBLEM 7-2

Statement: In Problem 7-1 let A and B represent points on separate, rotating bodies both having the given ω and α at $t = 0$, $\theta_A = 45$ deg, and $\theta_B = 120$ deg. Find their relative acceleration.

Given:

Initial rotation speed	$\omega := 100 \cdot \frac{rad}{sec}$	
Rotation acceleration	$\alpha := -500 \cdot \frac{rad}{sec^2}$	
Vector angles	$\theta_A := 45 \cdot deg$	$\theta_B := 120 \cdot deg$
Vector magnitude	$R := 6.5 \cdot in$	

Solution: See Mathcad file P0702.

1. Calculate the acceleration of points A and B using equation 7.3.

$$\mathbf{A_A} := R \cdot \alpha \cdot \left(-\sin\left(\theta_A\right) + j \cdot \cos\left(\theta_A\right)\right) - R \cdot \omega^2 \cdot \left(\cos\left(\theta_A\right) + j \cdot \sin\left(\theta_A\right)\right)$$

$$\mathbf{A_A} = -43664 - 48260j \frac{in}{sec^2}$$

$$\mathbf{A_B} := R \cdot \alpha \cdot \left(-\sin\left(\theta_B\right) + j \cdot \cos\left(\theta_B\right)\right) - R \cdot \omega^2 \cdot \left(\cos\left(\theta_B\right) + j \cdot \sin\left(\theta_B\right)\right)$$

$$\mathbf{A_B} = 35315 - 54667j \frac{in}{sec^2}$$

2. Calculate the relative acceleration of point B with respect to A using equation 7.4.

$$\mathbf{A_{BA}} := \mathbf{A_B} - \mathbf{A_A} \qquad \mathbf{A_{BA}} = 78978 - 6407j \frac{in}{sec^2}$$

Magnitude: $A_{BA} := \left|\mathbf{A_{BA}}\right|$ $\qquad A_{BA} = 79238 \frac{in}{sec^2}$

Direction: $\theta_{ABA} := \arg\left(\mathbf{A_{BA}}\right)$ $\qquad \theta_{ABA} = -4.638 \, deg$

PROBLEM 7-3a

Statement: The link lengths, coupler point location, and the values of θ_2, ω_2, and α_2 for the same fourbar linkages as used for position and velocity analysis in Chapters 4 and 6 are redefined in Table P7-1, which is the same as Table P6-1. The general linkage configuration and terminology are shown in Figure P7-1. For row *a*, draw the linkage to scale and graphically find the accelerations of points *A* and *B*. Then, calculate α_3 and α_4 and the acceleration of point *P*.

Given: Link lengths: Link 1 $d := 6 \cdot in$ Link 2 $a := 2 \cdot in$

Link 3 $b := 7 \cdot in$ Link 4 $c := 9 \cdot in$

Coupler point: $R_{PA} := 6 \cdot in$ $\delta_3 := 30 \cdot deg$

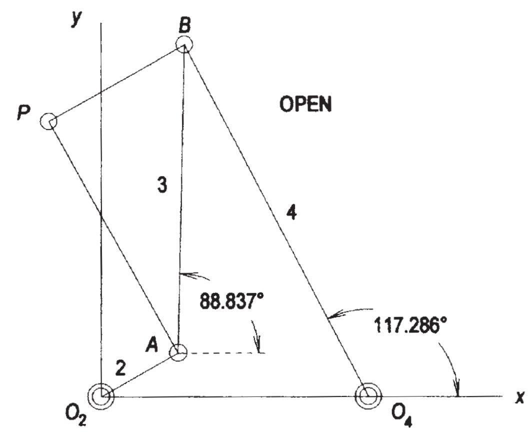
Link 2 position, velocity, and acceleration: $\theta_2 := 30 \cdot deg$ $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Solution: SeeFigure P7-1 and Mathcad file P0703a.

1. In order to solve for the accelerations at points *A* and *B*, we will need θ_3, ω_3, and θ_4. From the graphical position solution below,

$$\theta_3 := 88.837 \cdot deg$$

$$\theta_4 := 117.286 \cdot deg$$



Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{(\sin(\theta_3 - \theta_4))}$$

$$\omega_3 = -5.991\ rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{(\sin(\theta_4 - \theta_3))}$$

$$\omega_4 = -3.992\ rad \cdot sec^{-1}$$

2. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

3. For point *B*, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$A_{Bn} := c \cdot \omega_4{}^2$ $A_{Bn} = 143.403\ in \cdot sec^{-2}$

$\theta_{ABn} := \theta_4 + 180 \cdot deg$ $\theta_{ABn} = 297.286\ deg$

$A_{An} := a \cdot \omega_2{}^2$ $A_{An} = 200.000\ in \cdot sec^{-2}$

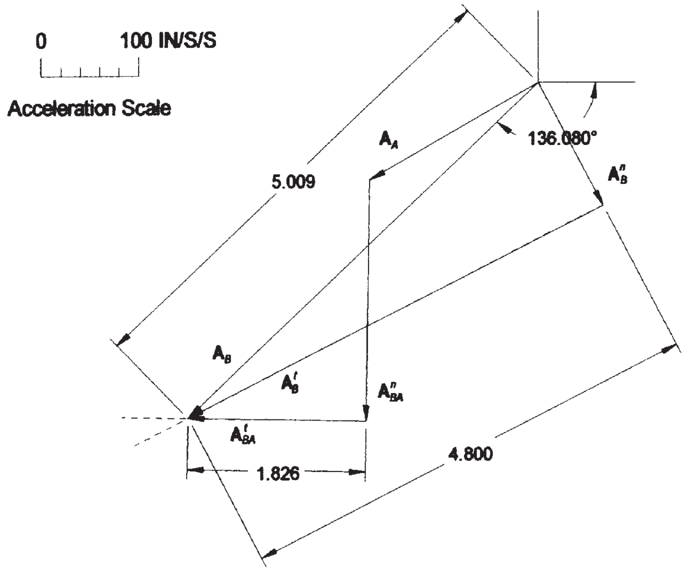
$\theta_{AAn} := \theta_2 + 180 \cdot deg$ $\theta_{AAn} = 210.000\ deg$

$A_{At} := a \cdot \alpha_2$ $A_{At} = 0.000\ in \cdot sec^{-2}$

$A_{BAn} := b \cdot \omega_3{}^2$ $A_{BAn} = 251.239\ in \cdot sec^{-2}$

$\theta_{ABAn} := \theta_3 + 180 \cdot deg$ $\theta_{ABAn} = 268.837\ deg$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of θ_4 + 180 deg and $\mathbf{A}_A{}^n$ at an angle of θ_2 + 180 deg. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_{BA}{}^n$ at an angle of θ_3 + 180 deg. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B{}^t$, $\mathbf{A}_{BA}{}^t$, and $\mathbf{A}_B$.



5. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 100 \cdot \dfrac{in}{sec^2}$

$A_{BAt} := 1.826 \cdot k_a \qquad A_{BAt} = 182.6\, in \cdot sec^{-2}$

$A_{Bt} := 4.800 \cdot k_a \qquad A_{Bt} = 480.0\, in \cdot sec^{-2}$

$A_B := 5.009 \cdot k_a \qquad A_B = 500.9\, in \cdot sec^{-2}$ at an angle of -136.08 deg

6. Calculate α_3 and α_4 using equation 7.6.

$\alpha_3 := \dfrac{A_{BAt}}{b} \qquad \alpha_3 = 26.086\, rad \cdot sec^{-2}$ CCW

$\alpha_4 := \dfrac{A_{Bt}}{c} \qquad \alpha_4 = 53.333\, rad \cdot sec^{-2}$ CCW

From the graphical solution we see that both of the tangential acceleration vectors for links 3 and 4 point to the left, so both of the angular accelerations are positive.

7. For point P, equation 7.4 becomes: $\qquad \mathbf{A}_P = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$, where

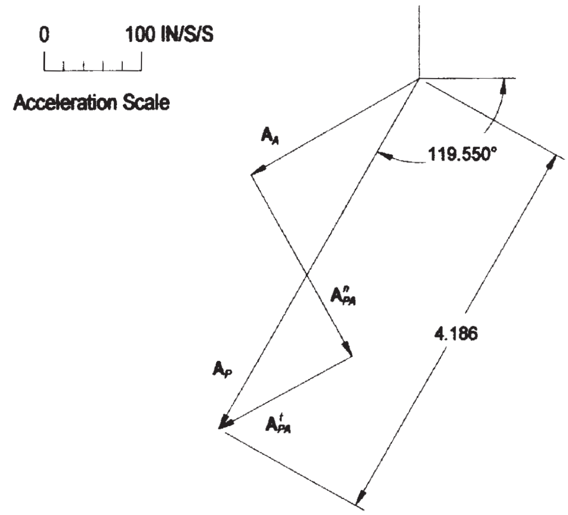
$A_{PAn} := R_{PA} \cdot \omega_3^2 \qquad A_{PAn} = 215.348\, in \cdot sec^{-2}$

$\theta_{APAn} := \theta_3 + \delta_3 + 180 \cdot deg \qquad \theta_{APAn} = 298.837\, deg$

$A_{PAt} := R_{PA} \cdot \alpha_3 \qquad A_{PAt} = 156.514\, in \cdot sec^{-2}$

$\theta_{APAt} := \theta_3 + \delta_3 + 90 \cdot deg \qquad \theta_{APAt} = 208.837\, deg$

8. Repeat proceedure of step 4 for the equation in step 7.



9. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 100 \cdot \dfrac{in}{sec^2}$

$A_P := 4.186 \cdot k_a \qquad A_P = 418.6\, in \cdot sec^{-2}$ at an angle of -119.6 deg

PROBLEM 7-4a

Statement: The link lengths, coupler point location, and the values of θ_2, ω_2, and α_2 for the same fourbar linkages as used for position and velocity analysis in Chapters 4 and 6 are redefined in Table P7-1, which is the same as Table P6-1. The general linkage configuration and terminology are shown in Figure P7-1. For row *a*, find the accelerations of points *A* and *B* using the analytic method. Then, calculate α_3 and α_4 and the acceleration of point *P*.

Given: Link lengths:

Link 1 $d := 6$ Link 2 $a := 2$

Link 3 $b := 7$ Link 4 $c := 9$

Coupler point: $R_{PA} := 6$ $\delta_3 := 30 \cdot deg$

Link 2 position, velocity, and acceleration: $\theta_2 := 30 \cdot deg$ $\omega_2 := 10$ $\alpha_2 := 0$

Solution: See Figure P7-1 and Mathcad file P0704a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 3.0000 \qquad K_2 = 0.6667 \qquad K_3 = 2.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \qquad A = -0.7113$$

$$B := -2 \cdot \sin(\theta_2) \qquad B = -1.0000$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \qquad C = 3.5566$$

2. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

Open: $$\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_{41} = -242.714\,deg$$

$$\theta_{41} := \theta_{41} - 360 \cdot deg \qquad \theta_{41} = -602.714\,deg$$

Crossed: $$\theta_{42} := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_{42} = 216.340\,deg$$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.8571 \qquad K_5 = -0.2857$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -1.6774$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 2.5906$$

4. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

Open: $$\theta_{31} := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_{31} = -271.163\,deg$$

$$\theta_{31} := \theta_{31} - 360 \cdot deg \qquad \theta_{31} = -631.163\,deg$$

Crossed: $\theta_{32} := 2\cdot\left(atan2\left(2\cdot D, -E + \sqrt{E^2 - 4\cdot D\cdot F}\right)\right)$ $\theta_{32} = 244.789\,deg$

5. Determine the angular velocity of links 3 and 4 for the open and crossed circuits using equations 6.18.

OPEN $\omega_{31} := \dfrac{a\cdot\omega_2}{b}\cdot\dfrac{sin(\theta_{41} - \theta_2)}{sin(\theta_{31} - \theta_{41})}$ $\omega_{31} = -5.991$

$\omega_{41} := \dfrac{a\cdot\omega_2}{c}\cdot\dfrac{sin(\theta_2 - \theta_{31})}{sin(\theta_{41} - \theta_{31})}$ $\omega_{41} = -3.992$

CROSS $\omega_{32} := \dfrac{a\cdot\omega_2}{b}\cdot\dfrac{sin(\theta_{42} - \theta_2)}{sin(\theta_{32} - \theta_{42})}$ $\omega_{32} = -0.662$

$\omega_{42} := \dfrac{a\cdot\omega_2}{c}\cdot\dfrac{sin(\theta_2 - \theta_{32})}{sin(\theta_{42} - \theta_{32})}$ $\omega_{42} = -2.662$

6. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Separate into real and imaginary parts to give the *x* and *y* components

$$\mathbf{A_A} = a\cdot\alpha_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) - a\cdot\omega_2^2\cdot(cos(\theta_2) + j\cdot sin(\theta_2))$$

$A_{Ax} := -a\cdot\alpha_2\cdot sin(\theta_2) - a\cdot\omega_2^2\cdot cos(\theta_2)$ $A_{Ax} = -173.205$

$A_{Ay} := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^2\cdot sin(\theta_2)$ $A_{Ay} = -100.000$

$A_A := \sqrt{A_{Ax}^2 + A_{Ay}^2}$ $\theta_{AA} := atan2(A_{Ax}, A_{Ay})$

The acceleration of pin A is $A_A = 200.000$ at $\theta_{AA} = -150\,deg$

7. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the open and crossed circuits.

OPEN $A := c\cdot sin(\theta_{41})$ $B := b\cdot sin(\theta_{31})$ $D := c\cdot cos(\theta_{41})$ $E := b\cdot cos(\theta_{31})$

$A = 7.999$ $B = 6.999$ $D = -4.126$ $E = 0.142$

$$C := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^2\cdot cos(\theta_2) + b\cdot\omega_{31}^2\cdot cos(\theta_{31}) - c\cdot\omega_{41}^2\cdot cos(\theta_{41})$$

$C = 244.045$

$$F := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^2\cdot sin(\theta_2) - b\cdot\omega_{31}^2\cdot sin(\theta_{31}) + c\cdot\omega_{41}^2\cdot sin(\theta_{41})$$

$F = -223.741$

$\alpha_{31} := \dfrac{C\cdot D - A\cdot F}{A\cdot E - B\cdot D}$ $\alpha_{31} = 26.080$ $\alpha_{41} := \dfrac{C\cdot E - B\cdot F}{A\cdot E - B\cdot D}$ $\alpha_{41} = 53.331$

CROSS $A := c\cdot sin(\theta_{42})$ $B := b\cdot sin(\theta_{32})$ $D := c\cdot cos(\theta_{42})$ $E := b\cdot cos(\theta_{32})$

$A = -5.333$ $B = -6.333$ $D = -7.250$ $E = -2.982$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_{32}^{\,2} \cdot \cos(\theta_{32}) - c \cdot \omega_{42}^{\,2} \cdot \cos(\theta_{42})$$

$$C = 223.253$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_{32}^{\,2} \cdot \sin(\theta_{32}) + c \cdot \omega_{42}^{\,2} \cdot \sin(\theta_{42})$$

$$F = -135.002$$

$$\alpha_{32} := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_{32} = 77.920 \qquad \alpha_{42} := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_{42} = 50.669$$

8. Use equation 7.13c to determine the acceleration of point B for the open and crossed circuits.

OPEN $A_{Bx1} := c \cdot \left(-\alpha_{41} \cdot \sin(\theta_{41}) - \omega_{41}^{\,2} \cdot \cos(\theta_{41})\right)$ $\qquad A_{Bx1} = -360.826$

$A_{By1} := c \cdot \left(\alpha_{41} \cdot \cos(\theta_{41}) - \omega_{41}^{\,2} \cdot \sin(\theta_{41})\right)$ $\qquad A_{By1} = -347.485$

$A_{B1} := \sqrt{A_{Bx1}^{\,2} + A_{By1}^{\,2}}$ $\qquad A_{B1} = 500.941$

$\theta_{AB1} := \mathrm{atan2}(A_{Bx1}, A_{By1})$ $\qquad \theta_{AB1} = -136.1\,deg$

CROSS $A_{Bx2} := c \cdot \left(-\alpha_{42} \cdot \sin(\theta_{42}) - \omega_{42}^{\,2} \cdot \cos(\theta_{42})\right)$ $\qquad A_{Bx2} = 321.587$

$A_{By2} := c \cdot \left(\alpha_{42} \cdot \cos(\theta_{42}) - \omega_{42}^{\,2} \cdot \sin(\theta_{42})\right)$ $\qquad A_{By2} = -329.551$

$A_{B2} := \sqrt{A_{Bx2}^{\,2} + A_{By2}^{\,2}}$ $\qquad A_{B2} = 460.459$

$\theta_{AB2} := \mathrm{atan2}(A_{Bx2}, A_{By2})$ $\qquad \theta_{AB2} = -45.7\,deg$

9. Use equations 7.32 to find the acceleration of the point P for the open and crossed circuits.

OPEN $\mathbf{A_A} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$

$$\mathbf{A_{PA1}} := R_{PA} \cdot \alpha_{31} \cdot (-\sin(\theta_{31} + \delta_3) + j \cdot \cos(\theta_{31} + \delta_3)) \ldots$$
$$+ -R_{PA} \cdot \omega_{31}^{\,2} \cdot (\cos(\theta_{31} + \delta_3) + j \cdot \sin(\theta_{31} + \delta_3))$$

$$\mathbf{A_{P1}} := \mathbf{A_A} + \mathbf{A_{PA1}}$$

$A_{P1} := |\mathbf{A_{P1}}|$ $\qquad A_{P1} = 418.556$ $\qquad \arg(\mathbf{A_{P1}}) = -119.548\,deg$

CROSS $\mathbf{A_A} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$

$$\mathbf{A_{PA2}} := R_{PA} \cdot \alpha_{32} \cdot (-\sin(\theta_{32} + \delta_3) + j \cdot \cos(\theta_{32} + \delta_3)) \ldots$$
$$+ -R_{PA} \cdot \omega_{32}^{\,2} \cdot (\cos(\theta_{32} + \delta_3) + j \cdot \sin(\theta_{32} + \delta_3))$$

$$\mathbf{A_{P2}} := \mathbf{A_A} + \mathbf{A_{PA2}}$$

$A_{P2} := |\mathbf{A_{P2}}|$ $\qquad A_{P2} = 298.225$ $\qquad \arg(\mathbf{A_{P2}}) = -11.282\,deg$

PROBLEM 7-5a

Statement: The link lengths and the values of θ_2, ω_2, and α_2 for the some non inverted offset fourbar slider-crank linkages are defined in Table P7-2. The general linkage configuration and terminology are shown in Figure P7-2. For row *a*, draw the linkage to scale and graphically find the accelerations of the pin joints *A* and *B* the acceleration of slip at the sliding joint.

Given:

Link lengths: Link 2 $a := 1.4 \cdot in$ Link 3 $b := 4 \cdot in$

Offset: $c := 1 \cdot in$

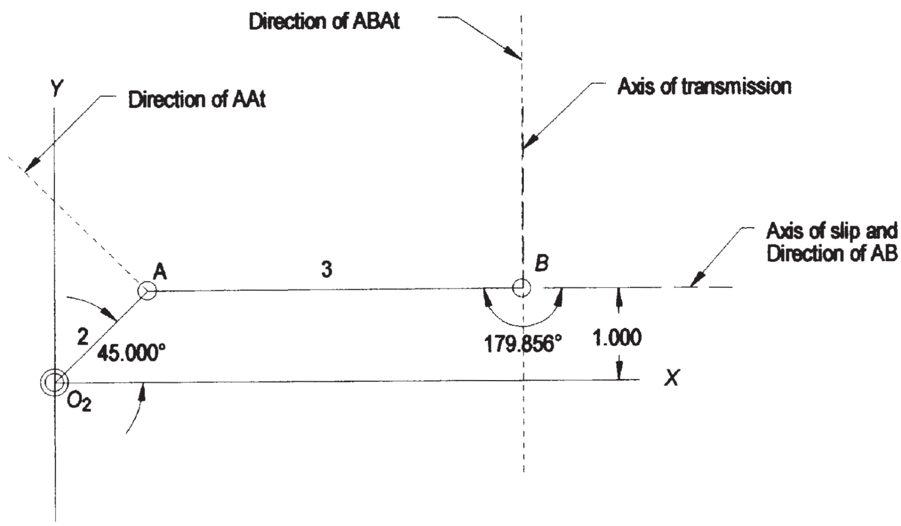
Link 2 position, velocity, and acceleration: $\theta_2 := 45 \cdot deg$ $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-2 and Mathcad file P0705a.

1. In order to solve for the accelerations at point *B*, we will need θ_3, and ω_3. From the graphical position solution below,

$\theta_3 := 360 \cdot deg - 179.856 \cdot deg$ $\theta_3 = 180.144\,deg$

Using equation 6.22a, $\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3)}$ $\omega_3 = -2.475\,rad \cdot sec^{-1}$



2. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

3. For point *B*, this becomes: $\mathbf{A}_B = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$A_{An} := a \cdot \omega_2{}^2$ $A_{An} = 140.000\,in \cdot sec^{-2}$

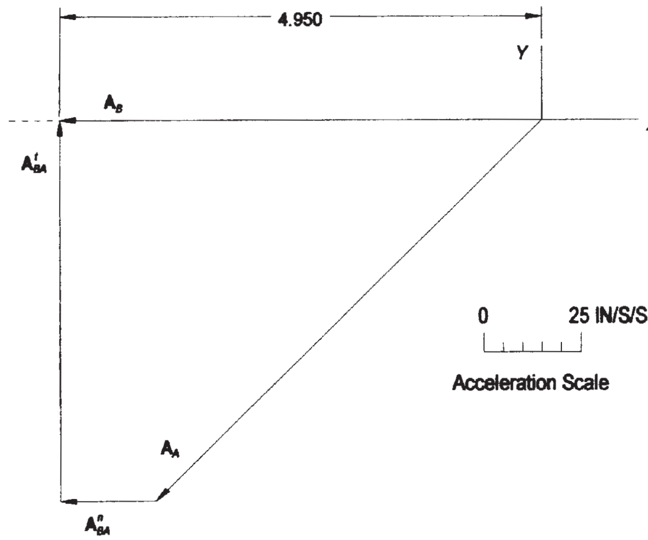
$\theta_{AAn} := \theta_2 + 180 \cdot deg$ $\theta_{AAn} = 225.000\,deg$

$A_{At} := a \cdot \alpha_2$ $A_{At} = 0.000\,in \cdot sec^{-2}$

$A_{BAn} := b \cdot \omega_3{}^2$ $A_{BAn} = 24.500\,in \cdot sec^{-2}$

$\theta_{ABAn} := \theta_3$ $\theta_{ABAn} = 180.144\,deg$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_A{}^n$ at an angle of θ_{AAn}. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_{BA}{}^n$ at an angle of θ_{ABAn}. Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B$ (horizontal) and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{BA}{}^t$, and $\mathbf{A}_B$.



5. From the graphical solution above,

Acceleration scale factor $k_a := 25 \cdot \dfrac{in}{sec^2}$

$A_B := 4.950 \cdot k_a$ $A_B = 123.8\, in \cdot sec^{-2}$ at an angle of 180 deg

6. The acceleration of slip is equal to $\mathbf{A}_B$ since link 1 is stationary.

PROBLEM 7-6a

Statement: The link lengths and the values of θ_2, ω_2, and α_2 for the some non inverted offset fourbar slider-crank linkages are defined in Table P7-2. The general linkage configuration and terminology are shown in Figure P7-2. For row *a*, draw the linkage to scale and find the accelerations of the pin joints *A* and *B* the acceleration of slip at the sliding joint using an analytical method.

Given: Link lengths:

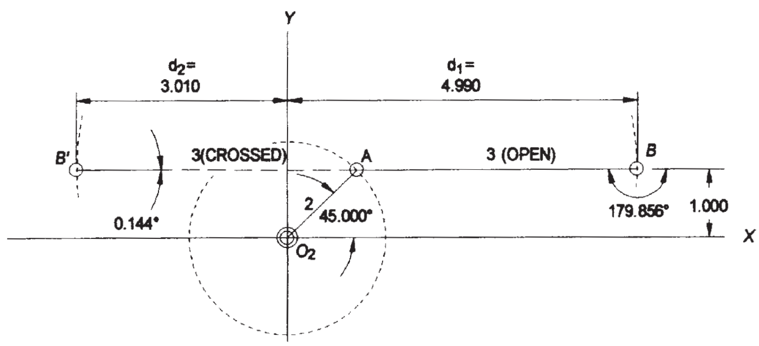
Link 2 $a := 1.4 \cdot in$ Link 3 $b := 4 \cdot in$

Offset: $c := 1 \cdot in$

Link 2 position, velocity, and acceleration: $\theta_2 := 45 \cdot deg$ $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-2 and Mathcad file P0706a.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and *d* using equations 4.16 and 4.17.

Open:

$$\theta_{32} := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_{32} = 180.144\,deg$$

$$d_2 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{32}) \qquad d_2 = 4.990\,in$$

Crossed:

$$\theta_{31} := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_{31} = -0.144\,deg$$

$$d_1 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{31}) \qquad d_1 = -3.010\,in$$

3. Determine the angular velocity of link 3 using equation 6.22a.

Open

$$\omega_{32} := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_{32})} \cdot \omega_2 \qquad \omega_{32} = -2.475\,\frac{rad}{sec}$$

Crossed

$$\omega_{31} := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_{31})} \cdot \omega_2 \qquad \omega_{31} = 2.475\,\frac{rad}{sec}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot \left(-\sin(\theta_2) + j \cdot \cos(\theta_2)\right) - a \cdot \omega_2^{\,2} \cdot \left(\cos(\theta_2) + j \cdot \sin(\theta_2)\right)$$

$$\mathbf{A_A} = -98.995 - 98.995j \,\frac{in}{sec^2} \qquad A_A := \left|\mathbf{A_A}\right| \qquad \theta_{AA} := arg\left(\mathbf{A_A}\right)$$

The acceleration of pin A is $\quad A_A = 140.0 \,\frac{in}{sec^2} \quad$ at $\quad \theta_{AA} = -135.0\,deg$

5. Determine the angular acceleration of link 3 using equation 7.16d.

Open $\quad \alpha_{32} := \dfrac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) + b \cdot \omega_{32}^{\,2} \cdot \sin(\theta_{32})}{b \cdot \cos(\theta_{32})}$

$$\alpha_{32} = 24.764 \,\frac{rad}{sec^2}$$

Crossed $\quad \alpha_{31} := \dfrac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) + b \cdot \omega_{31}^{\,2} \cdot \sin(\theta_{31})}{b \cdot \cos(\theta_{31})}$

$$\alpha_{31} = -24.764 \,\frac{rad}{sec^2}$$

6. Use equation 7.16e for the acceleration of pin B.

Open:

$$A_{B2} := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \alpha_{32} \cdot \sin(\theta_{32}) + b \cdot \omega_{32}^{\,2} \cdot \cos(\theta_{32})$$

$A_{B2} = -123.7 \,\frac{in}{sec^2}$ $\qquad$ A negative sign means that $\mathbf{A_B}$ is to the left

Crossed:

$$A_{B1} := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \alpha_{31} \cdot \sin(\theta_{31}) + b \cdot \omega_{31}^{\,2} \cdot \cos(\theta_{31})$$

$A_{B1} = -74.2 \,\frac{in}{sec^2}$ $\qquad$ A negative sign means that $\mathbf{A_B}$ is to the left

PROBLEM 7-7a

Statement: The link lengths and the values of θ_2, ω_2, and γ for an inverted fourbar slider-crank linkage are defined in Table P7-3, row *a*, and are given below. Find the acceleration of the pin joints *A* and *B* and the acceleration of slip at the sliding joint. Solve by the analytic vector loop method of Section 7.3 for the open configuration of the linkage.

Given: Link lengths:

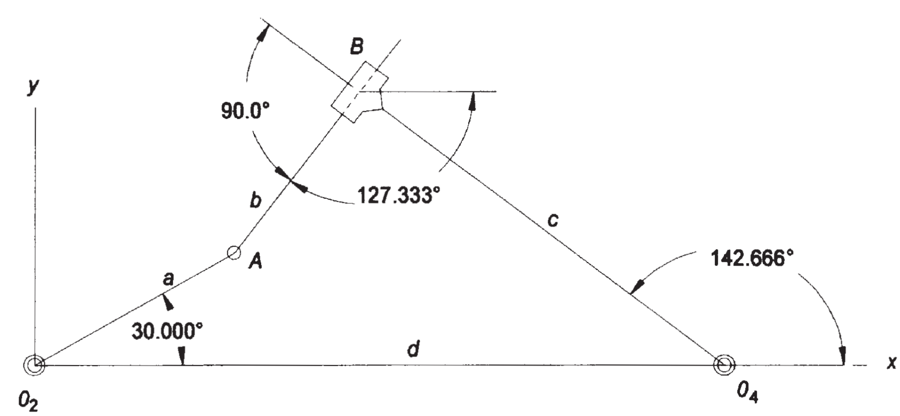
Link 2 $a := 2 \cdot in$ Link 4 $c := 4 \cdot in$ Link 1 $d := 6 \cdot in$

Angle between links 3 and 4 $\gamma := 90 \cdot deg$

Link 2 position, velocity, and accel. $\theta_2 := 30 \cdot deg$ $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := -25 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-3 and Mathcad file P0707a.

1. Draw the linkage to scale and label it.



2. Use the equations in Section 4.7 to solve for the positions of links 3 and 4 and for the length b.

$$P := a \cdot \sin(\theta_2) \cdot \sin(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \cos(\gamma) \qquad P = 1.000\,in$$

$$Q := -a \cdot \sin(\theta_2) \cdot \cos(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \sin(\gamma) \qquad Q = -4.268\,in$$

$$R := -c \cdot \sin(\gamma) \qquad S := R - Q \qquad T := 2 \cdot P \qquad U := Q + R$$

$$R = -4.000\,in \qquad S = 0.268\,in \qquad T = 2.000\,in \qquad U = -8.268\,in$$

$$\theta_4 := 2 \cdot atan2\left[(2 \cdot S), \left(-T + \sqrt{T^2 - 4 \cdot S \cdot U}\right)\right] \qquad \theta_4 = 142.667\,deg$$

$$\theta_3 := \theta_4 + \gamma \qquad \theta_3 = 232.667\,deg$$

$$b := \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_4)}{\sin(\theta_3)} \qquad b = 1.793\,in$$

3. Calculate the angular velocity of links 3 and 4 and the slip velocity using equations 6.30.

$$\omega_4 := \frac{a \cdot \omega_2 \cdot \cos(\theta_2 - \theta_3)}{(b + c \cdot \cos(\gamma))} \qquad \omega_4 = -10.292 \frac{rad}{sec}$$

$$bdot := \frac{-a \cdot \omega_2 \cdot \sin(\theta_2) + \omega_4 \cdot (b \cdot \sin(\theta_3) + c \cdot \sin(\theta_4))}{\cos(\theta_3)} \qquad bdot = 33.461 \frac{in}{sec}$$

$$\omega_3 := \omega_4$$

4. Solve for the accelerations using equations (7.26) and (7.27).

$$P := a \cdot \alpha_2 \cdot \cos(\theta_3 - \theta_2) \qquad P = 46.138\ in \cdot sec^{-2}$$

$$Q := a \cdot \omega_2^2 \cdot \sin(\theta_3 - \theta_2) \qquad Q = -77.075\ in \cdot sec^{-2}$$

$$R := c \cdot \omega_4^2 \cdot \sin(\theta_4 - \theta_3) \qquad R = -423.705\ in \cdot sec^{-2}$$

$$S := 2 \cdot bdot \cdot \omega_3 \qquad S = -688.757\ in \cdot sec^{-2}$$

$$T := b + c \cdot \cos(\theta_3 - \theta_4) \qquad T = 1.793\ in$$

$$\alpha_4 := \frac{P + Q + R - S}{T} \qquad \alpha_4 = 130.56\ rad \cdot sec^{-2}$$

$$K := a \cdot \omega_2^2 \cdot (b \cdot \cos(\theta_3 - \theta_2) + c \cdot \cos(\theta_4 - \theta_2)) \qquad K = -639.230\ in^2 \cdot sec^{-2}$$

$$L := a \cdot \alpha_2 \cdot (b \cdot \sin(\theta_2 - \theta_3) - c \cdot \sin(\theta_4 + \theta_2)) \qquad L = -9.025\ in^2 \cdot sec^{-2}$$

$$M := -\omega_4^2 \cdot (b^2 + c^2 + 2 \cdot b \cdot c \cdot \cos(\theta_4 - \theta_3)) \qquad M = -2.035 \times 10^3\ in^2 \cdot sec^{-2}$$

$$N := 2 \cdot bdot \cdot c \cdot \omega_4 \cdot \sin(\theta_4 - \theta_3) \qquad N = 2.755 \times 10^3\ in^2 \cdot sec^{-2}$$

$$bddot := -\frac{K + L + M + N}{T} \qquad bddot = -39.80 \frac{in}{sec^2}$$

The acceleration of slip is *bddot*. It is directed along link 3, positive inward from *B* towards *A*, so that its angle is θ_3. For the acceleration of the pin joints *A* and *B*,

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_A := |\mathbf{A_A}| \qquad A_A = 206.155 \frac{in}{sec^2} \qquad \theta_{AA} := arg(\mathbf{A_A}) \qquad \theta_{AA} = -135.964\ deg$$

$$\mathbf{A_B} := -c \cdot \alpha_4 \cdot (\sin(\theta_4) - j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$$A_B := |\mathbf{A_B}| \qquad A_B = 672.505 \frac{in}{sec^2} \qquad \theta_{AB} := arg(\mathbf{A_B}) \qquad \theta_{AB} = -88.280\ deg$$

PROBLEM 7-8a

Statement: The link lengths and the values of θ_2, ω_2, and γ for an inverted fourbar slider-crank linkage are defined in Table P7-3, row *a*, and are given below. Find the acceleration of the pin joints *A* and *B* and the acceleration of slip at the sliding joint. Solve by the analytic vector loop method of Section 7.3 for the crossed configuration of the linkage.

Given: Link lengths:

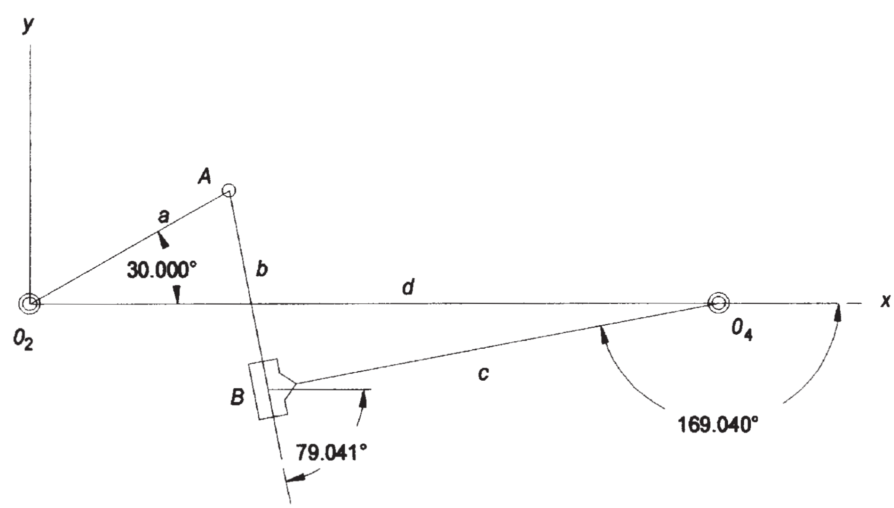
Link 2 $a := 2 \cdot in$ Link 4 $c := 4 \cdot in$ Link 1 $d := 6 \cdot in$

Angle between links 3 and 4 $\gamma := 90 \cdot deg$

Link 2 position, velocity, and accel. $\theta_2 := 30 \cdot deg$ $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := -25 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-3 and Mathcad file P0708a.

1. Draw the linkage to scale and label it.



2. Use the equations in Section 4.7 to solve for the positions of links 3 and 4 and for the length b.

$$P := a \cdot \sin(\theta_2) \cdot \sin(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \cos(\gamma) \qquad P = 1.000\, in$$

$$Q := -a \cdot \sin(\theta_2) \cdot \cos(\gamma) + (a \cdot \cos(\theta_2) - d) \cdot \sin(\gamma) \qquad Q = -4.268\, in$$

$$R := -c \cdot \sin(\gamma) \qquad S := R - Q \qquad T := 2 \cdot P \qquad U := Q + R$$

$$R = -4.000\, in \qquad S = 0.268\, in \qquad T = 2.000\, in \qquad U = -8.268\, in$$

$$\theta_4 := 2 \cdot atan2\left[(2 \cdot S), -T - \sqrt{T^2 - 4 \cdot S \cdot U}\right] \qquad \theta_4 = -169.041\, deg$$

$$\theta_3 := \theta_4 - \gamma \qquad \theta_3 = -259.041\, deg$$

$$b := \frac{a \cdot \sin(\theta_2) - c \cdot \sin(\theta_4)}{\sin(\theta_3)} \qquad b = 1.793\, in$$

3. Calculate the angular velocity of links 3 and 4 and the slip velocity using equations 6.30.

$$\omega_4 := \frac{a \cdot \omega_2 \cdot cos(\theta_2 - \theta_3)}{(b + c \cdot cos(\gamma))} \qquad \omega_4 = 3.639 \frac{rad}{sec}$$

$$bdot := \frac{-a \cdot \omega_2 \cdot sin(\theta_2) + \omega_4 \cdot (b \cdot sin(\theta_3) + c \cdot sin(\theta_4))}{cos(\theta_3)} \qquad bdot = 33.461 \frac{in}{sec}$$

$$\omega_3 := \omega_4$$

4. Solve for the accelerations using equations (7.26) and (7.27).

$$P := a \cdot \alpha_2 \cdot cos(\theta_3 - \theta_2) \qquad P = -16.312\, in \cdot sec^{-2}$$

$$Q := a \cdot \omega_2^{\,2} \cdot sin(\theta_3 - \theta_2) \qquad Q = 189.057\, in \cdot sec^{-2}$$

$$R := c \cdot \omega_4^{\,2} \cdot sin(\theta_4 - \theta_3) \qquad R = 52.961\, in \cdot sec^{-2}$$

$$S := 2 \cdot bdot \cdot \omega_3 \qquad S = 243.508\, in \cdot sec^{-2}$$

$$T := b + c \cdot cos(\theta_3 - \theta_4) \qquad T = 1.793\, in$$

$$\alpha_4 := \frac{P + Q + R - S}{T} \qquad \alpha_4 = -9.93\, rad \cdot sec^{-2}$$

$$K := a \cdot \omega_2^{\,2} \cdot (b \cdot cos(\theta_3 - \theta_2) + c \cdot cos(\theta_4 - \theta_2)) \qquad K = -639.230\, in^2 \cdot sec^{-2}$$

$$L := a \cdot \alpha_2 \cdot (b \cdot sin(\theta_2 - \theta_3) - c \cdot sin(\theta_4 + \theta_2)) \qquad L = -46.352\, in^2 \cdot sec^{-2}$$

$$M := -\omega_4^{\,2} \cdot \left(b^2 + c^2 + 2 \cdot b \cdot c \cdot cos(\theta_4 - \theta_3)\right) \qquad M = -254.418\, in^2 \cdot sec^{-2}$$

$$N := 2 \cdot bdot \cdot c \cdot \omega_4 \cdot sin(\theta_4 - \theta_3) \qquad N = 974.033\, in^2 \cdot sec^{-2}$$

$$bddot := -\frac{K + L + M + N}{T} \qquad bddot = -18.98 \frac{in}{sec^2}$$

The acceleration of slip is bddot. It is directed along link 3, positive inward from *B* towards *A*, so that its angle is θ_3. For the acceleration of the pin joints *A* and *B*,

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$A_A := |\mathbf{A_A}| \qquad A_A = 206.155 \frac{in}{sec^2} \qquad \theta_{AA} := arg(\mathbf{A_A}) \qquad \theta_{AA} = -135.964\, deg$$

$$\mathbf{A_B} := -c \cdot \alpha_4 \cdot (sin(\theta_4) - j \cdot cos(\theta_4)) - c \cdot \omega_4^{\,2} \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$$

$$A_B := |\mathbf{A_B}| \qquad A_B = 66.195 \frac{in}{sec^2} \qquad \theta_{AB} := arg(\mathbf{A_B}) \qquad \theta_{AB} = 47.822\, deg$$

PROBLEM 7-9a

Statement: The link lengths, gear ratio (λ), phase angle (ϕ), and the values of θ_2, ω_2, and α_2 for a geared fivebar from row *a* of Table P7-4 are given below. Find α_3 and α_4 and the linear acceleration of point *P*.

Given: Link lengths:

Link 1 $f := 6\cdot in$ Link 3 $b := 7\cdot in$ Link 5 $d := 4\cdot in$

Link 2 $a := 1\cdot in$ Link 4 $c := 9\cdot in$

Gear ratio, phase angle, and crank angle:

$\lambda := 2$ $\phi := 30\cdot deg$ $\theta_2 := 60\cdot deg$ $\omega_2 := 10\cdot rad\cdot sec^{-1}$ $\alpha_2 := 0\cdot rad\cdot sec^{-2}$

Coupler data: $R_{pa} := 6\cdot in$ $\delta_3 := 30\cdot deg$

Solution: See Figure P7-4 and Mathcad file P0709a.

1. Determine the angle of link 5 using the equation in Figure P6-4.

$\theta_5 := \lambda\cdot\theta_2 + \phi$ $\theta_5 = 150\,deg$

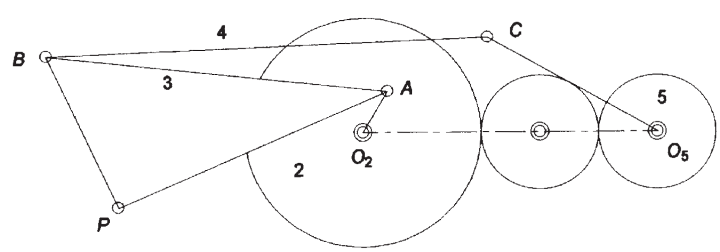
2. Choose the pitch radii of the gears. Since the gear ratio is positive, an idler must be used between gear 2 and gear 5. Let the idler be the same diameter as gear 5, and let all three gears be in line.

$f := r_2 + 3\cdot r_5$ $\lambda := \dfrac{r_2}{r_5}$

$r_5 := \dfrac{f}{\lambda + 3}$ $r_5 = 1.200\,in$

$r_2 := \lambda\cdot r_5$ $r_2 = 2.400\,in$

3. Draw the linkage to scale and label it.



4. Determine the values of the constants needed for finding θ_3 and θ_4 from equations 4.24h and 4.24i.

$A := 2\cdot c\cdot\left(d\cdot cos(\lambda\cdot\theta_2 + \phi) - a\cdot cos(\theta_2) + f\right)$ $A = 36.6462\,in^2$

$B := 2\cdot c\cdot\left(d\cdot sin(\lambda\cdot\theta_2 + \phi) - a\cdot sin(\theta_2)\right)$ $B = 20.412\,in^2$

$$C := \left(a^2 - b^2 + c^2 + d^2 + f^2\right) - 2\cdot a\cdot f\cdot cos(\theta_2) \ldots$$
$$+ -\left[2\cdot d\cdot\left(a\cdot cos(\theta_2) - f\right)\cdot cos(\lambda\cdot\theta_2 + \phi)\right] \ldots$$
$$+ -2\cdot a\cdot d\cdot sin(\theta_2)\cdot sin(\lambda\cdot\theta_2 + \phi)$$

$C = 37.4308\,in^2$

$D := C - A$ $D = 0.78461\,in^2$

$E := 2\cdot B$ $E = 40.823\,in^2$

$$F := A + C \qquad F = 74.077\ in^2$$

$$G := 2 \cdot b \cdot \left[-\left(d \cdot \cos\left(\lambda \cdot \theta_2 + \phi\right)\right) + a \cdot \cos\left(\theta_2\right) - f \right] \qquad G = -28.503\ in^2$$

$$H := 2 \cdot b \cdot \left[-\left(d \cdot \sin\left(\lambda \cdot \theta_2 + \phi\right)\right) + a \cdot \sin\left(\theta_2\right) \right] \qquad H = -15.876\ in^2$$

$$K := \left(a^2 + b^2 - c^2 + d^2 + f^2\right) - 2 \cdot a \cdot f \cdot \cos\left(\theta_2\right) \ldots$$
$$+ -\left[2 \cdot d \cdot \left(a \cdot \cos\left(\theta_2\right) - f\right) \cdot \cos\left(\lambda \cdot \theta_2 + \phi\right)\right] \ldots$$
$$+ -2 \cdot a \cdot d \cdot \sin\left(\theta_2\right) \cdot \sin\left(\lambda \cdot \theta_2 + \phi\right) \qquad K = -26.569\ in^2$$

$$L := K - G \qquad L = 1.933\ in^2$$

$$M := 2 \cdot H \qquad M = -31.751\ in^2$$

$$N := G + K \qquad N = -55.072\ in^2$$

5. Use equations 4.24h and 4.24i to find values of θ_3 and θ_4 for the open and crossed circuits.

OPEN $$\theta_{31} := 2 \cdot \left(atan2\left(2 \cdot L, -M + \sqrt{M^2 - 4 \cdot L \cdot N}\right)\right) \qquad \theta_{31} = 173.642\ deg$$

$$\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_{41} = -177.715\ deg$$

CROSSED $$\theta_{32} := 2 \cdot \left(atan2\left(2 \cdot L, -M - \sqrt{M^2 - 4 \cdot L \cdot N}\right)\right) \qquad \theta_{32} = -115.407\ deg$$

$$\theta_{42} := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_{42} = -124.050\ deg$$

6. Use equation 6.32c to find ω_5. $\omega_5 := \lambda \cdot \omega_2$

7. Calculate ω_3 and ω_4 using equations 6.33.

OPEN $$\omega_{31} := \frac{-\left[2 \cdot \sin\left(\theta_{41}\right) \cdot \left(a \cdot \omega_2 \cdot \sin\left(\theta_2 - \theta_{41}\right) + d \cdot \omega_5 \cdot \sin\left(\theta_{41} - \theta_5\right)\right)\right]}{b \cdot \left(\cos\left(\theta_{31} - 2 \cdot \theta_{41}\right) - \cos\left(\theta_{31}\right)\right)}$$

$$\omega_{31} = 32.585\ \frac{rad}{sec}$$

$$\omega_{41} := \frac{a \cdot \omega_2 \cdot \sin\left(\theta_2\right) + b \cdot \omega_{31} \cdot \sin\left(\theta_{31}\right) - d \cdot \omega_5 \cdot \sin\left(\theta_5\right)}{c \cdot \sin\left(\theta_{41}\right)}$$

$$\omega_{41} = 16.948\ \frac{rad}{sec}$$

CROSSED $$\omega_{32} := \frac{-\left[2 \cdot \sin\left(\theta_{42}\right) \cdot \left(a \cdot \omega_2 \cdot \sin\left(\theta_2 - \theta_{42}\right) + d \cdot \omega_5 \cdot \sin\left(\theta_{42} - \theta_5\right)\right)\right]}{b \cdot \left(\cos\left(\theta_{32} - 2 \cdot \theta_{42}\right) - \cos\left(\theta_{32}\right)\right)}$$

$$\omega_{32} = -75.191\ \frac{rad}{sec}$$

$$\omega_{42} := \frac{a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{32} \cdot \sin(\theta_{32}) - d \cdot \omega_5 \cdot \sin(\theta_5)}{c \cdot \sin(\theta_{42})}$$

$$\omega_{42} = -59.554 \frac{rad}{sec}$$

8. Use equation 7.28c to find α_5. $\alpha_5 := \lambda \cdot \alpha_2$

9. Calculate α_3 and α_4 using equations 7.29.

OPEN
$$\alpha_{31} := \frac{\begin{pmatrix} -a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{41}) - a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{41}) \ldots \\ + -b \cdot \omega_{31}^2 \cdot \cos(\theta_{31} - \theta_{41}) + d \cdot \omega_5^2 \cdot \cos(\theta_5 - \theta_{41}) \ldots \\ + d \cdot \alpha_5 \cdot \sin(\theta_5 - \theta_{41}) + c \cdot \omega_{41}^2 \end{pmatrix}}{b \cdot \sin(\theta_{31} - \theta_{41})}$$

$$\alpha_{31} = 3191 \frac{rad}{sec^2}$$

$$\alpha_{41} := \frac{\begin{pmatrix} a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{31}) + a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{31}) \ldots \\ + -c \cdot \omega_{41}^2 \cdot \cos(\theta_{31} - \theta_{41}) - d \cdot \omega_5^2 \cdot \cos(\theta_{31} - \theta_5) \ldots \\ + d \cdot \alpha_5 \cdot \sin(\theta_{31} - \theta_5) + b \cdot \omega_{31}^2 \end{pmatrix}}{c \cdot \sin(\theta_{41} - \theta_{31})}$$

$$\alpha_{41} = 2492 \frac{rad}{sec^2}$$

CROSSED
$$\alpha_{32} := \frac{\begin{pmatrix} -a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{42}) - a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{42}) \ldots \\ + -b \cdot \omega_{32}^2 \cdot \cos(\theta_{32} - \theta_{42}) + d \cdot \omega_5^2 \cdot \cos(\theta_5 - \theta_{42}) \ldots \\ + d \cdot \alpha_5 \cdot \sin(\theta_5 - \theta_{42}) + c \cdot \omega_{42}^2 \end{pmatrix}}{b \cdot \sin(\theta_{32} - \theta_{42})}$$

$$\alpha_{32} = -6648 \frac{rad}{sec^2}$$

$$\alpha_{42} := \frac{\begin{pmatrix} a \cdot \alpha_2 \cdot \sin(\theta_2 - \theta_{32}) + a \cdot \omega_2^2 \cdot \cos(\theta_2 - \theta_{32}) \ldots \\ + -c \cdot \omega_{42}^2 \cdot \cos(\theta_{32} - \theta_{42}) - d \cdot \omega_5^2 \cdot \cos(\theta_{32} - \theta_5) \ldots \\ + d \cdot \alpha_5 \cdot \sin(\theta_{32} - \theta_5) + b \cdot \omega_{32}^2 \end{pmatrix}}{c \cdot \sin(\theta_{42} - \theta_{32})}$$

$$\alpha_{42} = -5950 \frac{rad}{sec^2}$$

PROBLEM 7-10

Statement: An automobile driver took a curve too fast. The car spun out of control about its CG and slid off the road in a northeasterly direction. The friction of the skidding tires provided a 0.25g linear deceleration. The car rotated at 100 rpm. When the car hit the tree head-on at 30 mph, it took 0.1 sec to come to rest.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$ $\quad mph := mi\cdot hr^{-1}$

Given: $V := 30\cdot mph$ $\quad A_C := -0.25\cdot g$ $\quad \omega := 100\cdot rpm$

Solution: See Mathcad file P0710.

a. **What was the acceleration experienced by the child seated on the middle of the rear seat, 2 ft behind the car CG, just prior to impact?**

$r := 2\cdot ft$ $\quad \omega = 10.472\, rad\cdot sec^{-1}$ $\quad A_C = -8.044\, ft\cdot sec^{-2}$

We want to solve the vector equation $\mathbf{A_p} = \mathbf{A_c} + \mathbf{A_{pc}}^n + \mathbf{A_{pc}}^t$ where P is the position of the child and C is the CG. Since α is zero, $\mathbf{A_{pc}}^t$ is zero. $\mathbf{A_c}$ and $\mathbf{A_{pc}}^n$ both have the same direction, but are opposite in sense (see diagram).

$A_{pcn} := r\cdot\omega^2$

$A_{pcn} = 6.817\, g$

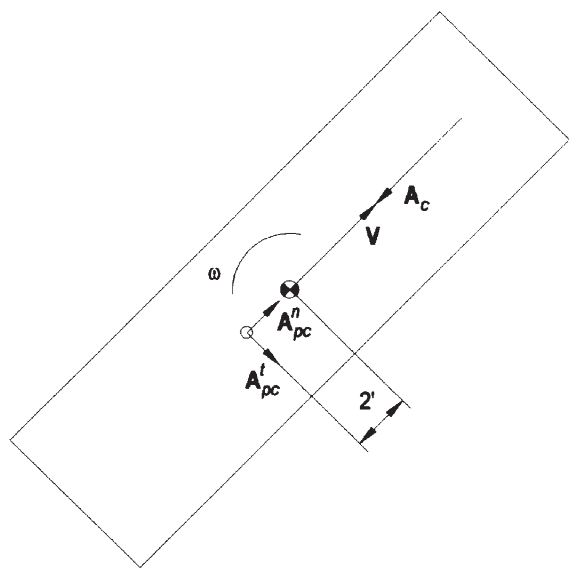
$A_P := A_C + A_{pcn}$

$A_P = 6.567\, g$

This acceleration is toward the tree.

$A_P = 211.3\, ft\cdot sec^{-2}$

$A_P = 2535\, in\cdot sec^{-2}$



b. **What force did the 100 lb child exert on her seat belt harness as a result of the acceleration just prior to impact?**

$W := 100\cdot lbf$

The seat back must provide a force of $\quad F := \frac{W}{g}\cdot A_P \quad F = 657\, lbf$

c. **Assuming a constant deceleration during the impact, what was the magnitude of the average deceleration fe by the passengers in that interval?**

$t := 0.1\cdot sec$

Assume that the rotation has ceased. Then, the average velocity during impact is

$V_{avg} := \frac{V}{2}$ $\quad V_{avg} = 22.0\,\frac{ft}{sec}$

and the average deceleration is $A_{avg} := \dfrac{V_{avg}}{t}$ $\quad A_{avg} = 6.838\,g$ $\quad A_{avg} = 220.0\,ft \cdot sec^{-2}$

Adding the deceleration due to skidding, $A_{dec} := A_c - A_{avg}$

$A_{dec} = -7.088\,g$ $\quad A_{dec} = -228\,ft \cdot sec^{-2}$ $\quad A_{dec} = -2737\,in \cdot sec^{-2}$

PROBLEM 7-11a

Statement: For row *a* in Table P7-1, find the angular jerk of links 3 and 4 and the linear jerk of the pin between links 3 and 4 (point *B*). Assume an angular jerk of zero on link 2. The linkage configuration and terminology are shown in Figure P7-1.

Given: Link lengths:

Link 1 $d := 6$ Link 2 $a := 2$

Link 3 $b := 7$ Link 4 $c := 9$

Coupler point: $R_{PA} := 6$ $\delta_3 := 30 \cdot deg$

Link 2 position, velocity, and acceleration: $\theta_2 := 30 \cdot deg$ $\omega_2 := 10$ $\alpha_2 := 0$ $\phi_2 := 0$

Solution: See Figure P7-1 and Mathcad file P0711a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 3.0000 \qquad K_2 = 0.6667 \qquad K_3 = 2.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3 \qquad A = -0.7113$$

$$B := -2 \cdot \sin(\theta_2) \qquad B = -1.0000$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3 \qquad C = 3.5566$$

2. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

Open: $$\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_{41} = -242.714\,deg$$

$$\theta_{41} := \theta_{41} - 360 \cdot deg \qquad \theta_{41} = -602.714\,deg$$

Crossed: $$\theta_{42} := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_{42} = 216.340\,deg$$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.8571 \qquad K_5 = -0.2857$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -1.6774$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 2.5906$$

4. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

Open: $$\theta_{31} := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_{31} = -271.163\,deg$$

$$\theta_{31} := \theta_{31} - 360 \cdot deg \qquad \theta_{31} = -631.163\,deg$$

Crossed: $$\theta_{32} := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_{32} = 244.789\,deg$$

5. Determine the angular velocity of links 3 and 4 for the open and crossed circuits using equations 6.18.

OPEN $\omega_{31} := \frac{a\cdot\omega_2}{b}\cdot\frac{\sin(\theta_{41}-\theta_2)}{\sin(\theta_{31}-\theta_{41})}$ $\omega_{31} = -5.991$

$\omega_{41} := \frac{a\cdot\omega_2}{c}\cdot\frac{\sin(\theta_2-\theta_{31})}{\sin(\theta_{41}-\theta_{31})}$ $\omega_{41} = -3.992$

CROSS $\omega_{32} := \frac{a\cdot\omega_2}{b}\cdot\frac{\sin(\theta_{42}-\theta_2)}{\sin(\theta_{32}-\theta_{42})}$ $\omega_{32} = -0.662$

$\omega_{42} := \frac{a\cdot\omega_2}{c}\cdot\frac{\sin(\theta_2-\theta_{32})}{\sin(\theta_{42}-\theta_{32})}$ $\omega_{42} = -2.662$

6. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the open and crossed circuits.

OPEN $A := c\cdot\sin(\theta_{41})$ $B := b\cdot\sin(\theta_{31})$ $D := c\cdot\cos(\theta_{41})$ $E := b\cdot\cos(\theta_{31})$

$A = 7.999$ $B = 6.999$ $D = -4.126$ $E = 0.142$

$C := a\cdot\alpha_2\cdot\sin(\theta_2) + a\cdot\omega_2{}^2\cdot\cos(\theta_2) + b\cdot\omega_{31}{}^2\cdot\cos(\theta_{31}) - c\cdot\omega_{41}{}^2\cdot\cos(\theta_{41})$

$C = 244.045$

$F := a\cdot\alpha_2\cdot\cos(\theta_2) - a\cdot\omega_2{}^2\cdot\sin(\theta_2) - b\cdot\omega_{31}{}^2\cdot\sin(\theta_{31}) + c\cdot\omega_{41}{}^2\cdot\sin(\theta_{41})$

$F = -223.741$

$\alpha_{31} := \frac{C\cdot D - A\cdot F}{A\cdot E - B\cdot D}$ $\alpha_{31} = 26.080$ $\alpha_{41} := \frac{C\cdot E - B\cdot F}{A\cdot E - B\cdot D}$ $\alpha_{41} = 53.331$

CROSS $A := c\cdot\sin(\theta_{42})$ $B := b\cdot\sin(\theta_{32})$ $D := c\cdot\cos(\theta_{42})$ $E := b\cdot\cos(\theta_{32})$

$A = -5.333$ $B = -6.333$ $D = -7.250$ $E = -2.982$

$C := a\cdot\alpha_2\cdot\sin(\theta_2) + a\cdot\omega_2{}^2\cdot\cos(\theta_2) + b\cdot\omega_{32}{}^2\cdot\cos(\theta_{32}) - c\cdot\omega_{42}{}^2\cdot\cos(\theta_{42})$

$C = 223.253$

$F := a\cdot\alpha_2\cdot\cos(\theta_2) - a\cdot\omega_2{}^2\cdot\sin(\theta_2) - b\cdot\omega_{32}{}^2\cdot\sin(\theta_{32}) + c\cdot\omega_{42}{}^2\cdot\sin(\theta_{42})$

$F = -135.002$

$\alpha_{32} := \frac{C\cdot D - A\cdot F}{A\cdot E - B\cdot D}$ $\alpha_{32} = 77.920$ $\alpha_{42} := \frac{C\cdot E - B\cdot F}{A\cdot E - B\cdot D}$ $\alpha_{42} = 50.669$

7. Use equations 7.36 and 7.37 to determine the angular jerk of links 3 and 4 for the open and crossed circuits.

OPEN $A := a\cdot\omega_2{}^3\cdot\sin(\theta_2)$ $D := b\cdot\omega_{31}{}^3\cdot\sin(\theta_{31})$ $G := 3\cdot c\cdot\omega_{41}\cdot\alpha_{41}\cdot\cos(\theta_{41})$

$B := 3\cdot a\cdot\omega_2\cdot\alpha_2\cdot\cos(\theta_2)$ $E := 3\cdot b\cdot\omega_{31}\cdot\alpha_{31}\cdot\cos(\theta_{31})$ $H := c\cdot\sin(\theta_{41})$

$C := a\cdot\phi_2\cdot\sin(\theta_2)$ $F := c\cdot\omega_{41}{}^3\cdot\sin(\theta_{41})$ $K := b\cdot\sin(\theta_{31})$

$L := a \cdot \omega_2{}^3 \cdot cos(\theta_2)$ $\quad P := b \cdot \omega_{31}{}^3 \cdot cos(\theta_{31})$ $\quad S := c \cdot \omega_{41}{}^3 \cdot cos(\theta_{41})$

$M := 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot sin(\theta_2)$ $\quad Q := 3 \cdot b \cdot \omega_{31} \cdot \alpha_{31} \cdot sin(\theta_{31})$ $\quad T := 3 \cdot c \cdot \omega_{41} \cdot \alpha_{41} \cdot sin(\theta_{41})$

$N := a \cdot \phi_2 \cdot cos(\theta_2)$ $\quad R := b \cdot cos(\theta_{31})$ $\quad U := c \cdot cos(\theta_{41})$

$$\phi_{41} := \frac{\begin{bmatrix} K \cdot (N - L - M - P - Q + S + T) \ldots \\ + R \cdot (A - B - C + D - E - F + G) \end{bmatrix}}{K \cdot U - H \cdot R} \qquad \phi_{41} = 749.012$$

$$\phi_{31} := \frac{A - B - C + D - E - F + G + H \cdot \phi_{41}}{K} \qquad \phi_{31} = 1242.6$$

$$\mathbf{J_A} := -a \cdot \omega_2{}^3 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2)) \ldots \\ + a \cdot \phi_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2))$$

$$\mathbf{J_{BA1}} := -b \cdot \omega_{31}{}^3 \cdot (-sin(\theta_{31}) + j \cdot cos(\theta_{31})) - 3 \cdot b \cdot \omega_{31} \cdot \alpha_{31} \cdot (cos(\theta_{31}) + j \cdot sin(\theta_{31})) \ldots \\ + b \cdot \phi_{31} \cdot (-sin(\theta_{31}) + j \cdot cos(\theta_{31}))$$

$\mathbf{J_{B1}} := \mathbf{J_A} + \mathbf{J_{BA1}}$

$J_{B1} := |\mathbf{J_{B1}}|$ $\quad J_{B1x} := Re(\mathbf{J_{B1}})$ $\quad J_{B1y} := Im(\mathbf{J_{B1}})$

$J_{B1} = 9301.9$ $\quad J_{B1x} = -9134.7$ $\quad J_{B1y} = 1755.5$

CROSSED $A := a \cdot \omega_2{}^3 \cdot sin(\theta_2)$ $\quad D := b \cdot \omega_{32}{}^3 \cdot sin(\theta_{32})$ $\quad G := 3 \cdot c \cdot \omega_{42} \cdot \alpha_{42} \cdot cos(\theta_{42})$

$B := 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot cos(\theta_2)$ $\quad E := 3 \cdot b \cdot \omega_{32} \cdot \alpha_{32} \cdot cos(\theta_{32})$ $\quad H := c \cdot sin(\theta_{42})$

$C := a \cdot \phi_2 \cdot sin(\theta_2)$ $\quad F := c \cdot \omega_{42}{}^3 \cdot sin(\theta_{42})$ $\quad K := b \cdot sin(\theta_{32})$

$L := a \cdot \omega_2{}^3 \cdot cos(\theta_2)$ $\quad P := b \cdot \omega_{32}{}^3 \cdot cos(\theta_{32})$ $\quad S := c \cdot \omega_{42}{}^3 \cdot cos(\theta_{42})$

$M := 3 \cdot a \cdot \omega_2 \cdot \alpha_2 \cdot sin(\theta_2)$ $\quad Q := 3 \cdot b \cdot \omega_{32} \cdot \alpha_{32} \cdot sin(\theta_{32})$ $\quad T := 3 \cdot c \cdot \omega_{42} \cdot \alpha_{42} \cdot sin(\theta_{42})$

$N := a \cdot \phi_2 \cdot cos(\theta_2)$ $\quad R := b \cdot cos(\theta_{32})$ $\quad U := c \cdot cos(\theta_{42})$

$$\phi_{42} := \frac{\begin{bmatrix} K \cdot (N - L - M - P - Q + S + T) \ldots \\ + R \cdot (A - B - C + D - E - F + G) \end{bmatrix}}{K \cdot U - H \cdot R} \qquad \phi_{42} = -246.639$$

$$\phi_{32} := \frac{A - B - C + D - E - F + G + H \cdot \phi_{42}}{K} \qquad \phi_{32} = -740.2$$

$$\mathbf{J_{BA2}} := -b \cdot \omega_{32}{}^3 \cdot (-sin(\theta_{32}) + j \cdot cos(\theta_{32})) - 3 \cdot b \cdot \omega_{32} \cdot \alpha_{32} \cdot (cos(\theta_{32}) + j \cdot sin(\theta_{32})) \ldots \\ + b \cdot \phi_{32} \cdot (-sin(\theta_{32}) + j \cdot cos(\theta_{32}))$$

$\mathbf{J_{B2}} := \mathbf{J_A} + \mathbf{J_{BA2}}$

$J_{B2} := |\mathbf{J_{B2}}|$ $\quad J_{B2x} := Re(\mathbf{J_{B2}})$ $\quad J_{B2y} := Im(\mathbf{J_{B2}})$

$J_{B2} = 4178.7$ $\quad J_{B2x} = -4147.9$ $\quad J_{B2y} = -506.4$

PROBLEM 7-12

Statement: You are riding on a carousel that is rotating at a constant 15 rpm. It has an inside radius of 3 ft and an outside radius of 10 ft. You begin to run from the inside to the outside along a radius. Your peak velocity with respect to the carousel is 5 mph and occurs at a radius of 7 ft. What is your maximum Coriolis acceleration magnitude and its direction with respect to the carousel?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given:

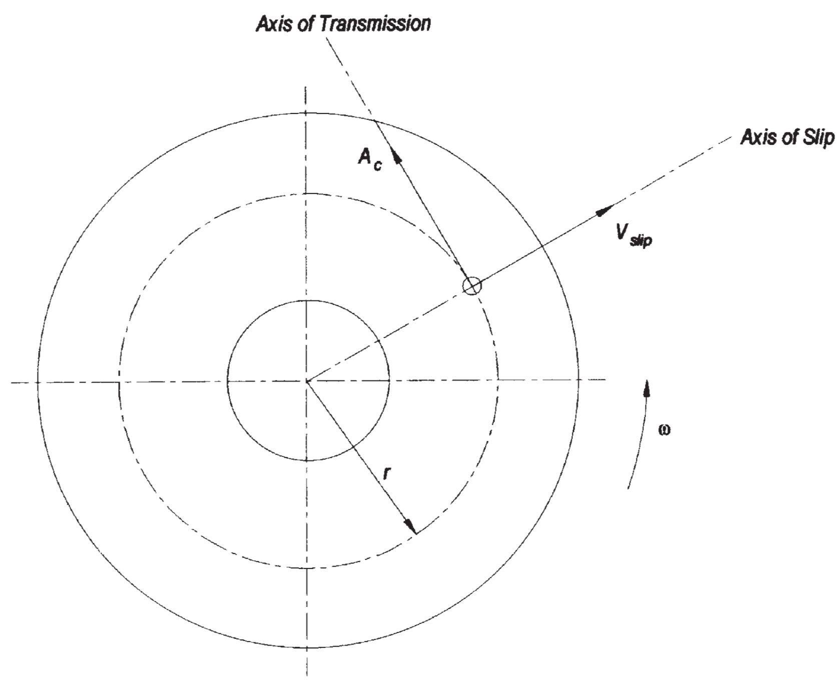
Carousel angular velocity $\omega := 15\cdot rpm$ $\omega = 1.571\dfrac{rad}{sec}$

Peak velocity $V_{slip} := 5\cdot mph$ $V_{slip} = 7.333\dfrac{ft}{sec}$

Radius at peak velocity $r := 7\cdot ft$

Solution: See Mathcad file P0712.

1. Draw a plan view of the carousel floor showing your position at peak velocity and the velocity and Coriolis acceleration vectors.



2. The direction of your path defines the axis of slip. The transmission axis is perpendicular to the axis of slip and positive in the direction of the tangential velocity of the carousel. Thus, the direction of the Coriolis acceleration vector is along the positive transmission axis.

3. Use equation 7.19 to calculate the magnitude of the Coriolis component of your acceleration.

$A_c := 2\cdot V_{slip}\cdot\omega$ $A_c = 23.038\dfrac{ft}{sec^2}$ $A_c = 276.46\dfrac{in}{sec^2}$

PROBLEM 7-13a

Statement: The linkage in Figure P7-5a has the dimensions and crank angle given below. Find α_3, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\omega_2 = 15$ rad/sec and $\alpha_2 = 10$ rad/sec^2 in the direction shown. Use the acceleration difference graphical method.

Given:

Link lengths: Link 2 $a := 0.8 \cdot in$ Link 3 $b := 1.93 \cdot in$

Offset: $c := -0.38 \cdot in$

Coupler point data: $p := 1.33 \cdot in$ $\delta_3 := 38.6 \cdot deg$

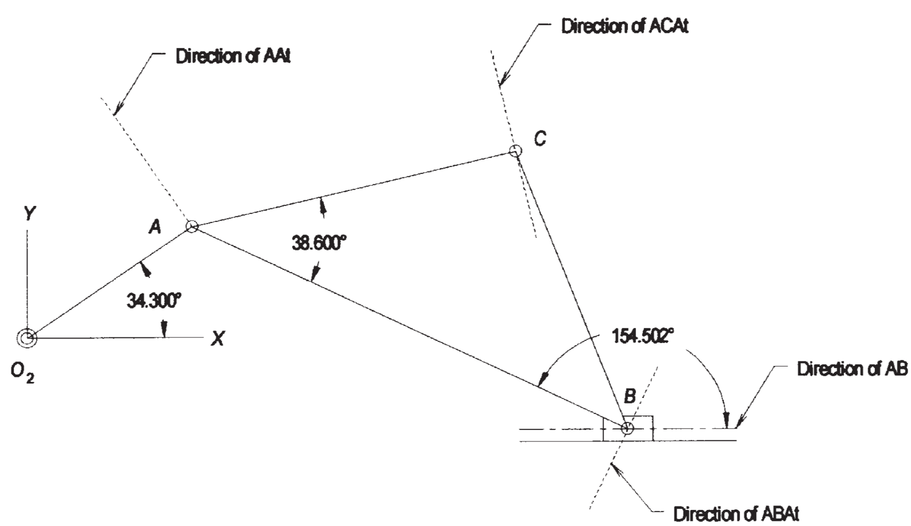
Link 2 position, velocity, and acceleration: $\theta_2 := 34.3 \cdot deg$ $\omega_2 := 15 \cdot \dfrac{rad}{sec}$ $\alpha_2 := 10 \cdot \dfrac{rad}{sec^2}$

Solution: See Figure P7-5a and Mathcad file P0713a.

1. In order to solve for the accelerations at points *A, B* and *C*, we will need θ_3, and ω_3. From the graphical position solution below,

$$\theta_3 := 154.502 \cdot deg$$

Using equation 6.22a, $\omega_3 := \dfrac{a \cdot \omega_2}{b} \cdot \dfrac{cos(\theta_2)}{cos(\theta_3)}$ $\omega_3 = -5.691\, rad \cdot sec^{-1}$



2. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

3. For point *B*, this becomes: $\mathbf{A}_B = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$A_{An} := a \cdot \omega_2{}^2$ $A_{An} = 180.000\, in \cdot sec^{-2}$

$\theta_{AAn} := \theta_2 + 180 \cdot deg$ $\theta_{AAn} = 214.300\, deg$

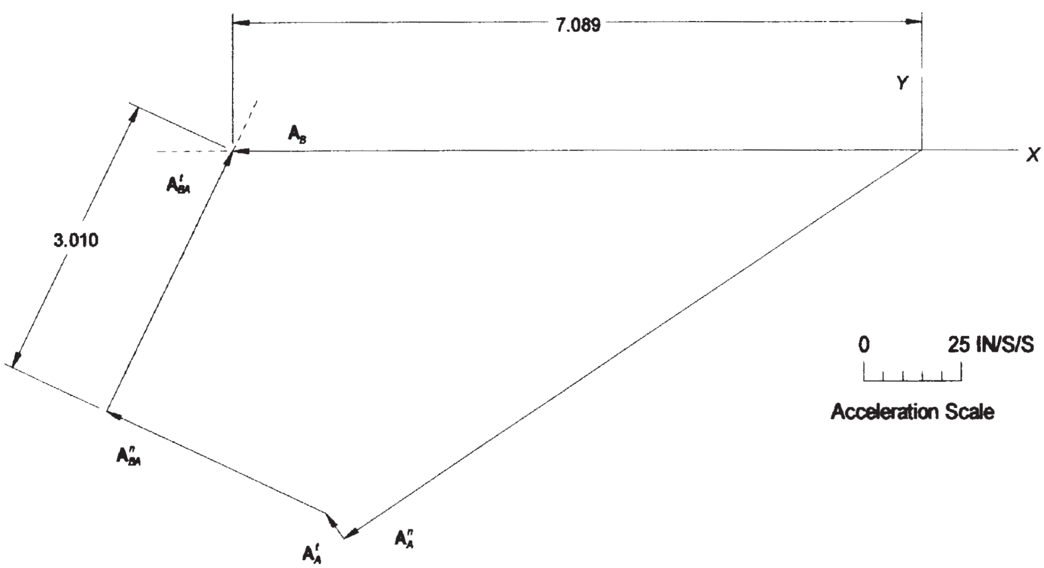
$A_{At} := a \cdot \alpha_2$ $A_{At} = 8.000\, in \cdot sec^{-2}$

$\theta_{AAt} := \theta_2 + 90 \cdot deg$ $\theta_{AAt} = 124.300\, deg$

$A_{BAn} := b \cdot \omega_3^{\,2}$ $\qquad A_{BAn} = 62.500\, in \cdot sec^{-2}$

$\theta_{ABAn} := \theta_3$ $\qquad \theta_{ABAn} = 154.502\, deg$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_A{}^n$ at an angle of θ_{AAn}. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of θ_{AAt}. From the tip of $\mathbf{A}_A{}^t$, draw $\mathbf{A}_{BA}{}^n$ at an angle of θ_{ABAn}. Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B$ (horizontal) and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{BA}{}^t$, and $\mathbf{A}_B$.



5. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 25 \cdot \dfrac{in}{sec^2}$

$A_B := 7.089 \cdot k_a$ $\qquad A_B = 177.2\, in \cdot sec^{-2}$ $\qquad$ at an angle of 180 deg

$A_{BAt} := 3.010 \cdot k_a$ $\qquad A_{BAt} = 75.3\, in \cdot sec^{-2}$

6. Calculate α_3 using equation 7.6.

$\alpha_3 := \dfrac{A_{BAt}}{b}$ $\qquad \alpha_3 = 38.990\, rad \cdot sec^{-2}$ $\quad$ CCW

7. For point C, equation 7.4 becomes: $\mathrm{AC} = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{CA}{}^t + \mathbf{A}_{CA}{}^n)$, where

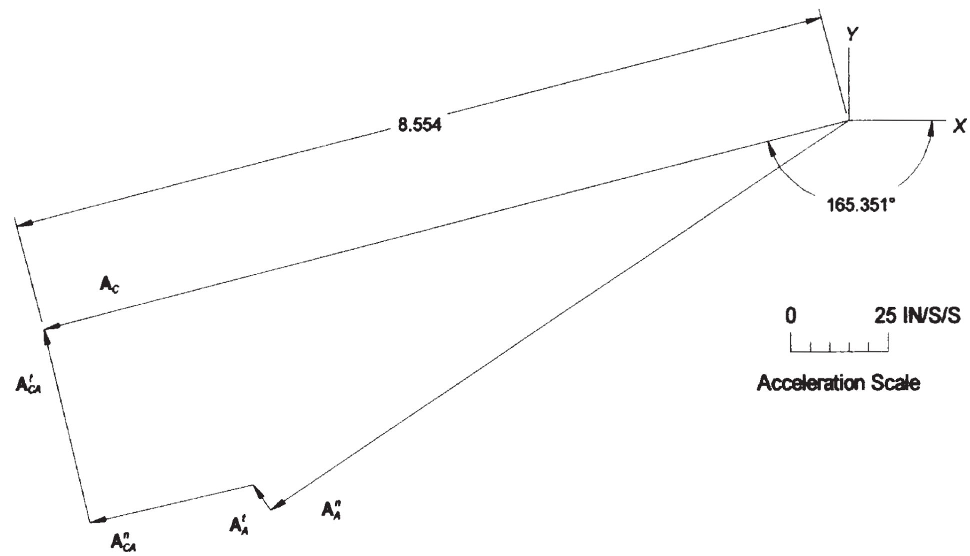
$A_{CAn} := p \cdot \omega_3^{\,2}$ $\qquad A_{CAn} = 43.070\, in \cdot sec^{-2}$

$\theta_{APAn} := \theta_3 + \delta_3$ $\qquad \theta_{APAn} = 193.102\, deg$

$A_{CAt} := p \cdot \alpha_3$ $\qquad A_{CAt} = 51.856\, in \cdot sec^{-2}$

$\theta_{APAt} := (\theta_3 + \delta_3) - 90 \cdot deg$ $\qquad \theta_{APAt} = 103.102\, deg$

8. Repeat procedure of step 4 for the equation in step 7.



9. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 25 \cdot \dfrac{in}{sec^2}$

$A_C := 8.554 \cdot k_a \qquad A_C = 213.9\, in \cdot sec^{-2} \qquad$ at an angle of -165.35 deg

PROBLEM 7-13b

Statement: The linkage in Figure P7-5a has the dimensions and crank angle given below. Find α_3, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\omega_2 = 15$ rad/sec and $\alpha_2 = 10$ rad/sec^2 in the direction shown. Use an analytical method.

Given: Link lengths:

Link 2 $a := 0.8 \cdot in$ Link 3 $b := 1.93 \cdot in$

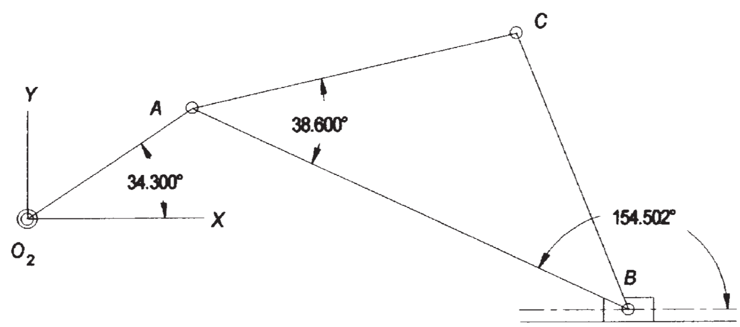
Offset: $c := -0.38 \cdot in$

Coupler point data: $p := 1.33 \cdot in$ $\delta_3 := 38.6 \cdot deg$

Link 2 position, velocity, and acceleration: $\theta_2 := 34.3 \cdot deg$ $\omega_2 := 15 \cdot \frac{rad}{sec}$ $\alpha_2 := 10 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-5a and Mathcad file P0713b.

1. Draw the linkage to scale and label it.



2. Determine θ_3 and d using equations 4.16 and 4.17.

$$\theta_3 := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_3 = 154.502\,deg$$

$$d := a \cdot cos(\theta_2) - b \cdot cos(\theta_3) \qquad d = 2.403\,in$$

3. Determine the angular velocity of link 3 using equation 6.22a.

$$\omega_3 := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3)} \cdot \omega_2 \qquad \omega_3 = -5.691\,\frac{rad}{sec}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$\mathbf{A_A} = -153.206 - 94.826j\,\frac{in}{sec^2} \qquad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := arg(\mathbf{A_A})$$

The acceleration of pin A is $A_A = 180\,\frac{in}{sec^2}$ at $\theta_{AA} = -148.2\,deg$

5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) + b \cdot \omega_3^{\,2} \cdot \sin(\theta_3)}{b \cdot \cos(\theta_3)} \qquad \alpha_3 = 38.990 \frac{rad}{sec^2}$$

6. Use equation 7.16e for the acceleration of pin *B*.

$$A_B := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \alpha_3 \cdot \sin(\theta_3) + b \cdot \omega_3^{\,2} \cdot \cos(\theta_3)$$

$$A_B = -177.2 \frac{in}{sec^2}$$ A negative sign means that $\mathbf{A_B}$ is to the left

7. Determine the acceleration of the coupler point *C* using equations 7.32. Note that θ_3 is defined from point *B* in Figure 7-6 and from point *A* in Figure 7-9. To use equation 7.32 for a slider-crank we must redefine θ_3.

$$\theta_3 := \theta_3 - 180 \cdot deg \qquad \theta_3 = -25.498\, deg$$

$$\mathbf{A_{CA}} := p \cdot \alpha_3 \cdot \left(-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)\right) \ldots$$
$$+ -p \cdot \omega_3^{\,2} \cdot \left(\cos(\theta_3 + \delta_3) + j \cdot \sin(\theta_3 + \delta_3)\right)$$

$$\mathbf{A_C} := \mathbf{A_A} + \mathbf{A_{CA}}$$

$$\mathbf{A_C} = -206.910 - 54.083j \frac{in}{sec^2} \qquad A_C := |\mathbf{A_C}| \qquad \theta_{AC} := arg(\mathbf{A_C})$$

The acceleration of point *C* is $A_C = 213.861 \frac{in}{sec^2}$ at $\theta_{AC} = -165.352\, deg$

PROBLEM 7-14a

Statement: The linkage in Figure P7-5b has the dimensions and effective crank angle given below. Find α_3, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for ω_2 = 15 rad/sec and α_2 = 10 rad/sec^2 in the directions shown. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (point of contact to *A*)	$a := 0.75 \cdot in$
Link 3 (*A* to *B*)	$b := 1.5 \cdot in$
Link 4 (point of contact to *B*)	$c := 0.75 \cdot in$
Link 1 (between contact points)	$d := 1.5 \cdot in$

Coupler point:

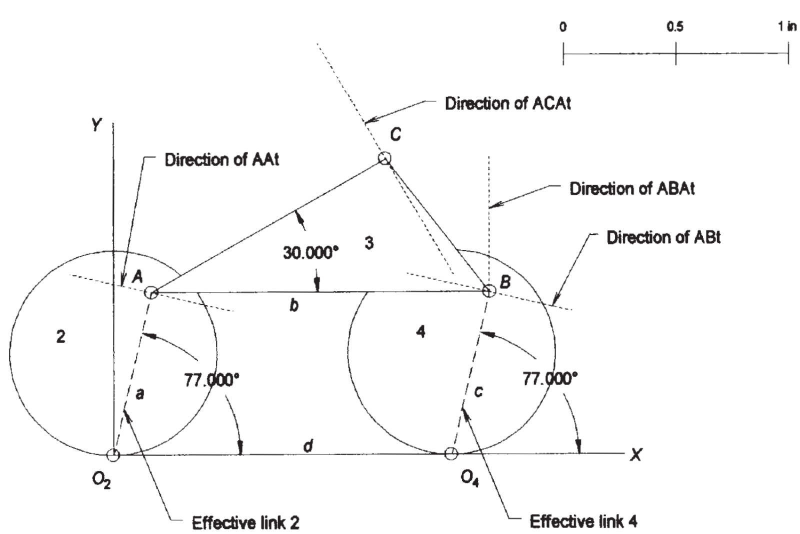
Distance *A* to *C*	$p := 1.2 \cdot in$
Angle *BAC*	$\delta := 30 \cdot deg$

Crank angle: $\theta_2 := 77 \cdot deg$

Input crank angular velocity $\omega_2 := 15 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 10 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-5b and Mathcad file P0714a.

1. Although the mechanism shown in Figure P6-5b is not entirely pin-jointed, it can be analyzed for the position shown by its effective pin-jointed fourbar, which is shown below. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points *A* and *B*, we will need θ_3, ω_3, and θ_4. From the graphical position solution above,

$$\theta_3 := 0.0 \cdot deg \qquad \theta_4 := 77.0 \cdot deg$$

This is a special-case Grashof in the parallelogram configuration. Therefore,

$$\omega_3 := 0.0 \cdot rad \cdot sec^{-1} \qquad \omega_4 := \omega_2$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

4. For point *B*, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$$A_{Bn} := c\cdot\omega_4{}^2 \qquad A_{Bn} = 168.750\, in\cdot sec^{-2}$$

$$\theta_{ABn} := \theta_4 + 180\cdot deg \qquad \theta_{ABn} = 257.000\, deg$$

$$A_{An} := a\cdot\omega_2{}^2 \qquad A_{An} = 168.750\, in\cdot sec^{-2}$$

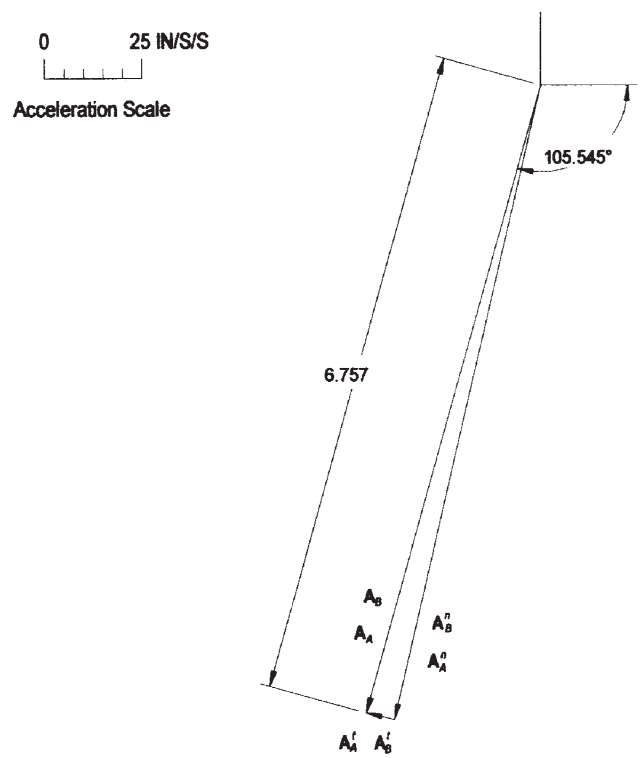
$$\theta_{AAn} := \theta_2 + 180\cdot deg \qquad \theta_{AAn} = 257.000\, deg$$

$$A_{At} := a\cdot\alpha_2 \qquad A_{At} = 7.500\, in\cdot sec^{-2}$$

$$\theta_{At} := \theta_2 + 90\cdot deg \qquad \theta_{At} = 167.000\, deg$$

$$A_{BAn} := b\cdot\omega_3{}^2 \qquad A_{BAn} = 0.000\, in\cdot sec^{-2}$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of θ_4 + 180 deg and $\mathbf{A}_A{}^n$ at an angle of θ_2 + 180 deg. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of θ_2 + 90 deg. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_A{}^t$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B{}^t$, $\mathbf{A}_{BA}{}^t$, and $\mathbf{A}_B$.



From the layout $\mathbf{A}_B{}^t = \mathbf{A}_A{}^t$ and $\mathbf{A}_{BA}{}^t$ = 0. Also, $\mathbf{A}_B = \mathbf{A}_A$.

Acceleration scale factor

$$k_a := 25\cdot\frac{in}{sec^2}$$

$$A_B := 6.757\cdot k_a$$

$$A_B = 168.9\, in\cdot sec^{-2}$$

at an angle of -105.55 deg

6. Since $\mathbf{A}_{BA}{}^t = 0$, $\alpha_3 = 0$ and, since $\mathbf{A}_B = \mathbf{A}_A$, $\alpha_4 = \alpha_2$.

7. For point *P*, equation 7.4 becomes: $\mathbf{A}_P = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$, where $\mathbf{A}_{PA}{}^t$ and $\mathbf{A}_{PA}{}^n$ are both zero since α_3 and ω_3 are zero. Therefore, $\mathbf{A}_P = \mathbf{A}_A$.

PROBLEM 7-14b

Statement: The linkage in Figure P7-5b has the dimensions and effective crank angle given below. Find α_3, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\omega_2 = 15$ rad/sec and $\alpha_2 = 10$ rad/sec² in the directions shown. Use an analytical method.

Given: Link lengths:

Link 2 (point of contact to A) $a := 0.75 \cdot in$ Link 3 (A to B) $b := 1.5 \cdot in$

Link 4 (point of contact to B) $c := 0.75 \cdot in$ Link 1 (between contact points) $d := 1.5 \cdot in$

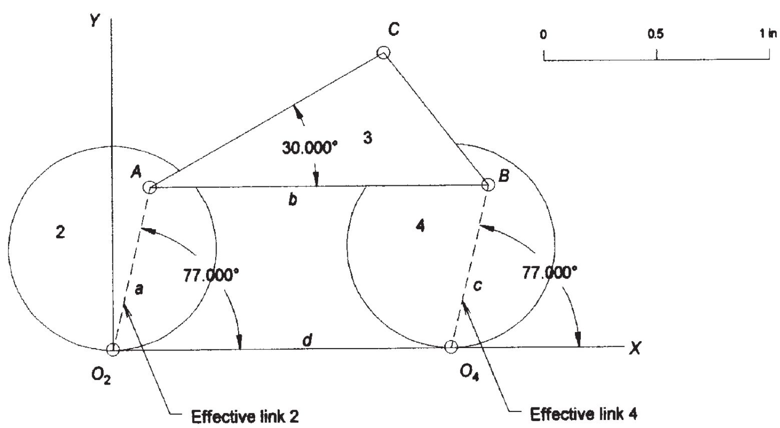
Coupler point:

Distance A to C $R_{ca} := 1.2 \cdot in$ Crank angle: $\theta_2 := 77 \cdot deg$ $\alpha_2 := 10 \cdot rad \cdot sec^{-2}$

Angle BAC $\delta_3 := 30 \cdot deg$ Input crank angular velocity $\omega_2 := 15 \cdot rad \cdot sec^{-1}$

Solution: See Figure P7-5b and Mathcad file P0714b.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.0000 \qquad K_2 = 2.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -1.2250 \qquad B = -1.9487 \qquad C = 2.3251$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_4 = -283.000\,deg$$

$$\theta_4 := \theta_4 + 360 \cdot deg \qquad \theta_4 = 77.000\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.0000 \qquad K_5 = -2.0000$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -3.5501$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.9487$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 0.0000$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3 := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_3 = 360.000\,deg$$

$$\theta_3 := \theta_3 - 360 \cdot deg \qquad \theta_3 = 0.000\,deg$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_3 = 0.000 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \qquad \omega_4 = 15.000 \frac{rad}{sec}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A_A} = -45.268 - 162.738j \frac{in}{sec^2} \qquad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := \arg(\mathbf{A_A})$$

The acceleration of pin A is $A_A = 168.917 \dfrac{in}{sec^2}$ at $\theta_{AA} = -105.5\,deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c \cdot \sin(\theta_4) \qquad B := b \cdot \sin(\theta_3) \qquad D := c \cdot \cos(\theta_4) \qquad E := b \cdot \cos(\theta_3)$$

$$A = 0.731\,in \qquad B = 0.000\,in \qquad D = 0.169\,in \qquad E = 1.500\,in$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 7.308\,in \cdot sec^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = 1.687\,in \cdot sec^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_3 = 0.000 \frac{rad}{sec^2} \qquad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_4 = 10.000 \frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point B.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-\sin(\theta_4) + j \cdot \cos(\theta_4)\right) - c \cdot \omega_4^{\,2} \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right)$$

$\mathbf{A_B} = -45.268 - 162.738j \, \dfrac{in}{sec^2}$ $\qquad A_B := \left|\mathbf{A_B}\right|$ $\qquad \theta_{AB} := arg(\mathbf{A_B})$

The acceleration of pin B is $\quad A_B = 168.917 \dfrac{in}{sec^2}$ at $\quad \theta_{AB} = -105.5\, deg$

10. Use equations 7.32 to find the acceleration of the point C.

$$\mathbf{A_{CA}} := R_{ca} \cdot \alpha_3 \cdot \left(-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)\right) \ldots$$
$$+ -R_{ca} \cdot \omega_3^{\,2} \cdot \left(\cos(\theta_3 + \delta_3) + j \cdot \sin(\theta_3 + \delta_3)\right)$$

$$\mathbf{A_C} := \mathbf{A_A} + \mathbf{A_{CA}}$$

$A_C := \left|\mathbf{A_C}\right|$ $\qquad A_C = 168.917 \dfrac{in}{sec^2}$ $\qquad arg(\mathbf{A_C}) = -105.545\, deg$

PROBLEM 7-15a

Statement: The linkage in Figure P7-5c has the dimensions and coupler angle given below. Find α_3, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\mathbf{V}_A$ = 10 in/sec and $\mathbf{A}_A$ = 15 in/sec² in the directions shown. Use the acceleration difference graphical method.

Given: Link lengths and angles:

Link 3 (*A* to *B*) $b := 1.8 \cdot in$

Coupler angle $\theta_3 := 128 \cdot deg$ Slider 4 angle $\theta_4 := 59 \cdot deg$

Coupler point:

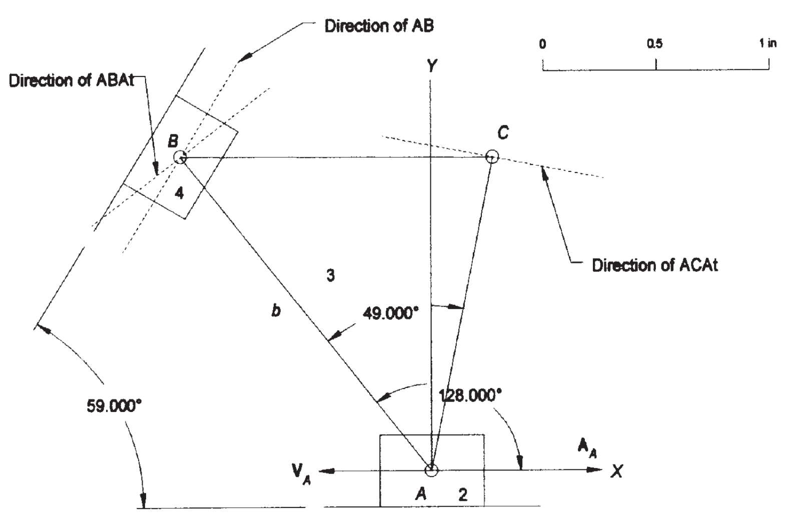
Distance *A* to *C* $p := 1.44 \cdot in$ Angle *BAC* $\delta_3 := -49 \cdot deg$

Input slider motion $V_A := -10 \cdot in \cdot sec^{-1}$ $A_A := 15 \cdot in \cdot sec^{-2}$

Solution: See Figure P7-5c and Mathcad file P0715a.

1. In order to solve for the accelerations at points *B* and *C*, we will need ω_3. From the layout below and Problem 6-18,

$$\theta_3 := 128 \cdot deg \qquad \omega_3 := -13.288 \cdot \frac{rad}{sec}$$



2. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

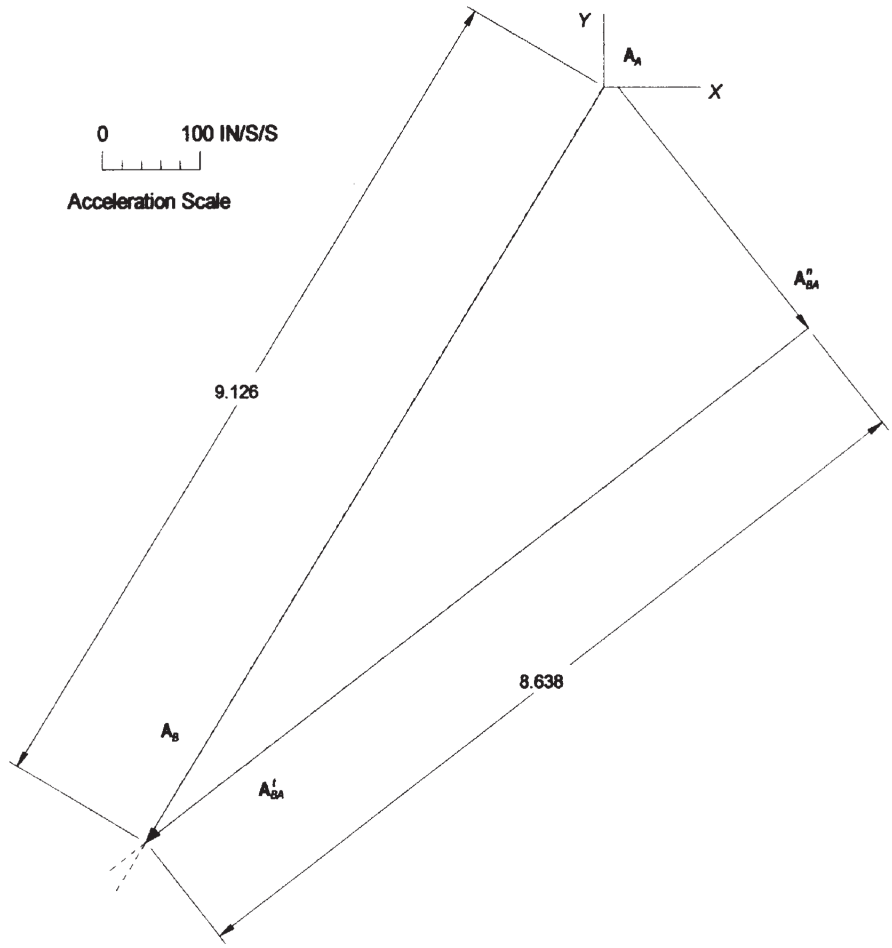
3. For point *B*, this becomes: $\mathbf{A}_B = \mathbf{A}_A + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$$A_A = 15.000\, in \cdot sec^{-2} \qquad \theta_{AA} := 0 \cdot deg$$

$$A_{BAn} := b \cdot \omega_3{}^2 \qquad A_{BAn} = 317.828\, in \cdot sec^{-2}$$

$$\theta_{ABAn} := \theta_3 + 180 \cdot deg \qquad \theta_{ABAn} = 308.000\, deg$$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_A$ at an angle of θ_{AA}. From the tip of $\mathbf{A}_A$, draw $\mathbf{A}_{BA}{}^n$ at an angle of θ_{ABAn}. Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{BA}{}^t$, and $\mathbf{A}_B$.



5. From the graphical solution above,

Acceleration scale factor $\quad k_a := 100\cdot\frac{in}{sec^2}$

$A_B := 9.126\cdot k_a \qquad A_B = 912.6\, in\cdot sec^{-2}$ at an angle of 239 deg

$A_{BAt} := 8.638\cdot k_a \qquad A_{BAt} = 863.8\, in\cdot sec^{-2}$

6. Calculate α_3 using equation 7.6.

$\alpha_3 := \frac{A_{BAt}}{b} \qquad \alpha_3 = 479.9\, rad\cdot sec^{-2}$ CW

7. For point C, equation 7.4 becomes: $\mathbf{A}_C = \mathbf{A}_A + (\mathbf{A}_{CA}{}^t + \mathbf{A}_{CA}{}^n)$, where

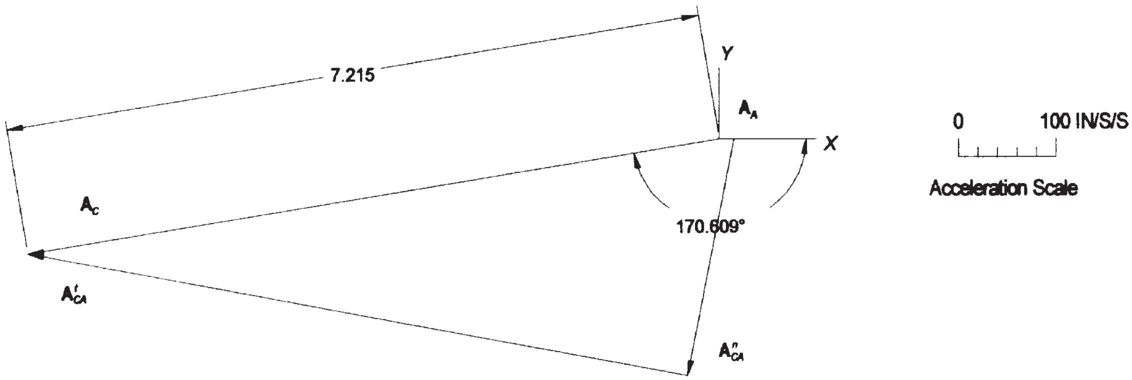
$A_{CAn} := p\cdot\omega_3^{\,2} \qquad A_{CAn} = 254.262\, in\cdot sec^{-2}$

$\theta_{APAn} := \theta_3 + \delta_3 + 180\cdot deg \qquad \theta_{APAn} = 259.000\, deg$

$A_{CAt} := p\cdot\alpha_3 \qquad A_{CAt} = 691.040\, in\cdot sec^{-2}$

$\theta_{APAt} := (\theta_3 + \delta_3) + 90\cdot deg \qquad \theta_{APAt} = 169.000\, deg$

8. Repeat procedure of step 4 for the equation in step 7.



9. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 100 \cdot \frac{in}{sec^2}$

$A_C := 7.215 \cdot k_a \qquad A_C = 721.5\, in \cdot sec^{-2}$ at an angle of -170.61 deg

PROBLEM 7-15b

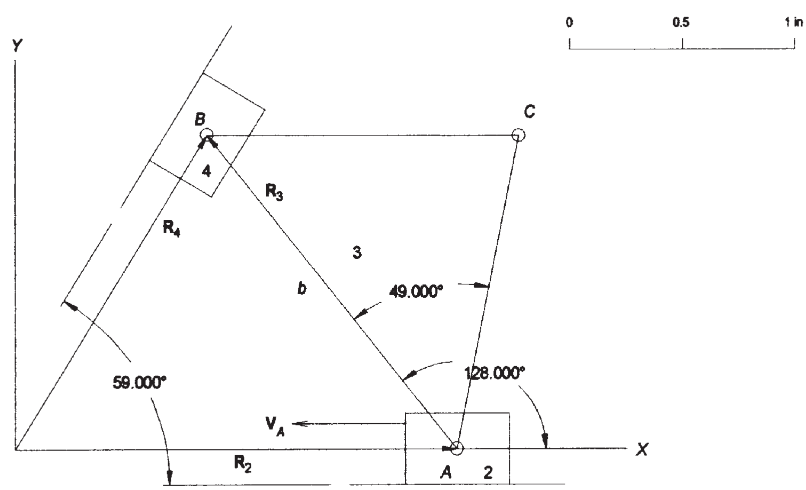
Statement: The linkage in Figure P7-5c has the dimensions and coupler angle given below. Find α_3, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\mathbf{V}_A$ = 10 in/sec and $\mathbf{A}_A$ = 15 in/sec^2 in the directions shown. Use an analytical method.

Given: Link lengths and angles:

Link 3 (*A* to *B*)	$b := 1.8 \cdot in$	
Coupler angle	$\theta_3 := 128 \cdot deg$	
Slider 4 angle	$\theta_4 := 59 \cdot deg$	
Coupler point:		
Distance *A* to *C*	$R_{ca} := 1.44 \cdot in$	
Angle *BAC*	$\delta_3 := -49 \cdot deg$	
Input slider motion	$V_A := -10 \cdot in \cdot sec^{-1}$	$A_A := 15 \cdot in \cdot sec^{-2}$

Solution: See Figure P7-5c and Mathcad file P0715b.

1. Draw the mechanism to scale and define a vector loop using the fourbar slider-crank derivation in Section 7.3 as a model.



2. Write the vector loop equation, differentiate it, expand the result and separate into real and imaginary parts to solve for ω_3 and $\mathbf{V}_B$.

$$\mathbf{R_2} + \mathbf{R_3} := \mathbf{R_4}$$

$$a \cdot e^{j \cdot \theta_2} + b \cdot e^{j \cdot \theta_3} := c \cdot e^{j \cdot \theta_4}$$

where a is the distance from the origin to point A, a variable; b is the distance from A to B, a constant; and c is the distance from the origin to point B, a variable. Angle θ_2 is zero, θ_3 is the angle that AB makes with the x axis, and θ_4 is the constant angle that slider 4 makes with the x axis. Differentiating,

$$\frac{d}{dt}a + j \cdot b \cdot \omega_3 \cdot e^{j \cdot \theta_3} := \left(\frac{d}{dt}c\right) \cdot e^{j \cdot \theta_4}$$

Substituting the Euler equivalents,

$$\frac{d}{dt}a + b\cdot\omega_3\cdot\left(-\sin(\theta_3) + j\cdot\cos(\theta_3)\right) := \left(\frac{d}{dt}c\right)\cdot\left(\cos(\theta_4) + j\cdot\sin(\theta_4)\right)$$

Separating into real and imaginary components and solving for ω_3. Note that $dc/dt = V_B$ and $da/dt = V_A$

$$\omega_3 := \frac{V_A\cdot\tan(\theta_4)}{b\cdot\left(\sin(\theta_3)\cdot\tan(\theta_4) + \cos(\theta_3)\right)} \qquad \omega_3 = -13.288\,\frac{rad}{sec}$$

3. Differentiate the velocity equation, expand it and solve for α_3 and A_B.

$$\frac{d^2}{dt^2}\left(a + b\cdot\alpha_3\cdot j\cdot e^{j\cdot\theta_3} + b\cdot\omega_3^{\,2}\cdot j\cdot e^{j\cdot\theta_3}\right) := \frac{d^2}{dt^2}c\cdot e^{j\cdot\theta_4}$$

Substituting the Euler equivalents,

$$\frac{d^2}{dt^2}a + b\cdot\alpha_3\left(-\sin(\theta_3) + j\cdot\cos(\theta_3)\right)\ \ldots \qquad := 0$$
$$+ -b\cdot\omega_3^{\,2}\cdot\left(\cos(\theta_3) + j\cdot\sin(\theta_3)\right) - \frac{d^2}{dt^2}c\cdot\left(\cos(\theta_4) + j\cdot\sin(\theta_4)\right)$$

Separating into real and imaginary components and solving for α_3 and A_B. Note that $d^2c/dt^2 = A_B$ and $d^2a/dt^2 = A_A$

$$\alpha_3 := \frac{A_A\cdot\sin(\theta_4) - b\cdot\omega_3^{\,2}\cdot\sin(\theta_4 - \theta_3)}{b\cdot\cos(\theta_4 - \theta_3)} \qquad \alpha_3 = 479.924\,\frac{rad}{sec^2}$$

$$A_B := \frac{b\cdot\alpha_3\cdot\cos(\theta_3) - b\cdot\omega_3^{\,2}\cdot\sin(\theta_3)}{\sin(\theta_4)} \qquad A_B = -912.662\,\frac{in}{sec^2}$$

4. Determine the acceleration of the coupler point C using equations 7.32.

$$\mathbf{A_{CA}} := R_{ca}\cdot\alpha_3\cdot\left(-\sin(\theta_3 + \delta_3) + j\cdot\cos(\theta_3 + \delta_3)\right) - R_{ca}\cdot\omega_3^{\,2}\cdot\left(\cos(\theta_3 + \delta_3) + j\cdot\sin(\theta_3 + \delta_3)\right)$$

$$\mathbf{A_{CA}} = -726.910 - 117.729j\,\frac{in}{sec^2}$$

$$\mathbf{A_A} := A_A \qquad \mathbf{A_C} := \mathbf{A_A} + \mathbf{A_{CA}}$$

$$\mathbf{A_C} = -711.910 - 117.729j\,\frac{in}{sec^2} \qquad \left|\mathbf{A_C}\right| = 721.579\,\frac{in}{sec^2} \qquad \arg(\mathbf{A_C}) = -170.610\,deg$$

PROBLEM 7-16

Statement: For the linkage shown in Figure P7-6a, write the vector loop equations; differentiate them, and do a complete position, velocity, and acceleration analysis of the linkage. Measure the linkage geometry from the figure. Assume $\omega_2 = 10$ rad/sec and $\alpha_2 = 20$ rad/sec^2.

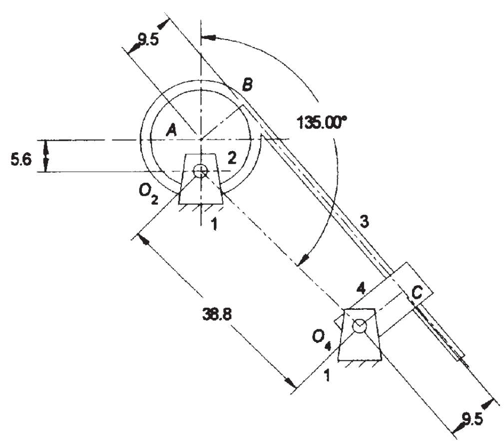
Given: Link lengths and angles:

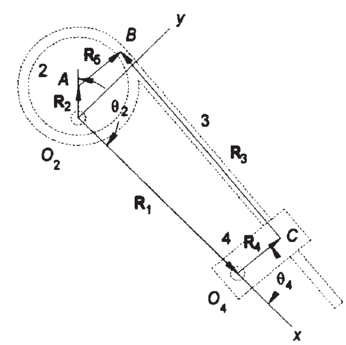
Link 2 (O_2 to A)	$a := 5.6 \cdot mm$	Link 4 (O_4 to C)	$c := 9.5 \cdot mm$
Link 3 offset (A to B)	$f := 9.5 \cdot mm$	Link 1 (O_2 to O_4)	$d := 38.8 \cdot mm$

Link 2 position, velocity and acceleration $\theta_2 := 135 \cdot deg$ $\quad \omega_2 := 10 \cdot \frac{rad}{sec}$ $\quad \alpha_2 := 20 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-6a and Mathcad file P0716.

1. Draw the mechanism to scale and define a vector loop using the fourbar inverted slider-crank derivation in Section 7.3 as a model.





2. Write the vector loop equation, expand the result and separate into real and imaginary parts to solve for θ_3 and b.

$$\mathbf{R_2} + \mathbf{R_5} := \mathbf{R_1} + \mathbf{R_4} + \mathbf{R_3}$$

$$a \cdot e^{j \cdot \theta_2} + f \cdot e^{j \cdot \theta_5} := d \cdot e^{j \cdot \theta_1} + c \cdot e^{j \cdot \theta_4} + b \cdot e^{j \cdot \theta_3}$$

where a is the distance from pivot O_2 to point A, a constant; f is the distance from A to B, a constant; b is the distance from B to C, a variable; and c is the distance from pivot O_4 to point C, a constant. Angle θ_2 is the input crank angle, θ_3 is the angle that BC makes with the x axis (measured from C), and θ_4 is the variable angle that link 4 makes with the x axis. The angular relationships are

$$\theta_5 := \theta_4 \qquad \text{and} \qquad \theta_3 := \theta_4 + 90 \cdot deg$$

Substituting the Euler equivalents,

$$a \cdot \left(\cos(\theta_2) + j \cdot \sin(\theta_2)\right) + f \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right) := d + c \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right) \ldots$$
$$+ b \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right)$$

If we stipulate that $f = c$, this reduces to

$$a \cdot \left(\cos(\theta_2) + j \cdot \sin(\theta_2)\right) := d + b \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right)$$

Separating into real and imaginary components and solving for θ_3 and b.

$$\theta_3 := atan2(a\cdot cos(\theta_2) - d, a\cdot sin(\theta_2)) \qquad \theta_3 = 174.7\,deg$$

$$\theta_4 := \theta_3 - 90\cdot deg \qquad \theta_4 = 84.7\,deg$$

$$b := \frac{a\cdot sin(\theta_2)}{sin(\theta_3)} \qquad b = 42.9\,mm$$

3. Differentiate the position equation to get the velocity equation.

Position (f and c terms cancel) $\quad a\cdot e^{j\cdot\theta_2} := d + b\cdot e^{j\cdot\theta_3}$

Velocity $\quad a\cdot j\cdot\omega_2\cdot e^{j\cdot\theta_2} := bdot\cdot e^{j\cdot\theta_3} + b\cdot j\cdot\omega_3\cdot e^{j\cdot\theta_3}$

Separating into real and imaginary components and solving for ω_3 and *bdot*.

$$\omega_3 := \frac{a\cdot cos(\theta_2 - \theta_3)}{b}\cdot\omega_2 \qquad \omega_3 = 1.00\frac{rad}{sec}$$

$$bdot := \frac{b\cdot\omega_3\cdot sin(\theta_3) - a\cdot\omega_2\cdot sin(\theta_2)}{cos(\theta_3)} \qquad bdot = 35.8\frac{mm}{sec}$$

4. Differentiate the velocity equation to get the acceleration equation.

Velocity $\quad a\cdot j\cdot\omega_2\cdot e^{j\cdot\theta_2} := bdot\cdot e^{j\cdot\theta_3} + b\cdot j\cdot\omega_3\cdot e^{j\cdot\theta_3}$

Acceleration $\quad a\cdot j\cdot\alpha_2\cdot e^{j\cdot\theta_2} + a\cdot j^2\cdot\omega_2{}^2\cdot e^{j\cdot\theta_2} := bddot\cdot e^{j\cdot\theta_3} + bdot\cdot j\cdot\omega_3\cdot e^{j\cdot\theta_3} \ldots$
$\quad + b\cdot j\cdot\alpha_3\cdot e^{j\cdot\theta_3} + b\cdot j^2\cdot\omega_3{}^2\cdot e^{j\cdot\theta_3}$

Separating into real and imaginary components and solving for α_3 and *bddot*.

$$\alpha_3 := \frac{1}{b}\cdot\left(a\cdot\alpha_2\cdot cos(\theta_2 - \theta_3) - a\cdot\omega_2{}^2\cdot sin(\theta_2 - \theta_3) - bdot\cdot\omega_3\right)$$

$$\alpha_3 = 9.50\frac{rad}{sec^2}$$

$$bddot := \frac{1}{sin(\theta_3)}\cdot\left[\begin{array}{l} a\cdot\left(\alpha_2\cdot cos(\theta_2) - \omega_2{}^2\cdot sin(\theta_2)\right) \ldots \\ + -bdot\cdot\omega_3\cdot cos(\theta_3) - b\cdot\left(\alpha_3\cdot cos(\theta_3) - \omega_3{}^2\cdot sin(\theta_3)\right)\end{array}\right]$$

$$bddot = -316.0\frac{mm}{sec^2}$$

PROBLEM 7-17

Statement: For the linkage shown in Figure P7-6b, write the vector loop equations; differentiate them, and do a complete position, velocity, and acceleration analysis of the linkage. Use the linkage geometry given below. Assume $\omega_2 = 10$ rad/sec and $\alpha_2 = 20$ rad/sec^2.

Given: Measured lengths and angles:

Link 1	$d := 61.9 \cdot mm$	Link 2	$a := 15.0 \cdot mm$	
Link 3	$b := 45.8 \cdot mm$	Link 4	$c := 18.1 \cdot mm$	
Link 5	$e := 23.1 \cdot mm$	Offset	$f := -2.6 \cdot mm$	from x' axis

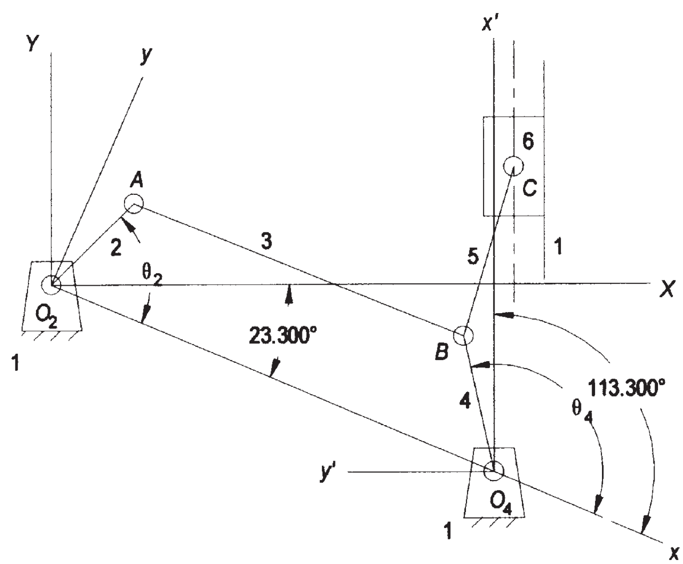
Crank angle: $\theta_{20} := 45 \cdot deg$ Global XY system

Coordinate rotation angles
$\alpha := -23.3 \cdot deg$ Global XY system to local xy system
$\beta := 113.3 \cdot deg$ Local xy system to local $x'y'$ system
$\gamma := 90 \cdot deg$ Global XY system to local $x'y'$ system

Link 2 position, velocity and acceleration $\theta_2 := 68.3 \cdot deg$ $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := 20 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-6b and Mathcad file P0717.

1. Draw the mechanism to scale and label it.



2. This mechanism can be analyzed as a pin-jointed fourbar (links 1, 2, 3, and 4) and a fourbar slider-crank (links 1, 4, 5, and 6). Link 4 is the common, non-stationary link in the two branches and is the input to the slider-crank. Since the vector loops are the same as those defined for the pin-jointed and slider-crank linkages, we can use the equations derived in the text with slight modification of variable names for the slider-crank.

   There are three coordinate systems: the global XY frame, the local xy frame for the pin-jointed fourbar, and the local $x'y'$ frame for the slider crank. Inputs to the fourbars must be in their respective local coordinates. **All output will be given in local coordinate system.**

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$K_1 := \frac{d}{a}$ $K_1 = 4.1267$ $K_2 := \frac{d}{c}$ $K_2 = 3.4199$

$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$ $K_3 = 4.2110$

$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$ $A = -0.8104$

$B := -2 \cdot \sin(\theta_2)$ $B = -1.8583$

$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$ $C = 6.7034$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \operatorname{atan2}\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right) + 360 \cdot deg \qquad \theta_4 = 125.692\,deg$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$K_4 := \frac{d}{b}$ $K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)}$ $K_4 = 1.3515$ $K_5 = -4.2406$

$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$ $D = -7.4978$

$E := -2 \cdot \sin(\theta_2)$ $E = -1.8583$

$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$ $F = 0.0160$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) + 360 \cdot deg \qquad \theta_3 = 0.955\,deg$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_3 = -3.357 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \qquad \omega_4 = 9.307 \frac{rad}{sec}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_B$. Determine the magnitude, and direction (in the local coordinate system).

$$\mathbf{A_B} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$\mathbf{A_B} = -833 - 1283j \frac{mm}{sec^2}$ $A_B := |\mathbf{A_B}|$ $\theta_{AB} := \arg(\mathbf{A_B})$

The acceleration of pin B is $A_B = 1530 \frac{mm}{sec^2}$ at $\theta_{AB} = -123.01\,deg$

9. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$A := c \cdot \sin(\theta_4)$ $B := b \cdot \sin(\theta_3)$ $D := c \cdot \cos(\theta_4)$ $E := b \cdot \cos(\theta_3)$

$A = 14.700\,mm$ $B = 0.763\,mm$ $D = -10.560\,mm$ $E = 45.794\,mm$

$$C := a\cdot\alpha_2\cdot\sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot\cos(\theta_2) + b\cdot\omega_3^{\,2}\cdot\cos(\theta_3) - c\cdot\omega_4^{\,2}\cdot\cos(\theta_4)$$

$$C = 2.264\times 10^3\,mm\cdot sec^{-2}$$

$$F := a\cdot\alpha_2\cdot\cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot\sin(\theta_2) - b\cdot\omega_3^{\,2}\cdot\sin(\theta_3) + c\cdot\omega_4^{\,2}\cdot\sin(\theta_4)$$

$$F = -18.160\,mm\cdot sec^{-2}$$

$$\alpha_3 := \frac{C\cdot D - A\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_3 = -34.706\,\frac{rad}{sec^2} \qquad \alpha_4 := \frac{C\cdot E - B\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_4 = 152.221\,\frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point C.

$$\mathbf{A_C} := c\cdot\alpha_4\cdot(-\sin(\theta_4) + j\cdot\cos(\theta_4)) - c\cdot\omega_4^{\,2}\cdot(\cos(\theta_4) + j\cdot\sin(\theta_4))$$

$$\mathbf{A_C} = -1323 - 2881j\,\frac{mm}{sec^2} \qquad A_C := |\mathbf{A_C}| \qquad \theta_{AC} := arg(\mathbf{A_C})$$

The acceleration of pin C is $A_C = 3170\,\frac{mm}{sec^2}$ at $\theta_{AC} = -114.7\,deg$

10. Determine θ_5 and the vertical distance from O_4 to C (d') for the slider-crank using equations 4.16 and 4.17.

$$\theta_5 := asin\left(\frac{c\cdot\sin(\theta_4 - \beta) - f}{e}\right) + \pi \qquad \theta_5 = 163.7\,deg$$

$$d' := c\cdot\cos(\theta_4 - \beta) - e\cdot\cos(\theta_5) \qquad d' = 39.8\,mm$$

11. Determine the angular velocity of link 5 using equation 6.22a.

$$\omega_5 := \frac{c}{e}\cdot\frac{\cos(\theta_4 - \beta)}{\cos(\theta_5)}\cdot\omega_4 \qquad \omega_5 = -7.421\,\frac{rad}{sec}$$

12. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{c\cdot\alpha_4\cdot\cos(\theta_4 - \beta) - c\cdot\omega_4^{\,2}\cdot\sin(\theta_4 - \beta) + e\cdot\omega_5^{\,2}\cdot\sin(\theta_5)}{e\cdot\cos(\theta_5)} \qquad \alpha_5 = -122.305\,\frac{rad}{sec^2}$$

13. Use equation 7.16e for the acceleration of pin C.

$$A_C := -c\cdot\alpha_4\cdot\sin(\theta_4 - \beta) - c\cdot\omega_4^{\,2}\cdot\cos(\theta_4 - \beta) + e\cdot\alpha_5\cdot\sin(\theta_5) + e\cdot\omega_5^{\,2}\cdot\cos(\theta_5)$$

$A_C = -162.9\,\frac{in}{sec^2}$ A negative sign means that $\mathbf{A_C}$ is downward

PROBLEM 7-18

Statement: For the linkage shown in Figure P7-6c, write the vector loop equations; differentiate them, and do a complete position, velocity, and acceleration analysis of the linkage. Use the linkage geometry given below. Assume $\omega_2 = 10$ rad/sec and $\alpha_2 = 20$ rad/sec^2.

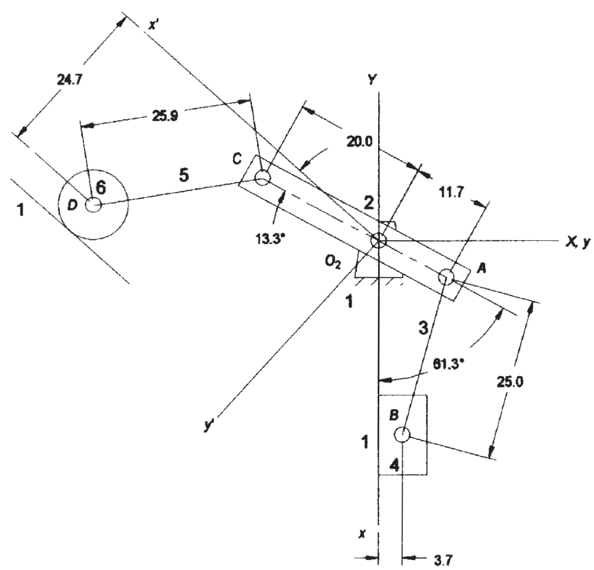
Given: Measured lengths and angles:

Link 21	$a1 := 11.7 \cdot mm$	Link 22	$a2 := 20.0 \cdot mm$
Link 3	$b1 := 25.0 \cdot mm$	Link 5	$b2 := 25.9 \cdot mm$
Offset	$c1 := 3.7 \cdot mm$	Offset'	$c2 := 24.7 \cdot mm$ from x' axis
Crank angle:	$\theta_{21} := 61.3 \cdot deg$		$\theta_{22} := 13.3 \cdot deg$

Link 2 velocity and acceleration $\quad \omega_2 := 10 \cdot \dfrac{rad}{sec} \quad \alpha_2 := 20 \cdot \dfrac{rad}{sec^2}$

Solution: See Figure P7-6c and Mathcad file P0718.

1. Draw the mechanism to scale and label it.



2. This mechanism can be analyzed as a fourbar slider-crank (links 1, 2, 3, and 4) and a fourbar slider-crank (links 1, 2, 5, and 6). Link 2 is the common, non-stationary link in the two branches and is the input to both slider-cranks. Since the vector loops are the same as those defined for the slider-crank linkage, we can use the equations derived in the text with slight modification of variable names.

   There are three coordinate systems: the global *XY* frame, the local *xy* frame for the first slider-crank, and the local *x'y'* frame for the second slider crank. Inputs to the fourbars must be in their respective local coordinates. **All output will be given in local coordinate system.**

3. Determine θ_3 and the vertical distance from O_2 to B $(d1)$ for the first slider-crank using equations 4.16 and 4.17.

$$\theta_3 := asin\left(-\frac{a1 \cdot sin(\theta_{21}) - c1}{b1}\right) + \pi \qquad \theta_3 = 164.8\,deg$$

$$d1 := a1 \cdot cos(\theta_{21}) - b1 \cdot cos(\theta_3) \qquad d1 = 29.7\,mm$$

4. Determine the angular velocity of link 3 using equation 6.22a.

$$\omega_3 := \frac{a1}{b1} \cdot \frac{\cos(\theta_{21})}{\cos(\theta_3)} \cdot \omega_2 \qquad \omega_3 = -2.329 \frac{rad}{sec}$$

5. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A_A} := a1 \cdot \alpha_2 \cdot (-\sin(\theta_{21}) + j \cdot \cos(\theta_{21})) - a1 \cdot \omega_2^{\,2} \cdot (\cos(\theta_{21}) + j \cdot \sin(\theta_{21}))$$

$$\mathbf{A_A} = -767.114 - 913.889j \frac{mm}{sec^2} \qquad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := arg(\mathbf{A_A})$$

The acceleration of pin A is $\quad A_A = 1193 \dfrac{mm}{sec^2} \qquad \theta_{AA} = \text{at}130.0\,deg$

6. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a1 \cdot \alpha_2 \cdot \cos(\theta_{21}) - a1 \cdot \omega_2^{\,2} \cdot \sin(\theta_{21}) + b1 \cdot \omega_3^{\,2} \cdot \sin(\theta_3)}{b1 \cdot \cos(\theta_3)} \qquad \alpha_3 = 36.4 \frac{rad}{sec^2}$$

7. Use equation 7.16e for the acceleration of pin B.

$$A_B := -a1 \cdot \alpha_2 \cdot \sin(\theta_{21}) - a1 \cdot \omega_2^{\,2} \cdot \cos(\theta_{21}) + b1 \cdot \alpha_3 \cdot \sin(\theta_3) + b1 \cdot \omega_3^{\,2} \cdot \cos(\theta_3)$$

$A_B = -659 \dfrac{mm}{sec^2}$ A negative sign means that $\mathbf{A_B}$ is upward

8. Determine θ_5 and the distance perpendicular to the x' axis from O_2 to BD ($d2$) for the second slider-crank using equations 4.16 and 4.17.

$$\theta_5 := asin\left(-\frac{a2 \cdot \sin(\theta_{22}) - c2}{b2}\right) + \pi \qquad \theta_5 = 230.9\,deg$$

$$d2 := a2 \cdot \cos(\theta_{22}) - b2 \cdot \cos(\theta_5) \qquad d2 = 35.8\,mm$$

9. Determine the angular velocity of link 5 using equation 6.22a.

$$\omega_5 := \frac{a2}{b2} \cdot \frac{\cos(\theta_{22})}{\cos(\theta_5)} \cdot \omega_2 \qquad \omega_5 = -11.915 \frac{rad}{sec}$$

10. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_C$, determine its magnitude, and direction.

$$\mathbf{A_C} := a2 \cdot \alpha_2 \cdot (-\sin(\theta_{22}) + j \cdot \cos(\theta_{22})) - a2 \cdot \omega_2^{\,2} \cdot (\cos(\theta_{22}) + j \cdot \sin(\theta_{22}))$$

$$\mathbf{A_C} = -2038.378 - 70.828j \frac{mm}{sec^2} \qquad A_C := |\mathbf{A_C}| \qquad \theta_{AC} := arg(\mathbf{A_C})$$

The acceleration of pin C is $\quad A_C = 2040 \dfrac{mm}{sec^2} \qquad \theta_{AC} = -178.0\,deg$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{a2 \cdot \alpha_2 \cdot cos\left(\theta_{22}\right) - a2 \cdot \omega_2^{\ 2} \cdot sin\left(\theta_{22}\right) + b2 \cdot \omega_5^{\ 2} \cdot sin\left(\theta_5\right)}{b2 \cdot cos\left(\theta_5\right)} \qquad \alpha_5 = 179.0 \frac{rad}{sec^2}$$

12. Use equation 7.16e for the acceleration of pin *D*.

$$A_D := -a2 \cdot \alpha_2 \cdot sin\left(\theta_{22}\right) - a2 \cdot \omega_2^{\ 2} \cdot cos\left(\theta_{22}\right) + b2 \cdot \alpha_5 \cdot sin\left(\theta_5\right) + b2 \cdot \omega_5^{\ 2} \cdot cos\left(\theta_5\right)$$

$$A_D = -7956 \frac{mm}{sec^2}$$

A negative sign means that $\mathbf{A_D}$ is downward and to the right

PROBLEM 7-19

Statement: For the linkage shown in Figure P7-6d, write the vector loop equations; differentiate them, and do a complete position, velocity, and acceleration analysis of the linkage. Use the linkage geometry given below. Assume $\omega_2 = 10$ rad/sec and $\alpha_2 = 20$ rad/sec^2.

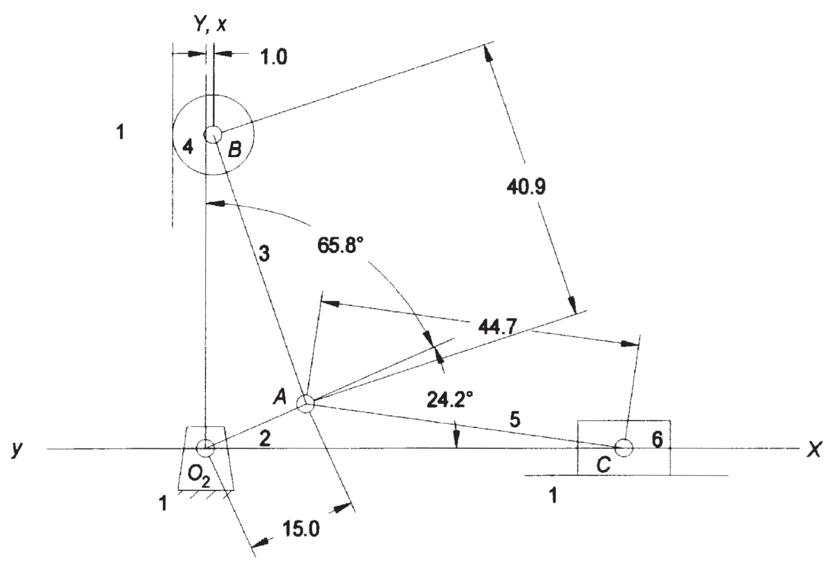
Given: Measured lengths and angles:

Link 21	$a1 := 15.0 \cdot mm$	Link 22	$a2 := 15 \cdot mm$	
Link 3	$b1 := 40.9 \cdot mm$	Link 5	$b2 := 44.7 \cdot mm$	
Offset	$c1 := -1.0 \cdot mm$	Offset'	$c2 := 0.0 \cdot mm$	from x' axis
Crank angle:	$\theta_{21} := -65.8 \cdot deg$		$\theta_{22} := 24.2 \cdot deg$	

Link 2 velocity and acceleration $\quad \omega_2 := 10 \cdot \frac{rad}{sec} \quad \alpha_2 := 20 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-6d and Mathcad file P0719.

1. Draw the mechanism to scale and label it.



2. This mechanism can be analyzed as a fourbar slider-crank (links 1, 2, 3, and 4) and a fourbar slider-crank (links 1, 2, 5, and 6). Link 2 is the common, non-stationary link in the two branches and is the input to both slider-cranks. Since the vector loops are the same as those defined for the slider-crank linkage, we can use the equations derived in the text with slight modification of variable names.

   There are two coordinate systems: the global *XY* frame and the local *xy* frame for the first slider-crank. Inputs to the fourbars must be in their respective local coordinates. **All output will be given in local coordinate system.**

3. Determine θ_3 and the vertical distance from O_2 to B ($d1$) for the first slider-crank using equations 4.16 and 4.17.

$$\theta_3 := asin\left(-\frac{a1 \cdot sin(\theta_{21}) - c1}{b1}\right) + \pi \qquad \theta_3 = 198.1\,deg$$

$$d1 := a1 \cdot cos(\theta_{21}) - b1 \cdot cos(\theta_3) \qquad d1 = 45.0\,mm$$

4. Determine the angular velocity of link 3 using equation 6.22a.

$$\omega_3 := \frac{a1}{b1} \cdot \frac{\cos(\theta_{21})}{\cos(\theta_3)} \cdot \omega_2 \qquad \omega_3 = -1.581 \frac{rad}{sec}$$

5. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its magnitude, and direction.

$$\mathbf{A_A} := a1 \cdot \alpha_2 \cdot (-\sin(\theta_{21}) + j \cdot \cos(\theta_{21})) - a1 \cdot \omega_2^{\,2} \cdot (\cos(\theta_{21}) + j \cdot \sin(\theta_{21}))$$

$$\mathbf{A_A} = -341.249 + 1491.157j \frac{mm}{sec^2} \qquad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := \arg(\mathbf{A_A})$$

The acceleration of pin A is $A_A = 1530 \frac{mm}{sec^2}$ at $\theta_{AA} = 102.9\,deg$

6. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a1 \cdot \alpha_2 \cdot \cos(\theta_{21}) - a1 \cdot \omega_2^{\,2} \cdot \sin(\theta_{21}) + b1 \cdot \omega_3^{\,2} \cdot \sin(\theta_3)}{b1 \cdot \cos(\theta_3)} \qquad \alpha_3 = -37.5 \frac{rad}{sec^2}$$

7. Use equation 7.16e for the acceleration of pin B.

$$A_B := -a1 \cdot \alpha_2 \cdot \sin(\theta_{21}) - a1 \cdot \omega_2^{\,2} \cdot \cos(\theta_{21}) + b1 \cdot \alpha_3 \cdot \sin(\theta_3) + b1 \cdot \omega_3^{\,2} \cdot \cos(\theta_3)$$

$$A_B = 37.5 \frac{mm}{sec^2}$$ A positive sign means that $\mathbf{A}_B$ is upward

8. Determine θ_5 and the distance perpendicular to the x' axis from O_2 to BD ($d2$) for the second slider-crank using equations 4.16 and 4.17.

$$\theta_5 := \operatorname{asin}\left(-\frac{a2 \cdot \sin(\theta_{22}) - c2}{b2}\right) + \pi \qquad \theta_5 = 172.1\,deg$$

$$d2 := a2 \cdot \cos(\theta_{22}) - b2 \cdot \cos(\theta_5) \qquad d2 = 58.0\,mm$$

9. Determine the angular velocity of link 5 using equation 6.22a.

$$\omega_5 := \frac{a2}{b2} \cdot \frac{\cos(\theta_{22})}{\cos(\theta_5)} \cdot \omega_2 \qquad \omega_5 = -3.090 \frac{rad}{sec}$$

10. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{a2 \cdot \alpha_2 \cdot \cos(\theta_{22}) - a2 \cdot \omega_2^{\,2} \cdot \sin(\theta_{22}) + b2 \cdot \omega_5^{\,2} \cdot \sin(\theta_5)}{b2 \cdot \cos(\theta_5)} \qquad \alpha_5 = 6.4 \frac{rad}{sec^2}$$

11. Use equation 7.16e for the acceleration of pin C.

$$A_C := -a2 \cdot \alpha_2 \cdot \sin(\theta_{22}) - a2 \cdot \omega_2^{\,2} \cdot \cos(\theta_{22}) + b2 \cdot \alpha_5 \cdot \sin(\theta_5) + b2 \cdot \omega_5^{\,2} \cdot \cos(\theta_5)$$

$$A_C = -1875 \frac{mm}{sec^2}$$ A negative sign means that $\mathbf{A}_C$ is to the left

PROBLEM 7-20

Statement: Figure P7-7 shows a sixbar linkage with the dimensions and crank angle given below. Find the angular acceleration of link 6 if ω_2 is a constant 1 rad/sec.

Given: Link lengths:

Link 2 $a := 1.00 \cdot in$ Link 3 $b := 5.00 \cdot in$ Offset: $c := 0.0 \cdot in$

Link 6 $d := 3.00 \cdot in$ Distance DB $L_{DB} := 1.50 \cdot in$

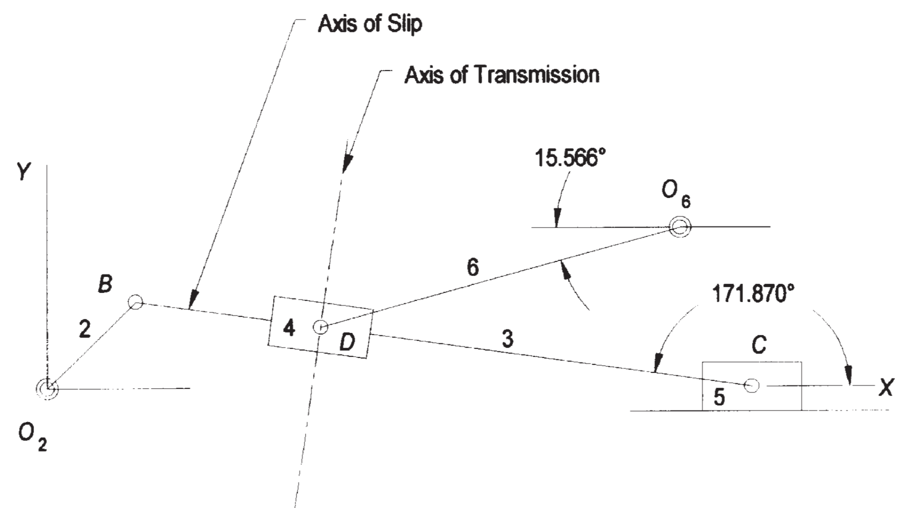
Link 2 position, velocity, and acceleration: $\theta_2 := 45 \cdot deg$ $\omega_2 := 1 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Solution: See Figure P7-7 and Mathcad file P0720.

1. Links 1, 2, 3, and 5 constitute a fourbar slider-crank. In order to solve for the accelerations at points *B*, *C* and *D*, we will need θ_3, and ω_3. From the graphical position solution below,

$$\theta_3 := 171.870 \cdot deg$$

Using equation 6.22a, $\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_3)}$ $\omega_3 = -0.143\, rad \cdot sec^{-1}$ CW



2. The graphical solution for accelerations of pin-jointed linkages uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

3. For point *C*, this becomes: $\mathbf{A}_C = (\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) + (\mathbf{A}_{CB}{}^t + \mathbf{A}_{CB}{}^n)$, where

$A_{Bn} := a \cdot \omega_2^{\,2}$ $A_{Bn} = 1.000\, in \cdot sec^{-2}$

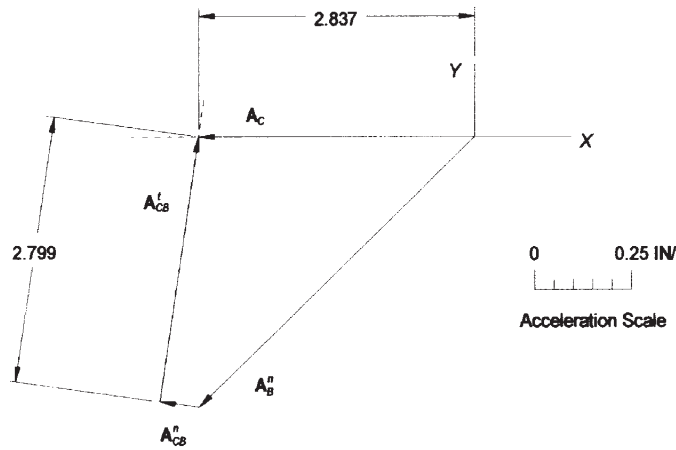
$\theta_{ABn} := \theta_2 + 180 \cdot deg$ $\theta_{ABn} = 225.000\, deg$

$A_{Bt} := a \cdot \alpha_2$ $A_{Bt} = 0.000\, in \cdot sec^{-2}$

$A_{CBn} := b \cdot \omega_3^{\,2}$ $A_{CBn} = 0.102\, in \cdot sec^{-2}$

$\theta_{ACBn} := \theta_3$ $\theta_{ACBn} = 171.870\, deg$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of θ_{ABn}. From the tip of $\mathbf{A}_B{}^n$, draw $\mathbf{A}_{CB}{}^n$ at an angle of θ_{ACBn}. Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{CB}{}^n$, draw construction lines in the directions of $\mathbf{A}_C$ (horizontal) and $\mathbf{A}_{CB}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{CB}{}^t$, and $\mathbf{A}_C$.



5. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 0.25 \cdot \dfrac{in}{sec^2}$

$A_C := 2.837 \cdot k_a \qquad A_C = 0.709\, in \cdot sec^{-2}$ at an angle of 180 deg

$A_{CBt} := 2.799 \cdot k_a \qquad A_{CBt} = 0.700\, in \cdot sec^{-2}$

6. Calculate α_3 using equation 7.6.

$\alpha_3 := \dfrac{A_{CBt}}{b} \qquad \alpha_3 = 0.140\, rad \cdot sec^{-2}$ CCW

7. In order to solve for the acceleration at points D, we will need θ_6, and ω_6. From the graphical position solution above,

$\theta_6 := 15.566 \cdot deg + 180 \cdot deg \qquad \theta_6 = 195.566\, deg$

From the vector diagram in Figure P7-7,

$V_D := 0.40 \cdot \dfrac{in}{sec}$ and $\omega_6 := \dfrac{V_D}{d} \qquad \omega_6 = 0.133 \dfrac{rad}{sec}$

$V_{slip} := 0.65 \cdot \dfrac{in}{sec}$

8. For point D, use equation 7.19, referenced to point B instead of O_2:
$(\mathbf{A}_D{}^t + \mathbf{A}_D{}^n) = \mathbf{A}_B + (\mathbf{A}_{DB}{}^t + \mathbf{A}_{DB}{}^n + \mathbf{A}_{DB}{}^{cor} + \mathbf{A}_{DB}{}^{slip})$, where

$A_{Dn} := d \cdot \omega_6{}^2 \qquad A_{Dn} = 0.053\, in \cdot sec^{-2}$

$\theta_{ADn} := \theta_6 - 180 \cdot deg \qquad \theta_{ADn} = 15.566\, deg$

$A_B := a \cdot \omega_2{}^2 \qquad A_B = 1.000\, in \cdot sec^{-2}$

$\theta_{AB} := \theta_2 + 180 \cdot deg$ $\qquad \theta_{AB} = 225.000\,deg$

$A_{DBt} := L_{DB} \cdot \alpha_3$ $\qquad A_{DBt} = 0.210\,in \cdot sec^{-2}$

$\theta_{ADBt} := \theta_3 - 90 \cdot deg$ $\qquad \theta_{ADBt} = 81.870\,deg$

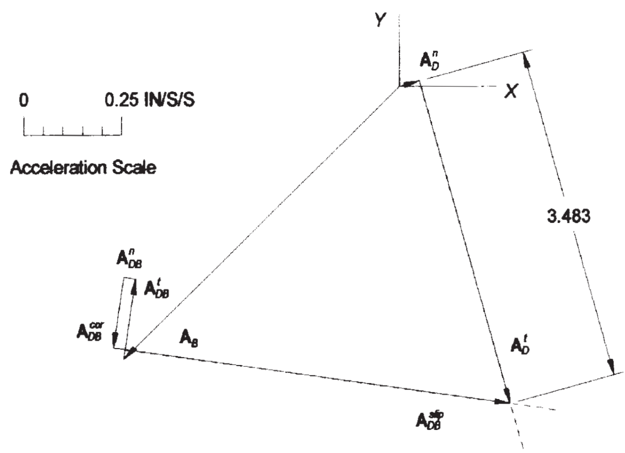
$A_{DBn} := L_{DB} \cdot \omega_3^{\,2}$ $\qquad A_{DBn} = 0.031\,in \cdot sec^{-2}$

$\theta_{ADBn} := \theta_3$ $\qquad \theta_{ADBn} = 171.870\,deg$

$A_{DBcor} := \left|2 \cdot V_{slip} \cdot \omega_3\right|$ $\qquad A_{DBcor} = 0.186\,in \cdot sec^{-2}$

$\theta_{ADBcor} := \theta_3 + 90 \cdot deg$ $\qquad \theta_{ADBcor} = 261.870\,deg$

9. Repeat procedure of step 4 for the equation in step 8.



10. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 0.25 \cdot \dfrac{in}{sec^2}$

$A_{Dt} := 3.483 \cdot k_a$ $\qquad A_{Dt} = 0.871\,in \cdot sec^{-2}$ at an angle of -74.434 deg

The angular acceleration of link 6 is

$\alpha_6 := \dfrac{A_{Dt}}{d}$ $\qquad \alpha_6 = 0.290\,\dfrac{rad}{sec^2}$ CCW

PROBLEM 7-21

Statement: The linkage in Figure P7-8a has the dimensions and crank angle given below. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for ω_2 = 15 rad/sec clockwise (CW) and α_2 = 25 rad/sec² CCW. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 116 \cdot mm$	Link 3 (A to B)	$b := 108 \cdot mm$
Link 4 (B to O_4)	$c := 110 \cdot mm$	Link 1 (O_2 to O_4)	$d := 174 \cdot mm$

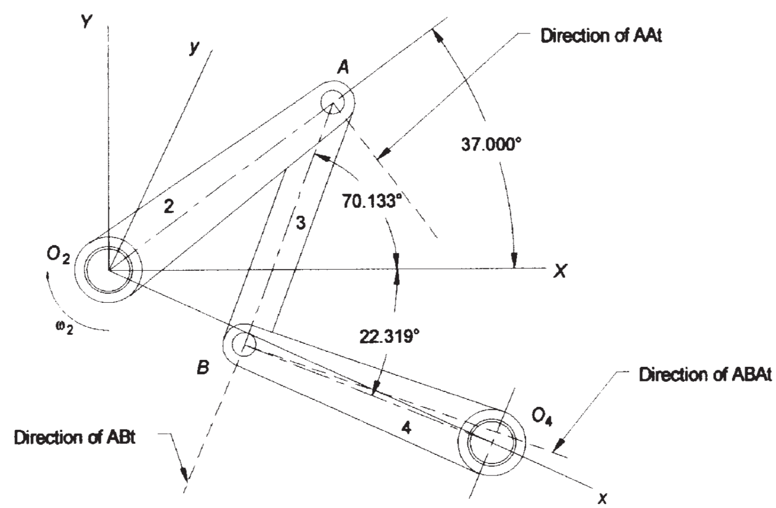
Crank angle: $\theta_2 := 62 \cdot deg$ Local xy system

Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 25 \cdot rad \cdot sec^{-2}$

Coordinate rotation angle $\alpha := -25 \cdot deg$ Global XY system to local xy system

Solution: See Figure P7-8a and Mathcad file P0721.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3, ω_3, and θ_4. From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 180 \cdot deg + 70.133 \cdot deg - \alpha \qquad \theta_3 = 275.133\, deg$$

$$\theta_4 := 180 \cdot deg - 22.319 \cdot deg - \alpha \qquad \theta_4 = 182.681\, deg$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{(\sin(\theta_3 - \theta_4))} \qquad \omega_3 = -13.869\, rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{(\sin(\theta_4 - \theta_3))} \qquad \omega_4 = 8.654\, rad \cdot sec^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

4. For point B, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$$A_{Bn} := c \cdot \omega_4^{\,2} \qquad A_{Bn} = 8237.9\, mm \cdot sec^{-2}$$

$$\theta_{ABn} := \theta_4 - 180 \cdot deg \qquad \theta_{ABn} = 2.681\, deg$$

$$A_{An} := a \cdot \omega_2^{\,2} \qquad A_{An} = 26100.0\, mm \cdot sec^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot deg \qquad \theta_{AAn} = 242.000\, deg$$

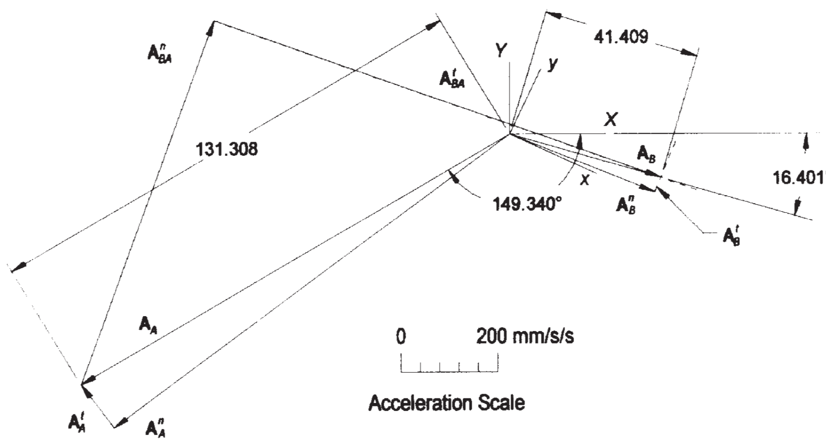
$$A_{At} := a \cdot \alpha_2 \qquad A_{At} = 2900.0\, mm \cdot sec^{-2}$$

$$\theta_{AAt} := \theta_2 + 90 \cdot deg \qquad \theta_{AAt} = 152.000\, deg$$

$$A_{BAn} := b \cdot \omega_3^{\,2} \qquad A_{BAn} = 20773\, mm \cdot sec^{-2}$$

$$\theta_{ABAn} := \theta_3 - 180 \cdot deg \qquad \theta_{ABAn} = 95.133\, deg$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A{}^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of $\theta_{AAt} + \alpha$. From the tip of $\mathbf{A}_A{}^t$, draw $\mathbf{A}_{BA}{}^n$ at an angle of $\theta_{BAn} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B{}^t$, $\mathbf{A}_B{}^t$, and $\mathbf{A}_{BA}{}^t$.



6. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 200 \cdot \dfrac{mm}{sec^2}$

$$A_A := 131.308 \cdot k_a \qquad A_A = 26262\, mm \cdot sec^{-2} \quad \text{at an angle of -149.34 deg}$$

$$A_B := 41.409 \cdot k_a \qquad A_B = 8282\, mm \cdot sec^{-2} \quad \text{at an angle of -16.40 deg}$$

PROBLEM 7-22

Statement: The linkage in Figure P7-8a has the dimensions and crank angle given below. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for ω_2 = 15 rad/sec clockwise (CW) and α_2 = 25 rad/sec^2 CCW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 116 \cdot mm$	Link 3 (A to B)	$b := 108 \cdot mm$
Link 4 (B to O_4)	$c := 110 \cdot mm$	Link 1 (O_2 to O_4)	$d := 174 \cdot mm$

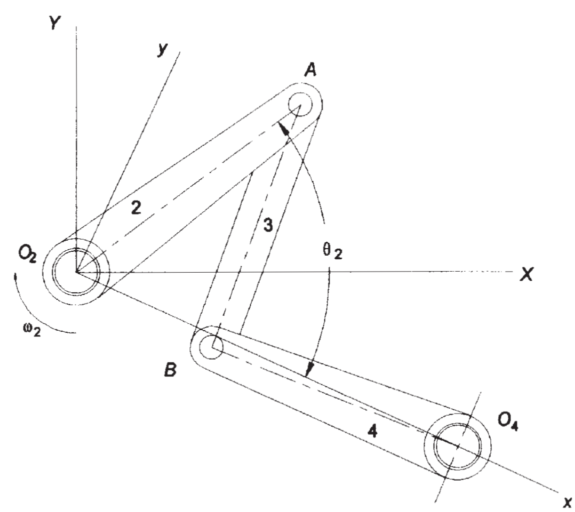
Crank angle: $\theta_2 := 62 \cdot deg$ Local xy system

Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 25 \cdot rad \cdot sec^{-2}$

Coordinate rotation angle $\beta := -25 \cdot deg$ Global XY system to local xy system

Solution: See Figure P7-8a and Mathcad file P0722.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.5000 \qquad K_2 = 1.5818$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.7307$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.0424 \qquad B = -1.7659 \qquad C = 2.0186$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2\cdot\left(atan2\left(2\cdot A, -B + \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) \qquad \theta_4 = 182.681\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.6111 \qquad K_5 = -1.7280$$

$$D := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad D = -2.0021$$

$$E := -2\cdot sin(\theta_2) \qquad E = -1.7659$$

$$F := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5 \qquad F = 0.0589$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2\cdot\left(atan2\left(2\cdot D, -E + \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_3 = 275.133\,deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4 - \theta_2)}{sin(\theta_3 - \theta_4)} \qquad \omega_3 = -13.869\,\frac{rad}{sec}$$

$$\omega_4 := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3)}{sin(\theta_4 - \theta_3)} \qquad \omega_4 = 8.654\,\frac{rad}{sec}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction (in the global coordinate system).

$$\mathbf{A_A} := a\cdot\alpha_2\cdot\left(-sin(\theta_2) + j\cdot cos(\theta_2)\right) - a\cdot\omega_2^{\,2}\cdot\left(cos(\theta_2) + j\cdot sin(\theta_2)\right)$$

$$\mathbf{A_A} = -14814 - 21683j\,\frac{mm}{sec^2} \qquad A_A := \left|\mathbf{A_A}\right| \qquad \theta_{AA} := arg(\mathbf{A_A}) + \beta$$

The acceleration of pin A is $A_A = 26261\,\frac{mm}{sec^2}$ at $\theta_{AA} = -149.3\,deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c\cdot sin(\theta_4) \qquad B := b\cdot sin(\theta_3) \qquad D := c\cdot cos(\theta_4) \qquad E := b\cdot cos(\theta_3)$$

$$A = -5.145\,mm \qquad B = -107.567\,mm \qquad D = -109.880\,mm \qquad E = 9.662\,mm$$

$$C := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot cos(\theta_2) + b\cdot\omega_3^{\,2}\cdot cos(\theta_3) - c\cdot\omega_4^{\,2}\cdot cos(\theta_4)$$

$$C = 2.490\times 10^4\,mm\cdot sec^{-2}$$

$$F := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) - b\cdot\omega_3^{\,2}\cdot sin(\theta_3) + c\cdot\omega_4^{\,2}\cdot sin(\theta_4)$$

$$F = -1.380\times 10^3\,mm\cdot sec^{-2}$$

$$\alpha_3 := \frac{C\cdot D - A\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_3 = 231.119\,\frac{rad}{sec^2} \qquad \alpha_4 := \frac{C\cdot E - B\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_4 = -7.768\,\frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point B for the crossed circuit.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-sin\left(\theta_4\right) + j \cdot cos\left(\theta_4\right)\right) - c \cdot \omega_4^{\,2} \cdot \left(cos\left(\theta_4\right) + j \cdot sin\left(\theta_4\right)\right)$$

$$\mathbf{A_B} = 8189 + 1239j \, \frac{mm}{sec^2} \qquad A_B := \left|\mathbf{A_B}\right| \qquad \theta_{AB} := arg\left(\mathbf{A_B}\right) + \beta$$

The acceleration of pin B is $A_B = 8282 \, \dfrac{mm}{sec^2}$ at $\theta_{AB} = -16.4\,deg$ (Global)

PROBLEM 7-23

Statement: The linkage in Figure P7-8a has the dimensions and crank angle given below. Find and plot α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 15$ rad/sec CW and $\alpha_2 = 25$ rad/sec^2 CCW.

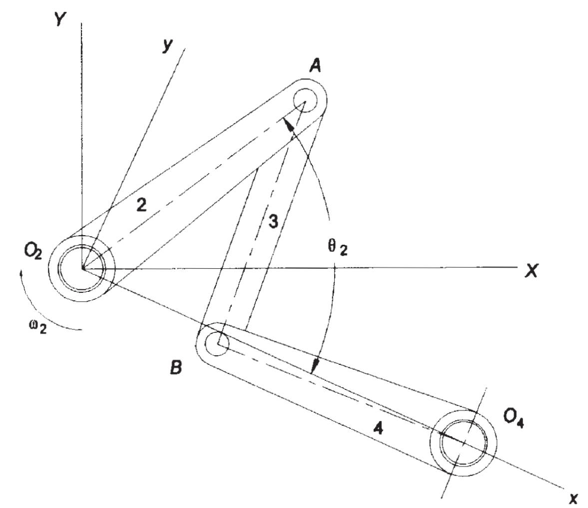
Given: Link lengths:

Link 2 (O_2 to A)	$a := 116 \cdot mm$	Link 3 (A to B)	$b := 108 \cdot mm$
Link 4 (B to O_4)	$c := 110 \cdot mm$	Link 1 (O_2 to O_4)	$d := 174 \cdot mm$

Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$ $\quad \alpha_2 := 25 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-8a and Mathcad file P0723.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 1.083$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = -0.094$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 95.4\,deg$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 1 \cdot deg .. \theta_{2toggle}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.5000 \qquad K_2 = 1.5818$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.7307$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.6111 \qquad K_5 = -1.7280$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the x and y components in the local coordinate system.

$$\mathbf{A_A}(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{Ax}(\theta_2) := Re(\mathbf{A_A}(\theta_2)) \qquad A_{Ay}(\theta_2) := Im(\mathbf{A_A}(\theta_2))$$

9. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \qquad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \qquad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

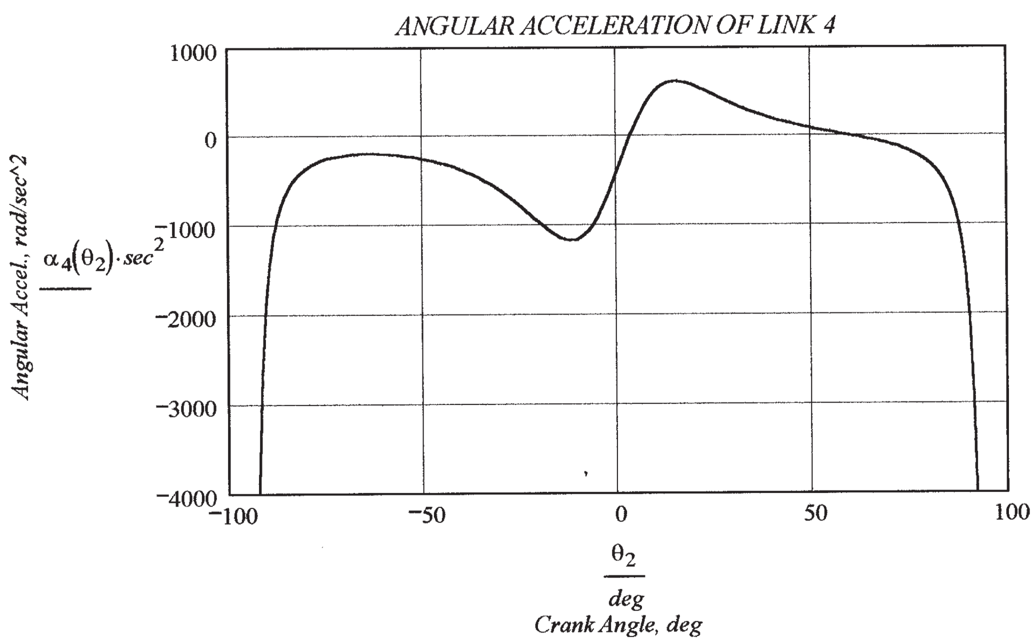
$$\alpha_4(\theta_2) := \frac{C(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot F(\theta_2)}{A(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot D(\theta_2)}$$

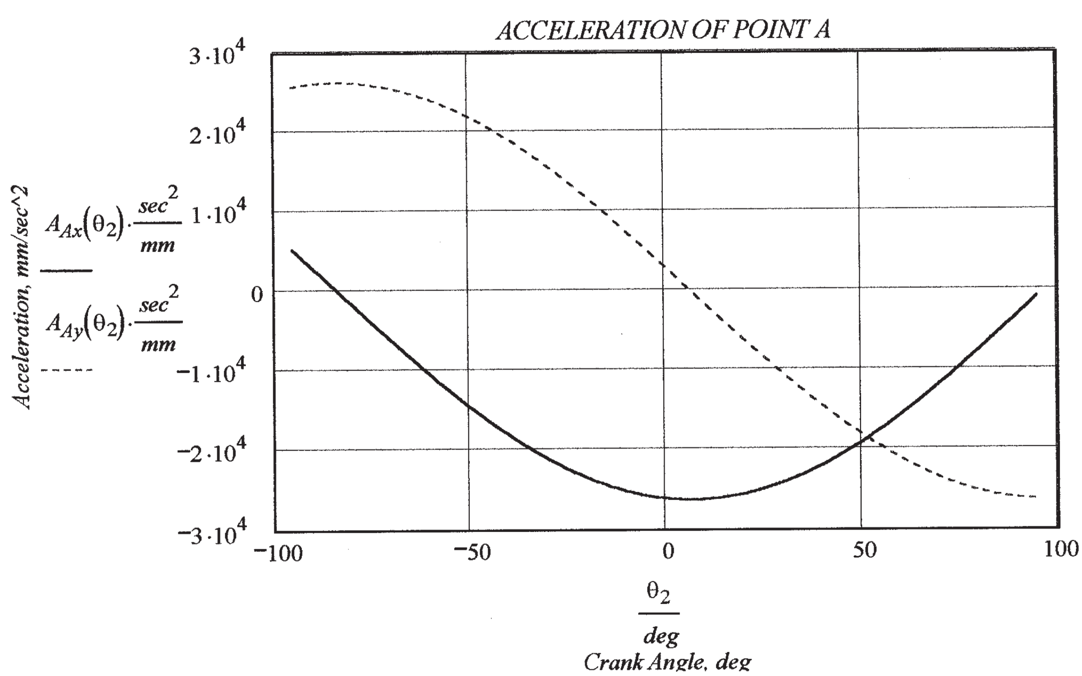
10. Use equation 7.13c to determine the acceleration of point B. Determine the x and y components in the local coordinate system.

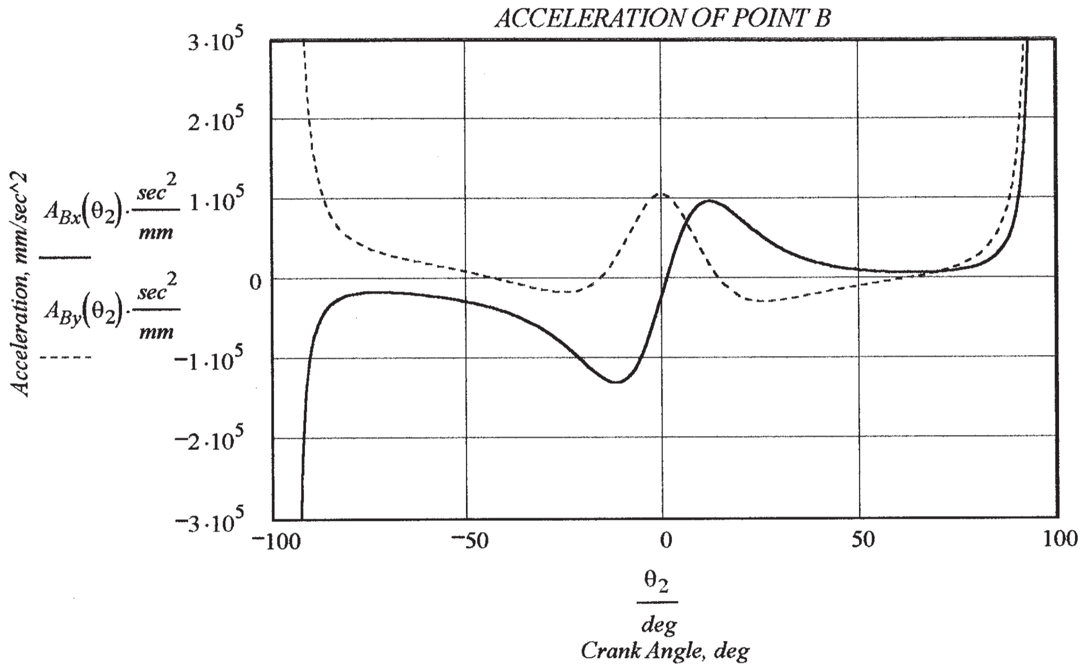
$$\mathbf{A_B}(\theta_2) := c\cdot\alpha_4(\theta_2)\cdot(-sin(\theta_4(\theta_2)) + j\cdot cos(\theta_4(\theta_2))) - c\cdot\omega_4(\theta_2)^2\cdot(cos(\theta_4(\theta_2)) + j\cdot sin(\theta_4(\theta_2)))$$

$$A_{Bx}(\theta_2) := Re(\mathbf{A_B}(\theta_2)) \qquad A_{By}(\theta_2) := Im(\mathbf{A_B}(\theta_2))$$

11. Plot the angular acceleration for link 4 and the acceleration components for pins B and C.






ACCELERATION OF POINT B
$3 \cdot 10^5$
$2 \cdot 10^5$
$1 \cdot 10^5$
0
$-1 \cdot 10^5$
$-2 \cdot 10^5$
$-3 \cdot 10^5$
$A_{Bx}(\theta_2) \cdot \frac{sec^2}{mm}$
$A_{By}(\theta_2) \cdot \frac{sec^2}{mm}$
Acceleration, mm/sec^2
−100
−50
0
50
100
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 7-24

Statement: The linkage in Figure P7-8b has the dimensions and crank angle given below. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for $\omega_2 = 20$ rad/sec CCW. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (*A* to *B*)	$a := 40 \cdot mm$	Link 3 (*B* to *C*)	$b := 96 \cdot mm$
Link 4 (*C* to *D*)	$c := 122 \cdot mm$	Link 1 (*A* to *D*)	$d := 162 \cdot mm$

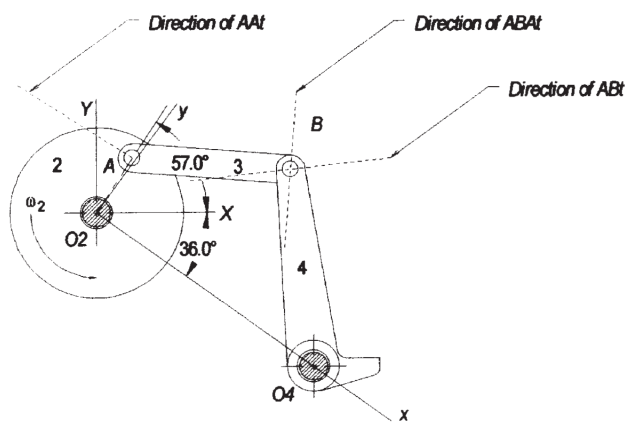
Crank angle: $\theta_2 := 93 \cdot deg$ Local *xy* system

Input crank angular velocity $\omega_2 := 20 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Coordinate rotation angle $\alpha := -36 \cdot deg$ Global *XY* system to local *xy* system

Solution: See Figure P7-8b and Mathcad file P0724.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points *A* and *B*, we will need θ_3, ω_3, and θ_4. From the graphical position solution above (in the local *xy* coordinate system),

$$\theta_3 := 31.486 \cdot deg$$

$$\theta_4 := 132.406 \cdot deg$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{(\sin(\theta_3 - \theta_4))} \qquad \omega_3 = -5.388\, rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{(\sin(\theta_4 - \theta_3))} \qquad \omega_4 = 5.870\, rad \cdot sec^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

4. For point *B*, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$$A_{Bn} := c \cdot \omega_4{}^2 \qquad A_{Bn} = 4203\, mm \cdot sec^{-2}$$

$\theta_{ABn} := \theta_4 - 180 \cdot deg$ $\theta_{ABn} = -47.594\, deg$

$A_{An} := a \cdot \omega_2^{\,2}$ $A_{An} = 16000\, mm \cdot sec^{-2}$

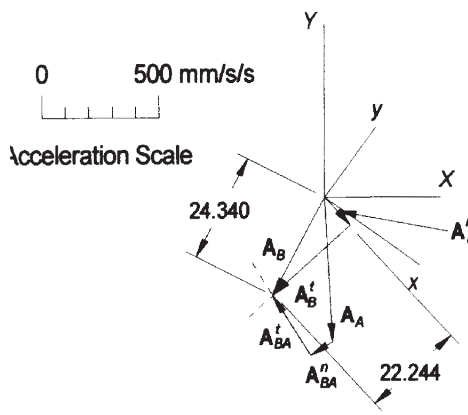
$\theta_{AAn} := \theta_2 + 180 \cdot deg$ $\theta_{AAn} = 273.000\, deg$

$A_{At} := a \cdot \alpha_2$ $A_{At} = 0.000\, mm \cdot sec^{-2}$

$A_{BAn} := b \cdot \omega_3^{\,2}$ $A_{BAn} = 2787\, mm \cdot sec^{-2}$

$\theta_{ABAn} := \theta_3 - 180 \cdot deg$ $\theta_{ABAn} = -148.514\, deg$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of θ_{ABn} and $\mathbf{A}_A{}^n$ at an angle of θ_{AAn}. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of θ_{AAt}. From the tip of $\mathbf{A}_A{}^t$, draw $\mathbf{A}_{BA}{}^n$ at an angle of θ_{BAn}. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B{}^t$, $\mathbf{A}_B{}^t$, and $\mathbf{A}_{BA}{}^t$.



6. From the graphical solution above,

Acceleration scale factor $k_a := 500 \cdot \dfrac{mm}{sec^2}$

$A_A := A_{An}$ $A_A = 16000\, mm \cdot sec^{-2}$ at an angle of 273.0 deg (Global)

$A_B := 24.340 \cdot k_a$ $A_B = 12170\, mm \cdot sec^{-2}$ at an angle of 242.610 deg

$A_{Bt} := 22.244 \cdot k_a$ $A_{Bt} = 11122\, mm \cdot sec^{-2}$

7. Calculate α_4 using equation 7.6.

$\alpha_4 := \dfrac{A_{Bt}}{c}$ $\alpha_4 = 91.2\, rad \cdot sec^{-2}$ CCW

PROBLEM 7-25

Statement: The linkage in Figure P7-8b has the dimensions and crank angle given below. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for $\omega_2 = 20$ rad/sec CCW constant. Use an analytical method.

Given: Link lengths:

Link 2 (*A* to *B*)	$a := 40 \cdot mm$	Link 3 (*B* to *C*)	$b := 96 \cdot mm$
Link 4 (*C* to *D*)	$c := 122 \cdot mm$	Link 1 (*A* to *D*)	$d := 162 \cdot mm$

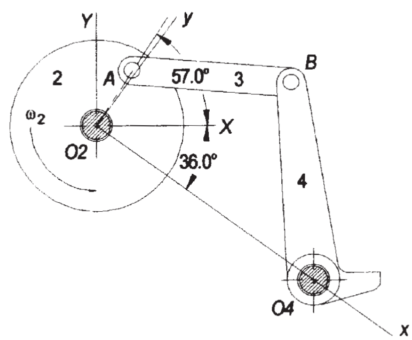
Crank angle: $\theta_2 := 93 \cdot deg$ Local *xy* system

Input crank angular velocity $\omega_2 := 20 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Coordinate rotation angle $\beta := -36 \cdot deg$ Global *XY* system to local *xy* system

Solution: See Figure P7-8b and Mathcad file P0725.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 4.0500 \qquad K_2 = 1.3279$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 3.4336$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.5992 \qquad B = -1.9973 \qquad C = 7.6054$$

3. Use equation 4.10b to find values of θ_4 for the open circuit in the local *xy* coordinate system.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) + 2 \cdot \pi \qquad \theta_4 = 132.386\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.6875 \qquad K_5 = -2.8875$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -7.0782$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.9973$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 1.1265$$

5. Use equation 4.13 to find values of θ_3 for the open circuit in the local *xy* coordinate system.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) + 2 \cdot \pi \qquad \theta_3 = 31.504\,deg$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_3 = -5.385\,\frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \qquad \omega_4 = 5.868\,\frac{rad}{sec}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction (in the global coordinate system).

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A_A} = 837 - 15978j\,\frac{mm}{sec^2} \qquad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := arg(\mathbf{A_A}) + \beta$$

The acceleration of pin A is $A_A = 16000\,\frac{mm}{sec^2}$ at $\theta_{AA} = -123.0\,deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot \sin(\theta_4) \qquad B := b \cdot \sin(\theta_3) \qquad D := c \cdot \cos(\theta_4) \qquad E := b \cdot \cos(\theta_3)$$

$$A = 90.111\,mm \qquad B = 50.166\,mm \qquad D = -82.244\,mm \qquad E = 81.850\,mm$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_4)$$

$$C = 4.368 \times 10^3\,mm \cdot sec^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_4)$$

$$F = -1.433 \times 10^4\,mm \cdot sec^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_3 = 81.037\,\frac{rad}{sec^2} \qquad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_4 = 93.586\,\frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point *B* for the open circuit.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot (-\sin(\theta_4) + j \cdot \cos(\theta_4)) - c \cdot \omega_4^2 \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4))$$

$\mathbf{A_B} = -5601 - 10800j \dfrac{mm}{sec^2}$ $\qquad A_B := \left|\mathbf{A_B}\right|$ $\qquad \theta_{AB} := arg\left(\mathbf{A_B}\right) + \beta$

The acceleration of pin *B* is $\quad A_B = 12166 \dfrac{mm}{sec^2}$ at $\theta_{AB} = -153.4\,deg$ (Global)

PROBLEM 7-26

Statement: The linkage in Figure P7-8b has the dimensions and crank angle given below. Find and plot α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 20$ rad/sec CCW constant.

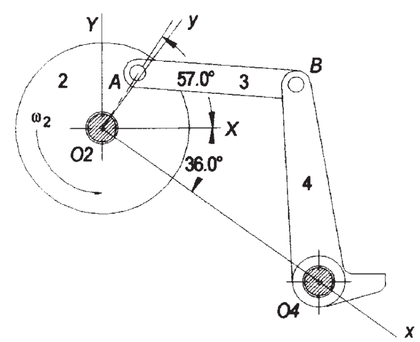
Given: Link lengths:

Link 2 (*A* to *B*)	$a := 40 \cdot mm$	Link 3 (*B* to *C*)	$b := 96 \cdot mm$
Link 4 (*C* to *D*)	$c := 122 \cdot mm$	Link 1 (*A* to *D*)	$d := 162 \cdot mm$

Input crank angular velocity $\omega_2 := 20 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-8b and Mathcad file P0726.

1. Draw the linkage to scale and label it.



2. The range of θ_2 for this Grashof crank-rocker is:

$$\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 4.0500 \qquad K_2 = 1.3279$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 3.4336$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.6875 \qquad K_5 = -2.8875$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right)\right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the x and y components in the local coordinate system.

$$\mathbf{A_A}(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{Ax}(\theta_2) := Re(\mathbf{A_A}(\theta_2)) \qquad A_{Ay}(\theta_2) := Im(\mathbf{A_A}(\theta_2))$$

9. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \qquad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \qquad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

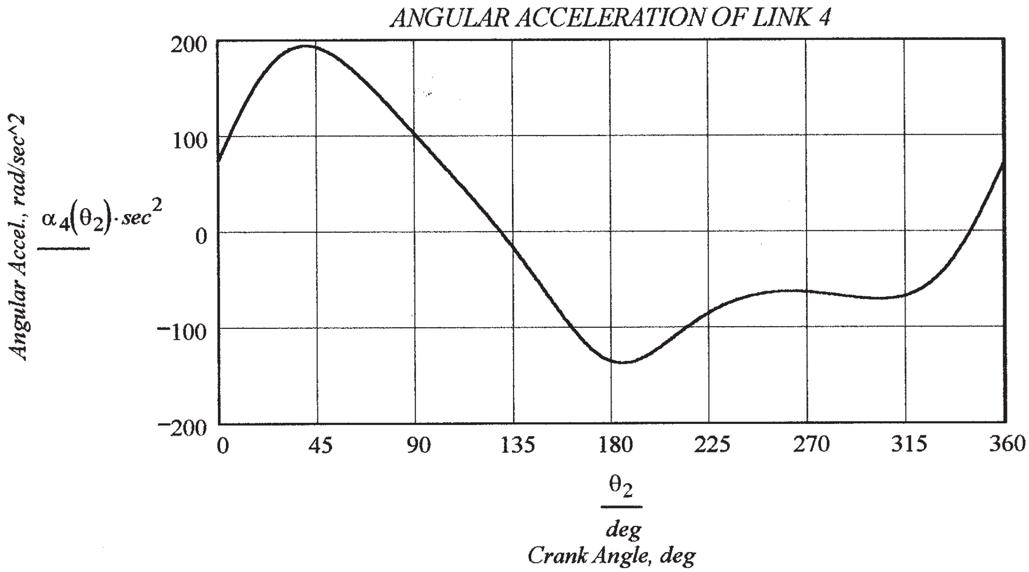
$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

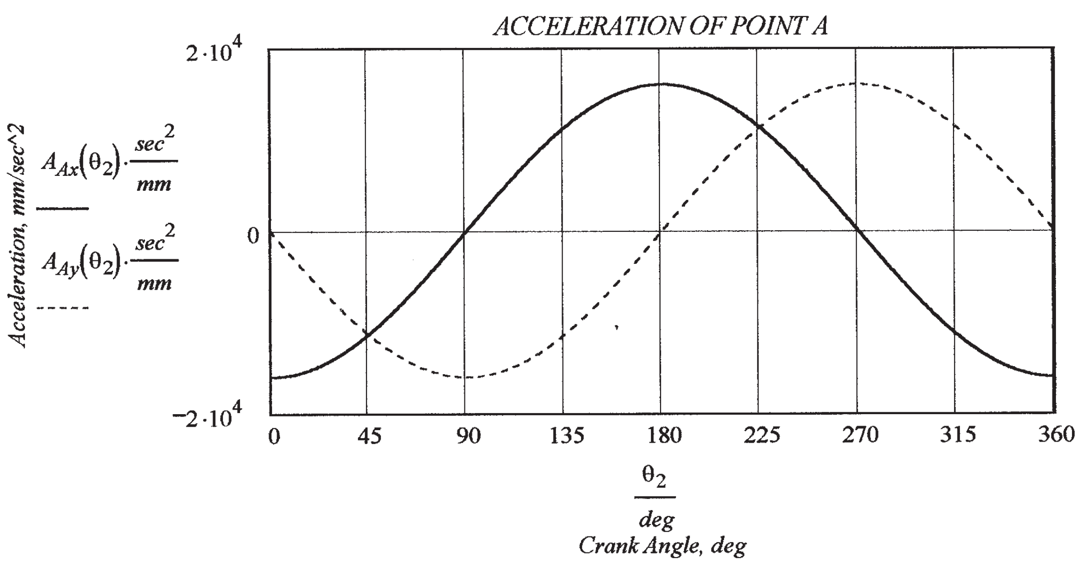
10. Use equation 7.13c to determine the acceleration of point B. Determine the x and y components in the local coordinate system.

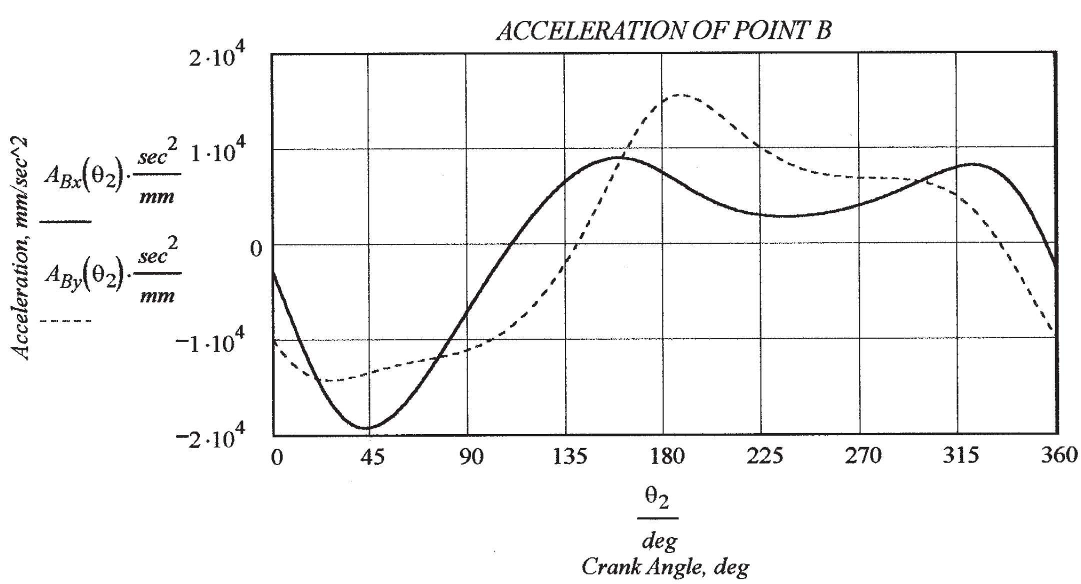
$$\mathbf{A_B}(\theta_2) := c \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$A_{Bx}(\theta_2) := Re(\mathbf{A_B}(\theta_2)) \qquad A_{By}(\theta_2) := Im(\mathbf{A_B}(\theta_2))$$

11. Plot the angular acceleration for link 4 and the acceleration components for pins A and B.


ANGULAR ACCELERATION OF LINK 4
200
100
0
−100
−200
Angular Accel., rad/sec^2
$\alpha_4(\theta_2)\cdot sec^2$
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg


ACCELERATION OF POINT A
$2\cdot10^4$
0
$-2\cdot10^4$
Acceleration, mm/sec^2
$A_{Ax}(\theta_2)\cdot\frac{sec^2}{mm}$
$A_{Ay}(\theta_2)\cdot\frac{sec^2}{mm}$
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg


ACCELERATION OF POINT B
$2\cdot10^4$
$1\cdot10^4$
0
$-1\cdot10^4$
$-2\cdot10^4$
Acceleration, mm/sec^2
$A_{Bx}(\theta_2)\cdot\frac{sec^2}{mm}$
$A_{By}(\theta_2)\cdot\frac{sec^2}{mm}$
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 7-27

Statement: The offset slider-crank linkage in Figure P7-8f has the dimensions and crank angle given below. Find $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for $\omega_2 = 25$ rad/sec CW, constant. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (D to E) $a := 63 \cdot mm$ Link 3 (E to F) $b := 130 \cdot mm$

Offset $c := -52 \cdot mm$

Link 2 position, velocity, and acceleration: $\theta_2 := 141 \cdot deg$ $\omega_2 := -25 \cdot \dfrac{rad}{sec}$ $\alpha_2 := 0 \cdot \dfrac{rad}{sec^2}$

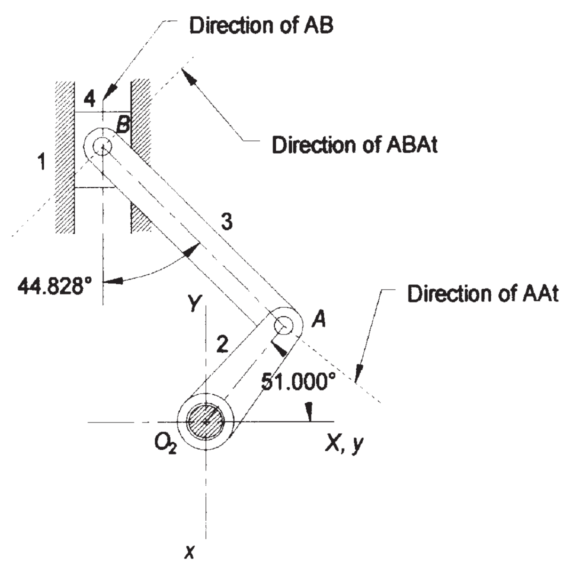
Coordinate rotation angle: $\alpha := -90 \cdot deg$

Solution: See Figure P7-8f and Mathcad file P0727.

1. In order to solve for the accelerations at point F, we will need θ_3, and ω_3. From the graphical position solution below (in the local coordinate system),

$$\theta_3 := 44.828 \cdot deg$$

Using equation 6.22a, $\omega_3 := \dfrac{a \cdot \omega_2}{b} \cdot \dfrac{\cos(\theta_2)}{\cos(\theta_3)}$ $\omega_3 = 13.276\, rad \cdot sec^{-1}$



2. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

3. For point F, this becomes: $\mathbf{A}_F = (\mathbf{A}_E{}^t + \mathbf{A}_E{}^n) + (\mathbf{A}_{FE}{}^t + \mathbf{A}_{FE}{}^n)$, where (angles in the global coordinate system)

$A_{En} := a \cdot \omega_2{}^2$ $A_{En} = 39375\, mm \cdot sec^{-2}$

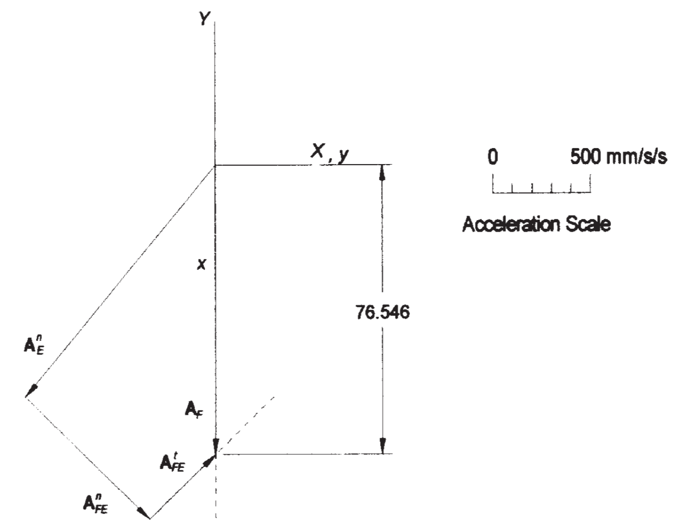
$\theta_{AEn} := \theta_2 + \alpha + 180 \cdot deg$ $\theta_{AEn} = 231.000\, deg$

$A_{Et} := a \cdot \alpha_2$ $A_{Et} = 0.000\, mm \cdot sec^{-2}$

$A_{FEn} := b \cdot \omega_3{}^2$ $A_{FEn} = 22911\, mm \cdot sec^{-2}$

$\theta_{AFEn} := \theta_3 + \alpha$ $\theta_{AFEn} = -45.172\, deg$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_E{}^n$ at an angle of θ_{AEn}. From the tip of $\mathbf{A}_E{}^n$, draw $\mathbf{A}_{FE}{}^n$ at an angle of θ_{AFEn}. Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{FE}{}^n$, draw construction lines in the directions of $\mathbf{A}_F$(vertical) and $\mathbf{A}_{FE}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{FE}{}^t$, and $\mathbf{A}_F$.



5. From the graphical solution above,

Acceleration scale factor $\quad k_a := 500 \cdot \dfrac{mm}{sec^2}$

$A_F := 76.546 \cdot k_a \qquad A_F = 38273\ mm \cdot sec^{-2}$ at an angle of -90 deg (global)

PROBLEM 7-28

Statement: The offset slider-crank linkage in Figure P7-8f has the dimensions and crank angle given below. Find $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown for $\omega_2 = 25$ rad/sec CW, constant. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A) $a := 63 \cdot mm$ Link 3 (A to B) $b := 130 \cdot mm$

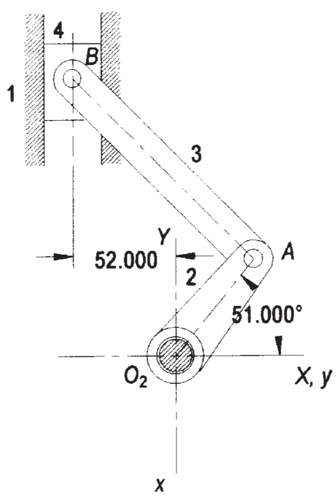
Offset $c := -52 \cdot mm$

Link 2 position, velocity, and acceleration: $\theta_2 := 141 \cdot deg$ $\omega_2 := -25 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Coordinate rotation angle: $\alpha := -90 \cdot deg$

Solution: See Figure P7-8f and Mathcad file P0728.

1. Draw the linkage to a convenient scale.



2. Determine θ_3 (in the local coordinate system) and d using equations 4.16 for the crossed circuit.

$$\theta_3 := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_3 = 44.828\, deg$$

$$d := a \cdot cos(\theta_2) - b \cdot cos(\theta_3) \qquad d = -141.160\, mm$$

3. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3 := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3)} \cdot \omega_2 \qquad \omega_3 = 13.276 \frac{rad}{sec}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_B$, determine its magnitude, and direction (global).

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$\mathbf{A_A} = 30600.122 - 24779.490j \frac{mm}{sec^2} \quad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := arg(\mathbf{A_A}) + \alpha$$

The acceleration of pin A is $A_A = 39375 \frac{mm}{sec^2}$ at $\theta_{AA} = -129.0\, deg$

5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) + b \cdot \omega_3^{\,2} \cdot \sin(\theta_3)}{b \cdot \cos(\theta_3)} \qquad \alpha_3 = -93.574 \frac{rad}{sec^2}$$

6. Use equation 7.16e for the acceleration of pin *B*.

$$A_B := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \alpha_3 \cdot \sin(\theta_3) + b \cdot \omega_3^{\,2} \cdot \cos(\theta_3)$$

$A_B = 38274 \frac{mm}{sec^2}$ A positive sign means that $\mathbf{A}_B$ is downward

PROBLEM 7-29

Statement: The offset slider-crank linkage in Figure P7-8f has the dimensions and crank angle given below. Find and plot $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 25$ rad/sec CW, constant..

Given: Link lengths:

Link 2 (O_2 to A) $a := 63 \cdot mm$ Link 3 (A to B) $b := 130 \cdot mm$

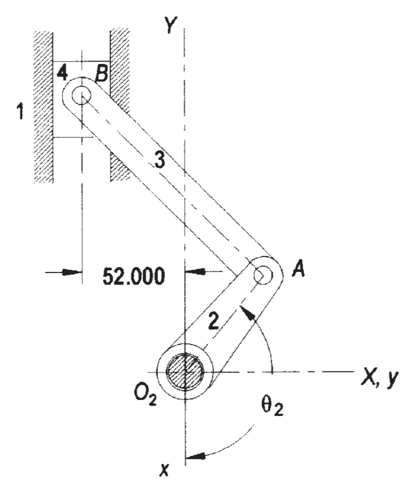
Offset $c := -52 \cdot mm$

Link 2 velocity, and acceleration: $\omega_2 := -25 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Coordinate rotation angle: $\alpha := -90 \cdot deg$

Solution: See Figure P7-8f and Mathcad file P0729.

1. Draw the linkage to a convenient scale.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine θ_3 using equations 4.16 for the crossed circuit.

$$\theta_3(\theta_2) := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right)$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3(\theta_2))} \cdot \omega_2$$

5. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its X and Y components in the global coordinate system.

$$\mathbf{A_A}(\theta_2) := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$A_{AX}(\theta_2) := Re(\mathbf{A_A}(\theta_2)) \cdot cos(\alpha) - Im(\mathbf{A_A}(\theta_2)) \cdot sin(\alpha)$$

$$A_{AY}(\theta_2) := Re(\mathbf{A_A}(\theta_2)) \cdot sin(\alpha) + Im(\mathbf{A_A}(\theta_2)) \cdot cos(\alpha)$$

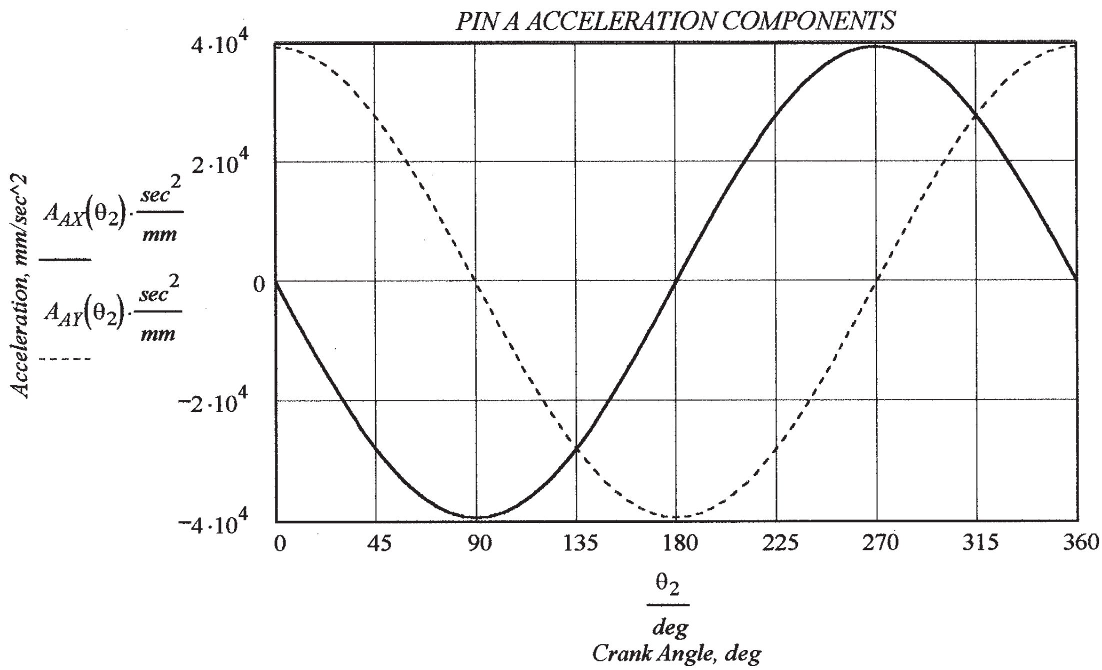
6. Determine the angular acceleration of link 3 using equation 7.16d.

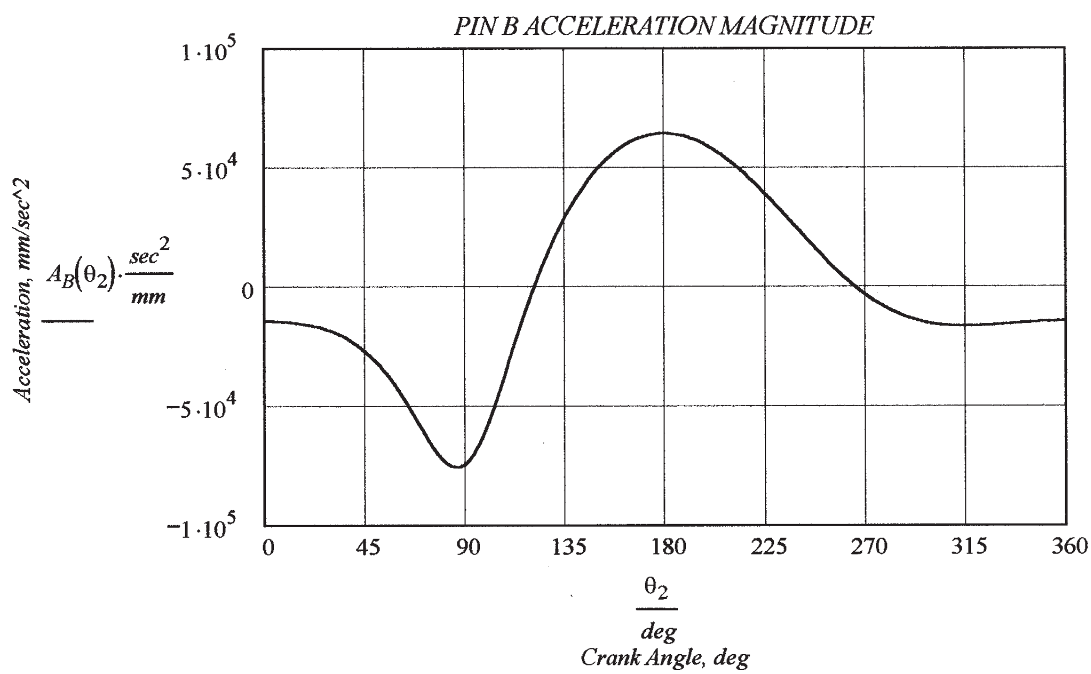
$$\alpha_3(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2))}{b \cdot \cos(\theta_3(\theta_2))}$$

7. Use equation 7.16e for the acceleration of pin B.

$$A_B(\theta_2) := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) \ldots$$
$$+ b \cdot \alpha_3(\theta_2) \cdot \sin(\theta_3(\theta_2)) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2))$$

8. Plot the components of $\mathbf{A}_A$ and the magnitude of $\mathbf{A}_B$. A positive value for $\mathbf{A}_B$ is downward, a negative value, upward.





PROBLEM 7-30

Statement: The linkage in Figure P7-8d has the dimensions and crank angle given below. Find $\mathbf{A}_4$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ in the global coordinate system for the position shown for $\omega_2 = 30$ rad/sec CW, constant. Use the acceleration difference graphical method.

Given: Link lengths:

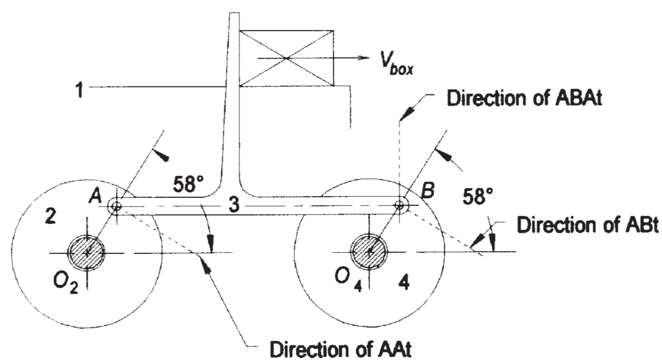
Link 2 (O_2 to A)	$a := 30 \cdot mm$	Link 3 (A to B)	$b := 150 \cdot mm$
Link 4 (O_4 to B)	$c := 30 \cdot mm$	Link 1 (O_2 to O_4)	$d := 150 \cdot mm$

Crank angle: $\theta_2 := 58 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := -30 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-8d and Mathcad file P0730.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3, ω_3, and θ_4. From the graphical position solution above (in the global XY coordinate system),

$\theta_3 := 0 \cdot deg$ $\qquad \theta_3 = 0.000\, deg$

$\theta_4 := \theta_2$ $\qquad \theta_4 = 58.000\, deg$

This is a special-case Grashof in the parallelogram configuration. Therefore,

$\omega_3 := 0.0 \cdot rad \cdot sec^{-1}$ $\qquad \omega_4 := \omega_2$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point B, this becomes: $(\mathbf{A}_B^t + \mathbf{A}_B^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n)$, where

$A_{Bn} := c \cdot \omega_4^{\,2}$ $\qquad A_{Bn} = 27000\, mm \cdot sec^{-2}$

$\theta_{ABn} := \theta_4 + 180 \cdot deg$ $\qquad \theta_{ABn} = 238.000\, deg$

$A_{An} := a \cdot \omega_2^{\,2}$ $\qquad A_{An} = 27000\, mm \cdot sec^{-2}$

$\theta_{AAn} := \theta_2 + 180 \cdot deg$ $\qquad \theta_{AAn} = 238.000\, deg$

$A_{At} := a \cdot \alpha_2$ $\qquad A_{At} = 0.0\, mm \cdot sec^{-2}$

$A_{BAn} := b \cdot \omega_3^{\,2}$ $\qquad A_{BAn} = 0\, mm \cdot sec^{-2}$

5. In this case, there is no need to draw an acceleration diagram since $\mathbf{A}_A^t$ and $\mathbf{A}_{BA}^n$ are zero. This means that $\mathbf{A}_{BA}^t$ and $\mathbf{A}_B^t$ will also be zero and $\mathbf{A}_A = \mathbf{A}_A^n$, $\mathbf{A}_B = \mathbf{A}_B^n$.

6. Since $\mathbf{A}_{BA}^t = 0$, $\alpha_3 = 0$ and, since $\mathbf{A}_B = \mathbf{A}_A$, $\alpha_4 = \alpha_2 = 0$.

7. For the box, equation 7.4 becomes: $\mathbf{A}_{box} = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{boxA}^t + \mathbf{A}_{boxA}^n)$, where $\mathbf{A}_{boxA}^t$ and $\mathbf{A}_{boxA}^n$ are both zer since α_3 and ω_3 are zero. Therefore, $\mathbf{A}_{box} = \mathbf{A}_{Ax}$.

PROBLEM 7-31

Statement: The linkage in Figure P7-8d has the dimensions and crank angle given below. Find $\mathbf{A}_4$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ in the global coordinate system for the position shown for ω_2 = 30 rad/sec CW, constant. Use an analytical method.

Given: Link lengths:

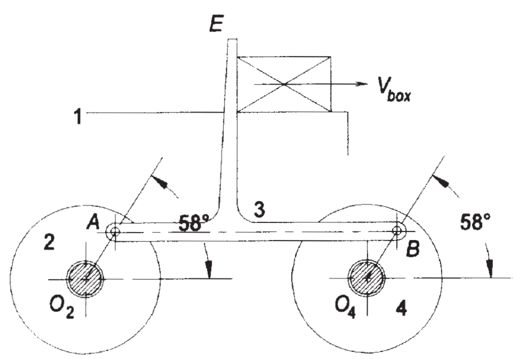
Link 2 (O_2 to A)	$a := 30 \cdot mm$	Link 3 (A to B)	$b := 150 \cdot mm$
Link 4 (O_4 to B)	$c := 30 \cdot mm$	Link 1 (O_2 to O_4)	$d := 150 \cdot mm$

Crank angle: $\theta_2 := 58 \cdot deg$

Input crank angular velocity $\omega_2 := -30 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-8d and Mathcad file P0731.

1. Draw the linkage to a convenient scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 5.0000 \qquad K_2 = 5.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.0000$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -6.1197 \qquad B = -1.6961 \qquad C = 2.8205$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_4 = -302.000\, deg$$

$$\theta_4 := \theta_4 + 360 \cdot deg \qquad \theta_4 = 58.000\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$K_4 := \dfrac{d}{b}$ $\qquad K_5 := \dfrac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)}$ $\qquad K_4 = 1.0000$ $\qquad K_5 = -5.0000$

$D := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$ $\qquad D = -8.9402$

$E := -2 \cdot sin(\theta_2)$ $\qquad E = -1.6961$

$F := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$ $\qquad F = 0.0000$

5. Use equation 4.13 to find values of θ_3 .

$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right)$ $\qquad \theta_3 = 360.000\,deg$

$\theta_3 := \theta_3 - 360 \cdot deg$ $\qquad \theta_3 = 0.000\,deg$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$\omega_3 := \dfrac{a \cdot \omega_2}{b} \cdot \dfrac{sin(\theta_4 - \theta_2)}{sin(\theta_3 - \theta_4)}$ $\qquad \omega_3 = 0.000 \dfrac{rad}{sec}$

$\omega_4 := \dfrac{a \cdot \omega_2}{c} \cdot \dfrac{sin(\theta_2 - \theta_3)}{sin(\theta_4 - \theta_3)}$ $\qquad \omega_4 = -30.000 \dfrac{rad}{sec}$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$

$\mathbf{A_A} = -14308 - 22897i \dfrac{mm}{sec^2}$ $\qquad A_A := |\mathbf{A_A}|$ $\qquad \theta_{AA} := arg(\mathbf{A_A})$

The acceleration of pin A is $\quad A_A = 27000 \dfrac{mm}{sec^2}$ at $\quad \theta_{AA} = -122.0\,deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$A := c \cdot sin(\theta_4)$ $\qquad B := b \cdot sin(\theta_3)$ $\qquad D := c \cdot cos(\theta_4)$ $\qquad E := b \cdot cos(\theta_3)$

$A = 1.002\,in$ $\qquad B = 0.000\,in$ $\qquad D = 0.626\,in$ $\qquad E = 5.906\,in$

$C := a \cdot \alpha_2 \cdot sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot cos(\theta_2) + b \cdot \omega_3^{\,2} \cdot cos(\theta_3) - c \cdot \omega_4^{\,2} \cdot cos(\theta_4)$

$C = 6.281 \times 10^{-11}\, mm \cdot sec^{-2}$

$F := a \cdot \alpha_2 \cdot cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot sin(\theta_2) - b \cdot \omega_3^{\,2} \cdot sin(\theta_3) + c \cdot \omega_4^{\,2} \cdot sin(\theta_4)$

$F = -1.733 \times 10^{-11}\, mm \cdot sec^{-2}$

$\alpha_3 := \dfrac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D}$ $\qquad \alpha_3 = 0.000 \dfrac{rad}{sec^2}$ $\qquad \alpha_4 := \dfrac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D}$ $\qquad \alpha_4 = 0.000 \dfrac{rad}{sec^2}$

9. Use equation 7.13c to determine the acceleration of point B.

$\mathbf{A_B} := c \cdot \alpha_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) - c \cdot \omega_4^{\,2} \cdot (cos(\theta_4) + j \cdot sin(\theta_4))$

$$\mathbf{A_B} = -14308 - 22897i \frac{mm}{sec^2} \qquad A_B := |\mathbf{A_B}| \qquad \theta_{AB} := arg(\mathbf{A_B})$$

The acceleration of pin B is $A_B = 27000 \frac{mm}{sec^2}$ at $\theta_{AB} = -122.0\, deg$

10. This is a special case Grashof in the parallelogram configuration. All points on link 3 have the same velocity and acceleration. The acceleration of the box will be equal to the X-component of the acceleration of any point on link 3.

$$A_{box} := Re(\mathbf{A_A})$$

The acceleration of the box is $A_{box} = -14308 \frac{mm}{sec^2}$ a negative sign means $\mathbf{A}_{box}$ is to the left

PROBLEM 7-32

Statement: The linkage in Figure P7-8d has the dimensions and crank angle given below. Write a computer program or use an equation solver to find and plot $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ in the global coordinate system for the maximum range of motion that this linkage allows if ω_2 = 30 rad/sec CW, constant .

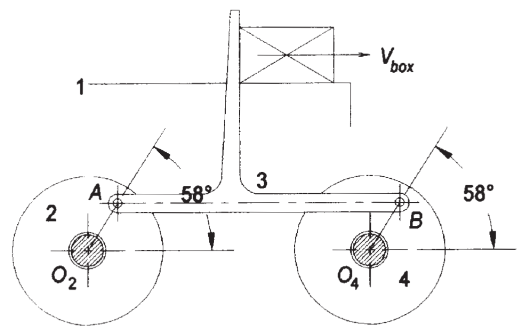
Given: Link lengths:

Link 2 (O_2 to A)	$a := 30 \cdot mm$	Link 3 (A to B)	$b := 150 \cdot mm$
Link 4 (O_4 to B)	$c := 30 \cdot mm$	Link 1 (O_2 to O_4)	$d := 150 \cdot mm$

Input crank angular velocity $\omega_2 := -30 \cdot rad \cdot sec^{-1}$ $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-8d and Mathcad file P0732.

1. Draw the linkage to a convenient scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 5.0000 \qquad K_2 = 5.0000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.0000$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.0000 \qquad K_5 = -5.0000$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4(\theta_2) - \theta_2)}{sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the XY components.

$$\mathbf{A_A}(\theta_2) := a\cdot\alpha_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) - a\cdot\omega_2{}^2\cdot(cos(\theta_2) + j\cdot sin(\theta_2))$$

$$A_{AX}(\theta_2) := Re(\mathbf{A_A}(\theta_2)) \qquad A_{AY}(\theta_2) := Im(\mathbf{A_A}(\theta_2))$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A'(\theta_2) := c\cdot sin(\theta_4(\theta_2)) \qquad B'(\theta_2) := b\cdot sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c\cdot cos(\theta_4(\theta_2)) \qquad E'(\theta_2) := b\cdot cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2{}^2\cdot cos(\theta_2) + b\cdot\omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2{}^2\cdot sin(\theta_2) - b\cdot\omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)^2\cdot sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2)\cdot D'(\theta_2) - A'(\theta_2)\cdot F'(\theta_2)}{A'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot D'(\theta_2)} \qquad \alpha_4(\theta_2) := \frac{C'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot F'(\theta_2)}{A'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot D'(\theta_2)}$$

9. Use equation 7.13c to determine the acceleration of point B.

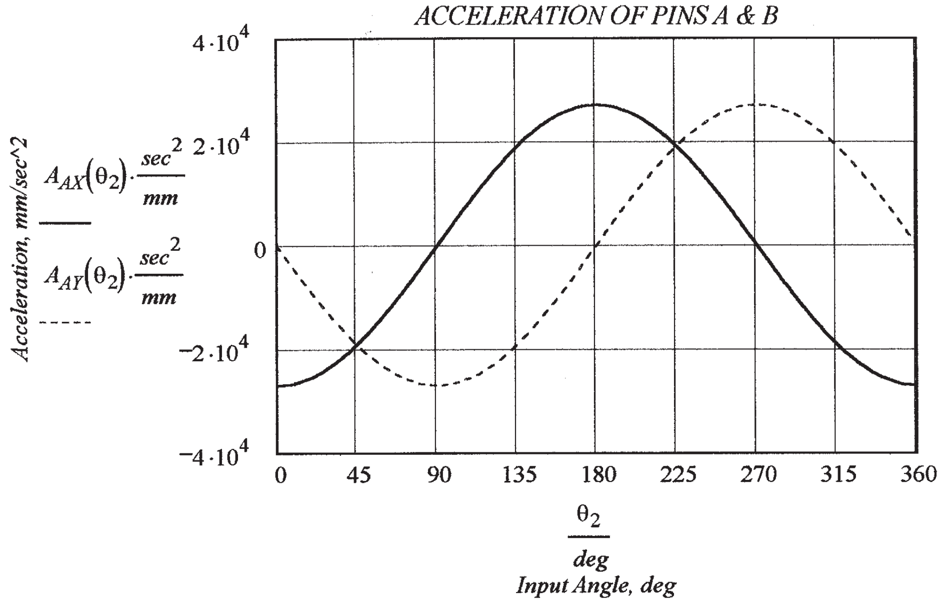
$$\mathbf{A_B}(\theta_2) := c\cdot\alpha_4(\theta_2)\cdot(-sin(\theta_4(\theta_2)) + j\cdot cos(\theta_4(\theta_2))) \ldots$$
$$+ -c\cdot\omega_4(\theta_2)^2\cdot(cos(\theta_4(\theta_2)) + j\cdot sin(\theta_4(\theta_2)))$$

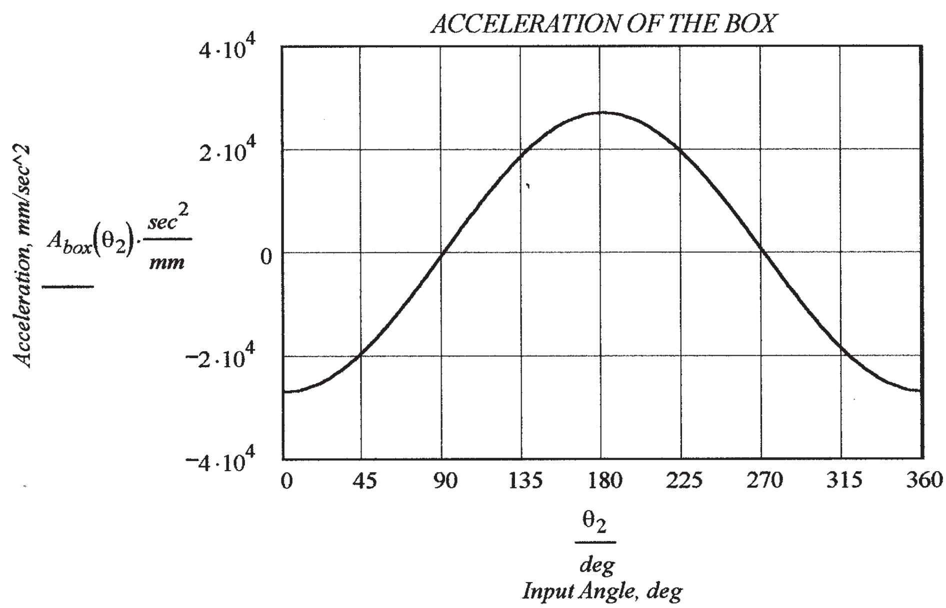
$$A_{BX}(\theta_2) := Re(\mathbf{A_B}(\theta_2)) \qquad A_{BY}(\theta_2) := Im(\mathbf{A_B}(\theta_2))$$

10. This is a special case Grashof in the parallelogram configuration. All points on link 3 have the same velocity and acceleration. The acceleration of the box will be equal to the X-component of the acceleration of any point on link 3.

$$A_{box}(\theta_2) := Re(\mathbf{A_A}(\theta_2))$$

11. Plot the accelerations over the range: $\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$


ACCELERATION OF PINS A & B
Acceleration, mm/sec^2
$A_{AX}(\theta_2)\cdot\frac{sec^2}{mm}$
$A_{AY}(\theta_2)\cdot\frac{sec^2}{mm}$
$4\cdot10^4$
$2\cdot10^4$
0
$-2\cdot10^4$
$-4\cdot10^4$
0 45 90 135 180 225 270 315 360
$\frac{\theta_2}{deg}$
Input Angle, deg


ACCELERATION OF THE BOX
Acceleration, mm/sec^2
$A_{box}(\theta_2)\cdot\frac{sec^2}{mm}$
$4\cdot10^4$
$2\cdot10^4$
0
$-2\cdot10^4$
$-4\cdot10^4$
0 45 90 135 180 225 270 315 360
$\frac{\theta_2}{deg}$
Input Angle, deg

PROBLEM 7-33

Statement: The linkage in Figure P7-8g has the dimensions and crank angle given below. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW, constant. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot mm$	Link 3 (A to B)	$b := 100 \cdot mm$
Link 4 (O_4 to B)	$c := 153 \cdot mm$	Link 1 (O_2 to O_4)	$d := 87 \cdot mm$

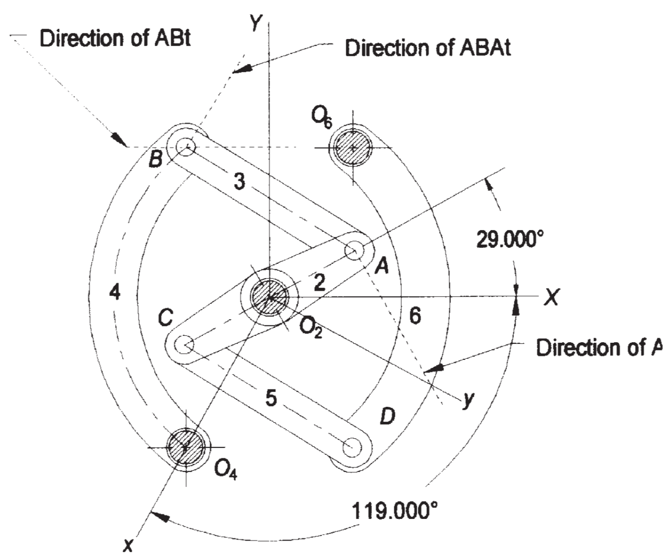
Crank angle: $\theta_2 := 148 \cdot deg$ Local xy system

Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Coordinate rotation angle $\beta := -119 \cdot deg$ Global XY system to local xy system

Solution: See Figure P7-8e and Mathcad file P0733.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3, ω_3, and θ_4. From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 266.812 \cdot deg \qquad \theta_4 := 208.876 \cdot deg$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{(\sin(\theta_3 - \theta_4))} \qquad \omega_3 = -7.576\, rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{(\sin(\theta_4 - \theta_3))} \qquad \omega_4 = -4.967\, rad \cdot sec^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

4. For point B, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where (angles in the global coordinate system)

$A_{Bn} := c \cdot \omega_4^{\,2}$ $\qquad A_{Bn} = 3775\, mm \cdot sec^{-2}$

$\theta_{ABn} := \theta_4 - 180 \cdot deg + \beta$ $\qquad \theta_{ABn} = -90.124\, deg$

$A_{An} := a \cdot \omega_2^{\,2}$ $\qquad A_{An} = 11025\, mm \cdot sec^{-2}$

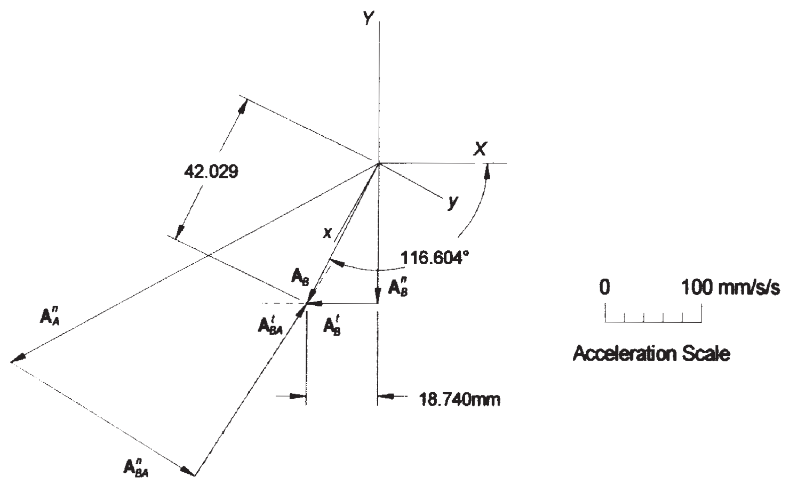
$\theta_{AAn} := \theta_2 + 180 \cdot deg + \beta$ $\qquad \theta_{AAn} = 209.000\, deg$

$A_{At} := a \cdot \alpha_2$ $\qquad A_{At} = 0.000\, mm \cdot sec^{-2}$

$A_{BAn} := b \cdot \omega_3^{\,2}$ $\qquad A_{BAn} = 5740\, mm \cdot sec^{-2}$

$\theta_{AABn} := \theta_3 - 180 \cdot deg + \beta$ $\qquad \theta_{AABn} = -32.188\, deg$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B^n$ at an angle of θ_{ABn} and $\mathbf{A}_A^n$ at an angle of θ_{AAn}. From the tip of $\mathbf{A}_A^n$, draw $\mathbf{A}_A^t$ at an angle of θ_{AAt}. From the tip of $\mathbf{A}_A^t$, draw $\mathbf{A}_{BA}^n$ at an angle of θ_{ABAn}. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B^n$ and $\mathbf{A}_{BA}^n$, draw construction lines in the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B^t$, $\mathbf{A}_B^t$, and $\mathbf{A}_{BA}^t$.



6. From the graphical solution above,

Acceleration scale factor $\quad k_a := 100 \cdot \dfrac{mm}{sec^2}$

$A_A := A_{An}$ $\qquad A_A = 11025\, mm \cdot sec^{-2}$ $\qquad$ at an angle of 209 deg

$A_B := 42.029 \cdot k_a$ $\qquad A_B = 4203\, mm \cdot sec^{-2}$ $\qquad$ at an angle of -116.6 deg

$A_{Bt} := 18.740 \cdot k_a$ $\qquad A_{Bt} = 1874\, mm \cdot sec^{-2}$

7. Calculate α_4 using equation 7.6.

$\alpha_4 := \dfrac{A_{Bt}}{c}$ $\qquad \alpha_4 = 12.2\, rad \cdot sec^{-2}$ $\qquad$ CCW

PROBLEM 7-34

Statement: The linkage in Figure P7-8g has the dimensions and crank angle given below. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW and α_2 = 10 rad/sec CCW, constant. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 49 \cdot mm$	Link 3 (A to B)	$b := 100 \cdot mm$
Link 4 (O_4 to B)	$c := 153 \cdot mm$	Link 1 (O_2 to O_4)	$d := 87 \cdot mm$

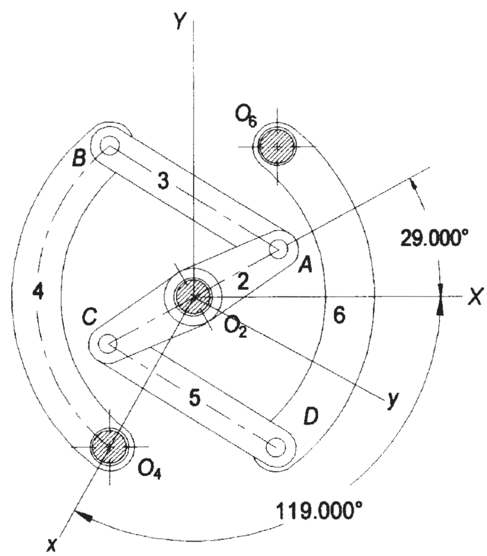
Crank angle: $\theta_2 := 148 \cdot deg$ Local *xy* system (see layout below)

Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 10 \cdot rad \cdot sec^{-2}$

Coordinate rotation angle $\beta := -119 \cdot deg$ Global *XY* system to local *xy* system

Solution: See Figure P7-8g and Mathcad file P0734.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.7755 \qquad K_2 = 0.5686$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.5592$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.5821 \qquad B = -1.0598 \qquad C = 4.6650$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_4 = 208.876\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.8700 \qquad K_5 = 0.3509$$

$$D := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5 \qquad D = -3.0104$$

$$E := -2 \cdot sin(\theta_2) \qquad E = -1.0598$$

$$F := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5 \qquad F = 2.2367$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_3 = 266.892\,deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{sin(\theta_4 - \theta_2)}{sin(\theta_3 - \theta_4)} \qquad \omega_3 = -7.570 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{sin(\theta_2 - \theta_3)}{sin(\theta_4 - \theta_3)} \qquad \omega_4 = -4.959 \frac{rad}{sec}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction (in the local coordinate system).

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$\mathbf{A_A} = 9090 - 6258j \frac{mm}{sec^2} \qquad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := arg(\mathbf{A_A}) - \beta$$

The acceleration of pin A is $\quad A_A = 11036 \dfrac{mm}{sec^2} \quad$ at $\quad \theta_{AA} = 84.46\,deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot sin(\theta_4) \qquad B := b \cdot sin(\theta_3) \qquad D := c \cdot cos(\theta_4) \qquad E := b \cdot cos(\theta_3)$$

$$A = -73.887\,mm \qquad B = -99.853\,mm \qquad D = -133.977\,mm \qquad E = -5.422\,mm$$

$$C := a \cdot \alpha_2 \cdot sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot cos(\theta_2) + b \cdot \omega_3^{\,2} \cdot cos(\theta_3) - c \cdot \omega_4^{\,2} \cdot cos(\theta_4)$$

$$C = -6.106 \times 10^3\, mm \cdot sec^{-2}$$

$$F := a \cdot \alpha_2 \cdot cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot sin(\theta_2) - b \cdot \omega_3^{\,2} \cdot sin(\theta_3) + c \cdot \omega_4^{\,2} \cdot sin(\theta_4)$$

$$F = -2.353 \times 10^3\, mm \cdot sec^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_3 = -49.646 \frac{rad}{sec^2} \qquad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_4 = 15.552 \frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point B for the crossed circuit.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-\sin(\theta_4) + j \cdot \cos(\theta_4)\right) - c \cdot \omega_4^{\,2} \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right)$$

$\mathbf{A_B} = 4444 - 267j \dfrac{mm}{sec^2}$ $\qquad A_B := |\mathbf{A_B}|$ $\qquad \theta_{AB} := \arg(\mathbf{A_B}) - \beta$

The acceleration of pin B is $\quad A_B = 4452 \dfrac{mm}{sec^2} \quad$ at $\quad \theta_{AB} = 115.6\,deg$

PROBLEM 7-35

Statement: At $t = 0$, the non-Grashof linkage in Figure P7-8g has the local axis at -119 deg and O_2A at 29 deg in the global XY coordinate system and $\omega_2 = 0$. Write a computer program or use an equation solver to find and plot ω_4, $\mathbf{V}_A$, $\mathbf{A}_A$, $\mathbf{V}_B$, and $\mathbf{A}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\alpha_2 = 15$ rad/sec CCW, constant.

Given: Link lengths:

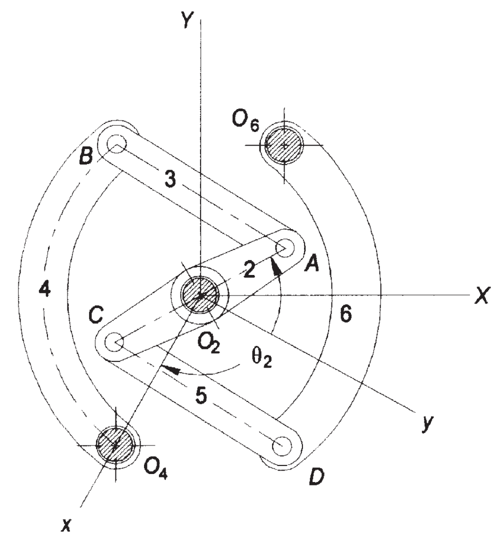
Link 2 (O_2 to A)	$a := 49 \cdot mm$	Link 3 (A to B)	$b := 100 \cdot mm$
Link 4 (O_4 to B)	$c := 153 \cdot mm$	Link 1 (O_2 to O_4)	$d := 87 \cdot mm$

Input crank angular velocity $\omega_2 := 0 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 15 \cdot rad \cdot sec^{-2}$

Initial crank angle: $\theta_{20} := 148 \cdot deg$ Local xy system (see layout below)

Solution: See Figure P7-8g and Mathcad file P0735.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 0.840$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = -6.338$$

$$\theta_{2toggle} := acos(arg_1) \qquad \theta_{2toggle} = 32.9\,deg$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := \theta_{20}, \theta_{20} + 1 \cdot deg .. 360 \cdot deg - \theta_{2toggle}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.7755 \qquad K_2 = 0.5686$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.5592$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.8700 \qquad K_5 = 0.3509$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 2, 3, and 4 using equations 6.18.

$$\omega_2(\theta_2) := \sqrt{2 \cdot (\theta_2 - \theta_{20}) \cdot \alpha_2}$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2(\theta_2)}{b} \cdot \frac{sin(\theta_4(\theta_2) - \theta_2)}{sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2(\theta_2)}{c} \cdot \frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. and determine its magnitude.

$$\mathbf{A_A}(\theta_2) := a \cdot \alpha_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) - a \cdot \omega_2(\theta_2)^2 \cdot (cos(\theta_2) + j \cdot sin(\theta_2))$$

$$A_A(\theta_2) := |\mathbf{A_A}(\theta_2)|$$

9. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot sin(\theta_4(\theta_2)) \qquad B(\theta_2) := b \cdot sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot cos(\theta_4(\theta_2)) \qquad E(\theta_2) := b \cdot cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot sin(\theta_2) + a \cdot \omega_2(\theta_2)^2 \cdot cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2(\theta_2)^2\cdot sin(\theta_2) - b\cdot\omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)^2\cdot sin(\theta_4(\theta_2))$$

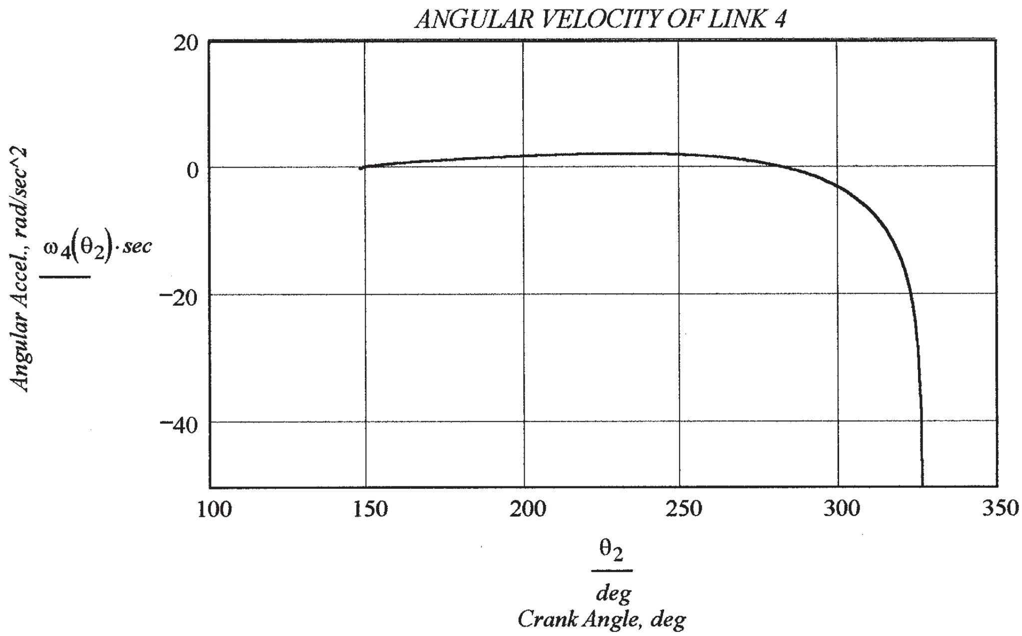
$$\alpha_4(\theta_2) := \frac{C(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot F(\theta_2)}{A(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot D(\theta_2)}$$

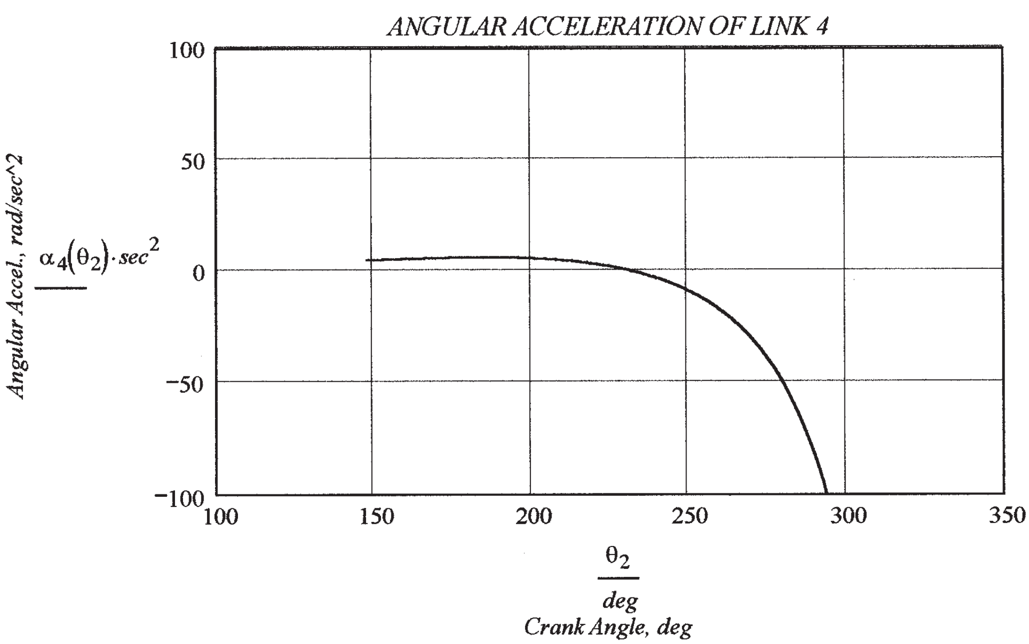
10. Use equation 7.13c to determine the acceleration of point B and determine its magnitude.

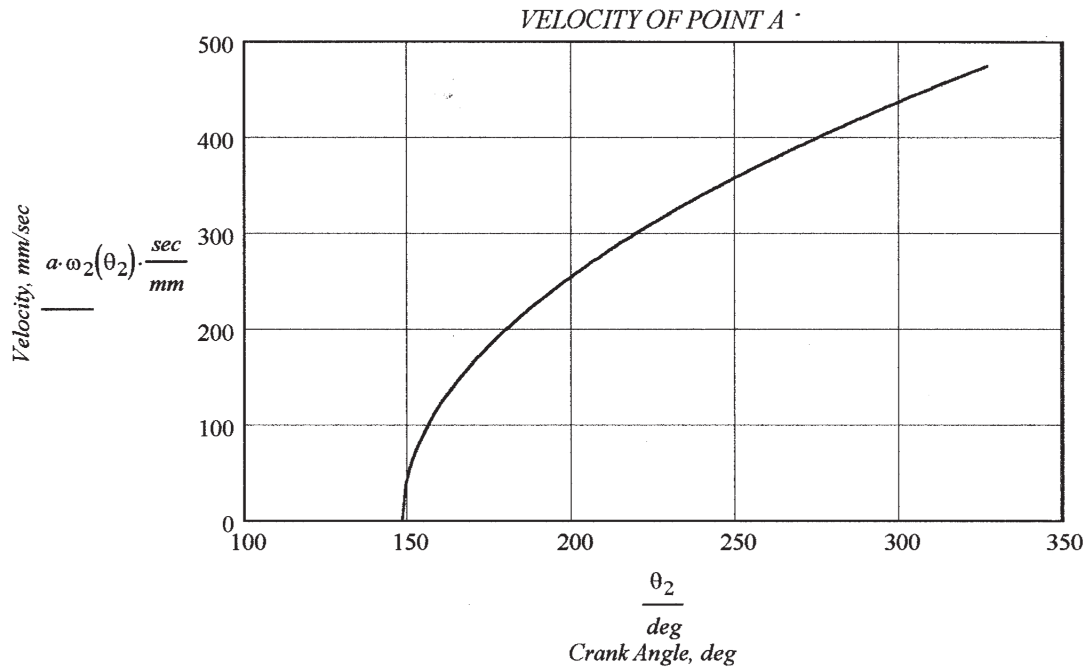
$$\mathbf{A_B}(\theta_2) := c\cdot\alpha_4(\theta_2)\cdot(-sin(\theta_4(\theta_2)) + j\cdot cos(\theta_4(\theta_2))) - c\cdot\omega_4(\theta_2)^2\cdot(cos(\theta_4(\theta_2)) + j\cdot sin(\theta_4(\theta_2)))$$

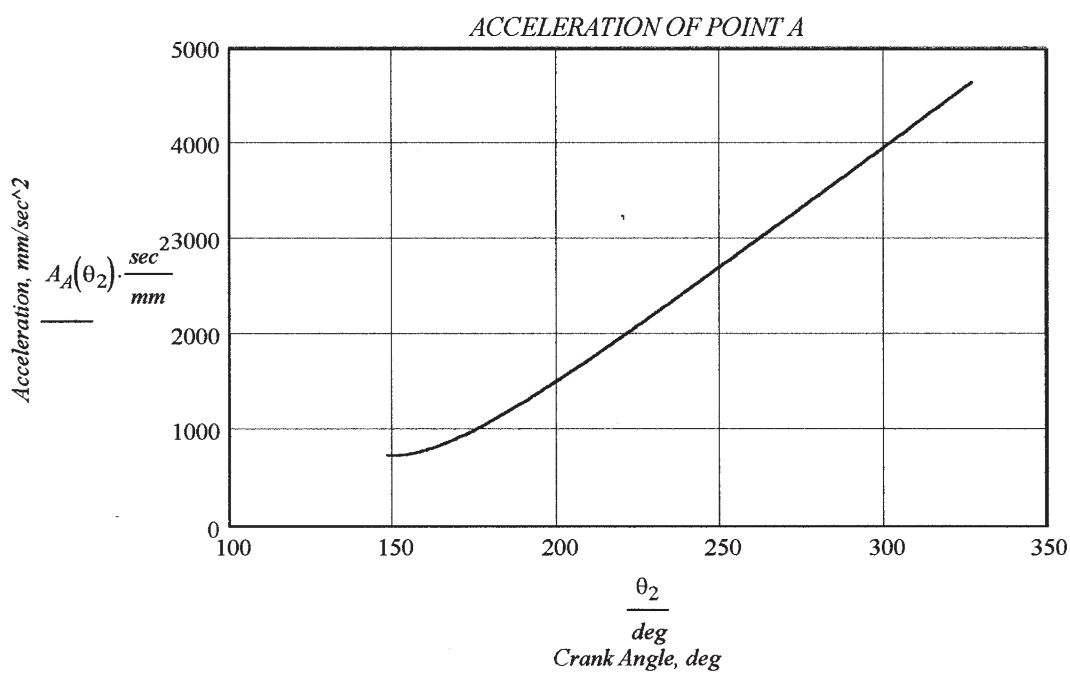
$$A_B(\theta_2) := |\mathbf{A_B}(\theta_2)|$$

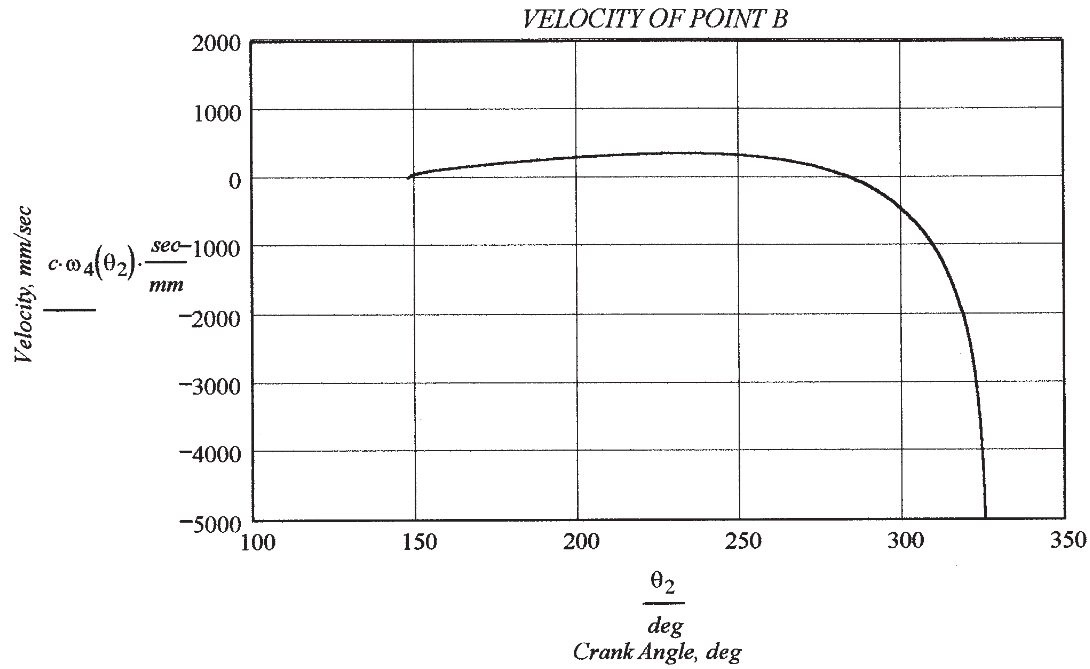
11. Plot the angular velocity and acceleration for link 4 and the velocity and acceleration components for pins A and B.

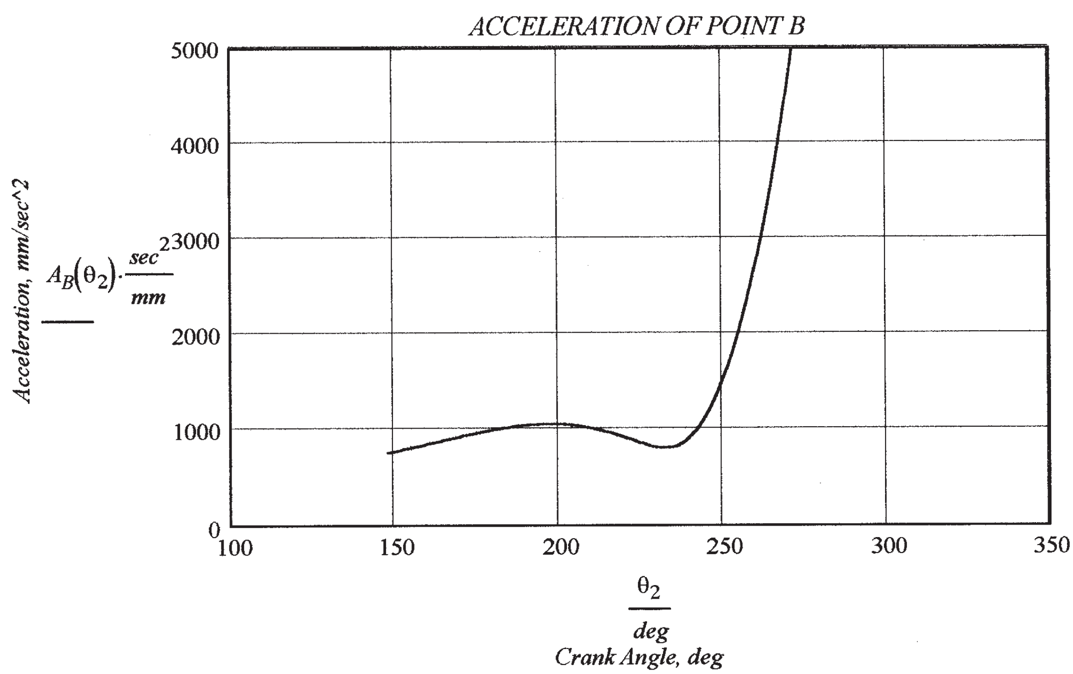





VELOCITY OF POINT A
500
400
300
200
100
0
100
150
200
250
300
350
Velocity, mm/sec
$a \cdot \omega_2(\theta_2) \cdot \frac{sec}{mm}$
$\frac{\theta_2}{deg}$
Crank Angle, deg


ACCELERATION OF POINT A
5000
4000
3000
2000
1000
0
100
150
200
250
300
350
Acceleration, mm/sec^2
$A_A(\theta_2) \cdot \frac{sec^2}{mm}$
$\frac{\theta_2}{deg}$
Crank Angle, deg


VELOCITY OF POINT B
2000
1000
0
−1000
−2000
−3000
−4000
−5000
100
150
200
250
300
350
Velocity, mm/sec
$c \cdot \omega_4(\theta_2) \cdot \frac{sec}{mm}$
$\frac{\theta_2}{deg}$
Crank Angle, deg


ACCELERATION OF POINT B
5000
4000
3000
2000
1000
0
100
150
200
250
300
350
Acceleration, mm/sec^2
$A_B(\theta_2) \cdot \frac{sec^2}{mm}$
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 7-36

Statement: The 3-cylinder radial compressor in Figure P7-8c has the dimensions and crank angle given below. Find the piston accelerations $\mathbf{A}_6$, $\mathbf{A}_7$, and $\mathbf{A}_8$ for the position shown for $\omega = 15$ rad/sec CW, constant. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 $a := 19 \cdot mm$ Link 3 $b := 70 \cdot mm$

Offset $c := 0 \cdot mm$

Link 2 position, velocity, and acceleration: $\theta_2 := -53 \cdot deg$ $\omega_2 := -15 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Cylinder angular spacing: $\beta := 120 \cdot deg$

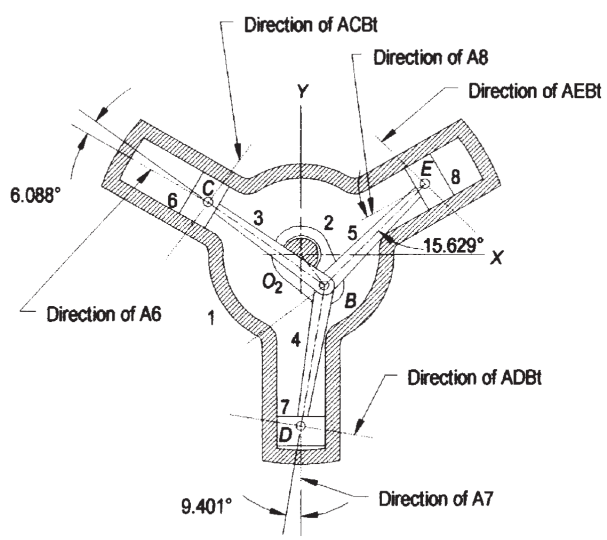
Solution: See Figure P7-8c and Mathcad file P0736.

1. In order to solve for the accelerations at piston 7, we will need θ_4, and ω_4. From the graphical position solution below (in the local coordinate system),

Coordinate system rotation angle: $\alpha := -90 \cdot deg$

$\theta_{21} := \theta_2 - \alpha$ $\theta_{21} = 37.000\,deg$ $\theta_4 := 180 \cdot deg - 9.401 \cdot deg$ $\theta_4 = 170.599\,deg$

Using equation 6.22a, $\omega_4 := \frac{a \cdot \omega_2}{b} \cdot \frac{\cos(\theta_{21})}{\cos(\theta_4)}$ $\omega_4 = 3.296\,rad \cdot sec^{-1}$



2. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

3. For point D, this becomes: $\mathbf{A}_D = (\mathbf{A}_B^t + \mathbf{A}_B^n) + (\mathbf{A}_{DB}^t + \mathbf{A}_{DB}^n)$, where (angles in the global coordinate system)

$A_{Bn} := a \cdot \omega_2^{\,2}$ $A_{Bn} = 4275\,mm \cdot sec^{-2}$

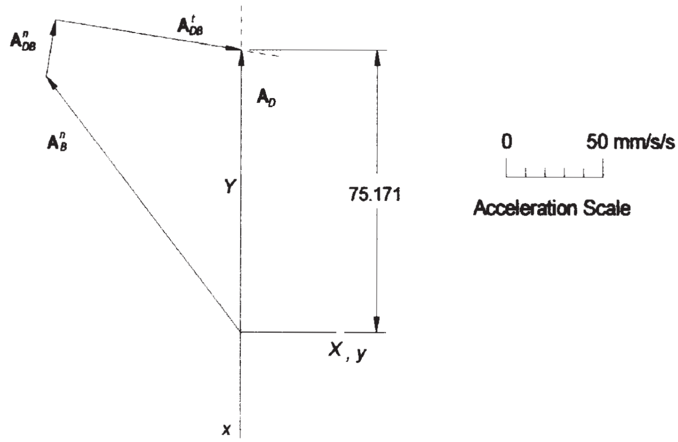
$\theta_{ABn} := \theta_{21} + \alpha + 180 \cdot deg$ $\theta_{ABn} = 127.000\,deg$

$A_{Bt} := a \cdot \alpha_2$ $A_{Bt} = 0.000\,mm \cdot sec^{-2}$

$A_{DBn} := b \cdot \omega_4^{\,2}$ $A_{DBn} = 760\,mm \cdot sec^{-2}$

$\theta_{ADBn} := \theta_4 + \alpha$ $\theta_{ADBn} = 80.599\,deg$

4. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of θ_{ABn}. From the tip of $\mathbf{A}_B{}^n$, draw $\mathbf{A}_{DB}{}^n$ at an angle of θ_{ADBn}. Now that the vectors with known magnitudes are drawn, from the origin and the tip of $\mathbf{A}_{DB}{}^n$, draw construction lines in the directions of $\mathbf{A}_D$ (vertical) and $\mathbf{A}_{DB}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_{DB}{}^t$, and $\mathbf{A}_D$.



5. From the graphical solution above,

Acceleration scale factor $\quad k_a := 50 \cdot \dfrac{mm}{sec^2}$

$A_D := 75.171 \cdot k_a \qquad A_D = 3759\, mm \cdot sec^{-2}$ at an angle of 90 deg (global)

6. In order to solve for the accelerations at piston 6, we will need θ_3, and ω_3. From the graphical position solution above (in the local coordinate system),

Coordinate system rotation angle: $\quad \alpha := -90 \cdot deg - \beta$

$\theta_{22} := \theta_2 - \alpha \qquad \theta_{22} = 157.000\, deg \qquad \theta_3 := 180 \cdot deg - 6.088 \cdot deg \qquad \theta_3 = 173.912\, deg$

Using equation 6.22a, $\quad \omega_3 := \dfrac{a \cdot \omega_2}{b} \cdot \dfrac{cos(\theta_{22})}{cos(\theta_3)} \qquad \omega_3 = -3.769\, rad \cdot sec^{-1}$

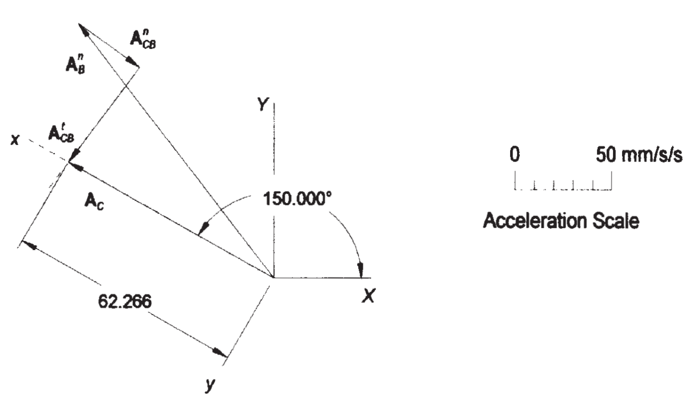
7. For point C, equation 7.4 becomes: $\mathbf{A}_C = (\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) + (\mathbf{A}_{CB}{}^t + \mathbf{A}_{CB}{}^n)$, where (angles in the global coordinate system)

$A_{CBn} := b \cdot \omega_3{}^2$

$A_{CBn} = 994.4\, mm \cdot sec^{-2}$

$\theta_{ACBn} := \theta_3 + \alpha$

$\theta_{ACBn} = -36.088\, deg$



8. Repeat the procedure of step 4 for the equation in step 7 using $\mathbf{A}_B{}^n$ and $\mathbf{A}_B{}^t$ from step 3.

9. From the graphical solution above,

Acceleration scale factor $k_a := 50 \cdot \dfrac{mm}{sec^2}$

$A_C := 62.266 \cdot k_a$ $A_C = 3113\, mm \cdot sec^{-2}$ at an angle of 150 deg (global)

10. In order to solve for the accelerations at piston 8, we will need θ_5, and ω_5. From the graphical position solution above (in the local coordinate system),

Coordinate system rotation angle: $\alpha := 360 \cdot deg + (-90 \cdot deg - 2 \cdot \beta)$

$\theta_{23} := \theta_2 - \alpha$ $\theta_{23} = -83.000\, deg$ $\theta_5 := 180 \cdot deg + 15.629 \cdot deg$ $\theta_5 = 195.629\, deg$

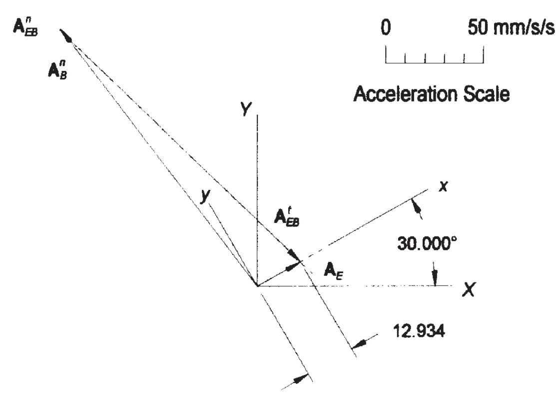
Using equation 6.22a, $\omega_5 := \dfrac{a \cdot \omega_2}{b} \cdot \dfrac{cos(\theta_{23})}{cos(\theta_5)}$ $\omega_5 = 0.515\, rad \cdot sec^{-1}$

11. For point E, equation 7.4 becomes: $\mathbf{A}_E = (\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) + (\mathbf{A}_{EB}{}^t + \mathbf{A}_{EB}{}^n)$, where (angles in the global coordinate system)

$A_{EBn} := b \cdot \omega_5^{\,2}$ $A_{EBn} = 18.583\, mm \cdot sec^{-2}$

$\theta_{AEBn} := \theta_5 + \alpha$ $\theta_{AEBn} = 225.629\, deg$

12. Repeat the procedure of step 4 for the equation in step 11 using $\mathbf{A}_B{}^n$ and $\mathbf{A}_B{}^t$ from step 3.



Note that, at the acceleration scale chosen, $\mathbf{A}_{EB}{}^n$ is so small that it can hardly be seen in the layout above.

13. From the graphical solution above,

Acceleration scale factor $k_a := 50 \cdot \dfrac{mm}{sec^2}$

$A_E := 12.934 \cdot k_a$ $A_E = 647\, mm \cdot sec^{-2}$ at an angle of 30 deg (global)

PROBLEM 7-37

Statement: The 3-cylinder radial compressor in Figure P7-8c has the dimensions and crank angle given below. Find the piston accelerations $\mathbf{A_6}$, $\mathbf{A_7}$, and $\mathbf{A_8}$ for the position shown for ω = 15 rad/sec CW, constant. Use an analytical method.

Given: Link lengths:

Link 2 $\quad a := 19 \cdot mm$ $\quad$ Link 3 $\quad b := 70 \cdot mm$

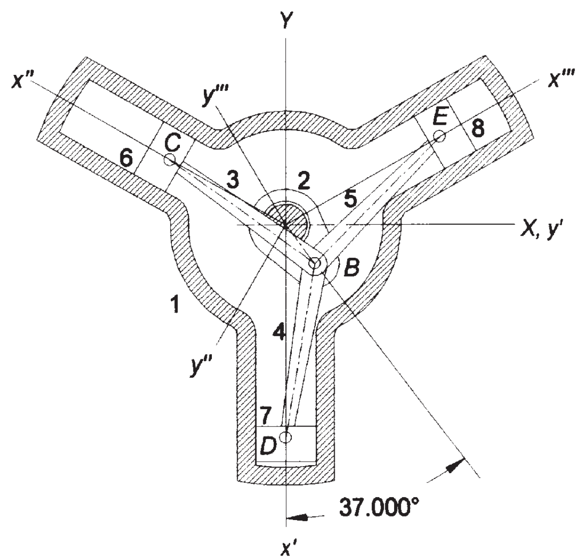
Offset $\quad c := 0 \cdot mm$

Link 2 position, velocity, and acceleration: $\quad \theta_2 := 37 \cdot deg \quad \omega_2 := -15 \cdot \frac{rad}{sec} \quad \alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Cylinder angular spacing: $\quad \beta := 120 \cdot deg$

Solution: See Figure P7-8c and Mathcad file P0737.

1. Draw the linkage to scale and label it.



2. Determine θ_4 and d' using equation 4.17. Coordinate rotation angle $\alpha := -90 \cdot deg$

$$\theta_4 := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_4 = 170.599\, deg \qquad (x'y' \text{ system})$$

$$d' := a \cdot cos(\theta_2) - b \cdot cos(\theta_4) \qquad d' = 3.316\, in$$

3. Determine the angular velocity of link 4 using equation 6.22a:

$$\omega_4 := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_4)} \cdot \omega_2 \qquad \omega_4 = 3.296 \frac{rad}{sec}$$

4. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_B$, determine its magnitude, and direction (global).

$$\mathbf{A_B} := a \cdot \alpha_2 \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right) - a \cdot \omega_2^{\,2} \cdot \left(cos(\theta_2) + j \cdot sin(\theta_2)\right)$$

$$\mathbf{A_B} = -3414.167 - 2572.759i \frac{mm}{sec^2} \qquad A_B := \left|\mathbf{A_B}\right| \qquad \theta_{AB} := arg(\mathbf{A_B}) + \alpha + 360 \cdot deg$$

The acceleration of pin B is $A_B = 4275 \dfrac{mm}{sec^2}$ at $\theta_{AB} = 127\,deg$

5. Determine the angular acceleration of link 4 using equation 7.16d.

$$\alpha_4 := \frac{a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) + b\cdot\omega_4^{\,2}\cdot sin(\theta_4)}{b\cdot cos(\theta_4)} \qquad \alpha_4 = 35.456\frac{rad}{sec^2}$$

6. Use equation 7.16e for the acceleration of pin D.

$$A_D := -a\cdot\alpha_2\cdot sin(\theta_2) - a\cdot\omega_2^{\,2}\cdot cos(\theta_2) + b\cdot\alpha_4\cdot sin(\theta_4) + b\cdot\omega_4^{\,2}\cdot cos(\theta_4)$$

$A_D = -3759\dfrac{mm}{sec^2}$ A negative sign means that $\mathbf{A}_D$ is inward

7. Determine θ_3 and d'' using equation 4.17.

$\theta_2 := \theta_2 + 120\cdot deg$ $\qquad \theta_2 = 157.000\,deg$ $\qquad$ ($x''y''$ system)

$\theta_3 := asin\left(\dfrac{a\cdot sin(\theta_2) - c}{b}\right) + \pi$ $\qquad \theta_3 = 173.912\,deg$

$d'' := a\cdot cos(\theta_2) - b\cdot cos(\theta_3)$ $\qquad d'' = 2.052\,in$

8. Determine the angular velocity of link 3 using equation 6.22a:

$\omega_3 := \dfrac{a}{b}\cdot\dfrac{cos(\theta_2)}{cos(\theta_3)}\cdot\omega_2$ $\qquad \omega_3 = -3.769\dfrac{rad}{sec}$

9. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3 := \frac{a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) + b\cdot\omega_3^{\,2}\cdot sin(\theta_3)}{b\cdot cos(\theta_3)} \qquad \alpha_3 = 22.483\frac{rad}{sec^2}$$

10. Use equation 7.16e for the acceleration of pin C.

$$A_C := -a\cdot\alpha_2\cdot sin(\theta_2) - a\cdot\omega_2^{\,2}\cdot cos(\theta_2) + b\cdot\alpha_3\cdot sin(\theta_3) + b\cdot\omega_3^{\,2}\cdot cos(\theta_3)$$

$A_C = 3113\dfrac{mm}{sec^2}$ A positive sign means that $\mathbf{A}_C$ is outward

11. Determine θ_5 and d''' using equation 4.17.

$\theta_2 := \theta_2 + 120\cdot deg$ $\qquad \theta_2 = 277.000\,deg$ $\qquad$ ($x'''y'''$ system)

$\theta_5 := asin\left(\dfrac{a\cdot sin(\theta_2) - c}{b}\right) + \pi$ $\qquad \theta_5 = 195.629\,deg$

$d''' := a\cdot cos(\theta_2) - b\cdot cos(\theta_5)$ $\qquad d''' = 2.745\,in$

12. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5 := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_5)} \cdot \omega_2 \qquad \omega_5 = 0.515 \frac{rad}{sec}$$

13. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) + b \cdot \omega_5^{\,2} \cdot \sin(\theta_5)}{b \cdot \cos(\theta_5)} \qquad \alpha_5 = -62.869 \frac{rad}{sec^2}$$

14. Use equation 7.16e for the acceleration of pin E.

$$A_E := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \alpha_5 \cdot \sin(\theta_5) + b \cdot \omega_5^{\,2} \cdot \cos(\theta_5)$$

$A_E = 646.72 \frac{mm}{sec^2}$ A positive sign means that $\mathbf{A}_E$ is outward

PROBLEM 7-38

Statement: The 3-cylinder radial compressor in Figure P7-8c has the dimensions and crank angle given below. Find and plot the piston accelerations $\mathbf{A}_6$, $\mathbf{A}_7$, and $\mathbf{A}_8$ for one revolution of the crank if $\omega = 15$ rad/sec CW, constant.

Given: Link lengths:

Link 2 $a := 19 \cdot mm$ Link 3 $b := 70 \cdot mm$

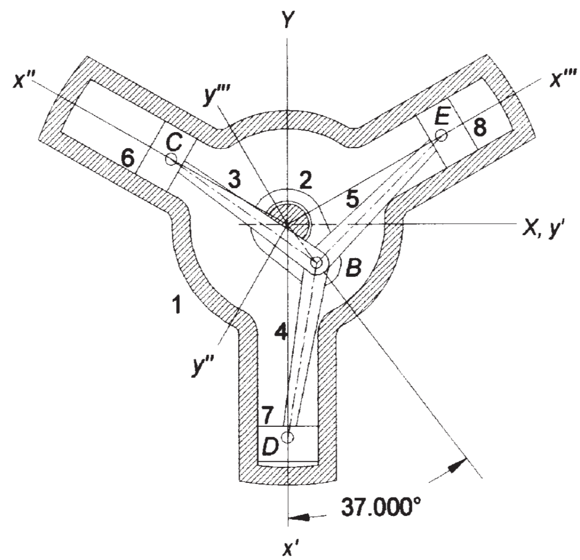
Offset $c := 0 \cdot mm$

Link 2 velocity, and acceleration: $\omega_2 := -15 \cdot \dfrac{rad}{sec}$ $\alpha_2 := 0 \cdot \dfrac{rad}{sec^2}$

Cylinder angular spacing: $\alpha := 120 \cdot deg$

Solution: See Figure P7-8c and Mathcad file P0738.

1. Draw the linkage to scale and label it. Note that there are three local coordinate systems.



2. Determine the range of motion for this slider-crank linkage. This will be the same in each coordinate frame.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine θ_4 using equation 4.17.

$$\theta_4(\theta_2) := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad (x'y' \text{ system})$$

4. Determine the angular velocity of link 4 using equation 6.22a:

$$\omega_4(\theta_2) := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_4(\theta_2))} \cdot \omega_2$$

5. Determine the angular acceleration of link 4 using equation 7.16d.

$$\alpha_4(\theta_2) := \frac{a \cdot \alpha_2 \cdot cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot sin(\theta_2) + b \cdot \omega_4(\theta_2)^2 \cdot sin(\theta_4(\theta_2))}{b \cdot cos(\theta_4(\theta_2))}$$

6. Use equation 7.16e for the acceleration of pin D.

$$A_D(\theta_2) := -a\cdot\alpha_2\cdot \sin(\theta_2) - a\cdot\omega_2{}^2\cdot \cos(\theta_2) \ldots$$
$$+ b\cdot\alpha_4(\theta_2)\cdot \sin(\theta_4(\theta_2)) + b\cdot\omega_4(\theta_2)^2\cdot \cos(\theta_4(\theta_2))$$

7. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := \operatorname{asin}\left(\frac{a\cdot \sin(\theta_2 + \alpha) - c}{b}\right) + \pi \qquad (x''y'' \text{ system})$$

8. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b}\cdot\frac{\cos(\theta_2 + \alpha)}{\cos(\theta_3(\theta_2))}\cdot\omega_2$$

9. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3(\theta_2) := \frac{a\cdot\alpha_2\cdot \cos(\theta_2 + \alpha) - a\cdot\omega_2{}^2\cdot \sin(\theta_2 + \alpha) + b\cdot\omega_3(\theta_2)^2\cdot \sin(\theta_3(\theta_2))}{b\cdot \cos(\theta_3(\theta_2))}$$

10. Use equation 7.16e for the acceleration of pin C.

$$A_C(\theta_2) := -a\cdot\alpha_2\cdot \sin(\theta_2 + \alpha) - a\cdot\omega_2{}^2\cdot \cos(\theta_2 + \alpha) \ldots$$
$$+ b\cdot\alpha_3(\theta_2)\cdot \sin(\theta_3(\theta_2)) + b\cdot\omega_3(\theta_2)^2\cdot \cos(\theta_3(\theta_2))$$

11. Determine θ_5 and d''' using equation 4.17.

$$\theta_5(\theta_2) := \operatorname{asin}\left(\frac{a\cdot \sin(\theta_2 + 2\cdot\alpha) - c}{b}\right) + \pi \qquad (x'''y''' \text{ system})$$

12. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5(\theta_2) := \frac{a}{b}\cdot\frac{\cos(\theta_2 + 2\cdot\alpha)}{\cos(\theta_5(\theta_2))}\cdot\omega_2$$

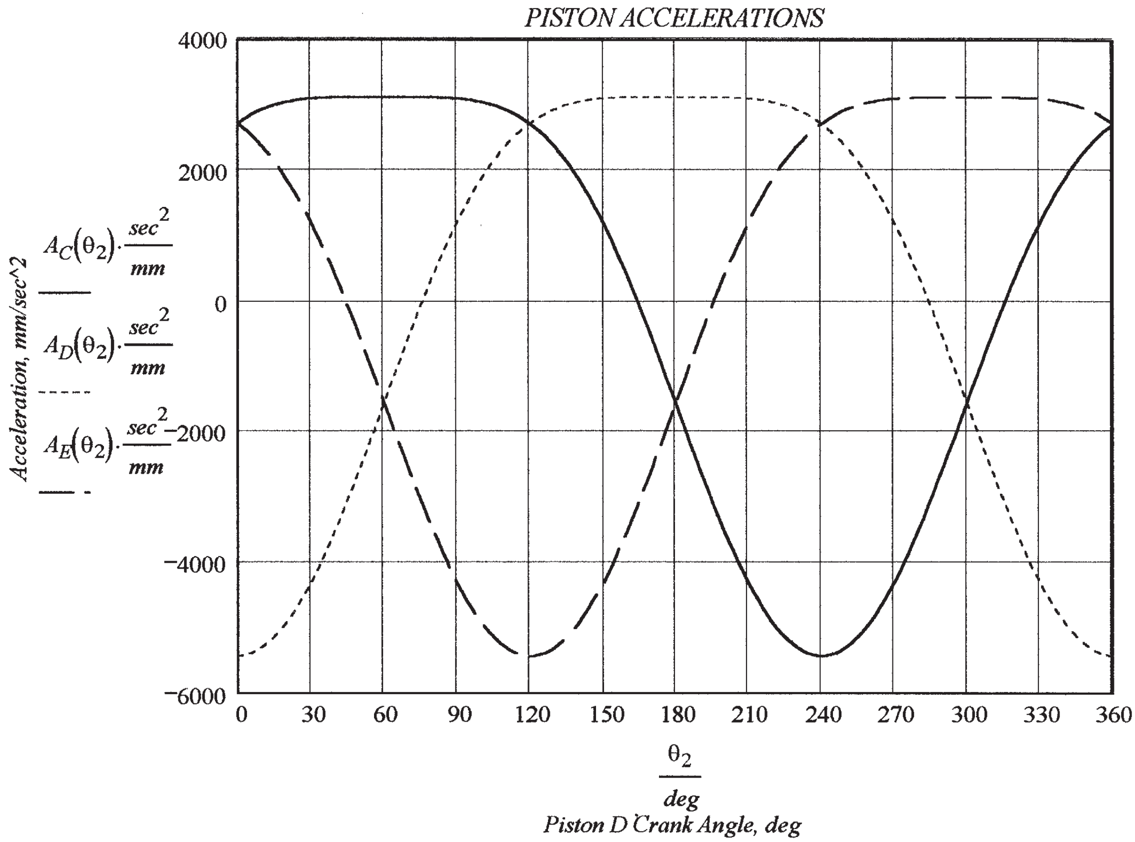
13. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5(\theta_2) := \frac{a\cdot\alpha_2\cdot \cos(\theta_2 + 2\cdot\alpha) - a\cdot\omega_2{}^2\cdot \sin(\theta_2 + 2\cdot\alpha) + b\cdot\omega_5(\theta_2)^2\cdot \sin(\theta_5(\theta_2))}{b\cdot \cos(\theta_5(\theta_2))}$$

14. Use equation 7.16e for the acceleration of pin E.

$$A_E(\theta_2) := -a\cdot\alpha_2\cdot \sin(\theta_2 + 2\cdot\alpha) - a\cdot\omega_2{}^2\cdot \cos(\theta_2 + 2\cdot\alpha) \ldots$$
$$+ b\cdot\alpha_5(\theta_2)\cdot \sin(\theta_5(\theta_2)) + b\cdot\omega_5(\theta_2)^2\cdot \cos(\theta_5(\theta_2))$$

15. Plot the piston accelerations.



PROBLEM 7-39

Statement: Figure P7-9 shows a linkage in one position. Find the instantaneous accelerations of points *A*, *B*, and *P* if link O_2A is rotating CW at 40 rad/sec.

Given: Link lengths:

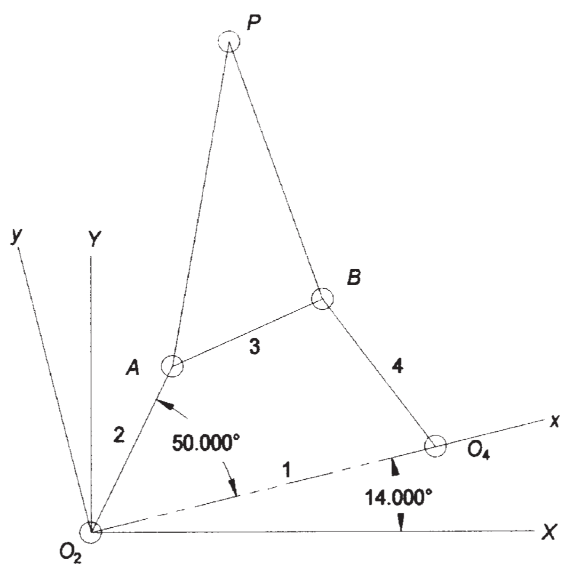
Link 2 (*A* to *B*)	$a := 5.00 \cdot in$	Link 3 (*B* to *C*)	$b := 4.40 \cdot in$
Link 4 (*C* to *D*)	$c := 5.00 \cdot in$	Link 1 (*A* to *D*)	$d := 9.50 \cdot in$

Coupler point: $R_{pa} := 8.90 \cdot in$ $\quad \delta_3 := 56 \cdot deg$

Crank angle and motion: $\theta_2 := 50 \cdot deg$ $\quad \omega_2 := -40 \cdot rad \cdot sec^{-1}$ $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-9 and Mathcad file P0739.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.9000 \qquad K_2 = 1.9000$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.4178$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.0607 \qquad B = -1.5321 \qquad C = 2.4537$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4 := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \qquad \theta_4 = -246.992\, deg$$

$\theta_4 := \theta_4 + 360 \cdot deg \qquad \theta_4 = 113.008\,deg$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 2.1591 \qquad K_5 = -2.4911$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -2.3605$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -1.5321$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 0.1539$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) \qquad \theta_3 = -349.895\,deg$$

$$\theta_3 := \theta_3 + 360 \cdot deg \qquad \theta_3 = 10.105\,deg$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_3 = 41.552\,\frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \qquad \omega_4 = -26.320\,\frac{rad}{sec}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A_A} = -5142.3 - 6128.4j\,\frac{in}{sec^2} \qquad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := arg(\mathbf{A_A})$$

The acceleration of pin A is $A_A = 8000\,\frac{in}{sec^2}$ at $\theta_{AA} = -130.0\,deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c \cdot \sin(\theta_4) \qquad B := b \cdot \sin(\theta_3) \qquad D := c \cdot \cos(\theta_4) \qquad E := b \cdot \cos(\theta_3)$$

$$A = 4.602\,in \qquad B = 0.772\,in \qquad D = -1.954\,in \qquad E = 4.332\,in$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_3^{\,2} \cdot \cos(\theta_3) - c \cdot \omega_4^{\,2} \cdot \cos(\theta_4)$$

$$C = 1.398 \times 10^4\,in \cdot sec^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_3^{\,2} \cdot \sin(\theta_3) + c \cdot \omega_4^{\,2} \cdot \sin(\theta_4)$$

$$F = -4.273 \times 10^3\,in \cdot sec^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_3 = -356.538\,\frac{rad}{sec^2} \qquad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_4 = 2.977 \times 10^3\,\frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point *B*.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-\sin(\theta_4) + j \cdot \cos(\theta_4)\right) - c \cdot \omega_4^{\,2} \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right)$$

$$\mathbf{A_B} = -12346.3 - 9005.8j \, \frac{in}{sec^2} \qquad A_B := \left|\mathbf{A_B}\right| \qquad \theta_{AB} := arg\left(\mathbf{A_B}\right)$$

The acceleration of pin *B* is $A_B = 15282 \, \frac{in}{sec^2}$ at $\theta_{AB} = -143.9 \, deg$

10. Use equations 7.32 to find the acceleration of the point *C*.

$$\mathbf{A_{CA}} := R_{pa} \cdot \alpha_3 \cdot \left(-\sin(\theta_3 + \delta_3) + j \cdot \cos(\theta_3 + \delta_3)\right) \ldots$$
$$+ -R_{pa} \cdot \omega_3^{\,2} \cdot \left(\cos(\theta_3 + \delta_3) + j \cdot \sin(\theta_3 + \delta_3)\right)$$

$$\mathbf{A_C} := \mathbf{A_A} + \mathbf{A_{CA}}$$

$$A_C := \left|\mathbf{A_C}\right| \qquad A_C = 23073 \, \frac{in}{sec^2} \qquad arg\left(\mathbf{A_C}\right) = -111.525 \, deg$$

PROBLEM 7-40

Statement: Figure P7-10 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point *P* at 2-deg increments of crank angle. Check your results with program FOURBAR.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to A) $a := 10 \cdot mm$ Link 3 (A to B) $b := 20.6 \cdot mm$

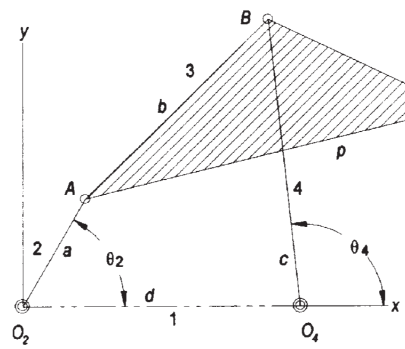
Link 4 (B to O_4) $c := 23.3 \cdot mm$ Link 1 (O_2 to O_4) $d := 22.2 \cdot mm$

Coupler point: $R_{pa} := 30.6 \cdot mm$ $\delta_3 := -31 \cdot deg$

Crank motion: $\omega_2 := 100 \cdot rpm$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-10 and Mathcad file P0740.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.2200 \qquad K_2 = 0.9528$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.5265$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.0777 \qquad K_5 = -1.1512$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$.

$$\mathbf{A_A}(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

8. Use equations 7.12 to determine the angular accelerations of link 3.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \qquad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \qquad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

9. Use equations 7.32 to find the acceleration of the point P.

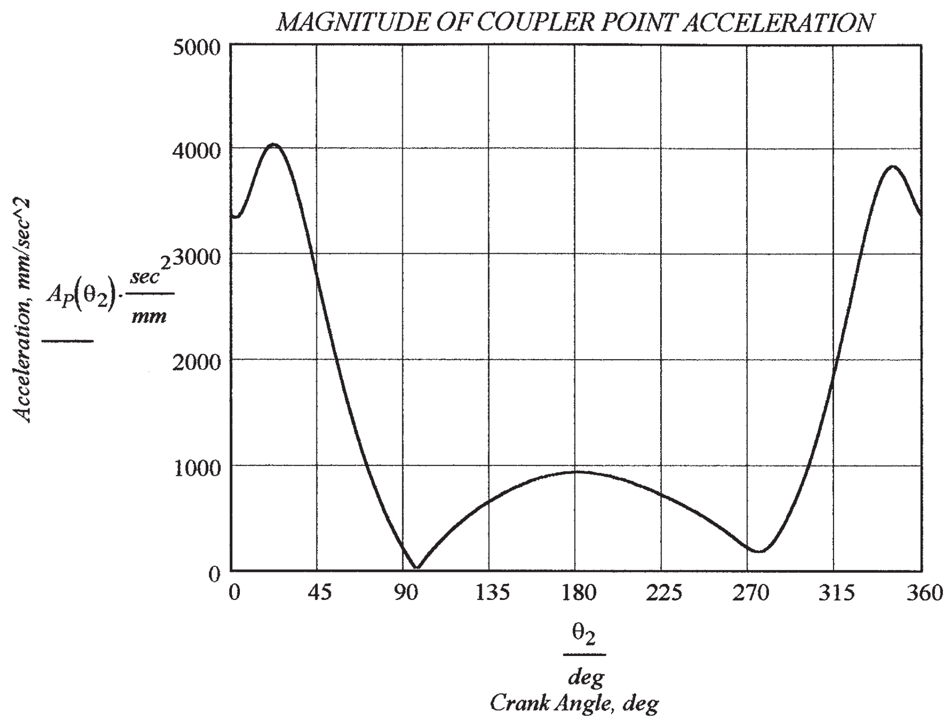
$$\mathbf{A_{PA}}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot (-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)) \;\ldots$$
$$+ -R_{pa} \cdot \omega_3(\theta_2)^2 \cdot (\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3))$$

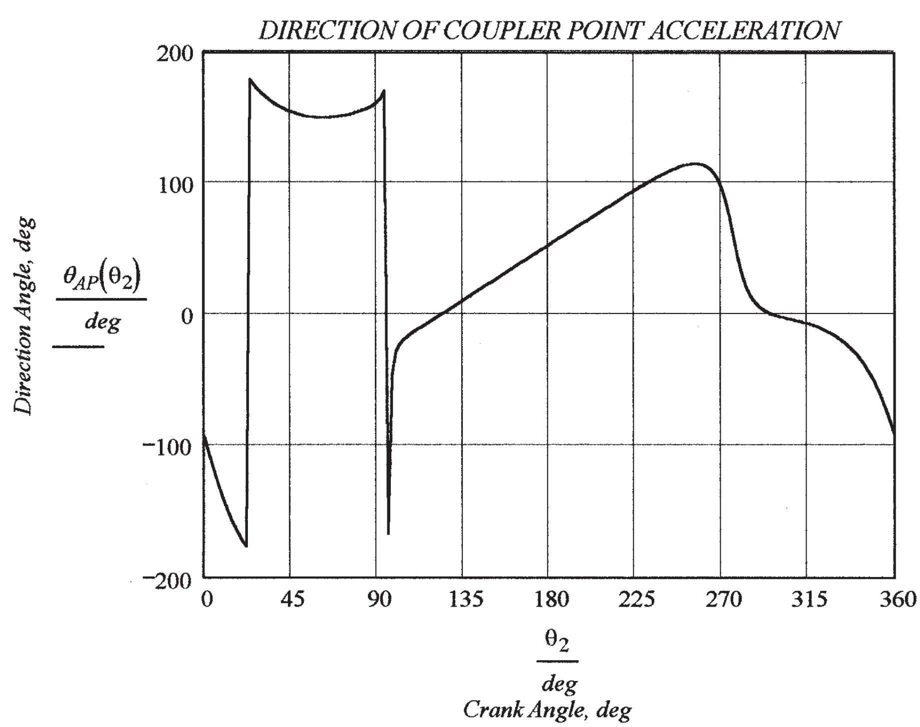
$$\mathbf{A_P}(\theta_2) := \mathbf{A_A}(\theta_2) + \mathbf{A_{PA}}(\theta_2)$$

$$A_P(\theta_2) := |\mathbf{A_P}(\theta_2)| \qquad \theta_{AP}(\theta_2) := \arg(\mathbf{A_P}(\theta_2))$$

10. Plot the magnitude and direction of the coupler point P. (See next page.)

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$


MAGNITUDE OF COUPLER POINT ACCELERATION
5000
4000
3000
2000
1000
0
0 45 90 135 180 225 270 315 360
Acceleration, mm/sec^2
$A_P(\theta_2)\cdot\frac{sec^2}{mm}$
$\frac{\theta_2}{deg}$
Crank Angle, deg


DIRECTION OF COUPLER POINT ACCELERATION
200
100
0
−100
−200
0 45 90 135 180 225 270 315 360
Direction Angle, deg
$\frac{\theta_{AP}(\theta_2)}{deg}$
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 7-41

Statement: Figure P7-11 shows a linkage that operates at 500 crank rpm. Find and plot the magnitude and direction of the acceleration of point B at 2-deg increments of crank angle. Check your result with program FOURBAR.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Link lengths:

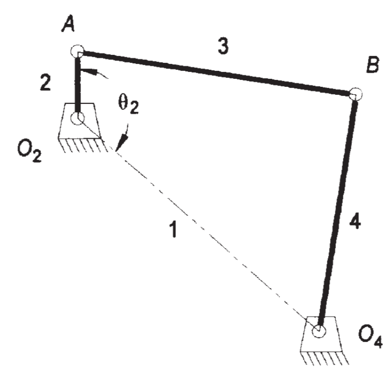
Link 2 (O_2 to A)	$a := 2.000\cdot in$	Link 3 (A to B)	$b := 8.375\cdot in$
Link 4 (B to O_4)	$c := 7.187\cdot in$	Link 1 (O_2 to O_4)	$d := 9.625\cdot in$

Input crank angular velocity $\omega_2 := 500\cdot rpm$ $\omega_2 = 52.360\,rad\cdot sec^{-1}$

$\alpha_2 := 0\cdot rad\cdot sec^{-2}$

Solution: See Figure P7-11 and Mathcad file P0741.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 4.8125 \qquad K_2 = 1.3392$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)} \qquad K_3 = 2.7186$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2\cdot\cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot\sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1)\cdot\cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.1493$$

$$K_5 = -3.4367$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \qquad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \qquad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

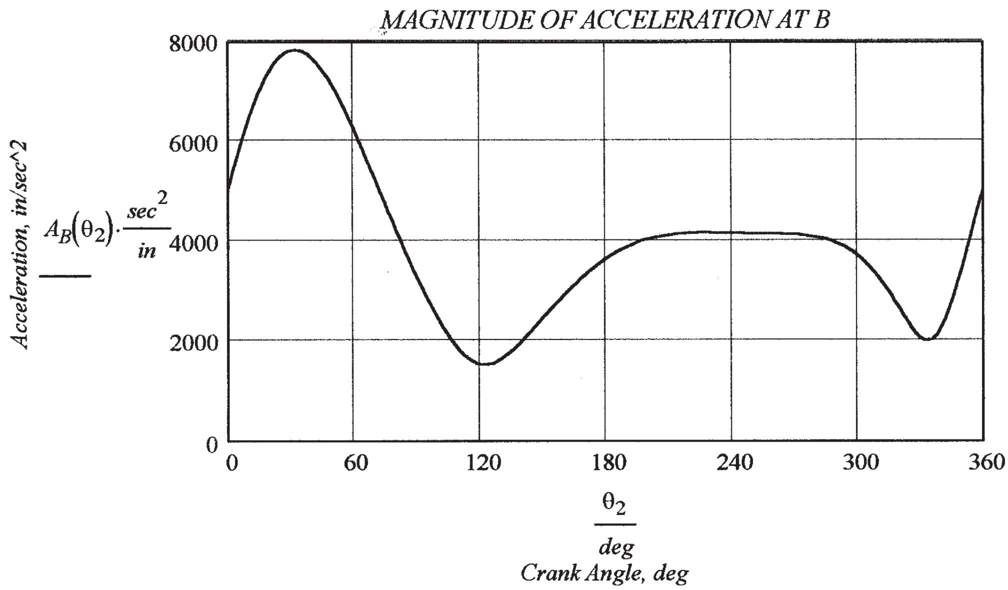
9. Determine the acceleration of point B using equations 7.13c.

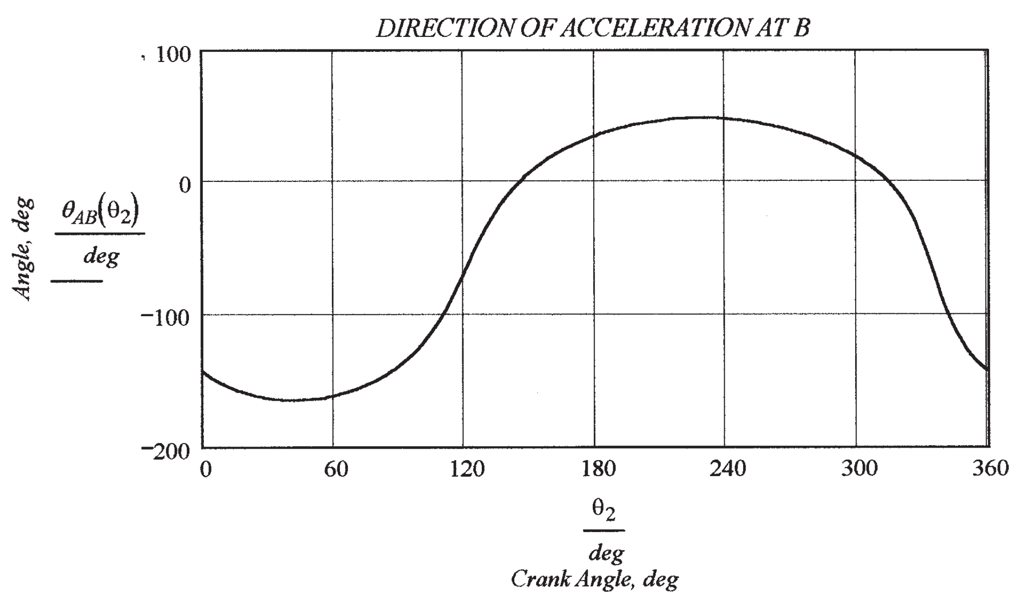
$$\mathbf{A_B}(\theta_2) := c \cdot \alpha_4(\theta_2) \cdot (-\sin(\theta_4(\theta_2)) + j \cdot \cos(\theta_4(\theta_2))) - c \cdot \omega_4(\theta_2)^2 \cdot (\cos(\theta_4(\theta_2)) + j \cdot \sin(\theta_4(\theta_2)))$$

$$A_B(\theta_2) := |\mathbf{A_B}(\theta_2)| \qquad \theta_{AB}(\theta_2) := arg(\mathbf{A_B}(\theta_2))$$

10. Plot the magnitude and angle of the acceleration at point B.

(See next page.)


MAGNITUDE OF ACCELERATION AT B
8000
6000
4000
2000
0
0
60
120
180
240
300
360
Acceleration, in/sec^2
$A_B(\theta_2)\cdot\frac{sec^2}{in}$
$\frac{\theta_2}{deg}$
Crank Angle, deg


DIRECTION OF ACCELERATION AT B
100
0
−100
−200
0
60
120
180
240
300
360
Angle, deg
$\frac{\theta_{AB}(\theta_2)}{deg}$
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 7-42

Statement: Figure P7-12 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

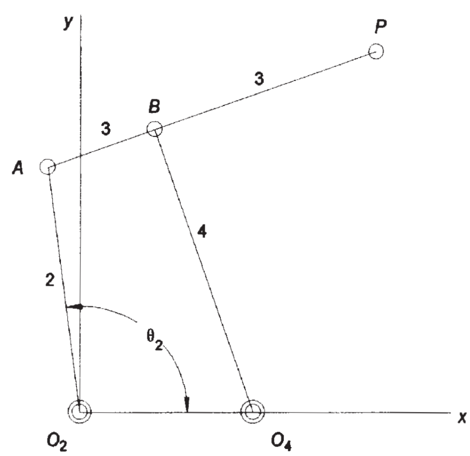
Link 2 (O_2 to A)	$a := 0.785 \cdot in$	Link 3 (A to B)	$b := 0.356 \cdot in$
Link 4 (B to O_4)	$c := 0.950 \cdot in$	Link 1 (O_2 to O_4)	$d := 0.544 \cdot in$

Coupler point: $R_{pa} := 1.09 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

Crank speed: $\omega_2 := 20 \cdot rpm$ $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-12 and Mathcad file P0742.

1. Draw the linkage to scale and label it.



2. Using the geometry defined in Figure 3-1a in the text, determine the input crank angles (relative to the line O_2O_4) at which links 2 and 3, and 3 and 4 are in toggle.

$$\theta_{20} := acos\left[\frac{a^2 + d^2 - (b + c)^2}{2 \cdot a \cdot d}\right] \qquad \theta_{20} = 158.286\,deg$$

$$\theta_2 := -\theta_{20}, -\theta_{20} + 2 \cdot deg .. \theta_{20}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.6930 \qquad K_2 = 0.5726$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.1317$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.5281 \qquad K_5 = -0.2440$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$.

$$\mathbf{A_A}(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2{}^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

9. Use equations 7.12 to determine the angular accelerations of link 3.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \qquad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \qquad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2{}^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2{}^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$

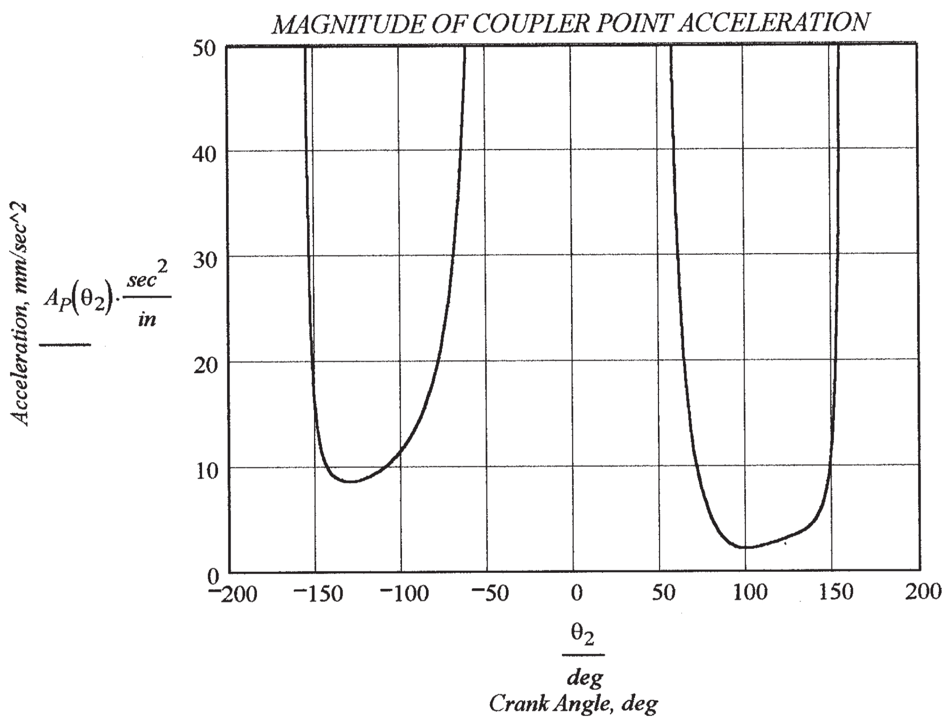
10. Use equations 7.32 to find the acceleration of the point P.

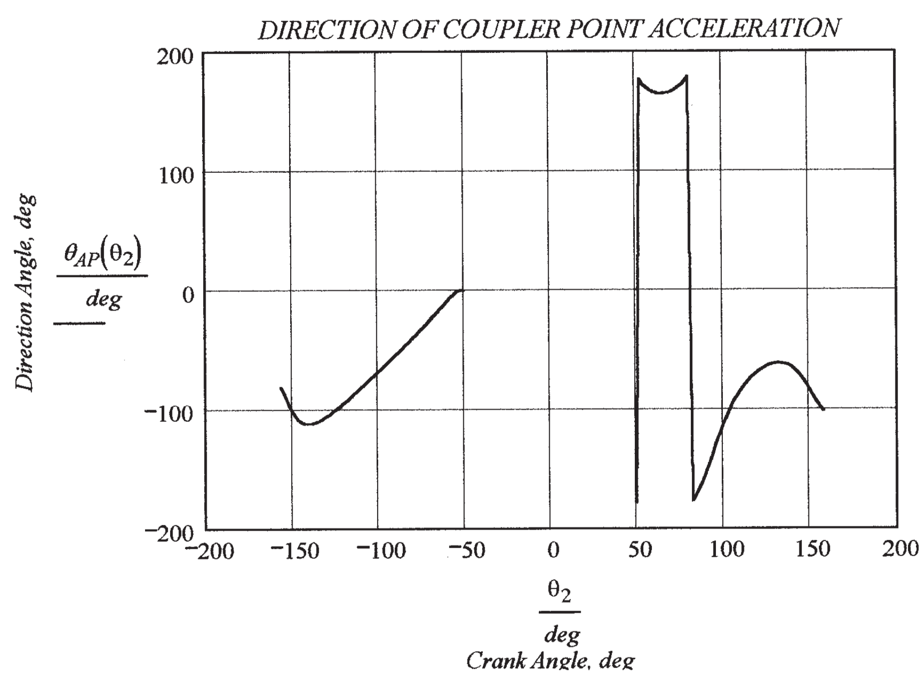
$$\mathbf{A_{PA}}(\theta_2) := R_{pa}\cdot\alpha_3(\theta_2)\cdot\left(-sin\left(\theta_3(\theta_2)+\delta_3\right)+\mathrm{j}\cdot cos\left(\theta_3(\theta_2)+\delta_3\right)\right)\ ...$$
$$+-R_{pa}\cdot\omega_3(\theta_2)^2\cdot\left(cos\left(\theta_3(\theta_2)+\delta_3\right)+\mathrm{j}\cdot sin\left(\theta_3(\theta_2)+\delta_3\right)\right)$$

$$\mathbf{A_P}(\theta_2) := \mathbf{A_A}(\theta_2)+\mathbf{A_{PA}}(\theta_2)$$

$$A_P(\theta_2) := \left|\mathbf{A_P}(\theta_2)\right| \qquad \theta_{AP}(\theta_2) := arg\left(\mathbf{A_P}(\theta_2)\right)$$

11. Plot the magnitude and direction of the coupler point P.





PROBLEM 7-43

Statement: Figure P7-13 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to A) $a := 0.86 \cdot in$ Link 3 (A to B) $b := 1.85 \cdot in$

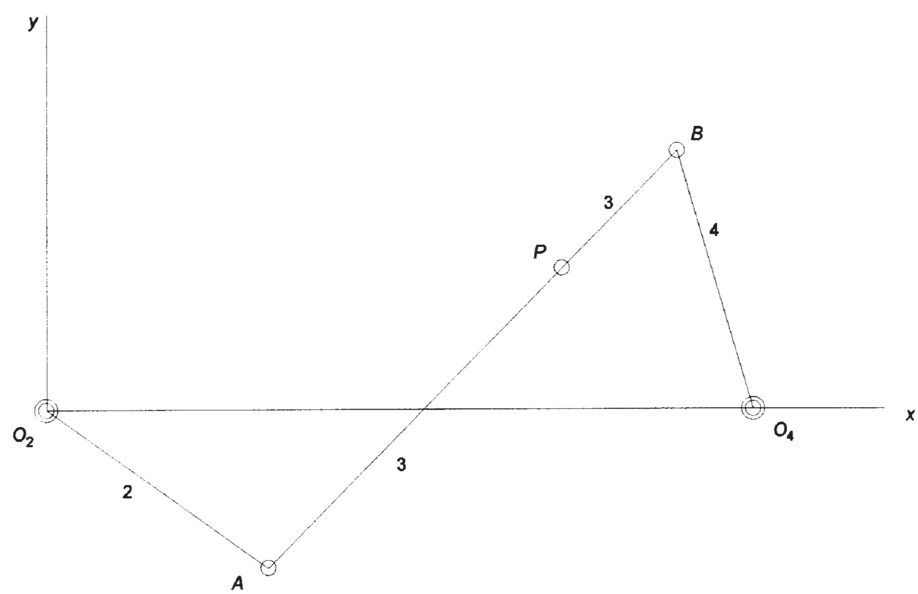
Link 4 (B to O_4) $c := 0.86 \cdot in$ Link 1 (O_2 to O_4) $d := 2.22 \cdot in$

Coupler point: $R_{pa} := 1.33 \cdot in$ $\delta_3 := 0 \cdot deg$

Crank motion: $\omega_2 := 80 \cdot rpm$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-13 and Mathcad file P0743.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} + \frac{b \cdot c}{(a \cdot d)} \qquad arg_1 = 1.228$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2 \cdot a \cdot d)} - \frac{b \cdot c}{(a \cdot d)} \qquad arg_2 = -0.439$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 116.0\, deg$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 1 \cdot deg .. \theta_{2toggle}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.5814 \qquad K_2 = 2.5814$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.0181$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.2000 \qquad K_5 = -2.6244$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$.

$$\mathbf{A}_{\mathbf{A}}(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

9. Use equations 7.12 to determine the angular accelerations of link 3.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \qquad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \qquad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot cos(\theta_2) + b\cdot\omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) - b\cdot\omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)^2\cdot sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2)\cdot D'(\theta_2) - A'(\theta_2)\cdot F'(\theta_2)}{A'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot D'(\theta_2)}$$

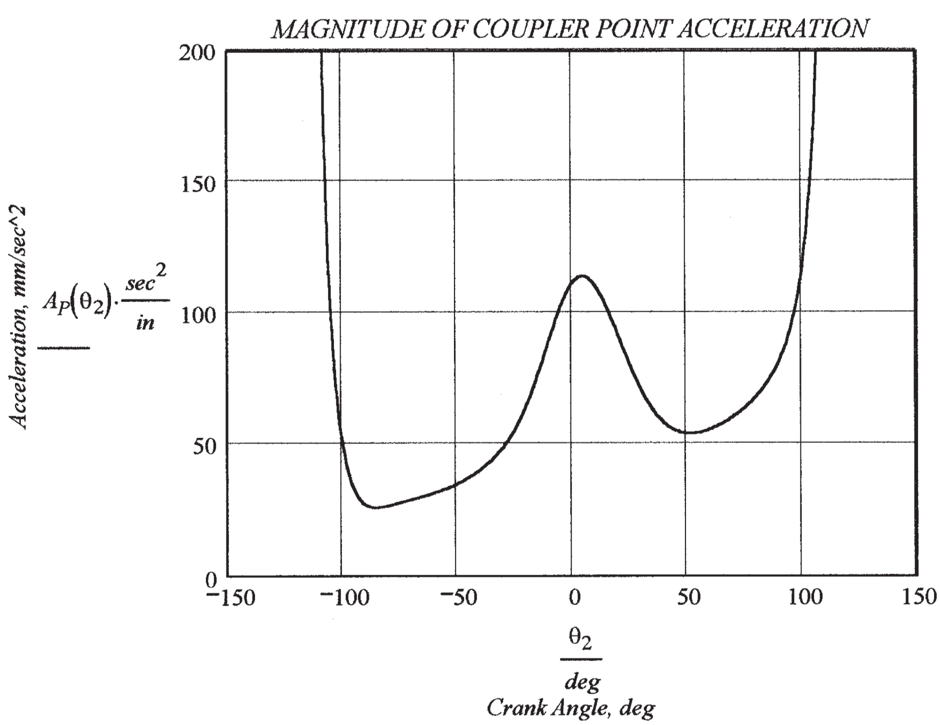
10. Use equations 7.32 to find the acceleration of the point *P*.

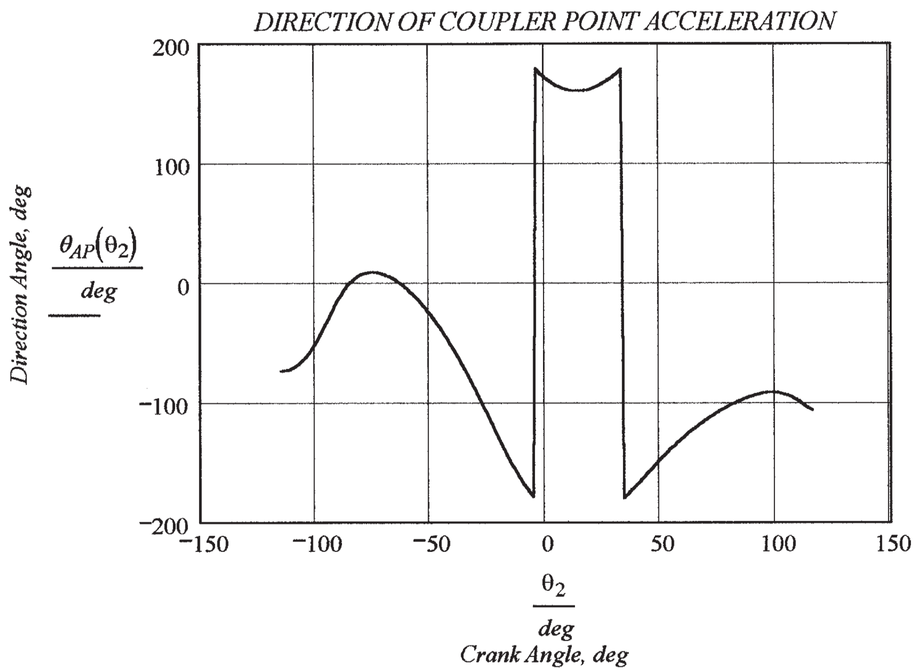
$$\mathbf{A_{PA}}(\theta_2) := R_{pa}\cdot\alpha_3(\theta_2)\cdot\left(-sin(\theta_3(\theta_2) + \delta_3) + j\cdot cos(\theta_3(\theta_2) + \delta_3)\right) \ldots$$
$$+ -R_{pa}\cdot\omega_3(\theta_2)^2\cdot\left(cos(\theta_3(\theta_2) + \delta_3) + j\cdot sin(\theta_3(\theta_2) + \delta_3)\right)$$

$$\mathbf{A_P}(\theta_2) := \mathbf{A_A}(\theta_2) + \mathbf{A_{PA}}(\theta_2)$$

$$A_P(\theta_2) := \left|\mathbf{A_P}(\theta_2)\right| \qquad \theta_{AP}(\theta_2) := arg(\mathbf{A_P}(\theta_2))$$

11. Plot the magnitude and direction of the coupler point *P*.




DIRECTION OF COUPLER POINT ACCELERATION
200
100
0
−100
−200
−150
−100
−50
0
50
100
150
Direction Angle, deg
$\frac{\theta_{AP}(\theta_2)}{deg}$
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 7-44

Statement: Figure P7-14 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2-deg increments of crank angle over the maximum range of motion possible. Check your results with program FOURBAR.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Link lengths:

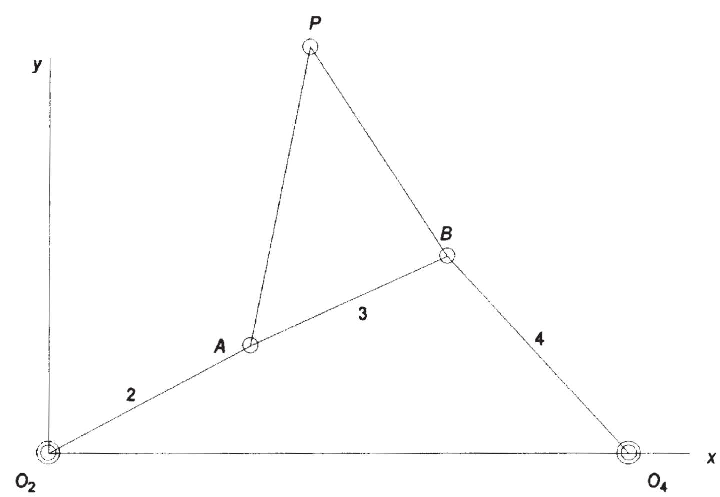
Link 2 (O_2 to A)	$a := 0.72\cdot in$	Link 3 (A to B)	$b := 0.68\cdot in$
Link 4 (B to O_4)	$c := 0.85\cdot in$	Link 1 (O_2 to O_4)	$d := 1.82\cdot in$

Coupler point: $R_{pa} := 0.97\cdot in$ $\quad \delta_3 := 54\cdot deg$

Crank motion: $\omega_2 := 80\cdot rpm$ $\quad \alpha_2 := 0\cdot rad\cdot sec^{-2}$

Solution: See Figure P7-14 and Mathcad file P0744.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this non-Grashof triple rocker using equations 4.33.

$$arg_1 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} + \frac{b\cdot c}{(a\cdot d)} \qquad arg_1 = 1.451$$

$$arg_2 := \frac{(a)^2 + (d)^2 - (b)^2 - (c)^2}{(2\cdot a\cdot d)} - \frac{b\cdot c}{(a\cdot d)} \qquad arg_2 = 0.568$$

$$\theta_{2toggle} := acos(arg_2) \qquad \theta_{2toggle} = 55.4\,deg$$

The other toggle angle is the negative of this. Thus,

$$\theta_2 := -\theta_{2toggle}, -\theta_{2toggle} + 1\cdot deg .. \theta_{2toggle}$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$K_1 = 2.5278 \qquad K_2 = 2.1412$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 3.3422$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 2.6765 \qquad K_5 = -3.6465$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 .

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$.

$$\mathbf{A_A}(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

9. Use equations 7.12 to determine the angular accelerations of link 3.

$$A'(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \qquad B'(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \qquad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2)\cdot D'(\theta_2) - A'(\theta_2)\cdot F'(\theta_2)}{A'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot D'(\theta_2)}$$

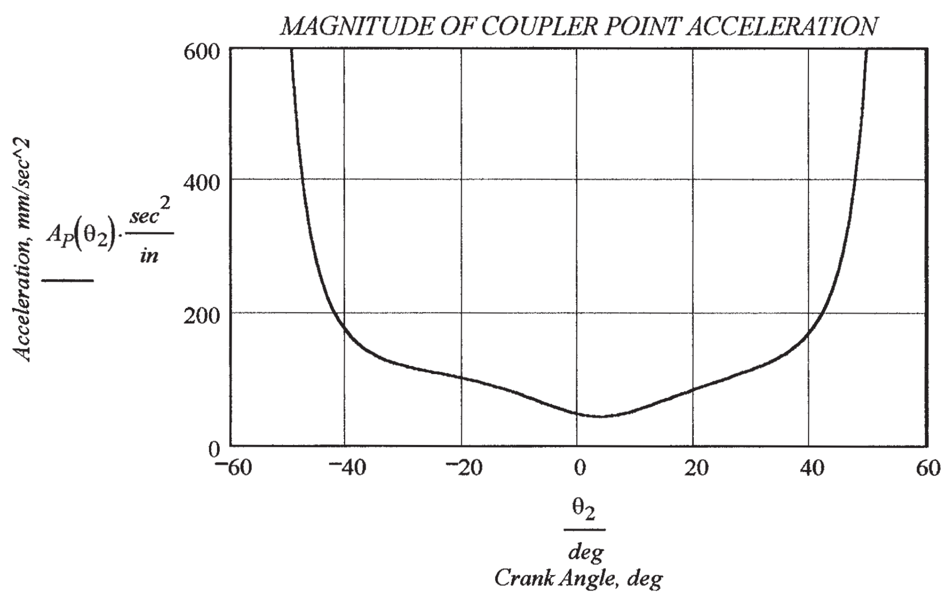
10. Use equations 7.32 to find the acceleration of the point *P*.

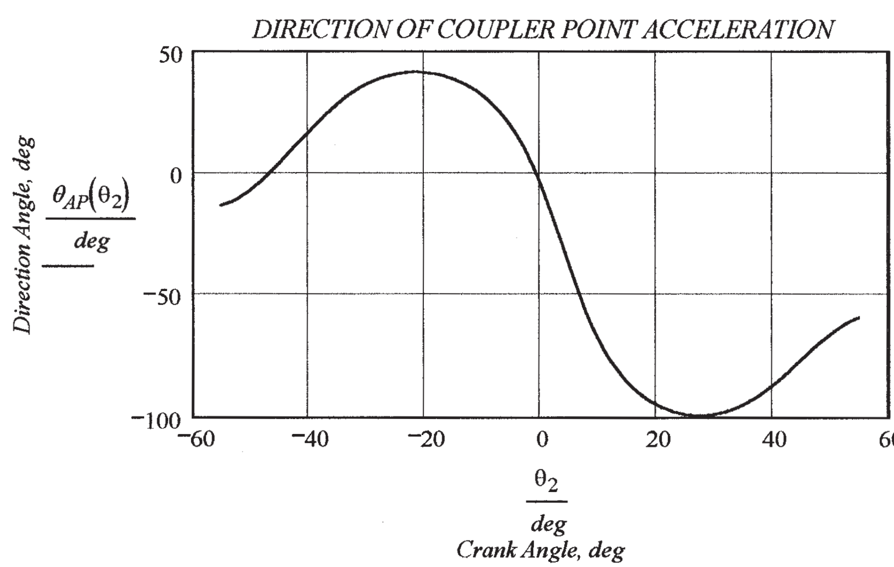
$$\mathbf{A_{PA}}(\theta_2) := R_{pa}\cdot\alpha_3(\theta_2)\cdot\left(-sin\left(\theta_3(\theta_2)+\delta_3\right)+j\cdot cos\left(\theta_3(\theta_2)+\delta_3\right)\right)\ ...$$
$$+-R_{pa}\cdot\omega_3(\theta_2)^2\cdot\left(cos\left(\theta_3(\theta_2)+\delta_3\right)+j\cdot sin\left(\theta_3(\theta_2)+\delta_3\right)\right)$$

$$\mathbf{A_P}(\theta_2) := \mathbf{A_A}(\theta_2) + \mathbf{A_{PA}}(\theta_2)$$

$$A_P(\theta_2) := \left|\mathbf{A_P}(\theta_2)\right| \qquad \theta_{AP}(\theta_2) := arg\left(\mathbf{A_P}(\theta_2)\right)$$

11. Plot the magnitude and direction of the coupler point *P*.





PROBLEM 7-45

Statement: Figure P7-15 shows a power hacksaw that is an offset slider-crank mechanism that has the dimensions given below. Draw an equivalent linkage diagram and then calculate and plot the acceleration of the saw blade with respect to the piece being cut over one revolution of the crank, which rotates at 50 rpm.

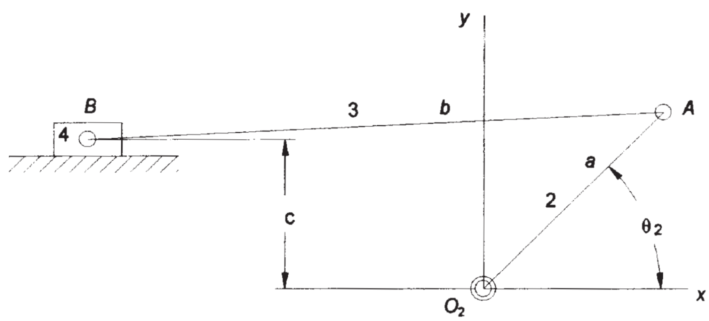
Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$a := 75\cdot mm$	Offset	$c := 45\cdot mm$
Link 3 (A to B)	$b := 170\cdot mm$		
Input crank angular velocity	$\omega_2 := 50\cdot rpm$	$\alpha_2 := 0\cdot rad\cdot sec^{-2}$	

Solution: See Figure P7-15 and Mathcad file P0745.

1. Draw the equivalent linkage to a convenient scale and label it.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$$

3. Determine θ_3 using equations 4.16 for the crossed circuit.

$$\theta_3(\theta_2) := asin\left(\frac{a\cdot sin(\theta_2) - c}{b}\right)$$

4. Determine the angular velocity of link 3 using equation 6.22a:

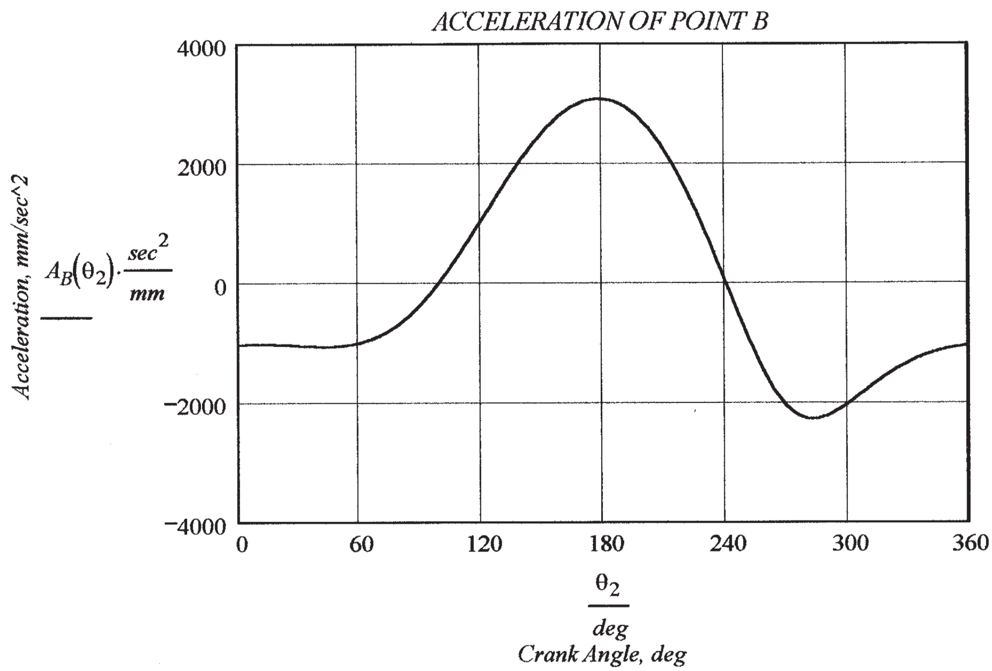
$$\omega_3(\theta_2) := \frac{a}{b}\cdot\frac{cos(\theta_2)}{cos(\theta_3(\theta_2))}\cdot\omega_2$$

5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3(\theta_2) := \frac{a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) + b\cdot\omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2))}{b\cdot cos(\theta_3(\theta_2))}$$

6. Use equation 7.16e for the acceleration of pin B.

$$A_B(\theta_2) := -a\cdot\alpha_2\cdot sin(\theta_2) - a\cdot\omega_2^{\,2}\cdot cos(\theta_2) \;...$$
$$+\, b\cdot\alpha_3(\theta_2)\cdot sin(\theta_3(\theta_2)) + b\cdot\omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2))$$


ACCELERATION OF POINT B
4000
2000
0
−2000
−4000
Acceleration, mm/sec^2
$A_B(\theta_2)\cdot\frac{sec^2}{mm}$
0
60
120
180
240
300
360
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 7-46

Statement: Figure P7-16 shows a walking beam indexing and pick-and-place mechanism that can be analyzed as two fourbar linkages driven by a common crank. Calculate and plot the relative acceleration between points *E* and *P* for one revolution of gear 2.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Link lengths (walking-beam linkage):

Link 2 (O_2 to A)	$a := 40\cdot mm$	Link 3 (A to D)	$b' := 108\cdot mm$
Link 4 (O_4 to D)	$c' := 40\cdot mm$	Link 1 (O_2 to O_4)	$d' := 108\cdot mm$

Link lengths (pick and place linkage):

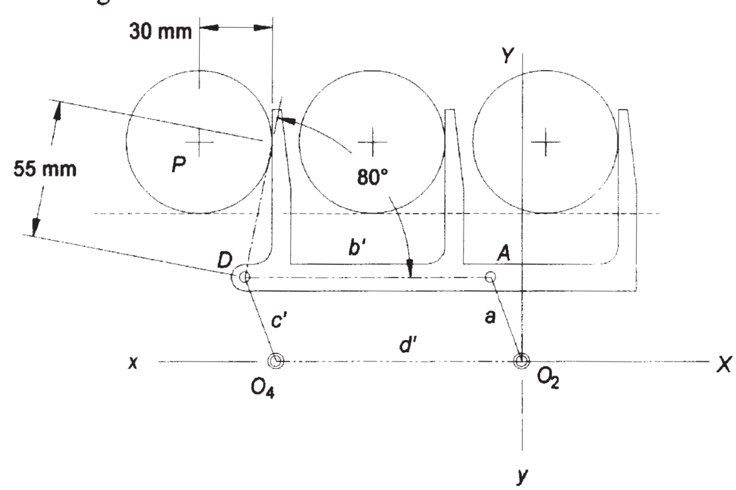
Link 2 (O_2 to B)	$a := 32\cdot mm$	Link 5 (B to C)	$b := 260\cdot mm$
Link 4 (C to O_6)	$c := 96\cdot mm$	Link 1 (O_2 to O_6)	$d := 200\cdot mm$
Rocker point E:	$u := 160\cdot mm$	$\delta_4 := -75\cdot deg$	

Crank speed: $\omega_2 := 100\cdot rpm$ $\alpha_2 := 0\cdot rad\cdot sec^{-2}$

Crank-pin phase angle $\phi := 120\cdot deg$

Solution: See Figure P7-16 and Mathcad file P0746.

1. Draw the walking-beam linkage to scale and label it.



2. Determine the range of motion for this mechanism.

$$\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg \qquad \text{(local } x'y' \text{ coordinate system)}$$

3. This part of the mechanism is a special-case Grashof in the parallelogram configuration. As such, the coupler does not rotate, but has curvilinear motion with every point on it having the same velocity. Therefore, it is only necessary to calculate the *X*-component of the acceleration at point *A* in order to determine the acceleration of the cylinder center, *P*.

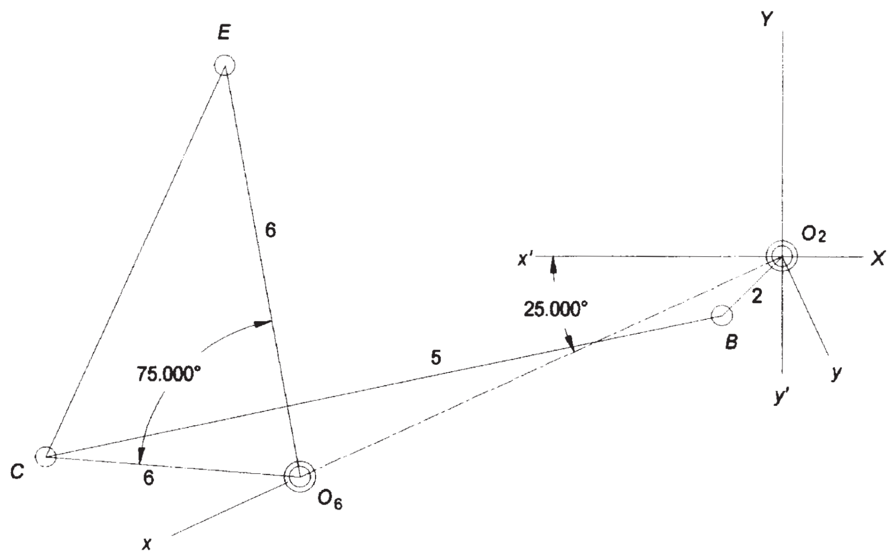
$$\mathbf{A_A}(\theta_2) := a\cdot\alpha_2\cdot\left(-\sin(\theta_2) + j\cdot\cos(\theta_2)\right) - a\cdot\omega_2^{\,2}\cdot\left(\cos(\theta_2) + j\cdot\sin(\theta_2)\right)$$

$$A_{Ax}(\theta_2) := Re(\mathbf{A_A}(\theta_2))$$

In the global coordinate frame,

$$\mathbf{A_P}(\theta_2) := -A_{Ax}(\theta_2)$$

4. Draw the pick-and-place linkage to scale and label it.



Coordinate rotation angle: $\alpha := 25 \cdot deg$

5. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 6.2500 \qquad K_2 = 2.0833$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = -2.8255$$

$$A(\theta_2) := \cos(\theta_2 + \phi - \alpha) - K_1 - K_2 \cdot \cos(\theta_2 + \phi - \alpha) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2 + \phi - \alpha)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2 + \phi - \alpha) + K_3$$

6. Use equation 4.10b to find values of θ_6 for the crossed circuit.

$$\theta_6(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)}\right)\right)$$

7. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.7692 \qquad K_5 = -5.9740$$

$$D(\theta_2) := \cos(\theta_2 + \phi - \alpha) - K_1 + K_4 \cdot \cos(\theta_2 + \phi - \alpha) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2 + \phi - \alpha)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2 + \phi - \alpha) + K_5$$

8. Use equation 4.13 to find values of θ_5 for the crossed circuit.

$$\theta_5(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right)\right)$$

9. Determine the angular velocity of links 5 and 6 using equations 6.18.

$$\omega_5(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin[\theta_6(\theta_2) - (\theta_2 + \phi - \alpha)]}{\sin(\theta_5(\theta_2) - \theta_6(\theta_2))}$$

$$\omega_6(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 + \phi - \alpha - \theta_5(\theta_2))}{\sin(\theta_6(\theta_2) - \theta_5(\theta_2))}$$

10. Use equations 7.12 to determine the angular acceleration of link 6.

$$A(\theta_2) := c \cdot \sin(\theta_6(\theta_2)) \qquad B(\theta_2) := b \cdot \sin(\theta_5(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_6(\theta_2)) \qquad E(\theta_2) := b \cdot \cos(\theta_5(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2 + \phi - \alpha) + a \cdot \omega_2{}^2 \cdot \cos(\theta_2 + \phi - \alpha) \ldots$$
$$+ b \cdot \omega_5(\theta_2 + \phi - \alpha)^2 \cdot \cos(\theta_5(\theta_2)) - c \cdot \omega_6(\theta_2)^2 \cdot \cos(\theta_6(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2 + \phi - \alpha) - a \cdot \omega_2{}^2 \cdot \sin(\theta_2 + \phi - \alpha) \ldots$$
$$+ -b \cdot \omega_5(\theta_2 + \phi - \alpha)^2 \cdot \sin(\theta_5(\theta_2)) + c \cdot \omega_6(\theta_2)^2 \cdot \sin(\theta_6(\theta_2))$$

$$\alpha_6(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

11. Determine the acceleration of the rocker point E using equations 7.13c.

$$\mathbf{A_E}(\theta_2) := c \cdot \alpha_6(\theta_2) \cdot (-\sin(\theta_6(\theta_2)) + j \cdot \cos(\theta_6(\theta_2))) - c \cdot \omega_6(\theta_2)^2 \cdot (\cos(\theta_6(\theta_2)) + j \cdot \sin(\theta_6(\theta_2)))$$

12. Transform this into the global XY system.

$$A_{Ex}(\theta_2) := \mathrm{Re}(\mathbf{A_E}(\theta_2)) \qquad A_{Ey}(\theta_2) := \mathrm{Im}(\mathbf{A_E}(\theta_2))$$

$$A_{EX}(\theta_2) := A_{Ex}(\theta_2) \cdot \cos(\alpha) - A_{Ey}(\theta_2) \cdot \sin(\alpha)$$

$$A_{EY}(\theta_2) := A_{Ex}(\theta_2) \cdot \sin(\alpha) + A_{Ey}(\theta_2) \cdot \cos(\alpha)$$

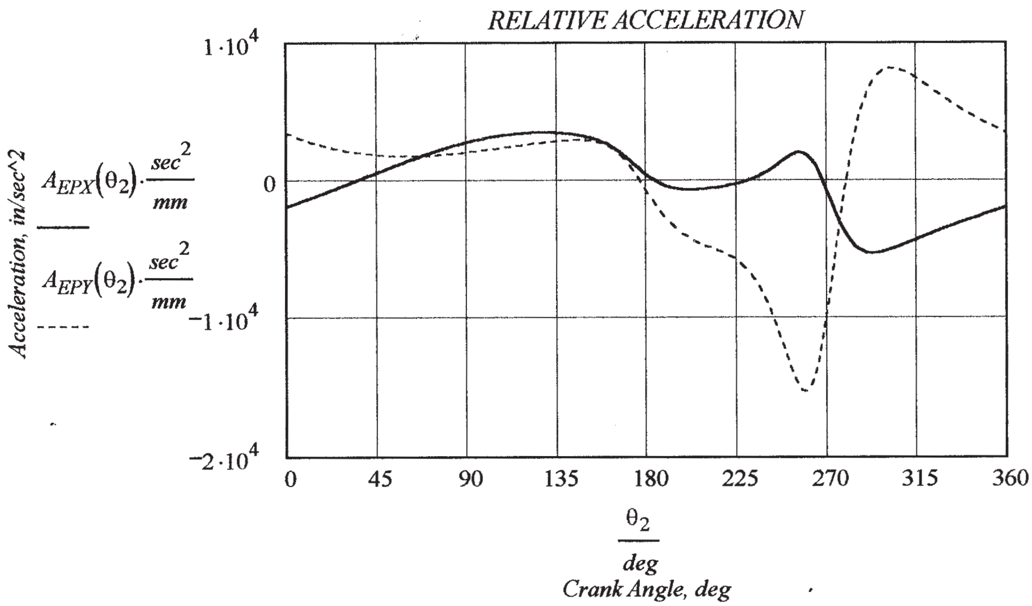
$$\mathbf{A_{EXY}}(\theta_2) := A_{EX}(\theta_2) + \mathrm{j} \cdot A_{EY}(\theta_2)$$

13. Calculate and plot the acceleration of E relative to P.

$$\mathbf{A_{EP}}(\theta_2) := \mathbf{A_{EXY}}(\theta_2) - \mathbf{A_P}(\theta_2)$$

$$A_{EPX}(\theta_2) := \mathrm{Re}(\mathbf{A_{EP}}(\theta_2)) \qquad A_{EPY}(\theta_2) := \mathrm{Im}(\mathbf{A_{EP}}(\theta_2))$$

(See next page.)


RELATIVE ACCELERATION
1·10^4
0
−1·10^4
−2·10^4
Acceleration, in/sec^2
$A_{EPX}(\theta_2)\cdot\frac{sec^2}{mm}$
$A_{EPY}(\theta_2)\cdot\frac{sec^2}{mm}$
0
45
90
135
180
225
270
315
360
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 7-47

Statement: Figure P6-17 shows a paper roll off-loading mechanism driven by an air cylinder. In the position shown, it has the dimensions given below. The V-links are rigidly attached to O_4A. The air cylinder is retracted at a constant acceleration of 0.1 m/sec^2. Draw a kinematic diagram of the mechanism, write the necessary equations, and calculate and plot the angular acceleration of the paper roll and the linear acceleration of its center as it rotates through 90 deg CCW from the position shown.

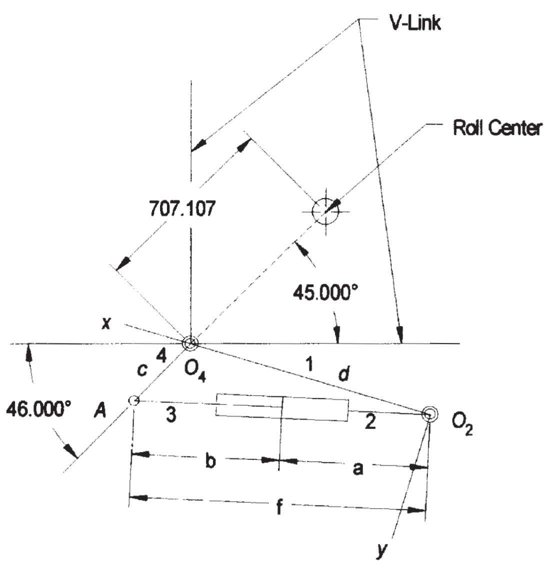
Given: Link lengths and angles:

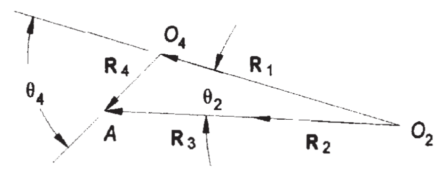
Link 4 (O_4 to A)	$c := 300 \cdot mm$	Paper roll location from O_4: $u := 707.1 \cdot mm$
Link 1 (O_2 to O_4)	$d := 930 \cdot mm$	$\delta_4 := -181 \cdot deg$
Link 4 initial angle	$\theta_{40} := 62.8 \cdot deg$	with respect to local x axis

Input cylinder acceleration $addot := -100 \cdot mm \cdot sec^{-2}$

Solution: See Figure P7-17 and Mathcad file P0747.

1. Draw the mechanism to scale and define a vector loop using the fourbar derivation in Section 6.7 as a model.





2. Write the vector loop equation, differentiate it, expand the result and separate into real and imaginary parts to solve for f, θ_2, and ω_4.

$$\mathbf{R_1} + \mathbf{R_4} := \mathbf{R_2} + \mathbf{R_3} \qquad (a)$$

$$d \cdot e^{j \cdot \theta_1} + c \cdot e^{j \cdot \theta_4} := a \cdot e^{j \cdot \theta_2} + b \cdot e^{j \cdot \theta_2} \qquad (b)$$

where a is the distance from the origin to the cylinder piston, a variable; b is the distance from the cylinder piston to A, a constant; and c is the distance from θ_4 to point A, a constant. Angle θ_1 is zero, $\theta_3 = \theta_2$, and θ_4 is the variable angle that the rocker arm makes with the x axis. Solving the position equations:

Let $f := a + b$ then, making this substitution and substituting the Euler equivalents,

$$d + c \cdot (\cos(\theta_4) + j \cdot \sin(\theta_4)) := f \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) \qquad (c)$$

Separating into real and imaginary components and solving for θ_2 and f,

$$\theta_2(\theta_4) := atan2(d + c \cdot \cos(\theta_4), c \cdot \sin(\theta_4)) \qquad (d)$$

$$f(\theta_4) := \frac{c \cdot sin(\theta_4)}{sin(\theta_2(\theta_4))} \tag{e}$$

For constant deceleration of the hydraulic cylinder rod, the velocity of the rod is

$$adot(\theta_4) := addot \cdot \sqrt{\frac{2 \cdot (f(\theta_4) - f(\theta_{40}))}{addot}}$$

Differentiate equation *b*.

$$j \cdot c \cdot \omega_4 \cdot e^{j \cdot \theta_4} := \left(\frac{d}{dt} f\right) \cdot e^{j \cdot \theta_2} + j \cdot f \cdot \omega_2 \cdot e^{j \cdot \theta_2} \tag{f}$$

Substituting the Euler equivalents,

$$c \cdot \omega_4 \cdot (-sin(\theta_4) + j \cdot cos(\theta_4)) := \left(\frac{d}{dt} f\right) \cdot (cos(\theta_2) + j \cdot sin(\theta_2)) + f \cdot \omega_2 \cdot (-sin(\theta_2) + j \cdot cos(\theta_2)) \tag{g}$$

Separating into real and imaginary components and solving for ω_4 and ω_2. Note that $df/dt = adot$.

$$\omega_4(\theta_4) := \frac{adot(\theta_4)}{c \cdot sin(\theta_2(\theta_4) - \theta_4)} \tag{h}$$

$$\omega_2(\theta_4) := \frac{c \cdot \omega_4(\theta_4) + adot(\theta_4) \cdot cos(\theta_2(\theta_4))}{f(\theta_4) \cdot sin(\theta_2(\theta_4))} \tag{i}$$

3. Differentiate equation *f*, expand the result and separate into real and imaginary parts to solve for α_4.

$$j \cdot c \cdot \alpha_4 \cdot e^{j \cdot \theta_4} + j^2 \cdot c \cdot \omega_4^{\,2} \cdot e^{j \cdot \theta_4} := addot \cdot e^{j \cdot \theta_2} + j \cdot adot \cdot \omega_2 \cdot e^{j \cdot \theta_2} \ldots$$
$$+ j \cdot adot \cdot \omega_2 \cdot e^{j \cdot \theta_2} + j \cdot f \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + j^2 \cdot f \cdot \omega_2^{\,2} \cdot e^{j \cdot \theta_2} \tag{f}$$

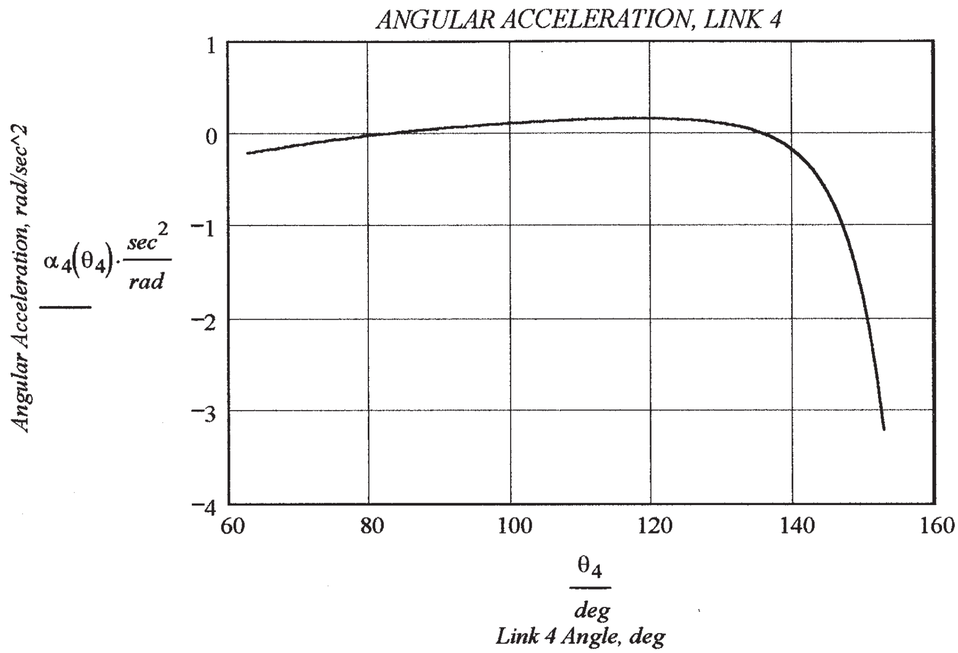
Separating into real and imaginary components and solving for α_4.

$$\alpha_4(\theta_4) := \frac{\left(addot + 2 \cdot adot(\theta_4) \cdot \omega_2(\theta_4) - f(\theta_4) \cdot \omega_2(\theta_4)^2\right) \cdot sin(2 \cdot \theta_2(\theta_4)) \ldots + c \cdot \omega_4(\theta_4)^2 \cdot (sin(\theta_2(\theta_4) + \theta_4))}{c \cdot sin(\theta_4 - \theta_2(\theta_4))}$$

4. Plot α_4 over a range of θ_4 of

$$\theta_4 := \theta_{40}, \theta_{40} + 1 \cdot deg .. \theta_{40} + 90 \cdot deg$$

(See next page.)

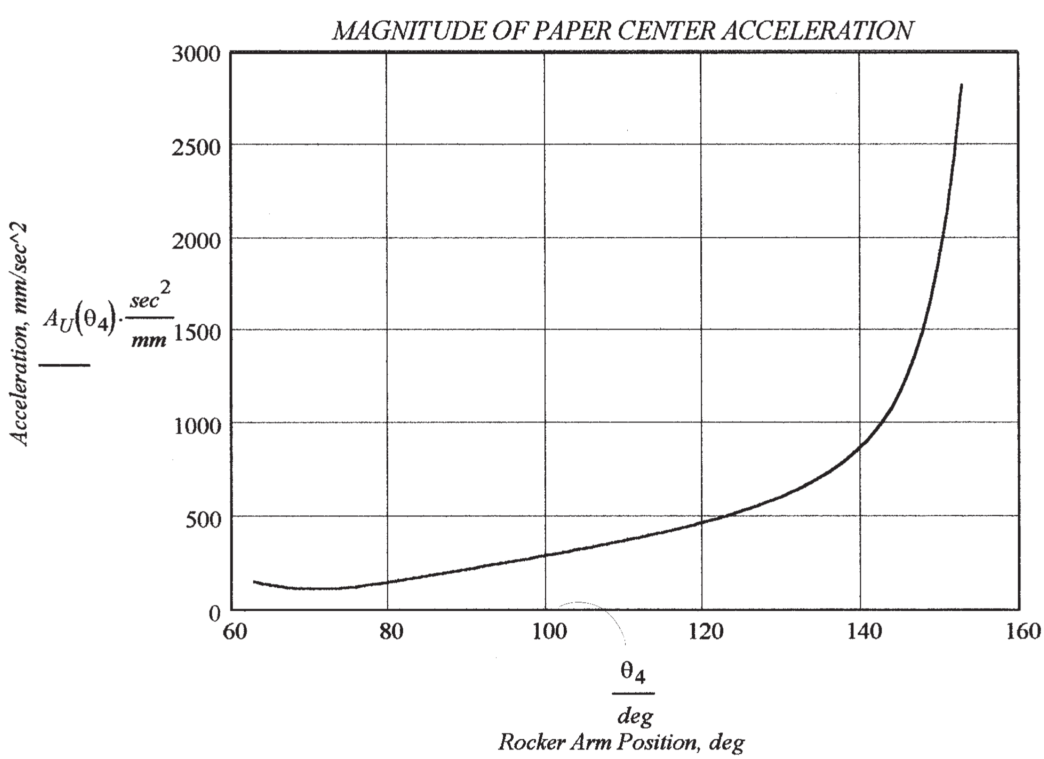


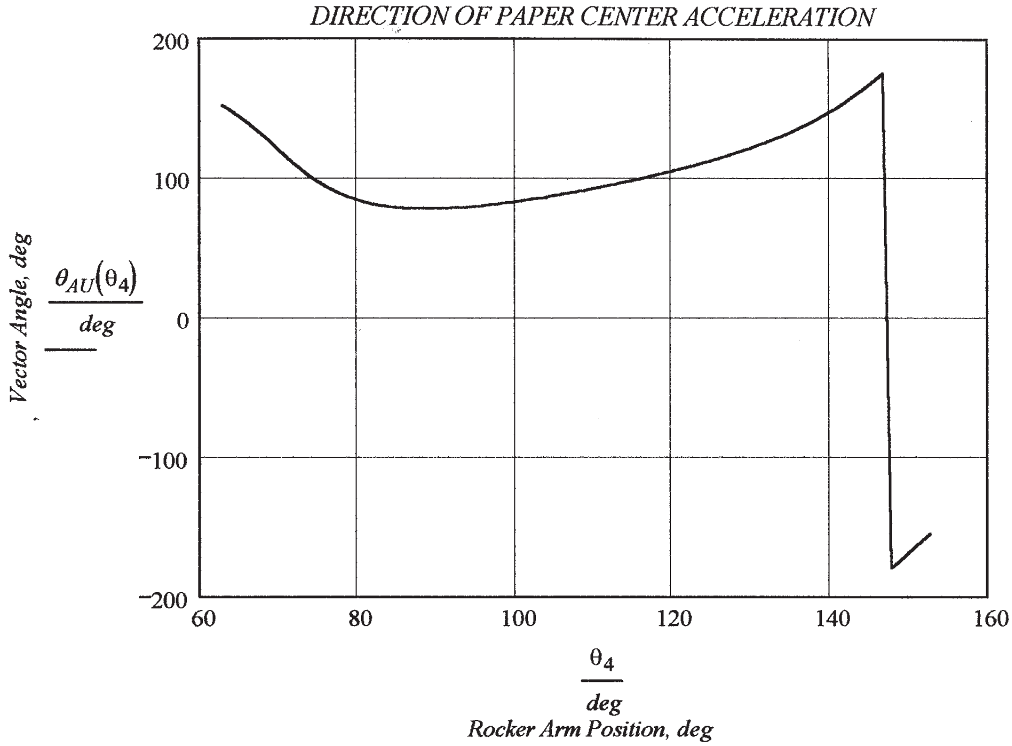
5. Determine the acceleration of the center of the paper roll using equation 7.31. The direction is in the local *xy* coordinate system.

$$\mathbf{A_U}(\theta_4) := u\cdot\alpha_4(\theta_4)\cdot(-sin(\theta_4+\delta_4)+j\cdot cos(\theta_4+\delta_4)) - u\cdot\omega_4(\theta_4)^2\cdot(cos(\theta_4+\delta_4)+j\cdot sin(\theta_4+\delta_4))$$

$$A_U(\theta_4) := |\mathbf{A_U}(\theta_4)| \qquad \theta_{AU}(\theta_4) := arg(\mathbf{A_U}(\theta_4))$$

6. Plot the magnitude and direction of the acceleration of the paper roll center.




DIRECTION OF PAPER CENTER ACCELERATION
200
100
0
−100
−200
60
80
100
120
140
160
Vector Angle, deg
$\frac{\theta_{AU}(\theta_4)}{deg}$
$\frac{\theta_4}{deg}$
Rocker Arm Position, deg

PROBLEM 7-48

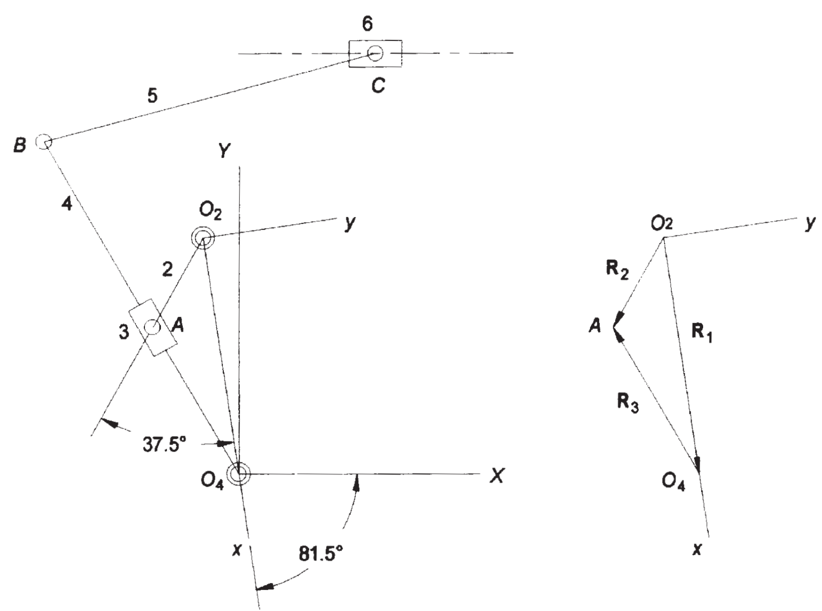
Statement: Figure P7-18 shows a mechanism and its dimensions. Find the accelerations of points A, B, and C for the position shown.

Given: Lengths and angles:

Link 2 (O_2 to A)	$a := 0.80 \cdot in$	Link 1 (O_2 to O_4)	$d := 1.85 \cdot in$
Link 4 (O_4 to B)	$a' := 2.97 \cdot in$	Link 5 (B to C)	$b' := 2.61 \cdot in$
Crank angle	$\theta_2 := -37.5 \cdot deg$	Local xy system	
Coordinate rotation angle	$\beta := -81.5 \cdot deg$	Slider-crank offset	$c' := 3.25 \cdot in$
Input crank motion	$\omega_2 := 40 \cdot \frac{rad}{min}$	$\alpha_2 := -1500 \cdot \frac{rad}{min^2}$	

Solution: See Figure P7-18 and Mathcad file P0748.

1. Draw the mechanism to scale and define a vector loop for the input portion (links 1, 2, 3, and 4) using the fourbar slider-crank derivation in Section 7.3 as a model.



2. Write the vector loop equation, expand the result and separate into real and imaginary parts to solve for θ_3 and b (distance from O_4 to A).

$$\mathbf{R_2} := \mathbf{R_1} + \mathbf{R_3}$$

$$a \cdot e^{j \cdot \theta_2} := d + b \cdot e^{j \cdot \theta_3}$$

Substituting the Euler equivalents,

$$a \cdot \left(\cos(\theta_2) + j \cdot \sin(\theta_2)\right) := d + b\left(\cos(\theta_3) + j \cdot \sin(\theta_3)\right)$$

Separating into real and imaginary components and solving for θ_3 and b.

$$\theta_3 := atan2\left(a \cdot \cos(\theta_2) - d, a \cdot \sin(\theta_2)\right) \qquad \theta_3 = -158.2\,deg$$

$$b := \frac{a \cdot sin(\theta_2)}{sin(\theta_3)} \qquad b = 1.31\,in$$

3. Differentiate the position equation, expand it and solve for ω_3 and *bdot*.

$$a \cdot j \cdot \omega_2 \cdot e^{j \cdot \theta_2} := b \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + bdot \cdot e^{j \cdot \theta_3}$$

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot cos(\theta_2 - \theta_3) \qquad \omega_3 = -0.208\,\frac{rad}{sec}$$

$$bdot := \frac{a \cdot \omega_2 \cdot cos(\theta_2) - b \cdot \omega_3 \cdot cos(\theta_3)}{sin(\theta_3)} \qquad bdot = -0.459\,\frac{in}{sec}$$

4. Differentiate the velocity equation, expand it and solve for α_3.

$$a \cdot j \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + a \cdot j^2 \cdot \omega_2^{\,2} \cdot e^{j \cdot \theta_2} := bdot \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b \cdot j \cdot \alpha_3 \cdot e^{j \cdot \theta_3} \ldots$$
$$+ b \cdot j^2 \cdot \omega_3^{\,2} \cdot e^{j \cdot \theta_3} + bddot \cdot e^{j \cdot \theta_3} + bdot \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3}$$

$$\alpha_3 := \frac{1}{b} \cdot \left(a \cdot \alpha_2 \cdot cos(\theta_2 - \theta_3) + a \cdot \omega_2^{\,2} \cdot sin(\theta_3 - \theta_2) - 2 \cdot bdot \cdot \omega_3\right)$$

$$\alpha_3 = -0.249\,\frac{rad}{sec^2}$$

5. Determine the acceleration of point A on link 2 (in the local xy coordinate system) using equations 7.13a.

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot \left(-sin(\theta_2) + j \cdot cos(\theta_2)\right) - a \cdot \omega_2^{\,2} \cdot \left(cos(\theta_2) + j \cdot sin(\theta_2)\right)$$

$$A_A := \left|\mathbf{A_A}\right| \qquad A_A = 0.487\,\frac{in}{sec^2} \qquad \theta_{AA} := arg(\mathbf{A_A}) \qquad \theta_{AA} = -174.348\,deg$$

6. Determine the acceleration of points A and B on link 3 (in the Global XY coordinate system) using equations 7.13a.

Transform θ_3 to the global coordinate frame: $\theta_3 := \theta_3 + \beta + 360 \cdot deg$ $\qquad \theta_3 = 120.337\,deg$

$$\mathbf{A_{A3}} := b \cdot \alpha_3 \cdot \left(-sin(\theta_3) + j \cdot cos(\theta_3)\right) - b \cdot \omega_3^{\,2} \cdot \left(cos(\theta_3) + j \cdot sin(\theta_3)\right)$$

$$A_{A3} := \left|\mathbf{A_{A3}}\right| \qquad A_{A3} = 0.331\,\frac{in}{sec^2} \qquad \theta_{AA3} := arg(\mathbf{A_{A3}}) \qquad \theta_{AA3} = 20.518\,deg$$

$$\mathbf{A_B} := a' \cdot \alpha_3 \cdot \left(-sin(\theta_3) + j \cdot cos(\theta_3)\right) - a' \cdot \omega_3^{\,2} \cdot \left(cos(\theta_3) + j \cdot sin(\theta_3)\right)$$

$$A_B := \left|\mathbf{A_B}\right| \qquad A_B = 0.752\,\frac{in}{sec^2} \qquad \theta_{AB} := arg(\mathbf{A_B}) \qquad \theta_{AB} = 20.518\,deg$$

7. Determine θ_5 using equation 4.16.

$$\theta_5 := asin\left(\frac{a' \cdot sin(\theta_3) - c'}{b'}\right) + \pi \qquad \theta_5 = 195.254\, deg$$

8. Determine the angular velocity of link 5 using equation 6.22a.

$$\omega_5 := \frac{a'}{b'} \cdot \frac{cos(\theta_3)}{cos(\theta_5)} \cdot \omega_3 \qquad \omega_5 = -0.124 \frac{rad}{sec}$$

9. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{a' \cdot \alpha_3 \cdot cos(\theta_3) - a' \cdot \omega_3^{\,2} \cdot sin(\theta_3) + b' \cdot \omega_5^{\,2} \cdot sin(\theta_5)}{b' \cdot cos(\theta_5)} \qquad \alpha_5 = -0.100 \frac{rad}{sec^2}$$

10. Use equation 7.16e for the acceleration of pin *C*.

$$A_C := -a' \cdot \alpha_3 \cdot sin(\theta_3) - a' \cdot \omega_3^{\,2} \cdot cos(\theta_3) + b' \cdot \alpha_5 \cdot sin(\theta_5) + b' \cdot \omega_5^{\,2} \cdot cos(\theta_5)$$

$$A_C = 0.73 \frac{in}{sec^2}$$

A positive sign means that A_C is to the right

PROBLEM 7-49

Statement: Figure P7-19 shows a walking beam mechanism. Calculate and plot the acceleration A_{out} for one revolution of the input crank 2 rotating at 100 rpm.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Link lengths:

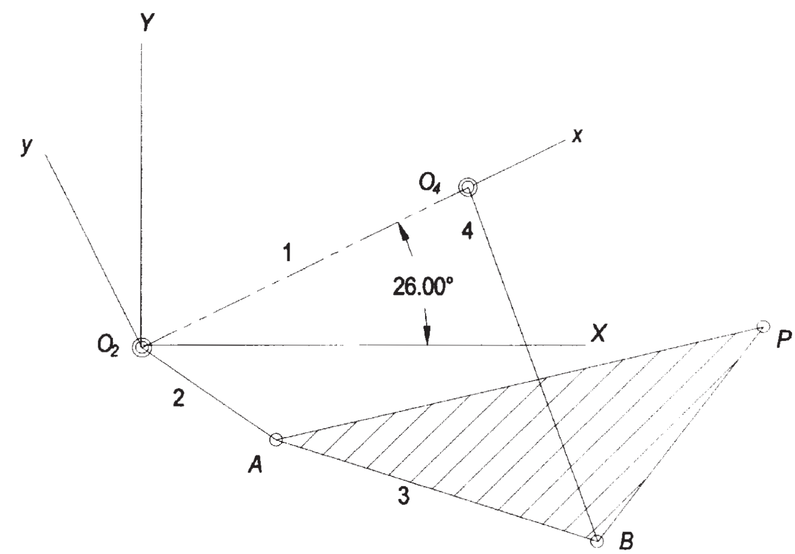
Link 2 (O_2 to A)	$a := 1.00\cdot in$	Link 3 (A to B)	$b := 2.06\cdot in$
Link 4 (B to O_4)	$c := 2.33\cdot in$	Link 1 (O_2 to O_4)	$d := 2.22\cdot in$

Coupler point: $p := 3.06$ $\delta_3 := 31\cdot deg$

Crank speed: $\omega_2 := 100\cdot rpm$ $\alpha_2 := 0\cdot rad\cdot sec^{-2}$

Solution: See Figure P7-19 and Mathcad file P0749.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 2.2200 \qquad K_2 = 0.9528$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)} \qquad K_3 = 1.5265$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2\cdot\cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot\sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1)\cdot\cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.0777 \qquad K_5 = -1.1512$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4(\theta_2) - \theta_2)}{sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Determine the acceleration of point A using equation 7.13a.

$$\mathbf{A_A}(\theta_2) := a\cdot\alpha_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) - a\cdot\omega_2^2\cdot(cos(\theta_2) + j\cdot sin(\theta_2))$$

9. Use equations 7.12 to determine the angular acceleration of link 3.

$$A(\theta_2) := c\cdot sin(\theta_4(\theta_2)) \qquad B(\theta_2) := b\cdot sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c\cdot cos(\theta_4(\theta_2)) \qquad E(\theta_2) := b\cdot cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^2\cdot cos(\theta_2) + b\cdot\omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^2\cdot sin(\theta_2) - b\cdot\omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)^2\cdot sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C(\theta_2)\cdot D(\theta_2) - A(\theta_2)\cdot F(\theta_2)}{A(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot D(\theta_2)}$$

10. Determine the acceleration of the coupler point P using equations 7.32.

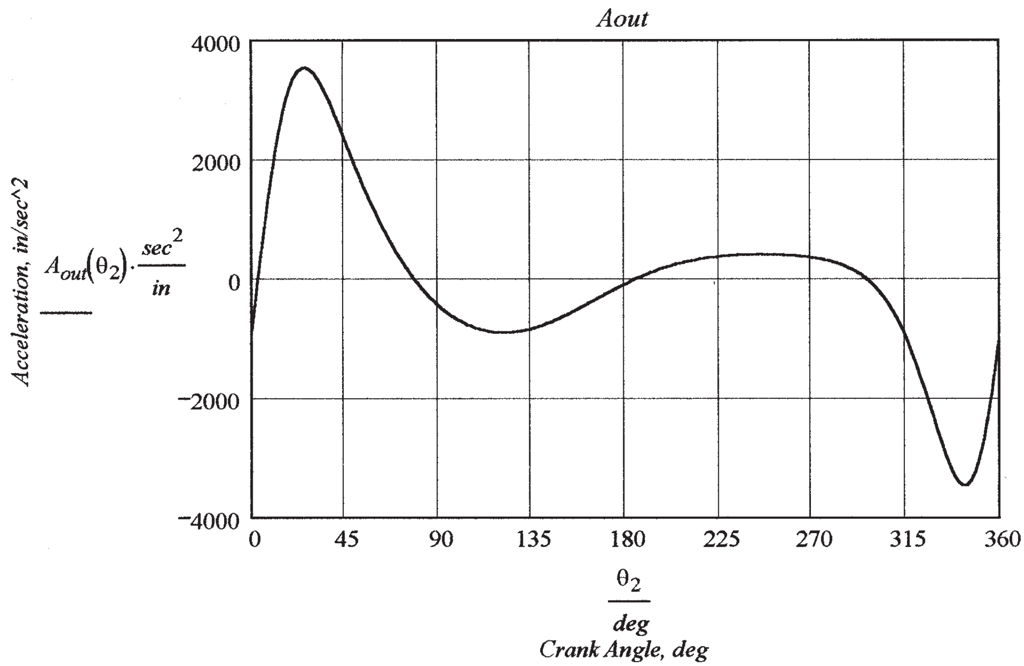
$$\mathbf{A_{PA}}(\theta_2) := p\cdot\alpha_3(\theta_2)\cdot(-sin(\theta_3(\theta_2) + \delta_3) + j\cdot cos(\theta_3(\theta_2) + \delta_3)) \ldots$$
$$+ -p\cdot\omega_3(\theta_2)^2\cdot(cos(\theta_3(\theta_2) + \delta_3) + j\cdot sin(\theta_3(\theta_2) + \delta_3))$$

$$\mathbf{A_P}(\theta_2) := \mathbf{A_A}(\theta_2) + \mathbf{A_{PA}}(\theta_2)$$

11. Plot the X-component (global coordinate system) of the acceleration of the coupler point P.

Coordinate rotation angle: $\alpha := 26 \cdot deg$

$$A_{out}(\theta_2) := Re(\mathbf{A_P}(\theta_2)) \cdot cos(\alpha) - Im(\mathbf{A_P}(\theta_2)) \cdot sin(\alpha)$$



PROBLEM 7-50

Statement: Figure P7-20 shows a surface grinder. The workpiece is oscillated under the spinning grinding wheel by the slider-crank linkage that has the dimensions given below. Calculate and plot the acceleration of the grinding wheel contact point relative to the workpiece over one revolution of the crank.

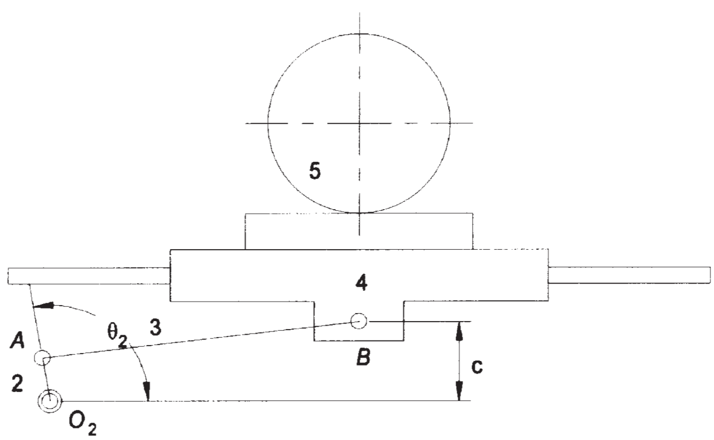
Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

Link 2 (O_2 to A)	$a := 22 \cdot mm$	Offset	$c := 40 \cdot mm$
Link 3 (A to B)	$b := 157 \cdot mm$		
Grinding wheel diameter	$d := 90 \cdot mm$		
Input crank angular velocity	$\omega_2 := 30 \cdot rpm$	CCW	$\alpha_2 := 0 \cdot rad \cdot sec^{-2}$
Grinding wheel angular velocity	$\omega_5 := 3450 \cdot rpm$	CCW	

Solution: See Figure P7-20 and Mathcad file P0750.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this slider-crank linkage.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine θ_3 using equation 4.17.

$$\theta_3(\theta_2) := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi$$

4. Determine the angular velocity of link 3 using equation 6.22a:

$$\omega_3(\theta_2) := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_3(\theta_2))} \cdot \omega_2$$

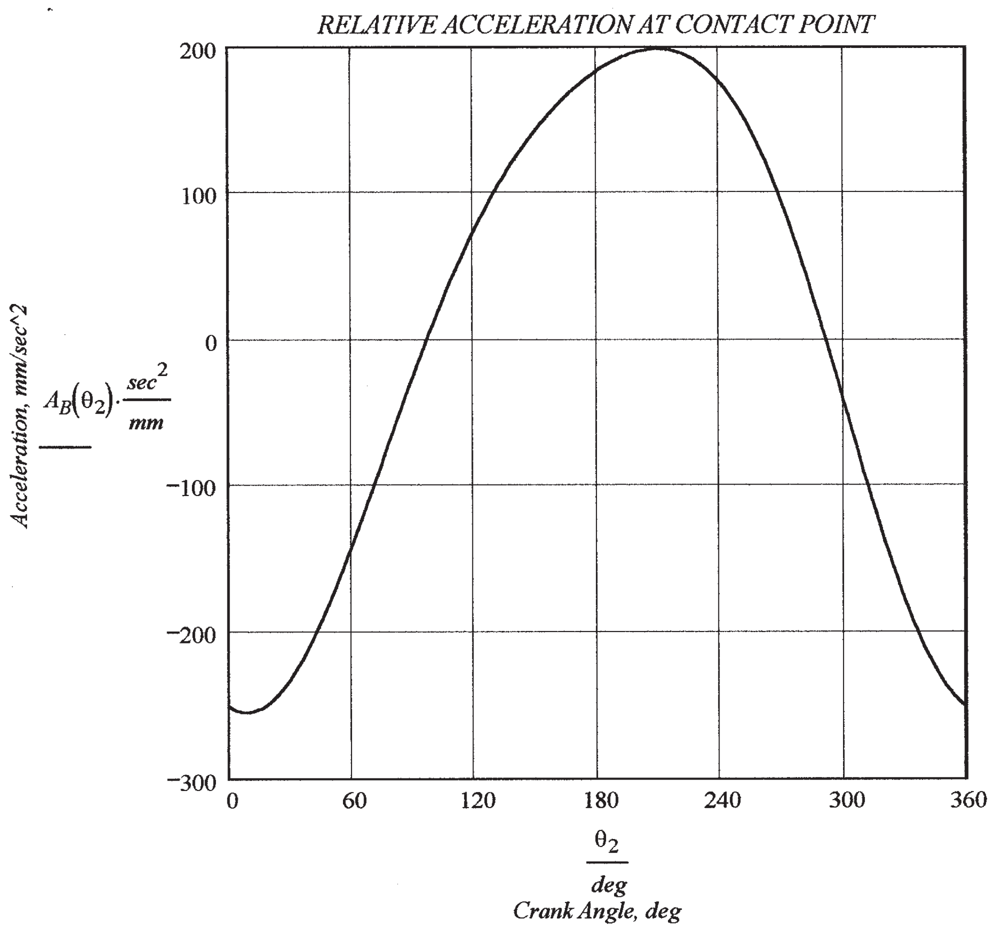
5. Determine the angular acceleration of link 3 using equation 7.16d.

$$\alpha_3(\theta_2) := \frac{a \cdot \alpha_2 \cdot cos(\theta_2) - a \cdot \omega_2^2 \cdot sin(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot sin(\theta_3(\theta_2))}{b \cdot cos(\theta_3(\theta_2))}$$

6. Use equation 7.16e for the acceleration of pin B.

$$A_B(\theta_2) := -a\cdot\alpha_2\cdot sin(\theta_2) - a\cdot\omega_2^{\,2}\cdot cos(\theta_2) \;... \\ + b\cdot\alpha_3(\theta_2)\cdot sin(\theta_3(\theta_2)) + b\cdot\omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2))$$

7. The acceleration of the grinding wheel contact point relative to the workpiece, which has zero tangential acceleration due to constant angular velocity, is A_B (positive to the right).



PROBLEM 7-51

Statement: Figure P7-21 shows a drag-link mechanism. Scale the diagram for dimensions and calculate and plot the angular acceleration of link 4 for an input of $\omega_2 = 1$ rad/sec. Comment on the uses for this mechanism.

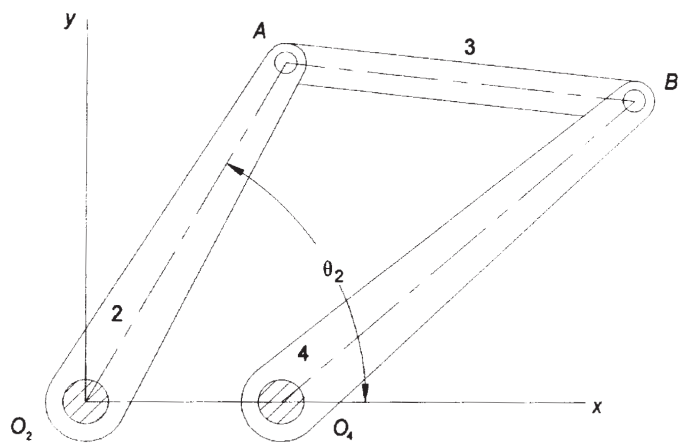
Given: Link lengths:

Link 2 (L_2)	$a := 1.38 \cdot in$	Link 3 (L_3)	$b := 1.22 \cdot in$
Link 4 (L_4)	$c := 1.62 \cdot in$	Link 1 (L_1)	$d := 0.68 \cdot in$

Input crank angular velocity $\omega_2 := -1 \cdot rad \cdot sec^{-1}$ CW $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-21 and Mathcad file P0751.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof double crank.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.4928 \qquad K_2 = 0.4198$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.7834$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

PROBLEM 7-51

Statement: Figure P7-21 shows a drag-link mechanism. Scale the diagram for dimensions and calculate and plot the angular acceleration of link 4 for an input of $\omega_2 = 1$ rad/sec. Comment on the uses for this mechanism.

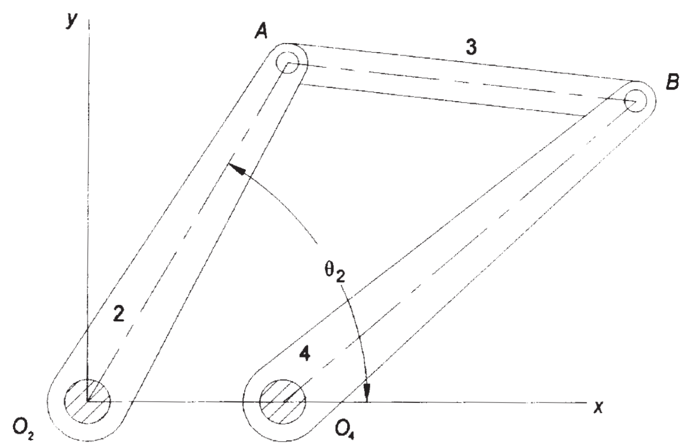
Given: Link lengths:

Link 2 (L_2)	$a := 1.38 \cdot in$	Link 3 (L_3)	$b := 1.22 \cdot in$
Link 4 (L_4)	$c := 1.62 \cdot in$	Link 1 (L_1)	$d := 0.68 \cdot in$

Input crank angular velocity $\omega_2 := -1 \cdot rad \cdot sec^{-1}$ CW $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-21 and Mathcad file P0751.

1. Draw the linkage to scale and label it.



2. Determine the range of motion for this Grashof double crank.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.4928 \qquad K_2 = 0.4198$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.7834$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.5574 \qquad K_5 = -0.3655$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

7. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c \cdot \sin(\theta_4(\theta_2)) \qquad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

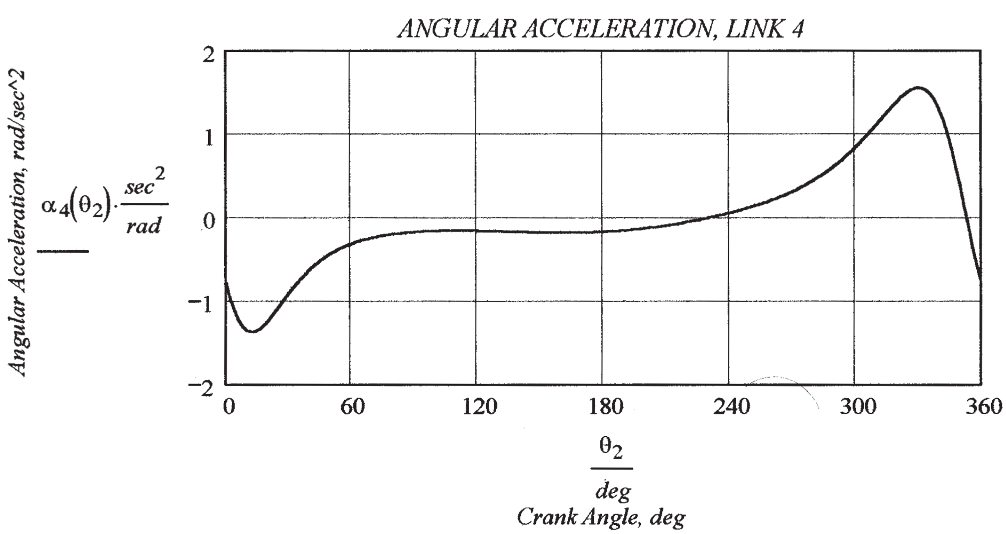
$$D(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \qquad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$

9. Plot the angular acceleration of link 4.



PROBLEM 7-52

Statement: Figure P7-22 shows a mechanism. Scale the diagram for dimensions and use a graphical method to calculate the accelerations of points A, B, and C for the position shown.

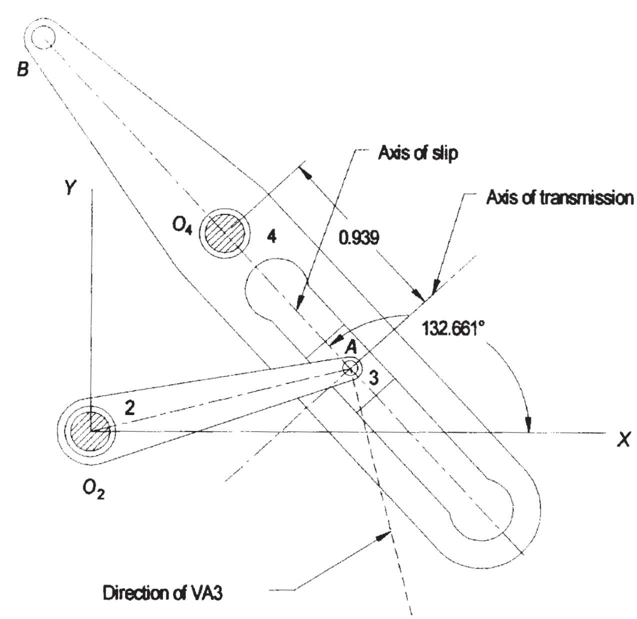
Given: Link lengths and angles:

Link 1 (O_2O_4)	$d := 1.22 \cdot in$	Angle O_2O_4 makes with X axis	$\theta_1 := 56.5 \cdot deg$
Link 2 (O_2A)	$a := 1.35 \cdot in$	Angle O_2A makes with X axis	$\theta_2 := 14 \cdot deg$
Link 4 (O_4B)	$e := 1.36 \cdot in$		
Link 5 (BC)	$f := 2.69 \cdot in$		
Link 6 (CO_6)	$g := 1.80 \cdot in$	Angle CO_6 makes with X axis	$\theta_6 := 88 \cdot deg$

Motion of link 2 $\quad \omega_2 := 20 \cdot rad \cdot sec^{-1}$ CW $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-22 and Mathcad file P0752.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point A on links 2 and 3.

$$V_{A3} := a \cdot \omega_2 \qquad V_{A3} = 27.000 \frac{in}{sec} \qquad \theta_{VA3} := \theta_2 - 90 \cdot deg \qquad \theta_{VA3} = -76.0\, deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point A on link 3. The equation to be solved graphically is

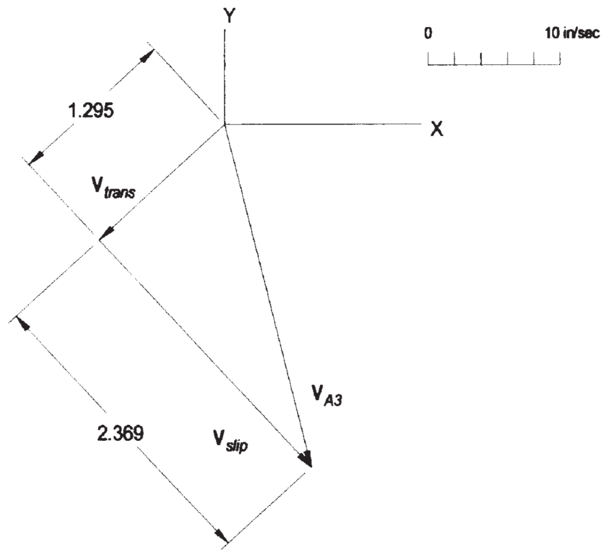
$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.

b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.

c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \frac{10 \cdot in \cdot sec^{-1}}{in}$

$V_{slip} := 2.369 \cdot in \cdot k_v$ $\quad V_{slip} = 23.690 \frac{in}{sec}$

$V_{trans} := 1.295 \cdot in \cdot k_v$ $\quad V_{trans} = 12.950 \frac{in}{sec}$

5. The true velocity of point A on link 4 is V_{trans},

$V_{A4} := V_{trans}$ $\quad V_{A4} = 12.95 \frac{in}{sec}$

6. Determine the angular velocity of link 4 using equation 6.7.

From the linkage layout above: $c := 0.939 \cdot in$ and $\theta_4 := 132.661 \cdot deg$

$\omega_4 := \frac{V_{A4}}{c}$ $\quad \omega_4 = 13.791 \frac{rad}{sec}$ CW

7. Determine the magnitude and sense of the vector $\mathbf{V}_B$ using equation 6.7.

$V_B := e \cdot \omega_4$ $\quad V_B = 18.756 \frac{in}{sec}$

$\theta_{VA4} := \theta_4 - 90 \cdot deg$ $\quad \theta_{VA4} = 42.661 \, deg$

8. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

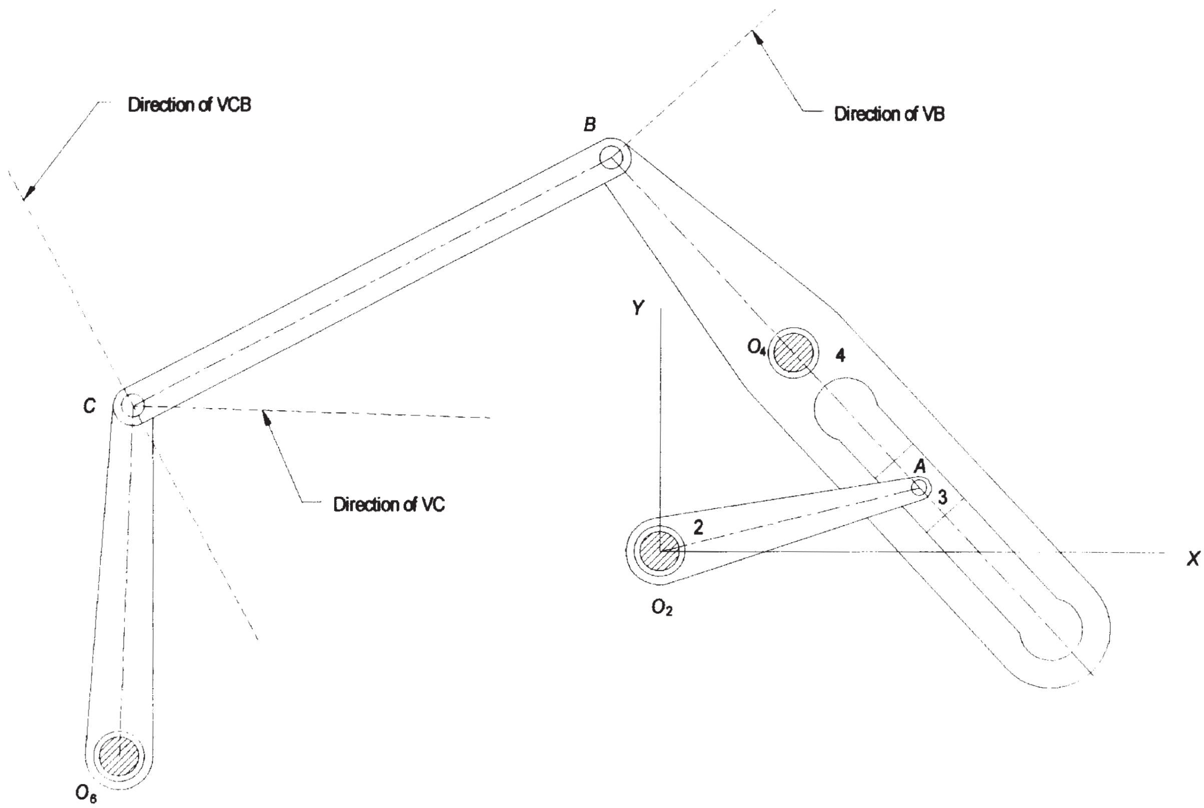
a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.

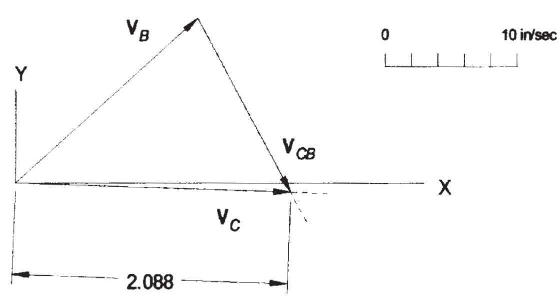
b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.

c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.

Draw links 1, 4, 5, and 6 to a convenient scale. Indicate the directions of the velocity vectors of interest.





9. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{10 \cdot in \cdot sec^{-1}}{in}$ $\qquad \theta_{VC} := \theta_6 - 90 \cdot deg$

$V_C := 2.088 \cdot in \cdot k_v \qquad V_C = 20.9 \dfrac{in}{sec} \qquad \theta_{VC} = -2.0\,deg$

$V_{CB} := 1.519 \cdot in \cdot k_v \qquad V_{CB} = 15.2 \dfrac{in}{sec}$

10. Determine the angular velocity of links 5 and 6 using equation 6.7.

$\omega_5 := \dfrac{V_{CB}}{f} \qquad \omega_5 = 5.647 \dfrac{rad}{sec}$ CCW

$\omega_6 := \dfrac{V_C}{g} \qquad \omega_6 = 11.600 \dfrac{rad}{sec}$ CCW

11. For the acceleration of point A, use equation 7.19: $(\mathbf{A}_{A2}{}^t + \mathbf{A}_{A2}{}^n) = \mathbf{A}_{A4}{}^t + \mathbf{A}_{A4}{}^n + \mathbf{A}_A{}^{cor} + \mathbf{A}_A{}^{slip}$, where

$$A_{A2n} := a \cdot \omega_2^2 \qquad A_{A2n} = 540.000\ in \cdot sec^{-2}$$

$$\theta_{AA} := \theta_2 + 180 \cdot deg \qquad \theta_{AA} = 194.000\ deg$$

$$A_{A2t} := a \cdot \alpha_2 \qquad A_{A2t} = 0.000\ in \cdot sec^{-2}$$

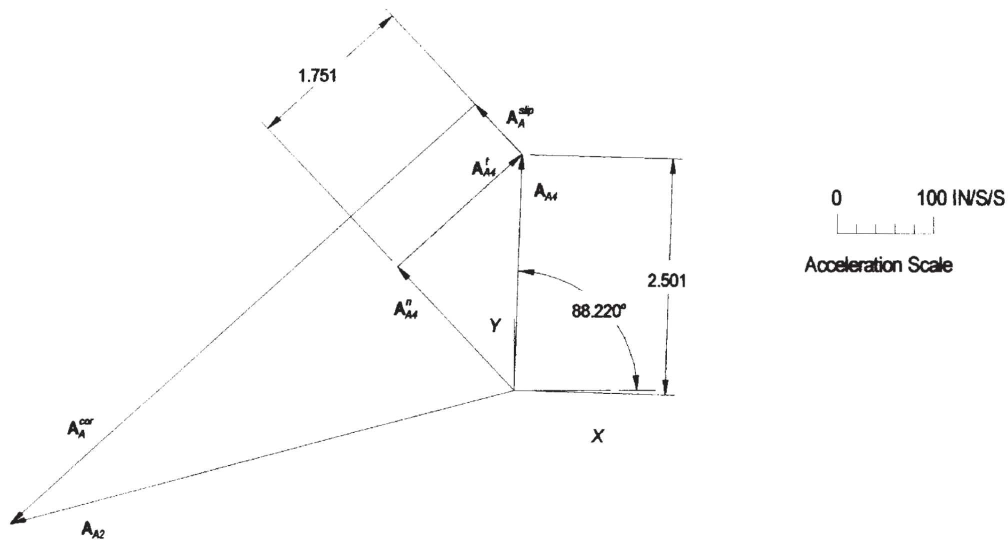
$$A_{A4n} := c \cdot \omega_4^2 \qquad A_{A4n} = 178.6\ in \cdot sec^{-2}$$

$$\theta_{AA4n} := \theta_4 \qquad \theta_{AA4n} = 132.661\ deg$$

$$A_{Acor} := \left|2 \cdot V_{slip} \cdot \omega_4\right| \qquad A_{Acor} = 653.430\ in \cdot sec^{-2}$$

$$\theta_{AAcor} := \theta_4 + 90 \cdot deg \qquad \theta_{AAcor} = 222.661\ deg$$

12. Repeat procedure of steps 3 and 7 for the equation in step 11.



13. From the acceleration polygon above,

Acceleration scale factor $\qquad k_a := 100 \cdot \frac{in}{sec^2}$

$$A_{A4} := 2.501 \cdot k_a \qquad A_{A4} = 250.1\ in \cdot sec^{-2} \quad \text{at an angle of 88.22 deg}$$

$$A_{A4t} := 1.751 \cdot k_a \qquad A_{A4t} = 175.1\ in \cdot sec^{-2}$$

The angular acceleration of link 4 is

$$\alpha_4 := \frac{A_{A4t}}{c} \qquad \alpha_4 = 186.5 \frac{rad}{sec^2} \quad \text{CCW}$$

14. The graphical solution for accelerations in pin-jointed fourbars uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

15. For point C, this becomes: $(\mathbf{A}_C^t + \mathbf{A}_C^n) = (\mathbf{A}_B^t + \mathbf{A}_B^n) + (\mathbf{A}_{CB}^t + \mathbf{A}_{CB}^n)$, where

$A_{Cn} := g \cdot \omega_6^{\,2}$ $\qquad A_{Cn} = 242.2\, in \cdot sec^{-2}$

$\theta_{ACn} := \theta_6 + 180 \cdot deg$ $\qquad \theta_{ACn} = 268.000\, deg$

$A_{Bn} := e \cdot \omega_4^{\,2}$ $\qquad A_{Bn} = 258.7\, in \cdot sec^{-2}$

$\theta_{ABn} := \theta_4 + 180 \cdot deg$ $\qquad \theta_{ABn} = 312.661\, deg$

$A_{Bt} := e \cdot \alpha_4$ $\qquad A_{Bt} = 253.6\, in \cdot sec^{-2}$

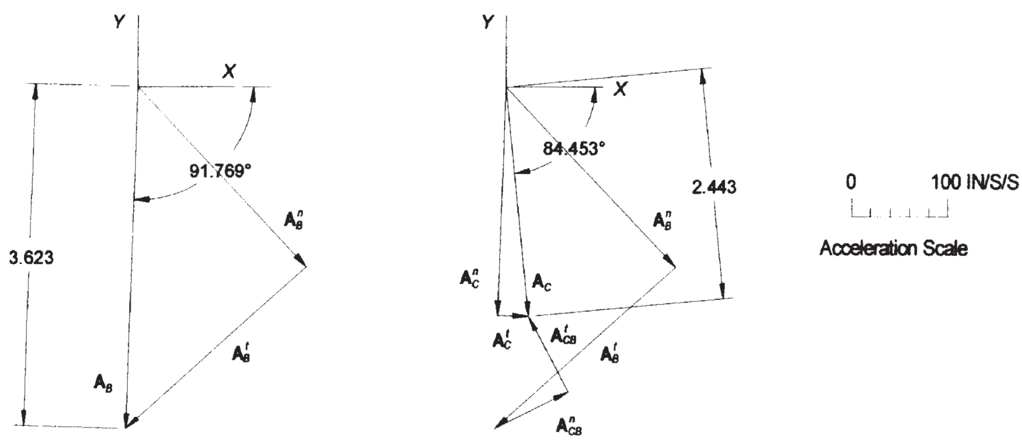
$\theta_{ABt} := \theta_4 + 90 \cdot deg$ $\qquad \theta_{ABt} = 222.661\, deg$

$A_{CBn} := f \cdot \omega_5^{\,2}$ $\qquad A_{CBn} = 85.78\, in \cdot sec^{-2}$

$\theta_5 := 27.5 \cdot deg$

$\theta_{ACBn} := \theta_5$ $\qquad \theta_{ACBn} = 27.500\, deg$

16. Repeat procedure of step 12 for the equation in step 15.



17. From the acceleration polygon above,

Acceleration scale factor $\qquad k_a := 100 \cdot \dfrac{in}{sec^2}$

$A_B := 3.623 \cdot k_a$ $\qquad A_B = 362.3\, in \cdot sec^{-2}$ $\qquad$ at an angle of -91.8 deg

$A_C := 2.443 \cdot k_a$ $\qquad A_C = 244.3\, in \cdot sec^{-2}$ $\qquad$ at an angle of -84.5 deg

PROBLEM 7-53

Statement: Figure P7-23 shows a quick-return mechanism with dimensions. Use a graphical method to calculate the accelerations of points *A*, *B*, and *C* for the position shown.

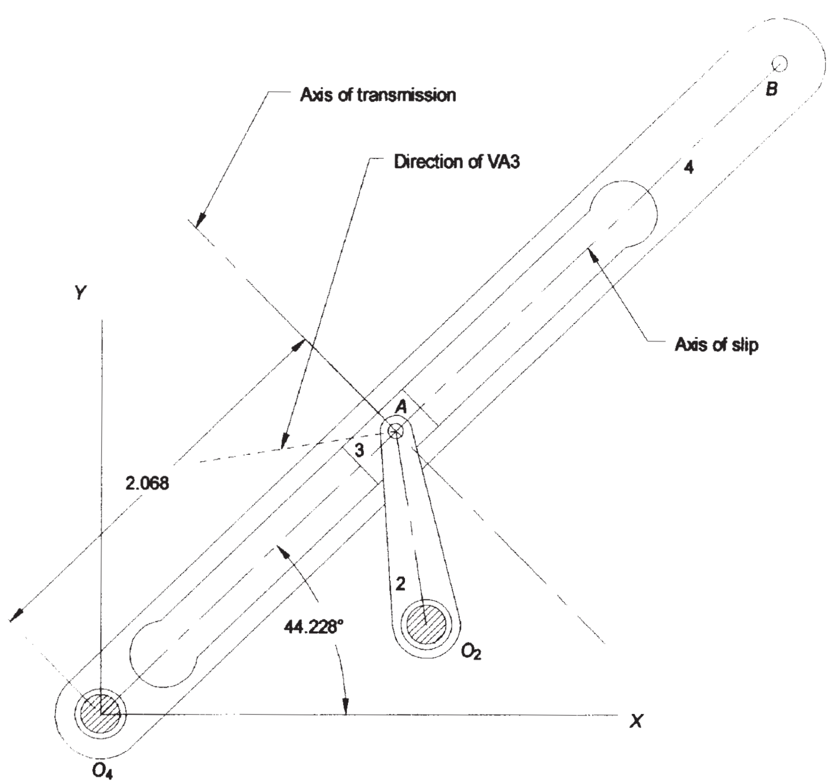
Given: Link lengths and angles:

Link 1 (O_2O_4)	$d := 1.69 \cdot in$	Angle O_2O_4 makes with X axis	$\theta_1 := 195.5 \cdot deg$
Link 2 (L_2)	$a := 1.00 \cdot in$	Angle link 2 makes with X axis	$\theta_2 := 99 \cdot deg$
Link 4 (L_4)	$e := 4.76 \cdot in$		
Link 5 (L_5)	$f := 4.55 \cdot in$		
Offset (O_2C)	$g := 2.86 \cdot in$		

Angular velocity of link 2 $\quad \omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-23 and Mathcad file P0753.

1. Draw the linkage to scale and indicate the axes of slip and transmission as well as the directions of velocities of interest.



2. Use equation 6.7 to calculate the magnitude of the velocity at point *A* on links 2 and 3.

$$V_{A3} := a \cdot \omega_2 \qquad V_{A3} = 10.000 \frac{in}{sec} \qquad \theta_{VA3} := \theta_2 + 90 \cdot deg \qquad \theta_{VA3} = 189.0\, deg$$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity components at point *A* on link 3. The equation to be solved graphically is

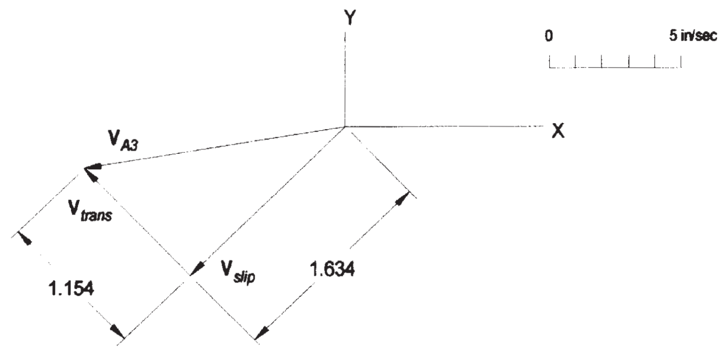
$$\mathbf{V}_{A3} = \mathbf{V}_{trans} + \mathbf{V}_{slip}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_{A3}$.

b. From the tip of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{slip}$, magnitude unknown.

c. From the tail of $\mathbf{V}_{A3}$, draw a construction line with the direction of $\mathbf{V}_{trans}$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{slip}$ from the tip of $\mathbf{V}_{trans}$ to the tip of $\mathbf{V}_{A3}$ and drawing $\mathbf{V}_{trans}$ from the tail of $\mathbf{V}_{A3}$ to the intersection of the $\mathbf{V}_{slip}$ construction line.



4. From the velocity triangle we have:

Velocity scale factor: $k_v := \dfrac{5 \cdot in \cdot sec^{-1}}{in}$

$V_{slip} := 1.634 \cdot in \cdot k_v$ $\qquad V_{slip} = 8.170 \dfrac{in}{sec}$

$V_{trans} := 1.154 \cdot in \cdot k_v$ $\qquad V_{trans} = 5.770 \dfrac{in}{sec}$

5. The true velocity of point A on link 4 is V_{trans},

$V_{A4} := V_{trans}$ $\qquad V_{A4} = 5.77 \dfrac{in}{sec}$

6. Determine the angular velocity of link 4 using equation 6.7.

From the linkage layout above: $c := 2.068 \cdot in$ and $\theta_4 := 44.228 \cdot deg$

$\omega_4 := \dfrac{V_{A4}}{c}$ $\qquad \omega_4 = 2.790 \dfrac{rad}{sec}$ CCW

7. Draw links 1, 4, 5, and 6 to a convenient scale. Indicate the directions of the velocity vectors of interest. (See next page.)

8. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point C. The equation to be solved graphically is

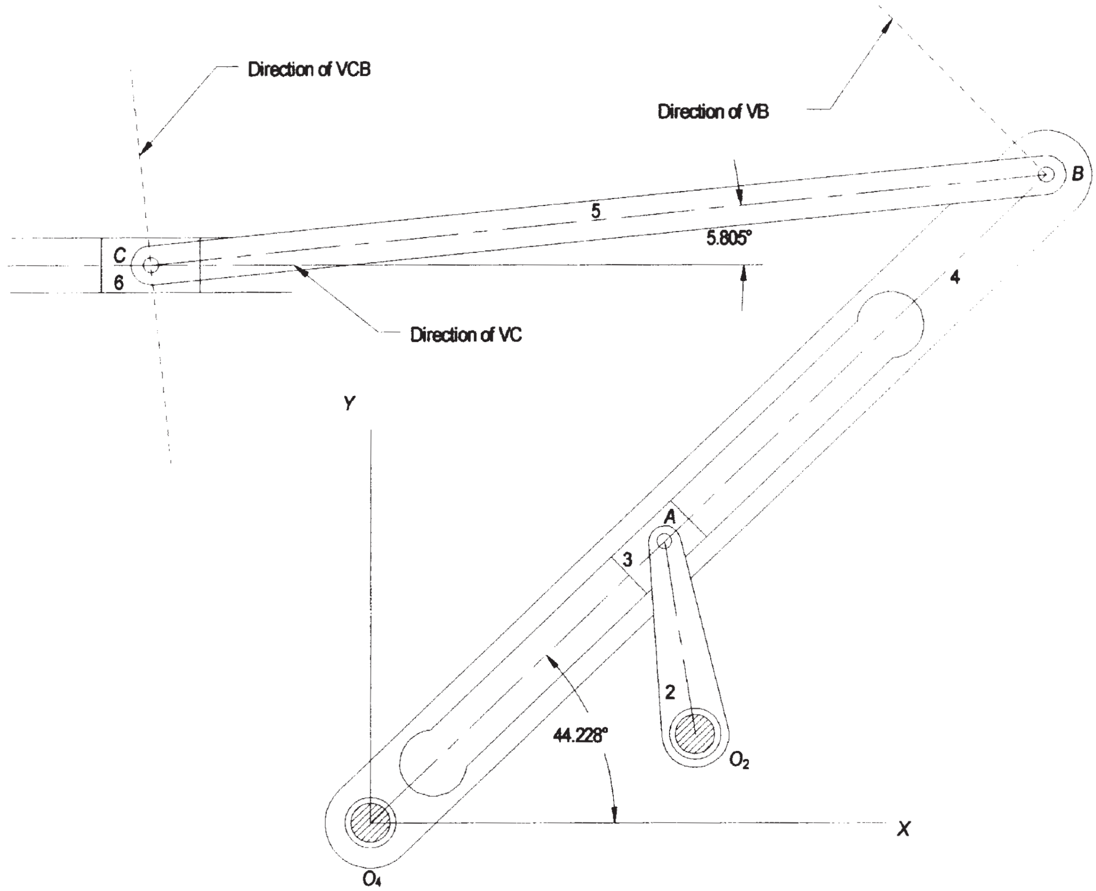
$$\mathbf{V}_C = \mathbf{V}_B + \mathbf{V}_{CB}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_B$.

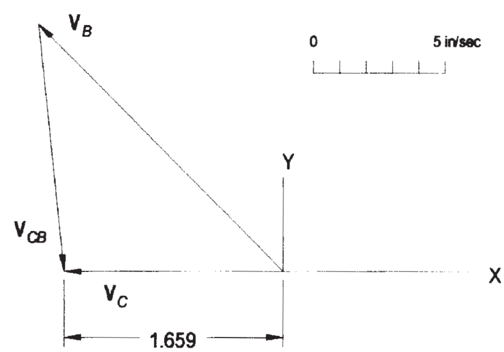
b. From the tip of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_{CB}$, magnitude unknown.

c. From the tail of $\mathbf{V}_B$, draw a construction line with the direction of $\mathbf{V}_C$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{CB}$ from the tip of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_C$ construction line and drawing $\mathbf{V}_C$ from the tail of $\mathbf{V}_B$ to the intersection of the $\mathbf{V}_{CB}$ construction line.



9. From the velocity triangle we have:



Velocity scale factor: $k_v := \dfrac{5 \cdot in \cdot sec^{-1}}{in}$

$V_C := 1.659 \cdot in \cdot k_v \qquad V_C = 8.30 \dfrac{in}{sec}$

$\theta_{VC} := 180 \cdot deg$

$V_{CB} := 1.913 \cdot in \cdot k_v \qquad V_{CB} = 9.56 \dfrac{in}{sec}$

10. Determine the angular velocity of link 5 using equation 6.7.

From the linkage layout above: $\theta_5 := 5.805 \cdot deg$

$\omega_5 := \dfrac{V_{CB}}{f} \qquad \omega_5 = 2.102 \dfrac{rad}{sec}$ CCW

11. For the acceleration of point A, use equation 7.19: $(\mathbf{A}_{A2}{}^t + \mathbf{A}_{A2}{}^n) = \mathbf{A}_{A4}{}^t + \mathbf{A}_{A4}{}^n + \mathbf{A}_A{}^{cor} + \mathbf{A}_A{}^{slip}$, where

$A_{A2n} := a \cdot \omega_2^{\,2} \qquad A_{A2n} = 100.000\, in \cdot sec^{-2}$

$\theta_{AA} := \theta_2 + 180 \cdot deg \qquad \theta_{AA} = 279.000\, deg$

$A_{A2t} := a \cdot \alpha_2$ $\qquad A_{A2t} = 0.000\ in \cdot sec^{-2}$

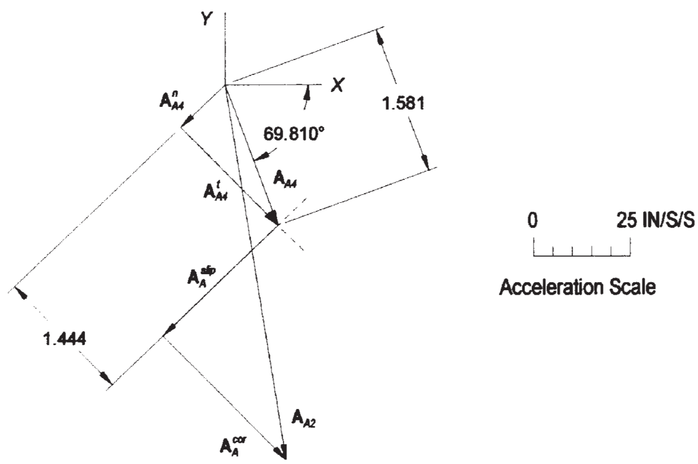
$A_{A4n} := c \cdot \omega_4^{\ 2}$ $\qquad A_{A4n} = 16.099\ in \cdot sec^{-2}$

$\theta_{AA4n} := \theta_4 + 180 \cdot deg$ $\qquad \theta_{AA4n} = 224.228\ deg$

$A_{Acor} := \left|2 \cdot V_{slip} \cdot \omega_4\right|$ $\qquad A_{Acor} = 45.591\ in \cdot sec^{-2}$

$\theta_{AAcor} := \theta_4 - 90 \cdot deg$ $\qquad \theta_{AAcor} = -45.772\ deg$ $\qquad$ (V_{slip} is negative)

12. Repeat procedure of steps 3 and 7 for the equation in step 11.



13. From the acceleration polygon above,

Acceleration scale factor $\qquad k_a := 25 \cdot \dfrac{in}{sec^2}$

$A_{A4} := 1.581 \cdot k_a$ $\qquad A_{A4} = 39.5\ in \cdot sec^{-2}$ $\qquad$ at an angle of -69.81 deg

$A_{A4t} := 1.444 \cdot k_a$ $\qquad A_{A4t} = 36.1\ in \cdot sec^{-2}$

The angular acceleration of link 4 is

$\alpha_4 := \dfrac{A_{A4t}}{c}$ $\qquad \alpha_4 = 17.5 \dfrac{rad}{sec^2}$ $\qquad$ CW

14. The graphical solution for accelerations in pin-jointed fourbars uses equation 7.4:
$(\mathbf{A}_P^{\,t} + \mathbf{A}_P^{\,n}) = (\mathbf{A}_A^{\,t} + \mathbf{A}_A^{\,n}) + (\mathbf{A}_{PA}^{\,t} + \mathbf{A}_{PA}^{\,n})$

15. For point C, this becomes: $\mathbf{A}_C = (\mathbf{A}_B^{\,t} + \mathbf{A}_B^{\,n}) + (\mathbf{A}_{CB}^{\,t} + \mathbf{A}_{CB}^{\,n})$, where

$A_{Bn} := e \cdot \omega_4^{\ 2}$ $\qquad A_{Bn} = 37.056\ in \cdot sec^{-2}$

$\theta_{ABn} := \theta_4 + 180 \cdot deg$ $\qquad \theta_{ABn} = 224.228\ deg$

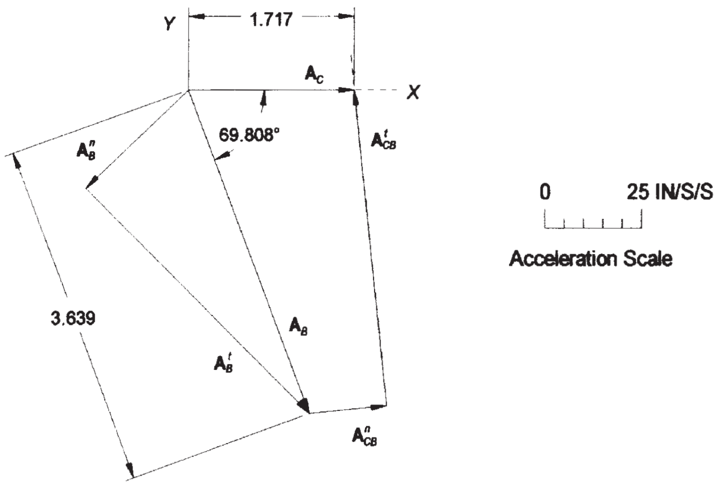
$A_{Bt} := e \cdot \alpha_4$ $\qquad A_{Bt} = 83.09\ in \cdot sec^{-2}$

$\theta_{ABt} := \theta_4 - 90 \cdot deg \qquad \theta_{ABt} = -45.772\,deg$

$A_{CBn} := f \cdot \omega_5^{\,2} \qquad A_{CBn} = 20.108\,in \cdot sec^{-2}$

$\theta_{ACBn} := \theta_5 \qquad \theta_{ACBn} = 5.805\,deg$

16. Repeat procedure of step 12 for the equation in step 15.



17. From the acceleration polygon above,

Acceleration scale factor $\qquad k_a := 25 \cdot \dfrac{in}{sec^2}$

$A_B := 3.639 \cdot k_a \qquad A_B = 91.0\,in \cdot sec^{-2}$ at an angle of -69.81 deg

$A_C := 1.717 \cdot k_a \qquad A_C = 42.9\,in \cdot sec^{-2}$ at an angle of 0 deg

PROBLEM 7-54

Statement: Figure P7-23 shows a quick-return mechanism with dimensions. Use an analytical method to calculate the accelerations of points *A*, *B*, and *C* for one revolution of the input link.

Given: Link lengths and angles:

Link 1 (O_2O_4)	$d := 1.69 \cdot in$	Link 4 (L_4)	$a' := 4.76 \cdot in$
Link 2 (L_2)	$a := 1.00 \cdot in$	Link 5 (L_5)	$b' := 4.55 \cdot in$

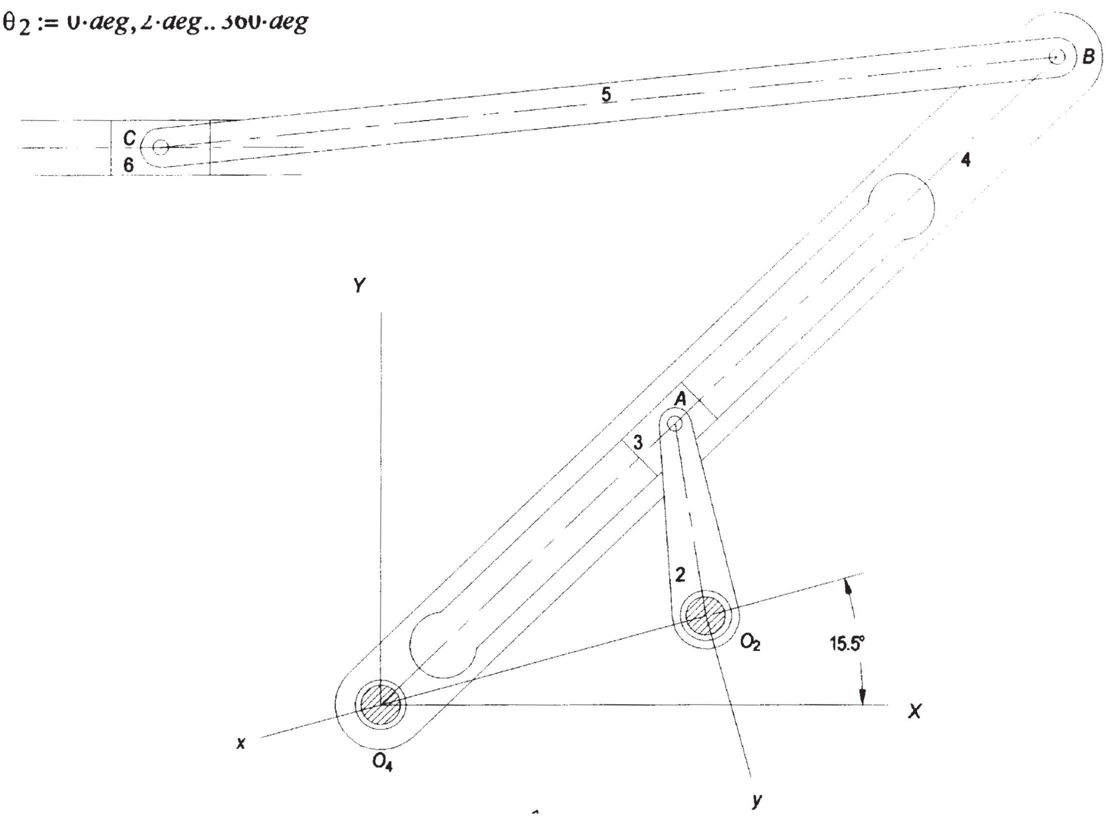
Input crank motion $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

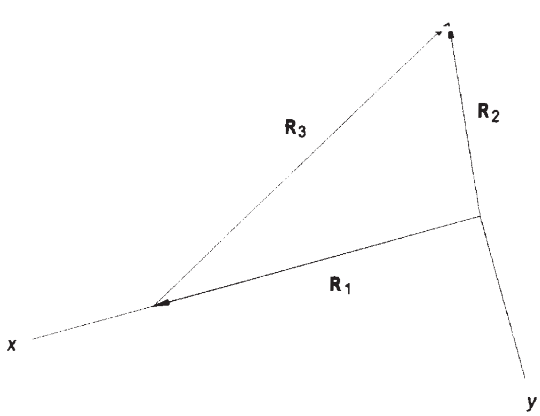
Coordinate rotation angle $\beta := -164.5 \cdot deg$ Offset (AE) $c' := 2.86 \cdot in$

Solution: See Figure P7-23 and Mathcad file P0754.

1. Draw the mechanism to scale and define a vector loop for the input portion (links 1, 2, 3, and 4) using the fourbar slider-crank derivation in Section 7.3 as a model.

$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$





2. Write the vector loop equation, expand the result and separate into real and imaginary parts to solve for θ_3 and b (distance from O_4 to A).

$$\mathbf{R_2} := \mathbf{R_1} + \mathbf{R_3}$$

$$a \cdot e^{j \cdot \theta_2} := d + b \cdot e^{j \cdot \theta_3}$$

Substituting the Euler equivalents,

$$a \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2)) := d + b(\cos(\theta_3) + j \cdot \sin(\theta_3))$$

Separating into real and imaginary components and solving for θ_3 and b.

$$\theta_3(\theta_2) := atan2(a \cdot \cos(\theta_2) - d, a \cdot \sin(\theta_2))$$

$$b(\theta_2) := \frac{a \cdot \sin(\theta_2)}{\sin(\theta_3(\theta_2))}$$

3. Differentiate the position equation, expand it and solve for ω_3 and *bdot*.

$$a \cdot j \cdot \omega_2 \cdot e^{j \cdot \theta_2} := b \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + bdot \cdot e^{j \cdot \theta_3}$$

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b(\theta_2)} \cdot \cos(\theta_2 - \theta_3(\theta_2))$$

$$bdot(\theta_2) := \frac{a \cdot \omega_2 \cdot \cos(\theta_2) - b(\theta_2) \cdot \omega_3(\theta_2) \cdot \cos(\theta_3(\theta_2))}{\sin(\theta_3(\theta_2))}$$

4. Differentiate the velocity equation, expand it and solve for α_3.

$$a \cdot j \cdot \alpha_2 \cdot e^{j \cdot \theta_2} + a \cdot j^2 \cdot \omega_2{}^2 \cdot e^{j \cdot \theta_2} := bdot \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3} + b \cdot j \cdot \alpha_3 \cdot e^{j \cdot \theta_3} \ldots$$
$$+ b \cdot j^2 \cdot \omega_3{}^2 \cdot e^{j \cdot \theta_3} + bddot \cdot e^{j \cdot \theta_3} + bdot \cdot j \cdot \omega_3 \cdot e^{j \cdot \theta_3}$$

$$\alpha_3(\theta_2) := \frac{1}{b(\theta_2)} \cdot \left(a \cdot \alpha_2 \cdot \cos(\theta_2 - \theta_3(\theta_2)) + a \cdot \omega_2{}^2 \cdot \sin(\theta_3(\theta_2) - \theta_2) - 2 \cdot bdot(\theta_2) \cdot \omega_3(\theta_2)\right)$$

5. Determine the acceleration of point A on link 2 (in the global XY coordinate system) using equations 7.13a.

$$\mathbf{A_{A2}}(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2{}^2 \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{A2}(\theta_2) := |\mathbf{A_{A2}}(\theta_2)| \qquad \theta_{AA2}(\theta_2) := arg(\mathbf{A_{A2}}(\theta_2)) + \beta$$

6. Determine the acceleration of points A and B on link 4 (in the Global XY coordinate system) using equations 7.13a.

Transform and rename θ_3 to the global coordinate frame: $\theta_4(\theta_2) := \theta_3(\theta_2) + \beta$

Rename α_3 to α_4: $\alpha_4(\theta_2) := \alpha_3(\theta_2)$

$$\omega_4(\theta_2) := \omega_3(\theta_2)$$

$$\mathbf{A_{A4}}(\theta_2) := b(\theta_2)\cdot\alpha_4(\theta_2)\cdot\left(-\sin(\theta_4(\theta_2)) + j\cdot\cos(\theta_4(\theta_2))\right) \ldots$$
$$+ -b(\theta_2)\cdot\omega_4(\theta_2)^2\cdot\left(\cos(\theta_4(\theta_2)) + j\cdot\sin(\theta_4(\theta_2))\right)$$

$$A_{A4}(\theta_2) := \left|\mathbf{A_{A4}}(\theta_2)\right| \qquad \theta_{AA4}(\theta_2) := \arg(\mathbf{A_{A4}}(\theta_2))$$

$$\mathbf{A_B}(\theta_2) := a'\cdot\alpha_4(\theta_2)\cdot\left(-\sin(\theta_4(\theta_2)) + j\cdot\cos(\theta_4(\theta_2))\right) \ldots$$
$$+ -a'\cdot\omega_4(\theta_2)^2\cdot\left(\cos(\theta_4(\theta_2)) + j\cdot\sin(\theta_4(\theta_2))\right)$$

$$A_B(\theta_2) := \left|\mathbf{A_B}(\theta_2)\right| \qquad \theta_{AB}(\theta_2) := \arg(\mathbf{A_B}(\theta_2))$$

7. Determine θ_5 using equation 4.16.

$$\theta_5(\theta_2) := \operatorname{asin}\left(\frac{a'\cdot\sin(\theta_4(\theta_2)) - c'}{b'}\right)$$

8. Determine the angular velocity of link 5 using equation 6.22a:

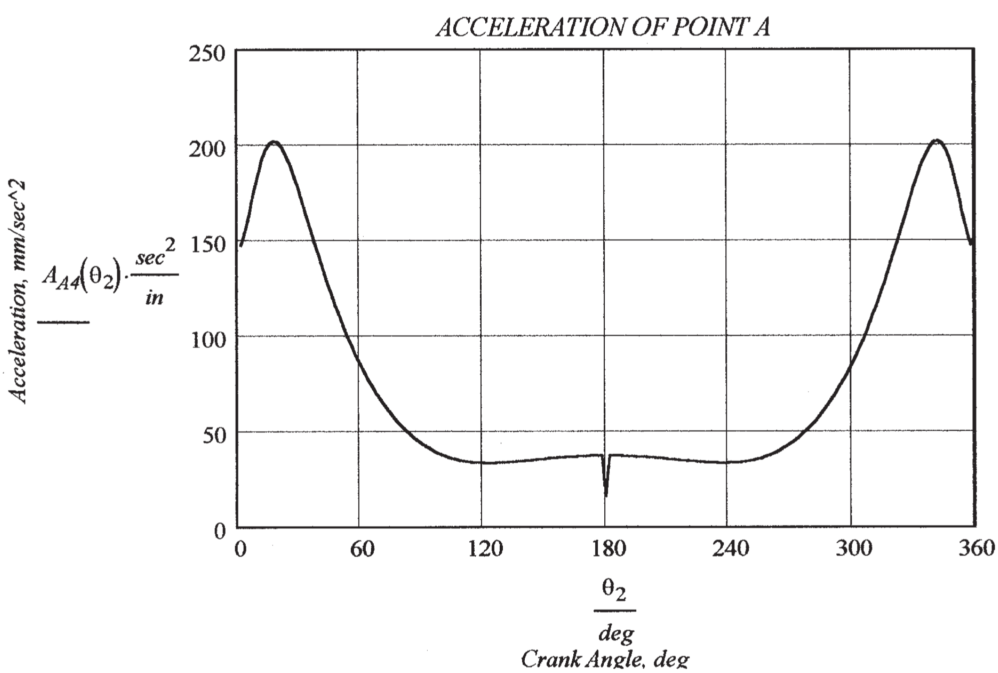
$$\omega_5(\theta_2) := \frac{a'}{b'}\cdot\frac{\cos(\theta_4(\theta_2))}{\cos(\theta_5(\theta_2))}\cdot\omega_4(\theta_2)$$

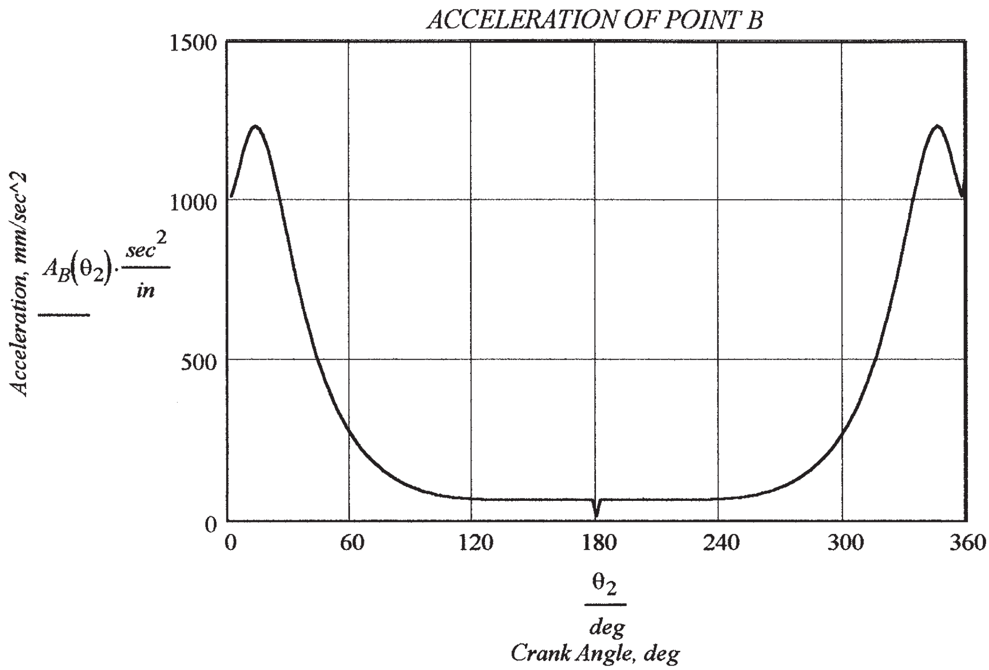
9. Determine the angular acceleration of link 5 using equation 7.16d.

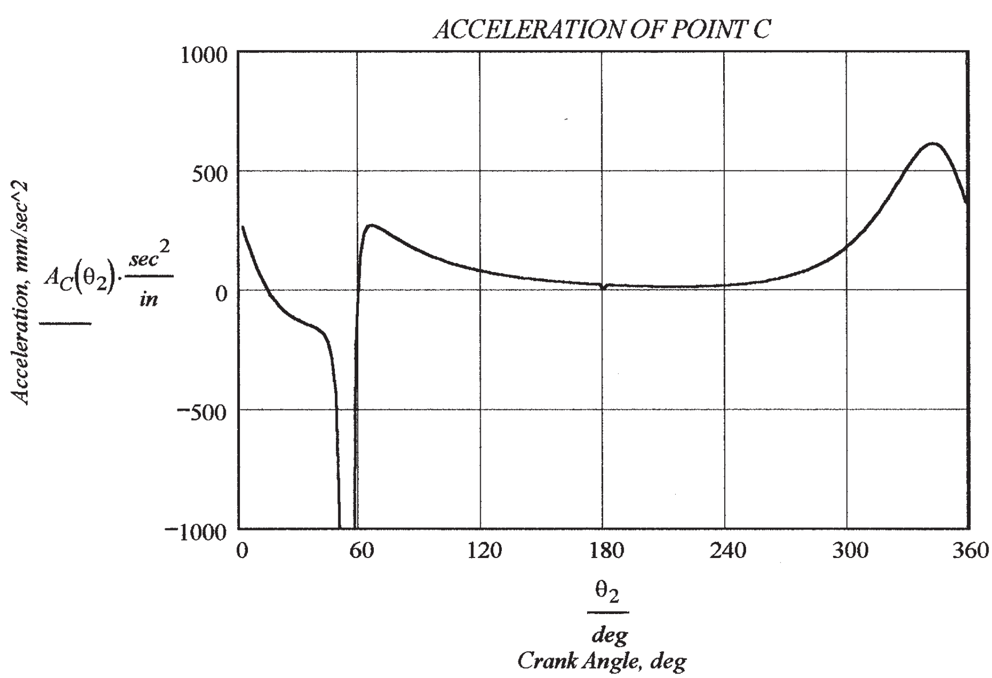
$$\alpha_5(\theta_2) := \frac{a'\cdot\alpha_4(\theta_2)\cdot\cos(\theta_4(\theta_2)) - a'\cdot\omega_4(\theta_2)^2\cdot\sin(\theta_4(\theta_2)) + b'\cdot\omega_5(\theta_2)^2\cdot\sin(\theta_5(\theta_2))}{b'\cdot\cos(\theta_5(\theta_2))}$$

10. Use equation 7.16e for the acceleration of pin C.

$$A_C(\theta_2) := -a'\cdot\alpha_4(\theta_2)\cdot\sin(\theta_4(\theta_2)) - a'\cdot\omega_4(\theta_2)^2\cdot\cos(\theta_4(\theta_2)) \ldots$$
$$+ b'\cdot\alpha_5(\theta_2)\cdot\sin(\theta_5(\theta_2)) + b'\cdot\omega_5(\theta_2)^2\cdot\cos(\theta_5(\theta_2))$$




ACCELERATION OF POINT B
Acceleration, mm/sec^2
$A_B(\theta_2)\cdot\frac{sec^2}{in}$
1500
1000
500
0
0
60
120
180
240
300
360
$\frac{\theta_2}{deg}$
Crank Angle, deg


ACCELERATION OF POINT C
Acceleration, mm/sec^2
$A_C(\theta_2)\cdot\frac{sec^2}{in}$
1000
500
0
−500
−1000
0
60
120
180
240
300
360
$\frac{\theta_2}{deg}$
Crank Angle, deg

PROBLEM 7-55

Statement: Figure P7-24 shows a drum pedal mechanism . For the dimensions given below, find and plot the output acceleration of the linkage over its range of motion if the input velocity V_{in} is a constant.

Given: Link lengths:

Link 2 (O_2A)	$a := 100 \cdot mm$	Link 3 (AB)	$b := 28 \cdot mm$
Link 4 (O_4B)	$c := 64 \cdot mm$	Link 1 (O_2O_4)	$d := 56 \cdot mm$

Distance to force application:

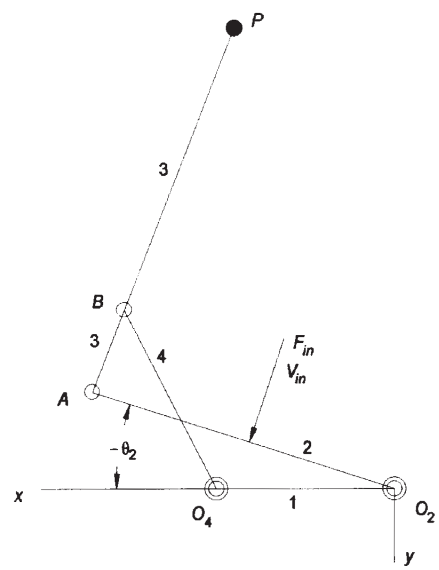
Link 2	$r_{in} := 48 \cdot mm$	Link 3 (AP)	$r_{out} := 124 \cdot mm$

Input velocity: $V_{in} := 3 \cdot m \cdot sec^{-1}$ $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$ $\quad \delta_3 := 0 \cdot deg$

Range of positions of link 2: $\theta_{20} := 162 \cdot deg$ $\quad \theta_{21} := 171 \cdot deg$

Solution: See Figure P7-24 and Mathcad file P0755.

1. Draw the mechanism to scale and label it.



2. Calculate the range of θ_2 in the local coordinate system (required to calculate θ_3 and θ_4).

Rotation angle of local xy system to global XY system: $\alpha := 180 \cdot deg$

$$\theta_2 := \theta_{20} - \alpha, (\theta_{20} - \alpha) + 1 \cdot deg .. \theta_{21} - \alpha$$

3. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.5600 \qquad K_2 = 0.8750$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.2850$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

4. Use equation 4.10b to find value of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2\cdot\left[atan2\left[2\cdot A(\theta_2), -B(\theta_2) - \sqrt{(B(\theta_2))^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right]\right]$$

5. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 2.0000 \qquad K_5 = -1.7543$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

6. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2\cdot\left[atan2\left[2\cdot D(\theta_2), -E(\theta_2) - \sqrt{(E(\theta_2))^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right]\right]$$

7. Determine the angular velocity of link 2 using equation 6.7 and links 3 and 4 using equations 6.18.

$$\omega_2 := \frac{V_{in}}{r_{in}} \qquad \omega_2 = 62.500\,\frac{rad}{sec}$$

$$\omega_3(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4(\theta_2) - \theta_2)}{sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Determine the acceleration of point A using equation 7.13a.

$$\mathbf{A_A}(\theta_2) := a\cdot\alpha_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) - a\cdot\omega_2^{\,2}\cdot(cos(\theta_2) + j\cdot sin(\theta_2))$$

9. Use equations 7.12 to determine the angular acceleration of link 3.

$$A(\theta_2) := c\cdot sin(\theta_4(\theta_2)) \qquad B(\theta_2) := b\cdot sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c\cdot cos(\theta_4(\theta_2)) \qquad E(\theta_2) := b\cdot cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot cos(\theta_2) + b\cdot\omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) - b\cdot\omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)^2\cdot sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C(\theta_2)\cdot D(\theta_2) - A(\theta_2)\cdot F(\theta_2)}{A(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot D(\theta_2)}$$

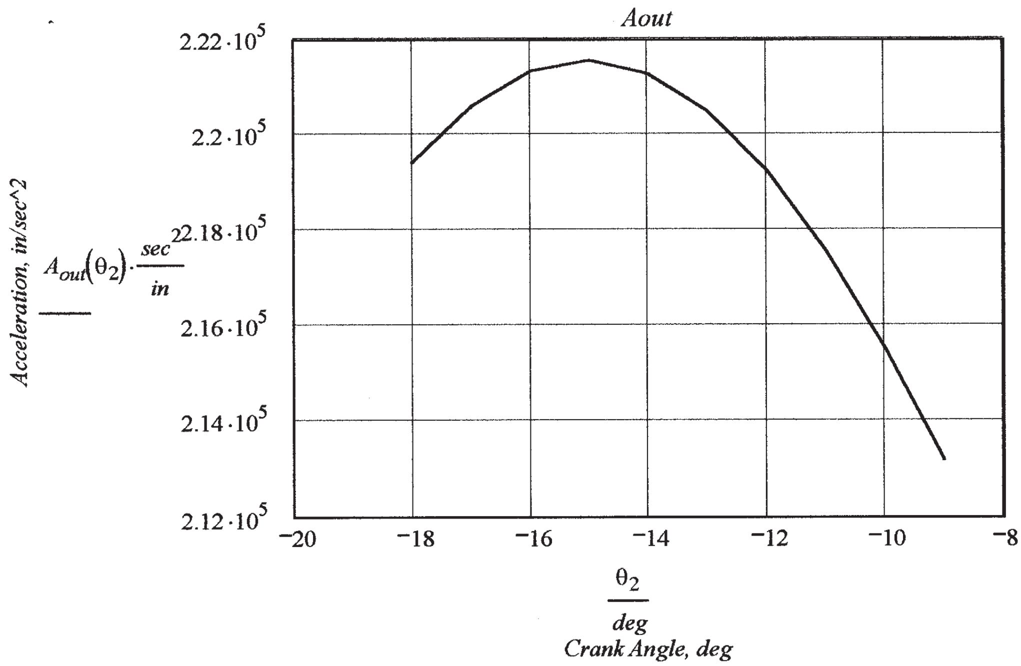
10. Determine the acceleration of the coupler point P using equations 7.32.

$$\mathbf{A_{PA}}(\theta_2) := r_{out}\cdot\alpha_3(\theta_2)\cdot(-sin(\theta_3(\theta_2)+\delta_3)+j\cdot cos(\theta_3(\theta_2)+\delta_3)) \ldots$$
$$+ -r_{out}\cdot\omega_3(\theta_2)^2\cdot(cos(\theta_3(\theta_2)+\delta_3)+j\cdot sin(\theta_3(\theta_2)+\delta_3))$$

$$\mathbf{A_P}(\theta_2) := \mathbf{A_A}(\theta_2) + \mathbf{A_{PA}}(\theta_2)$$

11. Plot the magnitude of the acceleration of the coupler point P.

$$A_{out}(\theta_2) := |\mathbf{A_P}(\theta_2)|$$



Note the acceleration scale values. If the scale went to zero, A_{out} would be a horizontal (constant) line over the range of θ_2 plotted.

PROBLEM 7-56

Statement: A tractor-trailer tipped over while negotiating an on-ramp to the New York Thruway. The road has a 50-ft radius at that point and tilts 3 deg toward the outside of the curve. The 45-ft-long by 8-ft-wide by 8.5-ft-high trailer box (13 ft from ground to top) was loaded with 44 415 lb of paper rolls in two rows by two high as shown in Figure P7-25. The rolls are 40-in-dia by 38-in-long and weigh about 900 lb each. They are wedged against rolling backward but not against sliding sidewards. The empty trailer weighed 14 000 lb. The driver claims that he was traveling at less than 15 mph and that the load of paper shifted inside the trailer, struck the trailer sidewall, and tipped the truck. The paper company that loaded the truck claims the load was properly stowed and would not shift at that speed. Independent test of the coefficient of friction between similar paper rolls and a similar trailer floor give a value of 0.43 +/- 0.08. The composite center of gravity of the loaded trailer is estimated to be 7.5 ft above the road. Determine the truck speed that would cause the truck to just begin to tip and the speed at which the rolls will just begin to slide sidways. What do you think caused the accident?

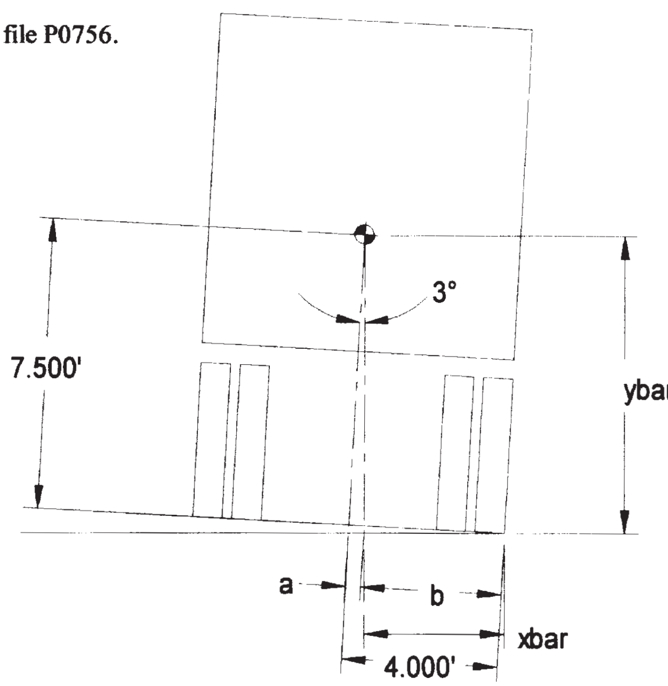
Given:

Weight of paper	$W_p := 44415 \cdot lbf$
Weight of trailer	$W_t := 14000 \cdot lbf$
Radius of curve	$r := 50 \cdot ft$
Nominal coefficient of friction	$\mu_{nom} := 0.43$
Coefficient of friction uncertainty	$u_\mu := 0.08$
Trailer width	$w := 8 \cdot ft$
Height of CG from pavement	$h := 7.5 \cdot ft$

Assumptions:
1. The paper rolls act as a monolith since they are tightly strapped together with steel bands.
2. The tractor has about 15 deg of potential roll freedom versus the trailer due to their relative angle in plan during the turn combined with the substantial pitch freedom in the fifth wheel. So the trailer can tip independently of the tractor.
3. The outside track width of the trailer tires is equal to the width of the trailer.

Solution: See Figure P7-25 and Mathcad file P0756.

1. First, calculate the location of the trailer's CG with respect to the outside wheel when it is on the reverse-banked curve. From the figure at right,



Tilt angle $\theta := 3 \cdot deg$

$a := h \cdot tan(\theta)$ $\quad a = 0.393\, ft$

$b := \dfrac{w}{2} - a$ $\quad b = 3.607\, ft$

$xbar := b \cdot cos(\theta)$ $\quad xbar = 3.602\, ft$

$$ybar := b \cdot sin(\theta) + \frac{h}{cos(\theta)}$$

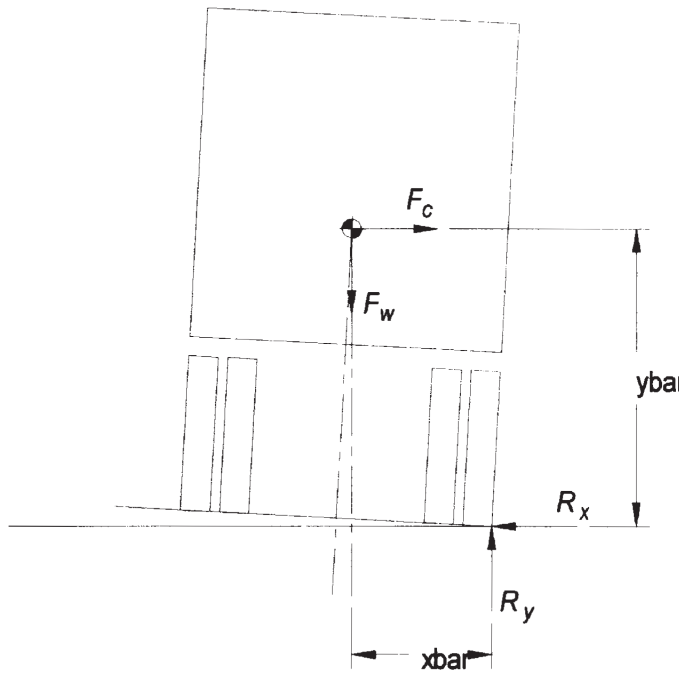
$ybar = 7.699\, ft$

The coordinates of the CG of the loaded trailer with respect to the lower outside corner of the tires are:

$xbar = 3.602\, ft$ $\quad ybar = 7.699\, ft$

2. The trailer is on the verge of tipping over when the couple due to centrifugal force is equal to the couple formed by the weight of the loaded trailer acting through its CG and the vertical reaction at the outside edge of the tires. At this instant, it is assumed that the entire weight of the trailer is reacted at the outside tires with the inside tires carrying none of the weight. Any increase in tangential velocity of the tractor and trailer will result in tipping. Summing moments about the tire edge (see figure below),

$$\Sigma M \qquad F_w \cdot xbar - F_c \cdot ybar = 0 \qquad (1)$$



where F_c is the centrifugal force due to the normal acceleration as the tractor and trailer go through the curve. The normal acceleration is

$$a_{tip} = \frac{v_{tip}^2}{r} \qquad (2)$$

and the force necessary to keep the tractor trailer following a circular path is

$$F_c = m_{tot} \cdot a_{tip} \qquad (3)$$

where m_{tot} is the total mass of the trailer and its payload. Combining equations (2) and (3) and solving for v_{tip}, we have

$$v_{tip} = \sqrt{\frac{F_c \cdot r}{m_{tot}}} \qquad (4)$$

or,

$$v_{tip} = \sqrt{\frac{F_c \cdot r \cdot g}{F_w}} \qquad (5)$$

3. Calculate the minimum tipping velocity of the tractor/trailer. From equations (1) and (5),

Total weight	$F_w := W_t + W_p$	$F_w = 58415\, lbf$
Centrifugal force required to tip the trailer	$F_c := \frac{xbar}{ybar} \cdot F_w$	$F_c = 27329\, lbf$
Minimum tipping speed	$v_{tip} := \sqrt{\frac{F_c \cdot r \cdot g}{F_w}}$	$v_{tip} = 18.7\, mph$

Thus, with the assumptions that we have made, the trailer would not begin to tip over until it reached a speed of

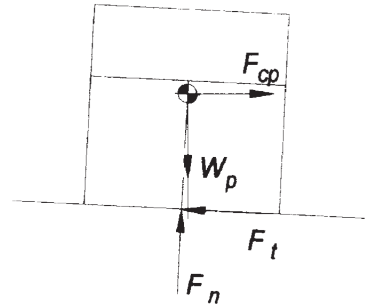
$$v_{tip} = 18.7\, mph$$

4. The load will slip when the friction force between the paper rolls and the trailer floor is no longer sufficient to react the centrifugal force on the paper rolls. Looking at the FBD of the paper rolls below, we see that

Normal force between paper and floor $\qquad F_n = W_p \cdot cos(\theta) - F_{cp} \cdot sin(\theta) \qquad (6)$

Tangential force tending to slide the paper

$$F_t = W_p \cdot sin(\theta) + F_{cp} \cdot cos(\theta) \qquad (7)$$



Centrifugal force on the paper

$$F_{cp} = \frac{W_p}{g} \cdot a_s = \frac{W_p}{g} \cdot \frac{v_s^2}{r} \qquad (8)$$

But, the maximum friction force is $F_f = \mu \cdot F_n = F_t$ (9)

Substituting equation (9) into (7), then combining (6) and (7) to eliminate F_n, and solving for F_{cp} yields

$$F_{cp} = \frac{W_p \cdot (\mu \cdot cos(\theta) - sin(\theta))}{\mu \cdot sin(\theta) + cos(\theta)} \qquad (10)$$

Substituting equation (10) into (8), to eliminate F_{cp}, and solving for v_s yields

$$v_s = \sqrt{\left[\frac{(\mu \cdot cos(\theta) - sin(\theta))}{\mu \cdot sin(\theta) + cos(\theta)}\right] \cdot r \cdot g} \qquad (11)$$

5. Use the upper and lower limit on the coefficient of friction to determine an upper and lower limit on the speed necessary to cause sliding.

Maximum coefficient $\mu_{max} := \mu_{nom} + u_\mu$ $\mu_{max} = 0.510$

Minimum coefficient $\mu_{min} := \mu_{nom} - u_\mu$ $\mu_{min} = 0.350$

Maximum velocity to cause sliding

$$v_{smax} := \sqrt{\left[\frac{(\mu_{max} \cdot cos(\theta) - sin(\theta))}{\mu_{max} \cdot sin(\theta) + cos(\theta)}\right] \cdot r \cdot g} \qquad v_{smax} = 18.3\, mph$$

Minimum velocity to cause sliding

$$v_{smin} := \sqrt{\left[\frac{(\mu_{min} \cdot cos(\theta) - sin(\theta))}{\mu_{min} \cdot sin(\theta) + cos(\theta)}\right] \cdot r \cdot g} \qquad v_{smin} = 14.8\, mph$$

6. This very rough analysis shows that, if the coefficient of friction was at or near the low end of its measured value, the paper load could slide at a tractor/trailer speed of 15 mph, which would lead to the trailer tipping over. In any case, it appears that the paper load would slide before the truck would tip with the load in place.

PROBLEM 7-57

Statement: Figure P7-26 shows a V-belt drive. The sheaves (pulleys) have pitch diameters of 150 and 300 mm, respectively. The smaller sheave is driven at a constant 1750 rpm. For a cross-sectional, differential element of the belt, write the equations of its acceleration for one complete trip around both pulleys including its travel between the pulleys. Compute and plot the acceleration of the differential element versus time for one circuit around the belt path. What does your analysis tell you about the dynamic behavior of the belt?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

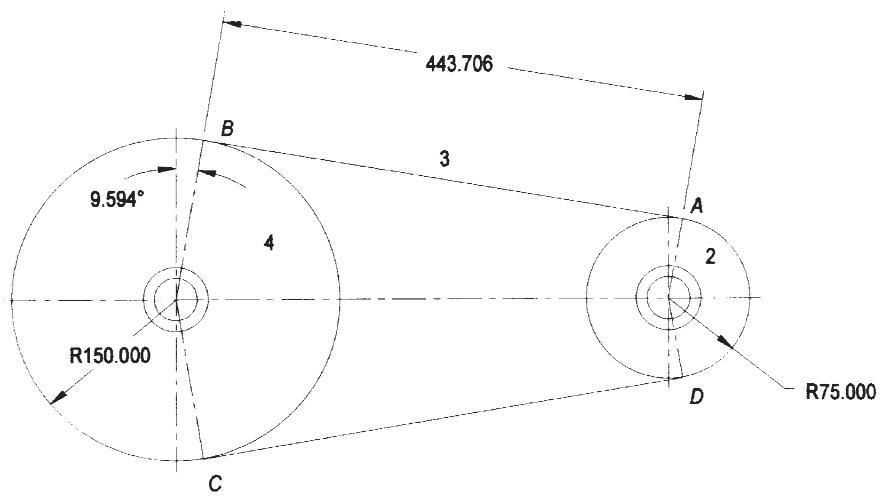
Given: Sheave radii and speed:

$r_2 := 75 \cdot mm \qquad r_4 := 150 \cdot mm \qquad \omega_2 := 1750 \cdot rpm$

Assumptions: Center distance: $C := 450 mm$

Solution: See Figure P7-26 and Mathcad file P0757.

1. Draw a schematic representation of the V-belt drive to scale and label it.



From the layout,

Angle to point of tangency	$\beta := 9.594 \cdot deg$	
Distance from A to B	$L_{AB} := 443.706 \cdot mm$	
Distance from B to C	$L_{BC} := (\pi + 2 \cdot \beta) \cdot r_4$	$L_{BC} = 521.473\,mm$
Distance from C to D	$L_{CD} := 443.706 \cdot mm$	
Distance from D to A	$L_{DA} := (\pi - 2 \cdot \beta) \cdot r_2$	$L_{DA} = 210.502\,mm$
Total path length	$L := L_{AB} + L_{BC} + L_{CD} + L_{DA}$	$L = 1619.387\,mm$

2. Calculate the angular velocity of each sheave.

Sheave 2 $\qquad \omega_2 = 183.260 \dfrac{rad}{sec}$

Sheave 4 $\qquad \omega_4 := \dfrac{r_2}{r_4} \cdot \omega_2 \qquad \omega_4 = 91.630 \dfrac{rad}{sec}$

3. Calculate the acceleration of an element on the belt for each portion of the path, starting at A.

From A to B the acceleration is zero since the belt velocity is constant $\quad A_{AB} := 0 \cdot \frac{mm}{sec^2}$

From B to C the tangential acceleration is zero since the belt velocity is constant The normal component is

$$A_{BC} := r_4 \cdot \omega_4^{\,2} \qquad A_{BC} = 1.259 \times 10^6 \, \frac{mm}{sec^2}$$

From C to D the acceleration is zero since the belt velocity is constant $\quad A_{CD} := 0 \cdot \frac{mm}{sec^2}$

From D to A the tangential acceleration is zero since the belt velocity is constant The normal component is

$$A_{DA} := r_2 \cdot \omega_2^{\,2} \qquad A_{DA} = 2.519 \times 10^6 \, \frac{mm}{sec^2}$$

4. Plot the acceleration over the entire path using a path variable of s.

$$s := 0 \cdot mm, 5 \cdot mm .. L$$

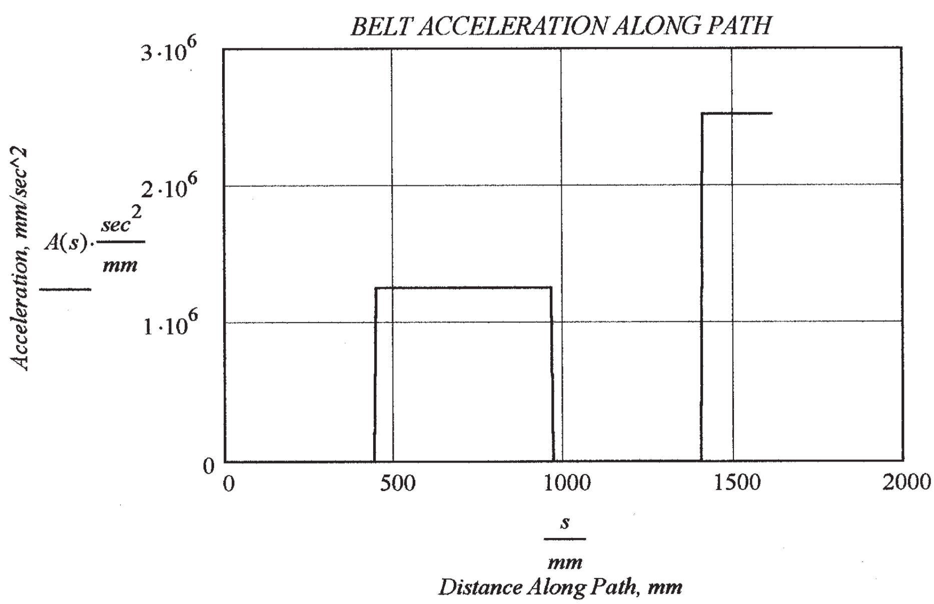
Define a range function that will plot a variable only between two limits using the Heaviside step function Φ.

$$R(s,a,b) := \Phi(s-a) - \Phi(s-b)$$

The acceleration function for the entire path is.

Let $\quad s_1 := L_{AB} \qquad s_2 := L_{AB} + L_{BC} \qquad s_3 := L_{AB} + L_{BC} + L_{CD}$

$$A(s) := A_{AB} \cdot R(s, 0 \cdot mm, s_1) + A_{BC} \cdot R(s, s_1, s_2) + A_{CD} \cdot R(s, s_2, s_3) + A_{DA} \cdot R(s, s_3, L)$$



5. The graph shows the sudden change in acceleration as the belt enters and leaves a sheave. This results in infinite jerk at these points. This results in belt "hop" at these points, a condition that will cause eventual fatigue failure and wear in the belt. Because of the belt hop caused by infinite jerk at the initial belt/sheave tangency points the span of the belt is continuously vibrating.

PROBLEM 7-58

Statement: Write a program using an equation solver or any computer language to solve for the displacements, velocities, and accelerations in an offset slider-crank linkage as shown in Figure P7-2. Plot the variation in all link's angular and pin's linear positions, velocities, and accelerations with a constant angular velocity input to the crank over one revolution for both open and crossed configurations of the linkage. To test the program, use data from row *a* of Table P7-2. Check your results with program SLIDER.

Enter: Link lengths:

Link 2 $a := 1.4 \cdot in$ Link 3 $b := 4 \cdot in$

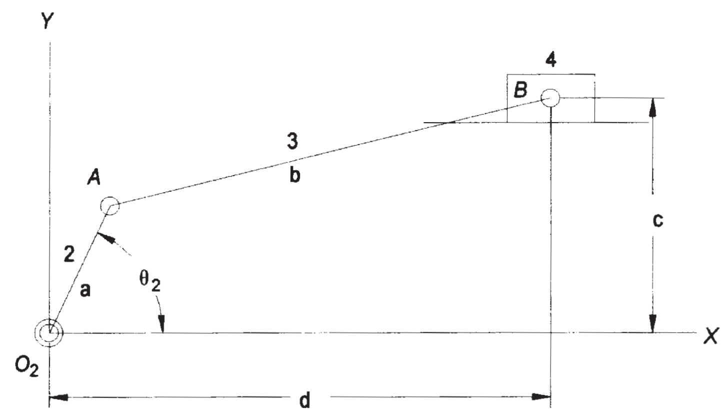
Offset: $c := 1 \cdot in$

Link 2 velocity and acceleration: $\omega_2 := 10 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Crank position range: $\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$

Solution: See Figure P7-2 and Mathcad file P0758.

1. Draw the linkage and label it.



2. Determine θ_3 and d using equations 4.16 and 4.17.

Open:
$$\theta_{32}(\theta_2) := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi$$

$$d_2(\theta_2) := a \cdot cos(\theta_2) - b \cdot cos(\theta_{32}(\theta_2))$$

Crossed:
$$\theta_{31}(\theta_2) := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right)$$

$$d_1(\theta_2) := a \cdot cos(\theta_2) - b \cdot cos(\theta_{31}(\theta_2))$$

3. Determine the angular velocity of link 3 using equation 6.22a.

Open
$$\omega_{32}(\theta_2) := \frac{a}{b} \cdot \frac{cos(\theta_2)}{cos(\theta_{32}(\theta_2))} \cdot \omega_2$$

Crossed $$\omega_{31}(\theta_2) := \frac{a}{b} \cdot \frac{\cos(\theta_2)}{\cos(\theta_{31}(\theta_2))} \cdot \omega_2$$

4. Determine the velocity of pin *A* using equation 6.23a:

$$\mathbf{V_A}(\theta_2) := a \cdot \omega_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2))$$

$$V_{AX}(\theta_2) := Re(\mathbf{V_A}(\theta_2)) \qquad V_{AY}(\theta_2) := Im(\mathbf{V_A}(\theta_2))$$

5. Determine the velocity of pin *B* using equation 6.22b:

Open $$V_{B1}(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{31}(\theta_2) \cdot \sin(\theta_{31}(\theta_2))$$

Crossed $$V_{B2}(\theta_2) := -a \cdot \omega_2 \cdot \sin(\theta_2) + b \cdot \omega_{32}(\theta_2) \cdot \sin(\theta_{32}(\theta_2))$$

6. Using the Euler identity to expand equation 7.15b for $\mathbf{A}_A$, determine its *x* and *y* components.

$$\mathbf{A_A}(\theta_2) := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^{2} \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$A_{AX}(\theta_2) := Re(\mathbf{A_A}(\theta_2)) \qquad A_{AY}(\theta_2) := Im(\mathbf{A_A}(\theta_2))$$

7. Determine the angular acceleration of link 3 using equation 7.16d.

Open $$\alpha_{32}(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{2} \cdot \sin(\theta_2) + b \cdot \omega_{32}(\theta_2)^{2} \cdot \sin(\theta_{32}(\theta_2))}{b \cdot \cos(\theta_{32}(\theta_2))}$$

Crossed $$\alpha_{31}(\theta_2) := \frac{a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{2} \cdot \sin(\theta_2) + b \cdot \omega_{31}(\theta_2)^{2} \cdot \sin(\theta_{31}(\theta_2))}{b \cdot \cos(\theta_{31}(\theta_2))}$$

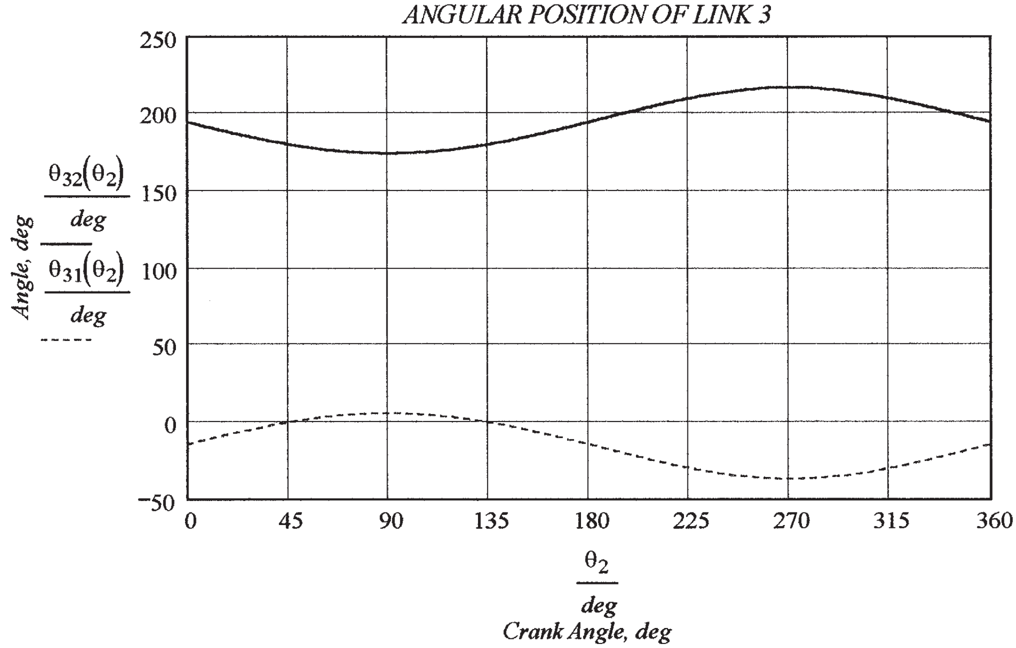
8. Use equation 7.16e for the acceleration of pin *B*.

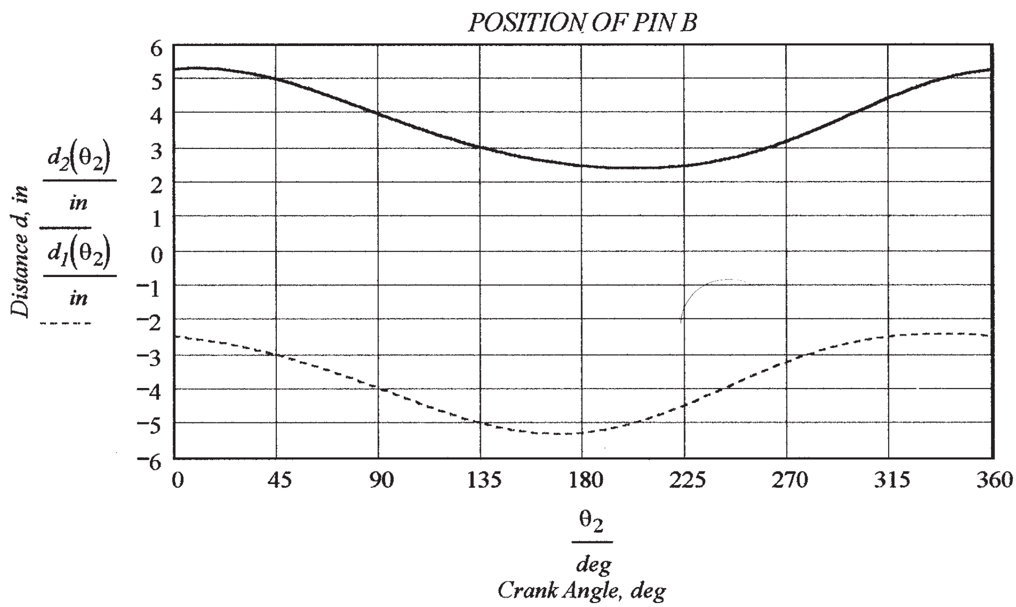
Open: $$A_{B2}(\theta_2) := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^{2} \cdot \cos(\theta_2) \ldots \\ + b \cdot \alpha_{32}(\theta_2) \cdot \sin(\theta_{32}(\theta_2)) + b \cdot \omega_{32}(\theta_2)^{2} \cdot \cos(\theta_{32}(\theta_2))$$

Crossed: $$A_{B1}(\theta_2) := -a \cdot \alpha_2 \cdot \sin(\theta_2) - a \cdot \omega_2^{2} \cdot \cos(\theta_2) \ldots \\ + b \cdot \alpha_{31}(\theta_2) \cdot \sin(\theta_{31}(\theta_2)) + b \cdot \omega_{31}(\theta_2)^{2} \cdot \cos(\theta_{31}(\theta_2))$$

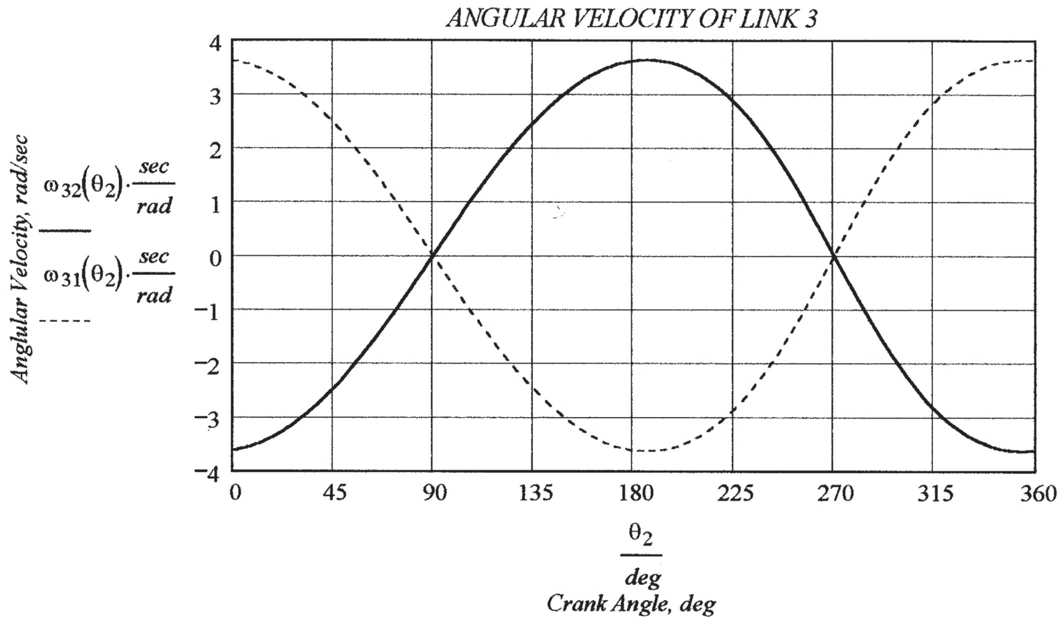
(See plots on the following pages.)

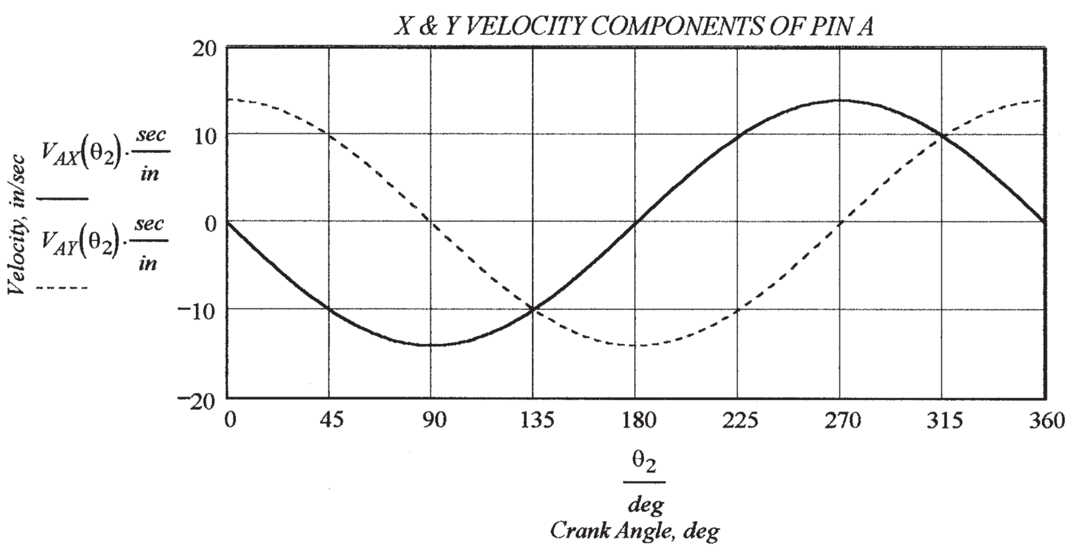
9. Plot the angular position of link 3 and the distance of pin *B* from the *y* axis for open (solid) and crossed (dashed) configurations.

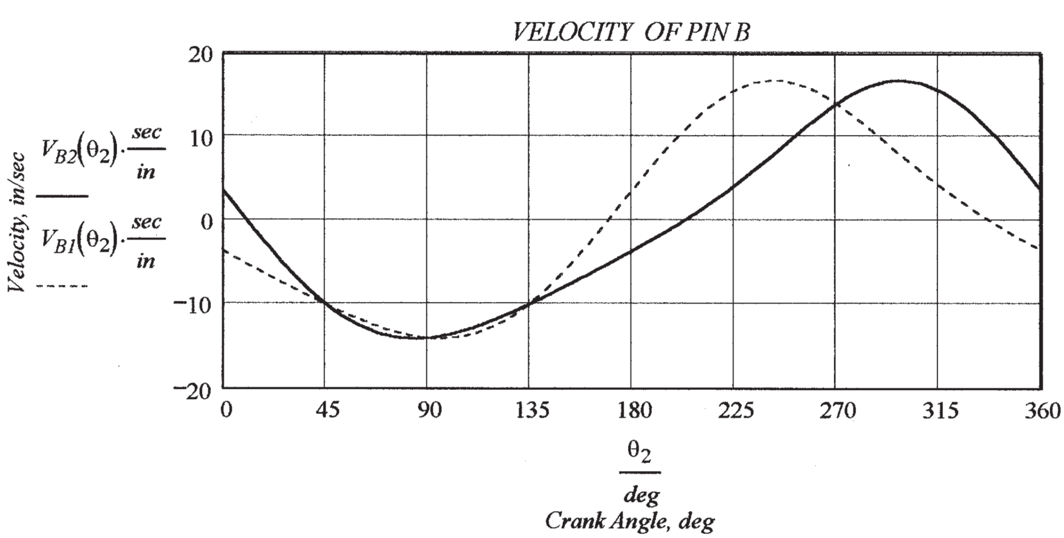




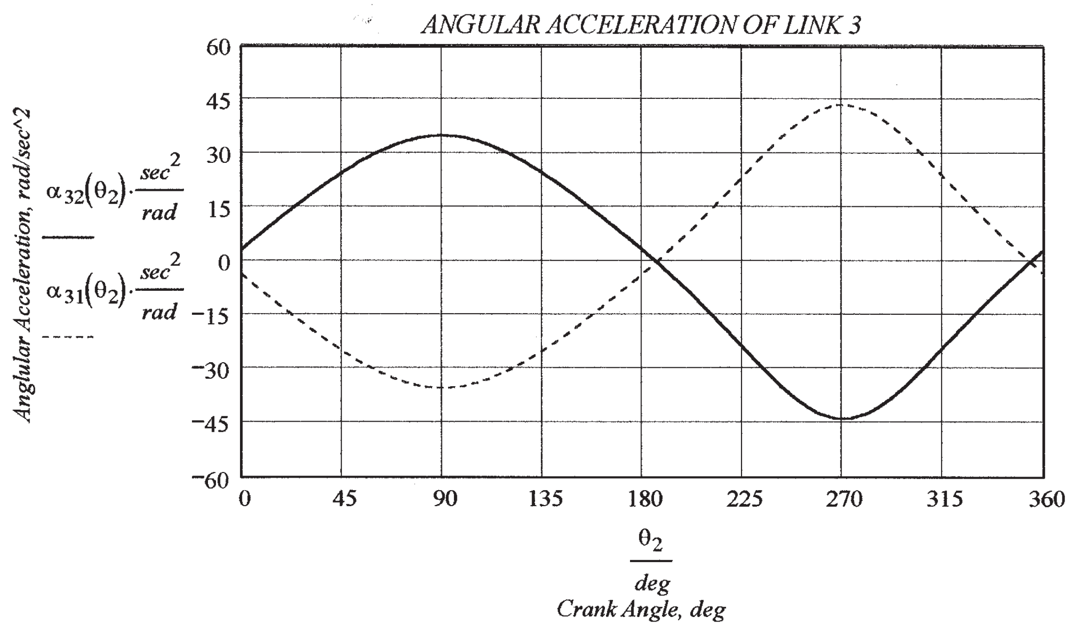
10. Plot the angular velocity of link 3 and the velocities of pins A and B for open (solid) and crossed (dashed) configurations.

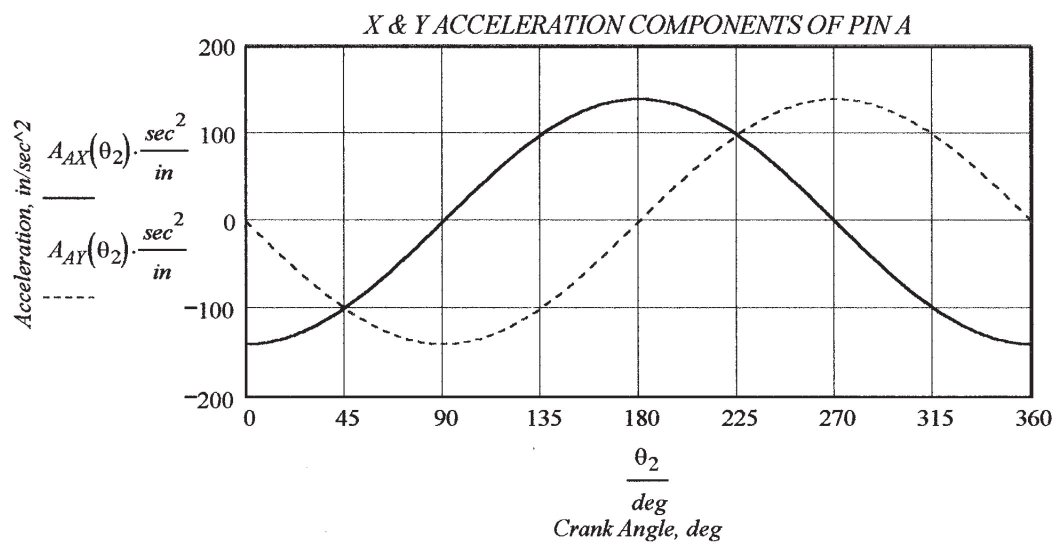


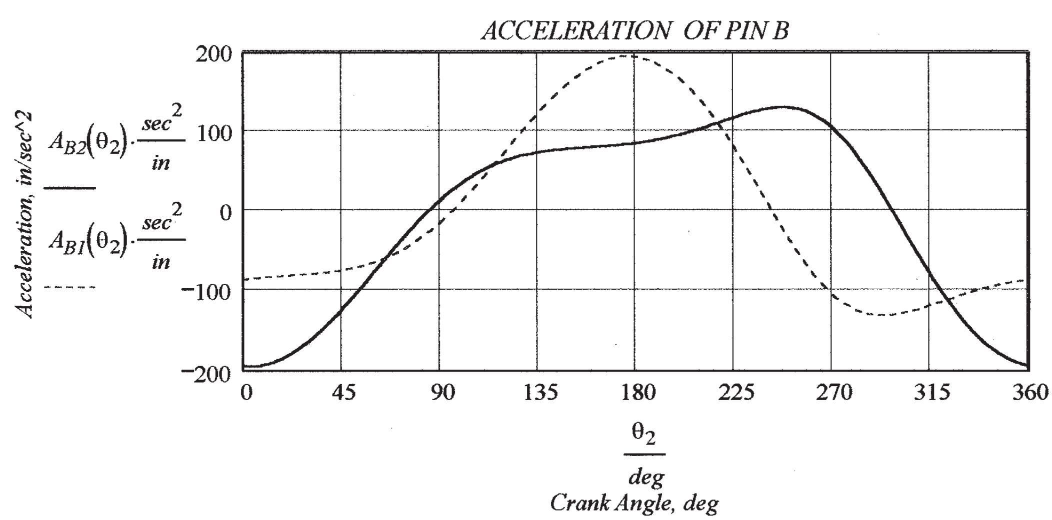




11. Plot the angular acceleration of link 3 and the accelerations of pins A and B for open (solid) and crossed (dashed) configurations.







PROBLEM 7-59

Statement: Write a program using an equation solver or any computer language to solve for the displacements, velocities, and accelerations in an inverted slider-crank linkage as shown in Figure P7-3. Plot the variation in all link's angular and pin's linear positions, velocities, and accelerations with a constant angular velocity input to the crank over one revolution for both open and crossed configurations of the linkage. To test the program, use data from row *e* of Table P7-3 except for the value of $\alpha 2$, which will be set to zero for this exercise.

Enter: Link lengths:

Link 2 $a := 4 \cdot in$ Link 4 $c := 2 \cdot in$ Link 1 $d := 8 \cdot in$

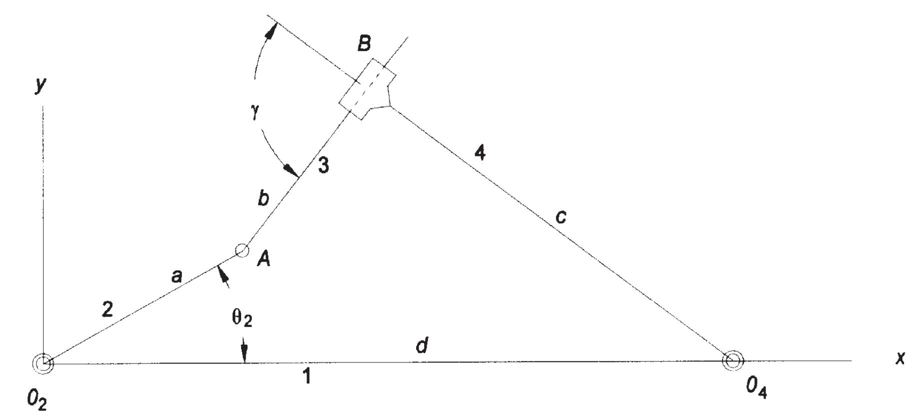
Angle between links 3 and 4 $\gamma := 30 \cdot deg$

Link 2 velocity and accel. $\omega_2 := -45 \cdot \frac{rad}{sec}$ $\alpha_2 := 0 \cdot \frac{rad}{sec^2}$

Crank position range: $\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$

Solution: See Figure P7-3 and Mathcad file P0759.

1. Draw the linkage and label it.



2. Use the equations in Section 4.7 to solve for the positions of links 3 and 4 and for the length b.

$$P(\theta_2) := a \cdot sin(\theta_2) \cdot sin(\gamma) + (a \cdot cos(\theta_2) - d) \cdot cos(\gamma)$$

$$Q(\theta_2) := -a \cdot sin(\theta_2) \cdot cos(\gamma) + (a \cdot cos(\theta_2) - d) \cdot sin(\gamma)$$

$$R := -c \cdot sin(\gamma) \qquad S(\theta_2) := R - Q(\theta_2) \qquad T(\theta_2) := 2 \cdot P(\theta_2) \qquad U(\theta_2) := Q(\theta_2) + R$$

Open: $$\theta_{41}(\theta_2) := 2 \cdot atan2\left[(2 \cdot S(\theta_2)), \left(-T(\theta_2) + \sqrt{T(\theta_2)^2 - 4 \cdot S(\theta_2) \cdot U(\theta_2)}\right)\right]$$

$$\theta_{31}(\theta_2) := \theta_{41}(\theta_2) + \gamma$$

$$b_1(\theta_2) := \frac{a \cdot sin(\theta_2) - c \cdot sin(\theta_{41}(\theta_2))}{sin(\theta_{31}(\theta_2))}$$

Crossed: $\theta_{42}(\theta_2) := 2 \cdot atan2\left[(2 \cdot S(\theta_2)), -T(\theta_2) - \sqrt{T(\theta_2)^2 - 4 \cdot S(\theta_2) \cdot U(\theta_2)}\right]$

$$\theta_{32}(\theta_2) := \theta_{42}(\theta_2) - \gamma$$

$$b_2(\theta_2) := \frac{a \cdot sin(\theta_2) - c \cdot sin(\theta_{42}(\theta_2))}{sin(\theta_{32}(\theta_2))}$$

3. Calculate the angular velocity of links 3 and 4 and the slip velocity using equations 6.30.

Open: $\omega_{41}(\theta_2) := \dfrac{a \cdot \omega_2 \cdot cos(\theta_2 - \theta_{31}(\theta_2))}{(b_1(\theta_2) + c \cdot cos(\gamma))}$

$$bdot_1(\theta_2) := \frac{-a \cdot \omega_2 \cdot sin(\theta_2) + \omega_{41}(\theta_2) \cdot (b_1(\theta_2) \cdot sin(\theta_{31}(\theta_2)) + c \cdot sin(\theta_{41}(\theta_2)))}{cos(\theta_{31}(\theta_2))}$$

$$\omega_{31}(\theta_2) := \omega_{41}(\theta_2)$$

Crossed: $\omega_{42}(\theta_2) := \dfrac{a \cdot \omega_2 \cdot cos(\theta_2 - \theta_{32}(\theta_2))}{(b_2(\theta_2) + c \cdot cos(\gamma))}$

$$bdot_2(\theta_2) := \frac{-a \cdot \omega_2 \cdot sin(\theta_2) + \omega_{42}(\theta_2) \cdot (b_2(\theta_2) \cdot sin(\theta_{32}(\theta_2)) + c \cdot sin(\theta_{42}(\theta_2)))}{cos(\theta_{32}(\theta_2))}$$

$$\omega_{32}(\theta_2) := \omega_{42}(\theta_2)$$

4. Solve for the accelerations using equations (7.26) and (7.27).

Open:

$$P_1(\theta_2) := a \cdot \alpha_2 \cdot cos(\theta_{31}(\theta_2) - \theta_2) \qquad Q_1(\theta_2) := a \cdot \omega_2{}^2 \cdot sin(\theta_{31}(\theta_2) - \theta_2)$$

$$R_1(\theta_2) := c \cdot \omega_{41}(\theta_2)^2 \cdot sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2)) \qquad S_1(\theta_2) := 2 \cdot bdot_1(\theta_2) \cdot \omega_{31}(\theta_2)$$

$$T_1(\theta_2) := b_1(\theta_2) + c \cdot cos(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))$$

$$\alpha_{41}(\theta_2) := \frac{P_1(\theta_2) + Q_1(\theta_2) + R_1(\theta_2) - S_1(\theta_2)}{T_1(\theta_2)}$$

$$K_1(\theta_2) := a \cdot \omega_2{}^2 \cdot (b_1(\theta_2) \cdot cos(\theta_{31}(\theta_2) - \theta_2) + c \cdot cos(\theta_{41}(\theta_2) - \theta_2))$$

$$L_1(\theta_2) := a \cdot \alpha_2 \cdot (b_1(\theta_2) \cdot sin(\theta_2 - \theta_{31}(\theta_2)) - c \cdot sin(\theta_{41}(\theta_2) + \theta_2))$$

$$M_1(\theta_2) := -\omega_{41}(\theta_2)^2 \cdot \left(b_1(\theta_2)^2 + c^2 + 2 \cdot b_1(\theta_2) \cdot c \cdot cos(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))\right)$$

$$N_1(\theta_2) := 2 \cdot bdot_1(\theta_2) \cdot c \cdot \omega_{41}(\theta_2) \cdot sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))$$

$$bddot_1(\theta_2) := -\frac{K_1(\theta_2) + L_1(\theta_2) + M_1(\theta_2) + N_1(\theta_2)}{T_1(\theta_2)}$$

$$\mathbf{A_A}(\theta_2) := a\cdot\alpha_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) - a\cdot\omega_2{}^2\cdot(cos(\theta_2) + j\cdot sin(\theta_2))$$

$$A_{AX}(\theta_2) := Re(\mathbf{A_A}(\theta_2)) \qquad A_{AY}(\theta_2) := Im(\mathbf{A_A}(\theta_2))$$

$$\mathbf{A_{B1}}(\theta_2) := -c\cdot\alpha_{41}(\theta_2)\cdot(sin(\theta_{41}(\theta_2)) - j\cdot cos(\theta_{41}(\theta_2))) \ldots$$
$$+ -c\cdot\omega_{41}(\theta_2)^2\cdot(cos(\theta_{41}(\theta_2)) + j\cdot sin(\theta_{41}(\theta_2)))$$

$$A_{B1}(\theta_2) := |\mathbf{A_{B1}}(\theta_2)| \qquad \theta_{AB1}(\theta_2) := arg(\mathbf{A_{B1}}(\theta_2))$$

Crossed:

$$P_2(\theta_2) := a\cdot\alpha_2\cdot cos(\theta_{32}(\theta_2) - \theta_2) \qquad Q_2(\theta_2) := a\cdot\omega_2{}^2\cdot sin(\theta_{32}(\theta_2) - \theta_2)$$

$$R_2(\theta_2) := c\cdot\omega_{42}(\theta_2)^2\cdot sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2)) \qquad S_2(\theta_2) := 2\cdot bdot_2(\theta_2)\cdot\omega_{32}(\theta_2)$$

$$T_2(\theta_2) := b_2(\theta_2) + c\cdot cos(\theta_{32}(\theta_2) - \theta_{42}(\theta_2))$$

$$\alpha_{42}(\theta_2) := \frac{P_2(\theta_2) + Q_2(\theta_2) + R_2(\theta_2) - S_2(\theta_2)}{T_2(\theta_2)}$$

$$K_2(\theta_2) := a\cdot\omega_2{}^2\cdot(b_2(\theta_2)\cdot cos(\theta_{32}(\theta_2) - \theta_2) + c\cdot cos(\theta_{42}(\theta_2) - \theta_2))$$

$$L_2(\theta_2) := a\cdot\alpha_2\cdot(b_2(\theta_2)\cdot sin(\theta_2 - \theta_{32}(\theta_2)) - c\cdot sin(\theta_{42}(\theta_2) + \theta_2))$$

$$M_2(\theta_2) := -\omega_{42}(\theta_2)^2\cdot\left(b_2(\theta_2)^2 + c^2 + 2\cdot b_2(\theta_2)\cdot c\cdot cos(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))\right)$$

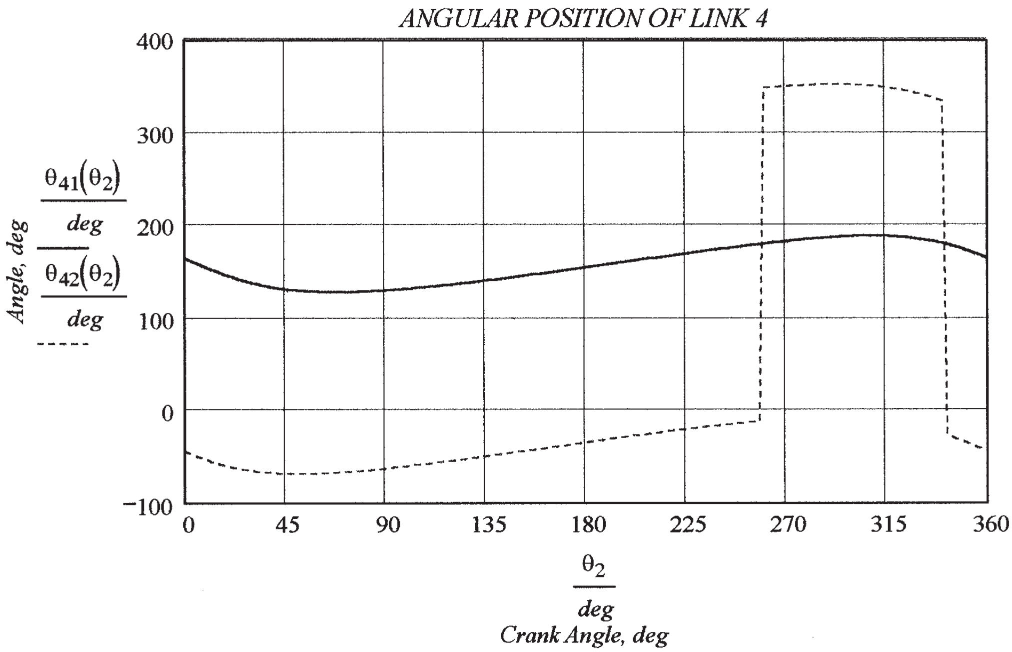
$$N_2(\theta_2) := 2\cdot bdot_2(\theta_2)\cdot c\cdot\omega_{42}(\theta_2)\cdot sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))$$

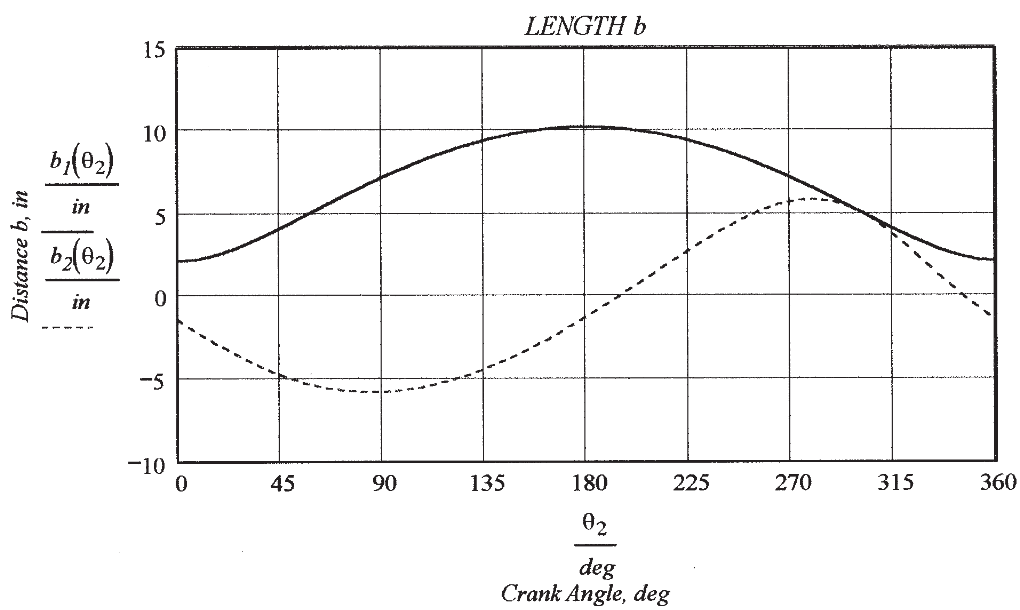
$$bddot_2(\theta_2) := -\frac{K_2(\theta_2) + L_2(\theta_2) + M_2(\theta_2) + N_2(\theta_2)}{T_2(\theta_2)}$$

$$\mathbf{A_{B2}}(\theta_2) := -c\cdot\alpha_{42}(\theta_2)\cdot(sin(\theta_{42}(\theta_2)) - j\cdot cos(\theta_{42}(\theta_2))) \ldots$$
$$+ -c\cdot\omega_{42}(\theta_2)^2\cdot(cos(\theta_{42}(\theta_2)) + j\cdot sin(\theta_{42}(\theta_2)))$$

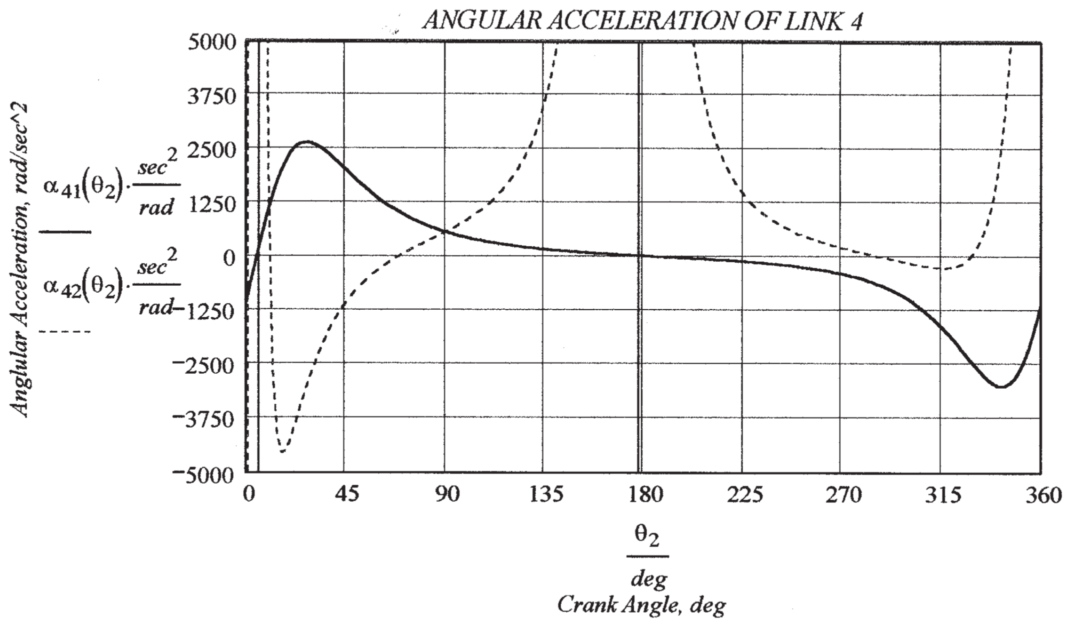
$$A_{B2}(\theta_2) := |\mathbf{A_{B2}}(\theta_2)| \qquad \theta_{AB2}(\theta_2) := arg(\mathbf{A_{B2}}(\theta_2))$$

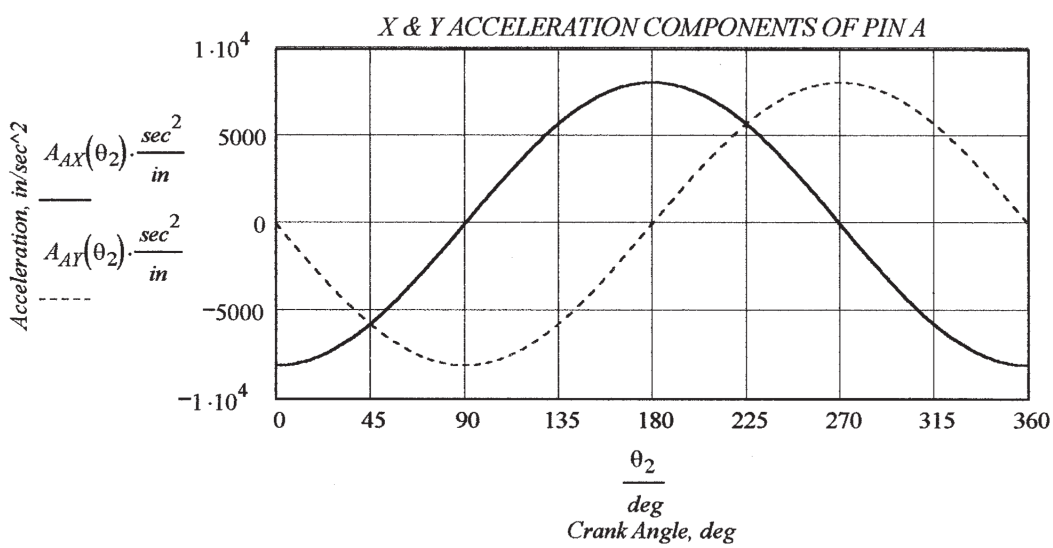
5. Plot the angular position of link 4 and the distance b for open (solid) and crossed (dashed) configurations.

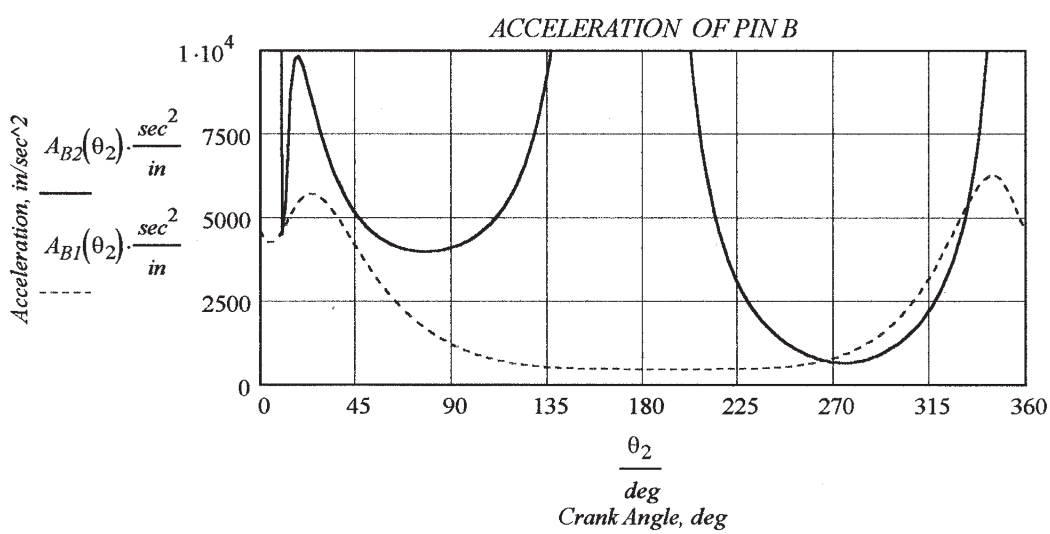




6. Plot the angular acceleration of link 4 and the accelerations of points A and B for open (solid) and crossed (dashed) configurations.







PROBLEM 7-60

Statement: Write a program using an equation solver or any computer language to solve for the displacements, velocities, and accelerations in a geared fivebar linkage as shown in Figure P7-4. Plot the variation in all link's angular and pin's linear positions, velocities, and accelerations with a constant angular velocity input to the crank over one revolution for both open and crossed configurations of the linkage. To test the program, use data from row *a* of Table P7-4. Check your results with program FIVEBAR.

Enter: Link lengths:

Link 1 $f := 6 \cdot in$ Link 3 $b := 7 \cdot in$ Link 5 $d := 4 \cdot in$

Link 2 $a := 1 \cdot in$ Link 4 $c := 9 \cdot in$

Gear ratio, phase angle, and crank velocity and acceleration:

$\lambda := 2$ $\phi := 30 \cdot deg$ $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Coupler data: $R_{pa} := 6 \cdot in$ $\delta_3 := 30 \cdot deg$

Solution: See Figure P7-4 and Mathcad file P0760.

1. Determine the angle of link 5 using the equation in Figure P6-4.

$$\theta_5(\theta_2) := \lambda \cdot \theta_2 + \phi$$

2. Determine the values of the constants needed for finding θ_3 and θ_4 from equations 4.24h and 4.24i.

$$A(\theta_2) := 2 \cdot c \cdot \left(d \cdot cos(\lambda \cdot \theta_2 + \phi) - a \cdot cos(\theta_2) + f\right)$$

$$B(\theta_2) := 2 \cdot c \cdot \left(d \cdot sin(\lambda \cdot \theta_2 + \phi) - a \cdot sin(\theta_2)\right)$$

$$\begin{aligned} C(\theta_2) := & \left(a^2 - b^2 + c^2 + d^2 + f^2\right) - 2 \cdot a \cdot f \cdot cos(\theta_2) \ldots \\ & + -\left[2 \cdot d \cdot \left(a \cdot cos(\theta_2) - f\right) \cdot cos(\lambda \cdot \theta_2 + \phi)\right] \ldots \\ & + -2 \cdot a \cdot d \cdot sin(\theta_2) \cdot sin(\lambda \cdot \theta_2 + \phi) \end{aligned}$$

$$D(\theta_2) := C(\theta_2) - A(\theta_2) \qquad E(\theta_2) := 2 \cdot B(\theta_2) \qquad F(\theta_2) := A(\theta_2) + C(\theta_2)$$

$$G(\theta_2) := 2 \cdot b \cdot \left[-\left(d \cdot cos(\lambda \cdot \theta_2 + \phi)\right) + a \cdot cos(\theta_2) - f\right]$$

$$H(\theta_2) := 2 \cdot b \cdot \left[-\left(d \cdot sin(\lambda \cdot \theta_2 + \phi)\right) + a \cdot sin(\theta_2)\right]$$

$$\begin{aligned} K(\theta_2) := & \left(a^2 + b^2 - c^2 + d^2 + f^2\right) - 2 \cdot a \cdot f \cdot cos(\theta_2) \ldots \\ & + -\left[2 \cdot d \cdot \left(a \cdot cos(\theta_2) - f\right) \cdot cos(\lambda \cdot \theta_2 + \phi)\right] \ldots \\ & + -2 \cdot a \cdot d \cdot sin(\theta_2) \cdot sin(\lambda \cdot \theta_2 + \phi) \end{aligned}$$

$$L(\theta_2) := K(\theta_2) - G(\theta_2) \qquad M(\theta_2) := 2 \cdot H(\theta_2) \qquad N(\theta_2) := G(\theta_2) + K(\theta_2)$$

3. Use equations 4.24h and 4.24i to find values of θ_3 and θ_4 for the open and crossed circuits.

OPEN $$\theta_{31}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot L(\theta_2), -M(\theta_2) + \sqrt{M(\theta_2)^2 - 4 \cdot L(\theta_2) \cdot N(\theta_2)}\right)\right)$$

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)}\right)\right)$$

CROSSED $\theta_{32}(\theta_2) := 2\cdot\left(atan2\left(2\cdot L(\theta_2), -M(\theta_2) - \sqrt{M(\theta_2)^2 - 4\cdot L(\theta_2)\cdot N(\theta_2)}\right)\right)$

$$\theta_{42}(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

4. Use equation 6.32c to find ω_5. $\omega_5 := \lambda\cdot\omega_2$

5. Calculate ω_3 and ω_4 and the pin velocities using equations 6.33.

OPEN

$$\omega_{31}(\theta_2) := \frac{-\left[2\cdot sin(\theta_{41}(\theta_2))\cdot\left(a\cdot\omega_2\cdot sin(\theta_2 - \theta_{41}(\theta_2)) + d\cdot\omega_5\cdot sin(\theta_{41}(\theta_2) - \theta_5(\theta_2))\right)\right]}{b\cdot\left(cos(\theta_{31}(\theta_2) - 2\cdot\theta_{41}(\theta_2)) - cos(\theta_{31}(\theta_2))\right)}$$

$$\omega_{41}(\theta_2) := \frac{a\cdot\omega_2\cdot sin(\theta_2) + b\cdot\omega_{31}(\theta_2)\cdot sin(\theta_{31}(\theta_2)) - d\cdot\omega_5\cdot sin(\theta_5(\theta_2))}{c\cdot sin(\theta_{41}(\theta_2))}$$

$$\mathbf{V_A}(\theta_2) := a\cdot\omega_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2))$$

$V_{Ax}(\theta_2) := Re(\mathbf{V_A}(\theta_2))$ $\qquad$ $V_{Ay}(\theta_2) := Im(\mathbf{V_A}(\theta_2))$

$$\mathbf{V_{BA1}}(\theta_2) := b\cdot\omega_{31}(\theta_2)\cdot(-sin(\theta_{31}(\theta_2)) + j\cdot cos(\theta_{31}(\theta_2)))$$

$$\mathbf{V_C}(\theta_2) := d\cdot\omega_5\cdot(-sin(\theta_5(\theta_2)) + j\cdot cos(\theta_5(\theta_2)))$$

$$\mathbf{V_{B1}}(\theta_2) := \mathbf{V_A}(\theta_2) + \mathbf{V_{BA1}}(\theta_2)$$

$V_{B1x}(\theta_2) := Re(\mathbf{V_{B1}}(\theta_2))$ $\qquad$ $V_{B1y}(\theta_2) := Im(\mathbf{V_{B1}}(\theta_2))$

CROSSED

$$\omega_{32}(\theta_2) := \frac{-\left[2\cdot sin(\theta_{42}(\theta_2))\cdot\left(a\cdot\omega_2\cdot sin(\theta_2 - \theta_{42}(\theta_2)) + d\cdot\omega_5\cdot sin(\theta_{42}(\theta_2) - \theta_5(\theta_2))\right)\right]}{b\cdot\left(cos(\theta_{32}(\theta_2) - 2\cdot\theta_{42}(\theta_2)) - cos(\theta_{32}(\theta_2))\right)}$$

$$\omega_{42}(\theta_2) := \frac{a\cdot\omega_2\cdot sin(\theta_2) + b\cdot\omega_{32}(\theta_2)\cdot sin(\theta_{32}(\theta_2)) - d\cdot\omega_5\cdot sin(\theta_5(\theta_2))}{c\cdot sin(\theta_{42}(\theta_2))}$$

$$\mathbf{V_{BA2}}(\theta_2) := b\cdot\omega_{32}(\theta_2)\cdot(-sin(\theta_{32}(\theta_2)) + j\cdot cos(\theta_{32}(\theta_2)))$$

$$\mathbf{V_{B2}}(\theta_2) := \mathbf{V_A}(\theta_2) + \mathbf{V_{BA2}}(\theta_2)$$

$V_{B2x}(\theta_2) := Re(\mathbf{V_{B2}}(\theta_2))$ $\qquad$ $V_{B2y}(\theta_2) := Im(\mathbf{V_{B2}}(\theta_2))$

6. Use equation 7.28c to find α_5. $\alpha_5 := \lambda\cdot\alpha_2$

7. Calculate α_3 and α_4 and the pin accelerations using equations 7.29.

OPEN

$$\alpha_{31}(\theta_2) := \frac{\begin{pmatrix} -a\cdot\alpha_2\cdot sin(\theta_2 - \theta_{41}(\theta_2)) - a\cdot\omega_2{}^2\cdot cos(\theta_2 - \theta_{41}(\theta_2)) \ldots \\ + -b\cdot\omega_{31}(\theta_2)^2\cdot cos(\theta_{31}(\theta_2) - \theta_{41}(\theta_2)) + d\cdot\omega_5{}^2\cdot cos(\theta_5(\theta_2) - \theta_{41}(\theta_2)) \ldots \\ + d\cdot\alpha_5\cdot sin(\theta_5(\theta_2) - \theta_{41}(\theta_2)) + c\cdot\omega_{41}(\theta_2)^2 \end{pmatrix}}{b\cdot sin(\theta_{31}(\theta_2) - \theta_{41}(\theta_2))}$$

$$\alpha_{41}(\theta_2) := \frac{\begin{pmatrix} a\cdot\alpha_2\cdot sin(\theta_2 - \theta_{31}(\theta_2)) + a\cdot\omega_2{}^2\cdot cos(\theta_2 - \theta_{31}(\theta_2)) \ldots \\ + -c\cdot\omega_{41}(\theta_2)^2\cdot cos(\theta_{31}(\theta_2) - \theta_{41}(\theta_2)) - d\cdot\omega_5{}^2\cdot cos(\theta_{31}(\theta_2) - \theta_5(\theta_2)) \ldots \\ + d\cdot\alpha_5\cdot sin(\theta_{31}(\theta_2) - \theta_5(\theta_2)) + b\cdot\omega_{31}(\theta_2)^2 \end{pmatrix}}{c\cdot sin(\theta_{41}(\theta_2) - \theta_{31}(\theta_2))}$$

$$\mathbf{A_A}(\theta_2) := a\cdot\alpha_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) - a\cdot\omega_2{}^2\cdot(cos(\theta_2) + j\cdot sin(\theta_2))$$

$$A_{Ax}(\theta_2) := Re(\mathbf{A_A}(\theta_2)) \qquad A_{Ay}(\theta_2) := Im(\mathbf{A_A}(\theta_2))$$

$$\mathbf{A_{BA1}}(\theta_2) := b\cdot\alpha_{31}(\theta_2)\cdot(-sin(\theta_{31}(\theta_2)) + j\cdot cos(\theta_{31}(\theta_2))) \ldots \\ + -b\cdot\omega_{31}(\theta_2)^2\cdot(cos(\theta_{31}(\theta_2)) + j\cdot sin(\theta_{31}(\theta_2)))$$

$$\mathbf{A_{B1}}(\theta_2) := \mathbf{A_A}(\theta_2) + \mathbf{A_{BA1}}(\theta_2)$$

$$A_{B1x}(\theta_2) := Re(\mathbf{A_{B1}}(\theta_2)) \qquad A_{B1y}(\theta_2) := Im(\mathbf{A_{B1}}(\theta_2))$$

$$\mathbf{A_C}(\theta_2) := c\cdot\alpha_5\cdot(-sin(\theta_5(\theta_2)) + j\cdot cos(\theta_5(\theta_2))) \ldots \\ + -c\cdot\omega_5{}^2\cdot(cos(\theta_5(\theta_2)) + j\cdot sin(\theta_5(\theta_2)))$$

$$A_{Cx}(\theta_2) := Re(\mathbf{A_C}(\theta_2)) \qquad A_{Cy}(\theta_2) := Im(\mathbf{A_C}(\theta_2))$$

CROSSED

$$\alpha_{32}(\theta_2) := \frac{\begin{pmatrix} -a\cdot\alpha_2\cdot sin(\theta_2 - \theta_{42}(\theta_2)) - a\cdot\omega_2{}^2\cdot cos(\theta_2 - \theta_{42}(\theta_2)) \ldots \\ + -b\cdot\omega_{32}(\theta_2)^2\cdot cos(\theta_{32}(\theta_2) - \theta_{42}(\theta_2)) + d\cdot\omega_5{}^2\cdot cos(\theta_5(\theta_2) - \theta_{42}(\theta_2)) \ldots \\ + d\cdot\alpha_5\cdot sin(\theta_5(\theta_2) - \theta_{42}(\theta_2)) + c\cdot\omega_{42}(\theta_2)^2 \end{pmatrix}}{b\cdot sin(\theta_{32}(\theta_2) - \theta_{42}(\theta_2))}$$

$$\alpha_{42}(\theta_2) := \frac{\begin{pmatrix} a\cdot\alpha_2\cdot sin(\theta_2 - \theta_{32}(\theta_2)) + a\cdot\omega_2{}^2\cdot cos(\theta_2 - \theta_{32}(\theta_2)) \ldots \\ + -c\cdot\omega_{42}(\theta_2)^2\cdot cos(\theta_{32}(\theta_2) - \theta_{42}(\theta_2)) - d\cdot\omega_5{}^2\cdot cos(\theta_{32}(\theta_2) - \theta_5(\theta_2)) \ldots \\ + d\cdot\alpha_5\cdot sin(\theta_{32}(\theta_2) - \theta_5(\theta_2)) + b\cdot\omega_{32}(\theta_2)^2 \end{pmatrix}}{c\cdot sin(\theta_{42}(\theta_2) - \theta_{32}(\theta_2))}$$

$$\mathbf{A_{BA2}}(\theta_2) := b\cdot\alpha_{32}(\theta_2)\cdot(-sin(\theta_{32}(\theta_2)) + j\cdot cos(\theta_{32}(\theta_2))) \ldots$$
$$+ -b\cdot\omega_{32}(\theta_2)^2\cdot(cos(\theta_{32}(\theta_2)) + j\cdot sin(\theta_{32}(\theta_2)))$$

$$\mathbf{A_{B2}}(\theta_2) := \mathbf{A_A}(\theta_2) + \mathbf{A_{BA2}}(\theta_2)$$

$$A_{B2x}(\theta_2) := Re(\mathbf{A_{B2}}(\theta_2)) \qquad A_{B2y}(\theta_2) := Im(\mathbf{A_{B2}}(\theta_2))$$

PROBLEM 7-61a

Statement: Find the acceleration of the slider in Figure 3-33 for $\theta_2 = 110$ deg with respect to the global X axis assuming a constant $\omega_2 = 1$ rad/sec CW. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.170 \cdot in$	Link 3 (A to B)	$b := 2.067 \cdot in$
Link 4 (O_4 to B)	$c := 2.310 \cdot in$	Link 1 (O_2 to O_4)	$d := 1.000 \cdot in$
Link 5 (B to C)	$e := 5.400 \cdot in$		

Coordinate angle $\delta := -102 \cdot deg$ Crank angle: $\theta_{2XY} := 110 \cdot deg$

Input crank angular velocity $\omega_2 := -1 \cdot rad \cdot sec^{-1}$ CW

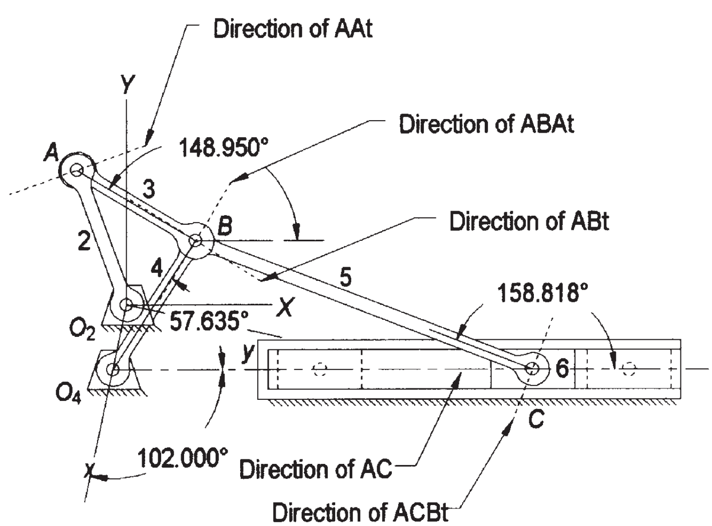
Input crank angular acceleration $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P6-33 and Mathcad file P0761a.

1. Draw the linkage to scale and label it.



$\theta_2 := \theta_{2XY} - \delta \qquad \theta_2 = 212.000\,deg$

2. In order to solve for the acceleration at point B, we will need θ_3, ω_3, and θ_4. From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 148.950 \cdot deg - 180 \cdot deg - \delta \qquad \theta_3 = 70.950\,deg$$

$$\theta_4 := 57.635 \cdot deg - \delta \qquad \theta_4 = 159.635\,deg$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{(\sin(\theta_3 - \theta_4))} \qquad \omega_3 = -0.832\,rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{(\sin(\theta_4 - \theta_3))} \qquad \omega_4 = -0.591\,rad \cdot sec^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

4. For point *B*, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$$A_{Bn} := c \cdot \omega_4^2 \qquad A_{Bn} = 0.806\,in \cdot sec^{-2}$$

$$\theta_{ABn} := \theta_4 - 180 \cdot deg + \delta \qquad \theta_{ABn} = -122.365\,deg$$

$$A_{An} := a \cdot \omega_2^2 \qquad A_{An} = 2.170\,in \cdot sec^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot deg + \delta \qquad \theta_{AAn} = 290.000\,deg$$

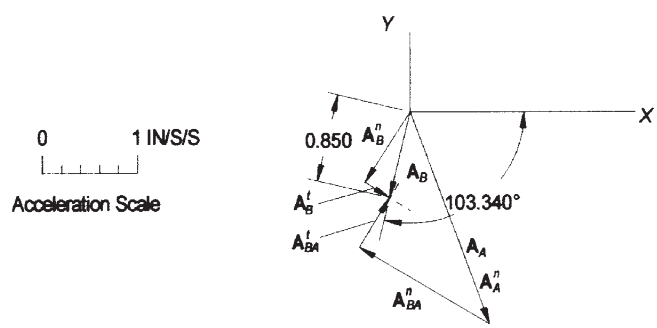
$$A_{At} := a \cdot \alpha_2 \qquad A_{At} = 0.0\,in \cdot sec^{-2}$$

$$\theta_{AAt} := \theta_2 + 90 \cdot deg + \delta \qquad \theta_{AAt} = 200.000\,deg$$

$$A_{BAn} := b \cdot \omega_3^2 \qquad A_{BAn} = 1.429\,in \cdot sec^{-2}$$

$$\theta_{ABAn} := \theta_3 + 180 \cdot deg + \delta \qquad \theta_{ABAn} = 148.950\,deg$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A{}^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of $\theta_{AAt} + \alpha$. From the tip of $\mathbf{A}_A{}^t$, draw $\mathbf{A}_{BA}{}^n$ at an angle of $\theta_{BAn} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B{}^t$, $\mathbf{A}_B{}^t$, and $\mathbf{A}_{BA}{}^t$.



6. From the graphical solution above,

Acceleration scale factor $\quad k_a := 1.000 \cdot \dfrac{in}{sec^2}$

$A_A := A_{An} \qquad A_A = 2.170\,in \cdot sec^{-2}$ at an angle of 290.0 deg

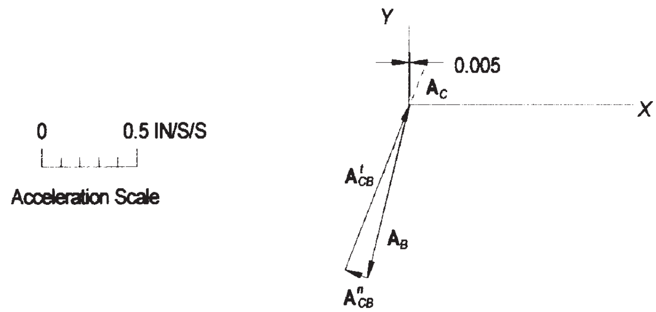
$A_B := 0.850 \cdot k_a \qquad A_B = 0.850\,in \cdot sec^{-2}$ at an angle of -103.340 deg

7. From Problem 6-70a, the angular velocity of link 5 is $\omega_5 := 0.145 \cdot rad \cdot sec^{-1}$ CCW. From the graphical layout, $\theta_5 := 158.818 \cdot deg$ in the global *XY* coordinate system.

8. For point C, equation 7.4 becomes: $\mathbf{A}_C = \mathbf{A}_B + (\mathbf{A}_{CB}{}^t + \mathbf{A}_{CB}{}^n)$, where $\mathbf{A}_B$ is known and

$A_{CBn} := e \cdot \omega_5^{\,2}$ $\qquad$ $A_{CBn} = 0.114\,in \cdot sec^{-2}$

$\theta_{ACBn} := \theta_5$ $\qquad$ $\theta_{ACBn} = 158.818\,deg$



9. From the graphical solution above,

Acceleration scale factor $\qquad$ $k_a := 0.50 \cdot \dfrac{in}{sec^2}$

$A_C := 0.005 \cdot k_a$ $\qquad$ $A_C = 0.0025\,in \cdot sec^{-2}$ $\qquad$ at an angle of 0.000 deg

PROBLEM 7-61b

Statement: Find the acceleration of the slider in Figure 3-33 for $\theta_2 = 110$ deg with respect to the global X axis assuming a constant $\omega_2 = 1$ rad/sec CW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.170 \cdot in$	Link 3 (A to B)	$b := 2.067 \cdot in$
Link 4 (O_4 to B)	$c := 2.310 \cdot in$	Link 1 (O_2 to O_4)	$d := 1.000 \cdot in$
Link 5 (B to C)	$e := 5.400 \cdot in$		

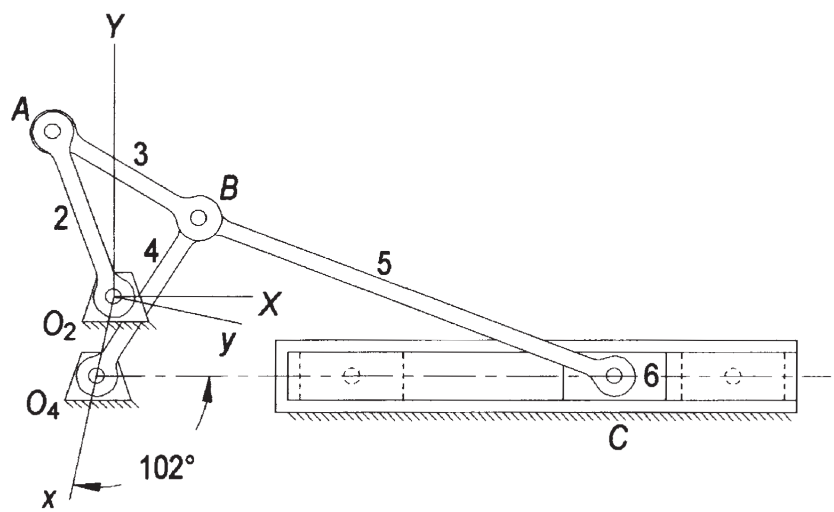
Coordinate angle $\delta := -102 \cdot deg$ Crank angle: $\theta_{2XY} := 110 \cdot deg$

Input crank angular velocity $\omega_2 := -1 \cdot rad \cdot sec^{-1}$ CW

Input crank angular acceleration $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P6-33 and Mathcad file P0761b.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

Transform crank angle to the local xy coordinate system:

$$\theta_2 := \theta_{2XY} - \delta \qquad \theta_2 = 212.000\,deg$$

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.4608 \qquad K_2 = 0.4329$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.6755$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -0.2662 \qquad B = 1.0598 \qquad C = 2.3515$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{4xy} := 2\cdot\left(atan2\left(2\cdot A, -B - \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) \qquad \theta_{4xy} = -200.365\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.4838$$

$$K_5 = -0.5178$$

$$D := \cos(\theta_2) - K_1 + K_4\cdot\cos(\theta_2) + K_5 \qquad D = -2.2370$$

$$E := -2\cdot\sin(\theta_2) \qquad E = 1.0598$$

$$F := K_1 + (K_4 - 1)\cdot\cos(\theta_2) + K_5 \qquad F = 0.3808$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{3xy} := 2\cdot\left(atan2\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_{3xy} = -289.050\,deg$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3 := \frac{a\cdot\omega_2}{b}\cdot\frac{\sin(\theta_{4xy} - \theta_2)}{\sin(\theta_{3xy} - \theta_{4xy})} \qquad \omega_3 = -0.832\,\frac{rad}{sec}$$

$$\omega_4 := \frac{a\cdot\omega_2}{c}\cdot\frac{\sin(\theta_2 - \theta_{3xy})}{\sin(\theta_{4xy} - \theta_{3xy})} \qquad \omega_4 = -0.591\,\frac{rad}{sec}$$

7. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c\cdot\sin(\theta_{4xy}) \qquad B := b\cdot\sin(\theta_{3xy}) \qquad D := c\cdot\cos(\theta_{4xy}) \qquad E := b\cdot\cos(\theta_{3xy})$$

$$A = 0.804\,in \qquad B = 1.954\,in \qquad D = -2.166\,in \qquad E = 0.675\,in$$

$$C := a\cdot\alpha_2\cdot\sin(\theta_2) + a\cdot\omega_2^2\cdot\cos(\theta_2) + b\cdot\omega_3^2\cdot\cos(\theta_{3xy}) - c\cdot\omega_4^2\cdot\cos(\theta_{4xy})$$

$$C = -0.618\,\frac{in}{sec^2}$$

$$F := a\cdot\alpha_2\cdot\cos(\theta_2) - a\cdot\omega_2^2\cdot\sin(\theta_2) - b\cdot\omega_3^2\cdot\sin(\theta_{3xy}) + c\cdot\omega_4^2\cdot\sin(\theta_{4xy})$$

$$F = 0.079\,\frac{in}{sec^2}$$

$$\alpha_3 := \frac{C\cdot D - A\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_3 = 0.267\,\frac{rad}{sec^2} \qquad \alpha_4 := \frac{C\cdot E - B\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_4 = -0.120\,\frac{rad}{sec^2}$$

8. Transform θ_4 back to the global XY system.

$$\theta_4 := \theta_{4xy} + \delta \qquad \theta_4 = -302.365\,deg$$

9. Determine θ_5 using equation 4.17.

Offset: $cc := 0\cdot in$

$$\theta_5 := asin\left(-\frac{c \cdot sin(\theta_4) - cc}{e}\right) + \pi \qquad \theta_5 = 158.818\, deg$$

10. Determine the angular velocity of link 5 using equation 6.22a:

$$\omega_5 := \frac{c}{e} \cdot \frac{cos(\theta_4)}{cos(\theta_5)} \cdot \omega_4 \qquad \omega_5 = 0.145 \frac{rad}{sec}$$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5 := \frac{c \cdot \alpha_4 \cdot cos(\theta_4) - c \cdot \omega_4^{\,2} \cdot sin(\theta_4) + e \cdot \omega_5^{\,2} \cdot sin(\theta_5)}{e \cdot cos(\theta_5)} \qquad \alpha_5 = 0.156 \frac{rad}{sec^2}$$

12. Use equation 7.16e for the acceleration of pin C.

$$A_C := -c \cdot \alpha_4 \cdot sin(\theta_4) - c \cdot \omega_4^{\,2} \cdot cos(\theta_4) + e \cdot \alpha_5 \cdot sin(\theta_5) + e \cdot \omega_5^{\,2} \cdot cos(\theta_5)$$

$$A_C = 0.00161 \frac{in}{sec^2}$$

A positive sign means that $\mathbf{A}_C$ is to the right.

PROBLEM 7-62

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular acceleration of link 4 and the linear acceleration of slider 6 in the sixbar slider-crank linkage of Figure 3-33 as a function of the angle of input link 2 for a constant ω_2 = 1 rad/sec CW. Plot A_c both as a function of θ_2 and separately as a function of slider position as shown in the figure.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.170 \cdot in$	Link 3 (A to B)	$b := 2.067 \cdot in$
Link 4 (O_4 to B)	$c := 2.310 \cdot in$	Link 1 (O_2 to O_4)	$d := 1.000 \cdot in$
Link 5 (B to C)	$e := 5.400 \cdot in$		

Coordinate angle $\delta := -102 \cdot deg$ Crank angle: $\theta_{2XY} := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

Input crank angular velocity $\omega_2 := -1 \cdot rad \cdot sec^{-1}$ CW

Input crank angular acceleration $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure 3-33 and Mathcad file P0762.

1. Transform crank angle to the local xy coordinate system: $\theta_2(\theta_{2XY}) := \theta_{2XY} - \delta$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_1 = 0.4608 \qquad K_2 := \frac{d}{c} \qquad K_2 = 0.4329$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.6755$$

$$A(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 - K_2 \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

$$B(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$C(\theta_{2XY}) := K_1 - (K_2 + 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{4xy}(\theta_{2XY}) := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot A(\theta_{2XY}), -B(\theta_{2XY}) - \sqrt{B(\theta_{2XY})^2 - 4 \cdot A(\theta_{2XY}) \cdot C(\theta_{2XY})} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.4838 \qquad K_5 = -0.5178$$

$$D(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 + K_4 \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$E(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$F(\theta_{2XY}) := K_1 + (K_4 - 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{3xy}(\theta_{2XY}) := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot D(\theta_{2XY}), -E(\theta_{2XY}) - \sqrt{E(\theta_{2XY})^2 - 4 \cdot D(\theta_{2XY}) \cdot F(\theta_{2XY})} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_{2XY}) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{4xy}(\theta_{2XY}) - \theta_2(\theta_{2XY}))}{\sin(\theta_{3xy}(\theta_{2XY}) - \theta_{4xy}(\theta_{2XY}))}$$

$$\omega_4(\theta_{2XY}) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2(\theta_{2XY}) - \theta_{3xy}(\theta_{2XY}))}{\sin(\theta_{4xy}(\theta_{2XY}) - \theta_{3xy}(\theta_{2XY}))}$$

7. Use equations 7.12 to determine the angular acceleration of link 4.

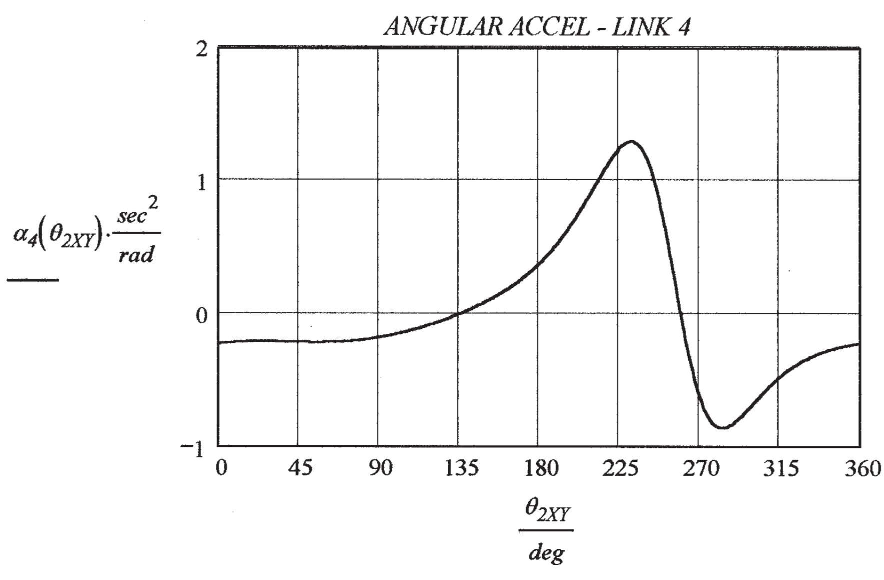
$$A(\theta_{2XY}) := c \cdot \sin(\theta_{4xy}(\theta_{2XY})) \qquad B(\theta_{2XY}) := b \cdot \sin(\theta_{3xy}(\theta_{2XY}))$$

$$D(\theta_{2XY}) := c \cdot \cos(\theta_{4xy}(\theta_{2XY})) \qquad E(\theta_{2XY}) := b \cdot \cos(\theta_{3xy}(\theta_{2XY}))$$

$$C(\theta_{2XY}) := a \cdot \alpha_2 \cdot \sin(\theta_2(\theta_{2XY})) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2(\theta_{2XY})) \ldots$$
$$+ b \cdot \omega_3(\theta_{2XY})^2 \cdot \cos(\theta_{3xy}(\theta_{2XY})) - c \cdot \omega_4(\theta_{2XY})^2 \cdot \cos(\theta_{4xy}(\theta_{2XY}))$$

$$F(\theta_{2XY}) := a \cdot \alpha_2 \cdot \cos(\theta_2(\theta_{2XY})) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2(\theta_{2XY})) \ldots$$
$$+ -b \cdot \omega_3(\theta_{2XY})^2 \cdot \sin(\theta_{3xy}(\theta_{2XY})) + c \cdot \omega_4(\theta_{2XY})^2 \cdot \sin(\theta_{4xy}(\theta_{2XY}))$$

$$\alpha_4(\theta_{2XY}) := \frac{C(\theta_{2XY}) \cdot E(\theta_{2XY}) - B(\theta_{2XY}) \cdot F(\theta_{2XY})}{A(\theta_{2XY}) \cdot E(\theta_{2XY}) - B(\theta_{2XY}) \cdot D(\theta_{2XY})}$$



8. Transform θ_4 back to the global *XY* system.

$$\theta_4(\theta_{2XY}) := \theta_{4xy}(\theta_{2XY}) + \delta$$

9. Determine θ_5 using equation 4.17. Offset: $cc := 0 \cdot in$

$$\theta_5(\theta_{2XY}) := \operatorname{asin}\left(-\frac{c \cdot \sin(\theta_4(\theta_{2XY})) - cc}{e}\right) + \pi$$

10. Determine the angular velocity of link 5 using equation 6.22a:

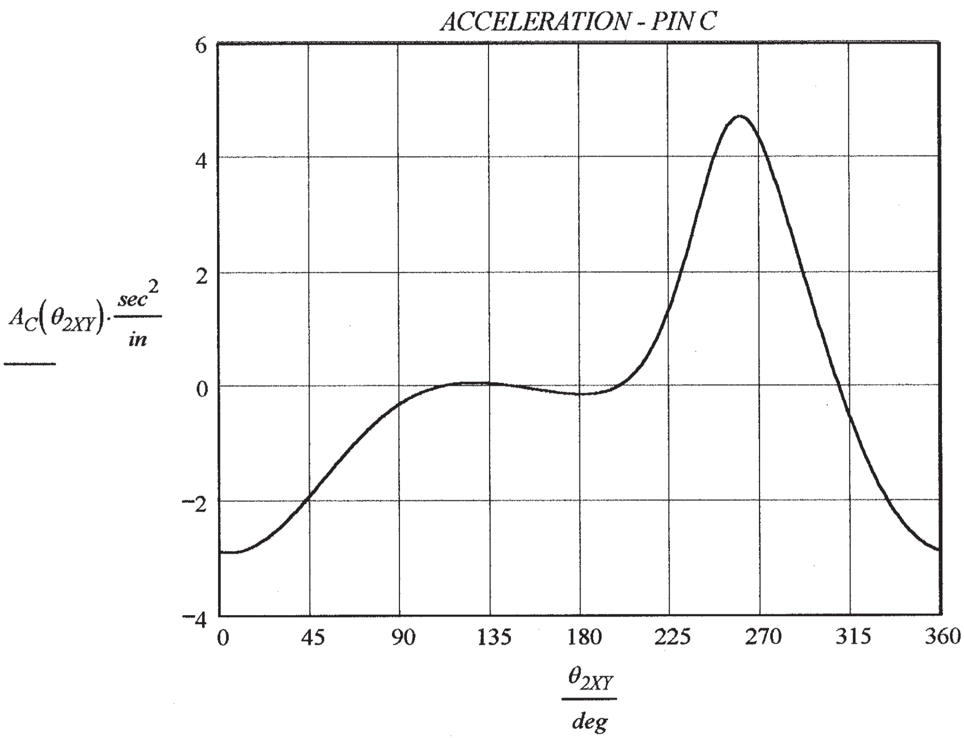
$$\omega_5(\theta_{2XY}) := \frac{c}{e}\cdot\frac{\cos(\theta_4(\theta_{2XY}))}{\cos(\theta_5(\theta_{2XY}))}\cdot\omega_4(\theta_{2XY})$$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5(\theta_{2XY}) := \frac{c\cdot\alpha_4(\theta_{2XY})\cdot\cos(\theta_4(\theta_{2XY})) - c\cdot\omega_4(\theta_{2XY})^2\cdot\sin(\theta_4(\theta_{2XY}))\ ... \\ + e\cdot\omega_5(\theta_{2XY})^2\cdot\sin(\theta_5(\theta_{2XY}))}{e\cdot\cos(\theta_5(\theta_{2XY}))}$$

12. Use equation 7.16e for the acceleration of pin C.

$$A_C(\theta_{2XY}) := -c\cdot\alpha_4(\theta_{2XY})\cdot\sin(\theta_4(\theta_{2XY})) - c\cdot\omega_4(\theta_{2XY})^2\cdot\cos(\theta_4(\theta_{2XY}))\ ... \\ + e\cdot\alpha_5(\theta_{2XY})\cdot\sin(\theta_5(\theta_{2XY})) + e\cdot\omega_5(\theta_{2XY})^2\cdot\cos(\theta_5(\theta_{2XY}))$$



PROBLEM 7-63a

Statement: Find the angular acceleration of link 6 in Figure 3-34b for $\theta_6 = 90$ deg with respect to the global X axis assuming constant $\omega_2 = 10$ rad/sec CW. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot in$	Link 3 (A to B)	$f := 4.248 \cdot in$
Link 4 (O_4 to C)	$c := 2.125 \cdot in$	Link 5 (C to D)	$b := 2.158 \cdot in$
Link 6 (O_6 to D)	$a := 1.542 \cdot in$	Link 5 (B to D)	$p := 3.274 \cdot in$
Link 1 X-offset	$d_X := 3.259 \cdot in$	Link 1 Y-offset	$d_Y := -2.905 \cdot in$

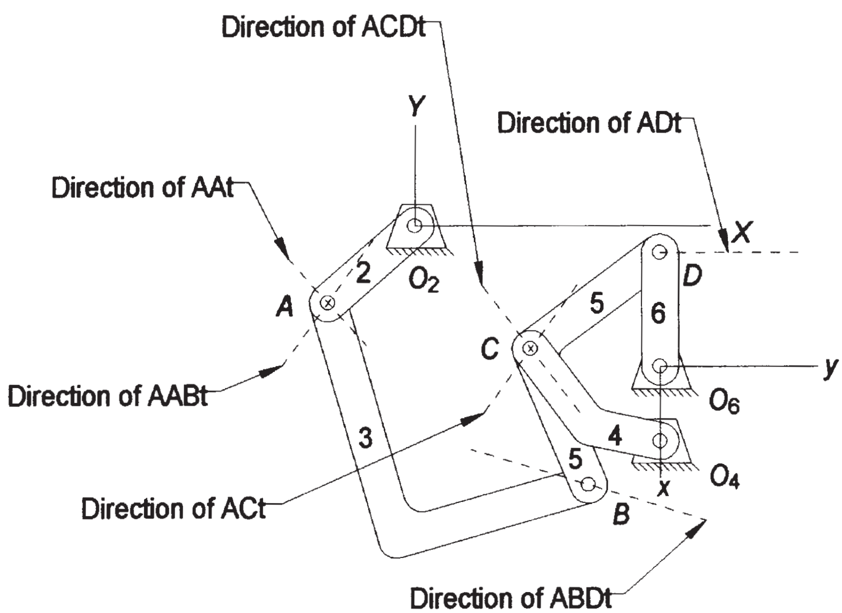
Angle CDB $\delta_5 := 36.0 \cdot deg$

Output rocker angle: $\theta_6 := 90 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CW

Solution: See Figure 3-34b and Mathcad file P0763a.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. Since this linkage is a Stephenson's II sixbar, we will have to start at link 6 and work back to link 2. We will assume a value for α_6 and eventually find a value for α_2. We will then multiply the magnitudes of all velocities by the ratio of the actual α_2 to the found α_2. Use Problem 6-73 for the link angular velocities.

Assume: $\alpha_6 := 100 \cdot rad \cdot sec^{-2}$ CCW

From Problem 6-73 and the graphical layout (angles in the global XY coordinate system):

$\omega_6 := 24.124 \cdot rad \cdot sec^{-1}$ CW $\theta_6 := 90.000 \cdot deg$

$\omega_5 := 14.64 \cdot rad \cdot sec^{-1}$ CW $\theta_{52} := 217.003 \cdot deg$ $\theta_{51} := 253.003 \cdot deg$

$\omega_4 := 14.64 \cdot rad \cdot sec^{-1}$ CW $\theta_4 := 144.195 \cdot deg$

$\omega_3 := 3.016 \cdot rad \cdot sec^{-1}$ CW $\theta_3 := 144.628 \cdot deg$

$\omega_2 := 10.000 \cdot rad \cdot sec^{-1}$ CW $\qquad \theta_2 := 221.690 \cdot deg$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$
4. For point C, this becomes: $(\mathbf{A}_C{}^t + \mathbf{A}_C{}^n) = (\mathbf{A}_D{}^t + \mathbf{A}_D{}^n) + (\mathbf{A}_{CD}{}^t + \mathbf{A}_{CD}{}^n)$, where

$$A_{Cn} := c \cdot \omega_4{}^2 \qquad A_{Cn} = 455.450\ in \cdot sec^{-2}$$

$$\theta_{ACn} := \theta_4 + 180 \cdot deg \qquad \theta_{ACn} = 324.195\ deg$$

$$A_{Dn} := a \cdot \omega_6{}^2 \qquad A_{Dn} = 897.394\ in \cdot sec^{-2}$$

$$\theta_{ADn} := \theta_6 + 180 \cdot deg \qquad \theta_{ADn} = 270.000\ deg$$

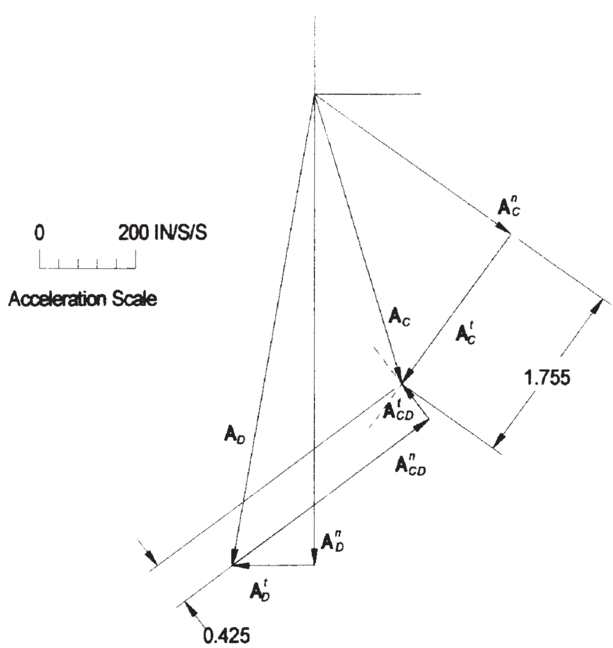
$$A_{Dt} := a \cdot \alpha_6 \qquad A_{Dt} = 154.200\ in \cdot sec^{-2}$$

$$\theta_{ADt} := \theta_6 + 90 \cdot deg \qquad \theta_{ADt} = 180.000\ deg$$

$$A_{CDn} := b \cdot \omega_5{}^2 \qquad A_{CDn} = 462.523\ in \cdot sec^{-2}$$

$$\theta_{ACDn} := \theta_{52} - 180 \cdot deg \qquad \theta_{ACDn} = 37.003\ deg$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_C{}^n$ at an angle of θ_{ACn} and $\mathbf{A}_D{}^n$ at an angle of θ_{ADn}. From the tip of $\mathbf{A}_D{}^n$, draw $\mathbf{A}_D{}^t$ at an angle of $\theta_{ADt} + \alpha$. From the tip of $\mathbf{A}_D{}^t$, draw $\mathbf{A}_{CD}{}^n$ at an angle of θ_{CDn}. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_C{}^n$ and $\mathbf{A}_{CD}{}^n$, draw construction lines in the directions of $\mathbf{A}_C{}^t$ and $\mathbf{A}_{CD}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_C{}^t$, $\mathbf{A}_C{}^t$, and $\mathbf{A}_{CD}{}^t$.



6. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 200 \cdot \dfrac{in}{sec^2}$

$$A_{Ct} := 1.755 \cdot k_a \qquad A_{Ct} = 351.00\ in \cdot sec^{-2}$$

$$\alpha_4 := \frac{A_{Ct}}{c} \qquad \alpha_4 = 165.176\ rad \cdot sec^{-2} \quad \text{CW}$$

$A_{CDt} := 0.425 \cdot k_a \qquad A_{CDt} = 85.00\, in \cdot sec^{-2}$

$\alpha_5 := \dfrac{A_{CDt}}{b} \qquad \alpha_5 = 39.388\, rad \cdot sec^{-2}$ CW

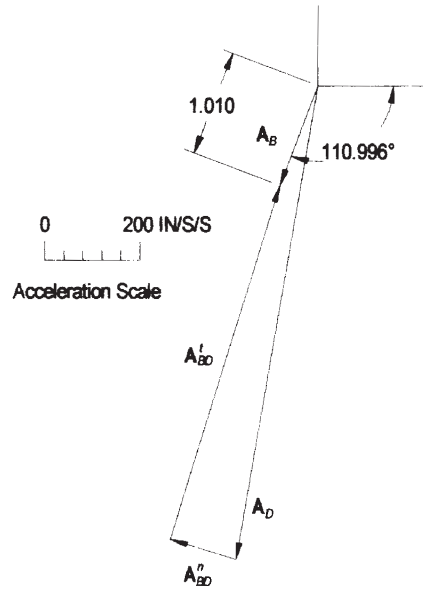
7. For point B, equation 7.4 becomes: $\mathbf{A}_B = \mathbf{A}_D + (\mathbf{A}_{BD}^t + \mathbf{A}_{BD}^n)$, where

$A_{BDn} := p \cdot \omega_5^2 \qquad A_{BDn} = 701.715\, in \cdot sec^{-2}$

$\theta_{ABDn} := \theta_{51} - 180 \cdot deg \qquad \theta_{ABDn} = 73.003\, deg$

$A_{BDt} := p \cdot \alpha_5 \qquad A_{BDt} = 128.957\, in \cdot sec^{-2}$

$\theta_{ABDt} := \theta_{ABDn} + 90 \cdot deg \qquad \theta_{ABDt} = 163.003\, deg$



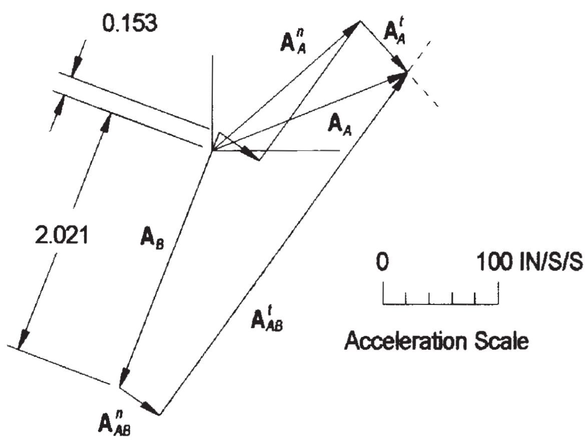
8. For point A, equation 7.4 becomes:
$(\mathbf{A}_A^t + \mathbf{A}_A^n) = \mathbf{A}_B + (\mathbf{A}_{AB}^t + \mathbf{A}_{AB}^n)$, where

$A_{An} := g \cdot \omega_2^2 \qquad A_{An} = 155.600\, in \cdot sec^{-2}$

$\theta_{AAn} := \theta_2 - 180 \cdot deg \qquad \theta_{AAn} = 41.690\, deg$

$A_{ABn} := f \cdot \omega_3^2 \qquad A_{ABn} = 38.641\, in \cdot sec^{-2}$

$\theta_{AABn} := \theta_3 - 180 \cdot deg \qquad \theta_{AABn} = -35.372\, deg$



9. The requirement for this problem is that $\alpha_2 = 0$ (constant ω_2). For this to be so $\mathbf{A}_A^t$ must be zero. To do this the magnitude of $\mathbf{A}_B$ must be smaller as shown above.

Correction ratio: $\quad r := \dfrac{-0.153}{2.021} \qquad r = -0.076$

Corrected value of α_6: $\quad \alpha 6 := r \cdot \alpha_6 \qquad \alpha 6 = -7.571 \dfrac{rad}{sec^2}$

PROBLEM 7-63b

Statement: Find the angular acceleration of link 6 in Figure 3-34b for $\theta_6 = 90$ deg with respect to the global X axis assuming constant $\omega_2 = 10$ rad/sec CW. Use an analytic method.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot in$	Link 3 (A to B)	$f := 4.248 \cdot in$
Link 4 (O_4 to C)	$c := 2.125 \cdot in$	Link 5 (C to D)	$b := 2.158 \cdot in$
Link 6 (O_6 to D)	$a := 1.542 \cdot in$	Link 5 (B to D)	$p := 3.274 \cdot in$
Link 1 X-offset	$d_X := 3.259 \cdot in$	Link 1 Y-offset	$d_Y := -2.905 \cdot in$

Angle CDB $\delta_5 := 36.0 \cdot deg$

Output rocker angle: $\theta_6 := 90 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := -10 \cdot rad \cdot sec^{-1}$ CW

Input crank angular acceleration $\alpha_2 := 0.0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P6-34b and Mathcad file P0763b.

1. Since this linkage is a Stephenson's II sixbar an iterative solution must be used. This problem is suited to a longer-term project rather than a daily homework problem.

PROBLEM 7-64

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular acceleration of link 6 in Figure 3-34b as a function of θ_2 for a constant ω_2 = 10 rad/sec CW.

Given: Link lengths:

Link 2 (O_2 to A)	$g := 1.556 \cdot in$	Link 3 (A to B)	$f := 4.248 \cdot in$
Link 4 (O_4 to C)	$c := 2.125 \cdot in$	Link 5 (C to D)	$b := 2.158 \cdot in$
Link 6 (O_6 to D)	$a := 1.542 \cdot in$	Link 5 (B to D)	$p := 3.274 \cdot in$
Link 1 X-offset	$d_X := 3.259 \cdot in$	Link 1 Y-offset	$d_Y := -2.905 \cdot in$

Angle CDB $\delta_5 := 36.0 \cdot deg$

Output rocker angle: $\theta_6 := 90 \cdot deg$ Global XY system

Input crank angular velocity $\omega_2 := -10 \cdot rad \cdot sec^{-1}$ CW

Input crank angular acceleration $\alpha_2 := 0.0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P6-34b and Mathcad file P0764.

1. Since this linkage is a Stephenson's II sixbar an iterative solution must be used. This problem is suited to a longer-term project rather than a daily homework problem.

PROBLEM 7-65a

Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular acceleration of link 6 assuming a constant ω_2 = 10 rad/sec CCW. Use a graphical method.

Given: Link lengths:

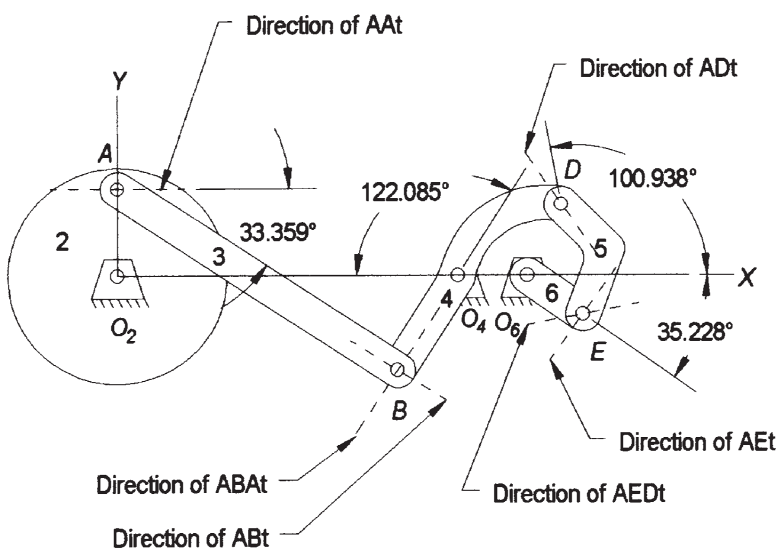
Link 2 (O_2 to A)	$a := 1.000 \cdot in$	Link 3 (A to B)	$b := 3.800 \cdot in$
Link 4 (O_4 to B)	$c := 1.286 \cdot in$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot in$
Link 4 (O_4 to D)	$e := 1.429 \cdot in$	Link 5 (D to E)	$f := 1.286 \cdot in$
Link 6 (O_6 to E)	$g := 0.771 \cdot in$	Link 1 (O_4 to O_6)	$h := 0.786 \cdot in$
Crank angle:	$\theta_2 := 90 \cdot deg$	Angle BO_4D	$\delta_4 := 157 \cdot deg$

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Input crank angular acceleration $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure 3-35 and Mathcad file P0765a.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3, ω_3, and θ_4. From the graphical position solution above,

$\theta_3 := -33.359 \cdot deg$ $\qquad \theta_3 = -33.359\,deg$

$\theta_{41} := -122.085 \cdot deg$ $\qquad \theta_{41} = -122.085\,deg$

$\theta_{42} := -122.085 \cdot deg + \delta_4$ $\qquad \theta_{42} = 34.915\,deg$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{(\sin(\theta_3 - \theta_{41}))} \qquad \omega_3 = 1.398\,rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{(\sin(\theta_{41} - \theta_3))} \qquad \omega_4 = -6.496\,rad \cdot sec^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

4. For point *B*, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$$A_{Bn} := c\cdot\omega_4^{\,2} \qquad A_{Bn} = 54.275\ in\cdot sec^{-2}$$

$$\theta_{ABn} := \theta_{41} + 180\cdot deg \qquad \theta_{ABn} = 57.915\ deg$$

$$A_{An} := a\cdot\omega_2^{\,2} \qquad A_{An} = 100.000\ in\cdot sec^{-2}$$

$$\theta_{AAn} := \theta_2 + 180\cdot deg \qquad \theta_{AAn} = 270.000\ deg$$

$$A_{At} := \left|a\cdot\alpha_2\right| \qquad A_{At} = 0.000\ in\cdot sec^{-2}$$

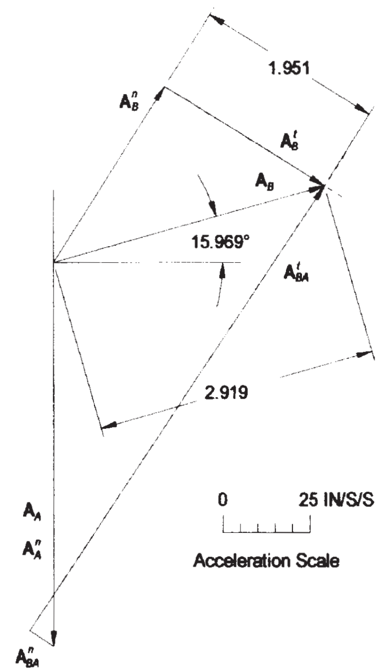
$$\theta_{AAt} := \theta_2 - 90\cdot deg \qquad \theta_{AAt} = 0.000\ deg$$

$$A_{BAn} := b\cdot\omega_3^{\,2} \qquad A_{BAn} = 7.429\ in\cdot sec^{-2}$$

$$\theta_{ABAn} := \theta_3 + 180\cdot deg \qquad \theta_{ABAn} = 146.641\ deg$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of θ_{ABn} and $\mathbf{A}_A{}^n$ at an angle of θ_{AAn}. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of $\theta_{AAt} + \alpha$. From the tip of $\mathbf{A}_A{}^t$, draw $\mathbf{A}_{BA}{}^n$ at an angle of θ_{BAn}. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B$, $\mathbf{A}_B{}^t$, and $\mathbf{A}_{BA}{}^t$.

6. From the graphical solution at right,



Acceleration scale factor $\qquad k_a := 25\cdot\dfrac{in}{sec^2}$

$$A_B := 2.919\cdot k_a \qquad A_B = 73.0\ in\cdot sec^{-2}$$

at an angle of 15.969 deg

$$A_{Bt} := 1.951\cdot k_a \qquad A_{Bt} = 48.8\ in\cdot sec^{-2}$$

$$\alpha_4 := \frac{A_{Bt}}{c} \qquad \alpha_4 = 37.928\ rad\cdot sec^{-2} \qquad \text{CCW}$$

6. From the graphical layout,

$$\theta_5 := 100.938\cdot deg - 180\cdot deg \qquad \theta_5 = -79.062\ deg$$

$$\theta_6 := -35.228\cdot deg \qquad \theta_6 = -35.228\ deg$$

Using equation (6.18),

$$\omega_5 := \frac{e\cdot\omega_4}{f}\cdot\frac{\sin(\theta_6 - \theta_{42})}{(\sin(\theta_5 - \theta_6))} \qquad \omega_5 = -9.804\ rad\cdot sec^{-1}$$

$$\omega_6 := \frac{e\cdot\omega_4}{g}\cdot\frac{\sin(\theta_{42} - \theta_5)}{(\sin(\theta_6 - \theta_5))} \qquad \omega_6 = -15.885\ rad\cdot sec^{-1}$$

7. For point E, this becomes: $(\mathbf{A}_E^t + \mathbf{A}_E^n) = (\mathbf{A}_D^t + \mathbf{A}_D^n) + (\mathbf{A}_{ED}^t + \mathbf{A}_{ED}^n)$, where

$A_{En} := g \cdot \omega_6^2 \qquad A_{En} = 194.560\ in \cdot sec^{-2}$

$\theta_{AEn} := \theta_6 + 180 \cdot deg \qquad \theta_{AEn} = 144.772\ deg$

$A_{Dn} := e \cdot \omega_4^2 \qquad A_{Dn} = 60.310\ in \cdot sec^{-2}$

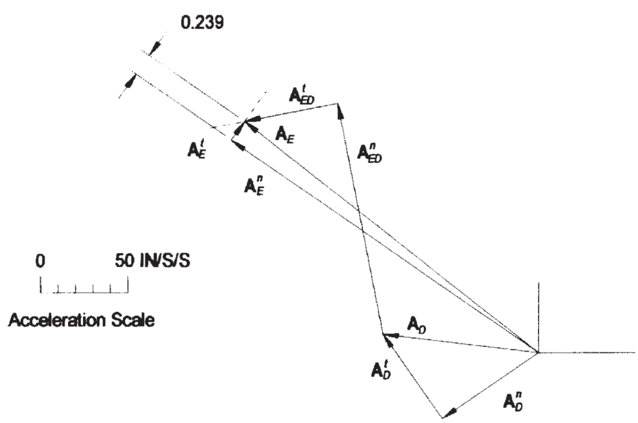
$\theta_{ADn} := \theta_{42} + 180 \cdot deg \qquad \theta_{ADn} = 214.915\ deg$

$A_{Dt} := e \cdot \alpha_4 \qquad A_{Dt} = 54.199\ in \cdot sec^{-2}$

$\theta_{ADt} := \theta_{42} + 90 \cdot deg \qquad \theta_{ADt} = 124.915\ deg$

$A_{EDn} := f \cdot \omega_5^2 \qquad A_{EDn} = 123.597\ in \cdot sec^{-2}$

$\theta_{AEDn} := \theta_5 + 180 \cdot deg \qquad \theta_{AEDn} = 100.938\ deg$



8. From the graphical solution above,

Acceleration scale factor $\quad k_a := 50 \cdot \dfrac{in}{sec^2}$

$A_{Et} := 0.239 \cdot k_a \qquad A_{Et} = 11.950\ in \cdot sec^{-2}$

$\alpha_6 := \dfrac{A_{Et}}{g} \qquad \alpha_6 = 15.499\ rad \cdot sec^{-2} \qquad$ CCW

PROBLEM 7-65b

Statement: Use a compass and straightedge to draw the linkage in Figure 3-35 with link 2 at 90 deg and find the angular acceleration of link 6 assuming a constant ω_2 = 10 rad/sec CCW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000 \cdot in$	Link 3 (A to B)	$b := 3.800 \cdot in$
Link 4 (O_4 to B)	$c := 1.286 \cdot in$	Link 1 (O_2 to O_4)	$d := 3.857 \cdot in$
Link 4 (O_4 to D)	$e := 1.429 \cdot in$	Link 5 (D to E)	$f := 1.286 \cdot in$
Link 6 (O_6 to E)	$g := 0.771 \cdot in$	Link 1 (O_4 to O_6)	$h := 0.786 \cdot in$

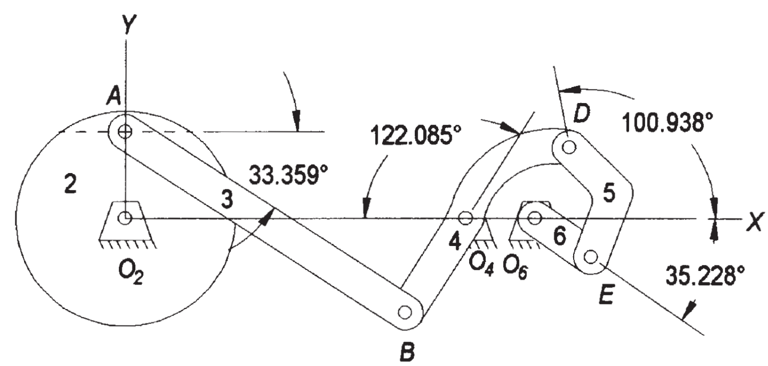
Crank angle: $\theta_2 := 90 \cdot deg$ Angle BO_4D $\delta_4 := 157 \cdot deg$

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW

Input crank angular acceleration $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P6-35 and Mathcad file P0765b.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 3.8570 \qquad K_2 = 2.9992$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.2015$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = -2.6555 \qquad B = -2.0000 \qquad C = 5.0585$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_{41} = 237.915\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.0150 \qquad K_5 = -3.7714$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -7.6284$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = -2.0000$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = 0.0856$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right) \qquad \theta_3 = 326.641\ deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_3 - \theta_{41})} \qquad \omega_3 = 1.398 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_{41} - \theta_3)} \qquad \omega_4 = -6.496 \frac{rad}{sec}$$

7. Link 4 will be the input to the second fourbar (links 4, 5, 6, and 1). Let the angle that link 4 makes in the second fourbar be

$$\theta_{42} := \theta_{41} + 157 \cdot deg - 360 \cdot deg \qquad \theta_{42} = 34.915\ deg$$

8. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{h}{e} \qquad K_2 := \frac{h}{g}$$

$$K_1 = 0.5500 \qquad K_2 = 1.0195$$

$$K_3 := \frac{e^2 - f^2 + g^2 + h^2}{(2 \cdot e \cdot g)} \qquad K_3 = 0.7263$$

$$A := \cos(\theta_{42}) - K_1 - K_2 \cdot \cos(\theta_{42}) + K_3$$

$$B := -2 \cdot \sin(\theta_{42})$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_{42}) + K_3$$

$$A = 0.1603 \qquad B = -1.1447 \qquad C = -0.3796$$

9. Use equation 4.10b to find values of θ_6 for the open circuit.

$$\theta_6 := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C} \right) \right) \qquad \theta_6 = -35.228\ deg$$

10. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$$K_4 := \frac{h}{f} \qquad K_5 := \frac{g^2 - h^2 - e^2 - f^2}{(2 \cdot e \cdot f)} \qquad K_4 = 0.6112$$

$$K_5 = -1.0119$$

$$D := \cos(\theta_{42}) - K_1 + K_4 \cdot \cos(\theta_{42}) + K_5 \qquad D = -0.2408$$

$$E := -2 \cdot \sin(\theta_{42}) \qquad E = -1.1447$$

$$F := K_1 + (K_4 - 1)\cdot\cos(\theta_{42}) + K_5 \qquad F = -0.7807$$

11. Use equation 4.13 to find values of θ_5 for the open circuit.

$$\theta_5 := 2\cdot\left(\mathrm{atan2}\left(2\cdot D, -E - \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_5 = 280.938\ deg$$

12. Determine the angular velocity of links 5 and 6 for the open circuit using equations 6.18.

$$\omega_5 := \left(\frac{e\cdot\omega_4}{f}\right)\cdot\frac{\sin(\theta_6 - \theta_{42})}{\sin(\theta_5 - \theta_6)} \qquad \omega_5 = -9.804\,\frac{rad}{sec}$$

$$\omega_6 := \left(\frac{e\cdot\omega_4}{g}\right)\cdot\frac{\sin(\theta_{42} - \theta_5)}{\sin(\theta_6 - \theta_5)} \qquad \omega_6 = -15.885\,\frac{rad}{sec}$$

13. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c\cdot\sin(\theta_{41}) \qquad B := b\cdot\sin(\theta_3) \qquad D := c\cdot\cos(\theta_{41}) \qquad E := b\cdot\cos(\theta_3)$$

$$A = -1.090\ in \qquad B = -2.090\ in \qquad D = -0.683\ in \qquad E = 3.174\ in$$

$$C := a\cdot\alpha_2\cdot\sin(\theta_2) + a\cdot\omega_2^{\ 2}\cdot\cos(\theta_2) + b\cdot\omega_3^{\ 2}\cdot\cos(\theta_3) - c\cdot\omega_4^{\ 2}\cdot\cos(\theta_{41})$$

$$C = 35.034\,\frac{in}{sec^2}$$

$$F := a\cdot\alpha_2\cdot\cos(\theta_2) - a\cdot\omega_2^{\ 2}\cdot\sin(\theta_2) - b\cdot\omega_3^{\ 2}\cdot\sin(\theta_3) + c\cdot\omega_4^{\ 2}\cdot\sin(\theta_{41})$$

$$F = -141.900\,\frac{in}{sec^2}$$

$$\alpha_3 := \frac{C\cdot D - A\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_3 = 36.545\,\frac{rad}{sec^2} \qquad \alpha_4 := \frac{C\cdot E - B\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_4 = 37.931\,\frac{rad}{sec^2}$$

14. Use equations 7.12 to determine the angular accelerations of links 5 and 6 for the open circuit. Use link 4 as the input to the second fourbar stage.

$$A := g\cdot\sin(\theta_6) \qquad B := f\cdot\sin(\theta_5) \qquad D := g\cdot\cos(\theta_6) \qquad E := f\cdot\cos(\theta_5)$$

$$A = -0.037\ \mathrm{ft} \qquad B = -0.105\ \mathrm{ft} \qquad D = 0.052\ \mathrm{ft} \qquad E = 0.020\ \mathrm{ft}$$

$$C := e\cdot\alpha_4\cdot\sin(\theta_{42}) + e\cdot\omega_4^{\ 2}\cdot\cos(\theta_{42}) + f\cdot\omega_5^{\ 2}\cdot\cos(\theta_5) - g\cdot\omega_6^{\ 2}\cdot\cos(\theta_6)$$

$$C = -4.583\,\frac{\mathrm{ft}}{sec^2}$$

$$F := e\cdot\alpha_4\cdot\cos(\theta_{42}) - e\cdot\omega_4^{\ 2}\cdot\sin(\theta_{42}) - f\cdot\omega_5^{\ 2}\cdot\sin(\theta_5) + g\cdot\omega_6^{\ 2}\cdot\sin(\theta_6)$$

$$F = 1.588\,\frac{\mathrm{ft}}{\mathrm{sec}^2}$$

$$\alpha_5 := \frac{C\cdot D - A\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_5 = -38.104\,\frac{rad}{sec^2} \qquad \alpha_6 := \frac{C\cdot E - B\cdot F}{A\cdot E - B\cdot D} \qquad \alpha_6 = 15.486\,\frac{rad}{sec^2}$$

PROBLEM 7-66

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular acceleration of link 6 in the sixbar linkage of Figure 3-35 as a function of θ_2 for a constant ω_2 = 1 rad/sec CCW.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 1.000\cdot in$	Link 3 (A to B)	$b := 3.800\cdot in$
Link 4 (O_4 to B)	$c := 1.286\cdot in$	Link 1 (O_2 to O_4)	$d := 3.857\cdot in$
Link 4 (O_4 to D)	$e := 1.429\cdot in$	Link 5 (D to E)	$f := 1.286\cdot in$
Link 6 (O_6 to E)	$g := 0.771\cdot in$	Link 1 (O_4 to O_6)	$h := 0.786\cdot in$

Input crank angular velocity $\omega_2 := 1\cdot rad\cdot sec^{-1}$ CCW $\alpha_2 := 0\cdot rad\cdot sec^{-2}$

Solution: See Figure 3-35 and Mathcad file P0766.

1. Define the range of the input angle: $\theta_2 := 0\cdot deg, 1\cdot deg .. 360 deg$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 3.8570 \qquad K_2 = 2.9992$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)} \qquad K_3 = 1.2015$$

$$A(\theta_2) := cos(\theta_2) - K_1 - K_2\cdot cos(\theta_2) + K_3$$

$$B(\theta_2) := -2\cdot sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 1.0150 \qquad K_5 = -3.7714$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Link 4 will be the input to the second fourbar (links 4, 5, 6, and 1). Let the angle that link 4 makes in the second fourbar be

$$\theta_{42}(\theta_2) := \theta_4(\theta_2) + 157 \cdot deg - 360 \cdot deg$$

8. Determine the values of the constants needed for finding θ_6 from equations 4.8a and 4.10a.

$$K_1 := \frac{h}{e} \qquad K_2 := \frac{h}{g}$$

$$K_1 = 0.5500 \qquad K_2 = 1.0195$$

$$K_3 := \frac{e^2 - f^2 + g^2 + h^2}{(2 \cdot e \cdot g)} \qquad K_3 = 0.7263$$

$$A'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K_1 - K_2 \cdot \cos(\theta_{42}(\theta_2)) + K_3$$

$$B'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

$$C'(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_{42}(\theta_2)) + K_3$$

9. Use equation 4.10b to find values of θ_6 for the open circuit.

$$\theta_6(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A'(\theta_2), -B'(\theta_2) - \sqrt{B'(\theta_2)^2 - 4 \cdot A'(\theta_2) \cdot C'(\theta_2)} \right) \right)$$

10. Determine the values of the constants needed for finding θ_5 from equations 4.11b and 4.12.

$$K_4 := \frac{h}{f} \qquad K_5 := \frac{g^2 - h^2 - e^2 - f^2}{(2 \cdot e \cdot f)} \qquad K_4 = 0.6112 \qquad K_5 = -1.0119$$

$$D'(\theta_2) := \cos(\theta_{42}(\theta_2)) - K_1 + K_4 \cdot \cos(\theta_{42}(\theta_2)) + K_5$$

$$E'(\theta_2) := -2 \cdot \sin(\theta_{42}(\theta_2))$$

$$F'(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_{42}(\theta_2)) + K_5$$

11. Use equation 4.13 to find values of θ_5 for the open circuit.

$$\theta_5(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D'(\theta_2), -E'(\theta_2) - \sqrt{E'(\theta_2)^2 - 4 \cdot D'(\theta_2) \cdot F'(\theta_2)} \right) \right)$$

12. Determine the angular velocity of link 6 for the open circuit using equations 6.18.

$$\omega_5(\theta_2) := \left(\frac{e \cdot \omega_4(\theta_2)}{f} \right) \cdot \frac{\sin(\theta_6(\theta_2) - \theta_{42}(\theta_2))}{\sin(\theta_5(\theta_2) - \theta_6(\theta_2))}$$

$$\omega_6(\theta_2) := \left(\frac{e \cdot \omega_4(\theta_2)}{g} \right) \cdot \frac{\sin(\theta_{42}(\theta_2) - \theta_5(\theta_2))}{\sin(\theta_6(\theta_2) - \theta_5(\theta_2))}$$

13. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_2) := c\cdot sin(\theta_4(\theta_2)) \qquad B(\theta_2) := b\cdot sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c\cdot cos(\theta_4(\theta_2)) \qquad E(\theta_2) := b\cdot cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot cos(\theta_2) \;...$$
$$+ b\cdot\omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot cos(\theta_4(\theta_2))$$

$$F(\theta_2) := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) \;...$$
$$+ -b\cdot\omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)^2\cdot sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot F(\theta_2)}{A(\theta_2)\cdot E(\theta_2) - B(\theta_2)\cdot D(\theta_2)}$$

14. Use equations 7.12 to determine the angular accelerations of links 5 and 6 for the open circuit. Use link 4 as the input to the second fourbar stage.

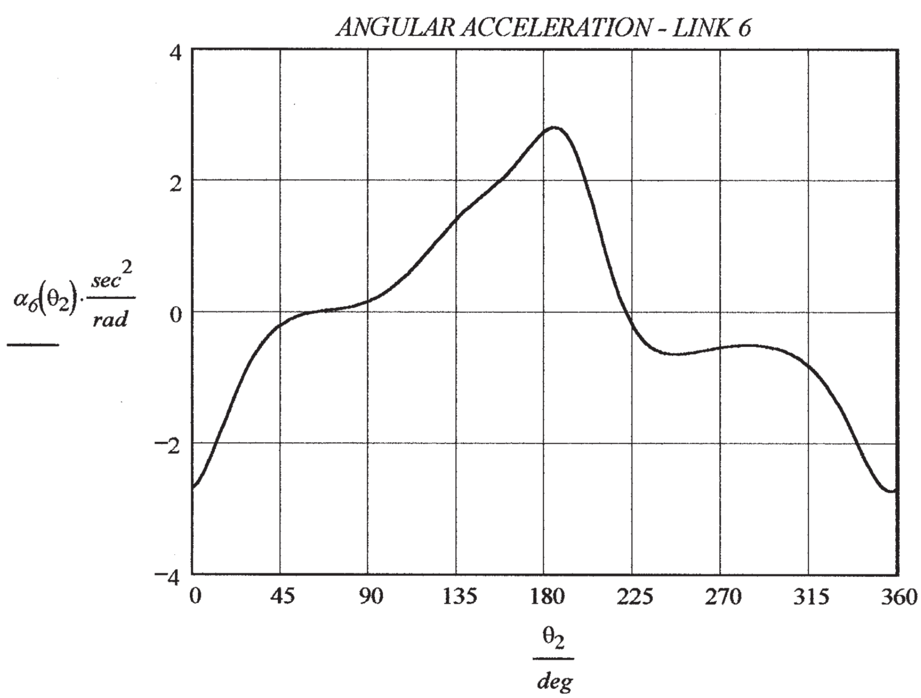
$$A'(\theta_2) := g\cdot sin(\theta_6(\theta_2)) \qquad B'(\theta_2) := f\cdot sin(\theta_5(\theta_2))$$

$$D'(\theta_2) := g\cdot cos(\theta_6(\theta_2)) \qquad E'(\theta_2) := f\cdot cos(\theta_5(\theta_2))$$

$$C'(\theta_2) := e\cdot\alpha_4(\theta_2)\cdot sin(\theta_{42}(\theta_2)) + e\cdot\omega_4(\theta_2)^2\cdot cos(\theta_{42}(\theta_2)) \;...$$
$$+ f\cdot\omega_5(\theta_2)^2\cdot cos(\theta_5(\theta_2)) - g\cdot\omega_6(\theta_2)^2\cdot cos(\theta_6(\theta_2))$$

$$F'(\theta_2) := e\cdot\alpha_4(\theta_2)\cdot cos(\theta_{42}(\theta_2)) - e\cdot\omega_4(\theta_2)^2\cdot sin(\theta_{42}(\theta_2)) \;...$$
$$+ -f\cdot\omega_5(\theta_2)^2\cdot sin(\theta_5(\theta_2)) + g\cdot\omega_6(\theta_2)^2\cdot sin(\theta_6(\theta_2))$$

$$\alpha_6(\theta_2) := \frac{C'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot F'(\theta_2)}{A'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot D'(\theta_2)}$$



PROBLEM 7-67

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the angular acceleration of link 8 in the linkage of Figure 3-36 as a function of θ_2 for a constant ω_2 = 1 rad/sec CCW.

Given: Link lengths:

Input crank (L_2)	$a := 0.450$	First coupler (L_3)	$b := 0.990$
Common rocker (O_4B)	$c := 0.590$	First ground link (O_2O_4)	$d := 1.000$
Common rocker (O_4C)	$a' := 0.590$	Second coupler (CD)	$b' := 0.325$
Output rocker (L_6)	$c' := 0.325$	Second ground link (O_4O_6)	$d' := 0.419$
Link 7 (L_7)	$e := 0.938$	Link 8 (L_8)	$f := 0.572$
Link 5 extension (DE)	$p := 0.823$	Angle DCE	$\delta := 7.0 \cdot deg$
Angle BO_4C	$\alpha := 128.6 \cdot deg$		

Input crank angular velocity $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CCW

Solution: See Figure 3-36 and Mathcad file P0767.

1. See problem 4-43 for the position solution. The velocity and acceleration solutions will use the same vector loop equations for links 5, 6, 7, and 8, differentiated with respect to time. This problem is suitable for a project assignment and is probably too long for an overnight assignment.

PROBLEM 7-68

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the magnitude and direction of the acceleration of point P in Figure 3-37a as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CCW. Also calculate and plot the acceleration of point P versus point A.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 0.136$	Link 3 (A to B)	$b := 1.000$
Link 4 (B to O_4)	$c := 1.000$	Link 1 (O_2 to O_4)	$d := 1.414$

Coupler point: $R_{pa} := 2.000$ $\quad \delta_3 := 0 \cdot deg$

Crank speed: $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure 3-37a and Mathcad file P0768.

1. Determine the range of motion for this Grashof crank rocker.

$$\theta_2 := 0 \cdot deg, 2 \cdot deg .. 360 \cdot deg$$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 10.3971 \qquad K_2 = 1.4140$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 7.4187$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.4140 \qquad K_5 = -7.4187$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

itude, and direction.

$sin(\theta_2))$

$_2))$

$_2))$

$\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot cos(\theta_4(\theta_2))$

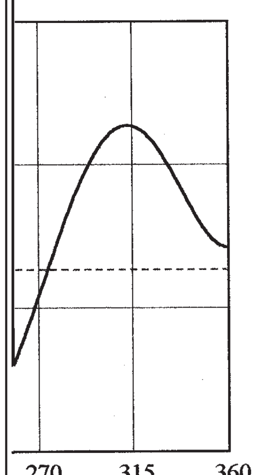
$_2)) + c \cdot \omega_4(\theta_2)^2 \cdot sin(\theta_4(\theta_2))$

$\dfrac{C'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$

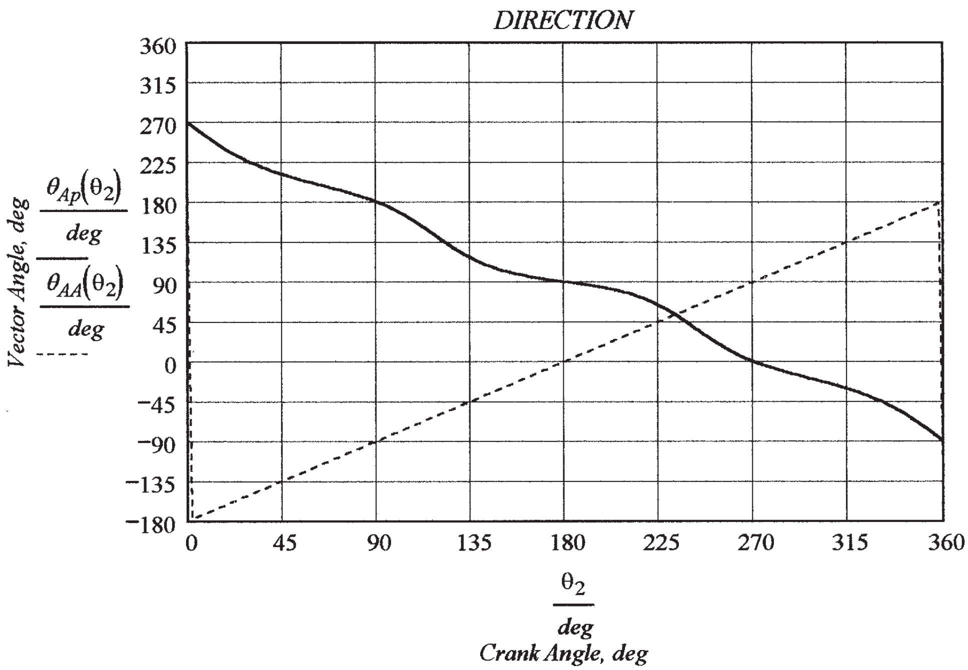
$_2) + \delta_3))$ ...

$_3(\theta_2) + \delta_3))$

$A_A(\theta_2) := |\mathbf{A_A}(\theta_2)|$



$$\theta_{AA}(\theta_2) := arg(\mathbf{A_A}(\theta_2))$$



PROBLEM 7-69

Statement: Write a computer program or use an equation solver such as Mathcad, Matlab, or TKSolver to calculate and plot the magnitude and the direction of the acceleration of point *P* in Figure 3-37b as a function of θ_2. Also calculate and plot the velocity of point *P* versus point *A*.

Given: Link lengths:

Input crank (L_2)	$a := 0.50$	First coupler (AB)	$b := 1.00$
Rocker 4 (O_4B)	$c := 1.00$	Rocker 5 (L_5)	$c' := 1.00$
Ground link (O_2O_4)	$d := 0.75$	Second coupler 6 (CD)	$b' := 1.00$
Coupler point (DP)	$p := 1.00$	Distance to O_P (O_2O_P)	$d' := 1.50$

Crank speed: $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ $\quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure 3-37b and Mathcad file P07-69.

1. Links 4, 5, *BC*, and *CD* form a parallelogram whose opposite sides remain parallel throughout the motion of the fourbar 1, 2, *AB*, 4. Define a position vector whose tail is at point *D* and whose tip is at point *P* and another whose tail is at O_4 and whose tip is at point *D*. Then, since $\mathbf{R}_5 = \mathbf{R}_{AB}$ and $\mathbf{R}_{DP} = -\mathbf{R}_4$, the position vector from O_2 to *P* is $\mathbf{P} = \mathbf{R}_1 + \mathbf{R}_{AB} - \mathbf{R}_4$. Separating this vector equation into real and imaginary parts gives the equations for the *X* and *Y* coordinates of the coupler point *P*.

$$X_P = d + b \cdot \cos(\theta_3) - c \cdot \cos(\theta_4) \qquad Y_P = b \cdot \sin(\theta_3) - c \cdot \sin(\theta_4)$$

2. Define one revolution of the input crank: $\theta_2 := 0 \cdot deg, 0.5 \cdot deg .. 360 \cdot deg$

3. Use equations 4.8a and 4.10 to calculate θ_4 as a function of θ_2 (for the crossed circuit).

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$$

$$K_1 = 1.5000 \qquad K_2 = 0.7500 \qquad K_3 = 0.8125$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2) \qquad C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$\theta_4(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right) - 2 \cdot \pi$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.7500 \qquad K_5 = -0.8125$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right) - 2 \cdot \pi$$

6. Define a local xy coordinate system with origin at O_P and with the positive x axis to the right. The coordinates of P are transformed to $x_P = X_P - d'$, $y_P = Y_P$.

$$xP(\theta_2) := d + b\cdot cos(\theta_3(\theta_2)) - c\cdot cos(\theta_4(\theta_2)) - d'$$

$$yP(\theta_2) := b\cdot sin(\theta_3(\theta_2)) - c\cdot sin(\theta_4(\theta_2))$$

7. Use equations 6.18 to calculate ω_3 and ω_4.

$$\omega_3(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4(\theta_2) - \theta_2)}{sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A'(\theta_2) := c\cdot sin(\theta_4(\theta_2)) \qquad B'(\theta_2) := b\cdot sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c\cdot cos(\theta_4(\theta_2)) \qquad E'(\theta_2) := b\cdot cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot cos(\theta_2) + b\cdot\omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) - b\cdot\omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)^2\cdot sin(\theta_4(\theta_2))$$

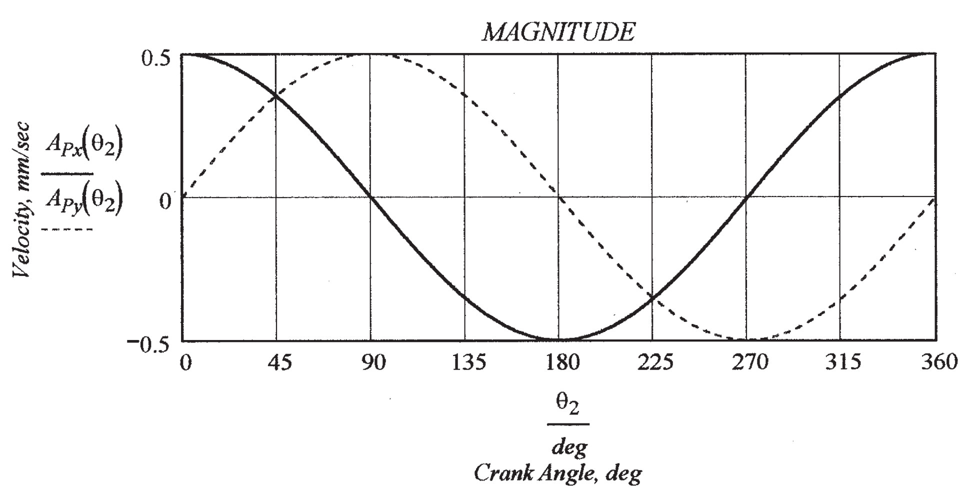
$$\alpha_3(\theta_2) := \frac{C'(\theta_2)\cdot D'(\theta_2) - A'(\theta_2)\cdot F'(\theta_2)}{A'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot D'(\theta_2)} \qquad \alpha_4(\theta_2) := \frac{C'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot F'(\theta_2)}{A'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot D'(\theta_2)}$$

9. Differentiate the position equations twice with respect to time to get the acceleration components.

$$\begin{aligned} A_{Px}(\theta_2) := &-b\cdot\left(\alpha_3(\theta_2)\cdot sin(\theta_3(\theta_2)) + \omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2))\right) \ldots \\ &+c\cdot\left(\alpha_4(\theta_2)\cdot sin(\theta_4(\theta_2)) + \omega_4(\theta_2)^2\cdot cos(\theta_4(\theta_2))\right) \end{aligned}$$

$$\begin{aligned} A_{Py}(\theta_2) := &\ b\cdot\left(\alpha_3(\theta_2)\cdot cos(\theta_3(\theta_2)) - \omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2))\right) \ldots \\ &+-c\cdot\left(\alpha_4(\theta_2)\cdot cos(\theta_4(\theta_2)) - \omega_4(\theta_2)^2\cdot sin(\theta_4(\theta_2))\right) \end{aligned}$$

$$A_P(\theta_2) := \sqrt{(A_{Px}(\theta_2))^2 + (A_{Py}(\theta_2))^2}$$



PROBLEM 7-70a

Statement: Find the angular accelerations of links 3 and 4 and the linear accelerations of points A, B and P_1 in the XY coordinate system for the linkage in Figure P7-27 in the position shown. Assume that θ_2 = 45 deg in the XY coordinate system and ω_2 = 10 rad/sec, constant. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use a graphical method.

Given: Link lengths:

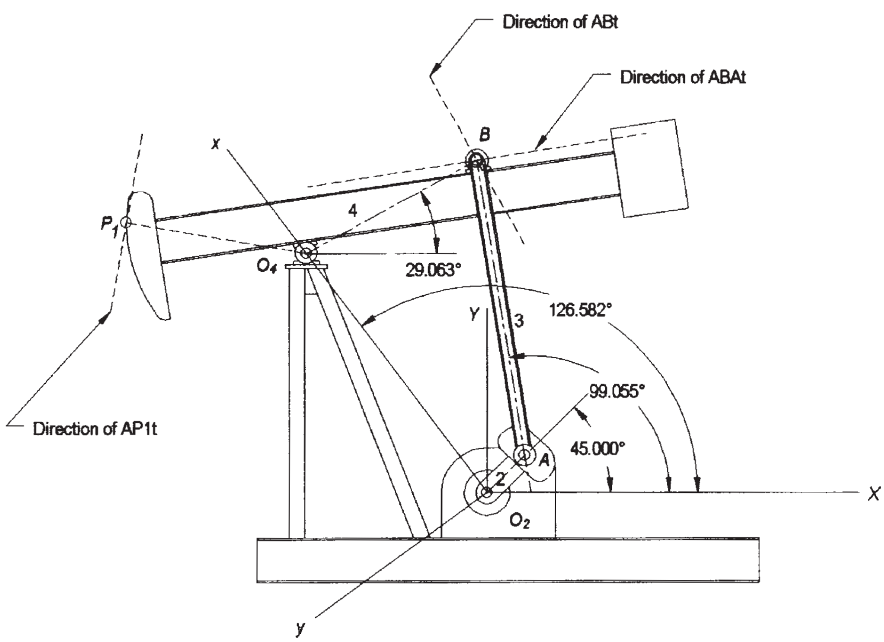
Link 2 (O_2 to A)	$a := 14.00 \cdot in$	Link 3 (A to B)	$b := 80.00 \cdot in$
Link 4 (O_4 to B)	$c := 51.26 \cdot in$		
Link 1 X-offset	$d_X := 47.5 \cdot in$	Link 1 Y-offset	$d_Y := 76.00 \cdot in - 12.00 \cdot in$
Coupler point x-offset	$p_x := 114.68 \cdot in$	Coupler point y-offset	$p_y := 33.19 \cdot in$
Crank angle:	$\theta_2 := 45 \cdot deg$		

Coordinate rotation angle $\alpha := 126.582 \cdot deg$ Global XY system to local xy system

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-27 and Mathcad file P0770a.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity acceleration of interest.



$$\theta_2 := 45 \cdot deg - \alpha$$

$$\theta_2 = -81.582\,deg$$

2. In order to solve for the accelerations at points A and B, we will need θ_3, ω_3, and θ_4. From the graphical position solution above (in the local xy coordinate system),

$$\theta_3 := 99.055 \cdot deg - \alpha \qquad \theta_3 = -27.527\,deg$$

$$\theta_4 := 29.063 \cdot deg - \alpha \qquad \theta_4 = -97.519\,deg$$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{(\sin(\theta_3 - \theta_4))} \qquad \omega_3 = -0.511\,rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a\cdot\omega_2}{c}\cdot\frac{\sin(\theta_2-\theta_3)}{(\sin(\theta_4-\theta_3))} \qquad \omega_4 = 2.353\,rad\cdot sec^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

4. For point B, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$$A_{Bn} := c\cdot\omega_4^{\,2} \qquad A_{Bn} = 283.838\,in\cdot sec^{-2}$$

$$\theta_{ABn} := \theta_4 - 180\cdot deg \qquad \theta_{ABn} = -277.519\,deg$$

$$A_{An} := a\cdot\omega_2^{\,2} \qquad A_{An} = 1400.0\,in\cdot sec^{-2}$$

$$\theta_{AAn} := \theta_2 + 180\cdot deg \qquad \theta_{AAn} = 98.418\,deg$$

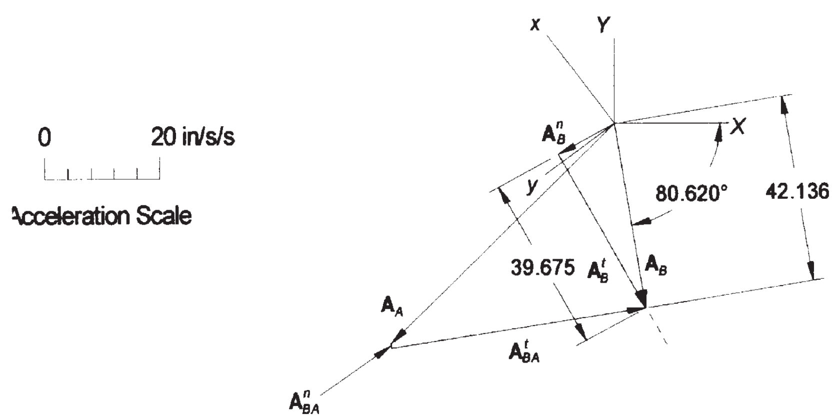
$$A_{At} := a\cdot\alpha_2 \qquad A_{At} = 0.0\,in\cdot sec^{-2}$$

$$\theta_{AAt} := \theta_2 + 90\cdot deg \qquad \theta_{AAt} = 8.418\,deg$$

$$A_{BAn} := b\cdot\omega_3^{\,2} \qquad A_{BAn} = 20.921\,in\cdot sec^{-2}$$

$$\theta_{ABAn} := \theta_3 - 180\cdot deg \qquad \theta_{ABAn} = -207.527\,deg$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A{}^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of $\theta_{AAt} + \alpha$. From the tip of $\mathbf{A}_A{}^t$, draw $\mathbf{A}_{BA}{}^n$ at an angle of $\theta_{BAn} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B{}^t$, $\mathbf{A}_B{}^t$, and $\mathbf{A}_{BA}{}^t$.



6. From the graphical solution above,

Acceleration scale factor $\quad k_a := 20.0\cdot\frac{in}{sec^2}$

$A_A := A_{An} \qquad A_A = 1400\,in\cdot sec^{-2}$ at an angle of -135.00 deg

$A_B := 42.136\cdot k_a \qquad A_B = 842.7\,in\cdot sec^{-2}$ at an angle of -80.62 deg

$A_{Bt} := 39.675\cdot k_a \qquad A_{Bt} = 793.500\,in\cdot sec^{-2}$ at an angle of -60.937 deg

$$\alpha_4 := \frac{A_{Bt}}{c} \qquad \alpha_4 = 15.480\,rad \cdot sec^{-2}$$

7. Determine the distance from O_4 to P_1 and the angle O_4P_1 makes with the x axis.

Distance O_2O_4: $d := \sqrt{d_X^{\,2} + d_Y^{\,2}}$ $\qquad d = 79.701\,in$

Distance P_1O_4: $u := \sqrt{(p_x - d)^2 + p_y^{\,2}}$ $\qquad u = 48.219\,in$

Angle O_4P_1: $\delta_4 := atan\left(\frac{p_y}{p_x - d}\right)$ $\qquad \delta_4 = 43.497\,deg$

4. Determine the acceleration at point P_1.

$A_{P1t} := u \cdot \alpha_4$ $\qquad A_{P1t} = 746.431\,in \cdot sec^{-2}$ $\qquad$ at an angle of 80.079 deg

$A_{P1n} := u \cdot \omega_4^{\,2}$ $\qquad A_{P1n} = 267.001\,in \cdot sec^{-2}$ $\qquad$ at an angle of -9.921 deg

$A_{P1} := \sqrt{A_{P1t}^{\,2} + A_{P1n}^{\,2}}$ $\qquad A_{P1} = 792.748\,in \cdot sec^{-2}$ $\qquad$ at an angle of 60.397 deg

$$atan\left(\frac{A_{P1t}}{A_{P1n}}\right) - 9.921 \cdot deg = 60.397\,deg$$

PROBLEM 7-70b

Statement: Find the angular accelerations of links 3 and 4 and the linear accelerations of points A, B and P_1 in the XY coordinate system for the linkage in Figure P7-27 in the position shown. Assume that θ_2 = 45 deg in the XY coordinate system and ω_2 = 10 rad/sec, constant. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the xy coordinate system. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A) $a := 14.00 \cdot in$ Link 3 (A to B) $b := 80.00 \cdot in$

Link 4 (O_4 to B) $c := 51.26 \cdot in$

Link 1 X-offset $d_X := 47.5 \cdot in$ Link 1 Y-offset $d_Y := 76.00 \cdot in - 12.00 \cdot in$

Coupler point x-offset $p_x := 114.68 \cdot in$ Coupler point y-offset $p_y := 33.19 \cdot in$

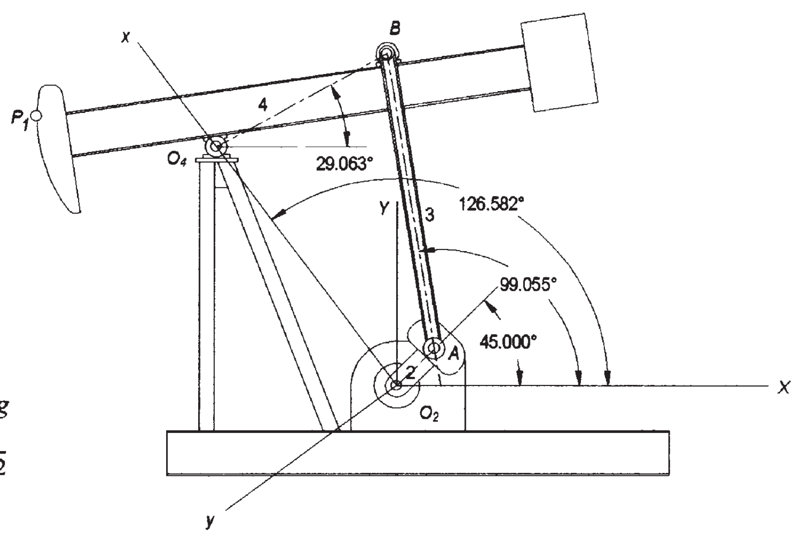
Crank angle: $\theta_{2XY} := 45 \cdot deg$

Coordinate rotation angle $\alpha := 126.582 \cdot deg$ Global XY system to local xy system

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-27 and Mathcad file P0770b.

1. Draw the linkage to scale and label it.



$\theta_2 := \theta_{2XY} - \alpha$

$\theta_2 = -81.582\,deg$

$d := \sqrt{d_X^2 + d_Y^2}$

$d = 79.701\,in$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$K_1 := \dfrac{d}{a}$ $K_2 := \dfrac{d}{c}$

$K_1 = 5.6929$ $K_2 = 1.5548$

$K_3 := \dfrac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)}$ $K_3 = 1.9340$

$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$

$B := -2 \cdot \sin(\theta_2)$

$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$

$A = -3.8401 \quad B = 1.9785 \quad C = 7.2529$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2\cdot\left(atan2\left(2\cdot A, -B + \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) \qquad \theta_4 = 262.482\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.9963 \qquad K_5 = -4.6074$$

$$D := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad D = -10.0081$$

$$E := -2\cdot sin(\theta_2) \qquad E = 1.9785$$

$$F := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5 \qquad F = 1.0849$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3 := 2\cdot\left(atan2\left(2\cdot D, -E + \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_3 = 332.475\,deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4 - \theta_2)}{sin(\theta_3 - \theta_4)} \qquad \omega_3 = -0.511\,\frac{rad}{sec}$$

$$\omega_4 := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3)}{sin(\theta_4 - \theta_3)} \qquad \omega_4 = 2.353\,\frac{rad}{sec}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction (in the global coordinate system).

$$\mathbf{A_A} := a\cdot\alpha_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) - a\cdot\omega_2^{\,2}\cdot(cos(\theta_2) + j\cdot sin(\theta_2))$$

$$\mathbf{A_A} = -205 + 1385i\,\frac{in}{sec^2} \qquad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := arg(\mathbf{A_A}) + \alpha$$

The acceleration of pin A is $\quad A_A = 1400\,\frac{in}{sec^2} \quad$ at $\quad \theta_{AA} = 225\,deg$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c\cdot sin(\theta_4) \qquad B := b\cdot sin(\theta_3) \qquad D := c\cdot cos(\theta_4) \qquad E := b\cdot cos(\theta_3)$$

$$A = -1.291\times 10^3\,mm \quad B = -939.044\,mm \quad D = -170.343\,mm \quad E = 1.802\times 10^3\,mm$$

$$C := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot cos(\theta_2) + b\cdot\omega_3^{\,2}\cdot cos(\theta_3) - c\cdot\omega_4^{\,2}\cdot cos(\theta_4)$$

$$C = 260.638\,in\cdot sec^{-2}$$

$$F := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) - b\cdot\omega_3^{\,2}\cdot sin(\theta_3) + c\cdot\omega_4^{\,2}\cdot sin(\theta_4)$$

$$F = 1.113\times 10^3\,in\cdot sec^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_3 = -14.227 \frac{rad}{sec^2} \qquad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_4 = -15.479 \frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point B for the crossed circuit.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-\sin(\theta_4) + j \cdot \cos(\theta_4)\right) - c \cdot \omega_4^2 \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right)$$

$$\mathbf{A_B} = -749 + 385i \frac{in}{sec^2} \qquad A_B := \left|\mathbf{A_B}\right| \qquad \theta_{AB} := arg(\mathbf{A_B}) + \alpha$$

The acceleration of pin B is $A_B = 842.7 \frac{in}{sec^2}$ at $\theta_{AB} = 279.4\,deg$ (Global)

10. Use equation 7.31 to determine the acceleration of the point P_1 on link 4.

$$u := \sqrt{(p_x - d)^2 + p_y^2} \qquad u = 48.219\,in$$

$$\delta_4 := 360 \cdot deg - \theta_4 + atan\left(\frac{p_y}{p_x - d}\right) \qquad \delta_4 = 141.014\,deg$$

$$\mathbf{A_{P1}} := u \cdot \alpha_4 \cdot \left(-\sin(\theta_4 + \delta_4) + j \cdot \cos(\theta_4 + \delta_4)\right) - u \cdot \omega_4^2 \cdot \left(\cos(\theta_4 + \delta_4) + j \cdot \sin(\theta_4 + \delta_4)\right)$$

$$\mathbf{A_{P1}} = 320 - 725i \frac{in}{sec^2} \qquad A_{P1} := \left|\mathbf{A_{P1}}\right| \qquad \theta_{AP1} := arg(\mathbf{A_{P1}}) + \alpha$$

The acceleration of point P_1 is $A_{P1} = 792.7 \frac{in}{sec^2}$ at $\theta_{AP1} = 60.39\,deg$ (Global)

PROBLEM 7-71

Statement: Find the angular accelerations of links 3 and 4 and the linear accelerations of points *A*, *B* and P_1 in the *XY* coordinate system for the linkage in Figure P7-27 in the position shown. Assume that $\theta_2 =$ 45 deg in the *XY* coordinate system and $\omega_2 = 10$ rad/sec, constant. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the *xy* coordinate system. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A) $a := 14.00 \cdot in$ Link 3 (A to B) $b := 80.00 \cdot in$

Link 4 (O_2 to B) $c := 51.26 \cdot in$

Link 1 X-offset $d_X := 47.5 \cdot in$ Link 1 Y-offset $d_Y := 76.00 \cdot in - 12.00 \cdot in$

Link 1 (O_2 to O_2) $d := \sqrt{d_X^{\,2} + d_Y^{\,2}}$ $d = 79.701\ in$

Coupler point x-offset $p_x := 114.68 \cdot in$ Coupler point y-offset $p_y := 33.19 \cdot in$

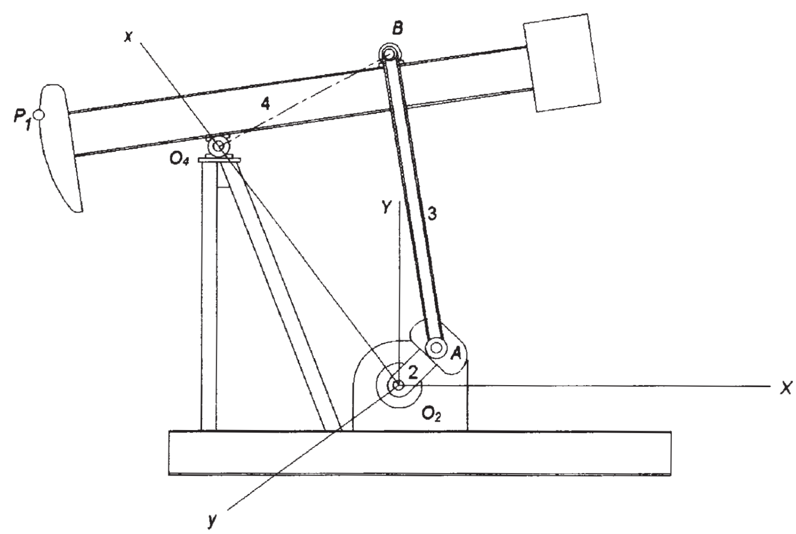
Crank angle: $\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

Coordinate rotation angle $\alpha := 126.582 \cdot deg$ Global *XY* system to local *xy* system

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ CCW $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-27 and Mathcad file P0771.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 5.6929 \qquad K_2 = 1.5548$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.9340$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.9963 \qquad K_5 = -4.6074$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the crossed circuit.

$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4(\theta_2) - \theta_2)}{sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$

7. Use equations 7.12 to determine the angular acceleration of link 4.

$$A'(\theta_2) := c\cdot sin(\theta_4(\theta_2)) \qquad B'(\theta_2) := b\cdot sin(\theta_3(\theta_2))$$

$$D'(\theta_2) := c\cdot cos(\theta_4(\theta_2)) \qquad E'(\theta_2) := b\cdot cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2^{\,2}\cdot cos(\theta_2) + b\cdot\omega_3(\theta_2)^2\cdot cos(\theta_3(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2^{\,2}\cdot sin(\theta_2) - b\cdot\omega_3(\theta_2)^2\cdot sin(\theta_3(\theta_2)) + c\cdot\omega_4(\theta_2)^2\cdot sin(\theta_4(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot F'(\theta_2)}{A'(\theta_2)\cdot E'(\theta_2) - B'(\theta_2)\cdot D'(\theta_2)}$$

8. Use equation 7.31 to determine and plot the acceleration of the point P_1 on link 4.

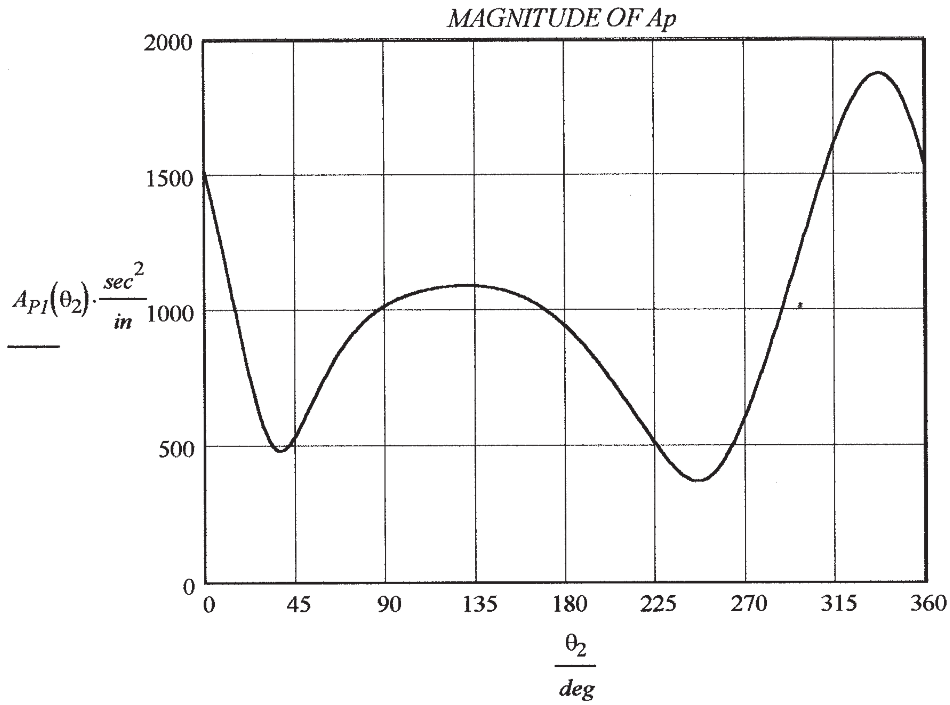
$$u := \sqrt{(p_x - d)^2 + p_y^{\,2}} \qquad u = 48.219\,in$$

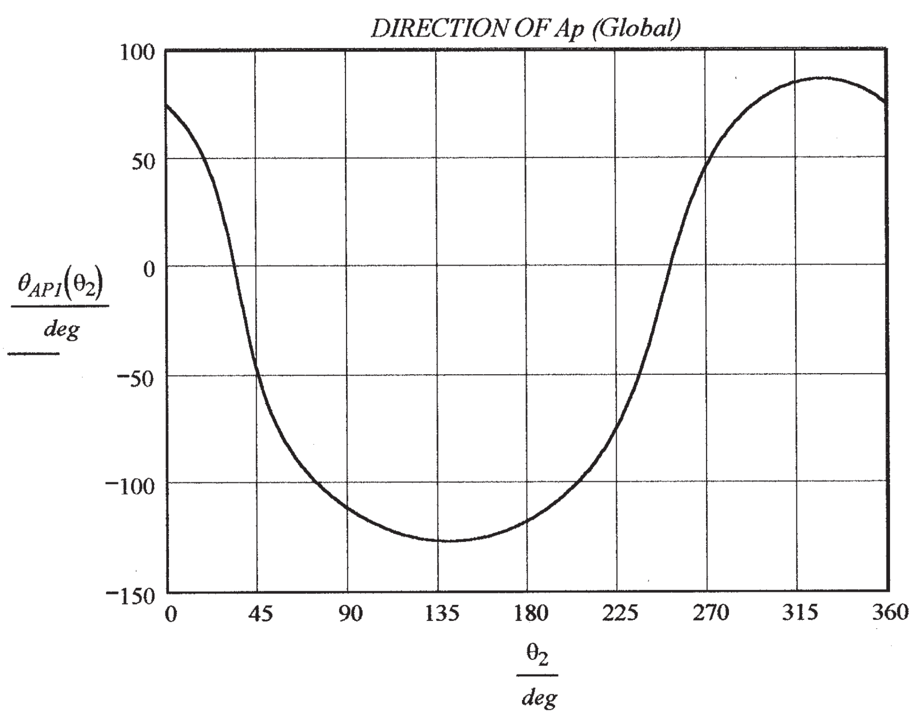
$$\delta_4 := 141.067\cdot deg$$

$$\mathbf{A_{P1}}(\theta_2) := u\cdot\alpha_4(\theta_2)\cdot(-sin(\theta_4(\theta_2) + \delta_4) + j\cdot cos(\theta_4(\theta_2) + \delta_4)) \ldots$$
$$+ -u\cdot\omega_4(\theta_2)^2\cdot(cos(\theta_4(\theta_2) + \delta_4) + j\cdot sin(\theta_4(\theta_2) + \delta_4))$$

$A_{P1}(\theta_2) := |\mathbf{A_{P1}}(\theta_2)| \qquad \theta_{AP1}(\theta_2) := arg(\mathbf{A_{P1}}(\theta_2)) + \alpha$

$\theta_{AP1}(\theta_2) := if(\theta_{AP1}(\theta_2) > 100 \cdot deg, \theta_{AP1}(\theta_2) - 2 \cdot \pi, \theta_{AP1}(\theta_2))$





$A_{P1}(-81.582 \cdot deg) = 792.706 \frac{in}{sec^2} \qquad \theta_{AP1}(-81.582 \cdot deg) = 60.447\, deg$

PROBLEM 7-72a

Statement: Find the angular accelerations of links 3 and 4 and the linear acceleration of point P in the XY coordinate system for the linkage in Figure P7-28 in the position shown. Assume that $\theta_2 = -94.121$ deg in the XY coordinate system, $\omega_2 = 1$ rad/sec, and $\alpha_2 = 10$ rad/sec^2. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0$ deg. Use a graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 9.17 \cdot in$	Link 3 (A to B)	$b := 12.97 \cdot in$
Link 4 (O_4 to B)	$c := 9.57 \cdot in$		
Link 1 X-offset	$d_X := 2.79 \cdot in$	Link 1 Y-offset	$d_Y := -6.95 \cdot in$

Coupler point data: $p := 15.00 \cdot in$ $\delta_3 := 0 \cdot deg$

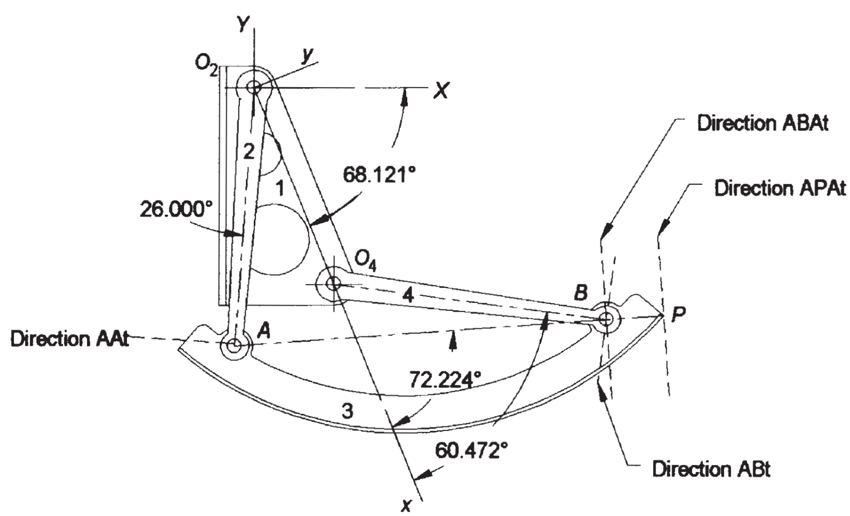
Crank angle: $\theta_2 := -26.000 \cdot deg$

Coordinate rotation angle $\alpha := -68.121 \cdot deg$ Global XY system to local xy system

Input crank angular velocity $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CW $\alpha_2 := 10 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-28 and Mathcad file P0772a.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A and B, we will need θ_3, ω_3, and θ_4. From the graphical position solution above (in the local xy coordinate system),

$\theta_3 := 72.224 \cdot deg$ $\theta_3 = 72.224\, deg$

$\theta_4 := 60.472 \cdot deg$ $\theta_4 = 60.472\, deg$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{(\sin(\theta_3 - \theta_4))} \qquad \omega_3 = 3.465\, rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{(\sin(\theta_4 - \theta_3))} \qquad \omega_4 = 4.656\, rad \cdot sec^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

4. For point B, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$$A_{Bn} := c \cdot \omega_4{}^2 \qquad A_{Bn} = 207.476\, in \cdot sec^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot deg + \alpha \qquad \theta_{ABn} = 172.351\, deg$$

$$A_{An} := a \cdot \omega_2{}^2 \qquad A_{An} = 9.170\, in \cdot sec^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot deg + \alpha \qquad \theta_{AAn} = 85.879\, deg$$

$$A_{At} := a \cdot \alpha_2 \qquad A_{At} = 91.700\, in \cdot sec^{-2}$$

$$\theta_{AAt} := \theta_2 + 90 \cdot deg + \alpha \qquad \theta_{AAt} = -4.121\, deg$$

$$A_{BAn} := b \cdot \omega_3{}^2 \qquad A_{BAn} = 155.694\, in \cdot sec^{-2}$$

$$\theta_{ABAn} := \theta_3 + 180 \cdot deg + \alpha \qquad \theta_{ABAn} = 184.103\, deg$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A{}^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of $\theta_{AAt} + \alpha$. From the tip of $\mathbf{A}_A{}^t$, draw $\mathbf{A}_{BA}{}^n$ at an angle of $\theta_{BAn} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B{}^t$, $\mathbf{A}_B{}^t$, and $\mathbf{A}_{BA}{}^t$.

6. From the graphical solution at right,

Acceleration scale factor $\qquad k_a := 10.0 \cdot \dfrac{in}{sec^2}$

$$A_A := 0.922 \cdot k_a \qquad A_A = 9.220\, in \cdot sec^{-2}$$

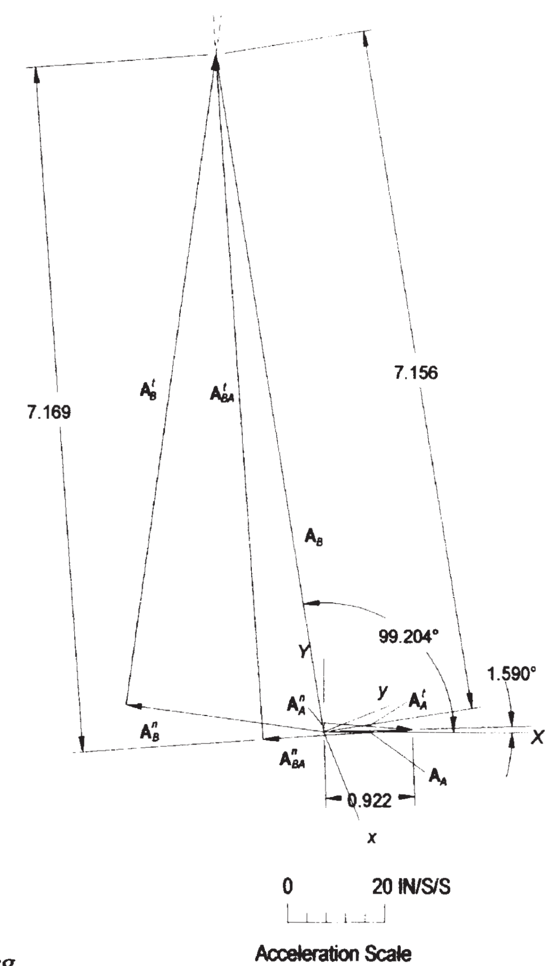
at an angle of 1.590 deg

$$A_B := 7.156 \cdot k_a \qquad A_B = 71.560\, in \cdot sec^{-2}$$

at an angle of 99.204 deg

$$A_{BAt} := 7.169 \cdot k_a \qquad A_{BAt} = 71.690\, in \cdot sec^{-2}$$

$$\alpha_3 := \frac{A_{BAt}}{b} \qquad \alpha_3 = 5.527\, rad \cdot sec^{-2}$$



7. For point P, equation 7.4 becomes:
$\mathbf{A}_P = \mathbf{A}_A + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$, where

$$A_{PAn} := p \cdot \omega_3{}^2 \qquad A_{PAn} = 180.062\, in \cdot sec^{-2}$$

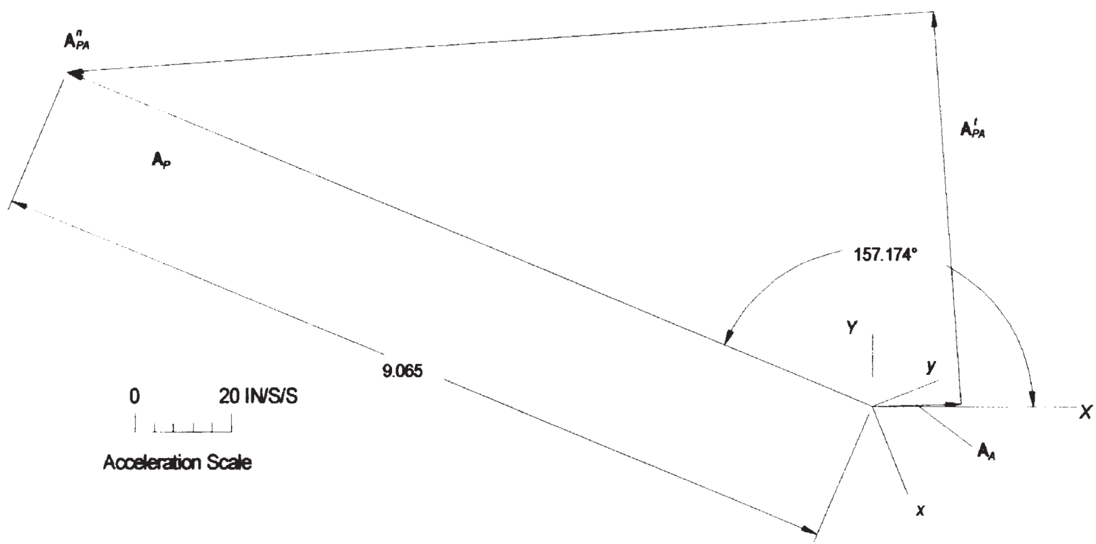
$$\theta_{APAn} := \theta_3 + \delta_3 + 180 \cdot deg + \alpha$$

$$\theta_{APAn} = 184.103\, deg$$

$$A_{PAt} := p \cdot \alpha_3 \qquad A_{PAt} = 82.911\, in \cdot sec^{-2}$$

$$\theta_{APAt} := \theta_{APAn} - 90 \cdot deg \qquad \theta_{APAt} = 94.103\, deg$$

8. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction.



9. From the graphical solution above,

Acceleration scale factor $k_a := 10.0 \cdot \frac{in}{sec^2}$

$A_P := 9.065 \cdot k_a$ $A_P = 90.650\ in \cdot sec^{-2}$ at an angle of 157.174 deg

PROBLEM 7-72b

Statement: Find the angular accelerations of links 3 and 4 and the linear acceleration of point P in the XY coordinate system for the linkage in Figure P7-28 in the position shown. Assume that $\theta_2 = -94.121$ deg in the XY coordinate system, $\omega_2 = 1$ rad/sec, and $\alpha_2 = 10$ rad/sec^2. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0$ deg. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A) $a := 9.174 \cdot in$ Link 3 (A to B) $b := 12.971 \cdot in$

Link 4 (O_4 to B) $c := 9.573 \cdot in$

Link 1 X-offset $d_X := 2.790 \cdot in$ Link 1 Y-offset $d_Y := -6.948 \cdot in$

Link 1(O_2 to O_4) $d := \sqrt{d_X^{\,2} + d_Y^{\,2}}$ $d = 7.487\,in$

Coupler point data: $R_{pa} := 15.00 \cdot in$ $\delta_3 := 0 \cdot deg$

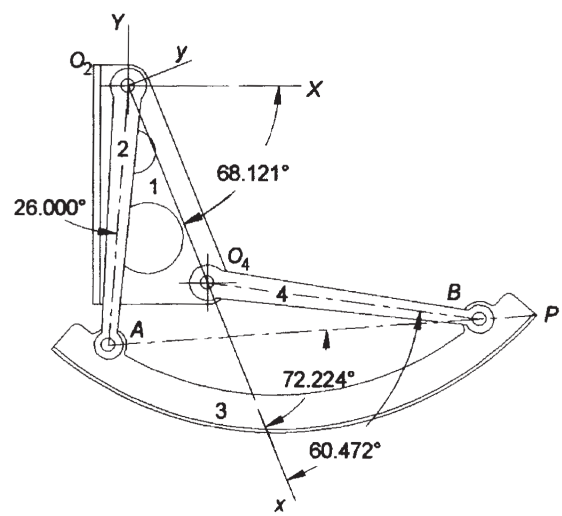
Crank angle: $\theta_2 := -26.000 \cdot deg$

Coordinate rotation angle $\alpha := -68.121 \cdot deg$ Global XY system to local xy system

Input crank angular velocity $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ CW $\alpha_2 := 10 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-28 and Mathcad file P0772b.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.8161 \qquad K_2 = 0.7821$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.3622$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$A = -0.2581 \qquad B = 0.8767 \qquad C = -0.4234$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4 := 2\cdot\left(atan2\left(2\cdot A, -B + \sqrt{B^2 - 4\cdot A\cdot C}\right)\right) \qquad \theta_4 = -299.512\,deg$$

$$\theta_4 := \theta_4 + 360\cdot deg \qquad \theta_4 = 60.488\,deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.5772 \qquad K_5 = -0.9111$$

$$D := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5 \qquad D = -0.3096$$

$$E := -2\cdot sin(\theta_2) \qquad E = 0.8767$$

$$F := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5 \qquad F = -0.4749$$

5. Use equation 4.13 to find values of θ_3 .

$$\theta_3 := 2\cdot\left(atan2\left(2\cdot D, -E + \sqrt{E^2 - 4\cdot D\cdot F}\right)\right) \qquad \theta_3 = -287.764\,deg$$

$$\theta_3 := \theta_3 + 360\cdot deg \qquad \theta_3 = 72.236\,deg$$

6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_3 := \frac{a\cdot\omega_2}{b}\cdot\frac{sin(\theta_4 - \theta_2)}{sin(\theta_3 - \theta_4)} \qquad \omega_3 = 3.467\frac{rad}{sec}$$

$$\omega_4 := \frac{a\cdot\omega_2}{c}\cdot\frac{sin(\theta_2 - \theta_3)}{sin(\theta_4 - \theta_3)} \qquad \omega_4 = 4.658\frac{rad}{sec}$$

7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A_A} := a\cdot\alpha_2\cdot\left(-sin(\theta_2) + j\cdot cos(\theta_2)\right) - a\cdot\omega_2{}^2\cdot\left(cos(\theta_2) + j\cdot sin(\theta_2)\right)$$

$$\mathbf{A_A} = 32.0 + 86.5j\,\frac{in}{sec^2} \qquad A_A := \left|\mathbf{A_A}\right| \qquad \theta_{AA} := arg(\mathbf{A_A}) + \alpha$$

The acceleration of pin A is $A_A = 92.198\,\frac{in}{sec^2}$ at $\theta_{AA} = 1.590\,deg$ (Global)

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$$A := c\cdot sin(\theta_4) \qquad B := b\cdot sin(\theta_3) \qquad D := c\cdot cos(\theta_4) \qquad E := b\cdot cos(\theta_3)$$

$$A = 8.331\,in \qquad B = 12.353\,in \qquad D = 4.716\,in \qquad E = 3.957\,in$$

$$C := a\cdot\alpha_2\cdot sin(\theta_2) + a\cdot\omega_2{}^2\cdot cos(\theta_2) + b\cdot\omega_3{}^2\cdot cos(\theta_3) - c\cdot\omega_4{}^2\cdot cos(\theta_4)$$

$$C = -86.722\,in\cdot sec^{-2}$$

$$F := a\cdot\alpha_2\cdot cos(\theta_2) - a\cdot\omega_2{}^2\cdot sin(\theta_2) - b\cdot\omega_3{}^2\cdot sin(\theta_3) + c\cdot\omega_4{}^2\cdot sin(\theta_4)$$

$$F = 118.753\,in\cdot sec^{-2}$$

$$\alpha_3 := \frac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_3 = 55.307 \frac{rad}{sec^2} \qquad \alpha_4 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_4 = 71.596 \frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point *B*.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-sin\left(\theta_4\right) + j \cdot cos\left(\theta_4\right)\right) - c \cdot \omega_4^{\,2} \cdot \left(cos\left(\theta_4\right) + j \cdot sin\left(\theta_4\right)\right)$$

$$\mathbf{A_B} = -698.8 + 156.9j \frac{in}{sec^2} \qquad A_B := \left|\mathbf{A_B}\right| \qquad \theta_{AB} := arg\left(\mathbf{A_B}\right) + \alpha$$

The acceleration of pin *B* is $A_B = 716.18 \frac{in}{sec^2}$ at $\theta_{AB} = 99.228\,deg$

10. Use equations 7.32 to find the acceleration of the point *P*.

$$\mathbf{A_{PA}} := R_{pa} \cdot \alpha_3 \cdot \left(-sin\left(\theta_3 + \delta_3\right) + \mathrm{j} \cdot cos\left(\theta_3 + \delta_3\right)\right) \ldots$$
$$+ -R_{pa} \cdot \omega_3^{\,2} \cdot \left(cos\left(\theta_3 + \delta_3\right) + \mathrm{j} \cdot sin\left(\theta_3 + \delta_3\right)\right)$$

$$\mathbf{A_P} := \mathbf{A_A} + \mathbf{A_{PA}}$$

$$A_P := \left|\mathbf{A_P}\right| \qquad A_P = 830 \frac{in}{sec^2} \qquad arg\left(\mathbf{A_P}\right) + \alpha = 100.214\,deg$$

PROBLEM 7-73

Statement: For the linkage in Figure P7-28, write a computer program or use an equation solver to calculate and plot the angular velocity and acceleration of links 3 and 4, and the magnitude and direction of the velocity and acceleration of point P as a function of θ_2 through its possible range of motion starting at the position shown. The position of the coupler point P on link 3 with respect to point A is: $p = 15.00$, $\delta_3 = 0$ deg. Assume that, at t = 0, $\theta_2 = -94.121$ deg in the XY coordinate system, $\omega_2 = 0$, and $\alpha_2 = 10$ rad/sec^2, constant.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 9.174 \cdot in$	Link 3 (A to B)	$b := 12.971 \cdot in$
Link 4 (O_4 to B)	$c := 9.573 \cdot in$		
Link 1 X-offset	$d_X := 2.790 \cdot in$	Link 1 Y-offset	$d_Y := -6.948 \cdot in$
Link 1(O_2 to O_4)	$d := \sqrt{d_X^{\,2} + d_Y^{\,2}}$	$d = 7.487\,in$	

Coupler point data: $R_{pa} := 15.00 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

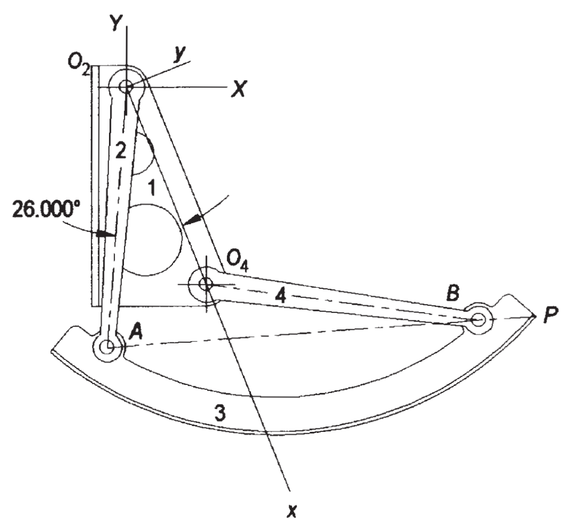
Crank angle: $\theta_2 := -26.000 \cdot deg, -25.9 \cdot deg .. -15 \cdot deg$ $\quad \theta_{20} := -26 \cdot deg$

Coordinate rotation angle $\alpha := -68.121 \cdot deg$ Global XY system to local xy system

Input crank angular velocity $\omega_{20} := 0 \cdot rad \cdot sec^{-1}$ $\quad \alpha_2 := 10 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-28 and Mathcad file P0773.

1. Draw the linkage to scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.8161 \qquad K_2 = 0.7821$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.3622$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1)\cdot cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_4(\theta_2) := 2\cdot\left(atan2\left(2\cdot A(\theta_2), -B(\theta_2) + \sqrt{B(\theta_2)^2 - 4\cdot A(\theta_2)\cdot C(\theta_2)}\right)\right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2\cdot a\cdot b)} \qquad K_4 = 0.5772 \qquad K_5 = -0.9111$$

$$D(\theta_2) := cos(\theta_2) - K_1 + K_4\cdot cos(\theta_2) + K_5$$

$$E(\theta_2) := -2\cdot sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1)\cdot cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 .

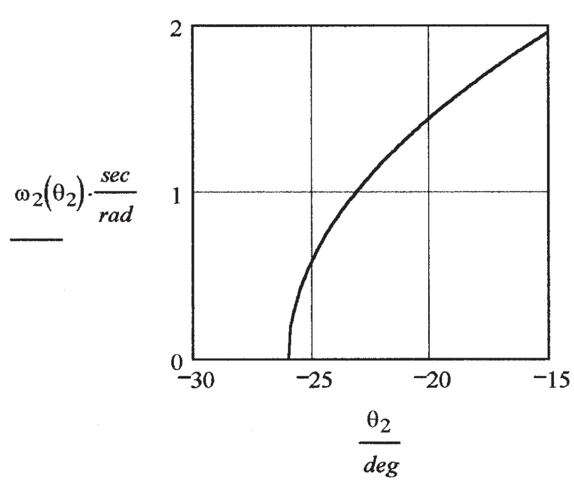
$$\theta_3(\theta_2) := 2\cdot\left(atan2\left(2\cdot D(\theta_2), -E(\theta_2) + \sqrt{E(\theta_2)^2 - 4\cdot D(\theta_2)\cdot F(\theta_2)}\right)\right)$$

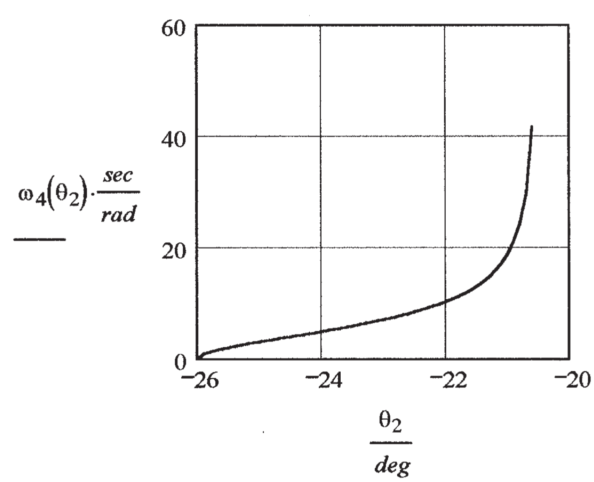
6. Determine the angular velocity of links 3 and 4 using equations 6.18.

$$\omega_2(\theta_2) := \sqrt{2\cdot(\theta_2 - \theta_{20})\cdot\alpha_2}$$

$$\omega_3(\theta_2) := \frac{a\cdot\omega_2(\theta_2)}{b}\cdot\frac{sin(\theta_4(\theta_2) - \theta_2)}{sin(\theta_3(\theta_2) - \theta_4(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a\cdot\omega_2(\theta_2)}{c}\cdot\frac{sin(\theta_2 - \theta_3(\theta_2))}{sin(\theta_4(\theta_2) - \theta_3(\theta_2))}$$





7. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A_A}(\theta_2) := a\cdot\alpha_2\cdot(-sin(\theta_2) + j\cdot cos(\theta_2)) - a\cdot\omega_2(\theta_2)^2\cdot(cos(\theta_2) + j\cdot sin(\theta_2))$$

8. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

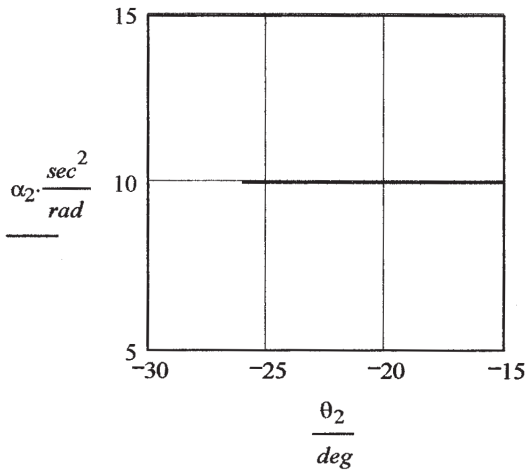
$$A'(\theta_2) := c\cdot sin(\theta_4(\theta_2)) \qquad B'(\theta_2) := b\cdot sin(\theta_3(\theta_2))$$

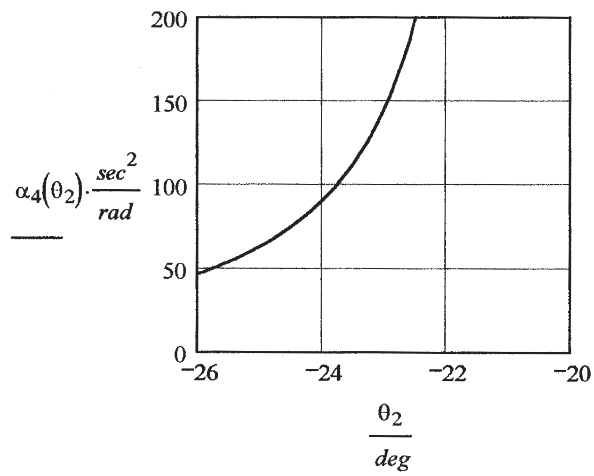
$$D'(\theta_2) := c \cdot \cos(\theta_4(\theta_2)) \qquad E'(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C'(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2(\theta_2)^2 \cdot \cos(\theta_2) + b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_4(\theta_2))$$

$$F'(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2(\theta_2)^2 \cdot \sin(\theta_2) - b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_4(\theta_2))$$

$$\alpha_3(\theta_2) := \frac{C'(\theta_2) \cdot D'(\theta_2) - A'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)} \qquad \alpha_4(\theta_2) := \frac{C'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot F'(\theta_2)}{A'(\theta_2) \cdot E'(\theta_2) - B'(\theta_2) \cdot D'(\theta_2)}$$





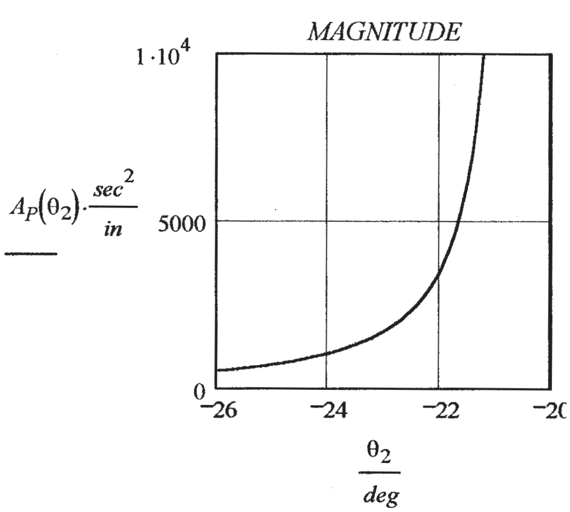
9. Use equations 7.32 to find the acceleration of the point P.

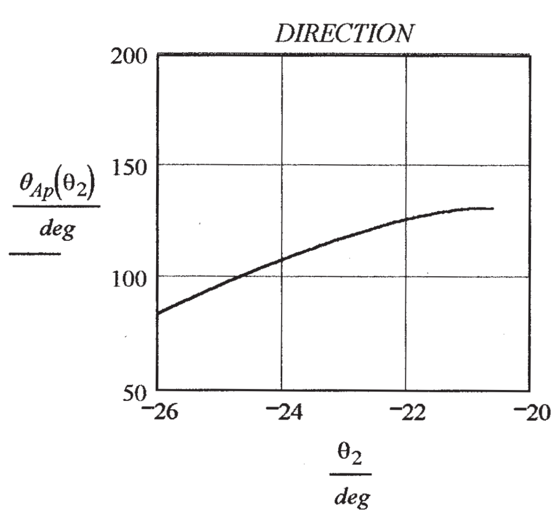
$$\mathbf{A_{PA}}(\theta_2) := R_{pa} \cdot \alpha_3(\theta_2) \cdot \left(-\sin(\theta_3(\theta_2) + \delta_3) + j \cdot \cos(\theta_3(\theta_2) + \delta_3)\right) \ldots$$
$$+ -R_{pa} \cdot \omega_3(\theta_2)^2 \cdot \left(\cos(\theta_3(\theta_2) + \delta_3) + j \cdot \sin(\theta_3(\theta_2) + \delta_3)\right)$$

$$\mathbf{A_P}(\theta_2) := \mathbf{A_A}(\theta_2) + \mathbf{A_{PA}}(\theta_2)$$

$$A_P(\theta_2) := \left|\mathbf{A_P}(\theta_2)\right| \qquad \theta_{Ap}(\theta_2) := \arg(\mathbf{A_P}(\theta_2)) + \alpha$$

$$\theta_{Ap}(\theta_2) := if\left(\theta_{Ap}(\theta_2) < 0, \theta_{Ap}(\theta_2) + 2 \cdot \pi, \theta_{Ap}(\theta_2)\right)$$



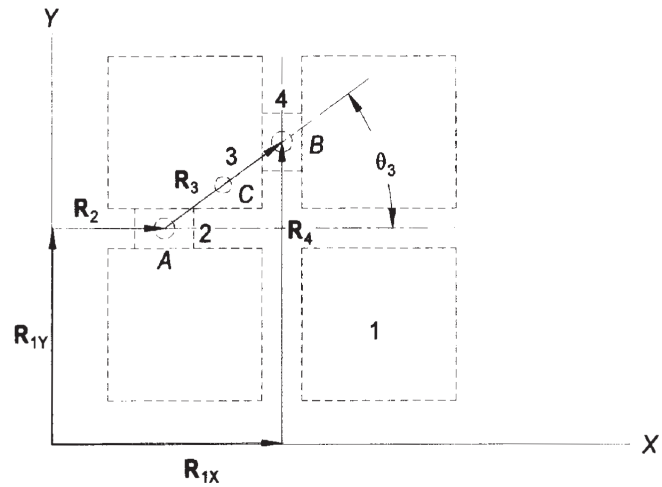


PROBLEM 7-74

Statement: Derive analytical expressions for the accelerations of points A and B in Figure P7-29 as a function of θ_3, ω_3, α_3 and the length AB of link 3. Use a vector loop equation.

Solution: See Figure P7-29 and Mathcad file P0774.

1. Establish the global XY system such that the coordinates of the intersection of the slot centerlines is at (d_X, d_Y). Then, define position vectors $\mathbf{R}_{1X}$, $\mathbf{R}_{1Y}$, $\mathbf{R}_2$, $\mathbf{R}_3$, and $\mathbf{R}_4$ as shown below.



2. Write the vector loop equation: $\mathbf{R}_{1Y} + \mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_{1X} - \mathbf{R}_4 = 0$ then substitute the complex number notation for each position vector. The equation then becomes:

$$d_Y \cdot e^{j\left(\frac{\pi}{2}\right)} + a \cdot e^{j(0)} + c \cdot e^{j(\theta_3)} - d_X \cdot e^{j(0)} - b \cdot e^{j\left(\frac{\pi}{2}\right)} = 0$$

3. Differentiate this equation with respect to time.

$$\frac{d}{dt}a + j \cdot c \cdot \omega_3 \cdot e^{j(\theta_3)} - \frac{d}{dt}b \cdot e^{j\left(\frac{\pi}{2}\right)} = 0$$

3. Substituting the Euler identity into this equation gives:

$$V_a + jc \cdot \omega_3 \cdot \left(\cos(\theta_3) + j \cdot \sin(\theta_3)\right) - V_b \cdot j = 0$$

4. Separate this equation into its real (x component) and imaginary (y component) parts, setting each equal to zero.

$$V_a - c \cdot \omega_3 \cdot \sin(\theta_3) = 0 \qquad c \cdot \omega_3 \cdot \cos(\theta_3) - V_b = 0$$

5. Solve for the two unknowns V_a and V_b in terms of the independent variables θ_3 and ω_3

$$V_a = c \cdot \omega_3 \cdot \sin(\theta_3) \qquad V_b = c \cdot \omega_3 \cdot \cos(\theta_3)$$

6. Differentiate again with respect to time to get the accelerations.

$$A_a = c \cdot \omega_3^{\,2} \cdot \cos(\theta_3) \qquad A_b = -c \cdot \omega_3^{\,2} \cdot \sin(\theta_3)$$

PROBLEM 7-75

Statement: The linkage in Figure P7-30a has link 2 at 120 deg in the global *XY* coordinate system. Find α_6 and $\mathbf{A}_D$ in the global *XY* coordinate system for the position shown if ω_2 = 10 rad/sec CCW and α_2 = 50 rad/sec^2 CW. Use the acceleration difference graphical method.

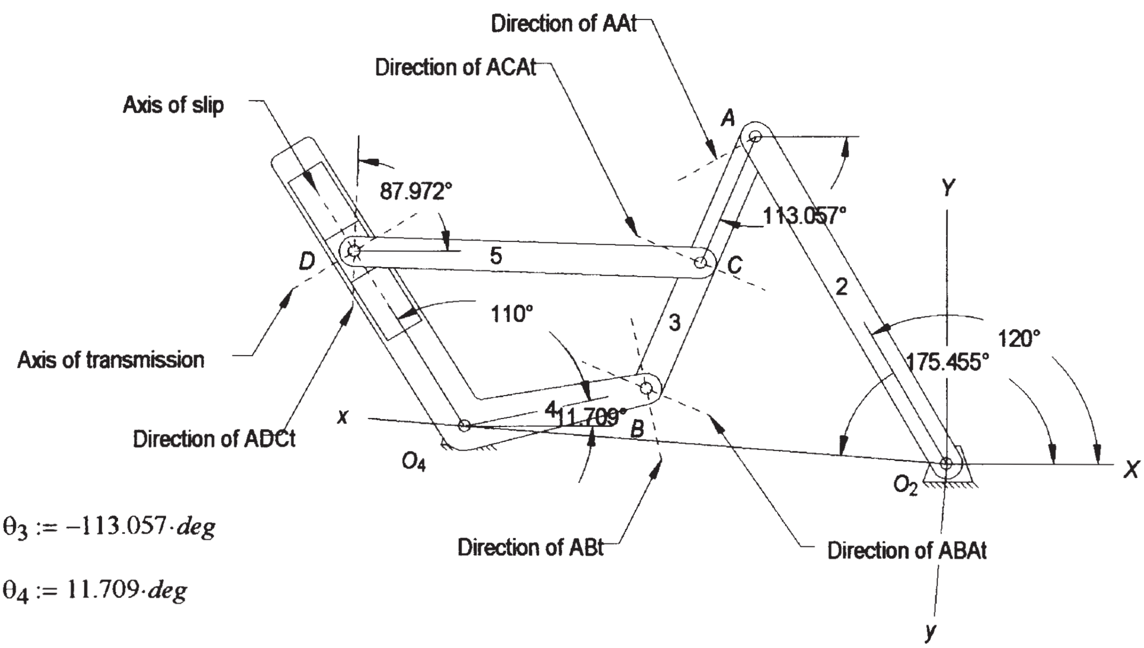
Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot in$	Link 3 (A to B)	$b := 4.50 \cdot in$
Link 4 (O_4 to B)	$c := 3.00 \cdot in$	Link 5 (C to D)	$e := 5.60 \cdot in$
Link 3 (A to C)	$p := 2.25 \cdot in$	Link 4 (O_4 to D)	$f := 3.382 \cdot in$
Link 1 X-offset	$d_X := -7.80 \cdot in$	Link 1 Y-offset	$d_Y := 0.62 \cdot in$

Angle ACB	$\delta_3 := 0.0 \cdot deg$	Angle BO_4D	$\delta_4 := 110.0 \cdot deg$
Input rocker angle:	$\theta_2 := 120 \cdot deg$	Global XY system	
Input crank angular velocity	$\omega_2 := 10 \cdot rad \cdot sec^{-1}$	CCW	$\alpha_2 := -50 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-30a and Mathcad file P0775.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



$\theta_3 := -113.057 \cdot deg$

$\theta_4 := 11.709 \cdot deg$

Coordinate rotation angle: $\delta := 175.455 \cdot deg$

2. In order to solve for the accelerations at points A and B, we will need θ_3, ω_3, and θ_4. From the graphical position solution above (in the local *xy* coordinate system),

$\theta_{2xy} := \theta_2 + \delta$ $\qquad \theta_{2xy} = 295.455\,deg$

$\theta_{3xy} := \theta_3 + \delta$ $\qquad \theta_{3xy} = 62.398\,deg$

$\theta_{4xy} := \theta_4 + \delta$ $\qquad \theta_{4xy} = 187.164\,deg$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{4xy} - \theta_{2xy})}{(\sin(\theta_{3xy} - \theta_{4xy}))} \qquad \omega_3 = 15.924\,rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_{2xy} - \theta_{3xy})}{(\sin(\theta_{4xy} - \theta_{3xy}))} \qquad \omega_4 = -20.107\, rad \cdot sec^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$
4. For point B, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$$A_{Bn} := c \cdot \omega_4{}^2 \qquad A_{Bn} = 1212.852\, in \cdot sec^{-2}$$

$$\theta_{ABn} := \theta_4 + 180 \cdot deg \qquad \theta_{ABn} = 191.709\, deg$$

$$A_{An} := a \cdot \omega_2{}^2 \qquad A_{An} = 620.000\, in \cdot sec^{-2}$$

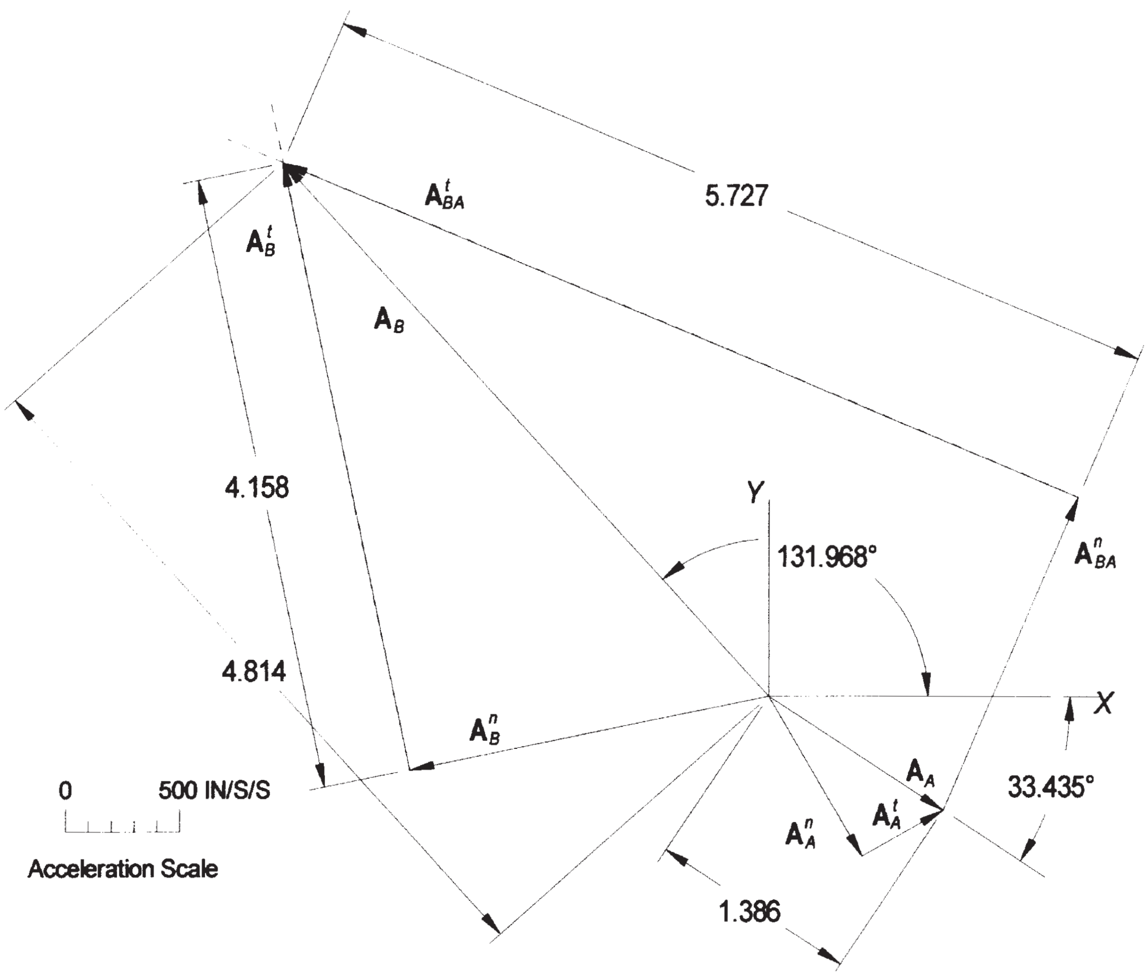
$$\theta_{AAn} := \theta_2 + 180 \cdot deg \qquad \theta_{AAn} = 300.000\, deg$$

$$A_{At} := |a \cdot \alpha_2| \qquad A_{At} = 310.000\, in \cdot sec^{-2}$$

$$\theta_{AAt} := \theta_2 - 90 \cdot deg \qquad \theta_{AAt} = 30.000\, deg$$

$$A_{BAn} := b \cdot \omega_3{}^2 \qquad A_{BAn} = 1141.131\, in \cdot sec^{-2}$$

$$\theta_{ABAn} := \theta_3 + 180 \cdot deg \qquad \theta_{ABAn} = 66.943\, deg$$



6. From the graphical solution above,

Acceleration scale factor $k_a := 500 \cdot \frac{in}{sec^2}$

$A_A := 1.386 \cdot k_a$ $A_A = 693.0\, in \cdot sec^{-2}$ at an angle of -33.435 deg

$A_B := 4.814 \cdot k_a$ $A_B = 2407.0\, in \cdot sec^{-2}$ at an angle of 131.968 deg

$A_{BAt} := 5.727 \cdot k_a$ $A_{BAt} = 2863.5\, in \cdot sec^{-2}$

$\alpha_3 := \frac{A_{BAt}}{b}$ $\alpha_3 = 636.333\, rad \cdot sec^{-2}$ CW

$A_{Bt} := 4.158 \cdot k_a$ $A_{Bt} = 2079.0\, in \cdot sec^{-2}$

$\alpha_4 := \frac{A_{Bt}}{c}$ $\alpha_4 = 693.000\, rad \cdot sec^{-2}$ CCW

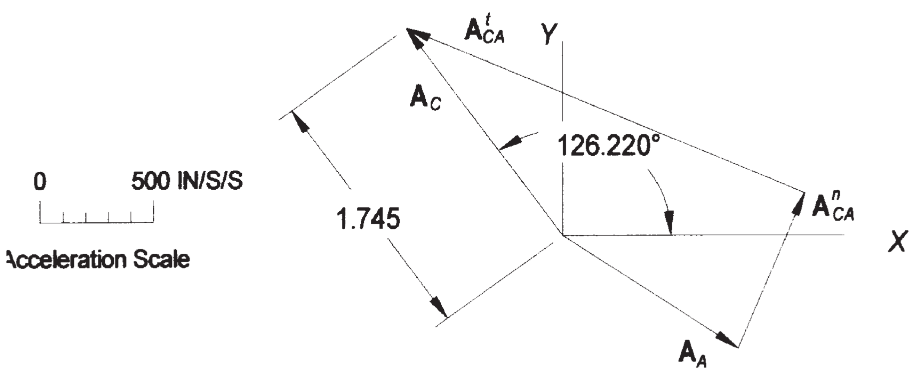
7. For point C, equation 7.4 becomes: $\mathbf{A}_C = \mathbf{A}_A + (\mathbf{A}_{CA}{}^t + \mathbf{A}_{CA}{}^n)$, where

$A_{CAn} := p \cdot \omega_3^{\,2}$ $A_{CAn} = 570.566\, in \cdot sec^{-2}$

$\theta_{ACAn} := \theta_3 + 180 \cdot deg$ $\theta_{ACAn} = 66.943\, deg$

$A_{CAt} := p \cdot \alpha_3$ $A_{CAt} = 1431.8\, in \cdot sec^{-2}$

$\theta_{ACAt} := \theta_{ACAn} + 90 \cdot deg$ $\theta_{ACAt} = 156.943\, deg$



8. From the graphical solution above,

Acceleration scale factor $k_a := 500 \cdot \frac{in}{sec^2}$

$A_C := 1.745 \cdot k_a$ $A_C = 872.50\, in \cdot sec^{-2}$ at an angle of 126.220 deg

9. Determining the acceleration of point D requires the (graphical) solution of two equations simultaneously. They are: $\mathbf{A}_D = \mathbf{A}_C + (\mathbf{A}_{DC}{}^t + \mathbf{A}_{DC}{}^n)$ and $\mathbf{A}_D = \mathbf{A}_D{}^t + \mathbf{A}_D{}^n + \mathbf{A}_D{}^{cor} + \mathbf{A}_D{}^{slip}$, where $\mathbf{A}_C$ is known from the above analysis and

From the velocity analysis (see Problem 6-91) $\omega_5 := -35.13 \cdot rad \cdot sec^{-1}$ $V_{slip} := 136.20 \cdot in \cdot sec^{-1}$

$A_{DCn} := e \cdot \omega_5^{\,2}$ $A_{DCn} = 6911\, in \cdot sec^{-2}$

$\theta_{ADCn} := 177.972 \cdot deg + 180 \cdot deg$ $\theta_{ADCn} = 357.972\, deg$

$A_{Dt} := f \cdot \alpha_4$ $\qquad A_{Dt} = 2343.7\ in \cdot sec^{-2}$

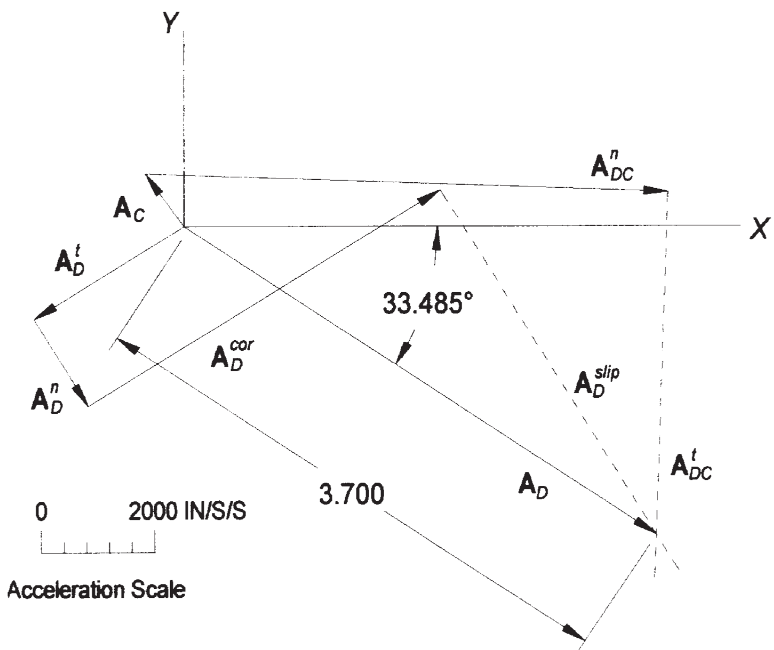
$\theta_{ADt} := \theta_4 + \delta_4 + 90 \cdot deg$ $\qquad \theta_{ADt} = 211.709\ deg$

$A_{Dn} := f \cdot \omega_4^{\ 2}$ $\qquad A_{Dn} = 1367.3\ in \cdot sec^{-2}$

$\theta_{ADn} := \theta_{ADt} + 90 \cdot deg$ $\qquad \theta_{ADn} = 301.709\ deg$

$A_{Dcor} := \left| 2 \cdot V_{slip} \cdot \omega_4 \right|$ $\qquad A_{Dcor} = 5477.1\ in \cdot sec^{-2}$

$\theta_{ADcor} := \theta_{ADt} - 180 \cdot deg$ $\qquad \theta_{ADcor} = 31.709\ deg$



8. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 2000 \cdot \dfrac{in}{sec^2}$

$A_D := 3.700 \cdot k_a$ $\qquad A_D = 7400\ in \cdot sec^{-2}$ $\qquad$ at an angle of 33.485 deg

$\alpha_6 := \alpha_4$ $\qquad \alpha_6 = 693.0\ rad \cdot sec^{-2}$

PROBLEM 7-76

Statement: The linkage in Figure P7-30a has link 2 at 120 deg in the global *XY* coordinate system. Find α_6 and $\mathbf{A}_D$ in the global *XY* coordinate system for the position shown if ω_2 = 10 rad/sec CCW and α_2 = 50 rad/sec^2 CW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 6.20 \cdot in$	Link 3 (A to B)	$b := 4.50 \cdot in$
Link 4 (O_4 to B)	$c := 3.00 \cdot in$	Link 5 (C to D)	$e := 5.60 \cdot in$
Link 3 (A to C)	$p := 2.25 \cdot in$	Link 4 (O_4 to D)	$f := 3.382 \cdot in$
Link 1 X-offset	$d_X := -7.80 \cdot in$	Link 1 Y-offset	$d_Y := 0.62 \cdot in$

Angle ACB $\quad \delta_3 := 0.0 \cdot deg$ $\quad$ Angle BO_4D $\quad \delta_4 := 110.0 \cdot deg$

Input rocker angle: $\quad \theta_{2XY} := 120 \cdot deg$ $\quad$ Global XY system

Coordinate transformation angle: $\quad \delta := 175.455 \cdot deg$

Input crank angular velocity $\quad \omega_2 := 10 \cdot rad \cdot sec^{-1}$ $\quad$ CCW

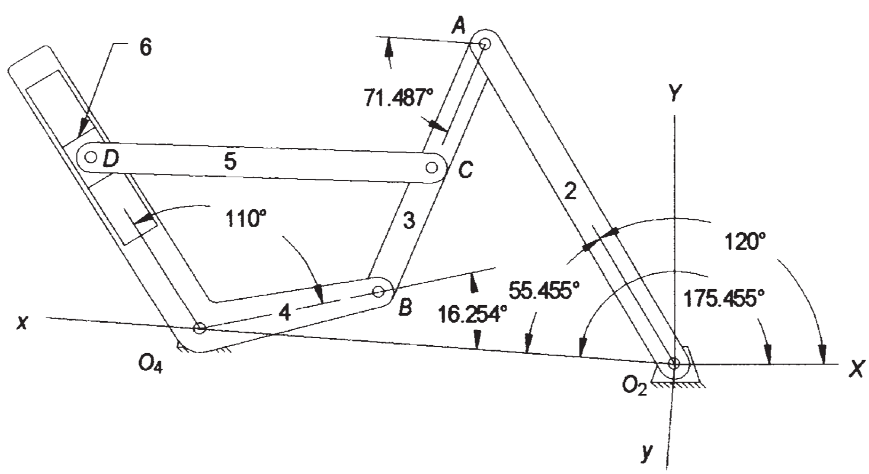
Input crank angular acceleration $\quad \alpha_2 := 10 \cdot rad \cdot sec^{-2}$ $\quad$ CW

Two argument inverse tangent

$$atan2(x,y) := \begin{array}{|l} return\ 0.5 \cdot \pi \ \ if\ (x = 0 \wedge y > 0) \\ return\ 1.5 \cdot \pi \ \ if\ (x = 0 \wedge y < 0) \\ return\ atan\left(\left(\frac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi \ \ otherwise \end{array}$$

Solution: See Figure P7-30a and Mathcad file P0776.

1. Draw the linkage to scale and label it.



Calculate the distance O_2O_4: $\quad d := \sqrt{d_X^{\,2} + d_Y^{\,2}}$ $\quad d = 7.825\ in$

Transform θ_{2XY} into the local coordinate system: $\quad \theta_2 := \theta_{2XY} - \delta$ $\quad \theta_2 = -55.455\ deg$

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 1.2620 \qquad K_2 = 2.6082$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.3767$$

$$A := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B := -2 \cdot \sin(\theta_2)$$

$$C := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

$$A = 0.2028 \qquad B = 1.6474 \qquad C = 1.5927$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right) \qquad \theta_{41} = -163.746\, deg$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.7388$$

$$K_5 = -1.9877$$

$$D := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5 \qquad D = -1.6967$$

$$E := -2 \cdot \sin(\theta_2) \qquad E = 1.6474$$

$$F := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5 \qquad F = -0.3067$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3 := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F}\right)\right) - 2 \cdot \pi \qquad \theta_3 = 71.488\, deg$$

6. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41} - \theta_2)}{\sin(\theta_3 - \theta_{41})} \qquad \omega_3 = 15.924 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_{41} - \theta_3)} \qquad \omega_4 = -20.107 \frac{rad}{sec}$$

7. Use equations 7.12 to determine the angular accelerations of links 3 and 4 for the crossed circuit.

$$A := c \cdot \sin(\theta_{41}) \qquad B := b \cdot \sin(\theta_3) \qquad D := c \cdot \cos(\theta_{41}) \qquad E := b \cdot \cos(\theta_3)$$

$$A = -21.328\, mm \qquad B = 108.386\, mm \qquad D = -73.154\, mm \qquad E = 36.291\, mm$$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^2 \cdot \cos(\theta_2) + b \cdot \omega_3^2 \cdot \cos(\theta_3) - c \cdot \omega_4^2 \cdot \cos(\theta_{41})$$

$$C = 1.827 \times 10^3\, in \cdot sec^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^2 \cdot \sin(\theta_2) - b \cdot \omega_3^2 \cdot \sin(\theta_3) + c \cdot \omega_4^2 \cdot \sin(\theta_{41})$$

$F = -875.698\ in \cdot sec^{-2}$

$\alpha_3 := \dfrac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D}$ $\qquad \alpha_3 = -540.822 \dfrac{rad}{sec^2}$ $\qquad \alpha_4 := \dfrac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D}$ $\qquad \alpha_4 = 572.346 \dfrac{rad}{sec^2}$

8. Use equation 7.13c to determine the acceleration of point B for the crossed circuit.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-\sin\left(\theta_{41}\right) + j \cdot \cos\left(\theta_{41}\right)\right) - c \cdot \omega_4^{\,2} \cdot \left(\cos\left(\theta_{41}\right) + j \cdot \sin\left(\theta_{41}\right)\right)$$

$\mathbf{A_B} = 1645 - 1309i \dfrac{in}{sec^2}$ $\qquad A_B := \left|\mathbf{A_B}\right|$ $\qquad \theta_{AB} := arg\left(\mathbf{A_B}\right)$

The acceleration of pin B is $\quad A_B = 2102.2 \dfrac{in}{sec^2}$ at $\theta_{AB} = -38.5\ deg$ (Global)

9. To proceed from here, define a vector loop for links 4, 5, and 6; write the loop equations and differentiate following the development for the inverted slider-crank linkage.

PROBLEM 7-77

Statement: The linkage in Figure P7-30b has link 3 perpendicular to the X-axis and links 2 and 4 are parallel to each other. Find α_4, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_P$ if ω_2 = 15 rad/sec CW and α_2 = 100 rad/sec2 CW. Use the acceleration difference graphical method.

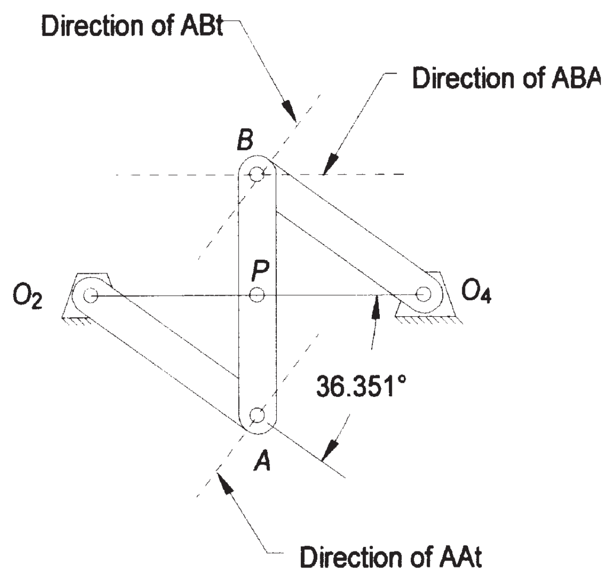
Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.75 \cdot in$	Link 3 (A to B)	$b := 3.26 \cdot in$
Link 4 (O_4 to B)	$c := 2.75 \cdot in$	Link 1 (O_4 to B)	$d := 4.43 \cdot in$
Coupler point data:	$p := 1.63 \cdot in$	$\delta_3 := 0 \cdot deg$	

Input crank angular velocity $\omega_2 := -15 \cdot rad \cdot sec^{-1}$ CW $\qquad \alpha_2 := -100 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-30b and Mathcad file P0777.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



$\theta_2 := -36.352 \cdot deg$

2. In order to solve for the accelerations at points A and B, we will need θ_3, ω_3, and θ_4. From the graphical position solution above (in the local xy coordinate system),

$\theta_3 := 90.000 \cdot deg \qquad \theta_3 = 90.000\, deg$

$\theta_4 := 180 \cdot deg + \theta_2 \qquad \theta_4 = 143.648\, deg$

Using equation (6.18),

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{(\sin(\theta_3 - \theta_4))} \qquad \omega_3 = -0.000\, rad \cdot sec^{-1}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{(\sin(\theta_4 - \theta_3))} \qquad \omega_4 = 15.000\, rad \cdot sec^{-1}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

4. For point B, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$A_{Bn} := c \cdot \omega_4{}^2 \qquad A_{Bn} = 618.750\, in \cdot sec^{-2}$

$\theta_{ABn} := \theta_4 + 180 \cdot deg$ $\theta_{ABn} = 323.648\,deg$

$A_{An} := a \cdot \omega_2^{\,2}$ $A_{An} = 618.750\,in \cdot sec^{-2}$

$\theta_{AAn} := \theta_2 + 180 \cdot deg$ $\theta_{AAn} = 143.648\,deg$

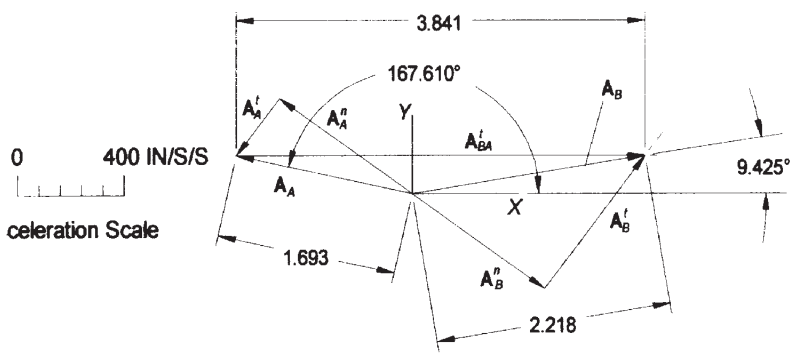
$A_{At} := \left| a \cdot \alpha_2 \right|$ $A_{At} = 275.000\,in \cdot sec^{-2}$

$\theta_{AAt} := \theta_2 - 90 \cdot deg$ $\theta_{AAt} = -126.352\,deg$

$A_{BAn} := b \cdot \omega_3^{\,2}$ $A_{BAn} = 0.000\,in \cdot sec^{-2}$

$\theta_{ABAn} := \theta_3 + 180 \cdot deg$ $\theta_{ABAn} = 270.000\,deg$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of $\theta_{ABn} + \alpha$ and $\mathbf{A}_A{}^n$ at an angle of $\theta_{AAn} + \alpha$. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of $\theta_{AAt} + \alpha$. From the tip of $\mathbf{A}_A{}^t$, draw $\mathbf{A}_{BA}{}^n$ at an angle of $\theta_{BAn} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B{}^t$, $\mathbf{A}_B{}^t$, and $\mathbf{A}_{BA}{}^t$.



6. From the graphical solution above,

Acceleration scale factor $k_a := 400 \cdot \dfrac{in}{sec^2}$

$A_A := 1.693 \cdot k_a$ $A_A = 677.2\,in \cdot sec^{-2}$ at an angle of 167.610 deg

$A_B := 2.218 \cdot k_a$ $A_B = 887.2\,in \cdot sec^{-2}$ at an angle of 9.425 deg

$A_{BAt} := 3.841 \cdot k_a$ $A_{BAt} = 1536.4\,in \cdot sec^{-2}$

$\alpha_3 := \dfrac{A_{BAt}}{b}$ $\alpha_3 = 471.288\,rad \cdot sec^{-2}$ CW

7. For point P, equation 7.4 becomes: $\mathbf{A}_P = \mathbf{A}_A + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$, where

$A_{PAn} := p \cdot \omega_3^{\,2}$ $A_{PAn} = 0.000\,in \cdot sec^{-2}$

$\theta_{APAn} := \theta_3 + \delta_3 + 180 \cdot deg$ $\theta_{APAn} = 270.000\,deg$

$A_{PAt} := p \cdot \alpha_3$ $A_{PAt} = 768.2\,in \cdot sec^{-2}$

$\theta_{APAt} := \theta_{APAn} + 90 \cdot deg$ $\theta_{APAt} = 360.000\,deg$

8. Since $\omega_3 = 0$ the acceleration at P is one-half that at B and is in the same direction as $\mathbf{A}_B$.

PROBLEM 7-78

Statement: The linkage in Figure P6-33b has link 3 perpendicular to the *X*-axis and links 2 and 4 are parallel to each other. Find α_4, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_P$ if ω_2 = 15 rad/sec CW and α_2 = 100 rad/sec^2 CW. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 2.75 \cdot in$	Link 3 (A to B)	$b := 3.26 \cdot in$
Link 4 (O_4 to B)	$c := 2.75 \cdot in$	Link 1 (O_4 to B)	$d := 4.43 \cdot in$

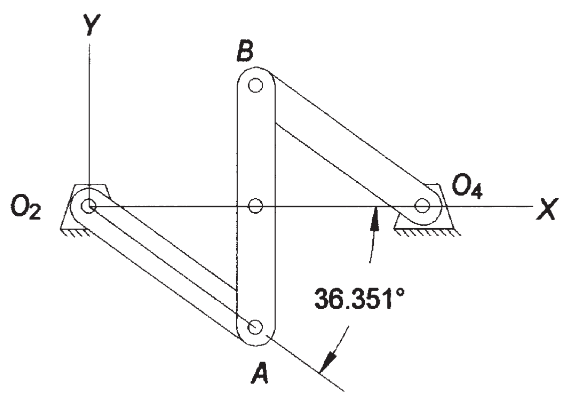
Coupler point data: $R_{PA} := 1.63 \cdot in$ $\quad \delta_3 := 0 \cdot deg$

Input crank angular velocity $\quad \omega_2 := -15 \cdot rad \cdot sec^{-1}$ CW

Input crank angular acceleration $\quad \alpha_2 := -100 \cdot rad \cdot sec^{-2}$ CW

Solution: See Figure P6-33b and Mathcad file P0778.

1. Draw the linkage to scale and label it.



From the figure, $\quad \theta_2 := 36.351 \cdot deg$

2. From the problem statement we have:

$\theta_3 := 90 \cdot deg \qquad \theta_4 := \theta_2 + 180 \cdot deg \qquad \theta_4 = 216.351\,deg$

3. Determine the angular velocity of links 3 and 4 for the crossed circuit using equations 6.18.

$$\omega_3 := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \qquad \omega_3 = -0.000 \frac{rad}{sec}$$

$$\omega_4 := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \qquad \omega_4 = 15.000 \frac{rad}{sec} \quad \text{CCW}$$

4. Using the Euler identity to expand equation 7.13a for $\mathbf{A}_A$. Determine the magnitude, and direction.

$$\mathbf{A_A} := a \cdot \alpha_2 \cdot (-\sin(\theta_2) + j \cdot \cos(\theta_2)) - a \cdot \omega_2^{\,2} \cdot (\cos(\theta_2) + j \cdot \sin(\theta_2))$$

$$\mathbf{A_A} = -335 - 588i \frac{in}{sec^2} \qquad A_A := |\mathbf{A_A}| \qquad \theta_{AA} := arg(\mathbf{A_A})$$

The acceleration of pin A is $\quad A_A = 677.1 \dfrac{in}{sec^2} \quad$ at $\quad \theta_{AA} = -119.7\,deg$

5. Use equations 7.12 to determine the angular accelerations of links 3 and 4.

$A := c \cdot \sin(\theta_4)$ $\quad B := b \cdot \sin(\theta_3)$ $\quad D := c \cdot \cos(\theta_4)$ $\quad E := b \cdot \cos(\theta_3)$

$A = -1.630\ in$ $\quad B = 3.260\ in$ $\quad D = -2.215\ in$ $\quad E := 0.0 \cdot in$

$$C := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\ 2} \cdot \cos(\theta_2) + b \cdot \omega_3^{\ 2} \cdot \cos(\theta_3) - c \cdot \omega_4^{\ 2} \cdot \cos(\theta_4)$$

$$C = 833.683\ in \cdot sec^{-2}$$

$$F := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\ 2} \cdot \sin(\theta_2) - b \cdot \omega_3^{\ 2} \cdot \sin(\theta_3) + c \cdot \omega_4^{\ 2} \cdot \sin(\theta_4)$$

$$F = -954.989\ in \cdot sec^{-2}$$

$\alpha_3 := \dfrac{C \cdot D - A \cdot F}{A \cdot E - B \cdot D}$ $\quad \alpha_3 = -471.320 \dfrac{rad}{sec^2}$ $\quad \alpha_4 := \dfrac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D}$ $\quad \alpha_4 = 431.175 \dfrac{rad}{sec^2}$

6. Use equation 7.13c to determine the acceleration of point *B*.

$$\mathbf{A_B} := c \cdot \alpha_4 \cdot \left(-\sin(\theta_4) + j \cdot \cos(\theta_4)\right) - c \cdot \omega_4^{\ 2} \cdot \left(\cos(\theta_4) + j \cdot \sin(\theta_4)\right)$$

$\mathbf{A_B} = 30509 - 14941i \dfrac{mm}{sec^2}$ $\quad A_B := |\mathbf{A_B}|$ $\quad \theta_{AB} := arg(\mathbf{A_B})$

The acceleration of pin *B* is $\quad A_B = 1337.5 \dfrac{in}{sec^2}$ $\quad$ at $\quad \theta_{AB} = -26.092\ deg$

7. Use equations 7.32 to find the acceleration of the point *P*.

$$\mathbf{A_{PA}} := R_{PA} \cdot \alpha_3 \cdot \left(-\sin(\theta_3 + \delta_3) + \mathrm{j} \cdot \cos(\theta_3 + \delta_3)\right) \ldots$$
$$+ -R_{PA} \cdot \omega_3^{\ 2} \cdot \left(\cos(\theta_3 + \delta_3) + \mathrm{j} \cdot \sin(\theta_3 + \delta_3)\right)$$

$$\mathbf{A_P} := \mathbf{A_A} + \mathbf{A_{PA}}$$

$A_P := |\mathbf{A_P}|$ $\quad A_P = 730.37 \dfrac{in}{sec^2}$ $\quad arg(\mathbf{A_P}) = -53.649\ deg$

PROBLEM 7-79

Statement: The crosshead linkage shown in Figure P7-30c has 2 *DOF* with inputs at cross heads 2 and 5. Find $\mathbf{A}_B$, $\mathbf{A}_{P3}$, and $\mathbf{A}_{P4}$ if the crossheads are each moving toward the origin of the *XY* coordinate system with a speed of 20 in/sec and are decelerating at 75 in/sec^2. Use a graphical method.

Given: Link lengths:

Link 3 (*A* to *B*)	$b := 34.32 \cdot in$	Link 4 (*B* to *C*)	$c := 50.4 \cdot in$
Link 2 *Y*-offset	$a := 59.5 \cdot in$	Link 5 *X*-offset	$d := 57 \cdot in$

Coupler point data: $AP_3 := 31.5 \cdot in$ $\quad BP_3 := 22.2 \cdot in$

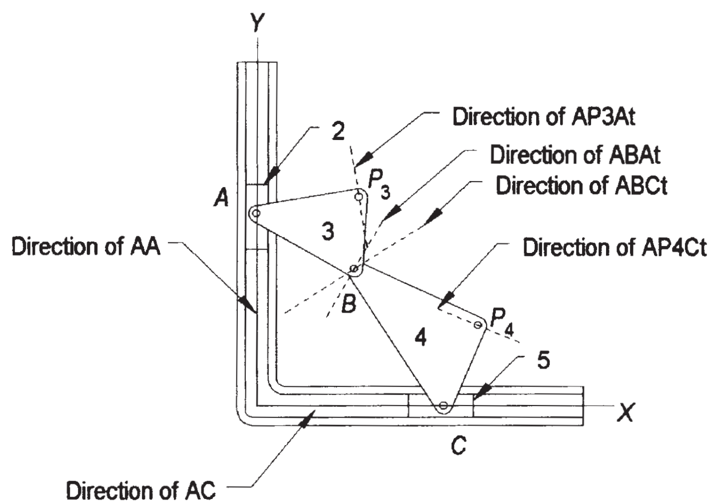
$BP_4 := 41.52 \cdot in$ $\quad CP_4 := 27.0 \cdot in$

Input velocities: $V_{2Y} := -20 \cdot in \cdot sec^{-1}$ $\quad V_{5X} := -20 \cdot in \cdot sec^{-1}$

Input accelerations $A_A := -75 \cdot in \cdot sec^{-2}$ $\quad A_C := -75 \cdot in \cdot sec^{-2}$

Solution: See Figure P7-30c and Mathcad file P0779.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at point *B*, we will need θ_3, ω_3, θ_4, and ω_3. From the velocity solution (Problem 6-98),

$\theta_3 := 320.123 \cdot deg$ $\qquad \theta_4 := 122.718 \cdot deg$

$\omega_3 := 1.749 \cdot rad \cdot sec^{-1}$ CCW $\qquad \omega_4 := -1.177 \cdot rad \cdot sec^{-1}$ CW

3. Use equation 7.4 to (graphically) determine the magnitude of the acceleration at point *B*, the magnitude of the relative accelerations $\mathbf{A}_{BA}$, $\mathbf{A}_{BC}$ and the angular accelerations of links 3 and 4. The equations to be solved (simultaneously) graphically are

$$\mathbf{A}_B = \mathbf{A}_A + \mathbf{A}_{BA} \qquad \mathbf{A}_B = \mathbf{A}_C + \mathbf{A}_{BC}$$

Eliminating $\mathbf{A}_B$ by combing equations and expanding,

$$\mathbf{A}_A + (\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n) = \mathbf{A}_C + (\mathbf{A}_{BC}^t + \mathbf{A}_{BC}^n)$$

$|A_A| = 75.000\ in \cdot sec^{-2}$ $\qquad \theta_{AA} := 270 \cdot deg$

$|A_C| = 75.000\ in \cdot sec^{-2}$ $\qquad \theta_{AC} := 180 \cdot deg$

$A_{BAn} := b \cdot \omega_3^2 \qquad A_{BAn} = 104.985\ in \cdot sec^{-2}$

$\theta_{ABAn} := \theta_3 - 180 \cdot deg \qquad \theta_{ABAn} = 140.123\ deg$

$A_{BCn} := c \cdot \omega_4^2 \qquad A_{BCn} = 69.821\ in \cdot sec^{-2}$

$\theta_{ABCn} := \theta_4 - 180 \cdot deg \qquad \theta_{ABCn} = -57.282\ deg$

a. Choose a convenient velocity scale and layout the known vectors $\mathbf{A}_A$ and $\mathbf{A}_C$

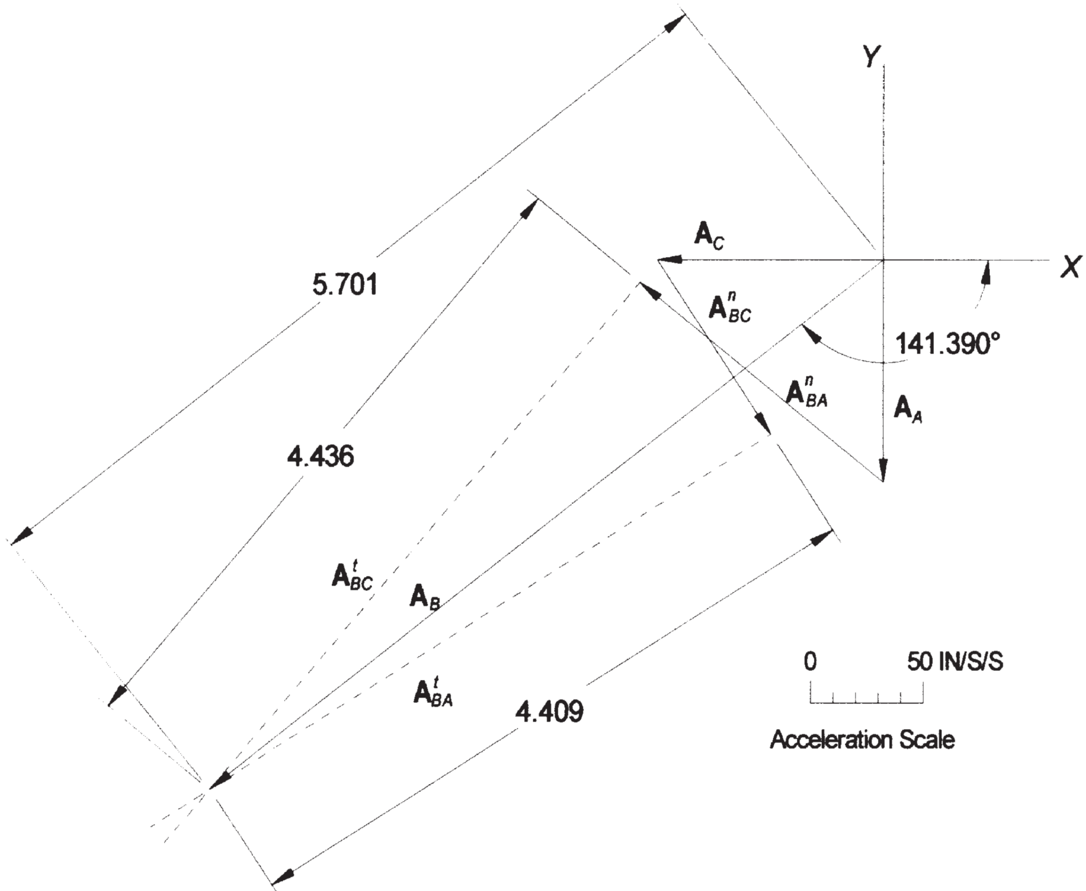
b. From the tip of $\mathbf{A}_A$, draw known vector $\mathbf{A}_{BA}{}^n$.

c. From the tip of $\mathbf{A}_C$, draw known vector $\mathbf{A}_{BC}{}^n$.

d. From the tips of $\mathbf{A}_{BA}{}^n$ and $\mathbf{A}_{BC}{}^n$ draw construction lines in the known directions of these vectors.

e. From the origin, draw a construction line to the intersection of the two construction lines in step d.

f. Complete the vector triangle by drawing $\mathbf{A}_{BA}$ from the tip of $\mathbf{A}_A$ to the intersection of the $\mathbf{A}_B$ construction line and drawing $\mathbf{A}_B$ from the tail of $\mathbf{A}_A$ to the intersection of the $\mathbf{A}_{BA}$ construction line. And then do the same with the $\mathbf{A}_{CB}$ vector



6. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 50 \cdot \dfrac{in}{sec^2}$

$A_B := 5.701 \cdot k_a \qquad A_B = 285.0\ in \cdot sec^{-2} \qquad$ at an angle of -141.390 deg

$A_{BAt} := 4.409 \cdot k_a \qquad A_{BAt} = 220.5\ in \cdot sec^{-2}$

$\alpha_3 := \dfrac{A_{BAt}}{b}$ $\qquad \alpha_3 = 6.423\, rad\cdot sec^{-2}$ $\qquad$ CW

$A_{BCt} := 4.436\cdot k_a$ $\qquad A_{BCt} = 221.8\, in\cdot sec^{-2}$

$\alpha_4 := \dfrac{A_{BCt}}{c}$ $\qquad \alpha_4 = 4.401\, rad\cdot sec^{-2}$ $\qquad$ CCW

7. From the graphical solution, the angles from A to P_3 and C to P_4 are

$\theta_{P3} := 9.250\cdot deg \qquad \theta_{P4} := 67.313\cdot deg$

8. The accelerations of points P_3 and P_4 with respect to A are:

$A_{P3An} := AP_3\cdot\omega_3^2 \qquad A_{P3An} = 96.359\, in\cdot sec^{-2} \qquad \theta_{AP3An} := \theta_{P3} - 180\cdot deg \qquad \theta_{AP3An} = -170.750\, deg$

$A_{P3At} := AP_3\cdot\alpha_3 \qquad A_{P3At} = 202.336\, in\cdot sec^{-2} \qquad \theta_{AP3At} := \theta_{P3} - 90\cdot deg \qquad \theta_{AP3At} = -80.750\, deg$

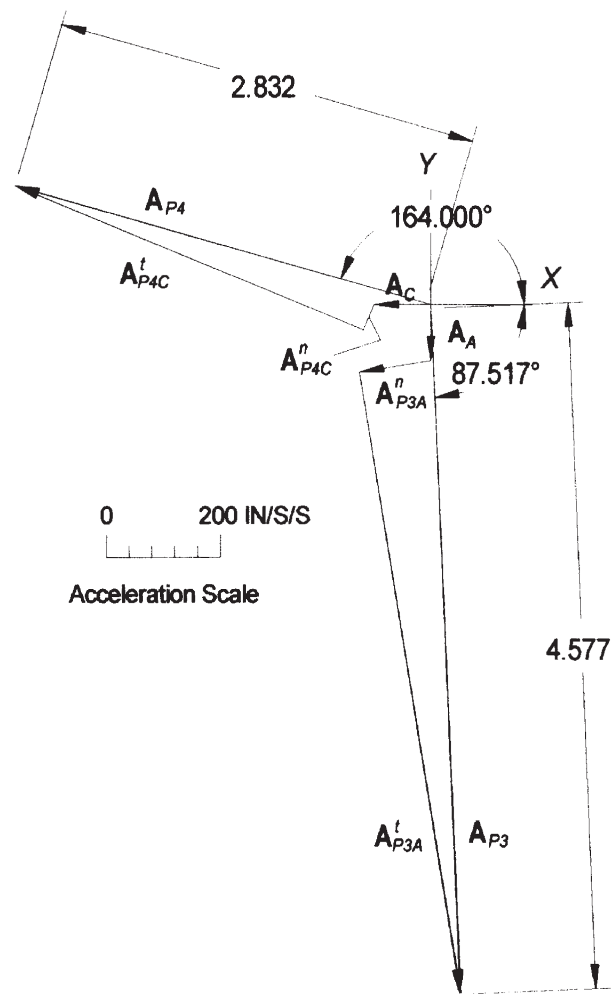
$A_{P4Cn} := CP_4\cdot\omega_4^2 \qquad A_{P4Cn} = 37.404\, in\cdot sec^{-2} \qquad \theta_{AP4Cn} := \theta_{P4} - 180\cdot deg \qquad \theta_{AP4Cn} = -112.687\, deg$

$A_{P4Ct} := CP_4\cdot\alpha_4 \qquad A_{P4Ct} = 118.821\, in\cdot sec^{-2} \qquad \theta_{AP4Ct} := \theta_{P4} + 90\cdot deg \qquad \theta_{AP4Ct} = 157.313\, deg$

3. Use equation 7.4 to (graphically) determine the magnitude of the acceleration at points P_3 and P_4.

$$\mathbf{A}_{P3} = \mathbf{A}_A + (\mathbf{A}_{P3A}{}^t + \mathbf{A}_{P3A}{}^n)$$

$$\mathbf{A}_{P4} = \mathbf{A}_C + (\mathbf{A}_{P4C}{}^t + \mathbf{A}_{P4C}{}^n)$$



6. From the graphical solution at right,

Acceleration scale factor

$k_a := 200\cdot\dfrac{in}{sec^2}$

$A_{P3} := 4.577\cdot k_a \qquad A_{P3} = 915.4\, in\cdot sec^{-2}$

at an angle of -87.517 deg

$A_{P4} := 2.832\cdot k_a \qquad A_{P4} = 566.4\, in\cdot sec^{-2}$

at an angle of 164.000 deg

PROBLEM 7-80

Statement: The crosshead linkage shown in Figure P7-30c has 2 *DOF* with inputs at cross heads 2 and 5. Find $\mathbf{A}_B$, $\mathbf{A}_{P3}$, and $\mathbf{A}_{P4}$ if the crossheads are each moving toward the origin of the *XY* coordinate system with a speed of 20 in/sec and are decelerating at 75 in/sec². Use an analytical method.

Given: Link lengths:

Link 3 (*A* to *B*)	$b := 34.32 \cdot in$	Link 4 (*B* to *C*)	$c := 50.4 \cdot in$
Link 2 *Y*-offset	$a := 59.5 \cdot in$	Link 5 *X*-offset	$d := 57 \cdot in$

Coupler point data: $AP_3 := 31.5 \cdot in$ $\quad BP_3 := 22.2 \cdot in$

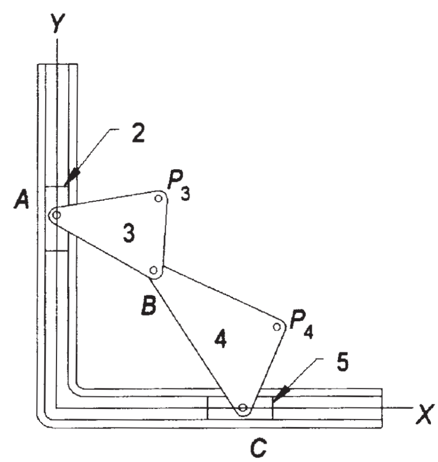
$BP_4 := 41.52 \cdot in$ $\quad CP_4 := 27.0 \cdot in$

Input velocities: $V_{2Y} := -20 \cdot in \cdot sec^{-1}$ $\quad V_{5X} := -20 \cdot in \cdot sec^{-1}$

Input accelerations $A_A := -75 \cdot in \cdot sec^{-2}$ $\quad A_C := -75 \cdot in \cdot sec^{-2}$

Solution: See Figure P7-30c and Mathcad file P0780.

1. Draw the linkage to a convenient scale and lable it.



2. In order to solve for the accelerations at point *B*, we will need θ_3, ω_3, θ_4, and ω_3. From the velocity solution (Problem 6-98),

$\theta_3 := 320.123 \cdot deg$ $\qquad \theta_4 := 122.718 \cdot deg$

$\omega_3 := 1.749 \cdot rad \cdot sec^{-1}$ CCW $\qquad \omega_4 := -1.177 \cdot rad \cdot sec^{-1}$ CW

3. Use equation 7.4 to (analytically) determine the magnitude of the acceleration at point *B*, the magnitude of the relative accelerations $\mathbf{A}_{BA}$, $\mathbf{A}_{BC}$ and the angular accelerations of links 3 and 4. The equations to be solved simultaneously are

$$\mathbf{A}_B = \mathbf{A}_A + \mathbf{A}_{BA} \qquad \mathbf{A}_B = \mathbf{A}_C + \mathbf{A}_{BC}$$

Eliminating $\mathbf{A}_B$ by combing equations and expanding,

$$\mathbf{A}_A + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n) = \mathbf{A}_C + (\mathbf{A}_{BC}{}^t + \mathbf{A}_{BC}{}^n)$$

$|A_A| = 75.000\, in \cdot sec^{-2}$ $\qquad \theta_{AA} := 270 \cdot deg$

$|A_C| = 75.000\, in \cdot sec^{-2}$ $\qquad \theta_{AC} := 180 \cdot deg$

$A_{BAn} := b \cdot \omega_3^{\,2}$ $\qquad A_{BAn} = 104.985\ in \cdot sec^{-2}$

$\theta_{ABAn} := \theta_3 - 180 \cdot deg$ $\qquad \theta_{ABAn} = 140.123\ deg$

$A_{BCn} := c \cdot \omega_4^{\,2}$ $\qquad A_{BCn} = 69.821\ in \cdot sec^{-2}$

$\theta_{ABCn} := \theta_4 - 180 \cdot deg$ $\qquad \theta_{ABCn} = -57.282\ deg$

Let $A_{Ay} := -|A_A|$ $\qquad A_{Ay} = -75.000\ in \cdot sec^{-2}$

$A_{BAnx} := A_{BAn} \cdot \cos(\theta_{ABAn})$ $\qquad A_{BAnx} = -80.568\ in \cdot sec^{-2}$

$A_{BAny} := A_{BAn} \cdot \sin(\theta_{ABAn})$ $\qquad A_{BAny} = 67.310\ in \cdot sec^{-2}$

$A_{Cx} := -|A_C|$ $\qquad A_{Cx} = -75.000\ in \cdot sec^{-2}$

$A_{BCnx} := A_{BCn} \cdot \cos(\theta_{ABCn})$ $\qquad A_{BCnx} = 37.738\ in \cdot sec^{-2}$

$A_{BCny} := A_{BCn} \cdot \sin(\theta_{ABCn})$ $\qquad A_{BCny} = -58.743\ in \cdot sec^{-2}$

$\theta_{ABAt} := \theta_{ABAn} - 90 \cdot deg$ $\qquad \theta_{ABAt} = 50.123\ deg$

$\theta_{ABCt} := \theta_{ABCn} + 90 \cdot deg$ $\qquad \theta_{ABCt} = 32.718\ deg$

$m_{BCt} := \tan(\theta_{ABCt})$ $\qquad m_{BCt} = 0.642$

$m_{BAt} := \tan(\theta_{ABAt})$ $\qquad m_{BAt} = 1.197$

Write the equations for the two lines that represent the vectors $\mathbf{A}_{BAt}$ and $\mathbf{A}_{BCt}$ and solve them simultaneously to find the coordinates of $\mathbf{A}_B$:

$$A_{Bx} := \frac{m_{BCt} \cdot (A_{Cx} + A_{BCnx}) - m_{BAt} \cdot A_{BAnx} - A_{BCny} + (A_{Ay} + A_{BAny})}{m_{BCt} - m_{BAt}}$$

$$A_{By} := m_{BCt} \cdot \left[A_{Bx} - (A_{Cx} + A_{BCnx})\right] + A_{BCny}$$

$A_{Bx} = -222.804\ in \cdot sec^{-2}$ $\qquad A_{By} = -177.941\ in \cdot sec^{-2}$

$A_B := \sqrt{A_{Bx}^{\,2} + A_{By}^{\,2}}$ $\qquad A_B = 285.140\ in \cdot sec^{-2}$

$\theta_{AB} := \operatorname{atan2}(A_{Bx}, A_{By})$ $\qquad \theta_{AB} = -141.388\ deg$

A similar approach can be taken to find $\mathbf{A}_{P3}$ and $\mathbf{A}_{P4}$.

PROBLEM 7-81

Statement: The crosshead linkage shown in Figure P7-30c has 2 *DOF* with inputs at cross heads 2 and 5. At t = 0, crosshead 2 is at rest at the origin of the global XY coordinate system and crosshead 5 is at rest at (70,0). Write a computer program to find and plot $\mathbf{A}_{P3}$ and $\mathbf{A}_{P4}$ for the first 5 sec of motion if $\mathbf{A}_2 = 0.5$ in/sec^2 upward and $\mathbf{A}_5 = 0.5$ in/sec^2 to the left.

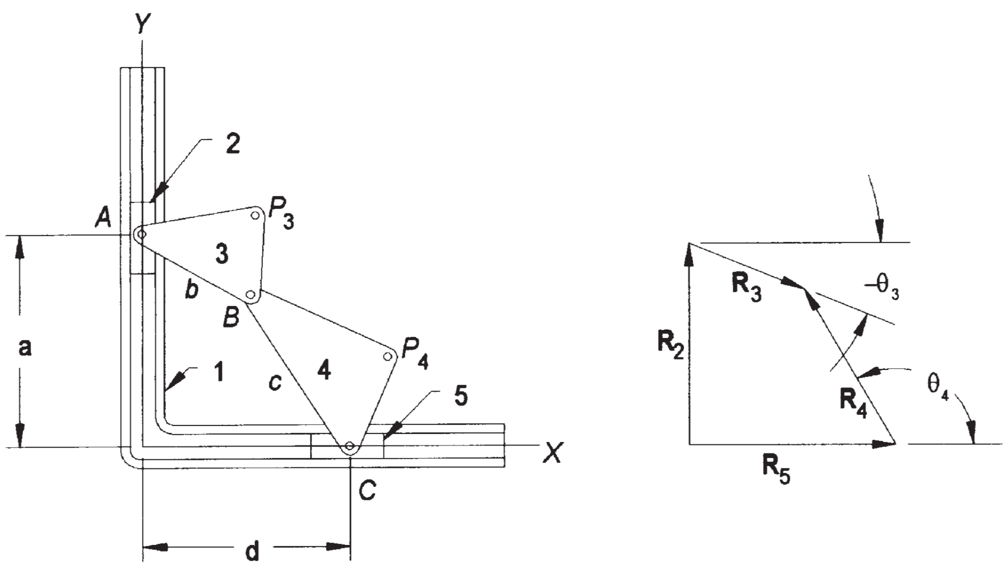
Given: Link lengths:

Link 3 (*A* to *B*)	$b := 34.32 \cdot in$	Link 4 (*B* to *C*)	$c := 50.4 \cdot in$
Link 2 *Y*-offset	$a := 59.5 \cdot in$	Link 5 *X*-offset	$d := 57 \cdot in$
Coupler point data:	$AP_3 := 31.5 \cdot in$	$BP_3 := 22.2 \cdot in$	$\delta_3 := 39.128 \cdot deg$
	$BP_4 := 41.52 \cdot in$	$CP_4 := 27.0 \cdot in$	$\delta_4 := -55.405 \cdot deg$

Input accelerations $A_2 := 0.5 \cdot in \cdot sec^{-2}$ $A_5 := -0.5 \cdot in \cdot sec^{-2}$

Solution: See Figure P7-30c and Mathcad file P0781.

1. Draw the linkage to a convenient scale and lable it.



$t := 0 \cdot sec, 0.1 \cdot sec .. 5 \cdot sec$ $a_0 := 0 \cdot in$ $d_0 := 70 \cdot in$

$a(t) := a_0 + 0.5 \cdot A_2 \cdot t^2$ $d(t) := d_0 + 0.5 A_5 \cdot t^2$

$V_2(t) := A_2 \cdot t$ $V_5(t) := A_5 \cdot t$

2. Define a vector loop as shown above. Summing the vectors around the loop,

$$\mathbf{R_2} + \mathbf{R_3} - \mathbf{R_4} - \mathbf{R_5} = 0$$

$$b \cdot cos(\theta_3) - c \cdot cos(\theta_4) - d = 0$$

$$a + b \cdot sin(\theta_3) - c \cdot sin(\theta_4) = 0$$

3. Substitute the polar notation and Euler equivalents and solve for θ_3 and θ_4 using the method of Section 4.5 and the identities in equations 4.9.

$$K_1(t) := \frac{a(t)^2 - b^2 + c^2 + d(t)^2}{2 \cdot c}$$

$$A(t) := K_1(t) - d(t) \qquad B(t) := -2 \cdot a(t) \qquad C(t) := K_1(t) + d(t)$$

$$\theta_4(t) := 2 \cdot atan2\left(2 \cdot A(t), -B(t) - \sqrt{B(t)^2 - 4 \cdot A(t) \cdot C(t)}\right)$$

$$K_2(t) := -\frac{a(t)^2 + b^2 - c^2 + d(t)^2}{2 \cdot b}$$

$$D(t) := K_2(t) - d(t) \qquad E(t) := -2 \cdot a(t) \qquad F(t) := K_2(t) + d(t)$$

$$\theta_3(t) := 2 \cdot atan2\left(2 \cdot D(t), -E(t) - \sqrt{E(t)^2 - 4 \cdot D(t) \cdot F(t)}\right)$$

4. Differentiate the position equations with respect to time and solve for ω_3 and ω_4.

$$-b \cdot \omega_3 \cdot sin(\theta_3) - c \cdot \omega_4 \cdot sin(\theta_4) - V_5 = 0$$

$$V_2 + b \cdot \omega_3 \cdot cos(\theta_3) - c \cdot \omega_4 \cdot cos(\theta_4) = 0$$

$$\omega_3(t) := -\frac{V_2(t) \cdot sin(\theta_4(t)) + V_5(t) \cdot cos(\theta_4(t))}{b \cdot sin(\theta_3(t) + \theta_4(t))}$$

$$\omega_4(t) := \frac{V_2(t) \cdot sin(\theta_3(t)) - V_5(t) \cdot cos(\theta_3(t))}{c \cdot sin(\theta_3(t) + \theta_4(t))}$$

5. Differentiate the velocity equations with respect to time and solve for α_3 and α_4.

$$-b \cdot \left(\alpha_3 \cdot sin(\theta_3) + \omega_3^2 \cdot cos(\theta_3)\right) - c \cdot \left(\alpha_4 \cdot sin(\theta_4) + \omega_4^2 \cdot cos(\theta_4)\right) - A_5 = 0$$

$$A_2 + b \cdot \left(\alpha_3 \cdot cos(\theta_3) - \omega_3^2 \cdot sin(\theta_3)\right) - c \cdot \left(\alpha_4 \cdot cos(\theta_4) - \omega_4^2 \cdot sin(\theta_4)\right) = 0$$

$$A'(t) := -b \cdot sin(\theta_3(t)) \qquad B'(t) := -c \cdot sin(\theta_4(t))$$

$$C'(t) := -b \cdot \omega_3(t)^2 \cdot cos(\theta_3(t)) - c \cdot \omega_3(t)^2 \cdot cos(\theta_4(t)) - A_5$$

$$D'(t) := b \cdot cos(\theta_3(t)) \qquad E'(t) := -c \cdot cos(\theta_4(t))$$

$$F'(t) := -b \cdot \omega_3(t)^2 \cdot sin(\theta_3(t)) + c \cdot \omega_3(t)^2 \cdot sin(\theta_4(t)) + A_2$$

$$\alpha_3(t) := \frac{B'(t) \cdot F'(t) - C'(t) \cdot E'(t)}{A'(t) \cdot E'(t) - B'(t) \cdot D'(t)} \qquad \alpha_4(t) := \frac{C'(t) \cdot D'(t) - A'(t) \cdot F'(t)}{A'(t) \cdot E'(t) - B'(t) \cdot D'(t)}$$

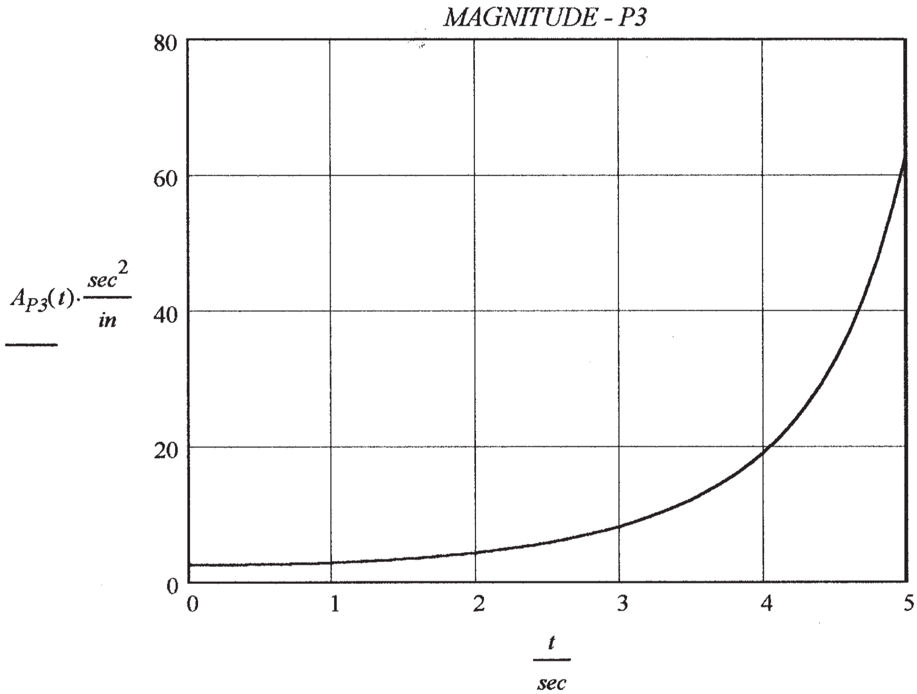
6. Using equations 6.32, solve for the accelerations at P_3.

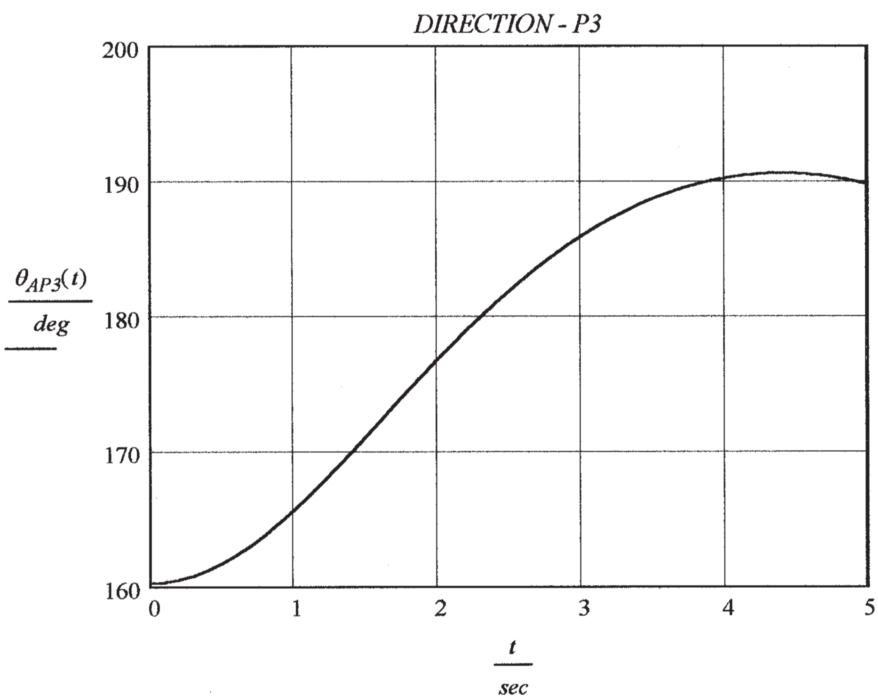
$$\mathbf{A_{P3A}}(t) := AP_3 \cdot \left[\begin{array}{l} \alpha_3(t) \cdot \left(-sin(\theta_3(t) + \delta_3) + j \cdot cos(\theta_3(t) + \delta_3)\right) \ldots \\ + -\omega_3(t)^2 \cdot \left(cos(\theta_3(t) + \delta_3) + j \cdot sin(\theta_3(t) + \delta_3)\right) \end{array}\right]$$

$$\mathbf{A_{P3}}(t) := j \cdot A_2 + \mathbf{A_{P3A}}(t) \qquad A_{P3}(t) := \left|\mathbf{A_{P3}}(t)\right| \qquad \theta_{AP3}(t) := arg(\mathbf{A_{P3}}(t))$$

$$\theta_{AP3}(t) := if\left(\theta_{AP3}(t) \le 0, \theta_{AP3}(t) + 2 \cdot \pi, \theta_{AP3}(t)\right)$$

7. Plot the magnitude and direction of the acceleration at P_3.





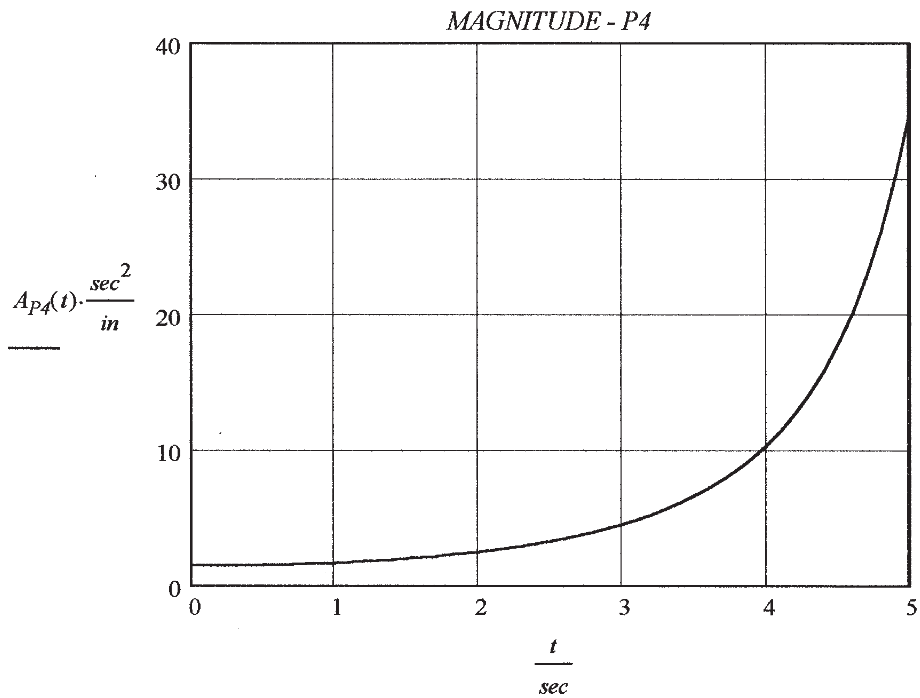
8. Using equations 6.32, solve for the accelerations at P_4.

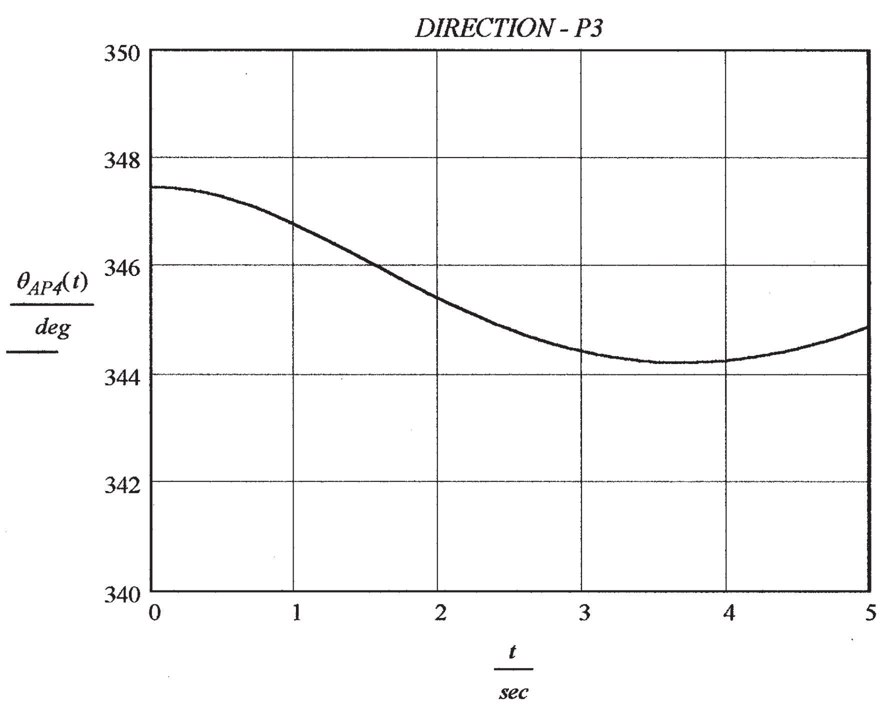
$$\mathbf{A_{P4A}}(t) := CP_4\cdot\left[\begin{array}{l}\alpha_4(t)\cdot\left(-sin\left(\theta_4(t)+\delta_4\right)+j\cdot cos\left(\theta_4(t)+\delta_4\right)\right)\ ...\\ +-\omega_4(t)^2\cdot\left(cos\left(\theta_4(t)+\delta_4\right)+j\cdot sin\left(\theta_4(t)+\delta_4\right)\right)\end{array}\right]$$

$$\mathbf{A_{P4}}(t) := j\cdot A_5 + \mathbf{A_{P4A}}(t) \qquad A_{P4}(t) := \left|\mathbf{A_{P4}}(t)\right| \qquad \theta_{AP4}(t) := arg\left(\mathbf{A_{P4}}(t)\right)$$

$$\theta_{AP4}(t) := if\left(\theta_{AP4}(t) \le 0, \theta_{AP4}(t) + 2\cdot\pi, \theta_{AP4}(t)\right)$$

9. Plot the magnitude and direction of the acceleration at P_4.





PROBLEM 7-82

Statement: The linkage in Figure P7-30d has the path of slider 6 perpendicular to the global X-axis and link 2 aligned with the global X-axis. Find α_2 and $\mathbf{A}_4$ in the position shown if the velocity of the slider is constant at 20 in/sec downward. Use the acceleration difference graphical method.

Given: Link lengths:

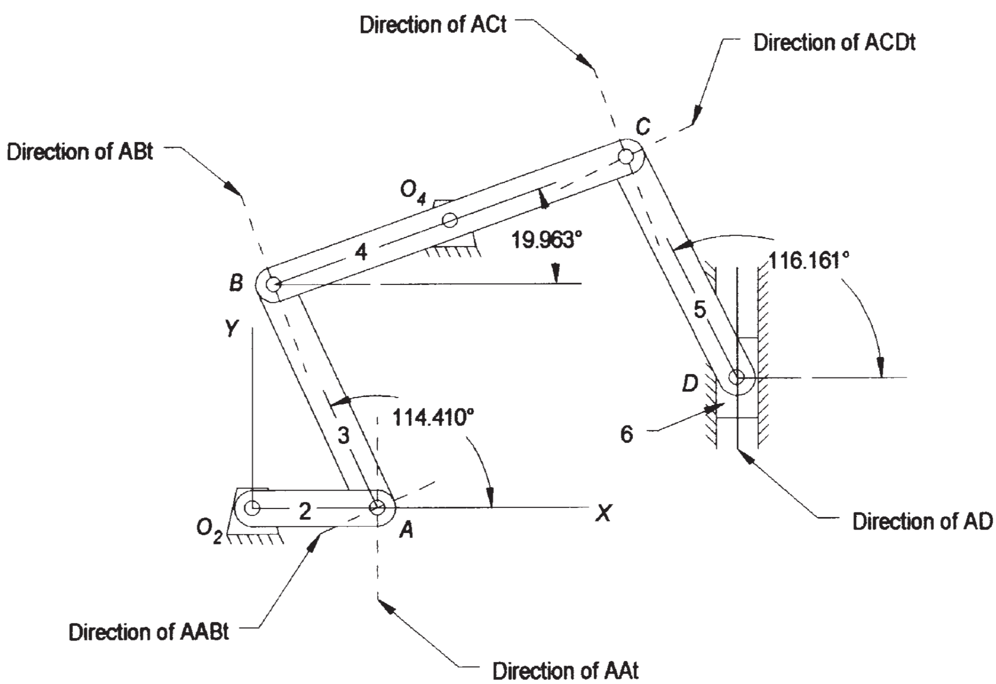
Link 2 (O_2 to A)	$a := 12 \cdot in$	Link 3 (A to B)	$b := 24 \cdot in$
Link 4 (O_4 to B)	$c := 18 \cdot in$	Link 5 (C to D)	$f := 24 \cdot in$
Link 4 (O_4 to C)	$e := 18 \cdot in$		
Link 1 X-offset	$d_X := 19 \cdot in$	Link 1 Y-offset	$d_Y := 28 \cdot in$

Angle BO_4C $\quad \delta_4 := 0.0 \cdot deg$

Input slider velocity $\quad V_D := 20 \cdot in \cdot sec^{-1} \quad \theta_{VD} := -90.0 \cdot deg$

Solution: See Figure P7-30d and Mathcad file P0782.

1. Draw the linkage to a convenient scale. Indicate the directions of the acceleration vectors of interest.



2. In order to solve for the accelerations at points A, B and C, we will need θ_2, ω_2, θ_3, ω_3, and θ_4 ω_4. From the graphical position solution above and the velocity solution (see Problem 6-99),

$$\theta_2 := 0.000 \cdot deg \qquad \omega_2 := 1.648 \cdot rad \cdot sec^{-1} \quad \text{CCW}$$

$$\theta_3 := 114.410 \cdot deg \qquad \omega_3 := 0.282 \cdot rad \cdot sec^{-1} \quad \text{CCW}$$

$$\theta_{41} := 199.963 \cdot deg \qquad \omega_4 := 1.003 \cdot rad \cdot sec^{-1} \quad \text{CW}$$

$$\theta_{42} := 19.963 \cdot deg$$

$$\theta_5 := 116.161 \cdot deg \qquad \omega_5 := 0.286 \cdot rad \cdot sec^{-1} \quad \text{CW}$$

3. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P^t + \mathbf{A}_P^n) = (\mathbf{A}_A^t + \mathbf{A}_A^n) + (\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n)$

4. For point C, this becomes: $(\mathbf{A}_C^t + \mathbf{A}_C^n) = \mathbf{A}_D + (\mathbf{A}_{CD}^t + \mathbf{A}_{CD}^n)$, where

$$A_{Cn} := e \cdot \omega_4^2 \qquad A_{Cn} = 18.108\ in \cdot sec^{-2}$$

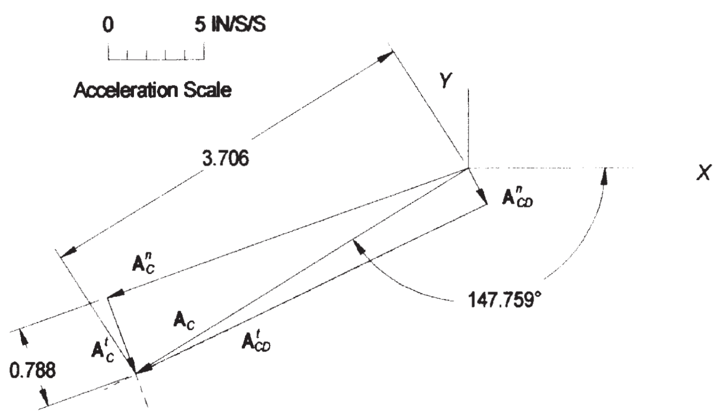
$$\theta_{ACn} := \theta_{42} + 180 \cdot deg \qquad \theta_{ACn} = 199.963\ deg$$

$$A_D := 0 \cdot in \cdot sec^{-2}$$

$$A_{CDn} := f \cdot \omega_5^2 \qquad A_{CDn} = 1.963\ in \cdot sec^{-2}$$

$$\theta_{ACDn} := \theta_5 + 180 \cdot deg \qquad \theta_{ACDn} = 296.161\ deg$$

5. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_C^n$ at an angle of $\theta_{ACn} + \alpha$ and $\mathbf{A}_D^n$ at an angle of $\theta_{ADn} + \alpha$. From the tip of $\mathbf{A}_D^n$, draw $\mathbf{A}_D^t$ at an angle of $\theta_{ADt} + \alpha$. From the tip of $\mathbf{A}_D^t$, draw $\mathbf{A}_{CD}^n$ at an angle of $\theta_{CDn} + \alpha$. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_C^n$ and $\mathbf{A}_{CD}^n$, draw construction lines in the directions of $\mathbf{A}_C^t$ and $\mathbf{A}_{CD}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_C^t$, $\mathbf{A}_C$, and $\mathbf{A}_{CD}^t$.



6. From the graphical solution above,

Acceleration scale factor $\qquad k_a := 5.0 \cdot \dfrac{in}{sec^2}$

$$A_C := 3.706 \cdot k_a \qquad A_C = 18.530\ in \cdot sec^{-2} \qquad \text{at an angle of } -147.759 \text{ deg}$$

$$A_{Ct} := 0.788 \cdot k_a \qquad A_{Ct} = 3.940\ in \cdot sec^{-2}$$

$$\alpha_4 := \frac{A_{Ct}}{e} \qquad \alpha_4 = 0.219\ rad \cdot sec^{-2} \qquad \text{CW}$$

Since link 4 is straight and the distance from O_4 to B is the same as to C,

$$A_B := A_C \qquad A_B = 18.530\ in \cdot sec^{-2} \qquad \text{at an angle of } 32.241 \text{ deg}$$

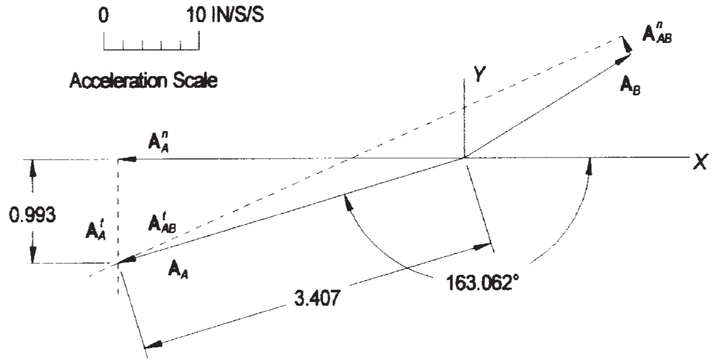
7. For point A, equation 7.4 becomes: $(\mathbf{A}_A^t + \mathbf{A}_A^n) = \mathbf{A}_B + (\mathbf{A}_{AB}^t + \mathbf{A}_{AB}^n)$, where

$$A_{An} := a \cdot \omega_2^2 \qquad A_{An} = 32.591\ in \cdot sec^{-2}$$

$$\theta_{AAn} := \theta_2 + 180 \cdot deg \qquad \theta_{AAn} = 180.000\ deg$$

$$A_{ABn} := b \cdot \omega_3^2 \qquad A_{ABn} = 1.909\ in \cdot sec^{-2}$$

$$\theta_{AABn} := \theta_3 \qquad \theta_{AABn} = 114.410\ deg$$



8. From the graphical solution above,

Acceleration scale factor $\quad k_a := 10 \cdot \dfrac{in}{sec^2}$

$A_A := 3.407 \cdot k_a \qquad A_A = 34.070\ in \cdot sec^{-2}$ at an angle of -163.062 deg

$A_{At} := 0.993 \cdot k_a \qquad A_{At} = 9.930\ in \cdot sec^{-2}$

$\alpha_2 := \dfrac{A_{At}}{a} \qquad \alpha_2 = 0.828\ rad \cdot sec^{-2}$ CW

PROBLEM 7-83

Statement: The linkage in Figure P7-30d has the path of slider 6 perpendicular to the global X-axis and link 2 aligned with the global X-axis. Find α_2 and $\mathbf{A}_A$ in the position shown if the velocity of the slider is constant at 20 in/sec downward. Use an analytical method.

Given: Link lengths:

Link 2 (O_2 to A)	$c := 12 \cdot in$	Link 3 (A to B)	$b := 24 \cdot in$
Link 4 (O_4 to B)	$a := 18 \cdot in$	Link 5 (C to D)	$f := 24 \cdot in$
Link 4 (O_4 to C)	$e := 18 \cdot in$		
Link 1 X-offset	$d_X := 19 \cdot in$	Link 1 Y-offset	$d_Y := 28 \cdot in$
Link 1 (O_2 to O_4)	$d := \sqrt{d_X^{\,2} + d_Y^{\,2}}$	$d = 33.838\, in$	

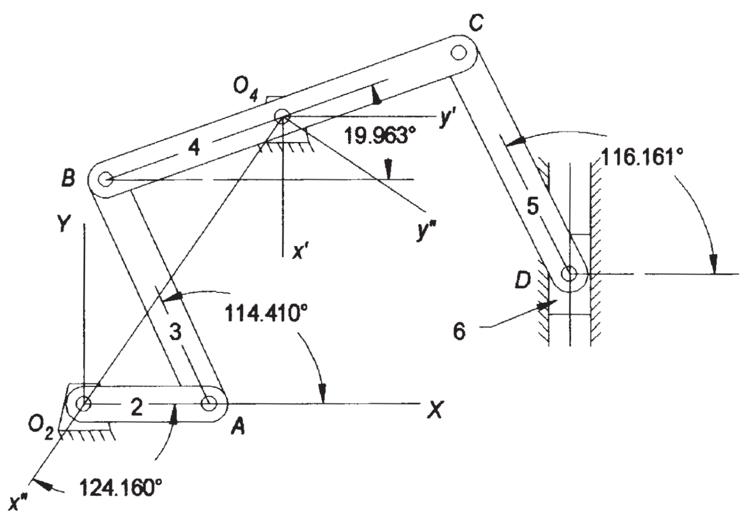
Axis rotation angle: $\delta := -124.160 \cdot deg$

Input slider velocity $V_D := 20 \cdot in \cdot sec^{-1}$ Downward (positive x' direction)

Input slider acceleration $A_D := 0 \cdot in \cdot sec^{-2}$

Solution: See Figure P7-30d and Mathcad file P0783.

1. Draw the linkage to a convenient scale and label it.



2. From the graphical position solution above and the velocity solution (see Problem 6-99),

Local $x''y''$ coordinate system:

Output	$\theta_2 := 124.160 \cdot deg$	$\omega_2 := 1.648 \cdot rad \cdot sec^{-1}$	CCW
	$\theta_3 := 58.570 \cdot deg$	$\omega_3 := 0.282 \cdot rad \cdot sec^{-1}$	CCW
Input	$\theta_{41} := 324.123 \cdot deg$	$\omega_4 := -1.003 \cdot rad \cdot sec^{-1}$	CW

Local $x'y'$ coordinate system:

$\theta_{42} := 109.963 \cdot deg$	$\omega_4 := -1.003 \cdot rad \cdot sec^{-1}$	CW
$\theta_5 := 206.161 \cdot deg$	$\omega_5 := -0.286 \cdot rad \cdot sec^{-1}$	CW

3. Solve equations 7.16b and c for α_2 (which is α_4 in this case):

$$\alpha_4 := \frac{A_D \cdot \cos(\theta_5) + e \cdot \omega_4^{\,2} \cdot \cos(\theta_5 - \theta_{42}) - f \cdot \omega_5^{\,2}}{e \cdot \sin(\theta_5 - \theta_{42})} \qquad \alpha_4 = -0.219\,rad \cdot sec^{-2}$$

4. Use equations 7.12 to determine the angular accelerations of link 2.

$$A := c \cdot \sin(\theta_2) \qquad B := b \cdot \sin(\theta_3) \qquad D := c \cdot \cos(\theta_2) \qquad E := b \cdot \cos(\theta_3)$$

$$A = 9.930\,in \qquad B = 20.479\,in \qquad D = -6.738\,in \qquad E = 12.515\,in$$

$$C := a \cdot \alpha_4 \cdot \sin(\theta_{41}) + a \cdot \omega_4^{\,2} \cdot \cos(\theta_{41}) + b \cdot \omega_3^{\,2} \cdot \cos(\theta_3) - c \cdot \omega_2^{\,2} \cdot \cos(\theta_2)$$

$$C = 36.278\,in \cdot sec^{-2}$$

$$F := a \cdot \alpha_4 \cdot \cos(\theta_{41}) - a \cdot \omega_4^{\,2} \cdot \sin(\theta_{41}) - b \cdot \omega_3^{\,2} \cdot \sin(\theta_3) + c \cdot \omega_2^{\,2} \cdot \sin(\theta_2)$$

$$F = 32.758\,in \cdot sec^{-2}$$

$$\alpha_2 := \frac{C \cdot E - B \cdot F}{A \cdot E - B \cdot D} \qquad \alpha_2 = -0.827\,\frac{rad}{sec^2}$$

9. Use equation 7.13c to determine the acceleration of point A.

$$\mathbf{A_A} := c \cdot \alpha_2 \cdot \left(-\sin(\theta_2) + j \cdot \cos(\theta_2)\right) - c \cdot \omega_2^{\,2} \cdot \left(\cos(\theta_2) + j \cdot \sin(\theta_2)\right)$$

$$\mathbf{A_A} = 26.510 - 21.397j\,\frac{in}{sec^2} \qquad A_A := \left|\mathbf{A_A}\right| \qquad \theta_{AA} := \arg(\mathbf{A_A}) + \delta$$

The acceleration of pin A is $A_A = 34.068\,\frac{in}{sec^2}$ at $\theta_{AA} = -163.068\,deg$ (Global)

PROBLEM 7-84

Statement: The linkage in Figure P7-30a has the path of slider 6 perpendicular to the global *X*-axis and link 2 aligned with the global *X*-axis at t = 0. Write a computer program or use an equation solver to find and plot $\mathbf{A}_D$ as a function of θ_2, over the possible range of motion of link 2, in the global *XY* coordinate system.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 12 \cdot in$	Link 3 (A to B)	$b := 24 \cdot in$
Link 4 (O_4 to B)	$c := 18 \cdot in$	Link 5 (C to D)	$f := 24 \cdot in$
Link 4 (O_4 to C)	$e := 18 \cdot in$		
Link 1 *X*-offset	$d_X := 19 \cdot in$	Link 1 *Y*-offset	$d_Y := 28 \cdot in$
Link 1 (O_2 to O_4)	$d := \sqrt{d_X^{\,2} + d_Y^{\,2}}$	$d = 33.838\,in$	

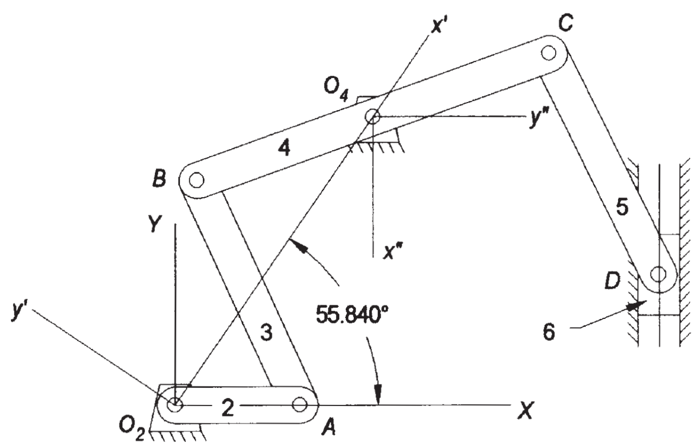
Axis rotation angle: $\delta_1 := 55.840 \cdot deg$ $\qquad \delta_2 := -90.000 \cdot deg$

Input velocity & accel: $\omega_2 := 1 \cdot rad \cdot sec^{-1}$ $\qquad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Crank angle: $\theta_{2XY} := -70 \cdot deg, -69 \cdot deg .. 180 \cdot deg$ $\qquad \theta_2(\theta_{2XY}) := \theta_{2XY} - \delta_1$

Solution: See Figure P7-30d and Mathcad file P0784.

1. Draw the linkage to a convenient scale and label it.



2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_1 = 2.8198 \qquad K_2 := \frac{d}{c} \qquad K_2 = 1.8799$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 2.4005$$

$$A(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 - K_2 \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

$$B(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$C(\theta_{2XY}) := K_1 - (K_2 + 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{4xy}(\theta_{2XY}) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_{2XY}), -B(\theta_{2XY}) - \sqrt{B(\theta_{2XY})^2 - 4 \cdot A(\theta_{2XY}) \cdot C(\theta_{2XY})}\right)\right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 1.4099 \qquad K_5 = -2.6753$$

$$D(\theta_{2XY}) := \cos(\theta_2(\theta_{2XY})) - K_1 + K_4 \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

$$E(\theta_{2XY}) := -2 \cdot \sin(\theta_2(\theta_{2XY}))$$

$$F(\theta_{2XY}) := K_1 + (K_4 - 1) \cdot \cos(\theta_2(\theta_{2XY})) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_{3xy}(\theta_{2XY}) := 2 \cdot \left(\operatorname{atan2}\left(2 \cdot D(\theta_{2XY}), -E(\theta_{2XY}) - \sqrt{E(\theta_{2XY})^2 - 4 \cdot D(\theta_{2XY}) \cdot F(\theta_{2XY})}\right)\right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_{2XY}) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{4xy}(\theta_{2XY}) - \theta_2(\theta_{2XY}))}{\sin(\theta_{3xy}(\theta_{2XY}) - \theta_{4xy}(\theta_{2XY}))}$$

$$\omega_4(\theta_{2XY}) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2(\theta_{2XY}) - \theta_{3xy}(\theta_{2XY}))}{\sin(\theta_{4xy}(\theta_{2XY}) - \theta_{3xy}(\theta_{2XY}))}$$

7. Use equations 7.12 to determine the angular acceleration of link 4.

$$A(\theta_{2XY}) := c \cdot \sin(\theta_{4xy}(\theta_{2XY})) \qquad B(\theta_{2XY}) := b \cdot \sin(\theta_{3xy}(\theta_{2XY}))$$

$$D(\theta_{2XY}) := c \cdot \cos(\theta_{4xy}(\theta_{2XY})) \qquad E(\theta_{2XY}) := b \cdot \cos(\theta_{3xy}(\theta_{2XY}))$$

$$C(\theta_{2XY}) := a \cdot \alpha_2 \cdot \sin(\theta_2(\theta_{2XY})) + a \cdot \omega_2^2 \cdot \cos(\theta_2(\theta_{2XY})) \ldots$$
$$+ b \cdot \omega_3(\theta_{2XY})^2 \cdot \cos(\theta_{3xy}(\theta_{2XY})) - c \cdot \omega_4(\theta_{2XY})^2 \cdot \cos(\theta_{4xy}(\theta_{2XY}))$$

$$F(\theta_{2XY}) := a \cdot \alpha_2 \cdot \cos(\theta_2(\theta_{2XY})) - a \cdot \omega_2^2 \cdot \sin(\theta_2(\theta_{2XY})) \ldots$$
$$+ -b \cdot \omega_3(\theta_{2XY})^2 \cdot \sin(\theta_{3xy}(\theta_{2XY})) + c \cdot \omega_4(\theta_{2XY})^2 \cdot \sin(\theta_{4xy}(\theta_{2XY}))$$

$$\alpha_4(\theta_{2XY}) := \frac{C(\theta_{2XY}) \cdot E(\theta_{2XY}) - B(\theta_{2XY}) \cdot F(\theta_{2XY})}{A(\theta_{2XY}) \cdot E(\theta_{2XY}) - B(\theta_{2XY}) \cdot D(\theta_{2XY})}$$

8. Transform θ_4 to the local $x''y''$ coordinate system and add 180 deg to get the link from O_4 to C, which is the input angle to the crank-slider portion of the linkage.

$$\theta_4(\theta_{2XY}) := \theta_{4xy}(\theta_{2XY}) + 180 \cdot deg + \delta_1 - \delta_2$$

9. Determine θ_5 in the x"y" coordinate system using equation 4.17. Offset: $cc := 27.5 in$

$$\theta_5(\theta_{2XY}) := \operatorname{asin}\left(-\frac{e \cdot \sin(\theta_4(\theta_{2XY})) - cc}{f}\right) + \pi$$

10. Determine the angular velocity of link 5 using equation 6.22a:

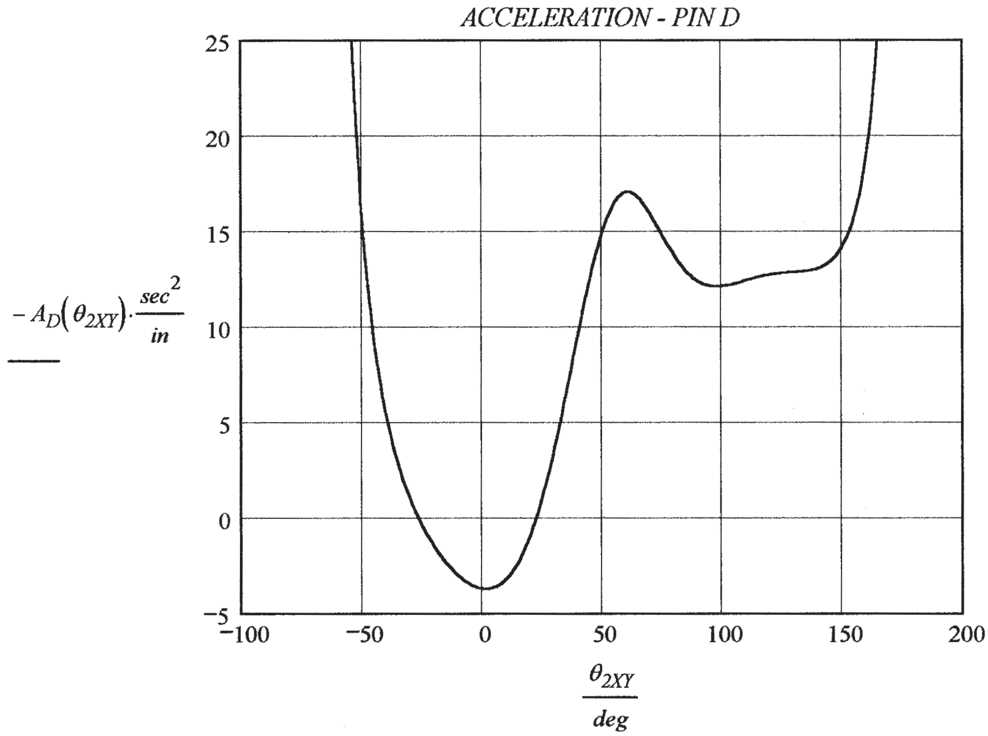
$$\omega_5(\theta_{2XY}) := \frac{e}{f} \cdot \frac{\cos(\theta_4(\theta_{2XY}))}{\cos(\theta_5(\theta_{2XY}))} \cdot \omega_4(\theta_{2XY})$$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5(\theta_{2XY}) := \frac{e\cdot\alpha_4(\theta_{2XY})\cdot\cos(\theta_4(\theta_{2XY})) - e\cdot\omega_4(\theta_{2XY})^2\cdot\sin(\theta_4(\theta_{2XY})) \ldots + f\cdot\omega_5(\theta_{2XY})^2\cdot\sin(\theta_5(\theta_{2XY}))}{f\cdot\cos(\theta_5(\theta_{2XY}))}$$

12. Use equation 7.16e for the acceleration of pin *D*.

$$A_D(\theta_{2XY}) := -e\cdot\alpha_4(\theta_{2XY})\cdot\sin(\theta_4(\theta_{2XY})) - e\cdot\omega_4(\theta_{2XY})^2\cdot\cos(\theta_4(\theta_{2XY})) \ldots + f\cdot\alpha_5(\theta_{2XY})\cdot\sin(\theta_5(\theta_{2XY})) + f\cdot\omega_5(\theta_{2XY})^2\cdot\cos(\theta_5(\theta_{2XY}))$$



Positive is up (positive *Y* direction in the global coordinate system).

PROBLEM 7-85

Statement: For the linkage of Figure P7-30e, write a computer program or use an equation solver to find and plot $\mathbf{A}_D$ in the global coordinate system for one revolution of link 2 if ω_2 is constant at 10 rad/sec CW.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 5.0 \cdot in$	Link 3 (A to B)	$b := 5.0 \cdot in$
Link 4 (O_4 to B)	$c := 6.0 \cdot in$	Link 1 (O_2 to O_4)	$d := 2.5 \cdot in$
Link 5 (B to C)	$e := 15 \cdot in$	Angle BO_4C	$\delta := 83.621 \cdot deg$

Crank angle: $\theta_2 := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$

Input crank angular velocity $\omega_2 := 10 \cdot rad \cdot sec^{-1}$ $\alpha_2 := 0 \cdot rad \cdot sec^{-2}$

Solution: See Figure P7-30e and Mathcad file P0785.

1. This sixbar drag-link mechanism can be analyzed as a fourbar Grashof double crank in series with a slider-crank mechanism using the output of the fourbar, link 4, as the input to the slider-crank.

2. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_1 = 0.5000 \qquad K_2 := \frac{d}{c} \qquad K_2 = 0.4167$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 0.7042$$

$$A(\theta_2) := \cos(\theta_2) - K_1 - K_2 \cdot \cos(\theta_2) + K_3$$

$$B(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$C(\theta_2) := K_1 - (K_2 + 1) \cdot \cos(\theta_2) + K_3$$

3. Use equation 4.10b to find values of θ_4 for the open circuit.

$$\theta_{41}(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot A(\theta_2), -B(\theta_2) - \sqrt{B(\theta_2)^2 - 4 \cdot A(\theta_2) \cdot C(\theta_2)} \right) \right)$$

4. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{(2 \cdot a \cdot b)} \qquad K_4 = 0.5000 \qquad K_5 = -0.4050$$

$$D(\theta_2) := \cos(\theta_2) - K_1 + K_4 \cdot \cos(\theta_2) + K_5$$

$$E(\theta_2) := -2 \cdot \sin(\theta_2)$$

$$F(\theta_2) := K_1 + (K_4 - 1) \cdot \cos(\theta_2) + K_5$$

5. Use equation 4.13 to find values of θ_3 for the open circuit.

$$\theta_3(\theta_2) := 2 \cdot \left(atan2\left(2 \cdot D(\theta_2), -E(\theta_2) - \sqrt{E(\theta_2)^2 - 4 \cdot D(\theta_2) \cdot F(\theta_2)} \right) \right)$$

6. Determine the angular velocity of links 3 and 4 for the open circuit using equations 6.18.

$$\omega_3(\theta_2) := \frac{a \cdot \omega_2}{b} \cdot \frac{\sin(\theta_{41}(\theta_2) - \theta_2)}{\sin(\theta_3(\theta_2) - \theta_{41}(\theta_2))}$$

$$\omega_4(\theta_2) := \frac{a \cdot \omega_2}{c} \cdot \frac{\sin(\theta_2 - \theta_3(\theta_2))}{\sin(\theta_{41}(\theta_2) - \theta_3(\theta_2))}$$

7. Use equations 7.12 to determine the angular acceleration of link 4.

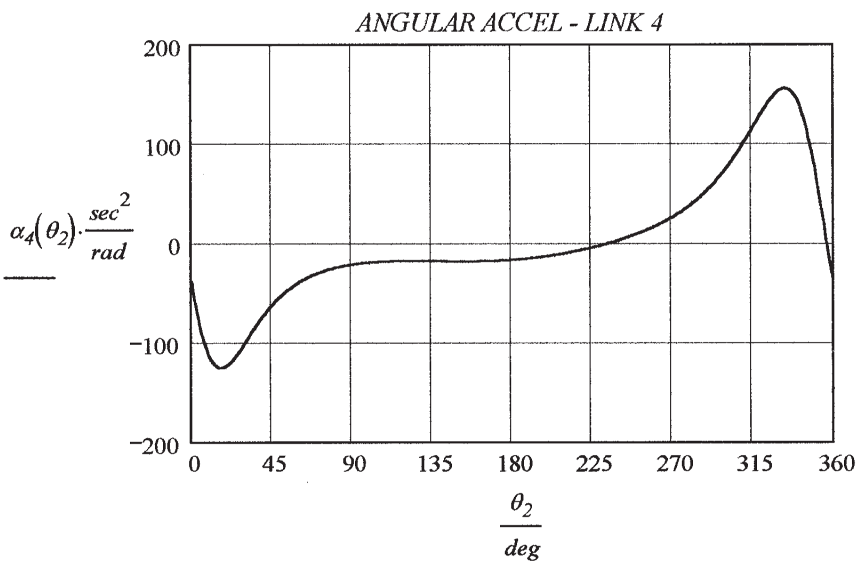
$$A(\theta_2) := c \cdot \sin(\theta_{41}(\theta_2)) \qquad B(\theta_2) := b \cdot \sin(\theta_3(\theta_2))$$

$$D(\theta_2) := c \cdot \cos(\theta_{41}(\theta_2)) \qquad E(\theta_2) := b \cdot \cos(\theta_3(\theta_2))$$

$$C(\theta_2) := a \cdot \alpha_2 \cdot \sin(\theta_2) + a \cdot \omega_2^{\,2} \cdot \cos(\theta_2) \ldots$$
$$+ b \cdot \omega_3(\theta_2)^2 \cdot \cos(\theta_3(\theta_2)) - c \cdot \omega_4(\theta_2)^2 \cdot \cos(\theta_{41}(\theta_2))$$

$$F(\theta_2) := a \cdot \alpha_2 \cdot \cos(\theta_2) - a \cdot \omega_2^{\,2} \cdot \sin(\theta_2) \ldots$$
$$+ -b \cdot \omega_3(\theta_2)^2 \cdot \sin(\theta_3(\theta_2)) + c \cdot \omega_4(\theta_2)^2 \cdot \sin(\theta_{41}(\theta_2))$$

$$\alpha_4(\theta_2) := \frac{C(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot F(\theta_2)}{A(\theta_2) \cdot E(\theta_2) - B(\theta_2) \cdot D(\theta_2)}$$



8. Transform θ_4 to the slider-crank coordinate system (origin at O_4) system.

$$\theta_{42}(\theta_2) := \theta_{41}(\theta_2) - \delta$$

9. Determine θ_5 using equation 4.17. Offset: $cc := 0 \cdot in$

$$\theta_5(\theta_2) := \operatorname{asin}\left(-\frac{c \cdot \sin(\theta_{42}(\theta_2)) - cc}{e}\right) + \pi$$

10. Determine the angular velocity of link 5 using equation 6.22a:

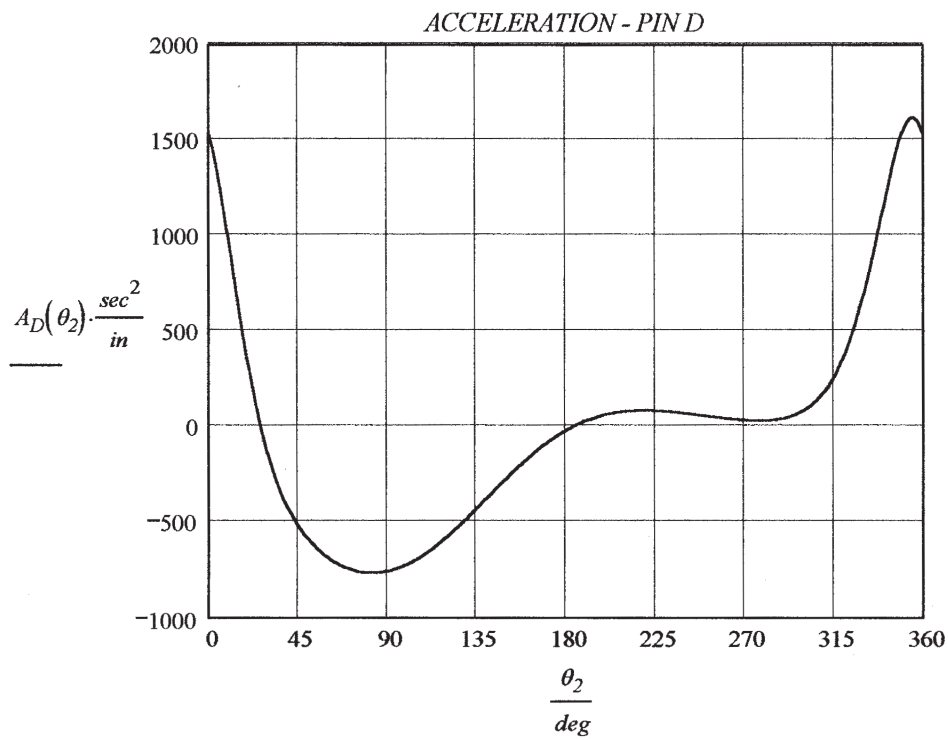
$$\omega_5(\theta_2) := \frac{c}{e} \cdot \frac{\cos(\theta_{42}(\theta_2))}{\cos(\theta_5(\theta_2))} \cdot \omega_4(\theta_2)$$

11. Determine the angular acceleration of link 5 using equation 7.16d.

$$\alpha_5(\theta_2) := \frac{c\cdot\alpha_4(\theta_2)\cdot\cos(\theta_{42}(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot\sin(\theta_{42}(\theta_2)) \ldots + e\cdot\omega_5(\theta_2)^2\cdot\sin(\theta_5(\theta_2))}{e\cdot\cos(\theta_5(\theta_2))}$$

12. Use equation 7.16e for the acceleration of pin D.

$$A_D(\theta_2) := -c\cdot\alpha_4(\theta_2)\cdot\sin(\theta_{42}(\theta_2)) - c\cdot\omega_4(\theta_2)^2\cdot\cos(\theta_{42}(\theta_2)) \ldots$$
$$+ e\cdot\alpha_5(\theta_2)\cdot\sin(\theta_5(\theta_2)) + e\cdot\omega_5(\theta_2)^2\cdot\cos(\theta_5(\theta_2))$$



PROBLEM 7-86

Statement: The linkage of Figure P6-33f has link 2 at 130 deg in the global XY coordinate system. Find $\mathbf{A}_D$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW and α_2 = 50 rad/sec² CW. Use the acceleration difference graphical method.

Given: Link lengths:

Link 2 (O_2 to A)	$a := 5.0 \cdot in$	Link 3 (A to B)	$b := 8.4 \cdot in$
Link 4 (O_4 to B)	$c := 2.5 \cdot in$	Link 1 (O_2 to O_4)	$d_1 := 12.5 \cdot in$
Link 5 (C to E)	$e := 8.9 \cdot in$	Link 5 (C to D)	$h := 5.9 \cdot in$
Link 6 (O_6 to E)	$f := 3.2 \cdot in$	Link 7 (D to F)	$k := 6.4 \cdot in$
Link 1 (O_2 to O_6)	$d_2 := 10.5 \cdot in$	Link 3 (A to C)	$g := 2.4 \cdot in$

Crank angle: $\theta_2 := 130 \cdot deg$

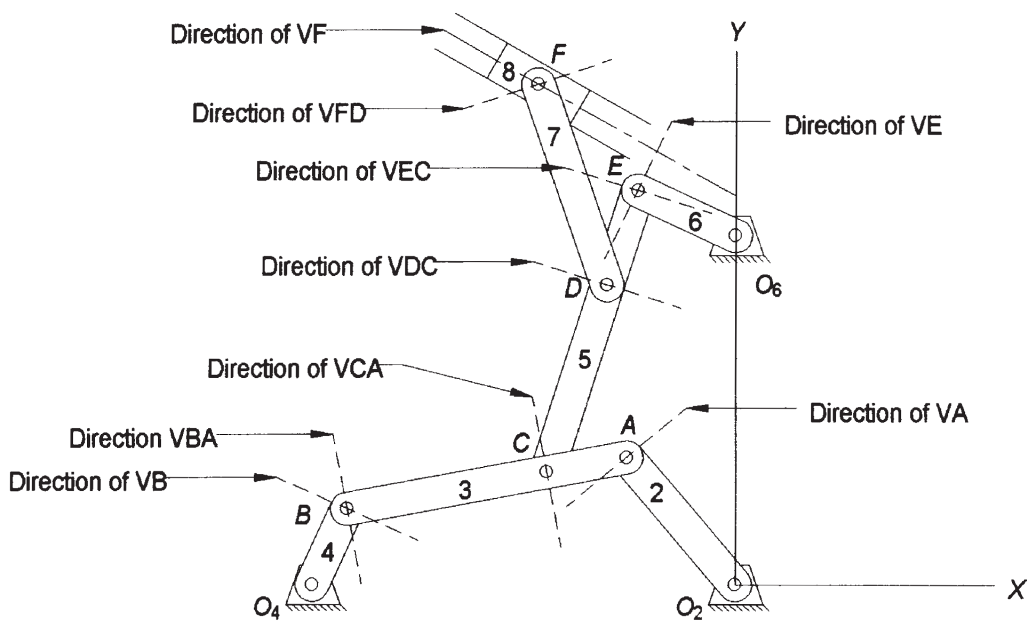
Slider axis offset and angle: $s := 11.7 \cdot in$ $\quad \delta_7 := 150 \cdot deg$

Input crank angular velocity $\quad \omega_2 := 15 \cdot rad \cdot sec^{-1}$ CW

Input crank angular acceleration $\quad \alpha_2 := 50 \cdot rad \cdot sec^{-2}$ CW

Solution: See Figure P6-33f and Mathcad file P0786.

1. Draw the linkage to a convenient scale. Indicate the directions of the velocity vectors of interest.



Angles measured from layout:

$\theta_{VB} := -24.351 \cdot deg$ $\quad \theta_{VBA} := -79.348 \cdot deg$ $\quad \theta_{VCA} := \theta_{VBA}$

$\theta_{VE} := 64.594 \cdot deg$ $\quad \theta_{VEC} := 162.461 \cdot deg$ $\quad \theta_{VDC} := \theta_{VEC}$

$\theta_{VF} := 150.00 \cdot deg$ $\quad \theta_{VFD} := 198.690 \cdot deg$ $\quad \theta_3 := -169.348 \cdot deg$

$\theta_4 := 65.649 \cdot deg$ $\quad \theta_5 := 72.461 \cdot deg$ $\quad \theta_6 := 154.594 \cdot deg$ $\quad \theta_7 := 108.690 \cdot deg$

2. Use equation 6.7 to calculate the magnitude of the velocity at point A.

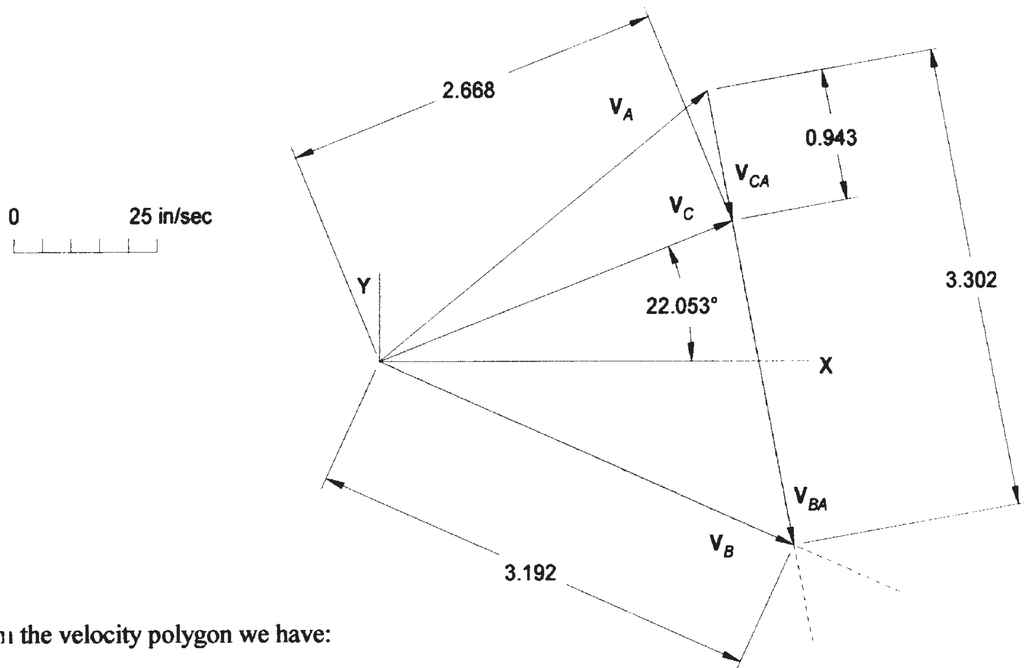
$V_A := a \cdot \omega_2 \qquad V_A = 75.000 \frac{in}{sec}$

$\theta_{VA} := \theta_2 - 90 \cdot deg \qquad \theta_{VA} = 40.000\, deg$

3. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *B*, the magnitude of the relative velocity $\mathbf{V}_{BA}$, and the angular velocity of link 3. The equations to be solved graphically are

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \quad \text{and} \quad \mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_A$.
b. From the tip of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_{BA}$, magnitude unknown.
c. From the tail of $\mathbf{V}_A$, draw a construction line with the direction of $\mathbf{V}_B$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{BA}$ from the tip of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_B$ construction line and drawing $\mathbf{V}_B$ from the tail of $\mathbf{V}_A$ to the intersection of the $\mathbf{V}_{BA}$ construction line.



4. From the velocity polygon we have:

Velocity scale factor: $k_v := \frac{25 \cdot in \cdot sec}{in}$

$V_B := 3.192 \cdot in \cdot k_v \qquad V_B = 79.800 \frac{in}{sec} \qquad \theta_{VB} = -24.351\, deg$

$\omega_4 := \frac{V_B}{c} \qquad \omega_4 = 31.920 \frac{rad}{sec}$ CW

$V_{BA} := 3.302 \cdot in \cdot k_v \qquad V_{BA} = 82.550 \frac{in}{sec} \qquad \theta_{VBA} = -79.348\, deg$

$\omega_3 := \frac{V_{BA}}{b} \qquad \omega_3 = 9.827 \frac{rad}{sec}$ CCW

5. Calculate the magnitude and direction of $\mathbf{V}_{CA}$ and determine the magnitude and velocity of $\mathbf{V}_C$ from the velocity polygon above.

$V_{CA} := g \cdot \omega_3 \qquad V_{CA} = 23.586 \frac{in}{sec} \qquad \theta_{VCA} = -79.348\, deg$

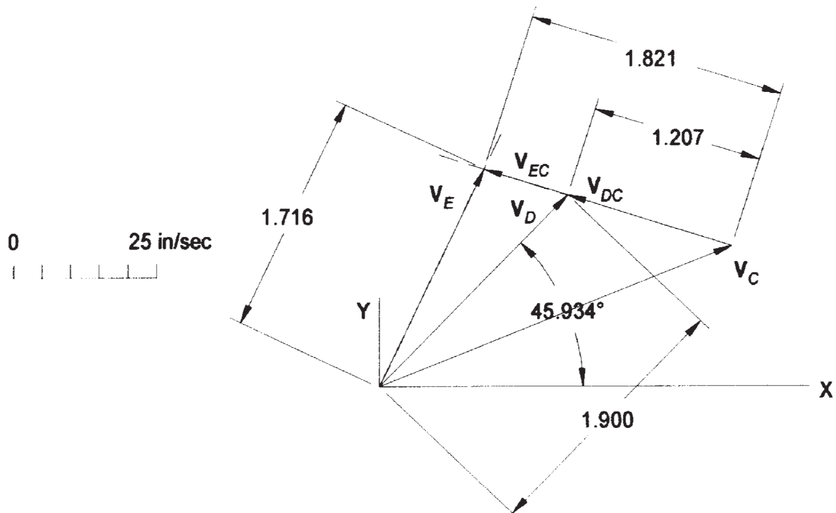
Length of $\mathbf{V}_{CA}$ on velocity polygon: $v_{CA} := \frac{V_{CA}}{k_v} \qquad v_{CA} = 0.943\, in$

$V_C := 2.668 \cdot in \cdot k_v$ $\qquad V_C = 66.700\,\dfrac{in}{sec}$ $\qquad \theta_{VC} := 22.053 deg$

6. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *E*, the magnitude of the relative velocity $\mathbf{V}_{ED}$, and the angular velocity of link 5. The equations to be solved graphically are

$$\mathbf{V}_E = \mathbf{V}_C + \mathbf{V}_{EC} \qquad \text{and} \qquad \mathbf{V}_D = \mathbf{V}_C + \mathbf{V}_{DC}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_C$.
b. From the tip of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_{EC}$, magnitude unknown.
c. From the tail of $\mathbf{V}_C$, draw a construction line with the direction of $\mathbf{V}_E$, magnitude unknown.
d. Complete the vector triangle by drawing $\mathbf{V}_{EC}$ from the tip of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_E$ construction line and drawing $\mathbf{V}_E$ from the tail of $\mathbf{V}_C$ to the intersection of the $\mathbf{V}_{EC}$ construction line.



7. From the velocity polygon we have:

Velocity scale factor: $\qquad k_v := \dfrac{25 \cdot in \cdot sec^{-1}}{in}$

$V_E := 1.716 \cdot in \cdot k_v$ $\qquad V_E = 42.900\,\dfrac{in}{sec}$ $\qquad \theta_{VE} = 64.594\,deg$

$\omega_6 := \dfrac{V_E}{f}$ $\qquad \omega_6 = 13.406\,\dfrac{rad}{sec}$ CW

$V_{EC} := 1.821 \cdot in \cdot k_v$ $\qquad V_{EC} = 45.525\,\dfrac{in}{sec}$ $\qquad \theta_{VEC} = 162.461\,deg$

$\omega_5 := \dfrac{V_{EC}}{e}$ $\qquad \omega_5 = 5.115\,\dfrac{rad}{sec}$ CCW

8. Calculate the magnitude and direction of $\mathbf{V}_{DE}$ and determine the magnitude and velocity of $\mathbf{V}_D$ from the velocity polygon above.

$V_{DC} := h \cdot \omega_5$ $\qquad V_{DC} = 30.179\,\dfrac{in}{sec}$ $\qquad \theta_{VDC} = 162.461\,deg$

Length of $\mathbf{V}_{DC}$ on velocity polygon: $\qquad v_{DC} := \dfrac{V_{DC}}{k_v}$ $\qquad v_{DC} = 1.207\,in$

$V_D := 1.900 \cdot in \cdot k_v$ $\qquad V_D = 47.500\,\dfrac{in}{sec}$ $\qquad \theta_{VD} := 45.934 deg$

9. Use equation 6.5 to (graphically) determine the magnitude of the velocity at point *F*, the magnitude of the relative velocity $\mathbf{V}_{FD}$, and the angular velocity of link 5. The equation to be solved graphically is

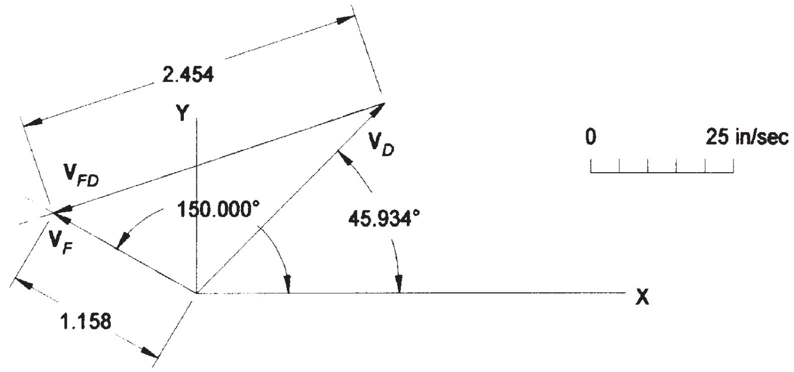
$$\mathbf{V}_F = \mathbf{V}_D + \mathbf{V}_{FD}$$

a. Choose a convenient velocity scale and layout the known vector $\mathbf{V}_D$.

b. From the tip of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_{FD}$, magnitude unknown.

c. From the tail of $\mathbf{V}_D$, draw a construction line with the direction of $\mathbf{V}_F$, magnitude unknown.

d. Complete the vector triangle by drawing $\mathbf{V}_{FD}$ from the tip of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_F$ construction line and drawing $\mathbf{V}_F$ from the tail of $\mathbf{V}_D$ to the intersection of the $\mathbf{V}_{FD}$ construction line.



10. From the velocity polygon we have:

Velocity scale factor: $k_v := \dfrac{25 \cdot in \cdot sec^{-1}}{in}$

$V_F := 1.158 \cdot in \cdot k_v$ $\quad V_F = 28.950 \dfrac{in}{sec}$ $\quad \theta_{VF} = 150.000\,deg$

$V_{FD} := 2.454 \cdot in \cdot k_v$ $\quad V_{FD} = 61.350 \dfrac{in}{sec}$

$\omega_7 := \dfrac{V_{FD}}{k}$ $\quad \omega_7 = 9.586 \dfrac{rad}{sec}$ CCW

11. The graphical solution for accelerations uses equation 7.4: $(\mathbf{A}_P{}^t + \mathbf{A}_P{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{PA}{}^t + \mathbf{A}_{PA}{}^n)$

12. For point B, this becomes: $(\mathbf{A}_B{}^t + \mathbf{A}_B{}^n) = (\mathbf{A}_A{}^t + \mathbf{A}_A{}^n) + (\mathbf{A}_{BA}{}^t + \mathbf{A}_{BA}{}^n)$, where

$A_{Bn} := c \cdot \omega_4{}^2$ $\quad A_{Bn} = 2547.2\,in \cdot sec^{-2}$

$\theta_{ABn} := \theta_4 - 180 \cdot deg$ $\quad \theta_{ABn} = -114.351\,deg$

$A_{An} := a \cdot \omega_2{}^2$ $\quad A_{An} = 1125.0\,in \cdot sec^{-2}$

$\theta_{AAn} := \theta_2 - 180 \cdot deg$ $\quad \theta_{AAn} = -50.000\,deg$

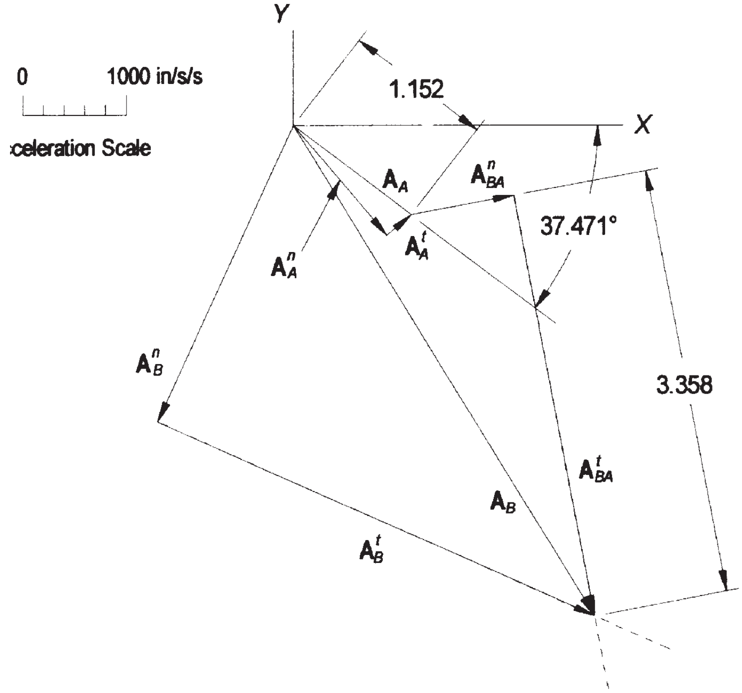
$A_{At} := a \cdot \alpha_2$ $\quad A_{At} = 250.0\,in \cdot sec^{-2}$

$\theta_{AAt} := \theta_2 - 90 \cdot deg$ $\quad \theta_{AAt} = 40.000\,deg$

$A_{BAn} := b \cdot \omega_3{}^2$ $\quad A_{BAn} = 811.3\,in \cdot sec^{-2}$

$\theta_{ABAn} := \theta_3 - 180 \cdot deg$ $\quad \theta_{ABAn} = -349.348\,deg$

13. Choose a convenient acceleration scale and draw the vectors with known magnitude and direction. From the origin, draw $\mathbf{A}_B{}^n$ at an angle of θ_{ABn} and $\mathbf{A}_A{}^n$ at an angle of θ_{AAn}. From the tip of $\mathbf{A}_A{}^n$, draw $\mathbf{A}_A{}^t$ at an angle of θ_{AAt}. From the tip of $\mathbf{A}_A{}^t$, draw $\mathbf{A}_{BA}{}^n$ at an angle of θ_{BAn}. Now that the vectors with known magnitudes are drawn, from the tips of $\mathbf{A}_B{}^n$ and $\mathbf{A}_{BA}{}^n$, draw construction lines in the directions of $\mathbf{A}_B{}^t$ and $\mathbf{A}_{BA}{}^t$, respectively. The intersection of these two lines are the tips of $\mathbf{A}_B$, $\mathbf{A}_B{}^t$, and $\mathbf{A}_{BA}{}^t$.



14. From the graphical solution above,

Acceleration scale factor $\quad k_a := 1000 \cdot \dfrac{in}{sec^2}$

$A_A := 1.152 \cdot k_a \qquad A_A = 1152\,in \cdot sec^{-2} \qquad \theta_{AA} := -37.471 \cdot deg$

$A_{BAt} := 3.358 \cdot k_a \qquad A_{BAt} = 3358\,in \cdot sec^{-2}$

$\alpha_3 := \dfrac{A_{BAt}}{b} \qquad \alpha_3 = 399.762\,\dfrac{rad}{sec^2} \qquad$ CW

15. For point C, : $\mathbf{A}_C = \mathbf{A}_A + (\mathbf{A}_{CA}{}^t + \mathbf{A}_{CA}{}^n)$, where

$A_A = 1.152 \times 10^3\,in \cdot sec^{-2} \qquad \theta_{AA} = -37.471\,deg$

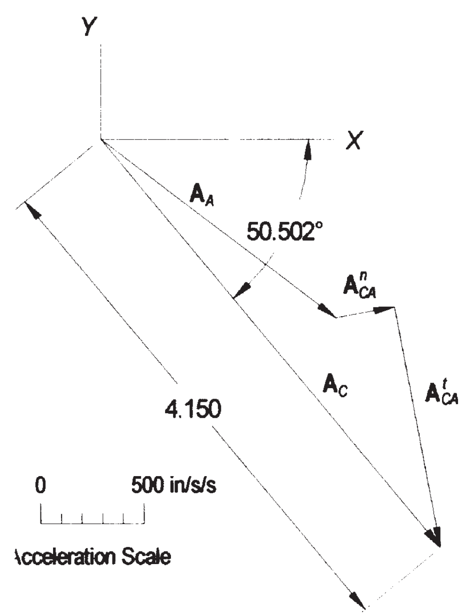
$\theta_{AAn} := \theta_2 - 180 \cdot deg \qquad \theta_{AAn} = -50.000\,deg$

$A_{CAt} := g \cdot \alpha_3 \qquad A_{CAt} = 959.4\,in \cdot sec^{-2}$

$\theta_{ACAt} := \theta_3 + 90 \cdot deg \qquad \theta_{ACAt} = -79.348\,deg$

$A_{CAn} := g \cdot \omega_3{}^2 \qquad A_{CAn} = 231.8\,in \cdot sec^{-2}$

$\theta_{ACAn} := \theta_3 - 180 \cdot deg \qquad \theta_{ACAn} = -349.348\,deg$



16. From the graphical solution above,

Acceleration scale factor $k_a := 500 \cdot \dfrac{in}{sec^2}$

$A_C := 4.150 \cdot k_a$ $\quad A_C = 2075\ in \cdot sec^{-2}$ $\quad \theta_{AC} := -50.502 \cdot deg$

17. For point *E*: $(\mathbf{A}_E{}^t + \mathbf{A}_E{}^n) = \mathbf{A}_C + (\mathbf{A}_{EC}{}^t + \mathbf{A}_{EC}{}^n)$, where

$A_{En} := f \cdot \omega_6{}^2$ $\quad A_{En} = 575.1\ in \cdot sec^{-2}$

$\theta_{AEn} := \theta_6 - 180 \cdot deg$ $\quad \theta_{AEn} = -25.406\ deg$

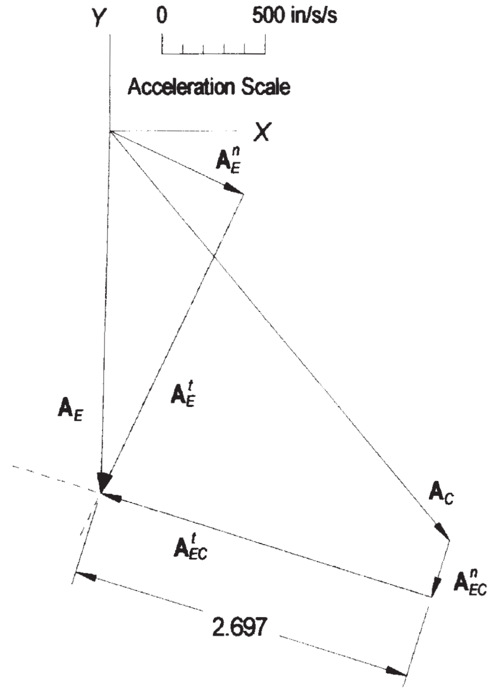
$A_{ECn} := e \cdot \omega_5{}^2$ $\quad A_{ECn} = 232.9\ in \cdot sec^{-2}$

$\theta_{AECn} := \theta_5 - 180 \cdot deg$ $\quad \theta_{AECn} = -107.539\ deg$

Acceleration scale factor $k_a := 500 \cdot \dfrac{in}{sec^2}$

$A_{ECt} := 2.697 \cdot k_a$ $\quad A_{ECt} = 1349\ in \cdot sec^{-2}$

$\alpha_5 := \dfrac{A_{ECt}}{e}$ $\quad \alpha_5 = 151.517 \dfrac{rad}{sec^2}$ CCW



18. For point *D*, : $\mathbf{A}_D = \mathbf{A}_C + (\mathbf{A}_{DC}{}^t + \mathbf{A}_{DC}{}^n)$, where

$A_C = 2075.0\ in \cdot sec^{-2}$ $\quad \theta_{AC} = -50.502\ deg$

$A_{DCt} := h \cdot \alpha_5$ $\quad A_{DCt} = 893.9\ in \cdot sec^{-2}$

$\theta_{ADCt} := \theta_5 + 90 \cdot deg$ $\quad \theta_{ADCt} = 162.461\ deg$

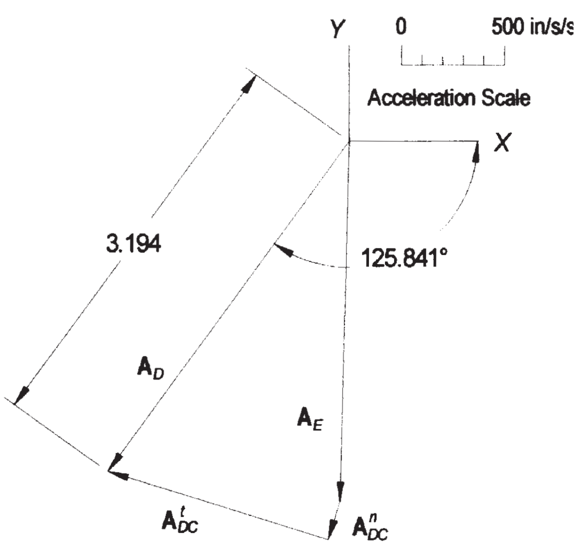
$A_{DCn} := h \cdot \omega_5{}^2$ $\quad A_{DCn} = 154.4\ in \cdot sec^{-2}$

$\theta_{ADCn} := \theta_5 - 180 \cdot deg$ $\quad \theta_{ADCn} = -107.539\ deg$

Acceleration scale factor $k_a := 500 \cdot \dfrac{in}{sec^2}$

$A_D := 3.194 \cdot k_a$ $\quad A_C = 2075\ in \cdot sec^{-2}$

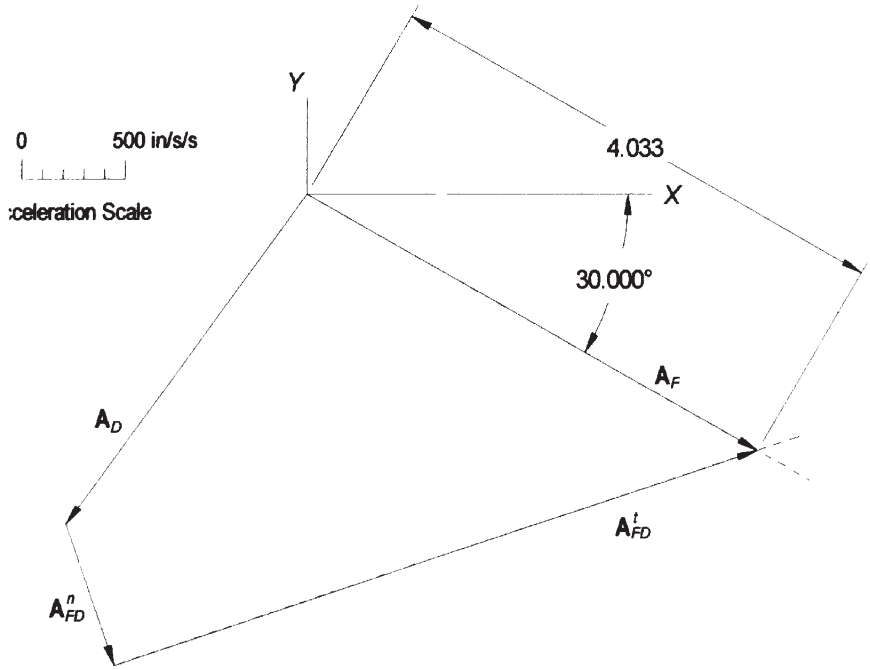
$\theta_{AD} := -125.841 \cdot deg$



19. For point *F*, : $\mathbf{A}_F = \mathbf{A}_D + (\mathbf{A}_{FD}{}^t + \mathbf{A}_{FD}{}^n)$, where

$A_{FDn} := k \cdot \omega_7{}^2$ $\quad A_{FDn} = 588.10\ in \cdot sec^{-2}$

$\theta_{AFDn} := \theta_7 - 180 \cdot deg$ $\quad \theta_{AFDn} = -71.310\ deg$



Acceleration scale factor $k_a := 500 \cdot \dfrac{in}{sec^2}$

$A_F := 4.033 \cdot k_a$ $A_F = 2017\ in \cdot sec^{-2}$ $\theta_{AD} := -30.00 \cdot deg$

PROBLEMS 7-3 AND 7-4

Row	Open α_3	Open α_4	A_P Mag	A_P Ang	Crossed α_3	Crossed α_4	A_P Mag	A_P Ang
a	26.1	53.3	419	240.4	77.9	50.7	298	-11.3
b	-29826	11390	273911	241.7	29792	-11425	275939	62.3
c	-154.4	-71.6	4400	238.9	-65.2	-148.0	3554	100.6
d	1025.6	1489.4	8085.2	188.9	1771.0	1307.4	6466.4	63.2
e	331.9	275.6	10260	264.8	1287.7	1344.1	19340	-65.5
f	-7518.2	-5343.2	90564	-64.7	-4158.2	-6333.2	139742	84.3
g	-23510	-19783	172688	191	-43709	-47436	273634	-63
h	-625.3	7229.9	63043	226.5	7229.9	-652.3	5663.5	116.1
i	-344.6	505.3	9492	-81.1	121.9	-728.0	27871	150
j	-1475.6	2225.2	9437.6	220.5	3017.5	-683.3	3062.5	214.4
k	-2693	-4054	56271	220.2	311.0	1672.1	27759	-39.1
l	-1565.6	-2462.2	49368	236.7	7876.7	8773.3	86597	218.2
m	680.8	149.2	35149	261.5	9266.1	10303	63831	103.9
n	-6.95	-127.3	1765.9	-46.5	-26.2	94.2	1776.8	263.8

PROBLEMS 7-5 AND 7-6

Row	A_A Mag	A_A Ang	Open α_3	A_B Mag	A_B Ang	Crossed α_3	A_B Mag	A_B Ang
a	140	-135	24.8	123.7	180	-24.8	74.3	180
b	288	-122	52.7	57.6	0	-52.7	362.9	180
c	676	153	-29.0	709.1	180	29.0	490.0	180
d	2016	300	173.8	1192.9	0	-173.8	847.3	0
e	12500	45	-447.1	6652.7	0	447.1	11095.7	0
f	6077	-81.4	473.8	2263.7	0	-473.8	449.3	180
g	70000	150	-1136.0	62688	180	1136.0	58430	180

PROBLEMS 7-7 AND 7-8

Row	Open α_3	Open α_4	Open A_{slip}	Crossed α_3	Crossed α_4	Crossed A_{slip}
a	130.5	130.5	-3111.7	-9.9	-9.9	1125.2
b	-1.31	-1.31	1129.3	-113.3	-113.3	748.12
c	-212.9	-212.9	3327.9	-217.8	-217.8	6776.4
d	9523.1	9523.1	-7462.1	2564.5	2564.5	13136
e	896.3	896.3	2020.1	595.6	595.6	-10444
f	1942	1942	75148	-2645.7	-2645.7	9459

PROBLEM 7-9

Row	Open α_3	Open α_4	Crossed α_3	Crossed α_4
a	3191	2492	-6648	-5949
b	549	-6997	476	289
c	314	228	87.2	147
d	960	-1397	37.1	-584.6
e	2171	-6524	7781	5414
f	248	2699	3271	-183.5
g	-22064	-23717	-5529	-29133
h	18218	-42880	-4373	7133
i	-5697	-3380	-2593	-7184