PROBLEM 8-32

Statement: Write a computer program or use an equation solver such as Mathcad or TKSolver to calculate and plot the pressure angle and radius of curvature for a simple harmonic displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over an interval of 60 deg at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20 deg. What is the minimum diameter roller follower needed to avoid undercutting with these data?

Enter:

Lift: $h := 20 \cdot mm$ Eccentricity $\varepsilon := 4 \cdot mm$

Duration: $\beta := 60 \cdot deg$ Prime circle radius $R_p := 50 \cdot mm$

Solution: See Mathcad file P0832.

1. The simple harmonic motion (SHM) is defined in local coordinates by equations 8.6. They are:

$$s(\theta) := \frac{h}{2} \cdot \left(1 - cos\left(\pi \cdot \frac{\theta}{\beta}\right)\right) \qquad v(\theta) := \frac{\pi}{\beta} \cdot \frac{h}{2} \cdot sin\left(\pi \cdot \frac{\theta}{\beta}\right)$$

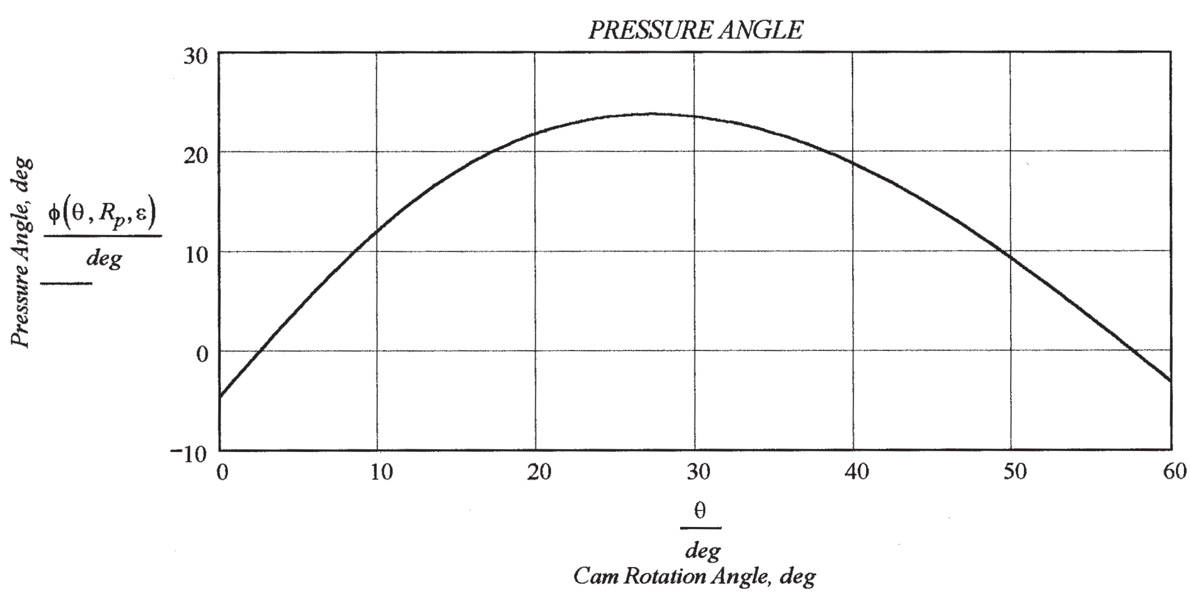
$$a(\theta) := \frac{\pi^2}{\beta^2} \cdot \frac{h}{2} \cdot cos\left(\pi \cdot \frac{\theta}{\beta}\right) \qquad j(\theta) := \frac{\pi^3}{\beta^3} \cdot \frac{h}{2} \cdot sin\left(\pi \cdot \frac{\theta}{\beta}\right)$$

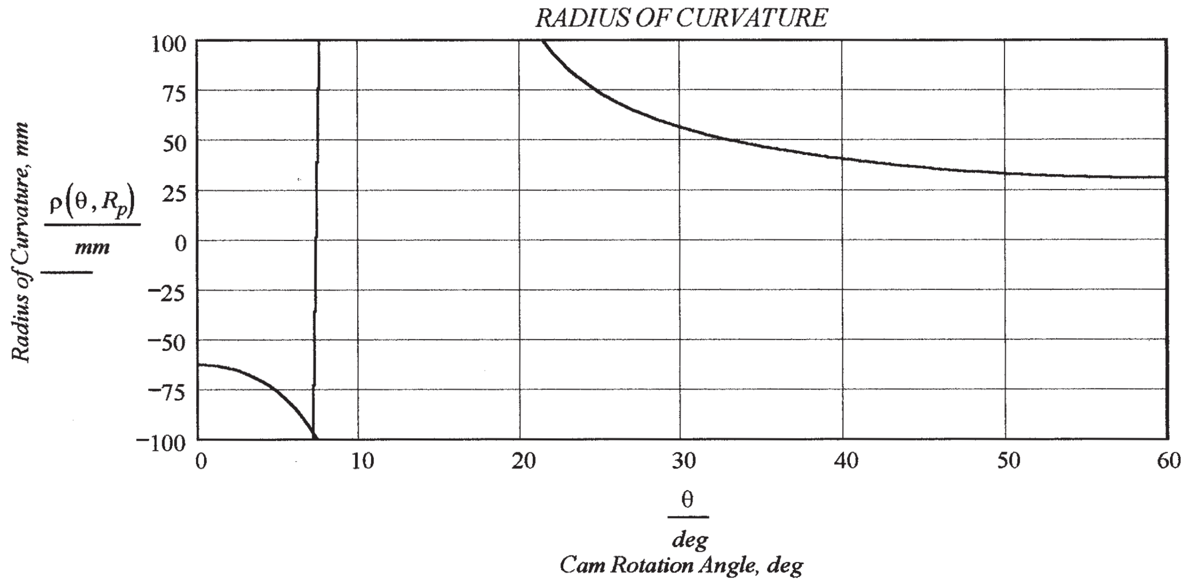
2. Using equations 8.31d and 8.33, write the pressure angle and radius of curvature functions.

$$\phi(\theta, R_p, \varepsilon) := atan\left(\frac{v(\theta) - \varepsilon}{s(\theta) + \sqrt{R_p^2 - \varepsilon^2}}\right)$$

$$\rho(\theta, R_p) := \frac{\left[\left(R_p + s(\theta)\right)^2 + v(\theta)^2\right]^{\frac{3}{2}}}{\left(R_p + s(\theta)\right)^2 + 2 \cdot v(\theta)^2 - a(\theta) \cdot \left(R_p + s(\theta)\right)}$$

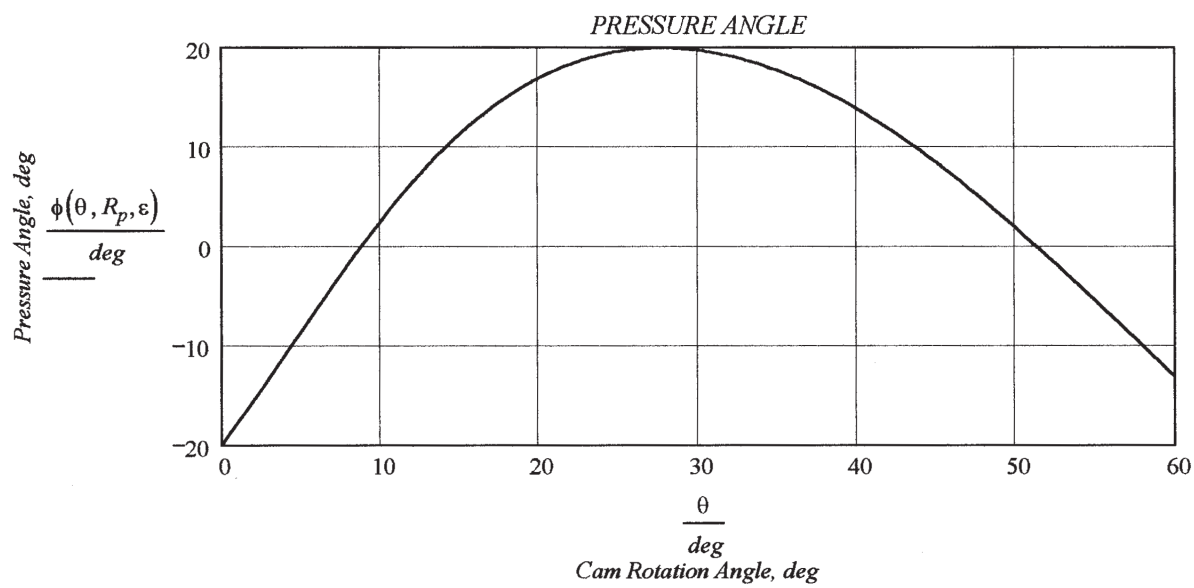
3. Plot the pressure angle and radius of curvature functions over the lift interval: $\theta := 0 \cdot deg, 0.5 \cdot deg .. \beta$





4. The graphs above show the pressure angle and radius of curvature for the values of R_p and ε entered on the first page. These values will be iterated below to obtain a balanced pressure angle whose absolute value is not greater than 20 deg.

$R_p := 39 \cdot mm$ $\qquad$ $\varepsilon := 13.3 \cdot mm$



5. From the graph of radius of curvature above, we see that the minimum value occurs at a cam angle of about 60 deg.

$\rho_{min} := \rho(60 \cdot deg, R_p)$ $\qquad$ $\rho_{min} = 23.362\,mm$

Using a multiple of 3, the maximum roller follower radius is $R_f := \dfrac{\rho_{min}}{3}$ $\qquad$ $R_f = 7.8\,mm$

PROBLEM 8-33

Statement: Derive equation 8.25 for the 4-5-6-7 polynomial.

Solution: See Mathcad file P0833.

1. There are eight boundary conditions. They are:

At $\theta = 0$:	$s = 0$,	$v = 0$,	$a = 0$,	$j = 0$
At $\theta = \beta$:	$s = h$,	$v = 0$,	$a = 0$,	$j = 0$

2. Write equation 8.23 in the form of equation *c* in Example 8-5, but with eight terms. Differentiate it repeatedly to get equations for v, a, and j that are similar to equations *d* and *e*. Write eight equations in the unknown coefficients C_0 through C_7, applying the boundary conditions above. These eight equations are.

For $\theta = 0$: $s = 0, v = 0, a = 0, j = 0$

$0 := C_0$ $\quad C_0 := 0$

$0 := C_1$ $\quad C_1 := 0$

$0 := 2 \cdot C_2$ $\quad C_2 := 0$

$0 := 6 \cdot C_3$ $\quad C_3 := 0$

For $\theta = \beta$: $s = h, v = 0, a = 0, j = 0$

$$h := C_4 + C_5 + C_6 + C_7$$

$$0 := 4 \cdot C_4 + 5 \cdot C_5 + 6 \cdot C_6 + 7 \cdot C_7$$

$$0 := 12 \cdot C_4 + 20 \cdot C_5 + 30 \cdot C_6 + 42 \cdot C_7$$

$$0 := 24 \cdot C_4 + 60 \cdot C_5 + 120 \cdot C_6 + 210 \cdot C_7$$

3. Solve the last four equations for the four unknown coefficients C_4, C_5, C_6, and C_7. Set $h = 1$ then multiply all coefficients by h.

$$C := \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 5 & 6 & 7 \\ 12 & 20 & 30 & 42 \\ 24 & 60 & 120 & 210 \end{pmatrix} \qquad H := \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} C_4 \\ C_5 \\ C_6 \\ C_7 \end{pmatrix} := C^{-1} \cdot H$$

$C_4 = 35$ $\qquad C_5 = -84$ $\qquad C_6 = 70$ $\qquad C_7 = -20$

PROBLEM 8-34

Statement: Derive an expression for the pressure angle of a barrel cam with zero eccentricity.

Solution: See Mathcad file P0834.

1. A barrel cam is shown in Figure 8-4. The motion of the follower is parallel to the axis of rotation of the cam. The transmission axis is the common normal between the follower and the groove in which it travels. The common normal will go through and be perpendicular to the center of rotation of the follower and will be normal to the wall of the groove at the point of contact. Let

   s = axial displacement of the follower from some reference point, length units
   v = velocity of the follower, length/rad units
   r = mean radius of the groove, length units

   then, for an infinitesimal rotation of the cam, the follower will advance a distance *ds* while the distance along the groove will have advanced an amount $rd\theta$. These two quantities form the legs of a right triangle whose hypotenuse is perpendicular to the common normal. Because of this relationship, we have

$$\tan(\phi) = \frac{1}{r}\cdot\left(\frac{d}{d\theta}s\right) = \frac{v}{r} \qquad \phi := \operatorname{atan}\left(\frac{v}{r}\right)$$

PROBLEM 8-35

Statement: Design a radial plate cam to move a translating roller follower through 30 mm in 30 deg, dwell for 100 deg, fall 10 mm in 10 deg, dwell for 20 deg, fall 20 mm in 20 deg, and dwell for the remainder. Camshaft ω = 200 rpm. Minimize the follower's peak velocity and determine the minimum prime circle radius that will give a maximum 25-deg pressure angle. Determine the minimum radii of curvature on the pitch curve.

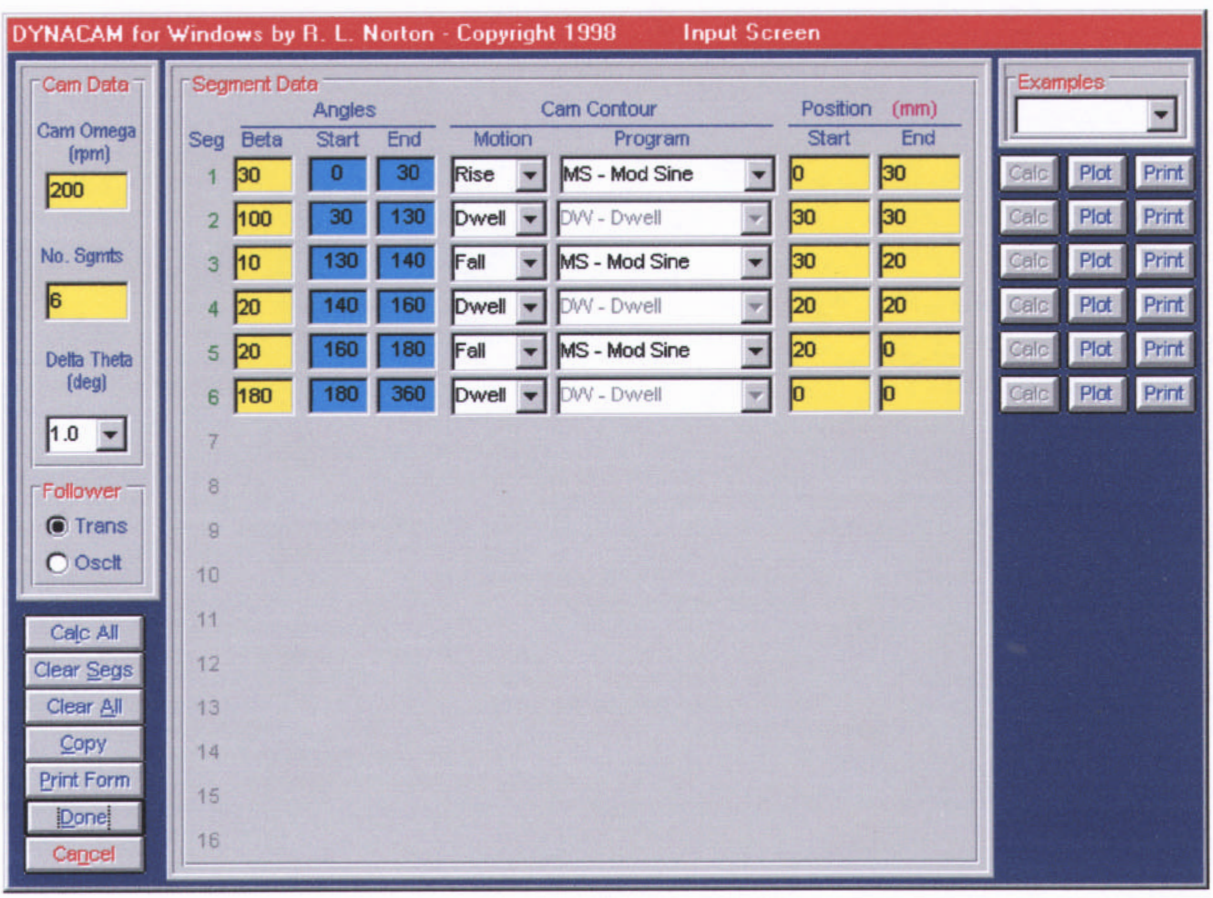
Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given:

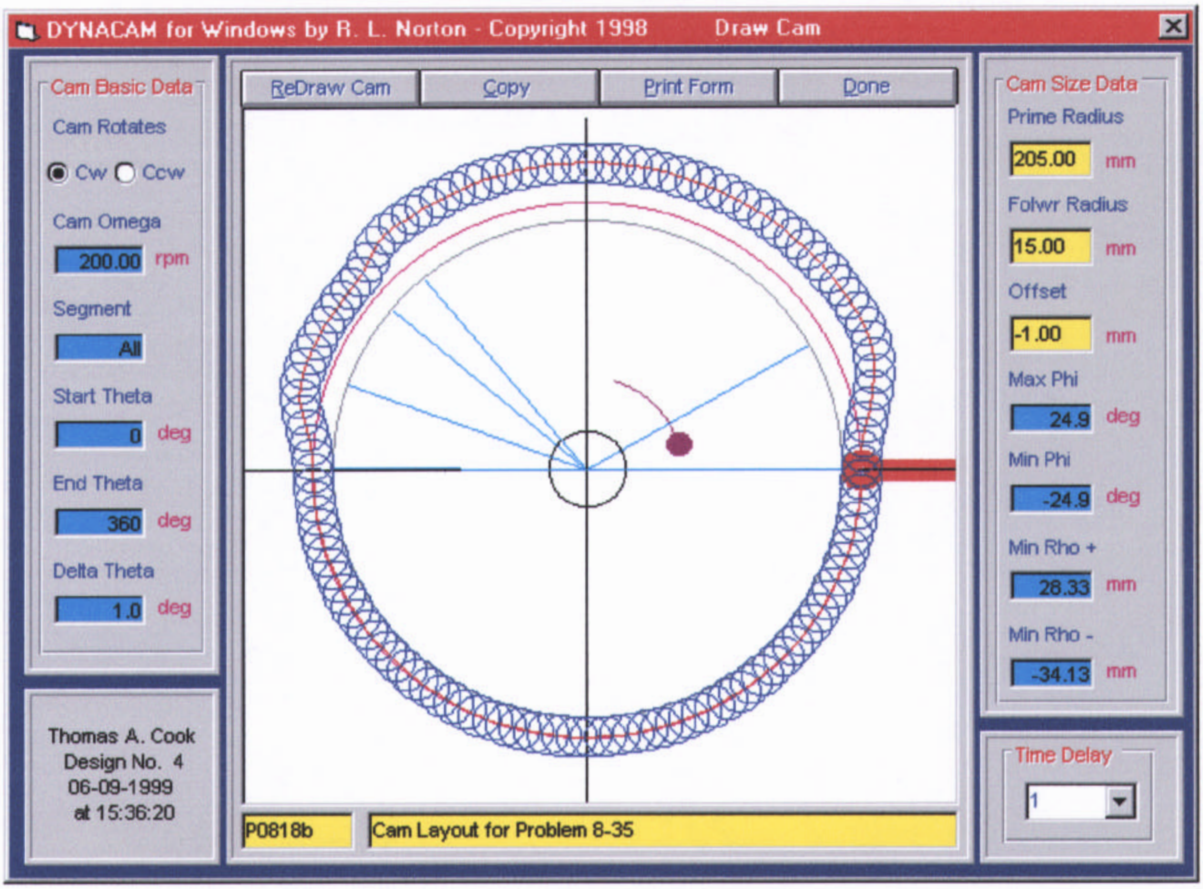
RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 30\cdot deg$	$\beta_2 := 100\cdot deg$	$\beta_3 := 10\cdot deg$	$\beta_4 := 20\cdot deg$
$h_1 := 30\cdot mm$	$h_2 := 0.0\cdot in$	$h_3 := 10\cdot mm$	$h_4 := 0.0\cdot in$
$\beta_5 := 20\cdot deg$	$\beta_6 := 180\cdot deg$		
$h_5 := 20\cdot mm$	$h_6 := 0.0\cdot in$		
Shaft speed	$\omega := 200\cdot rpm$		

Solution: See Mathcad file P0835.

1. From Table 8-3, the motion program with lowest velocity that does not have infinite jerk is the modified sinusoidal. Use the modified sine for segments 1, 3, and 5. Segments 2, 4, and 6 are dwells. Enter the above data into program DYNACAM. The input screen is shown below.



2. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.



3. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius	$R_p := 205 \cdot mm$
Roller follower radius	$R_f := 15 \cdot mm$
Follower eccentricity	$\varepsilon := -1.00 \cdot mm$

PROBLEM 8-36

Statement: Design a radial plate cam to move a translating roller follower through 30 mm in 30 deg, dwell for 100 deg, fall 10 mm in 10 deg, dwell for 20 deg, fall 20 mm in 20 deg, and dwell for the remainder. Camshaft ω = 200 rpm. Minimize the follower's peak acceleration and determine the minimum prime circle radius that will give a maximum 25-deg pressure angle. Determine the minimum radii of curvature on the pitch curve.

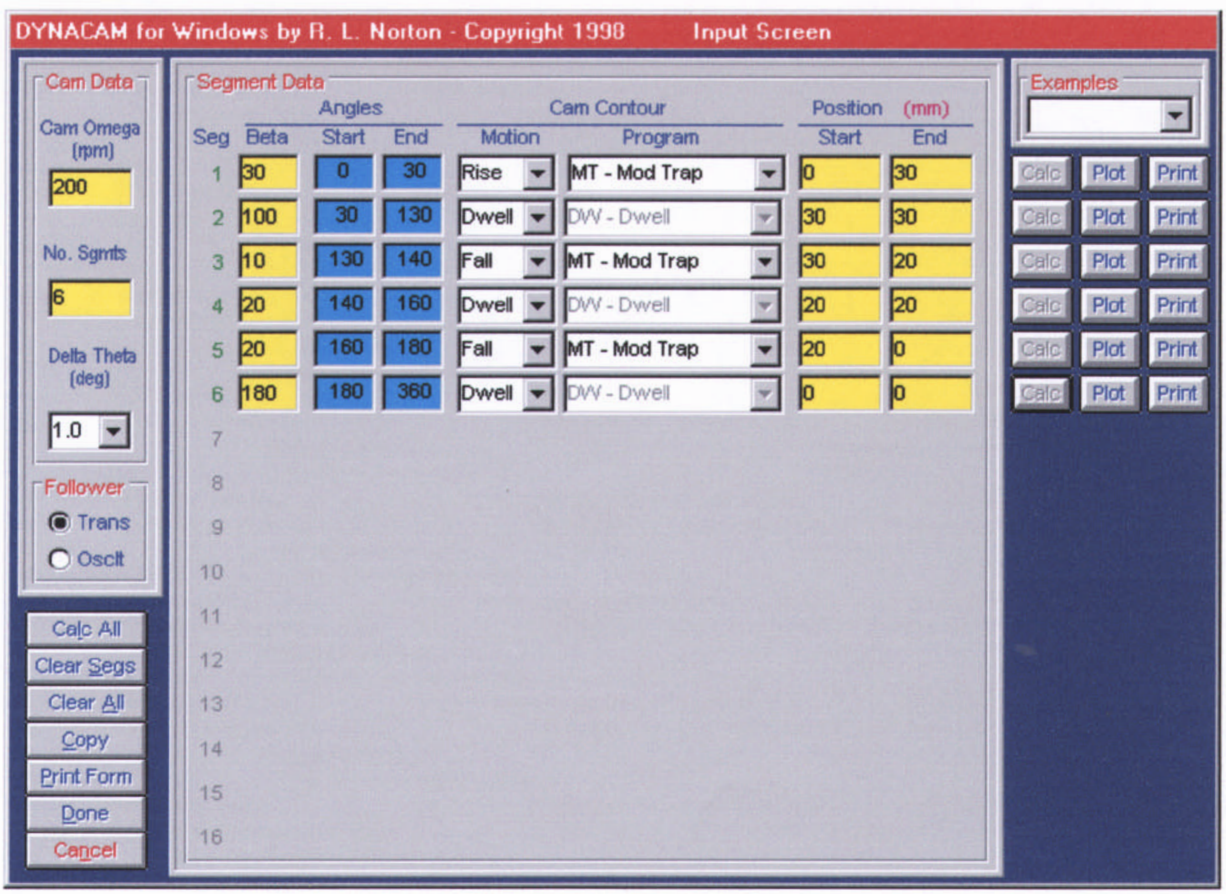
Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

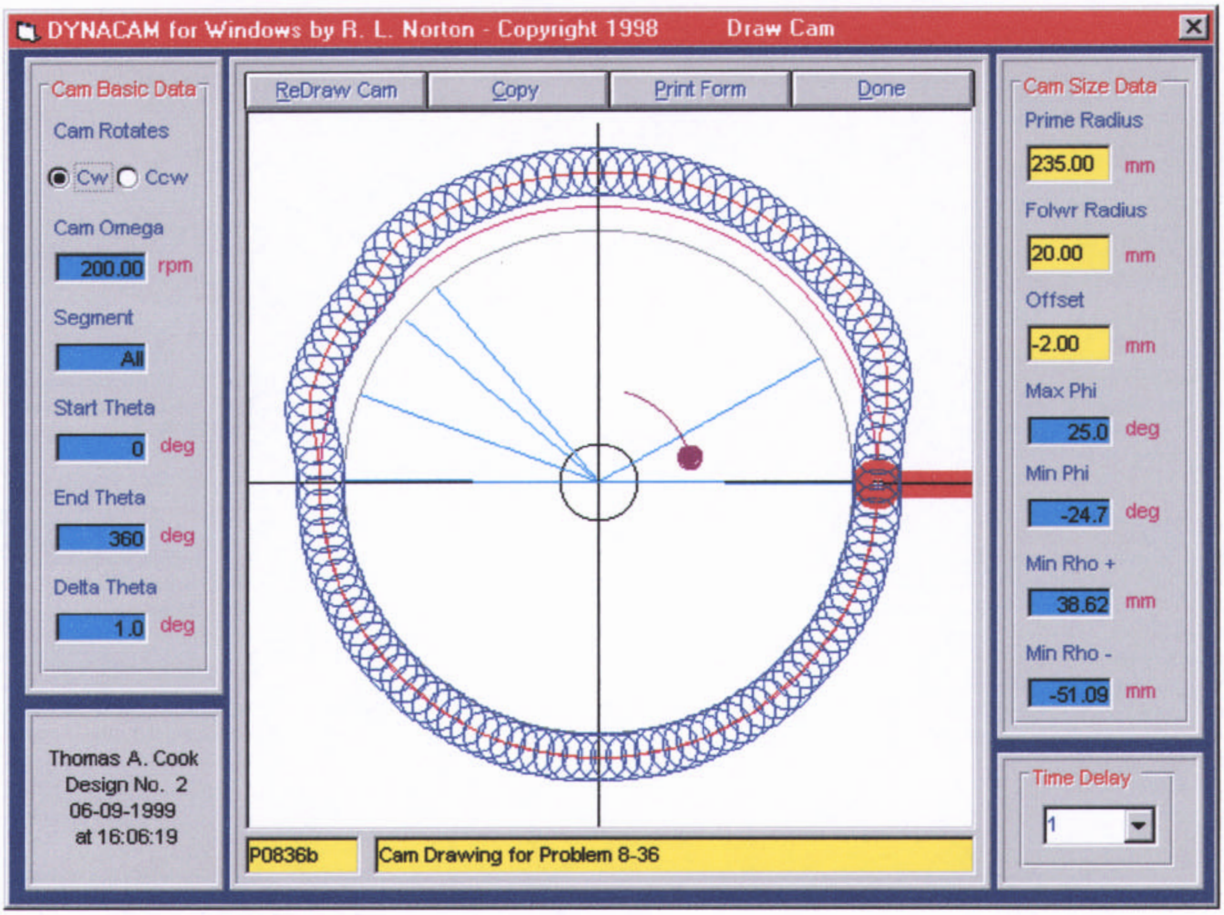
RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 30 \cdot deg$	$\beta_2 := 100 \cdot deg$	$\beta_3 := 10 \cdot deg$	$\beta_4 := 20 \cdot deg$
$h_1 := 30 \cdot mm$	$h_2 := 0.0 \cdot in$	$h_3 := 10 \cdot mm$	$h_4 := 0.0 \cdot in$
$\beta_5 := 20 \cdot deg$	$\beta_6 := 180 \cdot deg$		
$h_5 := 20 \cdot mm$	$h_6 := 0.0 \cdot in$		
Shaft speed	$\omega := 200 \cdot rpm$		

Solution: See Mathcad file P0836.

1. From Table 8-3, the motion program with lowest acceleration that does not have infinite jerk is the modified trapezoidal. Use the modified trapezoidal for segments 1, 3, and 5. Segments 2, 4, and 6 are dwells. Enter the above data into program DYNACAM. The input screen is shown below.



2. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.



3. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius $R_p := 235 \cdot mm$

Roller follower radius $R_f := 20 \cdot mm$

Follower eccentricity $\varepsilon := -2.00 \cdot mm$

PROBLEM 8-37

Statement: Design a radial plate cam to move a translating roller follower through 30 mm in 30 deg, dwell for 100 deg, fall 10 mm in 10 deg, dwell for 20 deg, fall 20 mm in 20 deg, and dwell for the remainder. Camshaft ω = 200 rpm. Minimize the follower's peak jerk and determine the minimum prime circle radius that will give a maximum 25-deg pressure angle. Determine the minimum radii of curvature on the pitch curve.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

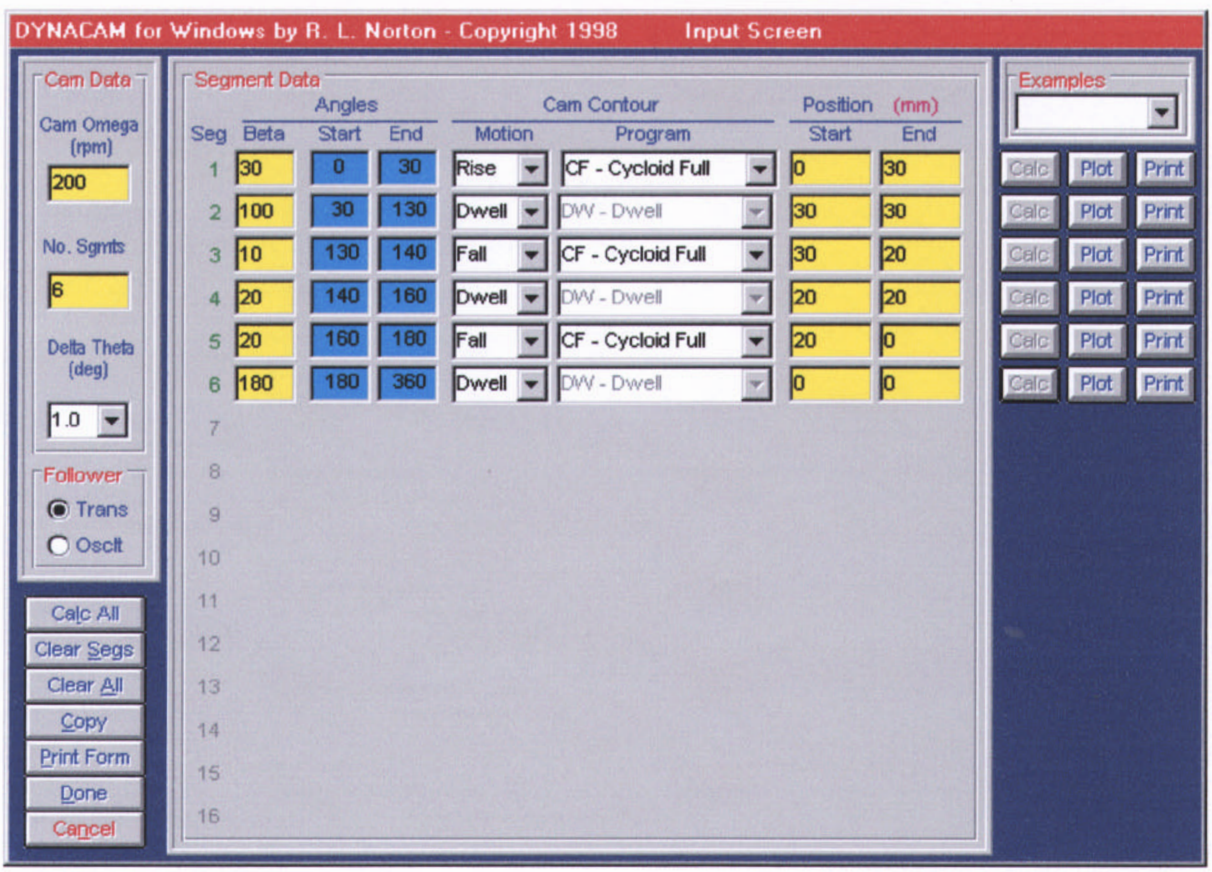
Given:

RISE/FALL	DWELL	FALL	DWELL
$\beta_1 := 30 \cdot deg$	$\beta_2 := 100 \cdot deg$	$\beta_3 := 10 \cdot deg$	$\beta_4 := 20 \cdot deg$
$h_1 := 30 \cdot mm$	$h_2 := 0.0 \cdot in$	$h_3 := 10 \cdot mm$	$h_4 := 0.0 \cdot in$
$\beta_5 := 20 \cdot deg$	$\beta_6 := 180 \cdot deg$		
$h_5 := 20 \cdot mm$	$h_6 := 0.0 \cdot in$		

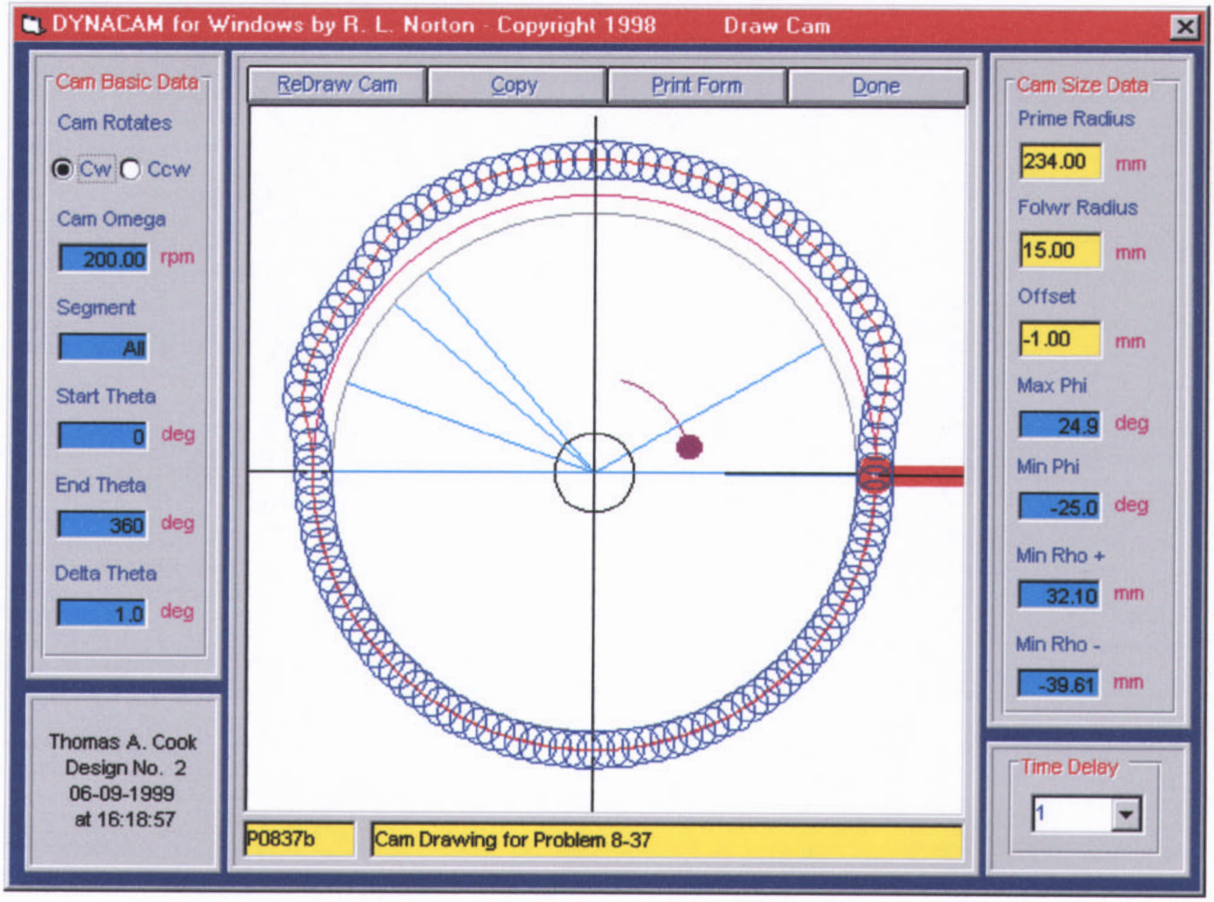
Shaft speed $\omega := 200 \cdot rpm$

Solution: See Mathcad file P0837.

1. From Table 8-3, the motion program with lowest jerk that does not have infinite jerk is the cycloidal displacement. Use the cycloidal for segments 1, 3, and 5. Segments 2, 4, and 6 are dwells. Enter the above data into program DYNACAM. The input screen is shown below.



2. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.



3. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius	$R_p := 234 \cdot mm$
Roller follower radius	$R_f := 15 \cdot mm$
Follower eccentricity	$\varepsilon := -1.00 \cdot mm$

PROBLEM 8-38

Statement: Design a radial plate cam to lift a translating roller follower through 10 mm in 65 deg, return to 0 in 65 deg, and dwell for the remainder. Camshaft ω = 3500 rpm. Minimize the cam size while not exceeding a 25-deg pressure angle. What size roller follower is needed?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

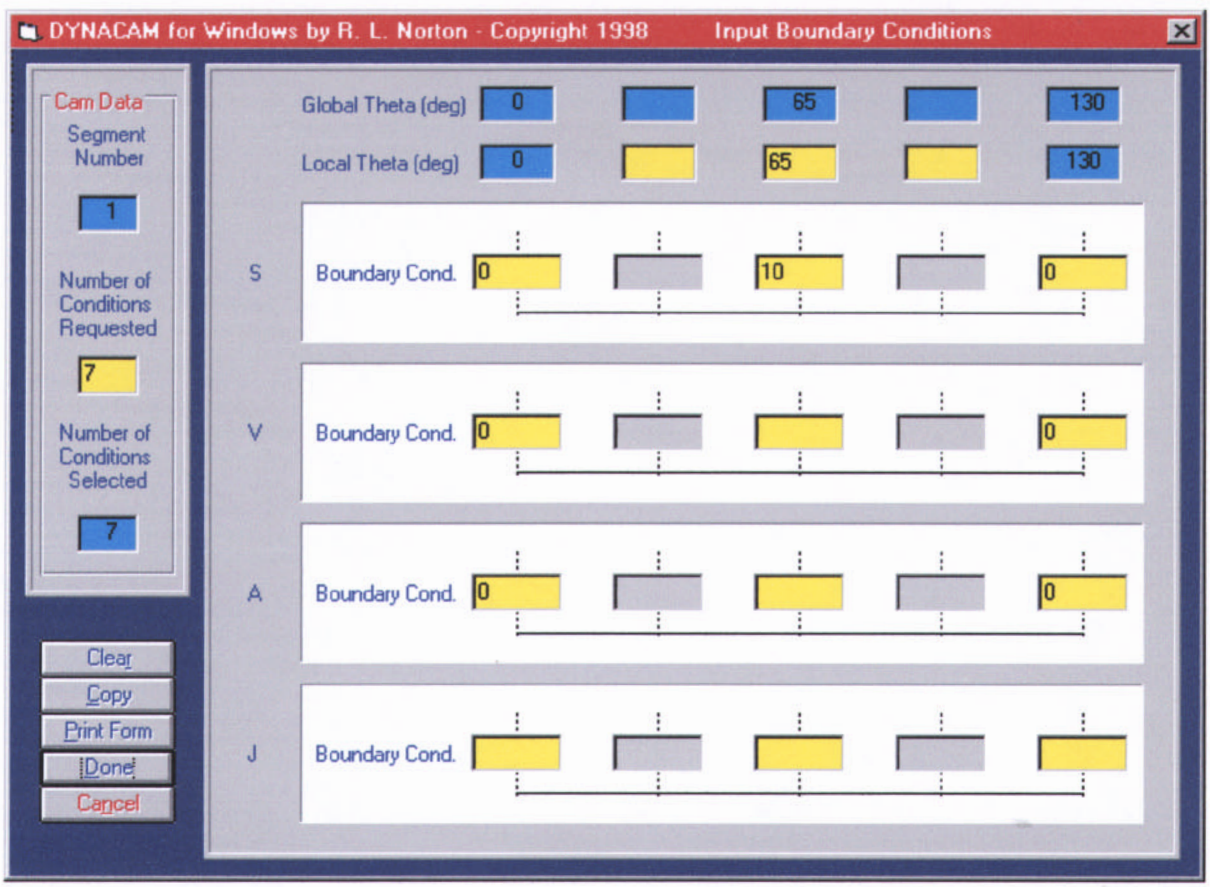
Given:

RISE/FALL	DWELL
$\beta_1 := 130 \cdot deg$	$\beta_2 := 230 \cdot deg$
$h_1 := 10 \cdot mm$	$h_2 := 0 \cdot mm$

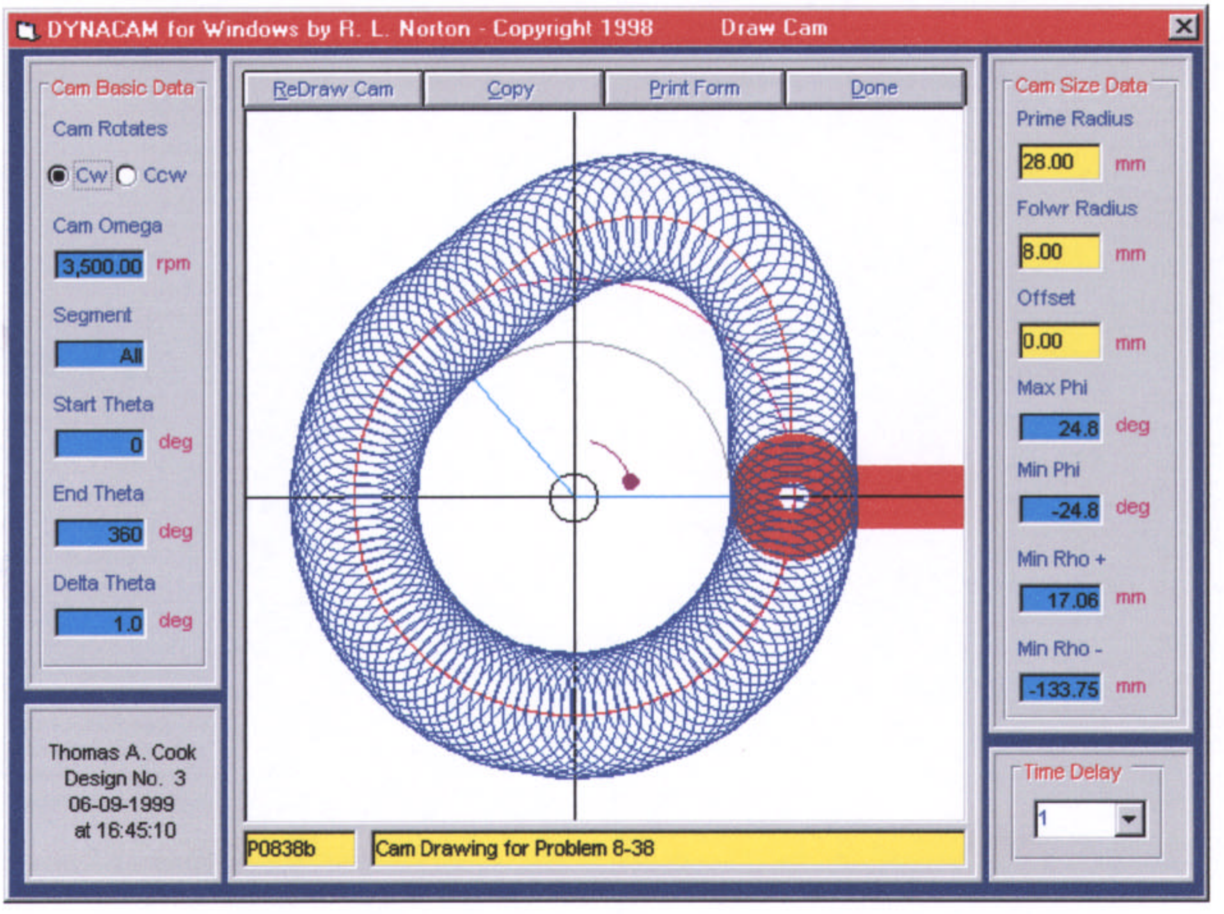
Shaft speed $\omega := 3500 \cdot rpm$

Solution: See Mathcad file P0838.

1. Use a two-segment polynomial as described in Example 8-8. This will result in a 6-deg polynomial requiring 7 boundary conditions. Enter the above data into program DYNACAM. The input boundary conditions screen for the polynomial segment is shown below.



2. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.



3. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius $R_p := 28 \cdot mm$

Roller follower radius $R_f := 8.0 \cdot mm$

Follower eccentricity $\varepsilon := 0.0 \cdot mm$

PROBLEM 8-39

Statement: Design a cam-driven quick-return mechanism for a 3:1 time ratio. The translating roller follower should move forward and back 50 mm and dwell in the back position for 80 deg. It should take one-third the time to return as to move forward. Camshaft ω = 100 rpm. Minimize the package size while maintaining a 25-deg maximum pressure angle. Draw a sketch of your design and provide *svaj*, ϕ, and ρ diagrams.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given:

Time ratio	$t_r := 3$	Dwell interval	$\beta_2 := 80\cdot deg$
Lift	$L := 50\cdot mm$	Shaft speed	$\omega := 100\cdot rpm$

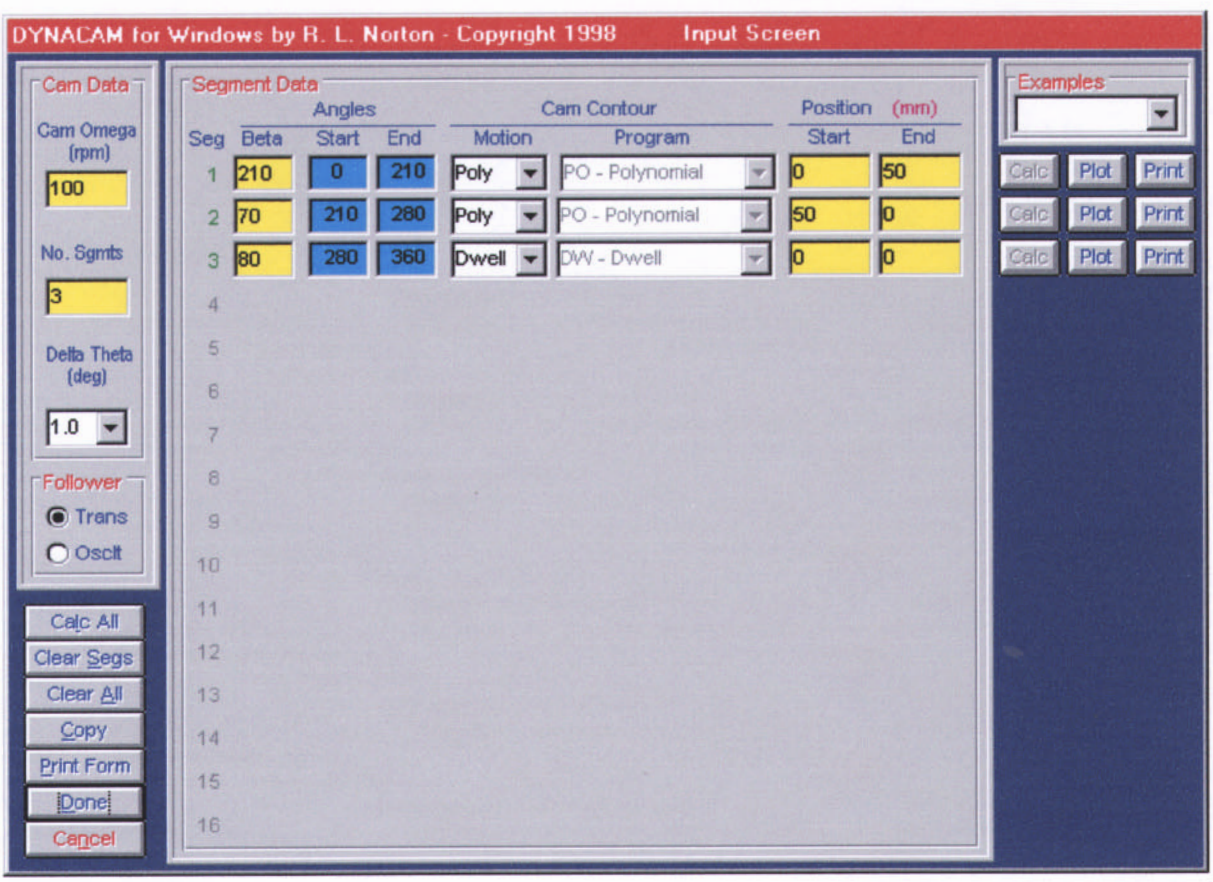
Solution: See Mathcad file P0839.

1. Calculate the rise/fall interval widths.

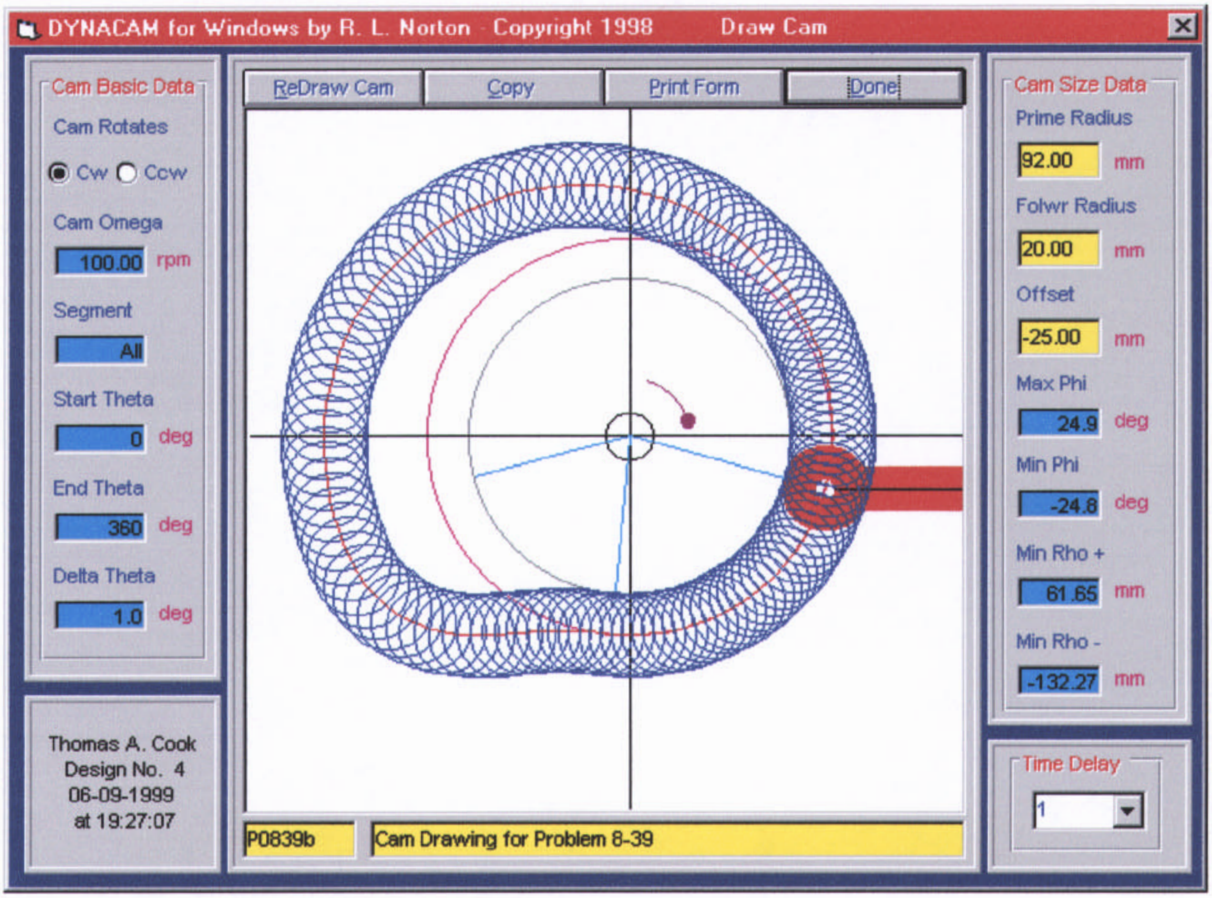
First segment interval $\beta_1 := 360\cdot deg - \beta_2$ $\beta_1 = 280.000\,deg$

Rise subinterval $\beta_r := \beta_1\cdot\dfrac{t_r}{t_r + 1}$ $\beta_r = 210.000\,deg$

2. Use a three-segment polynomial. Enter the above data into program DYNACAM. The input screen is shown below.



3. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.



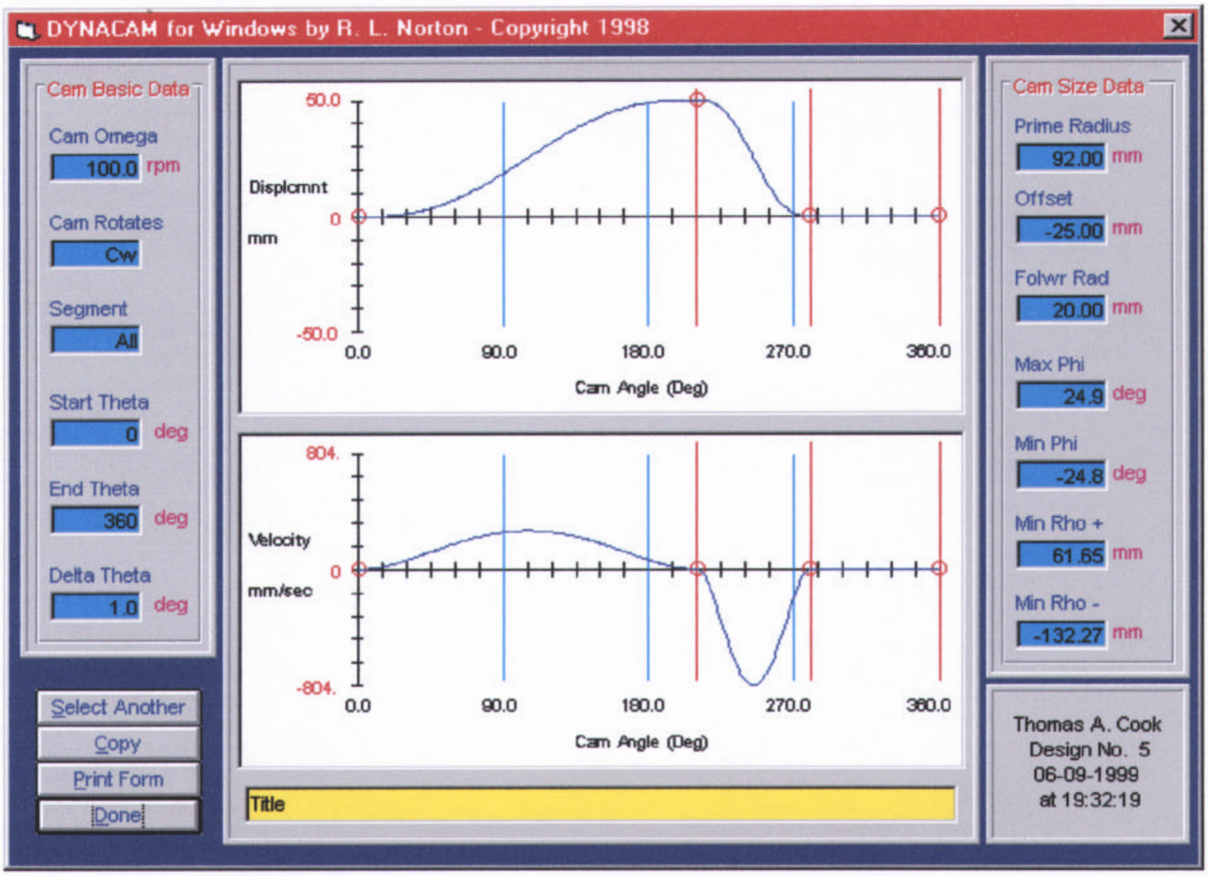
4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

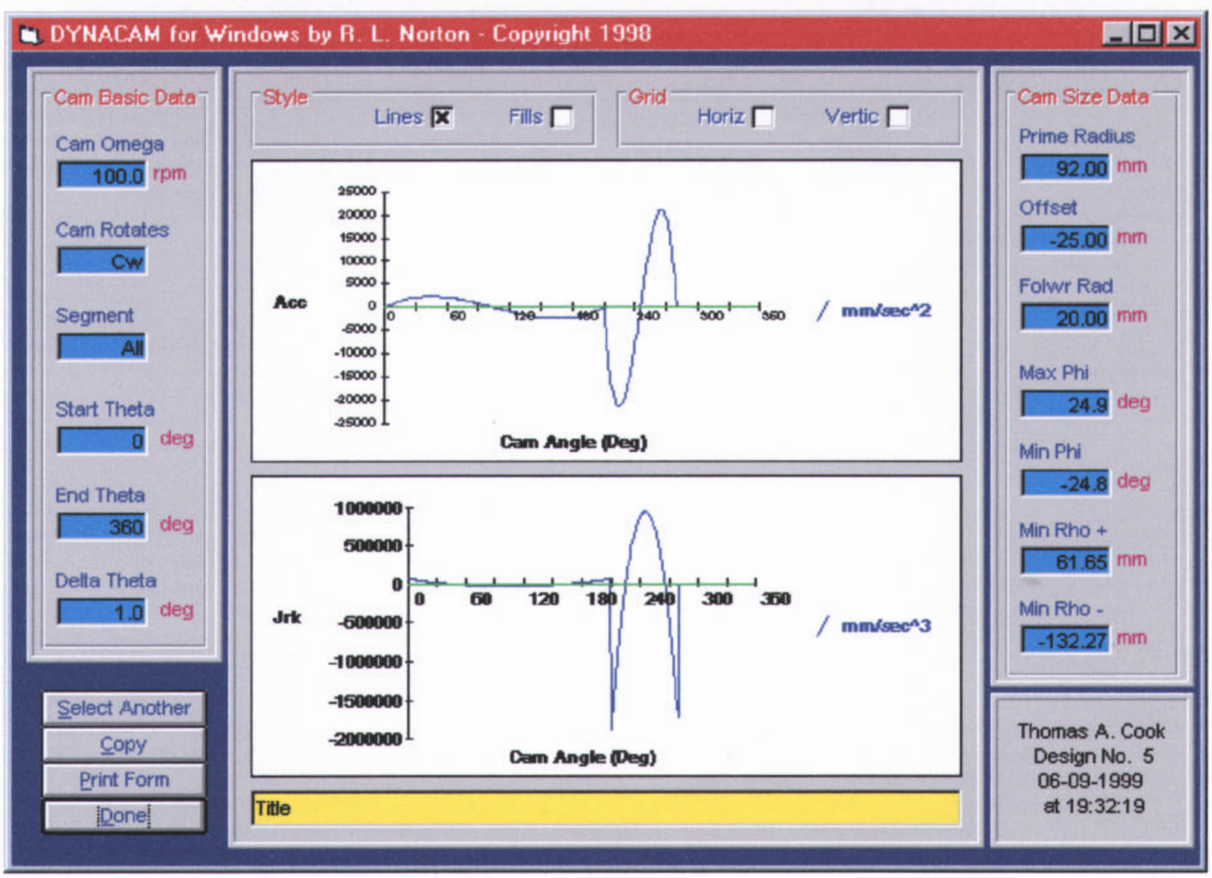
Prime circle radius $R_p := 92 \cdot mm$

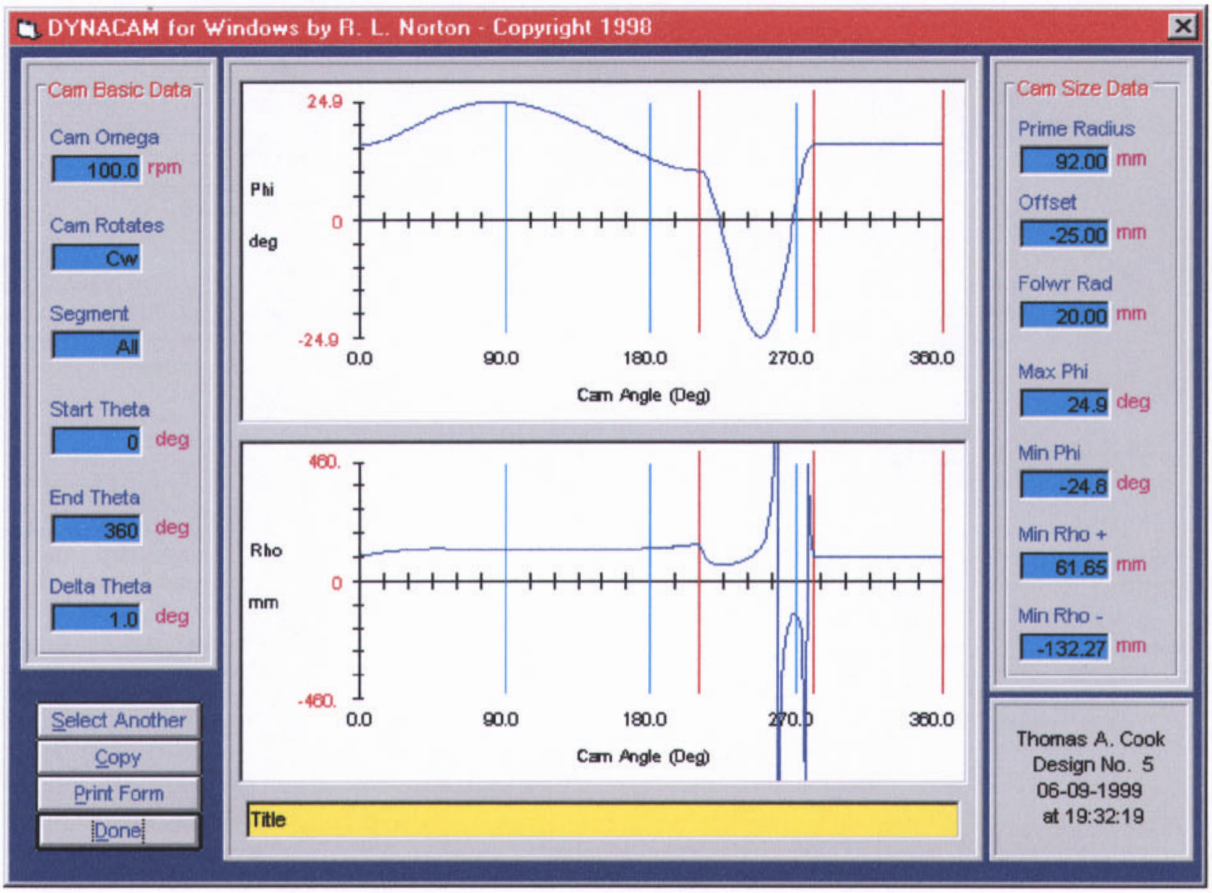
Roller follower radius $R_f := 20.0 \cdot mm$

Follower eccentricity $\varepsilon := -25.0 \cdot mm$

5. Graphs of *s, v, a, j*, ϕ, and ρ are shown on the following pages.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega 100.0 rpm
Cam Rotates Cw
Segment All
Start Theta 0 deg
End Theta 360 deg
Delta Theta 1.0 deg
Select Another
Copy
Print Form
Done
Displcmnt mm
50.0
0
-50.0
Velocity mm/sec
804.
-804.
0.0 90.0 180.0 270.0 360.0
Cam Angle (Deg)
Title
Cam Size Data
Prime Radius 92.00 mm
Offset -25.00 mm
Folwr Rad 20.00 mm
Max Phi 24.9 deg
Min Phi -24.8 deg
Min Rho + 61.65 mm
Min Rho - -132.27 mm
Thomas A. Cook
Design No. 5
06-09-1999
at 19:32:19


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega 100.0 rpm
Cam Rotates Cw
Segment All
Start Theta 0 deg
End Theta 360 deg
Delta Theta 1.0 deg
Select Another
Copy
Print Form
Done
Style Lines Fills
Grid Horiz Vertic
Acc / mm/sec^2
25000 20000 15000 10000 5000 0 -5000 -10000 -15000 -20000 -25000
0 60 120 180 240 300 360
Cam Angle (Deg)
Jrk / mm/sec^3
1000000 500000 0 -500000 -1000000 -1500000 -2000000
0 60 120 180 240 300 360
Cam Angle (Deg)
Title
Cam Size Data
Prime Radius 92.00 mm
Offset -25.00 mm
Folwr Rad 20.00 mm
Max Phi 24.9 deg
Min Phi -24.8 deg
Min Rho + 61.65 mm
Min Rho - -132.27 mm
Thomas A. Cook
Design No. 5
06-09-1999
at 19:32:19


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
100.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
Phi
deg
24.9
0
-24.9
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
Rho
mm
460.
0
-460.
Title
Cam Size Data
Prime Radius
92.00 mm
Offset
-25.00 mm
Folwr Rad
20.00 mm
Max Phi
24.9 deg
Min Phi
-24.8 deg
Min Rho +
61.65 mm
Min Rho -
-132.27 mm
Thomas A. Cook
Design No. 5
06-09-1999
at 19:32:19

PROBLEM 8-40

Statement: Design a cam-follower system to drive a linear translating piston at a constant velocity for 200 deg through a stroke of 100 mm at 60 rpm. Minimize the package size while maintaining a 25-deg maximum pressure angle. Draw a sketch of your design and provide *svaj*, ϕ, and ρ diagrams.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Constant velocity interval width and stroke $\beta_1 := 200 deg$ $h_1 := 100 mm$

Cam rotation speed $\omega := 60 \cdot rpm$

Assumptions: A roller follower can be attached to the translating piston to interface with the cam.

Solution: See Mathcad file P0840.

1. Calculate the cycle time and time for the cam to rotate through the CV segment.

$$t_{cycle} := \frac{2 \cdot \pi \cdot rad}{\omega} \qquad t_{cycle} = 1.000\,sec$$

$$t_1 := \frac{\beta_1}{360 \cdot deg} \cdot t_{cycle} \qquad t_1 = 0.556\,sec$$

2. Calculate the required constant velocity.

$$V_{cv} := \frac{h_1}{t_1} \qquad V_{cv} = 180.000\,\frac{mm}{sec}$$

3. Use a two-segment cam with both segments polynomials. Let the first segment be a 1-deg polynomial and the second be a 7-deg (8 boundary conditions). There are two design choices to be made concerning the cam motion. They are:

Lift at end of second segment (and beginning of first) $L_2 := 10 \cdot mm$

Subinterval of second segment at which $s = 0$ $\beta_f := 120 \cdot deg$

4. Enter the above data into program DYNACAM. The segment segment boundary condition input screen is shown on the next page followed by a cam drawing for the design choices made.

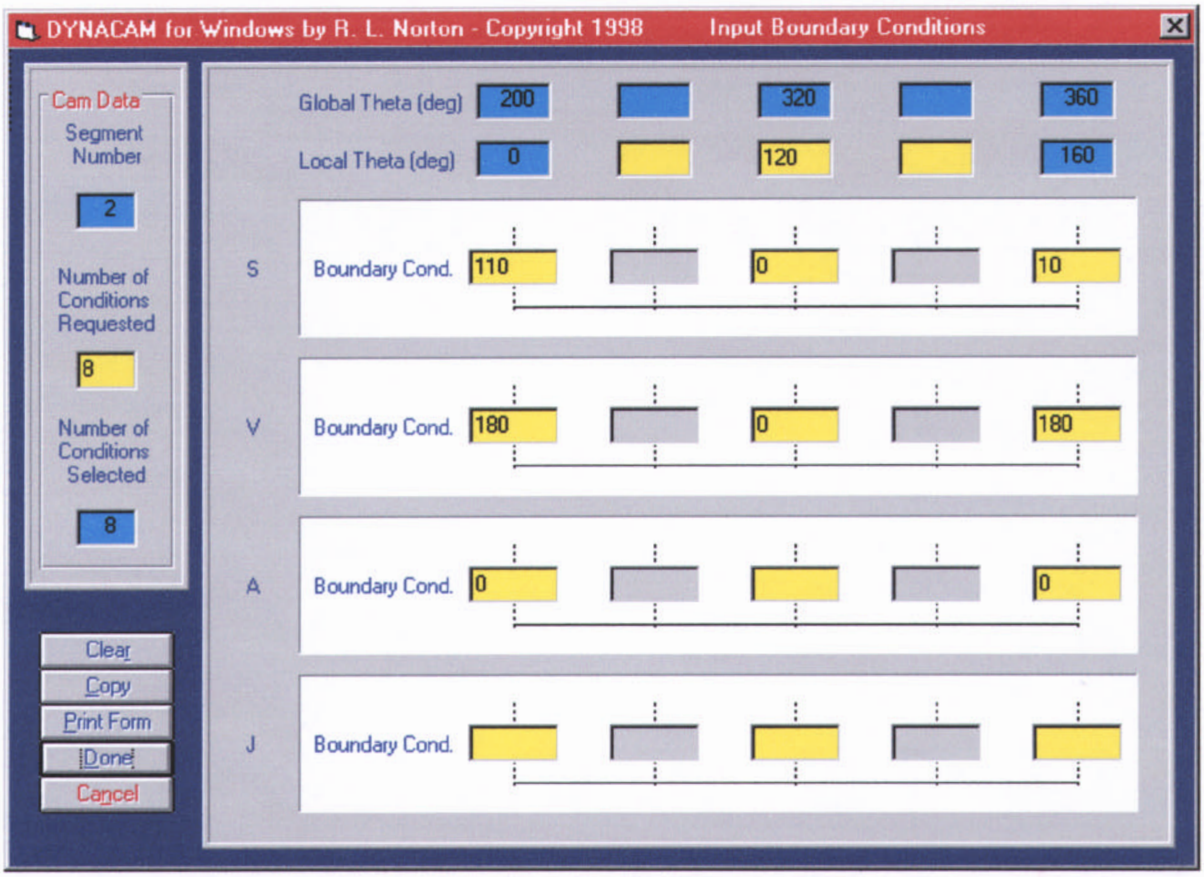
5. The cam was sized iteratively to balance the positive and negative extreme values of the pressure angle by varying the eccentricity and to reduce the maximum absolute pressure angle to 25 deg or less by increasing the prime circle radius. The resulting cam is shown below.

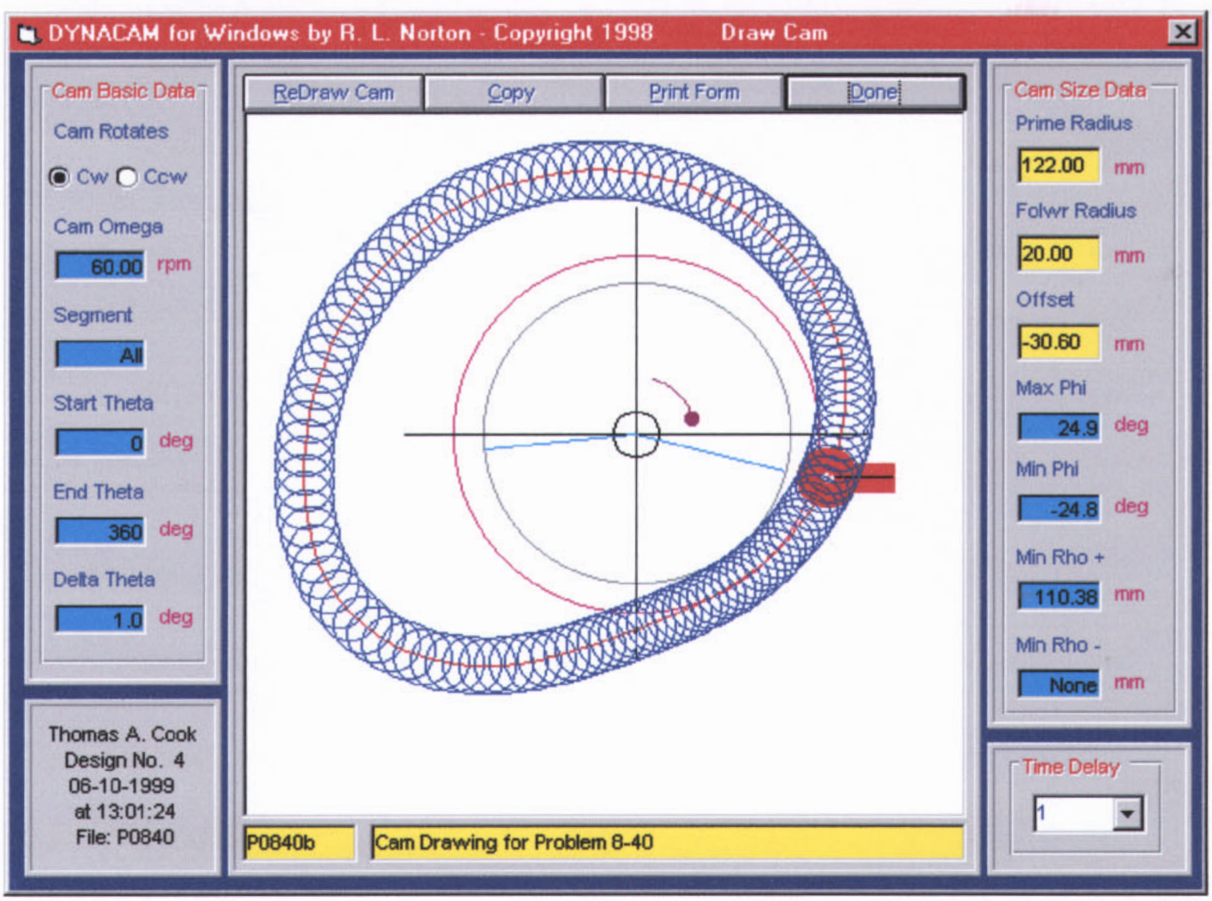
6. The minimum and maximum pressure angles and radius of curvature are shown in the cam drawing. The design has the following dimensions:

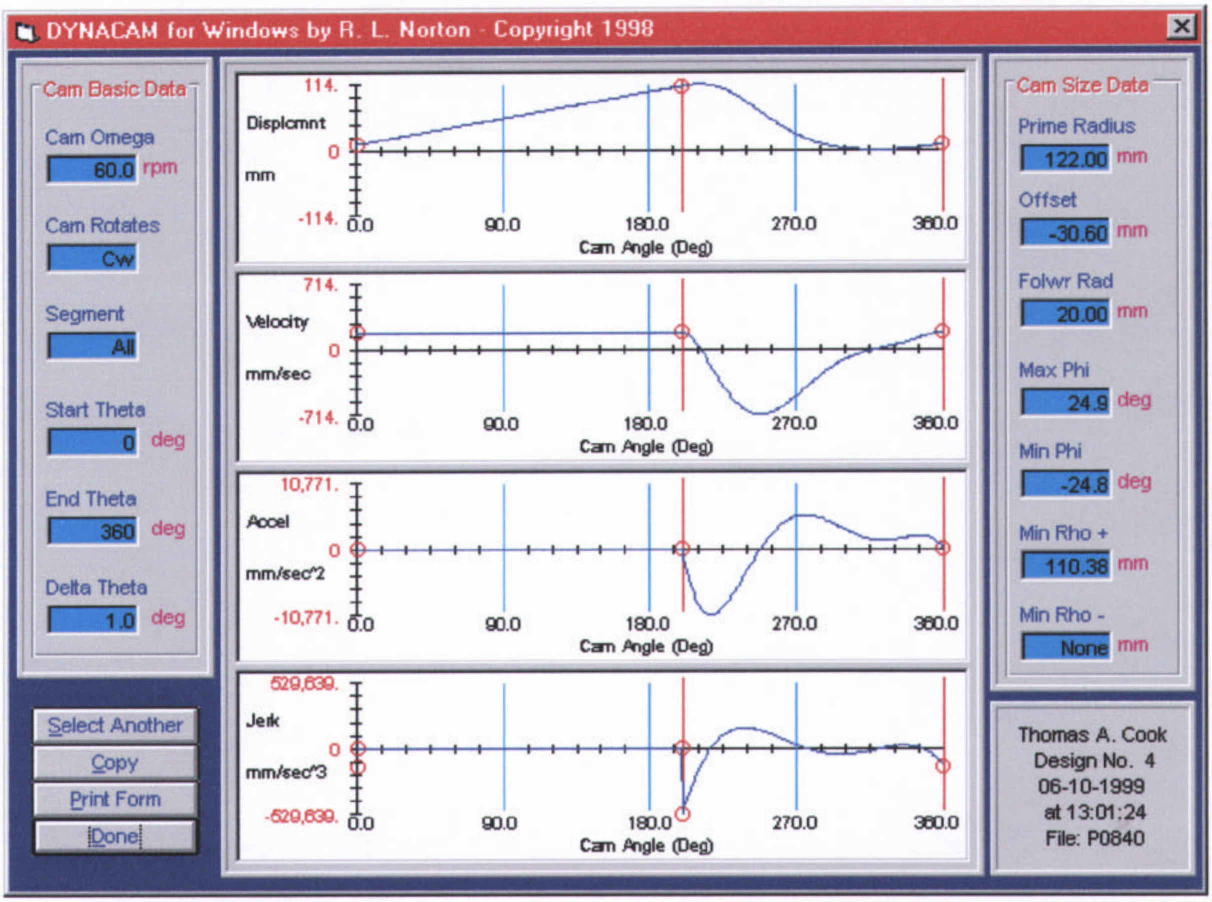
Prime circle radius $R_p := 122 \cdot mm$

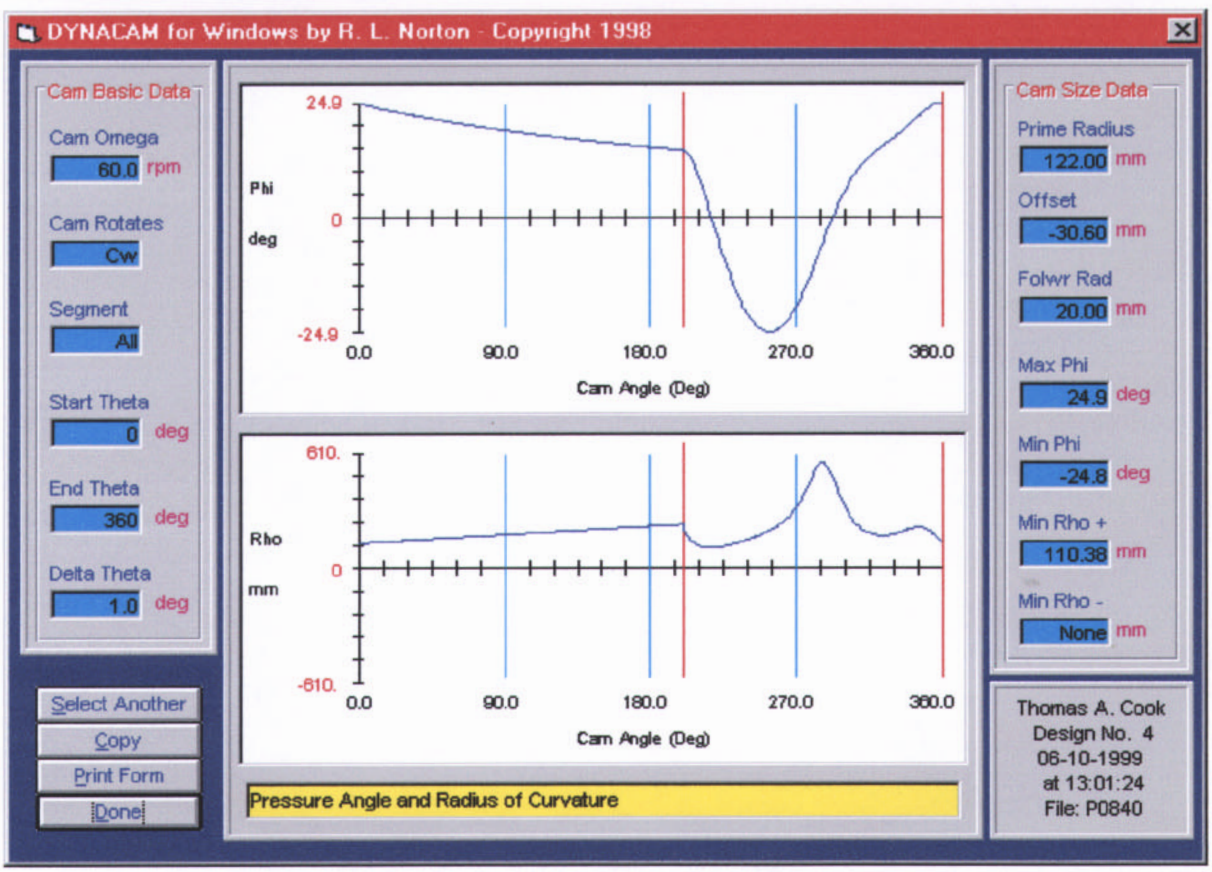
Roller follower radius $R_f := 20 \cdot mm$

Follower eccentricity $\varepsilon := -30.6 \cdot mm$


DYNACAM for Windows by R. L. Norton - Copyright 1998
Input Boundary Conditions
Cam Data
Segment Number
2
Number of Conditions Requested
8
Number of Conditions Selected
8
Clear
Copy
Print Form
Done
Cancel
Global Theta (deg)
200
320
360
Local Theta (deg)
0
120
160
S
Boundary Cond.
110
0
10
V
Boundary Cond.
180
0
180
A
Boundary Cond.
0
0
J
Boundary Cond.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Draw Cam
Cam Basic Data
Cam Rotates
Cw Ccw
Cam Omega
60.00 rpm
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Thomas A. Cook
Design No. 4
06-10-1999
at 13:01:24
File: P0840
ReDraw Cam
Copy
Print Form
Done
P0840b
Cam Drawing for Problem 8-40
Cam Size Data
Prime Radius
122.00 mm
Folwr Radius
20.00 mm
Offset
-30.60 mm
Max Phi
24.9 deg
Min Phi
-24.8 deg
Min Rho +
110.38 mm
Min Rho -
None mm
Time Delay
1


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
60.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
Displcmnt
mm
114.
0
-114.
Velocity
mm/sec
714.
-714.
Accel
mm/sec^2
10,771.
-10,771.
Jerk
mm/sec^3
529,639.
-529,639.
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
Cam Size Data
Prime Radius
122.00 mm
Offset
-30.60 mm
Folwr Rad
20.00 mm
Max Phi
24.9 deg
Min Phi
-24.8 deg
Min Rho +
110.38 mm
Min Rho -
None mm
Thomas A. Cook
Design No. 4
06-10-1999
at 13:01:24
File: P0840


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
60.0 rpm
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1.0 deg
Select Another
Copy
Print Form
Done
Phi
deg
24.9
0
-24.9
Rho
mm
610.
-610.
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
Pressure Angle and Radius of Curvature
Cam Size Data
Prime Radius
122.00 mm
Offset
-30.60 mm
Folwr Rad
20.00 mm
Max Phi
24.9 deg
Min Phi
-24.8 deg
Min Rho +
110.38 mm
Min Rho -
None mm
Thomas A. Cook
Design No. 4
06-10-1999
at 13:01:24
File: P0840

PROBLEM 8-41

Statement: Design a cam-follower system to rise 20 mm in 80 deg, fall 10 mm in 100 deg, dwell at 10 mm for 10 deg, fall 10 mm in 50 deg, and dwell at 0 for 30 deg. The total cycle must take 4 sec. Avoid unnecessary returns to zero acceleration. Minimize the package size and maximize the roller follower diameter while maintaining a 25 deg maximum pressure angle. Draw a sketch of your design and provide *s v a j*, ϕ, and ρ diagrams.

Given:

RISE	FALL	DWELL	FALL
$\beta_1 := 80 \cdot deg$	$\beta_2 := 100 \cdot deg$	$\beta_3 := 100 \cdot deg$	$\beta_4 := 50 \cdot deg$
$h_1 := 20 \cdot mm$	$h_2 := 10 \cdot mm$	$h_3 := 10 \cdot mm$	$h_4 := 0 \cdot mm$

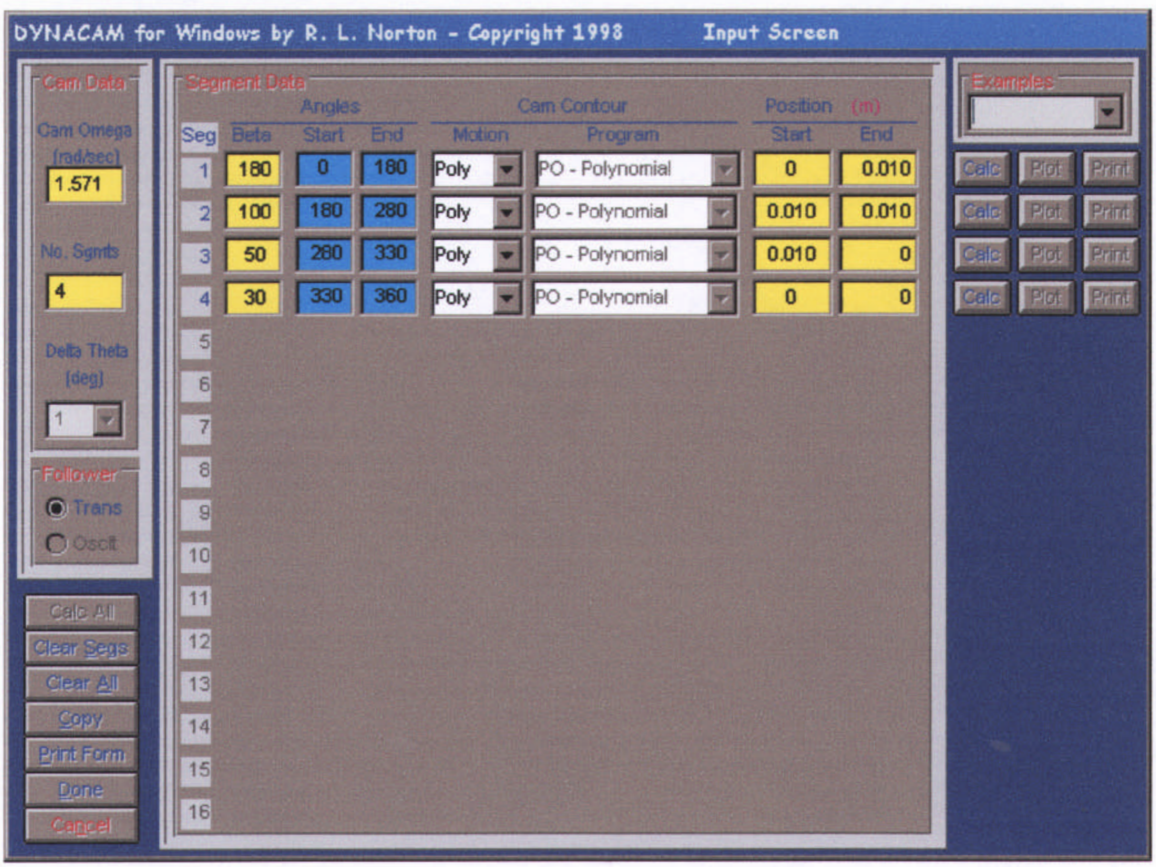
Cycle time: $\tau := 4 \cdot sec$

Solution: See Mathcad file P0841.

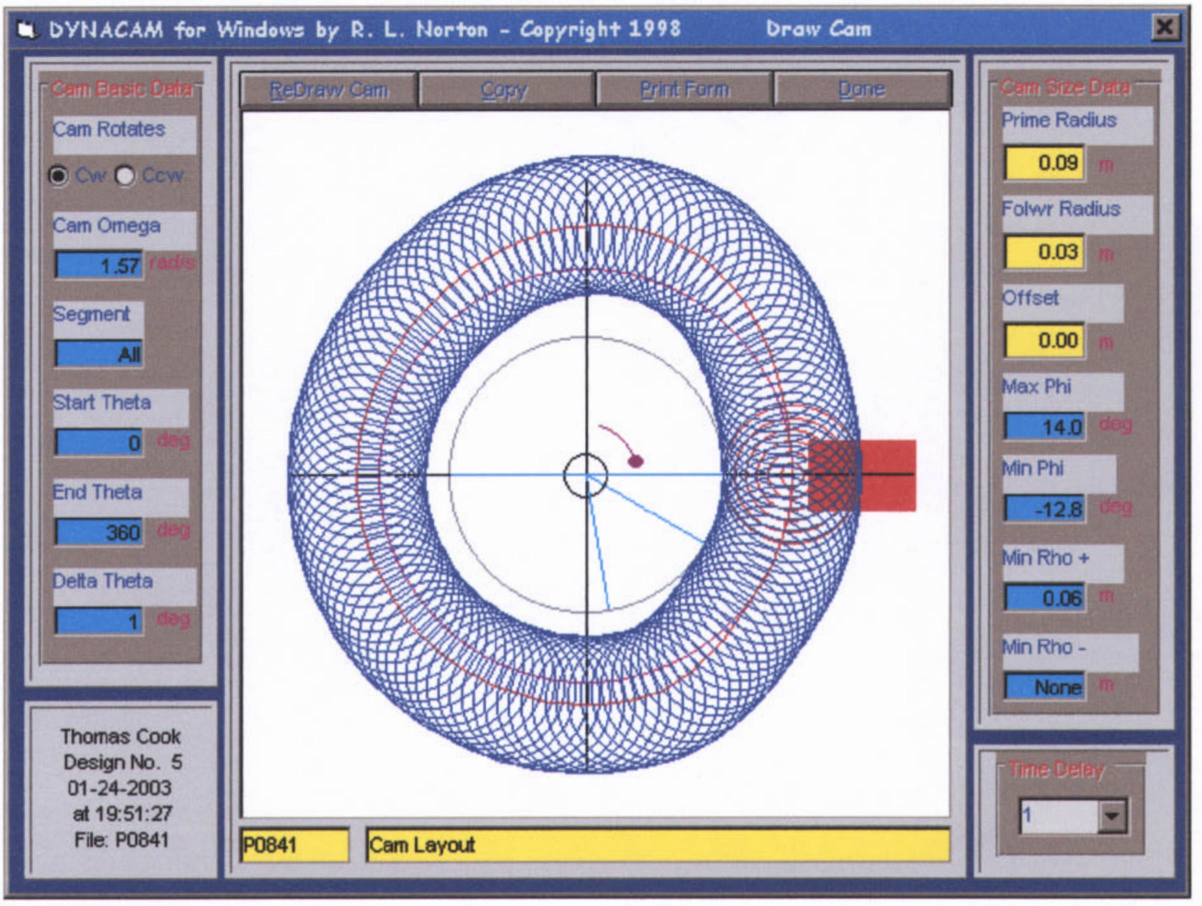
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.571 \frac{rad}{sec}$$

2. Use a four-segment for the for the rise, fall, dwell, fall, and dwell. Enter the above data into program DYNACAM. The input screen is shown below.



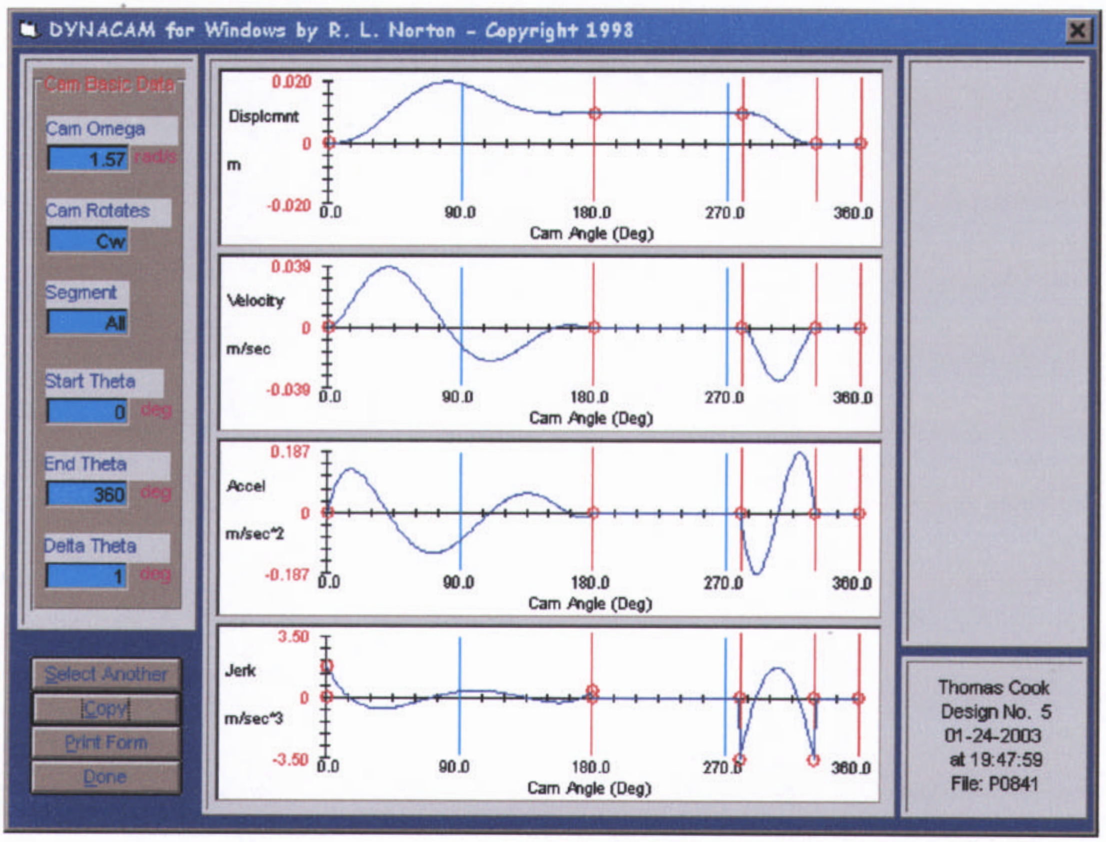
3. The cam was sized iteratively to reduce package size while keeping the follower radius large, the pressure angle low, and the follower radius at least two times the radius of curvature . The resulting cam is shown below.

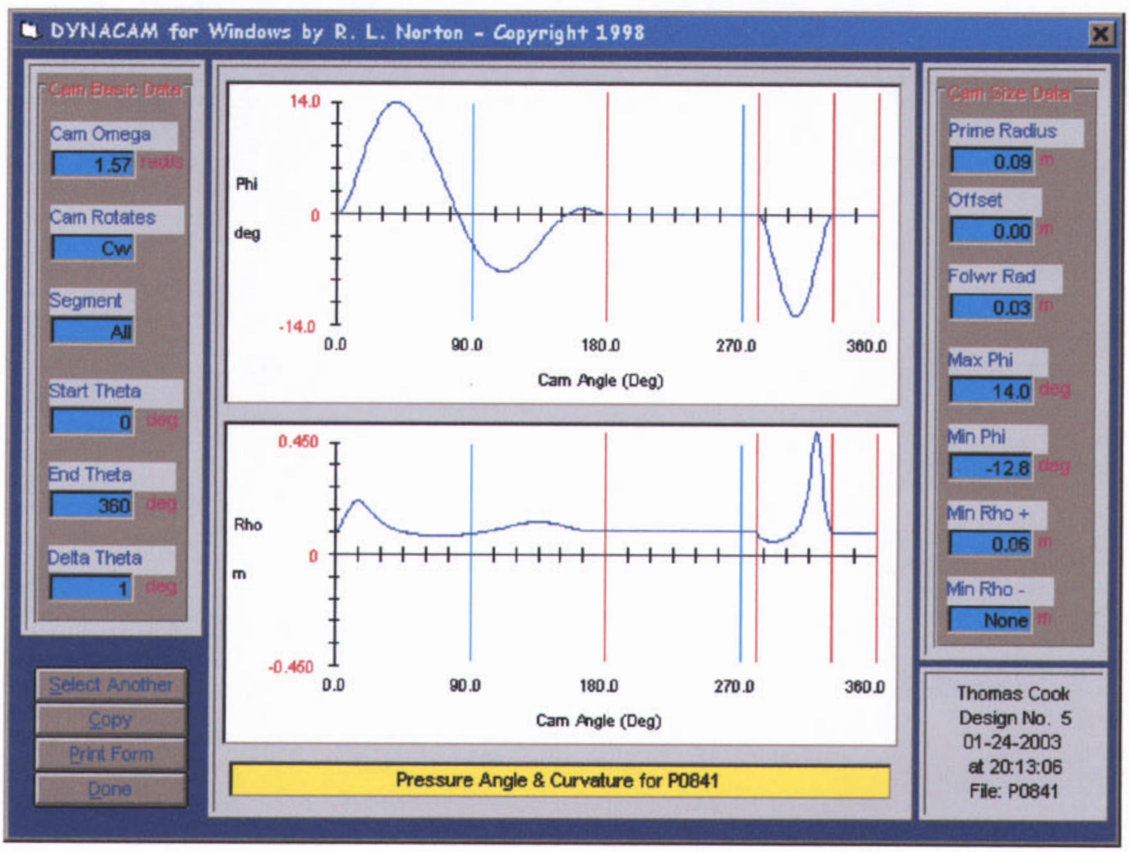


4. The minimum and maximum pressure angles and radius of curvature are shown in the figure above. The design has the following dimensions:

Prime circle radius	$R_p := 90 \cdot mm$
Roller follower radius	$R_f := 30 \cdot mm$
Follower eccentricity	$\varepsilon := 0 \cdot mm$

5. Graphs of *svaj,* ϕ and ρ for the roller follower are shown on the following page.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
1.57 rad/s
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1 deg
Select Another
Copy
Print Form
Done
Displcmnt
m
0.020
0
-0.020
Velocity
m/sec
0.039
-0.039
Accel
m/sec^2
0.187
-0.187
Jerk
m/sec^3
3.50
-3.50
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
Thomas Cook
Design No. 5
01-24-2003
at 19:47:59
File: P0841


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
1.57 rad/s
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1 deg
Select Another
Copy
Print Form
Done
Phi
deg
14.0
0
-14.0
Rho
m
0.450
-0.450
0.0
90.0
180.0
270.0
360.0
Cam Angle (Deg)
Pressure Angle & Curvature for P0841
Cam Size Data
Prime Radius
0.09 m
Offset
0.00 m
Folwr Rad
0.03 m
Max Phi
14.0 deg
Min Phi
-12.8 deg
Min Rho +
0.06 m
Min Rho -
None m
Thomas Cook
Design No. 5
01-24-2003
at 20:13:06
File: P0841

PROBLEM 8-42

Statement: Design a single-dwell cam to move a follower from 0 to 35 mm in 75 deg, fall 35 mm in 120 deg and dwell for the remainder. The total cycle must take 3 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the *SVAJ* diagrams.

Given:

RISE/FALL	DWELL
$\beta := 195 \cdot deg$	$\beta_3 := 165 \cdot deg$
$h := 35 \cdot mm$	$h_3 := 0.0 \cdot mm$

Cycle time: $\tau := 3 \cdot sec$

Solution: See Mathcad file P0842.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 2.094 \frac{rad}{sec}$$

2. Use a two-segment polynomial. Let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

at $\theta = 0$: $s = 0$, $v = 0$, $a = 0$
$\theta = \beta_1$: $s = h$, $v = 0$
$\theta = \beta$: $s = 0$, $v = 0$, $a = 0$

This is a minimum set of 8 BCs. The v = 0 condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, h. Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

Total lift: $h = 35.000\, mm$

Rise interval: $\beta_1 := 75 \cdot deg$ $\qquad A := \frac{\beta_1}{\beta}$ $\qquad A = 0.385$

Fall interval: $\beta_2 := 120 \cdot deg$

3. Use the 8 BCs and equation 8.23 to write 8 equations in s, v, and a similar to those in example 8-9 but with 8 terms in the equation for s (the highest term will be seventh degree).

For $\theta = 0$: $s = v = a = 0$

$$0 := c_0 \qquad 0 := c_1 \qquad 0 := c_2$$

For $\theta = \beta_1$: $s = h$, $v = 0$

$$h := c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5 + c_6 \cdot A^6 + c_7 \cdot A^7$$

$$0 := 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 + 6 \cdot c_6 \cdot A^5 + 7 \cdot c_7 \cdot A^6$$

For $\theta = \beta$: $s = v = a = 0$

$$0 := c_3 + c_4 + c_5 + c_6 + c_7$$

$$0 := 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 + 6 \cdot c_6 + 7 \cdot c_7$$

$$0 := 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5 + 30 \cdot c_6 + 42 \cdot c_7$$

4. Solve for the unknown polynomial coefficients. Note that C_0 through C_2 are zero

$$C := \begin{pmatrix} A^3 & A^4 & A^5 & A^6 & A^7 \\ 3\cdot A^2 & 4\cdot A^3 & 5\cdot A^4 & 6\cdot A^5 & 7\cdot A^6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{pmatrix} \qquad H := \begin{pmatrix} h \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix} := C^{-1}\cdot H$$

$c_3 = 220.838\,in$ $\quad c_4 = -966.49\,in$ $\quad c_5 = 1574.444\,in$

$c_6 = -1132.768\,in$ $\quad c_7 = 303.977\,in$

5. Write the *SVAJ* equations for the rise/fall segment.

$$S(\theta) := c_3\cdot\left(\frac{\theta}{\beta}\right)^3 + c_4\cdot\left(\frac{\theta}{\beta}\right)^4 + c_5\cdot\left(\frac{\theta}{\beta}\right)^5 + c_6\cdot\left(\frac{\theta}{\beta}\right)^6 + c_7\cdot\left(\frac{\theta}{\beta}\right)^7$$

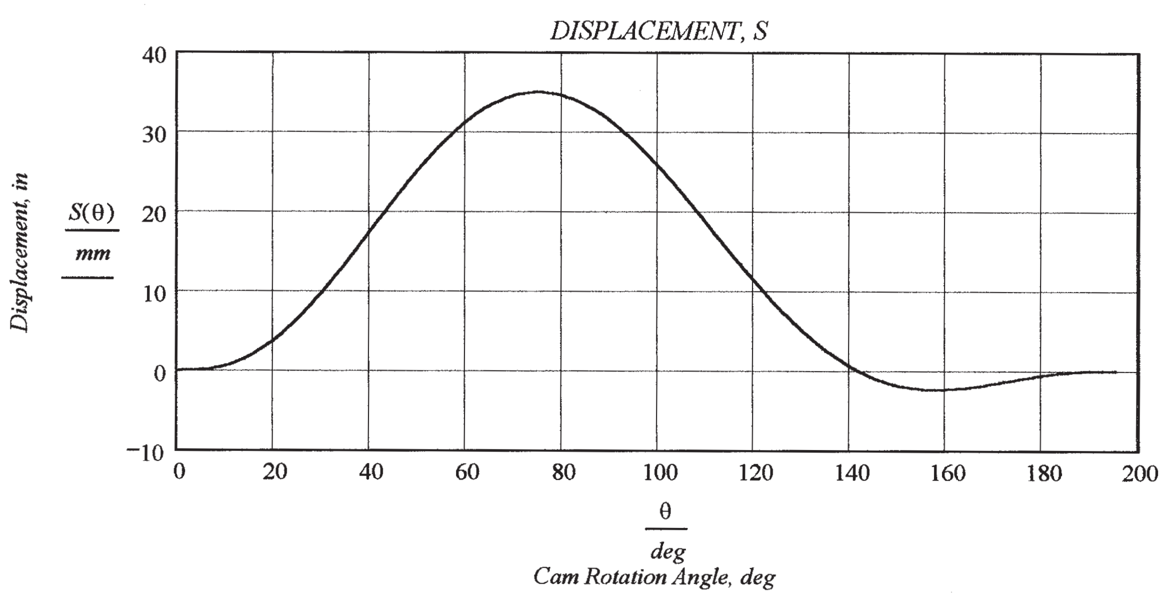
$$V(\theta) := \frac{1}{\beta}\cdot\left[3\cdot c_3\cdot\left(\frac{\theta}{\beta}\right)^2 + 4\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^3 + 5\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^4 + 6\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^5 + 7\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^6\right]$$

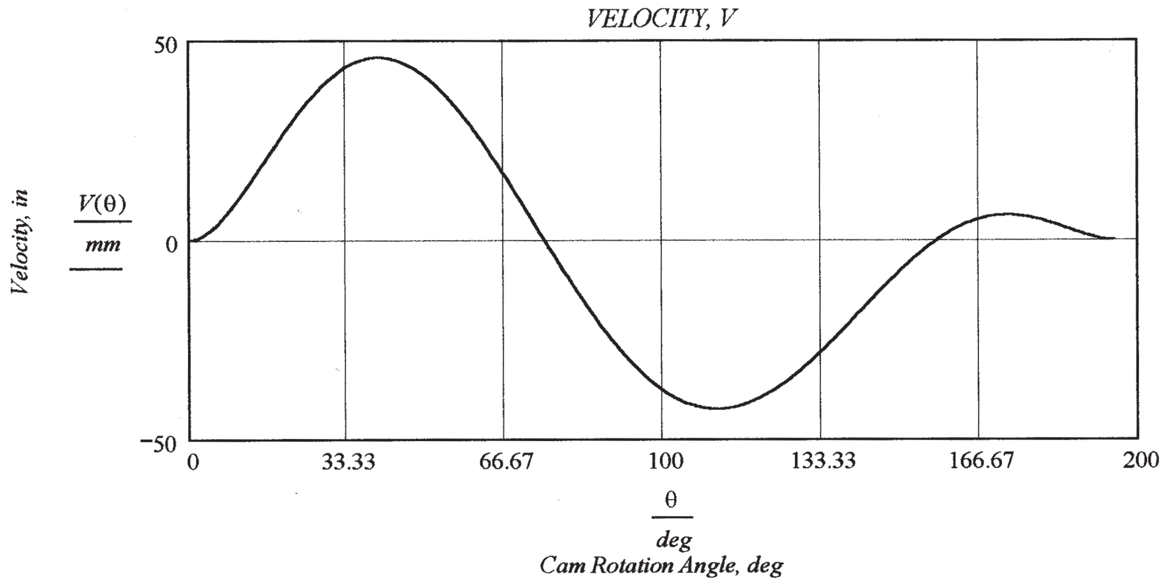
$$A(\theta) := \frac{1}{\beta^2}\cdot\left[6\cdot c_3\cdot\left(\frac{\theta}{\beta}\right) + 12\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^2 + 20\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^3 + 30\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^4 + 42\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^5\right]$$

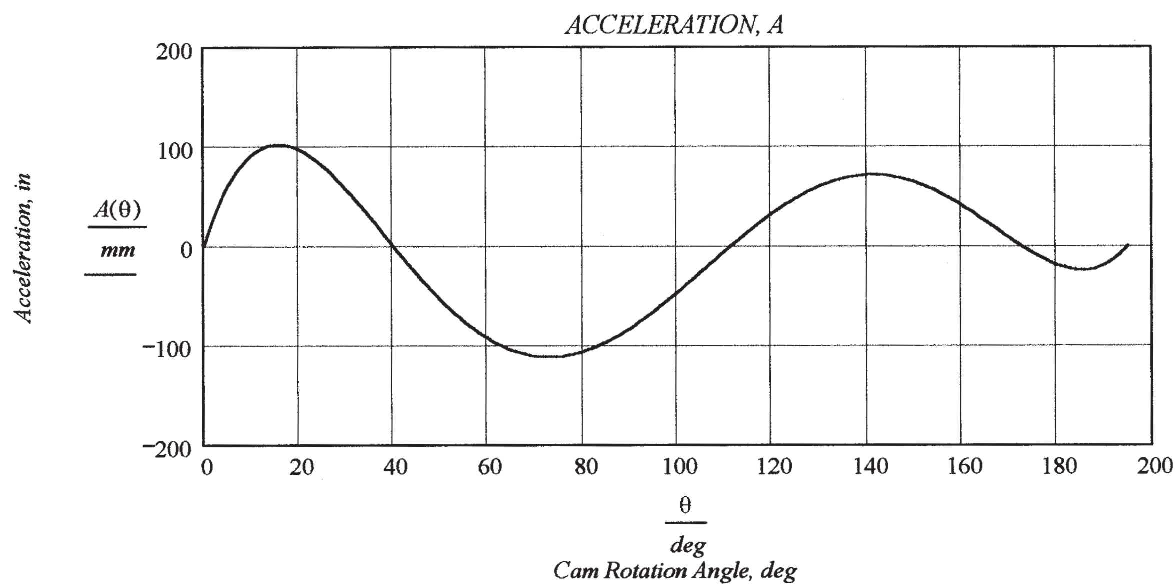
$$J(\theta) := \frac{1}{\beta^3}\cdot\left[6\cdot c_3 + 24\cdot c_4\cdot\left(\frac{\theta}{\beta}\right) + 60\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^2 + 120\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^3 + 210\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^4\right]$$

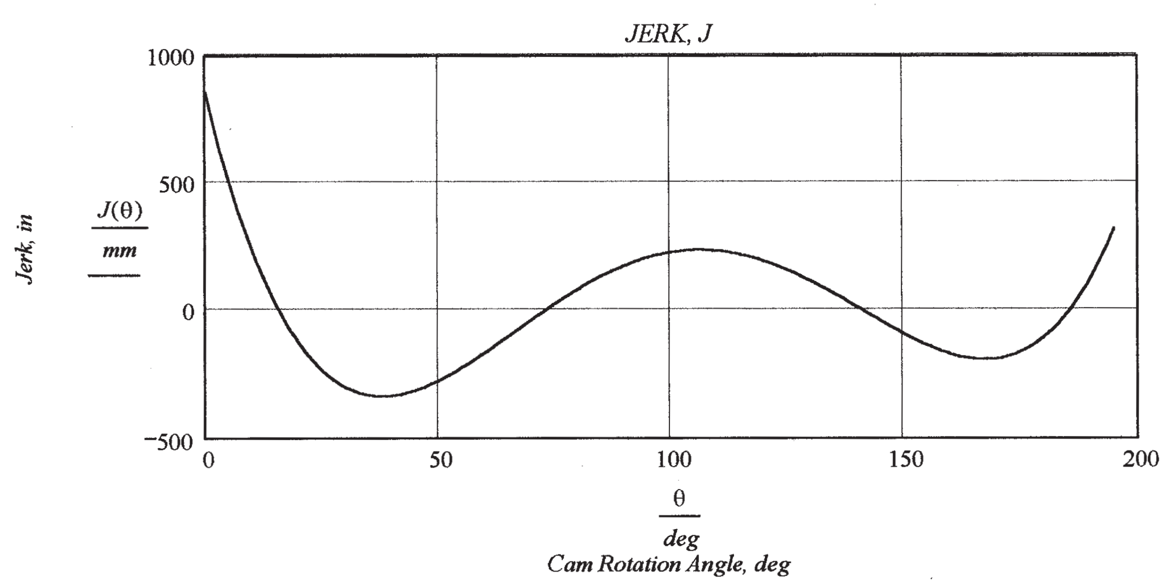
6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 <= \theta <= \beta$.

$\theta := 0\cdot deg, 1\cdot deg .. \beta$




VELOCITY, V
50
0
−50
Velocity, in
V(θ)
mm
0
33.33
66.67
100
133.33
166.67
200
θ
deg
Cam Rotation Angle, deg


ACCELERATION, A
200
100
0
−100
−200
Acceleration, in
A(θ)
mm
0
20
40
60
80
100
120
140
160
180
200
θ
deg
Cam Rotation Angle, deg


JERK, J
1000
500
0
−500
Jerk, in
J(θ)
mm
0
50
100
150
200
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-43

Statement: Design a cam to move a follower at a constant velocity of 100 mm/sec for 2 sec then return to its starting position with a total cycle time of 3 sec.

Given: Constant velocity: $v_c := 100\,mm\cdot sec^{-1}$

Time duration of cv segment: $t_{cv} := 2\cdot sec$ Cycle time: $\tau := 3\cdot sec$

Solution: See Mathcad file P0843.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2\cdot\pi\cdot rad}{\tau} \qquad \omega = 2.094\,\frac{rad}{sec}$$

2. Use a two-segment polynomial as demonstrated in Example 8-12. The lift during the first segment and the *svaj* equations for the first segment are:

Normalized velocity: $v_{cv} := \dfrac{v_c}{\omega}$ $\qquad v_{cv} = 47.746\,mm$

$h_{cv} := v_c\cdot t_{cv}$ $\qquad h_{cv} = 200.000\,mm$ $\qquad \beta_1 := \dfrac{t_{cv}}{\tau}\cdot 360\cdot deg$ $\qquad \beta_1 = 240\,deg$

$s_1(\theta) := h_{cv}\cdot\dfrac{\theta}{\beta_1}$ $\qquad v_1(\theta) := v_{cv}$ $\qquad a_1(\theta) := 0\cdot mm$ $\qquad j_1(\theta) := 0\cdot mm$

2. The boundary conditions for the second segment are:

at $\theta = \beta_1$: $\quad s = h_{cv},\ v = v_{cv}\ \ a = 0$

$\theta = 360$ deg: $\quad s = 0,\ \ v = v_{cv},\ \ a = 0$

This is a minimum set of 6 BCs. Define the total interval and the constant velocity interval, and the ratio of constant velocity interval to the total interval.

Total interval: $\beta := 360\cdot deg$

CV interval: $\beta_1 = 240\,deg$ $\qquad A := \dfrac{\beta_1}{\beta}$ $\qquad A = 0.667$

3. Use the 6 BCs and equation 8.23 to write 6 equations in *s*, *v*, and *a* similar to those in example 8-9 but with 6 terms in the equation for *s* (the highest term will be fifth degree).

For $\theta = \beta 1$: $\quad s = h_{cv},\ \ v = v_{cv}\ \ a = 0$

$$h_{cv} = c_0 + c_1\cdot A + c_2\cdot A^2 + c_3\cdot A^3 + c_4\cdot A^4 + c_5\cdot A^5$$

$$v_{cv} = \frac{1}{\beta}\cdot\left(c_1 + 2\cdot c_2\cdot A + 3\cdot c_3\cdot A^2 + 4\cdot c_4\cdot A^3 + 5\cdot c_5\cdot A^4\right)$$

$$0 = 2\cdot c_2 + 6\cdot c_3\cdot A + 12\cdot c_4\cdot A^2 + 20\cdot c_5\cdot A^3$$

For $\theta = \beta$: $\quad s = 0,\ \ v = v_{cv},\ \ a = 0$

$$0 = c_0 + c_1 + c_2 + c_3 + c_4 + c_5$$

$$0 = \frac{1}{\beta}\cdot\left(c_1 + 2\cdot c_2 + 3\cdot c_3 + 4\cdot c_4 + 5\cdot c_5\right)$$

$$0 = 2\cdot c_2 + 6\cdot c_3 + 12\cdot c_4 + 20\cdot c_5$$

4. Solve for the unknown polynomial coefficients.

$$C := \begin{pmatrix} 1 & A & A^2 & A^3 & A^4 & A^5 \\ 0 & 1 & 2\cdot A & 3\cdot A^2 & 4\cdot A^3 & 5\cdot A^4 \\ 0 & 0 & 2 & 6\cdot A & 12\cdot A^2 & 20\cdot A^3 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 6 & 12 & 20 \end{pmatrix} \qquad H := \begin{pmatrix} h_{cv} \\ \beta\cdot v_{cv} \\ 0 \\ 0 \\ \beta\cdot v_{cv} \\ 0 \end{pmatrix} \qquad \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} := C^{-1}\cdot H$$

$c_0 = 1.536 \times 10^5\ mm$ $\qquad c_1 = -9.717 \times 10^5\ mm$ $\qquad c_2 = 2.430 \times 10^6\ mm$

$c_3 = -2.997 \times 10^6\ mm$ $\qquad c_4 = 1.823 \times 10^6\ mm$ $\qquad c_5 = -4.374 \times 10^5\ mm$

5. Write the *svaj* equations for the second segment.

$$s_2(\theta) := c_0 + c_1\cdot\left(\frac{\theta}{\beta}\right) + c_2\cdot\left(\frac{\theta}{\beta}\right)^2 + c_3\cdot\left(\frac{\theta}{\beta}\right)^3 + c_4\cdot\left(\frac{\theta}{\beta}\right)^4 + c_5\cdot\left(\frac{\theta}{\beta}\right)^5$$

$$v_2(\theta) := \frac{1}{\beta}\cdot\left[c_1 + 2\cdot c_2\cdot\left(\frac{\theta}{\beta}\right) + 3\cdot c_3\cdot\left(\frac{\theta}{\beta}\right)^2 + 4\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^3 + 5\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^4\right]$$

$$a_2(\theta) := \frac{1}{\beta^2}\cdot\left[2\cdot c_2 + 6\cdot c_3\cdot\left(\frac{\theta}{\beta}\right) + 12\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^2 + 20\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^3\right]$$

$$j_2(\theta) := \frac{1}{\beta^3}\cdot\left[6\cdot c_3 + 24\cdot c_4\cdot\left(\frac{\theta}{\beta}\right) + 60\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^2\right]$$

4. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(\theta,a,b) := if[(\theta > a) \wedge (\theta \le b), 1, 0]$$

$$S(\theta) := R(\theta,0,\beta_1)\cdot s_1(\theta) + R(\theta,\beta_1,\beta)\cdot s_2(\theta)$$

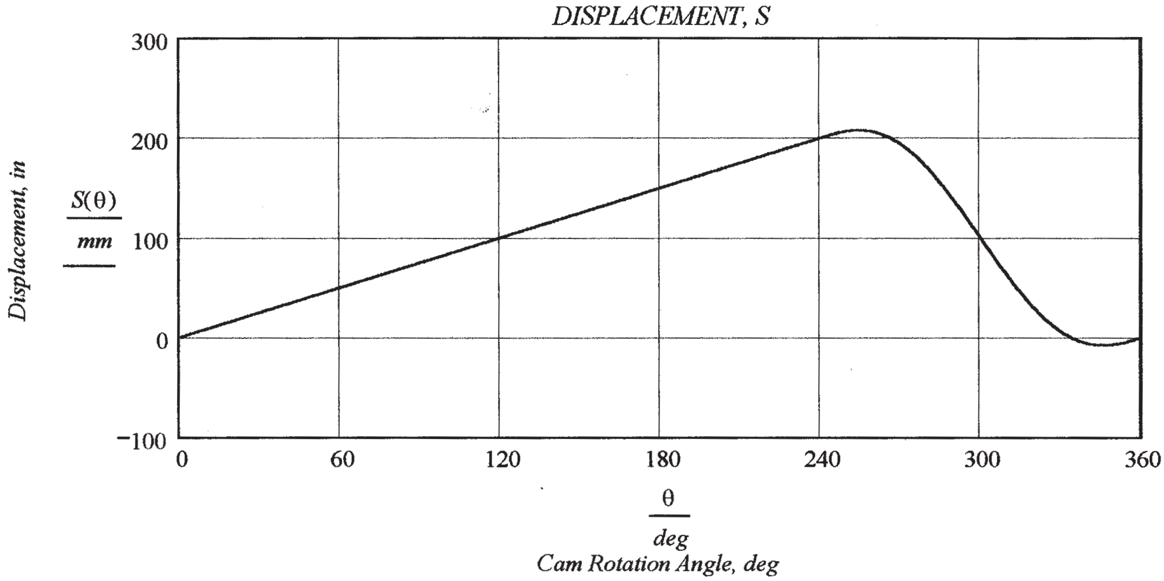
$$V(\theta) := R(\theta,0,\beta_1)\cdot v_1(\theta) + R(\theta,\beta_1,\beta)\cdot v_2(\theta)$$

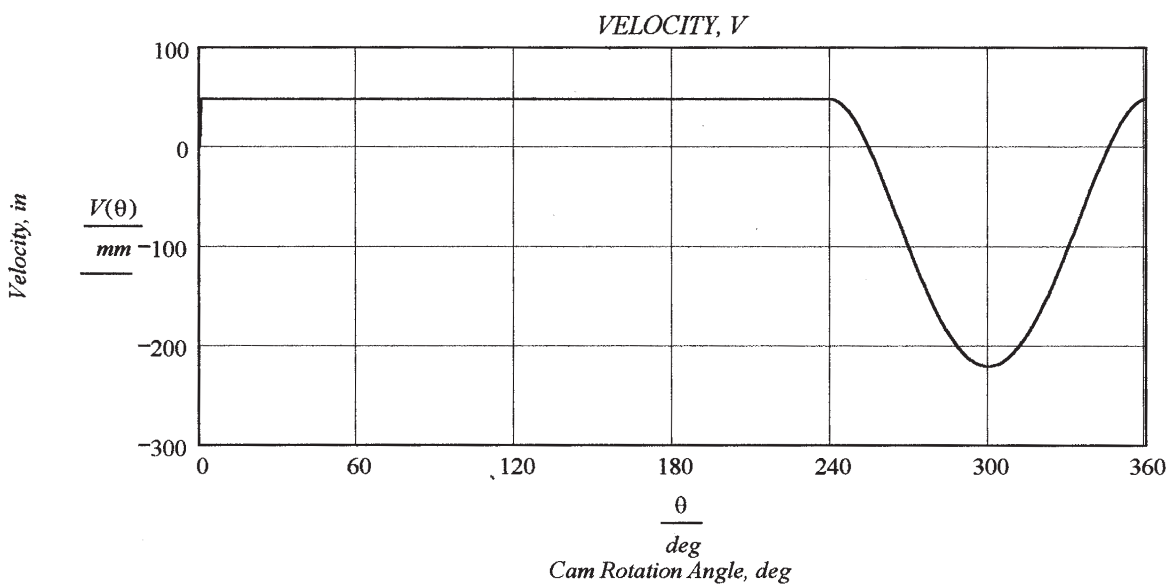
$$A(\theta) := R(\theta,0,\beta_1)\cdot a_1(\theta) + R(\theta,\beta_1,\beta)\cdot a_2(\theta)$$

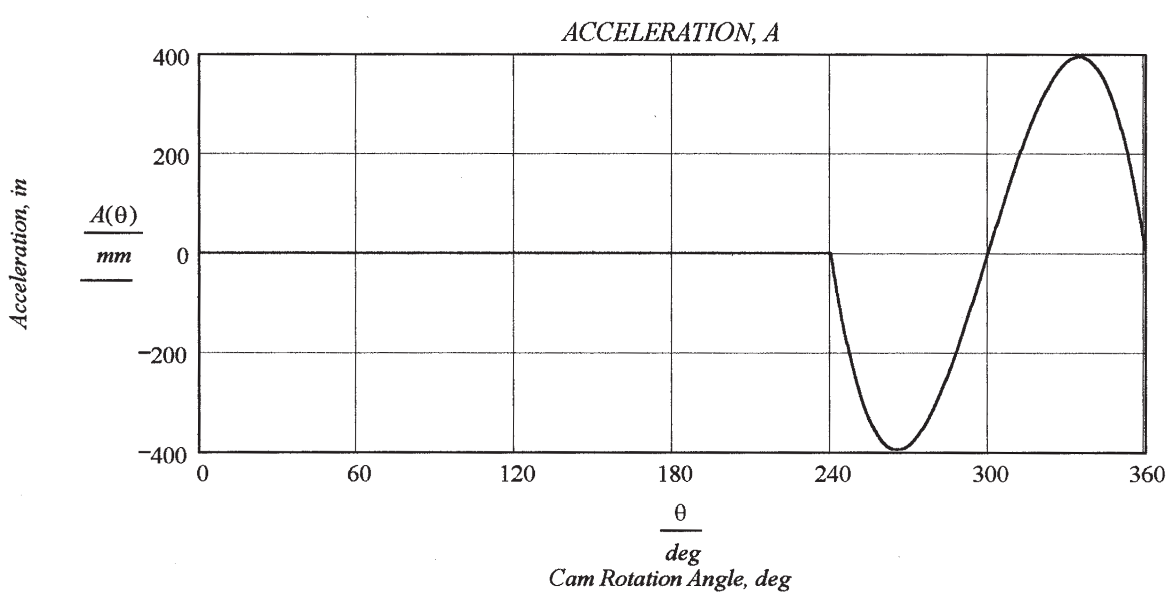
$$J(\theta) := R(\theta,0,\beta_1)\cdot j_1(\theta) + R(\theta,\beta_1,\beta)\cdot j_2(\theta)$$

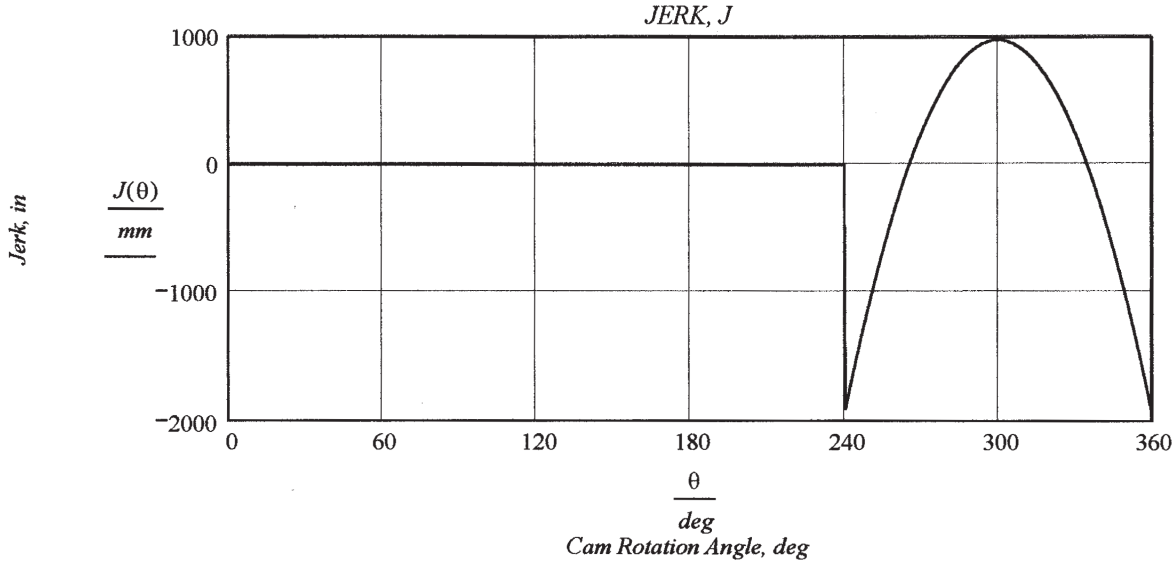
6. Plot the displacement, velocity, acceleration, and jerk over the interval 0 <= θ <= β.

$$\theta := 0\cdot deg, 0.5\cdot deg .. \beta$$


DISPLACEMENT, S
300
200
100
0
−100
Displacement, in
S(θ)
mm
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg


VELOCITY, V
100
0
−100
−200
−300
Velocity, in
V(θ)
mm
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg


ACCELERATION, A
400
200
0
−200
−400
Acceleration, in
A(θ)
mm
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg


JERK, J
1000
0
−1000
−2000
J(θ)
mm
Jerk, in
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-44

Statement: Design a double-dwell cam to move a follower from 0 to 50 mm in 75 deg, dwell for 75 deg, fall 50 mm in 75 deg and dwell for the remainder. The total cycle must take 5 sec. Use a modified trapezoidal function for rise and fall and plot the *s v a j* diagrams.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 75 \cdot deg$	$\beta_2 := 75 \cdot deg$	$\beta_3 := 75 \cdot deg$	$\beta_4 := 135 \cdot deg$
$h_1 := 50 \cdot mm$	$h_2 := 0 \cdot mm$	$h_3 := 50 \cdot mm$	$h_4 := 0 \cdot mm$

Cycle time: $\tau := 5 \cdot sec$

Solution: See Mathcad file P0844.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.257 \frac{rad}{sec}$$

2. The modified trapezoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$b := 0.25 \qquad c := 0.50 \qquad d := 0.25$$

$$C_v := 2.0000 \qquad C_a := 4.8881 \qquad C_j := 61.426$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 <= x <= b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot cos\left(\frac{\pi}{b} \cdot x\right)$$

for $b/2 <= x <= (1 - d)/2$

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for $(1 - d)/2 <= x <= (1 + d)/2$

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1 - d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1 - d}{2}\right)\right] \right]$$

$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1 - d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1 - d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1 - d}{2}\right)\right]$$

for (1 + *d*)/2 <= *x* <= 1 - *b*/2

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right) \cdot x + \left(2 \cdot d^2 - b^2\right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2}\right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for 1 - *b*/2 <= *x* <= 1

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2\right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \cdot sin\left[\frac{\pi}{b} \cdot (x - 1)\right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - cos\left[\frac{\pi}{b} \cdot (x - 1)\right] \right]$$

$$y''_5(x) := C_a \cdot sin\left[\frac{\pi}{b} \cdot (x - 1)\right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot cos\left[\frac{\pi}{b} \cdot (x - 1)\right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(x,x1,x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range 0 <= θ <= 360 deg.

6. Write the local *svaj* equations for the first interval, 0 <= θ <= β_1. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For 0 <= θ <= β_1 (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 <= \theta <= \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 <= \theta <= \beta_1 + \beta_2$

$$s_2(x) := h_1 \qquad v_2(x) := 0 \qquad a_2(x) := 0 \qquad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 <= \theta <= \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 <= \theta <= \beta_1 + \beta_2 + \beta_3$

$$s_3(x) := h_3\left[1 - \left(R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \right.\right.$$
$$\left.\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x)\right)\right]$$

$$v_3(x) := \frac{h_3}{\beta_3}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x)\right)$$

$$a_3(x) := \frac{h_3}{\beta_3^{\,2}}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x)\right)$$

$$j_3(x) := \frac{h_3}{\beta_3^{\,3}}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x)\right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4$

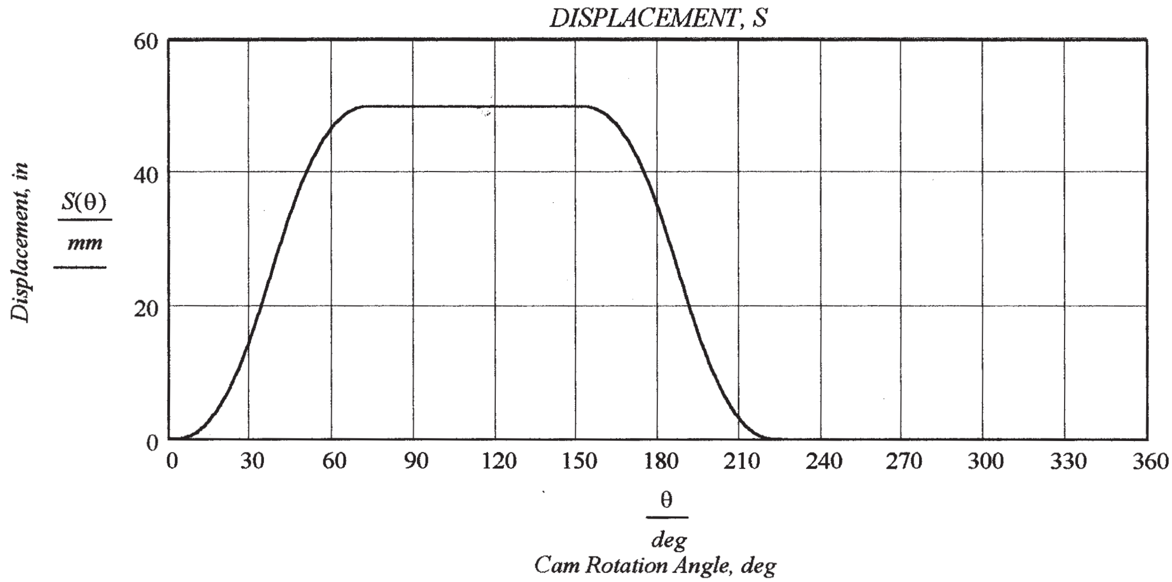
$$s_4(x) := 0 \qquad v_4(x) := 0 \qquad a_4(x) := 0 \qquad j_4(x) := 0$$

10. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

Let $\theta_1 := \beta_1 \qquad \theta_2 := \theta_1 + \beta_2 \qquad \theta_3 := \theta_2 + \beta_3 \qquad \theta_4 := \theta_3 + \beta_4$

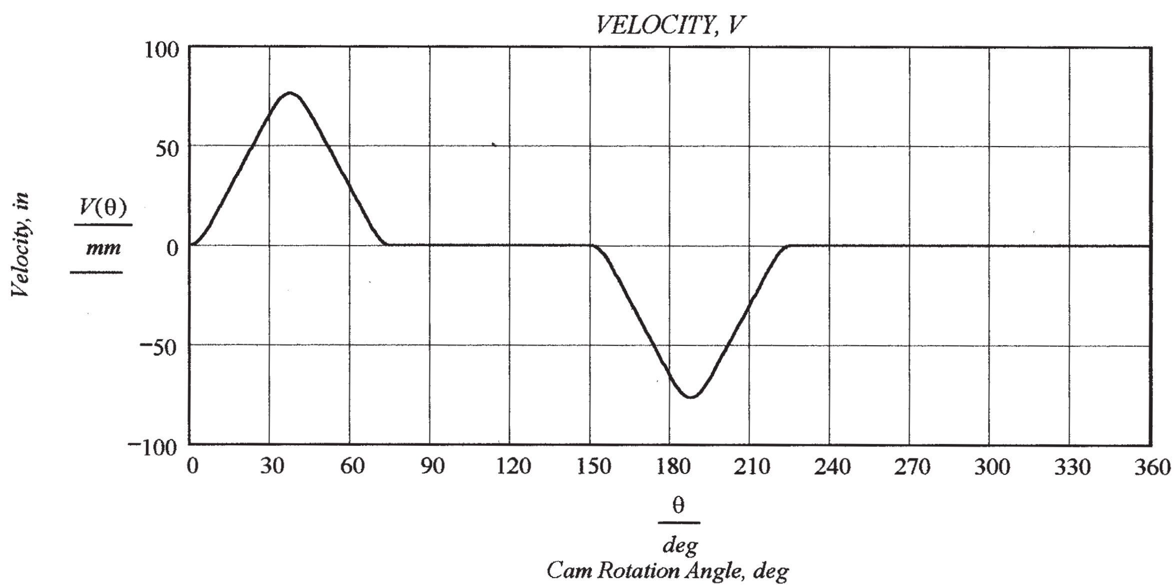
$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot s_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot s_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot s_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$

$$\theta := 0\cdot deg, 0.5\cdot deg .. 360\cdot deg$$



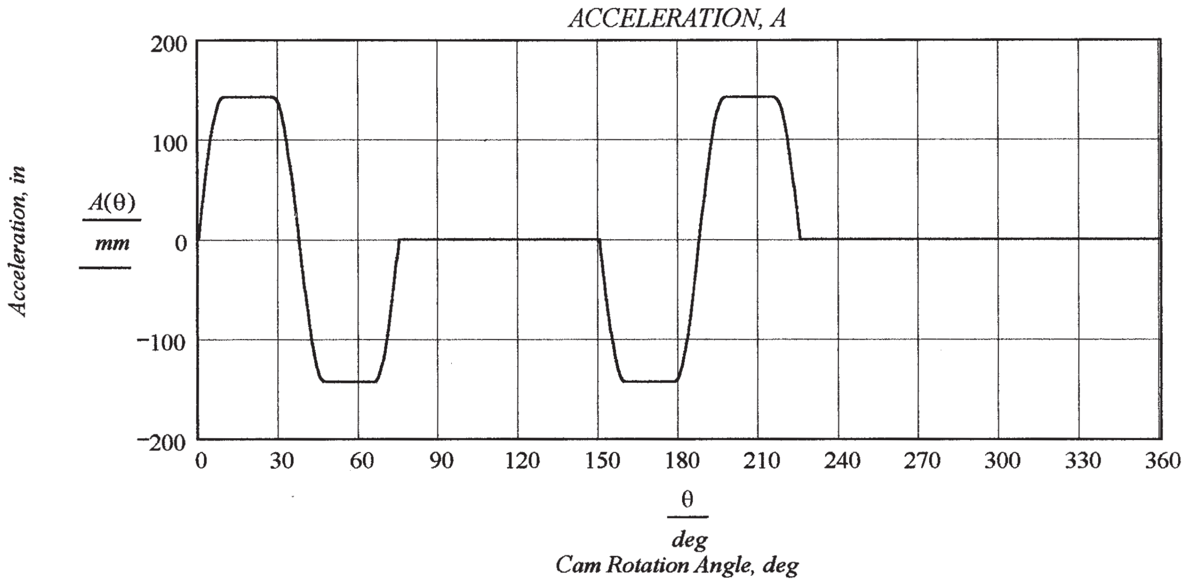
11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot v_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+R(\theta,\theta_2,\theta_3)\cdot v_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot v_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



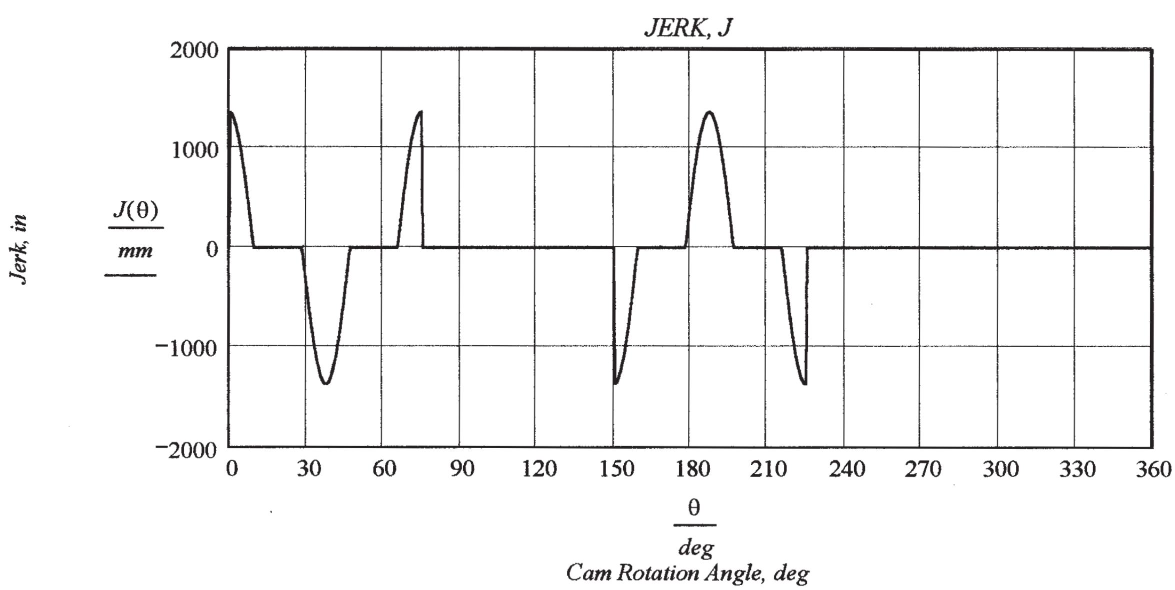
12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot a_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+R(\theta,\theta_2,\theta_3)\cdot a_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot a_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot j_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot j_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot j_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



PROBLEM 8-45

Statement: Design a double-dwell cam to move a follower from 0 to 50 mm in 75 deg, dwell for 75 deg, fall 50 mm in 75 deg and dwell for the remainder. The total cycle must take 5 sec. Use a modified sinusoidal function for rise and fall and plot the *s v a j* diagrams.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 75\cdot deg$	$\beta_2 := 75\cdot deg$	$\beta_3 := 75\cdot deg$	$\beta_4 := 135\cdot deg$
$h_1 := 50\cdot mm$	$h_2 := 0\cdot mm$	$h_3 := 50\cdot mm$	$h_4 := 0\cdot mm$

Cycle time: $\tau := 5\cdot sec$

Solution: See Mathcad file P0845.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2\cdot\pi\cdot rad}{\tau} \qquad \omega = 1.257\,\frac{rad}{sec}$$

2. The modified sinusoidal motion is defined in local coordinates by equations 8.15 through 8.19. The numerical constants in these SCCA equations are given in Table 8-2.

$$b := 0.25 \qquad c := 0.00 \qquad d := 0.75$$

$$C_a := 5.5280$$

3. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 <= x <= b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a\cdot\left[x\cdot\frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2\cdot\sin\left(\frac{\pi}{b}\cdot x\right)\right] \qquad y'_1(x) := C_a\cdot\frac{b}{\pi}\cdot\left(1 - \cos\left(\frac{\pi}{b}\cdot x\right)\right)$$

$$y''_1(x) := C_a\cdot\sin\left(\frac{\pi}{b}\cdot x\right) \qquad y'''_1(x) := C_a\cdot\frac{\pi}{b}\cdot\cos\left(\frac{\pi}{b}\cdot x\right)$$

for $b/2 <= x <= (1 - d)/2$

$$y_2(x) := C_a\cdot\left[\frac{x^2}{2} + b\cdot\left(\frac{1}{\pi} - \frac{1}{2}\right)\cdot x + b^2\cdot\left(\frac{1}{8} - \frac{1}{\pi^2}\right)\right] \qquad y'_2(x) := C_a\cdot\left[x + b\cdot\left(\frac{1}{\pi} - \frac{1}{2}\right)\right]$$

$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for $(1 - d)/2 <= x <= (1 + d)/2$

$$y_3(x) := C_a\cdot\left[\left(\frac{b}{\pi} + \frac{c}{2}\right)\cdot x + \left(\frac{d}{\pi}\right)^2 + b^2\cdot\left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2\cdot\cos\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right]\right]$$

$$y'_3(x) := C_a\cdot\left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi}\cdot\sin\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right]\right]$$

$$y''_3(x) := C_a\cdot\cos\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a\cdot\frac{\pi}{d}\cdot\sin\left[\frac{\pi}{d}\cdot\left(x - \frac{1-d}{2}\right)\right]$$

for $(1 + d)/2 <= x <= 1 - b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + \left(2 \cdot d^2 - b^2 \right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for $1 - b/2 <= x <= 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2 \right)}{\pi^2} + \frac{(1 - b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b} \cdot (x - 1) \right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b} \cdot (x - 1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(x, x1, x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 <= \theta <= 360$ deg.

6. Write the local *svaj* equations for the first interval, $0 <= \theta <= \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 <= \theta <= \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

for $(1 + d)/2 <= x <= 1 - b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + \left(2 \cdot d^2 - b^2 \right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for $1 - b/2 <= x <= 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2 \right)}{\pi^2} + \frac{(1 - b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b} \cdot (x - 1) \right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b} \cdot (x - 1) \right]$$

4. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of a and b and zero elsewhere.

$$R(x, x1, x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

5. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range $0 <= \theta <= 360$ deg.

6. Write the local *svaj* equations for the first interval, $0 <= \theta <= \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 <= \theta <= \beta_1$ (Rise)

$$s_1(x) := h_1 \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h_1}{\beta_1} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h_1}{\beta_1^2} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h_1}{\beta_1^3} \cdot \left(R\left(x, 0, \frac{b}{2}\right) \cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right) \cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right) \cdot y'''_5(x) \right)$$

7. Write the local *svaj* equations for the second interval, $\beta_1 <= \theta <= \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 <= \theta <= \beta_1 + \beta_2$

$$s_2(x) := h_1 \qquad v_2(x) := 0 \qquad a_2(x) := 0 \qquad j_2(x) := 0$$

8. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 <= \theta <= \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 <= \theta <= \beta_1 + \beta_2 + \beta_3$

$$s_3(x) := h_3\left[1 - \left(R\left(x,0,\frac{b}{2}\right)\cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y_3(x) \ldots \right.\right.$$
$$\left.\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y_5(x)\right)\right]$$

$$v_3(x) := -\frac{h_3}{\beta_3}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'_5(x)\right)$$

$$a_3(x) := -\frac{h_3}{\beta_3^{\,2}}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y''_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y''_5(x)\right)$$

$$j_3(x) := -\frac{h_3}{\beta_3^{\,3}}\cdot\left(R\left(x,0,\frac{b}{2}\right)\cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right)\cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right)\cdot y'''_5(x)\right)$$

9. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4$

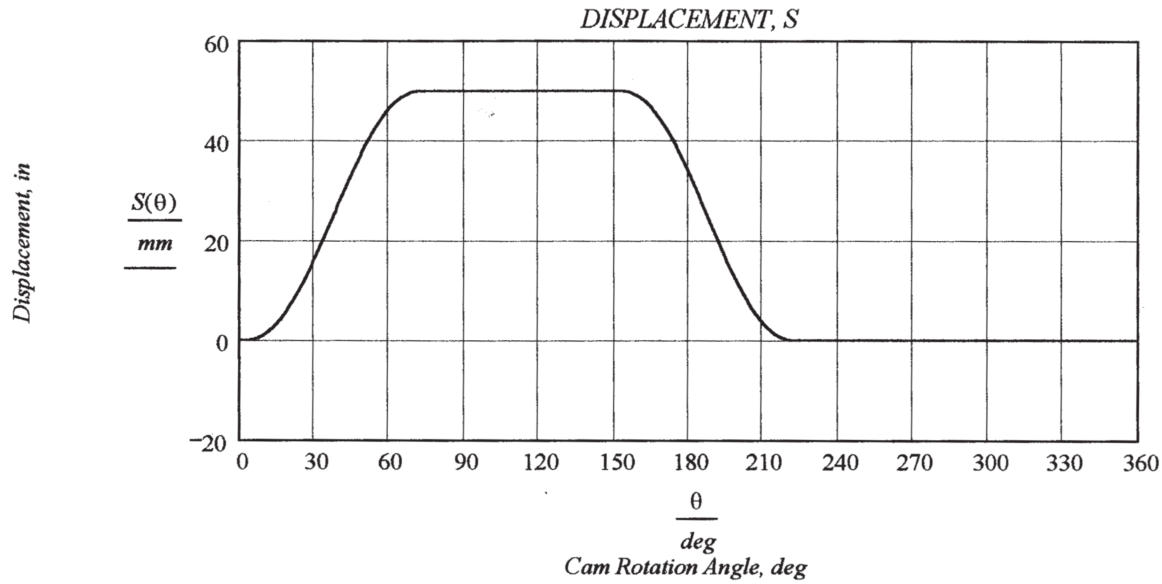
$$s_4(x) := 0 \qquad v_4(x) := 0 \qquad a_4(x) := 0 \qquad j_4(x) := 0$$

10. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

Let $\theta_1 := \beta_1 \qquad \theta_2 := \theta_1 + \beta_2 \qquad \theta_3 := \theta_2 + \beta_3 \qquad \theta_4 := \theta_3 + \beta_4$

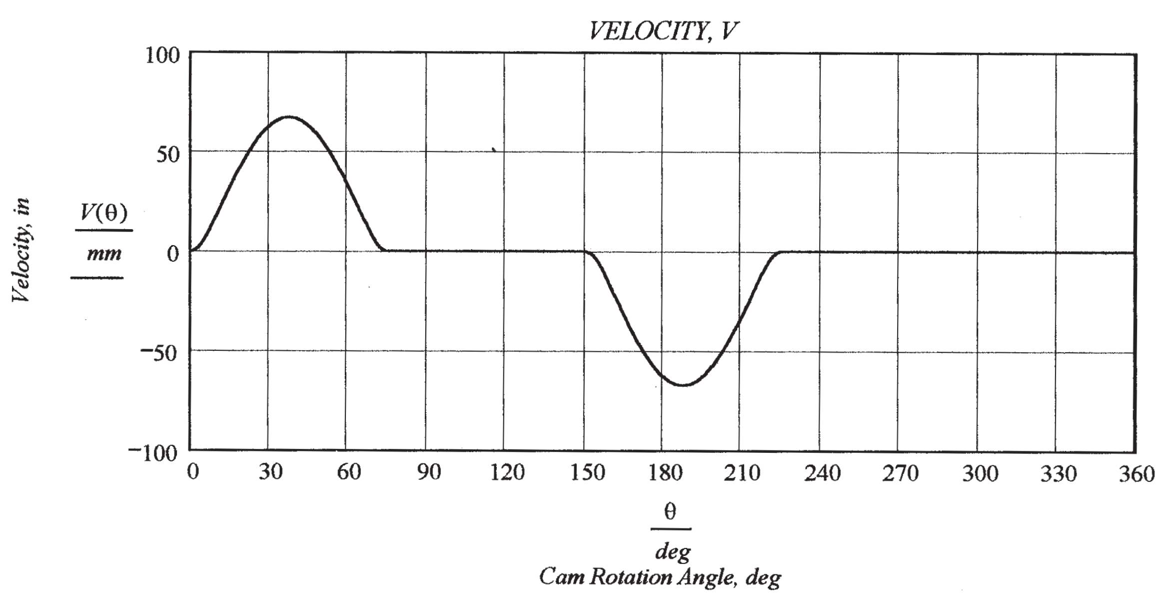
$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot s_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot s_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot s_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$

$$\theta := 0\cdot deg, 0.5\cdot deg .. 360\cdot deg$$



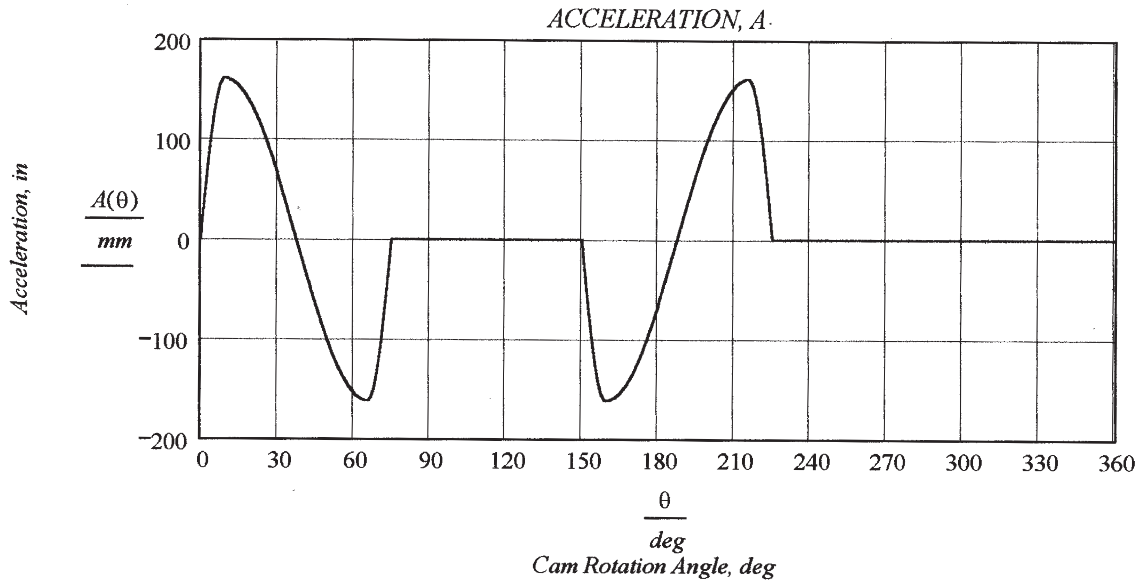
11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot v_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+R(\theta,\theta_2,\theta_3)\cdot v_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot v_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



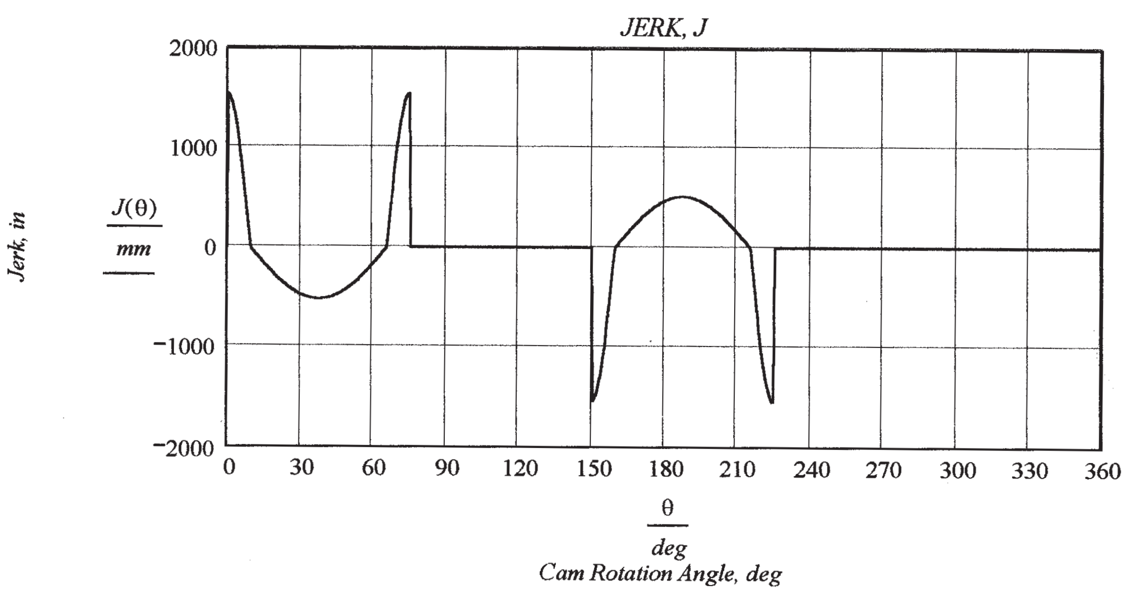
12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot a_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+R(\theta,\theta_2,\theta_3)\cdot a_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot a_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot j2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot j3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot j4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



PROBLEM 8-46

Statement: Design a double-dwell cam to move a follower from 0 to 50 mm in 75 deg, dwell for 75 deg, fall 50 mm in 75 deg and dwell for the remainder. The total cycle must take 5 sec. Use a 4-5-6-7 polynomial function for rise and fall and plot the *s v a j* diagrams.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 75 \cdot deg$	$\beta_2 := 75 \cdot deg$	$\beta_3 := 75 \cdot deg$	$\beta_4 := 135 \cdot deg$
$h_1 := 50 \cdot mm$	$h_2 := 0 \cdot mm$	$h_3 := 50 \cdot mm$	$h_4 := 0 \cdot mm$

Cycle time: $\tau := 5 \cdot sec$

Solution: See Mathcad file P0846.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.257 \frac{rad}{sec}$$

2. The 4-5-6-7 polynomial is defined in local coordinates by equation 8.25. Differentiate it to get *v*, *a*, and *j*.

$$s(\theta, \beta, h) := h \cdot \left[35 \cdot \left(\frac{\theta}{\beta}\right)^4 - 84 \cdot \left(\frac{\theta}{\beta}\right)^5 + 70 \cdot \left(\frac{\theta}{\beta}\right)^6 - 20 \cdot \left(\frac{\theta}{\beta}\right)^7 \right]$$

$$v(\theta, \beta, h) := \frac{h}{\beta} \cdot \left[140 \cdot \left(\frac{\theta}{\beta}\right)^3 - 420 \cdot \left(\frac{\theta}{\beta}\right)^4 + 420 \cdot \left(\frac{\theta}{\beta}\right)^5 - 140 \cdot \left(\frac{\theta}{\beta}\right)^6 \right]$$

$$a(\theta, \beta, h) := \frac{h}{\beta^2} \cdot \left[420 \cdot \left(\frac{\theta}{\beta}\right)^2 - 1680 \cdot \left(\frac{\theta}{\beta}\right)^3 + 2100 \cdot \left(\frac{\theta}{\beta}\right)^4 - 840 \cdot \left(\frac{\theta}{\beta}\right)^5 \right]$$

$$j(\theta, \beta, h) := \frac{h}{\beta^3} \cdot \left[840 \cdot \left(\frac{\theta}{\beta}\right) - 5040 \cdot \left(\frac{\theta}{\beta}\right)^2 + 8400 \cdot \left(\frac{\theta}{\beta}\right)^3 - 4200 \cdot \left(\frac{\theta}{\beta}\right)^4 \right]$$

3. The above equations can be used for a rise or fall by inserting the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(\theta, x, y) := if[(\theta > x) \cdot (\theta \le y), 1, 0]$$

4. Write the global *SVAJ* equations for the first interval, $0 <= \theta <= \beta_1$. For this interval, the local and global frames are coincident so the local equations can be used as written, substituting only for h_1 for *h* and β_1 for β. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 <= \theta <= \beta_1$

$$S_1(\theta) := R(\theta, 0, \beta_1) \cdot s(\theta, \beta_1, h_1) \qquad V_1(\theta) := R(\theta, 0, \beta_1) \cdot v(\theta, \beta_1, h_1)$$

$$A_1(\theta) := R(\theta, 0, \beta_1) \cdot a(\theta, \beta_1, h_1) \qquad J_1(\theta) := R(\theta, 0, \beta_1) \cdot j(\theta, \beta_1, h_1)$$

5. Write the global *SVAJ* equations for the second interval, $\beta_1 <= \theta <= \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 <= \theta <= \beta_1 + \beta_2$

$$S_2(\theta) := R(\theta, \beta_1, \beta_1 + \beta_2) \cdot h_1 \qquad V_2(\theta) := R(\theta, \beta_1, \beta_1 + \beta_2) \cdot 0 \cdot mm$$

$$A_2(\theta) := R(\theta, \beta_1, \beta_1 + \beta_2) \cdot 0 \cdot mm \qquad J_2(\theta) := R(\theta, \beta_1, \beta_1 + \beta_2) \cdot 0 \cdot mm$$

6. Write the global *SVAJ* equations for the third interval, $\beta_1 + \beta_2 <= \theta <= \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 <= \theta <= \beta_1 + \beta_2 + \beta_3$ Let $\alpha := \beta_1 + \beta_2$

$$S_3(\theta) := R(\theta, \alpha, \alpha + \beta_3) \cdot (h_1 - s(\theta - \alpha, \beta_3, h_3)) \qquad V_3(\theta) := -R(\theta, \alpha, \alpha + \beta_3) \cdot v(\theta - \alpha, \beta_3, h_3)$$

$$A_3(\theta) := -R(\theta, \alpha, \alpha + \beta_3) \cdot a(\theta - \alpha, \beta_3, h_3) \qquad J_3(\theta) := -R(\theta, \alpha, \alpha + \beta_3) \cdot j(\theta - \alpha, \beta_3, h_3)$$

7. Write the global *SVAJ* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S, V, A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4$ Let $\alpha := \beta_1 + \beta_2 + \beta_3$

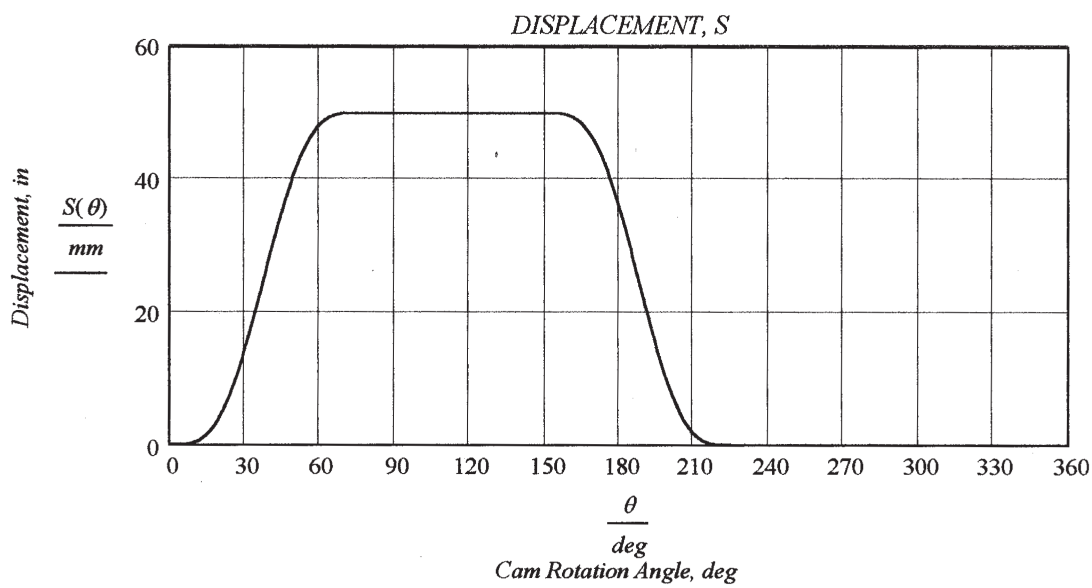
$$S_4(\theta) := R(\theta, \alpha, \alpha + \beta_4) \cdot 0 \cdot mm \qquad V_4(\theta) := R(\theta, \alpha, \alpha + \beta_4) \cdot 0 \cdot mm$$

$$A_4(\theta) := R(\theta, \alpha, \alpha + \beta_4) \cdot 0 \cdot mm \qquad J_4(\theta) := R(\theta, \alpha, \alpha + \beta_3) \cdot 0 \cdot mm$$

8. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

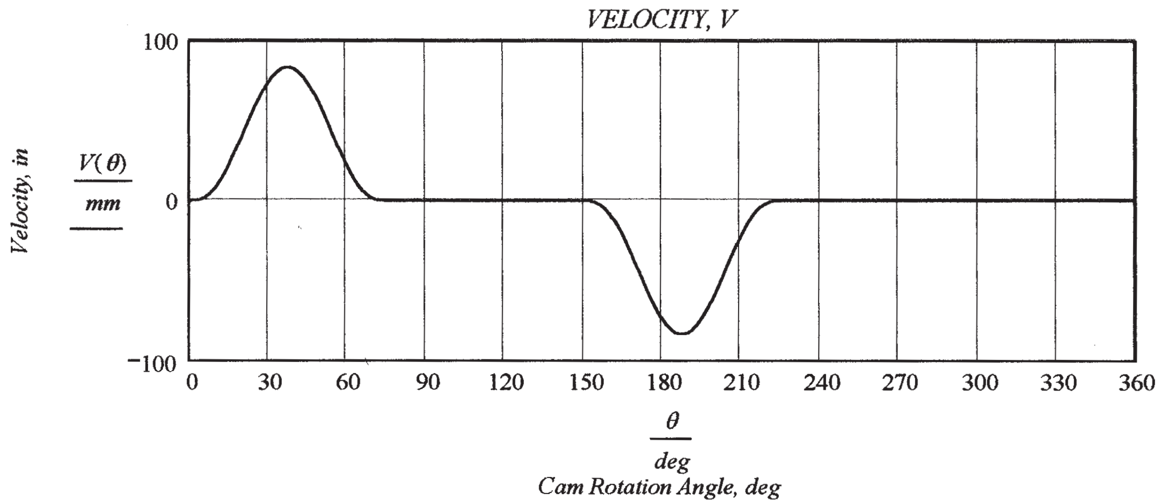
$$S(\theta) := S_1(\theta) + S_2(\theta) + S_3(\theta) + S_4(\theta)$$

$$\theta := 0 \cdot deg, 0.5 \cdot deg .. 360 \cdot deg$$



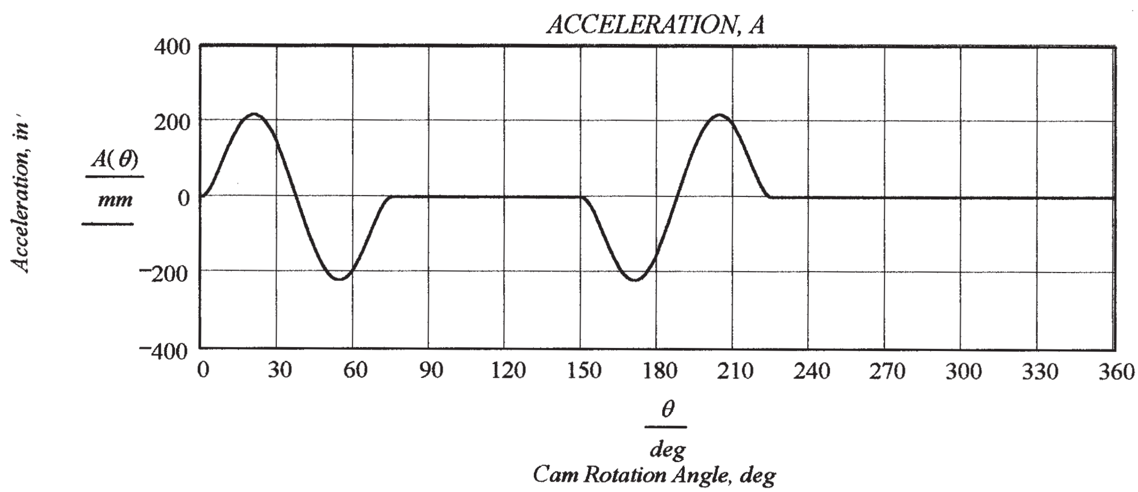
9. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := V_1(\theta) + V_2(\theta) + V_3(\theta) + V_4(\theta)$$



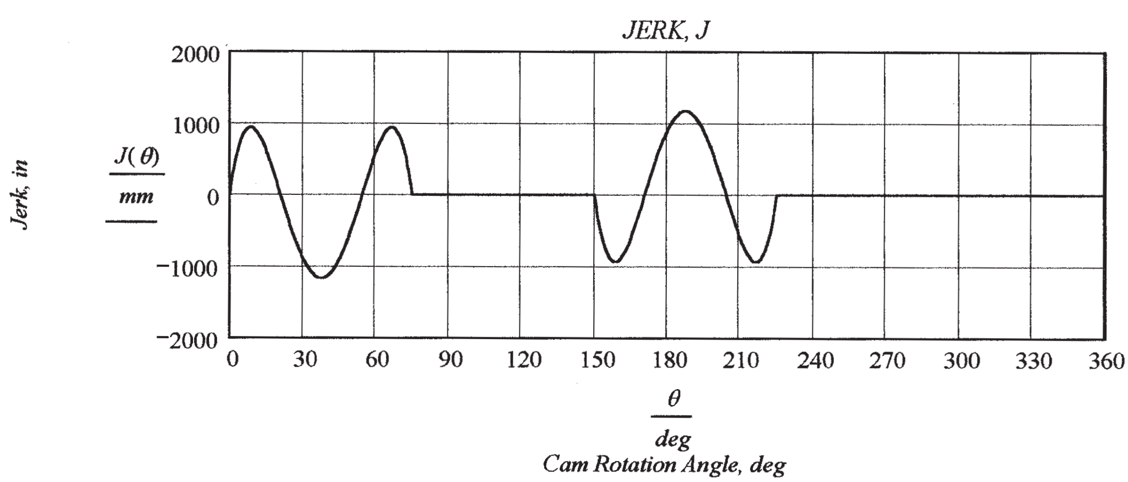
10. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := A_1(\theta) + A_2(\theta) + A_3(\theta) + A_4(\theta)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := J_1(\theta) + J_2(\theta) + J_3(\theta) + J_4(\theta)$$



PROBLEM 8-47

Statement: Design a single-dwell cam to move a follower from 0 to 65 mm in 90 deg, fall 65 mm in 180 deg and dwell for the remainder. The total cycle must take 2 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the *SVAJ* diagrams.

Given:

RISE/FALL	DWELL
$\beta := 270 \cdot deg$	$\beta_3 := 90 \cdot deg$
$h := 65 \cdot mm$	$h_3 := 0.0 \cdot mm$

Cycle time: $\tau := 2 \cdot sec$

Solution: See Mathcad file P0847.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 3.142 \frac{rad}{sec}$$

2. Use a two-segment polynomial. Let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

at
$\theta = 0$: $s = 0$, $v = 0$, $a = 0$
$\theta = \beta_1$: $s = h$, $v = 0$
$\theta = \beta$: $s = 0$, $v = 0$, $a = 0$

This is a minimum set of 8 BCs. The $v = 0$ condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, h. Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

Total lift: $h = 65.000\,mm$

Rise interval: $\beta_1 := 90 \cdot deg$ $\qquad A := \dfrac{\beta_1}{\beta}$ $\qquad A = 0.333$

Fall interval: $\beta_2 := 180 \cdot deg$

3. Use the 8 BCs and equation 8.23 to write 8 equations in *s*, *v*, and *a* similar to those in example 8-9 but with 8 terms in the equation for *s* (the highest term will be seventh degree).

For $\theta = 0$: $s = v = a = 0$

$$0 := c_0 \qquad 0 := c_1 \qquad 0 := c_2$$

For $\theta = \beta_1$: $s = h$, $v = 0$

$$h := c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5 + c_6 \cdot A^6 + c_7 \cdot A^7$$

$$0 := 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 + 6 \cdot c_6 \cdot A^5 + 7 \cdot c_7 \cdot A^6$$

For $\theta = \beta$: $s = v = a = 0$

$$0 := c_3 + c_4 + c_5 + c_6 + c_7$$

$$0 := 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 + 6 \cdot c_6 + 7 \cdot c_7$$

$$0 := 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5 + 30 \cdot c_6 + 42 \cdot c_7$$

4. Solve for the unknown polynomial coefficients. Note that C_0 through C_2 are zero

$$C := \begin{pmatrix} A^3 & A^4 & A^5 & A^6 & A^7 \\ 3\cdot A^2 & 4\cdot A^3 & 5\cdot A^4 & 6\cdot A^5 & 7\cdot A^6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{pmatrix} \qquad H := \begin{pmatrix} h \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix} := C^{-1}\cdot H$$

$c_3 = 582.985\,in$ $\quad c_4 = -2798.327\,in$ $\quad c_5 = 4897.072\,in$

$c_6 = -3731.102\,in$ $\quad c_7 = 1.049 \times 10^3\,in$

5. Write the *SVAJ* equations for the rise/fall segment.

$$S(\theta) := c_3\cdot\left(\frac{\theta}{\beta}\right)^3 + c_4\cdot\left(\frac{\theta}{\beta}\right)^4 + c_5\cdot\left(\frac{\theta}{\beta}\right)^5 + c_6\cdot\left(\frac{\theta}{\beta}\right)^6 + c_7\cdot\left(\frac{\theta}{\beta}\right)^7$$

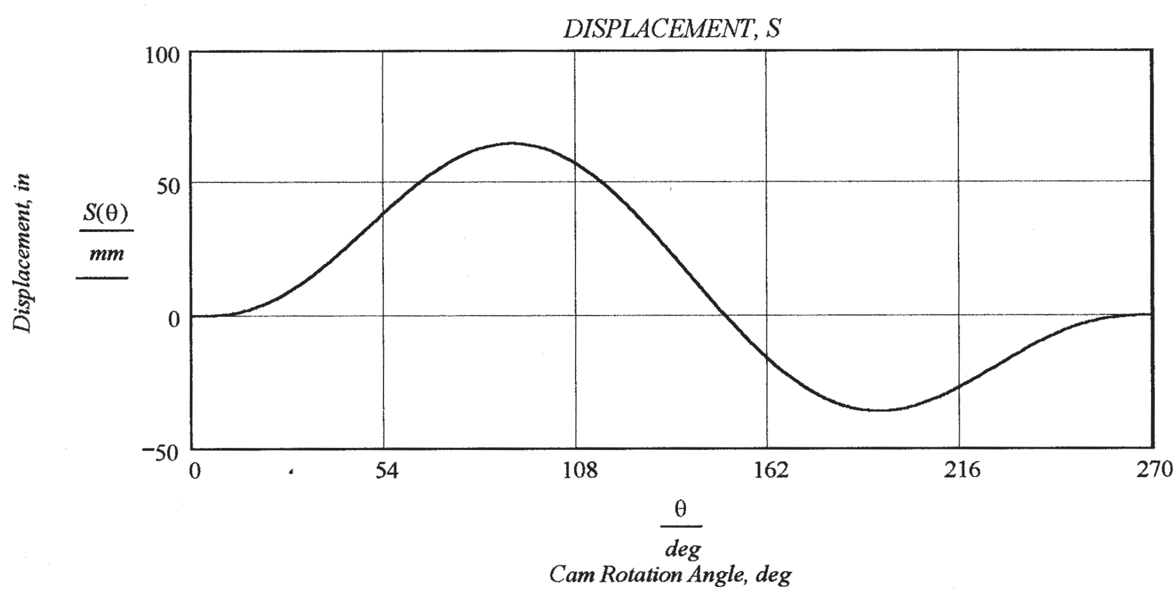
$$V(\theta) := \frac{1}{\beta}\cdot\left[3\cdot c_3\cdot\left(\frac{\theta}{\beta}\right)^2 + 4\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^3 + 5\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^4 + 6\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^5 + 7\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^6\right]$$

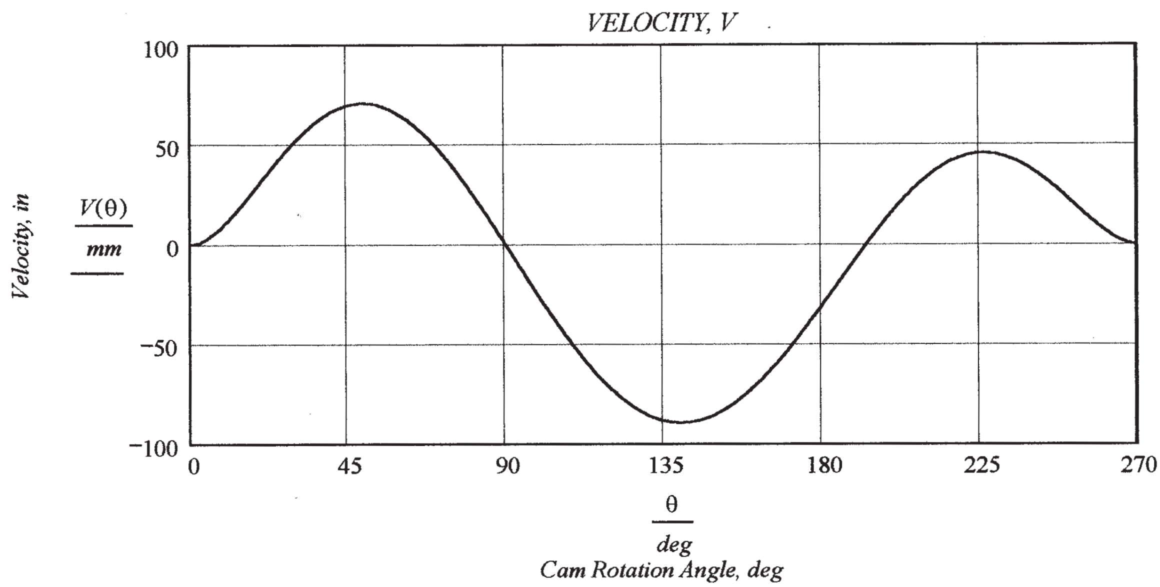
$$A(\theta) := \frac{1}{\beta^2}\cdot\left[6\cdot c_3\cdot\left(\frac{\theta}{\beta}\right) + 12\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^2 + 20\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^3 + 30\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^4 + 42\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^5\right]$$

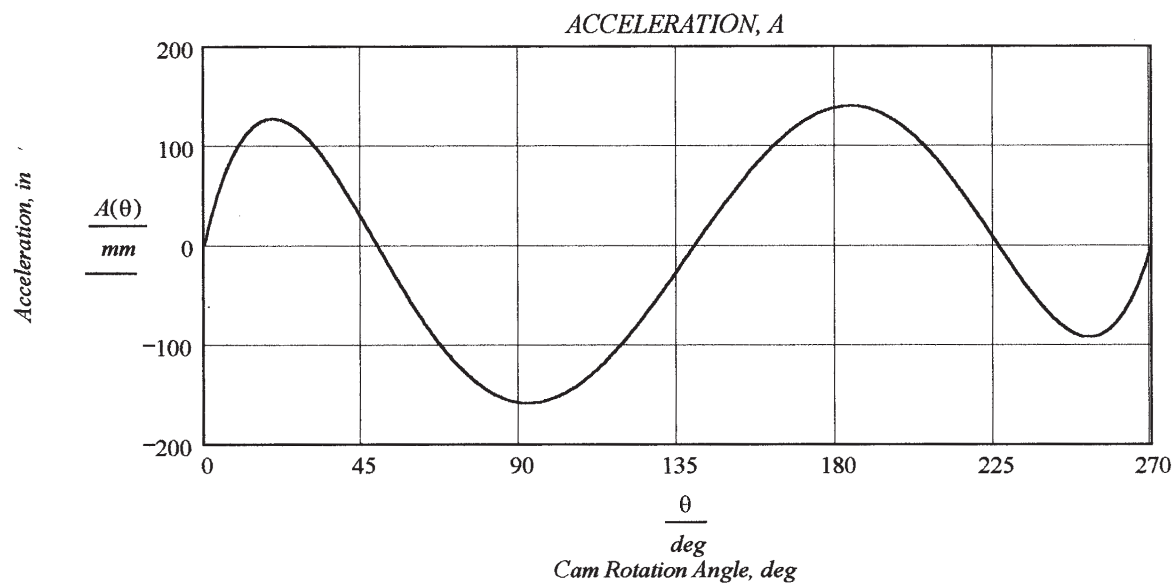
$$J(\theta) := \frac{1}{\beta^3}\cdot\left[6\cdot c_3 + 24\cdot c_4\cdot\left(\frac{\theta}{\beta}\right) + 60\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^2 + 120\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^3 + 210\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^4\right]$$

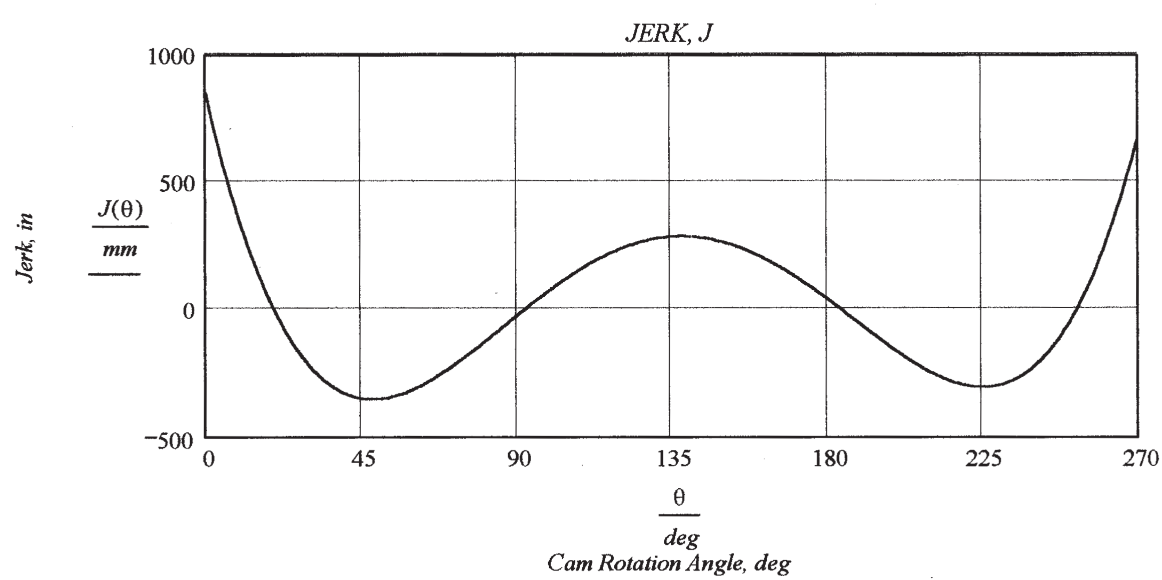
6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 <= \theta <= \beta$.

$\theta := 0\cdot deg, 1\cdot deg .. \beta$




VELOCITY, V
100
50
0
−50
−100
Velocity, in
V(θ)
mm
0
45
90
135
180
225
270
θ
deg
Cam Rotation Angle, deg


ACCELERATION, A
200
100
0
−100
−200
Acceleration, in
A(θ)
mm
0
45
90
135
180
225
270
θ
deg
Cam Rotation Angle, deg


JERK, J
1000
500
0
−500
Jerk, in
J(θ)
mm
0
45
90
135
180
225
270
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-48

Statement: Design a cam to move a follower at a constant velocity of 200 mm/sec for 3 sec then return to its starting position with a total cycle time of 6 sec.

Given: Constant velocity: $v_c := 200 \, mm \cdot sec^{-1}$

Time duration of cv segment: $t_{cv} := 3 \cdot sec$ Cycle time: $\tau := 6 \cdot sec$

Solution: See Mathcad file P0848.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.047 \frac{rad}{sec}$$

2. Use a two-segment polynomial as demonstrated in Example 8-12. The lift during the first segment and the *svaj* equations for the first segment are:

Normalized velocity: $v_{cv} := \frac{v_c}{\omega}$ $\qquad v_{cv} = 190.986 \, mm$

$$h_{cv} := v_c \cdot t_{cv} \qquad h_{cv} = 600.000 \, mm \qquad \beta_1 := \frac{t_{cv}}{\tau} \cdot 360 \cdot deg \qquad \beta_1 = 180 \, deg$$

$$s_1(\theta) := h_{cv} \cdot \frac{\theta}{\beta_1} \qquad v_1(\theta) := v_{cv} \qquad a_1(\theta) := 0 \cdot mm \qquad j_1(\theta) := 0 \cdot mm$$

2. The boundary conditions for the second segment are:

at $\theta = \beta_1$: $\quad s = h_{cv}, \quad v = v_{cv} \quad a = 0$

$\theta = 360$ deg: $\quad s = 0, \quad v = v_{cv}, \quad a = 0$

This is a minimum set of 6 BCs. Define the total interval and the constant velocity interval, and the ratio of constant velocity interval to the total interval.

Total interval: $\beta := 360 \cdot deg$

CV interval: $\beta_1 = 180 \, deg$ $\qquad A := \frac{\beta_1}{\beta}$ $\qquad A = 0.500$

3. Use the 6 BCs and equation 8.23 to write 6 equations in *s*, *v*, and *a* similar to those in example 8-9 but with 6 terms in the equation for *s* (the highest term will be fifth degree).

For $\theta = \beta 1$: $\quad s = h_{cv}, \quad v = v_{cv} \quad a = 0$

$$h_{cv} = c_0 + c_1 \cdot A + c_2 \cdot A^2 + c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5$$

$$v_{cv} = \frac{1}{\beta} \cdot \left(c_1 + 2 \cdot c_2 \cdot A + 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 \right)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 \cdot A + 12 \cdot c_4 \cdot A^2 + 20 \cdot c_5 \cdot A^3$$

For $\theta = \beta$: $\quad s = 0, \quad v = v_{cv}, \quad a = 0$

$$0 = c_0 + c_1 + c_2 + c_3 + c_4 + c_5$$

$$0 = \frac{1}{\beta} \cdot \left(c_1 + 2 \cdot c_2 + 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 \right)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5$$

4. Solve for the unknown polynomial coefficients.

$$C := \begin{pmatrix} 1 & A & A^2 & A^3 & A^4 & A^5 \\ 0 & 1 & 2\cdot A & 3\cdot A^2 & 4\cdot A^3 & 5\cdot A^4 \\ 0 & 0 & 2 & 6\cdot A & 12\cdot A^2 & 20\cdot A^3 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 6 & 12 & 20 \end{pmatrix} \qquad H := \begin{pmatrix} h_{cv} \\ \beta\cdot v_{cv} \\ 0 \\ 0 \\ \beta\cdot v_{cv} \\ 0 \end{pmatrix} \qquad \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} := C^{-1}\cdot H$$

$c_0 = 3.720 \times 10^4\,mm$ $\quad c_1 = -2.868 \times 10^5\,mm$ $\quad c_2 = 8.640 \times 10^5\,mm$

$c_3 = -1.248 \times 10^6\,mm$ $\quad c_4 = 8.64 \times 10^5\,mm$ $\quad c_5 = -2.304 \times 10^5\,mm$

5. Write the *svaj* equations for the second segment.

$$s_2(\theta) := c_0 + c_1\cdot\left(\frac{\theta}{\beta}\right) + c_2\cdot\left(\frac{\theta}{\beta}\right)^2 + c_3\cdot\left(\frac{\theta}{\beta}\right)^3 + c_4\cdot\left(\frac{\theta}{\beta}\right)^4 + c_5\cdot\left(\frac{\theta}{\beta}\right)^5$$

$$v_2(\theta) := \frac{1}{\beta}\cdot\left[c_1 + 2\cdot c_2\cdot\left(\frac{\theta}{\beta}\right) + 3\cdot c_3\cdot\left(\frac{\theta}{\beta}\right)^2 + 4\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^3 + 5\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^4\right]$$

$$a_2(\theta) := \frac{1}{\beta^2}\cdot\left[2\cdot c_2 + 6\cdot c_3\cdot\left(\frac{\theta}{\beta}\right) + 12\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^2 + 20\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^3\right]$$

$$j_2(\theta) := \frac{1}{\beta^3}\cdot\left[6\cdot c_3 + 24\cdot c_4\cdot\left(\frac{\theta}{\beta}\right) + 60\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^2\right]$$

4. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(\theta,a,b) := if[(\theta > a) \wedge (\theta \le b), 1, 0]$$

$$S(\theta) := R(\theta,0,\beta_1)\cdot s_1(\theta) + R(\theta,\beta_1,\beta)\cdot s_2(\theta)$$

$$V(\theta) := R(\theta,0,\beta_1)\cdot v_1(\theta) + R(\theta,\beta_1,\beta)\cdot v_2(\theta)$$

$$A(\theta) := R(\theta,0,\beta_1)\cdot a_1(\theta) + R(\theta,\beta_1,\beta)\cdot a_2(\theta)$$

$$J(\theta) := R(\theta,0,\beta_1)\cdot j_1(\theta) + R(\theta,\beta_1,\beta)\cdot j_2(\theta)$$

6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 <= \theta <= \beta$.

$$\theta := 0\cdot deg, 0.5\cdot deg .. \beta$$

PROBLEM 8-49

Statement: Size the cam from Problem 8-42 for a flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

RISE/FALL	DWELL
$\beta := 195 \cdot deg$	$\beta_3 := 165 \cdot deg$
$h := 35 \cdot mm$	$h_3 := 0.0 \cdot mm$

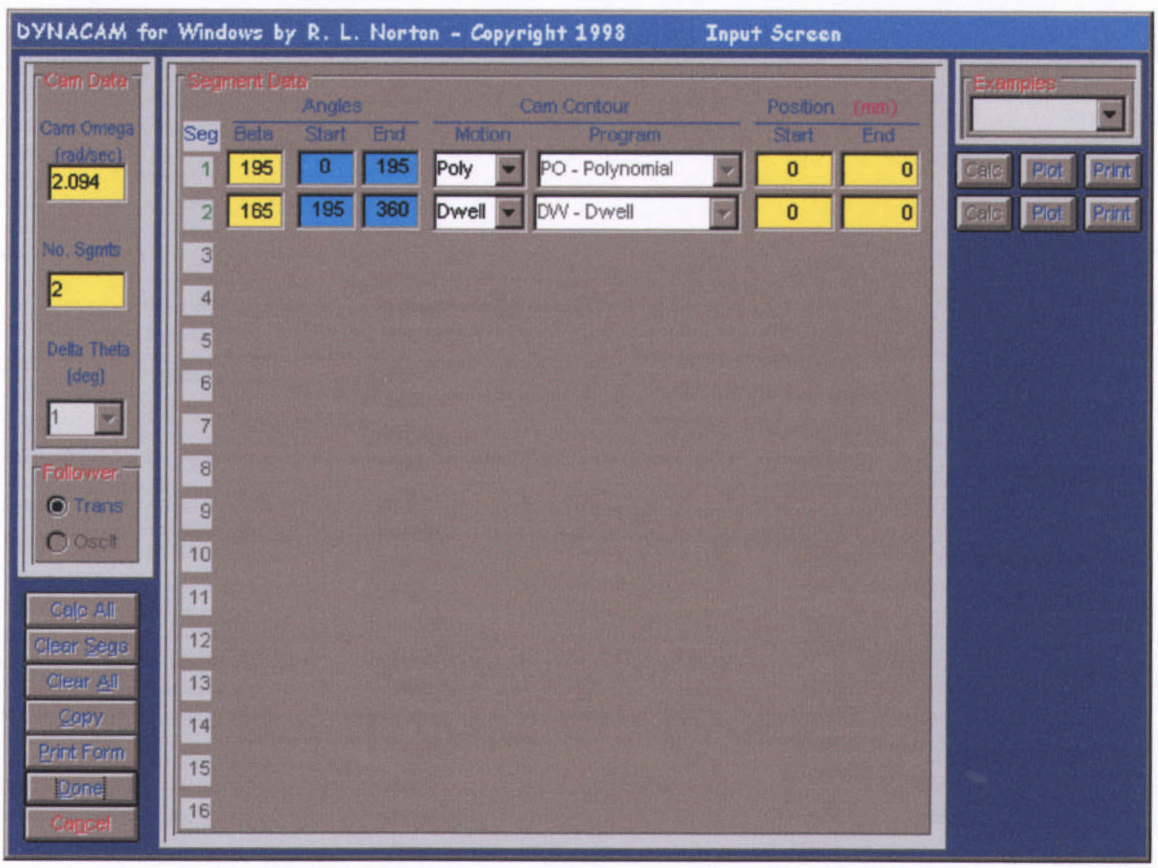
Cycle time: $\tau := 3 \cdot sec$

Solution: See Mathcad file P0849.

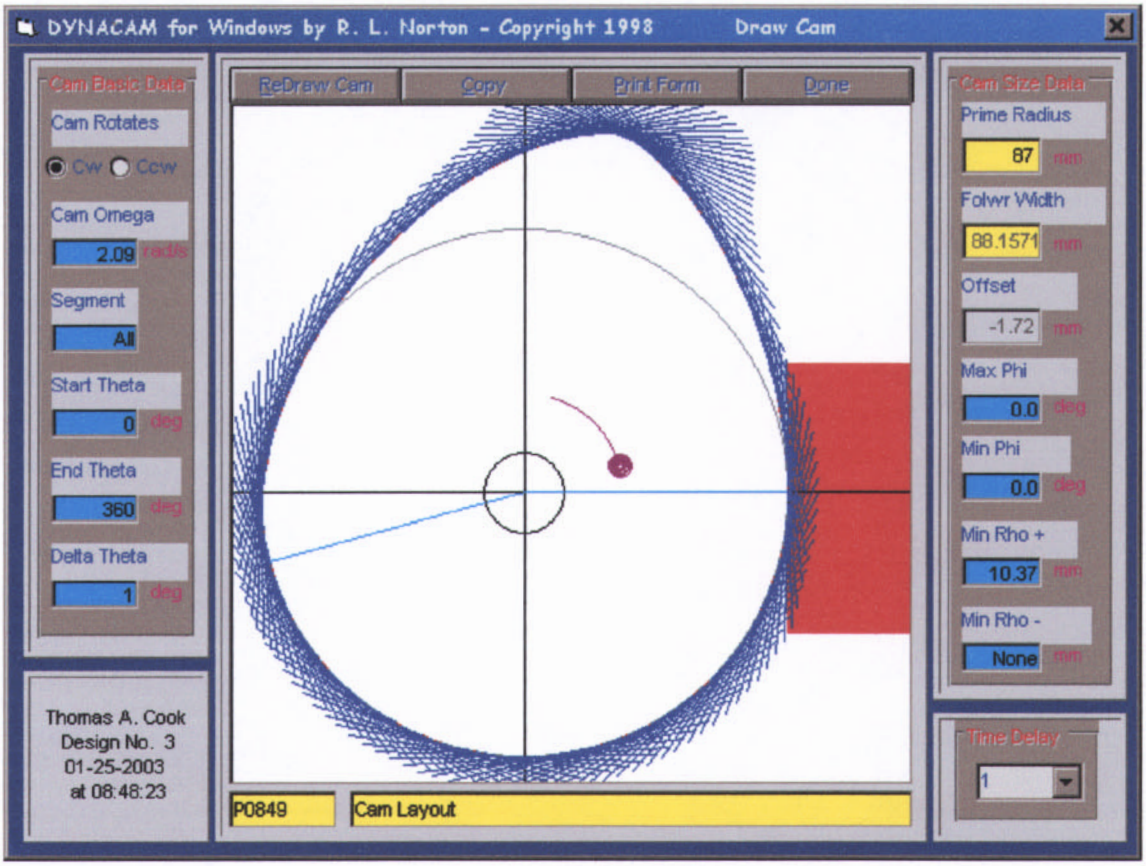
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 2.094 \frac{rad}{sec} \qquad \omega = 20.000\, rpm$$

2. Problem 8-42 used a two-segment polynomial with the rise and fall together in one segment and the dwell in the second segment. Enter the above data into program DYNACAM. The input screen is shown below.



3. The cam was sized iteratively to have the smallest base circle diameter while having no negative curvature and at least a 10 mm positive curvature. The resulting cam is shown below.

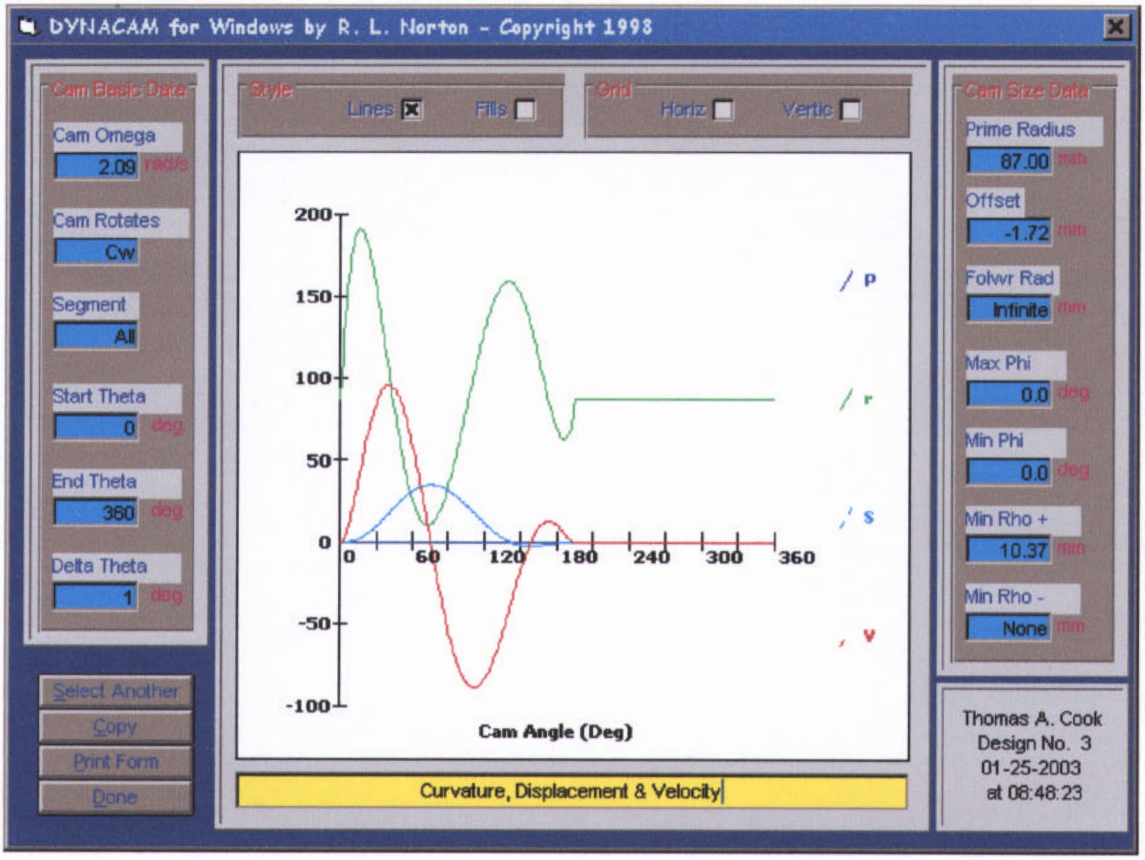


4. The minimum and maximum radius of curvature are shown in the figure above. The design has the following dimensions:

Base circle radius $R_b := 87 \cdot mm$

Follower width $facewidth := 89 \cdot mm$

5. Graphs of ρ, s, and v for the flat-faced follower are shown on the following page.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
2.09 rad/s
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1 deg
Select Another
Copy
Print Form
Done
Style
Lines
Fills
Grid
Horiz
Vertic
200
150
100
50
0
-50
-100
0
60
120
180
240
300
360
Cam Angle (Deg)
p
r
s
v
Curvature, Displacement & Velocity
Cam Size Data
Prime Radius
87.00 mm
Offset
-1.72 mm
Folwr Rad
Infinite mm
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
10.37 mm
Min Rho -
None mm
Thomas A. Cook
Design No. 3
01-25-2003
at 08:48:23

PROBLEM 8-50

Statement: Size the cam from Problem 8-44 for a translating flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 75 \cdot deg$	$\beta_2 := 75 \cdot deg$	$\beta_3 := 75 \cdot deg$	$\beta_4 := 135 \cdot deg$
$h_1 := 50 \cdot mm$	$h_2 := 0 \cdot mm$	$h_3 := 50 \cdot mm$	$h_4 := 0 \cdot mm$

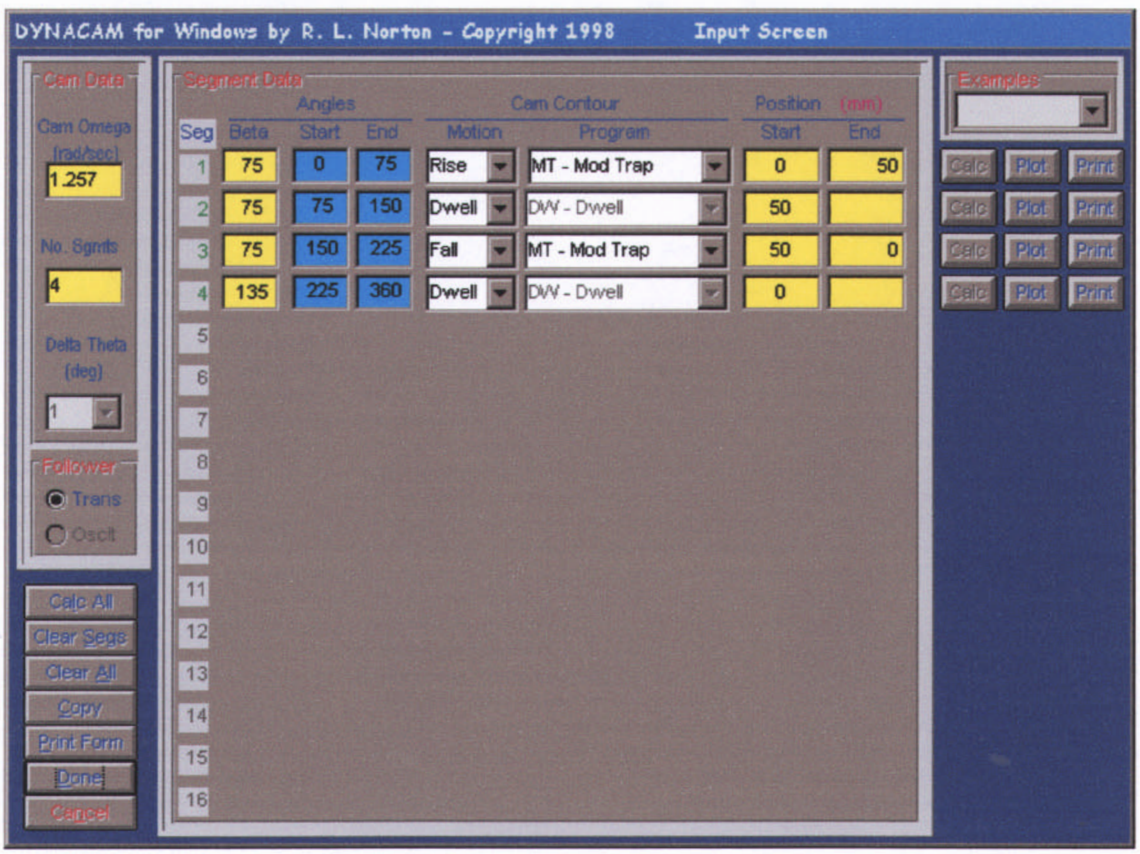
Cycle time: $\tau := 5 \cdot sec$

Solution: See Mathcad file P0850.

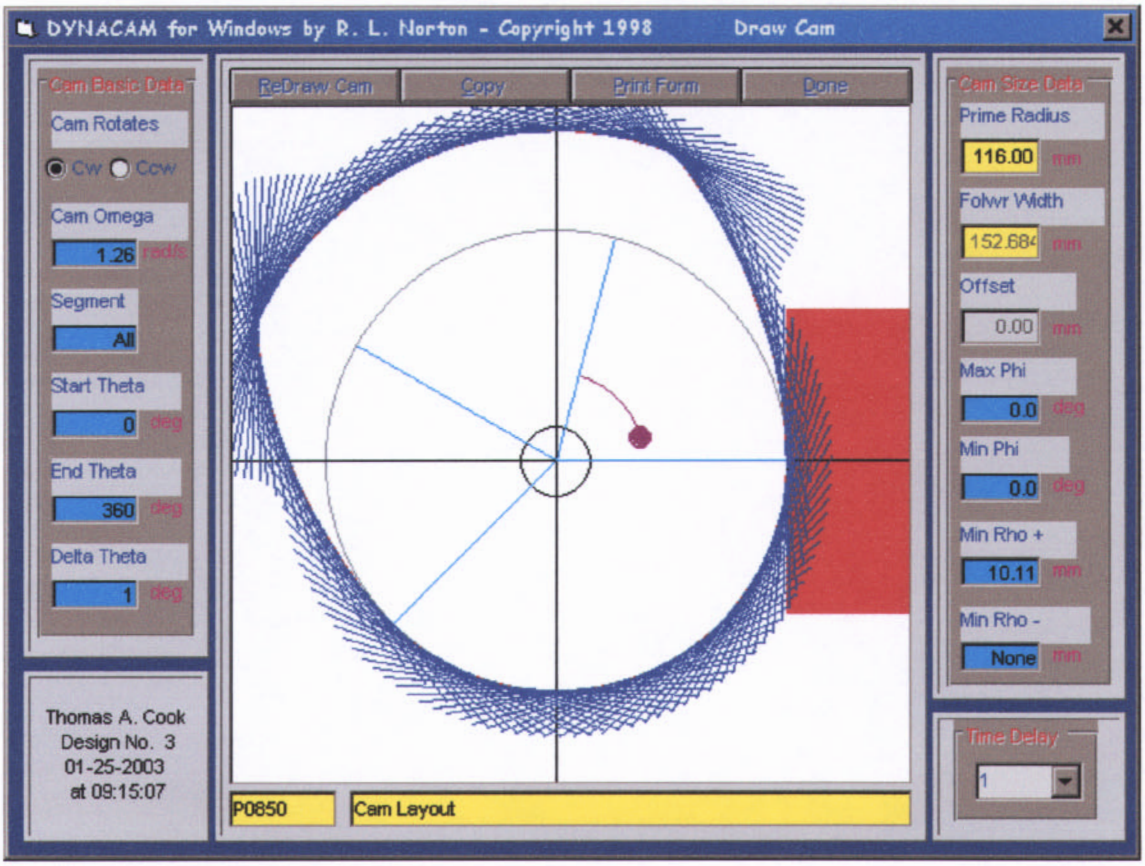
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.257 \frac{rad}{sec}$$

2. Problem 8-44 used modified trapezoidal rise and fall segments with two dwell segments. Enter the above data into program DYNACAM. The input screen is shown below.



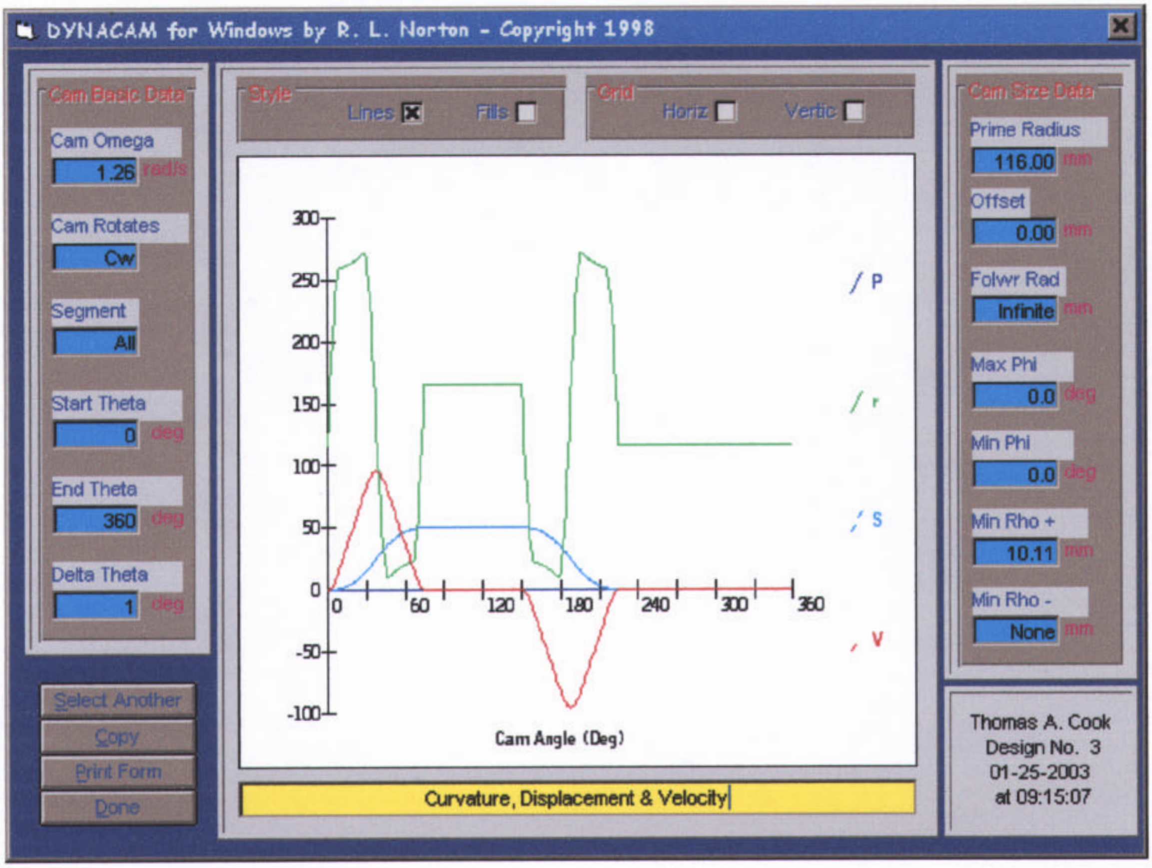
3. The cam was sized iteratively to have the smallest base circle diameter while having no negative curvature and at least a 10 mm positive curvature. The resulting cam is shown below.



4. The minimum and maximum radius of curvature are shown in the figure above. The design has the following dimensions:

Base circle radius	$R_b := 116 \cdot mm$
Follower width	$facewidth := 153 \cdot mm$

5. Graphs of ρ, s, and v for the flat-faced follower are shown on the following page.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
1.26 rad/s
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1 deg
Select Another
Copy
Print Form
Done
Style
Lines
Fills
Grid
Horiz
Vertic
Cam Angle (Deg)
P
r
S
V
Curvature, Displacement & Velocity
Cam Size Data
Prime Radius
116.00 mm
Offset
0.00 mm
Folwr Rad
Infinite mm
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
10.11 mm
Min Rho -
None mm
Thomas A. Cook
Design No. 3
01-25-2003
at 09:15:07

PROBLEM 8-51

Statement: Size the cam from Problem 8-45 for a translating flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 75 \cdot deg$	$\beta_2 := 75 \cdot deg$	$\beta_3 := 75 \cdot deg$	$\beta_4 := 135 \cdot deg$
$h_1 := 50 \cdot mm$	$h_2 := 0 \cdot mm$	$h_3 := 50 \cdot mm$	$h_4 := 0 \cdot mm$

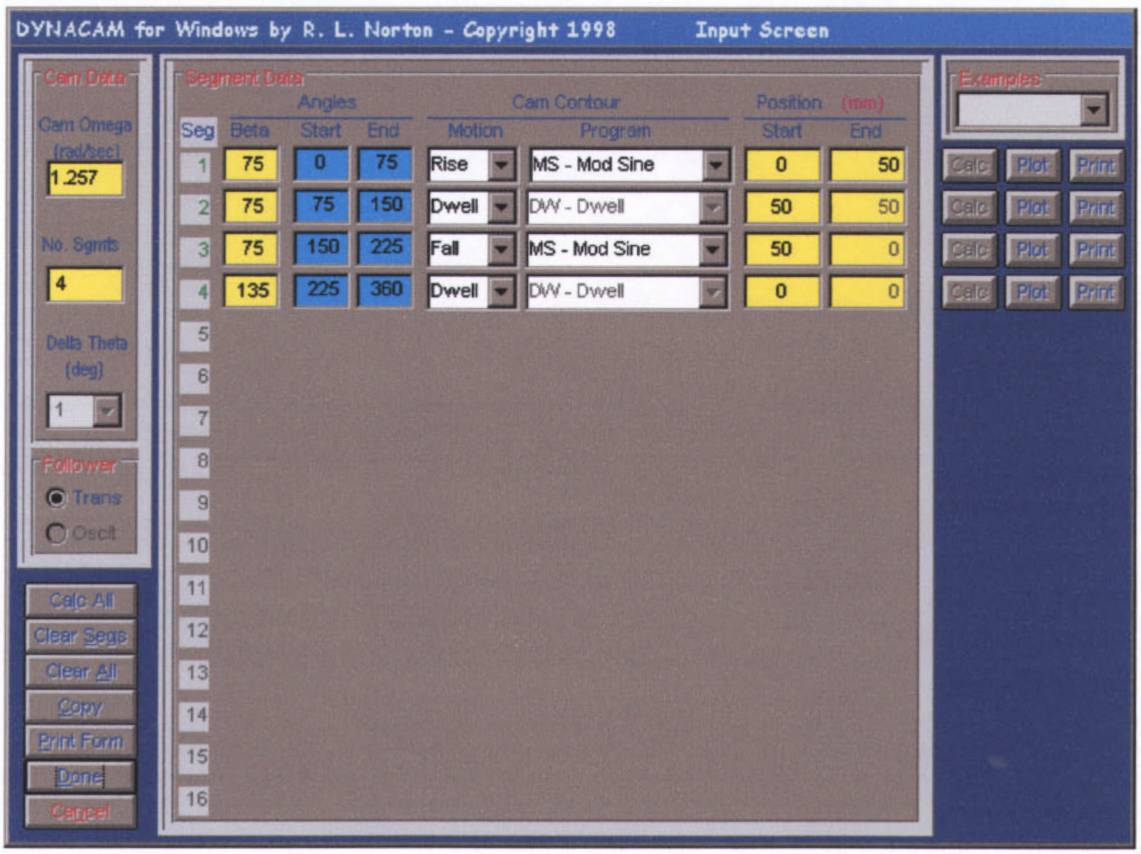
Cycle time: $\tau := 5 \cdot sec$

Solution: See Mathcad file P0851.

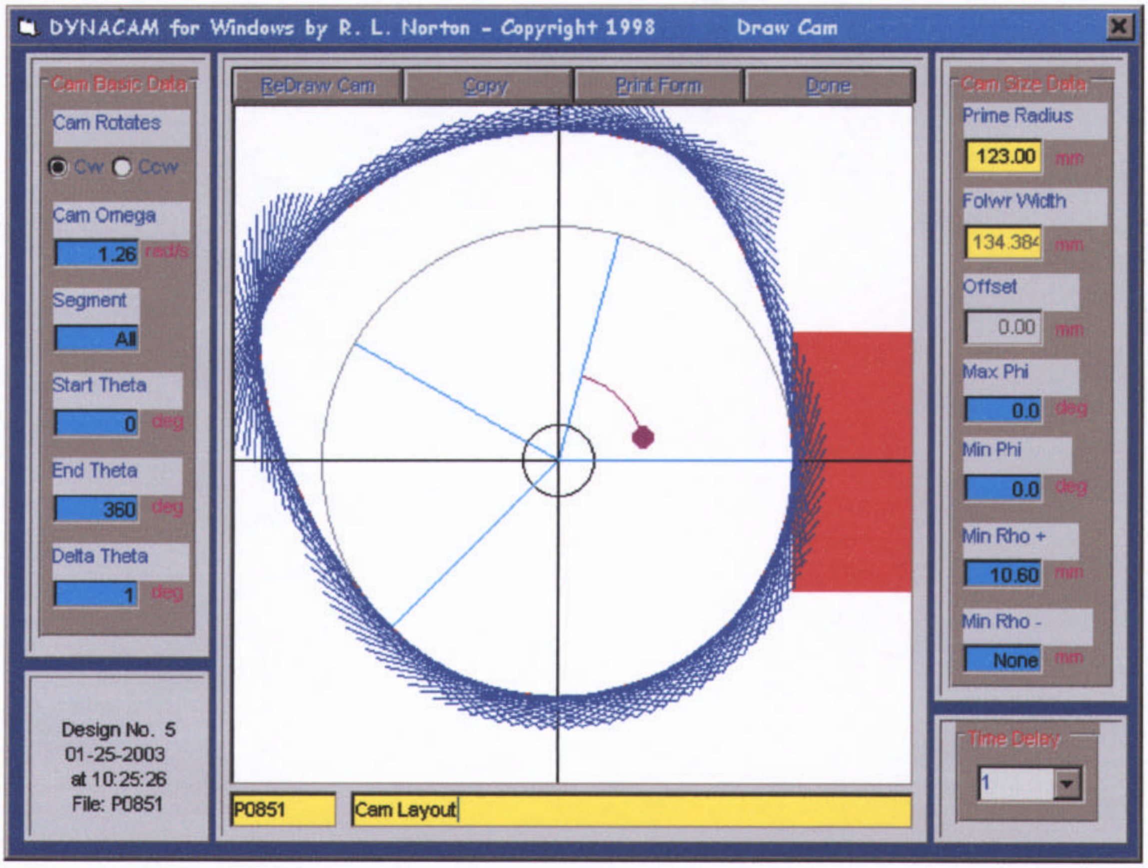
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.257 \frac{rad}{sec}$$

2. Problem 8-45 used modified sinusoidal rise and fall segments with two dwell segments. Enter the above data into program DYNACAM. The input screen is shown below.



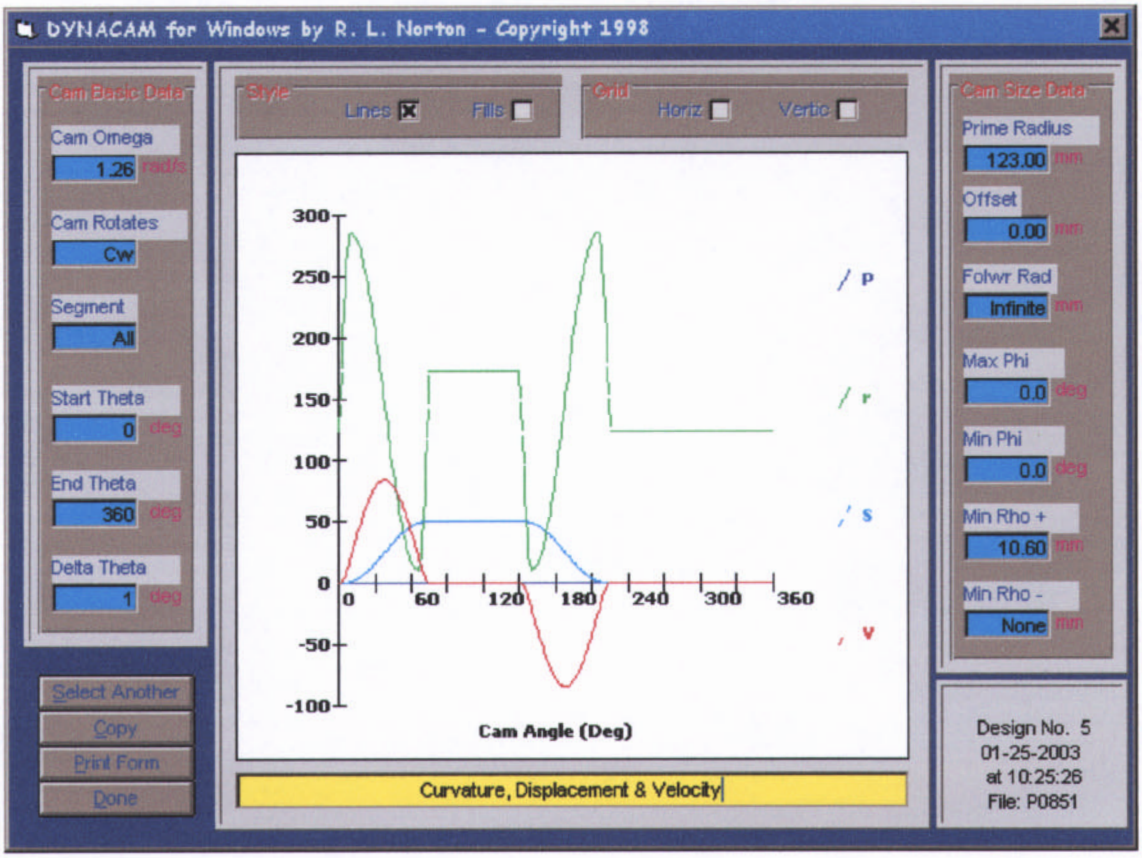
3. The cam was sized iteratively to have the smallest base circle diameter while having no negative curvature and at least a 10 mm positive curvature. The resulting cam is shown below.



4. The minimum and maximum radius of curvature are shown in the figure above. The design has the following dimensions:

Base circle radius	$R_b := 123 \cdot mm$
Follower width	$facewidth := 135 \cdot mm$

5. Graphs of ρ, s, and v for the flat-faced follower are shown on the following page.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
1.26 rad/s
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1 deg
Select Another
Copy
Print Form
Done
Style
Lines
Fills
Grid
Horiz
Vertic
300
250
200
150
100
50
0
-50
-100
0
60
120
180
240
300
360
Cam Angle (Deg)
p
r
s
v
Curvature, Displacement & Velocity
Cam Size Data
Prime Radius
123.00 mm
Offset
0.00 mm
Folwr Rad
Infinite mm
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
10.60 mm
Min Rho -
None mm
Design No. 5
01-25-2003
at 10:25:26
File: P0851

PROBLEM 8-52

Statement: Size the cam from Problem 8-46 for a translating flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

Given:

RISE	DWELL	FALL	DWELL
$\beta_1 := 75\cdot deg$	$\beta_2 := 75\cdot deg$	$\beta_3 := 75\cdot deg$	$\beta_4 := 135\cdot deg$
$h_1 := 50\cdot mm$	$h_2 := 0\cdot mm$	$h_3 := 50\cdot mm$	$h_4 := 0\cdot mm$

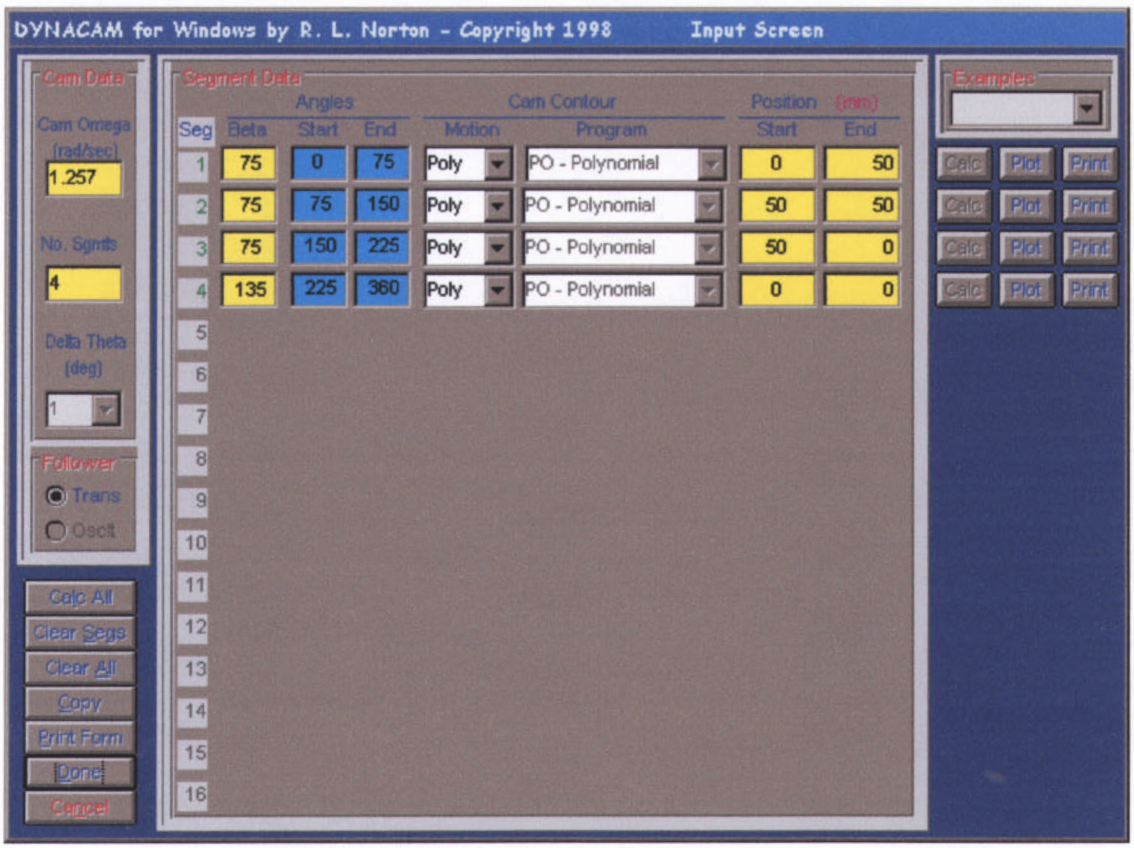
Cycle time: $\tau := 5\cdot sec$

Solution: See Mathcad file P0852.

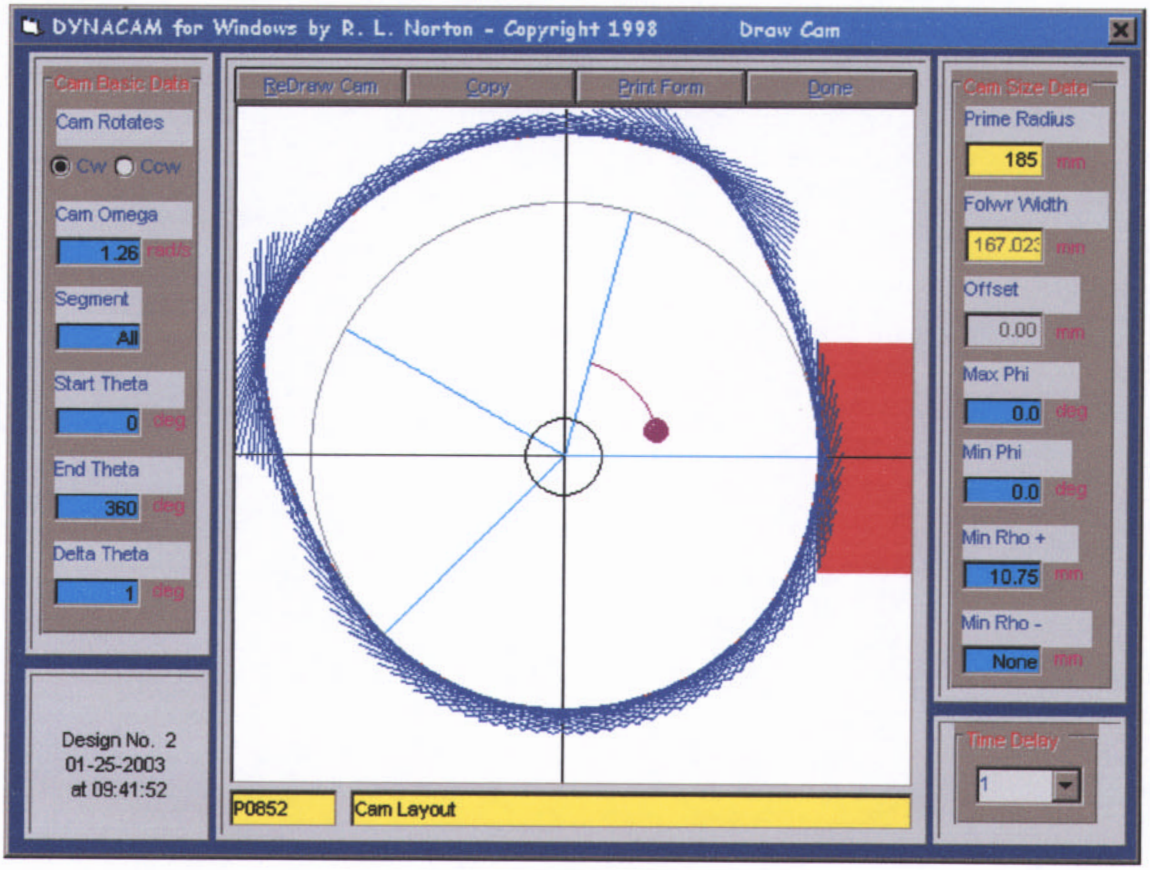
1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2\cdot\pi\cdot rad}{\tau} \qquad \omega = 1.257\,\frac{rad}{sec}$$

2. Problem 8-46 used 4-5-6-7 polynomials for the rise and fall segments with two dwell segments. Enter the above data into program DYNACAM. The input screen is shown below.



3. The cam was sized iteratively to have the smallest base circle diameter while having no negative curvature and at least a 10 mm positive curvature. The resulting cam is shown below.

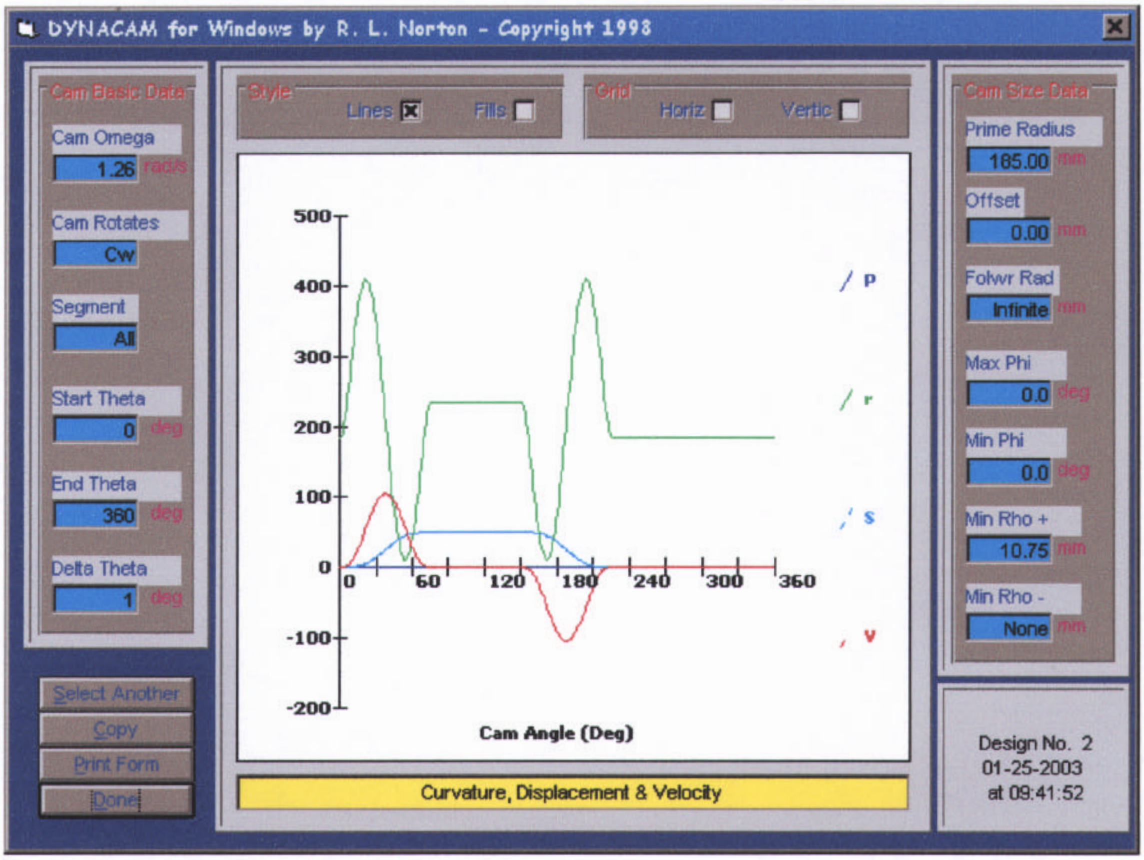


4. The minimum and maximum radius of curvature are shown in the figure above. The design has the following dimensions:

Base circle radius $R_b := 185 \cdot mm$

Follower width $facewidth := 168 \cdot mm$

5. Graphs of ρ, s, and v for the flat-faced follower are shown on the following page.


DYNACAM for Windows by R. L. Norton - Copyright 1998
Cam Basic Data
Cam Omega
1.26 rad/s
Cam Rotates
Cw
Segment
All
Start Theta
0 deg
End Theta
360 deg
Delta Theta
1 deg
Select Another
Copy
Print Form
Done
Style
Lines
Fills
Grid
Horiz
Vertic
500
400
300
200
100
0
-100
-200
0
60
120
180
240
300
360
Cam Angle (Deg)
p
r
s
v
Curvature, Displacement & Velocity
Cam Size Data
Prime Radius
185.00 mm
Offset
0.00 mm
Folwr Rad
Infinite mm
Max Phi
0.0 deg
Min Phi
0.0 deg
Min Rho +
10.75 mm
Min Rho -
None mm
Design No. 2
01-25-2003
at 09:41:52

PROBLEM 8-53

Statement: Design a single-dwell cam to move a follower from 0 to 50 mm in 100 deg, fall 50 mm in 120 deg and dwell for the remainder. The total cycle must take 1 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the *SVAJ* diagrams.

Given:

RISE/FALL	DWELL
$\beta := 220 \cdot deg$	$\beta_3 := 140 \cdot deg$
$h := 50 \cdot mm$	$h_3 := 0.0 \cdot mm$

Cycle time: $\tau := 1 \cdot sec$

Solution: See Mathcad file P0853.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 6.283 \frac{rad}{sec}$$

2. Use a two-segment polynomial. Let the rise and fall, together, be one segment and the dwell be the second segment. Then, the boundary conditions are:

at
$\theta = 0$: $s = 0$, $v = 0$, $a = 0$
$\theta = \beta_1$: $s = h$, $v = 0$
$\theta = \beta$: $s = 0$, $v = 0$, $a = 0$

This is a minimum set of 8 BCs. The v = 0 condition at $\theta = \beta_1$ is required to keep the displacement from overshooting the lift, *h*. Define the total lift, the rise interval, the fall interval, and the ratio of rise to the total interval.

Total lift: $h = 50.000\ mm$

Rise interval: $\beta_1 := 100 \cdot deg$ $\qquad A := \frac{\beta_1}{\beta}$ $\qquad A = 0.455$

Fall interval: $\beta_2 := 120 \cdot deg$

3. Use the 8 BCs and equation 8.23 to write 8 equations in *s*, *v*, and *a* similar to those in example 8-9 but with 8 terms in the equation for *s* (the highest term will be seventh degree).

For $\theta = 0$: $s = v = a = 0$

$$0 := c_0 \qquad 0 := c_1 \qquad 0 := c_2$$

For $\theta = \beta_1$: $s = h$, $v = 0$

$$h := c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5 + c_6 \cdot A^6 + c_7 \cdot A^7$$

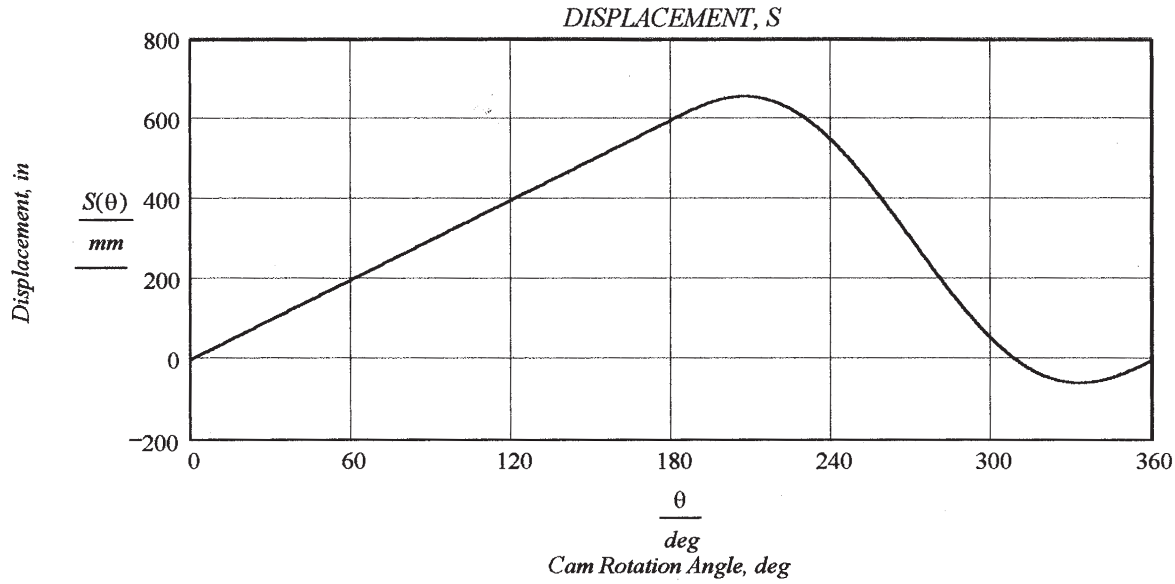
$$0 := 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4 + 6 \cdot c_6 \cdot A^5 + 7 \cdot c_7 \cdot A^6$$

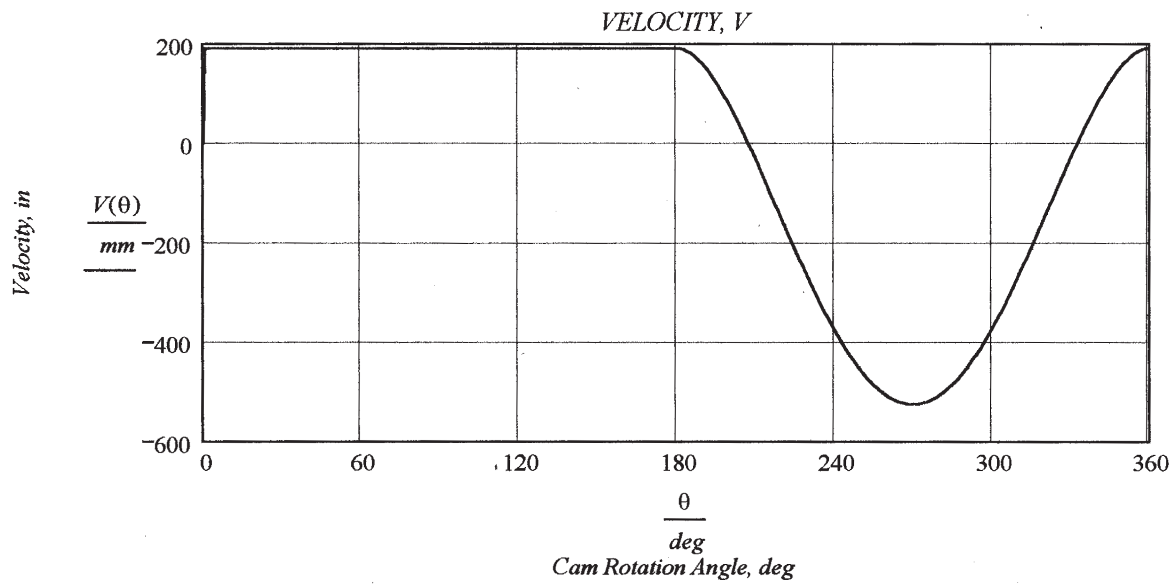
For $\theta = \beta$: $s = v = a = 0$

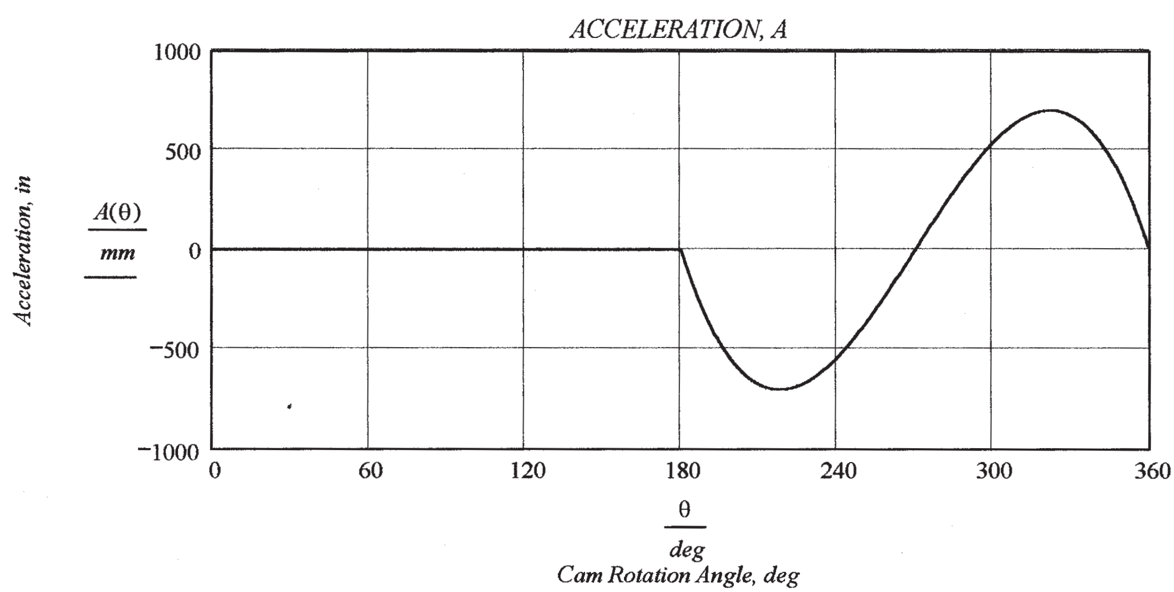
$$0 := c_3 + c_4 + c_5 + c_6 + c_7$$

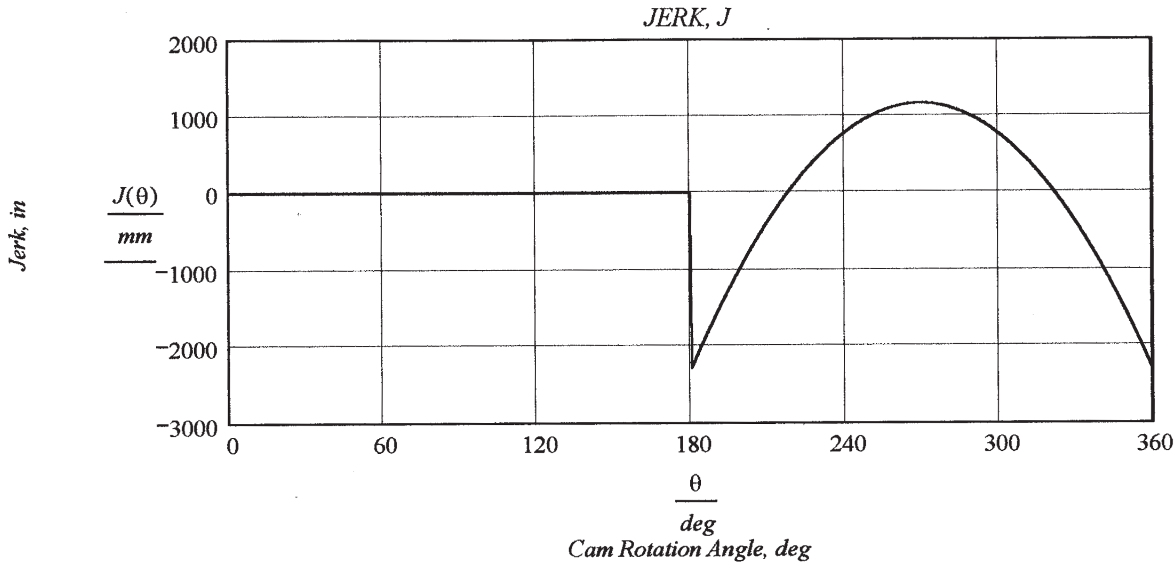
$$0 := 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5 + 6 \cdot c_6 + 7 \cdot c_7$$

$$0 := 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5 + 30 \cdot c_6 + 42 \cdot c_7$$


DISPLACEMENT, S
Displacement, in
S(θ)
mm
800
600
400
200
0
−200
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg


VELOCITY, V
Velocity, in
V(θ)
mm
200
0
−200
−400
−600
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg


ACCELERATION, A
Acceleration, in
A(θ)
mm
1000
500
0
−500
−1000
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg


JERK, J
2000
1000
0
-1000
-2000
-3000
J(θ)
mm
Jerk, in
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg

4. Solve for the unknown polynomial coefficients. Note that C_0 through C_2 are zero

$$C := \begin{pmatrix} A^3 & A^4 & A^5 & A^6 & A^7 \\ 3\cdot A^2 & 4\cdot A^3 & 5\cdot A^4 & 6\cdot A^5 & 7\cdot A^6 \\ 1 & 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 6 & 12 & 20 & 30 & 42 \end{pmatrix} \qquad H := \begin{pmatrix} h \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \end{pmatrix} := C^{-1}\cdot H$$

$c_3 = 193.740\,in$ $\quad c_4 = -723.297\,in$ $\quad c_5 = 1007.449\,in$

$c_6 = -619.969\,in$ $\quad c_7 = 142.076\,in$

5. Write the *SVAJ* equations for the rise/fall segment.

$$S(\theta) := c_3\cdot\left(\frac{\theta}{\beta}\right)^3 + c_4\cdot\left(\frac{\theta}{\beta}\right)^4 + c_5\cdot\left(\frac{\theta}{\beta}\right)^5 + c_6\cdot\left(\frac{\theta}{\beta}\right)^6 + c_7\cdot\left(\frac{\theta}{\beta}\right)^7$$

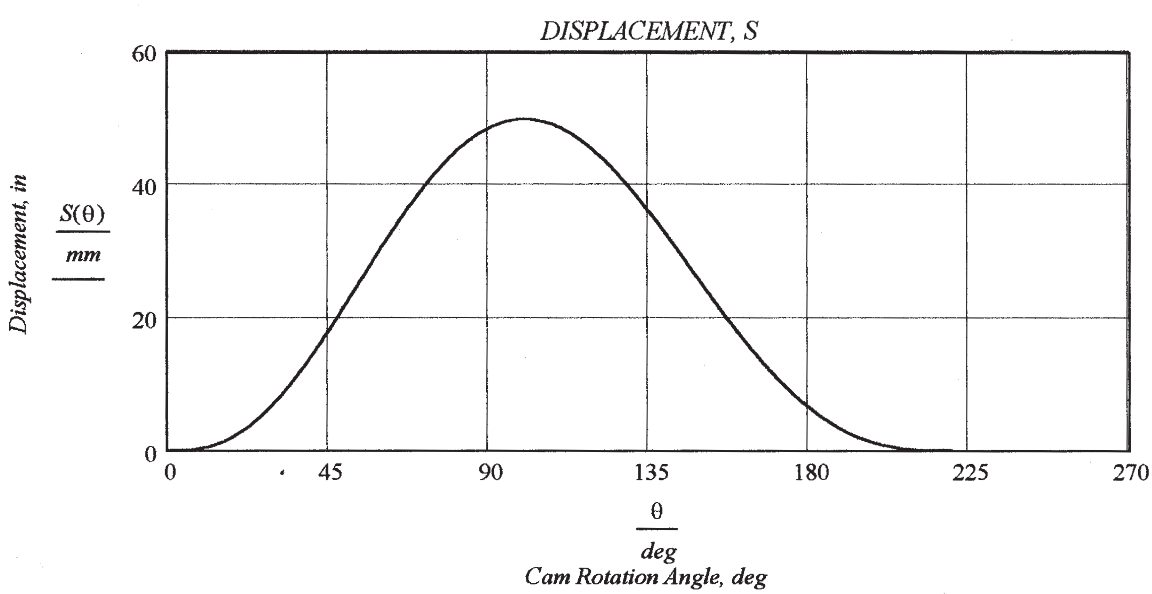
$$V(\theta) := \frac{1}{\beta}\cdot\left[3\cdot c_3\cdot\left(\frac{\theta}{\beta}\right)^2 + 4\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^3 + 5\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^4 + 6\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^5 + 7\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^6\right]$$

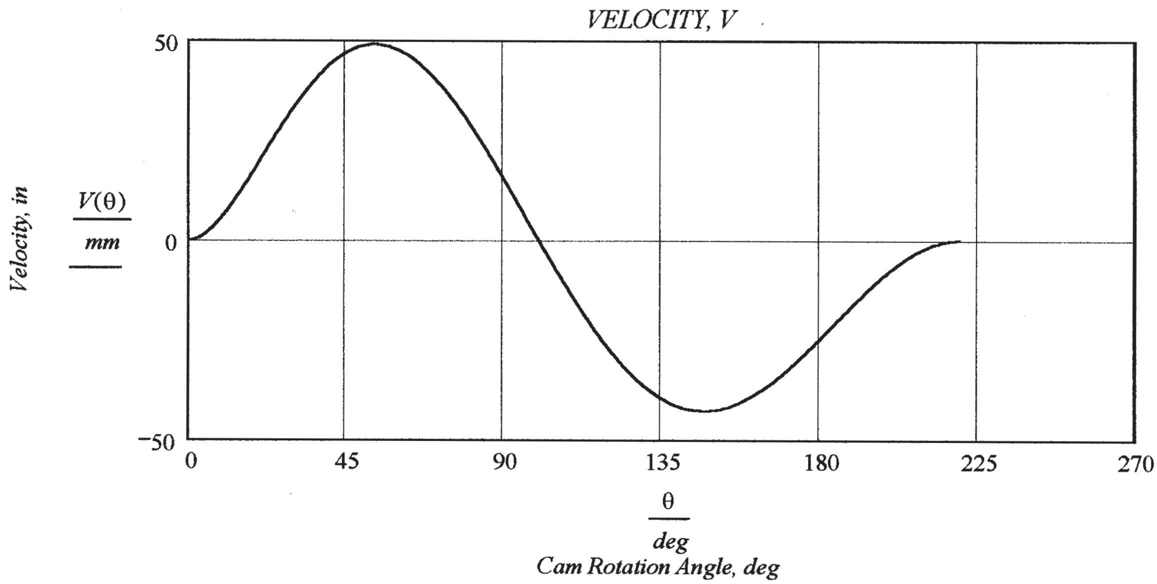
$$A(\theta) := \frac{1}{\beta^2}\cdot\left[6\cdot c_3\cdot\left(\frac{\theta}{\beta}\right) + 12\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^2 + 20\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^3 + 30\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^4 + 42\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^5\right]$$

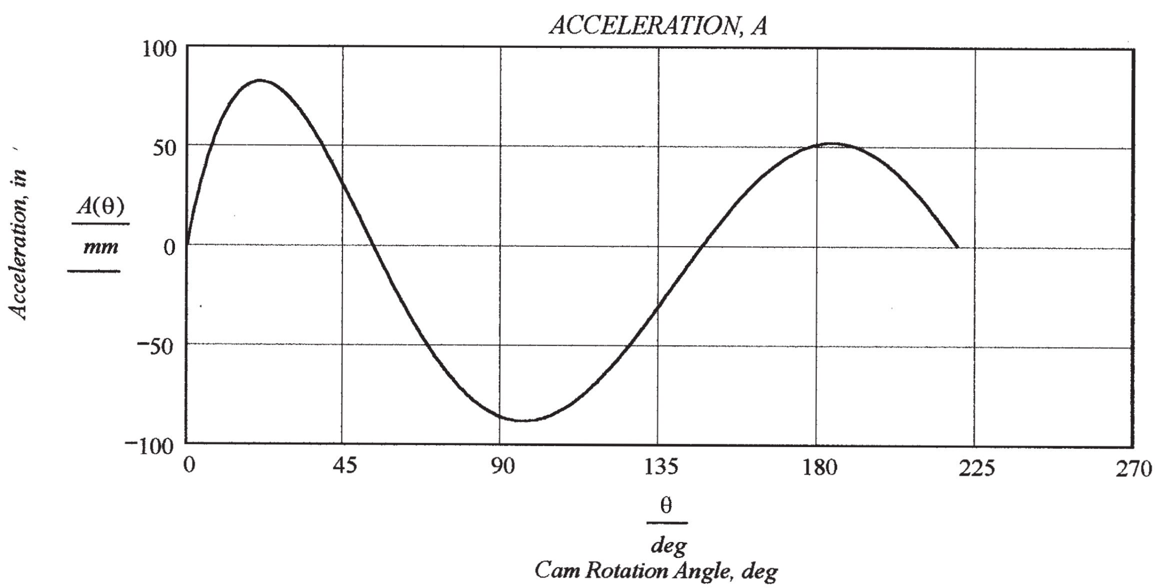
$$J(\theta) := \frac{1}{\beta^3}\cdot\left[6\cdot c_3 + 24\cdot c_4\cdot\left(\frac{\theta}{\beta}\right) + 60\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^2 + 120\cdot c_6\cdot\left(\frac{\theta}{\beta}\right)^3 + 210\cdot c_7\cdot\left(\frac{\theta}{\beta}\right)^4\right]$$

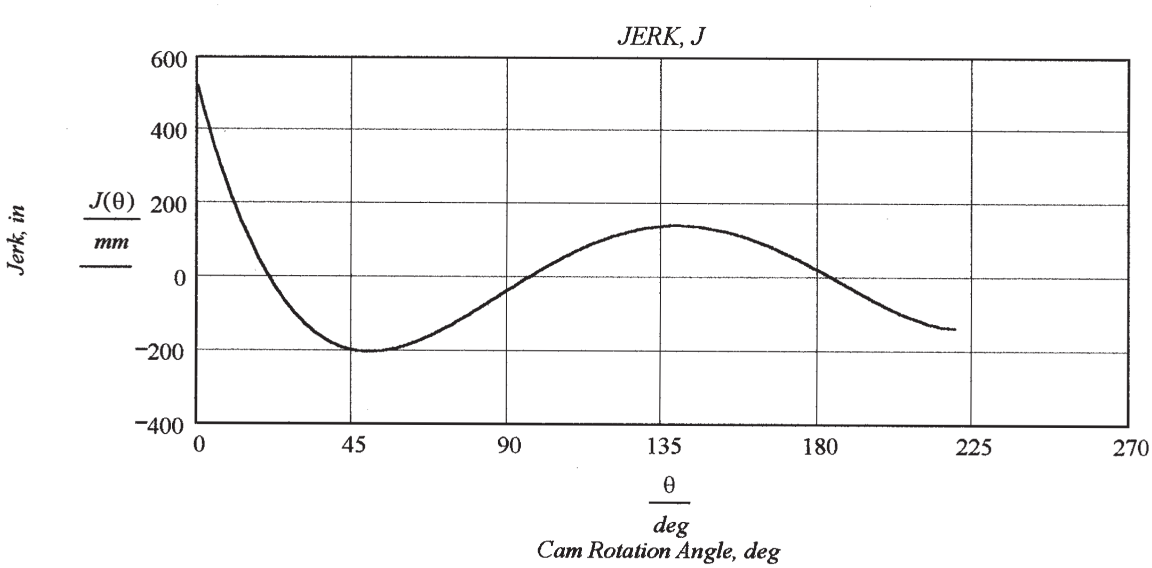
6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 <= \theta <= \beta$.

$\theta := 0\cdot deg, 1\cdot deg .. \beta$




VELOCITY, V
50
0
−50
Velocity, in
V(θ)
mm
0
45
90
135
180
225
270
θ
deg
Cam Rotation Angle, deg


ACCELERATION, A
100
50
0
−50
−100
Acceleration, in
A(θ)
mm
0
45
90
135
180
225
270
θ
deg
Cam Rotation Angle, deg


JERK, J
600
400
200
0
−200
−400
Jerk, in
J(θ)
mm
0
45
90
135
180
225
270
θ
deg
Cam Rotation Angle, deg

PROBLEM 8-54

Statement: Design a cam to move a follower at a constant velocity of 300 mm/sec for 2 sec then return to its starting position with a total cycle time of 4 sec.

Given: Constant velocity: $v_c := 300 \cdot mm \cdot sec^{-1}$

Time duration of cv segment: $t_{cv} := 2 \cdot sec$ Cycle time: $\tau := 4 \cdot sec$

Solution: See Mathcad file P0854.

1. The camshaft turns 2π rad during the time for one cycle. Thus, its speed is

$$\omega := \frac{2 \cdot \pi \cdot rad}{\tau} \qquad \omega = 1.571 \frac{rad}{sec}$$

2. Use a two-segment polynomial as demonstrated in Example 8-12. The lift during the first segment and the *svaj* equations for the first segment are:

Normalized velocity: $v_{cv} := \frac{v_c}{\omega}$ $\quad v_{cv} = 190.986\,mm$

$$h_{cv} := v_c \cdot t_{cv} \qquad h_{cv} = 600.000\,mm \qquad \beta_1 := \frac{t_{cv}}{\tau} \cdot 360 \cdot deg \qquad \beta_1 = 180\,deg$$

$$s_1(\theta) := h_{cv} \cdot \frac{\theta}{\beta_1} \qquad v_1(\theta) := v_{cv} \qquad a_1(\theta) := 0 \cdot mm \qquad j_1(\theta) := 0 \cdot mm$$

2. The boundary conditions for the second segment are:

at $\theta = \beta_1$: $\quad s = h_{cv}, \quad v = v_{cv} \quad a = 0$

$\theta = 360$ deg: $\quad s = 0, \quad v = v_{cv}, \quad a = 0$

This is a minimum set of 6 BCs. Define the total interval and the constant velocity interval, and the ratio of constant velocity interval to the total interval.

Total interval: $\beta := 360 \cdot deg$

CV interval: $\beta_1 = 180\,deg$ $\qquad A := \frac{\beta_1}{\beta}$ $\qquad A = 0.500$

3. Use the 6 BCs and equation 8.23 to write 6 equations in *s*, *v*, and *a* similar to those in example 8-9 but with 6 terms in the equation for *s* (the highest term will be fifth degree).

For $\theta = \beta 1$: $\quad s = h_{cv}, \quad v = v_{cv} \quad a = 0$

$$h_{cv} = c_0 + c_1 \cdot A + c_2 \cdot A^2 + c_3 \cdot A^3 + c_4 \cdot A^4 + c_5 \cdot A^5$$

$$v_{cv} = \frac{1}{\beta} \cdot \left(c_1 + 2 \cdot c_2 \cdot A + 3 \cdot c_3 \cdot A^2 + 4 \cdot c_4 \cdot A^3 + 5 \cdot c_5 \cdot A^4\right)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 \cdot A + 12 \cdot c_4 \cdot A^2 + 20 \cdot c_5 \cdot A^3$$

For $\theta = \beta$: $\quad s = 0, \quad v = v_{cv}, \quad a = 0$

$$0 = c_0 + c_1 + c_2 + c_3 + c_4 + c_5$$

$$0 = \frac{1}{\beta} \cdot \left(c_1 + 2 \cdot c_2 + 3 \cdot c_3 + 4 \cdot c_4 + 5 \cdot c_5\right)$$

$$0 = 2 \cdot c_2 + 6 \cdot c_3 + 12 \cdot c_4 + 20 \cdot c_5$$

4. Solve for the unknown polynomial coefficients.

$$C := \begin{pmatrix} 1 & A & A^2 & A^3 & A^4 & A^5 \\ 0 & 1 & 2\cdot A & 3\cdot A^2 & 4\cdot A^3 & 5\cdot A^4 \\ 0 & 0 & 2 & 6\cdot A & 12\cdot A^2 & 20\cdot A^3 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 2 & 6 & 12 & 20 \end{pmatrix} \qquad H := \begin{pmatrix} h_{cv} \\ \beta\cdot v_{cv} \\ 0 \\ 0 \\ \beta\cdot v_{cv} \\ 0 \end{pmatrix} \qquad \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{pmatrix} := C^{-1}\cdot H$$

$c_0 = 3.720 \times 10^4\ mm$ $\quad c_1 = -2.868 \times 10^5\ mm$ $\quad c_2 = 8.640 \times 10^5\ mm$

$c_3 = -1.248 \times 10^6\ mm$ $\quad c_4 = 8.64 \times 10^5\ mm$ $\quad c_5 = -2.304 \times 10^5\ mm$

5. Write the *svaj* equations for the second segment.

$$s_2(\theta) := c_0 + c_1\cdot\left(\frac{\theta}{\beta}\right) + c_2\cdot\left(\frac{\theta}{\beta}\right)^2 + c_3\cdot\left(\frac{\theta}{\beta}\right)^3 + c_4\cdot\left(\frac{\theta}{\beta}\right)^4 + c_5\cdot\left(\frac{\theta}{\beta}\right)^5$$

$$v_2(\theta) := \frac{1}{\beta}\cdot\left[c_1 + 2\cdot c_2\cdot\left(\frac{\theta}{\beta}\right) + 3\cdot c_3\cdot\left(\frac{\theta}{\beta}\right)^2 + 4\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^3 + 5\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^4\right]$$

$$a_2(\theta) := \frac{1}{\beta^2}\cdot\left[2\cdot c_2 + 6\cdot c_3\cdot\left(\frac{\theta}{\beta}\right) + 12\cdot c_4\cdot\left(\frac{\theta}{\beta}\right)^2 + 20\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^3\right]$$

$$j_2(\theta) := \frac{1}{\beta^3}\cdot\left[6\cdot c_3 + 24\cdot c_4\cdot\left(\frac{\theta}{\beta}\right) + 60\cdot c_5\cdot\left(\frac{\theta}{\beta}\right)^2\right]$$

4. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of *a* and *b* and zero elsewhere.

$$R(\theta,a,b) := if[(\theta > a) \wedge (\theta \le b),1,0]$$

$$S(\theta) := R(\theta,0,\beta_1)\cdot s_1(\theta) + R(\theta,\beta_1,\beta)\cdot s_2(\theta)$$

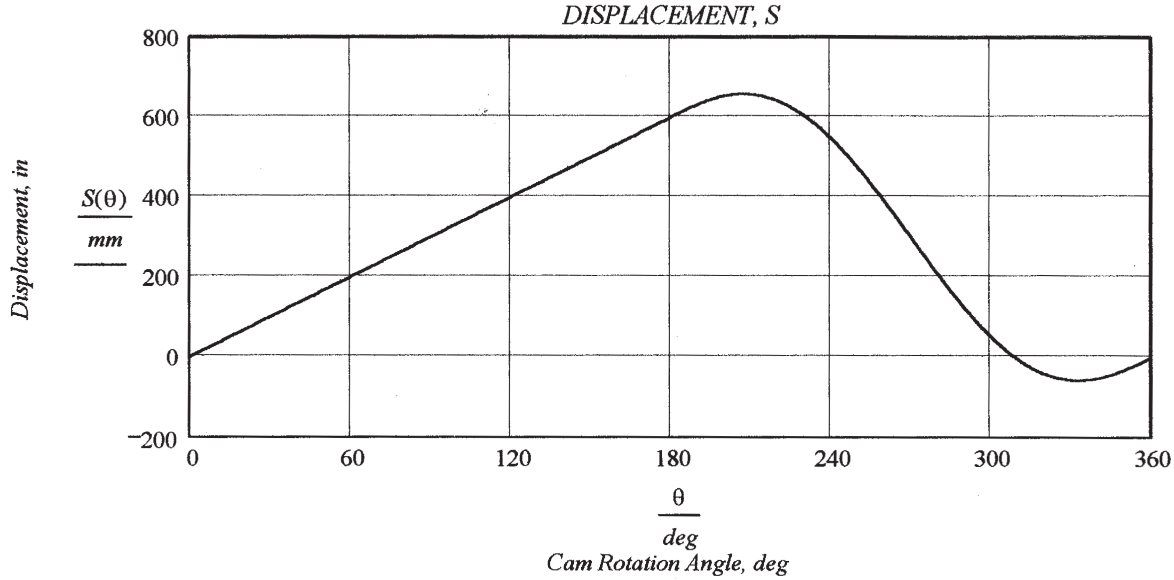
$$V(\theta) := R(\theta,0,\beta_1)\cdot v_1(\theta) + R(\theta,\beta_1,\beta)\cdot v_2(\theta)$$

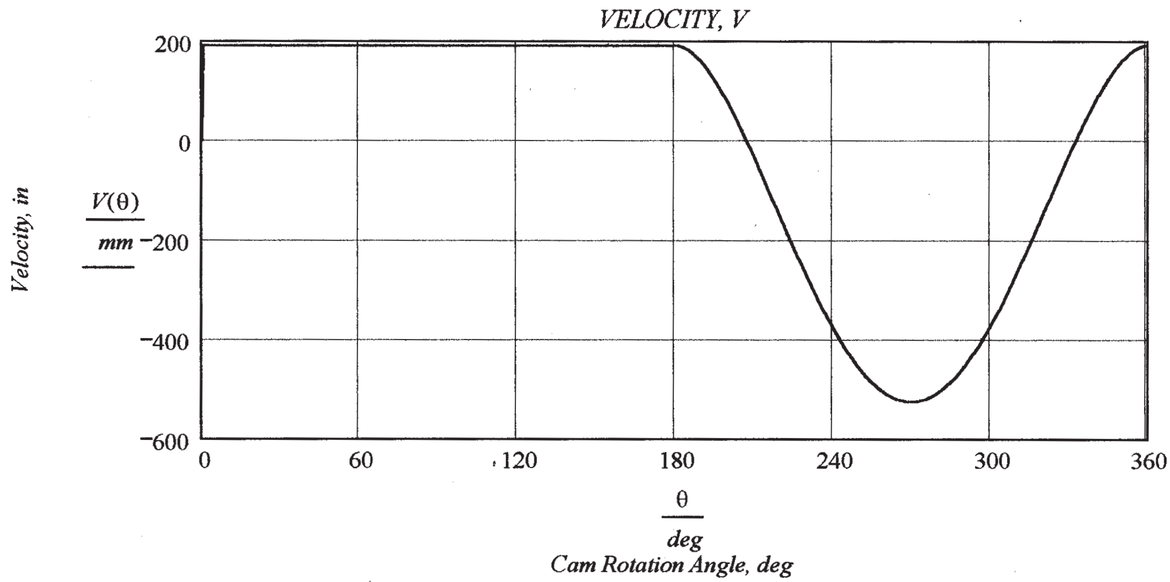
$$A(\theta) := R(\theta,0,\beta_1)\cdot a_1(\theta) + R(\theta,\beta_1,\beta)\cdot a_2(\theta)$$

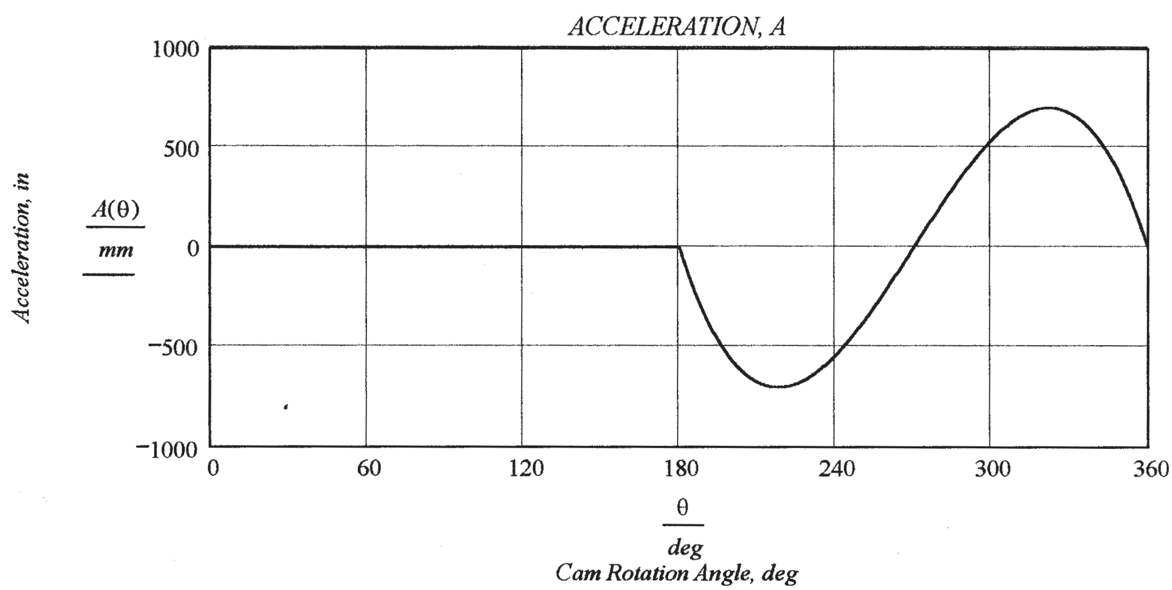
$$J(\theta) := R(\theta,0,\beta_1)\cdot j_1(\theta) + R(\theta,\beta_1,\beta)\cdot j_2(\theta)$$

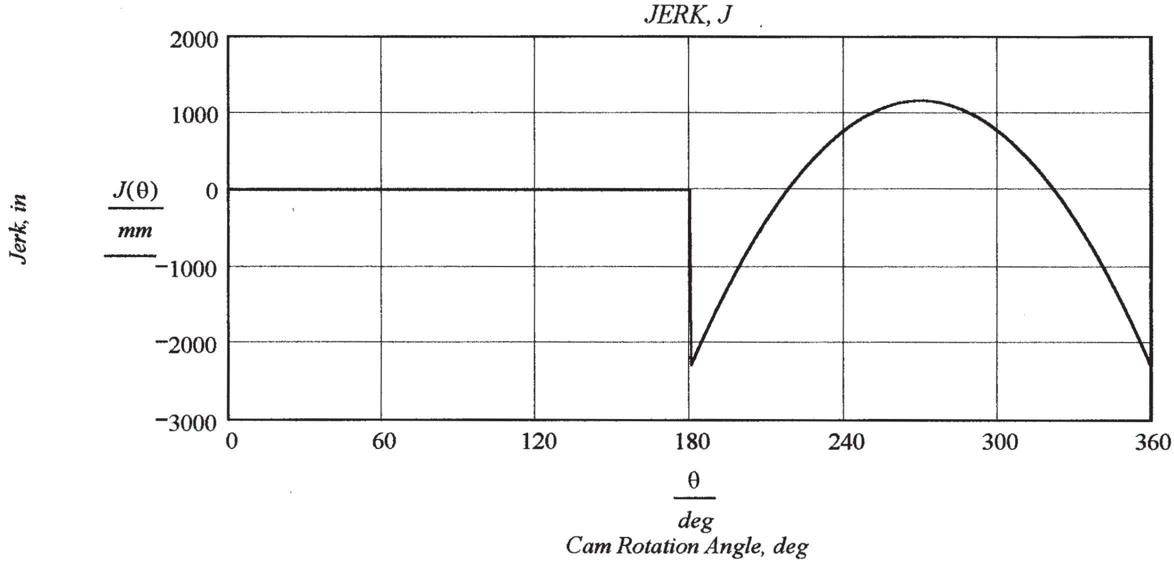
6. Plot the displacement, velocity, acceleration, and jerk over the interval $0 <= \theta <= \beta$.

$$\theta := 0\cdot deg, 0.5\cdot deg .. \beta$$


DISPLACEMENT, S
800
600
400
200
0
−200
Displacement, in
S(θ)
mm
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg


VELOCITY, V
200
0
−200
−400
−600
Velocity, in
V(θ)
mm
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg


ACCELERATION, A
1000
500
0
−500
−1000
Acceleration, in
A(θ)
mm
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg


JERK, J
2000
1000
0
−1000
−2000
−3000
Jerk, in
J(θ)
mm
0
60
120
180
240
300
360
θ
deg
Cam Rotation Angle, deg

for $(1 + d)/2 <= x <= 1 - b/2$

$$y_4(x) := C_a \cdot \left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2} \right) \cdot x + \left(2 \cdot d^2 - b^2 \right) \cdot \left(\frac{1}{\pi^2} - \frac{1}{8} \right) - \frac{1}{4} \right]$$

$$y'_4(x) := C_a \cdot \left(-x + \frac{b}{\pi} + 1 - \frac{b}{2} \right) \qquad y''_4(x) := -C_a \qquad y'''_4(x) := 0$$

for $1 - b/2 <= x <= 1$

$$y_5(x) := C_a \cdot \left[\frac{b}{\pi} \cdot x + \frac{2 \cdot \left(d^2 - b^2 \right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi} \right)^2 \cdot \sin\left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y'_5(x) := C_a \cdot \frac{b}{\pi} \cdot \left[1 - \cos\left[\frac{\pi}{b} \cdot (x - 1) \right] \right]$$

$$y''_5(x) := C_a \cdot \sin\left[\frac{\pi}{b} \cdot (x - 1) \right] \qquad y'''_5(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left[\frac{\pi}{b} \cdot (x - 1) \right]$$

2. The above equations can be used for a rise or fall by using the proper values of θ, β, and h. To plot the *SVAJ* curves, first define a range function that has a value of one between the values of x_1 and x_2 and zero elsewhere.

$$R(x,x1,x2) := if[(x > x1) \wedge (x \le x2), 1, 0]$$

3. The global *SVAJ* equations are composed of four intervals (rise, dwell, fall, and dwell). The local equations above must be assembled into a single equation each for *S*, *V*, *A*, and *J* that applies over the range 0 <= θ <= 360 deg.

4. Write the local *svaj* equations for the first interval, $0 <= \theta <= \beta_1$. Note that each subinterval function is multiplied by the range function so that it will have nonzero values only over its subinterval.

For $0 <= \theta <= \beta_1$ (Rise)

$$s_1(x) := h \cdot \left(R\left(x,0,\frac{b}{2}\right) \cdot y_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right) \cdot y_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right) \cdot y_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right) \cdot y_4(x) + R\left(x,1-\frac{b}{2},1\right) \cdot y_5(x) \right)$$

$$v_1(x) := \frac{h}{\beta_1} \cdot \left(R\left(x,0,\frac{b}{2}\right) \cdot y'_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right) \cdot y'_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right) \cdot y'_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right) \cdot y'_4(x) + R\left(x,1-\frac{b}{2},1\right) \cdot y'_5(x) \right)$$

$$a_1(x) := \frac{h}{\beta_1^2} \cdot \left(R\left(x,0,\frac{b}{2}\right) \cdot y''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right) \cdot y''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right) \cdot y''_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right) \cdot y''_4(x) + R\left(x,1-\frac{b}{2},1\right) \cdot y''_5(x) \right)$$

$$j_1(x) := \frac{h}{\beta_1^3} \cdot \left(R\left(x,0,\frac{b}{2}\right) \cdot y'''_1(x) + R\left(x,\frac{b}{2},\frac{1-d}{2}\right) \cdot y'''_2(x) + R\left(x,\frac{1-d}{2},\frac{1+d}{2}\right) \cdot y'''_3(x) \ldots \right.$$
$$\left. + R\left(x,\frac{1+d}{2},1-\frac{b}{2}\right) \cdot y'''_4(x) + R\left(x,1-\frac{b}{2},1\right) \cdot y'''_5(x) \right)$$

5. Write the local *svaj* equations for the second interval, $\beta_1 <= \theta <= \beta_1 + \beta_2$. For this interval, the value of *S* is the value of *S* at the end of the previous interval and the values of *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 <= \theta <= \beta_1 + \beta_2$

$$s_2(x) := h \qquad v_2(x) := 0 \qquad a_2(x) := 0 \qquad j_2(x) := 0$$

6. Write the local *svaj* equations for the third interval, $\beta_1 + \beta_2 <= \theta <= \beta_1 + \beta_2 + \beta_3$.

For $\beta_1 + \beta_2 <= \theta <= \beta_1 + \beta_2 + \beta_3$

$$s_3(x) := h\left[1 - \left(R\left(x, 0, \frac{b}{2}\right)\cdot y_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y_5(x)\right)\right]$$

$$v_3(x) := \frac{h}{\beta_3}\cdot\left(R\left(x, 0, \frac{b}{2}\right)\cdot y'_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y'_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y'_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y'_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y'_5(x)\right)$$

$$a_3(x) := \frac{h}{\beta_3^{\,2}}\cdot\left(R\left(x, 0, \frac{b}{2}\right)\cdot y''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y''_5(x)\right)$$

$$j_3(x) := \frac{h}{\beta_3^{\,3}}\cdot\left(R\left(x, 0, \frac{b}{2}\right)\cdot y'''_1(x) + R\left(x, \frac{b}{2}, \frac{1-d}{2}\right)\cdot y'''_2(x) + R\left(x, \frac{1-d}{2}, \frac{1+d}{2}\right)\cdot y'''_3(x) \ldots \atop + R\left(x, \frac{1+d}{2}, 1 - \frac{b}{2}\right)\cdot y'''_4(x) + R\left(x, 1 - \frac{b}{2}, 1\right)\cdot y'''_5(x)\right)$$

7. Write the local *svaj* equations for the fourth interval, $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4$. For this interval, the values of *S*, *V*, *A*, and *J* are zero because of the dwell.

For $\beta_1 + \beta_2 + \beta_3 <= \theta <= \beta_1 + \beta_2 + \beta_3 + \beta_4$

$$s_4(x) := 0 \qquad v_4(x) := 0 \qquad a_4(x) := 0 \qquad j_4(x) := 0$$

8. Write the complete global equation for the displacement and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

Let $\theta_1 := \beta_1 \qquad \theta_2 := \theta_1 + \beta_2 \qquad \theta_3 := \theta_2 + \beta_3 \qquad \theta_4 := \theta_3 + \beta_4$

$$S(\theta) := s_1\left(\frac{\theta}{\theta_1}\right) + R(\theta, \theta_1, \theta_2)\cdot s_2\left(\frac{\theta - \theta_1}{\theta_2 - \theta_1}\right) \ldots$$
$$+ R(\theta, \theta_2, \theta_3)\cdot s_3\left(\frac{\theta - \theta_2}{\theta_3 - \theta_2}\right) + R(\theta, \theta_3, \theta_4)\cdot s_4\left(\frac{\theta - \theta_3}{\theta_4 - \theta_3}\right)$$

$$\theta := 0\cdot deg, 0.5\cdot deg .. 360\cdot deg$$

PROBLEM 8-55

Statement: Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot *svaj* diagrams for the family of SCCA cam functions for any specified values of lift and duration. It should allow the user to change the values of the SCCA parameters b, c, d, and C_a to generate and plot any member of the family. Test it using cycloidal motion with a lift of 100 mm in 100 deg, a dwell of 80 deg, return to zero in 120 deg, and dwell for the remainder of the cycle at 1 rad/sec.

Input:

RISE	DWELL	FALL	DWELL
$\beta_1 := 100 \cdot deg$	$\beta_2 := 80 \cdot deg$	$\beta_3 := 120 \cdot deg$	$\beta_4 := 60 \cdot deg$

Total rise: $h := 100 \cdot mm$ Cam rotational velocity: $\omega := 1 \cdot rad \cdot sec^{-1}$

The numerical constants for the SCCA equations are given in Table 8-2 and listed below. Enter values of b, c, d, and C_a for the motion type that you desire.

Function	b	c	d	C_a
Modified trapezoidal	0.25	0.50	0.25	4.8881
Modified sine	0.25	0.00	0.75	5.5280
Cycloidal	0.50	0.00	0.50	6.2832

Enter values: $b := 0.50$ $c := 0.00$ $d := 0.50$ $C_a := 6.2832$

Program: See Mathcad file P0855.

1. The SCCA equations for the rise or fall interval (β) are divided into 5 subintervals. These are:

for $0 <= x <= b/2$ where, for these equations, x is a local coordinate that ranges from 0 to 1,

$$y_1(x) := C_a \cdot \left[x \cdot \frac{b}{\pi} - \left(\frac{b}{\pi}\right)^2 \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \right] \qquad y'_1(x) := C_a \cdot \frac{b}{\pi} \cdot \left(1 - \cos\left(\frac{\pi}{b} \cdot x\right)\right)$$

$$y''_1(x) := C_a \cdot \sin\left(\frac{\pi}{b} \cdot x\right) \qquad y'''_1(x) := C_a \cdot \frac{\pi}{b} \cdot \cos\left(\frac{\pi}{b} \cdot x\right)$$

for $b/2 <= x <= (1 - d)/2$

$$y_2(x) := C_a \cdot \left[\frac{x^2}{2} + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \cdot x + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) \right] \qquad y'_2(x) := C_a \cdot \left[x + b \cdot \left(\frac{1}{\pi} - \frac{1}{2}\right) \right]$$

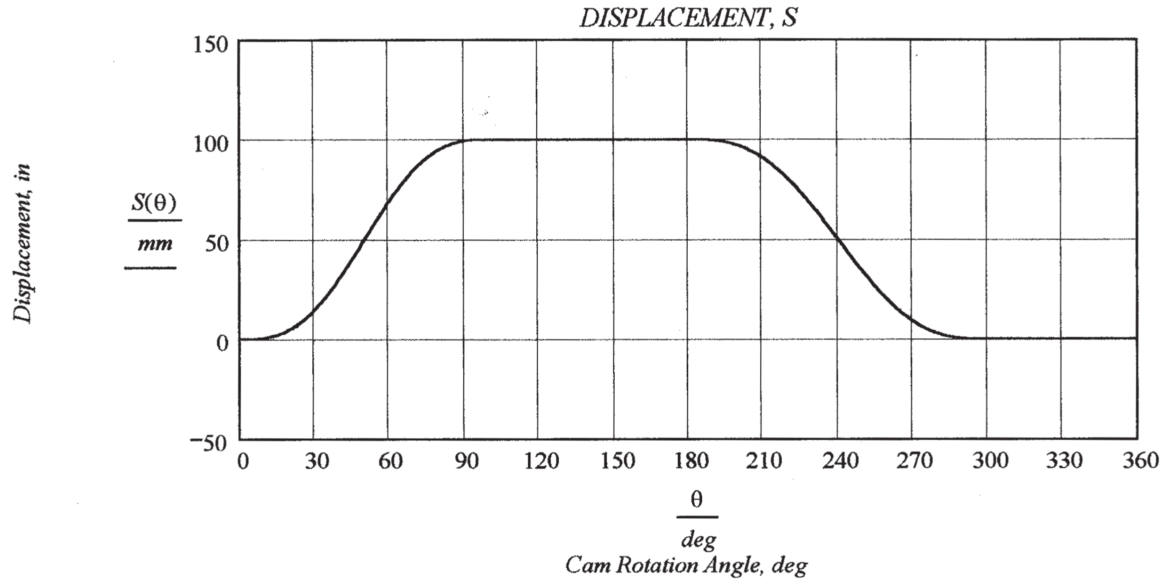
$$y''_2(x) := C_a \qquad y'''_2(x) := 0$$

for $(1 - d)/2 <= x <= (1 + d)/2$

$$y_3(x) := C_a \cdot \left[\left(\frac{b}{\pi} + \frac{c}{2}\right) \cdot x + \left(\frac{d}{\pi}\right)^2 + b^2 \cdot \left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

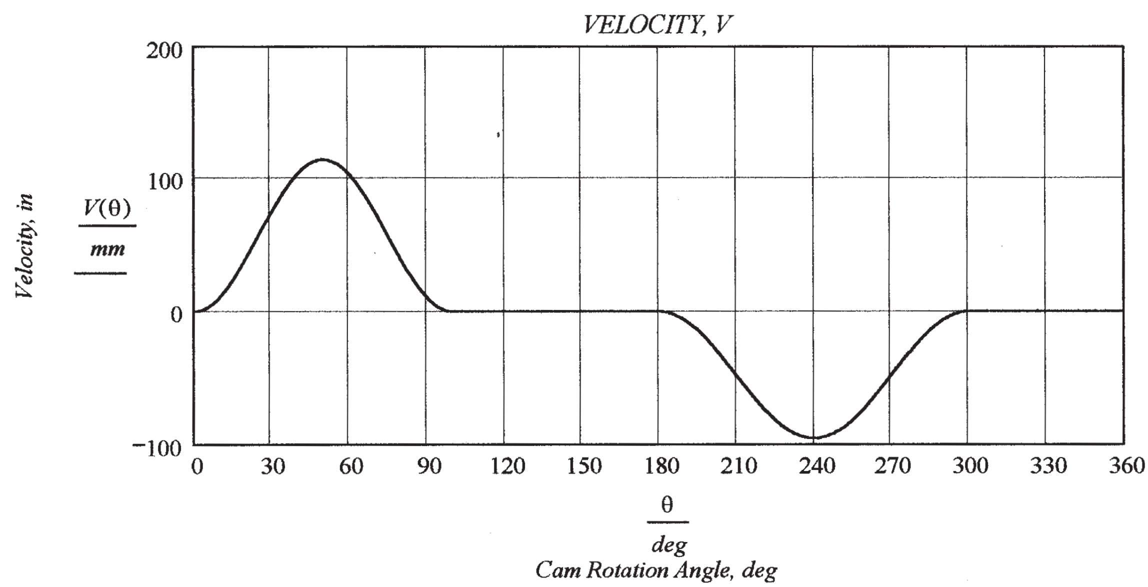
$$y'_3(x) := C_a \cdot \left[\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \right]$$

$$y''_3(x) := C_a \cdot \cos\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right] \qquad y'''_3(x) := -C_a \cdot \frac{\pi}{d} \cdot \sin\left[\frac{\pi}{d} \cdot \left(x - \frac{1-d}{2}\right)\right]$$



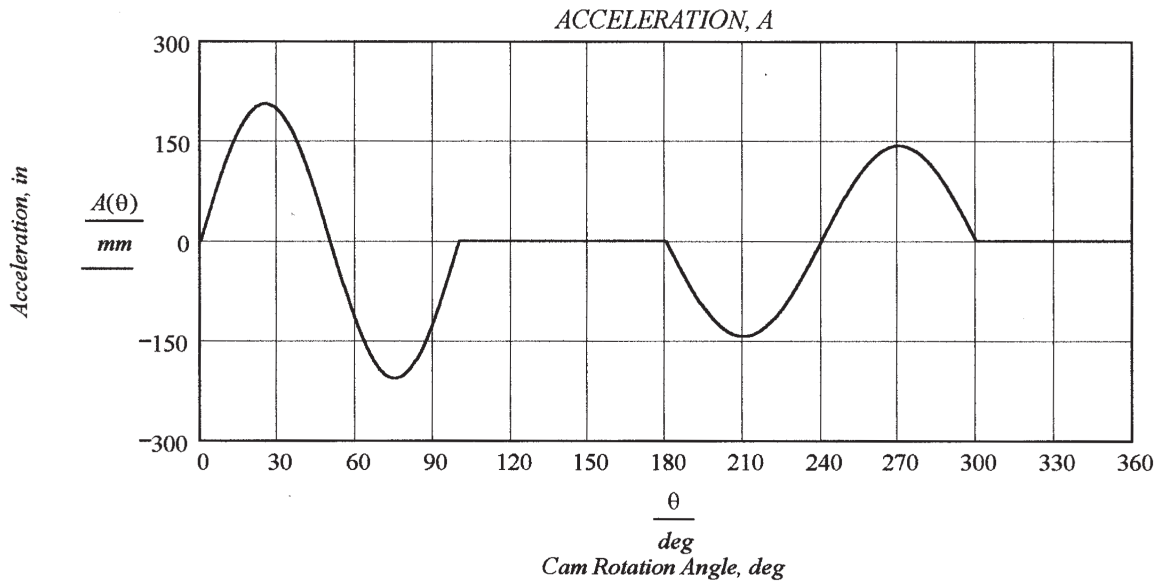
11. Write the complete global equation for the velocity and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$V(\theta) := v_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot v_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot v_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot v_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



12. Write the complete global equation for the acceleration and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$A(\theta) := a_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot a_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot a_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot a_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$



13. Write the complete global equation for the jerk and plot it over one rotation of the cam, which is the sum of the four intervals defined above.

$$J(\theta) := j_1\left(\frac{\theta}{\theta_1}\right) + R(\theta,\theta_1,\theta_2)\cdot j_2\left(\frac{\theta-\theta_1}{\theta_2-\theta_1}\right) \ldots$$
$$+ R(\theta,\theta_2,\theta_3)\cdot j_3\left(\frac{\theta-\theta_2}{\theta_3-\theta_2}\right) + R(\theta,\theta_3,\theta_4)\cdot j_4\left(\frac{\theta-\theta_3}{\theta_4-\theta_3}\right)$$