Chapter 9

GEAR TRAINS

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PROBLEM 9-1

Statement: A 22-tooth gear has AGMA standard full-depth involute teeth with diametral pitch of 4. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance.

Given: Tooth number $N := 22$ Diametral pitch $p_d := 4 \cdot in^{-1}$

Solution: See Table 9-1 and Mathcad file P0901.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter $d := \dfrac{N}{p_d}$ $d = 5.5000\ in$

Circular pitch $p_c := \dfrac{\pi}{p_d}$ $p_c = 0.7854\ in$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum $a := \dfrac{1.0000}{p_d}$ $a = 0.2500\ in$

Dedendum $b := \dfrac{1.2500}{p_d}$ $b = 0.3125\ in$

Tooth thickness $t := 0.5 \cdot p_c$ $t = 0.3927\ in$

Clearance $c := \dfrac{0.2500}{p_d}$ $c = 0.0625\ in$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions should be displayed to four decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in ten-thousandths of an inch..

PROBLEM 9-2

Statement: A 40-tooth gear has AGMA standard full-depth involute teeth with diametral pitch of 10. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance.

Given: Tooth number $N := 40$ Diametral pitch $p_d := 10 \cdot in^{-1}$

Solution: See Mathcad file P0902.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := \frac{N}{p_d}$	$d = 4.0000\ in$
Circular pitch	$p_c := \frac{\pi}{p_d}$	$p_c = 0.3142\ in$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := \frac{1.0000}{p_d}$	$a = 0.1000\ in$
Dedendum	$b := \frac{1.2500}{p_d}$	$b = 0.1250\ in$
Tooth thickness	$t := 0.5 \cdot p_c$	$t = 0.1571\ in$
Clearance	$c := \frac{0.2500}{p_d}$	$c = 0.0250\ in$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions should be displayed to four decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in ten-thousandths of an inch..

PROBLEM 9-3

Statement: A 30-tooth gear has AGMA standard full-depth involute teeth with diametral pitch of 12. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance.

Given: Tooth number $N := 30$ Diametral pitch $p_d := 12 \cdot in^{-1}$

Solution: See Mathcad file P0903.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter $d := \dfrac{N}{p_d}$ $d = 2.5000\ in$

Circular pitch $p_c := \dfrac{\pi}{p_d}$ $p_c = 0.2618\ in$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum $a := \dfrac{1.0000}{p_d}$ $a = 0.0833\ in$

Dedendum $b := \dfrac{1.2500}{p_d}$ $b = 0.1042\ in$

Tooth thickness $t := 0.5 \cdot p_c$ $t = 0.1309\ in$

Clearance $c := \dfrac{0.2500}{p_d}$ $c = 0.0208\ in$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions should be displayed to four decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in ten-thousandths of an inch..

PROBLEM 9-4

Statement: Using any available string, some tape, a pencil, and a drinking glass or tin can, generate and draw an involute curve on a piece of paper. With your protractor, show that all normals to the curve are tangent to the base circle.

Solution: This is a "hands-on" student demonstration project. The result should look like Figure 9-5.

PROBLEM 9-5

Statement: A spur gearset must have pitch diameters of 4.5 and 12 in. What is the largest standard tooth size, in terms of diametral pitch, that can be used without having any interference or undercutting and what are the number of teeth on each gear that result from using this diametral pitch? Assume that both gears are cut with a hob.

a. For a 20-deg pressure angle
b. For a 25-deg pressure angle

Given: Pitch diameters: $d_1 := 4.5 \cdot in$ $\quad d_2 := 12 \cdot in$

Solution: See Table 9-4 and Mathcad file P0905.

1. To avoid undercutting, use the minimum tooth numbers given in Table 9-4b.

a. Pressure angle of 20 deg.

$$N_{min} := 21 \qquad p_{dmin} := \frac{N_{min}}{d_1} \qquad p_{dmin} = 4.667\, in^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 5. But, since the pinion pitch diameter is not an integer, using 5 would result in a noninteger number of teeth. Therefore, we must go to the next larger (even) pitch (smaller tooth size) of $p_d := 6 \cdot in^{-1}$. The resulting tooth numbers are:

$$N_1 := p_d \cdot d_1 \qquad N_1 = 27 \qquad N_2 := p_d \cdot d_2 \qquad N_2 = 72$$

b. Pressure angle of 25 deg

$$N_{min} := 14 \qquad p_{dmin} := \frac{N_{min}}{d_1} \qquad p_{dmin} = 3.111\, in^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 4. $p_d := 4 \cdot in^{-1}$.
The resulting tooth numbers are:

$$N_1 := p_d \cdot d_1 \qquad N_1 = 18 \qquad N_2 := p_d \cdot d_2 \qquad N_2 = 48$$

PROBLEM 9-6

Statement: Design a simple, spur gear train for a ratio of -9:1 and a diametral pitch of 8. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 9$ Diametral pitch $p_d := 8 \cdot in^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth (see Table 9-4b) for a 20-deg pressure angle.

Design Choice: Pressure angle $\phi := 20 \cdot deg$

Solution: See Mathcad file P0906.

1. From inspection of Table 9-5a, we see that 17 teeth is the least number that the pinion can have for a gear ratio of 9. therefore, let the number of teeth on the pinion be

$$N_p := 17 \qquad \text{and} \qquad N_g := m_G \cdot N_p \qquad N_g = 153$$

2. Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

$$d_p := \frac{N_p}{p_d} \qquad d_p = 2.1250\,in \qquad d_g := \frac{N_g}{p_d} \qquad d_g = 19.1250\,in$$

3. Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

$$r_p := 0.5 \cdot d_p \qquad r_p = 1.0625\,in \qquad r_g := 0.5 \cdot d_g \qquad r_g = 9.5625\,in$$

$$a_p := \frac{1}{p_d} \qquad a_p = 0.1250\,in \qquad a_g := \frac{1}{p_d} \qquad a_g = 0.1250\,in$$

Center distance $C := r_p + r_g$ $C = 10.6250\,in$

$$Z := \sqrt{(r_p + a_p)^2 - (r_p \cdot cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot cos(\phi))^2} - C \cdot sin(\phi)$$

Contact ratio $m_p := \dfrac{p_d \cdot Z}{\pi \cdot cos(\phi)}$ $m_p = 1.704$

PROBLEM 9-7

Statement: Design a simple, spur gear train for a ratio of +8:1 and a diametral pitch of 6. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 8$ Diametral pitch $p_d := 6 \cdot in^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth for a 20-deg pressure angle (see Table 9-4b).

Design Choice: Pressure angle $\phi := 20 \cdot deg$

Solution: See Mathcad file P0907.

1. From inspection of Table 9-5a, we see that 17 teeth is the least number that the pinion can have for a gear ratio of 8. therefore, let the number of teeth on the pinion be

   $N_p := 17$ and $N_g := m_G \cdot N_p$ $N_g = 136$

2. Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

   $d_p := \frac{N_p}{p_d}$ $d_p = 2.8333\ in$ $d_g := \frac{N_g}{p_d}$ $d_g = 22.6667\ in$

3. Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

   $r_p := 0.5 \cdot d_p$ $r_p = 1.4167\ in$ $r_g := 0.5 \cdot d_g$ $r_g = 11.3333\ in$

   $a_p := \frac{1}{p_d}$ $a_p = 0.1667\ in$ $a_g := \frac{1}{p_d}$ $a_g = 0.1667\ in$

   Center distance $C := r_p + r_g$ $C = 12.7500\ in$

   $$Z := \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C \cdot \sin(\phi)$$

   Contact ratio $m_p := \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)}$ $m_p = 1.699$

4. An idler gear of any diameter is needed to get the positive ratio. If the idler is does not have the same number of teeth as the gear, the calculation of contact ratio (step 3) will not be correct.

PROBLEM 9-8

Statement: Design a simple, spur gear train for a ratio of -7:1 and a diametral pitch of 8. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 7$ Diametral pitch $p_d := 8 \cdot in^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth for a 20-deg pressure angle (see Table 9-4b).

Design Choice: Pressure angle $\phi := 20 \cdot deg$

Solution: See Mathcad file P0908.

1. From inspection of Table 9-5a, we see that 17 teeth is the least number that the pinion can have for a gear ratio of 7. therefore, let the number of teeth on the pinion be

   $N_p := 17$ and $N_g := m_G \cdot N_p$ $N_g = 119$

2. Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

   $d_p := \dfrac{N_p}{p_d}$ $d_p = 2.1250\ in$ $d_g := \dfrac{N_g}{p_d}$ $d_g = 14.8750\ in$

3. Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

   $r_p := 0.5 \cdot d_p$ $r_p = 1.0625\ in$ $r_g := 0.5 \cdot d_g$ $r_g = 7.4375\ in$

   $a_p := \dfrac{1}{p_d}$ $a_p = 0.1250\ in$ $a_g := \dfrac{1}{p_d}$ $a_g = 0.1250\ in$

   Center distance $C := r_p + r_g$ $C = 8.5000\ in$

   $$Z := \sqrt{\left(r_p + a_p\right)^2 - \left(r_p \cdot \cos(\phi)\right)^2} + \sqrt{\left(r_g + a_g\right)^2 - \left(r_g \cdot \cos(\phi)\right)^2} - C \cdot \sin(\phi)$$

   Contact ratio $m_p := \dfrac{p_d \cdot Z}{\pi \cdot \cos(\phi)}$ $m_p = 1.693$

PROBLEM 9-9

Statement: Design a simple, spur gear train for a ratio of +6.5:1 and a diametral pitch of 5. Specify pitch diameters and numbers of teeth. Calculate the contact ratio.

Given: Gear ratio $m_G := 6.5$ Diametral pitch $p_d := 5 \cdot in^{-1}$

Assumptions: The pinion is not cut by a hob and can, therefore, have fewer than 21 teeth for a 20-deg pressure angle (see Table 9-4b).

Design Choice: Pressure angle $\phi := 20 \cdot deg$

Solution: See Mathcad file P0909.

1. From inspection of Table 9-5a, we see that 17 teeth is the least number that the pinion can have for a gear ratio of 6.5. therefore, let the number of teeth on the pinion be (an even number so the gear tooth number will be an integer).

$$N_p := 18 \qquad \text{and} \qquad N_g := m_G \cdot N_p \qquad N_g = 117$$

2. Using equation 9.4c, calculate the pitch diameters of the pinion and gear.

$$d_p := \frac{N_p}{p_d} \qquad d_p = 3.6000\,in \qquad d_g := \frac{N_g}{p_d} \qquad d_g = 23.4000\,in$$

3. Calculate the contact ratio using equations 9.2 and 9.6b and those from Table 9-1.

$$r_p := 0.5 \cdot d_p \qquad r_p = 1.8000\,in \qquad r_g := 0.5 \cdot d_g \qquad r_g = 11.7000\,in$$

$$a_p := \frac{1}{p_d} \qquad a_p = 0.2000\,in \qquad a_g := \frac{1}{p_d} \qquad a_g = 0.2000\,in$$

Center distance $C := r_p + r_g$ $C = 13.5000\,in$

$$Z := \sqrt{(r_p + a_p)^2 - (r_p \cdot cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot cos(\phi))^2} - C \cdot sin(\phi)$$

Contact ratio $m_p := \dfrac{p_d \cdot Z}{\pi \cdot cos(\phi)}$ $m_p = 1.699$

4. An idler gear of any diameter is needed to get the positive ratio. If the idler is does not have the same number of teeth as the gear, the calculation of contact ratio (step 3) will not be correct.

PROBLEM 9-10

Statement: Design a compound, spur gear train for a ratio of -70:1 and diametral pitch of 10. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio: $m_G := 70$ Diametral pitch: $p_d := 10 \cdot in^{-1}$

Solution: See Mathcad file P0910.

1. Since the ratio is negative, we want to have an odd number of stages or an even number with an idler. Let the number of stages be

Possible number of stages $j := 2, 3 .. 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

then $j = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ $r(j) = \begin{pmatrix} 8.367 \\ 4.121 \\ 2.893 \end{pmatrix}$

2. Two stages would result in a stage ratio less than 10 but will require an idler, so we will use three stages. The average ratio for three stages is about 21:5. Using a pressure angle of 20 deg, let the stage ratios be

Stage 1 ratio $r_1 := \frac{21}{5}$ Stage 2 ratio $r_2 := \frac{20}{5}$ Stage 3 ratio $r_3 := \frac{25}{6}$

and let the driver gears have tooth numbers of

Tooth number index $i := 2, 3 .. 7$

$N_2 := 20$ $N_4 := 20$ $N_6 := 18$

then the driven gears will have tooth numbers of

$N_3 := r_1 \cdot N_2$ $N_5 := r_2 \cdot N_4$ $N_7 := r_3 \cdot N_6$

$N_3 = 84$ $N_5 = 80$ $N_7 = 75$

The pitch diameters are: $d_i := \frac{N_i}{Pd}$

Tooth numbers:

$i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix}$ $\frac{d_i}{in} = \begin{pmatrix} 2.0000 \\ 8.4000 \\ 2.0000 \\ 8.0000 \\ 1.8000 \\ 7.5000 \end{pmatrix}$ $i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix}$ $N_i = \begin{pmatrix} 20 \\ 84 \\ 20 \\ 80 \\ 18 \\ 75 \end{pmatrix}$

Checking the overall gear ratio: $\frac{N_3 \cdot N_5 \cdot N_7}{N_2 \cdot N_4 \cdot N_6} = 70.000$

✍ PROBLEM 9-11

Statement: Design a compound, spur gear train for a ratio of 50:1 and diametral pitch of 8. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 50$ Diametral pitch $p_d := 8 \cdot in^{-1}$

Solution: See Mathcad file P0911.

1. Since the ratio is positive, we want to have an even number of stages or an odd number with an idler. Let the number of stages be

Possible number of stages $j := 2, 3 .. 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

then $j = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ $r(j) = \begin{pmatrix} 7.071 \\ 3.684 \\ 2.659 \end{pmatrix}$

2. Two stages would result in a stage ratio less than 10 and about 7, so we will use two stages. The average ratio for two stages is about 50:7. Using a pressure angle of 20 deg, let the stage ratios be

Stage 1 ratio $r_1 := \frac{50}{7}$ Stage 2 ratio $r_2 := 7$

and let the driver gears have tooth numbers of (note that N_2 must be a multiple of 7)

Tooth number index $i := 2, 3 .. 5$

$N_2 := 21$ $N_4 := 18$

then the driven gears will have tooth numbers of

$N_3 := r_1 \cdot N_2$ $N_5 := r_2 \cdot N_4$

$N_3 = 150$ $N_5 = 126$

The pitch diameters are: $d_i := \frac{N_i}{pd}$

Tooth numbers:

$i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $\frac{d_i}{in} = \begin{pmatrix} 2.6250 \\ 18.7500 \\ 2.2500 \\ 15.7500 \end{pmatrix}$ $i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $N_i = \begin{pmatrix} 21 \\ 150 \\ 18 \\ 126 \end{pmatrix}$

Checking the overall gear ratio: $\frac{N_3 \cdot N_5}{N_2 \cdot N_4} = 50.000$

PROBLEM 9-12

Statement: Design a compound, spur gear train for a ratio of 150:1 and diametral pitch of 6. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 150$ Diametral pitch $p_d := 6 \cdot in^{-1}$

Solution: See Mathcad file P0912.

1. Since the ratio is positive, we want to have an even number of stages or an odd number with an idler. Let the number of stages be

   Possible number of stages $j := 2, 3 .. 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

   then $j = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ $r(j) = \begin{pmatrix} 12.247 \\ 5.313 \\ 3.500 \end{pmatrix}$

2. Two stages would result in a stage ratio greater than 10, so we will use three stages with an idler to get the required output direction. The average ratio for three stages is about 16:3. Using a pressure angle of 20 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 5$ Stage 2 ratio $r_2 := 5$ Stage 3 ratio $r_3 := 6$

   and let the driver gears have tooth numbers of

   Tooth number index $i := 2, 3 .. 8$

   $N_2 := 18$ $N_4 := 18$ $N_6 := 18$

   then the driven gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_5 := r_2 \cdot N_4$ $N_8 := r_3 \cdot N_6$

   $N_3 = 90$ $N_5 = 90$ $N_8 = 108$

   The idler gear will have $N_7 := 18$

   The pitch diameters are: $d_i := \frac{N_i}{p_d}$ Tooth numbers:

   $$i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{pmatrix} \quad \frac{d_i}{in} = \begin{pmatrix} 3.0000 \\ 15.0000 \\ 3.0000 \\ 15.0000 \\ 3.0000 \\ 3.0000 \\ 18.0000 \end{pmatrix} \qquad i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{pmatrix} \quad N_i = \begin{pmatrix} 18 \\ 90 \\ 18 \\ 90 \\ 18 \\ 18 \\ 108 \end{pmatrix}$$

   Checking the overall gear ratio: $\frac{N_3 \cdot N_5 \cdot N_8}{N_2 \cdot N_4 \cdot N_6} = 150.000$

PROBLEM 9-13

Statement: Design a compound, spur gear train for a ratio of -250:1 and diametral pitch of 9. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 250$ Diametral pitch $p_d := 9 \cdot in^{-1}$

Solution: See Mathcad file P0913.

1. Since the ratio is negative, we want to have an odd number of stages or an even number with an idler. Let the number of stages be

   Possible number of stages $j := 2, 3 .. 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

   then $j = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ $r(j) = \begin{pmatrix} 15.811 \\ 6.300 \\ 3.976 \end{pmatrix}$

2. Two stages would result in a stage ratio greater than 10 and will require an idler, so we will use three stages. The average ratio for three stages is about 20:3. Using a pressure angle of 20 deg, let the stage ratios be

   Stage 1 ratio $r_1 := \frac{20}{3}$ Stage 2 ratio $r_2 := \frac{18}{3}$ Stage 3 ratio $r_3 := \frac{25}{4}$

   and let the driver gears have tooth numbers of

   Tooth number index $i := 2, 3 .. 7$

   $N_2 := 18$ $N_4 := 18$ $N_6 := 20$

   then the driven gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_5 := r_2 \cdot N_4$ $N_7 := r_3 \cdot N_6$

   $N_3 = 120$ $N_5 = 108$ $N_7 = 125$

   The pitch diameters are: $d_i := \frac{N_i}{p_d}$

   Tooth numbers:

$$i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix} \quad \frac{d_i}{in} = \begin{pmatrix} 2.0000 \\ 13.3333 \\ 2.0000 \\ 12.0000 \\ 2.2222 \\ 13.8889 \end{pmatrix} \quad i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{pmatrix} \quad N_i = \begin{pmatrix} 18 \\ 120 \\ 18 \\ 108 \\ 20 \\ 125 \end{pmatrix}$$

   Checking the overall gear ratio: $\frac{N_3 \cdot N_5 \cdot N_7}{N_2 \cdot N_4 \cdot N_6} = 250.000$

PROBLEM 9-14

Statement: Design a compound, reverted, spur gear train for a ratio of 30:1 and diametral pitch of 10. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 30$ Diametral pitch $p_d := 10 \cdot in^{-1}$

Solution: See Mathcad file P0914.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be

   Possible number of stages $j := 2, 3 .. 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

   then $j = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ $r(j) = \begin{pmatrix} 5.477 \\ 3.107 \\ 2.340 \end{pmatrix}$

2. Two stages would result in a stage ratio less than 10, so we will use two stages. The average ratio for two stages is about 11:2. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 5$ Stage 2 ratio $r_2 := 6$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \frac{N_3}{N_2}$ $r_2 := \frac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 42.000$

   By iteration, find a multiple of K_{min} that will result in a minimum number of teeth on N_2 and N_4.

   $K := 2 \cdot K_{min}$ $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   $K = 84.000$ $N_2 = 14$ $N_4 = 12$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 70$ $N_5 := r_2 \cdot N_4$ $N_5 = 72$

   The pitch diameters are: $d_i := \frac{N_i}{pd}$ $i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $\frac{d_i}{in} = \begin{pmatrix} 1.4000 \\ 7.0000 \\ 1.2000 \\ 7.2000 \end{pmatrix}$ $N_i = \begin{pmatrix} 14 \\ 70 \\ 12 \\ 72 \end{pmatrix}$

   Checking the overall gear ratio: $\left(-\frac{N_3}{N_2}\right) \cdot \left(-\frac{N_5}{N_4}\right) = 30.000$

PROBLEM 9-15

Statement: Design a compound, reverted, spur gear train for a ratio of 40:1 and diametral pitch of 8. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 40$ Diametral pitch $p_d := 8 \cdot in^{-1}$

Solution: See Mathcad file P0915.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be

   Possible number of stages $j := 2, 3 .. 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

   then $j = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ $r(j) = \begin{pmatrix} 6.325 \\ 3.420 \\ 2.515 \end{pmatrix}$

2. Two stages would result in a stage ratio less than 10, so we will use two stages. The average ratio for two stages is about 20:3. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 5$ Stage 2 ratio $r_2 := 8$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \frac{N_3}{N_2}$ $r_2 := \frac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 54.000$

   By iteration, find a multiple of K_{min} that will result in a minimum number of teeth on N_2 and N_4.

   $K := 3 \cdot K_{min}$ $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   $K = 162.000$ $N_2 = 27$ $N_4 = 18$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 135$ $N_5 := r_2 \cdot N_4$ $N_5 = 144$

   The pitch diameters are: $d_i := \frac{N_i}{p_d}$ $i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $\frac{d_i}{in} = \begin{pmatrix} 3.3750 \\ 16.8750 \\ 2.2500 \\ 18.0000 \end{pmatrix}$ $N_i = \begin{pmatrix} 27 \\ 135 \\ 18 \\ 144 \end{pmatrix}$

   Checking the overall gear ratio: $\left(-\frac{N_3}{N_2}\right) \cdot \left(-\frac{N_5}{N_4}\right) = 40.000$

PROBLEM 9-16

Statement: Design a compound, reverted, spur gear train for a ratio of 75:1 and diametral pitch of 12. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 75$ Diametral pitch $p_d := 12 \cdot in^{-1}$

Solution: See Mathcad file P0916.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be

   Possible number of stages $j := 2, 3 .. 4$ Ideal, theoretical stage ratios $r(j) := m_G^{\frac{1}{j}}$

   then $j = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ $r(j) = \begin{pmatrix} 8.660 \\ 4.217 \\ 2.943 \end{pmatrix}$

2. Two stages would result in a stage ratio less than 10, so we will use two stages. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 7.5$ Stage 2 ratio $r_2 := 10$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \frac{N_3}{N_2}$ $r_2 := \frac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 93.500$

   By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4.

   $K := 2 \cdot K_{min}$ $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   $K = 187.000$ $N_2 = 22$ $N_4 = 17$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 165$ $N_5 := r_2 \cdot N_4$ $N_5 = 170$

   The pitch diameters are: $d_i := \frac{N_i}{pd}$ $i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $\frac{d_i}{in} = \begin{pmatrix} 1.8333 \\ 13.7500 \\ 1.4167 \\ 14.1667 \end{pmatrix}$ $N_i = \begin{pmatrix} 22 \\ 165 \\ 17 \\ 170 \end{pmatrix}$

   Checking the overall gear ratio: $\left(-\frac{N_3}{N_2}\right) \cdot \left(-\frac{N_5}{N_4}\right) = 75.000$

PROBLEM 9-17

Statement: Design a compound, reverted, spur gear train for a ratio of 7:1 and diametral pitch of 4. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 7$ Diametral pitch $p_d := 4 \cdot in^{-1}$

Solution: See Mathcad file P0917.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be 2.

2. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 3.5$ Stage 2 ratio $r_2 := 2$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \dfrac{N_3}{N_2}$ $r_2 := \dfrac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 13.500$

   By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4.

   $K := 6 \cdot K_{min}$ $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   $K = 81.000$ $N_2 = 18$ $N_4 = 27$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 63$ $N_5 := r_2 \cdot N_4$ $N_5 = 54$

   The pitch diameters are: $d_i := \dfrac{N_i}{pd}$

   $$i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \quad \frac{d_i}{in} = \begin{pmatrix} 4.5000 \\ 15.7500 \\ 6.7500 \\ 13.5000 \end{pmatrix} \quad N_i = \begin{pmatrix} 18 \\ 63 \\ 27 \\ 54 \end{pmatrix}$$

   Checking the overall gear ratio:

   $$\left(-\frac{N_3}{N_2}\right) \cdot \left(-\frac{N_5}{N_4}\right) = 7.000$$

PROBLEM 9-18

Statement: Design a compound, reverted, spur gear train for a ratio of 12:1 and diametral pitch of 6. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 12$ Diametral pitch $p_d := 6 \cdot in^{-1}$

Solution: See Mathcad file P0918.

1. Since the ratio is positive, we want to have an even number of stages. Let the number of stages be 2.

2. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 4$ Stage 2 ratio $r_2 := 3$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \dfrac{N_3}{N_2}$ $r_2 := \dfrac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 20.000$

   By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4.

   $K := 4 \cdot K_{min}$ $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   $K = 80.000$ $N_2 = 16$ $N_4 = 20$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 64$ $N_5 := r_2 \cdot N_4$ $N_5 = 60$

   The pitch diameters are: $d_i := \dfrac{N_i}{p_d}$

   $$i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \quad \frac{d_i}{in} = \begin{pmatrix} 2.6667 \\ 10.6667 \\ 3.3333 \\ 10.0000 \end{pmatrix} \quad N_i = \begin{pmatrix} 16 \\ 64 \\ 20 \\ 60 \end{pmatrix}$$

   Checking the overall gear ratio:

   $$\left(-\frac{N_3}{N_2} \right) \cdot \left(-\frac{N_5}{N_4} \right) = 12.000$$

PROBLEM 9-19

Statement: Design a compound, reverted, spur gear transmission that will give two shiftable ratios of +3:1 forward and -4.5:1 reverse with diametral pitch of 6. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 3$ Diametral pitch $p_d := 6 \cdot in^{-1}$

Solution: See Mathcad file P0919.

1. Since the forward ratio is positive, we want to have an even number of stages. Let the number of stages be 2.

2. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 2$ Stage 2 ratio $r_2 := 1.5$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \frac{N_3}{N_2}$ $r_2 := \frac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 7.500$

   By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4.

   $K := 6 \cdot K_{min}$ $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   $K = 45.000$ $N_2 = 15$ $N_4 = 18$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears for the forward train will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 30$ $N_5 := r_2 \cdot N_4$ $N_5 = 27$

   The pitch diameters are: $d_i := \frac{N_i}{Pd}$

   $$i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \quad \frac{d_i}{in} = \begin{pmatrix} 2.5000 \\ 5.0000 \\ 3.0000 \\ 4.5000 \end{pmatrix} \quad N_i = \begin{pmatrix} 15 \\ 30 \\ 18 \\ 27 \end{pmatrix}$$

   Checking the overall gear ratio:

   $$\left(\frac{N_3}{N_2}\right) \cdot \left(\frac{N_5}{N_4}\right) = 3.000$$

5. The reverse train will also have two stages and use the first forward stage and an idler gear to get the change in direction. Let the stage ratios be

   Stage 1 ratio $r_1 := 2$ Stage 2 ratio $r_2 := 2.25$

6. Let the number of teeth on the reverse stage driver gear be $N_6 := 12$ then the number of teeth on the driven gear will be

   Driven reverse gear $N_7 := r_2 \cdot N_6$ $N_7 = 27$

PROBLEM 9-20

Statement: Design a compound, reverted, spur gear transmission that will give two shiftable ratios of +5:1 forward and -3.5:1 reverse with diametral pitch of 6. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 5$ Diametral pitch $p_d := 6 \cdot in^{-1}$

Solution: See Mathcad file P0920.

1. Since the forward ratio is positive, we want to have an even number of stages. Let the number of stages be 2.
2. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := 2$ Stage 2 ratio $r_2 := 2.5$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \frac{N_3}{N_2}$ $r_2 := \frac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 10.500$

   By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4.

   $K := 4 \cdot K_{min}$ $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   $K = 42.000$ $N_2 = 14$ $N_4 = 12$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears for the forward train will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 28$ $N_5 := r_2 \cdot N_4$ $N_5 = 30$

   The pitch diameters are: $d_i := \frac{N_i}{p_d}$ $i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $\frac{d_i}{in} = \begin{pmatrix} 2.3333 \\ 4.6667 \\ 2.0000 \\ 5.0000 \end{pmatrix}$ $N_i = \begin{pmatrix} 14 \\ 28 \\ 12 \\ 30 \end{pmatrix}$

   Checking the overall gear ratio:

   $$\left(-\frac{N_3}{N_2}\right) \cdot \left(-\frac{N_5}{N_4}\right) = 5.000$$

5. The reverse train will also have two stages and use the first forward stage and an idler gear to get the change in direction. Let the stage ratios be

   Stage 1 ratio $r_1 := 2$ Stage 2 ratio $r_2 := 1.75$

6. Let the number of teeth on the reverse stage driver gear be $N_6 := 12$ then the number of teeth on the driven gear will be

   Driven reverse gear $N_7 := r_2 \cdot N_6$ $N_7 = 21$

PROBLEM 9-21

Statement: Design a compound, reverted, spur gear transmission that will give three shiftable ratios of +6:1, +3.5:1 forward and -4:1 reverse with diametral pitch of 8. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 6$ Diametral pitch $p_d := 8 \cdot in^{-1}$

Solution: See Mathcad file P0921.

1. Since the forward ratio is positive, we want to have an even number of stages. Let the number of stages be 2.
2. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := \frac{7}{3}$ Stage 2 ratio $r_2 := \frac{18}{7}$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \frac{N_3}{N_2}$ $r_2 := \frac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1)\cdot(r_2 + 1)$ $K_{min} = 11.905$

   By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4.

   $K := 4.2 \cdot K_{min}$ $N_2 := \frac{K}{r_1 + 1}$ $N_4 := \frac{K}{r_2 + 1}$

   $K = 50.000$ $N_2 = 15$ $N_4 = 14$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears for the forward train will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 35$ $N_5 := r_2 \cdot N_4$ $N_5 = 36$

   The pitch diameters are: $d_i := \frac{N_i}{pd}$ $i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $\frac{d_i}{in} = \begin{pmatrix} 1.8750 \\ 4.3750 \\ 1.7500 \\ 4.5000 \end{pmatrix}$ $N_i = \begin{pmatrix} 15 \\ 35 \\ 14 \\ 36 \end{pmatrix}$

   Checking the overall gear ratio:

   $$\left(-\frac{N_3}{N_2}\right)\cdot\left(-\frac{N_5}{N_4}\right) = 6.000$$

5. The second forward train will also have two stages and use the first forward stage to get the overall ratio. Let the stage ratios be

   Stage 1 ratio $r_1 := \frac{7}{3}$ Stage 2 ratio $r_2 := 1.5$

6. Let the number of teeth on the second forward stage driver gear be $N_6 := 20$ then the number of teeth on the driven gear will be

   Second forward driven gear $N_7 := r_2 \cdot N_6$ $N_7 = 30$

7. The reverse train will also have two stages and use the first forward stage and an idler gear to get the change in direction. Let the stage ratios be

Stage 1 ratio $r_1 := \frac{7}{3}$ Stage 2 ratio $r_2 := \frac{12}{7}$

8. Let the number of teeth on the reverse stage driver gear be $N_8 := 14$ then the number of teeth on the driven gear will be

Driven reverse gear $N_9 := r_2 \cdot N_8$ $N_9 = 24$

PROBLEM 9-22

Statement: Design a compound, reverted, spur gear transmission that will give three shiftable ratios of +4.5:1, +2.5:1 forward and -3.5:1 reverse with diametral pitch of 5. Specify pitch diameters and numbers of teeth. Sketch the train to scale.

Given: Gear ratio $m_G := 4.5$ Diametral pitch $p_d := 5 \cdot in^{-1}$

Solution: See Mathcad file P0922.

1. Since the forward ratio is positive, we want to have an even number of stages. Let the number of stages be 2.

2. Using a pressure angle of 25 deg, let the stage ratios be

   Stage 1 ratio $r_1 := \dfrac{9}{6}$ Stage 2 ratio $r_2 := \dfrac{m_G}{r_1}$ $r_2 = 3.000$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \dfrac{N_3}{N_2}$ $r_2 := \dfrac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 10.000$

   By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on N_2 and N_4.

   $K := 12 \cdot K_{min}$ $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   $K = 120.000$ $N_2 = 48$ $N_4 = 30$

   These are acceptable tooth numbers for gears with a 25-deg pressure angle that are cut by a hob.

4. The driven gears for the forward train will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 72$ $N_5 := r_2 \cdot N_4$ $N_5 = 90$

   The pitch diameters are: $d_i := \dfrac{N_i}{p_d}$ $i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $\dfrac{d_i}{in} = \begin{pmatrix} 9.6000 \\ 14.4000 \\ 6.0000 \\ 18.0000 \end{pmatrix}$ $N_i = \begin{pmatrix} 48 \\ 72 \\ 30 \\ 90 \end{pmatrix}$

   Checking the overall gear ratio:

   $$\left(-\frac{N_3}{N_2}\right) \cdot \left(-\frac{N_5}{N_4}\right) = 4.500$$

5. The second forward train will also have two stages and use the first forward stage to get the overall ratio. Let the stage ratios be

   Stage 1 ratio $r_1 = 1.500$ Stage 2 ratio $r_2 := \dfrac{2.5}{r_1}$ $r_2 = 1.667$

6. Let the number of teeth on the second forward stage driver gear be $N_6 := 45$ then the number of teeth on the driven gear will be

   Second forward driven gear $N_7 := r_2 \cdot N_6$ $N_7 = 75$

7. The reverse train will also have two stages and use the first forward stage and an idler gear to get the change in direction. Let the stage ratios be

Stage 1 ratio $r_1 = 1.500$ Stage 2 ratio $r_2 := \frac{3.5}{r_1}$ $r_2 = 2.333$

8. Let the number of teeth on the reverse stage driver gear be $N_8 := 12$ then the number of teeth on the driven gear will be

Driven reverse gear $N_9 := r_2 \cdot N_8$ $N_9 = 28$

PROBLEM 9-23

Statement: Design the rolling cones for a -3:1 ratio and a 60-deg included angle between shafts. Sketch the train to scale.

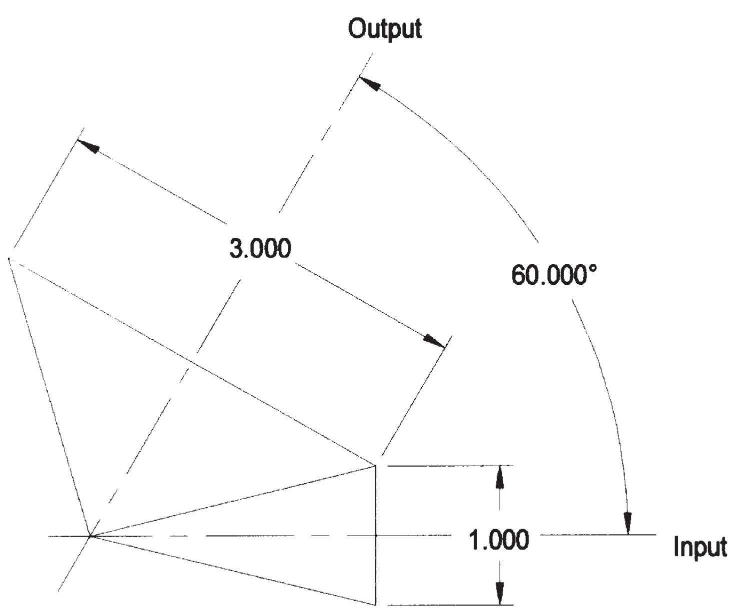
Given: Train ratio $m_T := -3$

Solution: See Figure 9-20 and Mathcad file P0923.

1. Choose a back-cone pitch diameter for the input cone of 1.000 in. Then, from equation 9.1,

$$d_{in} := 1.000 \cdot in \qquad d_{out} := |m_T| \cdot d_{in} \qquad d_{out} = 3.000\, in$$

2. Using these back-cone pitch diameters and the given shaft included angle, draw the cones to scale.



PROBLEM 9-24

Statement: Design the rolling cones for a -4.5:1 ratio and a 40-deg included angle between shafts. Sketch the train to scale.

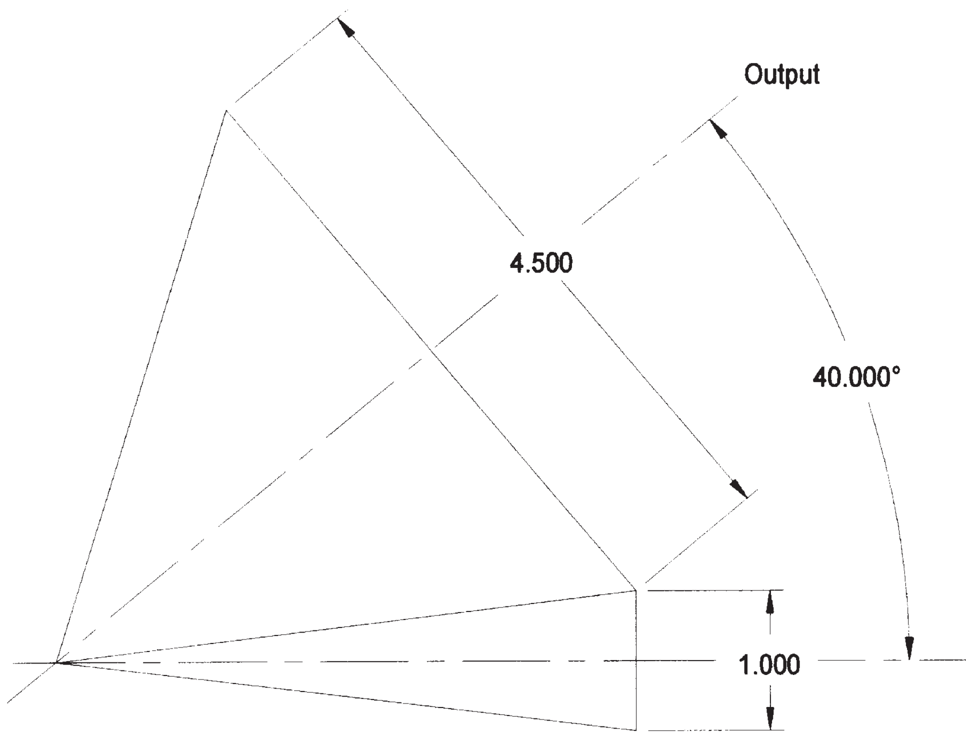
Given: Train ratio $m_T := -4.5$

Solution: See Figure 9-20 and Mathcad file P0924.

1. Choose a back-cone pitch diameter for the input cone of 1.000 in. Then, from equation 9.1,

$$d_{in} := 1.000 \cdot in \qquad d_{out} := \left| m_T \right| \cdot d_{in} \qquad d_{out} = 4.500\, in$$

2. Using these back-cone pitch diameters and the given shaft included angle, draw the cones to scale.



PROBLEM 9-25a

Statement: Figure P9-1 shows a compound planetary gear train (not to scale). Table P9-1 gives data for gear numbers of teeth and input velocities. For the data in row *a*, find ω_2.

Given: Tooth numbers:

$N_2 := 30$ $N_3 := 25$ $N_4 := 45$ $N_5 := 50$ $N_6 := 200$

Input velocities: $\omega_6 := 20$ $\omega_{arm} := -50$

Solution: See Figure P9-1 and Mathcad file P0925a.

1. The formula method will be used in this solution. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be 2 and last be 6. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_6 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

2. Calculate R using equation 9.14 and inspection of Figure P9-1.

$$R := \frac{N_2 \cdot N_3 \cdot N_5}{N_4 \cdot N_5 \cdot N_6} \qquad R = 0.08333$$

Note that N_5 appears both in the numerator (a driver) and the denominator (driven). It could be left out completely because it is an idler. R is positive in this case because gears 2 and 6 rotate in the same direction.

3. Solve the right-hand equation in step 1 for ω_2.

$$\omega_2 := \frac{\omega_6 - \omega_{arm}}{R} + \omega_{arm} \qquad \omega_2 = 790.000$$

ω_2 will be in the same direction as ω_6.

PROBLEM 9-26a

Statement: Figure P9-2 shows a compound planetary gear train (not to scale). Table P9-2 gives data for gear numbers of teeth and input velocities. For the data in row *a*, find ω_2.

Given: Tooth numbers:

$N_2 := 50$ $N_3 := 25$ $N_4 := 45$ $N_5 := 30$ $N_6 := 40$

Input velocities: $\omega_6 := 20$ $\omega_{arm} := -50$

Solution: See Figure P9-2 and Mathcad file P0926a.

1. The formula method will be used in this solution. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be 3 and last be 6. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_6 - \omega_{arm}}{\omega_3 - \omega_{arm}} = R$$

2. Calculate R using equation 9.14 and inspection of Figure P9-2.

$$R := \frac{N_3 \cdot N_5}{N_4 \cdot N_6} \qquad R = 0.4167$$

R is positive in this case because gears 3 and 6 rotate in the same direction.

3. Solve the right-hand equation in step 1 for ω_3.

$$\omega_3 := \frac{\omega_6 - \omega_{arm}}{R} + \omega_{arm} \qquad \omega_3 = 118.000$$

4. Solve for ω_2 using equation 9.7.

$$\omega_2 := -\frac{N_3}{N_2} \cdot \omega_3 \qquad \omega_2 = -59.000$$

ω_2 will be in the opposite direction as ω_6.

PROBLEM 9-27

Statement: Figure P9-3 shows a planetary gear train used in an automotive rear-end differential (not to scale). The car has wheels with a 15-in rolling radius and is moving forward in a straight line at 50 mph. The engine is turning 2000 rpm. The transmission is in direct drive (1:1) with the driveshaft.

a. What is the rear wheel's rpm and the gear ratio between ring and pinion?

b. As the car hits a patch of ice, the right wheel speeds up to 800 rpm. What is the speed of the left wheel? Hint: The average of both wheel's rpm is a constant.

c. Calculate the fundamental train value of the epicyclic stage.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: $r := 15 \cdot in \quad V := 50 \cdot mph \quad \omega_{eng} := 2000 rpm$

Solution: See Figure P9-3 and Mathcad file P0927.

1. Calculate the rear wheel rpm and the gear ratio between ring and pinion.

$$\omega_{wheel} := \frac{V}{r} \qquad \omega_{wheel} = 560.225\, rpm$$

$$m_G := \frac{\omega_{eng}}{\omega_{wheel}} \qquad m_G = 3.570$$

2. Using the hint given, calculate the speed of the left wheel when the right wheel hits the ice.

$$\omega_{avg} := \omega_{wheel} \qquad \omega_{right} := 800 \cdot rpm$$

$$\omega_{left} := 2 \cdot \omega_{avg} - \omega_{right} \qquad \omega_{left} = 320.451\, rpm$$

PROBLEM 9-28

Statement: Design a speed-reducing planetary gearbox to be used to lift a 5-ton load 50 ft with a motor that develops 20 lb-ft of torque at its operating speed of 1750 rpm. The available winch drum has no more than a 16-in diameter when full of its steel cable. The speed reducer should be no larger in diameter than the winch drum. Gears of no more than 75 teeth are desired, and diametral pitch needs to be no larger than 6 to stand the stresses. Make multiview sketches of your design and show all calculations. How long will it take to raise the load with your design?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given:

Load $W := 10000\cdot lbf$

Motor torque $T_m := 20\cdot ft\cdot lbf$ Motor speed $\omega_m := 1750\cdot rpm$

Maximum drum diameter $d_{max} := 16\cdot in$

Assumptions: Minimum drum diameter (with no cable) is $d_{min} := 8\cdot in$

Solution: See Figure 9-33 and Mathcad file P0901.

1. Determine the minimum gear ratio required.

Worst load torque: $T_L := W\cdot\dfrac{d_{max}}{2}$ $T_L = 6667\ ft\cdot lbf$

Minimum gear ratio required to lift load:

$$m_G := \frac{T_L}{T_m} \qquad m_G = 333.333$$

2. Use a combination of basic planetary gearsets, such as that shown in Figure 9-33, compounded in the following way. The input to each stage will be the sun gear. The output will be the arm, and the ring gear will be stationary. All stages will mesh with the same ring gear, which will be elongated to reach the planets of all stages. Except for the last stage, the arm of one stage will be coupled directly to the sun gear of the next stage. All stages will have the same number of teeth on the sun gears and the same (different from the sun gears) for the planets. Since they all mesh with the same ring gear, they must all have the same diametral pitch and pressure angle. Let the pressure angle be 25 deg.

3. Calculate the number of stages required to achieve the required gear ratio.

Let the diametral pitch be $p_d := 4\cdot in^{-1}$

Ring gear pitch diameter $d_r := 16\cdot in$ $N_r := p_d\cdot d_r$ $N_r = 64$

Number of teeth on each sun gear $N_s := 12$

Number of teeth on each planet gear $N_p := \dfrac{N_r - N_s}{2}$ $N_p = 26$

For a single stage, the gear ratio for the planetary configuration of Figure 9-33 is $-(N_r - N_s)/N_s$. For multiple stages,

$$m_G := \left(\frac{N_r - N_s}{N_s}\right)^n$$

where n is the number of stages. Solving for n,

$$n := ceil\left(\frac{log(m_G)}{log\left(\frac{N_r - N_s}{N_s}\right)}\right) \qquad n = 4 \quad \text{stages}$$

4. Calculate the actual gear ratio achieved.

$$m_G := \left(\frac{N_r - N_s}{N_s}\right)^n \qquad m_G = 352.605$$

5. Calculate the time required to raise the load $L := 50 \cdot ft$

Average drum diameter $\quad d_{avg} := \frac{1}{2} \cdot (d_{max} + d_{min}) \quad d_{avg} = 12.000\ in$

Average cable velocity $\quad V_{avg} := \frac{d_{avg}}{2} \cdot \frac{\omega_m}{m_G} \quad V_{avg} = 15.592 \frac{ft}{min}$

Average time to raise load $\quad t_{avg} := \frac{L}{V_{avg}} \quad t_{avg} = 3.2\ min$

PROBLEM 9-29

Statement: Determine all possible two-stage compound gear combinations that will give an approximation to the Naperian base 2.71828. Limit tooth numbers to between 18 and 80. Determine the arrangement that gives the smallest error.

Given:

$ratio := 2.71828$ $\quad N_{min} := 18$ $\quad N_{max} := 80$ $\quad err := 0.001\%$

Solution: See Mathcad file P0929.

1. Input the given data into the *TK-Solver* file *compound.tkw*. The $n := 8$ tooth number combinations given below are the result. Let $i := 1, 2 .. n$

$N2_i :=$	$N3_i :=$	$N4_i :=$	$N5_i :=$
25	67	70	71
29	57	47	65
30	32	31	79
30	64	62	79
31	48	45	79
31	64	60	79
31	79	75	80
35	67	50	71

$$least(r,n) := \left| \begin{array}{l} low \leftarrow r_1 \\ index \leftarrow 1 \\ for\ \ i \in 2..n \\ \quad \left| \begin{array}{l} index \leftarrow i \ \ if\ \ r_i < low \\ low \leftarrow r_i \ \ if\ \ r_i < low \end{array} \right. \\ index \end{array} \right.$$

2. Determine the stage ratios and the overall ratio for each set.

Stage 1 ratio $\quad ratio1_i := \dfrac{N3_i}{N2_i}$ $\qquad$ Stage 2 ratio $\quad ratio2_i := \dfrac{N5_i}{N4_i}$

Actual ratio for given set: $\quad Ratio_i := ratio1_i \cdot ratio2_i$

Error in ratio: $\quad Error_i := \left| ratio - Ratio_i \right|$

3. Find the set with the least error.

$R := least(Error, n)$ $\qquad R = 3$

$N2_R = 30$ $\qquad N3_R = 32$ $\qquad N4_R = 31$ $\qquad N5_R = 79$

$ratio1_R = 1.06667$ $\qquad ratio2_R = 2.54839$

$Ratio_R = 2.718280$ $\qquad Error_R = 4.30108 \times 10^{-7}$

PROBLEM 9-30

Statement: Determine all possible two-stage compound gear combinations that will give an approximation to 2π. Limit tooth numbers to between 18 and 80. Determine the arrangement that gives the smallest error.

Given:

$ratio := 6.283185$ $N_{min} := 15$ $N_{max} := 90$ $err := 0.001\%$

Solution: See Mathcad file P0930.

1. Input the given data into the *TK-Solver* file *compound.tkw*. The $n := 4$ tooth number combinations given below are the result. Let $i := 1, 2..n$

$N2_i :=$	$N3_i :=$	$N4_i :=$	$N5_i :=$
16	63	47	75
23	75	41	79
25	51	25	77
28	85	43	89

$least(r, n) :=$
```
low ← r_1
index ← 1
for i ∈ 2..n
    index ← i  if r_i < low
    low ← r_i  if r_i < low
index
```

2. Determine the stage ratios and the overall ratio for each set.

Stage 1 ratio $ratio1_i := \frac{N3_i}{N2_i}$ Stage 2 ratio $ratio2_i := \frac{N5_i}{N4_i}$

Actual ratio for given set: $Ratio_i := ratio1_i \cdot ratio2_i$

Error in ratio: $Error_i := |ratio - Ratio_i|$

3. Find the set with the least error.

$R := least(Error, n)$ $R = 3$

$N2_R = 25$ $N3_R = 51$ $N4_R = 25$ $N5_R = 77$

$ratio1_R = 2.04000$ $ratio2_R = 3.08000$

$Ratio_R = 6.283200$ $Error_R = 1.50000 \times 10^{-5}$

✍ PROBLEM 9-31

Statement: Determine all possible two-stage compound gear combinations that will give an approximation to $3\pi/2$. Limit tooth numbers to between 18 and 80. Determine the arrangement that gives the smallest error.

Given:

$ratio := 1.570796$ $N_{min} := 20$ $N_{max} := 100$ $err := 0.001\%$

Solution: See Mathcad file P0931.

1. Input the given data into the *TK-Solver* file *compound.tkw*. The $n := 29$ tooth number combinations given below are the result. Let $i := 1, 2 .. n$

$N2_i :=$	$N3_i :=$	$N4_i :=$	$N5_i :=$
22	31	61	68
22	34	61	62
22	49	95	67
25	51	100	77
29	44	85	88
32	63	94	75
33	34	61	93
33	51	61	62
36	55	71	73
37	59	67	66
38	49	55	67
41	57	77	87
41	75	92	79
43	50	57	77
43	55	57	70
43	58	79	92
44	62	61	68
44	67	95	98
45	64	67	74
46	75	82	79
47	63	64	75
47	65	81	92
50	51	50	77
55	62	61	85
55	67	76	98
56	85	86	89
57	77	86	100
58	88	85	88
61	68	66	93

```
least(r,n) := | low ← r_1
              | index ← 1
              | for i ∈ 2..n
              |    | index ← i  if r_i < low
              |    | low ← r_i  if r_i < low
              | index
```

2. Determine the stage ratios and the overall ratio for each set.

Stage 1 ratio $ratio1_i := \dfrac{N3_i}{N2_i}$ Stage 2 ratio $ratio2_i := \dfrac{N5_i}{N4_i}$

Actual ratio for given set: $Ratio_i := ratio1_i \cdot ratio2_i$

Error in ratio: $Error_i := \left| ratio - Ratio_i \right|$

3. Find the set with the least error.

$R := least(Error, n)$ $R = 12$

$N2_R = 41$ $N3_R = 57$ $N4_R = 77$ $N5_R = 87$

$ratio1_R = 1.39024$ $ratio2_R = 1.12987$

$Ratio_R = 1.570795$ $Error_R = 9.41400 \times 10^{-7}$

PROBLEM 9-32

Statement: Determine all possible two-stage compound gear combinations that will give an approximation to $3\pi/2$. Limit tooth numbers to between 18 and 80. Determine the arrangement that gives the smallest error.

Given:

$ratio := 4.71239 \qquad N_{min} := 20 \qquad N_{max} := 100 \qquad err := 0.001\%$

Solution: See Mathcad file P0932.

1. Input the given data into the *TK-Solver* file *compound.tkw*. The $n := 5$ tooth number combinations given below are the result. Let $i := 1, 2 .. n$

$N2_i :=$	$N3_i :=$	$N4_i :=$	$N5_i :=$
22	68	61	93
27	65	47	92
37	68	39	100
37	80	39	85
38	77	43	100

$least(r, n) :=$
```
low ← r_1
index ← 1
for i ∈ 2..n
    index ← i  if r_i < low
    low ← r_i  if r_i < low
index
```

2. Determine the stage ratios and the overall ratio for each set.

Stage 1 ratio $\quad ratio1_i := \dfrac{N3_i}{N2_i}$ $\qquad$ Stage 2 ratio $\quad ratio2_i := \dfrac{N5_i}{N4_i}$

Actual ratio for given set: $\quad Ratio_i := ratio1_i \cdot ratio2_i$

Error in ratio: $\quad Error_i := \left| ratio - Ratio_i \right|$

3. Find the set with the least error.

$R := least(Error, n) \qquad R = 4$

$N2_R = 37 \qquad N3_R = 80 \qquad N4_R = 39 \qquad N5_R = 85$

$ratio1_R = 2.16216 \qquad ratio2_R = 2.17949$

$Ratio_R = 4.712405 \qquad Error_R = 1.47124 \times 10^{-5}$

PROBLEM 9-33

Statement: Figure P-9-4a shows a reverted clock train. Design it using 25-deg nominal pressure angle gears of 24 p_d having between 12 and 150 teeth. Determine the tooth numbers and nominal center distance. If the center distance has a manufacturing tolerance of plus or minus 0.006 in, what will the pressure angle and the backlash at the minute hand be at each extreme of tolerance?

Given: Diametral pitch $p_d := 24 \cdot in^{-1}$ Tolerance (one side) $t := 0.006 in$

Solution: See Mathcad file P0933.

1. Write the speeds of the minute and hour hands and their ratio.

   Minute hand $\omega_1 := \dfrac{2 \cdot \pi \cdot rad}{hr}$ Hour hand $\omega_2 := \dfrac{2 \cdot \pi \cdot rad}{12 \cdot hr}$

   $ratio := \dfrac{\omega_1}{\omega_2}$ $ratio = 12.000$

   Assume some input speed ω_{in} to the shaft with gears B and C on it. Then,

   $\omega_1 := \dfrac{N_B}{N_A} \cdot \omega_{in}$ $\omega_2 := \dfrac{N_C}{N_E} \cdot \omega_{in}$ $\dfrac{\omega_1}{\omega_2} = \dfrac{N_B}{N_A} \cdot \dfrac{N_E}{N_C} = ratio$

   $\dfrac{N_B}{N_A} = \dfrac{N_3}{N_2} = r_1$ $\dfrac{N_E}{N_C} = \dfrac{N_5}{N_4} = r_2$ $ratio := r_1 \cdot r_2$

2. Let r_1 and r_2 be factors of ratio. Say,

   Minute ratio $r_1 := 3$ Hour ratio $r_2 := 4$

3. Following the procedure of Example 9-3,

   Tooth number index $i := 2, 3 .. 5$ $N_2 + N_3 = N_4 + N_5 = K$ and, $r_1 := \dfrac{N_3}{N_2}$ $r_2 := \dfrac{N_5}{N_4}$

   Solving independently for N_2 and N_4, $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   where $K_{min} := (r_1 + 1) \cdot (r_2 + 1)$ $K_{min} = 20.000$

   By iteration, find a multiple of K_{min} that will result in a minimum number of teeth on N_2 and N_4.

   $K := 3 \cdot K_{min}$ $N_2 := \dfrac{K}{r_1 + 1}$ $N_4 := \dfrac{K}{r_2 + 1}$

   $K = 60.000$ $N_2 = 15$ $N_4 = 12$

4. The other gears will have tooth numbers of

   $N_3 := r_1 \cdot N_2$ $N_3 = 45$ $N_5 := r_2 \cdot N_4$ $N_5 = 48$

The pitch diameters are: $d_i := \frac{N_i}{pd}$ $i = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ $\frac{d_i}{in} = \begin{pmatrix} 0.6250 \\ 1.8750 \\ 0.5000 \\ 2.0000 \end{pmatrix}$ $N_i = \begin{pmatrix} 15 \\ 45 \\ 12 \\ 48 \end{pmatrix}$ A B C E

5. The nominal center distance is $C := 0.5 \cdot (d_2 + d_3)$ $C = 1.250\, in$

6. Calculate the extreme values of center distance and their ratios with respect to the nominal center distance.

$C_{max} := C + t$ $C_{max} = 1.2560\, in$ $r_{max} := \frac{C_{max}}{C}$ $r_{max} = 1.0048$

$C_{min} := C - t$ $C_{min} = 1.2440\, in$ $r_{min} := \frac{C_{min}}{C}$ $r_{min} = 0.9952$

7. Using the equation in step 11 of Example 9-1, calculate the pressure angle that would result if the center distance were at its extreme values.

$\phi_{max} := acos\left(\frac{cos(25 \cdot deg)}{r_{max}}\right)$ $\phi_{max} = 25.581\, deg$

$\phi_{min} := acos\left(\frac{cos(25 \cdot deg)}{r_{min}}\right)$ $\phi_{min} = 24.401\, deg$

8. The backlash in the minute hand is found using equation 9.3,

$\theta_{Bmax} := 43200 \cdot t \cdot \frac{tan(\phi_{max})}{\pi \cdot d_2}$ $\theta_{Bmax} = 63$ Minutes of arc

Depending on the nominal clearance in the gearset, decreasing the center distance will eventually cause the teeth to jamb and the gears may not be capable of assembly.

PROBLEM 9-34

Statement: Figure P9-4b shows a three-speed shiftable transmission. Shaft F, with the cluster of gears E, G, and H, is capable of sliding left and right to engage and disengage the three gearsets in turn. Design the three reverted stages to give output speeds at shaft F of 150, 350, and 550 rpm for an input speed of 450 rpm on shaft D.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Output speeds:

$\omega_{F1} := 150\cdot rpm$ $\qquad \omega_{F2} := 350\cdot rpm$ $\qquad \omega_{F3} := 550\cdot rpm$

Input speed: $\omega_D := 450\cdot rpm$

Solution: See Figure P9-4*b* and Mathcad file P0934.

1. Determine the gearset ratios

$$r_1 := \frac{\omega_D}{\omega_{F1}} \qquad r_2 := \frac{\omega_D}{\omega_{F2}} \qquad r_3 := \frac{\omega_D}{\omega_{F3}}$$

$$r_1 = 3.0000 \qquad r_2 = 1.2857 \qquad r_3 = 0.8182$$

2. Following the procedure of Example 9-3,

$$N_A + N_E = N_B + N_G = N_C + N_H = K \text{ and,} \qquad r_1 := \frac{N_E}{N_A} \qquad r_2 := \frac{N_H}{N_C} \qquad r_3 := \frac{N_G}{N_B}$$

Solving independently for N_A, N_C and N_B, $\quad N_A := \frac{K}{r_1 + 1} \qquad N_C := \frac{K}{r_2 + 1} \qquad N_B := \frac{K}{r_3 + 1}$

where $\quad K_{min} := (r_1 + 1)\cdot(r_2 + 1)\cdot(r_3 + 1) \qquad K_{min} = 1.000\frac{1280}{77}$

By iteration, find a multiple of K_{min} that will result in a minimum, integer number of teeth on the drivers:

$$K := \frac{77}{16}\cdot K_{min} \qquad N_A := \frac{K}{r_1 + 1} \qquad N_C := \frac{K}{r_2 + 1} \qquad N_B := \frac{K}{r_3 + 1}$$

$$K = 80.000 \qquad N_A = 20 \qquad N_C = 35 \qquad N_B = 44$$

These are acceptable tooth numbers for gears with a 25- or 20-deg pressure angle that are cut by a hob.

3. The driven gears on shaft F will have tooth numbers of

$$N_E := r_1\cdot N_A \qquad N_E = 60 \qquad N_H := r_2\cdot N_C \qquad N_H = 45$$

$$N_G := r_3\cdot N_B \qquad N_G = 36$$

PROBLEM 9-35

Statement: Figure P9-5*a* shows an epicyclic train used to drive a winch drum. Gear A, on shaft 1, is driven at 20 rpm CW and gear D, on shaft 2, is fixed to ground. The tooth numbers are indicated in the figure. Determine the speed and direction of the drum. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$N_A := 72$ $N_B := 16$ $N_C := 48$ $N_D := 40$

Input speeds: $\omega_A := -20 \cdot rpm$ $\omega_D := 0 \cdot rpm$

Basic gearset efficiency: $E_0 := 0.98$

Solution: See Figure P9-5*a* and Mathcad file P0935.

1. Determine the speed of the arm using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-5a.

$$R := \left(-\frac{N_A}{N_B}\right) \cdot \left(-\frac{N_C}{N_D}\right) \qquad R = 5.40000$$

Solve the right-hand equation above for ω_{arm} with $\omega_D = 0$.

$$\omega_{arm} := \frac{R}{R-1} \cdot \omega_A \qquad \omega_{arm} = -24.55\, rpm$$

The arm drives the drum, so

$$\omega_{drum} := \omega_{arm} \qquad \omega_{drum} = -24.5\, rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := R \qquad \rho = 5.400$$

3. The combination of $\rho > 1$, shaft 2 fixed, and input to shaft 1 corresponds to Case 1 in Table 9-12 giving an efficiency of

$$\eta := \frac{\rho \cdot E_0 - 1}{\rho - 1} \qquad \eta = 0.975$$

PROBLEM 9-36

Statement: Figure P9-5*b* shows an epicyclic train. The arm is driven CCW at 20 rpm. Gear A is driven at 40 rpm CW. The tooth numbers are indicated in the figure. Find the speed of the ring gear D.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$N_A := 17 \qquad N_B := 28 \qquad N_C := 15 \qquad N_D := 60$

Input speeds: $\omega_A := -40 \cdot rpm \qquad \omega_{arm} := 20 \cdot rpm$

Solution: See Figure P9-5*b* and Mathcad file P0936.

1. Determine the speed of the ring gear using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-5*b*.

$$R := \left(-\frac{N_A}{N_B}\right) \cdot \left(\frac{N_C}{N_D}\right) \qquad R = -0.15179$$

Solve the right-hand equation above for ω_D.

$$\omega_D := R \cdot \left(\omega_A - \omega_{arm}\right) + \omega_{arm} \qquad \omega_D = 29.11\, rpm$$

PROBLEM 9-37

Statement: Figure P9-6*a* shows an epicyclic train. The arm is driven at 60 rpm CW and gear A, on shaft 1, is fixed to ground. The tooth numbers are indicated in the figure. Find the speed of gear D, on shaft 2. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$N_A := 108 \qquad N_B := 27 \qquad N_C := 100 \qquad N_D := 35$

Input speeds: $\omega_{arm} := -60 \cdot rpm \qquad \omega_A := 0 \cdot rpm$

Basic gearset efficiency: $E_0 := 0.98$

Solution: See Figure P9-6*a* and Mathcad file P0937.

1. Determine the speed of the sun gear using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-6*a*.

$$R := \left(-\frac{N_A}{N_B}\right) \cdot \left(-\frac{N_C}{N_D}\right) \qquad R = 11.42857$$

Solve the right-hand equation above for ω_D with $\omega_A = 0$.

$$\omega_D := (1 - R) \cdot \omega_{arm} \qquad \omega_D = 625.71\, rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := R \qquad \rho = 11.429$$

3. The combination of $\rho > 1$, shaft 1 fixed, and input to the arm corresponds to Case 4 in Table 9-12 giving an efficiency of

$$\eta := \frac{\rho - 1}{\rho - E_0} \qquad \eta = 0.998$$

PROBLEM 9-38

Statement: Figure P9-6*b* shows a differential. Gear A is driven at 10 rpm CCW and gear B is driven CW at 24 rpm. The tooth numbers are indicated in the figure. Find the speed of gear D.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Tooth numbers:

$N_{A'} := 18 \qquad N_{B'} := 18 \qquad N_C := 18 \qquad N_D := 30$

Input speeds: $\omega_A := 10\cdot rpm \qquad \omega_B := -24\cdot rpm$

Solution: See Figure P9-6*b* and Mathcad file P0938.

1. Determine the speed of the sun gear using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A' and last be B'. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_{B'} - \omega_{arm}}{\omega_{A'} - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-6*b*.

$$R := \left(-\frac{N_{A'}}{N_{B'}}\right) \qquad R = -1.00000$$

Solve the right-hand equation above for ω_D with $\omega_A = 0$.

$$\omega_{A'} := \omega_A \qquad \omega_{B'} := \omega_B$$

$$\omega_{arm} := \frac{R\cdot\omega_{A'} - \omega_{B'}}{R - 1} \qquad \omega_{arm} = -7.000\,rpm$$

Gear D is attached to the arm shaft so,

$$\omega_D := \omega_{arm} \qquad \omega_D = -7.00\,rpm$$

PROBLEM 9-39

Statement: Figure P9-7*a* shows a gear train containing both compound-reverted and epicyclic stages. The tooth numbers are indicated in the figure. The motor is driven CCW at 1750 rpm. Find the speeds of shafts 1 and 2.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$N_A := 15$ $N_B := 40$ $N_C := 20$ $N_D := 35$ $N_E := 48$

$N_F := 12$ $N_G := 72$ $N_H := 18$ $N_J := 42$ $N_K := 15$

Motor speed: $\omega_m := 1750 \cdot rpm$ CCW

Solution: See Figure P9-7*a* and Mathcad file P0939.

1. The motor furnishes two inputs to the epicyclic train, which is composed of gears E (sun), F (planets), and G (ring). One input is to the sun gear through the reverted train ABCD. The other is directly from the motor to the arm.

2. Determine the speed of the shaft containing gears D and E using equations 9.8.

$$\omega_D := \left(-\frac{N_A}{N_B}\right) \cdot \left(-\frac{N_C}{N_D}\right) \cdot \omega_m \qquad \omega_D = 375.000\,rpm$$

$$\omega_E := \omega_D \qquad \omega_E = 375.000\,rpm$$

3. Determine the speed of the ring gear G using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be E and last be G. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_G - \omega_{arm}}{\omega_E - \omega_{arm}} = R \qquad \omega_{arm} := \omega_m$$

Calculate R using equation 9.14 and inspection of Figure P9-7*a*.

$$R := -\frac{N_E}{N_G} \qquad R = -0.66667$$

Solve the right-hand equation above for ω_G.

$$\omega_G := R \cdot \left(\omega_E - \omega_{arm}\right) + \omega_{arm} \qquad \omega_G = 2666.67\,rpm$$

4. Gears G and H rotate at the same speed and gear H drives the two output shafts. Calculate their speed using equation 9.7.

$$\omega_1 := -\frac{N_H}{N_J} \cdot \omega_G \qquad \omega_1 = -1142.9\,rpm$$

$$\omega_2 := -\frac{N_H}{N_K} \cdot \omega_G \qquad \omega_2 = -3200.0\,rpm$$

PROBLEM 9-40

Statement: Figure P9-35*b* shows (schematically) a compound epicyclic train. The tooth numbers are 50, 25, 35, and 90 for gears 2, 3, 4, and 5, respectively. The arm is driven at 180 rpm CW and gear 5 is fixed to ground. Determine the speed and direction of gear 2. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Tooth numbers:

$N_2 := 50 \qquad N_3 := 25 \qquad N_4 := 35 \qquad N_5 := 90$

Input speeds: $\omega_{arm} := -180\cdot rpm \qquad \omega_5 := 0\cdot rpm$

Basic gearset efficiency: $E_0 := 0.98$

Solution: See Figure P9-7*b* and Mathcad file P0940.

1. Determine the speed of the sun gear 2 using the formula method for analyzing an epicyclic train. To start, choose a first and last gears that mesh with gears that have planetary motion. Let the first gear be 2 and last be 5. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-7*b*.

$$R := \left(-\frac{N_2}{N_3}\right)\cdot\left(\frac{N_4}{N_5}\right) \qquad R = -0.77778$$

Solve the right-hand equation above for ω_2 with $\omega_5 = 0$.

$$\omega_2 := \frac{(R-1)}{R}\cdot\omega_{arm} \qquad \omega_2 = -411.43\,rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := \frac{1}{R} \qquad \rho = -1.286$$

3. The combination of $\rho < -1$, shaft 1 fixed, and input to the arm corresponds to Case 8 in Table 9-12 giving an efficiency of

$$\eta := \frac{E_0\cdot(\rho - 1)}{\rho\cdot E_0 - 1} \qquad \eta = 0.991$$

PROBLEM 9-41

Statement: Figure P9-8 shows a compound epicyclic train. Gear 2 is driven at 800 rpm CCW and gear D is fixed to ground. The tooth numbers are indicated in the figure. Determine the speed and direction of gears 1 and 3.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$N_A := 24 \quad N_B := 30 \quad N_C := 72 \quad N_D := 60 \quad N_E := 120 \quad N_F := 100$

Gear 2 speed: $\omega_2 := 800 \cdot rpm$ CCW

Solution: See Figure P9-8 and Mathcad file P0941.

1. Determine the speed of the arm using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R \qquad \omega_A := \omega_2$$

Calculate R using equation 9.14 and inspection of Figure P9-8.

$$R := -\left(\frac{N_A}{N_B}\right) \cdot \left(\frac{N_C}{N_D}\right) \qquad R = -0.96000$$

Solve the right-hand equation above for ω_{arm} with $\omega_D = 0$.

$$\omega_{arm} := \frac{R}{R-1} \cdot \omega_A \qquad \omega_{arm} = 391.84\, rpm$$

The arm drives gear 1, so

$$\omega_1 := \omega_{arm} \qquad \omega_1 = 391.8\, rpm$$

2. Determine the speed of gear F using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be F. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_F - \omega_{arm}}{\omega_A - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-1.

$$R := \left(\frac{N_A}{N_B}\right) \cdot \left(\frac{N_E}{N_F}\right) \qquad R = 0.96000$$

Solve the right-hand equation above for ω_F.

$$\omega_F := R \cdot \left(\omega_A - \omega_{arm}\right) + \omega_{arm} \qquad \omega_F = 783.673\, rpm$$

Gear F drives gear 3, so

$$\omega_3 := \omega_F \qquad \omega_3 = 783.7\, rpm$$

PROBLEM 9-42

Statement: Figure P9-9*a* shows a compound epicyclic train. Shaft 1 is driven at 300 rpm CCW and gear A is fixed to ground. The tooth numbers are indicated in the figure. Determine the speed and direction of shaft 2.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Tooth numbers:

$N_A := 56 \quad N_B := 18 \quad N_C := 48 \quad N_D := 26 \quad N_E := 60 \quad N_F := 18 \quad N_G := 68$

Shaft 1 speed: $\omega_1 := 300\cdot rpm$ CCW

Solution: See Figure P9-9*a* and Mathcad file P0942.

1. Shaft 1 drives arm-1, the first stage arm, and arm-2, the second stage arm. The first stage is composed of gears A, B, C, and D, with gear A fixed. The second stage is composed of gears D, E, F, and G. Second stage inputs are from gear D and arm-2.

$$\omega_{arm} := \omega_1$$

2. Determine the speed of gear D using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be A and last be D. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_D - \omega_{arm}}{\omega_A - \omega_{arm}} = R \qquad \omega_A := 0\cdot rpm$$

Calculate R using equation 9.14 and inspection of Figure P9-9*a*.

$$R := \left(-\frac{N_A}{N_B}\right)\cdot\left(-\frac{N_C}{N_D}\right) \qquad R = 5.74359$$

Solve the right-hand equation above for ω_D with $\omega_A = 0$.

$$\omega_D := (1 - R)\cdot\omega_{arm} \qquad \omega_D = -1423.08\,rpm$$

3. Determine the speed of gear G using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be D and last be G. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_G - \omega_{arm}}{\omega_D - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-9*a*.

$$R := \left(-\frac{N_D}{N_E}\right)\cdot\left(-\frac{N_F}{N_G}\right) \qquad R = 0.11471$$

Solve the right-hand equation above for ω_G.

$$\omega_G := R\cdot\left(\omega_D - \omega_{arm}\right) + \omega_{arm} \qquad \omega_G = 102.4\,rpm$$

Gear G drives shaft 2, so

$$\omega_2 := \omega_G \qquad \omega_2 = 102.4\,rpm$$

PROBLEM 9-43

Statement: Figure P9-9*b* shows a compound epicyclic train. Shaft 1 is driven at 40 rpm. The tooth numbers are indicated in the figure. Determine the speed and direction of gears G and M.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$N_A := 36$ $N_B := 60$ $N_C := 40$ $N_{Dout} := 64$ $N_{Din} := 58$ $N_E := 18$ $N_F := 26$

$N_G := 66$ $N_H := 24$ $N_J := 54$ $N_K := 40$ $N_L := 38$ $N_M := 96$

Shaft 1 speed: $\omega_1 := 40 \cdot rpm$ CCW

Solution: See Figure P9-9*b* and Mathcad file P0943.

1. The motor furnishes inputs to both epicyclic trains. To the first stage, it drives the ring gear D through gear A, and it drives arm-1 through gears B and C. The second stage inputs are from shaft 1 through gears B and C to sun J, and from the output of the first stage through gear G to arm-2

2. Determine the speed of gear D using equations 9.7.

$$\omega_D := \left(-\frac{N_A}{N_{Dout}}\right) \cdot \omega_1 \qquad \omega_D = -22.500\, rpm$$

3. Determine the speed of arm-1 and gear J using equations 9.7.

$$\omega_{arm1} := \left(-\frac{N_B}{N_C}\right) \cdot \omega_1 \qquad \omega_{arm1} = -60.000\, rpm$$

$$\omega_J := \omega_{arm1}$$

4. Determine the speed of the ring gear G using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be D and last be G. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_G - \omega_{arm1}}{\omega_D - \omega_{arm1}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-9*b*.

$$R := \frac{N_{Din}}{N_E} \cdot \frac{N_F}{N_G} \qquad R = 1.26936$$

Solve the right-hand equation above for ω_G.

$$\omega_G := R \cdot \left(\omega_D - \omega_{arm1}\right) + \omega_{arm1} \qquad \omega_G = -12.40\, rpm$$

5. Determine the speed of gear L using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be J and last be L. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_L - \omega_{arm2}}{\omega_J - \omega_{arm2}} = R \qquad \omega_{arm2} := \omega_G$$

Calculate R using equation 9.14 and inspection of Figure P9-9b.

$$R := \left(-\frac{N_J}{N_H}\right)\cdot\left(-\frac{N_K}{N_L}\right) \qquad R = 2.36842$$

Solve the right-hand equation above for ω_L.

$$\omega_L := R\cdot\left(\omega_J - \omega_{arm2}\right) + \omega_{arm2} \qquad \omega_L = -125.14\,rpm$$

$$\omega_M := \omega_L \qquad \omega_M = -125.14\,rpm$$

PROBLEM 9-44

Statement: Calculate the ratios in the Model T transmission shown in Figure 9-46 (p. 476).

Given: Tooth numbers:

$N_3 := 27 \quad N_4 := 33 \quad N_5 := 24 \quad N_6 := 27 \quad N_7 := 21 \quad N_8 := 30$

Solution: See Figure P9-46 and Mathcad file P0944.

1. For low gear, the gearset consists of gears 3, 4, 6, and 7. Determine the ratio of input (arm) to output (gear 6) using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be gear 7 and last be gear 6. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_6 - \omega_{arm}}{\omega_7 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure 9-46.

$$R := \left(-\frac{N_3}{N_6}\right)\cdot\left(-\frac{N_7}{N_4}\right) \qquad R = 0.63636$$

Solve the right-hand equation above for the ratio $\omega_{arm}/\omega_6 = m_G$ for $\omega_7 = 0$.

$$m_{GL} := \frac{1}{(1 - R)} \qquad m_{GL} = 2.750$$

2. For reverse gear, the gearset consists of gears 3, 5, 6, and 8. Determine the ratio of input (arm) to output (gear 6) using the formula method for analyzing an epicyclic train. To start, choose a first and last gear that mesh with gears that have planetary motion. Let the first gear be gear 8 and last be gear 6. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_6 - \omega_{arm}}{\omega_8 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure 9-46.

$$R := \left(-\frac{N_3}{N_6}\right)\cdot\left(-\frac{N_8}{N_5}\right) \qquad R = 1.25000$$

Solve the right-hand equation above for the ratio $\omega_{arm}/\omega_6 = m_G$ for $\omega_8 = 0$.

$$m_{GR} := \frac{1}{(1 - R)} \qquad m_{GR} = -4.000$$

PROBLEM 9-45

Statement: Figure P7-26 shows a V-belt drive. The sheaves (pulleys) have pitch diameters of 150 and 300 mm, respectively. The smaller sheave is driven at a constant 1750 rpm. For a cross-sectional, differential element of the belt, write the equations of its acceleration for one complete trip around both pulleys including its travel between the pulleys. Compute and plot the acceleration of the differential element versus time for one circuit around the belt path. What does your analysis tell you about the dynamic behavior of the belt?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

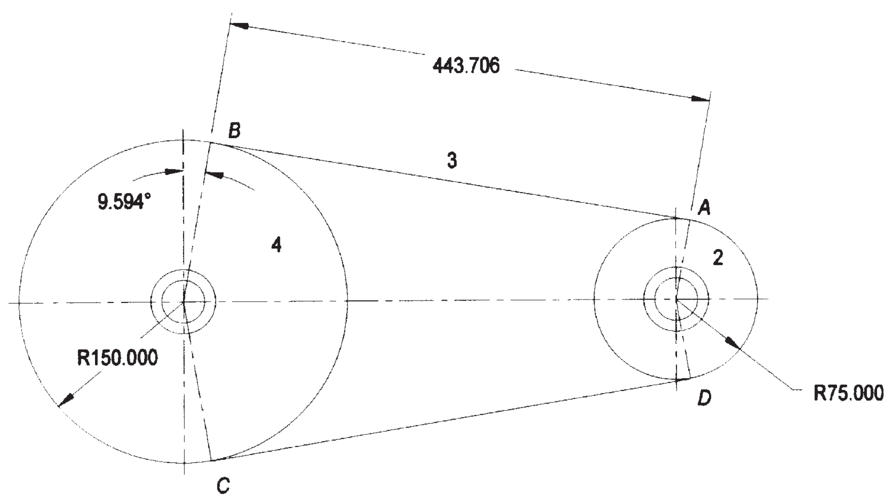
Given: Sheave radii and speed:

$r_2 := 75\cdot mm \qquad r_4 := 150\cdot mm \qquad \omega_2 := 1750\cdot rpm$

Assumptions: Center distance: $C := 450\, mm$

Solution: See Figure P7-26 and Mathcad file P0945.

1. Draw a schematic representation of the V-belt drive to scale and label it.



From the layout,

Angle to point of tangency	$\beta := 9.594\cdot deg$	
Distance from A to B	$L_{AB} := 443.706\cdot mm$	
Distance from B to C	$L_{BC} := (\pi + 2\cdot\beta)\cdot r_4$	$L_{BC} = 521.473\, mm$
Distance from C to D	$L_{CD} := 443.706\cdot mm$	
Distance from D to A	$L_{DA} := (\pi - 2\cdot\beta)\cdot r_2$	$L_{DA} = 210.502\, mm$
Total path length	$L := L_{AB} + L_{BC} + L_{CD} + L_{DA}$	$L = 1619.387\, mm$

2. Calculate the angular velocity of each sheave.

Sheave 2 $\qquad \omega_2 = 183.260\,\dfrac{rad}{sec}$

Sheave 4 $\qquad \omega_4 := \dfrac{r_2}{r_4}\cdot\omega_2 \qquad \omega_4 = 91.630\,\dfrac{rad}{sec}$

3. Calculate the acceleration of an element on the belt for each portion of the path, starting at A.

From A to B the acceleration is zero since the belt velocity is constant $\qquad A_{AB} := 0\cdot\frac{mm}{sec^2}$

From B to C the tangential acceleration is zero since the belt velocity is constant The normal component is

$$A_{BC} := r_4\cdot\omega_4^2 \qquad A_{BC} = 1.259\times 10^6\,\frac{mm}{sec^2}$$

From C to D the acceleration is zero since the belt velocity is constant $\qquad A_{CD} := 0\cdot\frac{mm}{sec^2}$

From D to A the tangential acceleration is zero since the belt velocity is constant The normal component is

$$A_{DA} := r_2\cdot\omega_2^2 \qquad A_{DA} = 2.519\times 10^6\,\frac{mm}{sec^2}$$

4. Plot the acceleration over the entire path using a path variable of s.

$$s := 0\cdot mm, 5\cdot mm..L$$

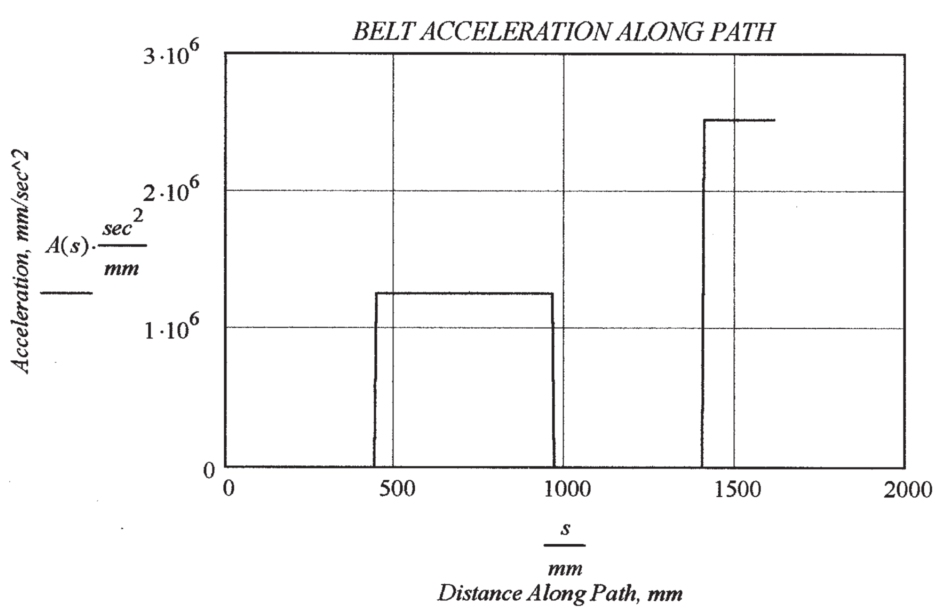
Define a range function that will plot a variable only between two limits using the Heaviside step function Φ.

$$R(s,a,b) := \Phi(s-a) - \Phi(s-b)$$

The acceleration function for the entire path is.

Let $\qquad s_1 := L_{AB} \qquad s_2 := L_{AB} + L_{BC} \qquad s_3 := L_{AB} + L_{BC} + L_{CD}$

$$A(s) := A_{AB}\cdot R(s,0\cdot mm,s_1) + A_{BC}\cdot R(s,s_1,s_2) + A_{CD}\cdot R(s,s_2,s_3) + A_{DA}\cdot R(s,s_3,L)$$



5. The graph shows the sudden change in acceleration as the belt enters and leaves a sheave. This results in infinite jerk at these points. This results in belt "hop" at these points, a condition that will cause eventual fatigue failure and wear in the belt. Because of the belt hop caused by infinite jerk at the initial belt/sheave tangency points the span of the belt is continuously vibrating.

PROBLEM 9-46

Statement: Figure P9-11 shows an involute that has been generated from a base circle of radius r_b. Point A is simultaneously on the base circle and on the involute. Point B is any point on the involute curve and point C is on the base circle where a line drawn from point B is tangent to the base circle. The angle ϕ_B (angle BOC) is known as the *involute pressure angle* corresponding to point B (not to be confused with the *pressure angle of two gears in mesh*, which is defined in the text). The angle AOB is known as the involute of ϕ_B and is often designated as *inv* ϕ_B. Using the definitions of the involute tooth form and Figure 9-5, derive an equation for *inv* ϕ_B as a function of ϕ_B alone.

Solution: See Figures P9-11 and 9-5 and Mathcad file P0946.

From Figure 9-5 and the definition of the involute curve, arc AC is equal to length BC. Therefore

Angle AOC = (arc AC) / OC = BC / OC and thus,

Angle $AOC = \tan \phi_B$ also,

Angle AOB = angle $AOC - \phi_B = \tan \phi_B - \phi_B$ thus,

$\text{inv } \phi_B = \tan \phi_B - \phi_B$

PROBLEM 9-47

Statement: Using the data and definitions from Problem 9-46, show that when the point B is at the pitch circle the *involute pressure angle* is equal to the *pressure angle of two gears in mesh.*

Solution: See Figures P9-11, 9-6 and 9-7; and Mathcad file P0947.

When the point B in Figure P9-11 is at the pitch point, points B and P (of Figures 9-6 and 9-7) are coincident. The angle between a line through the tangency point of the line of action with the base circle and point O (in Figure 9-6) is the *pressure angle of two gears in mesh*, ϕ. Looking at Figure P9-11, we see that this is also the angle designated as ϕ_B, which is the *involute pressure angle* of point B.

PROBLEM 9-48

Statement: Using the data and definitions from Problem 9-46 and with the point B at the pitch circle where the involute pressure angle ϕ_B is equal to the pressure angle ϕ of two gears in mesh, derive equation 9.4b.

Solution: See Figures P9-11, 9-6 and 9-9; and Mathcad file P0948.

From equation 9.4a and Figure 9-9 $p_c = \frac{\pi \cdot d}{N} = \frac{2\pi \cdot r_p}{N}$. Similarly, $p_b = \frac{\pi \cdot d_b}{N} = \frac{2\pi \cdot r_b}{N}$

Taking the ratio of p_b to p_c, $\frac{p_b}{p_c} = \frac{r_b}{r_c}$. But, from Figure 9-6, we see that $r_b = r_p \cdot cos(\phi)$. Thus,

$$\frac{p_b}{p_c} = \frac{r_p \cdot cos(\phi)}{r_p} \qquad \text{and} \qquad p_b = r_p \cdot cos(\phi)$$

PROBLEM 9-49

Statement: Using Figures 9-6 and 9-7, derive equation 9.2, which is used to calculate the length of action of a pair of meshing gears.

Solution: See Figures 9-6 and 9-7 and Mathcad file P0949.

1. Define points E_1 and E_2 as shown in the figure below.

   Note that the distance

   $O_1E_1 = r_g \cos\phi$ and $O_2E_2 = r_p \cos\phi$.

   $Z = AB = E_1B + E_2A - E_1E_2$

   $E_1E_2 = r_g \sin\phi + r_p \sin\phi = (r_g + r_p)\sin\phi = C\sin\phi$

   $O_1B = r_g + a_g$ and $O_2A = r_p + a_p$

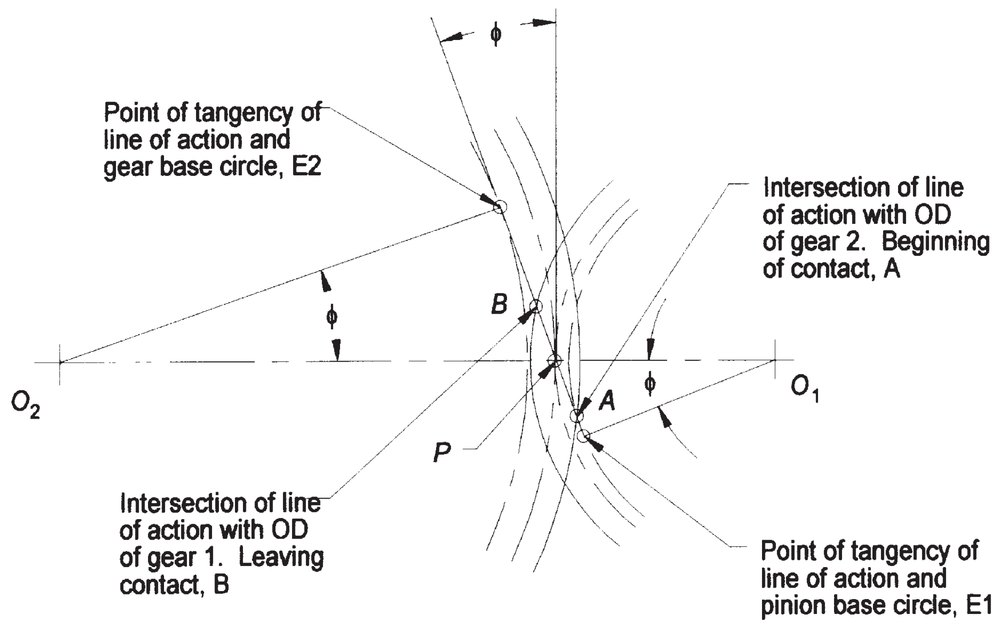
   From triangles

   O_1E_1B and O_2E_2A we have

   $$E1B = \sqrt{(r_g + a_g)^2 - (r_g \cdot cos(\phi))^2} \qquad E2A = \sqrt{(r_p + a_p)^2 - (r_p \cdot cos(\phi))^2}$$

   Thus,

   $$Z = \sqrt{(r_p + a_p)^2 - (r_p \cdot cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot cos(\phi))^2} - C \cdot sin(\phi)$$



PROBLEM 9-50

Statement: For lubrication purposes it is desired to have a backlash of 0.03 mm measured on the pitch circle of a 40-mm-diameter pinion in mesh with a 100-mm-diameter gear. if the gears are standard, full-depth, with 25 deg pressure angles, by how much should the center distance be increased?

Units: Arc minute: $arcmin := \frac{1}{60} \cdot deg$

Given: Pitch diameterss: $d_p := 40 \cdot mm \quad d_g := 100 \cdot mm$ Pressure angle: $\phi := 25 \cdot deg$

Desired backlash: $b := 0.03 \cdot mm$

Solution: See Figure 9-8 and Mathcad file P0950.

1. Convert the desired backlash from a linear to an angular measurement on the 40-mm-diameter pinion.

$$\theta_B := \frac{2 \cdot b}{d_p} \qquad \theta_B = 5.1566\, arcmin$$

2. Use equation 9.3 to calculate the change in center distance required.

$$\Delta C := \frac{\pi \cdot d_p \cdot \theta_B}{43200 \cdot tan(\phi) \cdot arcmin} \qquad \Delta C = 0.032\, mm$$

PROBLEM 9-51

Statement: For lubrication purposes it is desired to have a backlash of 0.0012 in measured on the pitch circle of a 2.000-in-diameter pinion in mesh with a 5.000-in-diameter gear. if the gears are standard, full-depth, with 25 deg pressure angles, by how much should the center distance be increased?

Units: Arc minute: $arcmin := \frac{1}{60} \cdot deg$

Given: Pitch diameterss: $d_p := 2.000 in$ $d_g := 5.000 in$ Pressure angle: $\phi := 25 \cdot deg$

Desired backlash: $b := 0.0012 in$

Solution: See Figure 9-8 and Mathcad file P0951.

1. Convert the desired backlash from a linear to an angular measurement on the 40-mm-diameter pinion.

$$\theta_B := \frac{2 \cdot b}{d_p} \qquad \theta_B = 4.1253\ arcmin$$

2. Use equation 9.3 to calculate the change in center distance required.

$$\Delta C := \frac{\pi \cdot d_p \cdot \theta_B}{43200 \cdot tan(\phi) \cdot arcmin} \qquad \Delta C = 1.287 \times 10^{-3}\ in$$

PROBLEM 9-52

Statement: A 22-tooth gear has standard full-depth involute teeth with module of 6. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance using the AGMA specifications in Table 9-1, substituting m for $1/p_d$.

Given: Tooth number $N := 22$ Module $m := 6 \cdot mm$

Solution: See Table 9-1 and Mathcad file P0952.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := N \cdot m$	$d = 132.00\, mm$
Circular pitch	$p_c := \pi \cdot m$	$p_c = 18.85\, mm$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := 1.0000 \cdot m$	$a = 6.00\, mm$
Dedendum	$b := 1.2500 \cdot m$	$b = 7.50\, mm$
Tooth thickness	$t := 0.5 \cdot p_c$	$t = 9.42\, mm$
Clearance	$c := 0.2500 \cdot m$	$c = 1.50\, mm$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions in mm should be displayed to two decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in hundredths of a millimeter.

✍ PROBLEM 9-53

Statement: A 40-tooth gear has standard full-depth involute teeth with module of 3. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance using the AGMA specifications in Table 9-1, substituting m for $1/p_d$.

Given: Tooth number $N := 40$ Module $m := 3\cdot mm$

Solution: See Table 9-1 and Mathcad file P0953.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter	$d := N\cdot m$	$d = 120.00\,mm$
Circular pitch	$p_c := \pi\cdot m$	$p_c = 9.42\,mm$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum	$a := 1.0000\cdot m$	$a = 3.00\,mm$
Dedendum	$b := 1.2500\cdot m$	$b = 3.75\,mm$
Tooth thickness	$t := 0.5\cdot p_c$	$t = 4.71\,mm$
Clearance	$c := 0.2500\cdot m$	$c = 0.75\,mm$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions in mm should be displayed to two decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in hundredths of a millimeter.

PROBLEM 9-54

Statement: A 30-tooth gear has standard full-depth involute teeth with module of 2. Calculate the pitch diameter, circular pitch, addendum, dedendum, tooth thickness, and clearance using the AGMA specifications in Table 9-1, substituting m for $1/p_d$.

Given: Tooth number $N := 30$ Module $m := 2 \cdot mm$

Solution: See Table 9-1 and Mathcad file P0954.

1. Calculate the pitch diameter using equation 9.4c and the circular pitch using equation 9.4d.

Pitch diameter $d := N \cdot m$ $d = 60.00\,mm$

Circular pitch $p_c := \pi \cdot m$ $p_c = 6.28\,mm$

2. Use the equations in Table 9-1 to calculate the addendum, dedendum, tooth thickness and clearance.

Addendum $a := 1.0000 \cdot m$ $a = 2.00\,mm$

Dedendum $b := 1.2500 \cdot m$ $b = 2.50\,mm$

Tooth thickness $t := 0.5 \cdot p_c$ $t = 3.14\,mm$

Clearance $c := 0.2500 \cdot m$ $c = 0.50\,mm$

Note: The circular tooth thickness is exactly half of the circular pitch, so the equation used above is more accurate than the one in Table 9-1. Also, all gear dimensions in mm should be displayed to two decimal places since manufacturing tolerances for gear teeth profiles are usually expressed in hundredths of a millimeter.

PROBLEM 9-55

Statement: Determine the minimum number of teeth on a pinion with a 20 deg pressure angle for the following gear-to-pinion ratios such that there will be no tooth-to-tooh interference: 1:1, 2:1, 3:1, 4:1, and 5:1.

Given: Pressure angle $\phi := 20 \cdot deg$

Solution: See Tables 9-4 and 9-5 and Mathcad file P0955.

1. From Table 9-5a, the minimum number of teeth on a 20-deg pressure angle pinion that can mesh with another gear is 13 if there are no more than 16 teeth on the other gear. So, for a 1:1 ratio, both gears can have 13 teeth. Looking further at Table 9-5a, we see that there must be more teeth on the pinion as the ratio increases. For instance, for ratios of 2:1 and 3:1, the pinion must have at least 15 teeth since a 15-tooth pinion can mesh with a gear with up to 45 teeth. If the raio is 4:1 or 5:1, the minimum number of teeth on the pinion is 16 since these ratios would require more than 45 teeth on the gear.

SUMMARY

Ratio	Min teeth on pinion	Number teeth on gear
1:1	13	13
2:1	15	30
3:1	15	45
4:1	16	64
5:1	16	80

However, from Table 9-4b, we see that none of these pinions could be cut with a hob without undercutting. They would have to be produced by other methods to avoid weakening the tooth by undercutting.

PROBLEM 9-56

Statement: Determine the minimum number of teeth on a pinion with a 25 deg pressure angle for the following gear-to-pinion ratios such that there will be no tooth-to-tooh interference: 1:1, 2:1, 3:1, 4:1, and 5:1.

Given: Pressure angle $\phi := 25 \cdot deg$

Solution: See Tables 9-4 and 9-5 and Mathcad file P0956.

1. From Table 9-5b, the minimum number of teeth on a 25-deg pressure angle pinion that can mesh with another gear is 9 if there are no more than 13 teeth on the other gear. So, for a 1:1 ratio, both gears can have 9 teeth. Looking further at Table 9-5b, we see that there must be more teeth on the pinion as the ratio increases. For instance, for ratios of 2:1 and 3:1, the pinion must have at least 10 teeth since a 10-tooth pinion can mesh with a gear with up to 32 teeth. If the raio is 4:1 or 5:1, the minimum number of teeth on the pinion is 11 since these ratios would require more than 32 teeth on the gear.

SUMMARY

Ratio	Min teeth on pinion	Number teeth on gear
1:1	9	9
2:1	10	20
3:1	10	30
4:1	11	44
5:1	11	55

However, from Table 9-4b, we see that none of these pinions could be cut with a hob without undercutting. They would have to be produced by other methods to avoid weakening the tooth by undercutting.

PROBLEM 9-57

Statement: A pinion with a 3.000-in pitch diameter is to mesh with a rack. What is the largest standard tooth size, in terms of diametral pitch, that can be used without having any interference?
a. For a 20-deg pressure angle
b. For a 25-deg pressure angle

Given: Pitch diameter: $d := 3.000\,in$

Solution: See Table 9-4 and Mathcad file P0957.

1. Assume that the pinion is generated by means other than being cut by a hob.

a. Pressure angle of 20 deg.

$$N_{min} := 18 \qquad p_{dmin} := \frac{N_{min}}{d} \qquad p_{dmin} = 6.000\,in^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 6. $p_d := 6 \cdot in^{-1}$.
The resulting number of teeth on the pinion is:

$$N := p_d \cdot d \qquad N = 18$$

b. Pressure angle of 25 deg

$$N_{min} := 12 \qquad p_{dmin} := \frac{N_{min}}{d} \qquad p_{dmin} = 4.000\,in^{-1}$$

From Table 9-2, the smallest standard diametral pitch (largest tooth size) that can be used is 4. $p_d := 4 \cdot in^{-1}$.
The resulting number of teeth on the pinion is:

$$N := p_d \cdot d \qquad N = 12$$

PROBLEM 9-58

Statement: A pinion with a 75-mm pitch diameter is to mesh with a rack. What is the largest standard tooth size, in terms of diametral pitch, that can be used without having any interference?

a. For a 20-deg pressure angle
b. For a 25-deg pressure angle

Given: Pitch diameter: $d := 75 \cdot mm$

Solution: See Table 9-4 and Mathcad file P0958.

1. Assume thatthe pinion is generated by means other than being cut by a hob.

a. Pressure angle of 20 deg.

$$N_{min} := 18 \qquad m := \frac{d}{N_{min}} \qquad m = 4.167\,mm$$

From Table 9-3, the largest standard module (largest tooth size) that can be used is 4 but, in order to have a whole number of teeth, the module must be 3 or 5. Since a module of 5 would result in fewer than 18 teeth, let $m := 3 \cdot mm$ The resulting number of teeth on the pinion is:

$$N := \frac{d}{m} \qquad N = 25$$

b. Pressure angle of 25 deg

$$N_{min} := 12 \qquad m := \frac{d}{N_{min}} \qquad m = 6.250\,mm$$

From Table 9-3, the largest standard module (largest tooth size) that can be used is 6 but, in order to have a whole number of teeth, the module must be 3 or 5. Since a module of 3 would result in more teeth than is necessary, let $m := 5 \cdot mm$ The resulting number of teeth on the pinion is:

$$N := \frac{d}{m} \qquad N = 15$$

PROBLEM 9-59

Statement: In order to have a smooth-running gearset it is desired to have a contact ratio of at least 1.5. If the gears have a pressure angle of 25 deg and gear ratio of 4, what is the minimum number of teeth on the pinion that will yield the required minimum contact ratio?

Given: Contact ratio: $m_p := 1.5$ Gear ratio: $m_G := 4$ Pressure angle: $\phi := 25 \cdot deg$

Solution: See Mathcad file P0959.

1. Write equations 9.6b and 9.2 for the contact ratio and length of action, respectively.

$$m_p = \frac{p_d \cdot Z}{\pi \cdot cos(\phi)} \qquad Z = \sqrt{(r_p + a_p)^2 - (r_p \cdot cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot cos(\phi))^2} - C \cdot sin(\phi)$$

Note that

$$p_d = \frac{N}{d} \qquad a_p = a_g = \frac{1}{p_d} \qquad r_g = m_G \cdot r_p \qquad r_p = \frac{d}{2} \qquad C = r_p + r_g = r_p \cdot (1 + m_G)$$

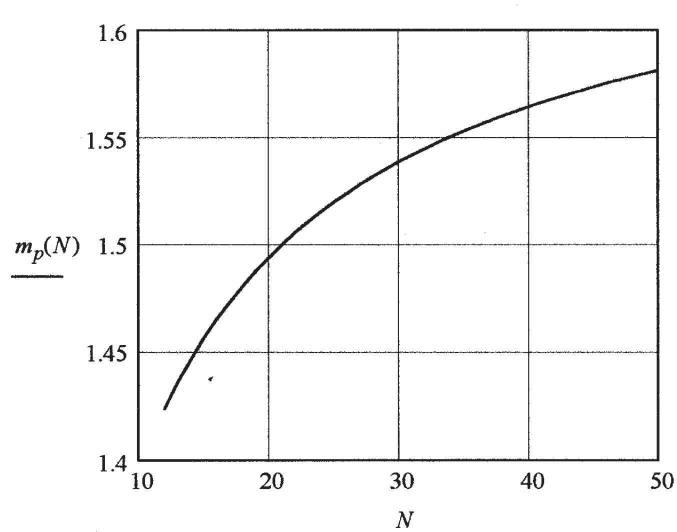
Substituting these identities into the equation for Z and collecting terms gives:

$$Z = \frac{1}{p_d} \cdot \left[\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2} \cdot cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2} \cdot cos(\phi)\right)^2} - \frac{N}{2} \cdot (1 + m_G) \cdot sin(\phi) \right]$$

Substituting this into the equation for m_p and we have:

$$m_p(N) := \frac{\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2} \cdot cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2} \cdot cos(\phi)\right)^2} - \frac{N}{2} \cdot (1 + m_G) \cdot sin(\phi)}{\pi \cdot cos(\phi)}$$

2. Notice that m_p is independent of p_d and depends only on the pressure angle, the number of teeth on the pinion, and the gear ratio. To determine the minimum number of teeth on the pinion, plot the function on the right side of the equation above over the range $N := 12, 13 .. 50$



From the graph we see that we need about 21 teeth. Try various values of N in the function until the calculated value of m_p is greater than or equal to the required value.

$$m_p(21) = 1.500$$

Let $N := 21$

PROBLEM 9-60

Statement: In order to have a smooth-running gearset it is desired to have a contact ratio of at least 1.5. If the gears have a pressure angle of 25 deg and 20-tooth pinion, what is the minimum gear ratio that will yield the required minimum contact ratio?

Given: Contact ratio: $m_p := 1.5$ Number of teeth: $N := 20$ Pressure angle: $\phi := 25\cdot deg$

Solution: See Mathcad file P0960.

1. Write equations 9.6b and 9.2 for the contact ratio and length of action, respectively.

$$m_p = \frac{p_d \cdot Z}{\pi \cdot \cos(\phi)} \qquad Z = \sqrt{(r_p + a_p)^2 - (r_p \cdot \cos(\phi))^2} + \sqrt{(r_g + a_g)^2 - (r_g \cdot \cos(\phi))^2} - C\cdot \sin(\phi)$$

Note that

$$p_d = \frac{N}{d} \qquad a_p = a_g = \frac{1}{p_d} \qquad r_g = m_G \cdot r_p \qquad r_p = \frac{d}{2} \qquad C = r_p + r_g = r_p \cdot (1 + m_G)$$

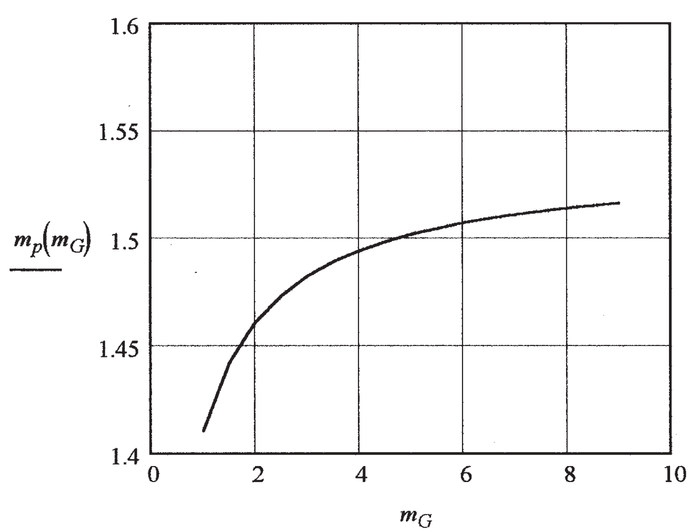
Substituting these identities into the equation for Z and collecting terms gives:

$$Z = \frac{1}{p_d} \cdot \left[\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2}\cdot \cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2}\cdot \cos(\phi)\right)^2} - \frac{N}{2}\cdot(1 + m_G)\cdot \sin(\phi) \right]$$

Substituting this into the equation for m_p and we have:

$$m_p(m_G) := \frac{\sqrt{\left(\frac{N}{2} + 1\right)^2 - \left(\frac{N}{2}\cdot \cos(\phi)\right)^2} + \sqrt{\left(\frac{m_G \cdot N}{2} + 1\right)^2 - \left(\frac{m_G \cdot N}{2}\cdot \cos(\phi)\right)^2} - \frac{N}{2}\cdot(1 + m_G)\cdot \sin(\phi)}{\pi \cdot \cos(\phi)}$$

2. Notice that m_p is independent of p_d and depends only on the pressure angle, the number of teeth on the pinion, and the gear ratio. To determine the minimum number of teeth on the pinion, plot the function on the right side of the equation above over the range $m_G := 1, 1.5 .. 9$.



From the graph we see that we need a gear ratio of about 5. Try various values of m_G in the function until the calculated value of m_p is greater than or equal to the required value.

$$m_p(5) = 1.502$$

Let $m_G := 5$

PROBLEM 9-61

Statement: Calculate and plot the train ratio of a noncircular gearset, as a function of input angle, that is based on the centrodes of Figure 6-15b. The link length ratios ar $L_1/L_2 = 1.60$, $L_3/L_2 = 1.60$, and $L_4/L_2 = 1.00$.

Given: Link lengths (assume $L_2 = 1.00$):

Link 2 (L_2)	$L_2 := 1.00$	Link 3 (L_3)	$L_3 := 1.60$
Link 4 (L_4)	$L_4 := 1.00$	Link 1 (L_1)	$L_1 := 1.60$

Solution: See Figure 6-15b and Mathcad file P0961.

1. In Figure 6-15b links 2 and 4 are the short links and 1 and 3 are the long links with link 1 fixed. To find the fixed and moving centrodes of I_{24} we will fix link 2 and find the intersection of links 1 and 3. The locus of these intersections is an ellipse that will be attached to link 2 when the linkage is reinverted such that link 1 is again grounded.

2. Invert the linkage, fixing link 2 to ground and determine the range of motion for this Grashof double crank.

$$a := L_3 \qquad b := L_4 \qquad c := L_1 \qquad d := L_2$$

$$a = 1.600 \qquad b = 1.000 \qquad c = 1.600 \qquad d = 1.000$$

$$\theta_3 := 0.5\cdot deg, 1\cdot deg .. 359.5\cdot deg$$

3. Determine the values of the constants needed for finding θ_4 on the inverted linkage (actually θ_1 on the original linkage) from equations 4.8a, 4.10a, 4.11b and 4.12.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.6250 \qquad K_2 = 0.6250$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2\cdot a\cdot c)} \qquad K_3 = 1.0000$$

$$A(\theta_3) := \cos(\theta_3) - K_1 - K_2\cdot\cos(\theta_3) + K_3$$

$$B(\theta_3) := -2\cdot\sin(\theta_3)$$

$$C(\theta_3) := K_1 - (K_2 + 1)\cdot\cos(\theta_3) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{11}(\theta_3) := 2\cdot\left[atan2\left[2\cdot A(\theta_3), -B(\theta_3) + \sqrt{(B(\theta_3))^2 - 4\cdot A(\theta_3)\cdot C(\theta_3)}\right]\right]$$

$$\theta_{12}(\theta_3) := 2\cdot\left[atan2\left[2\cdot A(\theta_3), -B(\theta_3) - \sqrt{(B(\theta_3))^2 - 4\cdot A(\theta_3)\cdot C(\theta_3)}\right]\right]$$

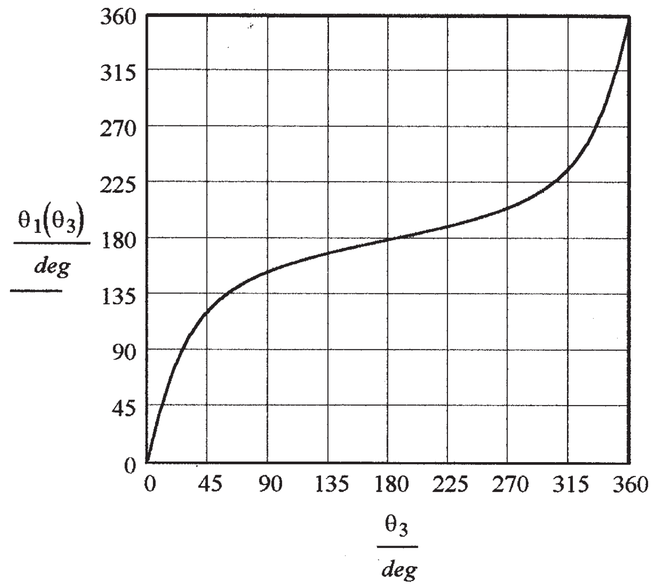
Use the crossed branch equation for the first 180 deg of crank motion and the open branch equation for the last 180 deg.

$$\theta_{10}(\theta_3) := if(\theta_3 \le \pi, \theta_{11}(\theta_3), \theta_{12}(\theta_3))$$

If the calculated angle is negative, make it positive.

$$\theta_1(\theta_3) := if(\theta_3 \le \pi, \theta_{10}(\theta_3), \theta_{10}(\theta_3) + 2\cdot\pi)$$

Plot θ1 as a function of θ3 (link 3 is the driving crank and link 1 is the driven crank in the inverted linkage).



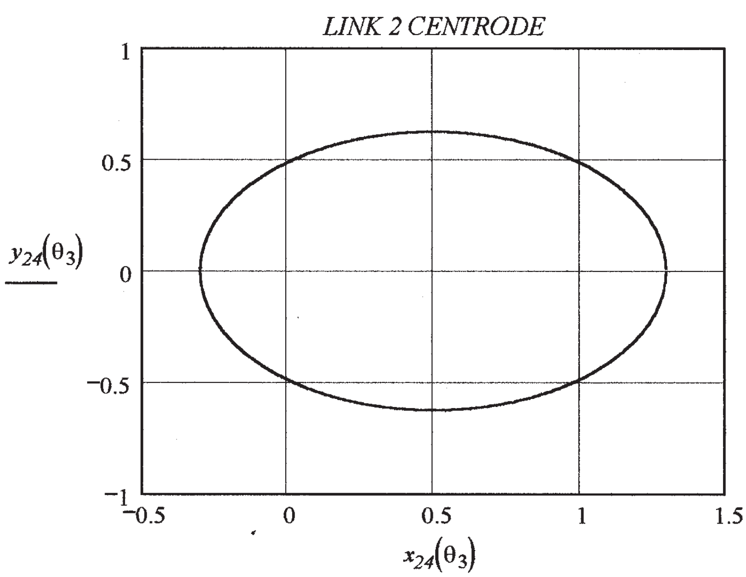
5. Define the coordinates of the intersection of links 1 and 3 for the inverted linkage.

Equation of link 3: $y = x \cdot m_3$, where $m_3(\theta_3) := tan(\theta_3)$

Equation of link 1: $y = (x - d) \cdot m_1$, where $m_1(\theta_3) := tan(\theta_1(\theta_3))$

The coordinates of the intersection (I_{24}) of these two lines are

$$x_{24}(\theta_3) := \frac{m_1(\theta_3)}{m_1(\theta_3) - m_3(\theta_3)} \cdot d \qquad y_{24}(\theta_3) := x_{24}(\theta_3) \cdot m_3(\theta_3)$$



$x_{24}(0.0001 \cdot deg) = 1.300$

$y_{24}(0.0001 \cdot deg) = 0.000$

$x_{24}(51.318 \cdot deg) = 0.500$

$y_{24}(51.318 \cdot deg) = 0.624$

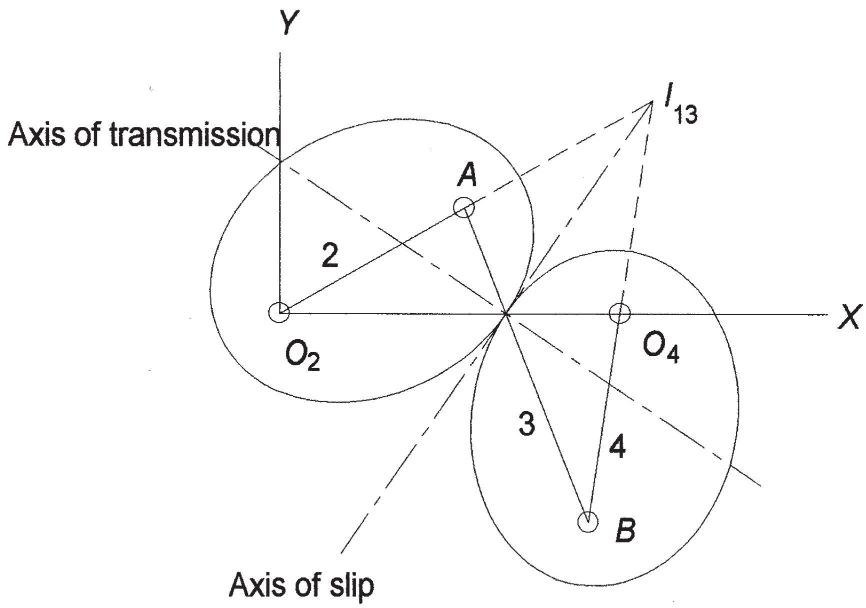
$x_{24}(179.0001 \cdot deg) = -0.300$

$y_{24}(179.0001 \cdot deg) = 0.005$

$\theta_1(51.318 \cdot deg) = 128.682\, deg$

The ellipse has a major axis of 1.600 and a minor axis of 1.248 and the origin on the graph above is the point O_2. The centrode for link 4 is a mirror image of this centrode.

6. The linkage is shown below, reinverted such that link 1 is fixed, with the centrodes attached to links 2 and 4. They are tangent to each other at the instant center I_{24}. The axis of slip is found by extending a line from the instant center I_{13} to I_{24}. The axis of transmission is perpendicular to this line as shown.



PROBLEM 9-62

Statement: Calculate and plot the train ratio of a noncircular gearset, as a function of input angle, that is based on the centrodes of Figure 6-15b. The link length ratios ar $L_1/L_2 = 1.80$, $L_3/L_2 = 1.80$, and $L_4/L_2 = 1.00$.

Given: Link lengths (assume $L_2 = 1.00$):

Link 2 (L_2)	$L_2 := 1.00$	Link 3 (L_3)	$L_3 := 1.80$
Link 4 (L_4)	$L_4 := 1.00$	Link 1 (L_1)	$L_1 := 1.80$

Solution: See Figure 6-15b and Mathcad file P0962.

1. In Figure 6-15b links 2 and 4 are the short links and 1 and 3 are the long links with link 1 fixed. To find the fixed and moving centrodes of I_{24} we will fix link 2 and find the intersection of links 1 and 3. The locus of these intersections is an ellipse that will be attached to link 2 when the linkage is reinverted such that link 1 is again grounded.

2. Invert the linkage, fixing link 2 to ground and determine the range of motion for this Grashof double crank.

$a := L_3 \quad b := L_4 \quad c := L_1 \quad d := L_2$

$a = 1.800 \quad b = 1.000 \quad c = 1.800 \quad d = 1.000$

$\theta_3 := 0.5 \cdot deg, 1 \cdot deg .. 359.5 \cdot deg$

3. Determine the values of the constants needed for finding θ_4 on the inverted linkage (actually θ_1 on the original linkage) from equations 4.8a, 4.10a, 4.11b and 4.12.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c}$$

$$K_1 = 0.5556 \qquad K_2 = 0.5556$$

$$K_3 := \frac{a^2 - b^2 + c^2 + d^2}{(2 \cdot a \cdot c)} \qquad K_3 = 1.0000$$

$$A(\theta_3) := \cos(\theta_3) - K_1 - K_2 \cdot \cos(\theta_3) + K_3$$

$$B(\theta_3) := -2 \cdot \sin(\theta_3)$$

$$C(\theta_3) := K_1 - (K_2 + 1) \cdot \cos(\theta_3) + K_3$$

4. Use equation 4.10b to find values of θ_4 for the crossed circuit.

$$\theta_{11}(\theta_3) := 2 \cdot \left[atan2\left[2 \cdot A(\theta_3), -B(\theta_3) + \sqrt{(B(\theta_3))^2 - 4 \cdot A(\theta_3) \cdot C(\theta_3)} \right] \right]$$

$$\theta_{12}(\theta_3) := 2 \cdot \left[atan2\left[2 \cdot A(\theta_3), -B(\theta_3) - \sqrt{(B(\theta_3))^2 - 4 \cdot A(\theta_3) \cdot C(\theta_3)} \right] \right]$$

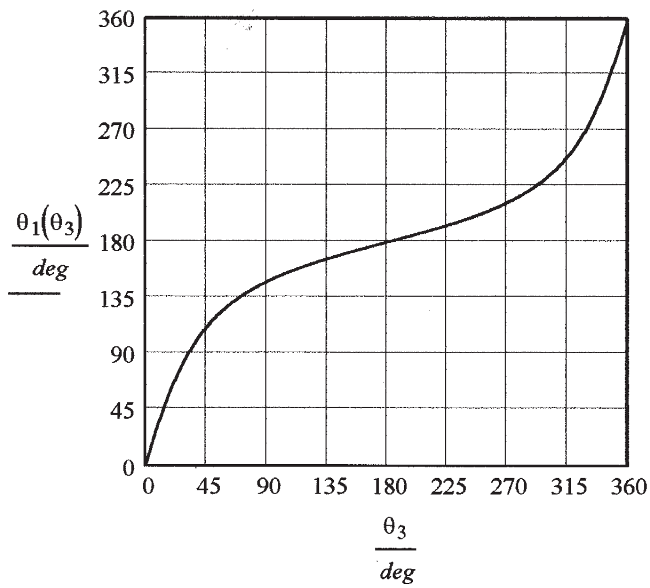
Use the crossed branch equation for the first 180 deg of crank motion and the open branch equation for the last 180 deg.

$$\theta_{10}(\theta_3) := if(\theta_3 \le \pi, \theta_{11}(\theta_3), \theta_{12}(\theta_3))$$

If the calculated angle is negative, make it positive.

$$\theta_1(\theta_3) := if(\theta_3 \le \pi, \theta_{10}(\theta_3), \theta_{10}(\theta_3) + 2 \cdot \pi)$$

Plot θ1 as a function of θ3 (link 3 is the driving crank and link 1 is the driven crank in the inverted linkage).



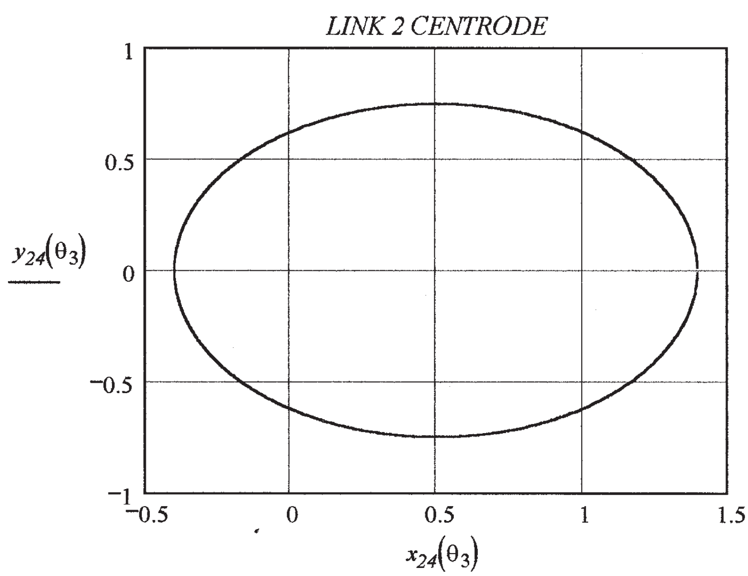
5. Define the coordinates of the intersection of links 1 and 3 for the inverted linkage.

Equation of link 3: $y = x \cdot m_3$, where $m_3(\theta_3) := tan(\theta_3)$

Equation of link 1: $y = (x - d) \cdot m_1$, where $m_1(\theta_3) := tan(\theta_1(\theta_3))$

The coordinates of the intersection (I_{24}) of these two lines are

$$x_{24}(\theta_3) := \frac{m_1(\theta_3)}{m_1(\theta_3) - m_3(\theta_3)} \cdot d \qquad y_{24}(\theta_3) := x_{24}(\theta_3) \cdot m_3(\theta_3)$$



$x_{24}(0.0001 \cdot deg) = 1.400$

$y_{24}(0.0001 \cdot deg) = 0.000$

$x_{24}(56.251 \cdot deg) = 0.500$

$y_{24}(56.251 \cdot deg) = 0.748$

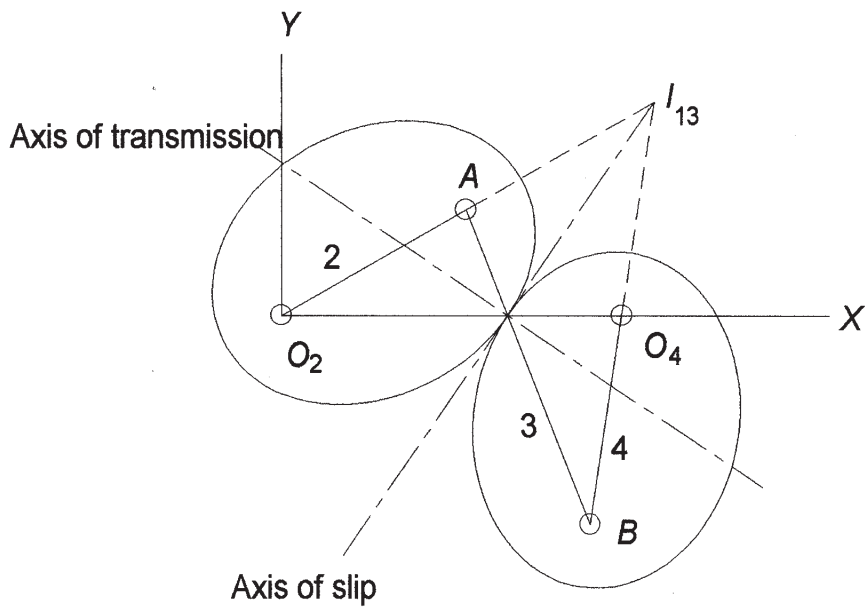
$x_{24}(179.0001 \cdot deg) = -0.400$

$y_{24}(179.0001 \cdot deg) = 0.007$

$\theta_1(56.251 \cdot deg) = 123.749\, deg$

The ellipse has a major axis of 1.800 and a minor axis of 1.496 and the origin on the graph above is the point O_2. The centrode for link 4 is a mirror image of this centrode.

6. The linkage is shown below, reinverted such that link 1 is fixed, with the centrodes attached to links 2 and 4. They are tangent to each other at the instant center I_{24}. The axis of slip is found by extending a line from the instant center I_{13} to I_{24}. The axis of transmission is perpendicular to this line as shown.



PROBLEM 9-63

Statement: Figure P9-35*b* shows (schematically) a compound epicyclic train. The tooth numbers are 50, 25, 35, and 90 for gears 2, 3, 4, and 5, respectively. The arm is driven at 180 rpm CW and gear 5 is fixed to ground. Determine the speed and direction of gear 2. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Tooth numbers:

$N_2 := 50 \qquad N_3 := 25 \qquad N_4 := 35 \qquad N_5 := 90$

Input speeds: $\omega_{arm} := -180\cdot rpm \qquad \omega_5 := 0\cdot rpm$

Basic gearset efficiency: $E_0 := 0.98$

Solution: See Figure P9-35*b* and Mathcad file P0963.

1. Determine the speed of the sun gear 2 using the formula method for analyzing an epicyclic train. To start, choose a first and last gears that mesh with gears that have planetary motion. Let the first gear be 2 and last be 5. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-7*b*.

$$R := \left(-\frac{N_2}{N_3}\right)\cdot\left(\frac{N_4}{N_5}\right) \qquad R = -0.77778$$

Solve the right-hand equation above for ω_2 with $\omega_5 = 0$.

$$\omega_2 := \frac{(R-1)}{R}\cdot\omega_{arm} \qquad \omega_2 = -411.43\, rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := \frac{1}{R} \qquad \rho = -1.286$$

3. The combination of $\rho < -1$, shaft 1 fixed, and input to the arm corresponds to Case 8 in Table 9-12 giving an efficiency of

$$\eta := \frac{E_0\cdot(\rho - 1)}{\rho\cdot E_0 - 1} \qquad \eta = 0.991$$

PROBLEM 9-64

Statement: Figure P9-35h shows (schematically) a compound epicyclic train. The tooth numbers are 80, 20, 25, and 85 for gears 2, 3, 4, and 5, respectively. Gear 2 is driven at 200 rpm CCW and gear 5 is fixed to ground. Determine the speed and direction of the arm. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Tooth numbers:

$N_2 := 80$ $N_3 := 20$ $N_4 := 25$ $N_5 := 85$

Input speeds: $\omega_2 := 200\cdot rpm$ $\omega_5 := 0\cdot rpm$

Basic gearset efficiency: $E_0 := 0.98$

Solution: See Figure P9-35h and Mathcad file P0964.

1. Determine the speed of the arm using the formula method for analyzing an epicyclic train. To start, choose first and last gears that mesh with gears that have planetary motion. Let the first gear be 2 and last be 5. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-7*b*.

$$R := \left(\frac{N_2}{N_3}\right)\cdot\left(\frac{N_4}{N_5}\right) \qquad R = 1.17647$$

Solve the right-hand equation above for ω_{arm} with $\omega_5 = 0$.

$$\omega_{arm} := \frac{R}{(R-1)}\cdot\omega_2 \qquad \omega_{arm} = 1333\,rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := R \qquad \rho = 1.176$$

3. The combination of $\rho > 1$, shaft 1 fixed, and input to gear 2 corresponds to Case 3 in Table 9-12 giving an efficiency of

$$\eta := \frac{\rho\cdot E_0 - 1}{E_0\cdot(\rho - 1)} \qquad \eta = 0.884$$

PROBLEM 9-65

Statement: Figure P9-35i shows (schematically) a compound epicyclic train. The tooth numbers are 24, 18, 20, and 90 for gears 2, 3, 4, and 5, respectively. The arm is driven at 100 rpm CCW and gear 2 is fixed to ground. Determine the speed and direction of gear 5. What is the efficiency of this train if the basic gearsets have $E_0 = 0.98$?

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Tooth numbers:

$N_2 := 24 \qquad N_3 := 18 \qquad N_4 := 20 \qquad N_5 := 90$

Input speeds: $\omega_{arm} := 100 \cdot rpm \qquad \omega_2 := 0 \cdot rpm$

Basic gearset efficiency: $E_0 := 0.98$

Solution: See Figure P9-35i and Mathcad file P0965.

1. Determine the speed of the arm using the formula method for analyzing an epicyclic train. To start, choose first and last gears that mesh with gears that have planetary motion. Let the first gear be 2 and last be 5. Then, using equation 9.13c, write the relationship among the first, last, and arm.

$$\frac{\omega_{Larm}}{\omega_{Farm}} = \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = R$$

Calculate R using equation 9.14 and inspection of Figure P9-7*b*.

$$R := \left(-\frac{N_2}{N_3}\right) \cdot \left(-\frac{N_3}{N_4}\right) \cdot \left(\frac{N_4}{N_5}\right) \qquad R = 0.26667$$

Solve the right-hand equation above for ω_5 with $\omega_2 = 0$.

$$\omega_5 := (R + 1) \cdot \omega_{arm} \qquad \omega_5 = 126.67\, rpm$$

2. Find the basic ratio ρ for the train using equation 9.15.

$$\rho := \frac{1}{R} \qquad \rho = 3.750$$

3. The combination of $\rho > 1$, shaft 2 fixed, and input to the arm corresponds to Case 2 in Table 9-12 giving an efficiency of

$$\eta := \frac{E_0 \cdot (\rho - 1)}{\rho - E_0} \qquad \eta = 0.973$$

PROBLEM 9-25

Row	ω_2	ω_6	ω_{arm}
a	790		
b		-80	
c			-4.54
d	814		
e		-61.98	
f			-5.81

PROBLEM 9-26

Row	ω_2	ω_6	ω_{arm}
a	-59		
b		-57.3	
c			61.54
d	-120		
e		-63.33	
f			128