Chapter 10

DYNAMICS FUNDAMENTALS

TOPIC/PROBLEM MATRIX

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PROBLEM 10-1

Statement: The mallet shown in Figure 10-2 has the specifications given below. Find the location of its composite CG, and its moment of inertia and radius of gyration about axis ZZ. Assume the wood has a density equal to 0.9 times that of water.

Given: Steel hammer head has specific weight: $\gamma_d := 0.3 \cdot lbf \cdot in^{-3}$

Wood handle has specific weight: $\gamma_h := 0.9 \cdot 0.036 \cdot lbf \cdot in^{-3}$

Head dimensions: $r_d := 0.5 \cdot in$ $\quad h_d := 3 \cdot in$

Handle dimensions: $r_{h1} := 0.625 \cdot in$ $\quad L_{h1} := 10 \cdot in$ $\quad r_{h2} := 0.3125 \cdot in$ $\quad L_{h2} := 2 \cdot r_d$

Solution: See Figure 10-2 and Mathcad file P1001.

1. Calculate the volume and weight of each component.

Head: $V_d := \pi \cdot r_d^2 \cdot h_d - \pi \cdot r_{h2}^2 \cdot L_{h2}$ $\qquad V_d = 2.049\ in^3$

$W_d := \gamma_d \cdot V_d$ $\qquad W_d = 0.615\ lbf$

Handle: $V_{h1} := \pi \cdot r_{h1}^2 \cdot L_{h1}$ $\qquad V_{h1} = 12.272\ in^3$

$V_{h2} := \pi \cdot r_{h2}^2 \cdot L_{h2}$ $\qquad V_{h2} = 0.307\ in^3$

$V_h := V_{h1} + V_{h2}$ $\qquad V_h = 12.579\ in^3$

$W_{h1} := \gamma_h \cdot V_{h1}$ $\qquad W_{h1} = 0.398\ lbf$

$W_{h2} := \gamma_h \cdot V_{h2}$ $\qquad W_{h2} = 9.940 \times 10^{-3}\ lbf$

$W_h := W_{h1} + W_{h2}$ $\qquad W_h = 0.408\ lbf$

2. The CG of each component will lie along the *XX* axis. Find the distance from the origin to the CGs (distance to axes *HH* and *DD*).

Head: $Xcg_d := L_{h1} + r_d$ $\qquad Xcg_d = 10.500\ in$

Handle: $$Xcg_h := \frac{0.5 \cdot L_{h1} \cdot V_{h1} + (L_{h1} + 0.5 \cdot L_{h2}) \cdot V_{h2}}{V_h}$$

$Xcg_h = 5.134\ in$

3. Find the location of the composite CG using equation 10.3d.

$$X_{Cg} := \frac{Xcg_d \cdot W_d + Xcg_h \cdot W_h}{W_d + W_h} \qquad X_{Cg} = 8.361\ in$$

4. Calculate the moment of inertia of the head with respect to the *ZZ* axis.

$$I_{DDd} := \frac{W_d}{12 \cdot g} \cdot \left(3 \cdot r_d^2 + h_d^2\right) \qquad I_{DDd} = 1.294 \times 10^{-3}\ lbf \cdot sec^2 \cdot in$$

$$I_{ZZd} := I_{DDd} + \frac{W_d}{g} \cdot Xcg_d^2 \qquad I_{ZZd} = 0.177\ lbf \cdot sec^2 \cdot in$$

5. Calculate the moment of inertia of the handle with respect to the *ZZ* axis.

$$I_{h1} := \frac{W_{h1}}{12 \cdot g} \cdot \left(3 \cdot r_{h1}^2 + L_{h1}^2\right) \qquad I_{h1} = 8.683 \times 10^{-3}\, lbf \cdot sec^2 \cdot in$$

$$I_{ZZh1} := I_{h1} + \frac{W_{h1}}{g} \cdot \left(\frac{L_{h1}}{2}\right)^2 \qquad I_{ZZh1} = 0.034\, lbf \cdot sec^2 \cdot in$$

$$I_{h2} := \frac{W_{h2}}{12 \cdot g} \cdot \left(3 \cdot r_{h2}^2 + L_{h2}^2\right) \qquad I_{h2} = 2.774 \times 10^{-6}\, lbf \cdot sec^2 \cdot in$$

$$I_{ZZh2} := I_{h2} + \frac{W_{h2}}{g} \cdot \left(L_{h1} + \frac{L_{h2}}{2}\right)^2 \qquad I_{ZZh2} = 2.841 \times 10^{-3}\, lbf \cdot sec^2 \cdot in$$

$$I_{ZZh} := I_{ZZh1} + I_{ZZh2} \qquad I_{ZZh} = 0.037\, lbf \cdot sec^2 \cdot in$$

6. Add the moments of the two components about the *ZZ* axis to get the moment of inertia of the mallet about the *ZZ* axis.

$$I_{ZZ} := I_{ZZd} + I_{ZZh} \qquad I_{ZZ} = 0.214\, lbf \cdot sec^2 \cdot in$$

7. Use equation 10.10b to calculate the radius of gyration about the *ZZ* axis.

$$k := \sqrt{\frac{I_{ZZ} \cdot g}{W_d + W_h}} \qquad k = 8.992\, in$$

PROBLEM 10-2

Statement: The mallet shown in Figure 10-2 (p. 496) has the specifications given below. Find the location of its composite CG, and its moment of inertia and radius of gyration about axis ZZ. Assume the wood has a density equal to 0.9 times that of water.

Given: Wood has specific weight: $\gamma := 0.9 \cdot 0.036 \cdot lbf \cdot in^{-3}$

Head dimensions: $r_d := 1.0 \cdot in$ $\quad h_d := 3 \cdot in$

Handle dimensions: $r_h := 0.625 \cdot in$ $\quad L_h := 10 \cdot in$

Solution: See Figure 10-2 and Mathcad file P1002.

1. Calculate the volume and weight of each component.

Head: $V_d := \pi \cdot r_d^{\,2} \cdot h_d$ $\qquad V_d = 9.425\ in^3$

$W_d := \gamma \cdot V_d$ $\qquad W_d = 0.305\ lbf$

Handle: $V_h := \pi \cdot r_h^{\,2} \cdot L_h$ $\qquad V_h = 12.272\ in^3$

$W_h := \gamma \cdot V_h$ $\qquad W_h = 0.398\ lbf$

2. The CG of each component will lie along the *XX* axis. Find the distance from the origin to the CGs (distance to axes *HH* and *DD*).

Head: $Xcg_d := L_h + r_d$ $\qquad Xcg_d = 11.000\ in$

Handle: $Xcg_h := \dfrac{L_h}{2}$ $\qquad Xcg_h = 5.000\ in$

3. Find the location of the composite CG using equation 10.3d.

$$X_{Cg} := \frac{Xcg_d \cdot W_d + Xcg_h \cdot W_h}{W_d + W_h} \qquad X_{Cg} = 7.606\ in$$

4. Calculate the moment of inertia of the head with respect to the ZZ axis.

$$I_{DDd} := \frac{W_d}{12 \cdot g} \cdot \left(3 \cdot r_d^{\,2} + h_d^{\,2}\right) \qquad I_{DDd} = 7.909 \times 10^{-4}\ lbf \cdot sec^2 \cdot in$$

$$I_{ZZd} := I_{DDd} + \frac{W_d}{g} \cdot Xcg_d^{\,2} \qquad I_{ZZd} = 0.096\ lbf \cdot sec^2 \cdot in$$

5. Calculate the moment of inertia of the handle with respect to the ZZ axis.

$$I_{HHh} := \frac{W_h}{12 \cdot g} \cdot \left(3 \cdot r_h^{\,2} + L_h^{\,2}\right) \qquad I_{HHh} = 8.683 \times 10^{-3}\ lbf \cdot sec^2 \cdot in$$

$$I_{ZZh} := I_{HHh} + \frac{W_h}{g} \cdot Xcg_h^{\,2} \qquad I_{ZZh} = 0.034\ lbf \cdot sec^2 \cdot in$$

6. Add the moments of the two components about the ZZ axis to get the moment of inertia of the mallet about the ZZ axis.

$$I_{ZZ} := I_{ZZd} + I_{ZZh} \qquad I_{ZZ} = 0.131\ lbf \cdot sec^2 \cdot in$$

7. Use equation 10.10b to calculate the radius of gyration about the ZZ axis.

$$k := \sqrt{\frac{I_{ZZ} \cdot g}{W_d + W_h}} \qquad k = 8.480\ in$$

✍ PROBLEM 10-3

Statement: Calculate the location of the composite center of gravity, the mass moment of inertia and the radius of gyration with respect to the specified axis, for whichever of the following commonly available items that are assigned. (Note these are not short problems).

a. A good-quality writing pen, about the pivot point at which you grip it to write. (How does placing the cap on the upper end of the pen affect these parameters when you write?)

b. Two table knives, one metal and one plastic, about the pivot axis when held for cutting. Compare the calculated results and comment on what they tell you about the dynamic usability of the two knives (ignore sharpness considerations).

c. A ball-peen hammer (available for inspection in any university machine shop), about the center of rotation (after you calculate it for the proper center of percussion).

d. A baseball bat (see the coach) about the center of rotation (after you calculate its location for the proper center of percussion).

e. A cylindrical coffee mug, about the handle hole.

Solution: No solution is provided here due to the wide variety of possibilities in this open-ended problem.

PROBLEM 10-4

Statement: Set up these equations in matrix form. Use program MATRIX, Mathcad, or a calculator that has matrix math capability to solve them.

a.	-5x	-2y	+12z	-w	=	-9
	x	+3y	-2z	+4w	=	10
	-x	-y	+z		=	-7
	3x	-3y	+7z	+9w	=	-6
b.	3x	-5y	+17z	-5w	=	-5
	-2x	+9y	-14z	+6w	=	22
	-x	-y		-2w	=	13
	4x	-7y	+8z	+4w	=	-9

Solution: See Mathcad file P1004.

1. Place the coefficients of the unknowns in a 4 x 4 matrix.

$$Ca := \begin{pmatrix} -5 & -2 & 12 & -1 \\ 1 & 3 & -2 & 4 \\ -1 & -1 & 1 & 0 \\ 3 & -3 & 7 & 9 \end{pmatrix} \qquad Cb := \begin{pmatrix} 3 & -5 & 17 & -5 \\ -2 & 9 & -14 & 6 \\ -1 & -1 & 0 & -2 \\ 4 & -7 & 8 & 4 \end{pmatrix}$$

2. Place the constants on the right-hand side of the equal sign in a 4 x 1 array.

$$Ba := \begin{pmatrix} -9 \\ 10 \\ -7 \\ -6 \end{pmatrix} \qquad Bb := \begin{pmatrix} -5 \\ 22 \\ 13 \\ -9 \end{pmatrix}$$

3. Get the solution array by premultiplying the constant array by the inverse of the coefficient matrix.

$$\begin{pmatrix} xa \\ ya \\ za \\ wa \end{pmatrix} := Ca^{-1} \cdot Ba \qquad xa = 3.547 \qquad ya = 4.884 \qquad za = 1.431 \qquad wa = -1.334$$

$$\begin{pmatrix} xb \\ yb \\ zb \\ wb \end{pmatrix} := Cb^{-1} \cdot Bb \qquad xb = -62.029 \qquad yb = 0.235 \qquad zb = 17.897 \qquad wb = 24.397$$

PROBLEM 10-5

Statement: Figure P10-1 shows a bracket made of steel.

a. Find the location of its centroid referred to point B.
b. Find its mass moment of inertia I_{xx} about the x axis through point B.
c. Find its mass moment of inertia I_{yy} about the y axis through point B.

Given: Dimensions in the figures below. Density: $\rho := 7800\,kg \cdot m^{-3}$

Solution: See Figure P10-1 and Mathcad file P1005.

1. Divide the bracket into five volumes, find the location of the CG and the mass moments for each of them and then add the results to get the CG and mass moments for the entire bracket.

$i := 1, 2..5$

2. Volume 1 is the rectangular prism with two negative cylinders shown below.

Dimensions:

$a := 64 \cdot mm$

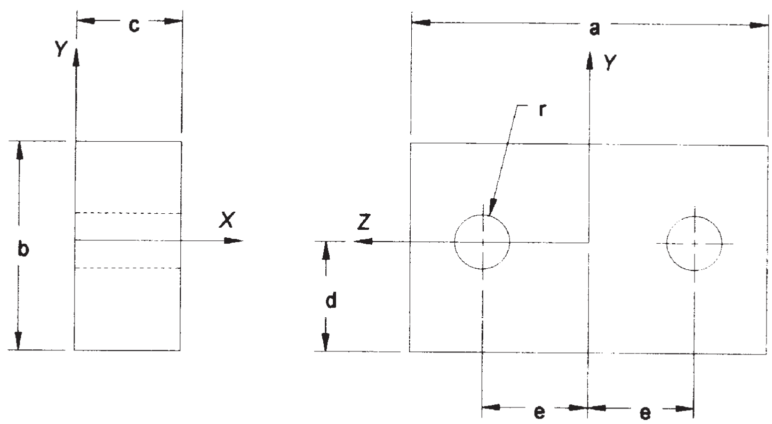
$b := 38 \cdot mm$

$c := 19 \cdot mm$

$d := 20 \cdot mm$

$e := 19 \cdot mm$

$r := 5 \cdot mm$



Determine the location of the CG in global coordinates.

$Xcg_1 := \frac{c}{2}$ $\quad Xcg_1 = 9.500\,mm$ $\quad Zcg_1 := 0 \cdot mm$

$$Ycg_1 := \frac{a \cdot b \cdot \frac{b}{2} - 2 \cdot \pi \cdot r^2 \cdot d}{a \cdot b - 2 \cdot \pi \cdot r^2} - d \qquad Ycg_1 = -1.069\,mm$$

Determine the volume and mass of this segment.

$V_1 := a \cdot b \cdot c - 2 \cdot \pi \cdot r^2 \cdot c$ $\quad V_1 = 4.322 \times 10^4\,mm^3$ $\quad M_1 := \rho \cdot V_1$ $\quad M_1 = 0.337\,kg$

Determine the mass moments about the local axes through the CGs.

Rectangular prism: $m_a := a \cdot b \cdot c \cdot \rho$ $\quad m_a = 0.360\,kg$

$$I_{ax} := \frac{m_a}{12} \cdot \left(a^2 + b^2\right) \qquad I_{ax} = 1.664 \times 10^{-4}\,kg \cdot m^2$$

$$I_{ay} := \frac{m_a}{12} \cdot \left(a^2 + c^2\right) \qquad I_{ay} = 1.339 \times 10^{-4}\,kg \cdot m^2$$

$$I_{az} := \frac{m_a}{12} \cdot \left(b^2 + c^2\right) \qquad I_{az} = 5.421 \times 10^{-5}\,kg \cdot m^2$$

Cylinder (one): $m_b := -\pi \cdot r^2 \cdot c \cdot \rho$ $\qquad m_b = -0.012\,kg$

$$I_{bx} := \frac{m_b \cdot r^2}{2} \qquad I_{bx} = -1.455 \times 10^{-7}\,kg \cdot m^2$$

$$I_{by} := \frac{m_b}{12} \cdot \left(3 \cdot r^2 + c^2\right) \qquad I_{by} = -4.229 \times 10^{-7}\,kg \cdot m^2$$

$$I_{bz} := \frac{m_b}{12} \cdot \left(3 \cdot r^2 + c^2\right) \qquad I_{bz} = -4.229 \times 10^{-7}\,kg \cdot m^2$$

Determine the mass moments about the global axes.

$$Ixx_1 := I_{ax} + m_a \cdot \left[\left(Ycg_1\right)^2 + \left(Zcg_1\right)^2\right] + 2 \cdot \left(I_{bx} + m_b \cdot e^2\right) \qquad Ixx_1 = 1.581 \times 10^{-4}\,kg \cdot m^2$$

$$Iyy_1 := I_{ay} + m_a \cdot \left[\left(Xcg_1\right)^2 + \left(Zcg_1\right)^2\right] + 2 \cdot \left[I_{by} + m_b \cdot \left(\frac{c}{2}\right)^2\right] \qquad Iyy_1 = 1.634 \times 10^{-4}\,kg \cdot m^2$$

$$Izz_1 := I_{az} + m_a \cdot \left[\left(Xcg_1\right)^2 + \left(Ycg_1\right)^2\right] + 2 \cdot \left(I_{bz}\right) \qquad Izz_1 = 8.631 \times 10^{-5}\,kg \cdot m^2$$

3. Volume 2 is the bend just above segment 1. It is a quarter hollow cylinder with dimensions shown below.

Dimensions:

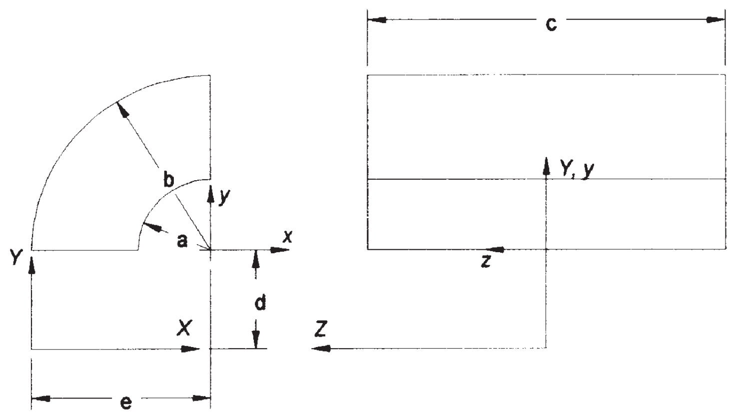
$a := 13 \cdot mm$

$b := 32 \cdot mm$

$c := 64 \cdot mm$

$d := 18 \cdot mm$

$e := 32 \cdot mm$



Determine the location of the CG in global coordinates.

$$Xcg_2 := e - \frac{4}{3 \cdot \pi} \cdot \frac{b^3 - a^3}{b^2 - a^2} \qquad Xcg_2 = 16.825\,mm \qquad Zcg_2 := 0 \cdot mm$$

$$Ycg_2 := d + \frac{4}{3 \cdot \pi} \cdot \frac{b^3 - a^3}{b^2 - a^2} \qquad Ycg_2 = 33.175\,mm$$

Determine the volume and mass of this segment.

$$V_2 := \frac{\pi \cdot b^2 \cdot c - \pi \cdot a^2 \cdot c}{4} \qquad V_2 = 4.298 \times 10^4\,mm^3 \qquad M_2 := \rho \cdot V_2 \qquad M_2 = 0.335\,kg$$

Determine the mass moments about the local axes noted on the drawing of the segment.

$$I_x := \frac{M_2}{12} \cdot \left(3 \cdot a^2 + 3 \cdot b^2 + c^2\right) \qquad I_x = 2.144 \times 10^{-4}\,kg \cdot m^2$$

$$I_y := \frac{M_2}{12}\cdot\left(3\cdot a^2 + 3\cdot b^2 + c^2\right) \qquad I_y = 2.144\times 10^{-4}\,kg\cdot m^2$$

$$I_z := \frac{M_2}{2}\cdot\left(a^2 + b^2\right) \qquad I_z = 2.000\times 10^{-4}\,kg\cdot m^2$$

Determine the mass moments about the global axes.

$$Ixx_2 := I_x + M_2\cdot d^2 \qquad Ixx_2 = 3.230\times 10^{-4}\,kg\cdot m^2$$

$$Iyy_1 := I_{ay} + M_2\cdot e^2 \qquad Iyy_1 = 4.771\times 10^{-4}\,kg\cdot m^2$$

$$Izz_1 := I_{az} + M_2\cdot\left(d^2 + e^2\right) \qquad Izz_1 = 5.061\times 10^{-4}\,kg\cdot m^2$$

4. Volume 3 is a rectangular prism with dimensions shown below.

Dimensions:

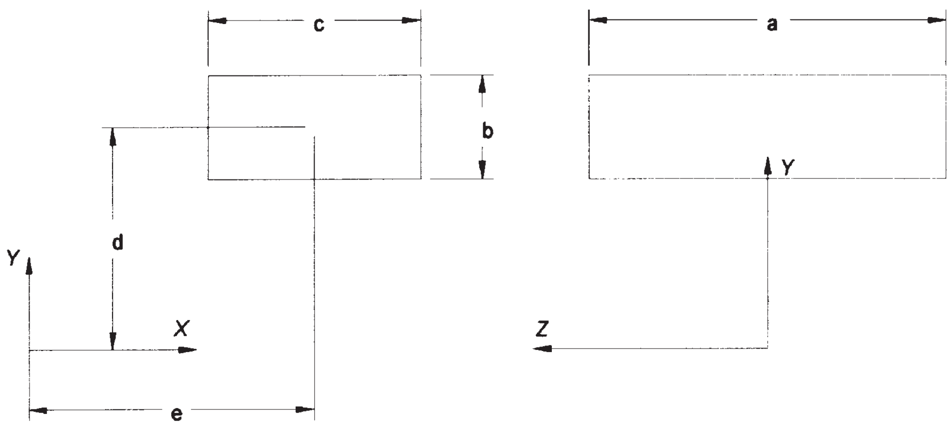
$a := 64\cdot mm$

$b := 19\cdot mm$

$c := 38\cdot mm$

$d := 40.5\cdot mm$

$e := 51\cdot mm$



Determine the location of the CG in global coordinates.

$Xcg_3 := e \qquad Xcg_3 = 51.000\,mm \qquad Ycg_3 := d \qquad Ycg_3 = 40.500\,mm$

$Zcg_3 := 0\cdot mm$

Determine the volume and mass of this segment.

$V_3 := a\cdot b\cdot c \qquad V_3 = 4.621\times 10^4\,mm^3 \qquad M_3 := \rho\cdot V_3 \qquad M_3 = 0.360\,kg$

Determine the mass moments about the local axes through the CGs.

$$I_x := \frac{M_3}{12}\cdot\left(a^2 + b^2\right) \qquad I_x = 1.339\times 10^{-4}\,kg\cdot m^2$$

$$I_y := \frac{M_3}{12}\cdot\left(a^2 + c^2\right) \qquad I_y = 1.664\times 10^{-4}\,kg\cdot m^2$$

$$I_z := \frac{M_3}{12}\cdot\left(b^2 + c^2\right) \qquad I_z = 5.421\times 10^{-5}\,kg\cdot m^2$$

Determine the mass moments about the global axes.

$$Ixx_3 := I_x + M_3\cdot\left[\left(Ycg_3\right)^2 + \left(Zcg_3\right)^2\right] \qquad Ixx_3 = 7.250\times 10^{-4}\,kg\cdot m^2$$

$$Iyy_3 := I_y + M_3 \cdot \left[\left(Xcg_3\right)^2 + \left(Zcg_3\right)^2 \right] \qquad Iyy_3 = 1.104 \times 10^{-3} \, kg \cdot m^2$$

$$Izz_3 := I_z + M_3 \cdot \left[\left(Xcg_3\right)^2 + \left(Ycg_3\right)^2 \right] \qquad Izz_1 = 5.061 \times 10^{-4} \, kg \cdot m^2$$

5. Volume 4 is a half cylinder with dimensions shown below.

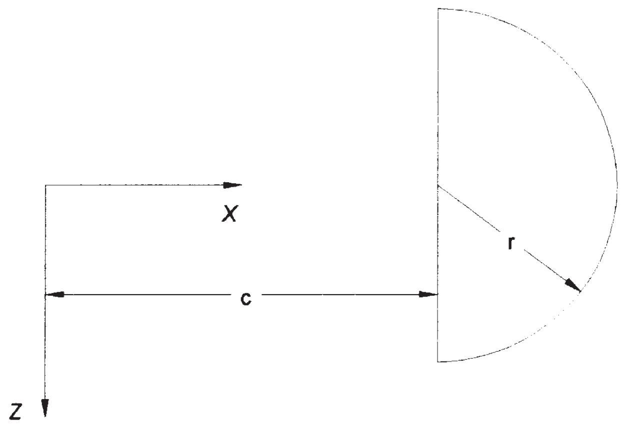
Dimensions:

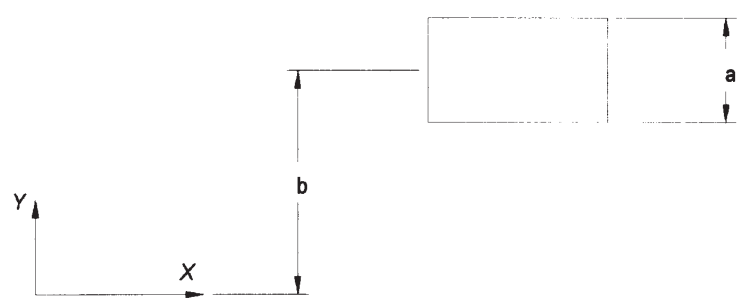
$a := 19 \cdot mm$

$b := 40.5 \cdot mm$

$c := 70 \cdot mm$

$r := 32 \cdot mm$





Determine the location of the CG in global coordinates.

$$Xcg_4 := c + \frac{4 \cdot r}{3 \cdot \pi} \qquad Xcg_4 = 83.581 \, mm \qquad Ycg_4 := b \qquad Ycg_3 = 40.500 \, mm$$

$$Zcg_4 := 0 \cdot mm$$

Determine the volume and mass of this segment.

$$V_4 := a \cdot \frac{\pi \cdot r^2}{2} \qquad V_4 = 3.056 \times 10^4 \, mm^3 \qquad M_4 := \rho \cdot V_4 \qquad M_4 = 0.238 \, kg$$

Determine the mass moments about the local axes of the segment.

$$I_x := \frac{M_4}{12} \cdot \left(3 \cdot r^2 + a^2\right) \qquad I_x = 6.820 \times 10^{-5} \, kg \cdot m^2$$

$$I_y := \frac{M_4}{2} \cdot r^2 \qquad I_y = 1.221 \times 10^{-4} \, kg \cdot m^2$$

$$I_z := \frac{M_4}{12} \cdot \left(3 \cdot r^2 + a^2\right) \qquad I_z = 6.820 \times 10^{-5} \, kg \cdot m^2$$

Determine the mass moments about the global axes.

$$Ixx_4 := I_x + M_4 \cdot \left[\left(Ycg_4\right)^2 + \left(Zcg_4\right)^2 \right] \qquad Ixx_4 = 4.592 \times 10^{-4}\, kg \cdot m^2$$

$$Iyy_4 := I_y + M_4 \cdot \left[\left(Xcg_4\right)^2 + \left(Zcg_4\right)^2 \right] \qquad Iyy_4 = 1.787 \times 10^{-3}\, kg \cdot m^2$$

$$Izz_4 := I_z + M_4 \cdot \left[\left(Xcg_4\right)^2 + \left(Ycg_4\right)^2 \right] \qquad Izz_4 = 2.124 \times 10^{-3}\, kg \cdot m^2$$

6. Volume 5 is a negative cylinder with dimensions shown below.

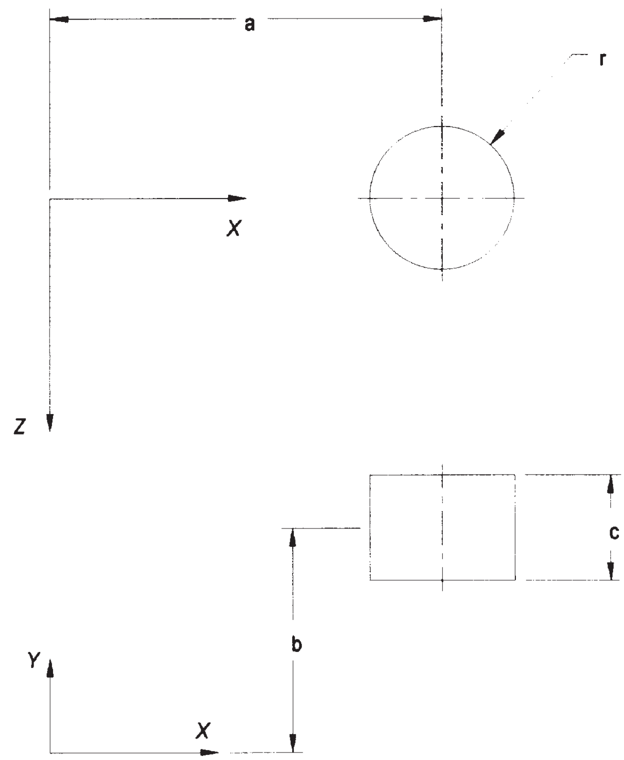
Dimensions:

$a := 70 \cdot mm$

$b := 40.5 \cdot mm$

$c := 19 \cdot mm$

$r := 13 \cdot mm$



Determine the location of the CG in global coordinates.

$Xcg_5 := a \qquad Xcg_5 = 70.000\, mm \qquad Ycg_5 := b \qquad Ycg_5 = 40.500\, mm$

$Zcg_5 := 0 \cdot mm$

Determine the volume and mass of this segment.

$V_5 := c \cdot \pi \cdot r^2 \qquad V_5 = 1.009 \times 10^4\, mm^3 \qquad M_5 := -\rho \cdot V_5 \qquad M_5 = -0.079\, kg$

Determine the mass moments about the local axes of the segment.

$$I_x := \frac{M_5}{12} \cdot \left(3 \cdot r^2 + c^2\right) \qquad I_x = -5.691 \times 10^{-6}\, kg \cdot m^2$$

$$I_y := \frac{M_5}{2} \cdot r^2 \qquad I_y = -6.649 \times 10^{-6}\, kg \cdot m^2$$

$$I_z := \frac{M_5}{12} \cdot \left(3 \cdot r^2 + c^2\right) \qquad I_z = -5.691 \times 10^{-6}\, kg \cdot m^2$$

Determine the mass moments about the global axes.

$$Ixx_5 := I_x + M_5 \cdot \left[\left(Ycg_5\right)^2 + \left(Zcg_5\right)^2 \right] \qquad Ixx_5 = -1.348 \times 10^{-4}\, kg \cdot m^2$$

$$Iyy_5 := I_y + M_5 \cdot \left[\left(Xcg_5\right)^2 + \left(Zcg_5\right)^2 \right] \qquad Iyy_5 = -3.922 \times 10^{-4}\, kg \cdot m^2$$

$$Izz_5 := I_z + M_5 \cdot \left[\left(Xcg_5\right)^2 + \left(Ycg_5\right)^2 \right] \qquad Izz_5 = -5.203 \times 10^{-4}\, kg \cdot m^2$$

a. Find the location of its centroid referred to point *B*.

$$X_{Cg} := \frac{\sum_i Xcg_i \cdot M_i}{\sum_i M_i} \qquad X_{Cg} = 34.919\, mm$$

$$Y_{Cg} := \frac{\sum_i Ycg_i \cdot M_i}{\sum_i M_i} \qquad Y_{Cg} = 26.688\, mm$$

$$Z_{Cg} := \frac{\sum_i Zcg_i \cdot M_i}{\sum_i M_i} \qquad Z_{Cg} = 0.000\, mm$$

b. Find its mass moment of inertia I_{xx} about the *X* axis through point *B*.

$$I_{XX} := \sum_i Ixx_i \qquad I_{XX} = 1.531 \times 10^{-3}\, kg \cdot m^2$$

c. Find its mass moment of inertia I_{yy} about the *Y* axis through point *B*.

$$I_{YY} := \sum_i Iyy_i \qquad I_{YY} = 2.976 \times 10^{-3}\, kg \cdot m^2$$

PROBLEM 10-6

Statement: Two springs are connected in series. One has a k of 34 and the other a k of 3.4. Calculate their effective spring constant. Which spring dominates? Repeat with the two springs in parallel. Which spring dominates? (Use any unit system.)

Given: Spring constants:

$$k_1 := 34 \cdot \frac{N}{mm} \qquad k_2 := 3.4 \cdot \frac{N}{mm}$$

Solution: See Mathcad file P1006.

1. For springs in series, use equation 10.19c to find the effective spring rate.

$$k_{eff} := \left(\frac{1}{k_1} + \frac{1}{k_2} \right)^{-1} \qquad k_{eff} = 3.091 \frac{N}{mm}$$

The spring with the smaller rate dominates.

2. For springs in parallel, use equation 10.20b to find the effective spring rate.

$$k_{eff} := k_1 + k_2 \qquad k_{eff} = 37.400 \frac{N}{mm}$$

The spring with the larger rate dominates.

PROBLEM 10-7

Statement: Two springs are connected in series. One has a *k* of 125 and the other a *k* of 25. Calculate their effective spring constant. Which spring dominates? Repeat with the two springs in parallel. Which spring dominates? (Use any unit system.)

Given: Spring constants:

$$k_1 := 125 \cdot \frac{N}{mm} \qquad k_2 := 25 \cdot \frac{N}{mm}$$

Solution: See Mathcad file P1007.

1. For springs in series, use equation 10.19c to find the effective spring rate.

$$k_{eff} := \left(\frac{1}{k_1} + \frac{1}{k_2} \right)^{-1} \qquad k_{eff} = 20.833 \frac{N}{mm}$$

The spring with the smaller rate dominates.

2. For springs in parallel, use equation 10.20b to find the effective spring rate.

$$k_{eff} := k_1 + k_2 \qquad k_{eff} = 150.000 \frac{N}{mm}$$

The spring with the larger rate dominates.

PROBLEM 10-8

Statement: Two springs are connected in series. One has a *k* of 125 and the other a *k* of 115. Calculate their effective spring constant. Which spring dominates? Repeat with the two springs in parallel. Which spring dominates? (Use any unit system.)

Given: Spring constants:

$$k_1 := 125 \cdot \frac{N}{mm} \qquad k_2 := 115 \cdot \frac{N}{mm}$$

Solution: See Mathcad file P1008.

1. For springs in series, use equation 10.19c to find the effective spring rate.

$$k_{eff} := \left(\frac{1}{k_1} + \frac{1}{k_2} \right)^{-1} \qquad k_{eff} = 59.896 \frac{N}{mm}$$

Neither spring dominates since they are both about equally stiff.

2. For springs in parallel, use equation 10.20b to find the effective spring rate.

$$k_{eff} := k_1 + k_2 \qquad k_{eff} = 240.000 \frac{N}{mm}$$

Neither spring dominates since they are both about equally stiff.

PROBLEM 10-9

Statement: Two dampers are connected in series. One has a damping factor of $c_1 = 12.5$ and the other, $c_2 = 1.2$. Calculate their effective damping constant. Which damper dominates? Repeat with the two dampers in parallel. Which damper dominates? (Use any unit system.)

Given: Damping factors:

$$c_1 := 12.5 \cdot \frac{N \cdot sec}{mm} \qquad c_2 := 1.2 \cdot \frac{N \cdot sec}{mm}$$

Solution: See Mathcad file P1009.

1. For dampers in series, use equation 10.18a to find the effective damping factor.

$$c_{eff} := \left(\frac{1}{c_1} + \frac{1}{c_2} \right)^{-1} \qquad c_{eff} = 1.095 \frac{N \cdot sec}{mm}$$

The softer damper dominates.

2. For dampers in parallel, use equation 10.18b to find the effective damping factor.

$$c_{eff} := c_1 + c_2 \qquad c_{eff} = 13.700 \frac{N \cdot sec}{mm}$$

The stiffer damper dominates.

PROBLEM 10-10

Statement: Two dampers are connected in series. One has a damping factor of $c_1 = 12.5$ and the other, $c_2 = 2.5$. Calculate their effective damping constant. Which damper dominates? Repeat with the two dampers in parallel. Which damper dominates? (Use any unit system.)

Given: Damping factors:

$$c_1 := 12.5 \cdot \frac{N \cdot sec}{mm} \qquad c_2 := 2.5 \cdot \frac{N \cdot sec}{mm}$$

Solution: See Mathcad file P1010.

1. For dampers in series, use equation 10.18a to find the effective damping factor.

$$c_{eff} := \left(\frac{1}{c_1} + \frac{1}{c_2} \right)^{-1} \qquad c_{eff} = 2.083 \frac{N \cdot sec}{mm}$$

The softer damper dominates.

2. For dampers in parallel, use equation 10.18b to find the effective damping factor.

$$c_{eff} := c_1 + c_2 \qquad c_{eff} = 15.000 \frac{N \cdot sec}{mm}$$

The stiffer damper dominates.

PROBLEM 10-11

Statement: Two dampers are connected in series. One has a damping factor of $c_1 = 12.5$ and the other, $c_2 = 10$. Calculate their effective damping constant. Which damper dominates? Repeat with the two dampers in parallel. Which damper dominates? (Use any unit system.)

Given: Damping factors:

$$c_1 := 12.5 \cdot \frac{N \cdot sec}{mm} \qquad c_2 := 10 \cdot \frac{N \cdot sec}{mm}$$

Solution: See Mathcad file P1011.

1. For dampers in series, use equation 10.18a to find the effective damping factor.

$$c_{eff} := \left(\frac{1}{c_1} + \frac{1}{c_2} \right)^{-1} \qquad c_{eff} = 5.556 \frac{N \cdot sec}{mm}$$

Neither damper dominates since they are both about equally stiff.

2. For dampers in parallel, use equation 10.18b to find the effective damping factor.

$$c_{eff} := c_1 + c_2 \qquad c_{eff} = 22.500 \frac{N \cdot sec}{mm}$$

Neither damper dominates since they are both about equally stiff.

PROBLEM 10-12

Statement: A mass of $m = 2.5$ and a spring with $k = 42$ are attached to one end of a lever at a radius of 4. Calculate the effective mass and effective spring constant at a radius of 12 on the same lever. (Use any unit system.)

Given:

Mass: $M := 2.5 \cdot kg$

Spring: $k := 42 \cdot \frac{N}{mm}$

Radius: $r_1 := 4 \cdot mm$ $\quad r_2 := 12 \cdot mm$

Solution: See Figure 10-8 and Mathcad file P1012.

1. The effective mass and spring must have the same energy as the original. Taking the mass first and using equation 10.22b,

$$m_{eff} := \left(\frac{r_1}{r_2}\right)^2 \cdot M \qquad m_{eff} = 0.278\,\text{kg}$$

2. To find the effective spring rate, use equation 10.23b.

$$k_{eff} := \left(\frac{r_1}{r_2}\right)^2 \cdot k \qquad k_{eff} = 4.667\,\frac{N}{mm}$$

PROBLEM 10-13

Statement: A mass of $m = 1.5$ and a spring with $k = 24$ are attached to one end of a lever at a radius of 3. Calculate the effective mass and effective spring constant at a radius of 10 on the same lever. (Use any unit system.)

Given:

Mass: $M := 1.5 \cdot kg$

Spring: $k := 24 \cdot \frac{N}{mm}$

Radius: $r_1 := 3 \cdot mm$ $\quad r_2 := 10 \cdot mm$

Solution: See Figure 10-8 and Mathcad file P1013.

1. The effective mass and spring must have the same energy as the original. Taking the mass first and using equation 10.22b,

$$m_{eff} := \left(\frac{r_1}{r_2}\right)^2 \cdot M \qquad m_{eff} = 0.135\,\text{kg}$$

2. To find the effective spring rate, use equation 10.23b.

$$k_{eff} := \left(\frac{r_1}{r_2}\right)^2 \cdot k \qquad k_{eff} = 2.160\,\frac{N}{mm}$$

PROBLEM 10-14

Statement: A mass of $m = 4.5$ and a spring with $k = 15$ are attached to one end of a lever at a radius of 12. Calculate the effective mass and effective spring constant at a radius of 3 on the same lever. (Use any unit system.)

Given:

Mass: $M := 4.5 \cdot kg$

Spring: $k := 15 \cdot \frac{N}{mm}$

Radius: $r_1 := 12 \cdot mm$ $\quad r_2 := 3 \cdot mm$

Solution: See Figure 10-8 and Mathcad file P1014.

1. The effective mass and spring must have the same energy as the original. Taking the mass first and using equation 10.22b,

$$m_{eff} := \left(\frac{r_1}{r_2}\right)^2 \cdot M \qquad m_{eff} = 72.000\ \text{kg}$$

2. To find the effective spring rate, use equation 10.23b.

$$k_{eff} := \left(\frac{r_1}{r_2}\right)^2 \cdot k \qquad k_{eff} = 240.000\ \frac{N}{mm}$$

PROBLEM 10-15

Statement: Refer to Figure 10-9 and Example 10-1. The data for the valve train are given below. Calculate the effective spring constant and effective mass of a single-DOF equivalent system placed on the cam side of the rocker arm. (Use ips unit system.)

Given: Tappet (solid cylinder): $d_{tp} := 0.75 \cdot in$ $\quad L_{tp} := 1.25 \cdot in$

Pushrod (hollow cylinder): $d_{pod} := 0.375 \cdot in$ $\quad d_{pid} := 0.250 \cdot in$ $\quad L_{pr} := 12 \cdot in$

Rocker arm: $w := 1.00 \cdot in$ $\quad h := 1.50 \cdot in$ $\quad a := 2.00 \cdot in$ $\quad b := 3.00 \cdot in$

Cam shaft (cam in center): $d_{cs} := 1.00 \cdot in$ $\quad L_{cs} := 3.00 \cdot in$

Valve spring: $k_{vs} := 200 \cdot lbf \cdot in^{-1}$

All parts are steel: Modulus of elasticity $E := 30 \cdot 10^{6} \cdot psi$ $\quad$ Spec. weight $\gamma := 0.3 \cdot \frac{lbf}{in^{3}}$

Solution: See Figure 10-9 and Mathcad file P1015.

1. Break the system into individual elements as shown in Figure 10-9b.
2. Define the individual spring constants of each of the six elements.

Cam shaft (simply-supported beam with central load):

Moment of inertia $\quad I_{cs} := \frac{\pi \cdot d_{cs}^{4}}{64}$

Spring constant $\quad k_{cs} := \frac{48 \cdot E \cdot I_{cs}}{L_{cs}^{3}}$ $\quad k_{cs} = 2.618 \times 10^{6} \frac{lbf}{in}$

Tappet (solid cylinder):

Area $\quad A_{tp} := \frac{\pi \cdot d_{tp}^{2}}{4}$

Spring constant $\quad k_{tp} := \frac{A_{tp} \cdot E}{L_{tp}}$ $\quad k_{tp} = 1.060 \times 10^{7} \frac{lbf}{in}$

Pushrod (hollow cylinder):

Area $\quad A_{pr} := \frac{\pi \cdot \left(d_{pod}^{2} - d_{pid}^{2}\right)}{4}$

Spring constant $\quad k_{pr} := \frac{A_{pr} \cdot E}{L_{pr}}$ $\quad k_{pr} = 1.534 \times 10^{5} \frac{lbf}{in}$

Rocker arm (side A):

Moment of inertia $\quad I_{r} := \frac{w \cdot h^{3}}{12}$

Spring constant $\quad k_{ra} := \frac{3 \cdot E \cdot I_{r}}{a^{3}}$ $\quad k_{ra} = 3.164 \times 10^{6} \frac{lbf}{in}$

Rocker arm (side B):

Spring constant $\quad k_{rb} := \frac{3 \cdot E \cdot I_{r}}{b^{3}}$ $\quad k_{rb} = 9.375 \times 10^{5} \frac{lbf}{in}$

3. Damping will be neglected.
4. Determine the mass of each of the elements.

Tappet (solid cylinder):

Volume $V_{tp} := A_{tp} \cdot L_{tp}$

Mass $m_{tp} := \dfrac{\gamma \cdot V_{tp}}{g}$ $\quad m_{tp} = 4.291 \times 10^{-4}\, lbf \cdot sec^2 \cdot in^{-1}$

Pushrod (hollow cylinder):

Volume $V_{pr} := A_{pr} \cdot L_{pr}$

Mass $m_{pr} := \dfrac{\gamma \cdot V_{pr}}{g}$ $\quad m_{pr} = 5.721 \times 10^{-4}\, lbf \cdot sec^2 \cdot in^{-1}$

Rocker arm (side *A*):

Volume $V_{ra} := w \cdot h \cdot a$

Mass $m_{ra} := \dfrac{\gamma \cdot V_{ra}}{g}$ $\quad m_{ra} = 2.331 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

Rocker arm (side *B*):

Volume $V_{rb} := w \cdot h \cdot b$

Mass $m_{rb} := \dfrac{\gamma \cdot V_{rb}}{g}$ $\quad m_{rb} = 3.497 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

Omit the valve and valve spring because no data are available.

5. Determine the effective mass and spring constant on either side of the rocker arm..

Left side: $m_L := m_{tp} + m_{pr} + m_{ra}$ $\quad m_L = 3.332 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

$$k_L := \left(\frac{1}{k_{cs}} + \frac{1}{k_{tp}} + \frac{1}{k_{pr}} + \frac{1}{k_{ra}} \right)^{-1} \qquad k_L = 1.368 \times 10^5 \frac{lbf}{in}$$

Right side: $m_R := m_{rb}$ $\quad m_R = 3.497 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

$$k_R := \left(\frac{1}{k_{rb}} + \frac{1}{k_{vs}} \right)^{-1} \qquad k_R = 199.957 \frac{lbf}{in}$$

6. Reduce the system to a single DOF.

$$m_{eff} := m_L + \left(\frac{b}{a} \right)^2 \cdot m_R \qquad m_{eff} = 0.011\, lbf \cdot sec^2 \cdot in^{-1}$$

$$k_{eff} := k_L + \left(\frac{b}{a} \right)^2 \cdot k_R \qquad k_{eff} = 1.372 \times 10^5 \frac{lbf}{in}$$

PROBLEM 10-16

Statement: Figure P10-2 shows a cam-follower system. The dimensions and other data are given below. Find the arm's mass, center of gravity location and mass moment of inertia about both its CG and arm pivot. Create a linear, one-DOF lumped mass model of the dynamic system referenced to the cam follower. Ignore damping.

Given:

Arm dimensions:		Cutout dimensions:	
Width (Z-direction):	$a := 2 \cdot in$	Width (Z-direction):	$a' := 1.5 \cdot in$
Height (Y-direction):	$b := 2.5 \cdot in$	Height (Y-direction):	$b' := 2.5 \cdot in$
Length (X-direction):	$c := 28 \cdot in$	Length (X-direction):	$c' := 3 \cdot in$

Distance from left end of arm to roller center: $L_r := 22 \cdot in$

Distance from left end of arm to arm pivot: $L_p := 10 \cdot in$

Specific weight of aluminum: $\gamma := 0.1 \cdot lbf \cdot in^{-3}$ $\quad E := 10.4 \cdot 10^6 \cdot psi$

Solution: See Figure P10-2 and Mathcad file P1016.

1. Determine the volume and weight of the arm.

$V_{solid} := a \cdot b \cdot c \qquad V_{solid} = 140.000\ in^3$

$V_{slot} := a' \cdot b' \cdot c' \qquad V_{slot} = 11.250\ in^3$

$V_a := V_{solid} - V_{slot} \qquad V_a = 128.750\ in^3$

$W_{solid} := \gamma \cdot V_{solid} \qquad W_{solid} = 14.000\ lbf$

$W_{slot} := -\gamma \cdot V_{slot} \qquad W_{slot} = -1.125\ lbf$

$W_a := W_{solid} + W_{slot} \qquad W_a = 12.875\ lbf$

2. Calculate the location of the arm CG with respect to the left end.

$$X_{Cg} := \frac{W_{solid} \cdot \frac{c}{2} + W_{slot} \cdot L_r}{W_a} \qquad X_{Cg} = 13.301\ in$$

3. Determine the moment of inertia of the arm about the its CG.

Solid portion about its own CG $\quad I_{zsolid} := \frac{W_{solid}}{12 \cdot g} \cdot \left(b^2 + c^2\right) \qquad I_{zsolid} = 2.388\ lbf \cdot sec^2 \cdot in$

Slot portion about its own CG $\quad I_{zslot} := \frac{W_{slot}}{12 \cdot g} \cdot \left(b^2 + c^2\right) \qquad I_{zslot} = -3.703 \times 10^{-3}\ lbf \cdot sec^2$

Solid portion about arm CG $\quad I_{Zsolid} := I_{zsolid} + \frac{W_{solid}}{g} \cdot \left(X_{Cg} - \frac{c}{2}\right)^2$

$I_{Zsolid} = 2.406\ lbf \cdot sec^2 \cdot in$

Slot portion about arm CG $I_{Zslot} := I_{zslot} + \frac{W_{slot}}{g} \cdot (L_r - X_{Cg})^2$

$I_{Zslot} = -0.224\ lbf \cdot sec^2 \cdot in$

Arm about its CG $I_{ZZ} := I_{Zsolid} + I_{Zslot}$ $I_{ZZ} = 2.181\ lbf \cdot sec^2 \cdot in$

4. Determine the moment of inertia of the arm about the arm pivot.

$$I_{pivot} := I_{ZZ} + \frac{W_a}{g} \cdot (X_{Cg} - L_p)^2 \qquad I_{pivot} = 2.545\ lbf \cdot sec^2 \cdot in$$

5. Determine the mass and spring constant of the arm on the left side of the pivot.

Moment of inertia $I_{arm} := \frac{a \cdot b^3}{12}$

Spring constant $k_L := \frac{3 \cdot E \cdot I_{arm}}{L_p^3}$ $k_L = 8.125 \times 10^4 \frac{lbf}{in}$

Volume $V_L := a \cdot b \cdot L_p$

Mass $m_L := \frac{\gamma \cdot V_L}{g}$ $m_L = 0.013\ lbf \cdot sec^2 \cdot in^{-1}$

6. Determine the mass and spring constant of the arm on the right side of the pivot.

Spring constant $k_R := \frac{3 \cdot E \cdot I_{arm}}{(c - L_p)^3}$ $k_R = 1.393 \times 10^4 \frac{lbf}{in}$

Volume $V_R := a \cdot b \cdot (c - L_p)$

Mass $m_R := \frac{\gamma \cdot V_R}{g}$ $m_R = 0.023\ lbf \cdot sec^2 \cdot in^{-1}$

6. Find the effective mass and spring constant referenced to the cam-follower..

$$m_{eff} := m_L + \left(\frac{L_r - L_p}{L_p}\right)^2 \cdot m_R \qquad m_{eff} = 0.04652\ lbf \cdot sec^2 \cdot in^{-1}$$

$$k_{eff} := k_L + \left(\frac{L_r - L_p}{L_p}\right)^2 \cdot k_R \qquad k_{eff} = 1.0131 \times 10^5 \frac{lbf}{in}$$

PROBLEM 10-17

Statement: Use the method of virtual work to find the torque required to rotate the cam in Figure P10-2 through one revolution. The cam is a pure eccentric.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given:

Cam eccentricity and speed: $e := 0.5\cdot in$ $\omega_2 := 500\cdot rpm$

Cam radius and thickness: $r_c := 3\cdot in$ $w_c := 0.75\cdot in$ (measured from Figure)

Spring rate and preload: $k := 123\cdot lbf\cdot in^{-1}$ $F_{50} := 173\cdot lbf$

Arm dimensions:		Cutout dimensions:	
Width (Z-direction):	$a := 2\cdot in$	Width (Z-direction):	$a' := 1.5\cdot in$
Height (Y-direction):	$b := 2.5\cdot in$	Height (Y-direction):	$b' := 2.5\cdot in$
Length (X-direction):	$c := 28\cdot in$	Length (X-direction):	$c' := 3\cdot in$

Roller follower dimensions: $r_f := 1\cdot in$ $w_f := 1.5\cdot in$

Distance from left end of arm to roller center: $L_r := 22\cdot in$

Distance from left end of arm to spring: $L_s := 29\cdot in$

Distance from left end of arm to pivot: $L_p := 10\cdot in$

Specific weight of aluminum: $\gamma_a := 0.1\cdot lbf\cdot in^{-3}$

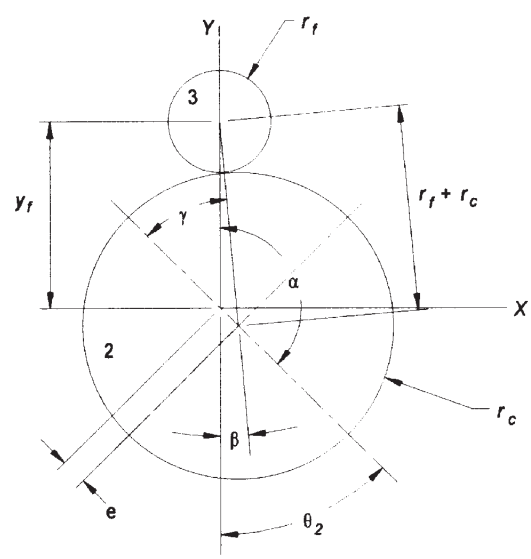
Specific weight of steel: $\gamma_s := 0.3\cdot lbf\cdot in^{-3}$

Assumptions: Small angle theory applies.

Solution: See Figure P10-2, Problem 10-16, and Mathcad file P1017.

1. Start with a kinematic analysis of the mechanism. Let the cam be link 2, the roller follower link 3, the arm link 4, and the spring link 5. Let the point where the spring connects to the arm be point B, and the center of the roller follower be point A.

2. Calculate the motion of the roller follower center with respect to the X axis, which goes through the center of rotation of the cam.



The cam and follower are shown at right. The origin of the coordinate frame is at the center of rotation. The cam, link 2, is shown rotated an amount θ_2 from the position at which the follower, link 3, is at its lowest position. A triangle is formed with sides e, $r_f + r_c$, and y_f. This triangle will be used to determine the displacement of the follower, y_f, with respect to the cam rotation angle, θ_2.

Using the law of sines,

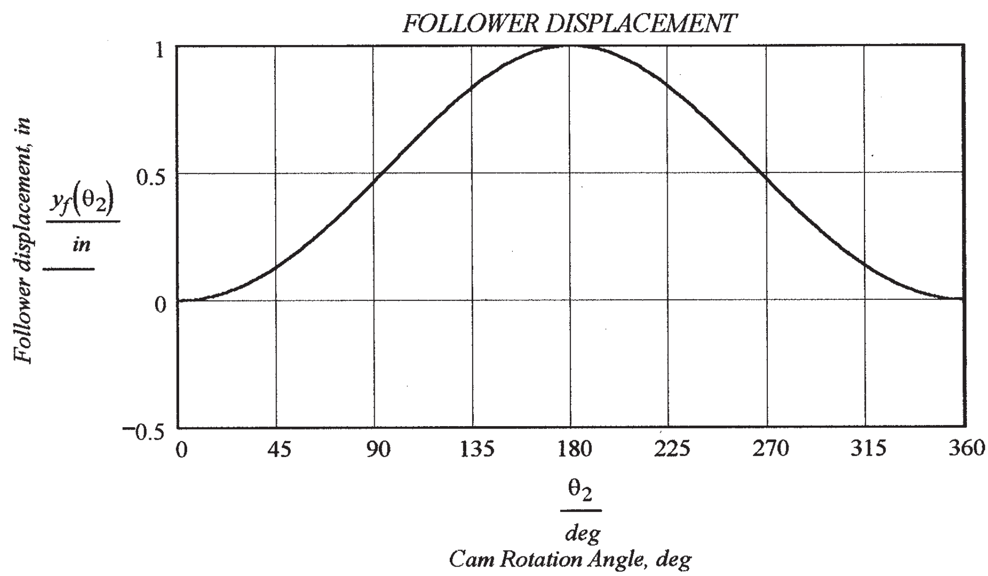
$$\beta(\theta_2) := asin\left(\frac{e}{r_f + r_c}\cdot sin(\theta_2)\right)$$

From the relationship among the angles of a triangle, $\gamma(\theta_2) := \theta_2 - \beta(\theta_2)$

And, using the law of cosines,

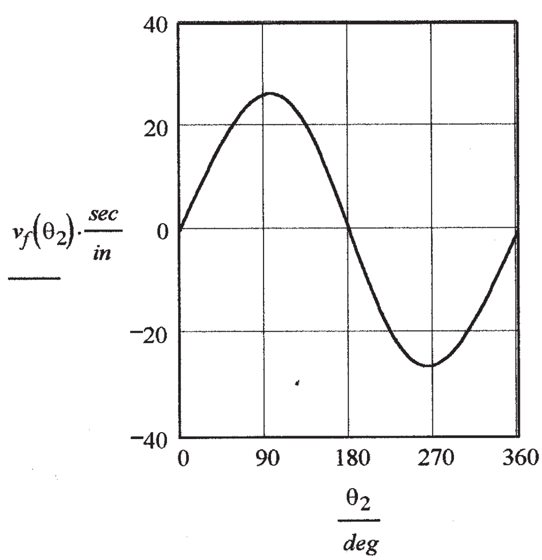
$$y_f(\theta_2) := \left[e^2 + (r_f + r_c)^2 - 2\cdot e\cdot(r_f + r_c)\cdot cos(\gamma(\theta_2))\right]^{\frac{1}{2}} \dots + e - (r_f + r_c)$$

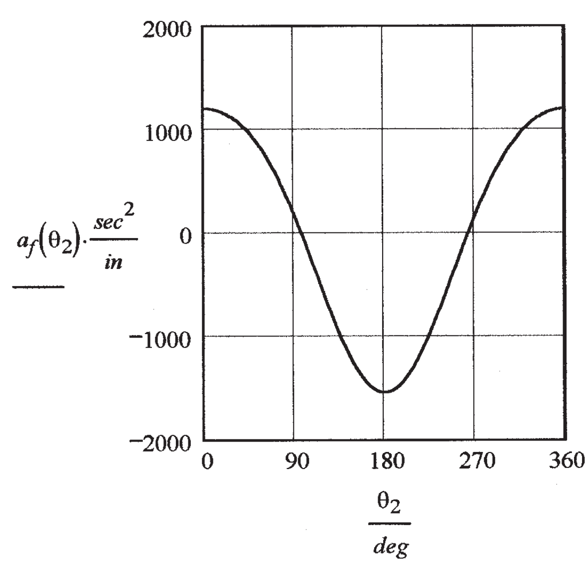
Plotting y_f as a function of θ_2: $\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$



3. Differentiate the displacement function twice to get the velocity and acceleration functions.

$$v_f(\theta_2) := \left(\frac{d}{d\theta_2} y_f(\theta_2)\right)\cdot\omega_2 \qquad a_f(\theta_2) := \left(\frac{d}{d\theta_2} v_f(\theta_2)\right)\cdot\omega_2$$





4. Determine the angular displacement, velocity, and acceleration of the arm, link 4.

$$\theta_4(\theta_2) := \frac{1}{L_r - L_p} \cdot y_f(\theta_2) \qquad \omega_4(\theta_2) := \frac{1}{L_r - L_p} \cdot v_f(\theta_2)$$

$$\alpha_4(\theta_2) := \frac{1}{L_r - L_p} \cdot a_f(\theta_2)$$

5. Determine the displacement and velocity of point *A*, where the spring attaches to the arm.

$$y_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot y_f(\theta_2) \qquad v_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot v_f(\theta_2)$$

6. Calculate the spring force as a function of θ_2.

$$F_5(\theta_2) := k \cdot y_5(\theta_2) + F_{50}$$

7. The spring force is the only external force on the linkage that is not applied at a joint and is, therefore, the only force to be considered in the first term of equation 10.28b. Determine the first term in the equation.

$$EF(\theta_2) := F_5(\theta_2) \cdot v_5(\theta_2)$$

8. Determine the components that make up the third term in equation 10.28b. Links 2 and 4 are rotating about fixed pivots so their input can be taken care of in the fourth term. The roller follower has both rotation and translation. The translation motion will be accounted for in this term.

Mass of follower: $m_f := \pi \cdot r_f^{\,2} \cdot w_f \cdot \frac{\gamma_s}{g}$ $\qquad m_f = 1.414 \text{ lb}$

$$IF(\theta_2) := m_f \cdot a_f(\theta_2) \cdot v_f(\theta_2)$$

9. Determine the components that make up the fourth term in equation 10.28b. There will be input from the rotation of the arm. The cam and follower have no angular acceleration so they have no contribution. From Problem 10-26, we have the following data:

Mass of roller and arm combined: $m_{ra} := \frac{14.289 \cdot lbf}{g}$ $\qquad m_{ra} = 14.289 \text{ lb}$

Distance from left end of arm to composite CG: $X_{Cg} := 14.162 \cdot in$

Composite moment of inertia with respect to CG: $I_{CG} := 2.433 \cdot lbf \cdot sec^2 \cdot in$

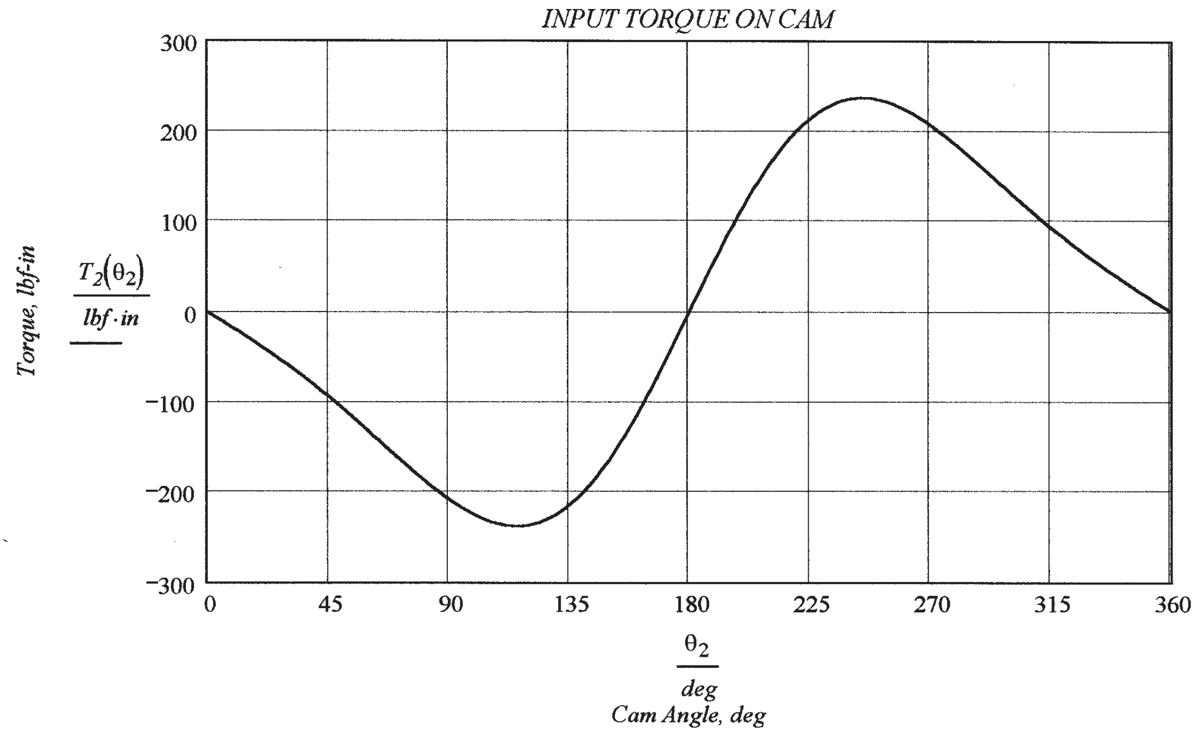
Moment with respect to the arm pivot:

$$I_4 := I_{CG} + m_{ra} \cdot (X_{Cg} - L_p)^2 \qquad I_4 = 3.074 \, lbf \cdot sec^2 \cdot in$$

$$IT(\theta_2) := I_4 \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)$$

10. Using equation 10.28b, solve for and plot the input torque as a function of cam angle.

$$T_2(\theta_2) := \frac{1}{\omega_2}\cdot\left(IF(\theta_2) + IT(\theta_2) - EF(\theta_2)\right)$$



PROBLEM 10-18

Statement: Use the method of virtual work to find the torque required to rotate the cam in Figure P10-3 through one revolution. The cam is a pure eccentric.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Cam eccentricity and speed: $e := 20\cdot mm$ $\quad \omega_2 := 200\cdot rpm$

Cam radius and thickness: $r_c := 3\cdot in$ $\quad w_c := 0.75\cdot in$ (measured from Figure)

Spring rate and preload: $k := 10\cdot N\cdot m^{-1}$ $\quad F_{50} := 0.2\cdot N$

Arm dimensions:		Cutout dimensions:	
Width (Z-direction):	$a := 2\cdot in$	Width (Z-direction):	$a' := 1.5\cdot in$
Height (Y-direction):	$b := 2.5\cdot in$	Height (Y-direction):	$b' := 2.5\cdot in$
Length (X-direction):	$c := 28\cdot in$	Length (X-direction):	$c' := 3\cdot in$

Roller follower dimensions: $r_f := 1\cdot in$ $\quad w_f := 1.5\cdot in$

Distance from left end of arm to roller center: $L_r := 22\cdot in$

Distance from left end of arm to spring: $L_s := 29\cdot in$

Distance from left end of arm to pivot: $L_p := 10\cdot in$

Specific weight of aluminum: $\gamma_a := 0.1\cdot lbf\cdot in^{-3}$

Specific weight of steel: $\gamma_s := 0.3\cdot lbf\cdot in^{-3}$

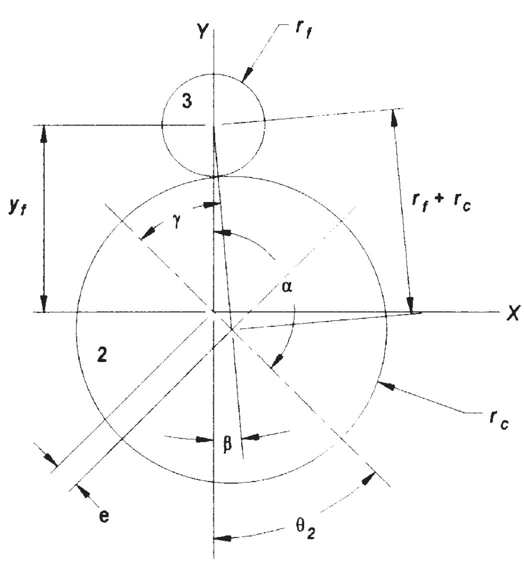
Assumptions: Small angle theory applies.

Solution: See Figure P10-3 and Mathcad file P1018.

1. Start with a kinematic analysis of the mechanism. Let the cam be link 2, the roller follower link 3, the arm link 4, and the spring link 5. Let the point where the spring connects to the arm be point *B*, and the center of the roller follower be point *A*.

2. Calculate the motion of the roller follower center with respect to the X axis, which goes through the center of rotation of the cam.

   The cam and follower are shown at right. The origin of the coordinate frame is at the center of rotation. The cam, link 2, is shown rotated an amount θ_2 from the position at which the follower, link 3, is at its lowest position. A triangle is formed with sides e, $r_f + r_c$, and y_f. This triangle will be used to determine the displacement of the follower, y_f, with respect to the cam rotation angle, θ_2.



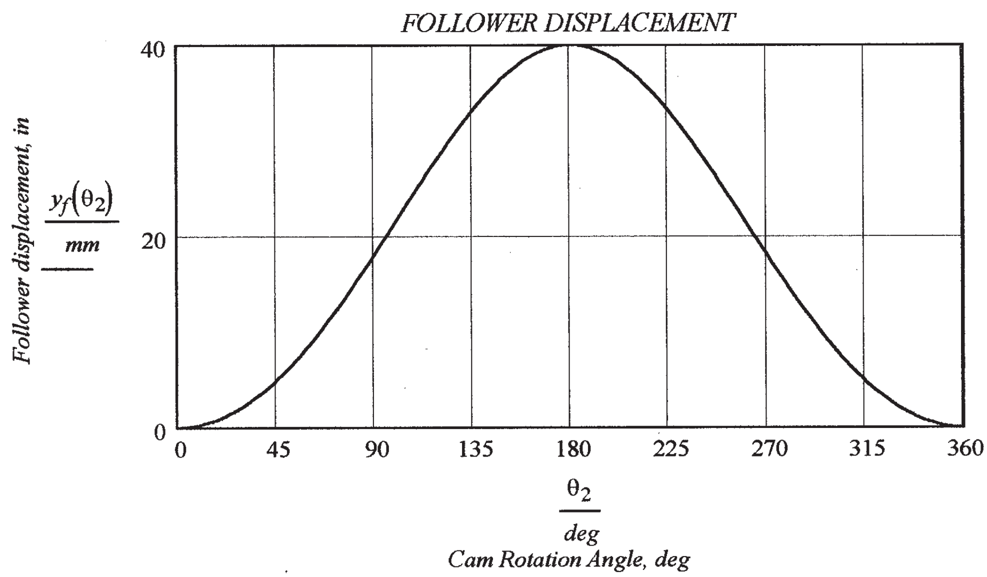
   Using the law of sines,

$$\beta(\theta_2) := asin\left(\frac{e}{r_f + r_c}\cdot sin(\theta_2)\right)$$

From the relationship among the angles of a triangle, $\gamma(\theta_2) := \theta_2 - \beta(\theta_2)$

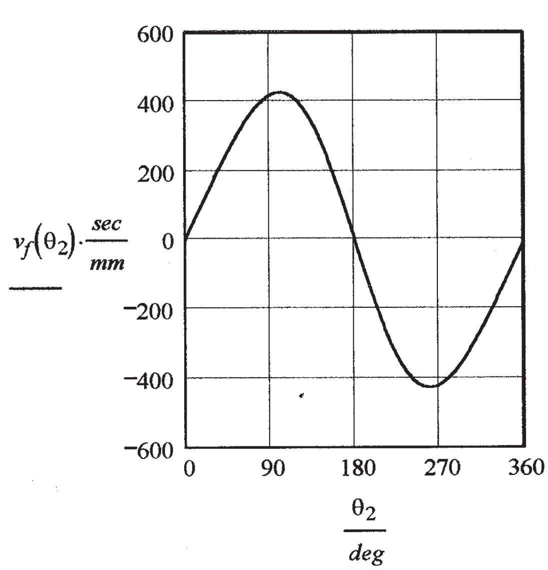
And, using the law of cosines, $y_f(\theta_2) := \left[e^2 + (r_f + r_c)^2 - 2\cdot e\cdot(r_f + r_c)\cdot cos(\gamma(\theta_2))\right]^{\frac{1}{2}} \ldots$
$+e - (r_f + r_c)$

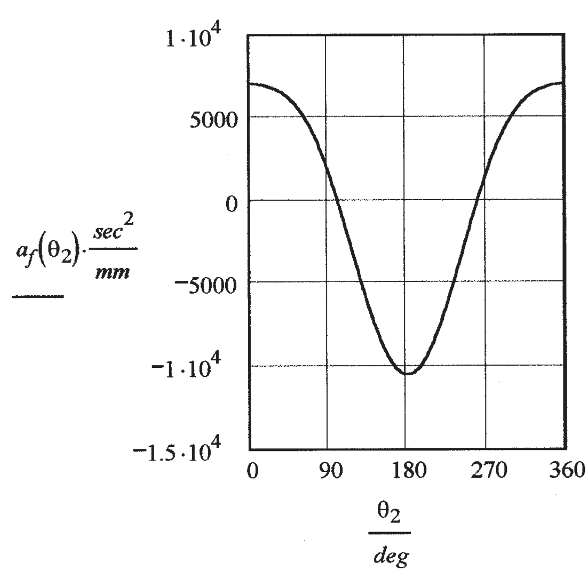
Plotting y_f as a function of θ_2: $\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$



3. Differentiate the displacement function twice to get the velocity and acceleration functions.

$$v_f(\theta_2) := \left(\frac{d}{d\theta_2} y_f(\theta_2)\right)\cdot\omega_2 \qquad a_f(\theta_2) := \left(\frac{d}{d\theta_2} v_f(\theta_2)\right)\cdot\omega_2$$





4. Determine the angular displacement, velocity, and acceleration of the arm, link 4.

$$\theta_4(\theta_2) := \frac{1}{L_r - L_p} \cdot y_f(\theta_2) \qquad \omega_4(\theta_2) := \frac{1}{L_r - L_p} \cdot v_f(\theta_2)$$

$$\alpha_4(\theta_2) := \frac{1}{L_r - L_p} \cdot a_f(\theta_2)$$

5. Determine the displacement and velocity of point A, where the spring attaches to the arm.

$$y_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot y_f(\theta_2) \qquad v_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot v_f(\theta_2)$$

6. Calculate the spring force as a function of θ_2.

$$F_5(\theta_2) := k \cdot y_5(\theta_2) + F_{50}$$

7. The spring force is the only external force on the linkage that is not applied at a joint and is, therefore, the only force to be considered in the first term of equation 10.28b. Determine the first term in the equation.

$$EF(\theta_2) := F_5(\theta_2) \cdot v_5(\theta_2)$$

8. Determine the components that make up the third term in equation 10.28b. Links 2 and 4 are rotating about fixed pivots so their input can be taken care of in the fourth term. The roller follower has both rotation and translation. The translation motion will be accounted for in this term.

Mass of follower: $m_f := 1 \cdot kg$

$$IF(\theta_2) := m_f \cdot a_f(\theta_2) \cdot v_f(\theta_2)$$

9. Determine the components that make up the fourth term in equation 10.28b. There will be input from the rotation of the arm. The cam and follower have no angular acceleration so they have no contribution. From Problem 10-26, we have the following data:

Mass of roller and arm combined: $m_{ra} := \dfrac{14.289 \cdot lbf}{g}$ $\qquad m_{ra} = 6.481\,kg$

Distance from left end of arm to composite CG: $X_{Cg} := 14.162 \cdot in$

Composite moment of inertia with respect to CG: $I_{CG} := 2.433 \cdot lbf \cdot sec^2 \cdot in$

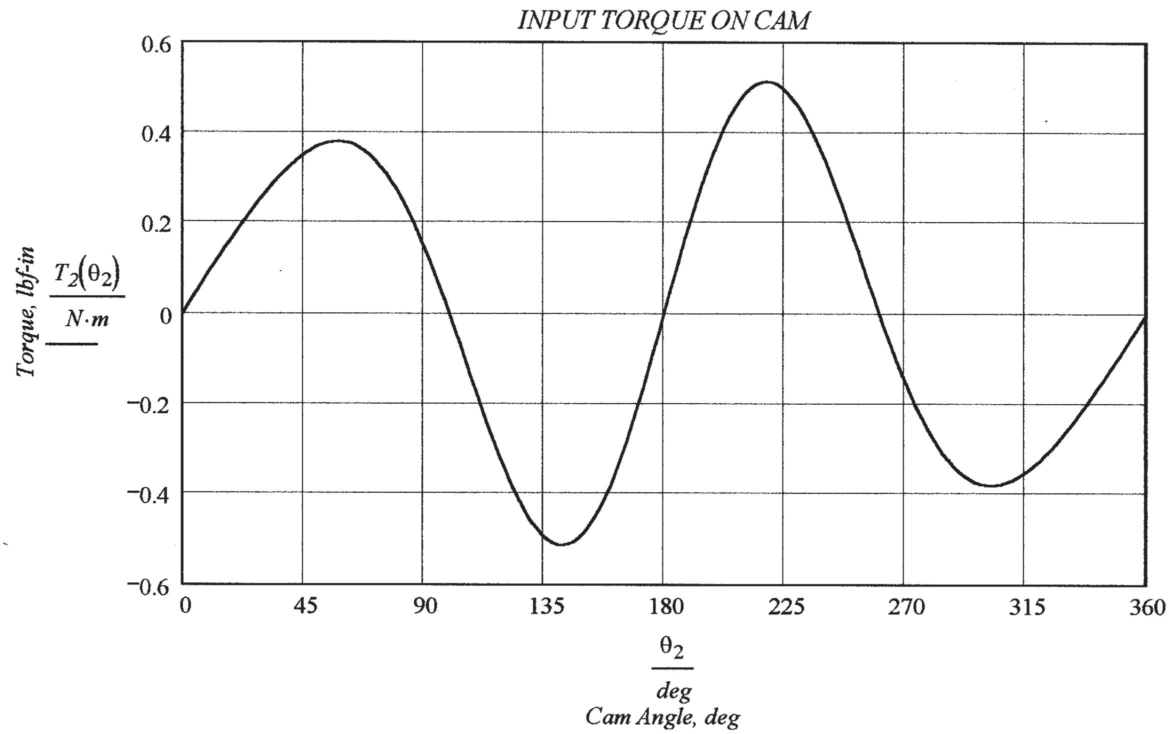
Moment with respect to the arm pivot:

$$I_4 := I_{CG} + m_{ra} \cdot (X_{Cg} - L_p)^2 \qquad I_4 = 0.347\,kg \cdot m^2$$

$$IT(\theta_2) := I_4 \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)$$

10. Using equation 10.28b, solve for and plot the input torque as a function of cam angle.

$$T_2(\theta_2) := \frac{1}{\omega_2} \cdot \left(IF(\theta_2) + IT(\theta_2) - EF(\theta_2) \right)$$



PROBLEM 10-19

Statement: Use the method of virtual work to find the torque required to rotate the cam in Figure P10-3 through one revolution. The cam motion is a double harmonic.

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Cam lift and speed: $h := 20\cdot mm$ $\quad \omega_2 := 200\cdot rpm$

Cam radius and thickness: $r_c := 3\cdot in$ $\quad w_c := 0.75\cdot in$ (measured from Figure)

Spring rate and preload: $k := 10\cdot N\cdot m^{-1}$ $\quad F_{50} := 0.2\cdot N$

Arm dimensions:		Cutout dimensions:	
Width (Z-direction):	$a := 2\cdot in$	Width (Z-direction):	$a' := 1.5\cdot in$
Height (Y-direction):	$b := 2.5\cdot in$	Height (Y-direction):	$b' := 2.5\cdot in$
Length (X-direction):	$c := 28\cdot in$	Length (X-direction):	$c' := 3\cdot in$

Roller follower dimensions: $r_f := 1\cdot in$ $\quad w_f := 1.5\cdot in$

Distance from left end of arm to roller center: $L_r := 22\cdot in$

Distance from left end of arm to spring: $L_s := 29\cdot in$

Distance from left end of arm to pivot: $L_p := 10\cdot in$

Specific weight of aluminum: $\gamma_a := 0.1\cdot lbf\cdot in^{-3}$

Specific weight of steel: $\gamma_s := 0.3\cdot lbf\cdot in^{-3}$

Solution: See Figure P10-3 and Mathcad file P1019.

1. Start with a kinematic analysis of the mechanism. Let the cam be link 2, the roller follower link 3, the arm link 4, and the spring link 5. Let the point where the spring connects to the arm be point *B*, and the center of the roller follower be point *A*.

2. Calculate the motion of the roller follower using equations 8.25. Define β and a range function so the rise and fall can be combined in a single equation.

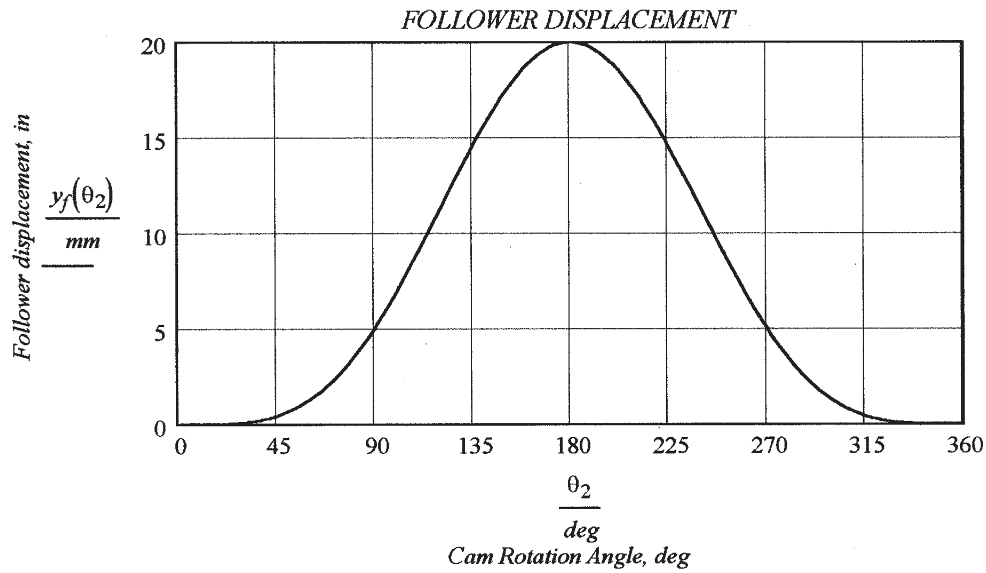
$$\beta := 180\cdot deg \qquad R(\theta, a, b) := if[(\theta > a)\cdot(\theta \le b), 1, 0]$$

$$y_f(\theta_2) := R(\theta_2, 0, \beta)\cdot\frac{h}{2}\cdot\left[\left(1 - \cos\left(\pi\cdot\frac{\theta_2}{\beta}\right)\right) - \frac{1}{4}\cdot\left(1 - \cos\left(2\cdot\pi\cdot\frac{\theta_2}{\beta}\right)\right)\right] \ldots$$
$$+ R(\theta_2, \beta, 2\cdot\pi)\cdot\frac{h}{2}\cdot\left[\left(1 + \cos\left(\pi\cdot\frac{\theta_2 - \beta}{\beta}\right)\right) - \frac{1}{4}\cdot\left(1 - \cos\left(2\cdot\pi\cdot\frac{\theta_2 - \beta}{\beta}\right)\right)\right]$$

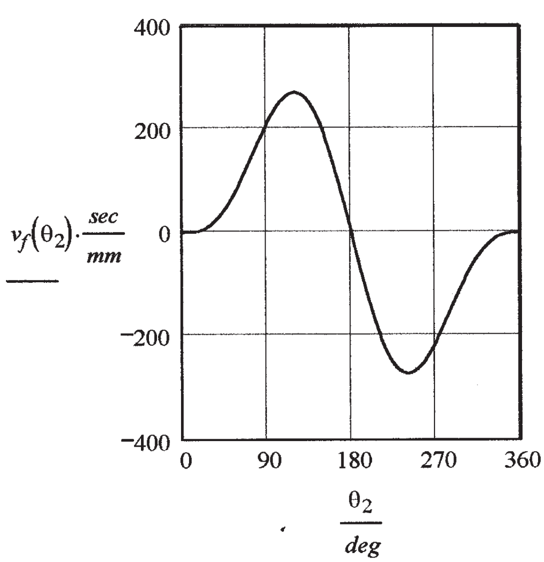
$$v_f(\theta_2) := \left[\begin{array}{l} R(\theta_2, 0, \beta)\cdot\frac{\pi}{\beta}\cdot\frac{h}{2}\cdot\left(\sin\left(\pi\cdot\frac{\theta_2}{\beta}\right) - \frac{1}{2}\cdot\sin\left(2\cdot\pi\cdot\frac{\theta_2}{\beta}\right)\right) \ldots \\ + -R(\theta_2, \beta, 2\cdot\pi)\cdot\frac{\pi}{\beta}\cdot\frac{h}{2}\cdot\left(\sin\left(\pi\cdot\frac{\theta_2 - \beta}{\beta}\right) + \frac{1}{2}\cdot\sin\left(2\cdot\pi\cdot\frac{\theta_2 - \beta}{\beta}\right)\right) \end{array}\right]\cdot\omega_2$$

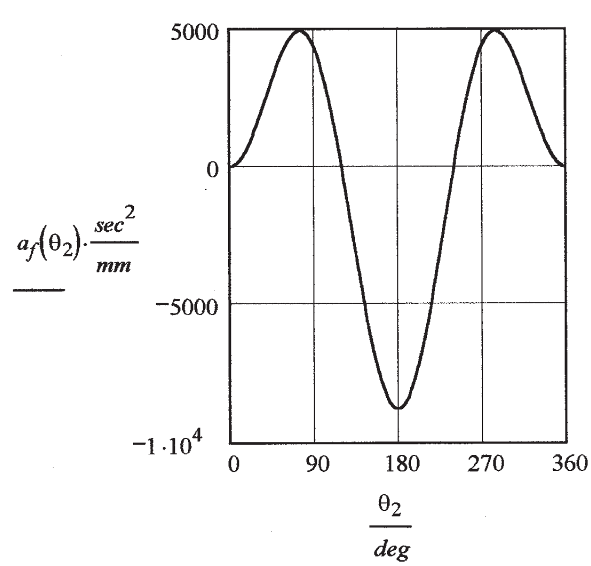
$$a_f(\theta_2) := \left[\begin{array}{l} R(\theta_2,0,\beta)\cdot\frac{\pi^2}{\beta^2}\cdot\frac{h}{2}\cdot\left(\cos\left(\pi\cdot\frac{\theta_2}{\beta}\right) - \cos\left(2\cdot\pi\cdot\frac{\theta_2}{\beta}\right)\right) \ldots \\ + -R(\theta_2,\beta,2\cdot\pi)\cdot\frac{\pi^2}{\beta^2}\cdot\frac{h}{2}\cdot\left(\cos\left(\pi\cdot\frac{\theta_2-\beta}{\beta}\right) + \cos\left(2\cdot\pi\cdot\frac{\theta_2-\beta}{\beta}\right)\right) \end{array} \right]\cdot\omega_2^2$$

Plotting y_f as a function of θ_2: $\theta_2 := 0\cdot deg, 2\cdot deg .. 360\cdot deg$



3. Plot the velocity and acceleration functions.





4. Determine the angular displacement, velocity, and acceleration of the arm, link 4.

$$\theta_4(\theta_2) := \frac{1}{L_r - L_p}\cdot y_f(\theta_2) \qquad \omega_4(\theta_2) := \frac{1}{L_r - L_p}\cdot v_f(\theta_2)$$

$$\alpha_4(\theta_2) := \frac{1}{L_r - L_p} \cdot a_f(\theta_2)$$

5. Determine the displacement and velocity of point A, where the spring attaches to the arm.

$$y_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot y_f(\theta_2) \qquad v_5(\theta_2) := \frac{L_s - L_p}{L_r - L_p} \cdot v_f(\theta_2)$$

6. Calculate the spring force as a function of θ_2.

$$F_5(\theta_2) := k \cdot y_5(\theta_2) + F_{50}$$

7. The spring force is the only external force on the linkage that is not applied at a joint and is, therefore, the only force to be considered in the first term of equation 10.28b. Determine the first term in the equation.

$$EF(\theta_2) := F_5(\theta_2) \cdot v_5(\theta_2)$$

8. Determine the components that make up the third term in equation 10.28b. Links 2 and 4 are rotating about fixed pivots so their input can be taken care of in the fourth term. The roller follower has both rotation and translation. The translation motion will be accounted for in this term.

Mass of follower: $m_f := 1 \cdot kg$

$$IF(\theta_2) := m_f \cdot a_f(\theta_2) \cdot v_f(\theta_2)$$

9. Determine the components that make up the fourth term in equation 10.28b. There will be input from the rotation of the arm. The cam and follower have no angular acceleration so they have no contribution. From Problem 10-26, we have the following data:

Mass of roller and arm combined: $m_{ra} := \frac{14.289 \cdot lbf}{g}$ $\quad m_{ra} = 6.481\ \text{kg}$

Distance from left end of arm to composite CG: $X_{Cg} := 14.162 \cdot in$

Composite moment of inertia with respect to CG: $I_{CG} := 2.433 \cdot lbf \cdot sec^2 \cdot in$

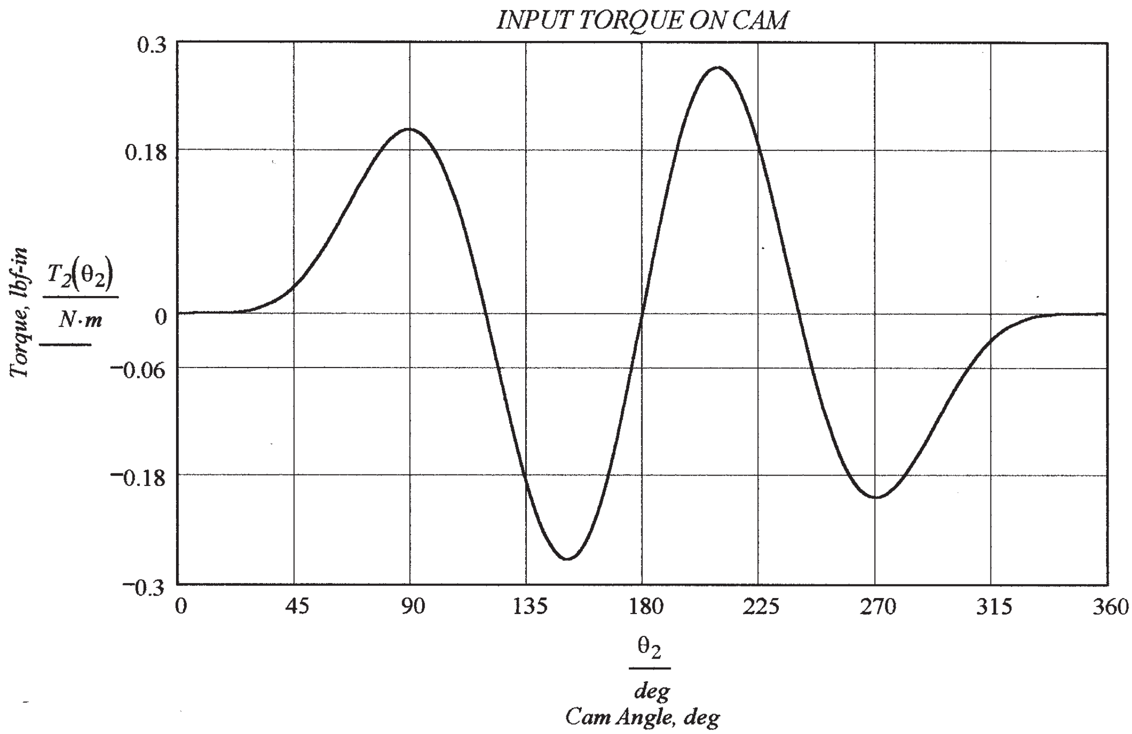
Moment with respect to the arm pivot:

$$I_4 := I_{CG} + m_{ra} \cdot (X_{Cg} - L_p)^2 \qquad I_4 = 0.347\ kg \cdot m^2$$

$$IT(\theta_2) := I_4 \cdot \alpha_4(\theta_2) \cdot \omega_4(\theta_2)$$

10. Using equation 10.28b, solve for and plot the input torque as a function of cam angle.

$$T_2(\theta_2) := \frac{1}{\omega_2} \cdot (IF(\theta_2) + IT(\theta_2) - EF(\theta_2))$$


INPUT TORQUE ON CAM
0.3
0.18
0
−0.06
−0.18
−0.3
Torque, lbf-in
T2(θ2)
N·m
0
45
90
135
180
225
270
315
360
θ2
deg
Cam Angle, deg

PROBLEM 10-20

Statement: A 3000-lb automobile has a final drive ratio of 1:3 and transmission gear ratios of 1:4, 1:3, 1:2, and 1:1 in first through fourth speeds, respectively. What is the effective mass of the vehicle as felt by the engine flywheel in each gear?

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Gear ratios:

Final $m_{Gf} := 3$ First $m_{G1} := 4$ Second $m_{G2} := 3$

Third $m_{G3} := 2$ Fourth $m_{G4} := 1$

Weight of automobile: $W := 3000 \cdot lbf$

Solution: See Mathcad file P1020.

1. Calculate the mass of the vehicle.

$$M := \frac{W}{g} \qquad M = 7.770\,blob$$

2. The effective mass of the vehicle at the flywheel while in first gear is,

$$m_{eff1} := \left(m_{Gf} \cdot m_{G1}\right)^{-2} \cdot M \qquad m_{eff1} = 0.0540\,blob$$

3. The effective mass of the vehicle at the flywheel while in second gear is,

$$m_{eff2} := \left(m_{Gf} \cdot m_{G2}\right)^{-2} \cdot M \qquad m_{eff2} = 0.0959\,blob$$

4. The effective mass of the vehicle at the flywheel while in third gear is,

$$m_{eff3} := \left(m_{Gf} \cdot m_{G3}\right)^{-2} \cdot M \qquad m_{eff3} = 0.216\,blob$$

5. The effective mass of the vehicle at the flywheel while in fourth gear is,

$$m_{eff4} := \left(m_{Gf} \cdot m_{G4}\right)^{-2} \cdot M \qquad m_{eff4} = 0.863\,blob$$

PROBLEM 10-21

Statement: Determine the effective spring constant and effective preload of the spring in Figure P10-2 as reflected back to the cam follower. See Problem 10-17 for additional data.

Given: Spring data: $k := 123 \cdot lbf \cdot in^{-1}$ $F_o := 173 \cdot lbf$

Distance from arm pivot to spring: $a := 19 \cdot in$

Distance from arm pivot to follower: $b := 12 \cdot in$

Solution: See Figure P10-2, Problem 10-17, and Mathcad file P1021.

1. The effective spring constant at the follower is,

$$k_{eff} := \left(\frac{a}{b}\right)^2 \cdot k \qquad k_{eff} = 308.354 \, \frac{lbf}{in}$$

2. The effective preload at the follower is,

$$F_{oeff} := \frac{a}{b} \cdot F_o \qquad F_{oeff} = 273.917 \, lbf$$

PROBLEM 10-22

Statement: What is the effective inertia of a load applied at the drum of Figure P9-5 as reflected back to gear *A*?

Given: From Problem 9-35:

Ratio of drum speed to speed of gear *A* $\quad ratio := \frac{5.4}{5.4 - 1} \quad ratio = 1.227$

Solution: See Figure P9-5, Problem 9-35, and Mathcad file P1022.

1. The energy of the effective inertia at gear *A* must be the same as that at the drum,

$$\frac{1}{2} \cdot I_{drum} \cdot \omega_{drum}^{2} := \frac{1}{2} \cdot I_{eff} \cdot \omega_{A}^{2}$$

$$\frac{I_{eff}}{I_{drum}} = \left(\frac{\omega_{drum}}{\omega_{A}}\right)^{2} = ratio^{2}$$

Thus, the ratio of the effective inertia to the drum inertia will be the square the ratio of drum speed to the speed of gear *A*.

$$Ratio_{inertia} := ratio^{2} \qquad Ratio_{inertia} = 1.506$$

PROBLEM 10-23

Statement: What is the effective inertia of a load applied at the drum of Figure P9-7 as reflected back to the arm?

Given: From Problem 9-40:

Ratio of drum speed to speed of the arm $ratio := 16.5$

Solution: See Figure P9-7, Problem 9-40, and Mathcad file P1023.

1. The energy of the effective inertia at the arm must be the same as that at the drum,

$$\frac{1}{2} \cdot I_{drum} \cdot \omega_{drum}^{2} := \frac{1}{2} \cdot I_{eff} \cdot \omega_{arm}^{2}$$

$$\frac{I_{eff}}{I_{drum}} = \left(\frac{\omega_{drum}}{\omega_{arm}} \right)^{2} = ratio^{2}$$

Thus, the ratio of the effective inertia to the drum inertia will be the square the ratio of drum speed to the speed of the arm.

$Ratio_{inertia} := ratio^{2}$ $\quad Ratio_{inertia} = 272.250$

PROBLEM 10-24

Statement: Refer to Figure 10-8a (p. 507). For the data given below, find the equivalent mass at point *A* and the equivalent spring at point *B*.

Given: $a := 100 \cdot mm$ $\quad b := 150 \cdot mm$ $\quad k_A := 2000 N \cdot m^{-1}$ $\quad m_B := 2 \cdot kg$

Solution: See Figure 10-8a and Mathcad file P1024.

1. Use equation 10.22b to calculate the equivalent mass at point *A*.

$$m_{eff} := \left(\frac{b}{a}\right)^2 \cdot m_B \qquad m_{eff} = 4.5\ \text{kg}$$

2. Use equation 10.23b to calculate the equivalent spring constant at point *B*.

$$k_{eff} := \left(\frac{a}{b}\right)^2 \cdot k_A \qquad k_{eff} = 888.9\,\frac{N}{m}$$

PROBLEM 10-25

Statement: Refer to Figure 10-8a (p. 507). For the data given below, find the equivalent mass at point *A* and the equivalent spring at point *B*.

Given: $a := 50 \cdot mm$ $\quad b := 150 \cdot mm$ $\quad k_A := 1000 N \cdot m^{-1}$ $\quad m_B := 3 \cdot kg$

Solution: See Figure 10-8a and Mathcad file P1025.

1. Use equation 10.22b to calculate the equivalent mass at point *A*.

$$m_{eff} := \left(\frac{b}{a}\right)^2 \cdot m_B \qquad m_{eff} = 27\,\mathrm{kg}$$

2. Use equation 10.23b to calculate the equivalent spring constant at point *B*.

$$k_{eff} := \left(\frac{a}{b}\right)^2 \cdot k_A \qquad k_{eff} = 111.1\,\frac{N}{m}$$

PROBLEM 10-26

Statement: For the cam-follower arm in Figure P10-2, determine the location of its fixed pivot that will have zero reaction force when the cam applies its force to the follower.

Given:

Arm dimensions:		Cutout dimensions:	
Width (Z-direction):	$a := 2 \cdot in$	Width (Z-direction):	$a' := 1.5 \cdot in$
Height (Y-direction):	$b := 2.5 \cdot in$	Height (Y-direction):	$b' := 2.5 \cdot in$
Length (X-direction):	$c := 28 \cdot in$	Length (X-direction):	$c' := 3 \cdot in$

Roller dimensions: $r := 1 \cdot in$ $\quad w := 1.5 \cdot in$

Distance from left end of arm to roller center: $L_r := 22 \cdot in$

Specific weight of aluminum: $\gamma_a := 0.1 \cdot lbf \cdot in^{-3}$

Specific weight of steel: $\gamma_s := 0.3 \cdot lbf \cdot in^{-3}$

Assumptions: The center of the steel roller is located on the horizontal centerline (X axis) of the arm.

Solution: See Figure P10-2 and Mathcad file P1026.

1. Determine the volume and weight of the roller and the arm.

Roller: $V_r := \pi \cdot r^2 \cdot w \qquad V_r = 4.712\, in^3$

$W_r := \gamma_s \cdot V_r \qquad W_r = 1.414\, lbf$

Arm: $V_{solid} := a \cdot b \cdot c \qquad V_{solid} = 140.000\, in^3$

$V_{slot} := a' \cdot b' \cdot c' \qquad V_{slot} = 11.250\, in^3$

$V_a := V_{solid} - V_{slot} \qquad V_a = 128.750\, in^3$

$W_{solid} := \gamma_a \cdot V_{solid} \qquad W_{solid} = 14.000\, lbf$

$W_{slot} := -\gamma_a \cdot V_{slot} \qquad W_{slot} = -1.125\, lbf$

$W_a := W_{solid} + W_{slot} \qquad W_a = 12.875\, lbf$

2. Calculate the location of the arm CG with respect to the left end.

$$Xcg_a := \frac{W_{solid} \cdot \frac{c}{2} + W_{slot} \cdot L_r}{W_a} \qquad Xcg_a = 13.301\, in$$

3. Calculate the location of the composite CG with respect to the left end.

$$Xcg := \frac{W_r \cdot L_r + W_a \cdot Xcg_a}{W_r + W_a} \qquad Xcg = 14.162\, in$$

4. Determine the moment of inertia of the arm about the composite CG

Solid portion about its own CG $I_{zsolid} := \frac{W_{solid}}{12 \cdot g} \cdot \left(b^2 + c^2\right)$ $I_{zsolid} = 2.388\, lbf \cdot sec^2 \cdot in$

Slot portion about its own CG $I_{zslot} := \frac{W_{slot}}{12 \cdot g} \cdot \left(b^2 + c^2\right)$ $I_{zslot} = -0.0037\, lbf \cdot sec^2 \cdot in$

Solid portion about composite CG $I_{Zsolid} := I_{zsolid} + \frac{W_{solid}}{g} \cdot \left(X_{Cg} - \frac{c}{2}\right)^2$

$I_{Zsolid} = 2.389\, lbf \cdot sec^2 \cdot in$

Slot portion about composite CG $I_{Zslot} := I_{zslot} + \frac{W_{slot}}{g} \cdot \left(L_r - X_{Cg}\right)^2$

$I_{Zslot} = -0.183\, lbf \cdot sec^2 \cdot in$

Arm about composite CG $I_{ZZa} := I_{Zsolid} + I_{Zslot}$

$I_{ZZa} = 2.206\, lbf \cdot sec^2 \cdot in$

5. Determine the moment of inertia of the roller about the composite CG

Roller about its own CG $I_{zr} := \frac{W_r}{2 \cdot g} \cdot r^2$ $I_{zr} = 1.831 \times 10^{-3}\, lbf \cdot sec^2 \cdot in$

Roller about composite CG $I_{ZZr} := I_{zr} + \frac{W_r}{g} \cdot \left(L_r - X_{Cg}\right)^2$

$I_{ZZr} = 0.227\, lbf \cdot sec^2 \cdot in$

6. Determine the moment of inertia of the assembly about the composite CG

$I_{ZZ} := I_{ZZa} + I_{ZZr}$ $I_{ZZ} = 2.433\, lbf \cdot sec^2 \cdot in$

7. Calculate the radius of gyration using equation 10.11b.

$k := \sqrt{\frac{I_{ZZ} \cdot g}{W_a + W_r}}$ $k = 8.108\, in$

8. Use equation 10.15b to calculate the location of the pivot point with respect to the left end.

$X_{pivot} := X_{Cg} - \frac{k^2}{L_r - X_{Cg}}$ $X_{pivot} = 5.775\, in$

PROBLEM 10-27

Statement: Figure P10-4 shows a fourbar mechanism. The crank is 1.00in wide by 0.5 in thick. The coupler and rocker are both 0.75 in wide by 0.5 in thick. All links are made from steel. The ends of the links have a full radius equal to one half of the link width. The pivot pins all have a diameter of 0.25 in. Find the moment of inertia of the crank and rocker about their fixed pivots and the moment of inertia of the coupler about its CG.

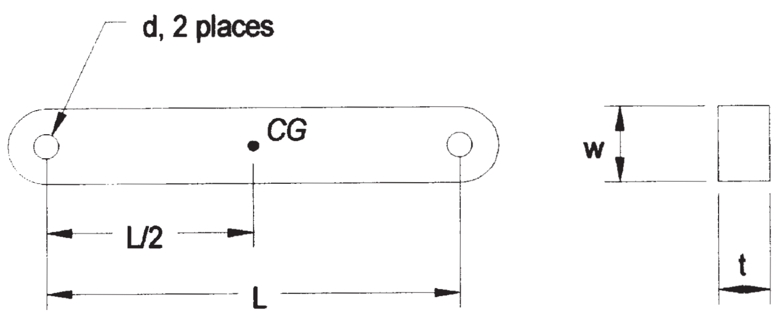
Given: Dimensions in the figure below. Specific weight of steel: $\gamma := 0.28 \, lbf \cdot in^{-3}$

Pivot pin diameter: $d := 0.25 \cdot in$

Solution: See Figure P10-4 and Mathcad file P1027.

1. Divide each link into 3 volumes: 1) the main, rectangular body; 2) the half-cylinrical ends; and 3) the pivot pin holes.

2. All links have the same basic shape, which is shown below.



2. Volume 1 is the main rectangular body of length L, width w, and thickness t.

Mass: $$m_1(L,w,t) := \frac{L \cdot w \cdot t \cdot \gamma}{g}$$

Moment of inertia about *CG*: $$I_1(L,w,t) := \frac{m_1(L,w,t)}{12} \cdot \left(L^2 + w^2\right)$$

3. Volume 2 is the half-cylinder of diameter w, and length t. There are two of these.

Mass: $$m_2(w,t) := \frac{\pi \cdot w^2 \cdot t \cdot \gamma}{8 \cdot g}$$

Moment of inertia about *CG*: $$I_2(L,w,t) := 2 \cdot \left[\frac{m_2(w,t) \cdot w^2}{8} + m_2(w,t) \cdot \left(\frac{L}{2}\right)^2\right]$$

4. Volume 3 is the cylinder of diameter d, and length t. There are two of these and they are negative.

Mass: $$m_3(t) := -\frac{\pi \cdot d^2 \cdot t \cdot \gamma}{8 \cdot g}$$

Moment of inertia about *CG*: $$I_3(L,t) := 2 \cdot \left[\frac{m_3(t) \cdot d^2}{8} + m_3(t) \cdot \left(\frac{L}{2}\right)^2\right]$$

5. For the crank: $L := 2.00 \cdot in,\ \ w := 1.00 \cdot in,\ \ t := 0.50 \cdot in$

Mass: $m_{crank} := m_1(L,w,t) + m_2(w,t) + m_3(t)$ $\qquad m_{crank} = 8.587 \times 10^{-4}\, lbf \cdot sec^2 \cdot in^{-1}$

Moment of inertia about CG:

$$I_{crankCG} := I_1(L,w,t) + I_2(L,w,t) + I_3(L,t) \qquad I_{crankCG} = 6.046 \times 10^{-4}\, lbf \cdot sec^2 \cdot in$$

Moment of inertia about pivot axis:

$$I_{crank} := I_{crankCG} + m_{crank} \cdot \left(\frac{L}{2}\right)^2 \qquad I_{crank} = 1.463 \times 10^{-3}\, lbf \cdot sec^2 \cdot in$$

6. For the rocker: $L := 7.187 \cdot in,\ \ w := 0.75 \cdot in,\ \ t := 0.50 \cdot in$

Mass: $m_{rocker} := m_1(L,w,t) + m_2(w,t) + m_3(t)$ $\qquad m_{rocker} = 2.026 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

Moment of inertia about CG:

$$I_{rockerCG} := I_1(L,w,t) + I_2(L,w,t) + I_3(L,t) \qquad I_{rockerCG} = 0.010\, lbf \cdot sec^2 \cdot in$$

Moment of inertia about pivot axis:

$$I_{rocker} := I_{rockerCG} + m_{rocker} \cdot \left(\frac{L}{2}\right)^2 \qquad I_{rocker} = 0.037\, lbf \cdot sec^2 \cdot in$$

7. For the coupler $L := 8.375 \cdot in,\ \ w := 0.75 \cdot in,\ \ t := 0.50 \cdot in$

Mass: $m_{coupler} := m_1(L,w,t) + m_2(w,t) + m_3(t)$ $\qquad m_{coupler} = 2.349 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

Moment of inertia about CG:

$$I_{couplerCG} := I_1(L,w,t) + I_2(L,w,t) + I_3(L,t) \qquad I_{couplerCG} = 0.016\, lbf \cdot sec^2 \cdot in$$

PROBLEM 10-28

Statement: The rocker in Figure 10-11a has the dimensions given below. When supported on knife-edges at *A* and *B*, the weights at the supports were found to be 4.3 N and 5.8 N, respectively. The rocker was supported at its pivot point with a low-friction ball bearing and the period of oscillation was found. What is the approximate moment of inertia of the rocker with respect to its pivot axis?

Given: Dimensions: $a := 50.8\,mm$ $b := 76.2\,mm$

Total weight: $W := 10.1 \cdot N$ $W_A := 4.3 \cdot N$ $W_B := 5.8 \cdot N$

Period of oscillation: $\tau := 0.75 \cdot sec$

Solution: See Figure 10-11a and Mathcad file P1028.

1. Determine the location of the *CG* by summing moments on the rocker about point *A*. The distance, r, from the pivot axis to the *CG* is:

$$W \cdot (a + r) - W_B \cdot (a + b) = 0$$

$$r := \frac{W_B \cdot (a + b) - W \cdot a}{W} \qquad r = 22.131\,mm$$

2. Use equation 10.10g to calculate the approximate moment of inertia with respect to the pivot axis.

$$I_{pivot} := W \cdot r \cdot \left(\frac{\tau}{2 \cdot \pi} \right)^2 \qquad I_{pivot} = 3185\,kg \cdot mm^2$$

PROBLEM 10-30

Statement: Figure P10-5 shows a cam-follower system that drives slider 6 through an external output arm 3. Arms 2 and 3 are both rigidly attached to the 0.75-in-dia shaft X-X, which rotates in bearings that are supported by the housing. The pin-to-pin dimensions of the links are shown. The cross-section of arms 2, 3, and 5 are solid, rectangular 1.5 x 0.75 in steel. The ends of these links have a full radius equal to one half the link width. Link 4 is 1-in-dia x 0.125 in-wall, round steel tubing. Link 6 is a 2-in-dia x 6-in long solid steel cylinder. Find the effective mass and effective spring constant of the follower train referenced to the cam-follower roller.

Given: Link 6 (solid cylinder): $d_6 := 2.00 in$ $L_6 := 6.00 in$

Rocker arm 5: $w := 0.75 \cdot in$ $h := 1.50 \cdot in$ $a := 10.0 \cdot in$ $b := 8.0 \cdot in$

Pushrod 4 (hollow cylinder): $d_{4od} := 1.00 \cdot in$ $d_{4id} := 0.75 \cdot in$ $L_4 := 22 \cdot in$

Output arm 3: $w := 0.75 \cdot in$ $h := 1.50 \cdot in$ $L_3 := 16 \cdot in$

Spring: $k_s := 150 \cdot lbf \cdot in^{-1}$

Roller arm 2: $w := 0.75 \cdot in$ $h := 1.50 \cdot in$ $L_2 := 8 \cdot in$

All parts are steel: Modulus of elasticity $E := 30 \cdot 10^6 \cdot psi$ Spec. weight $\gamma := 0.28 \cdot \frac{lbf}{in^3}$

Solution: See Figures P10-5 and 10-9, and Mathcad file P1030.

1. Break the system into individual elements as shown in Figure 10-9b.
2. Define the individual spring constants of each of the six elements.

Roller arm (cantilever beam with end load):

Moment of inertia $I_2 := \frac{w \cdot h^3}{12}$

Spring constant $k_2 := \frac{3 \cdot E \cdot I_2}{L_2^3}$ $k_2 = 3.708 \times 10^4 \frac{lbf}{in}$

Output arm:

Moment of inertia $I_3 := \frac{w \cdot h^3}{12}$

Spring constant $k_3 := \frac{3 \cdot E \cdot I_3}{L_3^3}$ $k_3 = 4.635 \times 10^3 \frac{lbf}{in}$

Pushrod (hollow cylinder):

Area $A_4 := \frac{\pi \cdot \left(d_{4od}^2 - d_{4id}^2\right)}{4}$

Spring constant $k_4 := \frac{A_4 \cdot E}{L_4}$ $k_4 = 4.686 \times 10^5 \frac{lbf}{in}$

Rocker arm 5 (side B):

Moment of inertia $I_5 := \frac{w \cdot h^3}{12}$

Spring constant $k_{5a} := \frac{3 \cdot E \cdot I_5}{a^3}$ $k_{5a} = 1.898 \times 10^4 \frac{lbf}{in}$

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Rocker arm 5 (side C):

Spring constant $k_{5b} := \frac{3 \cdot E \cdot I_5}{b^3}$ $k_{5b} = 3.708 \times 10^4 \frac{lbf}{in}$

3. Damping will be neglected.
4. Determine the mass of each of the elements.

Roller arm 2 (cantilever beam with end load):

Volume $V_2 := w \cdot h \cdot L_2$

Mass $m_2 := \frac{\gamma \cdot V_2}{g}$ $m_2 = 6.527 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

Outout arm 3 (cantilever beam with end load):

Volume $V_3 := w \cdot h \cdot L_3$

Mass $m_3 := \frac{\gamma \cdot V_3}{g}$ $m_3 = 0.013\, lbf \cdot sec^2 \cdot in^{-1}$

Pushrod 4 (hollow cylinder):

Volume $V_4 := A_4 \cdot L_4$

Mass $m_4 := \frac{\gamma \cdot V_4}{g}$ $m_4 = 5.482 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

Rocker arm 5 (side B):

Volume $V_{5a} := w \cdot h \cdot a$

Mass $m_{5a} := \frac{\gamma \cdot V_{5a}}{g}$ $m_{5a} = 8.159 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

Rocker arm 5 (side C):

Volume $V_{5b} := w \cdot h \cdot b$

Mass $m_{5b} := \frac{\gamma \cdot V_{5b}}{g}$ $m_{5b} = 6.527 \times 10^{-3}\, lbf \cdot sec^2 \cdot in^{-1}$

Cylinder 6:

Volume $V_6 := \frac{\pi \cdot d_6^2}{4} \cdot L_6$

Mass $m_6 := \frac{\gamma \cdot V_6}{g}$ $m_6 = 0.014\, lbf \cdot sec^2 \cdot in^{-1}$

Omit the spring because no data are available.

5. Determine the effective mass and spring constant on either side of the rocker arm 5, reflected to point A.

Upper side: $m_U := m_4 + m_{5a}$ $m_U = 0.014\, lbf \cdot sec^2 \cdot in^{-1}$

$$k_U := \left(\frac{1}{k_s} + \frac{1}{k_4} + \frac{1}{k_{5a}}\right)^{-1} \qquad k_U = 148.777\,\frac{lbf}{in}$$

Lower side:

$$m_L := m_6 + m_{5b} \qquad m_L = 0.020\,lbf\cdot sec^2\cdot in^{-1}$$

$$k_L := k_{5b} \qquad k_L = 3.708\times 10^4\,\frac{lbf}{in}$$

$$m_{Aeff} := m_U + \left(\frac{b}{a}\right)^2\cdot m_L \qquad m_{Aeff} = 0.027\,lbf\cdot sec^2\cdot in^{-1}$$

$$k_{Aeff} := k_U + \left(\frac{b}{a}\right)^2\cdot k_L \qquad k_{Aeff} = 2.388\times 10^4\,\frac{lbf}{in}$$

6. Determine the effective mass and spring constant on either side of the output and roller arms, reflected to the roller.

Output side:

$$m_O := m_{Aeff} + m_3 \qquad m_O = 0.040\,lbf\cdot sec^2\cdot in^{-1}$$

$$k_O := \left(\frac{1}{k_{Aeff}} + \frac{1}{k_3}\right)^{-1} \qquad k_O = 3.881\times 10^3\,\frac{lbf}{in}$$

Roller side:

$$m_R := m_2 \qquad m_R = 6.527\times 10^{-3}\,lbf\cdot sec^2\cdot in^{-1}$$

$$k_R := k_2 \qquad k_R = 3.708\times 10^4\,\frac{lbf}{in}$$

$$m_{eff} := m_R + \left(\frac{L_3}{L_2}\right)^2\cdot m_O \qquad m_{eff} = 0.165\,lbf\cdot sec^2\cdot in^{-1}$$

$$k_{eff} := k_R + \left(\frac{L_3}{L_2}\right)^2\cdot k_O \qquad k_{eff} = 5.260\times 10^4\,\frac{lbf}{in}$$

PROBLEM 10-31

Statement: The spring in Figure P10-5 has a rate of 150 lb/in with a preload of 60 lb. Determine the effective spring constant and preload of the spring as reflected back to the cam follower. See Problem 10-30 for a description of the system.

Given: Spring data: $k := 150 \cdot lbf \cdot in^{-1}$ $F_o := 60 \cdot lbf$

Distance from arm pivot to spring (L_3): $a := 16 \cdot in$

Distance from arm pivot to follower (L_2): $b := 8 \cdot in$

Solution: See Figure P10-5, Problem 10-30, and Mathcad file P1031.

1. The effective spring constant at the follower is,

$$k_{eff} := \left(\frac{a}{b}\right)^2 \cdot k \qquad k_{eff} = 600 \frac{lbf}{in}$$

2. The effective preload at the follower is,

$$F_{oeff} := \frac{a}{b} \cdot F_o \qquad F_{oeff} = 120\, lbf$$

PROBLEM 10-32

Statement: A company wants to manufacture chimes that are made from hollow tubes of various lengths. Regardless of length they will be hung from a hole that is 25 mm from one end of the tube. Develop an equation that will give the distance from this hole to the point where the chime should be struck such that there will be zero reaction force at the hole where the chime is hung. The distance should be a function of the length (L), outside diameter (OD), and inside diameter (ID) of the chime as well as the distance from the end to the hanging hole (25 mm) only. Solve your equation for the following dimensions: $L = 300$ mm, $OD = 35$ mm, $ID = 30$ mm.

Given: Distance to hanging hole from end of tube: $d := 25 \cdot mm$

Test dimensions: $L_t := 300 \cdot mm$ $\quad OD_t := 35 \cdot mm$ $\quad ID_t := 30 \cdot mm$

Solution: See Mathcad file P1032.

1. Refering to Figure 10-3, let the distance from the hanging hole to the point where the chime should be struck be L_s. Then,

$$L_s = x + l_p \quad \text{where} \quad x = \frac{I_G}{m \cdot l_p} \quad \text{from equation 10.15a}$$

2. The CG of the chime will be located at $L/2$ from either end. From Appendix C, for a hollow tube

$$\frac{I_G}{m} = \frac{0.75 \cdot ID^2 + 0.75 \cdot OD^2 + L^2}{12}$$

and, the distance from the CG to the hanging hole is $x = \frac{L}{2} - d$

3. Combine these equations to derive the equation for the strike distance, L_s, which is measured from the hanging hole.

$$L_s(L, ID, OD) := \frac{0.75 \cdot ID^2 + 0.75 \cdot OD^2 + L^2}{12 \cdot (0.5 \cdot L - d)} + \left(\frac{L}{2} - d \right) \quad \text{where} \quad d = 25\,mm$$

4. Test the equation with the given values.

$$L_{stest} := L_s(L_t, ID_t, OD_t) \qquad L_{stest} = 186.1\,mm$$

PROBLEM 10-33

Statement: What is the amount by which the roller arm of Problem 10-30 must be extended on the opposite side of the pivot axis O2 in order to make the pivot axis a *center of rotation* if the point where the cam-follower is mounted is a *center of percussion*?

Given: Length of roller arm without extension: $L_2 := 8 \cdot in$

Cross-section dimensions: $w := 1.50 in$ $\quad t := 0.125 in$

Solution: See Figure P10-5 and Mathcad file P1033.

1. Refering to Figure 10-3, let x be the distance from O_2 to the CG of the link after extension and l_p be the distance from the CG (after extension) to the roller axis. The amount by which the link is extended is ΔL The extended length of the link is $L_2 + \Delta L$.

$$L_2 = x + l_p \quad \text{where} \quad x = \frac{I_G}{m \cdot l_p} = \frac{L_2 + \Delta L}{2} \qquad \text{from equation 10.15a}$$

2. Eliminating x and l_p from these three equations gives (ignoring the rounded ends on the link)

$$\Delta L = \sqrt{L_2^{\,2} - 4 \cdot \frac{I_G}{m}} \qquad \text{where} \qquad \frac{I_G}{m} = \frac{\left(L_2 + \Delta L\right)^2 + w^2}{12} \qquad \text{(See Appendix } C\text{)}$$

3. Solve for ΔL iteratively.

$$f(\Delta L) := \sqrt{L_2^{\,2} - 4 \cdot \left[\frac{\left(L_2 + \Delta L\right)^2 + w^2}{12}\right]}$$

Try $\quad \Delta L := 3.953 \cdot in \qquad f(\Delta L) = 3.953\, in$

Let $\quad \Delta L := 4.0 \cdot in$

Then the new length of link 2 is $\quad L_{new} := L_2 + \Delta L \qquad L_{new} = 12.0\, in$