

Chapter **12**

BALANCING

TOPIC/PROBLEM MATRIX

| SECT | TOPIC | PROBLEMS |
|------|--------------------------------------|---|
| 12.1 | Static Balance | 12-1, 12-2, 12-3, 12-4 |
| 12.2 | Dynamic Balance | 12-5, 12-13, 12-14, 12-15, 12-16, 12-17, 12-18, 12-19 |
| 12.3 | Balancing Linkages | 12-8a, 12-12, 12-27, 12-29, 12-31, 12-33, 12-35 |
| 12.5 | Effect of Balancing on Input Torque | 12-8b, 12-9, 12-10, 12-11 |
| 12.6 | Balancing Shaking Moment in Linkages | 12-20, 12-21, 12-22, 12-23, 12-28, 12-30, 12-32, 12-34, 12-36 |
| 12.7 | Measuring and Correcting Imbalance | 12-6, 12-7, 12-24, 12-25, 12-26 |



PROBLEM 12-1a

Statement: A system of two coplanar arms on a common shaft, as shown in Figure 12-1, is to be designed. For row α in Table P12-1, find the shaking force of the linkage when run unbalanced and design a counterweight to statically balance the system. Work in any consistent unit system you prefer.

Given: Masses and radii:

$$m_1 := 0.20 \cdot \text{kg} \quad r_1 := 1.25 \cdot \text{m} \quad \theta_1 := 30 \cdot \text{deg}$$

$$m_2 := 0.40 \cdot \text{kg} \quad r_2 := 2.25 \cdot \text{m} \quad \theta_2 := 120 \cdot \text{deg}$$

Solution: See Figure 12-1, Table P12-1, and Mathcad file P1201a.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$R_{1x} := r_1 \cdot \cos(\theta_1) \quad R_{1x} = 1.083 \text{ m} \quad R_{1y} := r_1 \cdot \sin(\theta_1) \quad R_{1y} = 0.625 \text{ m}$$

$$R_{2x} := r_2 \cdot \cos(\theta_2) \quad R_{2x} = -1.125 \text{ m} \quad R_{2y} := r_2 \cdot \sin(\theta_2) \quad R_{2y} = 1.949 \text{ m}$$

2. Solve equation 12.2c for the mass-radius product components.

$$mR_{bx} := -m_1 \cdot R_{1x} - m_2 \cdot R_{2x} \quad mR_{bx} = 0.233 \text{ kg} \cdot \text{m}$$

$$mR_{by} := -m_1 \cdot R_{1y} - m_2 \cdot R_{2y} \quad mR_{by} = -0.904 \text{ kg} \cdot \text{m}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required.

$$\theta_b := \text{atan2}(mR_{bx}, mR_{by}) \quad \theta_b = -75.524 \text{ deg}$$

$$mR_b := \sqrt{mR_{bx}^2 + mR_{by}^2} \quad mR_b = 0.934 \text{ kg} \cdot \text{m}$$



PROBLEM 12-2

Statement: The minute hand on Big Ben weighs 40 lb and is 10 ft long. Its CG is 4 ft from the pivot. Calculate the mR product and angular location needed to statically balance this link and design a physical counterweight, positioned close to the center. Select material and design the detailed shape of the counterweight, which is of 2 in uniform thickness in the Z direction.

Given: Weight and radius data:

$$W_I := 40 \text{ lbf} \quad r_I := 4 \text{ ft}$$

Counterweight thickness: $t := 2 \text{ in}$

Design choices:

Counterweight material is steel: $\gamma := 0.28 \text{ lbf} \cdot \text{in}^{-3}$

Counter weight shape is cylindrical

Solution: See Mathcad file P1202.

1. Since there is only one mass to balance, the balancing mass will be on the same centerline as the minute hand, on the side opposite the center of rotation of the hand.
2. Solve equation 12.2c for the required mass-radius for balancing.

$$mR_b := \frac{W_I}{g} \cdot r_I \quad mR_b = 4.973 \text{ slug} \cdot \text{ft}$$

3. The mass of a solid cylinder is $m_b := \frac{\pi \cdot r_c^2 \cdot t \cdot \gamma}{g}$
4. Combine the equations in steps 2 and 3 and solve for the cylinder radius as a function of the balance radius. Plot the result over a suitable range of R_b to get an idea of the sizes involved.

$$r_c(R_b) := \sqrt{\frac{mR_b \cdot g}{\pi \cdot t \cdot R_b \cdot \gamma}} \quad R_{bal} := 5 \text{ in}, 6 \text{ in}.. 20 \text{ in}$$

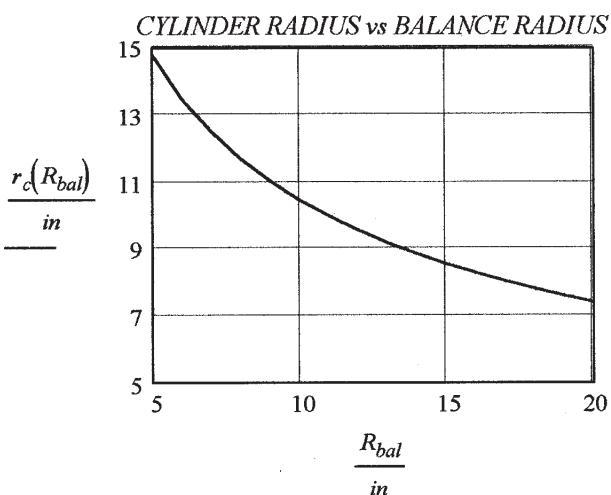
5. Choose a balance radius of $R_b := 15 \text{ in}$. Then the required mass is

$$m_b := \frac{mR_b}{R_b} \quad m_b = 3.978 \text{ slug}$$

$$W_b := m_b \cdot g \quad W_b = 128.000 \text{ lbf}$$

6. Calculate the required cylinder diameter.

$$d_c := 2 \cdot \sqrt{\frac{mR_b \cdot g}{\pi \cdot t \cdot R_b \cdot \gamma}} \quad d_c = 17.059 \text{ in}$$



 **PROBLEM 12-3**

Statement: A "V for victory" advertising sign is being designed to be oscillated about the apex of the V, on the billboard, as the rocker of a fourbar linkage. The angle between the legs of the V is 20 deg. Each leg is 8 ft long and 1.5 ft wide. Material is 0.25-in thick aluminum. Design the V-link for static balance.

Given: V dimensions:

$$\text{Length: } l_v := 8 \cdot \text{ft} \quad \text{Width: } w_v := 1.5 \cdot \text{ft} \quad \text{Thickness: } t_v := 0.25 \cdot \text{in}$$

$$\text{Specific weight of aluminum: } \gamma_a := 0.1 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Design choices:

$$\text{Counterweight material is steel: } \gamma_s := 0.28 \cdot \text{lbf} \cdot \text{in}^{-3}$$

$$\text{Counter weight shape is cylindrical disk with thickness } t_c := 3 \cdot \text{in}$$

Solution: See Mathcad file P1203.

1. Since the V is symmetrical in geometry and mass, the counterweight will be located on the centerline of the V opposite the pivot point from the V.
2. Assume that the centerlines of each leg of the V intersect at the pivot point. Then, the mass, distance to the CG, and angle with respect to the V centerline of each leg is:

$$\text{Mass of one leg} \quad m_{leg} := l_v \cdot w_v \cdot t_v \cdot \frac{\gamma_a}{g} \quad m_{leg} = 43.200 \text{ lb}$$

$$m_1 := m_{leg} \quad r_1 := 4 \cdot \text{ft} \quad \theta_1 := 10 \cdot \text{deg}$$

$$m_2 := m_{leg} \quad r_2 := 4 \cdot \text{ft} \quad \theta_2 := -10 \cdot \text{deg}$$

3. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$R_{Ix} := r_1 \cdot \cos(\theta_1) \quad R_{Ix} = 3.939 \text{ ft} \quad R_{Iy} := r_1 \cdot \sin(\theta_1) \quad R_{Iy} = 0.695 \text{ ft}$$

$$R_{2x} := r_2 \cdot \cos(\theta_2) \quad R_{2x} = 3.939 \text{ ft} \quad R_{2y} := r_2 \cdot \sin(\theta_2) \quad R_{2y} = -0.695 \text{ ft}$$

4. Solve equation 12.2c for the mass-radius product.

$$mR_{bx} := -m_1 \cdot R_{Ix} - m_2 \cdot R_{2x} \quad mR_{bx} = -4.084 \times 10^3 \text{ in-lb}$$

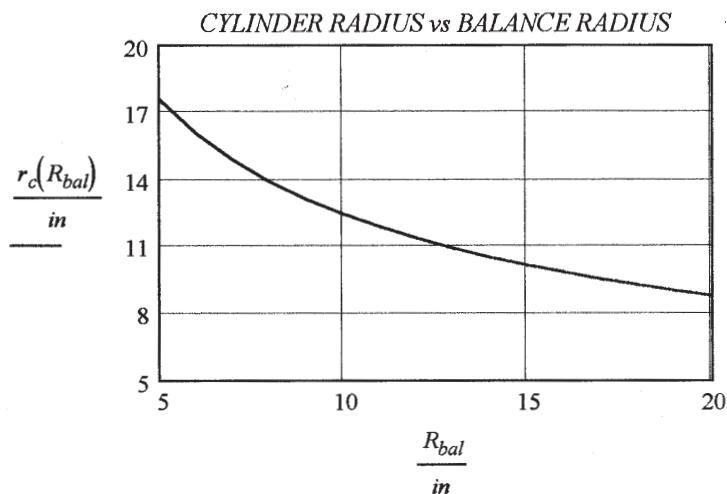
$$mR_{by} := -m_1 \cdot R_{Iy} - m_2 \cdot R_{2y} \quad mR_{by} = 0.000 \text{ in-lb}$$

$$mR_b := |mR_{bx}| \quad mR_b = 4084 \text{ in-lb}$$

5. The mass of a solid cylinder is $m_b := \pi \cdot r_c^2 \cdot t \cdot \frac{\gamma}{g}$

6. Combine the equations in steps 4 and 5 and solve for the cylinder radius as a function of the balance radius. Plot the result over a suitable range of R_b to get an idea of the sizes involved.

$$r_c(R_b) := \sqrt{\frac{mR_b \cdot g}{\pi \cdot t_c \cdot R_b \cdot \gamma_s}} \quad R_{bal} := 5 \cdot \text{in}, 6 \cdot \text{in} \dots 20 \cdot \text{in}$$



7. Choose a balance radius of $R_b := 18 \cdot in$. Then the required mass is

$$m_b := \frac{mR_b}{R_b} \quad m_b = 226.900 \text{ lb}$$

8. Calculate the required cylinder diameter.

$$d_c := 2 \cdot \sqrt{\frac{mR_b \cdot g}{\pi \cdot t_c \cdot R_b \cdot \gamma_s}} \quad d_c = 18.5 \text{ in}$$

 PROBLEM 12-4

Statement: A three-bladed ceiling fan has 1.5 ft by 0.25 ft equispaced rectangular blades that normally weigh 2 lb each. Manufacturing tolerances will cause the blade weight to vary up to plus or minus 5%. The mounting accuracy of the blades will vary the location of the CG versus the spin axis by plus or minus 10% of the blades' diameters. Calculate the weight of the largest steel counterweight needed at a 2-in radius to statically balance the worst-case blade assembly.

Given: Blade dimensions:

$$\text{Length: } l_b := 1.5 \cdot \text{ft} \quad \text{Width: } w_b := 0.25 \cdot \text{ft} \quad \text{Nominal weight: } W_{bnom} := 2 \cdot \text{lbf}$$

$$\text{Manufacturing tolerances: } \text{Weight} \quad t_w := 0.05 \quad \text{CG offset} \quad t_{CG} := 0.10$$

Assumptions: 1. The blades are held in place by a bracket such that their base is 6 in from the center of rotation making the tip 24 in from the center. Thus, the blade sweep diameter is 48 in.
2. There is one heavy (maximum weight) blade at 0 deg and two light (minimum weight) ones at 120 and 240 deg, respectively. Thus,

$$W_1 := (1 + t_w) \cdot W_{bnom} \quad W_1 = 2.100 \text{ lbf} \quad r_1 := 15 \cdot \text{in} \quad \theta_1 := 0 \cdot \text{deg}$$

$$W_2 := (1 - t_w) \cdot W_{bnom} \quad W_2 = 1.900 \text{ lbf} \quad r_2 := 15 \cdot \text{in} \quad \theta_2 := 120 \cdot \text{deg}$$

$$W_3 := (1 - t_w) \cdot W_{bnom} \quad W_3 = 1.900 \text{ lbf} \quad r_3 := 15 \cdot \text{in} \quad \theta_3 := 240 \cdot \text{deg}$$

Solution: See Mathcad file P1204.

1. There are two factors to be taken into account, the variation in blade weight and the error or eccentricity in the location of the global CG. The variation in blade weight about its spin axis will be considered first.
2. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$R_{1x} := r_1 \cdot \cos(\theta_1) \quad R_{1x} = 15.000 \text{ in} \quad R_{1y} := r_1 \cdot \sin(\theta_1) \quad R_{1y} = 0.000 \text{ in}$$

$$R_{2x} := r_2 \cdot \cos(\theta_2) \quad R_{2x} = -7.500 \text{ in} \quad R_{2y} := r_2 \cdot \sin(\theta_2) \quad R_{2y} = 12.990 \text{ in}$$

$$R_{3x} := r_3 \cdot \cos(\theta_3) \quad R_{3x} = -7.500 \text{ in} \quad R_{3y} := r_3 \cdot \sin(\theta_3) \quad R_{3y} = -12.990 \text{ in}$$

3. Solve equation 12.2c for the mass-radius product components.

$$mR_{bx} := \frac{(-W_1 \cdot R_{1x} - W_2 \cdot R_{2x} - W_3 \cdot R_{3x})}{g} \quad mR_{bx} = -3.000 \text{ in-lb}$$

$$mR_{by} := \frac{(-W_1 \cdot R_{1y} - W_2 \cdot R_{2y} - W_3 \cdot R_{3y})}{g} \quad mR_{by} = 0.000 \text{ in-lb}$$

4. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required.

$$\theta_b := \text{atan2}(mR_{bx}, mR_{by}) \quad \theta_b = 180.000 \text{ deg}$$

$$mR_{bb} := \sqrt{mR_{bx}^2 + mR_{by}^2} \quad mR_{bb} = 3.000 \text{ in-lb}$$

5. Now, account for the fact that the blades' spin axis can be eccentric from their CG.

$$\text{Maximum eccentricity: } r_e := 48 \cdot \text{in} \cdot t_{CG} \quad r_e = 4.800 \text{ in}$$

mR product due to eccentricity:

$$mR_{be} := \frac{W_1 + W_2 + W_3}{g} \cdot r_e \quad mR_{be} = 28.320 \text{ in} \cdot \text{lb}$$

6. Add the two *mR* products and divide by the 2-in radius specified for the counterweight to find the maximum weight required.

$$R_{cw} := 2.0 \cdot \text{in} \quad mR_b := mR_{bb} + mR_{be} \quad mR_b = 31.320 \text{ in} \cdot \text{lb}$$

$$m_{cw} := \frac{mR_b}{R_{cw}} \quad m_{cw} = 15.66 \text{ lb}$$

Note that 90% of the counterweight is required to balance the eccentricity. The manufacturer would be well advised to try to control this variation more tightly.



PROBLEM 12-5a

Statement: A system of three non coplanar weights is arranged on a shaft generally as shown in Figure 12-3. For row *a* in Table P12-2, find the shaking forces and shaking moment when run unbalanced at 100 rpm and specify the mR product and angle of the counterweights in planes *A* and *B* needed to dynamically balance the system. The correction planes are 20 units apart. Work in a any consistent unit system you prefer.

Given: Masses and radii:

$$\begin{array}{llll} m_1 := 0.20 \cdot \text{kg} & r_1 := 1.25 \cdot \text{m} & \theta_1 := 30 \cdot \text{deg} & l_1 := 2 \cdot \text{m} \\ m_2 := 0.40 \cdot \text{kg} & r_2 := 2.25 \cdot \text{m} & \theta_2 := 120 \cdot \text{deg} & l_2 := 8 \cdot \text{m} \\ m_3 := 1.24 \cdot \text{kg} & r_3 := 5.50 \cdot \text{m} & \theta_3 := -30 \cdot \text{deg} & l_3 := 17 \cdot \text{m} \end{array}$$

$$\text{Distance between correction planes: } l_B := 20 \cdot \text{m}$$

Solution: See Figure 12-3, Table P12-2, and Mathcad file P1205a.

1. Resolve the position vectors into *xy* components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$\begin{array}{llll} R_{1x} := r_1 \cdot \cos(\theta_1) & R_{1x} = 1.083 \text{ m} & R_{1y} := r_1 \cdot \sin(\theta_1) & R_{1y} = 0.625 \text{ m} \\ R_{2x} := r_2 \cdot \cos(\theta_2) & R_{2x} = -1.125 \text{ m} & R_{2y} := r_2 \cdot \sin(\theta_2) & R_{2y} = 1.949 \text{ m} \\ R_{3x} := r_3 \cdot \cos(\theta_3) & R_{3x} = 4.763 \text{ m} & R_{3y} := r_3 \cdot \sin(\theta_3) & R_{3y} = -2.750 \text{ m} \end{array}$$

2. Solve equations 12.4e for summation of moments about *O*.

$$\begin{aligned} mR_{Bx} &:= \frac{-(m_1 \cdot R_{1x}) \cdot l_1 - (m_2 \cdot R_{2x}) \cdot l_2 - (m_3 \cdot R_{3x}) \cdot l_3}{l_B} & mR_{Bx} &= -4.862 \text{ kg} \cdot \text{m} \\ mR_{By} &:= \frac{-(m_1 \cdot R_{1y}) \cdot l_1 - (m_2 \cdot R_{2y}) \cdot l_2 - (m_3 \cdot R_{3y}) \cdot l_3}{l_B} & mR_{By} &= 2.574 \text{ kg} \cdot \text{m} \end{aligned}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *B*.

$$\begin{array}{ll} \theta_B := \text{atan2}(mR_{Bx}, mR_{By}) & \theta_B = 152.101 \text{ deg} \\ mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} & mR_B = 5.501 \text{ kg} \cdot \text{m} \end{array}$$

4. Solve equations 12.4c for forces in *x* and *y* directions in plane *A*.

$$\begin{array}{ll} mR_{Ax} := -m_1 \cdot R_{1x} - m_2 \cdot R_{2x} - m_3 \cdot R_{3x} - mR_{Bx} & mR_{Ax} = -0.811 \text{ kg} \cdot \text{m} \\ mR_{Ay} := -m_1 \cdot R_{1y} - m_2 \cdot R_{2y} - m_3 \cdot R_{3y} - mR_{By} & mR_{Ay} = -0.069 \text{ kg} \cdot \text{m} \end{array}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *A*.

$$\begin{array}{ll} \theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) & \theta_A = -175.160 \text{ deg} \\ mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} & mR_A = 0.814 \text{ kg} \cdot \text{m} \end{array}$$



PROBLEM 12-6

Statement: A wheel and tire assembly has been run at 100 rpm on a dynamic balancing machine as shown in Figure 12-12. The force measured at the left bearing had a peak of 5 lb at a phase angle of 45 deg with respect to the zero reference angle on the tire. The force measured at the right bearing had a peak of 2 lb at a phase angle of -120 deg with respect to the zero reference angle on the tire. The center distance between the two bearings on the machine is 10 in. The left edge of the wheel rim is 4 in from the centerline of the closest bearing. The wheel is 7-in wide at the rim. Calculate the size and location with respect to the tire's zero reference angle, of balance weights needed on each side of the rim to dynamically balance the tire assembly. The wheel rim diameter is 15 in.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Bearing forces and plane locations with respect to correction plane A :

$$\text{Left: } F_1 := 5 \cdot lbf \quad \theta_1 := 45 \cdot \text{deg} \quad l_1 := 14 \cdot \text{in}$$

$$\text{Right: } F_2 := 2 \cdot lbf \quad \theta_2 := -120 \cdot \text{deg} \quad l_2 := 4 \cdot \text{in}$$

$$\text{Distance between correction planes: } l_B := 7 \cdot \text{in}$$

$$\text{Correction weight radii: } R_A := 7.5 \cdot \text{in} \quad R_B := 7.5 \cdot \text{in}$$

$$\text{Tire rotational speed: } \omega := 100 \cdot rpm$$

Solution: See Figure 12-12 and Mathcad file P1206.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{Ix} := F_1 \cdot \cos(\theta_1) \quad F_{Ix} = 3.536 \text{ lbf} \quad F_{Iy} := F_1 \cdot \sin(\theta_1) \quad F_{Iy} = 3.536 \text{ lbf}$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -1.000 \text{ lbf} \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = -1.732 \text{ lbf}$$

2. Solve equations 12.4e for summation of moments about O , modified for the bearing forces.

$$mR_{Bx} := \frac{F_{Ix} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{Bx} = 22.883 \text{ in} \cdot \text{lb}$$

$$mR_{By} := \frac{F_{Iy} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{By} = 21.411 \text{ in} \cdot \text{lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B . Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 43.096 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 31.338 \text{ in} \cdot \text{lb}$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 4.18 \text{ lbf}$$

4. Solve equations 12.4c for forces in x and y directions in plane A .

$$mR_{Ax} := \frac{F_{Ix} + F_{2x}}{2} - mR_{Bx} \quad mR_{Ax} = -13.956 \text{ in-lb}$$

$$mR_{Ay} := \frac{F_{Iy} + F_{2y}}{2} - mR_{By} \quad mR_{Ay} = -15.061 \text{ in-lb}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A .

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -132.820 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 20.533 \text{ in-lb}$$

$$W_A := \frac{mR_A \cdot g}{R_A} \quad W_A = 2.74 \text{ lbf}$$

 PROBLEM 12-7

Statement: A wheel and tire assembly has been run at 100 rpm on a dynamic balancing machine as shown in Figure 12-12. The force measured at the left bearing had a peak of 6 lb at a phase angle of -60 deg with respect to the zero reference angle on the tire. The force measured at the right bearing had a peak of 4 lb at a phase angle of 150 deg with respect to the zero reference angle on the tire. The center distance between the two bearings on the machine is 10 in. The left edge of the wheel rim is 4 in from the centerline of the closest bearing. The wheel is 7-in wide at the rim. Calculate the size and location with respect to the tire's zero reference angle, of balance weights needed on each side of the rim to dynamically balance the tire assembly. The wheel rim diameter is 16 in.

Units: $rpm := 2\pi \cdot rad \cdot min^{-1}$

Given: Bearing forces and plane locations with respect to correction plane A:

$$\text{Left: } F_1 := 6 \cdot lbf \quad \theta_1 := -60 \cdot \text{deg} \quad l_1 := 14 \cdot \text{in}$$

$$\text{Right: } F_2 := 4 \cdot lbf \quad \theta_2 := 150 \cdot \text{deg} \quad l_2 := 4 \cdot \text{in}$$

$$\text{Distance between correction planes: } l_B := 7 \cdot \text{in}$$

$$\text{Correction weight radii: } R_A := 8 \cdot \text{in} \quad R_B := 8 \cdot \text{in}$$

$$\text{Tire rotational speed: } \omega := 100 \cdot rpm$$

Solution: See Figure 12-12 and Mathcad file P1207.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{Ix} := F_1 \cdot \cos(\theta_1) \quad F_{Ix} = 3.000 \text{ lbf} \quad F_{Iy} := F_1 \cdot \sin(\theta_1) \quad F_{Iy} = -5.196 \text{ lbf}$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -3.464 \text{ lbf} \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = 2.000 \text{ lbf}$$

2. Solve equations 12.4e for summation of moments about O , modified for the bearing forces.

$$mR_{Bx} := \frac{F_{Ix} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{Bx} = 14.155 \text{ in} \cdot \text{lb}$$

$$mR_{By} := \frac{F_{Iy} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{By} = -32.565 \text{ in} \cdot \text{lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B . Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = -66.507 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 35.508 \text{ in} \cdot \text{lb}$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 4.44 \text{ lbf}$$

4. Solve equations 12.4c for forces in x and y directions in plane A.

$$mR_{Ax} := \frac{F_{Ix} + F_{2x}}{2} - mR_{Bx} \quad mR_{Ax} = -15.789 \text{ in-lb}$$

$$mR_{Ay} := \frac{F_{Iy} + F_{2y}}{2} - mR_{By} \quad mR_{Ay} = 21.312 \text{ in-lb}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A .

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = 126.533 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 26.523 \text{ in-lb}$$

$$W_A := \frac{mR_A \cdot g}{R_A} \quad W_A = 3.32 \text{ lbf}$$



PROBLEM 12-8a

Statement: Table P11-3 shows the geometry and kinematic data for several fourbar linkages. For row *a* in Table P11-3:

- Calculate the size and angular locations of the counterbalance mass-radius products needed on links 2 and 4 to completely force balance the linkage by the method of Berkof and Lowen. Check your manual calculation with program FOURBAR.
- Calculate the input torque for the linkage both with and without the added balance weights and compare the results. Use program FOURBAR.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (}O_2 \text{ to } A\text{)} \quad l_2 := 4.00 \cdot in \quad \text{Link 3 (}A \text{ to } B\text{)} \quad l_3 := 12.00 \cdot in$$

$$\text{Link 4 (}B \text{ to } O_4\text{)} \quad l_4 := 8.00 \cdot in \quad \text{Link 3 (}O_2 \text{ to } O_4\text{)} \quad l_1 := 15.00 \cdot in$$

$$\text{Link angles:} \quad \theta_2 := 45 \cdot deg \quad \theta_3 := 24.97 \cdot deg \quad \theta_4 := 99.30 \cdot deg$$

$$\text{Mass:} \quad m_2 := 0.002 \cdot blob \quad m_3 := 0.020 \cdot blob \quad m_4 := 0.100 \cdot blob$$

$$\text{Moment of inertia:} \quad I_{G2} := 0.10 \cdot blob \cdot in^2 \quad I_{G3} := 0.20 \cdot blob \cdot in^2 \quad I_{G4} := 0.50 \cdot blob \cdot in^2$$

$$\text{Mass center:} \quad b_2 := 2.00 \cdot in \quad \phi_2 := 0 \cdot deg \quad b_3 := 5.00 \cdot in \quad \phi_3 := 0 \cdot deg$$

$$b_4 := 4.00 \cdot in \quad \phi_4 := 30 \cdot deg$$

Solution: See Table P11-3 and Mathcad file P1208a.

- Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -0.0467 \text{ in} \cdot blob$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.0000 \text{ in} \cdot blob$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -0.0667 \text{ in} \cdot blob$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.0000 \text{ in} \cdot blob$$

- Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -0.0507 \text{ in} \cdot blob$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.0000 \text{ in} \cdot blob$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -0.4131 \text{ in} \cdot blob$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4)$$

$$mR_{4y} = -0.2000 \text{ in}\cdot\text{blob}$$

3. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

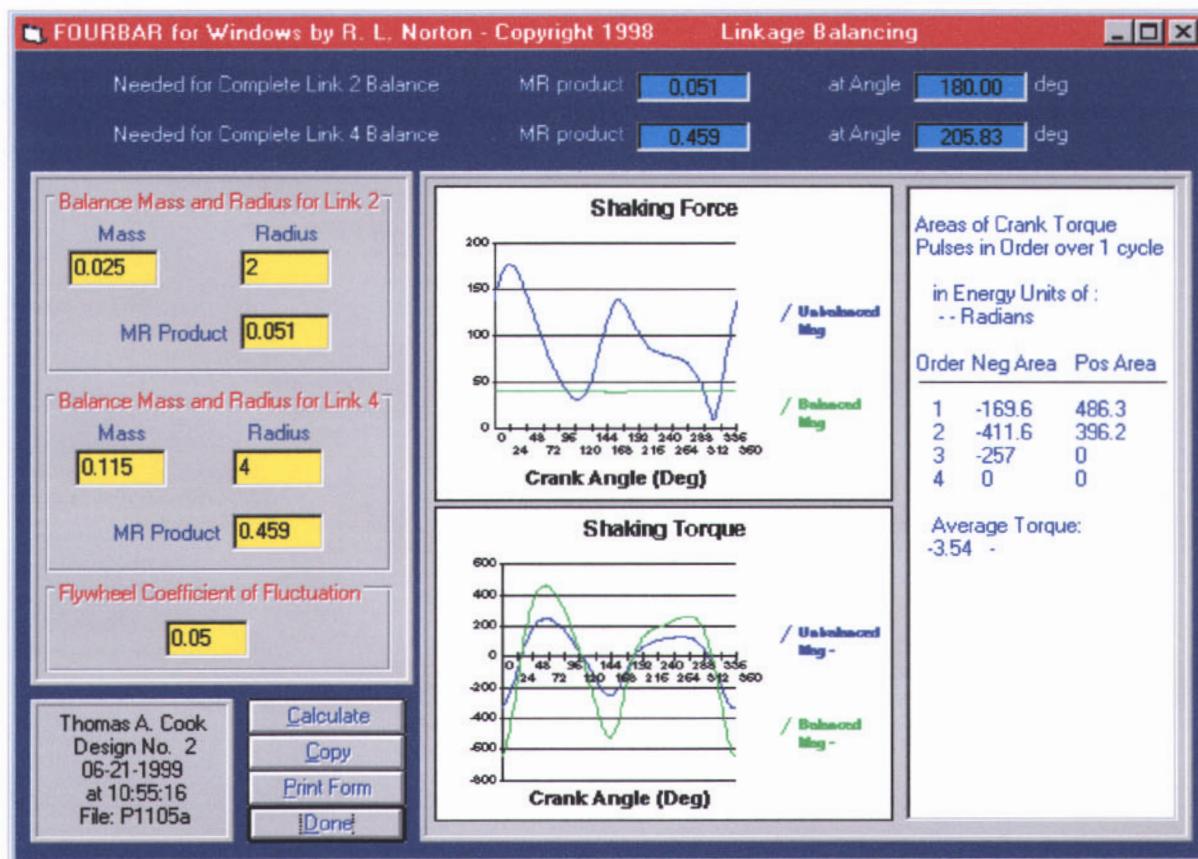
$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 0.0507 \text{ blob}\cdot\text{in}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = -154.165 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 0.459 \text{ blob}\cdot\text{in}$$

4. Check the result using program FOURBAR. The linkage balancing screen below confirms the calculations.



5. The maximum positive input torque without force balancing is 245.7 lb-in. After force balancing it is 460.7 lb-in.



PROBLEM 12-9

Statement: Link 2 in Figure P12-1 rotates at 500 rpm. The links are steel with cross-sections of 1 x 2 in. Half of the 29-lb weight of the laybar and reed are supported by the linkage at point B. Design counterweights to force balance the linkage and determine its change in peak torque versus unbalanced condition. See Problem 11-13 for more information on the overall mechanism.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Link lengths:

$$\text{Link 2 (A to B)} \quad l_2 := 2.00 \cdot in \quad \text{Link 3 (B to C)} \quad l_3 := 8.375 \cdot in$$

$$\text{Link 4 (C to D)} \quad l_4 := 7.187 \cdot in \quad \text{Link 1 (A to D)} \quad l_1 := 9.625 \cdot in$$

$$\text{Coupler point:} \quad R_{pa} := 0.0 \cdot in \quad \delta_3 := 0 \cdot deg$$

$$\text{Crank angle and motion:} \quad \omega_2 := 500 \cdot rpm \quad \alpha_2 := 0 \cdot rad \cdot sec^{-2}$$

$$\text{Link cross-section dims:} \quad w := 2.00 \cdot in \quad t := 1.00 \cdot in$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma := 0.28 \cdot lbf \cdot in^{-3}$$

Solution: See Figure P12-1, Problem 11-13, and Mathcad file P1209.

- Determine the distance to the CG in the LRCS on each of the three moving links. All three are located on the x' axis in the LRCS and their angle is zero deg.

$$b_2 := 0.5 \cdot l_2 \quad b_2 = 1.000 \text{ in} \quad b_3 := 0.5 \cdot l_3 \quad b_3 = 4.188 \text{ in}$$

$$\phi_2 := 0 \cdot deg \quad \phi_3 := 0 \cdot deg \quad \phi_4 := 0 \cdot deg$$

- Determine the mass and moment of inertia of each link.

$$m_2 := w \cdot t \cdot l_2 \cdot \frac{\gamma}{g} \quad m_3 := w \cdot t \cdot l_3 \cdot \frac{\gamma}{g} \quad m_{41} := w \cdot t \cdot l_4 \cdot \frac{\gamma}{g} \quad m_{42} := \frac{29 \cdot lbf}{2 \cdot g}$$

$$m_2 = 0.0029 \text{ blob} \quad m_3 = 0.012 \text{ blob} \quad m_{41} = 0.010 \text{ blob}$$

$$m_4 := m_{41} + m_{42} \quad m_4 = 0.048 \text{ blob} \quad b_4 := \frac{l_4}{m_4} \left(\frac{m_{41}}{2} + m_{42} \right) \quad b_4 = 6.406 \text{ in}$$

$$I_{G2} := \frac{m_2}{12} \cdot \left(w^2 + l_2^2 \right) \quad I_{G2} = 0.00193 \text{ blob} \cdot in^2$$

$$I_{G3} := \frac{m_3}{12} \cdot \left(w^2 + l_3^2 \right) \quad I_{G3} = 0.01792 \text{ blob} \cdot in^2$$

$$I_{G4} := \left(l_4 - b_4 \right)^2 \cdot m_{42} + \frac{m_{41}}{12} \cdot \left(w^2 + l_4^2 \right) + m_{41} \cdot \left(b_4 - \frac{l_4}{2} \right)^2 \quad I_{G4} = 0.154 \text{ blob} \cdot in^2$$

- Define any external forces, their locations and directions.

$$\text{Beat-up force} \quad F := 590 \cdot lbf \quad \text{acting on link 4 at a distance} \quad R := l_4 + 3.75 \cdot in$$

The angle in the CGS is 180 deg.

$$R = 10.937 \text{ in}$$

- Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -0.0121 \text{ in}\cdot\text{blob}$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.0000 \text{ in}\cdot\text{blob}$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -0.0437 \text{ in}\cdot\text{blob}$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.0000 \text{ in}\cdot\text{blob}$$

5. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -0.0150 \text{ in}\cdot\text{blob}$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.0000 \text{ in}\cdot\text{blob}$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -0.3510 \text{ in}\cdot\text{blob}$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.0000 \text{ in}\cdot\text{blob}$$

6. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

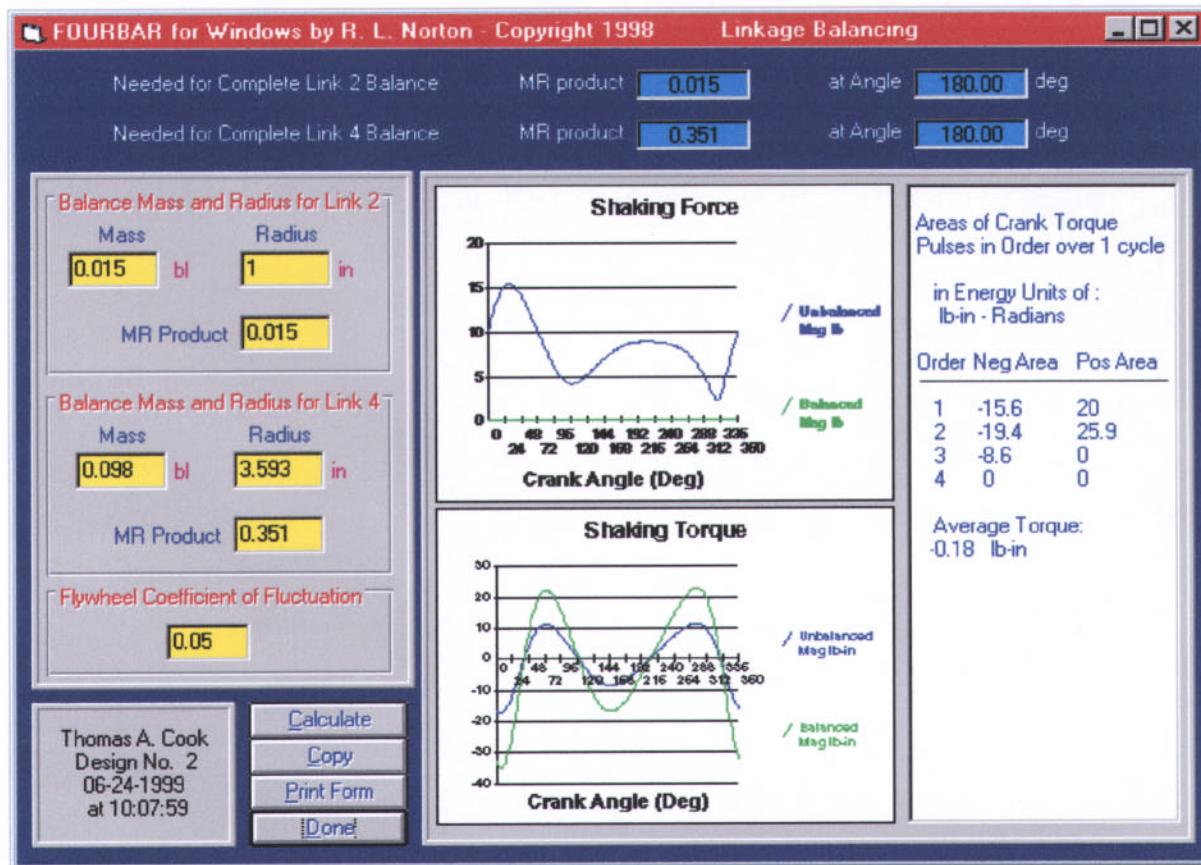
$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 0.0150 \text{ blob}\cdot\text{in}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 0.351 \text{ blob}\cdot\text{in}$$

7. Check the result using program FOURBAR. The linkage balancing screen below (on the next page) confirms the calculations.

8. The maximum peak input torque without force balancing is 17.3 lb-in. After force balancing it is 34.9 lb-in.



 **PROBLEM 12-10**

Statement: Figure P12-2a shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. The crank O_2A rotates at a constant speed of $\omega = 40 \text{ rad/sec}$. Design counterweights to force balance the linkage and determine its change in peak torque versus the unbalanced condition.

Given: Link lengths:

$$\text{Link 2 (}O_2\text{ to }A\text{)} \quad a := 1.00 \cdot m \quad \text{Link 3 (}A\text{ to }B\text{)} \quad b := 2.06 \cdot m$$

$$\text{Link 4 (}B\text{ to }O_4\text{)} \quad c := 2.33 \cdot m \quad \text{Link 1 (}O_2\text{ to }O_4\text{)} \quad d := 2.22 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 3.06 \cdot m \quad \delta_3 := -31 \cdot \text{deg}$$

$$\text{Crank angle and motion:} \quad \theta_2 := 60 \cdot \text{deg} \quad \omega_2 := 40 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P12-2a and Mathcad file P1210.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.500 \text{ m} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 1.165 \text{ m}$$

$$\text{Link 3:} \quad R_{CG3x} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x} = 1.561 \text{ m}$$

$$R_{CG3y} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y} = -0.525 \text{ m}$$

$$R_{CG3} := \sqrt{R_{CG3x}^2 + R_{CG3y}^2} \quad R_{CG3} = 1.647 \text{ m}$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x}, R_{CG3y}) \quad \delta_{33} = -18.600 \text{ deg}$$

2. Determine the mass of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 10.380 \text{ kg} \quad m_3 = 112.332 \text{ kg} \quad m_4 = 24.185 \text{ kg}$$

3. Convert the above data for use in equations 12.8c and 12.8d.

$$\text{Link lengths:} \quad l_2 := a \quad l_3 := b \quad l_4 := c$$

$$\text{CG position vectors:} \quad b_2 := R_{CG2} \quad b_3 := R_{CG3} \quad b_4 := R_{CG4}$$

$$\phi_2 := 0 \cdot \text{deg} \quad \phi_3 := \delta_{33} \quad \phi_4 := 0 \cdot \text{deg}$$

4. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -27.2117 \text{ kg}\cdot\text{m}$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = -28.6467 \text{ kg}\cdot\text{m}$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -198.3298 \text{ kg}\cdot\text{m}$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 66.7469 \text{ kg}\cdot\text{m}$$

5. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -32.4017 \text{ kg}\cdot\text{m}$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = -28.6467 \text{ kg}\cdot\text{m}$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -226.5057 \text{ kg}\cdot\text{m}$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 66.7469 \text{ kg}\cdot\text{m}$$

6. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = -138.520 \text{ deg}$$

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 43.25 \text{ kg}\cdot\text{m}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 163.581 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 236.14 \text{ kg}\cdot\text{m}$$

7. The maximum peak input torque without force balancing is found from program FOURBAR to be 464.9 kN-m. After force balancing it is 719.4 kN-m.

 **PROBLEM 12-11**

Statement: Figure P12-2b shows a fourbar linkage and its dimensions in meters. The steel crank and rocker have uniform cross sections 50 mm wide by 25 mm thick. The aluminum coupler is 25 mm thick. The crank O_2A rotates at a constant speed of $\omega = 50 \text{ rad/sec}$. Design counterweights to force balance the linkage and determine its change in peak torque versus the unbalanced condition.

Given: Link lengths:

$$\text{Link 2 (}O_2 \text{ to } A\text{)} \quad a := 0.72 \cdot m \quad \text{Link 3 (}A \text{ to } B\text{)} \quad b := 0.68 \cdot m$$

$$\text{Link 4 (}B \text{ to } O_4\text{)} \quad c := 0.85 \cdot m \quad \text{Link 1 (}O_2 \text{ to } O_4\text{)} \quad d := 1.82 \cdot m$$

$$\text{Coupler point:} \quad R_{pa} := 0.97 \cdot m \quad \delta_3 := 54 \cdot \text{deg}$$

$$\text{Crank angle and motion:} \quad \theta_2 := 30 \cdot \text{deg} \quad \omega_2 := 50 \cdot \text{rad} \cdot \text{sec}^{-1} \quad \alpha_2 := 0 \cdot \text{rad} \cdot \text{sec}^{-2}$$

Link cross-section dims:

$$w_2 := 50 \cdot \text{mm} \quad t_2 := 25 \cdot \text{mm} \quad t_3 := 25 \cdot \text{mm} \quad w_4 := 50 \cdot \text{mm} \quad t_4 := 25 \cdot \text{mm}$$

$$\text{Material specific weight:} \quad \text{steel} \quad \gamma_s := 0.3 \cdot \text{lbf} \cdot \text{in}^{-3} \quad \text{aluminum} \quad \gamma_a := 0.1 \cdot \text{lbf} \cdot \text{in}^{-3}$$

Solution: See Figure P12-2b and Mathcad file P1211.

1. Determine the distance to the CG in the LRCS on each of the three moving links.

$$\text{Links 2 and 4:} \quad R_{CG2} := 0.5 \cdot a \quad R_{CG2} = 0.360 \text{ m} \quad R_{CG4} := 0.5 \cdot c \quad R_{CG4} = 0.425 \text{ m}$$

$$\text{Link 3:} \quad R_{CG3x} := \frac{R_{pa} \cdot \cos(\delta_3) + b}{3} \quad R_{CG3x} = 0.417 \text{ m}$$

$$R_{CG3y} := \frac{R_{pa} \cdot \sin(\delta_3)}{3} \quad R_{CG3y} = 0.262 \text{ m}$$

$$R_{CG3} := \sqrt{R_{CG3x}^2 + R_{CG3y}^2} \quad R_{CG3} = 0.492 \text{ m}$$

At an angle with respect to the local x' axis of

$$\delta_{33} := \text{atan2}(R_{CG3x}, R_{CG3y}) \quad \delta_{33} = 32.117 \text{ deg}$$

2. Determine the mass of each link.

$$m_2 := w_2 \cdot t_2 \cdot a \cdot \frac{\gamma_s}{g} \quad m_3 := \frac{1}{2} \cdot b \cdot |R_{pa} \cdot \sin(\delta_3)| \cdot t_3 \cdot \frac{\gamma_a}{g} \quad m_4 := w_4 \cdot t_4 \cdot c \cdot \frac{\gamma_s}{g}$$

$$m_2 = 7.474 \text{ kg} \quad m_3 = 18.463 \text{ kg} \quad m_4 = 8.823 \text{ kg}$$

3. Convert the above data for use in equations 12.8c and 12.8d.

$$\text{Link lengths:} \quad l_2 := a \quad l_3 := b \quad l_4 := c$$

$$\text{CG position vectors:} \quad b_2 := R_{CG2} \quad b_3 := R_{CG3} \quad b_4 := R_{CG4}$$

$$\phi_2 := 0 \cdot \text{deg} \quad \phi_3 := \delta_{33} \quad \phi_4 := 0 \cdot \text{deg}$$

4. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -5.1471 \text{ kg}\cdot\text{m}$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 5.1138 \text{ kg}\cdot\text{m}$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -9.6175 \text{ kg}\cdot\text{m}$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = -6.0371 \text{ kg}\cdot\text{m}$$

5. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -7.8375 \text{ kg}\cdot\text{m}$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 5.1138 \text{ kg}\cdot\text{m}$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -13.3673 \text{ kg}\cdot\text{m}$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = -6.0371 \text{ kg}\cdot\text{m}$$

6. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 146.877 \text{ deg}$$

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 9.36 \text{ kg}\cdot\text{m}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = -155.694 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 14.67 \text{ kg}\cdot\text{m}$$

7. The maximum peak input torque without force balancing is found from program FOURBAR to be 5.34 MN-m. After force balancing it is 10.63 MN-m.

 **PROBLEM 12-12**

Statement: Write a computer program or use an equation solver such as *Mathcad*, *Matlab*, or *TKSolver* to solve for the mass-radius products that will force balance any fourbar linkage for which the geometry and mass properties are known.

Units: Use any set of compatible units.

Enter data: Link lengths:

| | | | |
|-------------------------------|--------------------------------|--|---------------------------------|
| Link 2 (O ₂ to A) | $l_2 := 4.00$ | Link 3 (A to B) | $l_3 := 12.00$ |
| Link 4 (B to O ₄) | $l_4 := 8.00$ | Link 1 (O ₂ to O ₄) | $l_1 := 15.00$ |
| Mass: | $m_2 := 0.002$ | $m_3 := 0.020$ | $m_4 := 0.100$ |
| Mass center: | $b_2 := 2.00$ | $b_3 := 5.00$ | $b_4 := 4.00$ |
| | $\phi_2 := 0 \cdot \text{deg}$ | $\phi_3 := 0 \cdot \text{deg}$ | $\phi_4 := 30 \cdot \text{deg}$ |

Solution: See Mathcad file P1212.

1. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \cos(\phi_3) - l_2 \right) \quad mb_{2x} = -0.0467$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{l_2}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -0.0667$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{l_4}{l_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000$$

2. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -0.0507$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -0.413$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = -0.200$$

3. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 0.0507$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 0.459$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = -154.165 \text{ deg}$$

 **PROBLEM 12-13**

Statement: Figure P12-3 shows a system with two weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$\begin{array}{llll} W_1 := 15 \cdot \text{lbf} & r_1 := 6 \cdot \text{in} & \theta_1 := 0 \cdot \text{deg} & l_1 := -7 \cdot \text{in} \\ W_2 := 20 \cdot \text{lbf} & r_2 := 5 \cdot \text{in} & \theta_2 := 270 \cdot \text{deg} & l_2 := 9 \cdot \text{in} \end{array}$$

$$\text{Distance between correction planes: } l_B := 9 \cdot \text{in}$$

$$\text{Correction weight radii: } \text{Plane 4} \quad R_A := 8 \cdot \text{in} \quad \text{Plane 3} \quad R_B := 5 \cdot \text{in}$$

Solution: See Figure P12-3 and Mathcad file P1213.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$\begin{array}{llll} R_{Ix} := r_1 \cdot \cos(\theta_1) & R_{Ix} = 6.000 \text{ in} & R_{Iy} := r_1 \cdot \sin(\theta_1) & R_{Iy} = 0.000 \text{ in} \\ R_{2x} := r_2 \cdot \cos(\theta_2) & R_{2x} = 0.000 \text{ in} & R_{2y} := r_2 \cdot \sin(\theta_2) & R_{2y} = -5.000 \text{ in} \end{array}$$

2. Solve equations 12.4e for summation of moments about O , which is at plane 4.

$$\begin{array}{ll} mR_{Bx} := \frac{-(W_1 \cdot R_{Ix}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2}{l_B \cdot g} & mR_{Bx} = 70.000 \text{ in-lb} \\ mR_{By} := \frac{-(W_1 \cdot R_{Iy}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2}{l_B \cdot g} & mR_{By} = 100.000 \text{ in-lb} \end{array}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the weight required at the given radius.

$$\begin{array}{ll} \theta_B := \text{atan2}(mR_{Bx}, mR_{By}) & \theta_B = 55.008 \text{ deg} \\ mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} & mR_B = 122.066 \text{ in-lb} \\ W_3 := \frac{mR_B \cdot g}{R_B} & W_3 = 24.4 \text{ lbf} \end{array}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (4).

$$\begin{array}{ll} mR_{Ax} := \frac{-W_1 \cdot R_{Ix} - W_2 \cdot R_{2x}}{g} - mR_{Bx} & mR_{Ax} = -160.000 \text{ in-lb} \\ mR_{Ay} := \frac{-W_1 \cdot R_{Iy} - W_2 \cdot R_{2y}}{g} - mR_{By} & mR_{Ay} = 0.000 \text{ in-lb} \end{array}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (4).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = 180.000 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 160.000 \text{ in-lb}$$

$$W_4 := \frac{mR_A \cdot g}{R_A} \quad W_4 = 20.0 \text{ lbf}$$

 **PROBLEM 12-14**

Statement: Figure P12-4 shows a system with two weights on a rotating shaft. For the given data below, determine the radii and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$W_1 := 15 \cdot \text{lbf} \quad r_1 := 4 \cdot \text{in} \quad \theta_1 := 30 \cdot \text{deg} \quad l_1 := 0 \cdot \text{in}$$

$$W_2 := 20 \cdot \text{lbf} \quad r_2 := 6 \cdot \text{in} \quad \theta_2 := 270 \cdot \text{deg} \quad l_2 := 9 \cdot \text{in}$$

Distance between correction planes: $l_B := 12 \cdot \text{in}$

Correction weights: Plane 3 $W_3 := 15 \cdot \text{lbf}$ Plane 4 $W_4 := 30 \cdot \text{lbf}$

Solution: See Figure P12-4 and Mathcad file P1214.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$R_{Ix} := r_1 \cdot \cos(\theta_1) \quad R_{Ix} = 3.464 \text{ in} \quad R_{Iy} := r_1 \cdot \sin(\theta_1) \quad R_{Iy} = 2.000 \text{ in}$$

$$R_{2x} := r_2 \cdot \cos(\theta_2) \quad R_{2x} = 0.000 \text{ in} \quad R_{2y} := r_2 \cdot \sin(\theta_2) \quad R_{2y} = -6.000 \text{ in}$$

2. Solve equations 12.4e for summation of moments about O , which is at plane 3.

$$mR_{Bx} := \frac{-(W_1 \cdot R_{Ix}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2}{l_B \cdot g} \quad mR_{Bx} = 0.000 \text{ in-lb}$$

$$mR_{By} := \frac{-(W_1 \cdot R_{Iy}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2}{l_B \cdot g} \quad mR_{By} = 90.000 \text{ in-lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (4). Also, solve for the radius required for the given weight.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 90.000 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 90.000 \text{ in-lb}$$

$$R_4 := \frac{mR_B \cdot g}{W_4} \quad R_4 = 3.00 \text{ in}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (3).

$$mR_{Ax} := \frac{-W_1 \cdot R_{Ix} - W_2 \cdot R_{2x}}{g} - mR_{Bx} \quad mR_{Ax} = -51.962 \text{ in-lb}$$

$$mR_{Ay} := \frac{-W_1 \cdot R_{Iy} - W_2 \cdot R_{2y}}{g} - mR_{By} \quad mR_{Ay} = 0.000 \text{ in-lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (3).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = 180.000 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 51.962 \text{ in-lb}$$

$$R_3 := \frac{mR_A \cdot g}{W_3} \quad R_3 = 3.46 \text{ in}$$



PROBLEM 12-15

Statement: Figure P12-5 shows a system with two weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$\begin{array}{llll} W_1 := 10 \cdot \text{lbf} & r_1 := 3 \cdot \text{in} & \theta_1 := 90 \cdot \text{deg} & l_1 := 3 \cdot \text{in} \\ W_2 := 15 \cdot \text{lbf} & r_2 := 3 \cdot \text{in} & \theta_2 := 240 \cdot \text{deg} & l_2 := 7 \cdot \text{in} \end{array}$$

Distance between correction planes: $l_B := 12 \cdot \text{in}$

Correction weight radii: Plane 4 $R_A := 3 \cdot \text{in}$ Plane 3 $R_B := 3 \cdot \text{in}$

Solution: See Figure P12-5 and Mathcad file P1215.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$\begin{array}{llll} R_{Ix} := r_1 \cdot \cos(\theta_1) & R_{Ix} = 0.000 \text{ in} & R_{Iy} := r_1 \cdot \sin(\theta_1) & R_{Iy} = 3.000 \text{ in} \\ R_{2x} := r_2 \cdot \cos(\theta_2) & R_{2x} = -1.500 \text{ in} & R_{2y} := r_2 \cdot \sin(\theta_2) & R_{2y} = -2.598 \text{ in} \end{array}$$

2. Solve equations 12.4e for summation of moments about O , which is at plane 4.

$$\begin{array}{ll} mR_{Bx} := \frac{-(W_1 \cdot R_{Ix}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2}{l_B \cdot g} & mR_{Bx} = 13.125 \text{ in-lb} \\ mR_{By} := \frac{-(W_1 \cdot R_{Iy}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2}{l_B \cdot g} & mR_{By} = 15.233 \text{ in-lb} \end{array}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 49.252 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 20.108 \text{ in-lb}$$

$$W_3 := \frac{mR_B \cdot g}{R_B} \quad W_3 = 6.70 \text{ lbf}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (4).

$$mR_{Ax} := \frac{-W_1 \cdot R_{Ix} - W_2 \cdot R_{2x}}{g} - mR_{Bx} \quad mR_{Ax} = 9.375 \text{ in-lb}$$

$$mR_{Ay} := \frac{-W_1 \cdot R_{Iy} - W_2 \cdot R_{2y}}{g} - mR_{By} \quad mR_{Ay} = -6.262 \text{ in-lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (4).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -33.741 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 11.274 \text{ in}\cdot\text{lb}$$

$$W_4 := \frac{mR_A \cdot g}{R_A} \quad W_4 = 3.76 \text{ lbf}$$



PROBLEM 12-16

Statement: Figure P12-6 shows a system with three weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$W_1 := 9 \cdot \text{lbf} \quad r_1 := 4 \cdot \text{in} \quad \theta_1 := 90 \cdot \text{deg} \quad l_1 := -14 \cdot \text{in}$$

$$W_2 := 9 \cdot \text{lbf} \quad r_2 := 6 \cdot \text{in} \quad \theta_2 := 225 \cdot \text{deg} \quad l_2 := 4 \cdot \text{in}$$

$$W_3 := 6 \cdot \text{lbf} \quad r_3 := 10 \cdot \text{in} \quad \theta_3 := 315 \cdot \text{deg} \quad l_3 := -4 \cdot \text{in}$$

$$\text{Distance between correction planes:} \quad l_B := 14 \cdot \text{in}$$

$$\text{Correction weight radii:} \quad \text{Plane 4} \quad R_A := 3 \cdot \text{in} \quad \text{Plane 5} \quad R_B := 3 \cdot \text{in}$$

Solution: See Figure P12-6 and Mathcad file P1216.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$R_{Ix} := r_1 \cdot \cos(\theta_1) \quad R_{Ix} = 0.000 \text{ in} \quad R_{Iy} := r_1 \cdot \sin(\theta_1) \quad R_{Iy} = 4.000 \text{ in}$$

$$R_{2x} := r_2 \cdot \cos(\theta_2) \quad R_{2x} = -4.243 \text{ in} \quad R_{2y} := r_2 \cdot \sin(\theta_2) \quad R_{2y} = -4.243 \text{ in}$$

$$R_{3x} := r_3 \cdot \cos(\theta_3) \quad R_{3x} = 7.071 \text{ in} \quad R_{3y} := r_3 \cdot \sin(\theta_3) \quad R_{3y} = -7.071 \text{ in}$$

2. Solve equations 12.4e for summation of moments about O , which is at plane 4.

$$mR_{Bx} := \frac{-(W_1 \cdot R_{Ix}) \cdot l_1 - (W_2 \cdot R_{2x}) \cdot l_2 - (W_3 \cdot R_{3x}) \cdot l_3}{l_B \cdot g} \quad mR_{Bx} = 23.031 \text{ in-lb}$$

$$mR_{By} := \frac{-(W_1 \cdot R_{Iy}) \cdot l_1 - (W_2 \cdot R_{2y}) \cdot l_2 - (W_3 \cdot R_{3y}) \cdot l_3}{l_B \cdot g} \quad mR_{By} = 34.788 \text{ in-lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (5). Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 56.493 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 41.721 \text{ in-lb}$$

$$W_5 := \frac{mR_B \cdot g}{R_B} \quad W_5 = 13.91 \text{ lbf}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (4).

$$mR_{Ax} := \frac{-W_1 \cdot R_{Ix} - W_2 \cdot R_{2x} - W_3 \cdot R_{3x}}{g} - mR_{Bx} \quad mR_{Ax} = -27.274 \text{ in-lb}$$

$$mR_{Ay} := \frac{-W_1 \cdot R_{Iy} - W_2 \cdot R_{2y} - W_3 \cdot R_{3y}}{g} - mR_{By} \quad mR_{Ay} = 9.822 \text{ in-lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *A* (4).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = 160.194 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 28.989 \text{ in-lb}$$

$$W_4 := \frac{mR_A \cdot g}{R_A} \quad W_4 = 9.66 \text{ lbf}$$



PROBLEM 12-17

Statement: Figure P12-6 shows a system with three weights on a rotating shaft. For the given data below, determine the magnitudes and angles of the balance weights needed to dynamically balance the system.

Given: Weights and radii:

$$W_2 := 10 \cdot \text{lbf} \quad r_2 := 3 \cdot \text{in} \quad \theta_2 := 90 \cdot \text{deg} \quad l_2 := 6 \cdot \text{in}$$

$$W_3 := 10 \cdot \text{lbf} \quad r_3 := 4 \cdot \text{in} \quad \theta_3 := 180 \cdot \text{deg} \quad l_3 := 12 \cdot \text{in}$$

$$W_4 := 8 \cdot \text{lbf} \quad r_4 := 4 \cdot \text{in} \quad \theta_4 := 315 \cdot \text{deg} \quad l_4 := 8 \cdot \text{in}$$

$$\text{Distance between correction planes:} \quad l_B := 8 \cdot \text{in}$$

$$\text{Correction weight radii:} \quad \text{Plane 1} \quad R_A := 4 \cdot \text{in} \quad \text{Plane 5} \quad R_B := 3 \cdot \text{in}$$

Solution: See Figure P12-7 and Mathcad file P1217.

1. Resolve the position vectors into xy components in the arbitrary coordinate system associated with the freeze-frame position of the linkage chosen for analysis.

$$R_{2x} := r_2 \cdot \cos(\theta_2) \quad R_{2x} = 0.000 \text{ in} \quad R_{2y} := r_2 \cdot \sin(\theta_2) \quad R_{2y} = 3.000 \text{ in}$$

$$R_{3x} := r_3 \cdot \cos(\theta_3) \quad R_{3x} = -4.000 \text{ in} \quad R_{3y} := r_3 \cdot \sin(\theta_3) \quad R_{3y} = 0.000 \text{ in}$$

$$R_{4x} := r_4 \cdot \cos(\theta_4) \quad R_{4x} = 2.828 \text{ in} \quad R_{4y} := r_4 \cdot \sin(\theta_4) \quad R_{4y} = -2.828 \text{ in}$$

2. Solve equations 12.4e for summation of moments about O , which is at plane 1.

$$mR_{Bx} := \frac{-(W_2 \cdot R_{2x}) \cdot l_2 - (W_3 \cdot R_{3x}) \cdot l_3 - (W_4 \cdot R_{4x}) \cdot l_4}{l_B \cdot g} \quad mR_{Bx} = 37.373 \text{ in-lb}$$

$$mR_{By} := \frac{-(W_2 \cdot R_{2y}) \cdot l_2 - (W_3 \cdot R_{3y}) \cdot l_3 - (W_4 \cdot R_{4y}) \cdot l_4}{l_B \cdot g} \quad mR_{By} = 0.127 \text{ in-lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (5). Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 0.195 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 37.373 \text{ in-lb}$$

$$W_5 := \frac{mR_B \cdot g}{R_B} \quad W_5 = 12.46 \text{ lbf}$$

4. Solve equations 12.4c for forces in x and y directions in plane A (1).

$$mR_{Ax} := \frac{-W_2 \cdot R_{2x} - W_3 \cdot R_{3x} - W_4 \cdot R_{4x}}{g} - mR_{Bx} \quad mR_{Ax} = -20.000 \text{ in-lb}$$

$$mR_{Ay} := \frac{-W_2 \cdot R_{2y} - W_3 \cdot R_{3y} - W_4 \cdot R_{4y}}{g} - mR_{By} \quad mR_{Ay} = -7.500 \text{ in-lb}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *A* (1).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -159.444 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 21.360 \text{ in-lb}$$

$$W_I := \frac{mR_A \cdot g}{R_A} \quad W_I = 5.34 \text{ lbf}$$



PROBLEM 12-18

Statement: The 400-mm-dia steel roller in Figure P12-8 has been tested on a dynamic balancing machine at 100 rpm and shows the unbalance forces given below. Determine the angular locations and required diameters of 25-mm deep holes drilled radially inward from the surface in planes 2 and 3 to dynamically balance the system.

Units: $rpm := 2\pi \cdot rad \cdot min^{-1}$

Given: Forces and locations:

$$F_1 := 0.291 \cdot N \quad \theta_1 := 45 \cdot deg \quad l_1 := -200 \cdot mm$$

$$F_4 := 0.514 \cdot N \quad \theta_4 := 210 \cdot deg \quad l_4 := 550 \cdot mm$$

Distance between correction planes: $l_B := 300 \cdot mm$

Correction weight radii: Plane 2 $R_A := 187.5 \cdot mm$ Plane 3 $R_B := 187.5 \cdot mm$

Test speed: $\omega := 100 \cdot rpm$

Solution: See Figure P12-8 and Mathcad file P1218.

1. Resolve the force vectors into xy components in the arbitrary coordinate system associated with the test apparatus.

$$F_{Ix} := F_1 \cdot \cos(\theta_1) \quad F_{Ix} = 0.206 \cdot N \quad F_{Iy} := F_1 \cdot \sin(\theta_1) \quad F_{Iy} = 0.206 \cdot N$$

$$F_{4x} := F_4 \cdot \cos(\theta_4) \quad F_{4x} = -0.445 \cdot N \quad F_{4y} := F_4 \cdot \sin(\theta_4) \quad F_{4y} = -0.257 \cdot N$$

2. Solve equations 12.3 for summation of moments about O , which is at plane 1. Negative mass is used.

$$mR_{Bx} := \frac{F_{Ix} \cdot l_1 + F_{4x} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{Bx} = -8.693 \times 10^{-3} \cdot kg \cdot m$$

$$mR_{By} := \frac{F_{Iy} \cdot l_1 + F_{4y} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{By} = -5.547 \times 10^{-3} \cdot kg \cdot m$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the weight and drill diameter required at the given radius.

$$\theta_B := atan2(mR_{Bx}, mR_{By}) \quad \theta_B = -147.455 \cdot deg$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 0.0103 \cdot kg \cdot m$$

$$W_3 := \frac{mR_B \cdot g}{R_B} \quad W_3 = 0.539 \cdot N \quad (\text{removed})$$

$$\rho := 7800 \cdot kg \cdot m^{-3} \quad depth := 25 \cdot mm$$

$$d_3 := 2 \cdot \sqrt{\frac{W_3}{\pi \cdot \rho \cdot depth \cdot g}} \quad d_3 = 18.9 \cdot mm$$

4. Solve equations 12.4c for forces in x and y directions in plane A (2).

$$mR_{Ax} := \frac{-F_{Ix} - F_{4x}}{2} - mR_{Bx} \quad mR_{Ax} = 0.0109 \text{ kg}\cdot\text{m}$$

$$mR_{Ay} := \frac{-F_{Iy} - F_{4y}}{2} - mR_{By} \quad mR_{Ay} = 6.015 \times 10^{-3} \text{ kg}\cdot\text{m}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (2).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = 28.944 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 0.0124 \text{ kg}\cdot\text{m}$$

$$W_2 := \frac{mR_A \cdot g}{R_A} \quad W_2 = 0.650 \text{ N} \quad (\text{removed})$$

$$d_2 := 2 \cdot \sqrt{\frac{W_2}{\pi \cdot \rho \cdot \text{depth} \cdot g}} \quad d_2 = 20.8 \text{ mm}$$



PROBLEM 12-19

Statement: The 500-mm-dia steel roller in Figure P12-8 has been tested on a dynamic balancing machine at 100 rpm and shows the unbalance forces given below. Determine the angular locations and required diameters of 25-mm deep holes drilled radially inward from the surface in planes 2 and 3 to dynamically balance the system.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Forces and locations:

$$F_I := 0.230 \cdot N \quad \theta_I := 30 \cdot deg \quad l_I := -200 \cdot mm$$

$$F_4 := 0.620 \cdot N \quad \theta_4 := 135 \cdot deg \quad l_4 := 550 \cdot mm$$

Distance between correction planes: $l_B := 300 \cdot mm$

Correction weight radii: Plane 2 $R_A := 237.5 \cdot mm$ Plane 3 $R_B := 237.5 \cdot mm$

Test speed: $\omega := 100 \cdot rpm$

Solution: See Figure P12-8 and Mathcad file P1219.

1. Resolve the force vectors into xy components in the arbitrary coordinate system associated with the test apparatus.

$$F_{Ix} := F_I \cdot \cos(\theta_I) \quad F_{Ix} = 0.199 \cdot N \quad F_{Iy} := F_I \cdot \sin(\theta_I) \quad F_{Iy} = 0.115 \cdot N$$

$$F_{4x} := F_4 \cdot \cos(\theta_4) \quad F_{4x} = -0.438 \cdot N \quad F_{4y} := F_4 \cdot \sin(\theta_4) \quad F_{4y} = 0.438 \cdot N$$

2. Solve equations 12.3 for summation of moments about O , which is at plane 1. Negative mass is used.

$$mR_{Bx} := \frac{F_{Ix} \cdot l_I + F_{4x} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{Bx} = -8.540 \times 10^{-3} \cdot kg \cdot m$$

$$mR_{By} := \frac{F_{Iy} \cdot l_I + F_{4y} \cdot l_4}{l_B \cdot \omega^2} \quad mR_{By} = 6.630 \times 10^{-3} \cdot kg \cdot m$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B (3). Also, solve for the weight and drill diameter required at the given radius.

$$\theta_B := atan2(mR_{Bx}, mR_{By}) \quad \theta_B = 142.176 \cdot deg$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 0.0108 \cdot kg \cdot m$$

$$W_3 := \frac{mR_B \cdot g}{R_B} \quad W_3 = 0.446 \cdot N \quad (\text{removed})$$

$$\rho := 7800 \cdot kg \cdot m^{-3} \quad depth := 25 \cdot mm$$

$$d_3 := 2 \cdot \sqrt{\frac{W_3}{\pi \cdot \rho \cdot depth \cdot g}} \quad d_3 = 17.241 \cdot mm$$

4. Solve equations 12.4c for forces in x and y directions in plane A (4).

$$mR_{Ax} := \frac{-F_{Ix} - F_{4x}}{2} - mR_{Bx} \quad mR_{Ax} = 0.0107 \text{ kg}\cdot\text{m}$$

$$mR_{Ay} := \frac{-F_{Iy} - F_{4y}}{2} - mR_{By} \quad mR_{Ay} = -0.012 \text{ kg}\cdot\text{m}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A (4).

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -47.441 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 0.0159 \text{ kg}\cdot\text{m}$$

$$W_4 := \frac{mR_A \cdot g}{R_A} \quad W_4 = 0.655 \text{ N} \quad (\text{removed})$$

$$d_4 := 2 \cdot \sqrt{\frac{W_4}{\pi \cdot \rho \cdot \text{depth} \cdot g}} \quad d_4 = 20.876 \text{ mm}$$

 PROBLEM 12-20

Statement: The linkage in Figure P12-9a has rectangular steel links of 20 x 10 mm cross-section similar to that shown in Figure 12-10a. Design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment. State all assumptions.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to } A\text{)} \quad a_2 := 78 \cdot \text{mm} \quad \text{Link 3 (A to } B\text{)} \quad a_3 := 109 \cdot \text{mm}$$

$$\text{Link 4 (B to } O_4\text{)} \quad a_4 := 121 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to } O_4\text{)} \quad a_1 := 54 \cdot \text{mm}$$

$$\text{Link cross-section dims: } h := 20 \cdot \text{mm} \quad t := 10 \cdot \text{mm}$$

$$\text{Material density: steel} \quad \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Figure P12-9a and Mathcad file P1220.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \sqrt{3 \left(\frac{a_3}{h} \right)^2 - 1} - \frac{a_3}{2} \quad e = 39.366 \text{ mm}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 187.731 \text{ mm}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 39.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 54.500 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 60.500 \text{ mm}$$

3. Determine the mass of each link.

$$m_2 := h \cdot t \cdot a_2 \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := h \cdot t \cdot a_4 \cdot \rho$$

$$m_2 = 0.122 \text{ kg} \quad m_3 = 0.293 \text{ kg} \quad m_4 = 0.189 \text{ kg}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 54.500 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 0.0114 \text{ kg} \cdot \text{m}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 0.0177 \text{ kg} \cdot \text{m}$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 6.6760 \times 10^{-3} \text{ kg} \cdot \text{m}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 6.2981 \times 10^{-3} \text{ kg} \cdot \text{m}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \cdot \text{mm} \quad R_4 := 70.8 \cdot \text{mm}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 0.0890 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 0.0890 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.211 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 54.2 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.278 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 63.8 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 1.161 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 74.244 \text{ mm}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 3.157 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 106.625 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2{}^2 + r_2{}^2 + a_2 \cdot r_2) \quad I_{cw2} = 2.671 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{cw4} := m'_4(k'_4{}^2 + r_4{}^2 + a_4 \cdot r_4) \quad I_{cw4} = 6.432 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$



PROBLEM 12-21

Statement: The linkage in Figure P12-9a has rectangular steel links of 20 x 10 mm cross-section with "dog bone" ends of 50 mm dia, similar to that shown in Figure 12-10b. Design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment. State all assumptions.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 78 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad a_3 := 109 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4 \text{)} \quad a_4 := 121 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4 \text{)} \quad a_1 := 54 \cdot \text{mm}$$

$$\text{Link cross-section dims: } h := 20 \cdot \text{mm} \quad t := 10 \cdot \text{mm} \quad c := 25 \cdot \text{mm}$$

$$\text{Material density: steel} \quad \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Assumption: Only link 3 has the "dog bone" configuration.

Solution: See Figure P12-9a and Mathcad file P1221.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equations 12.13.

$$B := 12 \cdot \left(\frac{a_3}{c} \right) + 24 \quad B = 76.320 \quad C := 24 \cdot \left(\frac{a_3}{c} \right) + 26 \quad C = 130.640$$

$$D := -2 \cdot \left(\frac{a_3}{c} \right)^3 + 13 \cdot \left(\frac{a_3}{c} \right) + 12 \cdot \pi - 10 \quad D = -81.385$$

$$v := \begin{pmatrix} D \\ C \\ B \\ 8 \end{pmatrix} \quad r := \text{polyroots}(v) \quad r = \begin{pmatrix} -6.999 \\ -3.022 \\ 0.481 \end{pmatrix} \quad \text{Choosing the positive root,}$$

$$e := h \cdot r_3 \quad e = 9.620 \text{ mm}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 39.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 54.500 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 60.500 \text{ mm}$$

3. Determine the mass of each link.

$$m_2 := h \cdot t \cdot a_2 \cdot \rho \quad m_3 := t \cdot \left[h \cdot (a_3 - 2 \cdot c + 2 \cdot e) + 2 \cdot \pi \cdot c^2 \right] \cdot \rho \quad m_4 := h \cdot t \cdot a_4 \cdot \rho$$

$$m_2 = 0.122 \text{ kg} \quad m_3 = 0.428 \text{ kg} \quad m_4 = 0.189 \text{ kg}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 54.500 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 0.0167 \text{ kg} \cdot \text{m}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 0.0259 \text{ kg}\cdot\text{m}$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 0.0120 \text{ kg}\cdot\text{m}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 0.0145 \text{ kg}\cdot\text{m}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \cdot \text{mm} \quad R_4 := 91 \cdot \text{mm}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 0.159 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 0.159 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.281 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 59.4 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.348 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 74.5 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2 \cdot (a_2^2 + h^2)}{12} + m_2 \cdot (r_2 + R_{CG2})^2 + m_{2b} \cdot (R_2 - r_2)^2 \quad I'_2 = 1.283 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 67.555 \text{ mm}$$

$$I'_4 := \frac{m_4 \cdot (a_4^2 + h^2)}{12} + m_4 \cdot (r_4 + R_{CG4})^2 + m_{4b} \cdot (R_4 - r_4)^2 \quad I'_4 = 3.718 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 103.358 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2 \cdot (k'_2)^2 + r_2^2 + a_2 \cdot r_2 \quad I_{cw2} = 3.579 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{cw4} := m'_4 \cdot (k'_4)^2 + r_4^2 + a_4 \cdot r_4 \quad I_{cw4} = 8.784 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

 **PROBLEM 12-22**

Statement: The linkage in Figure P12-9b has rectangular steel links of 20 x 10 mm cross-section similar to that shown in Figure 12-10a. Design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment. State all assumptions.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to } A\text{)} \quad a_2 := 50 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad a_3 := 185 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4\text{)} \quad a_4 := 90 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4\text{)} \quad a_1 := 172 \cdot \text{mm}$$

$$\text{Link cross-section dims: } h := 20 \cdot \text{mm} \quad t := 10 \cdot \text{mm}$$

$$\text{Material density: steel} \quad \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Figure P12-9b and Mathcad file P1222.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \sqrt{3 \left(\frac{a_3}{h} \right)^2 - 1} - \frac{a_3}{2} \quad e = 67.402 \text{ mm}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 319.805 \text{ mm}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 25.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 92.500 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 45.000 \text{ mm}$$

3. Determine the mass of each link.

$$m_2 := h \cdot t \cdot a_2 \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := h \cdot t \cdot a_4 \cdot \rho$$

$$m_2 = 0.078 \text{ kg} \quad m_3 = 0.499 \text{ kg} \quad m_4 = 0.140 \text{ kg}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 92.500 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 0.0125 \text{ kg} \cdot \text{m}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 0.0225 \text{ kg} \cdot \text{m}$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 0.0105 \text{ kg} \cdot \text{m}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 0.0161 \text{ kg} \cdot \text{m}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \text{ mm} \quad R_4 := 115 \text{ mm}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 0.140 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 0.140 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.218 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 57.1 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.281 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 80.0 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2 \cdot (r_2 + R_{CG2})^2 + m_{2b} \cdot (R_2 - r_2)^2 \quad I'_2 = 5.898 \times 10^{-4} \text{ kg}\cdot\text{m}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 51.980 \text{ mm}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4 \cdot (r_4 + R_{CG4})^2 + m_{4b} \cdot (R_4 - r_4)^2 \quad I'_4 = 2.465 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 93.707 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2 \cdot (k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 1.926 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{cw4} := m'_4 \cdot (k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 6.281 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$



PROBLEM 12-23

Statement: The linkage in Figure P12-9b has rectangular steel links of 20 x 10 mm cross-section with "dog bone" ends of 50 mm dia, similar to that shown in Figure 12-10b. Design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment. State all assumptions.

Given:

Link lengths:

$$\begin{array}{llll} \text{Link 2 (O}_2 \text{ to } A) & a_2 := 50 \cdot \text{mm} & \text{Link 3 (A to B)} & a_3 := 185 \cdot \text{mm} \\ \text{Link 4 (B to O}_4) & a_4 := 90 \cdot \text{mm} & \text{Link 1 (O}_2 \text{ to O}_4) & a_1 := 172 \cdot \text{mm} \\ \text{Link cross-section dims:} & h := 20 \cdot \text{mm} & t := 10 \cdot \text{mm} & c := 25 \cdot \text{mm} \\ \text{Material density: steel} & \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3} \end{array}$$

Assumption: Only link 3 has the "dog bone" configuration.

Solution: See Figure P12-9b and Mathcad file P1223.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equations 12.13.

$$B := 12 \cdot \left(\frac{a_3}{c} \right) + 24 \quad B = 112.800 \quad C := 24 \cdot \left(\frac{a_3}{c} \right) + 26 \quad C = 203.600$$

$$D := -2 \cdot \left(\frac{a_3}{c} \right)^3 + 13 \cdot \left(\frac{a_3}{c} \right) + 12 \cdot \pi - 10 \quad D = -686.549$$

$$v := \begin{pmatrix} D \\ C \\ B \\ 8 \end{pmatrix} \quad r := \text{polyroots}(v) \quad r = \begin{pmatrix} -11.116 \\ -4.646 \\ 1.662 \end{pmatrix} \quad \text{Choosing the positive root,}$$

$$e := h \cdot r_3 \quad e = 33.236 \text{ mm}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 25.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 92.500 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 45.000 \text{ mm}$$

3. Determine the mass of each link.

$$\begin{array}{lll} m_2 := h \cdot t \cdot a_2 \cdot \rho & m_3 := t \cdot [h \cdot (a_3 - 2 \cdot c + 2 \cdot e) + 2 \cdot \pi \cdot c^2] \cdot \rho & m_4 := h \cdot t \cdot a_4 \cdot \rho \\ m_2 = 0.078 \text{ kg} & m_3 = 0.621 \text{ kg} & m_4 = 0.140 \text{ kg} \end{array}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 92.500 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 0.0155 \text{ kg} \cdot \text{m}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 0.0279 \text{ kg}\cdot\text{m}$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 0.0136 \text{ kg}\cdot\text{m}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 0.0216 \text{ kg}\cdot\text{m}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \cdot \text{mm} \quad R_4 := 119.5 \cdot \text{mm}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 0.181 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 0.181 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.259 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 59.9 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.321 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 86.9 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2 \left(a_2^2 + h^2 \right)}{12} + m_2 \cdot (r_2 + R_{CG2})^2 + m_{2b} \cdot (R_2 - r_2)^2 \quad I'_2 = 6.226 \times 10^{-4} \text{ kg}\cdot\text{m}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 49.041 \text{ mm}$$

$$I'_4 := \frac{m_4 \left(a_4^2 + h^2 \right)}{12} + m_4 \cdot (r_4 + R_{CG4})^2 + m_{4b} \cdot (R_4 - r_4)^2 \quad I'_4 = 2.735 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 92.276 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2 \left(k'_2^2 + r_2^2 + a_2 \cdot r_2 \right) \quad I_{cw2} = 2.328 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{cw4} := m'_4 \left(k'_4^2 + r_4^2 + a_4 \cdot r_4 \right) \quad I_{cw4} = 7.677 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$



PROBLEM 12-24

Statement: The device in Figure P12-10 is used to balance fan blade/hub assemblies. The center distance between the two bearings on the machine is 250 mm. The left edge of the fan hub (plane A) is 100 mm from the centerline of the closest bearing (at F_2). The hub is 75 mm wide and has a diameter of 200 mm along the surfaces where balancing weights are fastened. The peak magnitude of force F_1 is 0.5 N at a phase angle of 30 deg with respect to the rotating x' axis. Force F_2 had a peak of 0.2 N at a phase angle of 130 deg. Calculate the magnitudes and locations, with respect to the x' axis, of balance weights placed in planes A and B of the hub to dynamically balance the fan assembly.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Bearing forces and plane locations with respect to correction plane A:

$$\text{Left: } F_1 := 0.5 \cdot N \quad \theta_1 := 30 \cdot \text{deg} \quad l_1 := 350 \cdot \text{mm}$$

$$\text{Right: } F_2 := 0.2 \cdot N \quad \theta_2 := 130 \cdot \text{deg} \quad l_2 := 100 \cdot \text{mm}$$

$$\text{Distance between correction planes: } l_B := 75 \cdot \text{mm}$$

$$\text{Correction weight radii: } R_A := 100 \cdot \text{mm} \quad R_B := 100 \cdot \text{mm}$$

$$\text{Fan rotational speed: } \omega := 600 \cdot rpm$$

Solution: See Figure P12-10 and Mathcad file P1224.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{Ix} := F_1 \cdot \cos(\theta_1) \quad F_{Ix} = 0.433 N \quad F_{Iy} := F_1 \cdot \sin(\theta_1) \quad F_{Iy} = 0.250 N$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -0.129 N \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = 0.153 N$$

2. Solve equations 12.4e for summation of moments about O , modified for the bearing forces.

$$mR_{Bx} := \frac{F_{Ix} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{Bx} = 0.468 \text{ mm} \cdot kg$$

$$mR_{By} := \frac{F_{Iy} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{By} = 0.347 \text{ mm} \cdot kg$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane B. Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 36.550 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 0.583 \text{ mm} \cdot kg$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 0.057 N$$

4. Solve equations 12.4c for forces in x and y directions in plane A.

$$mR_{Ax} := \frac{F_{Ix} + F_{2x}}{2} - mR_{Bx} \quad mR_{Ax} = -0.391 \text{ mm} \cdot kg$$

$$mR_{Ay} := \frac{F_{1y} + F_{2y}}{2} - mR_{By} \quad mR_{Ay} = -0.245 \text{ mm}\cdot\text{kg}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A.

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -147.936 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 0.462 \text{ mm}\cdot\text{kg}$$

$$W_A := \frac{mR_A \cdot g}{R_A} \quad W_A = 0.045 \text{ N}$$



PROBLEM 12-25

Statement: The device in Figure P12-10 is used to balance fan blade/hub assemblies. The center distance between the two bearings on the machine is 250 mm. The left edge of the fan hub (plane *A*) is 100 mm from the centerline of the closest bearing (at F_2). The hub is 55 mm wide and has a diameter of 150 mm along the surfaces where balancing weights are fastened. The peak magnitude of force F_1 is 1.5 N at a phase angle of 60 deg with respect to the rotating x' axis. Force F_2 had a peak of 2.0 N at a phase angle of -180 deg. Calculate the magnitudes and locations, with respect to the x' axis, of balance weights placed in planes *A* and *B* of the hub to dynamically balance the fan assembly.

Units: $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Bearing forces and plane locations with respect to correction plane *A*:

$$\text{Left: } F_1 := 1.5 \cdot N \quad \theta_1 := 60 \cdot \text{deg} \quad l_1 := 350 \cdot \text{mm}$$

$$\text{Right: } F_2 := 2.0 \cdot N \quad \theta_2 := -180 \cdot \text{deg} \quad l_2 := 100 \cdot \text{mm}$$

$$\text{Distance between correction planes: } l_B := 55 \cdot \text{mm}$$

$$\text{Correction weight radii: } R_A := 75 \cdot \text{mm} \quad R_B := 75 \cdot \text{mm}$$

$$\text{Fan rotational speed: } \omega := 600 \cdot rpm$$

Solution: See Figure P12-10 and Mathcad file P1225.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{Ix} := F_1 \cdot \cos(\theta_1) \quad F_{Ix} = 0.750 \cdot N \quad F_{Iy} := F_1 \cdot \sin(\theta_1) \quad F_{Iy} = 1.299 \cdot N$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -2.000 \cdot N \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = 1.531 \times 10^{-15} \cdot N$$

2. Solve equations 12.4e for summation of moments about *O*, modified for the bearing forces.

$$mR_{Bx} := \frac{F_{Ix} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{Bx} = 0.288 \cdot \text{mm} \cdot \text{kg}$$

$$mR_{By} := \frac{F_{Iy} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega^2} \quad mR_{By} = 2.094 \cdot \text{mm} \cdot \text{kg}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *B*. Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 82.173 \cdot \text{deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 2.114 \cdot \text{mm} \cdot \text{kg}$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 0.28 \cdot N$$

4. Solve equations 12.4c for forces in x and y directions in plane *A*.

$$mR_{Ax} := \frac{F_{Ix} + F_{2x}}{2} - mR_{Bx} \quad mR_{Ax} = -0.604 \cdot \text{mm} \cdot \text{kg}$$

$$mR_{Ay} := \frac{F_{Iy} + F_{2y}}{2} - mR_{By} \quad mR_{Ay} = -1.765 \text{ mm}\cdot\text{kg}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A.

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -108.906 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 1.866 \text{ mm}\cdot\text{kg}$$

$$W_A := \frac{mR_A \cdot g}{R_A} \quad W_A = 0.24 \text{ N}$$

 **PROBLEM 12-26**

Statement: The device in Figure P12-10 is used to balance fan blade/hub assemblies. The center distance between the two bearings on the machine is 250 mm. The left edge of the fan hub (plane *A*) is 100 mm from the centerline of the closest bearing (at F_2). The hub is 125 mm wide and has a diameter of 250 mm along the surfaces where balancing weights are fastened. The peak magnitude of force F_1 is 1.1 N at a phase angle of 120 deg with respect to the rotating x' axis. Force F_2 had a peak of 1.8 N at a phase angle of -93 deg. Calculate the magnitudes and locations, with respect to the x' axis, of balance weights placed in planes *A* and *B* of the hub to dynamically balance the fan assembly.

Units: $rpm := 2\pi \cdot rad \cdot min^{-1}$

Given: Bearing forces and plane locations with respect to correction plane *A*:

$$\text{Left: } F_1 := 1.1 \cdot N \quad \theta_1 := 120 \cdot \text{deg} \quad l_1 := 350 \cdot \text{mm}$$

$$\text{Right: } F_2 := 1.8 \cdot N \quad \theta_2 := -93 \cdot \text{deg} \quad l_2 := 100 \cdot \text{mm}$$

$$\text{Distance between correction planes: } l_B := 125 \cdot \text{mm}$$

$$\text{Correction weight radii: } R_A := 125 \cdot \text{mm} \quad R_B := 125 \cdot \text{mm}$$

$$\text{Fan rotational speed: } \omega := 600 \cdot rpm$$

Solution: See Figure P12-10 and Mathcad file P1226.

1. Resolve the force vectors into xy components with respect to the zero reference angle of the tire.

$$F_{Ix} := F_1 \cdot \cos(\theta_1) \quad F_{Ix} = -0.550 N \quad F_{Iy} := F_1 \cdot \sin(\theta_1) \quad F_{Iy} = 0.953 N$$

$$F_{2x} := F_2 \cdot \cos(\theta_2) \quad F_{2x} = -0.094 N \quad F_{2y} := F_2 \cdot \sin(\theta_2) \quad F_{2y} = -1.798 N$$

2. Solve equations 12.4e for summation of moments about *O*, modified for the bearing forces.

$$mR_{Bx} := \frac{F_{Ix} \cdot l_1 + F_{2x} \cdot l_2}{l_B \cdot \omega} \quad mR_{Bx} = -0.409 \text{ mm} \cdot \text{kg}$$

$$mR_{By} := \frac{F_{Iy} \cdot l_1 + F_{2y} \cdot l_2}{l_B \cdot \omega} \quad mR_{By} = 0.311 \text{ mm} \cdot \text{kg}$$

3. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane *B*. Also, solve for the weight required at the given radius.

$$\theta_B := \text{atan2}(mR_{Bx}, mR_{By}) \quad \theta_B = 142.728 \text{ deg}$$

$$mR_B := \sqrt{mR_{Bx}^2 + mR_{By}^2} \quad mR_B = 0.514 \text{ mm} \cdot \text{kg}$$

$$W_B := \frac{mR_B}{R_B} \cdot g \quad W_B = 0.040 N$$

4. Solve equations 12.4c for forces in x and y directions in plane *A*.

$$mR_{Ax} := \frac{F_{Ix} + F_{2x}}{2} - mR_{Bx} \quad mR_{Ax} = 0.246 \text{ mm} \cdot \text{kg}$$

$$mR_{Ay} := \frac{F_{Iy} + F_{2y}}{2} - mR_{By} \quad mR_{Ay} = -0.525 \text{ mm}\cdot\text{kg}$$

5. Solve equations 12.2d and 12.2e for the position angle and mass-radius product required in plane A .

$$\theta_A := \text{atan2}(mR_{Ax}, mR_{Ay}) \quad \theta_A = -64.911 \text{ deg}$$

$$mR_A := \sqrt{mR_{Ax}^2 + mR_{Ay}^2} \quad mR_A = 0.580 \text{ mm}\cdot\text{kg}$$

$$W_A := \frac{mR_A \cdot g}{R_A} \quad W_A = 0.046 \text{ N}$$



PROBLEM 12-27

Statement: Figure P12-11a shows a fourbar linkage and its dimensions in mm. All links are 4-mm-thick steel. Link 3 has a uniform cross section and is 20 mm wide with ends that extend 10 mm beyond the pivot hole. Links 2 and 4 have a 10-mm radius at each end. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Given:

Link lengths:

$$\text{Link 2 (O}_2 \text{ to } A\text{)} \quad L_2 := 116 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad L_3 := 108 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4\text{)} \quad L_4 := 110 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4\text{)} \quad L_1 := 174 \cdot \text{mm}$$

$$\text{Link cross section: } h := 20 \cdot \text{mm} \quad t := 4 \cdot \text{mm} \quad r := 10 \cdot \text{mm}$$

$$\text{Mass center: } b_2 := 0.5 \cdot L_2 \quad b_3 := 0.5 \cdot L_3 \quad b_4 := 0.5 \cdot L_4$$

$$\phi_2 := 0 \cdot \text{deg} \quad \phi_3 := 0 \cdot \text{deg} \quad \phi_4 := 0 \cdot \text{deg}$$

$$\text{Mass density of steel: } \rho := 7.8 \cdot 10^3 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Figure P12-11 and Mathcad file P1227.

1. Calculate the mass of each link.

$$m_2 := (L_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_2 = 0.082 \text{ kg}$$

$$m_3 := (L_3 + 2 \cdot r) \cdot h \cdot t \cdot \rho \quad m_3 = 0.080 \text{ kg}$$

$$m_4 := (L_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_4 = 0.078 \text{ kg}$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -4.633 \text{ kg} \cdot \text{mm}$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000 \text{ kg} \cdot \text{mm}$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -4.393 \text{ kg} \cdot \text{mm}$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000 \text{ kg} \cdot \text{mm}$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -9.399 \text{ kg} \cdot \text{mm}$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000 \text{ kg} \cdot \text{mm}$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -8.707 \text{ kg} \cdot \text{mm}$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.000 \text{ kg} \cdot \text{mm}$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 9.3994 \text{ kg} \cdot \text{mm}$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 8.707 \text{ kg} \cdot \text{mm}$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \text{ deg}$$



PROBLEM 12-28

Statement: Use the data of Problem 12-27 to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to } A\text{)} \quad a_2 := 116 \cdot \text{mm} \quad \text{Link 3 (A to B)} \quad a_3 := 108 \cdot \text{mm}$$

$$\text{Link 4 (B to O}_4\text{)} \quad a_4 := 110 \cdot \text{mm} \quad \text{Link 1 (O}_2 \text{ to O}_4\text{)} \quad a_1 := 174 \cdot \text{mm}$$

$$\text{Link cross-section dims: } h := 20 \cdot \text{mm} \quad t := 4 \cdot \text{mm}$$

$$\text{Material density: steel} \quad \rho := 7800 \cdot \text{kg} \cdot \text{m}^{-3}$$

Solution: See Figure P12-11a and Mathcad file P1228.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \sqrt{3 \left(\frac{a_3}{h} \right)^2 - 1} - \frac{a_3}{2} \quad e = 38.995 \text{ mm}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 185.989 \text{ mm}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 58.000 \text{ mm}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 54.000 \text{ mm}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 55.000 \text{ mm}$$

3. Determine the mass of each link.

$$m_2 := h \cdot t \cdot a_2 \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := h \cdot t \cdot a_4 \cdot \rho$$

$$m_2 = 0.072 \text{ kg} \quad m_3 = 0.116 \text{ kg} \quad m_4 = 0.069 \text{ kg}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 54.000 \text{ mm}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 6.731 \text{ kg} \cdot \text{mm}$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 6.383 \text{ kg} \cdot \text{mm}$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 2.533 \text{ kg} \cdot \text{mm}$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 2.608 \text{ kg} \cdot \text{mm}$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 75 \text{ mm}$$

$$R_4 := 76 \text{ mm}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 0.034 \text{ kg}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 0.034 \text{ kg}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 0.106 \text{ kg} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 63.4 \text{ mm}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 0.103 \text{ kg} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 62.0 \text{ mm}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 1.155 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 104.310 \text{ mm}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 1.018 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 99.429 \text{ mm}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 2.363 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 2.116 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$



PROBLEM 12-29

Statement: Figure P12-11b shows a fourbar linkage and its dimensions in inches. All links have a uniform 0.5-in wide x 0.2-in thick cross-section and are made from steel. Link 3 has squared ends that extend 0.25 in from the pivot point centers. Links 2 and 4 have rounded ends that have a radius of 0.25 in. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad L_2 := 2.75 \cdot in$$

$$\text{Link 3 (A to B)} \quad L_3 := 3.26 \cdot in$$

$$\text{Link 4 (B to O}_4 \text{)} \quad L_4 := 2.75 \cdot in$$

$$\text{Link 1 (O}_2 \text{ to O}_4 \text{)} \quad L_1 := 4.46 \cdot in$$

$$\text{Link cross section: } h := 0.50 \cdot in$$

$$t := 0.20 \cdot in$$

$$r := 0.25 \cdot in$$

$$\text{Mass center: } b_2 := 0.5 \cdot L_2$$

$$b_3 := 0.5 \cdot L_3$$

$$b_4 := 0.5 \cdot L_4$$

$$\phi_2 := 0 \cdot deg$$

$$\phi_3 := 0 \cdot deg$$

$$\phi_4 := 0 \cdot deg$$

$$\text{Mass density of steel: } \rho := 7.8 \cdot 10^3 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-11 and Mathcad file P1229.

1. Calculate the mass of each link.

$$m_2 := \left(L_2 \cdot h + \pi \cdot r^2 \right) \cdot t \cdot \rho \quad m_3 := \left(L_3 + 2 \cdot r \right) \cdot h \cdot t \cdot \rho \quad m_4 := \left(L_4 \cdot h + \pi \cdot r^2 \right) \cdot t \cdot \rho$$

$$m_2 = 2.294 \times 10^{-4} \text{ blob} \quad m_3 = 2.744 \times 10^{-4} \text{ blob} \quad m_4 = 2.294 \times 10^{-4} \text{ blob}$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -3.773 \times 10^{-4} \text{ blob} \cdot in$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000 \text{ blob} \cdot in$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -3.773 \times 10^{-4} \text{ blob} \cdot in$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000 \text{ blob} \cdot in$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -6.927 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000 \text{ blob} \cdot in$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -6.927 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.000 \text{ blob} \cdot in$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 6.927 \times 10^{-4} \text{ blob} \cdot in$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 6.927 \times 10^{-4} \text{ blob} \cdot in$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \text{ deg}$$



PROBLEM 12-30

Statement: Use the data of Problem 12-29 to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to } A\text{)} \quad a_2 := 2.75 \cdot in \quad \text{Link 3 (A to B)} \quad a_3 := 3.26 \cdot in$$

$$\text{Link 4 (B to O}_4\text{)} \quad a_4 := 2.75 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4\text{)} \quad a_1 := 4.46 \cdot in$$

$$\text{Link cross-section dims: } h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Material density: steel } \rho := 7800 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-11b and Mathcad file P1230.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h} \right)^2 - 1} - \frac{a_3}{2} \quad e = 1.182 \text{ in}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 5.624 \text{ in}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 1.375 \text{ in}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 1.630 \text{ in}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 1.375 \text{ in}$$

3. Determine the mass of each link.

$$m_2 := (a_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := (a_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho$$

$$m_2 = 2.294 \times 10^{-4} \text{ blob} \quad m_3 = 4.105 \times 10^{-4} \text{ blob} \quad m_4 = 2.294 \times 10^{-4} \text{ blob}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 1.630 \text{ in}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 5.644 \times 10^{-4} \text{ blob} \cdot in$$

$$mr_4 := m_4 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 5.644 \times 10^{-4} \text{ blob} \cdot in$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 2.490 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 2.490 \times 10^{-4} \text{ blob} \cdot in$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 2 \cdot \text{in} \quad R_4 := 2 \cdot \text{in}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 1.245 \times 10^{-4} \text{ blob}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 1.245 \times 10^{-4} \text{ blob}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 3.539 \times 10^{-4} \text{ blob} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 1.595 \text{ in}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 3.539 \times 10^{-4} \text{ blob} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 1.6 \text{ in}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 2.193 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 2.489 \text{ in}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 2.193 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 2.489 \text{ in}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 4.645 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 4.645 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

 **PROBLEM 12-31**

Statement: Figure P12-12 shows a fourbar linkage and its dimensions in inches. All links have a uniform 0.5-in wide x 0.2-in thick cross-section and are made from aluminum. Link 3 has squared ends that extend 0.25 in from the pivot point centers. Links 2 and 4 have rounded ends that have a radius of 0.25 in. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to } A\text{)} \quad L_2 := 3.44 \cdot in \quad \text{Link 3 (A to } B\text{)} \quad L_3 := 7.40 \cdot in$$

$$\text{Link 4 (B to } O_4\text{)} \quad L_4 := 3.44 \cdot in \quad \text{Link 1 (O}_2 \text{ to } O_4\text{)} \quad L_1 := 8.88 \cdot in$$

$$\text{Link cross section: } h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Mass center: } b_2 := 0.5 \cdot L_2 \quad b_3 := 0.5 \cdot L_3 \quad b_4 := 0.5 \cdot L_4$$

$$\phi_2 := 0 \cdot deg \quad \phi_3 := 0 \cdot deg \quad \phi_4 := 0 \cdot deg$$

$$\text{Mass density of aluminum: } \rho := 2.8 \cdot 10^3 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-12 and Mathcad file P1231.

1. Calculate the mass of each link.

$$m_2 := \left(L_2 \cdot h + \pi \cdot r^2 \right) \cdot t \cdot \rho \quad m_3 := \left(L_3 + 2 \cdot r \right) \cdot h \cdot t \cdot \rho \quad m_4 := \left(L_4 \cdot h + \pi \cdot r^2 \right) \cdot t \cdot \rho$$

$$m_2 = 1.004 \times 10^{-4} \text{ blob} \quad m_3 = 2.070 \times 10^{-4} \text{ blob} \quad m_4 = 1.004 \times 10^{-4} \text{ blob}$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -3.560 \times 10^{-4} \text{ blob} \cdot in$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000 \text{ blob} \cdot in$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -3.560 \times 10^{-4} \text{ blob} \cdot in$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000 \text{ blob} \cdot in$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -5.287 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000 \text{ blob} \cdot in$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -5.287 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.000 \text{ blob} \cdot in$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 5.287 \times 10^{-4} \text{ blob} \cdot in$$

$$\theta_{b2} := atan2(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 5.287 \times 10^{-4} \text{ blob} \cdot in$$

$$\theta_{b4} := atan2(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \text{ deg}$$

 **PROBLEM 12-32**

Statement: Use the data of Problem 12-31 to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to A)} \quad a_2 := 3.44 \cdot in \quad \text{Link 3 (A to B)} \quad a_3 := 7.40 \cdot in$$

$$\text{Link 4 (B to O}_4 \text{)} \quad a_4 := 3.44 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4 \text{)} \quad a_1 := 8.88 \cdot in$$

$$\text{Link cross-section dims: } h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Material density: alum } \rho := 2800 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-12 and Mathcad file P1232.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h} \right)^2 - 1} - \frac{a_3}{2} \quad e = 2.704 \text{ in}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 12.807 \text{ in}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 1.720 \text{ in}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 3.700 \text{ in}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 1.720 \text{ in}$$

3. Determine the mass of each link.

$$m_2 := (a_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := (a_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho$$

$$m_2 = 1.004 \times 10^{-4} \text{ blob} \quad m_3 = 3.356 \times 10^{-4} \text{ blob} \quad m_4 = 1.004 \times 10^{-4} \text{ blob}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 3.700 \text{ in}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 5.772 \times 10^{-4} \text{ blob} \cdot in$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 5.772 \times 10^{-4} \text{ blob} \cdot in$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 4.044 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 4.044 \times 10^{-4} \text{ blob} \cdot in$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 2 \cdot \text{in} \quad R_4 := 2 \cdot \text{in}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 2.022 \times 10^{-4} \text{ blob}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 2.022 \times 10^{-4} \text{ blob}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 3.026 \times 10^{-4} \text{ blob} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 1.907 \text{ in}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 3.026 \times 10^{-4} \text{ blob} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 1.907 \text{ in}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 1.424 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 2.169 \text{ in}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 1.424 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 2.169 \text{ in}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2^2 + r_2^2 + a_2 \cdot r_2) \quad I_{cw2} = 4.510 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$I_{cw4} := m'_4(k'_4^2 + r_4^2 + a_4 \cdot r_4) \quad I_{cw4} = 4.510 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

 PROBLEM 12-33

Statement: Figure P12-13 shows a fourbar linkage and its dimensions in inches. Links 2 and 4 are rectangular steel with a 1-in wide x 0.12-in thick cross-section and 0.5-in radius ends. The coupler is 0.25-in-thick aluminum with 0.5-in radii at points A, B, and P. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to } A\text{)} \quad L_2 := 5.00 \cdot in \quad \text{Link 3 (A to B)} \quad L_3 := 4.40 \cdot in$$

$$\text{Link 4 (B to O}_4\text{)} \quad L_4 := 5.00 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4\text{)} \quad L_1 := 9.50 \cdot in$$

$$\text{Link cross section: } h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Mass center: } b_2 := 0.5 \cdot L_2 \quad b_4 := 0.5 \cdot L_4 \quad \text{Coupler point data:}$$

$$\phi_2 := 0 \cdot deg \quad \phi_4 := 0 \cdot deg \quad p := 8.9 \cdot in \quad \delta := 56 \cdot deg$$

$$\text{Mass density of steel and aluminum: } \rho_1 := 7.8 \cdot 10^3 \cdot kg \cdot m^{-3} \quad \rho_2 := 2.8 \cdot 10^3 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-13 and Mathcad file P1233.

1. Determine the location of the mass center and the mass of link 3. Take AB as the base of the triangle, Then,

$$\text{The height is: } h := p \cdot \sin(\delta) \quad h = 7.378 \text{ in} \quad t_3 := 0.25 \cdot in$$

$$\text{The horizontal distance from } A \text{ to } P \text{ is: } AP_x := p \cdot \cos(\delta) \quad AP_x = 4.977 \text{ in}$$

$$xbar := \frac{L_3 + AP_x}{3} \quad xbar = 3.126 \text{ in} \quad ybar := \frac{h}{3} \quad ybar = 2.459 \text{ in}$$

$$b_3 := \sqrt{(xbar^2 + ybar^2)} \quad b_3 = 3.977 \text{ in} \quad \phi_3 := \text{atan}\left(\frac{ybar}{xbar}\right) \quad \phi_3 = 38.199 \text{ deg}$$

$$m_3 := \rho_2 \cdot t_3 \cdot \frac{L_3 \cdot h}{2} \quad m_3 = 1.063 \times 10^{-3} \text{ blob}$$

2. Calculate the mass of links 2 and 4.

$$m_2 := (L_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho_1 \quad m_4 := (L_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho_1$$

$$m_2 = 5.414 \times 10^{-3} \text{ blob} \quad m_4 = 5.414 \times 10^{-3} \text{ blob}$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -1.540 \times 10^{-3} \text{ blob} \cdot in$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 2.972 \times 10^{-3} \text{ blob} \cdot in$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -3.776 \times 10^{-3} \text{ blob} \cdot in$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = -2.972 \times 10^{-3} \text{ blob} \cdot in$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -0.015 \text{ blob} \cdot in$$

$$\begin{aligned}
 mR_{2y} &:= mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) & mR_{2y} &= 2.972 \times 10^{-3} \text{ blob.in} \\
 mR_{4x} &:= mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) & mR_{4x} &= -0.017 \text{ blob.in} \\
 mR_{4y} &:= mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) & mR_{4y} &= -2.972 \times 10^{-3} \text{ blob.in}
 \end{aligned}$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$\begin{aligned}
 mR_{b2} &:= \sqrt{mR_{2x}^2 + mR_{2y}^2} & mR_{b2} &= 0.0154 \text{ blob.in} \\
 \theta_{b2} &:= \text{atan2}(mR_{2x}, mR_{2y}) & \theta_{b2} &= 168.848 \text{ deg} \\
 mR_{b4} &:= \sqrt{mR_{4x}^2 + mR_{4y}^2} & mR_{b4} &= 0.0176 \text{ blob.in} \\
 \theta_{b4} &:= \text{atan2}(mR_{4x}, mR_{4y}) & \theta_{b4} &= -170.260 \text{ deg}
 \end{aligned}$$



PROBLEM 12-34

Statement: Use the data of Problem 12-33, changing link 3 to be steel with the same cross-section dimensions as links 2 and 4, to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to } A\text{)} \quad a_2 := 5.00 \cdot in \quad \text{Link 3 (A to B)} \quad a_3 := 4.40 \cdot in$$

$$\text{Link 4 (B to O}_4\text{)} \quad a_4 := 5.00 \cdot in \quad \text{Link 1 (O}_2 \text{ to O}_4\text{)} \quad a_1 := 9.50 \cdot in$$

$$\text{Link cross-section dims: } h := 0.50 \cdot in \quad t := 0.20 \cdot in \quad r := 0.25 \cdot in$$

$$\text{Material density: steel } \rho := 7800 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-13 and Mathcad file P1234.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h} \right)^2 - 1} - \frac{a_3}{2} \quad e = 1.602 \text{ in}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 7.605 \text{ in}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 2.500 \text{ in}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 2.200 \text{ in}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 2.500 \text{ in}$$

3. Determine the mass of each link.

$$m_2 := (a_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := (a_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho$$

$$m_2 = 3.936 \times 10^{-4} \text{ blob} \quad m_3 = 5.550 \times 10^{-4} \text{ blob} \quad m_4 = 3.936 \times 10^{-4} \text{ blob}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 2.200 \text{ in}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 1.388 \times 10^{-3} \text{ blob} \cdot in$$

$$mr_4 := m_3 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 1.388 \times 10^{-3} \text{ blob} \cdot in$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 4.036 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 4.036 \times 10^{-4} \text{ blob} \cdot in$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 2 \cdot \text{in} \quad R_4 := 2 \cdot \text{in}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 2.018 \times 10^{-4} \text{ blob}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 2.018 \times 10^{-4} \text{ blob}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 5.954 \times 10^{-4} \text{ blob} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 2.331 \text{ in}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 5.954 \times 10^{-4} \text{ blob} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 2.331 \text{ in}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2 \cdot (a_2^2 + h^2)}{12} + m_2 \cdot (r_2 + R_{CG2})^2 + m_{2b} \cdot (R_2 - r_2)^2 \quad I'_2 = 0.0100 \text{ blob} \cdot \text{in}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 4.105 \text{ in}$$

$$I'_4 := \frac{m_4 \cdot (a_4^2 + h^2)}{12} + m_4 \cdot (r_4 + R_{CG4})^2 + m_{4b} \cdot (R_4 - r_4)^2 \quad I'_4 = 0.0100 \text{ blob} \cdot \text{in}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 4.105 \text{ in}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2 \left(k'_2^2 + r_2^2 + a_2 \cdot r_2 \right) \quad I_{cw2} = 0.0202 \text{ blob} \cdot \text{in}^2$$

$$I_{cw4} := m'_4 \left(k'_4^2 + r_4^2 + a_4 \cdot r_4 \right) \quad I_{cw4} = 0.0202 \text{ blob} \cdot \text{in}^2$$



PROBLEM 12-35

Statement: Figure P12-14 shows a fourbar linkage and its dimensions in inches. All links are 0.08-in-thick steel and have a uniform cross-section of 0.26-in wide x 0.12-in thick. Links 2 and 4 have rounded ends with a 0.13-in radius. Link 3 has a squared ends that extend 0.13-in from the pivot point centers. Design counterweights to force balance the linkage using the method of Berkof and Lowen.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (O}_2 \text{ to } A\text{)} \quad L_2 := 5.52 \cdot in$$

$$\text{Link 3 (A to B)} \quad L_3 := 4.88 \cdot in$$

$$\text{Link 4 (B to O}_4\text{)} \quad L_4 := 6.48 \cdot in$$

$$\text{Link 1 (O}_2 \text{ to O}_4\text{)} \quad L_1 := 2.72 \cdot in$$

$$\text{Link cross section: } h := 0.26 \cdot in$$

$$t := 0.08 \cdot in$$

$$r := 0.13 \cdot in$$

$$\text{Mass center: } b_2 := 0.5 \cdot L_2$$

$$b_3 := 0.5 \cdot L_3$$

$$b_4 := 0.5 \cdot L_4$$

$$\phi_2 := 0 \cdot deg$$

$$\phi_3 := 0 \cdot deg$$

$$\phi_4 := 0 \cdot deg$$

$$\text{Mass density of steel: } \rho := 7.8 \cdot 10^3 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-14 and Mathcad file P1235.

1. Calculate the mass of each link.

$$m_2 := (L_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_3 := (L_3 + 2 \cdot r) \cdot h \cdot t \cdot \rho \quad m_4 := (L_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho$$

$$m_2 = 8.690 \times 10^{-5} \text{ blob} \quad m_3 = 7.803 \times 10^{-5} \text{ blob} \quad m_4 = 1.015 \times 10^{-4} \text{ blob}$$

2. Solve equations 12.8c and 12.8d for the total mR product components for links 2 and 4.

$$mb_{2x} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \cos(\phi_3) - L_2 \right) \quad mb_{2x} = -2.154 \times 10^{-4} \text{ blob} \cdot in$$

$$mb_{2y} := m_3 \cdot \left(b_3 \cdot \frac{L_2}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{2y} = 0.000 \text{ blob} \cdot in$$

$$mb_{4x} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \cos(\phi_3) \right) \quad mb_{4x} = -2.528 \times 10^{-4} \text{ blob} \cdot in$$

$$mb_{4y} := -m_3 \cdot \left(b_3 \cdot \frac{L_4}{L_3} \cdot \sin(\phi_3) \right) \quad mb_{4y} = 0.000 \text{ blob} \cdot in$$

3. Determine the additional mR product components for links 2 and 4.

$$mR_{2x} := mb_{2x} - m_2 \cdot b_2 \cdot \cos(\phi_2) \quad mR_{2x} = -4.552 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_{2y} := mb_{2y} - m_2 \cdot b_2 \cdot \sin(\phi_2) \quad mR_{2y} = 0.000 \text{ blob} \cdot in$$

$$mR_{4x} := mb_{4x} - m_4 \cdot b_4 \cdot \cos(\phi_4) \quad mR_{4x} = -5.816 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_{4y} := mb_{4y} - m_4 \cdot b_4 \cdot \sin(\phi_4) \quad mR_{4y} = 0.000 \text{ blob} \cdot in$$

4. Solve equations 12.2d and 12.2e for the position angle and additional mass-radius product required.

$$mR_{b2} := \sqrt{mR_{2x}^2 + mR_{2y}^2} \quad mR_{b2} = 4.552 \times 10^{-4} \text{ blob} \cdot in$$

$$\theta_{b2} := \text{atan2}(mR_{2x}, mR_{2y}) \quad \theta_{b2} = 180.000 \text{ deg}$$

$$mR_{b4} := \sqrt{mR_{4x}^2 + mR_{4y}^2} \quad mR_{b4} = 5.816 \times 10^{-4} \text{ blob} \cdot in$$

$$\theta_{b4} := \text{atan2}(mR_{4x}, mR_{4y}) \quad \theta_{b4} = 180.000 \text{ deg}$$

 **PROBLEM 12-36**

Statement: Use the data of Problem 12-35 to design the necessary balance weights and other features necessary to completely eliminate the shaking force and shaking moment the linkage exerts on the ground link.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Link lengths:

$$\text{Link 2 (}O_2 \text{ to } A\text{)} \quad a_2 := 5.52 \cdot in \quad \text{Link 3 (}A \text{ to } B\text{)} \quad a_3 := 4.88 \cdot in$$

$$\text{Link 4 (}B \text{ to } O_4\text{)} \quad a_4 := 6.48 \cdot in \quad \text{Link 1 (}O_2 \text{ to } O_4\text{)} \quad a_1 := 2.72 \cdot in$$

$$\text{Link cross-section dims: } h := 0.26 \cdot in \quad t := 0.12 \cdot in \quad r := 0.13 \cdot in$$

$$\text{Material density: steel } \rho := 7800 \cdot kg \cdot m^{-3}$$

Solution: See Figure P12-14 and Mathcad file P1236.

1. Determine the amount by which link 3 must be elongated to make it physically like a pendulum by using equation 12.12a.

$$e := \frac{h}{2} \cdot \sqrt{3 \cdot \left(\frac{a_3}{h} \right)^2 - 1} - \frac{a_3}{2} \quad e = 1.784 \text{ in}$$

$$l_3 := a_3 + 2 \cdot e \quad l_3 = 8.448 \text{ in}$$

2. Determine the distance to the CG in the LRCS on each of the three moving links.

$$R_{CG2} := 0.5 \cdot a_2 \quad R_{CG2} = 2.760 \text{ in}$$

$$r_3 := 0.5 \cdot a_3 \quad r_3 = 2.440 \text{ in}$$

$$R_{CG4} := 0.5 \cdot a_4 \quad R_{CG4} = 3.240 \text{ in}$$

3. Determine the mass of each link.

$$m_2 := (a_2 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho \quad m_3 := h \cdot t \cdot l_3 \cdot \rho \quad m_4 := (a_4 \cdot h + \pi \cdot r^2) \cdot t \cdot \rho$$

$$m_2 = 1.304 \times 10^{-4} \text{ blob} \quad m_3 = 1.924 \times 10^{-4} \text{ blob} \quad m_4 = 1.522 \times 10^{-4} \text{ blob}$$

4. Solve equations 12.17a and 12.17b for the total mR product components for links 2 and 4.

$$b_3 := a_3 - r_3 \quad b_3 = 2.440 \text{ in}$$

$$mr_2 := m_3 \cdot \left(b_3 \cdot \frac{a_2}{a_3} \right) \quad mr_2 = 5.310 \times 10^{-4} \text{ blob} \cdot in$$

$$mr_4 := m_4 \cdot \left(r_3 \cdot \frac{a_4}{a_3} \right) \quad mr_4 = 6.233 \times 10^{-4} \text{ blob} \cdot in$$

5. Determine the additional mR product components for links 2 and 4 to accomplish the force balance.

$$mR_2 := mr_2 - m_2 \cdot R_{CG2} \quad mR_2 = 1.712 \times 10^{-4} \text{ blob} \cdot in$$

$$mR_4 := mr_4 - m_4 \cdot R_{CG4} \quad mR_4 = 1.302 \times 10^{-4} \text{ blob} \cdot in$$

6. Let the distance to the force-balance mass from the pivot point on links 2 and 4 be

$$R_2 := 2 \cdot \text{in} \quad R_4 := 1.52 \cdot \text{in}$$

then the masses of the force balance weights will be (R_4 was chosen to make the weights the same)

$$m_{2b} := \frac{mR_2}{R_2} \quad m_{2b} = 8.561 \times 10^{-5} \text{ blob}$$

$$m_{4b} := \frac{mR_4}{R_4} \quad m_{4b} = 8.563 \times 10^{-5} \text{ blob}$$

7. Calculate the new, total mass of links 2 and 4 and the distance from the pivot point to the composite CG.

$$m'_2 := m_2 + m_{2b} \quad m'_2 = 2.160 \times 10^{-4} \text{ blob} \quad r_2 := \frac{mr_2}{m'_2} \quad r_2 = 2.459 \text{ in}$$

$$m'_4 := m_4 + m_{4b} \quad m'_4 = 2.378 \times 10^{-4} \text{ blob} \quad r_4 := \frac{mr_4}{m'_4} \quad r_4 = 2.621 \text{ in}$$

8. Calculate the new radius of gyration of links 2 and 4.

$$I'_2 := \frac{m_2(a_2^2 + h^2)}{12} + m_2(r_2 + R_{CG2})^2 + m_{2b}(R_2 - r_2)^2 \quad I'_2 = 3.900 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$k'_2 := \sqrt{\frac{I'_2}{m'_2}} \quad k'_2 = 4.250 \text{ in}$$

$$I'_4 := \frac{m_4(a_4^2 + h^2)}{12} + m_4(r_4 + R_{CG4})^2 + m_{4b}(R_4 - r_4)^2 \quad I'_4 = 5.865 \times 10^{-3} \text{ blob} \cdot \text{in}^2$$

$$k'_4 := \sqrt{\frac{I'_4}{m'_4}} \quad k'_4 = 4.966 \text{ in}$$

9. Solve equations 12.17d and 12.17e to determine the mass moment of inertia required for the two inertia counterweights that are geared to links 2 and 4..

$$I_{cw2} := m'_2(k'_2)^2 + r_2^2 + a_2 \cdot r_2 \quad I_{cw2} = 0.00814 \text{ blob} \cdot \text{in}^2$$

$$I_{cw4} := m'_4(k'_4)^2 + r_4^2 + a_4 \cdot r_4 \quad I_{cw4} = 0.0115 \text{ blob} \cdot \text{in}^2$$

PROBLEM 12-1

| Row | $m_b r_b$ | θ_b |
|-----|-----------|------------|
| a | 0.934 | -75.5 |
| b | 40.21 | 148.55 |
| c | 5.932 | 152.3 |
| d | 75.47 | 12.0 |
| e | 7.448 | -80.76 |

PROBLEM 12-5

| Row | $m_a r_a$ | θ_a | $m_b r_b$ | θ_b |
|-----|-----------|------------|-----------|------------|
| a | 0.814 | -175.2 | 5.50 | 152.1 |
| b | 25.13 | 140.23 | 19.32 | 83.89 |
| c | 7.48 | -154.4 | 7.993 | 176.3 |
| d | 31.91 | 12.83 | 37.52 | 74.82 |
| e | 6.254 | -84.5 | 3.671 | -73.9 |

PROBLEM 12-8

| Row | $m_{b2} R_{b2}$ | θ_{b2} | $m_{b4} R_{b4}$ | θ_{b4} |
|-----|-----------------|---------------|-----------------|---------------|
| a | 0.051 | 180.0 | 0.459 | -154.2 |
| b | 0.255 | -162.4 | 1.437 | -158.3 |
| c | 0.107 | 142.3 | 0.476 | -164.3 |
| d | 0.772 | 141.8 | 1.573 | -135.5 |
| e | 0.078 | 158.4 | 0.166 | 175.1 |
| f | 7.338 | 172.9 | 3.152 | -93.2 |
| g | 2.029 | -175.2 | 0.320 | 45.0 |

