Chapter 13

ENGINE DYNAMICS

TOPIC/PROBLEM MATRIX

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PROBLEM 13-1

Statement: A slider-crank linkage has the dimensions and speed at t = 0 given below. Its initial crank angle is zero. Calculate the piston acceleration at the time specified below. Use two methods, the exact solution, and the approximate Fourier series solution and compare the results.

Given: Link lengths: $r := 3 \cdot in$ $\quad l := 12 \cdot in$

Initial angular velocity: $\omega := 200 \cdot rad \cdot sec^{-1}$ Time span: $t := 1.0 \cdot sec$

Solution: See Mathcad file P1301.

1. Calculate the exact acceleration using equation 13.1f.

$$a_{exact} := -r \cdot \omega^2 \cdot \left[\cos(\omega \cdot t) - \frac{r \cdot \left[l^2 \cdot \left(1 - 2 \cdot \cos(\omega \cdot t)^2\right) - r^2 \cdot \sin(\omega \cdot t)^4 \right]}{\left(l^2 - r^2 \cdot \sin(\omega \cdot t)^2\right)^{\frac{3}{2}}} \right]$$

$$a_{exact} = -42679.3 \frac{in}{sec^2}$$

2. Calculate the approximate acceleration using equation 13.3e.

$$a_{approx} := -r \cdot \omega^2 \cdot \left(\cos(\omega \cdot t) + \frac{r}{l} \cdot \cos(2 \cdot \omega \cdot t) \right) \qquad a_{approx} = -42703.6 \frac{in}{sec^2}$$

3. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error := \frac{a_{approx} - a_{exact}}{a_{exact}} \qquad error = 0.057\,\%$$

The approximate solution is a little less than 0.06% high.

PROBLEM 13-2

Statement: A slider-crank linkage has the dimensions and speed at t = 0 given below. Its initial crank angle is zero. Calculate the piston acceleration at the time specified below. Use two methods, the exact solution, and the approximate Fourier series solution and compare the results.

Given: Link lengths: $r := 4 \cdot in$ $\quad l := 15 \cdot in$

Initial angular velocity: $\omega := 200 \cdot rad \cdot sec^{-1}$ $\quad$ Time span: $t := 0.9 \cdot sec$

Solution: See Mathcad file P1302.

1. Calculate the exact acceleration using equation 13.1f.

$$a_{exact} := -r \cdot \omega^2 \cdot \left[cos(\omega \cdot t) - \frac{r \cdot \left[l^2 \cdot \left(1 - 2 \cdot cos(\omega \cdot t)^2\right) - r^2 \cdot sin(\omega \cdot t)^4 \right]}{\left(l^2 - r^2 \cdot sin(\omega \cdot t)^2\right)^{\frac{3}{2}}} \right]$$

$$a_{exact} = 107395.7 \frac{in}{sec^2}$$

2. Calculate the approximate acceleration using equation 13.3e.

$$a_{approx} := -r \cdot \omega^2 \cdot \left(cos(\omega \cdot t) + \frac{r}{l} \cdot cos(2 \cdot \omega \cdot t) \right) \qquad a_{approx} = 107857.8 \frac{in}{sec^2}$$

3. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error := \frac{a_{approx} - a_{exact}}{a_{exact}} \qquad error = 0.430\,\%$$

The approximate solution is a little less than 0.5% high.

PROBLEM 13-3

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the gas force and gas torque at this position.

Given: Link lengths: $r := 3 \cdot in$ $\quad l := 12 \cdot in$

Piston bore: $B := 2 \cdot in$ $\quad$ Peak pressure: $p_g := 1000 \cdot psi$ $\quad$ Crank angle: $\theta := 10 \cdot deg$

Assumptions: An approximate solution is acceptable.

Solution: See Mathcad file P1303.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \qquad F_g = 3142\ lbf$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta)\right) \qquad T_{g21} = 2040\ in \cdot lbf$$

PROBLEM 13-4

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the gas force and gas torque at this position.

Given: Link lengths: $r := 4 \cdot in$ $\quad l := 15 \cdot in$

Piston bore: $B := 3 \cdot in$ $\quad$ Peak pressure: $p_g := 600 \cdot psi$ $\quad$ Crank angle: $\theta := 5 \cdot deg$

Assumptions: An approximate solution is acceptable.

Solution: See Mathcad file P1304.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \qquad F_g = 4241\ lbf$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21} := F_g \cdot r \cdot sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot cos(\theta)\right) \qquad T_{g21} = 1871\ in \cdot lbf$$

PROBLEM 13-5

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the exact gas torque at this position and compare it to that obtained by the approximate expression in equation 13.8b. What is the percent error?

Given: Link lengths: $r := 3 \cdot in$ $\quad l := 12 \cdot in$

Piston bore: $B := 2 \cdot in$ $\quad$ Peak pressure: $p_g := 1000 \cdot psi$ $\quad$ Crank angle: $\theta := 10 \cdot deg$

Solution: See Mathcad file P1305.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \qquad F_g = 3142\, lbf$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21a} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta)\right) \qquad T_{g21a} = 2039.53\, in \cdot lbf$$

3. Calculate the exact gas torque on the crank using equations 13.7b, 13.1d, and 13.6d.

$$\phi := \operatorname{atan}\left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta)\right)^2}}\right] \qquad \phi = 2.488\, deg$$

$$x := r \cdot \cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta)\right)^2} \qquad x = 14.943\, in$$

$$T_{g21e} := F_g \cdot \tan(\phi) \cdot x \qquad T_{g21e} = 2039.91\, in \cdot lbf$$

4. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error := \frac{T_{g21a} - T_{g21e}}{T_{g21e}} \qquad error = -0.0186\,\%$$

PROBLEM 13-6

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the exact gas torque at this position and compare it to that obtained by the approximate expression in equation 13.8b. What is the percent error?

Given: Link lengths: $r := 4 \cdot in$ $\quad l := 15 \cdot in$

Piston bore: $B := 3 \cdot in$ $\quad$ Peak pressure: $p_g := 600 \cdot psi$ $\quad$ Crank angle: $\theta := 5 \cdot deg$

Solution: See Mathcad file P1306.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \qquad F_g = 4241\ lbf$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21a} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta)\right) \qquad T_{g21a} = 1871.35\ in \cdot lbf$$

3. Calculate the exact gas torque on the crank using equations 13.7b, 13.1d, and 13.6d.

$$\phi := \operatorname{atan}\left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta)\right)^2}}\right] \qquad \phi = 1.332\ deg$$

$$x := r \cdot \cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta)\right)^2} \qquad x = 18.981\ in$$

$$T_{g21e} := F_g \cdot \tan(\phi) \cdot x \qquad T_{g21e} = 1871.45\ in \cdot lbf$$

4. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error := \frac{T_{g21a} - T_{g21e}}{T_{g21e}} \qquad error = -0.00567\ \%$$

PROBLEM 13-7

Statement: The dimensions and mass properties of a connecting rod are given below.
a. Calculate an exact dynamic model using two lumped masses, one at the wrist pin, point *B*, and one at whatever point is required. Define the lumped masses and their locations.
b. Calculate an approximate dynamic model using two lumped masses, one at the wrist pin, point *B*, and one at the crank pin, point *A*. Define the lumped masses and their locations.
c. Calculate the error in the mass moment of inertia of the approximate model as a percentage of the original mass moment of inertia.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 blob$

Mass moment of inertia: $I_{G3} := 0.620 \cdot blob \cdot in^2$

Distance to CG (as fraction of *l*): $r_a := 0.4$

Solution: See Mathcad file P1307.

1. Determine the exact model using equations 13.9d and 13.9e.

Distance from point *A* to CG: $l_a := r_a \cdot l$ $\quad l_a = 4.800\, in$

Distance from point *B* to CG: $l_b := l - l_a$ $\quad l_b = 7.200\, in$

Distance from CG to lumped mass at P: $l_p := \dfrac{I_{G3}}{m_3 \cdot l_b}$ $\quad l_p = 4.306\, in$

Masses: $m_p := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_p = 0.0125\, blob$

$m_b := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_b = 0.00748\, blob$

2. Determine the approximate model using equations 13.9d, letting $l_p = l_a$.

Let $l_p := l_a$

Masses: $m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.0120\, blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.00800\, blob$

3. Calculate the mass moment of inertia of the approximate model and compare it to the original.

$I_{G3approx} := m_{3a} \cdot l_a^2 + m_{3b} \cdot l_b^2$ $\quad I_{G3approx} = 0.691\, blob \cdot in^2$

$error := \dfrac{I_{G3approx} - I_{G3}}{I_{G3}}$ $\quad error = 11.48\,\%$

PROBLEM 13-8

Statement: The dimensions and mass properties of a connecting rod are given below.

a. Calculate an exact dynamic model using two lumped masses, one at the wrist pin, point *B*, and one at whatever point is required. Define the lumped masses and their locations.

b. Calculate an approximate dynamic model using two lumped masses, one at the wrist pin, point B, and one at the crank pin, point *A*. Define the lumped masses and their locations.

c. Calculate the error in the mass moment of inertia of the approximate model as a percentage of the original mass moment of inertia.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 blob$

Mass moment of inertia: $I_{G3} := 1.020 \cdot blob \cdot in^2$

Distance to CG (as fraction of *l*): $r_a := 0.25$

Solution: See Mathcad file P1308.

1. Determine the exact model using equations 13.9d and 13.9e.

Distance from point *A* to CG: $l_a := r_a \cdot l$ $l_a = 3.750\, in$

Distance from point *B* to CG: $l_b := l - l_a$ $l_b = 11.250\, in$

Distance from CG to lumped mass at P: $l_p := \dfrac{I_{G3}}{m_3 \cdot l_b}$ $l_p = 3.627\, in$

Masses: $m_p := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $m_p = 0.0189\, blob$

$m_b := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $m_b = 0.00609\, blob$

2. Determine the approximate model using equations 13.9d, letting $l_p = l_a$.

Let $l_p := l_a$

Masses: $m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $m_{3a} = 0.0188\, blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $m_{3b} = 0.00625\, blob$

3. Calculate the mass moment of inertia of the approximate model and compare it to the original.

$I_{G3approx} := m_{3a} \cdot l_a^2 + m_{3b} \cdot l_b^2$ $I_{G3approx} = 1.055\, blob \cdot in^2$

$error := \dfrac{I_{G3approx} - I_{G3}}{I_{G3}}$ $error = 3.40\,\%$

PROBLEM 13-9

Statement: The dimensions and mass properties of a crank are given below. Calculate a statically equivalent, two-lumped mass dynamic model with the lumps placed at the main pin and crankpin. What is the percent error in the model's moment of inertia about the crank pivot?

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 blob$

Mass moment of inertia: $I_2 := 0.300 \cdot blob \cdot in^2$

Distance to CG (as fraction of r): $r_a := 0.3$

Solution: See Mathcad file P1309.

1. Determine the statically equivalent model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_a \cdot r$ $r_{G2} = 1.050\,in$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $m_{2a} = 0.0180\,blob$

2. Calculate the mass moment of inertia of the statically equivalent model and compare it to the original.

$I_{O2model} := m_{2a} \cdot r^2$ $I_{O2model} = 0.2205\,blob \cdot in^2$

$error := \frac{I_{O2model} - I_2}{I_2}$ $error = -26.50\,\%$

PROBLEM 13-10

Statement: The dimensions and mass properties of a crank are given below. Calculate a statically equivalent, two-lumped mass dynamic model with the lumps placed at the main pin and crankpin. What is the percent error in the model's moment of inertia about the crank pivot?

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 blob$

Mass moment of inertia: $I_2 := 0.400 \cdot blob \cdot in^2$

Distance to CG (as fraction of r): $r_a := 0.4$

Solution: See Mathcad file P13010.

1. Determine the statically equivalent model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_a \cdot r$ $r_{G2} = 1.600\ in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $m_{2a} = 0.0200\ blob$

2. Calculate the mass moment of inertia of the statically equivalent model and compare it to the original.

$I_{O2model} := m_{2a} \cdot r^2$ $I_{O2model} = 0.3200\ blob \cdot in^2$

$error := \dfrac{I_{O2model} - I_2}{I_2}$ $error = -20.00\,\%$

PROBLEM 13-11

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf\cdot sec^2\cdot in^{-1}$ $\quad rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given:

Conrod length: $l := 12\cdot in$ mass: $m_3 := 0.020\, blob$

Distance to CG (as fraction of l): $r_{3a} := 0.4$

Crank length: $r := 3.5\cdot in$ mass: $m_2 := 0.060\cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.3$

Piston mass: $m_4 := 0.012\cdot blob$

Crank speed: $\omega := 2000\cdot rpm$ Crank angle: $\theta := 45\cdot deg$

Solution: See Mathcad file P1311.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a}\cdot l$ $\qquad l_a = 4.800\, in$

Distance from point B to CG: $l_b := l - l_a$ $\qquad l_b = 7.200\, in$

Let $l_p := l_a$ $\qquad m_{3a} := m_3\cdot\dfrac{l_b}{l_p + l_b}$ $\qquad m_{3a} = 0.0120\, blob$

$m_{3b} := m_3\cdot\dfrac{l_p}{l_p + l_b}$ $\qquad m_{3b} = 0.00800\, blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a}\cdot r$ $\qquad r_{G2} = 1.050\, in$

Mass: $m_{2a} := m_2\cdot\dfrac{r_{G2}}{r}$ $\qquad m_{2a} = 0.0180\, blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\qquad m_A = 0.0300\, blob$

$m_B := m_{3b} + m_4$ $\qquad m_B = 0.0200\, blob$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A\cdot\left(-r\cdot\omega^2\cdot cos(\theta)\right) - m_B\cdot\left[-r\cdot\omega^2\cdot\left(cos(\theta) + \frac{r}{l}\cdot cos(2\cdot\theta)\right)\right] \qquad F_{ix} = 5428\, lbf$$

$$F_{iy} := -m_A\cdot\left(-r\cdot\omega^2\cdot sin(\theta)\right) \qquad F_{iy} = 3257\, lbf$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_i = 6330\, lbf \quad \text{at} \quad atan2\left(F_{ix}, F_{iy}\right) = 30.964\, deg$$

$$T_{i21} := -m_B\cdot r^2\cdot\omega^2\cdot sin(\theta)\cdot\left(\frac{r}{2\cdot l} + cos(\theta) + \frac{3\cdot r}{2\cdot l}\cdot cos(2\cdot\theta)\right) \qquad T_{i21} = -6482\, in\cdot lbf$$

PROBLEM 13-12

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.4$

Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.019 \cdot blob$

Crank speed: $\omega := 3000 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1312.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.800\, in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 7.200\, in$

Let $l_p := l_a$ $\quad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.0120\, blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.00800\, blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.600\, in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\quad m_{2a} = 0.0200\, blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.0320\, blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.0270\, blob$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \qquad F_{ix} = 21948\, lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = 6317\, lbf$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_i = 22839\, lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 16.055\, deg$$

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot cos(2 \cdot \theta)\right) \qquad T_{i21} = -27345\, in \cdot lbf$$

PROBLEM 13-13

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.25$

Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.3$

Piston mass: $m_4 := 0.023 \cdot blob$

Crank speed: $\omega := 2500 \cdot rpm$ Crank angle: $\theta := 24 \cdot deg$

Solution: See Mathcad file P1313.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 3.750\ in$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 11.250\ in$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $m_{3a} = 0.0188\ blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $m_{3b} = 0.00625\ blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 1.050\ in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $m_{2a} = 0.0180\ blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.0367\ blob$

$m_B := m_{3b} + m_4$ $m_B = 0.0292\ blob$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \qquad F_{ix} = 15559\ lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = 3586\ lbf$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_i = 15967\ lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 12.978\ deg$$

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot cos(2 \cdot \theta)\right) \qquad T_{i21} = -12630\ in \cdot lbf$$

PROBLEM 13-14

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.25$

Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.015 \cdot blob$

Crank speed: $\omega := 4000 \cdot rpm$ Crank angle: $\theta := 18 \cdot deg$

Solution: See Mathcad file P1314.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 3.750\ in$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 11.250\ in$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $m_{3a} = 0.0188\ blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $m_{3b} = 0.00625\ blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 1.600\ in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $m_{2a} = 0.0200\ blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.0388\ blob$

$m_B := m_{3b} + m_4$ $m_B = 0.0213\ blob$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \qquad F_{ix} = 43267\ lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = 8404\ lbf$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_i = 44075\ lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 10.992\ deg$$

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot cos(2 \cdot \theta)\right) \qquad T_{i21} = -25956\ in \cdot lbf$$

PROBLEM 13-15

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 \, blob$

Distance to CG (as fraction of l): $r_{3a} := 0.4$

Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.3$

Piston mass: $m_4 := 0.022 \cdot blob$ Gas force: $F_g := 300 \cdot lbf$

Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 45 \cdot deg$

Solution: See Mathcad file P1315.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.800 \, in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 7.200 \, in$

Let $l_p := l_a$

$$m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \qquad m_{3a} = 0.0120 \, blob$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \qquad m_{3b} = 0.00800 \, blob$$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.050 \, in$

Mass: $$m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \qquad m_{2a} = 0.0180 \, blob$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \qquad m_A = 0.0300 \, blob$$

$$m_B := m_{3b} + m_4 \qquad m_B = 0.0300 \, blob$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := atan\left[\frac{r \cdot sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}}\right] \qquad \phi = 11.902 \, deg$$

$$a_B := -r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right) \qquad a_B = -108560.1 \frac{in}{sec^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -\left[\left(m_4 + m_{3b}\right) \cdot a_B + F_g\right] \cdot tan(\phi) \qquad F_{41} = 623.2 \, lbf$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$F_{34x} := F_g + m_4 \cdot a_B$ $\qquad F_{34x} = -2088.3\ lbf$

$F_{34y} := -\left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi)$ $\qquad F_{34y} = 623.2\ lbf$

$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2}$ $\qquad F_{34} = 2179.3\ lbf$

$\theta_{34} := atan2\left(F_{34x}, F_{34y}\right)$ $\qquad \theta_{34} = 163.384\ deg$

7. Determine the crankpin force F_{32} using equation 13.22.

$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot cos(\theta) - \left(m_{3b} + m_4\right) \cdot a_B - F_g$ $\qquad F_{32x} = 4259.5\ lbf$

$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot sin(\theta) + \left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi)$ $\qquad F_{32y} = 679.5\ lbf$

$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2}$ $\qquad F_{32} = 4313.4\ lbf$

$\theta_{32} := atan2\left(F_{32x}, F_{32y}\right)$ $\qquad \theta_{32} = 9.064\ deg$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot cos(\theta) + F_{32x}$ $\qquad F_{21x} = 6213.6\ lbf$

$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot sin(\theta) + F_{32y}$ $\qquad F_{21y} = 2633.6\ lbf$

$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2}$ $\qquad F_{21} = 6748.7\ lbf$

$\theta_{21} := atan2\left(F_{21x}, F_{21y}\right)$ $\qquad \theta_{21} = 22.969\ deg$

PROBLEM 13-16

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020\,blob$

Distance to CG (as fraction of *l*): $r_{3a} := 0.4$

Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$

Distance to CG (as fraction of *r*): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.019 \cdot blob$ Gas force: $F_g := 600 \cdot lbf$

Crank speed: $\omega := 3000 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1316.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point *A* to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.800\,in$

Distance from point *B* to CG: $l_b := l - l_a$ $\quad l_b = 7.200\,in$

Let $l_p := l_a$

$$m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \qquad m_{3a} = 0.0120\,blob$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \qquad m_{3b} = 0.00800\,blob$$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.600\,in$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $\quad m_{2a} = 0.0200\,blob$

3. Calculate the lumped masses at points *A* and *B* using equations 13.12.

$$m_A := m_{2a} + m_{3a} \qquad m_A = 0.0320\,blob$$

$$m_B := m_{3b} + m_4 \qquad m_B = 0.0270\,blob$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := atan\left[\frac{r \cdot sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}}\right] \qquad \phi = 9.594\,deg$$

$$a_B := -r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right) \qquad a_B = -407690.5\,\frac{in}{sec^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -\left[\left(m_4 + m_{3b}\right) \cdot a_B + F_g\right] \cdot tan(\phi) \qquad F_{41} = 1759.2\,lbf$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B \qquad F_{34x} = -7146.1\ lbf$$

$$F_{34y} := -\left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi) \qquad F_{34y} = 1759.2\ lbf$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2} \qquad F_{34} = 7359.5\ lbf$$

$$\theta_{34} := atan2\left(F_{34x}, F_{34y}\right) \qquad \theta_{34} = 166.170\ deg$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot cos(\theta) - \left(m_{3b} + m_4\right) \cdot a_B - F_g \qquad F_{32x} = 14510.4\ lbf$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot sin(\theta) + \left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi) \qquad F_{32y} = 609.5\ lbf$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2} \qquad F_{32} = 14523.2\ lbf$$

$$\theta_{32} := atan2\left(F_{32x}, F_{32y}\right) \qquad \theta_{32} = 2.405\ deg$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot cos(\theta) + F_{32x} \qquad F_{21x} = 21348.2\ lbf$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot sin(\theta) + F_{32y} \qquad F_{21y} = 4557.3\ lbf$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2} \qquad F_{21} = 21829.2\ lbf$$

$$\theta_{21} := atan2\left(F_{21x}, F_{21y}\right) \qquad \theta_{21} = 12.050\ deg$$

PROBLEM 13-17

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.25$

Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.3$

Piston mass: $m_4 := 0.032 \cdot blob$ Gas force: $F_g := 900 \cdot lbf$

Crank speed: $\omega := 2500 \cdot rpm$ Crank angle: $\theta := 24 \cdot deg$

Solution: See Mathcad file P1317.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 3.750\, in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 11.250\, in$

Let $l_p := l_a$ $\quad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.0188\, blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.00625\, blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.050\, in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\quad m_{2a} = 0.0180\, blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.0367\, blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.0383\, blob$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := atan\left[\frac{r \cdot \sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot \sin(\theta)\right)^2}}\right] \qquad \phi = 5.446\, deg$$

$$a_B := -r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta)\right) \qquad a_B = -256600.5 \frac{in}{sec^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -\left[\left(m_4 + m_{3b}\right) \cdot a_B + F_g\right] \cdot \tan(\phi) \qquad F_{41} = 849.9\, lbf$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B \qquad F_{34x} = -7311.2\ lbf$$

$$F_{34y} := -\left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi) \qquad F_{34y} = 849.9\ lbf$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2} \qquad F_{34} = 7360.5\ lbf$$

$$\theta_{34} := atan2\left(F_{34x}, F_{34y}\right) \qquad \theta_{34} = 173.369\ deg$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot cos(\theta) - \left(m_{3b} + m_4\right) \cdot a_B - F_g \qquad F_{32x} = 13024.0\ lbf$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot sin(\theta) + \left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi) \qquad F_{32y} = 979.5\ lbf$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2} \qquad F_{32} = 13060.8\ lbf$$

$$\theta_{32} := atan2\left(F_{32x}, F_{32y}\right) \qquad \theta_{32} = 4.301\ deg$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot cos(\theta) + F_{32x} \qquad F_{21x} = 16968.6\ lbf$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot sin(\theta) + F_{32y} \qquad F_{21y} = 2735.8\ lbf$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2} \qquad F_{21} = 17187.7\ lbf$$

$$\theta_{21} := atan2\left(F_{21x}, F_{21y}\right) \qquad \theta_{21} = 9.159\ deg$$

PROBLEM 13-18

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.25$

Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.014 \cdot blob$ Gas force: $F_g := 1200 \cdot lbf$

Crank speed: $\omega := 4000 \cdot rpm$ Crank angle: $\theta := 18 \cdot deg$

Solution: See Mathcad file P1318.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $l_a = 3.750\, in$

Distance from point B to CG: $l_b := l - l_a$ $l_b = 11.250\, in$

Let $l_p := l_a$ $m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $m_{3a} = 0.0188\, blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $m_{3b} = 0.00625\, blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $r_{G2} = 1.600\, in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $m_{2a} = 0.0200\, blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.0388\, blob$

$m_B := m_{3b} + m_4$ $m_B = 0.0203\, blob$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := atan\left[\frac{r \cdot sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}}\right] \qquad \phi = 4.727\, deg$$

$$a_B := -r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right) \qquad a_B = -818901.3 \frac{in}{sec^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -\left[\left(m_4 + m_{3b}\right) \cdot a_B + F_g\right] \cdot tan(\phi) \qquad F_{41} = 1271.9\, lbf$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B \qquad F_{34x} = -10264.6\ lbf$$

$$F_{34y} := -\left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot \tan(\phi) \qquad F_{34y} = 1271.9\ lbf$$

$$F_{34} := \sqrt{F_{34x}^{\ 2} + F_{34y}^{\ 2}} \qquad F_{34} = 10343.1\ lbf$$

$$\theta_{34} := \operatorname{atan2}\left(F_{34x}, F_{34y}\right) \qquad \theta_{34} = 172.936\ deg$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - \left(m_{3b} + m_4\right) \cdot a_B - F_g \qquad F_{32x} = 27898.2\ lbf$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + \left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot \tan(\phi) \qquad F_{32y} = 2794.6\ lbf$$

$$F_{32} := \sqrt{F_{32x}^{\ 2} + F_{32y}^{\ 2}} \qquad F_{32} = 28037.8\ lbf$$

$$\theta_{32} := \operatorname{atan2}\left(F_{32x}, F_{32y}\right) \qquad \theta_{32} = 5.720\ deg$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x} \qquad F_{21x} = 41247.9\ lbf$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y} \qquad F_{21y} = 7132.2\ lbf$$

$$F_{21} := \sqrt{F_{21x}^{\ 2} + F_{21y}^{\ 2}} \qquad F_{21} = 41860.0\ lbf$$

$$\theta_{21} := \operatorname{atan2}\left(F_{21x}, F_{21y}\right) \qquad \theta_{21} = 9.810\ deg$$

PROBLEM 13-19

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
a. Exactly balance the crank and recalculate the inertia force.
b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius -*r* on the crank and recalculate the inertia force.
c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020\,blob$

Distance to CG (as fraction of *l*): $r_{3a} := 0.4$

Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of *r*): $r_{2a} := 0.3$

Piston mass: $m_4 := 0.012 \cdot blob$

Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 45 \cdot deg$

Solution: See Mathcad file P1319.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point *A* to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.800\,in$

Distance from point *B* to CG: $l_b := l - l_a$ $\quad l_b = 7.200\,in$

Let $l_p := l_a$

$$m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \qquad m_{3a} = 0.0120\,blob$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \qquad m_{3b} = 0.00800\,blob$$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.050\,in$

Mass: $$m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \qquad m_{2a} = 0.0180\,blob$$

3. Calculate the lumped masses at points *A* and *B* using equations 13.12.

$$m_A := m_{2a} + m_{3a} \qquad m_A = 0.0300\,blob$$

$$m_B := m_{3b} + m_4 \qquad m_B = 0.0200\,blob$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta) \right) \right] \qquad F_{ix} = 2171\,lbf$$

$$F_{iy} := 0 \cdot lbf \qquad F_{iy} = 0\,lbf$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{ib} = 2171\,lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 0.000\,deg$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \qquad m_p = 6.6667 \times 10^{-3}\,blob$$

$$F_{ix} := -m_A\cdot\left(-r\cdot\omega^2\cdot cos(\theta)\right) - m_B\cdot\left[-r\cdot\omega^2\cdot\left(cos(\theta) + \frac{r}{l}\cdot cos(2\cdot\theta)\right)\right] \dots + \left(m_A + m_p\right)\cdot\left(-r\cdot\omega^2\cdot cos(\theta)\right) \qquad F_{ix} = 1447\,lbf$$

$$F_{iy} := -m_A\cdot\left(-r\cdot\omega^2\cdot sin(\theta)\right) + \left(m_A + m_p\right)\cdot\left(-r\cdot\omega^2\cdot sin(\theta)\right) \qquad F_{iy} = -724\,lbf$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{iob} = 1618\,lbf \quad \text{at} \quad atan2\left(F_{ix},F_{iy}\right) = -26.565\,deg$$

6. Compare the results to those for an unbalanced crank. From Problem 13-11, the inertia force for the unbalanced crank is $F_i := 6330\cdot lbf$. The percent differences for the balanced and overbalanced cranks are:

Exactly balanced: $\Delta := \frac{F_{ib} - F_i}{F_i}$ $\quad \Delta = -65.7\,\%$

Overbalanced: $\Delta := \frac{F_{iob} - F_i}{F_i}$ $\quad \Delta = -74.4\,\%$

✍ PROBLEM 13-20

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
a. Exactly balance the crank and recalculate the inertia force.
b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius *-r* on the crank and recalculate the inertia force.
c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 blob$

Distance to CG (as fraction of *l*): $r_{3a} := 0.4$

Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$

Distance to CG (as fraction of *r*): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.019 \cdot blob$

Crank speed: $\omega := 3000 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1320.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point *A* to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.800\, in$

Distance from point *B* to CG: $l_b := l - l_a$ $\quad l_b = 7.200\, in$

Let $l_p := l_a$

$$m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \qquad m_{3a} = 0.0120\, blob$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \qquad m_{3b} = 0.00800\, blob$$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.600\, in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\quad m_{2a} = 0.0200\, blob$

3. Calculate the lumped masses at points *A* and *B* using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.0320\, blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.0270\, blob$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta) \right) \right] \qquad F_{ix} = 11008\, lbf$$

$$F_{iy} := 0 \cdot lbf \qquad F_{iy} = 0\, lbf$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{ib} = 11008\, lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 0.000\, deg$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \qquad m_p = 9.0000 \times 10^{-3}\,blob$$

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \ldots + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) \qquad F_{ix} = 7931\,lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = -1777\,lbf$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{iob} = 8127\,lbf \quad \text{at} \quad atan2\left(F_{ix}, F_{iy}\right) = -12.626\,deg$$

6. Compare the results to those for an unbalanced crank. From Problem 13-12, the inertia force for the unbalanced crank is $F_i := 22839 \cdot lbf$. The percent differences for the balanced and overbalanced cranks are:

Exactly balanced: $$\Delta := \frac{F_{ib} - F_i}{F_i} \qquad \Delta = -51.8\,\%$$

Overbalanced: $$\Delta := \frac{F_{iob} - F_i}{F_i} \qquad \Delta = -64.4\,\%$$

PROBLEM 13-21

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.

a. Exactly balance the crank and recalculate the inertia force.

b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius -*r* on the crank and recalculate the inertia force.

c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 blob$

Distance to CG (as fraction of *l*): $r_{3a} := 0.25$

Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of *r*): $r_{2a} := 0.3$

Piston mass: $m_4 := 0.023 \cdot blob$

Crank speed: $\omega := 2500 \cdot rpm$ Crank angle: $\theta := 24 \cdot deg$

Solution: See Mathcad file P1321.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point *A* to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 3.750\, in$

Distance from point *B* to CG: $l_b := l - l_a$ $\quad l_b = 11.250\, in$

Let $l_p := l_a$

$$m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \qquad m_{3a} = 0.0188\, blob$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \qquad m_{3b} = 0.00625\, blob$$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.050\, in$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $\quad m_{2a} = 0.0180\, blob$

3. Calculate the lumped masses at points *A* and *B* using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.0367\, blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.0292\, blob$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta) \right) \right] \qquad F_{ix} = 7506\, lbf$$

$$F_{iy} := 0 \cdot lbf \qquad F_{iy} = 0\, lbf$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{ib} = 7506\, lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 0.000\, deg$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \qquad m_p = 9.7500 \times 10^{-3}\ blob$$

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \ldots + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) \qquad F_{ix} = 5369\ lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = -951\ lbf$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{iob} = 5453\ lbf \quad \text{at} \quad atan2\left(F_{ix}, F_{iy}\right) = -10.048\ deg$$

6. Compare the results to those for an unbalanced crank. From Problem 13-13, the inertia force for the unbalanced crank is $F_i := 15967 \cdot lbf$. The percent differences for the balanced and overbalanced cranks are:

Exactly balanced: $$\Delta := \frac{F_{ib} - F_i}{F_i} \qquad \Delta = -53.0\,\%$$

Overbalanced: $$\Delta := \frac{F_{iob} - F_i}{F_i} \qquad \Delta = -65.9\,\%$$

PROBLEM 13-22

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.

a. Exactly balance the crank and recalculate the inertia force.

b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius *-r* on the crank and recalculate the inertia force.

c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 15 \cdot in$ mass: $m_3 := 0.025 \, blob$

Distance to CG (as fraction of *l*): $r_{3a} := 0.25$

Crank length: $r := 4 \cdot in$ mass: $m_2 := 0.050 \cdot blob$

Distance to CG (as fraction of *r*): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.015 \cdot blob$

Crank speed: $\omega := 4000 \cdot rpm$ Crank angle: $\theta := 18 \cdot deg$

Solution: See Mathcad file P1322.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point *A* to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 3.750 \, in$

Distance from point *B* to CG: $l_b := l - l_a$ $\quad l_b = 11.250 \, in$

Let $l_p := l_a$

$$m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \qquad m_{3a} = 0.0188 \, blob$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \qquad m_{3b} = 0.00625 \, blob$$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.600 \, in$

Mass: $$m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \qquad m_{2a} = 0.0200 \, blob$$

3. Calculate the lumped masses at points *A* and *B* using equations 13.12.

$$m_A := m_{2a} + m_{3a} \qquad m_A = 0.0388 \, blob$$

$$m_B := m_{3b} + m_4 \qquad m_B = 0.0213 \, blob$$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta) \right) \right] \qquad F_{ix} = 17402 \, lbf$$

$$F_{iy} := 0 \cdot lbf \qquad F_{iy} = 0 \, lbf$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{ib} = 17402 \, lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 0.000 \, deg$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \qquad m_p = 7.0833 \times 10^{-3}\ blob$$

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \ldots$$
$$+ \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) \qquad F_{ix} = 12674\ lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = -1536\ lbf$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{iob} = 12766\ lbf \text{ at } atan2\left(F_{ix}, F_{iy}\right) = -6.911\ deg$$

6. Compare the results to those for an unbalanced crank. From Problem 13-14, the inertia force for the unbalanced crank is $F_i := 44075 \cdot lbf$. The percent differences for the balanced and overbalanced cranks are:

Exactly balanced: $$\Delta := \frac{F_{ib} - F_i}{F_i} \qquad \Delta = -60.5\%$$

Overbalanced: $$\Delta := \frac{F_{iob} - F_i}{F_i} \qquad \Delta = -71.0\%$$

PROBLEM 13-23

Statement: Combine the necessary equations to develop expressions that show how each of these dynamic parameters varies as a function of the crank/conrod ratio alone:

a. Piston acceleration.
b. Inertia force.
c. Inertia torque.
d. Pin forces.

Plot the functions. Check your conclusions with program ENGINE.

Solution: See Mathcad file P1323.

1. Note that the stroke volume *V* of the cylinder must be held at some constant value while varying the *r/l* ratio in order to isolate the effects of that parameter. We must hold all other factors constant for this analysis as well, such as all masses, crank angle, and bore/stroke ratio. We will assume that the crank radius *r* is held constant as we vary the *r/l* ratio in order to keep the stroke volume constant. Note that we can vary the *r/l* ratio independently of the crank radius *r* since for any *r* we can choose a value of *l* to maintain *r/l* constant.

2. (Part a.) The expression for the piston acceleration is shown in equation *a*. Note that the *r/l* ratio only appears in the second harmonic term. If we allow *r/l* to become very small (long conrod), then this term diminishes toward zero, leaving a pure harmonic primary component. Thus, as the conrod length approaches infinity, the acceleration approaches a pure cosine. The *r/l* term acts as a distortion factor that changes the shape of this harmonic function in a way that increases its peak negative value. The optimum value for *r/l* is zero. The longer the conrod, the lower the peak acceleration.

$$a_p := -r\cdot\omega^2\cdot\left(\cos(\omega\cdot t) + \frac{r}{l}\cdot\cos(2\cdot\omega\cdot t)\right) \qquad (a)$$

3. (Part b.) The expression for inertia force is shown in equation *b*. Note that only the *x*-component contains an *r/l* term and it is within the piston acceleration term. Thus the effects of the *r/l* ratio on inertia force are the same as those described for piston acceleration above. The optimum value for *r/l* is then zero. The longer the conrod, the lower the peak inertia force in the *x*-direction.

$$F_{ix} := -m_A\cdot\left(-r\cdot\omega^2\cdot\cos(\omega\cdot t)\right) - m_B\cdot\left[-r\cdot\omega^2\cdot\left[\cos(\theta) + \frac{r}{l}\cdot\cos[2\cdot(\omega\cdot t)]\right]\right]$$

$$F_{iy} := -m_A\cdot\left(-r\cdot\omega^2\cdot\sin(\omega\cdot t)\right) \qquad (b)$$

4. (Part c.) The expression for inertia torque is shown in equation *c*. Note that only the primary and tertiary components contain an *r/l* term. Thus, the effects of the r/l ratio on inertia torque are similar to those described for inertia force above. The dominant term in this equation is the second harmonic as it has the largest possible coefficient of 1. If the *r/l* ratio were zero (infinite conrod) then the inertia torque would be a pure harmonic of twice crank frequency. A nonzero *r/l* adds distortion to this curve in the form of the 1st and 3rd harmonics. The optimum value for *r/l* is zero. The longer the conrod, the lower the peak inertia torque.

$$T_{i21} := -m_B\cdot r^2\cdot\omega^2\cdot\sin(\omega\cdot t)\cdot\left[\frac{r}{2\cdot l} + \cos(\omega\cdot t) + \frac{3\cdot r}{2\cdot l}\cdot\cos[2\cdot(\omega\cdot t)]\right] \qquad (c)$$

5. (Part d.) The equation for the wrist pin force is shown in equation *d*. It contains both the acceleration of the piston (*e*) and tanϕ (*f*). These are substituted into *d* to get *g*. This expression shows that the wrist pin force is directly proportional to both *r/l* and the sum of that ratio squared to form the fourth power as shown in *h*. The optimum value of *r/l* is then zero. The longer the conrod, the lower the wrist pin force.

$$F_{34x} := F_g + m_4\cdot a_B \qquad (d)$$

$$F_{34y} := -\left[F_g + \left(m_4 + m_{3b}\right)\cdot a_B\right]\cdot\tan(\phi)$$

$$a_B := -r\cdot\omega^2\cdot\left[\cos(\omega\cdot t) + \frac{r}{l}\cdot\cos[2\cdot(\omega\cdot t)]\right] \qquad (e)$$

$$\phi := \operatorname{atan}\left[\frac{r\cdot\sin(\omega\cdot t)}{l\cdot\sqrt{1-\left(\frac{r}{l}\cdot\sin(\omega\cdot t)\right)^2}}\right] \qquad (f)$$

$$F_{34x} := F_g + m_4\cdot\left[-r\cdot\omega^2\cdot\left[\cos(\omega\cdot t) + \frac{r}{l}\cdot\cos[2\cdot(\omega\cdot t)]\right]\right] \qquad (g)$$

$$F_{34y} := -\left[F_g + (m_4 + m_{3b})\cdot\left[-r\cdot\omega^2\cdot\left[\cos(\omega\cdot t) + \frac{r}{l}\cdot\cos[2\cdot(\omega\cdot t)]\right]\right]\right]\cdot\left[\frac{r}{l}\cdot\frac{\sin(\omega\cdot t)}{\sqrt{1-\left(\frac{r}{l}\cdot\sin(\omega\cdot t)\right)^2}}\right]$$

F_{34x} proportional to $\frac{r}{l}$

F_{34y} proportional to $-\left(\frac{r^2}{l^2} + \frac{r^4}{l^4}\right)$ $\qquad (h)$

6. (Part d.) The equation for the crankpin force is shown in equation *i*. It contains both the acceleration of the piston (*e*) and tanϕ (*f*). These are substituted into *i* to get *j*. This expression shows that the crankpin force is directly proportional to both *r/l* and the sum of that ratio squared and to the fourth power as shown in *k*. The optimum value of *r/l* is then zero. The longer the conrod, the lower the crankpin force.

$$F_{32x} := m_{3a}\cdot r\cdot\omega^2\cdot\cos(\omega\cdot t) - (m_{3b} + m_4)\cdot a_B - F_g$$

$$F_{32y} := m_{3a}\cdot r\cdot\omega^2\cdot\sin(\omega\cdot t) + \left[F_g + (m_4 + m_{3b})\cdot a_B\right]\cdot\tan(\phi) \qquad (i)$$

$$F_{32x} := m_{3a}\cdot r\cdot\omega^2\cdot\cos(\omega\cdot t) - (m_{3b} + m_4)\cdot\left[-r\cdot\omega^2\cdot\left[\cos(\omega\cdot t) + \frac{r}{l}\cdot\cos[2\cdot(\omega\cdot t)]\right]\right] - F_g$$

$$F_{32y} := m_{3a}\cdot r\cdot\omega^2\cdot\sin(\omega\cdot t)\ \ldots \qquad (j)$$
$$+\left[F_g + (m_4 + m_{3b})\cdot\left[-r\cdot\omega^2\cdot\left[\cos(\omega\cdot t) + \frac{r}{l}\cdot\cos[2\cdot(\omega\cdot t)]\right]\right]\right]\cdot\left[\frac{r}{l}\cdot\frac{\sin(\omega\cdot t)}{\sqrt{1-\left(\frac{r}{l}\cdot\sin(\omega\cdot t)\right)^2}}\right]$$

F_{32x} proportional to $\frac{r}{l}$

F_{32y} proportional to $\left(\frac{r^2}{l^2} + \frac{r^4}{l^4}\right)$ $\qquad (k)$

7. (Part d.) The equation for the mainpin force is shown in equation *l*. It is equal to the crankpin force just analyzed above plus another term. Note that the second term does not contain *r/l*, so the mainpin force has the same dependence on *r/l* as does the crankpin force.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot cos(\omega \cdot t) + F_{32x}$$

(*l*)

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot sin(\omega \cdot t) + F_{32y}$$

F_{21x} proportional to $\frac{r}{l}$

(*m*)

F_{21y} proportional to $\left(\frac{r^2}{l^2} + \frac{r^4}{l^4}\right)$

PROBLEM 13-24

Statement: Combine the necessary equations to develop expressions that show how each of these dynamic parameters varies as a function of the bore/stroke ratio alone:

a. Gas force.
b. Gas torque.
c. Inertia force.
d. Inertia torque.
e. Pin forces.

Plot the functions. Check your conclusions with program ENGINE.

Solution: See Mathcad file P1324.

1. The stroke volume *V* of the cylinder must be held at some constant value as shown in equation *a* while varying the *B/S* ratio in order to isolate the effects of that parameter. This relationship is rearranged to introduce the bore/stroke ratio *B/S* in equation *b*. We must hold all other factors constant for this analysis as well, such as all masses, crank angle, and crank/conrod ratio. Note that we can vary the *B/S* ratio independently of the crank/ conrod ratio *r/l* since for any *r* we can choose a value of *l* to maintain *r/l* constant.

$$V := \frac{\pi}{4}\cdot B^2\cdot S = \text{constant} \qquad (a)$$

but, $B^2 := \left(\frac{B}{S}\right)^2\cdot S^2$ so, $V := \frac{\pi}{4}\cdot\left(\frac{B}{S}\right)^2\cdot S^3$ (*b*)

2. (Part a.) The gas force equation is shown in equation *c* and the expression for $(B/S)^2$ from *b* is substituted in equation *c*. Expressions *b* and *c* are each then solved for *S* and combined in *d*. Gas force F_g is proportional to *B/S* as shown in *e*. Any increase in the *B/S* ratio will increase the gas force and vice versa.

$$F_g := \frac{\pi}{4}\cdot p_g\cdot B^2 \quad \text{or} \quad F_g := \frac{\pi}{4}\cdot p_g\cdot\left(\frac{B}{S}\right)^2\cdot S^2 \qquad (c)$$

combine equations *b* and *c* to get:

$$F_g := 0.922635\cdot p_g\cdot\left[V\cdot\left(\frac{B}{S}\right)\right]^{\frac{2}{3}} \qquad (d)$$

so, for a given p_g and *V*, F_g is proportional to $(B/S)^{2/3}$. (*e*)

3. (Part b.) The gas torque equation is shown in equation *f*. Expressions for its F_g and *r* terms are shown in equation *g*. These are substituted into *f* and shown as *h*. The first expression in parentheses is the stroke volume *V*, which is constant as shown in *i*. Thus, as shown in *j*, the gas torque is independent of *B/S*.

$$T_{g21} := F_g\cdot r\cdot\sin(\omega\cdot t)\cdot\left(1 + \frac{r}{l}\cdot\cos(\omega\cdot t)\right) \qquad (f)$$

$$F_g := \frac{\pi}{4}\cdot p_g\cdot B^2 \qquad r := \frac{S}{2} \qquad (g)$$

$$T_{g21} := \frac{1}{2}\cdot p_g\cdot\left(\frac{\pi}{4}\cdot B^2\cdot s\right)\cdot\sin(\omega\cdot t)\cdot\left(1 + \frac{r}{l}\cdot\cos(\omega\cdot t)\right) \qquad (h)$$

$$V := \frac{\pi}{4}\cdot B^2\cdot S = \text{constant} \qquad (i)$$

$$T_{g21} := -\frac{1}{2}\cdot p_g\cdot V\cdot \sin(\omega\cdot t)\cdot\left(1+\frac{r}{l}\cdot\cos(\omega\cdot t)\right) \qquad (j)$$

4. (Part c.) The expression for inertia force is shown in equation *k* with the expression $S = 2r$ substituted and the S factored out. These are simplified by grouping all other factors as parameters K_1, K_2, and K_3 and the magnitude of the force found. The dimensionless parameter of *B/S* ratio is introduced in *l*. The stroke volume constraint is then introduced, combined with *l* and substituted in *k* to get the nonlinear inverse relationship of inertia force to the *B/S* ratio shown in *m*.

$$F_{ix} := S\cdot\left[\frac{\omega^2}{2}\cdot\left[m_A\cdot\cos(\omega\cdot t) + m_B\cdot\left(\cos(\theta) + \frac{r}{l}\cdot\cos(2\cdot\omega\cdot t)\right)\right]\right] = S\cdot K_1$$

$$F_{iy} := S\cdot\left(\frac{m_A}{2}\cdot\omega^2\cdot\sin(\omega\cdot t)\right) = S\cdot K_2$$

$$F_i = \sqrt{F_{ix}^2 + F_{iy}^2} = \sqrt{(S\cdot K_1)^2 (S\cdot K_2)^2} = \left(S\cdot\sqrt{K_1^2 + K_2^2} = S\cdot K_3\right) \qquad (k)$$

$$S := B\cdot\left(\frac{B}{S}\right)^{-1} \qquad F_i := K_3\cdot B\cdot\left(\frac{B}{S}\right)^{-1} \qquad (l)$$

introduce volume constraint,

$$V = \frac{\pi}{4}\cdot B^2\cdot S = \frac{\pi}{4}\cdot B^3\cdot\left(\frac{B}{S}\right)^{-1} \qquad B = \left[\frac{4}{\pi}\cdot V\cdot\left(\frac{B}{S}\right)\right]^{\frac{1}{3}} = 1.083852\cdot V^{\frac{1}{3}}\cdot\left(\frac{B}{S}\right)^{\frac{1}{3}}$$

substitute this into the force equation,

$$F_i := 1.083852\cdot K_3\cdot V^{\frac{1}{3}}\cdot\left(\frac{B}{S}\right)^{-\frac{2}{3}} \qquad (m)$$

5. (Part d.) The inertia torque equation is shown simplified in equation *n*. Equation *l* is substituted into *n* to show the relationship of inertia torque to the *B/S* ratio in *o*.

$$T_{i21} := S^2\cdot\left[\frac{m_B\cdot\omega^2}{8}\cdot\left(\frac{r}{2\cdot l}\cdot\sin(\omega\cdot t) - \sin(2\cdot\omega\cdot t) - \frac{3\cdot r}{2\cdot l}\cdot\sin(3\cdot\omega\cdot t)\right)\right] = S^2\cdot K_4 \qquad (n)$$

$$T_{i21} = K_4\cdot\left[B\cdot\left(\frac{B}{S}\right)^{-1}\right]^2 = K_4\cdot B^2\cdot\left(\frac{B}{S}\right)^{-2} = K_4\cdot\left[\frac{4}{\pi}\cdot V\cdot\left(\frac{B}{S}\right)\right]^{\frac{2}{3}}\cdot\left(\frac{B}{S}\right)^{-2} \qquad (o)$$

6. (Part e.) The equation for the wrist pin force is shown in equation *p*. It contains both the gas force (*c*) and the acceleration of the piston (*q*). These are substituted along with *l* into *p* to get *r*. This expression shows that the wrist pin force is directly proportional to both B^2 and S as shown in *s*. This means that there may be an optimum value of *B/S*, which will minimize the wrist pin force. Note that the $\tan\phi$ term contains only *r/l* terms so is constant in this analysis.

$$F_{34x} := F_g + m_4\cdot a_B \qquad (p)$$

$$F_{34y} := -\left[F_g + (m_4 + m_{3b})\cdot a_B\right]\cdot\tan(\phi)$$

$$a_B := -r\cdot\omega^2\cdot\left[cos(\omega\cdot t) + \frac{r}{l}\cdot cos[2\cdot(\omega\cdot t)]\right] \qquad (q)$$

$$F_{34x} := -\frac{\pi}{4}\cdot p_g\cdot B^2 + m_4\cdot\left[-\frac{S}{2}\cdot\omega^2\cdot\left[cos(\omega\cdot t) + \frac{r}{l}\cdot cos[2\cdot(\omega\cdot t)]\right]\right] \qquad (r)$$

$$F_{34y} := -\left[-\frac{\pi}{4}\cdot p_g\cdot B^2 + (m_4 + m_{3b})\cdot\left[-\frac{S}{2}\cdot\omega^2\left[cos(\omega\cdot t) + \frac{r}{l}\cdot cos[2\cdot(\omega\cdot t)]\right]\right]\cdot tan(\phi)\right]$$

F_{34} is proportional to $B^2 - S$ (s)

7. (Part e.) The equation for the crankpin force is shown in equation *t*. It contains both the gas force F_g (*c*) and the acceleration of the piston (*q*). These are substituted along with *l* into *t* to get *u*. This expression shows that the crankpin force is directly proportional to both B^2 and *S* as shown in *v*. This means that there may be an optimum value of *B/S*, which will minimize the crankpin force. Note that the $\tan\phi$ term contains only *r/l* terms so is constant in this analysis.

$$F_{32x} := m_{3a}\cdot r\cdot\omega^2\cdot cos(\omega\cdot t) - (m_{3b} + m_4)\cdot a_B - F_g$$

$$F_{32y} := m_{3a}\cdot r\cdot\omega^2\cdot sin(\omega\cdot t) + \left[F_g + (m_4 + m_{3b})\cdot a_B\right]\cdot tan(\phi) \qquad (t)$$

$$F_{32x} := \left[m_{3a}\cdot\frac{S}{2}\cdot\omega^2\cdot cos(\omega\cdot t) - (m_{3b} + m_4)\cdot\left[-\frac{S}{2}\cdot\omega^2\cdot\left[cos(\omega\cdot t) + \frac{r}{l}\cdot cos[2\cdot(\omega\cdot t)]\right]\right]\right] + \frac{\pi}{4}\cdot p_g\cdot B^2$$

$$F_{32y} := m_{3a}\cdot\frac{S}{2}\cdot\omega^2\cdot sin(\omega\cdot t) \ldots$$
$$+\left[-\frac{\pi}{4}\cdot p_g\cdot B^2 + (m_4 + m_{3b})\cdot\left[\frac{-S}{2}\cdot\omega^2\cdot\left[cos(\omega\cdot t) + \frac{r}{l}\cdot cos[2\cdot(\omega\cdot t)]\right]\right]\right]\cdot tan(\phi) \qquad (u)$$

F_{32x} proportional to $B^2 + S$

F_{32y} proportional to $-B^2$ (v)

8. (Part e.) The equation for the mainpin force is shown in equation *w*. It is equal to the crankpin force just analyzed above plus another term. The result from *u* is substituted along with *l* into *w* to get *x*. This expression shows that the crankpin force is directly proportional to both B^2 and *S* as shown in *x*. This means that there may be an optimum value of *B/S*, which will minimize the mainpin force.

$$F_{21x} := m_{2a}\cdot\frac{S}{2}\cdot\omega^2\cdot cos(\omega\cdot t) + F_{32x}$$

$$F_{21y} := m_{2a}\cdot\frac{S}{2}\cdot\omega^2\cdot sin(\omega\cdot t) + F_{32y} \qquad (w)$$

F_{21x} proportional to $B^2 + S$

F_{21y} proportional to $B^2 + S$ (x)

PROBLEM 13-25

Statement: Develop an expression to determine the optimum bore/stroke ratio to minimize the wrist pin force. Plot the function.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Assumptions: Use the data from Problem 13-15 for the unvarying parameters.

Conrod length: $l := 12 \cdot in$ mass: $m_3 := 0.020 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.4$

Crank length: $r := 3.5 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.3$

Piston mass: $m_4 := 0.022 \cdot blob$ Gas pressure: $p_g := 300 \cdot psi$

Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 45 \cdot deg$

Stroke volume: $V := 1 \cdot in^3$ R/L ratio: $RoverL := 0.2917$

Solution: See Problem 13-15 and Mathcad file P1325.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.800\ in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 7.200\ in$

Let $l_p := l_a$ $\quad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.0120\ blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.00800\ blob$

2. Determine the stroke and crank radius as functions of B/S

Stroke as function of B/S: $S(BoverS) := \left(\dfrac{4 \cdot V}{\pi}\right)^{\frac{1}{3}} \cdot BoverS^{-\frac{2}{3}}$

Crank length as a function of stroke: $r(BoverS) := 0.5 \cdot S(BoverS)$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := atan\left[\frac{RoverL \cdot sin(\theta)}{\sqrt{1 - (RoverL \cdot sin(\theta))^2}}\right] \qquad \phi = 11.903\ deg$$

$$a_B(BoverS) := -r(BoverS) \cdot \omega^2 \cdot (cos(\theta) + RoverL \cdot cos(2 \cdot \theta))$$

5. Write an expression for the gas force as a function of the B/S ratio.

$$F_g(BoverS) := \frac{\pi}{4} \cdot p_g \cdot BoverS^2 \cdot S(BoverS)^2$$

6. Determine the wrist pin force F_{34} using equation 13.21.

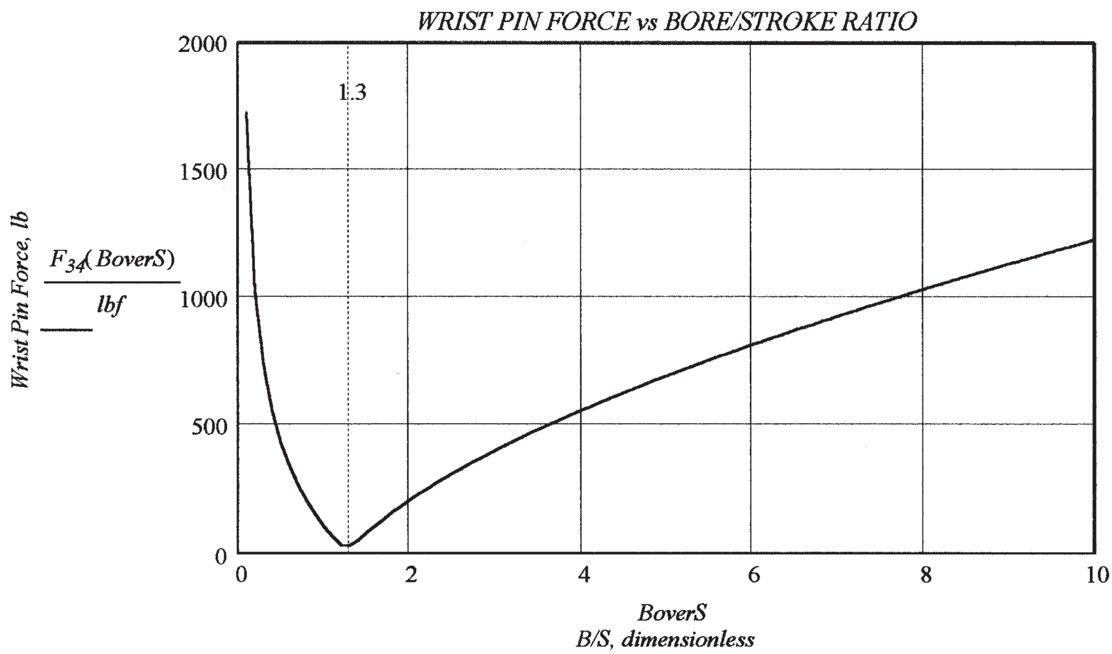
$$F_{34x}(BoverS) := F_g(BoverS) + m_4 \cdot a_B(BoverS)$$

$$F_{34y}(BoverS) := -\left[F_g(BoverS) + (m_4 + m_{3b}) \cdot a_B(BoverS)\right] \cdot tan(\phi)$$

$$F_{34}(BoverS) := \sqrt{F_{34x}(BoverS)^2 + F_{34y}(BoverS)^2}$$

7. Plot the wrist pin force F_{34} as a function of *B/S*.

$BoverS := 0.1, 0.2 .. 10$



8. The graph shows that there is an optimum value of B/S that will minimize the wrist pin force, which in this case, is about $B/S = 1.3$. However, the optimum value is dependent on the other design choices (especially peak gas pressure) and can vary considerably from this value. The bore, stroke, and crank radius that would result from choosing this value are:

$S_{opt} := S(1.3)$ $\quad S_{opt} = 0.910\,in$ $\quad r_{opt} := r(1.3)$ $\quad r_{opt} = 0.455\,in$

$B_{opt} := \sqrt{\dfrac{4 \cdot V}{\pi \cdot S_{opt}}}$ $\quad B_{opt} = 1.183\,in$

$l_{opt} := \dfrac{r_{opt}}{RoverL}$ $\quad l_{opt} = 1.560\,in$

This probably is not a practical design.

PROBLEM 13-26

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the main pin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

a. Piston, conrod and crank masses = 0.
b. Piston mass = 1 blob, conrod and crank masses = 0.
c. Conrod mass = 1 blob, piston and crank masses = 0.
d. Crank mass = 1 blob, conrod and piston masses = 0.

Place the CG of the crank at 0.5*r* and the conrod at 0.33*l*. Compare and explain the differences in the main pin force under these different conditions with reference to the governing equations.

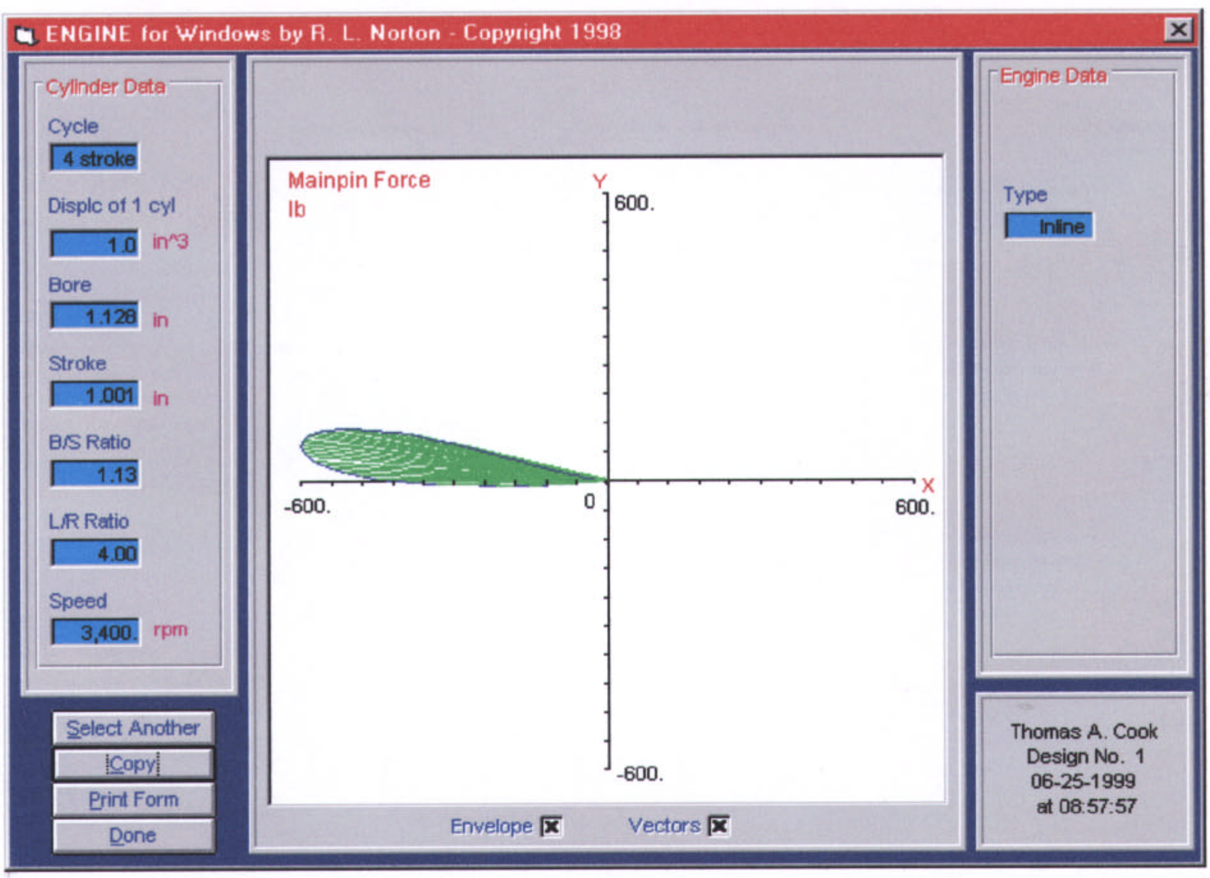
Assumptions:
1. Peak cylinder pressure is 600 psi.
2. Link length ratio is L/R = 4.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

Solution: See Mathcad file P1326.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

a. $m_2 := 0$ $m_3 := 0$ $m_4 := 0$ $F_{21max} := 604.3 \cdot lbf$

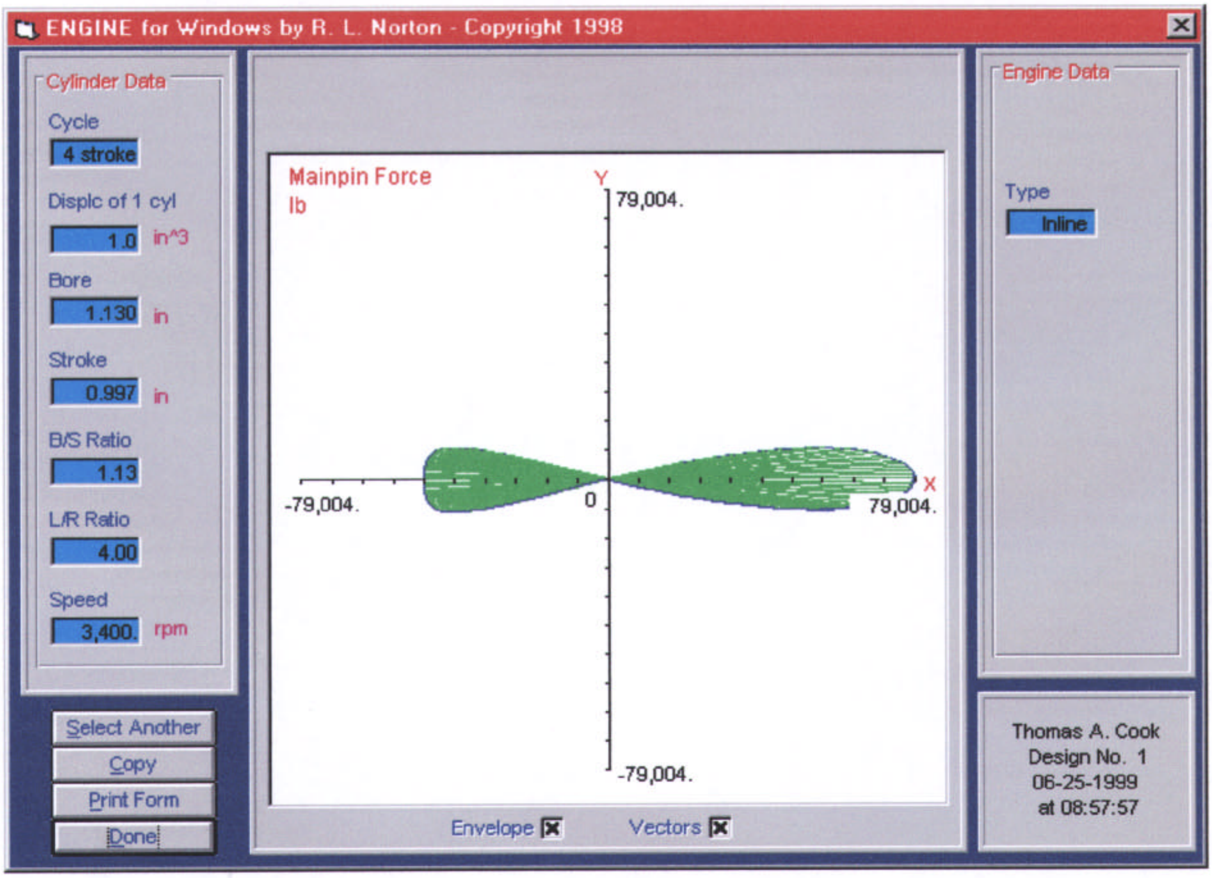
Polar plot of main pin force for part *a*.



2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

b. $m_2 := 0$ $m_3 := 0$ $m_4 := 1$ $F_{21max} := 79004 \cdot lbf$

Polar plot of main pin force for part *b*.



3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

$c. \quad m_2 := 0 \quad m_3 := 1 \quad m_4 := 0 \quad F_{21max} := 68417 \cdot lbf$

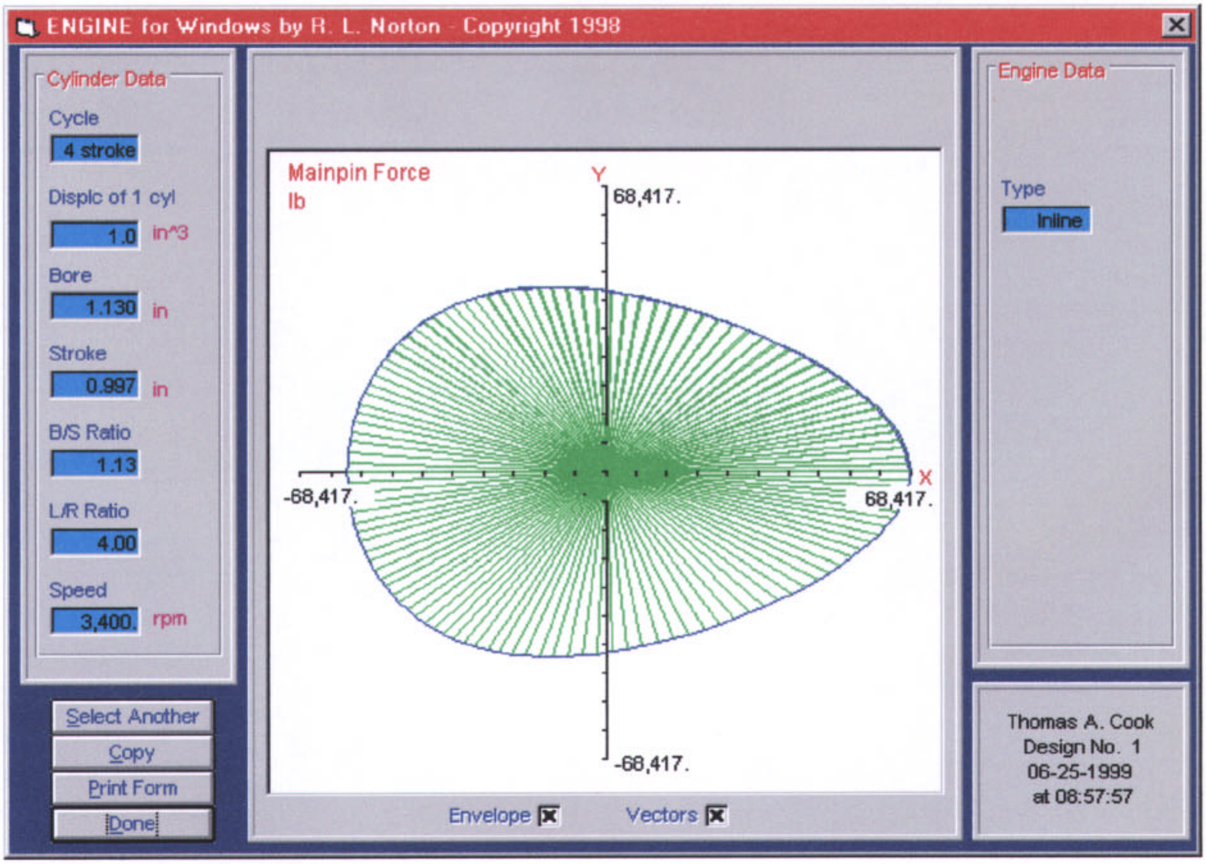
See polar plot on next page.

4. Enter the above data and the masses for part *d* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

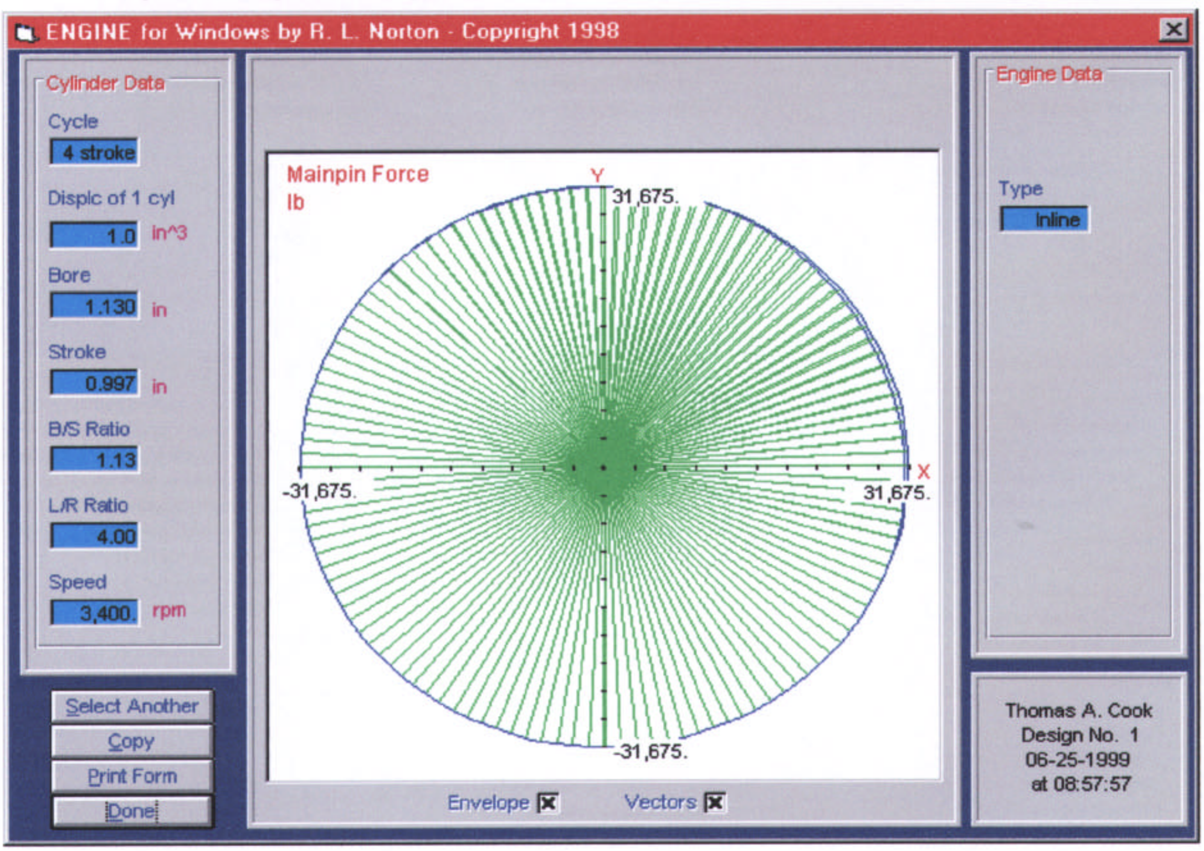
$d. \quad m_2 := 1 \quad m_3 := 0 \quad m_4 := 0 \quad F_{21max} := 31696 \cdot lbf$

See polar plot on next page.

Polar plot of main pin force for part *c*.



Polar plot of main pin force for part *d*.



PROBLEM 13-27

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the crankpin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

a. Piston, conrod and crank masses = 0.
b. Piston mass = 1 blob, conrod and crank masses = 0.
c. Conrod mass = 1 blob, piston and crank masses = 0.
d. Crank mass = 1 blob, conrod and piston masses = 0.

Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the crankpin force under these different conditions with reference to the governing equations.

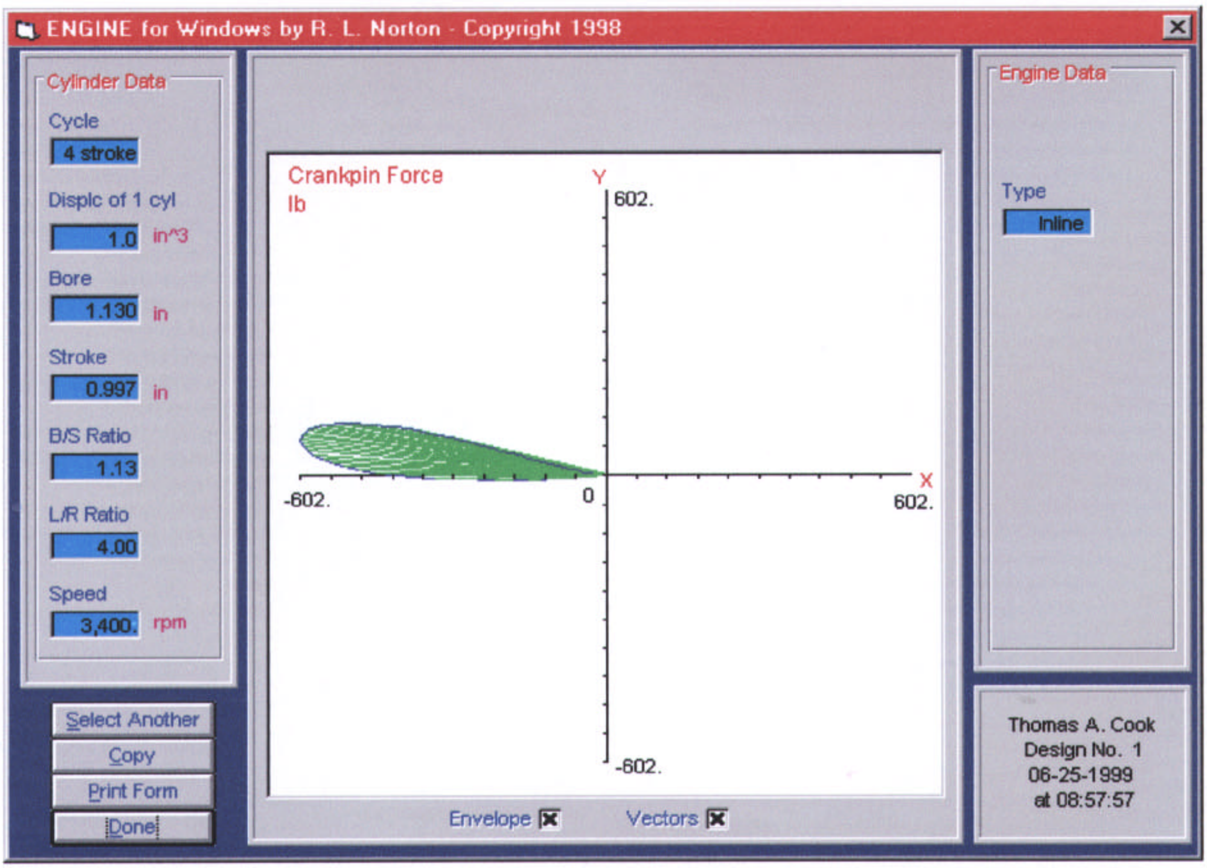
Assumptions:
1. Peak cylinder pressure is 600 psi.
2. Link length ratio is L/R = 4.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

Solution: See Mathcad file P1327.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

a. $m_2 := 0$ $m_3 := 0$ $m_4 := 0$ $F_{32max} := 602 \cdot lbf$

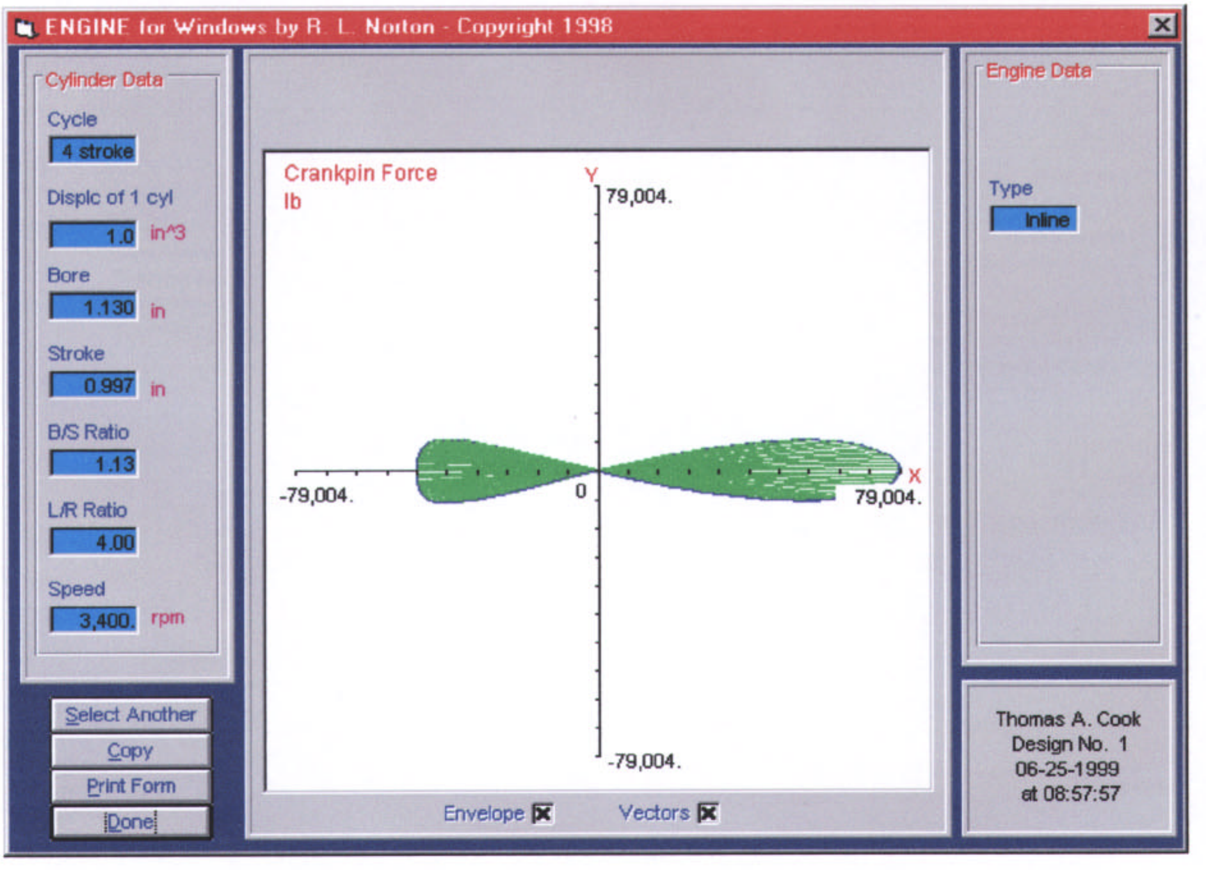
Polar plot of crankpin force for part *a*.



2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

b. $m_2 := 0$ $m_3 := 0$ $m_4 := 1$ $F_{32max} := 79004 \cdot lbf$

Polar plot of crankpin force for part *b*.



3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

c. $m_2 := 0$ $m_3 := 1$ $m_4 := 0$ $F_{32max} := 68417 \cdot lbf$

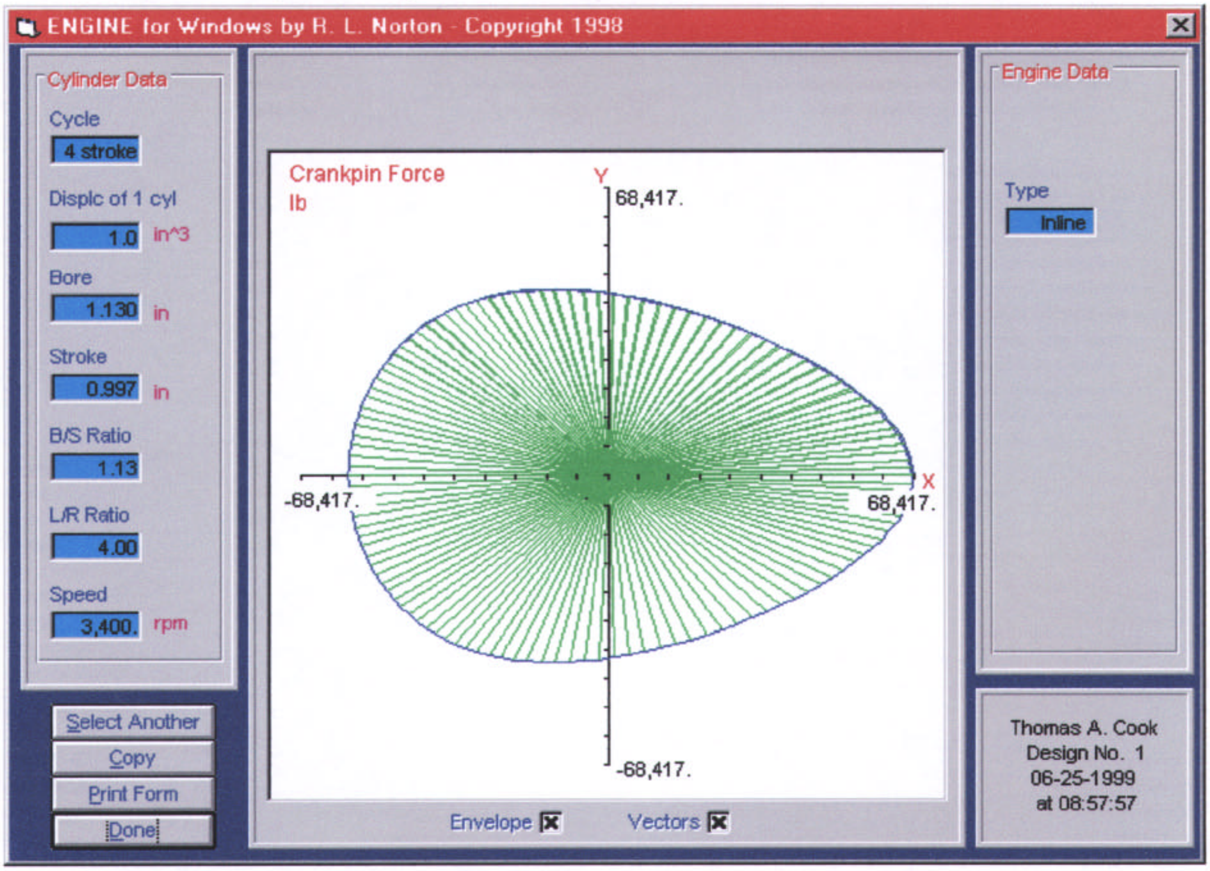
See polar plot on next page.

4. Enter the above data and the masses for part *d* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

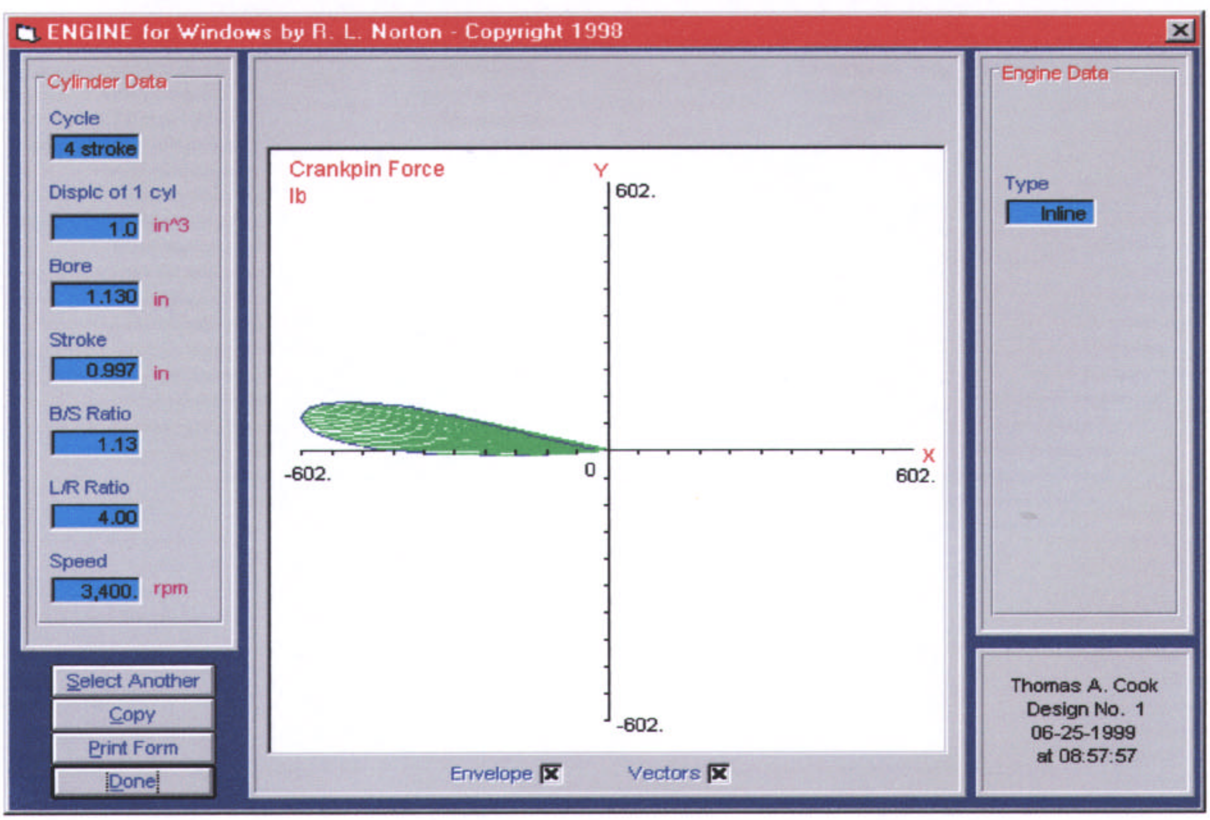
d. $m_2 := 1$ $m_3 := 0$ $m_4 := 0$ $F_{32max} := 602 \cdot lbf$

See polar plot on next page.

Polar plot of crankpin force for part *c*.



Polar plot of crankpin force for part *d*.



PROBLEM 13-28

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the wrist pin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

a. Piston, conrod and crank masses = 0.
b. Piston mass = 1 blob, conrod and crank masses = 0.
c. Conrod mass = 1 blob, piston and crank masses = 0.
d. Crank mass = 1 blob, conrod and piston masses = 0.

Place the CG of the crank at 0.5*r* and the conrod at 0.33*l*. Compare and explain the differences in the wrist pin force under these different conditions with reference to the governing equations.

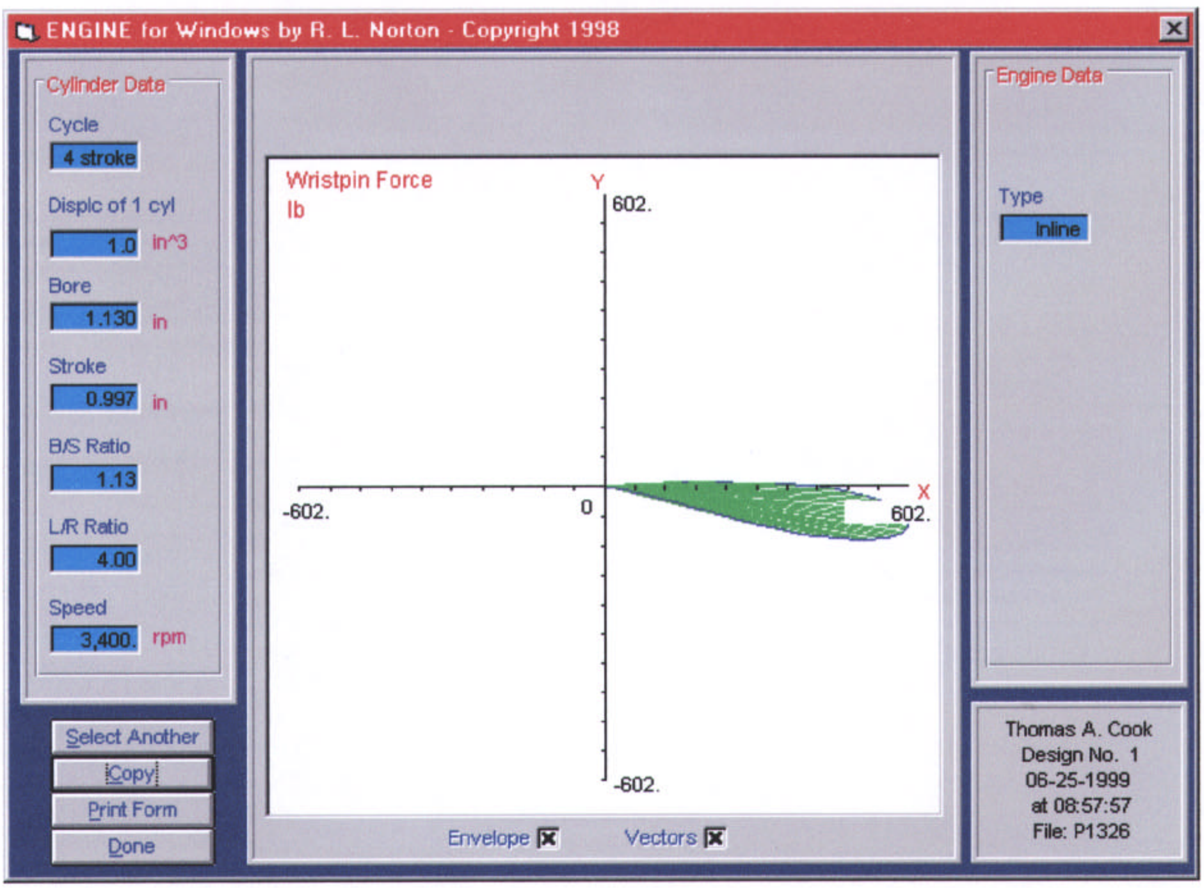
Assumptions:
1. Peak cylinder pressure is 600 psi.
2. Link length ratio is L/R = 4.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

Solution: See Mathcad file P1328.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

$a. \quad m_2 := 0 \quad m_3 := 0 \quad m_4 := 0 \quad F_{34max} := 602 \cdot lbf$

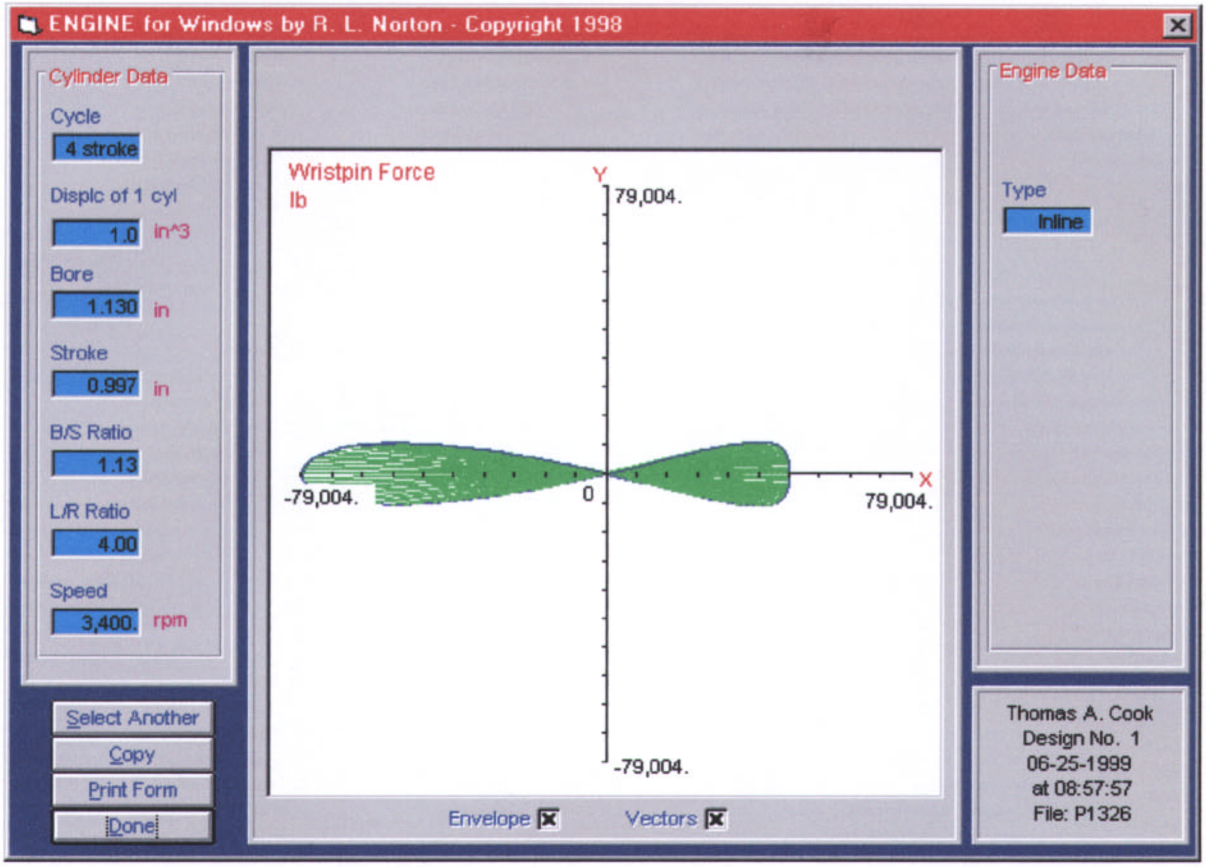
Polar plot of wrist pin force for part *a*.



2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

$b. \quad m_2 := 0 \quad m_3 := 0 \quad m_4 := 1 \quad F_{34max} := 79004 \cdot lbf$

Polar plot of wrist pin force for part *b*.



3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

$c.$ $m_2 := 0$ $m_3 := 1$ $m_4 := 0$ $F_{34max} := 2914 \cdot lbf$

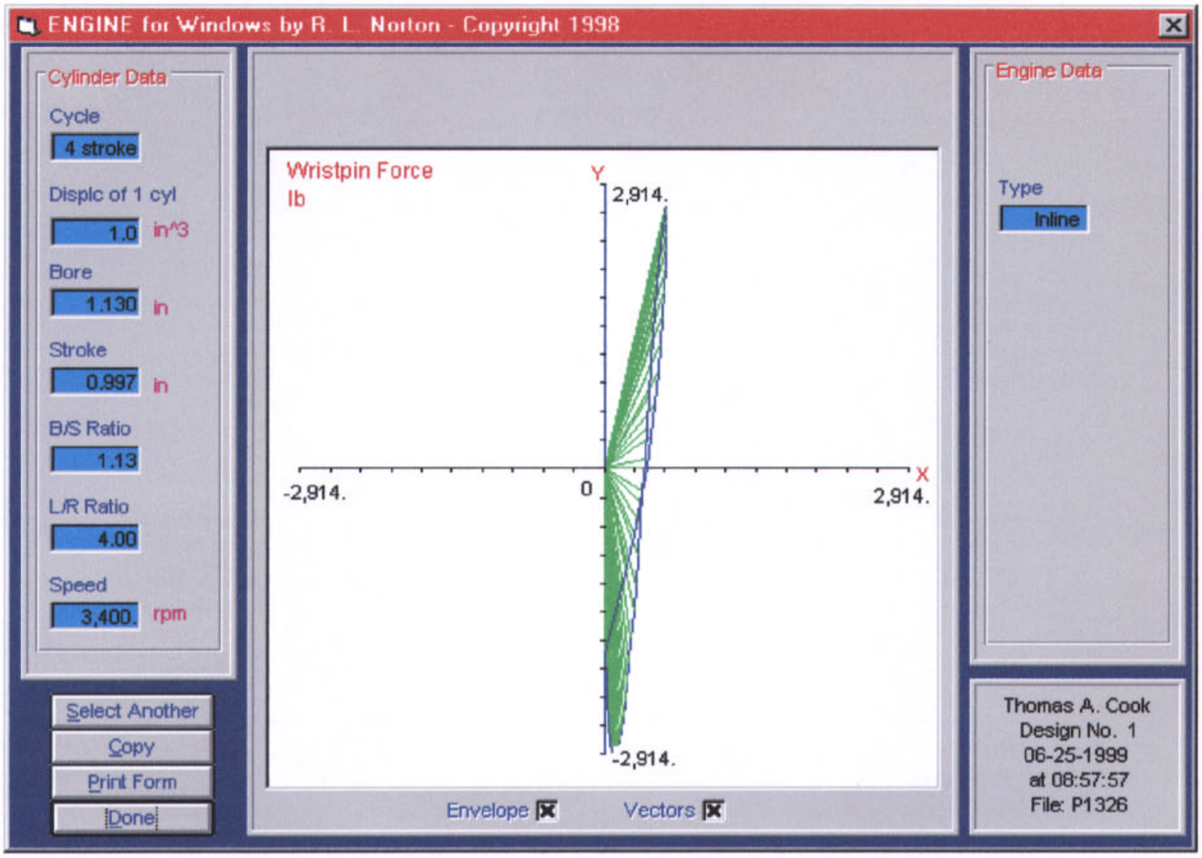
See polar plot on next page.

4. Enter the above data and the masses for part *d* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

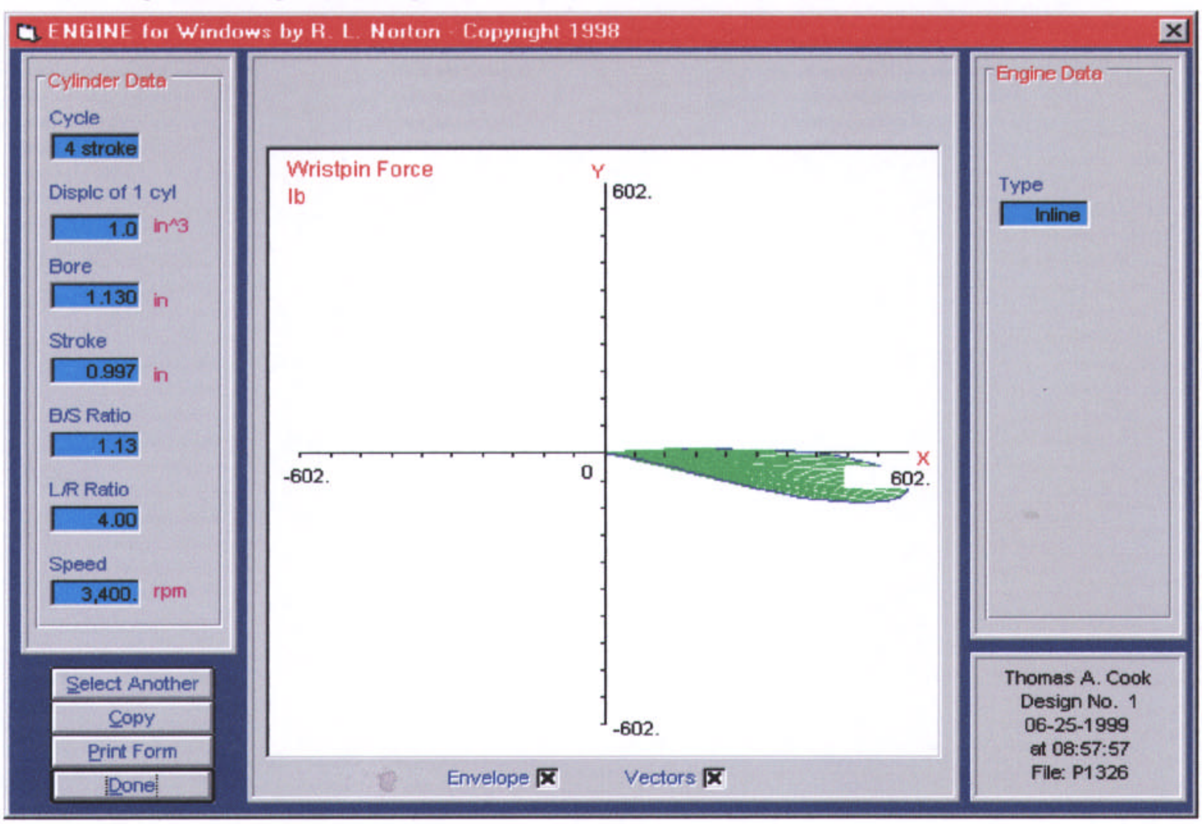
$d.$ $m_2 := 1$ $m_3 := 0$ $m_4 := 0$ $F_{34max} := 602 \cdot lbf$

See polar plot on next page.

Polar plot of wrist pin force for part *c*.



Polar plot of wrist pin force for part *d*.



PROBLEM 13-29

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the main pin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

a. Engine unbalanced.
b. Crank exactly balanced against mass at crankpin.
c. Crank optimally overbalanced against masses at crankpin and wrist pin.

Piston, conrod, and crank masses = 1. Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the main pin force under these different conditions with reference to the governing equations.

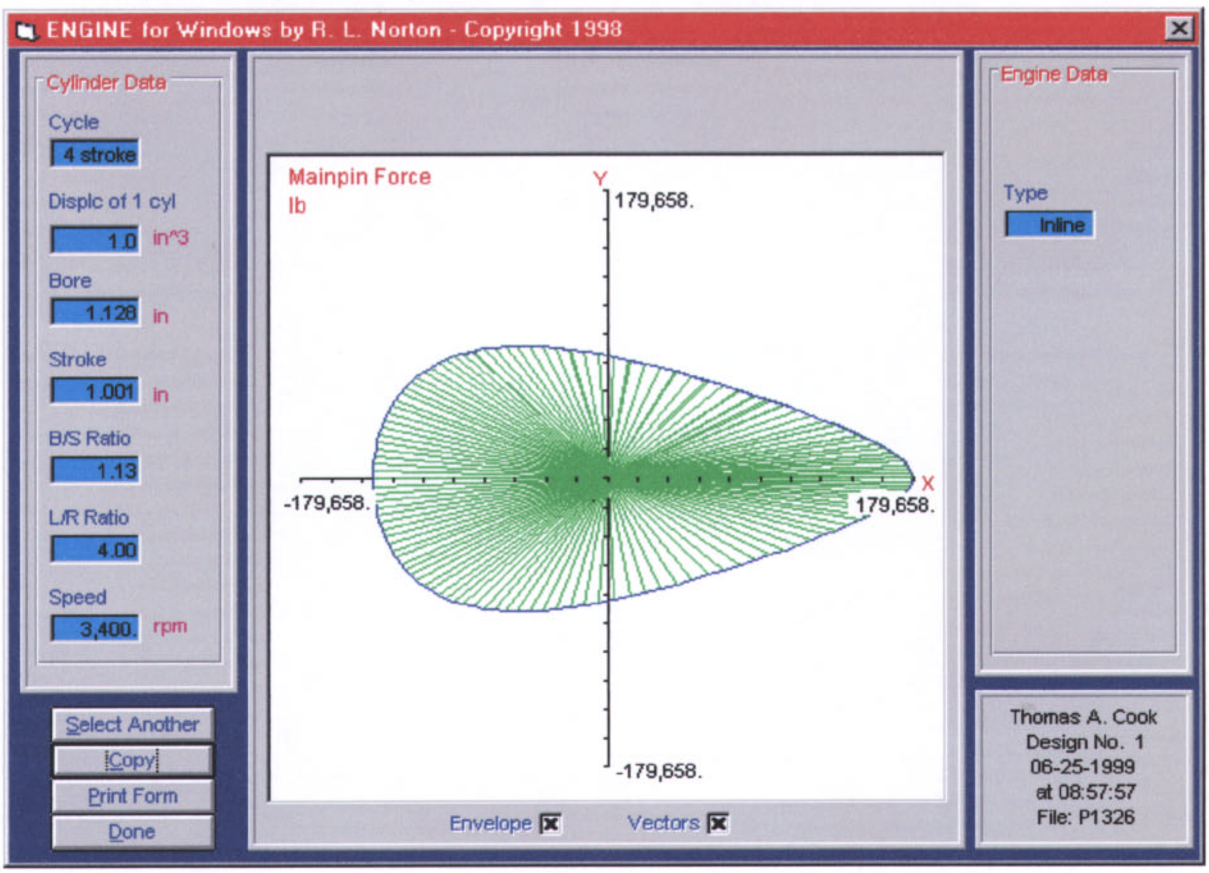
Assumptions:
1. Peak cylinder pressure is 600 psi.
2. Link length ratio is L/R = 4.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

Solution: See Mathcad file P1329.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

a. Engine unbalanced $F_{21max} := 179658 \cdot lbf$

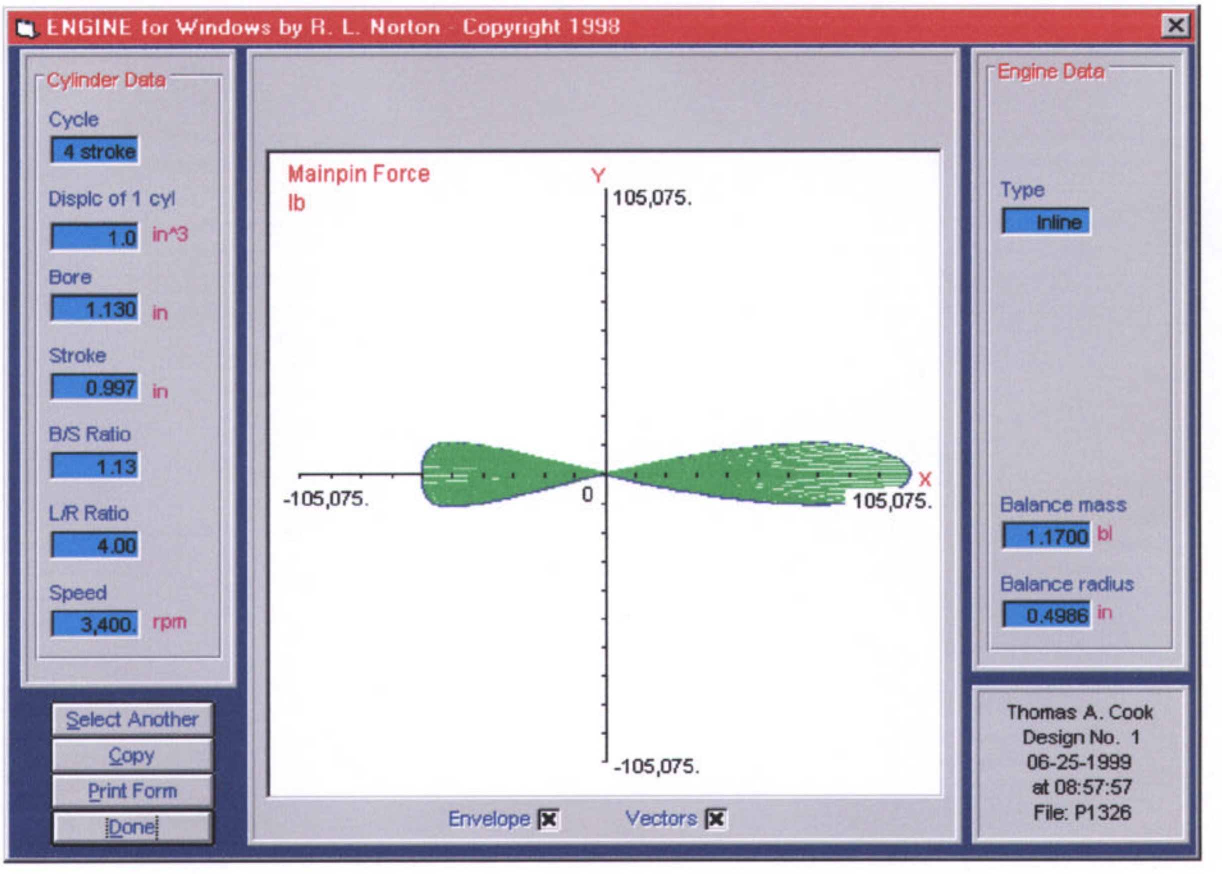
Polar plot of main pin force for part *a*.



2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

b. Crank exactly balanced against mass at crankpin $F_{21max} := 105075 \cdot lbf$

Polar plot of main pin force for part *b*.

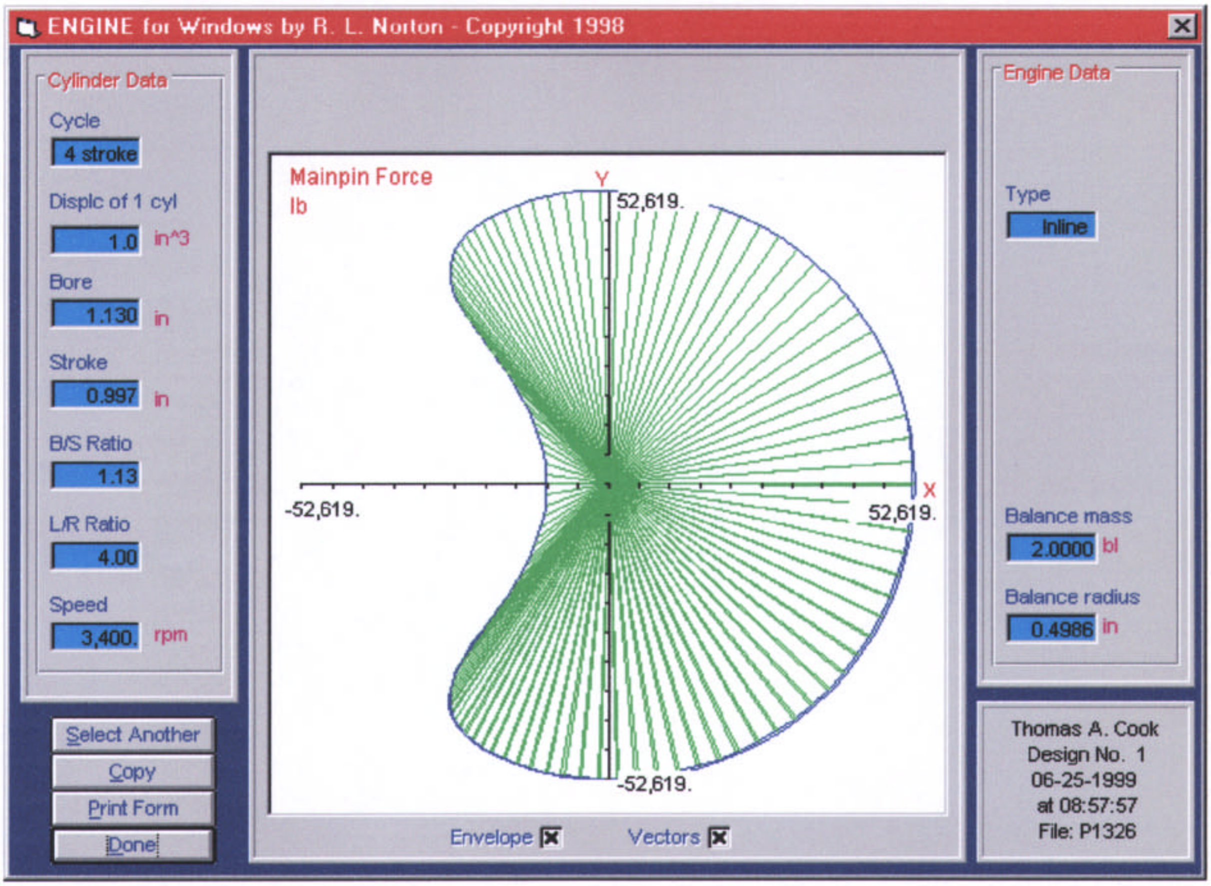


3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the force on the main pin and to get the polar plot of this force.

   c. Crank optimally overbalanced. $F_{21max} := 52619 \cdot lbf$

See polar plot on next page.

Polar plot of main pin force for part *c*.



PROBLEM 13-30

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the crankpin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

a. Engine unbalanced.
b. Crank exactly balanced against mass at crankpin.
c. Crank optimally overbalanced against masses at crankpin and wrist pin.

Piston, conrod, and crank masses = 1. Place the CG of the crank at 0.5*r* and the conrod at 0.33*l*. Compare and explain the differences in the crankpin force under these different conditions with reference to the governing equations.

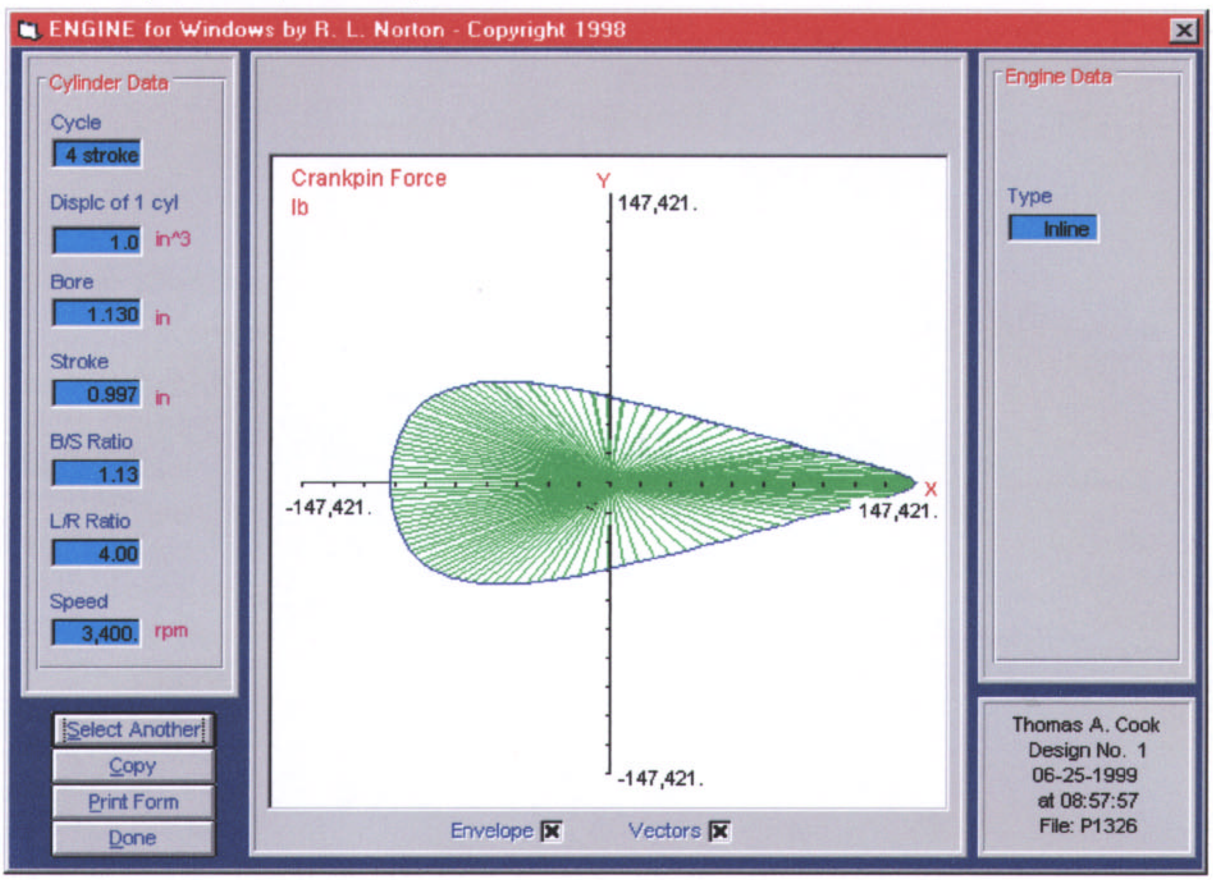
Assumptions:
1. Peak cylinder pressure is 600 psi.
2. Link length ratio is L/R = 4.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

Solution: See Mathcad file P1330.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

a. Engine unbalanced $F_{21max} := 147421 \cdot lbf$

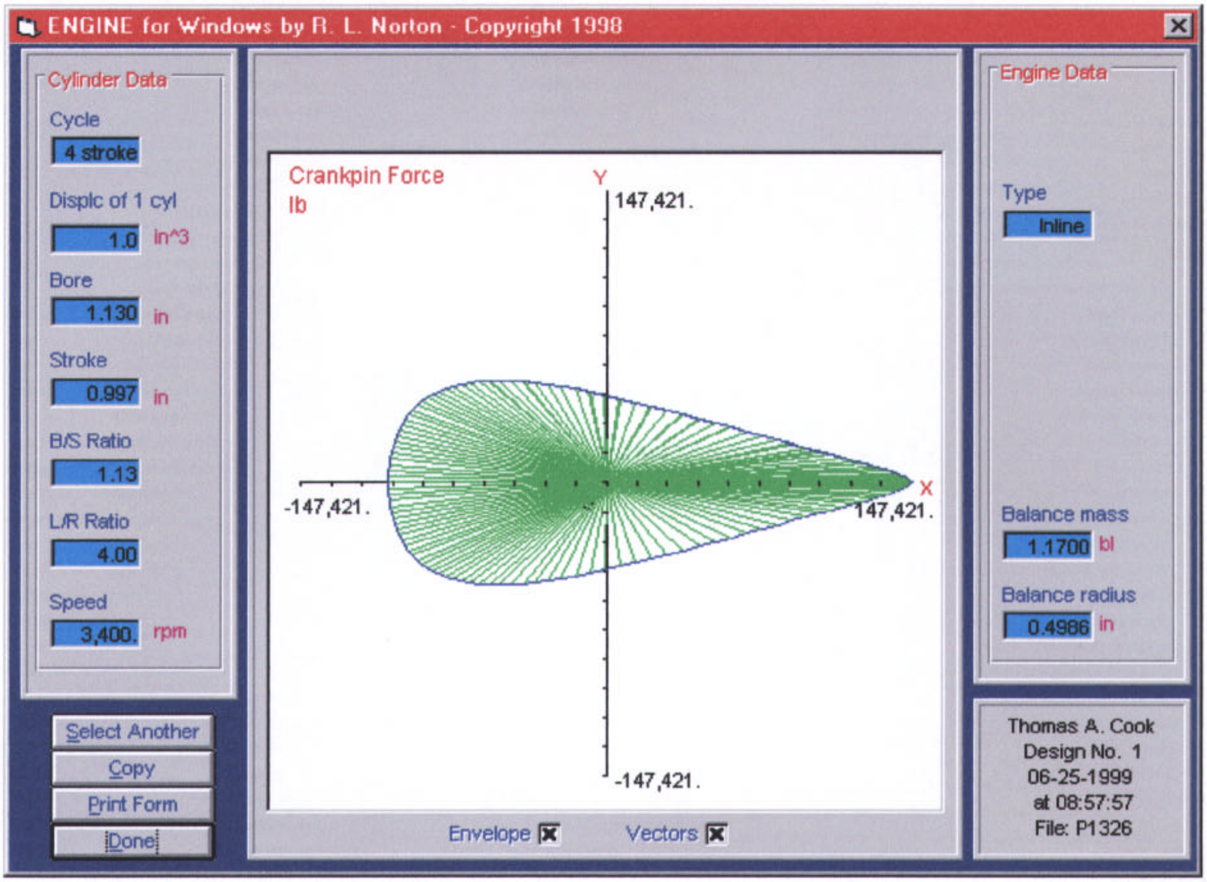
Polar plot of crankpin force for part *a*.



2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

b. Crank exactly balanced against mass at crankpin $F_{21max} := 147421 \cdot lbf$

Polar plot of crankpin force for part *b*.

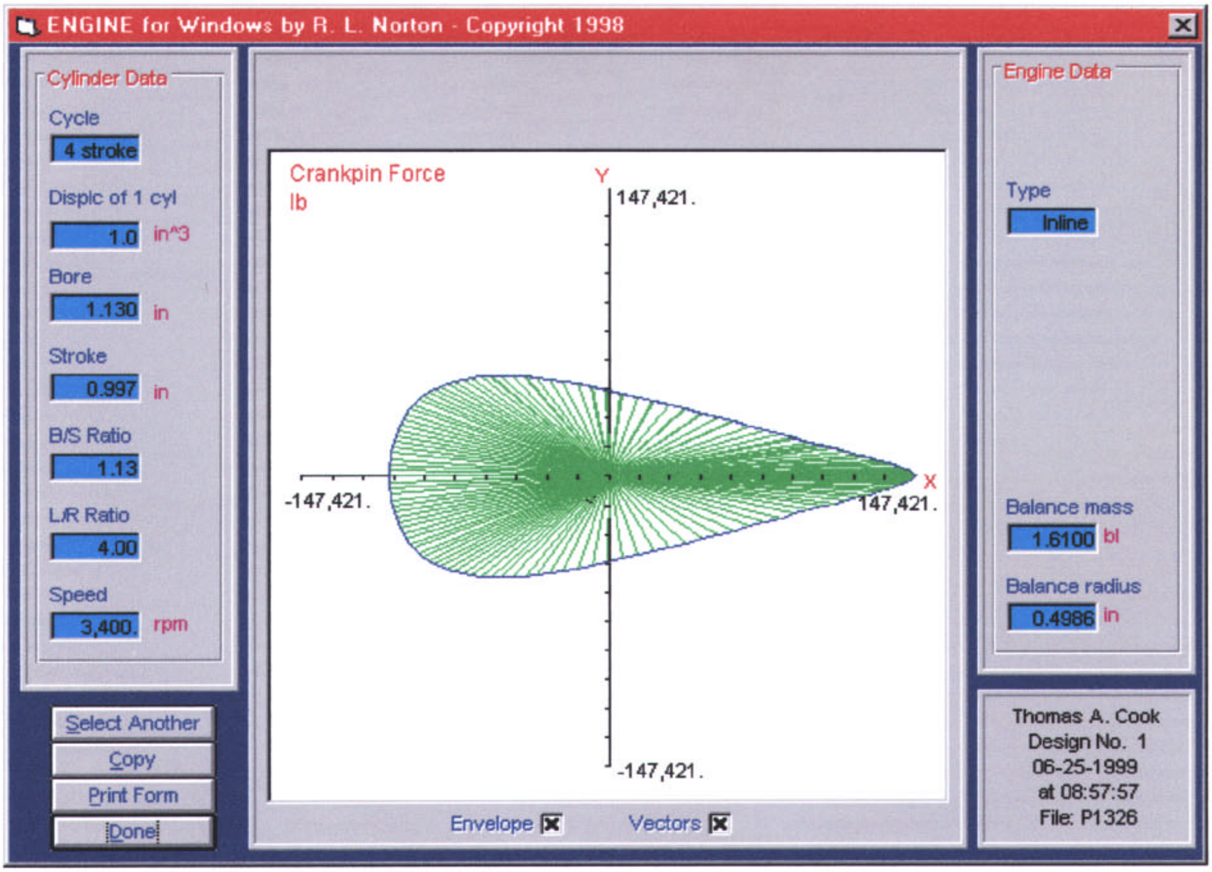


3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the force on the crankpin and to get the polar plot of this force.

c. Crank optimally overbalanced. $F_{21max} := 147421 \cdot lbf$

See polar plot on next page.

Polar plot of crankpin force for part *c*.



PROBLEM 13-31

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the wrist pin of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

a. Engine unbalanced.

b. Crank exactly balanced against mass at crankpin.

c. Crank optimally overbalanced against masses at crankpin and wrist pin.

Piston, conrod, and crank masses = 1. Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the wrist pin force under these different conditions with reference to the governing equations.

Assumptions:

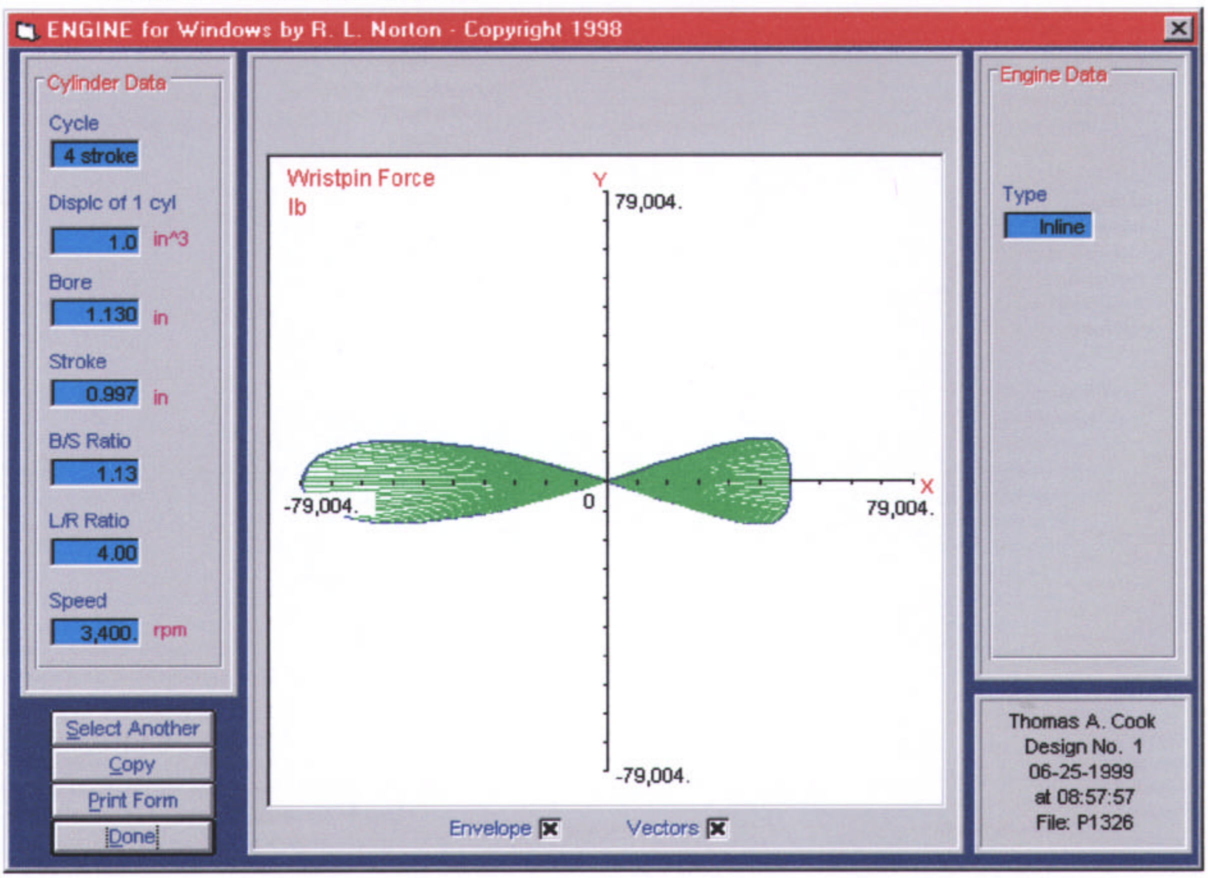
1. Peak cylinder pressure is 600 psi.
2. Link length ratio is L/R = 4.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

Solution: See Mathcad file P1331.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

a. Engine unbalanced $F_{21max} := 79004 \cdot lbf$

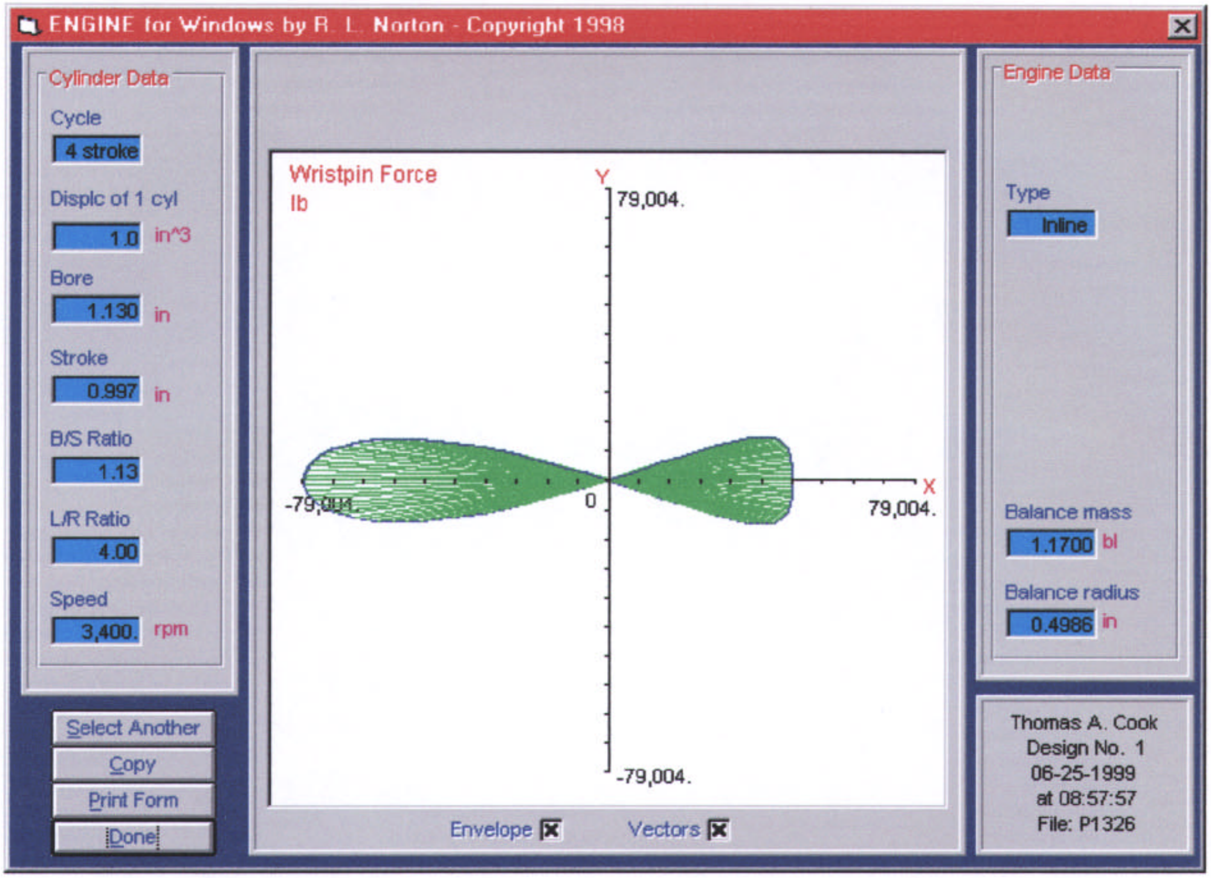
Polar plot of wrist pin force for part *a*.



2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

b. Crank exactly balanced against mass at crankpin $F_{21max} := 79004 \cdot lbf$

Polar plot of wrist pin force for part *b*.

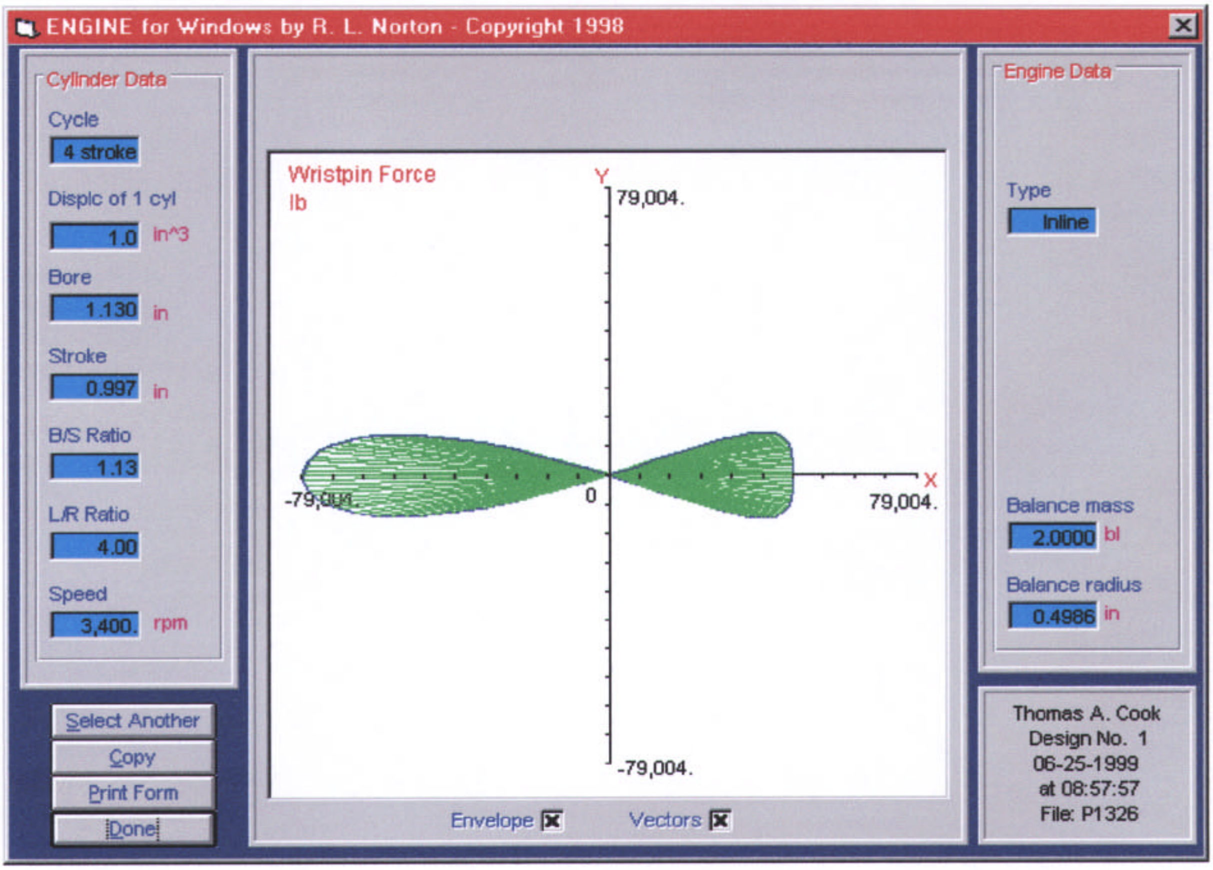


3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the force on the wrist pin and to get the polar plot of this force.

c. Crank optimally overbalanced. $F_{21max} := 79004 \cdot lbf$

See polar plot on next page.

Polar plot of wrist pin force for part *c*.



PROBLEM 13-32

Statement: Use program ENGINE, your own computer program, or an equation solver to calculate the maximum value and the polar plot shape of the force on the shaking force of a one-cubic-inch displacement, single-cylinder engine with bore = 1.12838 in for the following situations.

a. Engine unbalanced.
b. Crank exactly balanced against mass at crankpin.
c. Crank optimally overbalanced against masses at crankpin and wrist pin.

Piston, conrod, and crank masses = 1. Place the CG of the crank at $0.5r$ and the conrod at $0.33l$. Compare and explain the differences in the shaking force force under these different conditions with reference to the governing equations.

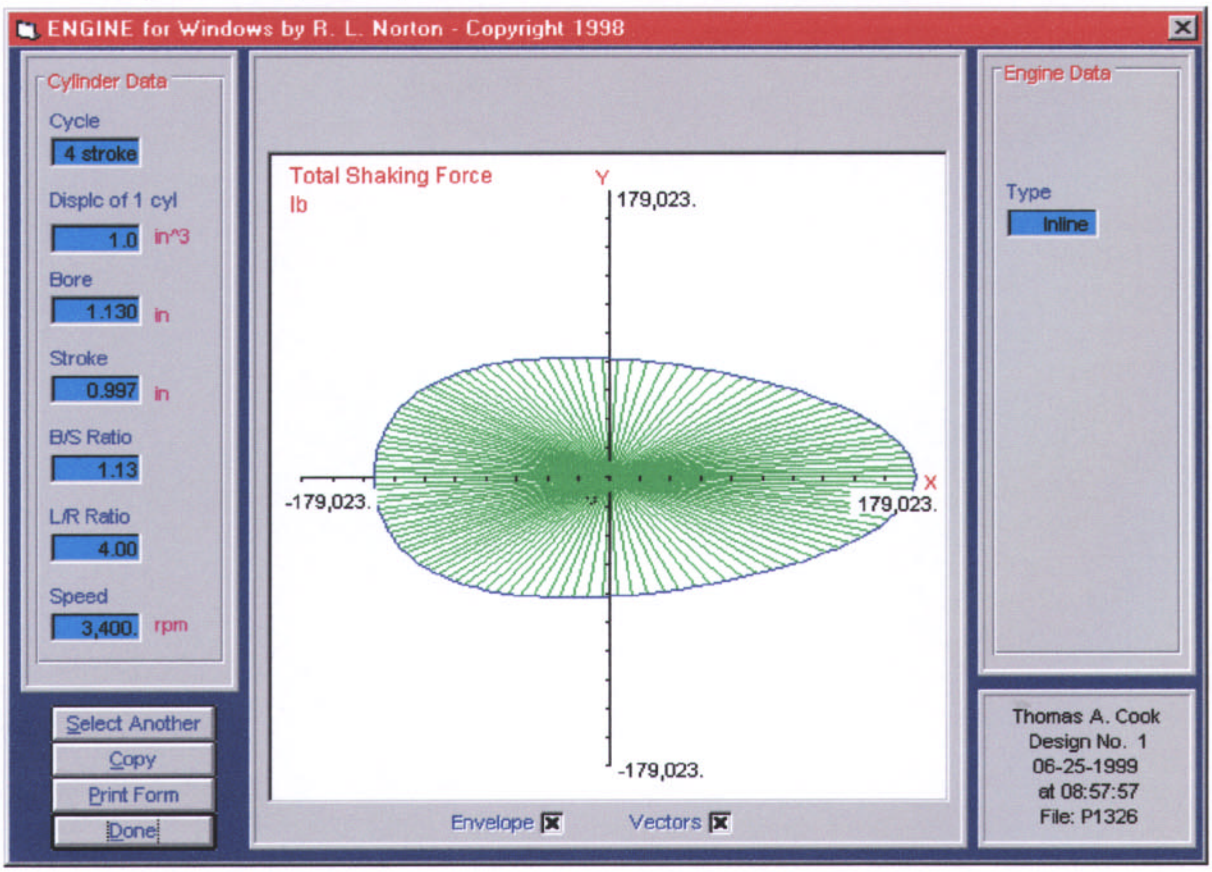
Assumptions:
1. Peak cylinder pressure is 600 psi.
2. Link length ratio is L/R = 4.
3. Main pin dia is 2.00 in.
4. Crankpin dia is 1.500 in.
5. Friction coefficient is zero.

Solution: See Mathcad file P1332.

1. Enter the above data and the masses for part *a* into program ENGINE to determine the maximum value of the shaking force and to get the polar plot of this force.

a. Engine unbalanced $F_{21max} := 179023 \cdot lbf$

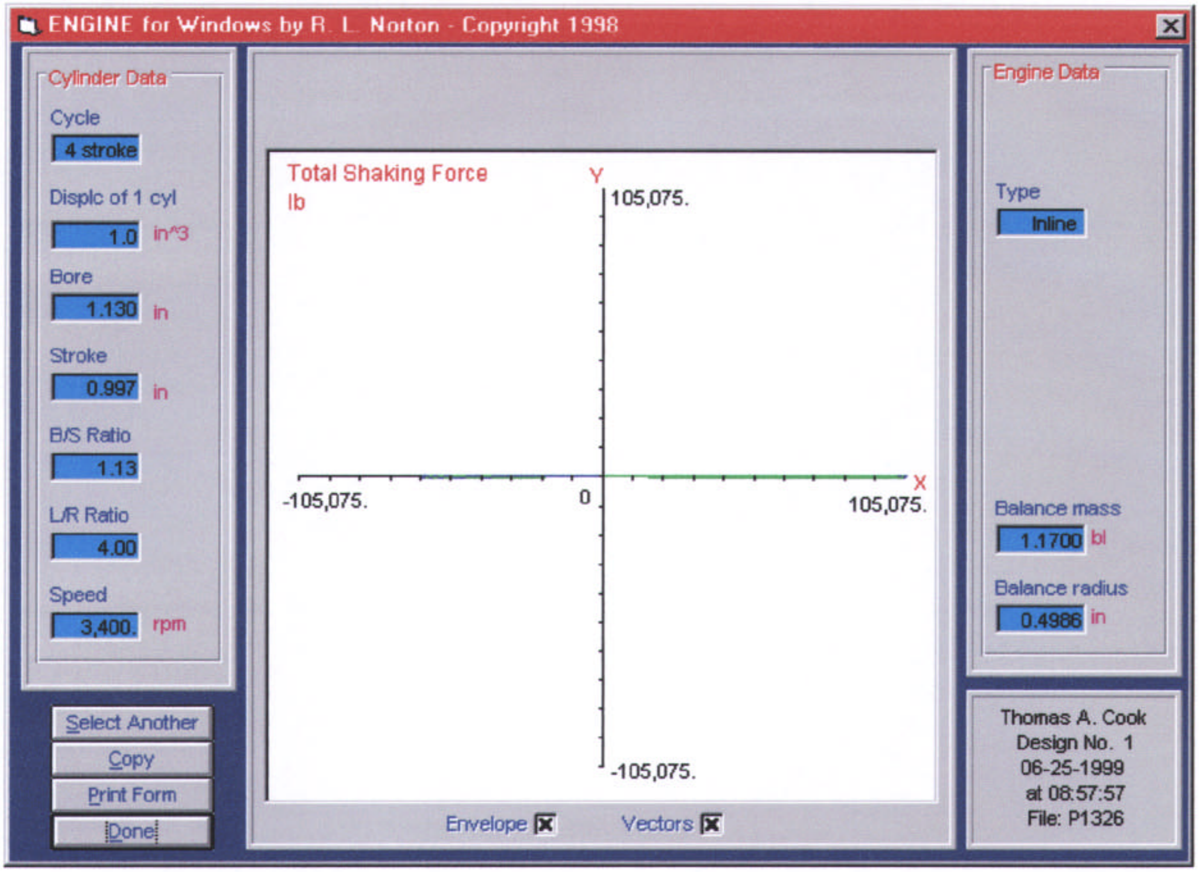
Polar plot of shaking force for part *a*.



2. Enter the above data and the masses for part *b* into program ENGINE to determine the maximum value of the shaking force and to get the polar plot of this force.

b. Crank exactly balanced against mass at crankpin $F_{21max} := 105075 \cdot lbf$

Polar plot of shaking force for part *b*.

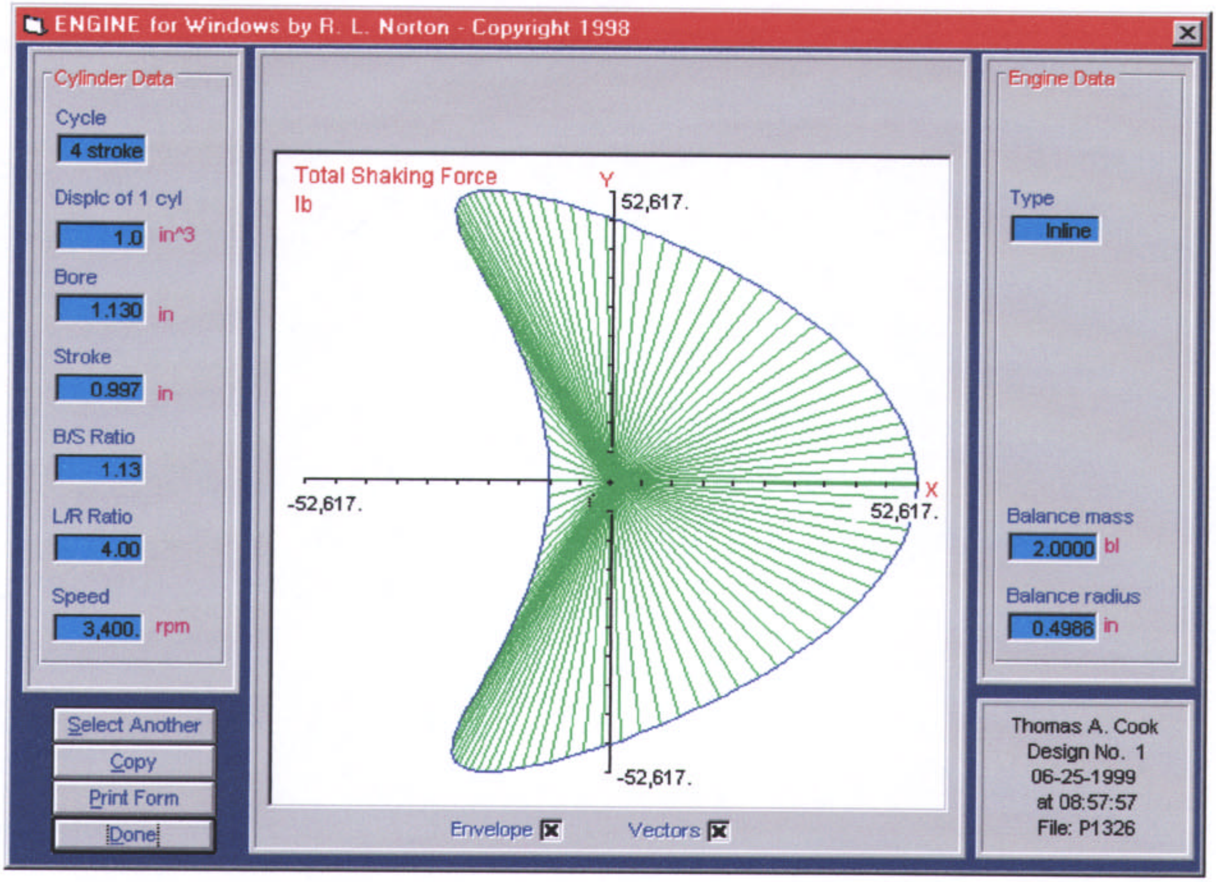


3. Enter the above data and the masses for part *c* into program ENGINE to determine the maximum value of the shaking force and to get the polar plot of this force.

c. Crank optimally overbalanced. $F_{21max} := 52617 \cdot lbf$

See polar plot on next page.

Polar plot of shaking force for part *c*.



PROBLEM 13-33

Statement: Figure P13-1 shows a single-cylinder air compressor stopped at top dead center (TDC). There is a static pressure P = 100 psi trapped in the 3-in-bore cylinder. The entire assembly weighs 30 lb. Draw the necessary free-body diagrams to determine the forces at points *A*, *B*, and *C*, and the supports R_1 and R_2, which are symmetrically located about the piston centerline. Assume that the piston remains stationary.

Given: Pressure: $p := 100\,psi$ Bore: $d := 3.00\,in$

Solution: See Figure P13-1 and Mathcad file P1333.

1. Calculate the gas force:

$$F_g := p\cdot\left(\frac{\pi\cdot d^2}{4}\right) \qquad F_g = 706.9\,lbf$$

2. Draw the FBDs. There will be four FBDs: the piston, the conrod, the crank, and the housing or body. The piston will have two downward forces; the gas force F_g and the piston weight; and one upward force F_{34} acting at point *C*, whose magnitude is the sum of the gas force and the piston weight. The conrod will have two downward forces; F_{43} and the conrod weight; and one upward force F_{23} acting at point *B*, whose magnitude is the sum of the gas force and the weights of the piston and the conrod. The crank will have two downward forces; F_{32} and the crank weight; and one upward force F_{12} acting at point *A*, whose magnitude is the sum of the gas force and the weights of the piston, conrod, and crank. The housing will have two downward forces; F_{21} acting at *A* and the weight of the housing acting at its CG; and three upward forces; F_g acting at the top of the cylinder, and the reaction forces R_1 and R_2, whose magnitudes will be one-half the total weight of the assembly, which is 15 lb each. The gas force causes compression in piston, conrod, and crank; and tension in the housing. The net force at the supports is the total assembly weight.

PROBLEM 13-34

Statement: Calculate and plot the position, velocity, and acceleration of a slider-crank linkage with $r = 3$, $l = 12$, and $\omega = 200$ rad/sec over one cycle using the exact solution and the approximate Fourier solution. Also, calculate and plot the percent difference between the exact and approximate solutions for acceleration.

Given: Link lengths: $r := 3$ $\quad l := 12$

Angular velocity: $\omega := 200 \cdot rad \cdot sec^{-1}$

Solution: See Mathcad file P1334.

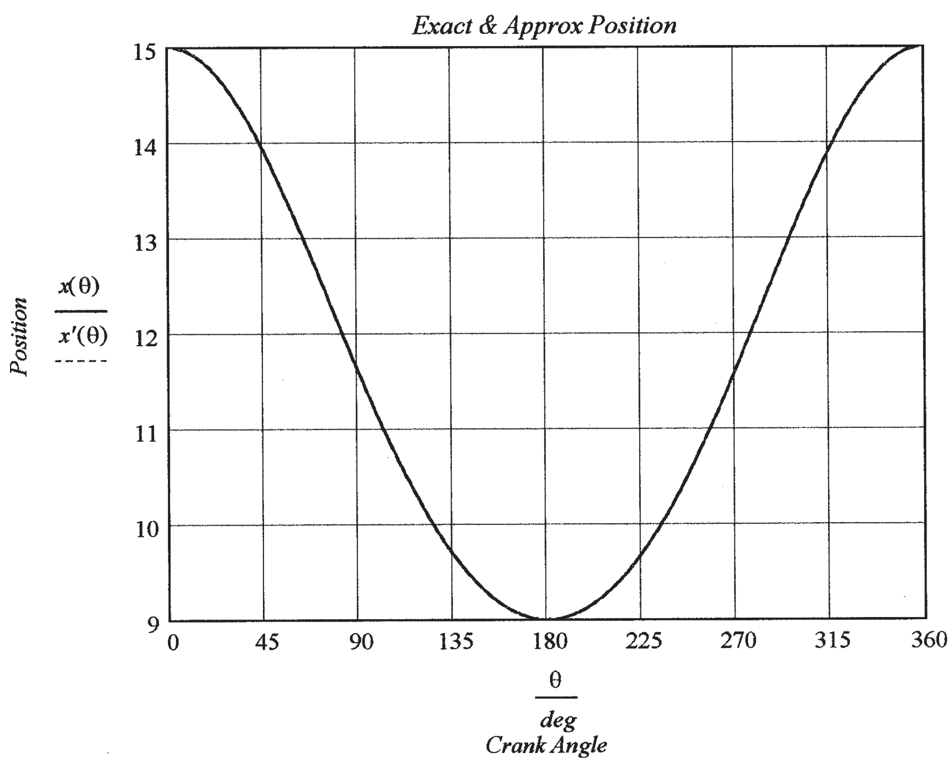
1. Define the crank angle over one cycle.

$$\theta := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

2. Calculate and plot the exact and approximate position using equations 13.1d and 13.3c.

Exact: $$x(\theta) := r \cdot cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}$$

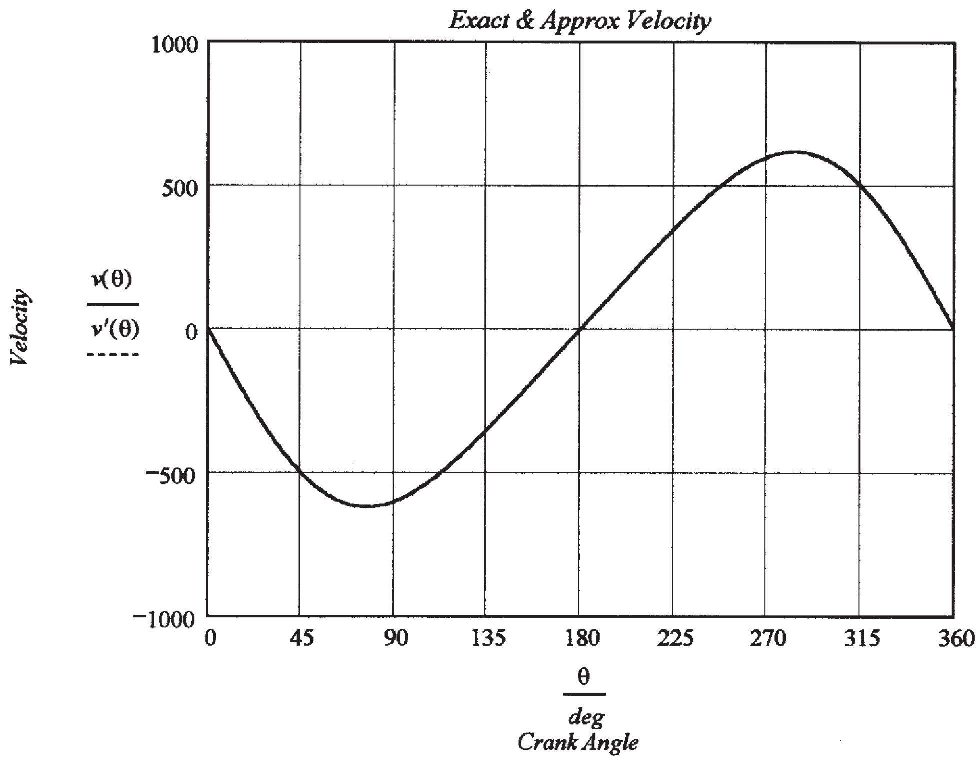
Approximate: $$x'(\theta) := l - \frac{r^2}{4 \cdot l} + r \cdot \left(cos(\theta) + \frac{r}{4 \cdot l} \cdot cos(2 \cdot \theta)\right)$$



3. Calculate and plot the exact and approximate velocity using equations 13.1e and 13.3d.

Exact: $$v(\theta) := -r \cdot \omega \cdot \left[sin(\theta) + \frac{r}{2 \cdot l} \cdot \frac{sin(2 \cdot \theta)}{\sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}} \right]$$

Approximate: $$v'(\theta) := -r\cdot\omega\cdot\left(\sin(\theta) + \frac{r}{2\cdot l}\cdot\sin(2\cdot\theta)\right)$$



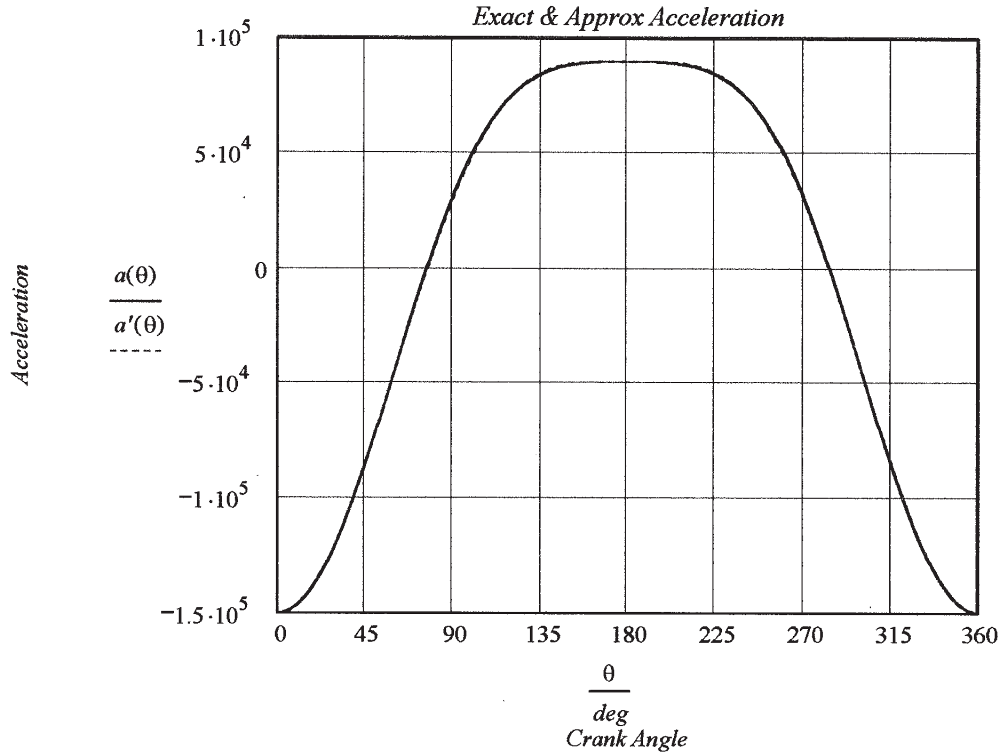
4. Calculate and plot the exact and approximate acceleration using equations 13.1f and 13.3e.

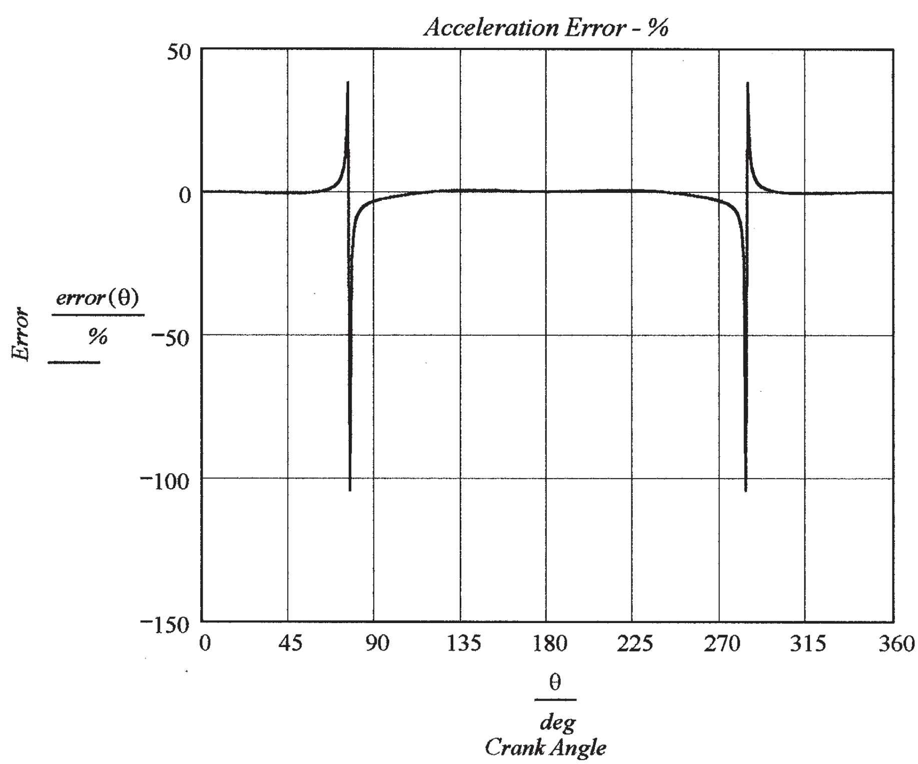
Exact: $$a(\theta) := -r\cdot\omega^2\cdot\left[\cos(\theta) - \frac{r\cdot\left[l^2\cdot\left(1 - 2\cdot\cos(\theta)^2\right) - r^2\cdot\sin(\theta)^4\right]}{\left(l^2 - r^2\cdot\sin(\theta)^2\right)^{\frac{3}{2}}}\right]$$

Approximate: $$a'(\theta) := -r\cdot\omega^2\cdot\left(\cos(\theta) + \frac{r}{l}\cdot\cos(2\cdot\theta)\right)$$

5. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error(\theta) := \frac{a'(\theta) - a(\theta)}{a(\theta)}$$


Exact & Approx Acceleration
1·10^5
5·10^4
0
−5·10^4
−1·10^5
−1.5·10^5
Acceleration
a(θ)
a'(θ)
0
45
90
135
180
225
270
315
360
θ
deg
Crank Angle


Acceleration Error - %
50
0
−50
−100
−150
Error
error(θ)
%
0
45
90
135
180
225
270
315
360
θ
deg
Crank Angle

PROBLEM 13-35

Statement: Calculate and plot the position, velocity, and acceleration of a slider-crank linkage with $r = 3$, $l = 15$, and $\omega = 100$ rad/sec over one cycle using the exact solution and the approximate Fourier solution. Also, calculate and plot the percent difference between the exact and approximate solutions for acceleration.

Given: Link lengths: $r := 3 \quad l := 15$

Angular velocity: $\omega := 100 \cdot rad \cdot sec^{-1}$

Solution: See Mathcad file P1335.

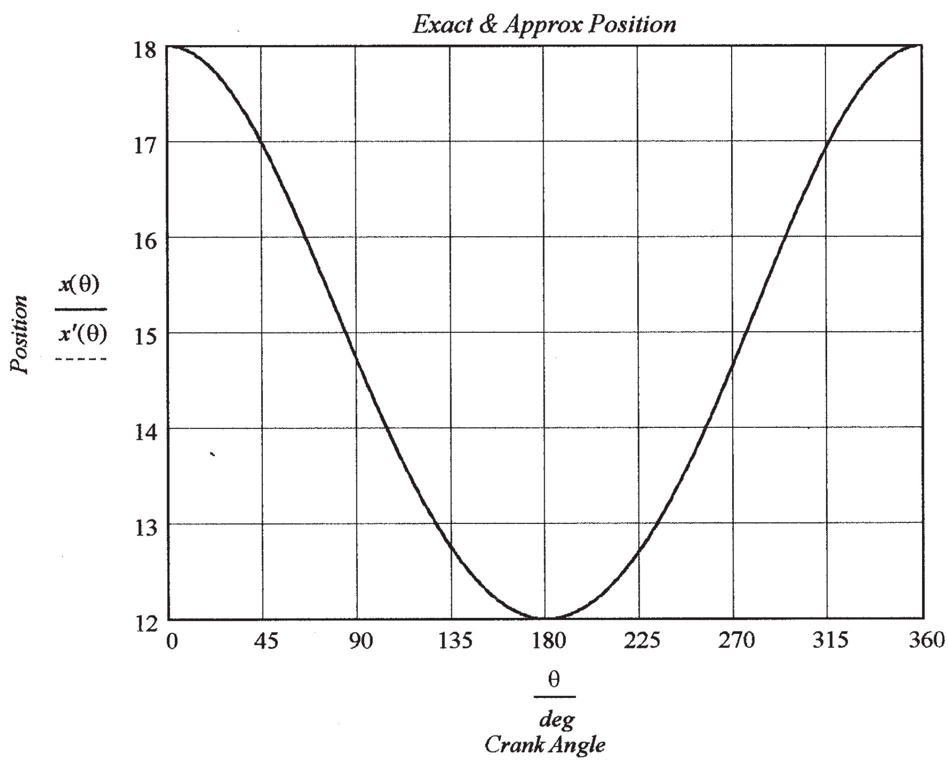
1. Define the crank angle over one cycle.

$$\theta := 0 \cdot deg, 1 \cdot deg .. 360 \cdot deg$$

2. Calculate and plot the exact and approximate position using equations 13.1d and 13.3c.

Exact: $$x(\theta) := r \cdot cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}$$

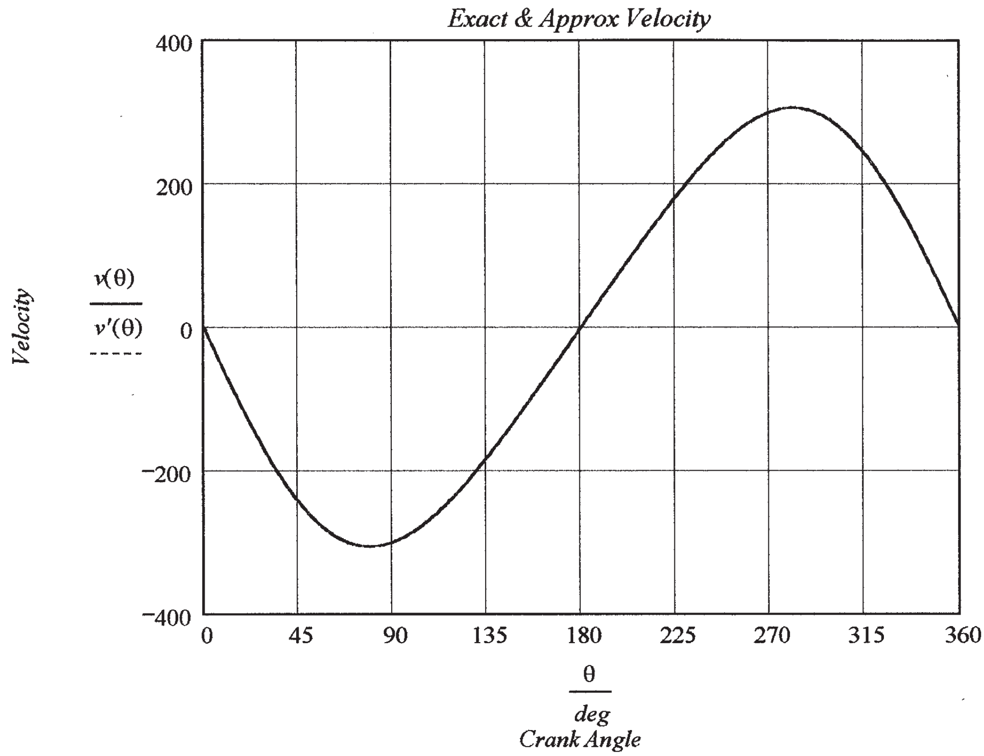
Approximate: $$x'(\theta) := l - \frac{r^2}{4 \cdot l} + r \cdot \left(cos(\theta) + \frac{r}{4 \cdot l} \cdot cos(2 \cdot \theta)\right)$$



3. Calculate and plot the exact and approximate velocity using equations 13.1e and 13.3d.

Exact: $$v(\theta) := -r \cdot \omega \cdot \left[sin(\theta) + \frac{r}{2 \cdot l} \cdot \frac{sin(2 \cdot \theta)}{\sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}} \right]$$

Approximate: $v'(\theta) := -r\cdot\omega\cdot\left(\sin(\theta) + \frac{r}{2\cdot l}\cdot\sin(2\cdot\theta)\right)$



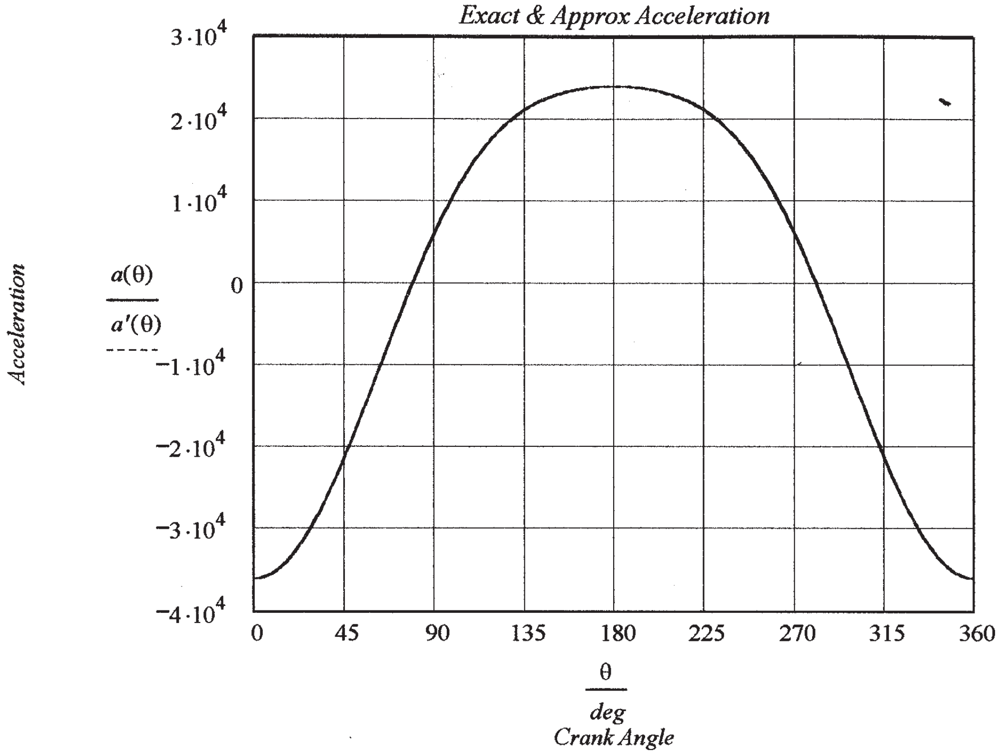
4. Calculate and plot the exact and approximate acceleration using equations 13.1f and 13.3e.

Exact: $$a(\theta) := -r\cdot\omega^2\cdot\left[\cos(\theta) - \frac{r\cdot\left[l^2\cdot\left(1 - 2\cdot\cos(\theta)^2\right) - r^2\cdot\sin(\theta)^4\right]}{\left(l^2 - r^2\cdot\sin(\theta)^2\right)^{\frac{3}{2}}}\right]$$

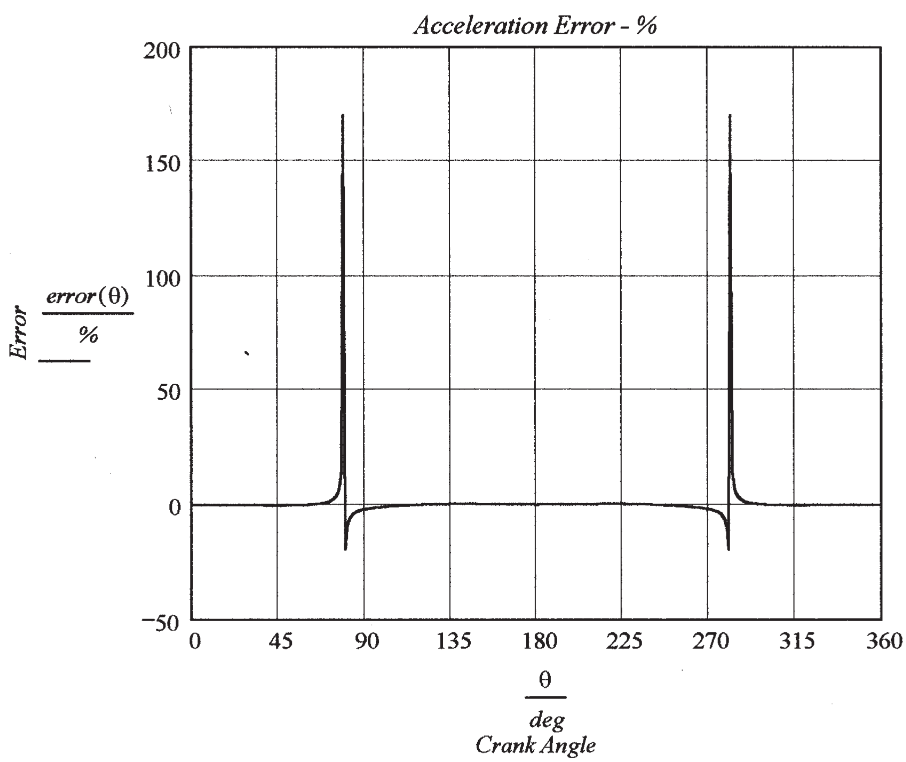
Approximate: $$a'(\theta) := -r\cdot\omega^2\cdot\left(\cos(\theta) + \frac{r}{l}\cdot\cos(2\cdot\theta)\right)$$

5. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error(\theta) := \frac{a'(\theta) - a(\theta)}{a(\theta)}$$



Exact & Approx Acceleration
3·10^4
2·10^4
1·10^4
0
−1·10^4
−2·10^4
−3·10^4
−4·10^4
Acceleration
a(θ)
a'(θ)
0
45
90
135
180
225
270
315
360
θ
deg
Crank Angle



Acceleration Error - %
200
150
100
50
0
−50
Error
error(θ)
%
0
45
90
135
180
225
270
315
360
θ
deg
Crank Angle

PROBLEM 13-36

Statement: A slider-crank linkage has the dimensions and speed at t = 0 given below. Its initial crank angle is zero. Calculate the piston acceleration at the time specified below. Use two methods, the exact solution, and the approximate Fourier series solution and compare the results.

Given: Link lengths: $r := 3 \cdot in$ $\quad l := 9 \cdot in$

Initial angular velocity: $\omega := 100 \cdot rad \cdot sec^{-1}$ $\quad$ Time span: $t := 0.01 \cdot sec$

Solution: See Mathcad file P1336.

1. Calculate the exact acceleration using equation 13.1f.

$$a_{exact} := -r \cdot \omega^2 \cdot \left[cos(\omega \cdot t) - \frac{r \cdot \left[l^2 \cdot \left(1 - 2 \cdot cos(\omega \cdot t)^2\right) - r^2 \cdot sin(\omega \cdot t)^4 \right]}{\left(l^2 - r^2 \cdot sin(\omega \cdot t)^2\right)^{\frac{3}{2}}} \right]$$

$$a_{exact} = -12133.3 \frac{in}{sec^2}$$

2. Calculate the approximate acceleration using equation 13.3e.

$$a_{approx} := -r \cdot \omega^2 \cdot \left(cos(\omega \cdot t) + \frac{r}{l} \cdot cos(2 \cdot \omega \cdot t) \right) \qquad a_{approx} = -12047.6 \frac{in}{sec^2}$$

3. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error := \frac{a_{approx} - a_{exact}}{a_{exact}} \qquad error = -0.706\,\%$$

PROBLEM 13-37

Statement: A slider-crank linkage has the dimensions and speed at t = 0 given below. Its initial crank angle is zero. Calculate the piston acceleration at the time specified below. Use two methods, the exact solution, and the approximate Fourier series solution and compare the results.

Given: Link lengths: $r := 3 \cdot in$ $\quad l := 15 \cdot in$

Initial angular velocity: $\omega := 100 \cdot rad \cdot sec^{-1}$ $\quad$ Time span: $t := 0.02 \cdot sec$

Solution: See Mathcad file P1337.

1. Calculate the exact acceleration using equation 13.1f.

$$a_{exact} := -r \cdot \omega^2 \cdot \left[cos(\omega \cdot t) - \frac{r \cdot \left[l^2 \cdot \left(1 - 2 \cdot cos(\omega \cdot t)^2\right) - r^2 \cdot sin(\omega \cdot t)^4 \right]}{\left(l^2 - r^2 \cdot sin(\omega \cdot t)^2\right)^{\frac{3}{2}}} \right]$$

$$a_{exact} = 16436.6 \frac{in}{sec^2}$$

2. Calculate the approximate acceleration using equation 13.3e.

$$a_{approx} := -r \cdot \omega^2 \cdot \left(cos(\omega \cdot t) + \frac{r}{l} \cdot cos(2 \cdot \omega \cdot t) \right) \qquad a_{approx} = 16406.3 \frac{in}{sec^2}$$

3. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error := \frac{a_{approx} - a_{exact}}{a_{exact}} \qquad error = -0.185\,\%$$

PROBLEM 13-38

Statement: The equation given below is an approximation of the gas force over 180 deg of crank angle. Using this equation with β = 15 deg and F_{gmax} = 1200 lb, calculate and plot the approximate gas torque for r = 4 in and l = 12 in. What is the total energy delivered by the gas force over the 180 deg of motion? What is the average power delivered if the crank rotates at a constant speed of 1500 rpm?

Units: $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Link lengths: $r := 4\cdot in$ $\quad l := 12\cdot in$ $\qquad$ Speed: $n := 1500\, rpm$

Maximum gas force: $F_{gmax} := 1200\cdot lbf$ $\qquad$ Gas force parameter: $\beta := 15\cdot deg$

Gas force equation:

$$F_{g1}(\theta) := F_{gmax}\cdot sin\left(\frac{\theta}{\beta}\cdot\frac{\pi}{2}\right) \qquad F_{g2}(\theta) := \frac{F_{gmax}}{2}\cdot\left(1 + cos\left(\pi\cdot\frac{\theta-\beta}{\pi-\beta}\right)\right)$$

$$F_g(\theta) := if\left(\theta \le \beta, F_{g1}(\theta), F_{g2}(\theta)\right)$$

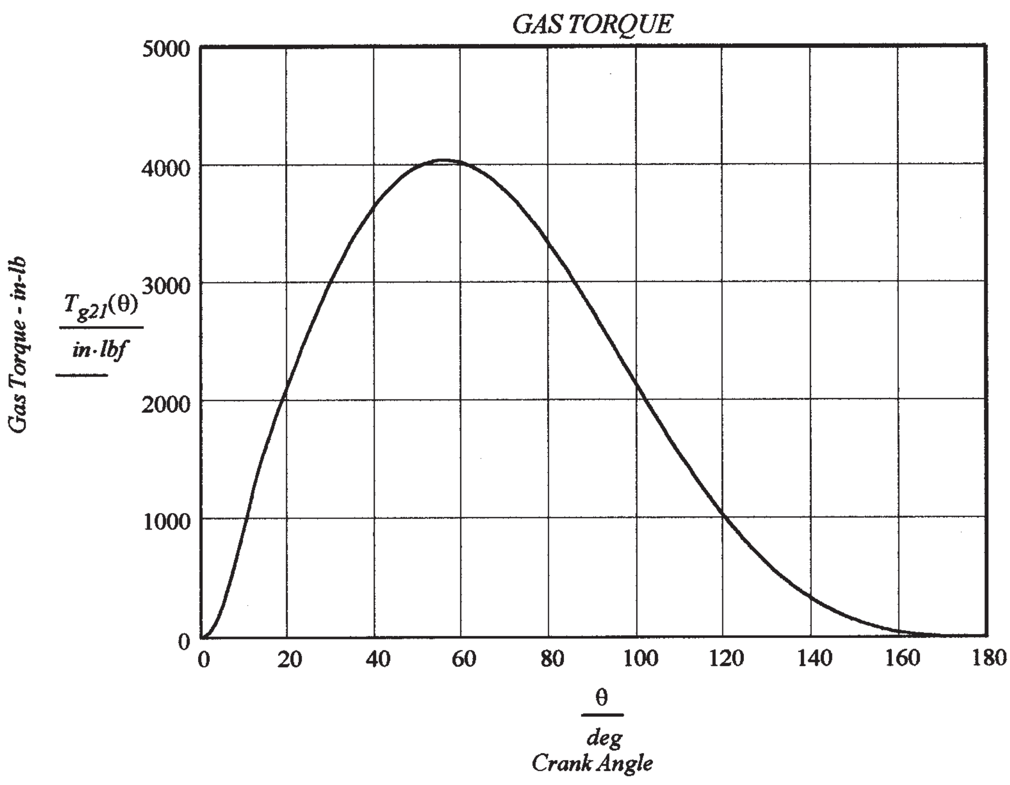
Solution: See Figure P2-17 and Mathcad file P0236.

1. Define the crank angle over 180 deg.

$$\theta := 0\cdot deg, 1\cdot deg .. 180\cdot deg$$

2. Using equation 13.8b, calculate and plot the approximate gas torque as a function of crank angle.

$$T_{g21}(\theta) := F_g(\theta)\cdot r\cdot sin(\theta)\cdot\left(1 + \frac{r}{l}\cdot cos(\theta)\right)$$



3. Integrate the gas torque over the range of crank motion (180 deg) to determine the energy delivered.

$$E := \int_0^{\pi} T_{g21}(\theta)\, d\theta \qquad E = 485.7\, lbf \cdot ft$$

4. Divide the energy delivered by the time for the crank to travel 180 deg to get the average power.

$$t := \frac{180 \cdot deg}{n} \qquad t = 0.020\ \mathrm{sec}$$

$$P_{avg} := \frac{E}{t} \qquad P_{avg} = 44.2\, hp$$

PROBLEM 13-39

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the gas force and gas torque at this position.

Given: Link lengths: $r := 3.75 \cdot in$ $\quad l := 11 \cdot in$

Piston bore: $B := 2.5 \cdot in$ $\quad$ Peak pressure: $p_g := 1150 \cdot psi$ $\quad$ Crank angle: $\theta := 12 \cdot deg$

Assumptions: An approximate solution is acceptable.

Solution: See Mathcad file P1339.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \qquad F_g = 5645\ lbf$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21} := F_g \cdot r \cdot \sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot \cos(\theta)\right) \qquad T_{g21} = 5869\ in \cdot lbf$$

PROBLEM 13-40

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the exact gas torque at this position and compare it to that obtained by the approximate expression in equation 13.8b. What is the percent error?

Given: Link lengths: $r := 3.75\cdot in$ $l := 11\cdot in$

Piston bore: $B := 2.5\cdot in$ Peak pressure: $p_g := 1150\cdot psi$ Crank angle: $\theta := 12\cdot deg$

Solution: See Mathcad file P1340.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4}\cdot p_g\cdot B^2 \qquad F_g = 5645\ lbf$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21a} := F_g\cdot r\cdot \sin(\theta)\cdot\left(1 + \frac{r}{l}\cdot\cos(\theta)\right) \qquad T_{g21a} = 5868.91\ in\cdot lbf$$

3. Calculate the exact gas torque on the crank using equations 13.7b, 13.1d, and 13.6d.

$$\phi := atan\left[\frac{r\cdot\sin(\theta)}{l\cdot\sqrt{1-\left(\frac{r}{l}\cdot\sin(\theta)\right)^2}}\right] \qquad \phi = 4.064\ deg$$

$$x := r\cdot\cos(\theta) + l\cdot\sqrt{1-\left(\frac{r}{l}\cdot\sin(\theta)\right)^2} \qquad x = 14.640\ in$$

$$T_{g21e} := F_g\cdot\tan(\phi)\cdot x \qquad T_{g21e} = 5872.61\ in\cdot lbf$$

4. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error := \frac{T_{g21a} - T_{g21e}}{T_{g21e}} \qquad error = -0.0630\ \%$$

✍ PROBLEM 13-41

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the gas force and gas torque at this position.

Given: Link lengths: $r := 4.12 \cdot in$ $\quad l := 14.5 \cdot in$

Piston bore: $B := 2.25 \cdot in$ Peak pressure: $p_g := 1325 \cdot psi$ Crank angle: $\theta := 9 \cdot deg$

Assumptions: An approximate solution is acceptable.

Solution: See Mathcad file P1341.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \qquad F_g = 5268\ lbf$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21} := F_g \cdot r \cdot sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot cos(\theta)\right) \qquad T_{g21} = 4348\ in \cdot lbf$$

PROBLEM 13-42

Statement: A slider-crank linkage has the dimensions given below. The peak gas pressure and crank angle are also given. Calculate the exact gas torque at this position and compare it to that obtained by the approximate expression in equation 13.8b. What is the percent error?

Given: Link lengths: $r := 4.12 \cdot in$ $l := 14.5 \cdot in$

Piston bore: $B := 2.25 \cdot in$ Peak pressure: $p_g := 1325 \cdot psi$ Crank angle: $\theta := 9 \cdot deg$

Solution: See Mathcad file P1342.

1. Calculate the gas force on the piston using equation 13.4.

$$F_g := \frac{\pi}{4} \cdot p_g \cdot B^2 \qquad F_g = 5268\, lbf$$

2. Calculate the approximate gas torque on the crank using equation 13.8b.

$$T_{g21a} := F_g \cdot r \cdot sin(\theta) \cdot \left(1 + \frac{r}{l} \cdot cos(\theta)\right) \qquad T_{g21a} = 4348.38\, in \cdot lbf$$

3. Calculate the exact gas torque on the crank using equations 13.7b, 13.1d, and 13.6d.

$$\phi := atan\left[\frac{r \cdot sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}}\right] \qquad \phi = 2.548\, deg$$

$$x := r \cdot cos(\theta) + l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2} \qquad x = 18.555\, in$$

$$T_{g21e} := F_g \cdot tan(\phi) \cdot x \qquad T_{g21e} = 4349.32\, in \cdot lbf$$

4. Compare the results by calculating the error in the approximation as a percent of the exact.

$$error := \frac{T_{g21a} - T_{g21e}}{T_{g21e}} \qquad error = -0.0217\,\%$$

PROBLEM 13-43

Statement: The dimensions and mass properties of a slider-crank linkage are given below. Determine the approximately dynamically equivalent two-mass lumped parameter model for this linkage with the masses placed at the crank and wrist pins.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Masses: $m_2 := 0.045\,blob$ $m_3 := 0.120\,blob$ $m_4 := 0.15\,blob$

Crank CG location as fraction of its length: $r_2 := 0.4$

Assumption: Two-thirds of the conrod mass is placed at the crank pin and one-third at the wrist pin.

Solution: See Mathcad file P1343.

1. Determine the lumped masses for link 3 using equations 13.10a and 13.10b and the assumption as to mass distribution.

Masses: $m_{3a} := m_3 \cdot \frac{2}{3}$ $m_{3a} = 0.0800\,blob$

$m_{3b} := m_3 \cdot \frac{1}{3}$ $m_{3b} = 0.04000\,blob$

2. Determine the lumped mass at point A due to the crank alone using equation 13.11.

Masses: $m_{2a} := m_2 \cdot r_2$ $m_{2a} = 0.0180\,blob$

3. Determine the two-mass model with masses placed at the crank and wrist pins using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.098\,blob$

$m_B := m_{3b} + m_4$ $m_B = 0.190\,blob$

PROBLEM 13-44

Statement: The dimensions and mass properties of a connecting rod are given below. Calculate the sizes of two dynamically equivalent masses and the location of one if the other is placed at point *B* (see Figure 13-10).

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 12.5 in$ mass: $m_3 := 0.120 blob$

Mass moment of inertia: $I_{G3} := 0.15 \cdot blob \cdot in^2$

Distance to CG : $l_a := 4.5 \cdot in$

Solution: See Mathcad file P1344.

1. Determine the exact model using equations 13.9d and 13.9e.

Distance from point *B* to CG: $l_b := l - l_a$ $\quad l_b = 8.000\, in$

Distance from CG to lumped mass at P: $l_p := \dfrac{I_{G3}}{m_3 \cdot l_b}$ $\quad l_p = 0.156\, in$

Masses: $m_p := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_p = 0.118\, blob$

$m_b := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_b = 0.00230\, blob$

PROBLEM 13-45

Statement: The dimensions and mass properties of a slider-crank linkage are given below. Determine the approximately dynamically equivalent two-mass lumped parameter model for this linkage with the masses placed at the crank and wrist pins.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Masses: $m_2 := 0.060\,blob$ $m_3 := 0.180\,blob$ $m_4 := 0.160\,blob$

Crank CG location as fraction of its length: $r_2 := 0.38$

Assumption: Two-thirds of the conrod mass is placed at the crank pin and one-third at the wrist pin.

Solution: See Mathcad file P1345.

1. Determine the lumped masses for link 3 using equations 13.10a and 13.10b and the assumption as to mass distribution.

Masses: $m_{3a} := m_3 \cdot \frac{2}{3}$ $m_{3a} = 0.1200\,blob$

$m_{3b} := m_3 \cdot \frac{1}{3}$ $m_{3b} = 0.06000\,blob$

2. Determine the lumped mass at point *A* due to the crank alone using equation 13.11.

Masses: $m_{2a} := m_2 \cdot r_2$ $m_{2a} = 0.0228\,blob$

3. Determine the two-mass model with masses at the crank and wrist pins using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.143\,blob$

$m_B := m_{3b} + m_4$ $m_B = 0.220\,blob$

PROBLEM 13-46

Statement: The dimensions and mass properties of a connecting rod are given below. Calculate the sizes of two dynamically equivalent masses and the location of one if the other is placed at point *B* (see Figure 13-10).

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$

Given: Conrod length: $l := 10.4 in$ mass: $m_3 := 0.180 blob$

Mass moment of inertia: $I_{G3} := 0.12 \cdot blob \cdot in^2$

Distance to CG : $l_a := 4.16 \cdot in$

Solution: See Mathcad file P1346.

1. Determine the exact model using equations 13.9d and 13.9e.

Distance from point *B* to CG: $l_b := l - l_a$ $l_b = 6.240\ in$

Distance from CG to lumped mass at P: $l_p := \dfrac{I_{G3}}{m_3 \cdot l_b}$ $l_p = 0.107\ in$

Masses: $m_p := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $m_p = 0.177\ blob$

$m_b := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $m_b = 0.00303\ blob$

PROBLEM 13-47

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 12.5 in$ mass: $m_3 := 0.120 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.36$

Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.045 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.015 \cdot blob$

Crank speed: $\omega := 1800 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1347.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.500\ in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 8.000\ in$

Let $l_p := l_a$

$$m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \qquad m_{3a} = 0.0768\ blob$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \qquad m_{3b} = 0.04320\ blob$$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.252\ in$

Mass: $m_{2a} := m_2 \cdot \frac{r_{G2}}{r}$ $\quad m_{2a} = 0.0180\ blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \qquad m_A = 0.0948\ blob$$

$$m_B := m_{3b} + m_4 \qquad m_B = 0.0582\ blob$$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \qquad F_{ix} = 15546\ lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = 5271\ lbf$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_i = 16415\ lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 18.731\ deg$$

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot cos(2 \cdot \theta)\right) \qquad T_{i21} = -11943\ in \cdot lbf$$

PROBLEM 13-48

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 10.4 in$ mass: $m_3 := 0.180 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.40$

Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.38$

Piston mass: $m_4 := 0.016 \cdot blob$

Crank speed: $\omega := 1850 \cdot rpm$ Crank angle: $\theta := 20 \cdot deg$

Solution: See Mathcad file P1348.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.160\ in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 6.240\ in$

Let $l_p := l_a$

$$m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \qquad m_{3a} = 0.1080\ blob$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \qquad m_{3b} = 0.07200\ blob$$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 0.988\ in$

Mass:

$$m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \qquad m_{2a} = 0.0228\ blob$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \qquad m_A = 0.1308\ blob$$

$$m_B := m_{3b} + m_4 \qquad m_B = 0.0880\ blob$$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \qquad F_{ix} = 21708\ lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = 4365\ lbf$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_i = 22143\ lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 11.371\ deg$$

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot cos(2 \cdot \theta)\right) \qquad T_{i21} = -10324\ in \cdot lbf$$

PROBLEM 13-49

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 10.4 in$ mass: $m_3 := 0.120 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.36$

Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.045 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.40$

Piston mass: $m_4 := 0.015 \cdot blob$

Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 25 \cdot deg$

Solution: See Mathcad file P1349.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 3.744 in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 6.656 in$

Let $l_p := l_a$ $\quad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.0768 blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.04320 blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.040 in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\quad m_{2a} = 0.0180 blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.0948 blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.0582 blob$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \qquad F_{ix} = 16881 lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = 4569 lbf$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_i = 17489 lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 15.145 deg$$

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot cos(2 \cdot \theta)\right) \qquad T_{i21} = -9280 in \cdot lbf$$

PROBLEM 13-50

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the inertia force and inertia torque for the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 12.5 in$ mass: $m_3 := 0.180 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.40$

Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.38$

Piston mass: $m_4 := 0.015 \cdot blob$

Crank speed: $\omega := 1500 \cdot rpm$ Crank angle: $\theta := 22 \cdot deg$

Solution: See Mathcad file P1350.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\qquad l_a = 5.000\ in$

Distance from point B to CG: $l_b := l - l_a$ $\qquad l_b = 7.500\ in$

Let $l_p := l_a$ $\qquad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\qquad m_{3a} = 0.1080\ blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\qquad m_{3b} = 0.07200\ blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\qquad r_{G2} = 1.189\ in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\qquad m_{2a} = 0.0228\ blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\qquad m_A = 0.1308\ blob$

$m_B := m_{3b} + m_4$ $\qquad m_B = 0.0870\ blob$

4. Calculate the inertia force and inertia torque using equations 13.14d and 13.15d.

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \qquad F_{ix} = 16806\ lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = 3784\ lbf$$

$$F_i := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_i = 17227\ lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 12.689\ deg$$

$$T_{i21} := -m_B \cdot r^2 \cdot \omega^2 \cdot sin(\theta) \cdot \left(\frac{r}{2 \cdot l} + cos(\theta) + \frac{3 \cdot r}{2 \cdot l} \cdot cos(2 \cdot \theta)\right) \qquad T_{i21} = -10419\ in \cdot lbf$$

PROBLEM 13-51

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces torque on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Conrod length: $l := 12.5 in$ mass: $m_3 := 0.120 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.36$

Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.045 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.40$

Piston mass: $m_4 := 0.150 \cdot blob$ Gas force: $F_g := 600 \cdot lbf$

Crank speed: $\omega := 1800 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Mathcad file P1351.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\qquad l_a = 4.500\ in$

Distance from point B to CG: $l_b := l - l_a$ $\qquad l_b = 8.000\ in$

Let $l_p := l_a$

$$m_{3a} := m_3 \cdot \frac{l_b}{l_p + l_b} \qquad m_{3a} = 0.0768\ blob$$

$$m_{3b} := m_3 \cdot \frac{l_p}{l_p + l_b} \qquad m_{3b} = 0.04320\ blob$$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\qquad r_{G2} = 1.252\ in$

Mass:
$$m_{2a} := m_2 \cdot \frac{r_{G2}}{r} \qquad m_{2a} = 0.0180\ blob$$

3. Calculate the lumped masses at points A and B using equations 13.12.

$$m_A := m_{2a} + m_{3a} \qquad m_A = 0.0948\ blob$$

$$m_B := m_{3b} + m_4 \qquad m_B = 0.1932\ blob$$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := atan\left[\frac{r \cdot sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}}\right] \qquad \phi = 7.192\ deg$$

$$a_B := -r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right) \qquad a_B = -110234.9 \frac{in}{sec^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -\left[(m_4 + m_{3b}) \cdot a_B + F_g\right] \cdot tan(\phi) \qquad F_{41} = 2612\ lbf$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$F_{34x} := F_g + m_4 \cdot a_B$ $\qquad F_{34x} = -15935\ lbf$

$F_{34y} := -\left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi)$ $\qquad F_{34y} = 2611.9\ lbf$

$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2}$ $\qquad F_{34} = 16147.9\ lbf$

$\theta_{34} := atan2\left(F_{34x}, F_{34y}\right)$ $\qquad \theta_{34} = 170.692\ deg$

7. Determine the crankpin force F_{32} using equation 13.22.

$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot cos(\theta) - \left(m_{3b} + m_4\right) \cdot a_B - F_g$ $\qquad F_{32x} = 28094\ lbf$

$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot sin(\theta) + \left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi)$ $\qquad F_{32y} = 1658.6\ lbf$

$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2}$ $\qquad F_{32} = 28143\ lbf$

$\theta_{32} := atan2\left(F_{32x}, F_{32y}\right)$ $\qquad \theta_{32} = 3.379\ deg$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot cos(\theta) + F_{32x}$ $\qquad F_{21x} = 29828\ lbf$

$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot sin(\theta) + F_{32y}$ $\qquad F_{21y} = 2659.5\ lbf$

$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2}$ $\qquad F_{21} = 29946\ lbf$

$\theta_{21} := atan2\left(F_{21x}, F_{21y}\right)$ $\qquad \theta_{21} = 5.095\ deg$

PROBLEM 13-52

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces torque on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 10.4 in$ mass: $m_3 := 0.180 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.40$

Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.38$

Piston mass: $m_4 := 0.160 \cdot blob$ Gas force: $F_g := 600 \cdot lbf$

Crank speed: $\omega := 1850 \cdot rpm$ Crank angle: $\theta := 20 \cdot deg$

Solution: See Mathcad file P1352.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.160 \, in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 6.240 \, in$

Let $l_p := l_a$ $\quad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.1080 \, blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.07200 \, blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 0.988 \, in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\quad m_{2a} = 0.0228 \, blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.1308 \, blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.2320 \, blob$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := atan\left[\frac{r \cdot sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}}\right] \qquad \phi = 4.905 \, deg$$

$$a_B := -r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right) \qquad a_B = -110386.2 \frac{in}{sec^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -\left[(m_4 + m_{3b}) \cdot a_B + F_g\right] \cdot tan(\phi) \qquad F_{41} = 2146 \, lbf$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B \qquad F_{34x} = -17062\ lbf$$

$$F_{34y} := -\left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi) \qquad F_{34y} = 2146.3\ lbf$$

$$F_{34} := \sqrt{F_{34x}^{\ 2} + F_{34y}^{\ 2}} \qquad F_{34} = 17196.3\ lbf$$

$$\theta_{34} := atan2\left(F_{34x}, F_{34y}\right) \qquad \theta_{34} = 172.830\ deg$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot cos(\theta) - \left(m_{3b} + m_4\right) \cdot a_B - F_g \qquad F_{32x} = 34913\ lbf$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot sin(\theta) + \left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi) \qquad F_{32y} = 1458.2\ lbf$$

$$F_{32} := \sqrt{F_{32x}^{\ 2} + F_{32y}^{\ 2}} \qquad F_{32} = 34943\ lbf$$

$$\theta_{32} := atan2\left(F_{32x}, F_{32y}\right) \qquad \theta_{32} = 2.392\ deg$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot cos(\theta) + F_{32x} \qquad F_{21x} = 37004\ lbf$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot sin(\theta) + F_{32y} \qquad F_{21y} = 2219.2\ lbf$$

$$F_{21} := \sqrt{F_{21x}^{\ 2} + F_{21y}^{\ 2}} \qquad F_{21} = 37070\ lbf$$

$$\theta_{21} := atan2\left(F_{21x}, F_{21y}\right) \qquad \theta_{21} = 3.432\ deg$$

PROBLEM 13-53

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces torque on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 10.4 in$ mass: $m_3 := 0.120 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.36$

Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.045 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.40$

Piston mass: $m_4 := 0.150 \cdot blob$ Gas force: $F_g := 350 \cdot lbf$

Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 25 \cdot deg$

Solution: See Mathcad file P1353.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 3.744\ in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 6.656\ in$

Let $l_p := l_a$ $\quad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.0768\ blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.04320\ blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.040\ in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\quad m_{2a} = 0.0180\ blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.0948\ blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.1932\ blob$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := atan\left[\frac{r \cdot sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}}\right] \qquad \phi = 6.065\ deg$$

$$a_B := -r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right) \qquad a_B = -121690.6 \frac{in}{sec^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -\left[(m_4 + m_{3b}) \cdot a_B + F_g\right] \cdot tan(\phi) \qquad F_{41} = 2461\ lbf$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B \qquad F_{34x} = -17904\ lbf$$

$$F_{34y} := -\left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi) \qquad F_{34y} = 2460.8\ lbf$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2} \qquad F_{34} = 18071.9\ lbf$$

$$\theta_{34} := atan2\left(F_{34x}, F_{34y}\right) \qquad \theta_{34} = 172.174\ deg$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot cos(\theta) - \left(m_{3b} + m_4\right) \cdot a_B - F_g \qquad F_{32x} = 31099\ lbf$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot sin(\theta) + \left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot tan(\phi) \qquad F_{32y} = 1240.9\ lbf$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2} \qquad F_{32} = 31124\ lbf$$

$$\theta_{32} := atan2\left(F_{32x}, F_{32y}\right) \qquad \theta_{32} = 2.285\ deg$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot cos(\theta) + F_{32x} \qquad F_{21x} = 32959\ lbf$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot sin(\theta) + F_{32y} \qquad F_{21y} = 2108.5\ lbf$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2} \qquad F_{21} = 33027\ lbf$$

$$\theta_{21} := atan2\left(F_{21x}, F_{21y}\right) \qquad \theta_{21} = 3.660\ deg$$

PROBLEM 13-54

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below. Calculate the pin forces torque on the linkage.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given:

Conrod length: $l := 12.5\,in$ mass: $m_3 := 0.180\,blob$

Distance to CG (as fraction of l): $r_{3a} := 0.40$

Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.060 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.38$

Piston mass: $m_4 := 0.150 \cdot blob$ Gas force: $F_g := 550 \cdot lbf$

Crank speed: $\omega := 1500 \cdot rpm$ Crank angle: $\theta := 22 \cdot deg$

Solution: See Mathcad file P1354.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 5.000\,in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 7.500\,in$

Let $l_p := l_a$ $\quad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.1080\,blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.07200\,blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.189\,in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\quad m_{2a} = 0.0228\,blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.1308\,blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.2220\,blob$

4. Calculate the conrod angle and acceleration of the piston using equations 13.16e and 13.3e, respectively.

$$\phi := atan\left[\frac{r \cdot sin(\theta)}{l \cdot \sqrt{1 - \left(\frac{r}{l} \cdot sin(\theta)\right)^2}}\right] \qquad \phi = 5.382\,deg$$

$$a_B := -r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right) \qquad a_B = -85516.9\,\frac{in}{sec^2}$$

5. Determine the sidewall force F_{41} using equation 13.20.

$$F_{41} := -\left[\left(m_4 + m_{3b}\right) \cdot a_B + F_g\right] \cdot tan(\phi) \qquad F_{41} = 1737\,lbf$$

6. Determine the wrist pin force F_{34} using equation 13.21.

$$F_{34x} := F_g + m_4 \cdot a_B \qquad F_{34x} = -12278\ lbf$$

$$F_{34y} := -\left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot \tan(\phi) \qquad F_{34y} = 1736.9\ lbf$$

$$F_{34} := \sqrt{F_{34x}^2 + F_{34y}^2} \qquad F_{34} = 12399.8\ lbf$$

$$\theta_{34} := atan2\left(F_{34x}, F_{34y}\right) \qquad \theta_{34} = 171.948\ deg$$

7. Determine the crankpin force F_{32} using equation 13.22.

$$F_{32x} := m_{3a} \cdot r \cdot \omega^2 \cdot \cos(\theta) - \left(m_{3b} + m_4\right) \cdot a_B - F_g \qquad F_{32x} = 26168\ lbf$$

$$F_{32y} := m_{3a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + \left[F_g + \left(m_4 + m_{3b}\right) \cdot a_B\right] \cdot \tan(\phi) \qquad F_{32y} = 1387.7\ lbf$$

$$F_{32} := \sqrt{F_{32x}^2 + F_{32y}^2} \qquad F_{32} = 26205\ lbf$$

$$\theta_{32} := atan2\left(F_{32x}, F_{32y}\right) \qquad \theta_{32} = 3.035\ deg$$

8. Determine the main pin force F_{21} using equations 13.19c and 13.22.

$$F_{21x} := m_{2a} \cdot r \cdot \omega^2 \cdot \cos(\theta) + F_{32x} \qquad F_{21x} = 27801\ lbf$$

$$F_{21y} := m_{2a} \cdot r \cdot \omega^2 \cdot \sin(\theta) + F_{32y} \qquad F_{21y} = 2047.3\ lbf$$

$$F_{21} := \sqrt{F_{21x}^2 + F_{21y}^2} \qquad F_{21} = 27876\ lbf$$

$$\theta_{21} := atan2\left(F_{21x}, F_{21y}\right) \qquad \theta_{21} = 4.212\ deg$$

PROBLEM 13-55

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
a. Exactly balance the crank and recalculate the inertia force.
b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius *-r* on the crank and recalculate the inertia force.
c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Conrod length: $l := 12.5 in$ mass: $m_3 := 0.120 blob$

Distance to CG (as fraction of *l*): $r_{3a} := 0.36$

Crank length: $r := 3.13 \cdot in$ mass: $m_2 := 0.045 \cdot blob$

Distance to CG (as fraction of *r*): $r_{2a} := 0.4$

Piston mass: $m_4 := 0.015 \cdot blob$

Crank speed: $\omega := 1800 \cdot rpm$ Crank angle: $\theta := 30 \cdot deg$

Solution: See Problem 13-47 and Mathcad file P1355.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point *A* to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 4.500\ in$

Distance from point *B* to CG: $l_b := l - l_a$ $\quad l_b = 8.000\ in$

Let $l_p := l_a$ $\quad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.0768\ blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.04320\ blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.252\ in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\quad m_{2a} = 0.0180\ blob$

3. Calculate the lumped masses at points *A* and *B* using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.0948\ blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.0582\ blob$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta) \right) \right] \qquad F_{ix} = 6416\ lbf$$

$$F_{iy} := 0 \cdot lbf \qquad F_{iy} = 0\ lbf$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{ib} = 6416\ lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 0.000\ deg$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \qquad m_p = 0.0194\,blob$$

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot \cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(\cos(\theta) + \frac{r}{l} \cdot \cos(2 \cdot \theta)\right)\right] \ldots + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot \cos(\theta)\right) \qquad F_{ix} = 4547\,lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot \sin(\theta)\right) + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot \sin(\theta)\right) \qquad F_{iy} = -1079\,lbf$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{iob} = 4673\,lbf \quad \text{at} \quad atan2\left(F_{ix}, F_{iy}\right) = -13.346\,deg$$

6. Compare the results to those for an unbalanced crank. From Problem 13-47, the inertia force for the unbalanced crank is $F_i := 16415 \cdot lbf$. The percent differences for the balanced and overbalanced cranks are:

Exactly balanced: $\Delta := \frac{F_{ib} - F_i}{F_i}$ $\quad \Delta = -60.9\,\%$

Overbalanced: $\Delta := \frac{F_{iob} - F_i}{F_i}$ $\quad \Delta = -71.5\,\%$

PROBLEM 13-56

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
a. Exactly balance the crank and recalculate the inertia force.
b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius -*r* on the crank and recalculate the inertia force.
c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf\cdot sec^2\cdot in^{-1}$ $rpm := 2\cdot\pi\cdot rad\cdot min^{-1}$

Given: Conrod length: $l := 10.4\,in$ mass: $m_3 := 0.180\,blob$

Distance to CG (as fraction of *l*): $r_{3a} := 0.40$

Crank length: $r := 2.60\cdot in$ mass: $m_2 := 0.060\cdot blob$

Distance to CG (as fraction of *r*): $r_{2a} := 0.38$

Piston mass: $m_4 := 0.016\cdot blob$

Crank speed: $\omega := 1850\cdot rpm$ Crank angle: $\theta := 20\cdot deg$

Solution: See Problem 13-48 and Mathcad file P1356.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point *A* to CG: $l_a := r_{3a}\cdot l$ $l_a = 4.160\,in$

Distance from point *B* to CG: $l_b := l - l_a$ $l_b = 6.240\,in$

Let $l_p := l_a$ $m_{3a} := m_3\cdot\dfrac{l_b}{l_p + l_b}$ $m_{3a} = 0.1080\,blob$

$m_{3b} := m_3\cdot\dfrac{l_p}{l_p + l_b}$ $m_{3b} = 0.07200\,blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a}\cdot r$ $r_{G2} = 0.988\,in$

Mass: $m_{2a} := m_2\cdot\dfrac{r_{G2}}{r}$ $m_{2a} = 0.0228\,blob$

3. Calculate the lumped masses at points *A* and *B* using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $m_A = 0.1308\,blob$

$m_B := m_{3b} + m_4$ $m_B = 0.0880\,blob$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B\cdot\left[-r\cdot\omega^2\cdot\left(cos(\theta) + \frac{r}{l}\cdot cos(2\cdot\theta)\right)\right] \qquad F_{ix} = 9714\,lbf$$

$$F_{iy} := 0\cdot lbf \qquad F_{iy} = 0\,lbf$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{ib} = 9714\,lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 0.000\,deg$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \qquad m_p = 0.0293\ blob$$

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \ldots$$
$$+ \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) \qquad F_{ix} = 7024\ lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = -979\ lbf$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{iob} = 7092\ lbf \quad \text{at} \quad atan2\left(F_{ix}, F_{iy}\right) = -7.935\ deg$$

6. Compare the results to those for an unbalanced crank. From Problem 13-48, the inertia force for the unbalanced crank is $F_i := 22143 \cdot lbf$. The percent differences for the balanced and overbalanced cranks are:

Exactly balanced: $\Delta := \frac{F_{ib} - F_i}{F_i} \qquad \Delta = -56.1\,\%$

Overbalanced: $\Delta := \frac{F_{iob} - F_i}{F_i} \qquad \Delta = -68.0\,\%$

PROBLEM 13-57

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
a. Exactly balance the crank and recalculate the inertia force.
b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius *-r* on the crank and recalculate the inertia force.
c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Conrod length: $l := 10.4 in$ mass: $m_3 := 0.120 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.36$

Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.045 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.40$

Piston mass: $m_4 := 0.015 \cdot blob$

Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 25 \cdot deg$

Solution: See Problem 13-49 and Mathcad file P1357.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\quad l_a = 3.744 in$

Distance from point B to CG: $l_b := l - l_a$ $\quad l_b = 6.656 in$

Let $l_p := l_a$ $\quad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\quad m_{3a} = 0.0768 blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\quad m_{3b} = 0.04320 blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\quad r_{G2} = 1.040 in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\quad m_{2a} = 0.0180 blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\quad m_A = 0.0948 blob$

$m_B := m_{3b} + m_4$ $\quad m_B = 0.0582 blob$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta) \right) \right] \qquad F_{ix} = 7082 lbf$$

$$F_{iy} := 0 \cdot lbf \qquad F_{iy} = 0 lbf$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{ib} = 7082 lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 0.000 deg$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \qquad m_p = 0.0194\,blob$$

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \ldots + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) \qquad F_{ix} = 5077\,lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = -935\,lbf$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{iob} = 5163\,lbf \quad \text{at} \quad atan2\left(F_{ix}, F_{iy}\right) = -10.435\,deg$$

6. Compare the results to those for an unbalanced crank. From Problem 13-49, the inertia force for the unbalanced crank is $F_i := 17489 \cdot lbf$. The percent differences for the balanced and overbalanced cranks are:

Exactly balanced: $\Delta := \frac{F_{ib} - F_i}{F_i}$ $\qquad \Delta = -59.5\,\%$

Overbalanced: $\Delta := \frac{F_{iob} - F_i}{F_i}$ $\qquad \Delta = -70.5\,\%$

PROBLEM 13-58

Statement: The dimensions and mass properties of a crank, connecting rod, and piston are given below.
a. Exactly balance the crank and recalculate the inertia force.
b. Overbalance the crank with approximately two-thirds of the mass at the wrist pin placed at radius $-r$ on the crank and recalculate the inertia force.
c. Compare these results to those for the unbalanced crank.

Units: $blob := lbf \cdot sec^2 \cdot in^{-1}$ $\quad rpm := 2 \cdot \pi \cdot rad \cdot min^{-1}$

Given: Conrod length: $l := 12.5 in$ mass: $m_3 := 0.180 blob$

Distance to CG (as fraction of l): $r_{3a} := 0.36$

Crank length: $r := 2.60 \cdot in$ mass: $m_2 := 0.045 \cdot blob$

Distance to CG (as fraction of r): $r_{2a} := 0.40$

Piston mass: $m_4 := 0.015 \cdot blob$

Crank speed: $\omega := 2000 \cdot rpm$ Crank angle: $\theta := 25 \cdot deg$

Solution: See Problem 13-50 and Mathcad file P1358.

1. Determine the approximate conrod model using equations 13.9d, letting $l_p = l_a$.

Distance from point A to CG: $l_a := r_{3a} \cdot l$ $\qquad l_a = 4.500 \, in$

Distance from point B to CG: $l_b := l - l_a$ $\qquad l_b = 8.000 \, in$

Let $l_p := l_a$ $\qquad m_{3a} := m_3 \cdot \dfrac{l_b}{l_p + l_b}$ $\qquad m_{3a} = 0.1152 \, blob$

$m_{3b} := m_3 \cdot \dfrac{l_p}{l_p + l_b}$ $\qquad m_{3b} = 0.06480 \, blob$

2. Determine the statically equivalent crank model using equations 13.11.

Distance from O_2 to CG: $r_{G2} := r_{2a} \cdot r$ $\qquad r_{G2} = 1.040 \, in$

Mass: $m_{2a} := m_2 \cdot \dfrac{r_{G2}}{r}$ $\qquad m_{2a} = 0.0180 \, blob$

3. Calculate the lumped masses at points A and B using equations 13.12.

$m_A := m_{2a} + m_{3a}$ $\qquad m_A = 0.1332 \, blob$

$m_B := m_{3b} + m_4$ $\qquad m_B = 0.0798 \, blob$

4. Calculate the inertia force for an exactly balanced crank using equations 13.14d and 13.15d.

$$F_{ix} := -m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta) \right) \right] \qquad F_{ix} = 9465 \, lbf$$

$$F_{iy} := 0 \cdot lbf \qquad F_{iy} = 0 \, lbf$$

$$F_{ib} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{ib} = 9465 \, lbf \quad \text{at} \quad atan2(F_{ix}, F_{iy}) = 0.000 \, deg$$

5. Calculate the inertia force for an overbalanced crank using equations 13.14d and 13.15d.

$$m_p := \frac{m_B}{3} \qquad m_p = 0.0266\, blob$$

$$F_{ix} := -m_A \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) - m_B \cdot \left[-r \cdot \omega^2 \cdot \left(cos(\theta) + \frac{r}{l} \cdot cos(2 \cdot \theta)\right)\right] \ldots + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot cos(\theta)\right) \qquad F_{ix} = 6716\, lbf$$

$$F_{iy} := -m_A \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) + \left(m_A + m_p\right) \cdot \left(-r \cdot \omega^2 \cdot sin(\theta)\right) \qquad F_{iy} = -1282\, lbf$$

$$F_{iob} := \sqrt{F_{ix}^2 + F_{iy}^2} \qquad F_{iob} = 6837\, lbf \quad \text{at} \quad atan2\left(F_{ix}, F_{iy}\right) = -10.808\, deg$$

6. Compare the results to those for an unbalanced crank. From Problem 13-50, the inertia force for the unbalanced crank is $F_i := 17227 \cdot lbf$. The percent differences for the balanced and overbalanced cranks are:

Exactly balanced: $\Delta := \frac{F_{ib} - F_i}{F_i}$ $\Delta = -45.1\,\%$

Overbalanced: $\Delta := \frac{F_{iob} - F_i}{F_i}$ $\Delta = -60.3\,\%$