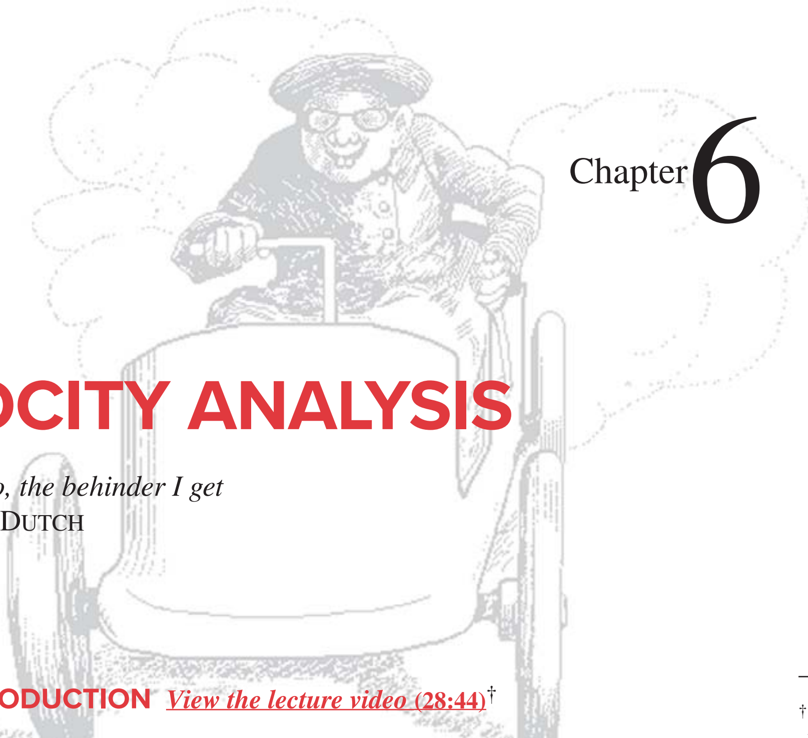
Chapter 6



VELOCITY ANALYSIS

The faster I go, the behinder I get
ANON. PENN. DUTCH

6.0 INTRODUCTION *View the lecture video (28:44)*†

Once a position analysis is done, the next step is to determine the velocities of all links and points of interest in the mechanism. We need to know the velocities in our mechanism or machine, both to calculate the stored kinetic energy from $mV^2/2$ and also as a step on the way to the determination of the link's accelerations that are needed for the dynamic force calculations. Many methods and approaches exist to find velocities in mechanisms. We will examine only a few of these methods here. We will first develop manual graphical methods, which are often useful as a check on the more complete and accurate analytical solution. We will also investigate the properties of the instant center of velocity which can shed much light on a mechanism's velocity behavior with very little effort. Finally, we will derive the analytical solution for the fourbar and inverted crank-slider as examples of the general vector loop equation solution to velocity analysis problems. From these calculations we will be able to establish some indices of merit to judge our designs while they are still on the drawing board (or in the computer).

† http://www.designofmachinery.com/DOM/Velocity_Analysis_with_ICs.mp4

6.1 DEFINITION OF VELOCITY

Velocity is defined as *the rate of change of position with respect to time*. Position ($\mathbf{R}$) is a vector quantity and so is velocity. Velocity can be **angular** or **linear**. **Angular velocity** will be denoted as ω and **linear velocity** as $\mathbf{V}$.

$$\omega = \frac{d\theta}{dt}; \qquad \mathbf{V} = \frac{d\mathbf{R}}{dt} \tag{6.1}$$

Figure 6-1 shows a link *PA* in pure rotation, pivoted at point *A* in the *xy* plane. Its position is defined by the position vector $\mathbf{R}_{PA}$. We are interested in the velocity of point

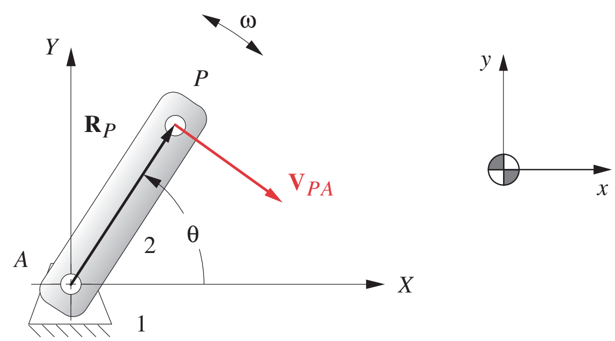


FIGURE 6-1

A link in pure rotation

P when the link is subjected to an angular velocity ω. If we represent the position vector $\mathbf{R}_{PA}$ as a complex number in polar form,

$$\mathbf{R}_{PA} = pe^{j\theta} \tag{6.2}$$

where p is the scalar length of the vector. We can easily differentiate it to obtain:

$$\mathbf{V}_{PA} = \frac{d\mathbf{R}_{PA}}{dt} = p\,je^{j\theta}\frac{d\theta}{dt} = p\omega\,je^{j\theta} \tag{6.3}$$

Compare the right side of equation 6.3 to the right side of equation 6.2. Note that as a result of the differentiation, the velocity expression has been multiplied by the (constant) complex operator j. This causes a rotation of this velocity vector through 90 degrees with respect to the original position vector. (See also Figure 4-8b.) This 90-degree rotation is positive, or counterclockwise. However, the velocity expression is also multiplied by ω, which may be either positive or negative. As a result, the velocity vector will be **rotated 90 degrees** from the angle θ of the position vector **in a direction dictated by the sign of** ω. This is just mathematical verification of what you already knew, namely that *velocity is always in a direction perpendicular to the radius of rotation and is tangent to the path of motion* as shown in Figure 6-1.

Substituting the Euler identity (equation 4.4a) into equation 6.3 gives us the real and imaginary (or x and y) components of the velocity vector.

$$\mathbf{V}_{PA} = p\omega\,j(\cos\theta + j\sin\theta) = p\omega(-\sin\theta + j\cos\theta) \tag{6.4}$$

Note that the sine and cosine terms have swapped positions between the real and imaginary terms, due to multiplying by the j coefficient. This is evidence of the 90-degree rotation of the velocity vector versus the position vector. The former x component has become the y component, and the former y component has become a minus x component. Study Figure 4-8b to review why this is so.

The velocity $\mathbf{V}_{PA}$ in Figure 6-1 can be referred to as an **absolute velocity** since it is referenced to A, which is the origin of the global coordinate axes in that system. As such, we could have referred to it as $\mathbf{V}_P$, with the absence of the second subscript imply-

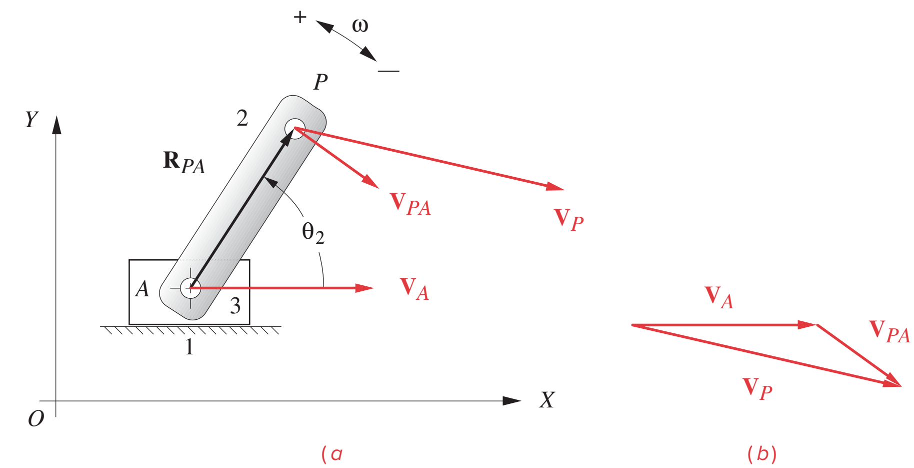


FIGURE 6-2

Velocity difference

ing reference to the global coordinate system. Figure 6-2a shows a different and slightly more complicated system in which the pivot A is no longer stationary. It has a known linear velocity $\mathbf{V}_A$ as part of the translating carriage, link 3. If ω is unchanged, the velocity of point P versus A will be the same as before, but $\mathbf{V}_{PA}$ can no longer be considered an absolute velocity. It is now a **velocity difference** and **must** carry the second subscript as $\mathbf{V}_{PA}$. The absolute velocity $\mathbf{V}_P$ must now be found from the **velocity difference equation** whose graphical solution is shown in Figure 6-2b:

$$\mathbf{V}_{PA} = \mathbf{V}_P - \mathbf{V}_A \tag{6.5a}$$

rearranging:

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA} \tag{6.5b}$$

Note the similarity of equations 6.5 to the **position difference equation** 4.1.

Figure 6-3 shows two independent bodies P and A, which could be two automobiles, moving in the same plane. If their independent velocities $\mathbf{V}_P$ and $\mathbf{V}_A$ are known, their **relative velocity** $\mathbf{V}_{PA}$ can be found from equations 6.5 arranged algebraically as:

$$\mathbf{V}_{PA} = \mathbf{V}_P - \mathbf{V}_A \tag{6.6}$$

The graphical solution to this equation is shown in Figure 6-3b. Note that it is similar to Figure 6-2b except for a different vector being the resultant.

As we did for position analysis, we give these two cases different names despite the fact that the same equation applies. Repeating the definition from Section 4.2, modified to refer to velocity:

CASE 1: *Two points in the same body* => **velocity difference**

CASE 2: *Two points in different bodies* => **relative velocity**

We will find use for this semantic distinction when we analyze both linkage velocities and the velocity of slip later in this chapter.

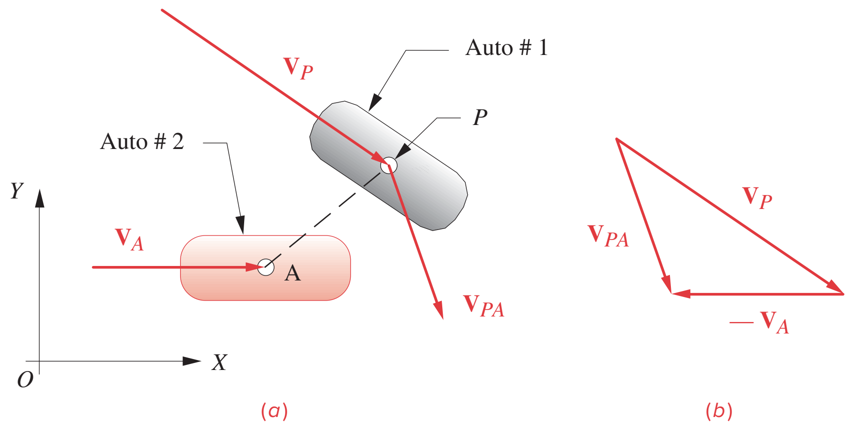


FIGURE 6-3

Relative velocity

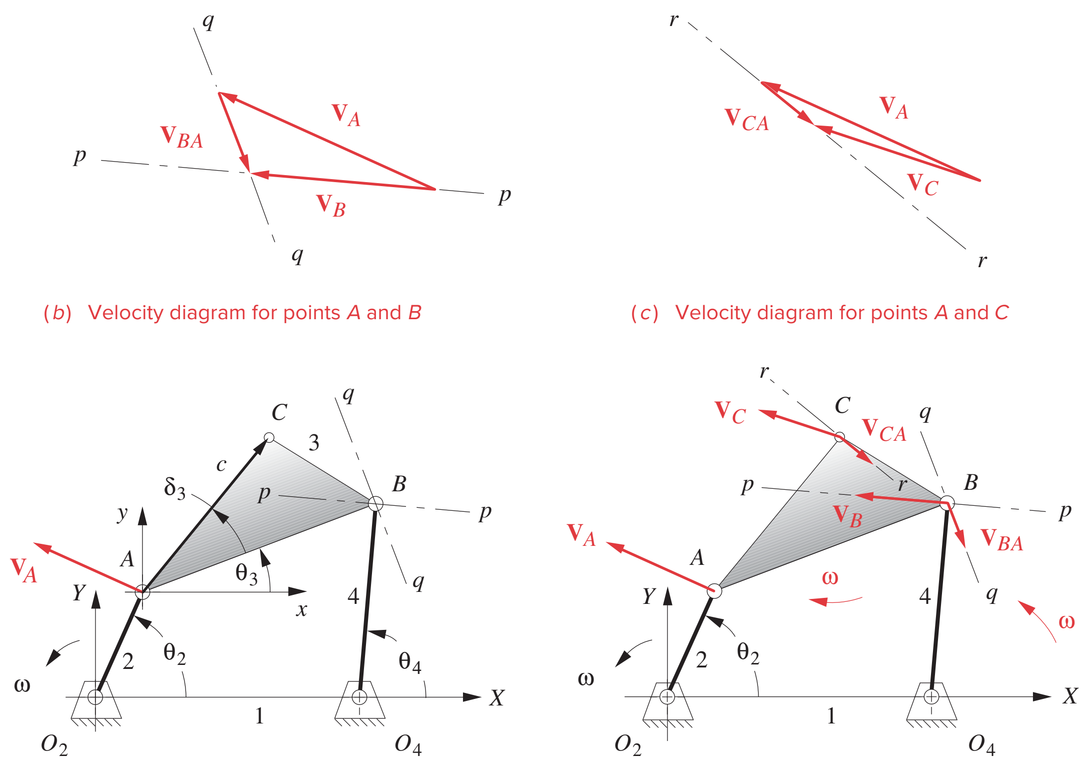
6.2 GRAPHICAL VELOCITY ANALYSIS

Before programmable calculators and computers became universally available to engineers, graphical methods were the only practical way to solve these velocity analysis problems. With some practice and with proper tools such as a drafting machine or CAD package, one can fairly rapidly solve for the velocities of particular points in a mechanism for any one input position by drawing vector diagrams. However, it is a tedious process if velocities for many positions of the mechanism are to be found, because each new position requires a completely new set of vector diagrams be drawn. Very little of the work done to solve for the velocities at position 1 carries over to position 2, etc. Nevertheless, this method still has more than historical value as it can provide a quick check on the results from a computer program solution. Such a check needs only be done for a few positions to prove the validity of the program. Also, graphical solutions provide the beginning student some visual feedback on the solution that can help develop an understanding of the underlying principles. It is principally for this last reason that graphical solutions are included in this text even in this "age of the computer."

To solve any velocity analysis problem graphically, we need only two equations, 6.5 and 6.7 (which is merely the scalar form of equation 6.3):

$$|\mathbf{V}| = v = r\omega \tag{6.7}$$

Note that the scalar equation 6.7 defines only the **magnitude** (v) of the velocity of any point on a body that is in pure rotation. In a graphical CASE 1 analysis, the **direction** of the vector due to the rotation component must be understood from equation 6.3 to be perpendicular to the radius of rotation. Thus, if the center of rotation is known, the direction of the velocity component due to that rotation is known and its sense will be consistent with the angular velocity ω of the body.



(*b*) Velocity diagram for points *A* and *B*

(*c*) Velocity diagram for points *A* and *C*

(*a*) Linkage showing velocity of point *A*

(*d*) Linkage showing velocities of points *A*, *B*, and *C*

FIGURE 6-4

Graphical solution for velocities in a pin-jointed linkage

Figure 6-4 shows a fourbar linkage in a particular position. We wish to solve for the angular velocities of links 3 and 4 (ω_3, ω_4) and the linear velocities of points *A*, *B*, and *C* ($\mathbf{V}_A$, $\mathbf{V}_B$, $\mathbf{V}_C$). Point *C* represents any general point of interest. Perhaps *C* is a coupler point. The solution method is valid for any point on any link. To solve this problem, we need to know the *lengths of all the links*, the *angular positions of all the links*, and the *instantaneous input velocity of any one driving link or driving point*. Assuming we have designed this linkage, we will know or can measure the link lengths. We must also first do a **complete position analysis** to find the link angles θ_3 and θ_4 given the input link's position θ_2. This can be done by any of the methods in Chapter 4. In general we must solve these problems in stages, first for link positions, then for velocities, and finally for accelerations. For the following example, we will assume that a complete position analysis has been done and that the input is to link 2 with known θ_2 and ω_2 for this one "freeze frame" position of the moving linkage.

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EXAMPLE 6-1

Graphical Velocity Analysis for One Position of Linkage.

Problem: Given θ_2, θ_3, θ_4, ω_2, find ω_3, ω_4, $\mathbf{V}_A$, $\mathbf{V}_B$, $\mathbf{V}_C$ by graphical methods.

Solution: (See Figure 6-4.)

1 Start at the end of the linkage about which you have the most information. Calculate the magnitude of the velocity of point A using scalar equation 6.7.

$$v_a = \left(AO_2\right)\omega_2 \tag{a}$$

2 Draw the velocity vector $\mathbf{V}_A$ with its length equal to its magnitude v_A at some convenient scale with its root at point A and its direction perpendicular to the radius AO_2. Its sense is the same as that of ω_2 as shown in Figure 6-4a.

3 Move next to a point about which you have some information. Note that the direction of the velocity of point B is predictable since it is pivoting in pure rotation about point O_4. Draw the construction line pp through point B perpendicular to BO_4, to represent the direction of $\mathbf{V}_B$ as shown in Figure 6-4a.

4 Write the velocity difference vector equation 6.5 for point B versus point A.

$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \tag{b}$$

We will use point A as the reference point to find $\mathbf{V}_B$ because A is in the same link as B and we have already solved for $\mathbf{V}_A$. Any two-dimensional vector equation can be solved for two unknowns. Each term has two parameters, namely magnitude and direction. There are then potentially six unknowns in this equation, two per term. We must know four of them to solve it. We know both magnitude and direction of $\mathbf{V}_A$ and the direction of $\mathbf{V}_B$. We need to know one more parameter.

5 The term $\mathbf{V}_{BA}$ represents the velocity of B with respect to A. If we assume that the link BA is rigid, then there can be no component of $\mathbf{V}_{BA}$ that is directed along the line BA, because point B cannot move toward or away from point A without shrinking or stretching the rigid link! Therefore, the direction of $\mathbf{V}_{BA}$ must be perpendicular to the line BA. Draw construction line qq through point B and perpendicular to BA to represent the direction of $\mathbf{V}_{BA}$, as shown in Figure 6-4a.

6 Now the vector equation can be solved graphically by drawing a vector diagram as shown in Figure 6-4b. Either drafting tools or a CAD package is needed for this step. Draw velocity vector $\mathbf{V}_A$ carefully to some scale, maintaining its direction. (It is drawn twice its size in the figure.) The equation in step 4 says to add $\mathbf{V}_{BA}$ to $\mathbf{V}_A$, so draw a line parallel to line qq across the tip of $\mathbf{V}_A$. The resultant, or left side of the equation, must close the vector diagram, from the tail of the first vector drawn ($\mathbf{V}_A$) to the tip of the last, so draw a line parallel to pp across the tail of $\mathbf{V}_A$. The intersection of these lines parallel to pp and qq defines the lengths of $\mathbf{V}_B$ and $\mathbf{V}_{BA}$. The senses of the vectors are determined from reference to the equation. $\mathbf{V}_\mathrm{A}$ was added to $\mathbf{V}_{BA}$, so they must be arranged tip to tail. $\mathbf{V}_B$ is the resultant, so it must be from the tail of the first to the tip of the last. The resultant vectors are shown in Figure 6-4b and d.

7 The angular velocities of links 3 and 4 can be calculated from equation 6.7:

$$\omega_4 = \frac{v_B}{BO_4} \qquad \text{and} \qquad \omega_3 = \frac{v_{BA}}{BA} \tag{c}$$

Note that the velocity difference term $\mathbf{V}_{BA}$ represents the rotational component of velocity of link 3 due to ω_3. This must be true if point *B* cannot move toward or away from point *A*. The only velocity difference they can have, one to the other, is due to rotation of the line connecting them. You may think of point *B* on the line *BA* rotating about point *A* as a center, or point *A* on the line *AB* rotating about *B* as a center. The rotational velocity ω of any body is a "free vector" that has no particular point of application to the body. It exists everywhere on the body.

8 Finally we can solve for $\mathbf{V}_C$, again using equation 6.5. We select any point in link 3 for which we know the absolute velocity to use as the reference, such as point *A*.

$$\mathbf{V}_C = \mathbf{V}_A + \mathbf{V}_{CA} \tag{d}$$

In this case, we can calculate the magnitude of $\mathbf{V}_{CA}$ from equation 6.7 as we have already found ω_3,

$$v_{ca} = c\omega_3 \tag{e}$$

Since both $\mathbf{V}_A$ and $\mathbf{V}_{CA}$ are known, the vector diagram can be directly drawn as shown in Figure 6-4c. $\mathbf{V}_C$ is the resultant that closes the vector diagram. Figure 6-4d shows the calculated velocity vectors on the linkage diagram. Note that the velocity difference vector $\mathbf{V}_{CA}$ is perpendicular to line *CA* (along line *rr*) for the same reasons as discussed in step 7 above.

The above example contains some interesting and significant principles that deserve further emphasis. Equation 6.5a is repeated here for discussion.

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA} \tag{6.5a}$$

This equation represents the *absolute* velocity V_P of some general point *P* referenced to the origin of the global coordinate system. The right side defines it as the sum of the absolute velocity V_A of some other reference point *A* in the same system and the velocity difference (or relative velocity) V_{PA} of point *P* versus point *A*. This equation could also be written:

Velocity = translation component + rotation component

These are the same two components of motion defined by Chasles' theorem, and introduced for displacement in Section 4.3. Chasles' theorem holds for velocity as well. These two components of motion, translation and rotation, are independent of one another. If either is zero in a particular example, the complex motion will reduce to one of the special cases of pure translation or pure rotation. When both are present, the total velocity is merely their vector sum.

Let us review what was done in Example 6-1 in order to extract the general strategy for solution of this class of problem. We started at the input side of the mechanism, as that is where the driving angular velocity is defined. We first looked for a point (*A*) for which the motion was pure rotation so that one of the terms in equation 6.5 would be zero. (We could as easily have looked for a point in pure translation to bootstrap the solution.)

We then solved for the absolute velocity of that point ($\mathbf{V}_A$) using equations 6.5 and 6.7. *(Steps 1 and 2)*

We then used the point (A) just solved for as a reference point to define the translation component in equation 6.5 written for a new point (B). Note that we needed to choose a second point (B) that was in the same rigid body as the reference point (A) which we had already solved and about which we could predict some aspect of the new point's (B's) velocity. In this example, we knew the direction of the velocity $\mathbf{V}_B$. In general this condition will be satisfied by any point on a link that is jointed to ground (as is link 4). In this example, we could not have solved for point C until we solved for B, because point C is on a floating link for which point we do not yet know the velocity direction. *(Steps 3 and 4)*

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To solve the equation for the second point (B), we also needed to recognize that the rotation component of velocity is directed perpendicular to the line connecting the two points in the link (B and A in the example). You **will always know the direction of the rotation component** in equation 6.5 **if it represents a velocity difference** (CASE 1) **situation**. *If the rotation component relates two points in the* **same rigid body,** *then that velocity difference component is always perpendicular to the line connecting those two points* (see Figure 6-2). This will be true regardless of the two points selected. But, *this is not true in a* CASE 2 *situation* (see Figure 6-3). *(Steps 5 and 6)*

Once we found the absolute velocity ($\mathbf{V}_B$) of a second point on the same link (CASE 1), we could solve for the angular velocity of that link. (Note that points A and B are both on link 3 and the velocity of point O_4 is zero.) Once the angular velocities of all the links were known, we could solve for the linear velocity of any point (such as C) in any link using equation 6.5. To do so, we had to understand the concept of angular velocity as a **free vector**, meaning that it exists everywhere on the link at any given instant. It has no particular center. *It has an infinity of potential centers*. The link simply *has an angular velocity*, just as does a frisbee thrown and spun across the lawn.

All points on a *frisbee*, if spinning while flying, obey equation 6.5. Left to its own devices, the frisbee will spin about its center of gravity (*CG*), which is close to the center of its circular shape. But if you are an expert frisbee player (and have rather pointed fingers), you can imagine catching that flying frisbee between your two index fingers in some off-center location (not at the *CG*), such that the frisbee continues to spin about your fingertips. In this somewhat far-fetched example of championship frisbee play, you will have taken the translation component of the frisbee's motion to zero, but its independent rotation component will still be present. Moreover, it will now be spinning about a different center (your fingers) than it was in flight (its *CG*). Thus this **free vector** of angular velocity (ω) is happy to attach itself to any point on the body. The body still has the same ω, regardless of the assumed center of rotation. It is this property that allows us to solve equation 6.5 for literally **any point** on a rigid body in complex motion **referenced to any other point** on that body. *(Steps 7 and 8)*

6.3 INSTANT CENTERS OF VELOCITY *View a tutorial video (28:55)*†

The definition of an **instant center** of velocity is *a point, common to two bodies in plane motion, which point has the same instantaneous velocity in each body*. Instant centers are sometimes also called *centros* or *poles*. Since it takes two bodies or links to create an

† http://www.designofmachinery.com/DOM/Instant_Centers_Tutorial.mp4

instant center (*IC*), we can easily predict the quantity of instant centers to expect from any collection of links. The combination formula for n things taken r at a time is:

$$C = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} \tag{6.8a}$$

For our case $r = 2$ and it reduces to:

$$C = \frac{n(n-1)}{2} \tag{6.8b}$$

From equation 6.8b we can see that a fourbar linkage has 6 instant centers, a sixbar has 15, and an eightbar has 28.

Figure 6-5 shows a fourbar linkage in an arbitrary position. It also shows a **linear graph**† that is useful for keeping track of which *ICs* have been found. This particular graph can be created by drawing a circle on which we mark off as many points as there are links in our assembly. We will then draw a line between the dots representing the link pairs each time we find an instant center. The resulting linear graph is the set of lines connecting the dots. It does not include the circle that was used only to place the dots. This graph is actually a geometric solution to equation 6.8b, since connecting all the points in pairs gives all the possible combinations of points taken two at a time.

Some *ICs* can be found by inspection, using only the definition of the instant center. Note in Figure 6-5a that the four pin joints each satisfy the definition. They clearly must have the same velocity in both links at all times. These have been labeled $I_{1,2}$, $I_{2,3}$, $I_{3,4}$, and $I_{1,4}$. The order of the subscripts is immaterial. Instant center $I_{2,1}$ is the same as $I_{1,2}$. These pin-joint *ICs* are sometimes called "permanent" instant centers as they remain in the same location for all positions of the linkage. In general, instant centers will move to new locations as the linkage changes position, thus the adjective *instant.* In this fourbar example there are two more *ICs* to be found. It will help to use the Aronhold-Kennedy theorem,‡ also called *Kennedy's rule,*[3] to locate them.

† Note that this *graph* is not a plot of points on an *x, y* coordinate system. Rather it is a *linear graph* from the fascinating branch of mathematics called *graph theory*, which is itself a branch of topology. Linear graphs are often used to depict interrelationships between various phenomena. They have many applications in kinematics especially as a way to classify linkages and to find isomers.

‡ Discovered independently by Aronhold in Germany, in 1872, and by Kennedy in England, in 1886. Kennedy[3] states in his preface, "The theorem of the three virtual (instant) centers … was first given, I believe, by Aronhold, although its previous publication was unknown to me until some years after I had given it in my lectures." It tends to be attributed to Kennedy in the English-speaking world and to Aronhold in the German-speaking world.

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Kennedy's rule:

Any three bodies in plane motion will have exactly three instant centers, and ***they will lie on the same straight line****.*

The first part of this rule is just a restatement of equation 6.8b for $n = 3$. It is the second clause in this rule that is most useful. Note that this rule does **not** require that the three bodies be connected in any way. We can use this rule, in conjunction with the linear graph, to find the remaining *ICs* that are not obvious from inspection. Figure 6.5b shows the construction necessary to find instant center $I_{1,3}$. Figure 6-5c shows the construction necessary to find instant center $I_{2,4}$. The following example describes the procedure in detail.

EXAMPLE 6-2

Finding All Instant Centers for a Fourbar Linkage.

Problem: Given a fourbar linkage in one position, find all *ICs* by graphical methods.

Solution: (See Figure 6-5 and the video *Instant Centers and Centrodes*.)

1 Draw a circle with all links numbered around the circumference as shown in Figure 6-5a.

2 Locate as many *ICs* as possible by inspection. All pin joints will be permanent *ICs*. Connect the link numbers on the circle to create a linear graph and record those *ICs* found, as shown in Figure 6-5a.

3 Identify a link combination on the linear graph for which the *IC* has not been found, and draw a dotted line connecting those two link numbers. Identify two triangles on the graph that each contain the dotted line and whose other two sides are solid lines representing *ICs* already found.

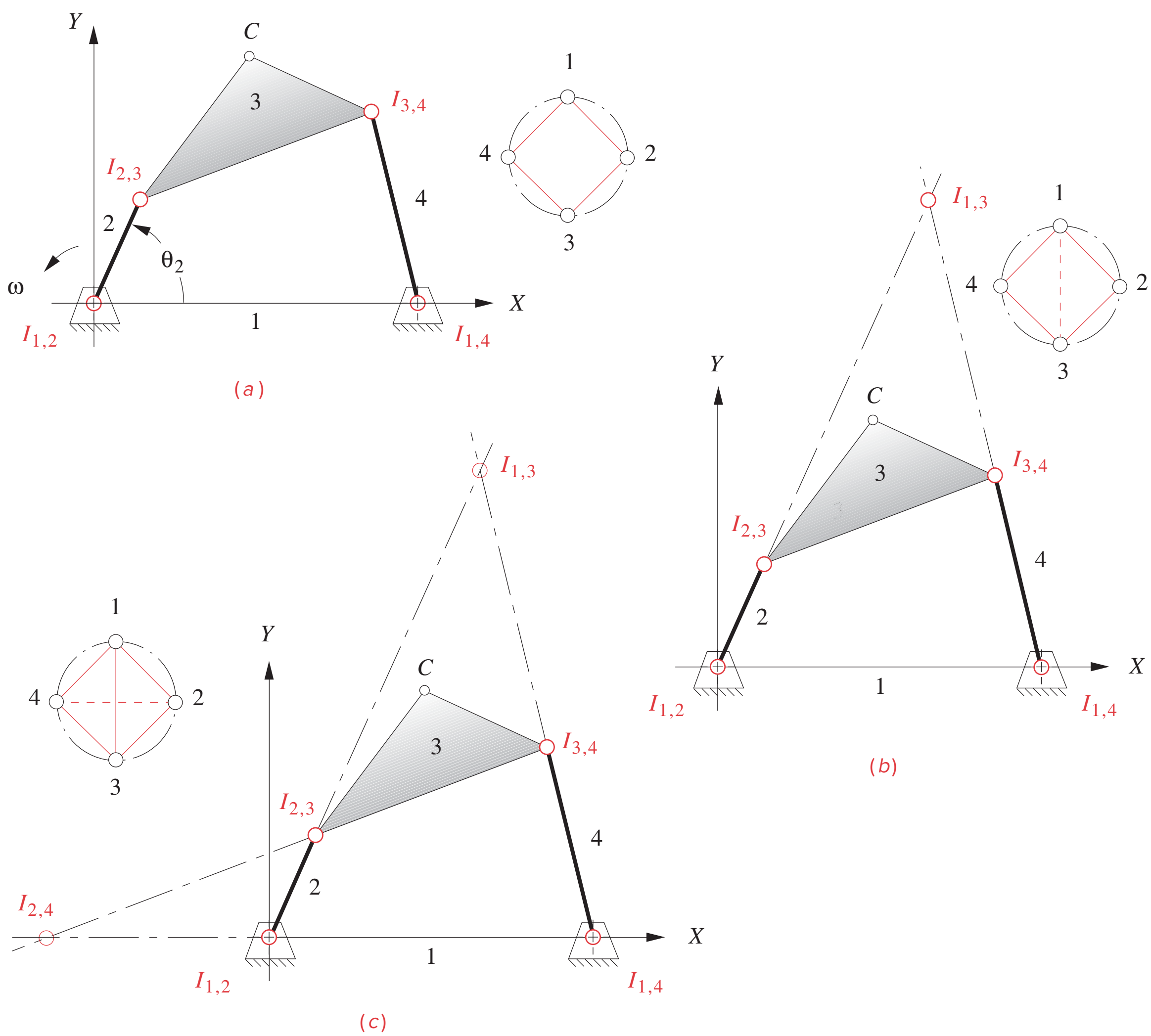

FIGURE 6-5

Locating instant centers in the pin-jointed linkage

On the graph in Figure 6-5b, link numbers 1 and 3 have been connected with a dotted line. This line forms one triangle with sides 13, 34, 14 and another with sides 13, 23, 12. These triangles define trios of *IC*s that obey **Kennedy's rule**. Thus *IC*s 13, 34, and 14 **must lie on the same straight line**. Also *IC*s 13, 23 and 12 will **lie on a different straight line**.

4 On the linkage diagram draw a line through the two known *IC*s that form a trio with the unknown *IC*. Repeat for the other trio. In Figure 6-5b, a line has been drawn through $I_{1,2}$ and $I_{2,3}$ and extended. $I_{1,3}$ must lie on this line. Another line has been drawn through $I_{1,4}$ and $I_{3,4}$ and extended to intersect the first line. By Kennedy's rule, instant center $I_{1,3}$ must also lie on this line, so their intersection is $I_{1,3}$.

5 Connect link numbers 2 and 4 with a dotted line on the linear graph as shown in Figure 6-5c. This line forms one triangle with sides 24, 23, 34 and another with sides 24, 12, 14. These sides represent trios of *IC*s that obey Kennedy's rule. Thus *IC*s 24, 23, and 34 must lie on the same straight line. Also *IC*s 24, 12, and 14 lie on a different straight line.

6 On the linkage diagram draw a line through the two known *IC*s that form a trio with the unknown *IC*. Repeat for the other trio. In Figure 6-5c, a line has been drawn through $I_{1,2}$ and $I_{1,4}$ and extended. $I_{2,4}$ must lie on this line. Another line has been drawn through $I_{2,3}$ and $I_{3,4}$ and extended to intersect the first line. By Kennedy's rule, instant center $I_{2,4}$ must also lie on this line, so their intersection is $I_{2,4}$.

7 If there were more links, this procedure would be repeated until all *IC*s were found.

The presence of slider joints makes finding the instant centers a little more subtle as is shown in the next example. Figure 6-6a shows a **fourbar crank-slider linkage**. Note that there are only three pin joints in this linkage. All pin joints are *permanent instant centers*. But the joint between links 1 and 4 is a rectilinear, sliding full joint. A sliding joint is kinematically equivalent to an infinitely long link, "pivoted" at infinity. Figure 6-6b shows a nearly equivalent pin-jointed version of the crank-slider in which link 4 is a very long rocker. Point *B* now swings through a shallow arc that is nearly a straight line. It is clear in Figure 6-6b that, in this linkage, $I_{1,4}$ is at pivot O_4. Now imagine increasing the length of this long, link 4 rocker even more. In the limit, link 4 approaches infinite length, the pivot O_4 approaches infinity along the line that was originally the long rocker, and the arc motion of point *B* approaches a straight line. Thus, *a slider joint will have its instant center at infinity along a line perpendicular to the direction of sliding* as shown in Figure 6-6a.

EXAMPLE 6-3

Finding All Instant Centers for a Crank-Slider Linkage.

Problem: Given a crank-slider linkage in one position, find all *ICs* by graphical methods.

Solution: (See Figure 6-7, and the video *Instant Centers and Centrodes*.)

1 Draw a circle with all links numbered around the circumference as shown in Figure 6-7a.

2 Locate all *ICs* possible by inspection. All pin joints will be permanent *ICs*. The slider joint's

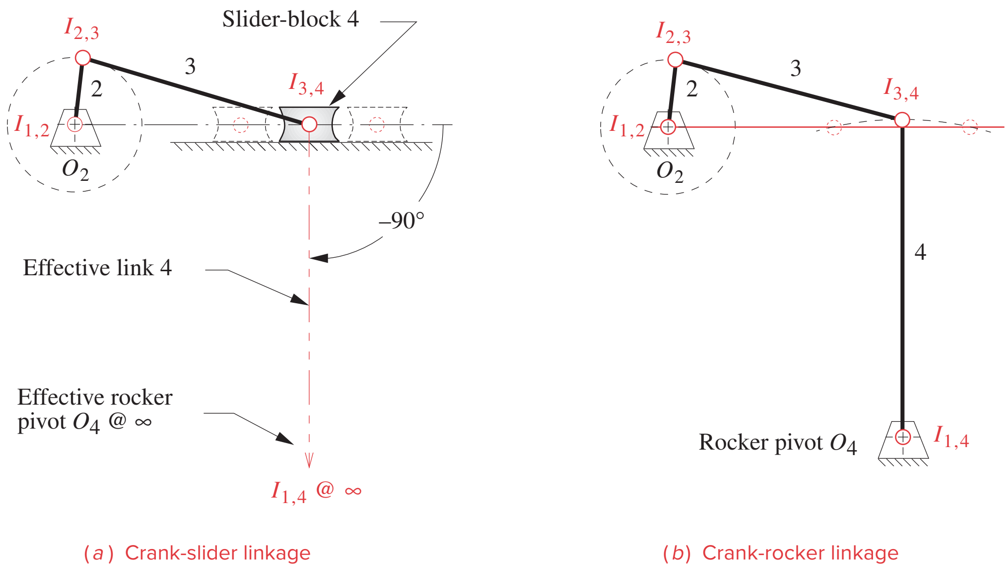

FIGURE 6-6

A rectilinear slider's instant center is at infinity

instant center will be at infinity along a line perpendicular to the axis of sliding. Connect the link numbers on the circle to create a linear graph and record those *ICs* found, as shown in Figure 6-7a.

3 Identify a link combination on the linear graph for which the *IC* has not been found, and draw a dotted line connecting those two link numbers. Identify two triangles on the graph that each contain the dotted line and whose other two sides are solid lines representing *ICs* already found. In the graph on Figure 6-7b, link numbers 1 and 3 have been connected with a dotted line. This line forms one triangle with sides 13, 34, 14 and another with sides 13, 23, 12. These sides represent trios of *ICs* that obey Kennedy's rule. Thus *ICs* 13, 34, and 14 must lie on the same straight line. Also *ICs* 13, 23, and 12 lie on a different straight line.

4 On the linkage diagram draw a line through the two known *ICs* that form a trio with the unknown *IC*. Repeat for the other trio. In Figure 6-7b, a line has been drawn from $I_{1,2}$ through $I_{2,3}$ and extended. $I_{1,3}$ must lie on this line. Another line has been drawn from $I_{1,4}$ (at infinity) through $I_{3,4}$ and extended to intersect the first line. By Kennedy's rule, instant center $I_{1,3}$ must also lie on this line, so their intersection is $I_{1,3}$.

5 Connect link numbers 2 and 4 with a dotted line on the graph as shown in Figure 6-7c. This line forms one triangle with sides 24, 23, 34 and another with sides 24, 12, 14. These sides also represent trios of *ICs* that obey Kennedy's rule. Thus *ICs* 24, 23, and 34 must lie on the same straight line. Also *ICs* 24, 12, and 14 lie on a different straight line.

6 On the linkage diagram draw a line through the two known *ICs* that form a trio with the unknown *IC*. Repeat for the other trio. In Figure 6-7c, a line has been drawn from $I_{1,2}$ to intersect $I_{1,4}$, and extended. Note that the only way to "intersect" $I_{1,4}$ at infinity is to draw a line parallel to the line $I_{3,4}\,I_{1,4}$ since all parallel lines intersect at infinity. Instant center $I_{2,4}$ must lie on this

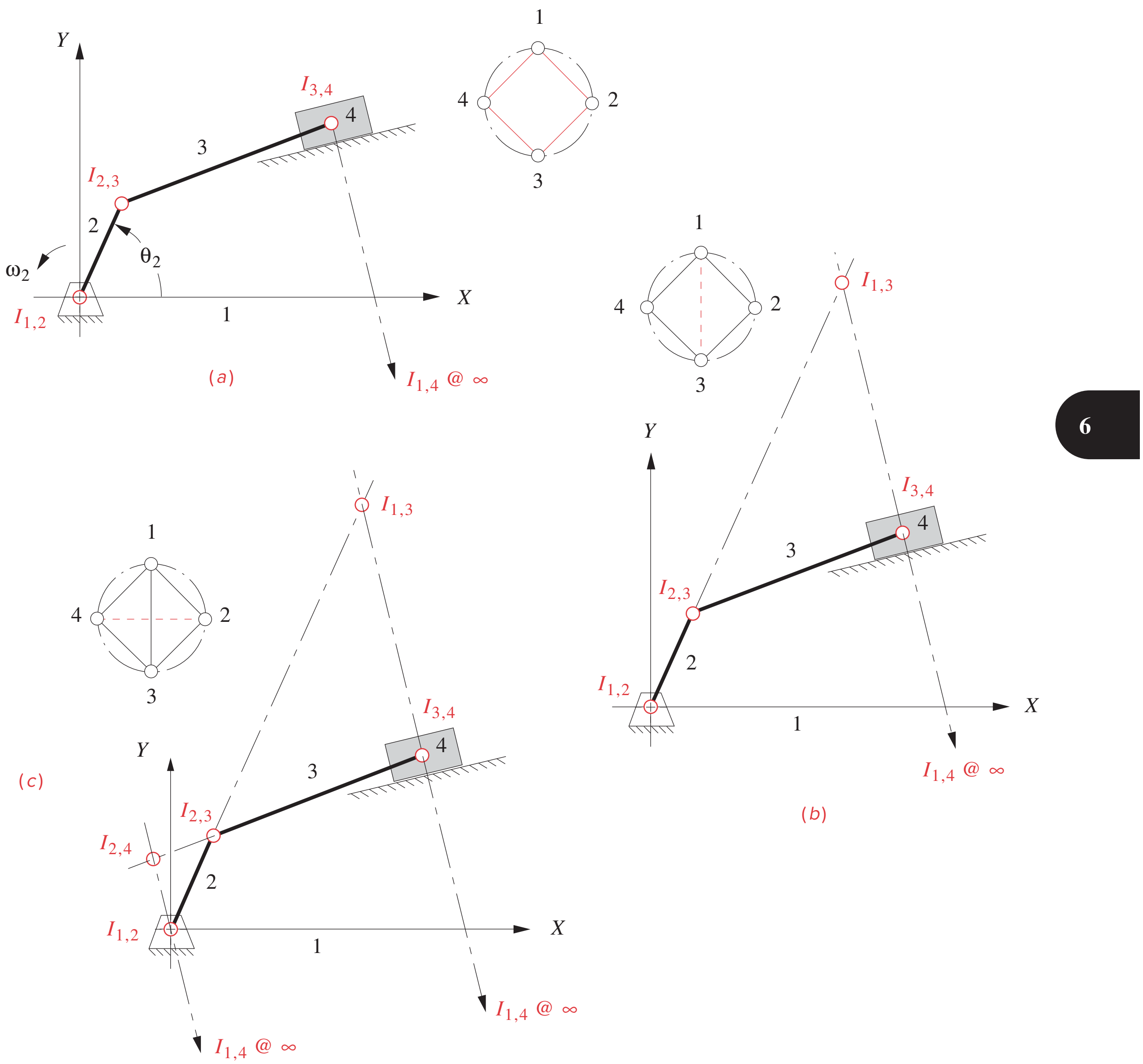

FIGURE 6-7

Locating instant centers in the slider-crank linkage

parallel line. Another line has been drawn through $I_{2,3}$ and $I_{3,4}$ and extended to intersect the first line. By Kennedy's rule, instant center $I_{2,4}$ must also lie on this line, so their intersection is $I_{2,4}$.

7 If there were more links, this procedure would be repeated until all *ICs* were found.

The procedure in this slider example is identical to that used in the pin-jointed fourbar, except that it is complicated by the presence of instant centers located at infinity.

In Section 2.10 and Figure 2-12c we showed that a cam-follower mechanism is really a fourbar linkage in disguise. As such it will also possess instant centers. The presence of the half joint in this, or any linkage, makes the location of the instant centers a little more complicated. We have to recognize that the instant center between any two links will be along a line that is perpendicular to the *relative velocity* vector between the links at the half joint, as shown in the following example. Figure 6-8 shows the same cam-follower mechanism as in Figure 2-12c. The effective links 2, 3, and 4 are also shown.

EXAMPLE 6-4

Finding All Instant Centers for a Cam-Follower Mechanism.

Problem: Given a cam and follower in one position, find all *IC*s by graphical methods.

Solution: (See Figure 6-8.)

1 Draw a circle with all links numbered around the circumference as shown in Figure 6-8b. In this case there are only three links and thus only three *IC*s to be found as shown by equation 6.8. Note that the links are numbered 1, 2, and 4. The missing link 3 is the variable-length effective coupler.

2 Locate all *IC*s possible by inspection. All pin joints will be permanent *IC*s. The two fixed pivots $I_{1,2}$ and $I_{1,4}$ are the only pin joints here. Connect the link numbers on the circle to create a linear graph and record those *IC*s found, as shown in Figure 6-8b. The only link combination on the linear graph for which the *IC* has not been found is $I_{2,4}$, so draw a dotted line connecting those two link numbers.

3 Kennedy's rule says that all three *ICs* must lie on the same straight line; thus the remaining instant center $I_{2,4}$ must lie on the line $I_{1,2}$ $I_{1,4}$ extended. Unfortunately in this example, we have too few links to find a second line on which $I_{2,4}$ must lie.

4 On the linkage diagram draw a line through the two known *ICs* that form a trio with the unknown *IC*. In Figure 6-8c, a line has been drawn from $I_{1,2}$ through $I_{1,4}$ and extended. This is, of course, link 1. By Kennedy's rule, $I_{2,4}$ must lie on this line.

5 Looking at Figure 6-8c that shows the effective links of the equivalent fourbar linkage for this position, we can extend effective link 3 until it intersects link 1 extended. Just as in the "pure" fourbar linkage, instant center 2,4 lies on the intersection of links 1 and 3 extended (see Example 6-2).

6 Figure 6-8d shows that it is not necessary to construct the effective fourbar linkage to find $I_{2,4}$. Note that the **common tangent** to links 2 and 4 at their contact point (the half joint) has been drawn. This line is also called the **axis of slip** because it is the line along which all relative (slip) velocity will occur between the two links. Thus the velocity of link 4 versus 2, $\mathbf{V}_{42}$, is directed along the axis of slip. Instant center $I_{2,4}$ must therefore lie along a line perpendicular to the common tangent, called the **common normal.** Note that this line is the same as the effective link 3 line in Figure 6-8c.

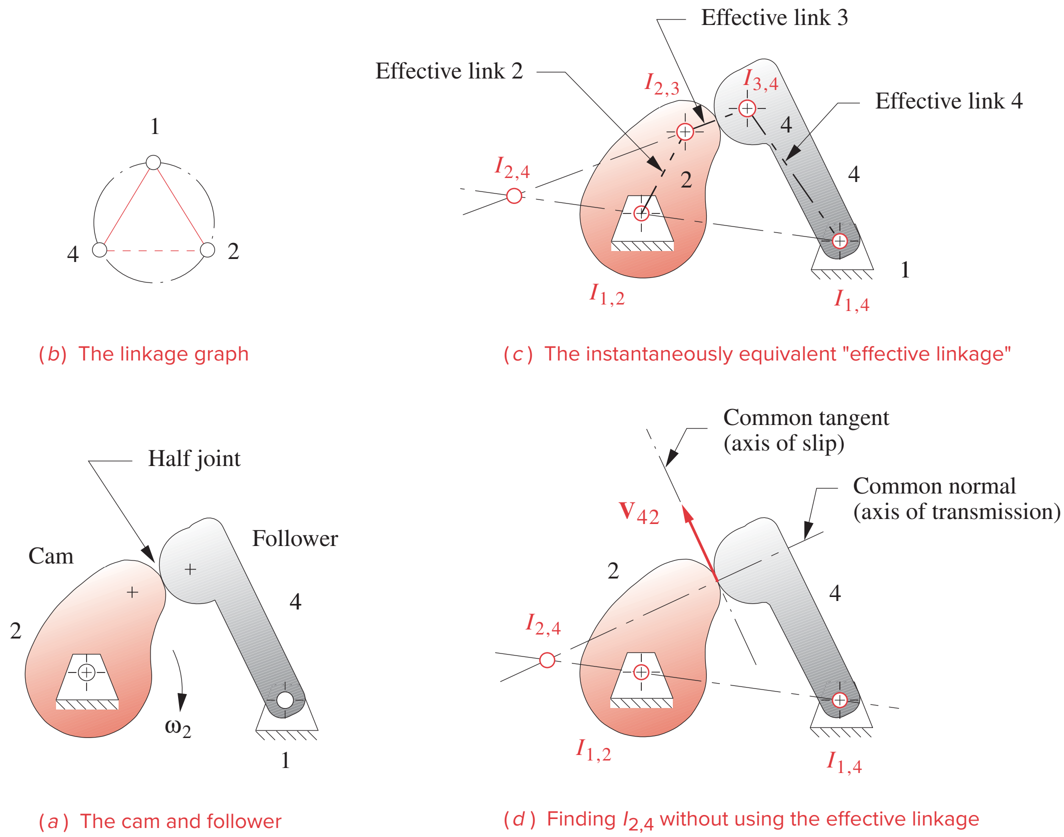

FIGURE 6-8

Locating instant centers in the cam-follower mechanism

6.4 VELOCITY ANALYSIS WITH INSTANT CENTERS

Once the *ICs* have been found, they can be used to do a very rapid graphical velocity analysis of the linkage. Note that, depending on the particular position of the linkage being analyzed, some of the *IC*s may be very far removed from the links. For example, if links 2 and 4 are nearly parallel, their extended lines will intersect at a point far away and not be practically available for velocity analysis. Figure 6-9 shows the same linkage as Figure 6-5 with $I_{1,3}$ located and labeled. From the definition of the instant center, both links sharing the instant center will have identical velocity at that point. Instant center $I_{1,3}$ involves the coupler (link 3), which is in complex motion, and the ground link 1, which is stationary. All points on link 1 have zero velocity in the global coordinate system, which is embedded in link 1. Therefore, $I_{1,3}$ must have zero velocity at this instant. If $I_{1,3}$ has zero velocity, then it can be considered to be an instantaneous "fixed pivot" about which link 3 is in pure rotation with respect to link 1. A moment later, $I_{1,3}$ will move to a new location and link 3 will be "pivoting" about a new instant center.

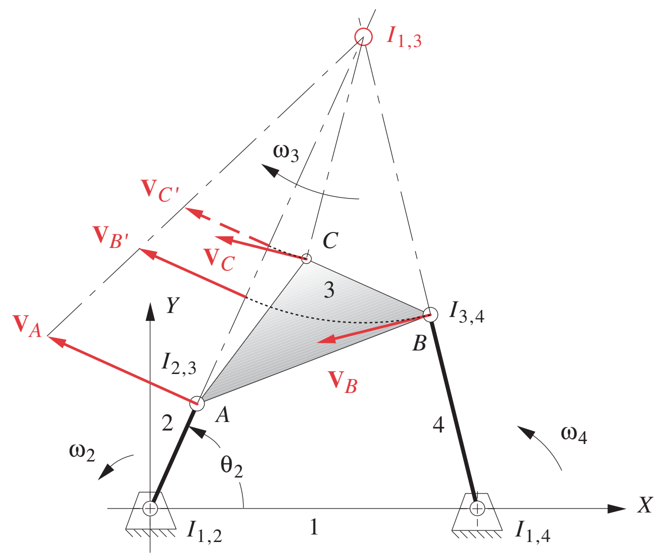

FIGURE 6-9

Velocity analysis using instant centers

The velocity of point A is shown on Figure 6-9. The magnitude of $\mathbf{V}_A$ can be computed from equation 6.7. Its direction and sense can be determined by inspection as was done in Example 6-1. Note that point A is also instant center $I_{2,3}$. It has the same velocity as part of link 2 and as part of link 3. Since link 3 is effectively pivoting about $I_{1,3}$ at this instant, the angular velocity ω_3 can be found by rearranging equation 6.7:

$$\omega_3 = \frac{v_A}{\left(AI_{1,3}\right)} \tag{6.9a}$$

Once ω_3 is known, the magnitude of $\mathbf{V}_B$ can also be found from equation 6.7:

$$v_B = \left(BI_{1,3}\right)\omega_3 \tag{6.9b}$$

Once $\mathbf{V}_B$ is known, ω_4 can also be found from equation 6.7:

$$\omega_4 = \frac{v_B}{\left(BO_4\right)} \tag{6.9c}$$

Finally, the magnitude of $\mathbf{V}_C$ (or the velocity of any other point on the coupler) can be found from equation 6.7:

$$v_C = \left(CI_{1,3}\right)\omega_3 \tag{6.9d}$$

Note that equations 6.7 and 6.9 provide only the **scalar magnitude** of these velocity vectors. We have to determine their **direction** from the information in the scale diagram (Figure 6-9). Since we know the location of $I_{1,3}$, which is an instantaneous "fixed" pivot for link 3, all of that link's absolute velocity vectors for this instant will be **perpendicular**

to their radii from $I_{1,3}$ to the point in question. $\mathbf{V}_B$ and $\mathbf{V}_C$ can be seen to be perpendicular to their radii from $I_{1,3}$. Note that $\mathbf{V}_B$ is also perpendicular to the radius from O_4 because B is also pivoting about that point as part of link 4.

A rapid graphical solution to equations 6.9 is shown in the figure. Arcs centered at $I_{1,3}$ are swung from points B and C to intersect line $AI_{1,3}$. The magnitudes of velocities $\mathbf{V}_{B'}$ and $\mathbf{V}_{C'}$ are found from the vectors drawn perpendicular to that line at the intersections of the arcs and line $AI_{1,3}$. The lengths of the vectors are defined by the line from the tip of $\mathbf{V}_A$ to the instant center $I_{1,3}$. These vectors can then be slid along their arcs back to points B and C, maintaining their tangency to the arcs.

Thus, we have in only a few steps found all the same velocities that were found using the more tedious method of Example 6-1. The instant center method is a quick graphical method to analyze velocities, but it will only work if the instant centers are in reachable locations for the particular linkage position analyzed. However, the graphical method using the velocity difference equation shown in Example 6-1 will always work, regardless of linkage position.

Angular Velocity Ratio

The **angular velocity ratio** m_V is defined as *the output angular velocity divided by the input angular velocity*. For a fourbar mechanism this is expressed as:

$$m_V = \frac{\omega_4}{\omega_2} \tag{6.10}$$

We can derive this ratio for any linkage by constructing a **pair of effective links** as shown in Figure 6-10a. The definition of **effective link pairs** is *two lines, mutually parallel, drawn through the fixed pivots and intersecting the coupler extended*. These are shown as O_2A' and O_4B' in Figure 6-10a. Note that there is an infinity of possible effective link pairs. They must be parallel to one another but may make any angle with link 3. In the figure they are shown perpendicular to link 3 for convenience in the derivation to follow. The angle between links 2 and 3 is shown as ν. The transmission angle between links 3 and 4 is μ. We will now derive an expression for the angular velocity ratio using these effective links, the actual link lengths, and angles ν and μ.

From geometry:

$$O_2A' = (O_2A)\sin\nu \qquad\qquad O_4B' = (O_4B)\sin\mu \tag{6.11a}$$

From equation 6.7

$$V_{A'} = (O_2A')\omega_2 \tag{6.11b}$$

The component of velocity $V_{A'}$ lies along the link AB. Just as with a two-force member in which a force applied at one end transmits only its component that lies along the link to the other end, this velocity component can be transmitted along the link to point B. This is sometimes called the **principle of transmissibility**. We can then equate these components at either end of the link.

$$V_{A'} = V_{B'} \tag{6.11c}$$

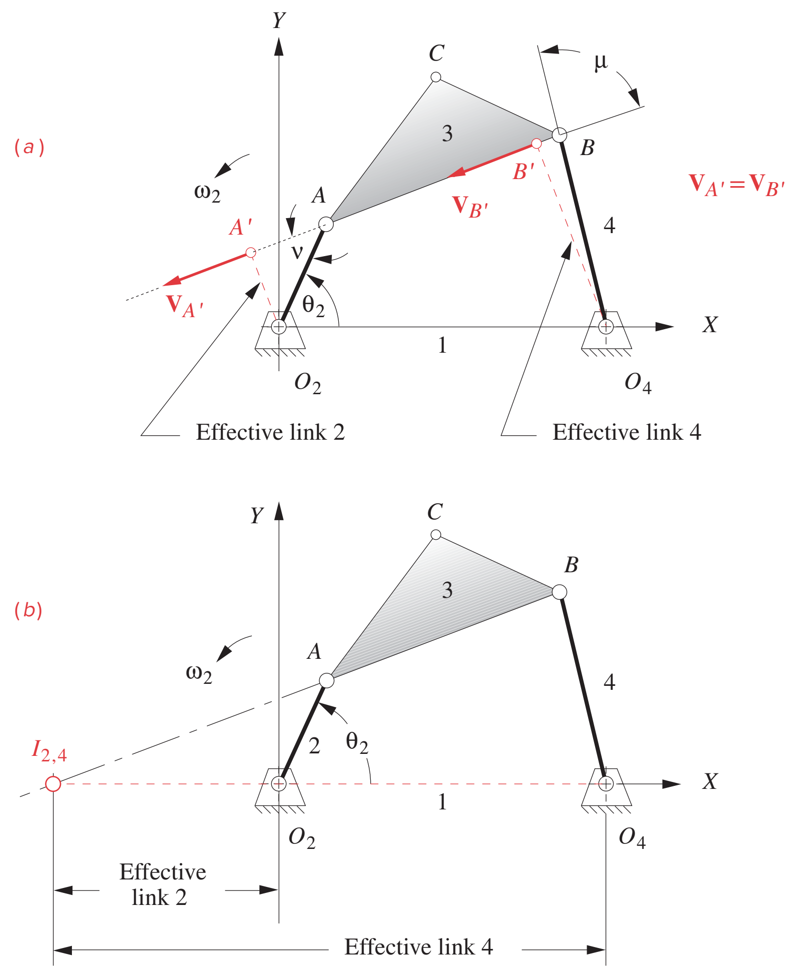

FIGURE 6-10

Effective links and the angular velocity ratio

Then:

$$O_2A'\omega_2 = O_4B'\omega_4 \tag{6.11d}$$

rearranging:

$$\frac{\omega_4}{\omega_2} = \frac{O_2A'}{O_4B'} \tag{6.11e}$$

and substituting:

$$\frac{\omega_4}{\omega_2} = \frac{O_2A\sin\nu}{O_4B\sin\mu} = m_V \tag{6.11f}$$

Note in equation 6.11f that as angle ν goes through zero, the angular velocity ratio will be zero regardless of the values of ω_2 or the link lengths, and thus ω_4 will be zero. When angle ν is zero, links 2 and 3 will be colinear and thus be in their toggle positions. We learned in Section 3.3 that the limiting positions of link 4 are defined by these toggle conditions. We should expect that the velocity of link 4 will be zero when it has come to the end of its travel. An even more interesting situation obtains if we allow angle μ to go to zero. Equation 6.11f shows that ω_4 **will go to infinity** when $\mu = 0$, regardless of the values of ω_2 or the link lengths. We clearly cannot allow μ to reach zero. In fact, we learned in Section 3.3 that we should keep this transmission angle μ above about 40 degrees to maintain good quality of motion and force transmission.*

Figure 6-10b shows the same linkage as in Figure 6-10a, but the effective links have now been drawn so that they are not only parallel but are also colinear, and thus lie on top of one another. Both intersect the extended coupler at the same point, which is instant center $I_{2,4}$. So, *A*' and *B*' of Figure 6-10a are now coincident at $I_{2,4}$. This allows us to write an equation for the **angular velocity ratio** in terms of the distances from the fixed pivots to instant center $I_{2,4}$.

$$m_V = \frac{\omega_4}{\omega_2} = \frac{O_2 I_{2,4}}{O_4 I_{2,4}} \tag{6.11g}$$

Thus, the instant center $I_{2,4}$ can be used to determine the **angular velocity ratio**.

Mechanical Advantage

The power P in a mechanical system can be defined as the dot or scalar product of the force vector **F** and the velocity vector **V** at any point:

$$P = \mathbf{F} \cdot \mathbf{V} = F_x V_x + F_y V_y \tag{6.12a}$$

For a rotating system, power P becomes the product of torque T and angular velocity ω that, in two dimensions, have the same (z) direction:

$$P = T\omega \tag{6.12b}$$

The power flows through a passive system and:

$$P_{in} = P_{out} + losses \tag{6.12c}$$

Mechanical efficiency can be defined as:

$$\varepsilon = \frac{P_{out}}{P_{in}} \tag{6.12d}$$

Linkage systems can be very efficient if they are well made with low friction bearings on all pivots. Losses are often less than 10%. For simplicity in the following analysis we will assume that the losses are zero (i.e., a conservative system). Then, letting T_{in} and ω_{in} represent input torque and angular velocity, and T_{out} and ω_{out} represent output torque and angular velocity,

* This limitation on transmission angle is only critical if the output load is applied to a link that is pivoted to ground (i.e., to link 4 in the case of a fourbar linkage). If the load is applied to a floating link (e.g., a coupler), then other measures of the quality of force transmission than the transmission angle are more appropriate, as discussed in Chapter 11, Section 11.12, where the joint force index is defined.

$$P_{in} = T_{in}\omega_{in}$$
$$P_{out} = T_{out}\omega_{out} \tag{6.12e}$$

and:

$$P_{out} = P_{in}$$
$$T_{out}\omega_{out} = T_{in}\omega_{in}$$
$$\frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}} \tag{6.12f}$$

Note that the **torque ratio** ($m_T = T_{out} / T_{in}$) is the inverse of the angular velocity ratio.

Mechanical advantage (m_A) can be defined as:

$$m_A = \frac{F_{out}}{F_{in}} \tag{6.13a}$$

6

Assuming that the input and output forces are applied at some radii r_{in} and r_{out}, perpendicular to their respective force vectors,

$$F_{out} = \frac{T_{out}}{r_{out}}$$
$$F_{in} = \frac{T_{in}}{r_{in}} \tag{6.13b}$$

substituting equations 6.13b in 6.13a gives an expression in terms of torque.

$$m_A = \left(\frac{T_{out}}{T_{in}}\right)\left(\frac{r_{in}}{r_{out}}\right) \tag{6.13c}$$

Substituting equation 6.12f in 6.13c gives

$$m_A = \left(\frac{\omega_{in}}{\omega_{out}}\right)\left(\frac{r_{in}}{r_{out}}\right) \tag{6.13d}$$

and substituting equation 6.11f gives

$$m_A = \left(\frac{O_4 B \sin\mu}{O_2 A \sin\nu}\right)\left(\frac{r_{in}}{r_{out}}\right) \tag{6.13e}$$

See Figure 6-11 and compare equation 6.13e to equation 6.11f and its discussion under **angular velocity ratio**. Equation 6.13e shows that for any choice of r_{in} and r_{out}, the mechanical advantage responds to changes in angles ν and μ in opposite fashion to that of the angular velocity ratio. If the transmission angle μ goes to zero (which we don't want it to do), the mechanical advantage also goes to zero regardless of the amount of input force or torque applied. But, when angle ν goes to zero (which it can and does, twice per cycle in a Grashof linkage), the mechanical advantage becomes infinite! This is the principle of a rock-crusher mechanism as shown in Figure 6-11. A quite moderate force applied to link 2 can generate a huge force on link 4 to crush the rock. Of course, we cannot expect to achieve the theoretical output of infinite force or torque magnitude, as the strengths of the links and joints will limit the maximum forces and torques obtainable. Another com-

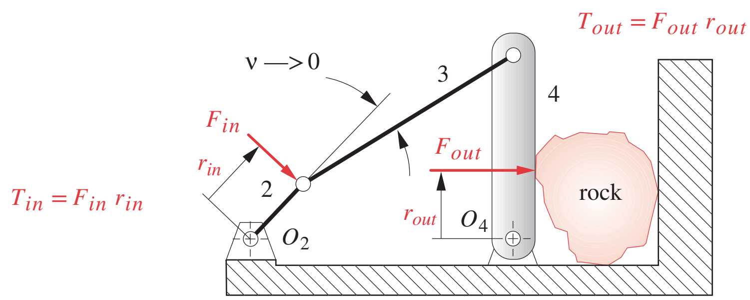

FIGURE 6-11

"Rock-crusher" toggle mechanism

mon example of a linkage that takes advantage of this theoretically infinite mechanical advantage at the toggle position is a ViseGrip locking pliers (see Figure P6-21).

These two ratios, **angular velocity ratio** and **mechanical advantage**, provide useful, dimensionless **indices of merit** by which we can judge the relative quality of various linkage designs that may be proposed as solutions.

Using Instant Centers in Linkage Design

In addition to providing a quick numerical velocity analysis, instant center analysis more importantly gives the designer a remarkable overview of the linkage's global behavior. It is quite difficult to mentally visualize the complex motion of a "floating" coupler link even in a simple fourbar linkage, unless you build a model or run a computer simulation. Because this complex coupler motion in fact reduces to an instantaneous pure rotation about the instant center $I_{1,3}$, finding that center allows the designer to visualize the motion of the coupler as a pure rotation. One can literally *see* the motion and the directions of velocities of any points of interest by relating them to the instant center. It is only necessary to draw the linkage in a few positions of interest, showing the instant center locations for each position.

Figure 6-12 shows a practical example of how this visual, qualitative analysis technique could be applied to the design of an automobile rear suspension system. Most automobile suspension mechanisms are either fourbar linkages or fourbar crank-sliders, with the wheel assembly carried on the coupler (as was also shown in Figure 3-19). Figure 6-12a shows a rear suspension design from a domestic car of 1970s vintage that was later redesigned because of a disturbing tendency to "bump steer," i.e., turn the rear axle when hitting a bump on one side of the car. The figure is a view looking from the center of the car outward, showing the fourbar linkage that controls the up and down motion of one side of the rear axle and one wheel. Links 2 and 4 are pivoted to the frame of the car which is link 1. The wheel and axle assembly is rigidly attached to the coupler, link 3. Thus the wheel assembly has complex motion in the vertical plane. Ideally, one would like the wheel to move up and down in a straight vertical line when hitting a bum Figure 6-12b shows the motion of the wheel and the new instant center ($I_{1,3}$) location for the situation when one wheel has hit a bum The velocity vector for the center of the wheel in each position is drawn perpendicular to its radius from $I_{1,3}$. You can see that the wheel center has a significant horizontal component of motion as it moves up over the bump.

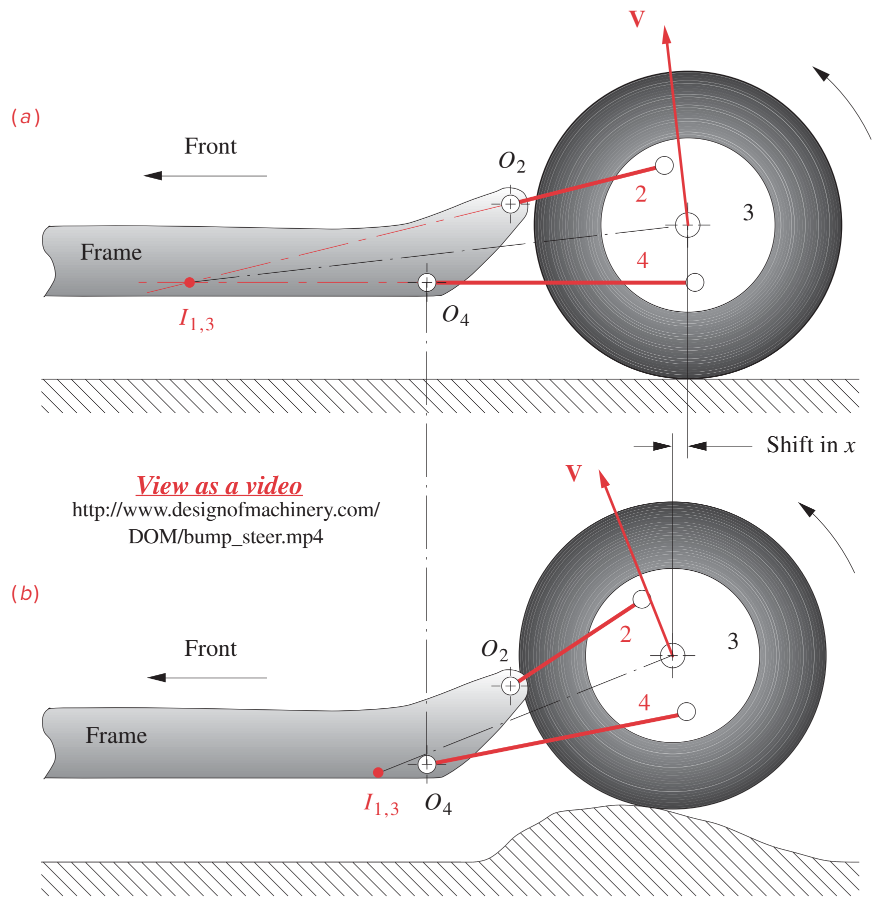

FIGURE 6-12

"Bump steer" due to shift in instant center location

This horizontal component causes the wheel center on that side of the car to move forward while it moves upward, thus turning the axle (about a vertical axis) and steering the car with the rear wheels in the same way that you steer a toy wagon. Viewing the path of the instant center over some range of motion gives a clear picture of the behavior of the coupler link. The undesirable behavior of this suspension linkage system could have been predicted from this simple instant center analysis before ever building the mechanism.

Another practical example of the effective use of instant centers in linkage design is shown in Figure 6-13, which is an optical adjusting mechanism used to position a mirror and allow a small amount of rotational adjustment.[1] A more detailed account of this design case study[2] is provided in Chapter 1. The designer, K. Towfigh, recognized that $I_{1,3}$ at point E is an instantaneous "fixed pivot" and will allow very small pure rotations about that point with very small translational error. He then designed a one-piece, plastic fourbar linkage whose "pin joints" are thin webs of plastic that flex to allow slight rota-

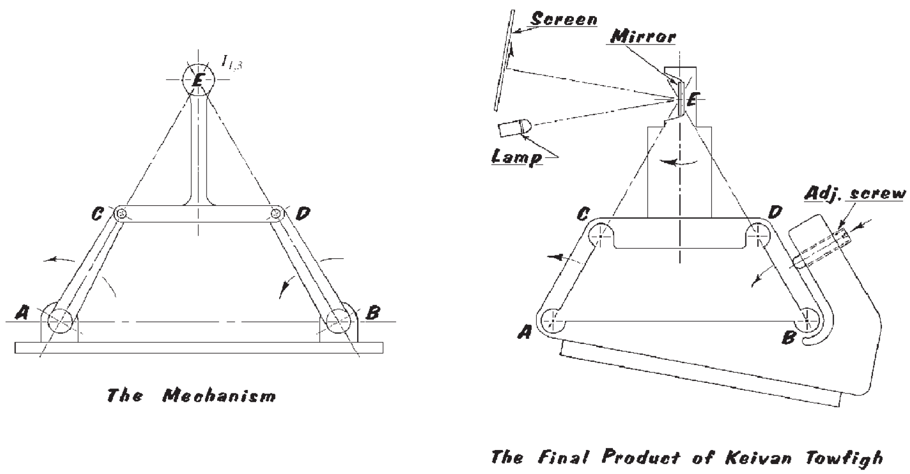

FIGURE 6-13

An optical adjustment compliant linkage *Reproduced from reference [2] with permission*

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tion. This is termed a **compliant linkage**,* one that uses elastic deformations of the links as hinges instead of pin joints. He then placed the mirror on the coupler at $I_{1,3}$. Even the fixed link 1 is the same piece as the "movable links" and has a small set screw to provide the adjustment. A simple and elegant design.

* See also Section 2.16 for more information on compliant mechanisms.

6.5 CENTRODES *View a tutorial video* (21:01)†

† http://www.designofmachinery.com/DOM/Centrodes.mp4

Figure 6-14 illustrates the fact that the successive positions of an instant center (or **centro**) form a path of their own. *This path, or locus, of the instant center is called the* **centrode**. Since there are two links needed to create an instant center, there will be two centrodes associated with any one instant center. These are formed by projecting the path of the instant center first on one link and then on the other. Figure 6-14a shows the locus of instant center $I_{1,3}$ as projected onto link 1. Because link 1 is stationary, or fixed, this is called the **fixed centrode**. By temporarily inverting the mechanism and fixing link 3 as the ground link, as shown in Figure 6-14b, we can move link 1 as the coupler and project the locus of $I_{1,3}$ onto link 3. In the original linkage, link 3 was the moving coupler, so this is called the **moving centrode**. Figure 6-14c shows the original linkage with both fixed and moving centrodes superposed.

The definition of the instant center says that both links have the same velocity at that point, at that instant. Link 1 has zero velocity everywhere, as does the fixed centrode. So, as the linkage moves, the moving centrode must roll against the fixed centrode without slipping. If you cut the fixed and moving centrodes out of metal, as shown in Figure 6-14d, and roll the moving centrode (which is link 3) against the fixed centrode (which is link 1), the complex motion of link 3 will be identical to that of the original linkage. *All of the coupler curves of points on link 3 will have the same path shapes as in the original linkage*. We now have, in effect, a "linkless" fourbar linkage, really one composed of two bodies that have these centrode shapes rolling against one another. Links 2 and 4 have been eliminated. Note that the example shown in Figure 6-14 is a non-Grashof fourbar. The lengths of its centrodes are limited by the double-rocker toggle positions.

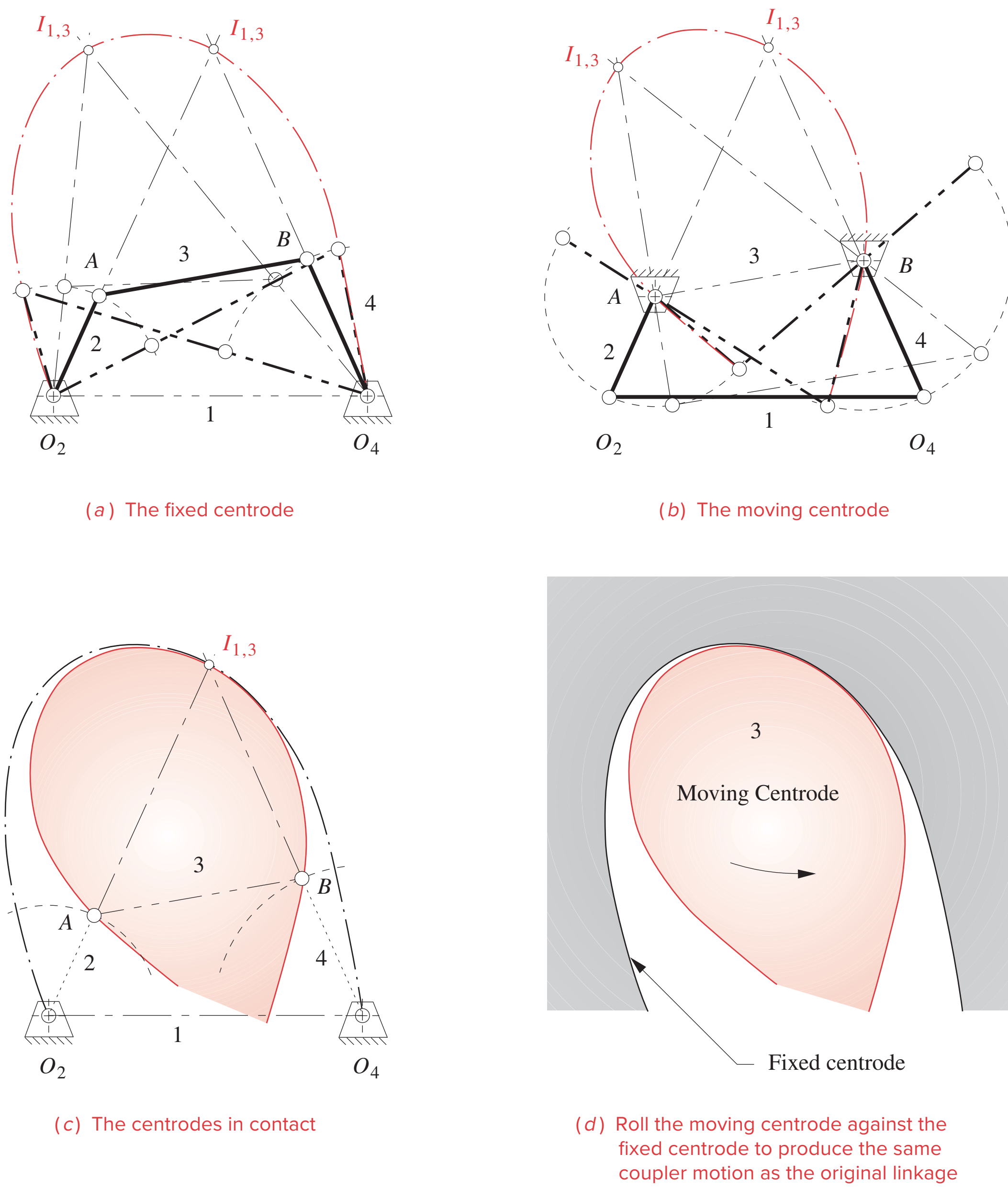
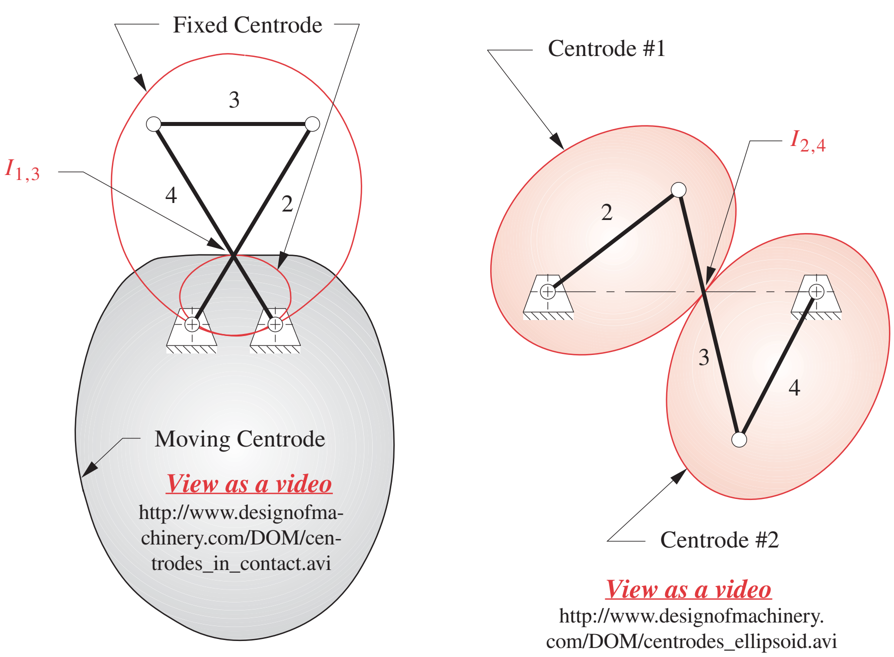


FIGURE 6-14

Open-loop fixed and moving centrodes (or polodes) of a fourbar linkage

All instant centers of a linkage will have centrodes.* If the links are directly connected by a joint, such as $I_{2,3}$, $I_{3,4}$, $I_{1,2}$, and $I_{1,4}$, their fixed and moving centrodes will degenerate to a point at that location on each link. The most interesting centrodes are those involving links not directly connected to one another such as $I_{1,3}$ and $I_{2,4}$. If we look at the double-crank linkage in Figure 6-15a in which links 2 and 4 both revolve fully, we see that the centrodes of $I_{1,3}$ form closed curves. The motion of link 3 with respect to link 1 could be duplicated by causing these two centrodes to roll against one another without slipping. Note that there are two loops to the moving centrode. Both must roll on the single-loop fixed centrode to complete the motion of the equivalent double-crank linkage.

We have so far dealt largely with the instant center $I_{1,3}$. Instant center $I_{2,4}$ involves two links that are each in pure rotation and not directly connected to one another. If we use a special-case Grashof linkage with the links crossed (sometimes called an **antiparallelogram** linkage), the centrodes of $I_{2,4}$ become ellipses as shown in Figure 6-15b. To guarantee no slip, it will probably be necessary to put meshing teeth on each centrode. We then will have a pair of elliptical, **noncircular gears**, or *gearset*, which gives the *same output motion as the original double-crank linkage* and will have the *same variations in the angular velocity ratio and mechanical advantage as the linkage* had. Thus we can see that *gearsets are also just fourbar linkages in disguise*. Noncircular gears find much use in machinery, such as printing presses, where rollers must be speeded and slowed with some pattern during each cycle or revolution. More complicated shapes of noncircular gears are analogous to cams and followers in that the equivalent fourbar linkage must



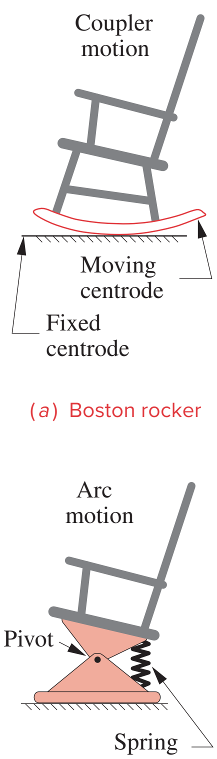
(*a*) Closed-loop centrodes of $I_{1,3}$ for a Grashof double-crank linkage

(*b*) Ellipsoidal centrodes of $I_{2,4}$ for a special-case Grashof anti-parallelogram linkage

FIGURE 6-15

Closed-loop fixed and moving centrodes

* Since instant centers are called *poles* as well as *centros*, *centrodes* are sometimes also called *polodes*. We will use the *centro* and *centrode* nomenclature in this text.



(*a*) Boston rocker

(*b*) Platform rocker

FIGURE 6-16

Some rocking chairs use centrodes of a fourbar linkage

have variable-length links. **Circular gears** are just a special case of noncircular gears that give a **constant angular velocity ratio** and are widely used in all machines. Gears and gearsets will be dealt with in greater detail in Chapter 9.

In general, centrodes of crank-rockers and double- or triple-rockers will be open curves with asymptotes. Centrodes of double-crank linkages will be closed curves. Program LINKAGES will calculate and draw the fixed and moving centrodes for any linkage input to it. Open the files F06-14.4br, F06-15a.4br, and F06-15b.4br in program LINKAGES to see the centrodes of these linkages drawn as the linkages rotate.

A "Linkless" Linkage

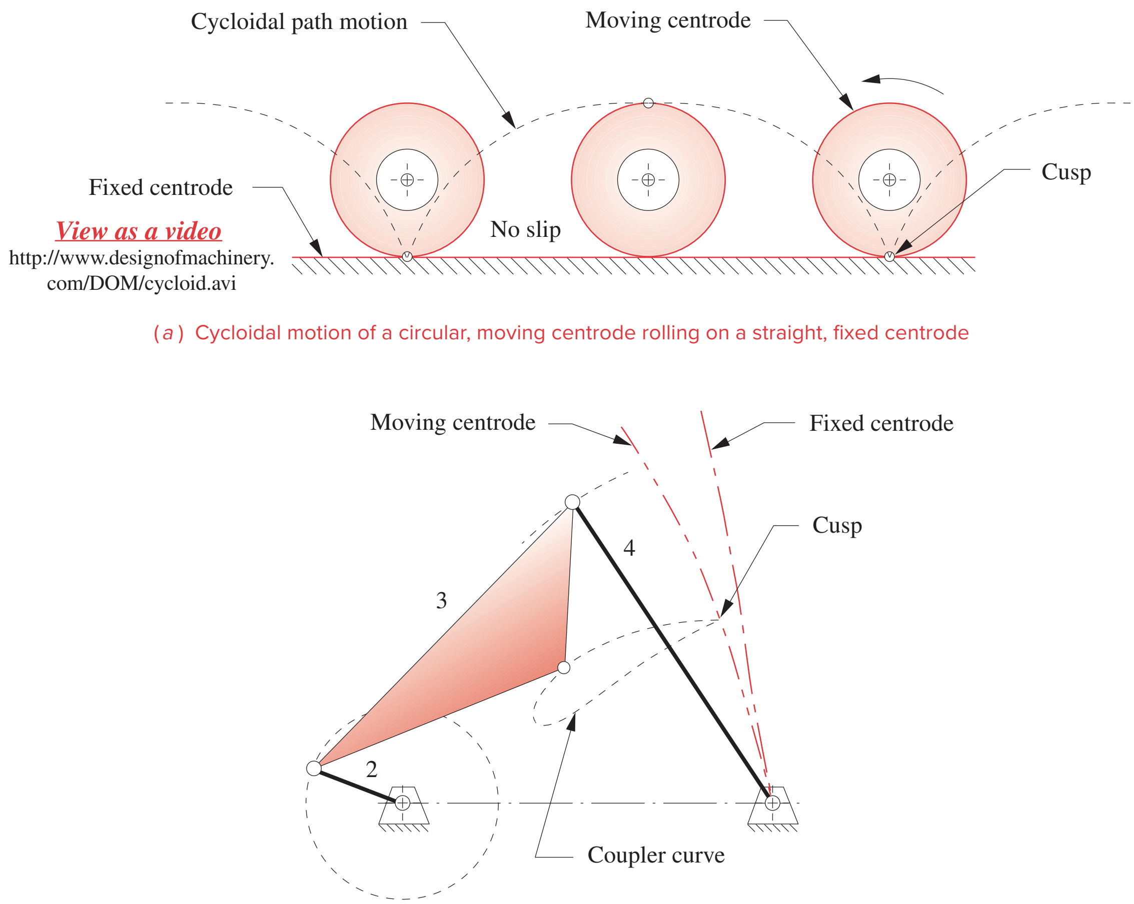
A common example of a mechanism made of centrodes is shown in Figure 6-16a. You have probably rocked in a *Boston* or *Hitchcock* rocking chair and experienced the soothing motions that it delivers to your body. You may have also rocked in a *platform* rocker as shown in Figure 6-16b and noticed that its motion did not feel as soothing.

There are good kinematic reasons for the difference. The platform rocker has a fixed pin joint between the seat and the base (floor). Thus all parts of your body are in pure rotation along concentric arcs. You are in effect riding on the rocker of a linkage.

The Boston rocker has a shaped (curved) base, or "runners," which rolls against the floor. These runners are usually *not* circular arcs. They have a higher-order curve contour. They are, in fact, **moving centrodes**. The floor is the **fixed centrode**. When one is rolled against the other, the chair and its occupant experience coupler curve motion. Every part of your body travels along a different sixth-order coupler curve that provides smooth accelerations and velocities and feels better than the cruder second-order (circular) motion of the platform rocker. Our ancestors, who carved these rocking chairs, probably had never heard of fourbar linkages and centrodes, but they knew intuitively how to create comfortable motions.

Cusps

Another example of a centrode that you probably use frequently is the path of the tire on your car or bicycle. As your tire rolls against the road without slipping, the road becomes a fixed centrode, and the circumference of the tire is the moving centrode. The tire is, in effect, the coupler of a linkless fourbar linkage. All points on the contact surface of the tire move along cycloidal coupler curves and pass through a cusp of zero velocity when they reach the fixed centrode at the road surface as shown in Figure 6-17a. All other points on the tire and wheel assembly travel along coupler curves that do not have cusps. This last fact is a clue to a means to identify coupler points that will have cusps in their coupler curve. *If a coupler point is chosen to be on the moving centrode at one extreme of its path motion* (*i.e., at one of the positions of* $I_{1,3}$), *then it will have a cusp in its coupler curve*. Figure 6-17b shows a coupler curve of such a point, drawn with program LINKAGES. The right end of the coupler path touches the moving centrode and as a result has a cusp at that point. So, if you desire a cusp in your coupler motion, many are available. Simply choose a coupler point on the moving centrode of link 3. Open the file F06-17b.4br in program LINKAGES to animate that linkage with its coupler curve or centrodes. Note in Figure 6-14 that choosing any location of instant center $I_{1,3}$ on the coupler as the coupler point will provide a cusp at that point.



(*a*) Cycloidal motion of a circular, moving centrode rolling on a straight, fixed centrode

(*b*) Coupler curve cusps exist only on the moving centrode

FIGURE 6-17

Examples of centrodes

6.6 VELOCITY OF SLIP

When there is a sliding joint between two links and neither one is the ground link, the velocity analysis is more complicated. Figure 6-18 shows an inversion of the fourbar crank-slider mechanism in which the sliding joint is floating, i.e., not grounded. To solve for the velocity at the sliding joint *A*, we have to recognize that there is more than one point *A* at that joint. There is a point *A* as part of link 2 (A_2), a point *A* as part of link 3 (A_3), and a point *A* as part of link 4 (A_4). This is a CASE 2 situation in which we have at least two points belonging to different links but occupying the same location at a given instant. Thus, the **relative velocity** equation 6.6 will apply. We can usually solve for the velocity of at least one of these points directly from the known input information using equation 6.7. It and equation 6.6 are all that is needed to solve for everything else. In

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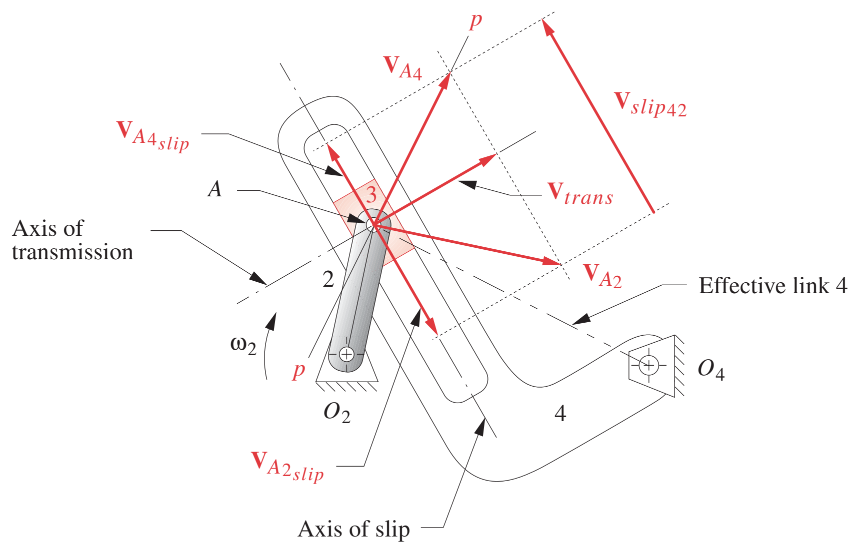

FIGURE 6-18

Velocity of slip and velocity of transmission (note that the applied ω is negative as shown)

this example, link 2 is the driver, and θ_2 and ω_2 are given for the "freeze frame" position shown. We wish to solve for ω_4, the angular velocity of link 4, and also for the velocity of slip at the joint labeled A.

In Figure 6-18 the **axis of slip** is shown to be tangent to the slider motion and is the line along which all sliding occurs between links 3 and 4. The **axis of transmission** is defined to be perpendicular to the axis of slip and pass through the slider joint at A. This *axis of transmission is the* **only line** *along which we can transmit motion or force across the slider joint, except for friction.* We will assume friction to be negligible in this example. Any force or velocity vector applied to point A can be resolved into two components along these two axes that provide a *translating and rotating, local coordinate system* for analysis at the joint. The component along the axis of transmission will do useful work at the joint. But, the component along the axis of slip does no work, except *friction work*.

EXAMPLE 6-5

Graphical Velocity Analysis at a Sliding Joint.

Problem: Given θ_2, θ_3, θ_4, ω_2, find ω_3, ω_4, $\mathbf{V}_A$, by graphical methods.

Solution: (See Figure 6-18.)

1 Start at the end of the linkage for which you have the most information. Calculate the magnitude of the velocity of **point *A* as part of link 2** (A_2) using scalar equation 6.7.

$$v_{A_2} = (AO_2)\omega_2 \tag{a}$$

2 Draw the velocity vector $\mathbf{V}_{A2}$ with its length equal to its magnitude v_{A2} at some convenient scale and with its root at point A and its direction perpendicular to the radius AO_2. Its sense is the same as that of ω_2 as is shown in Figure 6-18.

3 Draw the **axis of slip** and **axis of transmission** through point A.

4 Project $\mathbf{V}_{A2}$ onto the axis of slip and onto the axis of transmission to create the components $\mathbf{V}_{A2slip}$ and $\mathbf{V}_{trans}$ of $\mathbf{V}_{A2}$ on the axes of slip and transmission, respectively. Note that the **transmission component** is shared by all true velocity vectors at this point, as it is the only component that can transmit across the joint.

5 Note that link 3 is pin-jointed to link 2, so $\mathbf{V}_{A3} = \mathbf{V}_{A2}$.

6 Note that the direction of the velocity of point $\mathbf{V}_{A4}$ is predictable since all points on link 4 are pivoting in pure rotation about point O_4. Draw the line pp through point A and perpendicular to the effective link 4, AO_4. Line pp is the direction of velocity $\mathbf{V}_{A4}$.

7 Construct the true magnitude of velocity vector $\mathbf{V}_{A4}$ by extending the projection of the **transmission component** $\mathbf{V}_{trans}$ until it intersects line p

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8 Project $\mathbf{V}_{A4}$ onto the axis of slip to create the **slip component** $\mathbf{V}_{A4slip}$.

9 Write the relative velocity vector equation 6.6 for the **slip components** of point A_2 versus point A_4.

$$V_{slip_{42}} = V_{A_{4}slip} - V_{A_{2}slip} \tag{b}$$

10 The angular velocities of links 3 and 4 are identical because they share the slider joint and must rotate together. They can be calculated from equation 6.7:

$$\omega_4 = \omega_3 = \frac{V_{A_4}}{AO_4} \tag{c}$$

Instant center analysis also can be used to solve sliding-joint velocity problems.

EXAMPLE 6-6

Graphical Velocity Analysis of a Cam and Follower.

Problem: Given θ_2, ω_2, find ω_3, by graphical methods.

Solution: (See Figure 6-19.)

1 Construct the effective radius of the cam $R_{2\,eff}$ at the instantaneous point of contact with the follower for this position (point A in the figure). Its length is distance O_2A. Calculate the magnitude of the velocity of point A as part of link 2 (A_2) using scalar equation 6.7.

$$v_{A_2} = (AO_2)\omega_2 \tag{a}$$

2 Draw the velocity vector $\mathbf{V}_{A2}$ with its length equal to its magnitude v_{A2} at some convenient scale and with its root at point A and its direction perpendicular to the radius O_2A. Its sense is the same as that of ω_2 as is shown in Figure 6-19.

3 Construct the axis of slip (common tangent to cam and follower) and its normal, the axis of transmission, as shown in Figure 6-19.

4 Project $\mathbf{V}_{A2}$ onto the axis of transmission to create the component $\mathbf{V}_{trans}$. Note that the **transmission component** is shared by all true velocity vectors at this point, as it is the only component that can transmit across the joint.

5 Project $\mathbf{V}_{A2}$ onto the axis of slip to create the **slip component** $\mathbf{V}_{A2slip}$.

6 Note that the direction of the velocity of point $\mathbf{V}_{A3}$ is predictable since all points on link 3 are pivoting in pure rotation about point O_3. Construct the effective radius of the follower $R_{3\,eff}$ at the instantaneous point of contact with the follower for this position (point A in the figure). Its length is distance O_3A.

7 Construct a line in the direction of $\mathbf{V}_{A3}$ perpendicular to $R_{3\,eff}$. Construct the true magnitude of velocity vector $\mathbf{V}_{A3}$ by extending the projection of the transmission component $\mathbf{V}_{trans}$ until it intersects the line of $\mathbf{V}_{A3}$.

8 Project $\mathbf{V}_{A3}$ onto the axis of slip to create the **slip component** $\mathbf{V}_{A3slip}$.

9 The total slip velocity at A is the vector difference between the two slip components. Write the relative velocity vector equation 6.6 for the slip components of point A_3 versus A_2.

$$V_{slip_{32}} = V_{A_3 slip} - V_{A_2 slip} \tag{b}$$

10 The angular velocity of link 3 can be calculated from equation 6.7:

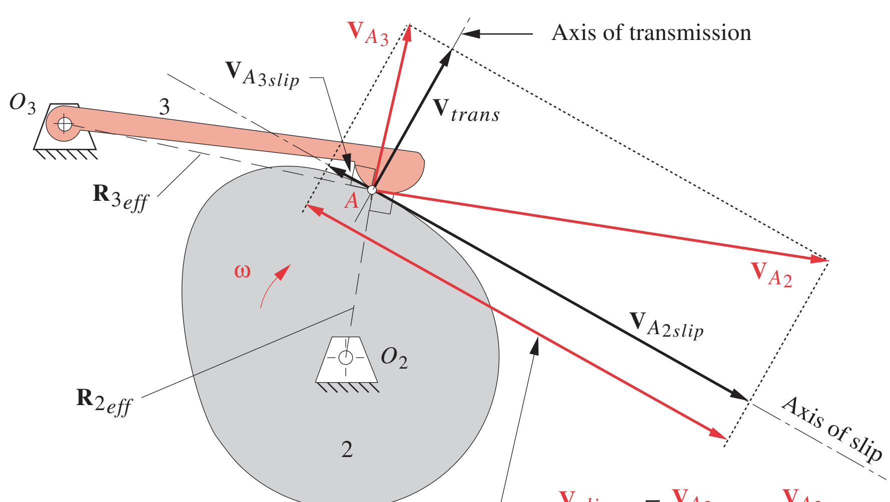

FIGURE 6-19

Graphical velocity analysis of a cam and follower

$$\omega_3 = \frac{V_{A_3}}{AO_3} \tag{c}$$

The above examples show how mechanisms with sliding or half joints can be solved graphically for velocities at one position. In the next section, we will develop the general solution using algebraic equations to solve similar problems.

6.7 ANALYTICAL SOLUTIONS FOR VELOCITY ANALYSIS

View the lecture video (46:41)†

† http://www.designofmachinery.com/DOM/Velocity_Analysis_with_Vectors.mp4

The Fourbar Pin-Jointed Linkage

The vector-loop position equations for the fourbar pin-jointed linkage were derived in Section 4.5. The linkage was shown in Figure 4-6 and is shown again in Figure 6-20 on which we also show an input angular velocity ω_2 applied to link 2. This ω_2 can be a time-varying input velocity. The vector loop equation is shown in equations 4.5a and 4.5c, repeated here for your convenience.

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0 \tag{4.5a}$$

As before, we substitute the complex number notation for the vectors, denoting their scalar lengths as *a, b, c, d* as shown in Figure 6-20a.

$$a e^{j\theta_2} + b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0 \tag{4.5c}$$

To get an expression for velocity, differentiate equation 4.5c with respect to time.

$$ja e^{j\theta_2}\frac{d\theta_2}{dt} + jb e^{j\theta_3}\frac{d\theta_3}{dt} - jc e^{j\theta_4}\frac{d\theta_4}{dt} = 0 \tag{6.14a}$$

But,

$$\frac{d\theta_2}{dt} = \omega_2; \qquad \frac{d\theta_3}{dt} = \omega_3; \qquad \frac{d\theta_4}{dt} = \omega_4 \tag{6.14b}$$

and:

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0 \tag{6.14c}$$

Note that the θ_1 term has dropped out because that angle is a constant, and thus its derivative is zero. Note also that equation 6.14 is, in fact, the **relative velocity** or **velocity difference equation**.

$$\mathbf{V}_A + \mathbf{V}_{BA} - \mathbf{V}_B = 0 \tag{6.15a}$$

where:

$$\begin{aligned} \mathbf{V}_A &= ja\omega_2 e^{j\theta_2} \\ \mathbf{V}_{BA} &= jb\omega_3 e^{j\theta_3} \\ \mathbf{V}_B &= jc\omega_4 e^{j\theta_4} \end{aligned} \tag{6.15b}$$

Compare equations 6.15 to equations 6.3, 6.5, and 6.6. This equation is solved graphically in the vector diagram of Figure 6-20b. Note the transmission angle μ drawn between

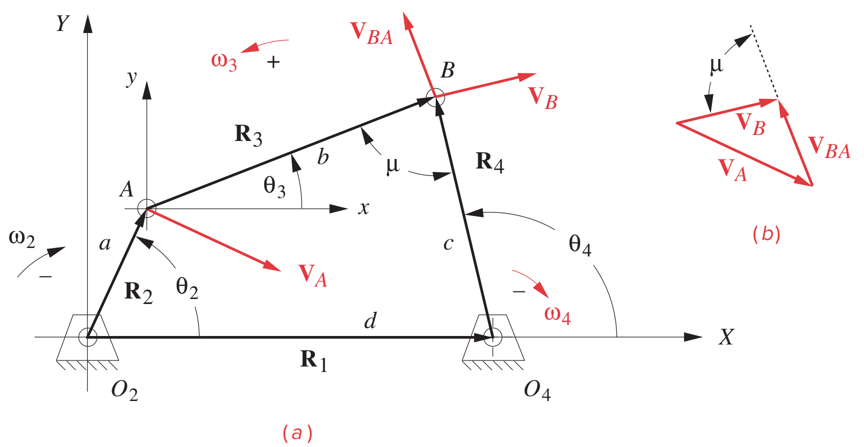

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FIGURE 6-20

Position vector loop for a fourbar linkage showing velocity vectors for a negative (*cw*) ω_2

links 3 and 4 and also between V_B and V_{BA}. This shows an alternate way to define the transmission angle using the velocity vectors at point *B*.

We now need to solve equation 6.14 for ω_3 and ω_4, knowing the input velocity ω_2, the link lengths, and all link angles. Thus the position analysis derived in Section 4.5 must be done first to determine the link angles before this velocity analysis can be completed. We wish to solve equation 6.14 to get expressions in this form:

$$\omega_3 = f\left(a, b, c, d, \theta_2, \theta_3, \theta_4, \omega_2\right) \qquad \omega_4 = g\left(a, b, c, d, \theta_2, \theta_3, \theta_4, \omega_2\right) \tag{6.16}$$

The strategy of solution will be the same as was done for the position analysis. First, substitute the Euler identity from equation 4.4a in each term of equation 6.14c:

$$\begin{aligned} ja\omega_2\left(\cos\theta_2 + j\sin\theta_2\right) + jb\omega_3\left(\cos\theta_3 + j\sin\theta_3\right) \\ -jc\omega_4\left(\cos\theta_4 + j\sin\theta_4\right) = 0 \end{aligned} \tag{6.17a}$$

Multiply through by the operator *j*:

$$\begin{aligned} a\omega_2\left(j\cos\theta_2 + j^2\sin\theta_2\right) + b\omega_3\left(j\cos\theta_3 + j^2\sin\theta_3\right) \\ -c\omega_4\left(j\cos\theta_4 + j^2\sin\theta_4\right) = 0 \end{aligned} \tag{6.17b}$$

The cosine terms have become the imaginary, or *y*-directed terms, and because $j^2 = -1$, the sine terms have become real or *x*-directed.

$$\begin{aligned} a\omega_2\left(-\sin\theta_2 + j\cos\theta_2\right) + b\omega_3\left(-\sin\theta_3 + j\cos\theta_3\right) \\ -c\omega_4\left(-\sin\theta_4 + j\cos\theta_4\right) = 0 \end{aligned} \tag{6.17c}$$

We can now separate this vector equation into its two components by collecting all real and all imaginary terms separately:

real part (x component):

$$-a\omega_2 \sin\theta_2 - b\omega_3 \sin\theta_3 + c\omega_4 \sin\theta_4 = 0 \tag{6.17d}$$

imaginary part (y component):

$$a\omega_2 \cos\theta_2 + b\omega_3 \cos\theta_3 - c\omega_4 \cos\theta_4 = 0 \tag{6.17e}$$

Note that the j's have canceled in equation 6.17e. We can solve these two equations, 6.17d and 6.17e, simultaneously by direct substitution to get:

$$\omega_3 = \frac{a\omega_2}{b} \frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} \tag{6.18a}$$

$$\omega_4 = \frac{a\omega_2}{c} \frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} \tag{6.18b}$$

Once we have solved for ω_3 and ω_4, we can then solve for the linear velocities by substituting the Euler identity into equations 6.15,

$$\mathbf{V}_A = ja\omega_2(\cos\theta_2 + j\sin\theta_2) = a\omega_2(-\sin\theta_2 + j\cos\theta_2) \tag{6.19a}$$

$$\mathbf{V}_{BA} = jb\omega_3(\cos\theta_3 + j\sin\theta_3) = b\omega_3(-\sin\theta_3 + j\cos\theta_3) \tag{6.19b}$$

$$\mathbf{V}_B = jc\omega_4(\cos\theta_4 + j\sin\theta_4) = c\omega_4(-\sin\theta_4 + j\cos\theta_4) \tag{6.19c}$$

where the real and imaginary terms are the x and y components, respectively. Equations 6.18 and 6.19 provide a complete solution for the angular velocities of the links and the linear velocities of the joints in the pin-jointed fourbar linkage. Note that there are also two solutions to this velocity problem, corresponding to the open and crossed circuits of the linkage. They are found by the substitution of the open or crossed circuit values of θ_3 and θ_4 obtained from equations 4.10 and 4.12-4.13 into equations 6.18 and 6.19. Figure 6-20a shows the open circuit.

EXAMPLE 6-7

Velocity Analysis of a Fourbar Linkage with the Vector Loop Method.

Problem: Given a fourbar linkage with the link lengths $L_1 = d = 100$ mm, $L_2 = a = 40$ mm, $L_3 = b = 120$ mm, $L_4 = c = 80$ mm. For $\theta_2 = 40°$ and $\omega_2 = 25$ rad/sec find the values of ω_3 and ω_4, V_A, V_{BA}, and V_B for the open circuit of the linkage. Use the angles found for the same linkage and position in Example 4-1.

Solution: (See Figure 6-20 for nomenclature.)

1 Example 4-1 found the link angles for the open circuit of this linkage to be $\theta_3 = 20.298°$ and $\theta_4 = 57.325°$.

2 Use these angles and equations 6.18 to find ω_3 and ω_4 for the open circuit.

$$\omega_3 = \frac{a\omega_2}{b}\frac{\sin(\theta_4 - \theta_2)}{\sin(\theta_3 - \theta_4)} = \frac{40(25)}{120}\frac{\sin(57.325° - 40°)}{\sin(20.298° - 57.325°)} = -4.121 \text{ rad/sec}$$

$$\omega_4 = \frac{a\omega_2}{c}\frac{\sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_3)} = \frac{40(25)}{80}\frac{\sin(40° - 20.298°)}{\sin(57.325° - 20.298°)} = 6.998 \text{ rad/sec} \tag{a}$$

3 Use the angular velocities and equations 6.19 to find the linear velocities of points A and B.

$$\begin{aligned}\mathbf{V}_A &= a\omega_2(-\sin\theta_2 + j\cos\theta_2)\\ &= 40(25)(-\sin 40° + j\cos 40°) = -642.79 + j766.04\end{aligned}$$

$$\mathbf{V}_{A_x} = -642.79; \quad \mathbf{V}_{A_y} = 766.04; \quad \mathbf{V}_{A_{mag}} = 1000 \text{ mm/sec}; \quad \mathbf{V}_{A_{ang}} = 130° \tag{b}$$

$$\begin{aligned}\mathbf{V}_{BA} &= b\omega_3(-\sin\theta_3 + j\cos\theta_3)\\ &= 120(-4.121)(-\sin 20.298° + j20.298°) = 171.55 - j463.80\end{aligned}$$

$$\mathbf{V}_{BA_x} = 171.55; \quad \mathbf{V}_{BA_y} = -463.80; \quad \mathbf{V}_{BA_{mag}} = 494.51 \text{ mm/sec}; \quad \mathbf{V}_{BA_{ang}} = -69.70° \tag{c}$$

$$\begin{aligned}\mathbf{V}_B &= c\omega_4(-\sin\theta_4 + j\cos\theta_4)\\ &= 80(6.998)(-\sin 57.325 + j\cos 57.325) = -471.242 + j302.243\end{aligned}$$

$$\mathbf{V}_{B_x} = -471.242; \quad \mathbf{V}_{B_y} = 302.243; \quad \mathbf{V}_{B_{mag}} = 559.84 \text{ mm/sec}; \quad \mathbf{V}_{B_{ang}} = 147.33° \tag{d}$$

4 As an exercise, repeat the above process to find the velocities for the crossed circuit of the linkage.

The Fourbar Crank-Slider

The position equations for the fourbar offset crank-slider linkage (inversion #1) were derived in Section 4.6. The linkage was shown in Figure 4-10 and is shown again in Figure 6-21a on which we also show an input angular velocity ω_2 applied to link 2. This ω_2 can be a time-varying input velocity. The vector loop equation 4.14 is repeated here for your convenience.

$$\mathbf{R}_2 - \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0 \tag{4.14a}$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0 \tag{4.14b}$$

Differentiate equation 4.14b with respect to time noting that a, b, c, θ_1, and θ_4 are constant but the length of link d varies with time in this inversion.

$$ja\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{d} = 0 \tag{6.20a}$$

The term $\dot{d}$ is the linear velocity of the slider block. Equation 6.20a is the velocity difference equation 6.5 and can be written in that form.

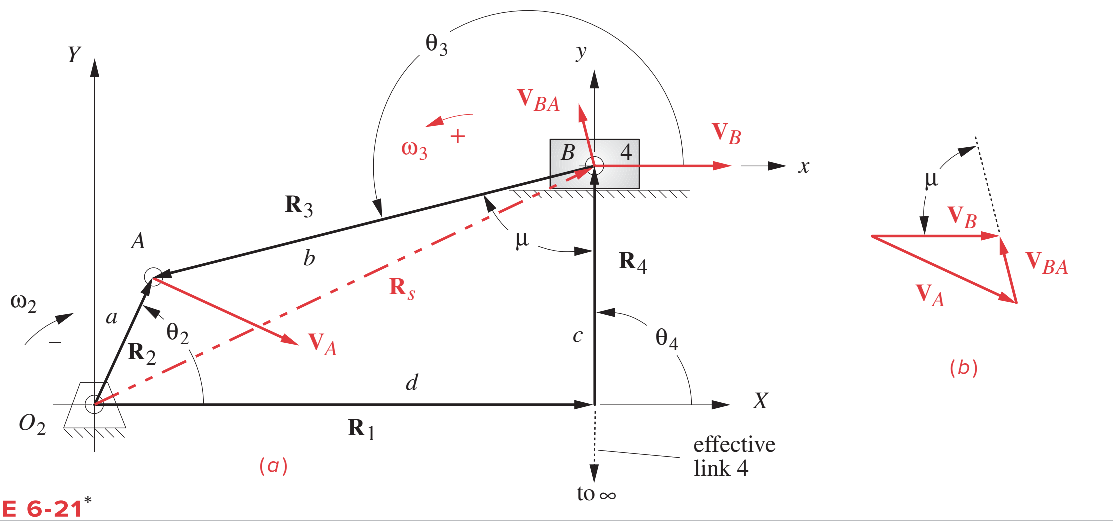

FIGURE 6-21*
Position vector loop for a fourbar crank-slider linkage showing velocity vectors for a negative (*CW*) ω_2

$$\mathbf{V}_A - \mathbf{V}_{AB} - \mathbf{V}_B = 0$$

or:
$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{AB}$$

but:
$$\mathbf{V}_{AB} = -\mathbf{V}_{BA}$$

then:
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \tag{6.20b}$$

Equation 6.20 is identical in form to equations 6.5 and 6.15a. Note that because we arranged the position vector $\mathbf{R}_3$ in Figure 4-10 and Figure 6-21 with its root at point *B*, directed from *B* to *A*, its derivative represents the velocity difference of point *A* with respect to point *B*, the opposite of that in the previous fourbar example. Compare this also to equation 6.15b noting that its vector $\mathbf{R}_3$ is directed from *A* to *B*. Figure 6-21b shows the vector diagram of the graphical solution to equation 6.20b.

Substitute the Euler equivalent, equation 4.4a, in equation 6.20a,

$$ja\omega_2\left(\cos\theta_2 + j\sin\theta_2\right) - jb\omega_3\left(\cos\theta_3 + j\sin\theta_3\right) - \dot{d} = 0 \tag{6.21a}$$

simplify,

$$a\omega_2\left(-\sin\theta_2 + j\cos\theta_2\right) - b\omega_3\left(-\sin\theta_3 + j\cos\theta_3\right) - \dot{d} = 0 \tag{6.21b}$$

and separate into real and imaginary components.

real part (*x* component):

$$-a\omega_2\sin\theta_2 + b\omega_3\sin\theta_3 - \dot{d} = 0 \tag{6.21c}$$

imaginary part (*y* component):

$$a\omega_2\cos\theta_2 - b\omega_3\cos\theta_3 = 0 \tag{6.21d}$$

* Note the transmission angle μ in Figure 6-21a drawn between link 3 and effective link 4 as previously defined. It is also shown drawn between vectors $\mathbf{V}_B$ and $\mathbf{V}_{BA}$ in Figure 6-21b, indicating an alternate way to define the transmission angle as *the acute angle between the absolute velocity and velocity difference vectors* at a point such as *B*. This approach does not require construction of the slider's effective link 4 to determine the transmission angle.

These are two simultaneous equations in the two unknowns, $\dot{d}$ and ω_3. Equation 6.21d can be solved for ω_3 and substituted into 6.21c to find $\dot{d}$.

$$\omega_3 = \frac{a}{b}\frac{\cos\theta_2}{\cos\theta_3}\omega_2 \tag{6.22a}$$

$$\dot{d} = -a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3 \tag{6.22b}$$

The absolute velocity of point A and the velocity difference of point A versus point B are found from equation 6.20:

$$\mathbf{V}_A = a\omega_2\left(-\sin\theta_2 + j\cos\theta_2\right) \tag{6.23a}$$

$$\mathbf{V}_{AB} = b\omega_3\left(-\sin\theta_3 + j\cos\theta_3\right) \tag{6.23b}$$

$$\mathbf{V}_{BA} = -\mathbf{V}_{AB} \tag{6.23c}$$

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EXAMPLE 6-8

Velocity Analysis of a Fourbar Crank-Slider Linkage with the Vector Loop Method.

Problem: Given a fourbar crank-slider linkage with the link lengths $L_2 = a = 40$ mm, $L_3 = b = 120$ mm, *offset* $= c = -20$ mm. For $\theta_2 = 60°$ and $\omega_2 = -30$ rad/sec, find ω_3 and linear velocities of points A and B for the open circuit. Use the angles and positions found for the same linkage and its link 2 position in Example 4-2.

Solution: (See Figure 6-21, for nomenclature.)

1 Example 4-2 found angle $\theta_3 = 152.91°$ and slider position $d = 126.84$ mm for the open circuit.

2 Using equation 6.22a and the data from step 1, calculate the coupler angular velocity ω_3.

$$\omega_3 = \frac{a}{b}\frac{\cos\theta_2}{\cos\theta_3}\omega_2 = \frac{40}{120}\frac{\cos 60°}{\cos 152.91°}\left(-30\right) = 5.616 \text{ rad/sec} \tag{a}$$

3 Using equation 6.22b and the data from steps 1 and 2, calculate the slider velocity $\dot{d}$.

$$\dot{d} = -a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3 = -40\left(-30\right)\sin 60° + 120\left(5.616\right)\sin 152.91° = 1346 \text{ mm/sec} \tag{b}$$

4 Using equation 6.23 and the result from step 2, calculate the linear velocities V_A and V_{BA}.

$$\begin{gathered}\mathbf{V}_A = a\omega_2\left(-\sin\theta_2 + j\cos\theta_2\right) = 40\left(-30\right)\left(-\sin 60° + j\cos 60°\right) = 1039.23 - j600 \\ \mathbf{V}_{A_x} = 1039.23; \quad \mathbf{V}_{A_y} = -600; \quad \mathbf{V}_{A_{mag}} = 1200 \text{ mm/sec}; \quad \mathbf{V}_{A_{ang}} = -30°\end{gathered} \tag{c}$$

$$\begin{gathered}\mathbf{V}_{AB} = b\omega_3\left(-\sin\theta_3 + j\cos\theta_3\right) \\ \mathbf{V}_{AB} = 120\left(5.616\right)\left(-\sin 152.91° + j\cos 152.91°\right) = -306.86 - j600 \\ \mathbf{V}_{BA} = -\mathbf{V}_{AB} = 306.86 + j600 \\ \mathbf{V}_{BA_x} = 306.86; \quad \mathbf{V}_{BA_y} = 600; \quad \mathbf{V}_{BA_{mag}} = 673.92 \text{ mm/sec}; \quad \mathbf{V}_{BA_{ang}} = 62.91°\end{gathered} \tag{d}$$

The Fourbar Slider-Crank

The *fourbar* **slider-crank** *linkage* has the same geometry as the *fourbar* **crank-slider** *linkage* that was analyzed in the previous section. The name change indicates that it will be driven with the slider as input and the crank as output. This is sometimes referred to as a "back-driven" crank-slider. We will use the term **slider-crank** to define it as slider-driven. This is a very commonly used linkage configuration. Every internal-combustion, piston engine has as many of these as it has cylinders. The vector loop is as shown in Figure 6-21 and the vector loop equation is identical to that of the crank-slider (equation 4.14a). The derivation for θ_2 as a function of slider position d was done in Section 4-7. Now we want to solve for ω_2 as a function of slider velocity $\dot{d}$ and the known link lengths and angles.

We can start with equations 6.21c and d, which also apply to this linkage:

$$-a\omega_2 \sin\theta_2 + b\omega_3 \sin\theta_3 - \dot{d} = 0 \tag{6.21c}$$

$$a\omega_2 \cos\theta_2 - b\omega_3 \cos\theta_3 = 0 \tag{6.21d}$$

Solve equation 6.21d for ω_3 in terms of ω_2.

$$\omega_3 = \frac{a\omega_2 \cos\theta_2}{b\cos\theta_3} \tag{6.24a}$$

Substitute equation 6.24a for ω_3 in equation 6.21c and solve for ω_2.

$$\omega_2 = \frac{\dot{d}\cos\theta_3}{a\left(\cos\theta_2 \sin\theta_3 - \sin\theta_2 \cos\theta_3\right)} \tag{6.24b}$$

The circuit of the linkage depends on the value of d chosen and the angular velocities will be for the circuit represented by the values of θ_2 and θ_3 used from equation 4.21.*

* The crank-slider and slider-crank linkage both have two circuits or configurations in which they can be independently assembled, sometimes called open and crossed. Because effective link 4 is always perpendicular to the slider axis, it is parallel to itself on both circuits. This results in the two circuits being mirror images of one another, mirrored about a line through the crank pivot and perpendicular to the slide axis. Thus, the choice of value of slider position d in the calculation of the slider-crank linkage determines which circuit is being analyzed. But, because of the change points at TDC and BDC, the slider crank has two branches on each circuit and the two solutions obtained from equation 4.21 represent the two branches on the one circuit being analyzed. In contrast, the crank-slider has only one branch per circuit because when the crank is driven, it can make a full revolution and there are no change points to separate branches. See Section 4.13 for a more complete discussion of circuits and branches in linkages.

6

EXAMPLE 6-9

Velocity Analysis of a Fourbar Slider-Crank Linkage with the Vector Loop Method.

Problem: Given a fourbar slider-crank linkage with the link lengths $L_2 = a = 40$ mm, $L_3 = b = 120$ mm, *offset* $= c = -20$ mm. For $d = 100$ mm and $\dot{d} = 1200$ mm/sec, find ω_2 and ω_3 for both branches of one circuit of the linkage. Use the angles found for the same linkage in Example 4-3.

Solution: (See Figure 6-21 for nomenclature.)

1 Example 4-3 found angles $\theta_{2_1} = 95.798°$, $\theta_{3_1} = 150.113°$ for branch 1 and $\theta_{2_2} = -118.418°$, $\theta_{3_2} = 187.267°$ for branch 2 of this linkage.

2 Using equation 6.24b and the data from step 1, calculate the crank angular velocity ω_{2_1}.

$$\omega_{2_1} = \frac{\dot{d}\cos\theta_{3_1}}{a\left(\cos\theta_{2_1}\sin\theta_{3_1} - \sin\theta_{2_1}\cos\theta_{3_1}\right)}$$

$$= \frac{1200\cos 150.113°}{40\left(\cos 95.798° \sin 150.113° - \sin 95.798° \cos 150.113°\right)} = -32.023 \text{ rad/sec} \tag{a}$$

3 Using equation 6.24a and data from steps 1 and 2, calculate coupler angular velocity ω_{3_1}.

$$\omega_{3_1} = \frac{a\omega_{2_1}\cos\theta_{2_1}}{b\cos\theta_{3_1}} = \frac{40(-32.023)\cos 95.798°}{120\cos 150.113°} = -1.244 \text{ rad/sec} \qquad (b)$$

4 Example 4-3 found $\theta_{2_2} = -118.418°$ and $\theta_{3_2} = 187.267°$ for branch 2 of this linkage.

5 Using equation 6.24b and the data from step 2, calculate the crank angular velocity ω_{2_2}.

$$\omega_{2_2} = \frac{\dot{d}\cos\theta_{3_2}}{a\left(\cos\theta_{2_2}\sin\theta_{3_2} - \sin\theta_{2_2}\cos\theta_{3_2}\right)}$$

$$= \frac{1200\cos(187.267°)}{40\left[\cos(-118.418°)\sin(187.267°) - \sin(-118.418°)\cos(187.267°)\right]} = 36.639 \text{ rad/sec} \qquad (c)$$

6

6 Using equation 6.24a and the data from steps 3 and 4, calculate coupler angular velocity ω_{3_2}.

$$\omega_{3_2} = \frac{a\omega_{2_2}\cos\theta_{2_2}}{b\cos\theta_{3_2}} = \frac{40(36.639)\cos(-118.418°)}{120\cos(187.267°)} = 5.859 \text{ rad/sec} \qquad (d)$$

The Fourbar Inverted Crank-Slider

The position equations for the fourbar inverted crank-slider linkage were derived in Section 4.8. The linkage was shown in Figure 4-13 and is shown again in Figure 6-22 on which we also show an input angular velocity ω_2 applied to link 2. This ω_2 can vary with time. The vector loop equations 4.14 are valid for this linkage as well.

All slider linkages will have at least one link whose effective length between joints varies as the linkage moves. In this inversion the length of link 3 between points *A* and *B*, designated as *b*, will change as it passes through the slider block on link 4. To get an expression for velocity, differentiate equation 4.14b with respect to time noting that *a*, *c*, *d*, and θ_1 are constant and *b* varies with time.

$$ja\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{b}e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0 \qquad (6.25a)$$

The value of *db/dt* will be one of the variables to be solved for in this case and is the $\dot{b}$ term in the equation. Another variable will be ω_4, the angular velocity of link 4. Note, however, that we also have an unknown in ω_3, the angular velocity of link 3. There is a total of three unknowns. Equation 6.25a can only be solved for two unknowns. Thus we require another equation to solve the system. There is a fixed relationship between angles θ_3 and θ_4, shown as γ in Figure 6-22 and defined in equation 4.22, repeated here:

$$\text{open configuration: } \theta_3 = \theta_4 + \gamma; \qquad \text{crossed configuration: } \theta_3 = \theta_4 + \gamma - \pi \qquad (4.22)$$

Differentiate it with respect to time to obtain:

$$\omega_3 = \omega_4 \qquad (6.25b)$$

We wish to solve equation 6.25a to get expressions in this form:

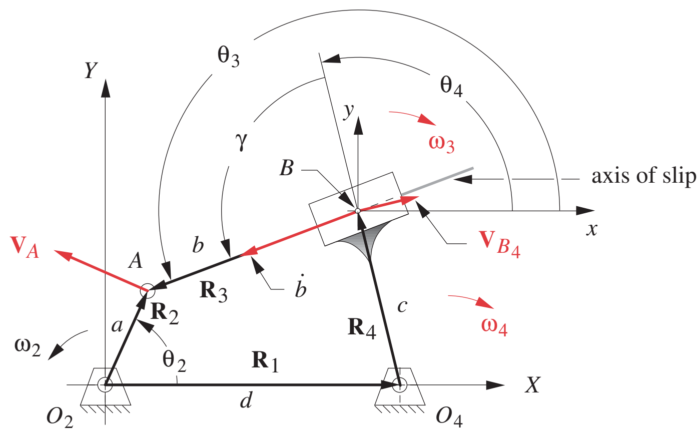

FIGURE 6-22

Velocity analysis of inversion #3 of the slider-crank fourbar linkage

$$\omega_3 = \omega_4 = f(a, b, c, d, \theta_2, \theta_3, \theta_4, \omega_2)$$
$$\frac{db}{dt} = \dot{b} = g(a, b, c, d, \theta_2, \theta_3, \theta_4, \omega_2) \tag{6.26}$$

Substitution of the Euler identity (equation 4.4a) into equation 6.25a yields:

$$\begin{aligned} &ja\omega_2(\cos\theta_2 + j\sin\theta_2) - jb\omega_3(\cos\theta_3 + j\sin\theta_3) \\ &\quad -\dot{b}(\cos\theta_3 + j\sin\theta_3) - jc\omega_4(\cos\theta_4 + j\sin\theta_4) = 0 \end{aligned} \tag{6.27a}$$

Multiply by the operator j and substitute ω_4 for ω_3 from equation 6.25b:

$$\begin{aligned} &a\omega_2(-\sin\theta_2 + j\cos\theta_2) - b\omega_4(-\sin\theta_3 + j\cos\theta_3) \\ &\quad -\dot{b}(\cos\theta_3 + j\sin\theta_3) - c\omega_4(-\sin\theta_4 + j\cos\theta_4) = 0 \end{aligned} \tag{6.27b}$$

We can now separate this vector equation into its two components by collecting all real and all imaginary terms separately:

real part (x component):

$$-a\omega_2\sin\theta_2 + b\omega_4\sin\theta_3 - \dot{b}\cos\theta_3 + c\omega_4\sin\theta_4 = 0 \tag{6.28a}$$

imaginary part (y component):

$$a\omega_2\cos\theta_2 - b\omega_4\cos\theta_3 - \dot{b}\sin\theta_3 - c\omega_4\cos\theta_4 = 0 \tag{6.28b}$$

Collect terms and rearrange equations 6.28 to isolate one unknown on the left side.

$$\dot{b}\cos\theta_3 = -a\omega_2\sin\theta_2 + \omega_4(b\sin\theta_3 + c\sin\theta_4) \tag{6.29a}$$
$$\dot{b}\sin\theta_3 = a\omega_2\cos\theta_2 - \omega_4(b\cos\theta_3 + c\cos\theta_4) \tag{6.29b}$$

Either equation can be solved for $\dot{b}$ and the result substituted in the other. Solving equation 6.29a:

$$\dot{b} = \frac{-a\omega_2 \sin\theta_2 + \omega_4 \left(b\sin\theta_3 + c\sin\theta_4 \right)}{\cos\theta_3} \tag{6.30a}$$

Substitute in equation 6.29b and simplify:

$$\omega_4 = \frac{a\omega_2 \cos\left(\theta_2 - \theta_3\right)}{b + c\cos\left(\theta_4 - \theta_3\right)} \tag{6.30b}$$

Equation 6.30a provides the **velocity of slip** at point B. Equation 6.30b gives the **angular velocity** of link 4. Note that we can substitute $-\gamma = \theta_4 - \theta_3$ from equation 4.18 (for an open linkage) into equation 6.30b to further simplify it. Note that $\cos(-\gamma) = \cos(\gamma)$.

6

$$\omega_4 = \frac{a\omega_2 \cos\left(\theta_2 - \theta_3\right)}{b + c\cos\gamma} \tag{6.30c}$$

The **velocity of slip** from equation 6.30a is always directed along the **axis of slip** as shown in Figure 6-22. There is also a component orthogonal to the axis of slip called the **velocity of transmission**. This lies along the **axis of transmission** which is the only line along which any useful work can be transmitted across the sliding joint. All energy associated with motion along the slip axis is converted to heat and lost.

The absolute linear velocity of point A is found from equation 6.23a. We can find the absolute velocity of point B on link 4 since ω_4 is now known. From equation 6.15b:

$$\mathbf{V}_{B_4} = jc\omega_4 e^{j\theta_4} = c\omega_4 \left(-\sin\theta_4 + j\cos\theta_4 \right) \tag{6.31a}$$

The velocity of transmission is the component of V_{b4} normal to the axis of slip. The absolute velocity of point B on link 3 is found from equation 6.5 as

$$\mathbf{V}_{B_3} = \mathbf{V}_{B_4} + \mathbf{V}_{B_{34}} = \mathbf{V}_{B_4} + \mathbf{V}_{slip_{34}} \tag{6.31b}$$

6.8 VELOCITY ANALYSIS OF THE GEARED FIVEBAR LINKAGE

The position loop equation for the geared fivebar mechanism was derived in Section 4.9 and is repeated here. See Figure P6-4 for notation.

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_5} - fe^{j\theta_1} = 0 \tag{4.27b}$$

Differentiate this with respect to time to get an expression for velocity.

$$a\omega_2 je^{j\theta_2} + b\omega_3 je^{j\theta_3} - c\omega_4 je^{j\theta_4} - d\omega_5 je^{j\theta_5} = 0 \tag{6.32a}$$

Substitute the Euler equivalents:

$$\begin{aligned} &a\omega_2 j\left(\cos\theta_2 + j\sin\theta_2\right) + b\omega_3 j\left(\cos\theta_3 + j\sin\theta_3\right) \\ &\quad -c\omega_4 j\left(\cos\theta_4 + j\sin\theta_4\right) - d\omega_5 j\left(\cos\theta_5 + j\sin\theta_5\right) = 0 \end{aligned} \tag{6.32b}$$

Note that the angle θ_5 is defined in terms of θ_2, the gear ratio λ, and the phase angle ϕ.

$$\theta_5 = \lambda\theta_2 + \phi \tag{4.27c}$$

Differentiate with respect to time:

$$\omega_5 = \lambda\omega_2 \tag{6.32c}$$

Since a complete position analysis must be done before a velocity analysis, we will assume that the values of θ_5 and ω_5 have been found and will leave these equations in terms of θ_5 and ω_5.

Separating the real and imaginary terms in equation 6.32b:

real: $$-a\omega_2\sin\theta_2 - b\omega_3\sin\theta_3 + c\omega_4\sin\theta_4 + d\omega_5\sin\theta_5 = 0 \tag{6.32d}$$

imaginary: $$a\omega_2\cos\theta_2 + b\omega_3\cos\theta_3 - c\omega_4\cos\theta_4 - d\omega_5\cos\theta_5 = 0 \tag{6.32e}$$

The only two unknowns are ω_3 and ω_4. Either equation 6.32d or 6.32e can be solved for one unknown and the result substituted in the other. The solution for ω_3 is:

$$\omega_3 = -\frac{2\sin\theta_4\left[a\omega_2\sin(\theta_2-\theta_4) + d\omega_5\sin(\theta_4-\theta_5)\right]}{b\left[\cos(\theta_3-2\theta_4) - \cos\theta_3\right]} \tag{6.33a}$$

The angular velocity ω_4 can be found from equation 6.32d using ω_3.

$$\omega_4 = \frac{a\omega_2\sin\theta_2 + b\omega_3\sin\theta_3 - d\omega_5\sin\theta_5}{c\sin\theta_4} \tag{6.33b}$$

With all link angles and angular velocities known, the linear velocities of the pin joints can be found from:

$$\mathbf{V}_A = a\omega_2(-\sin\theta_2 + j\cos\theta_2) \tag{6.33c}$$
$$\mathbf{V}_{BA} = b\omega_3(-\sin\theta_3 + j\cos\theta_3) \tag{6.33d}$$
$$\mathbf{V}_C = d\omega_5(-\sin\theta_5 + j\cos\theta_5) \tag{6.33e}$$
$$\mathbf{V}_B = \mathbf{V}_A + \mathbf{V}_{BA} \tag{6.33f}$$

6.9 VELOCITY OF ANY POINT ON A LINKAGE

Once the angular velocities of all the links are found, it is easy to define and calculate the velocity of *any point on any link* for any input position of the linkage. Figure 6-23 shows the fourbar linkage with its coupler, link 3, enlarged to contain a coupler point The crank and rocker have also been enlarged to show points *S* and *U* which might represent the centers of gravity of those links. We want to develop algebraic expressions for the velocities of these (or any) points on the links.

To find the velocity of point *S*, draw the position vector from the fixed pivot O_2 to point *S*. This vector, $\mathbf{R}_{SO_2}$ makes an angle δ_2 with the vector $\mathbf{R}_{AO_2}$. The angle δ_2 is completely defined by the geometry of link 2 and is constant. The position vector for point *S* is then:

$$\mathbf{R}_{SO_2} = \mathbf{R}_S = se^{j(\theta_2+\delta_2)} = s\left[\cos(\theta_2+\delta_2) + j\sin(\theta_2+\delta_2)\right] \tag{4.29}$$

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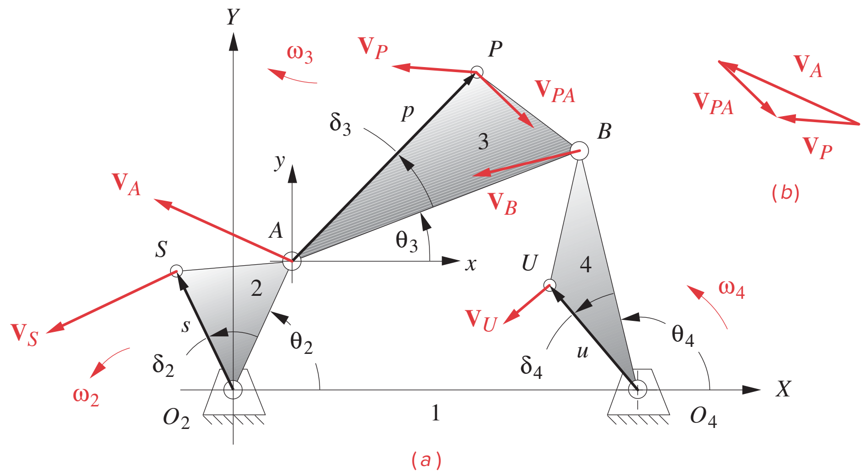


FIGURE 6-23

Finding the velocities of points on the links

6

Differentiate this position vector to find the velocity of that point.

$$\mathbf{V}_S = jse^{j(\theta_2+\delta_2)}\omega_2 = s\omega_2\left[-\sin(\theta_2+\delta_2)+j\cos(\theta_2+\delta_2)\right] \tag{6.34}$$

The position of point U on link 4 is found in the same way, using the angle δ_4 which is a constant angular offset within the link. The expression is:

$$\mathbf{R}_{UO_4} = ue^{j(\theta_4+\delta_4)} = u\left[\cos(\theta_4+\delta_4)+j\sin(\theta_4+\delta_4)\right] \tag{4.30}$$

Differentiate this position vector to find the velocity of that point.

$$\mathbf{V}_U = jue^{j(\theta_4+\delta_4)}\omega_4 = u\omega_4\left[-\sin(\theta_4+\delta_4)+j\cos(\theta_4+\delta_4)\right] \tag{6.35}$$

The velocity of point P on link 3 can be found from the addition of two velocity vectors, such as $\mathbf{V}_A$ and $\mathbf{V}_{PA}$. $\mathbf{V}_A$ is already defined from our analysis of the link velocities. $\mathbf{V}_{PA}$ is the velocity difference of point P with respect to point A. Point A is chosen as the reference point because angle θ_3 is defined in a LNCS and angle δ_3 is defined in a LRCS whose origins are both at A. Position vector $\mathbf{R}_{PA}$ is defined in the same way as $\mathbf{R}_S$ or $\mathbf{R}_U$ using the internal link offset angle δ_3 and the angle of link 3, θ_3. This was done in equations 4.31 (repeated here).

$$\mathbf{R}_{PA} = pe^{j(\theta_3+\delta_3)} = p\left[\cos(\theta_3+\delta_3)+j\sin(\theta_3+\delta_3)\right] \tag{4.31a}$$

$$\mathbf{R}_P = \mathbf{R}_A + \mathbf{R}_{PA} \tag{4.31b}$$

Differentiate equations 4.31 to find the velocity of point P.

$$\mathbf{V}_{PA} = jpe^{j(\theta_3+\delta_3)}\omega_3 = p\omega_3\left[-\sin(\theta_3+\delta_3)+j\cos(\theta_3+\delta_3)\right] \tag{6.36a}$$

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA} \tag{6.36b}$$

Please compare equations 6.36 with equations 6.5 and 6.15. It is, again, the velocity difference equation.

Note that if, for example, you wished to derive an equation for the velocity of a coupler point P on the crank-slider linkage as set up in Figure 6-21, or the inverted crank-slider of Figure 6-22, both of which have the vector for link 3 defined with its root at point B rather than at point A, you might want to use point B as the reference point rather than point A, making equation 6.36b become:

$$\mathbf{V}_P = \mathbf{V}_{B_3} + \mathbf{V}_{PB_3} \tag{6.36c}$$

Angle θ_3 would then be defined in a LNCS at point B, and δ_3 in a LRCS at point B.

6.10 REFERENCES

1 **Towfigh, K.** (1969). "The Fourbar Linkage as an Adjustment Mechanism." *Proc. of Applied Mechanism Conference*, Tulsa, OK, pp. 27-1 to 27-4.

2 **Wood, G. A.** (1977). "Educating for Creativity in Engineering." *Proc. of ASEE 85th Annual Conference*, University of North Dakota, pp. 1-13.

3 **Kennedy, A. B. W.** (1893). *Mechanics of Machinery*. Macmillan, London, pp. vii, 73.

6.11 PROBLEMS‡

6-1 Use the relative velocity equation and solve graphically or analytically.

a. A ship is steaming due north at 20 knots (nautical miles per hour). A submarine is laying in wait 1/2 mile due west of the ship. The sub fires a torpedo on a course of 85 degrees. The torpedo travels at a constant speed of 30 knots. Will it strike the ship? If not, by how many nautical miles will it miss?

b. A plane is flying due south at 500 mph at 35,000 ft altitude, straight and level. A second plane is initially 40 miles due east of the first plane, also at 35,000 feet altitude, flying straight and level and traveling at 550 mph. Determine the compass angle at which the second plane would be on a collision course with the first. How long will it take for the second plane to catch the first?

6-2 A point is at a 6.5 in radius on a body in pure rotation with $\omega = 100$ rad/sec. The rotation center is at the origin of a coordinate system. When the point is at position A, its position vector makes a 45° angle with the X axis. At position B, its position vector makes a 75° angle with the X axis. Draw this system to some convenient scale and:

a. Write an expression for the particle's velocity vector in position A using complex number notation, in both polar and cartesian forms.

b. Write an expression for the particle's velocity vector in position B using complex number notation, in both polar and cartesian forms.

c. Write a vector equation for the velocity difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the position difference numerically.

d. Check the result of part c with a graphical method.

6-3 Repeat Problem 6-2 considering points A and B to be on separate bodies rotating about the origin with ω's of –50 (A) and +75 rad/sec (B). Find their relative velocity.

*6-4 A general fourbar linkage configuration and its notation are shown in Figure P6-1. The link lengths, coupler point location, and the values of θ_2 and ω_2 for the same fourbar

‡ All problem figures are provided as PDF files, and some are also provided as animated Working Model files. PDF filenames are the same as the figure number. Run the file *Animations.html* to access and run the animations.

TABLE P6-0 Part 1
Topic/Problem Matrix

6.1 Definition of Velocity
6-1, 6-2, 6-3

6.2 Graphical Velocity Analysis
Pin-Jointed Fourbar
6-17a, 6-24, 6-28, 6-36, 6-39, 6-84a, 6-87a, 6-94
Fourbar Crank-Slider
6-16a, 6-32, 6-43[§]
Fourbar Slider-Crank
6-110, 6-111
Other Fourbar
6-18a, 6-98[§]
Geared Fivebar
6-10
Sixbar
6-70a, 6-73a, 6-76a, 6-99
Eightbar 6-103[§]

6.3 Instant Centers of Velocity
6-12, 6-13, 6-14, 6-15, 6-68, 6-72, 6-75, 6-78, 6-83, 6-86, 6-88, 6-97, 6-102, 6-104, 6-105

6.4 Velocity Analysis with Instant Centers
6-4, 6-16b, 6-17b, 6-18b, 6-25, 6-29, 6-33, 6-40, 6-70b, 6-73b, 6-76b, 6-84b, 6-87b, 6-92, 6-95, 6-100
Mech. Advantage
6-21a, 6-21b, 6-22a, 6-22b, 6-58

6.5 Centrodes
6-23, 6-63, 6-69, 6-89

6.6 Velocity of Slip
6-6, 6-8, 6-19, 6-20, 6-61, 6-64, 6-65, 6-66, 6-91, 6-106 to 6-109, 6-112, 6-113

[§]May be solved using either the velocity difference or instant center graphical method.

TABLE P6-1 Data for Problems 6-4 to 6-5[‡]

Row	Link 1	Link 2	Link 3	Link 4	θ_2	ω_2	R_{pa}	δ_3
a	6	2	7	9	30	10	6	30
b	7	9	3	8	85	–12	9	25
c	3	10	6	8	45	–15	10	80
d	8	5	7	6	25	24	5	45
e	8	5	8	6	75	–50	9	300
f	5	8	8	9	15	–45	10	120
g	6	8	8	9	25	100	4	300
h	20	10	10	10	50	–65	6	20
i	4	5	2	5	80	25	9	80
j	20	10	5	10	33	25	1	0
k	4	6	10	7	88	–80	10	330
l	9	7	10	7	60	–90	5	180
m	9	7	11	8	50	75	10	90
n	9	7	11	6	120	15	15	60

[‡]Drawings of these linkages are in the *PDF Problem Workbook* folder.

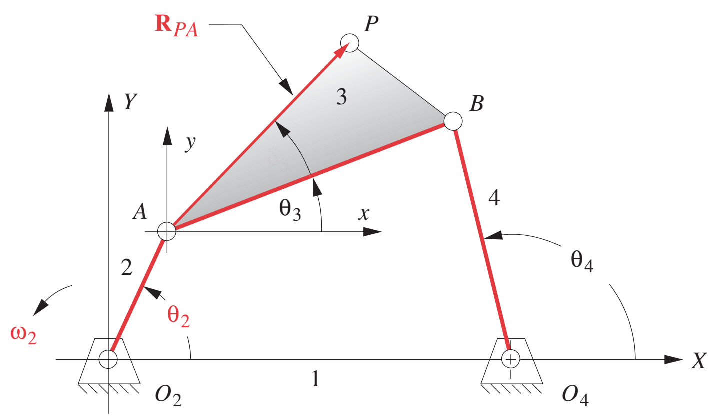


FIGURE P6-1
Configuration and terminology for the pin-jointed fourbar linkage of Problems 6-4 to 6-5

linkages as used for position analysis in Chapter 4 are redefined in Table P6-1, which is basically the same as Table P4-1. *For the row(s) assigned*, draw the linkage to scale and find the velocities of the pin joints *A* and *B* and of instant centers $I_{1,3}$ and $I_{2,4}$ using a graphical method. Then calculate ω_3 and ω_4 and find the velocity of point *P*.

*†6-5 Repeat Problem 6-4 using an analytical method. Draw the linkage to scale and label it before setting up the equations.

*6-6 The general linkage configuration and terminology for an offset fourbar crank-slider linkage are shown in Figure P6-2. The link lengths and the values of θ_2 and ω_2 are defined in Table P6-2. *For the row(s) assigned*, draw the linkage to scale and find the velocities of the pin joints *A* and *B* and the velocity of slip at the sliding joint using a graphical method.

*†6-7 Repeat Problem 6-6 using an analytical method. Draw the linkage to scale and label it before setting up the equations.

TABLE P6-2 Data for Problems 6-6 to 6-7‡

Row	Link 2	Link 3	Offset	θ_2	ω_2
a	1.4	4	1	45	10
b	2	6	–3	60	–12
c	3	8	2	–30	–15
d	3.5	10	1	120	24
e	5	20	–5	225	–50
f	3	13	0	100	–45
g	7	25	10	330	100

‡ Drawings of these linkages are in the *PDF Problem Workbook* folder.

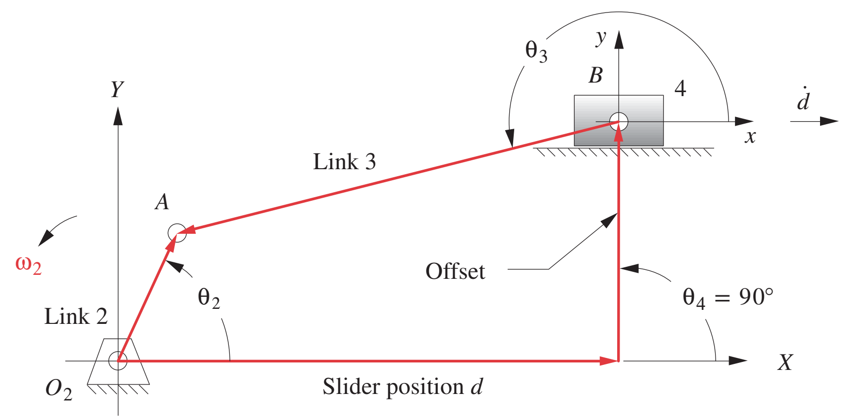


FIGURE P6-2

Configuration and terminology for Problems 6-6, 6-7, 6-110, 6-111

*6-8 The general linkage configuration and terminology for an inverted fourbar crank-slider linkage are shown in Figure P6-3. The link lengths and the values of θ_2, ω_2, and γ are defined in Table P6-3. *For the row(s) assigned*, draw the linkage to scale and find the velocities of points *A* and *B* and velocity of slip at the sliding joint using a graphical method.

*†6-9 Repeat Problem 6-8 using an analytical method. Draw the linkage to scale and label it before setting up the equations.

*6-10 The general linkage configuration and terminology for a geared fivebar linkage are shown in Figure P6-4. The link lengths, gear ratio (λ), phase angle (ϕ), and the values of θ_2 and ω_2 are defined in Table P6-4. *For the row(s) assigned*, draw the linkage to scale and find ω_3 and ω_4 using a graphical method.

*†6-11 Repeat Problem 6-10 using an analytical method. Draw the linkage to scale and label it before setting up the equations.

6-12 Find all the instant centers of the linkages shown in Figure P6-5.

6-13 Find all the instant centers of the linkages shown in Figure P6-6.

6-14 Find all the instant centers of the linkages shown in Figure P6-7.

TABLE P6-0 Part 2
Topic/Problem Matrix

6.7 Analytic Solutions for Velocity Analysis
6-90
Pin-Jointed Fourbar
6-26, 6-27, 6-30, 6-31, 6-37, 6-38, 6-41, 6-42, 6-48, 6-62
Fourbar Crank-Slider
6-7, 6-34, 6-35, 6-44, 6-45, 6-52, 6-60
Fourbar Inverted Crank-Slider
6-9
Sixbar
6-70c, 6-71, 6-73c, 6-74, 6-76c, 6-77, 6-93, 6-101
Eightbar
6-79
Mechanical Advantage
6-55a, 6-55b, 6-57a, 6-57b, 6-59a, 6-59b, 6-67

6.8 Velocity Analysis of Geared Fivebar
6-11

6.9 Velocity of Any Point on a Linkage
6-5, 6-16c, 6-17c, 6-18c, 6-46, 6-47, 6-49, 6-50, 6-51, 6-53, 6-54, 6-56, 6-80, 6-81, 6-82, 6-84c, 6-85, 6-87c, 6-96

* Answers in Appendix F.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

TABLE P6-3 Data for Problems 6-8 to 6-9

Row	Link 1	Link 2	Link 4	γ	θ_2	ω_2
a	6	2	4	90	30	10
b	7	9	3	75	85	–15
c	3	10	6	45	45	24
d	8	5	3	60	25	–50
e	8	4	2	30	75	–45
f	5	8	8	90	150	100

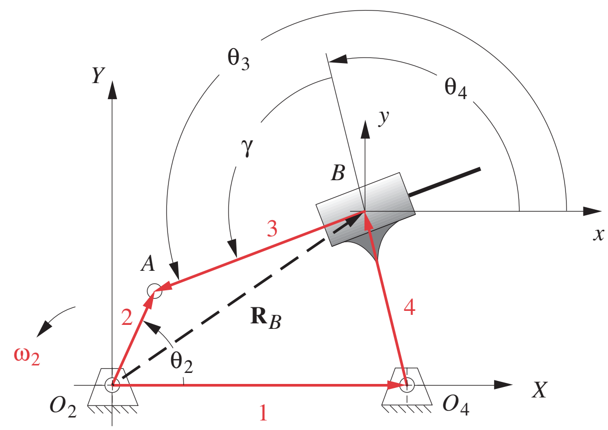


FIGURE P6-3

Configuration and terminology for Problems 6-8 to 6-9

6-15 Find all the instant centers of the linkages shown in Figure P6-8.

*6-16 The linkage in Figure P6-5a has $O_2A = 0.8$, $AB = 1.93$, $AC = 1.33$, and offset = 0.38 in. The crank angle in the position shown is 34.3° and angle $BAC = 38.6°$. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15$ rad/sec in the direction shown:

a. Using the velocity difference graphical method.
b. Using the instant center graphical method.
†c. Using an analytical method.

6-17 The linkage in Figure P6-5c has $I_{12}A = 0.75$, $AB = 1.5$, and $AC = 1.2$ in. The effective crank angle in the position shown is 77° and angle $BAC = 30°$. Find ω_3, ω_4, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\omega_2 = 15$ rad/sec in direction shown:

a. Using the velocity difference graphical method.
b. Using the instant center graphical method.
†c. Using an analytical method. (Hint: Create an effective linkage for the position shown and analyze as a pin-jointed fourbar.)

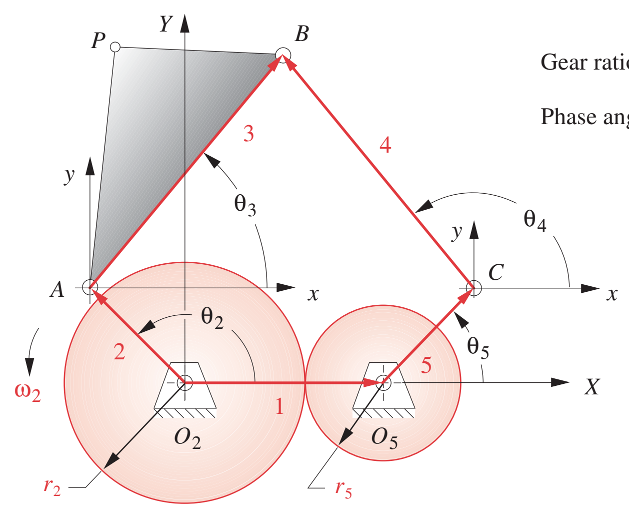
6-18 The linkage in Figure P6-5f has $AB = 1.8$ and $AC = 1.44$ in. The angle of AB in the position shown is 128° and angle $BAC = 49°$. The slider at B is at an angle of 59°. Find ω_3, $\mathbf{V}_B$, and $\mathbf{V}_C$ for the position shown for $\mathbf{V}_A = 10$ in/sec in the direction shown:

a. Using the velocity difference graphical method.
b. Using the instant center graphical method.
†c. Using an analytical method.

* Answers in Appendix F.

TABLE P6-4 Data for Problems 6-10 to 6-11

Row	Link 1	Link 2	Link 3	Link 4	Link 5	λ	φ	ω_2	θ_2
a	6	1	7	9	4	2.0	30	10	60
b	6	5	7	8	4	–2.5	60	–12	30
c	3	5	7	8	4	–0.5	0	–15	45
d	4	5	7	8	4	–1.0	120	24	75
e	5	9	11	8	8	3.2	–50	–50	–39
f	10	2	7	5	3	1.5	30	–45	120
g	15	7	9	11	4	2.5	–90	100	75
h	12	8	7	9	4	–2.0	60	–65	55
i	9	7	8	9	4	–4.0	120	25	100



Gear ratio $\lambda = \pm \dfrac{r_2}{r_5}$

Phase angle $\varphi = \theta_5 - \lambda\theta_2$

FIGURE P6-4

Configuration and terminology for Problems 6-10 and 6-11

6-19 The cam-follower in Figure P6-5d has $O_2A = 0.853$ in. Find $\mathbf{V}_4$, $\mathbf{V}_{trans}$, and $\mathbf{V}_{slip}$ for the position shown with $\omega_2 = 20$ rad/sec in the direction shown.

6-20 The cam-follower in Figure P6-5e has $O_2A = 0.980$ in and $O_3A = 1.344$ in. Find ω_3, $\mathbf{V}_{trans}$, and $\mathbf{V}_{slip}$ for the position shown for $\omega_2 = 10$ rad/sec in the direction shown.

6-21 The linkage in Figure P6-6b has $L_1 = 61.9$, $L_2 = 15$, $L_3 = 45.8$, $L_4 = 18.1$, $L_5 = 23.1$ mm. θ_2 is 68.3° in the xy coordinate system, which is at –23.3° in the XY coordinate system. The X component of O_2C is 59.2 mm. For the position shown, find the velocity ratio $V_{I_{5,6}} / V_{I_{2,3}}$ and the mechanical advantage from link 2 to link 6:

a. Using the velocity difference graphical method.
b. Using the instant center graphical method.

6-22 Repeat Problem 6-21 for the mechanism in Figure P6-6d, which has the dimensions: $L_2 = 15$, $L_3 = 40.9$, $L_5 = 44.7$ mm. θ_2 is 24.2° in the XY coordinate system.

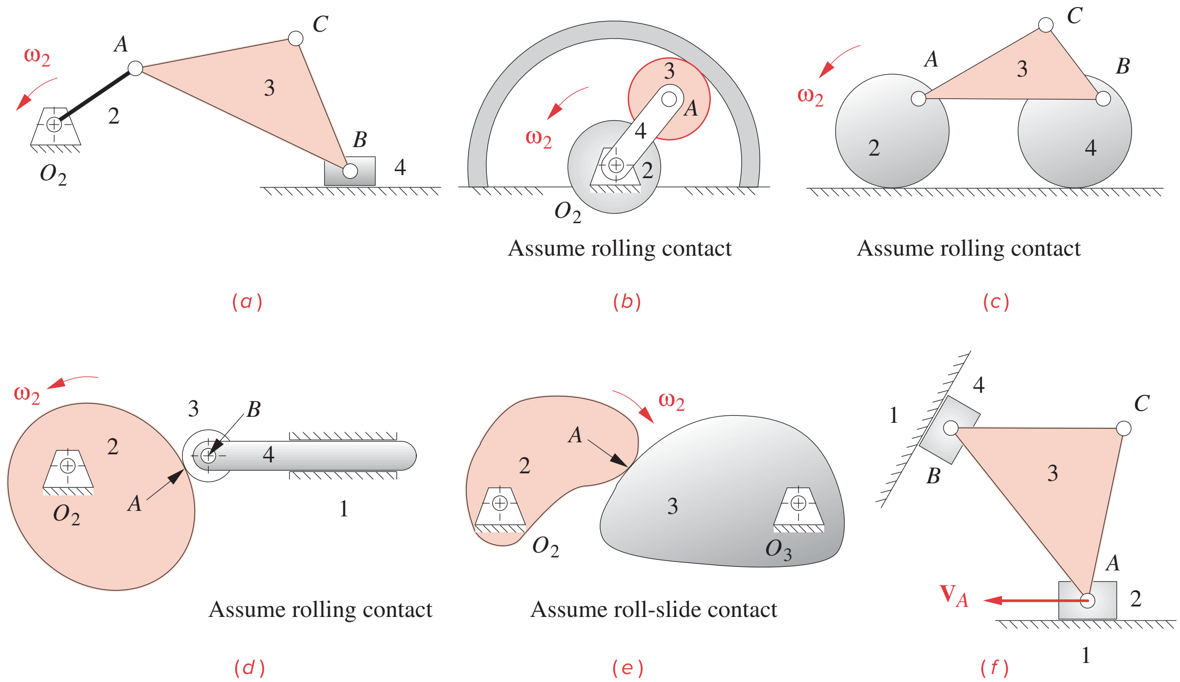


FIGURE P6-5

Velocity analysis and instant center problems. Problems 6-12 and 6-16 to 6-20

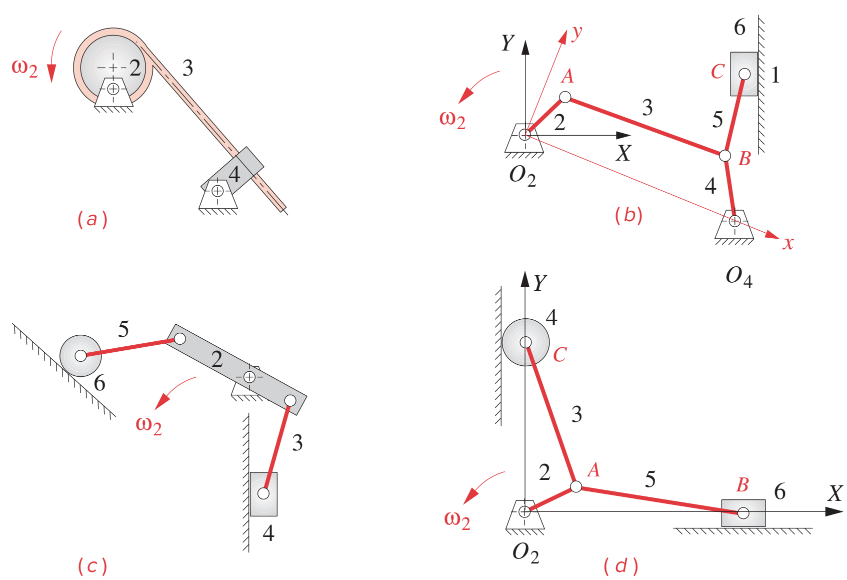


FIGURE P6-6

Problems 6-13, 6-21, and 6-22

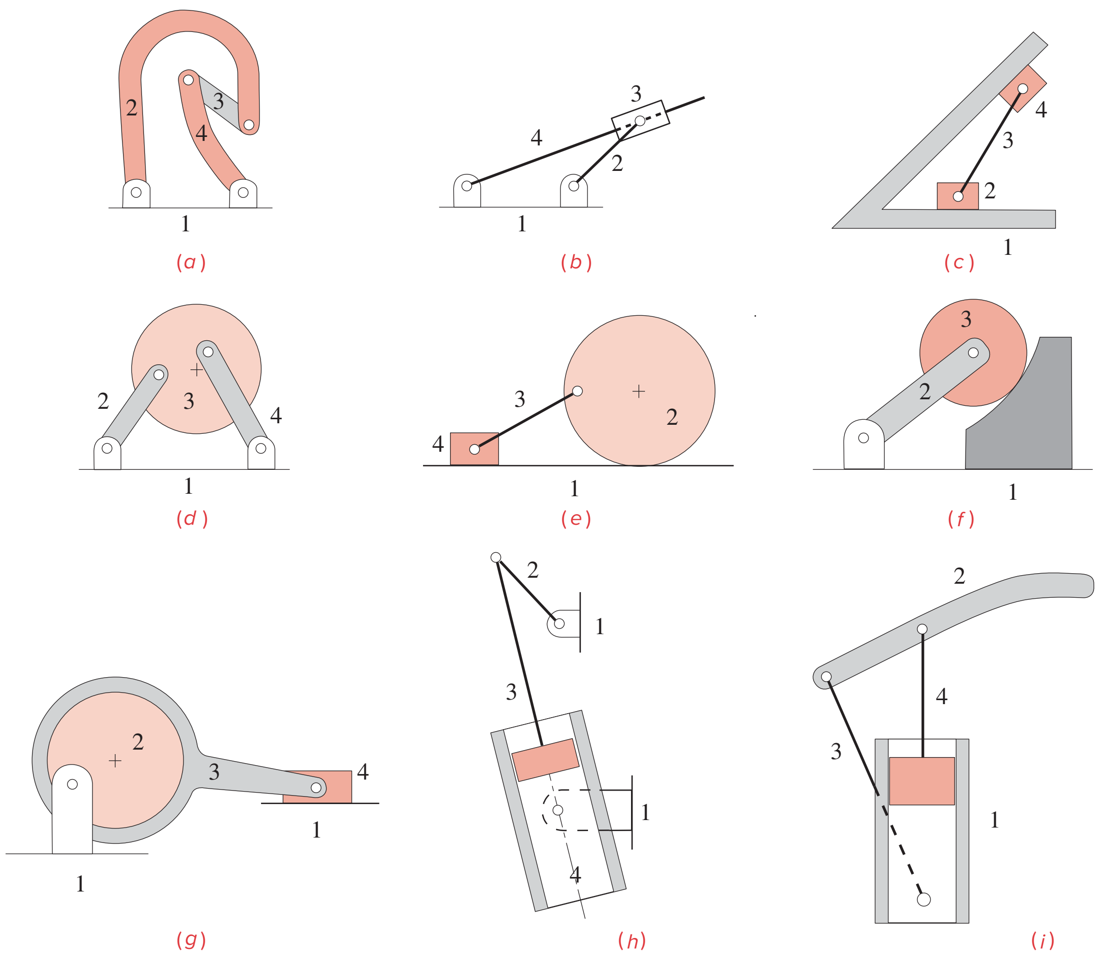


FIGURE P6-7

Problems 6-14 and 6-23.

†6-23 Generate and draw the fixed and moving centrodes of links 1 and 3 for the linkage in Figure P6-7a.

6-24 The linkage in Figure P6-8a has link 1 at –25° and O_2A at 37° in the global *XY* coordinate system. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW. Use the velocity difference graphical method. (Print the figure from its PDF file and draw on it.)

6-25 The linkage in Figure P6-8a has link 1 at –25° and O_2A at 37° in the global *XY* coordinate system. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW. Use the instant center graphical method. (Print the figure from its PDF file and draw on it.)

†6-26 The linkage in Figure P6-8a has θ_2 = 62° in the local *x'y'* coordinate system. The angle between the *X* and *x* axes is 25°. Find ω_4**,** $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the position shown if ω_2 = 15 rad/sec CW. Use an analytical method.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†6-27 For the linkage in Figure P6-8a, write a computer program or use an equation solver to find and plot ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if ω_2 = 15 rad/sec CW.

6-28 The linkage in Figure P6-8b has link 1 at –36° and link 2 at 57° in the global *XY* coordinate system. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 20 rad/sec CCW. Use the velocity difference graphical method. (Print the figure from its PDF file and draw on it.)

6-29 The linkage in Figure P6-8b has link 1 at –36° and link 2 at 57° in the global *XY* coordinate system. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 20 rad/sec CCW. Use the instant center graphical method. (Print the figure from its PDF file and draw on it.)

†6-30 The linkage in Figure P6-8b has link 1 at –36° and link 2 at 57° in the global *XY* coordinate system. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 20 rad/sec CCW. Use an analytical method.

†6-31 The linkage in Figure P6-8b has link 1 at –36° in the global *XY* coordinate system. Write a computer program or use an equation solver to find and plot ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if ω_2 = 20 rad/sec CCW.

6-32 The offset crank-slider linkage in Figure P6-8f has link 2 at 51° in the global *XY* coordinate system. Find $\mathbf{V}_A$ and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 25 rad/sec CW. Use the velocity difference graphical method. (Print the figure from its PDF file and draw on it.)

6-33 The offset crank-slider linkage in Figure P6-8f has link 2 at 51° in the global *XY* coordinate system. Find $\mathbf{V}_A$ and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 25 rad/sec CW. Use the instant center graphical method. (Print the figure from its PDF file and draw on it.)

†6-34 The offset crank-slider linkage in Figure P6-8f has link 2 at 51° in the global *XY* coordinate system. Find $\mathbf{V}_A$ and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 25 rad/sec CW. Use an analytical method.

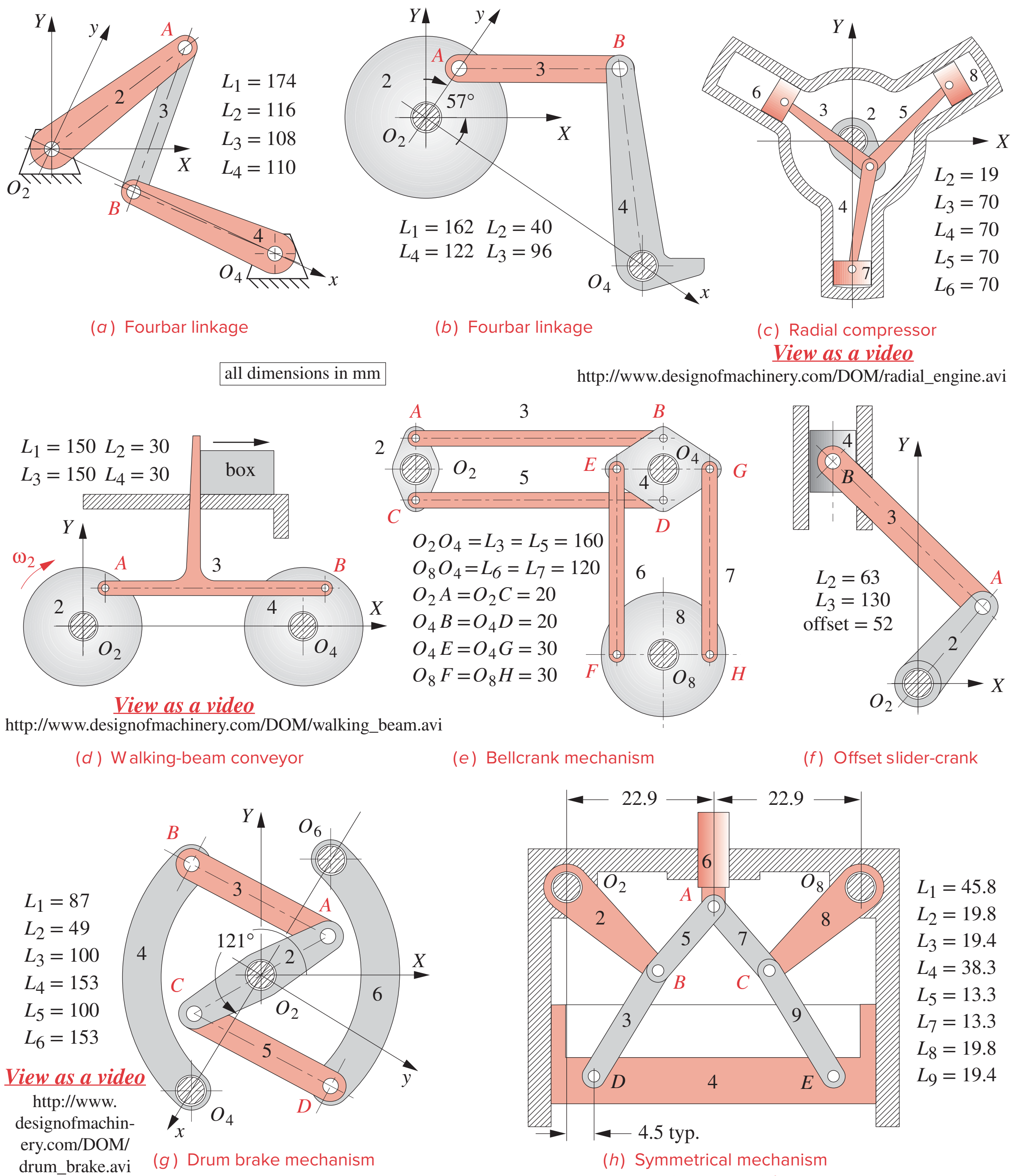
†6-35 For the offset crank-slider linkage in Figure P6-8f , write a computer program or use an equation solver to find and plot $\mathbf{V}_A$ and $\mathbf{V}_B$ in the global coordinate system for the maximum range of motion that this linkage allows if ω_2 = 25 rad/sec CW.

6-36 The linkage in Figure P6-8d has link 2 at 58° in the global *XY* coordinate system. Find $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ in the global coordinate system for the position shown if ω_2 = 30 rad/sec CW. Use the velocity difference graphical method. (Make a copy of the figure from its PDF file and draw on it.)

†6-37 The linkage in Figure P6-8d has link 2 at 58° in the global *XY* coordinate system. Find $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ in the global coordinate system for the position shown if ω_2 = 30 rad/sec CW. Use an analytical method.

†6-38 For the linkage in Figure P6-8d, write a computer program or use an equation solver to find and plot $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_{box}$ in the global coordinate system for the maximum range of motion that this linkage allows if ω_2 = 30 rad/sec CW.

6-39 The linkage in Figure P6-8g has the local *xy* axis at –119° and O_2A at 29° in the global *XY* coordinate system. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW. Use the velocity difference graphical method.



(*a*) Fourbar linkage

(*b*) Fourbar linkage

(*c*) Radial compressor

View as a video

http://www.designofmachinery.com/DOM/radial_engine.avi

View as a video

http://www.designofmachinery.com/DOM/walking_beam.avi

(*d*) Walking-beam conveyor

(*e*) Bellcrank mechanism

(*f*) Offset slider-crank

View as a video

http://www.designofmachinery.com/DOM/drum_brake.avi

(*g*) Drum brake mechanism

(*h*) Symmetrical mechanism

View as a video

http://www.designofmachinery.com/DOM/compression_chamber.avi

FIGURE P6-8

Problems 6-15 and 6-24 to 6-45

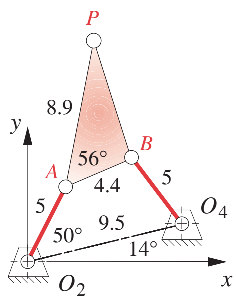
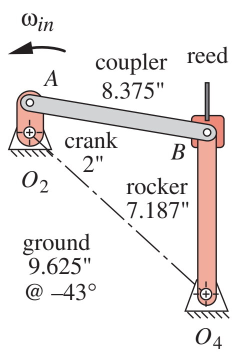


FIGURE P6-9

Problem 6-46

6



View as a video

http://www.designof-machinery.com/DOM/loom_laybar_drive.avi

FIGURE P6-11

Problem 6-48 Loom laybar drive

[*] Answers in Appendix F.

[†] These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

6-40 The linkage in Figure P6-8g has the local xy axis at –119° and O_2A at 29° in the global XY coordinate system. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW. Use the instant center graphical method. (Make a copy of the figure from its PDF file and draw on it.)

[†]6-41 The linkage in Figure P6-8g has the local xy axis at –119° and O_2A at 29° in the global XY coordinate system. Find ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW. Use an analytical method.

[†]6-42 The linkage in Figure P6-8g has the local xy axis at –119° in the global XY coordinate system. Write a computer program or use an equation solver to find and plot ω_4, $\mathbf{V}_A$, and $\mathbf{V}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if ω_2 = 15 rad/sec CW.

6-43 The 3-cylinder radial compressor in Figure P6-8c has its cylinders equispaced at 120°. Find the piston velocities $\mathbf{V}_6$, $\mathbf{V}_7$, $\mathbf{V}_8$ with the crank at –53° using a graphical method if ω_2 = 15 rad/sec CW. (Make a copy of the figure from its PDF file and draw on it.)

[†]6-44 The 3-cylinder radial compressor in Figure P6-8c has its cylinders equispaced at 120°. Find the piston velocities $\mathbf{V}_6$, $\mathbf{V}_7$, $\mathbf{V}_8$ with the crank at –53° using an analytical method if ω_2 = 15 rad/sec CW.

[†]6-45 The 3-cylinder radial compressor in Figure P6-8c has its cylinders equispaced at 120°. Write a program or use an equation solver to find and plot the piston velocities $\mathbf{V}_6$, $\mathbf{V}_7$, $\mathbf{V}_8$ for one revolution of the crank if ω_2 = 15 rad/sec CW.

6-46 Figure P6-9 shows a linkage in one position. Find the instantaneous velocities of points A, B, and P if link O_2A is rotating CW at 40 rad/sec.

[*†]6-47 Figure P6-10 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point P at 2° increments of crank angle for ω_2 = 100 rpm. Check your result with program LINKAGES.

[*†]6-48 Figure P6-11 shows a linkage that operates at 500 crank rpm. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of point B at 2° increments of crank angle. Check the result with program LINKAGES.

[*†]6-49 Figure P6-12 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of

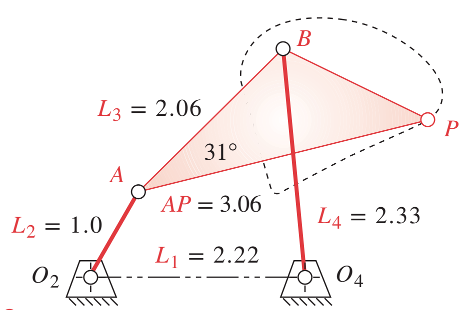


FIGURE P6-10

Problem 6-47 A fourbar linkage with a double straight-line coupler curve

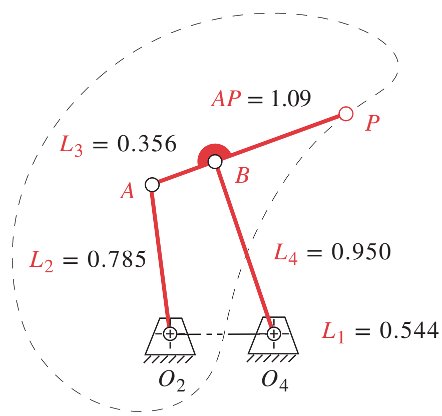


FIGURE P6-12

Problem 6-49

the coupler point P at 2° increments of crank angle for ω_2 = 20 rpm over the maximum range of motion possible. Check your result with program LINKAGES.

†6-50 Figure P6-13 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point P at 2° increments of crank angle for ω_2 = 80 rpm over the maximum range of motion possible. Check your result with program LINKAGES.

*†6-51 Figure P6-14 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the velocity of the coupler point P at 2° increments of crank angle for ω_2 = 80 rpm over the maximum range of motion possible. Check your result with program LINKAGES.

†6-52 Figure P6-15 shows a power hacksaw, used to cut metal. Link 5 pivots at O_5 and its weight forces the sawblade against the workpiece while the linkage moves the blade (link 4) back and forth on link 5 to cut the part. It is an offset crank-slider mechanism with the dimensions shown in the figure. Draw an equivalent linkage diagram; then calculate and plot the velocity of the sawblade with respect to the piece being cut over one revolution of the crank at 50 rpm.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

* Answers in Appendix F.

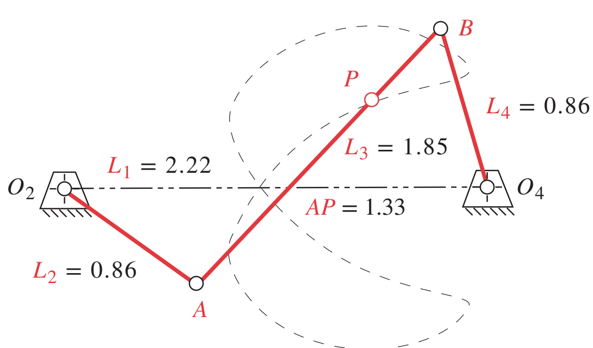


FIGURE P6-13

Problem 6-50

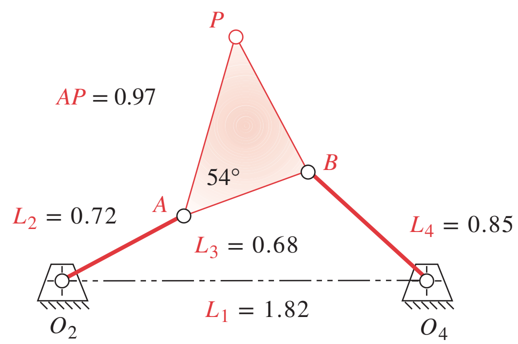


FIGURE P6-14

Problem 6-51

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†6-53 Figure P6-16 shows a walking-beam indexing and pick-and-place mechanism that can be analyzed as two fourbar linkages driven by a common crank. The link lengths are given in the figure. The phase angle between the two crankpins on links 4 and 5 is given. The product cylinders being pushed have 60-mm diameters. The point of contact between the left vertical finger and the leftmost cylinder in the position shown is 58 mm at 80° versus the left end of the parallelogram's coupler (point *D*). Calculate and plot the absolute velocities of points *E* and *P* and the relative velocity between points *E* and *P* for one revolution of gear 2.

†6-54 Figure P6-17 shows a paper roll off-loading mechanism driven by an air cylinder. In the position shown, $AO_2 = 1.1$ m at 178° and O_4A is 0.3 m at 226°. $O_2O_4 = 0.93$ m at 163°. The V-links are rigidly attached to O_4A. The air cylinder is retracted at a constant velocity of 0.2 m/sec. Draw a kinematic diagram of the mechanism, write the necessary equations, and calculate and plot the angular velocity of the paper roll and the linear velocity of its center as it rotates through 90° CCW from the position shown.

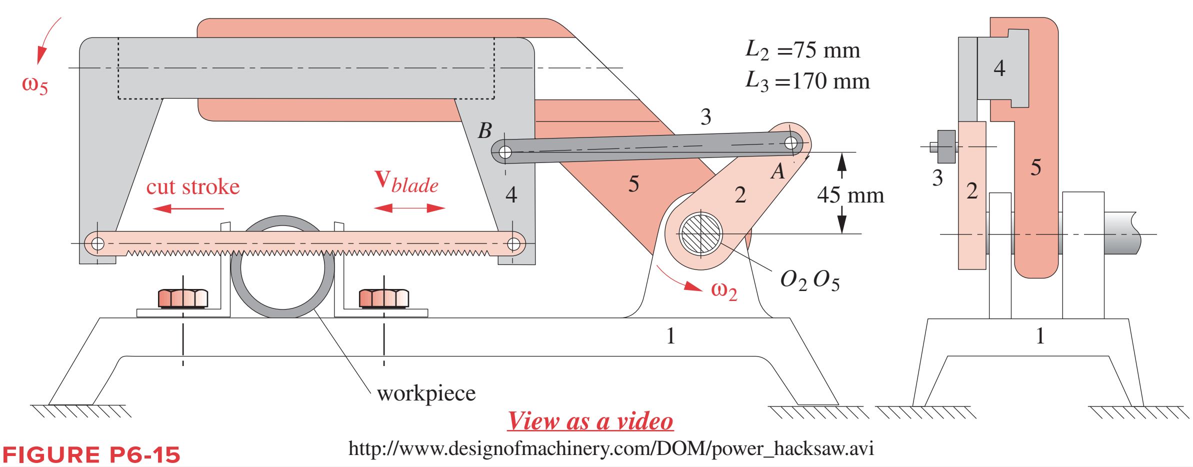


FIGURE P6-15

Problem 6-52 Power hacksaw

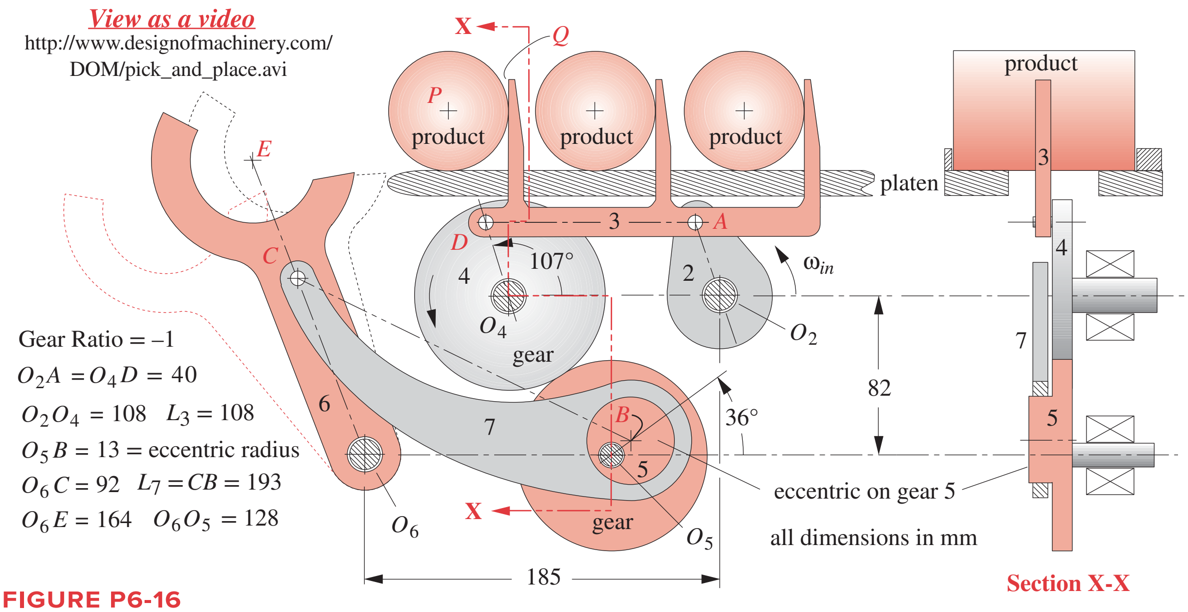


FIGURE P6-16

Problem 6-53 Walking-beam indexer with pick-and-place mechanism

†6-55 Figure P6-18 shows a powder compaction mechanism.

a. Calculate its mechanical advantage for the position shown.
b. Calculate and plot its mechanical advantage as a function of the angle of link *AC* as it rotates from 15 to 60°.

†6-56 Figure P6-19 shows a walking-beam mechanism. Calculate and plot the velocity V_{out} for one revolution of the input crank 2 rotating at 100 rpm.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

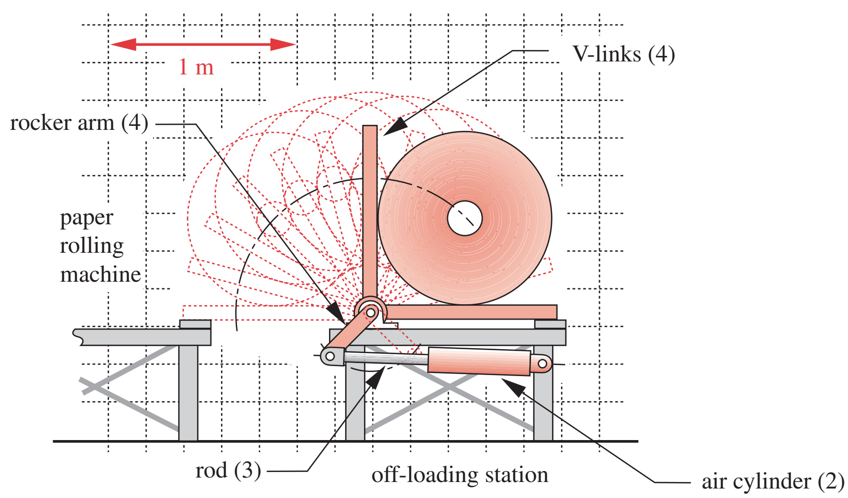


FIGURE P6-17

Problem 6-54

6

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http://www.designofmachinery.com/
DOM/powder_compacting_press.avi

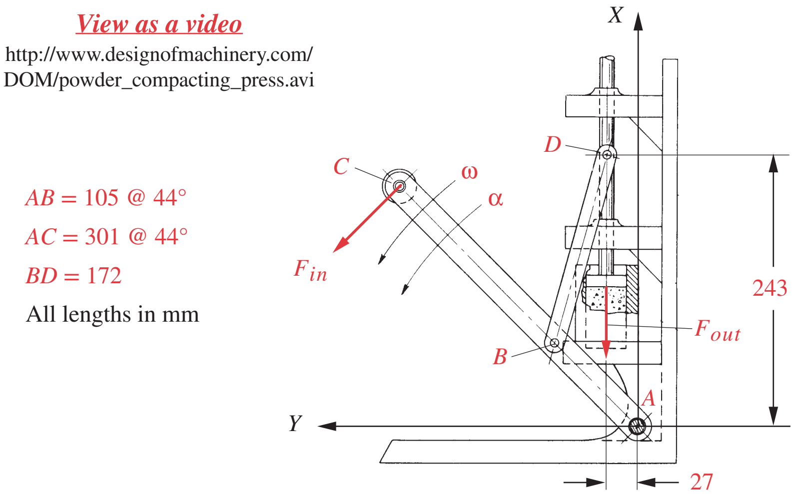

FIGURE P6-18

Problem 6-55 *Adapted from P. H. Hill and W. P. Rule. (1960). Mechanisms: Analysis and Design*

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†6-57 Figure P6-20 shows a crimping tool.

a. Calculate its mechanical advantage for the position shown.
b. Calculate and plot its mechanical advantage as a function of the angle of link *AB* as it rotates from 60 to 45°.

†6-58 Figure P6-21 shows a locking pliers. Calculate its mechanical advantage for the position shown. Scale the diagram for any needed dimensions.

†6-59 Figure P6-22 shows a fourbar toggle clamp used to hold a workpiece in place by clamping it at *D*. $O_2A = 70$, $O_2C = 138$, $AB = 35$, $O_4B = 34$, $O_4D = 82$, and $O_2O_4 = 48$ mm. At the position shown, link 2 is at 104°. Toggle occurs when link 2 reaches 90°.

a. Calculate its mechanical advantage for the position shown.
b. Calculate and plot its mechanical advantage as a function of the angle of link *AB* as link 2 rotates from 120 to 90°.

[View as a video](http://www.designofmachinery.com/DOM/walking_beam_eight-bar.avi)
http://www.designofmachinery.com/DOM/walking_beam_eight-bar.avi

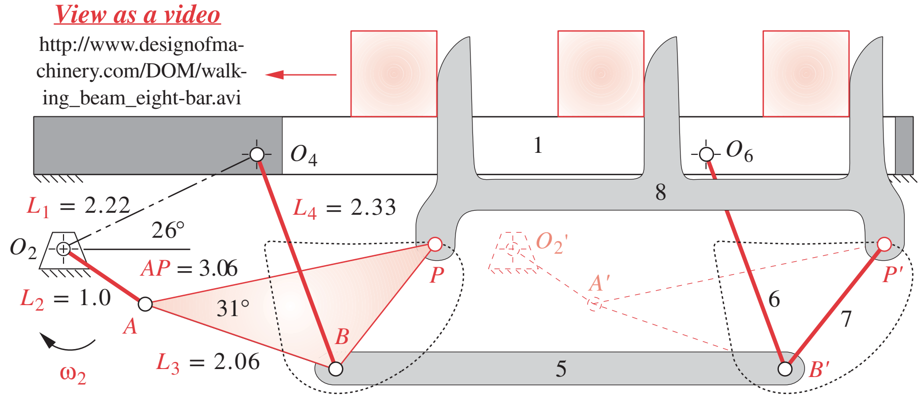

FIGURE P6-19

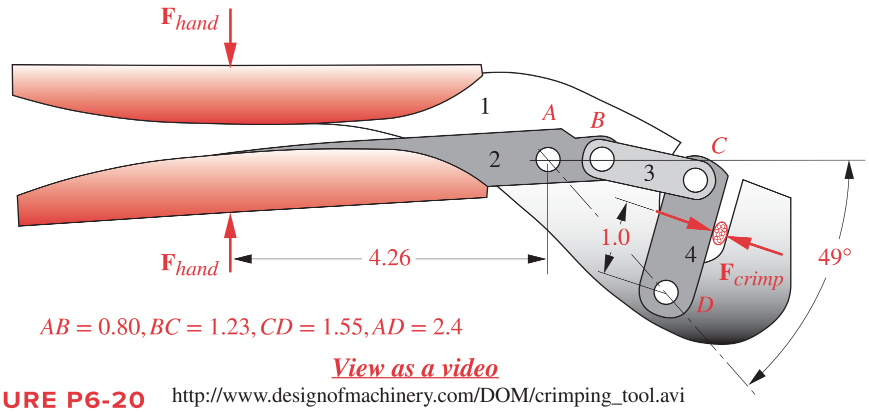

FIGURE P6-20

Problem 6-57

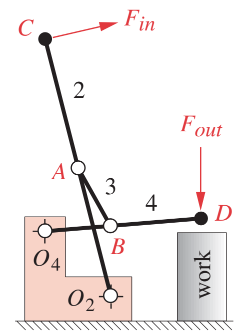

FIGURE P6-22

Problem 6-59

†6-60 Figure P6-23 shows a surface grinder. The workpiece is oscillated under the spinning 90-mm-diameter grinding wheel by the crank-slider linkage which has a 22-mm crank, a 157-mm connecting rod, and a 40-mm offset. The crank turns at 120 rpm, and the grinding wheel turns at 3450 rpm. Calculate and plot the velocity of the grinding wheel contact point relative to the workpiece over one revolution of the crank.

6-61 Figure P6-24 shows an inverted crank-slider mechanism. Link 2 is 2.5 in long. The distance O_4A is 4.1 in and O_2O_4 is 3.9 in. Find ω_2, ω_3, ω_4, V_{A4}, $\mathbf{V}_{trans}$, and $\mathbf{V}_{slip}$ for the position shown with $V_{A2} = 20$ in/sec in the direction shown.

*†6-62 Figure P6-25 shows a drag link mechanism with dimensions. Write the necessary equations, and solve them to calculate the angular velocity of link 4 for an input of ω_2 = 1 rad/sec. Comment on uses for this mechanism.

†6-63 Figure P6-25 shows a drag link mechanism with dimensions. Write the necessary equations, and solve them to calculate and plot the centrodes of instant center $I_{2,4}$.

* Answers in Appendix F.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

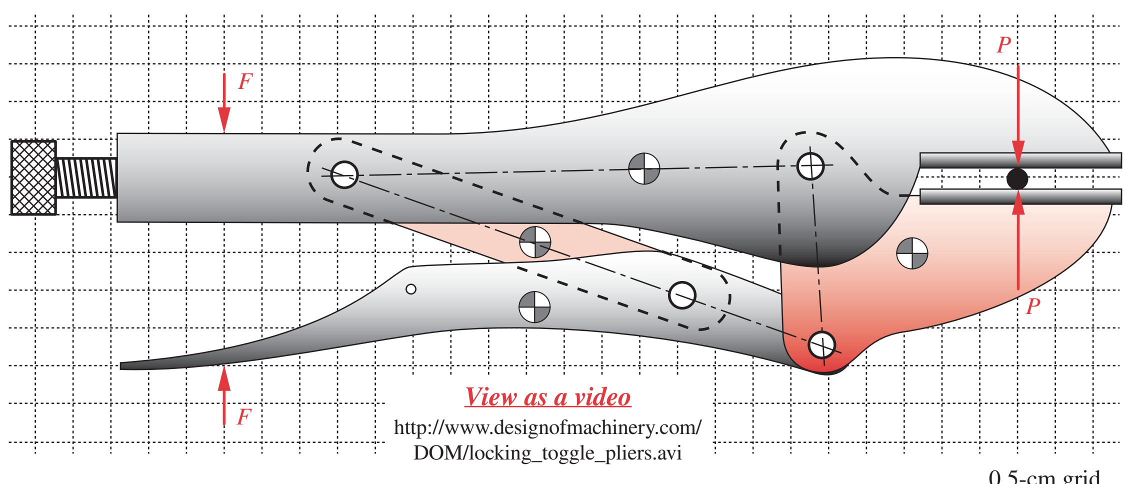

FIGURE P6-21

Problem 6-58

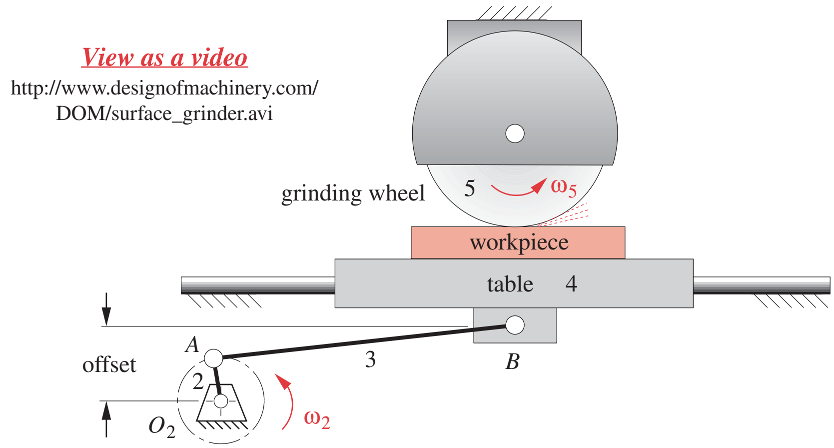

FIGURE P6-23

Problem 6-60 A surface grinder

6-64 Figure P6-26 shows a mechanism with dimensions. Use a graphical method to calculate the velocities of points A, B, and C and the velocity of slip for the position shown. $\omega_2 = 20$ rad/sec.

*6-65 Figure P6-27 shows a cam and follower. Distance $O_2A = 1.89$ in and $O_3B = 1.645$ in. Find the velocities of points A and B, the velocity of transmission, velocity of slip, and ω_3 if $\omega_2 = 50$ rad/sec. Use a graphical method.

6-66 Figure P6-28 shows a quick-return mechanism with dimensions. Use a graphical method to calculate the velocities of points A, B, and C and the velocity of slip for the position shown. $\omega_2 = 10$ rad/sec.

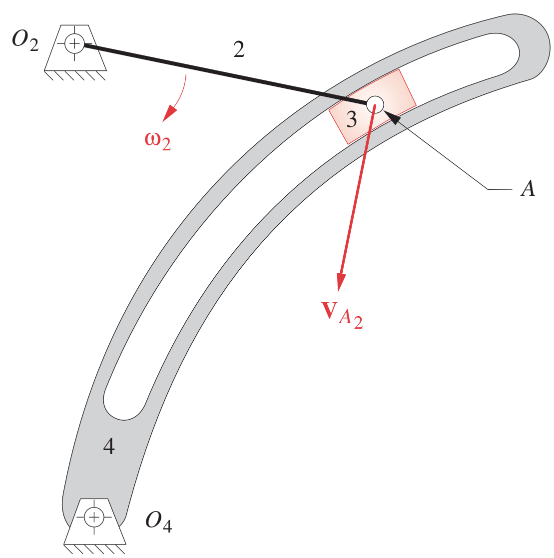

FIGURE P6-24

Problem 6-61

* Answers in Appendix F.

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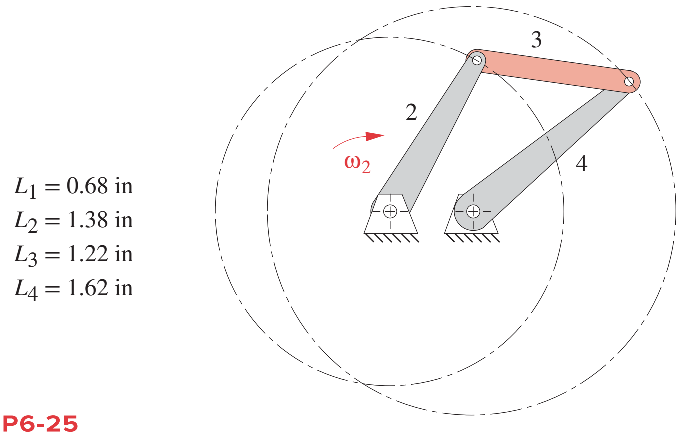


FIGURE P6-25

Problems 6-62 and 6-63

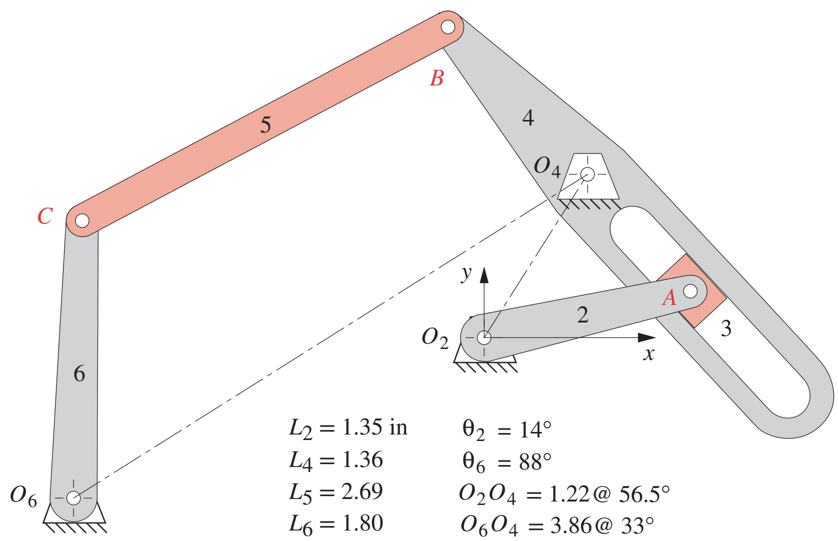


FIGURE P6-26

Problems 6-64, 6-106, 6-107

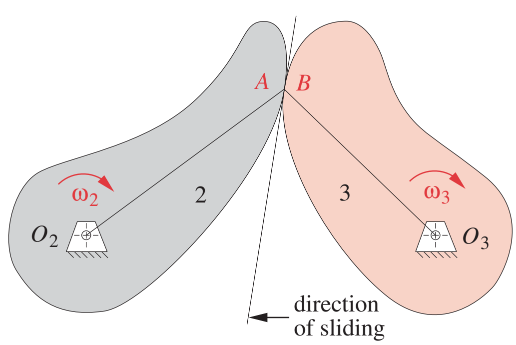
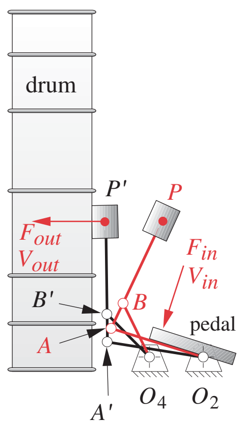


FIGURE P6-27

Problem 6-65



View as a video
http://www.designofmachinery.com/DOM/drum_pedal.avi

FIGURE P6-29
Problem 6-67

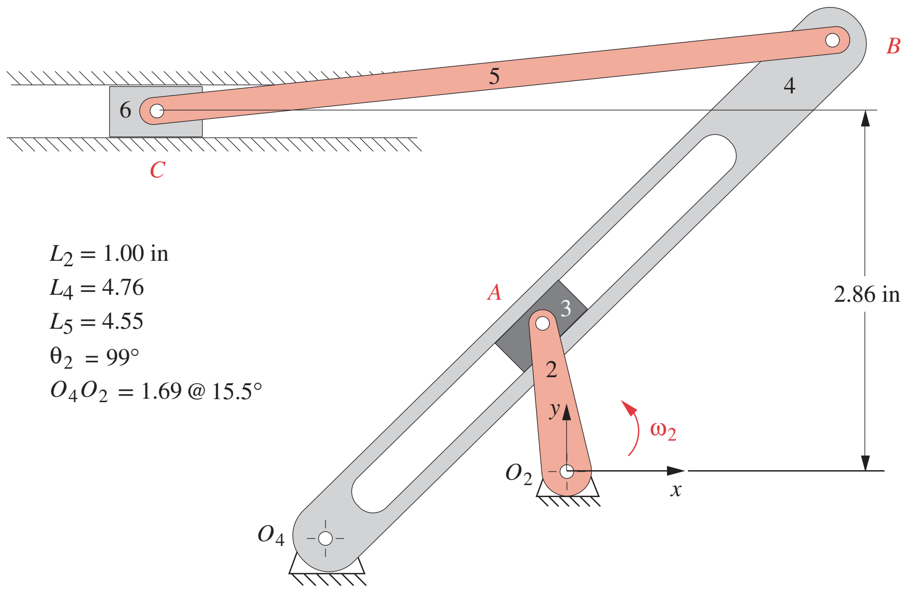


FIGURE P6-28
Problems 6-66, 6-108, 6-109

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†6-67 Figure P6-29 shows a drum pedal mechanism. $O_2A = 100$ mm at 162° and rotates to 171° at A'. $O_2O_4 = 56$ mm, $AB = 28$ mm, $AP = 124$ mm, and $O_4B = 64$ mm. The distance from O_4 to F_{in} is 48 mm. Find and plot the mechanical advantage and the velocity ratio of the linkage over its range of motion. If the input velocity V_{in} is a constant magnitude of 3 m/sec and F_{in} is constant at 50 N, find the output velocity and output force over the range of motion and the power in.

6-68 Figure 3-33 shows a sixbar slider-crank linkage. Find all its instant centers in the position shown.

†6-69 Calculate and plot the centrodes of instant center I_{24} of the linkage in Figure 3-33 so that a pair of noncircular gears can be made to replace the driver dyad 23.

6-70 Find the velocity of the slider in Figure 3-33 for the position shown if $\theta_2 = 110°$ with respect to the global X axis assuming $\omega_2 = 1$ rad/sec CW:

a. Using a graphical method.
b. Using the method of instant centers.
c. Using an analytical method.†

†6-71 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TKSolver* to calculate and plot the angular velocity of link 4 and the linear velocity of slider 6 in the sixbar crank-slider linkage of Figure 3-33 as a function of the angle of input link 2 for a constant $\omega_2 = 1$ rad/sec CW. Plot V_c both as a function of θ_2 and separately as a function of slider position as shown in the figure. Find the percent deviation from constant velocity over $240° < \theta_2 < 270°$ and over $190° < \theta_2 < 315°$.

6-72 Figure 3-34 shows Stephenson's sixbar mechanism. Find all its instant centers in the position shown:

a. In part (*a*) of the figure.
b. In part (*b*) of the figure.
c. In part (*c*) of the figure.

6-73 Find the angular velocity of link 6 of the linkage in Figure 3-34b for the position shown ($\theta_6 = 90°$ with respect to the *x* axis) assuming $\omega_2 = 10$ rad/sec CW:

a. Using a graphical method.
b. Using the method of instant centers.
c. Using an analytical method.†

†6-74 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TK-Solver* to calculate and plot the angular velocity of link 6 in the sixbar linkage of Figure 3-34 as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CW.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

6-75 Figure 3-35 shows a Watt II sixbar mechanism. Find all its instant centers in the position shown:

a. In part (*a*) of the figure.
b. In part (*b*) of the figure.

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6-76 Find the angular velocity of link 6 of the linkage in Figure 3-35 with $\theta_2 = 90°$ assuming $\omega_2 = 10$ rad/sec CCW:

a. Using a graphical method (use a compass and straightedge to draw the the linkage with link 2 at 90°).
b. Using the method of instant centers (use a compass and straightedge to draw the the linkage with link 2 at 90°).
c. Using an analytical method.†

†6-77 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TK-Solver* to calculate and plot the angular velocity of link 6 in the sixbar linkage of Figure 3-35 as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CCW.

6-78 Figure 3-36 shows an eightbar mechanism. Find all its instant centers in the position shown in part (*a*) of the figure.

†6-79 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TK-Solver* to calculate and plot the angular velocity of link 8 in the linkage of Figure 3-36 as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CCW.

†6-80 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TKSolver* to calculate and plot magnitude and direction of the velocity of point *P* in Figure 3-37a as a function of θ_2. Also calculate and plot the velocity of point *P* versus point *A*.

†6-81 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TK-Solver* to calculate the percent error of the deviation from a perfect circle for the path of point *P* in Figure 3-37a.

†6-82 Repeat Problem 6-80 for the linkage in Figure 3-37b.

6-83 Find all instant centers of the linkage in Figure P6-30 in the position shown.

6-84 Find the angular velocities of links 3 and 4 and the linear velocities of points *A*, *B* and P_1 in the *XY* coordinate system for the linkage in Figure P6-30 in the position shown. Assume that $\theta_2 = 45°$ in the *XY* coordinate system and $\omega_2 = 10$ rad/sec. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the *xy* coordinate system:

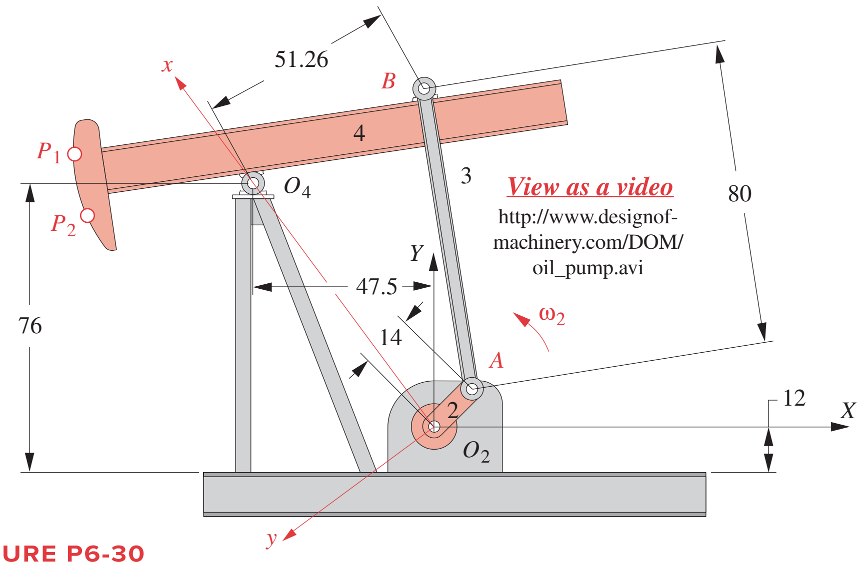


FIGURE P6-30

Problems 6-83 to 6-85 An oil field pump—dimensions in inches

a. Using a graphical method.
b. Using the method of instant centers.
c. Using an analytical method.†

§6-85 Using the data from Problem 6-84, write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TKSolver* to calculate and plot magnitude and direction of the absolute velocity of point P_1 in Figure P6-30 as a function of θ_2.

6-86 Find all instant centers of the linkage in Figure P6-31 in the position shown.

§ Note that these can be long problems to solve and may be more appropriate for a project assignment than an overnight problem. In most cases, the solution can be checked with program LINKAGES.

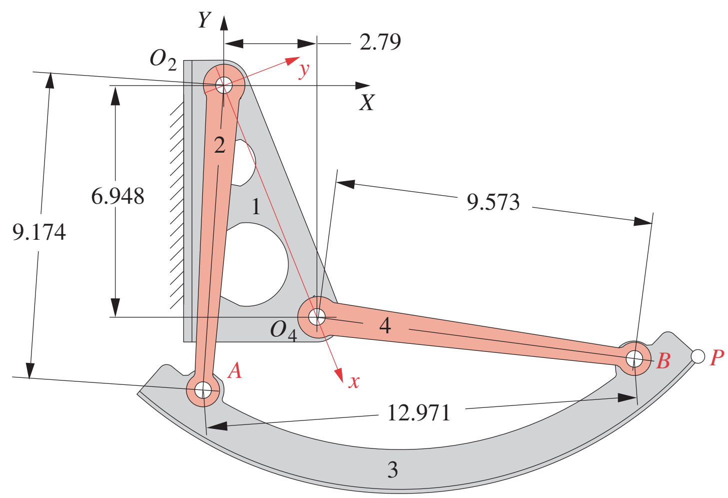


FIGURE P6-31

Problems 6-86 and 6-87 An aircraft overhead bin mechanism—dimensions in inches

6-87 Find the angular velocities of links 3 and 4, and the linear velocity of point *P* in the *XY* coordinate system for the linkage in Figure P6-31 in the position shown. Assume that $\theta_2 = -94.121°$ in the *XY* coordinate system and $\omega_2 = 1$ rad/sec. The position of the coupler point *P* on link 3 with respect to point *A* is: $p = 15.00$, $\delta_3 = 0°$:

a. Using a graphical method.
b. Using the method of instant centers.
c. Using an analytical method.†

6-88 Figure P6-32 shows a fourbar double slider known as an elliptical trammel. Find all its instant centers in the position shown.

6-89 The elliptical trammel in Figure P6-32 must be driven by rotating link 3 in a full circle. Points on line *AB* describe ellipses. Find and draw (manually or with a computer) the fixed and moving centrodes of instant center I_{13}. (Hint: These are called the Cardan circles.)

6-90 Derive analytical expressions for the velocities of points *A* and *B* in Figure P6-32 as a function of θ_3 , ω_3, and the length *AB* of link 3. Use a vector loop equation.

6-91 The linkage in Figure P6-33a has link 2 at 120° in the global *XY* coordinate system. Find ω_6 and $\mathbf{V}_D$ in the global coordinate system for the position shown if $\omega_2 = 10$ rad/sec CCW. Use the velocity difference graphical method. (Print the figure from its PDF file and draw on it.)

6-92 The linkage in Figure P6-33a has link 2 at 120° in the global *XY* coordinate system. Find ω_6 and $\mathbf{V}_D$ in the global coordinate system for the position shown if $\omega_2 = 10$ rad/sec CCW. Use the instant center graphical method. (Print the figure from its PDF file and draw on it.)

6-93 The linkage in Figure P6-33a has link 2 at 120° in the global *XY* coordinate system. Find ω_6 and $\mathbf{V}_D$ in the global coordinate system for the position shown if $\omega_2 = 10$ rad/sec CCW. Use an analytical method.

6-94 The linkage in Figure P6-33b has link 3 perpendicular to the *X* axis and links 2 and 4 are parallel to each other. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_P$ if $\omega_2 = 15$ rad/sec CW. Use the velocity difference graphical method. (Print the figure's PDF file and draw on it.)

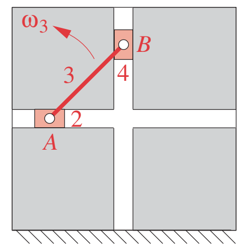
6-95 The linkage in Figure P6-33b has link 3 perpendicular to the *X* axis and links 2 and 4 are parallel to each other. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_P$ if $\omega_2 = 15$ rad/sec CW. Use the instant center graphical method. (Print the figure from its PDF file and draw on it.)

6-96 The linkage in Figure P6-33b has link 3 perpendicular to the *X* axis and links 2 and 4 are parallel to each other. Find ω_3, $\mathbf{V}_A$, $\mathbf{V}_B$, and $\mathbf{V}_P$ if $\omega_2 = 15$ rad/sec CW. Use an analytical method.

6-97 The crosshead linkage shown in Figure P6-33c has 2 *DOF* with inputs at crossheads 2 and 5. Find instant centers $I_{1,3}$ and $I_{1,4}$.

6-98 The crosshead linkage shown in Figure P6-33c has 2 *DOF* with inputs at crossheads 2 and 5. Find $\mathbf{V}_B$, $\mathbf{V}_{P3}$, and $\mathbf{V}_{P4}$ if the crossheads are each moving toward the origin of the *XY* coordinate system with a speed of 20 in/sec. Use a graphical method of your choice. (Print the figure from its PDF file and draw on it.)

6-99 The linkage in Figure P6-33d has the path of slider 6 perpendicular to the global *X* axis and link 2 aligned with the global *X* axis. Find $\mathbf{V}_A$ in the position shown if the velocity of the slider is 20 in/sec downward. Use the velocity difference graphical method. (Print the figure from its PDF file and draw on it.)



View as a video
http://www.designofmachinery.com/DOM/elliptic_trammel.avi

FIGURE P6-32

Elliptical trammel
Problems 6-88 to 6-90

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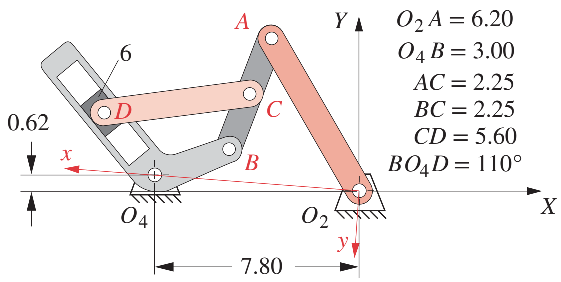
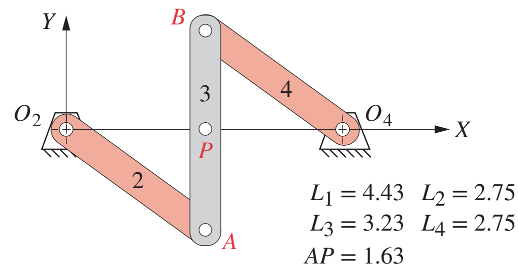
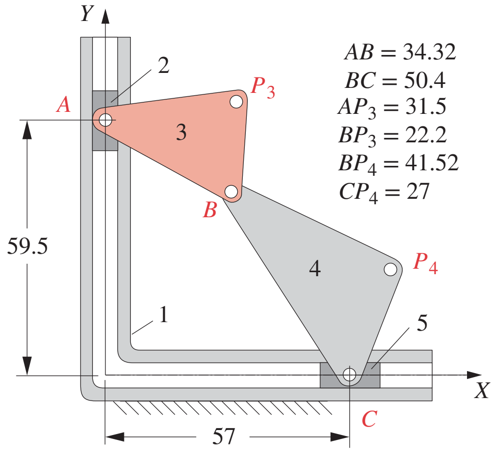


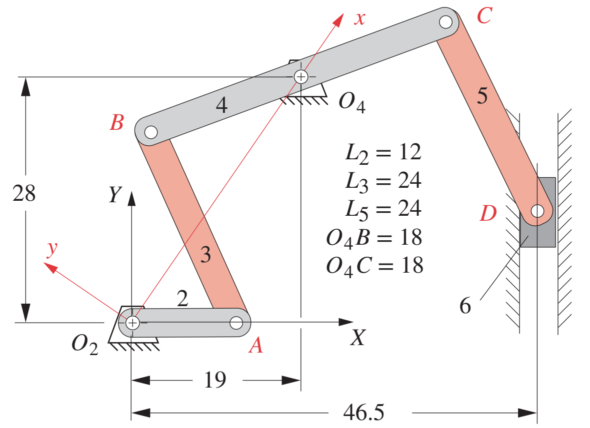
(a) Sixbar linkage



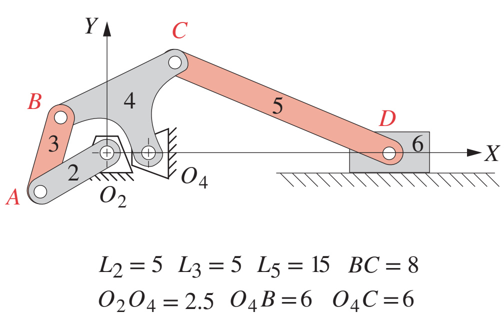
(b) Fourbar linkage



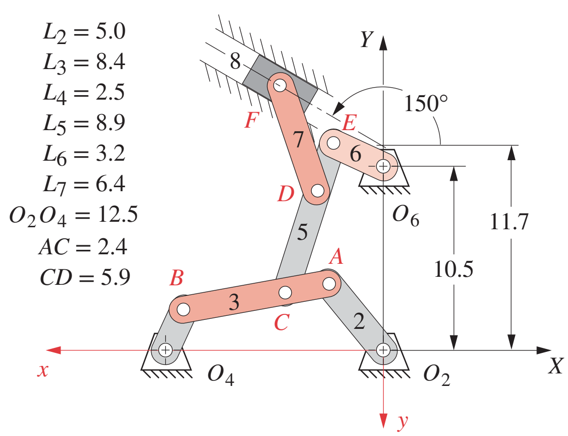
(c) Dual crosshead mechanism



(d) Sixbar linkage



(e) Drag link slider-crank



(f) Eightbar mechanism

FIGURE P6-33

Problems 6-91 to 6-103

6-100 The linkage in Figure P6-33d has the path of slider 6 perpendicular to the global X axis and link 2 aligned with the global X axis. Find $\mathbf{V}_A$ in the position shown if the velocity of the slider is 20 in/sec downward. Use the instant center graphical method. (Print the figure from its PDF file and draw on it.)

6-101 For the linkage of Figure P6-33e, write a computer program or use an equation solver to find and plot $\mathbf{V}_D$ in the global coordinate system for one revolution of link 2 if $\omega_2 =$ 10 rad/sec CW.

6-102 For the linkage of Figure P6-33f, locate and identify all instant centers.

6-103 The linkage of Figure P6-33f has link 2 at 130° in the global XY coordinate system. Find $\mathbf{V}_D$ in the global coordinate system for the position shown if $\omega_2 = 15$ rad/sec CW. Use any graphical method. (Print the figure from its PDF file and draw on it.)

6-104 For the linkage of Figure P6-34, locate and identify all instant centers. $O_2O_4 = AB = BC = DE = 1$. $O_2A = O_4B = BE = CD = 1.75$. $O_4C = AE = 2.60$.

6-105 For the linkage of Figure P6-34, show that $I_{1,6}$ is stationary for all positions of the linkage. $O_2O_4 = AB = BC = DE = 1$. $O_2A = O_4B = BE = CD = 1.75$. $O_4C = AE = 2.60$.

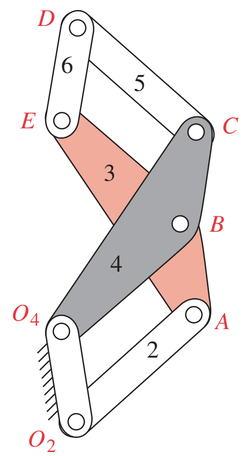


FIGURE P6-34
Problems 6-104, 6-105

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6-106 Figure P6-26 shows a mechanism with dimensions. Use a graphical method to determine the velocities of points A and B, and the velocity of slip for the position shown if $\omega_2 = 24$ rad/sec CW. Ignore links 5 and 6.

6-107 Repeat Problem 6-106 using an analytical method.

6-108 Figure P6-28 shows a quick-return mechanism with dimensions. Use a graphical method to determine the velocities of points A and B and the velocity of slip for the position shown if $\omega_2 = 16$ rad/sec CCW. Ignore links 5 and 6.

6-109 Repeat Problem 6-108 using an analytical method.

6-110 The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Figure P6-2. The link lengths and the values of d and $\dot{d}$ are defined in Table P6-5. For the row(s) assigned, find the velocity of the pin joint A and the angular velocity of the crank using a graphical method.

6-111 The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Figure P6-2. The link lengths and the values of d and $\dot{d}$ are defined in Table P6-5. For the rows assigned, find the velocity of pin joint A and the angular velocity of the crank using the analytic method. Draw the linkage to scale and label it before setting up the equations.

TABLE P6-5 Data for Problems 6-110 to 6-111‡

Row	Link 2	Link 3	Offset	d	$\dot{d}$
a	1.4	4	1	2.5	10
b	2	6	–3	5	–12
c	3	8	2	8	–15
d	3.5	10	1	–8	24
e	5	20	–5	15	–50
f	3	13	0	–12	–45
g	7	25	10	25	100

‡ Drawings of these linkages are in the *PDF Problem Workbook* folder.

6-112 Figure P6-7b shows an inversion of the fourbar crank-slider. Use a graphical method to calculate the velocity of the moving joint, the velocity of slip, and the angular velocity of link 4 for the position shown. $L_1 = 10.0$ in, $L_2 = 8.0$ in, and $\theta_2 =-140$ in the LCS determined by O_2 and O_4. $\omega_2 = 5$ rad/sec.

6-113 Figure P6-7b shows an inversion of the fourbar crank-slider. Use an analytical method to calculate and plot the angular velocity of link 4 as a function of the crank angle over its full 360° of motion. Use the dimensions given in Problem 6-112. $\omega_2 = 5$ rad/sec.