Chapter 7

ACCELERATION ANALYSIS

Take it to warp five, Mr. Sulu
CAPTAIN KIRK

7.0 INTRODUCTION *View the lecture video (41:39)*†

Once a velocity analysis is done, the next step is to determine the accelerations of all links and points of interest in the mechanism or machine. We need to know the accelerations to calculate the dynamic forces from $\mathbf{F} = m\mathbf{a}$. The dynamic forces will contribute to the stresses in the links and other components. Many methods and approaches exist to find accelerations in mechanisms. We will examine only a few of these methods here. We will first develop a manual graphical method, which is often useful as a check on the more complete and accurate analytical solution. Then we will derive the analytical solution for accelerations in the fourbar and inverted crank-slider linkages as examples of the general vector loop equation solution to acceleration analysis problems.

† http://www.designofmachinery.com/DOM/Acceleration_Analysis.mp4

7.1 DEFINITION OF ACCELERATION

Acceleration is defined as *the rate of change of velocity with respect to time*. Velocity ($\mathbf{V}$, ω) is a vector quantity and so is acceleration. Accelerations can be **angular** or **linear**. **Angular acceleration** will be denoted as α and **linear acceleration** as $\mathbf{A}$.

$$\alpha = \frac{d\omega}{dt}; \qquad \mathbf{A} = \frac{d\mathbf{V}}{dt} \tag{7.1}$$

Figure 7-1 shows a link PA in pure rotation, pivoted at point A in the xy plane. We are interested in the acceleration of point P when the link is subjected to an angular velocity ω and an angular acceleration α, which need not have the same sense. The link's position is defined by the position vector $\mathbf{R}$, and the velocity of point P is $\mathbf{V}_{PA}$. These vectors were defined in equations 6.2 and 6.3 which are repeated here for convenience. (See also Figure 6-1.)

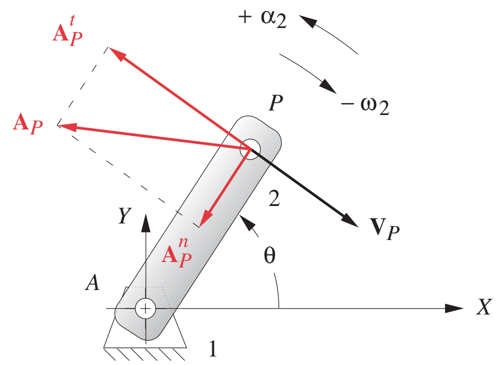

FIGURE 7-1

Acceleration of a link in pure rotation with a positive (CCW) α_2 and a negative (CW) ω_2

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$$\mathbf{R}_{PA} = pe^{j\theta} \tag{6.2}$$

$$\mathbf{V}_{PA} = \frac{d\mathbf{R}_{PA}}{dt} = p\,je^{j\theta}\frac{d\theta}{dt} = p\omega\,je^{j\theta} \tag{6.3}$$

where p is the scalar length of the vector $\mathbf{R}_{PA}$. We can easily differentiate equation 6.3 to obtain an expression for the acceleration of point P:

$$\begin{aligned}
\mathbf{A}_{PA} &= \frac{d\mathbf{V}_{PA}}{dt} = \frac{d\left(p\omega\,je^{j\theta}\right)}{dt}\\
\mathbf{A}_{PA} &= j\,p\left(e^{j\theta}\frac{d\omega}{dt} + \omega\,je^{j\theta}\frac{d\theta}{dt}\right)\\
\mathbf{A}_{PA} &= p\alpha\,je^{j\theta} - p\omega^2 e^{j\theta}\\
\mathbf{A}_{PA} &= \mathbf{A}^t_{PA} + \mathbf{A}^n_{PA}
\end{aligned} \tag{7.2}$$

Note that there are two functions of time in equation 6.3, θ and ω. Thus there are two terms in the expression for acceleration, the tangential component of acceleration involving α and the normal (or centripetal) component $\mathbf{A}^n_A$ involving ω^2. As a result of the differentiation, the tangential component is multiplied by the (constant) complex operator j. This causes a rotation of this acceleration vector through 90° with respect to the original position vector. (See also Figure 4-8b.) This 90° rotation is nominally positive, or counterclockwise (CCW). However, the tangential component is also multiplied by α, which may be either positive or negative. As a result, the tangential component of acceleration will be **rotated 90°** from the angle θ of the *position vector* **in a direction dictated by the sign of** α. This is just mathematical verification of what you already knew, namely that *tangential acceleration is always in a direction perpendicular to the radius of rotation and is thus tangent to the path of motion* as shown in Figure 7-1. The normal, or centripetal, acceleration component is multiplied by j^2, or –1. This directs *the centripetal component at 180° to the angle* θ *of the original position vector,* i.e., toward the center (centripetal means *toward the center*). The total acceleration $\mathbf{A}_{PA}$ of point P is the vector sum of the tangential $\mathbf{A}^t_A$ and normal $\mathbf{A}^n_A$ components as shown in Figure 7-1 and equation 7.2.

Substituting the Euler identity (equation 4.4a) into equations 7.2 gives us the real and imaginary (or x and y) components of the acceleration vector.

$$\mathbf{A}_{PA} = p\alpha(-\sin\theta + j\cos\theta) - p\omega^2(\cos\theta + j\sin\theta) \tag{7.3}$$

The acceleration $\mathbf{A}_{PA}$ in Figure 7-1 can be referred to as an **absolute acceleration** since it is referenced to A, which is the origin of the global coordinate axes in that system. As such, we could have referred to it as $\mathbf{A}_P$, with the absence of the second subscript implying reference to the global coordinate system.

Figure 7-2a shows a different and slightly more complicated system in which the pivot A is no longer stationary. It has a known linear acceleration $\mathbf{A}_A$ as part of the translating carriage, link 3. If α is unchanged, the acceleration of point P versus A will be the same as before, but $\mathbf{A}_{PA}$ can no longer be considered an absolute acceleration. It is now an **acceleration difference** and **must** carry the second subscript as $\mathbf{A}_{PA}$. The absolute acceleration $\mathbf{A}_P$ must now be found from the **acceleration difference** equation whose graphical solution is shown in Figure 7-2b:

$$\begin{aligned}\mathbf{A}_P &= \mathbf{A}_A + \mathbf{A}_{PA}\\ \left(\mathbf{A}_P^t + \mathbf{A}_P^n\right) &= \left(\mathbf{A}_A^t + \mathbf{A}_A^n\right) + \left(\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n\right)\end{aligned} \tag{7.4}$$

Note the similarity of equations 7.4 to the **velocity difference equation** (equation 6.5). Note also that the solution for $\mathbf{A}_P$ in equation 7.4 can be found by adding either the resultant vector $\mathbf{A}_{PA}$ or its normal and tangential components $\mathbf{A}_{PA}^n$ and $\mathbf{A}_{PA}^t$ to the vector $\mathbf{A}_A$ in Figure 7-2b. The vector $\mathbf{A}_A$ has a zero normal component in this example because link 3 is in pure translation.

Figure 7-3 shows two independent bodies P and A, which could be two automobiles, moving in the same plane. Auto #1 is turning and accelerating into the path of auto #2, that is decelerating to avoid a crash. If their independent accelerations $\mathbf{A}_P$ and $\mathbf{A}_A$ are known, their **relative acceleration** $\mathbf{A}_{PA}$ can be found from equation 7.4 arranged algebraically as:

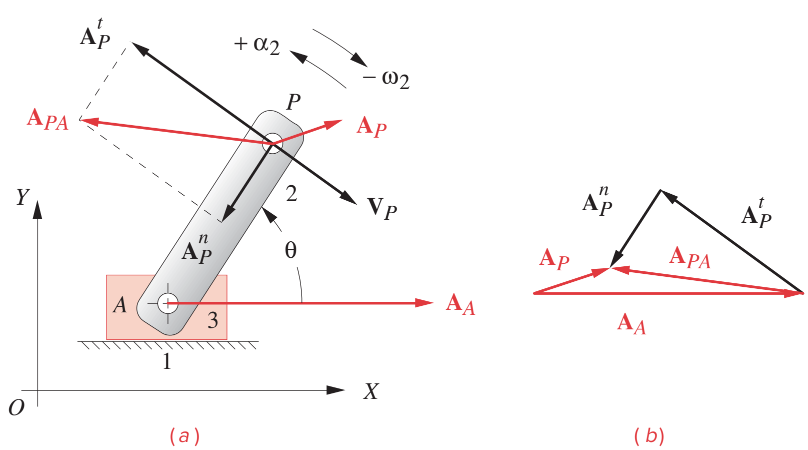

FIGURE 7-2

Acceleration difference in a system with a positive (CCW) α_2 and a negative (CW) ω

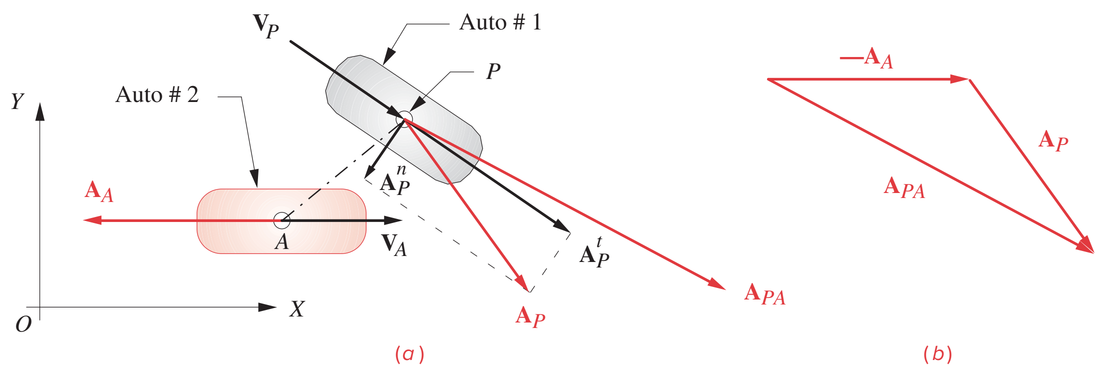

FIGURE 7-3
Relative acceleration

$$\mathbf{A}_{PA} = \mathbf{A}_P - \mathbf{A}_A \tag{7.5}$$

The graphical solution to this equation is shown in Figure 7-3b.

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As we did for velocity analysis, we give these two cases different names despite the fact that the same equation applies. Repeating the definition from Section 6.1, modified to refer to acceleration:

Case 1: Two points in the same body => ***acceleration difference***

Case 2: Two points in different bodies => ***relative acceleration***

7.2 GRAPHICAL ACCELERATION ANALYSIS

The comments made in regard to graphical velocity analysis in Section 6.2 apply as well to graphical acceleration analysis. Historically, graphical methods were the only practical way to solve these acceleration analysis problems. With some practice, and with proper tools such as a drafting machine, drafting instruments, or a CAD package, one can fairly rapidly solve for the accelerations of particular points in a mechanism for any one input position by drawing vector diagrams. However, if accelerations for many positions of the mechanism are to be found, each new position requires a completely new set of vector diagrams be drawn. Very little of the work done to solve for the accelerations at position 1 carries over to position 2, etc. This is an even more tedious process than that for graphical velocity analysis because there are more components to draw. Nevertheless, this method still has more than historical value as it can provide a quick check on the results from a computer program solution. Such a check only needs to be done for a few positions to prove the validity of the program.

To solve any acceleration analysis problem graphically, we need only three equations, equation 7.4 and equations 7.6 (which are merely the scalar magnitudes of the terms in equation 7.2):

$$\begin{aligned} \left|\mathbf{A}^t\right| &= A^t = r\alpha \\ \left|\mathbf{A}^n\right| &= A^n = r\omega^2 \end{aligned} \tag{7.6}$$

Note that the scalar equations 7.6 define only the **magnitudes** (A^t, A^n) of the components of acceleration of any point in rotation. In a CASE 1 graphical analysis, the **directions** of the vectors due to the centripetal and tangential components of the acceleration difference must be understood from equation 7.2 to be perpendicular to and along the radius of rotation, respectively. Thus, if the center of rotation is known or assumed, the directions of the acceleration difference components due to that rotation are known and their senses will be consistent with the angular velocity ω and angular acceleration α of the body.

Figure 7-4 shows a fourbar linkage in one particular position. We wish to solve for the angular accelerations of links 3 and 4 (α_3, α_4) and the linear accelerations of points A, B, and C ($\mathbf{A}_A$, $\mathbf{A}_B$, $\mathbf{A}_C$). Point C represents any general point of interest such as a coupler point. The solution method is valid for any point on any link. To solve this problem, we need to know the *lengths of all the links*, the *angular positions of all the links*, the *angular velocities of all the links,* and the *instantaneous input acceleration of any one driving link or driving point*. Assuming that we have designed this linkage, we will know or can measure the link lengths. We must also first do a **complete position and velocity analysis** to find the link angles θ_3 and θ_4 and angular velocities ω_3 and ω_4 given the input link's position θ_2, input angular velocity ω_2, and input acceleration α_2. This can be done by any of the methods in Chapters 4 and 6. In general we must solve these problems in stages, first for link positions, then for velocities, and finally for accelerations. For the following example, we will assume that a complete position and velocity analysis has been done and that the input is to link 2 with known θ_2, ω_2, and α_2 for this one "freeze-frame" position of the moving linkage.

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EXAMPLE 7-1

Graphical Acceleration Analysis for One Position of a Fourbar Linkage.

Problem: Given θ_2, θ_3, θ_4, ω_2, ω_3, ω_4, α_2, find α_3, α_4, $\mathbf{A}_A$, $\mathbf{A}_B$, $\mathbf{A}_P$ by graphical methods.

Solution: (See Figure 7-4.)

1 Start at the end of the linkage about which you have the most information. Calculate the magnitudes of the centripetal and tangential components of acceleration of point A using scalar equations 7.6.

$$A_A^n = (AO_2)\omega_2^2; \qquad A_A^t = (AO_2)\alpha_2 \tag{a}$$

2 On the linkage diagram, Figure 7-4a, draw the acceleration component vectors $\mathbf{A}_A^n$ and $\mathbf{A}_A^t$ with their lengths equal to their magnitudes at some convenient scale. Place their roots at point A with their directions respectively along and perpendicular to the radius AO_2. The sense of $\mathbf{A}_A^t$ is defined by that of α_2 (according to the right-hand rule), and the sense of $\mathbf{A}_A^n$ is the opposite of that of the position vector $\mathbf{R}_A$ as shown in Figure 7-4a.

3 Move next to a point about which you have some information, such as B on link 4. Note that the directions of the tangential and normal components of acceleration of point B are predictable since this link is in pure rotation about point O_4. Draw the construction line pp through point B perpendicular to BO_4, to represent the direction of $\mathbf{A}_B^t$ as shown in Figure 7-4a.

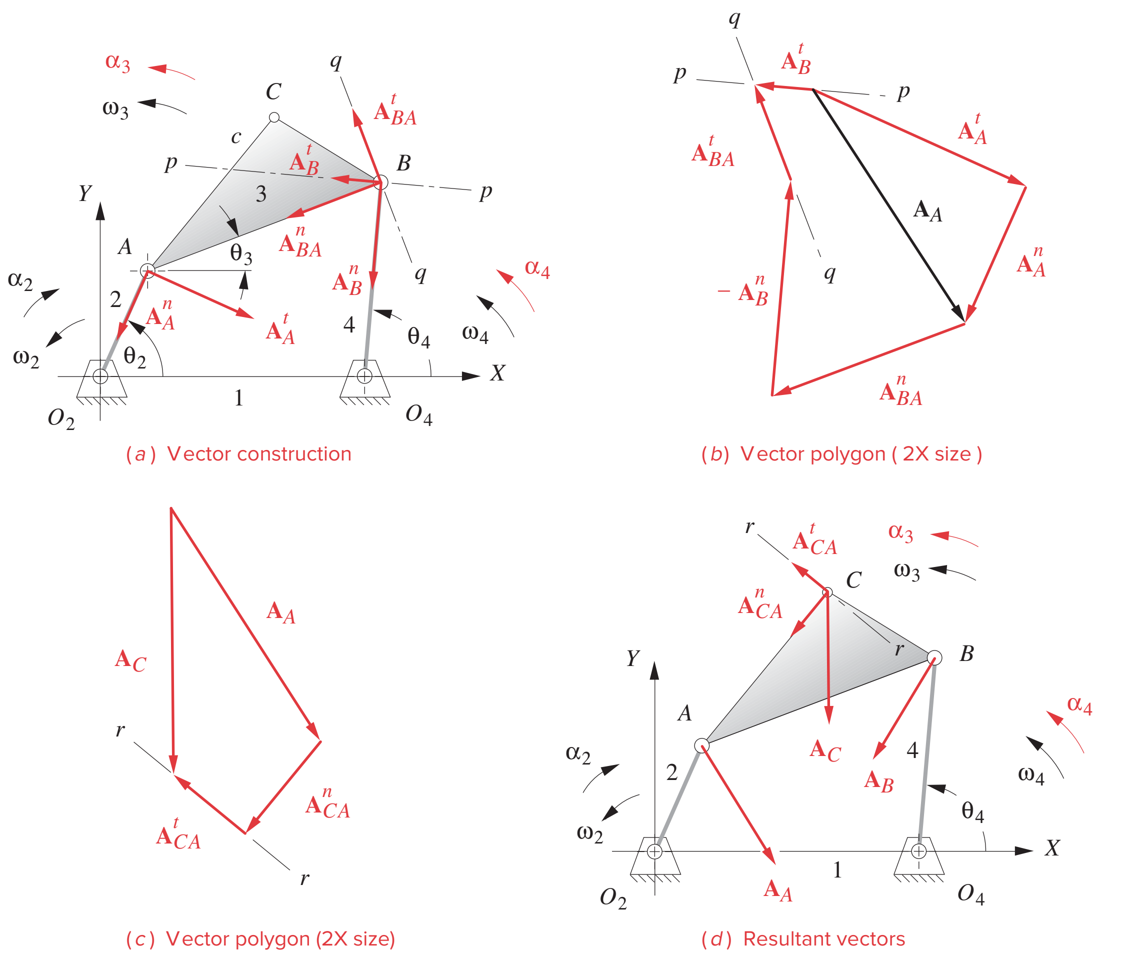

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FIGURE 7-4

Graphical solution for acceleration in a pin-jointed linkage with a negative (CW) α_2 and a positive (CCW) ω_2

4 Write the acceleration difference vector equation 7.4 for point B versus point A.

$$\mathbf{A}_B = \mathbf{A}_A + \mathbf{A}_{BA} \tag{b}$$

Substitute the normal and tangential components for each term:

$$\left(\mathbf{A}_B^t + \mathbf{A}_B^n\right) = \left(\mathbf{A}_A^t + \mathbf{A}_A^n\right) + \left(\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n\right) \tag{c}$$

We will use point A as the reference point to find $\mathbf{A}_B$ because A is in the same link as B and we have already solved for $\mathbf{A}_A^t$ and $\mathbf{A}_A^n$. Any two-dimensional vector equation can be solved for two unknowns. Each term has two parameters, namely magnitude and direction. There are then potentially twelve unknowns in this equation, two per term. We must know ten of them to solve it. We know both the magnitudes and directions of $\mathbf{A}_A^t$ and $\mathbf{A}_A^n$ and the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_B^n$ that are along line pp and line BO_4, respectively. We can also calculate the magnitude of $\mathbf{A}_B^n$ from equation 7.6 since we know ω_4. This provides seven known values. We need to know three more parameters to solve the equation.

5 The term $\mathbf{A}_{BA}$ represents the acceleration difference of B with respect to A. This has two components. The normal component $\mathbf{A}_{BA}^n$ is directed along the line BA because we are using point A as the reference center of rotation for the free vector ω_3, and its magnitude can be calculated from equation 7.6. The direction of $\mathbf{A}_{BA}^t$ must then be perpendicular to the line BA. Draw construction line qq through point B and perpendicular to BA to represent the direction of $\mathbf{A}_{BA}^t$ as shown in Figure 7-4a. The calculated magnitude and direction of component $\mathbf{A}_{BA}^n$ and the known direction of $\mathbf{A}_{BA}^t$ provide the needed additional three parameters.

6 Now the vector equation can be solved graphically by drawing a vector diagram as shown in Figure 7-4b. Either drafting tools or a CAD package is necessary for this step. The strategy is to first draw all vectors for which we know both magnitude and direction, being careful to arrange their senses according to equation 7.4.

First draw acceleration vectors ($\mathbf{A}_A^t$) and ($\mathbf{A}_A^n$) tip to tail, carefully to some scale, maintaining their directions. (They are drawn twice size in the figure.) Note that the sum of these two components is the vector $\mathbf{A}_A$. The equation in step 4 says to add $\mathbf{A}_{BA}$ to $\mathbf{A}_A$. We know $\mathbf{A}_{BA}^n$, so we can draw that component at the end of $\mathbf{A}_A$. We also know $\mathbf{A}_B^n$, but this component is on the left side of equation 7.4, so we must subtract it. Draw the negative (opposite sense) of $\mathbf{A}_B^n$ at the end of $\mathbf{A}_{BA}^n$.

This exhausts our supply of components for which we know both magnitude and direction. Our two remaining knowns are the directions of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$ that lie along the lines pp and qq, respectively. Draw a line parallel to line qq across the tip of the vector representing *minus* $\mathbf{A}_B^n$. The resultant, or left side of the equation, must close the vector diagram, from the tail of the first vector drawn ($\mathbf{A}_A$) to the tip of the last, so draw a line parallel to pp across the tail of $\mathbf{A}_A$. The intersection of these lines parallel to pp and qq defines the lengths of $\mathbf{A}_B^t$ and $\mathbf{A}_{BA}^t$. The senses of these vectors are determined from reference to equation 7.4. Vector $\mathbf{A}_A$ was added to $\mathbf{A}_{BA}$, so their components must be arranged tip to tail. Vector $\mathbf{A}_B$ is the resultant, so its component $\mathbf{A}_B^t$ must be from the tail of the first to the tip of the last. The resultant vectors are shown in Figure 7-4b and d.

7 The angular accelerations of links 3 and 4 can be calculated from equation 7.6:

$$\alpha_4 = \frac{A_B^t}{BO_4} \qquad\qquad \alpha_3 = \frac{A_{BA}^t}{BA} \tag{d}$$

Note that the acceleration difference term $\mathbf{A}_{BA}^t$ represents the rotational component of acceleration of link 3 due to α_3. The rotational acceleration α of any body is a "**free vector**" which has no particular point of application to the body. It exists everywhere on the body.

8 Finally we can solve for $\mathbf{A}_C$ using equation 7.4 again. We select any point in link 3 for which we know the absolute velocity to use as the reference, such as point A.

$$\mathbf{A}_C = \mathbf{A}_A + \mathbf{A}_{CA} \tag{e}$$

In this case, we can calculate the magnitude of $\mathbf{A}_{CA}^t$ from equation 7.6 as we have already found α_3,

$$A_{CA}^t = c\alpha_3 \tag{f}$$

The magnitude of the component $\mathbf{A}_{CA}^{n}$ can be found from equation 7.6 using ω_3.

$$A_{CA}^{n} = c\omega_3^2 \tag{g}$$

Since both $\mathbf{A}_A$ and $\mathbf{A}_{CA}$ are known, the vector diagram can be directly drawn as shown in Figure 7-4c. Vector $\mathbf{A}_C$ is the resultant that closes the vector diagram. Figure 7-4d shows the calculated acceleration vectors on the linkage diagram.

The above example contains some interesting and significant principles that deserve further emphasis. Equations 7.4 are repeated here for discussion.

$$\begin{gathered}\mathbf{A}_P = \mathbf{A}_A + \mathbf{A}_{PA} \\ \left(\mathbf{A}_P^t + \mathbf{A}_P^n\right) = \left(\mathbf{A}_A^t + \mathbf{A}_A^n\right) + \left(\mathbf{A}_{PA}^t + \mathbf{A}_{PA}^n\right)\end{gathered} \tag{7.4}$$

These equations represent the *absolute* acceleration of some general point P referenced to the origin of the global coordinate system. The right side defines it as the sum of the absolute acceleration of some other reference point A in the same system and the acceleration difference (or relative acceleration) of point P versus point A. These terms are then further broken down into their normal (centripetal) and tangential components that have definitions as shown in equation 7.2.

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Let us review what was done in Example 7-1 in order to extract the general strategy for solution of this class of problem. We started at the input side of the mechanism, as that is where the driving angular acceleration α_2 was defined. We first looked for a point (A) for which the motion was pure rotation. We then solved for the absolute acceleration of that point ($\mathbf{A}_A$) using equations 7.4 and 7.6 by breaking $\mathbf{A}_A$ into its normal and tangential components. *(Steps 1 and 2)*

We then used the point (A) just solved for as a reference point to define the translation component in equation 7.4 written for a new point (B). Note that we needed to choose a second point (B) in the same rigid body as the reference point (A) that we had already solved, and about which we could predict some aspect of the new point's (B's) acceleration components. In this example, we knew the direction of the component $\mathbf{A}_B^t$, though we did not yet know its magnitude. We could also calculate both magnitude and direction of the centripetal component, $\mathbf{A}_B^n$, since we knew ω_4 and the link length. In general this situation will obtain for any point on a link that is jointed to ground (as is link 4). In this example, we could not have solved for point C until we solved for B, because point C is on a floating link for which we do not yet know the angular acceleration or absolute acceleration direction. *(Steps 3 and 4)*

To solve the equation for the second point (B), we also needed to recognize that the tangential component of the acceleration difference $\mathbf{A}_{BA}^t$ is always directed perpendicular to the line connecting the two related points in the link (B and A in the example). In addition, you will always know the magnitude and direction of the centripetal acceleration components in equation 7.4 ***if* it represents an acceleration difference** (CASE 1) **situation**. *If the two points are in the* ***same rigid*** **body,** *then that acceleration difference centripetal component has a magnitude of* $r\omega^2$ *and is always directed along the line connecting the two points, pointing toward the reference point as the center* (see Figure 7-2). These observations will be true regardless of the two points selected. But, *note this is not* *true in a* CASE 2 *situation* as shown in Figure 7-3a where the normal component of acceleration of auto #2 is **not** directed along the line connecting points *A* and *P*. *(Steps 5 and 6)*

Once we found the absolute acceleration of point *B* ($\mathbf{A}_B$), we could solve for α_4, the angular acceleration of link 4 using the tangential component of $\mathbf{A}_B$ in equation (*d*). Because points *A* and *B* are both on link 3, we could also determine the angular acceleration of link 3 using the tangential component of the acceleration difference $\mathbf{A}_{BA}$ between points *B* and *A*, in equation (*d*). Once the angular accelerations of all the links were known, we could then solve for the linear acceleration of any point (such as *C*) in any link using equation 7.4. To do so, we had to understand the concept of angular acceleration as a **free vector**, which means that it exists everywhere on the link at any given instant. It has no particular center. *It has an infinity of potential centers*. The link simply *has an angular acceleration*. It is this property that allows us to solve equation 7.4 for literally **any point** on a rigid body in complex motion **referenced to any other point** on that body. *(Steps 7 and 8)*

7.3 ANALYTICAL SOLUTIONS FOR ACCELERATION ANALYSIS

7

The Fourbar Pin-Jointed Linkage

The position equations for the fourbar pin-jointed linkage were derived in Section 4.5. The linkage was shown in Figure 4-6 and is shown again in Figure 7-5a on which we also show an input angular acceleration α_2 applied to link 2. This input angular acceleration α_2 may vary with time. The vector loop equation was shown in equations 4.5a and c, repeated here for your convenience.

$$\mathbf{R}_2 + \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0 \tag{4.5a}$$

As before, we substitute the complex number notation for the vectors, denoting their scalar lengths as *a, b, c, d* as shown in Figure 7-5.

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0 \tag{4.5c}$$

In Section 6.7, we differentiated equation 4.5c versus time to get an expression for velocity which is repeated here.

$$ja\omega_2 e^{j\theta_2} + jb\omega_3 e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0 \tag{6.14c}$$

We will now differentiate equation 6.14c versus time to obtain an expression for accelerations in the linkage. Each term in equation 6.14c contains two functions of time, θ and ω. Differentiating with the chain rule in this example will result in two terms in the acceleration expression for each term in the velocity equation.

$$\left(j^2 a\omega_2^2 e^{j\theta_2} + ja\alpha_2 e^{j\theta_2}\right) + \left(j^2 b\omega_3^2 e^{j\theta_3} + jb\alpha_3 e^{j\theta_3}\right) - \left(j^2 c\omega_4^2 e^{j\theta_4} + jc\alpha_4 e^{j\theta_4}\right) = 0 \tag{7.7a}$$

Simplifying and grouping terms:

$$\left(a\alpha_2 je^{j\theta_2} - a\omega_2^2 e^{j\theta_2}\right) + \left(b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3}\right) - \left(c\alpha_4 je^{j\theta_4} - c\omega_4^2 e^{j\theta_4}\right) = 0 \tag{7.7b}$$

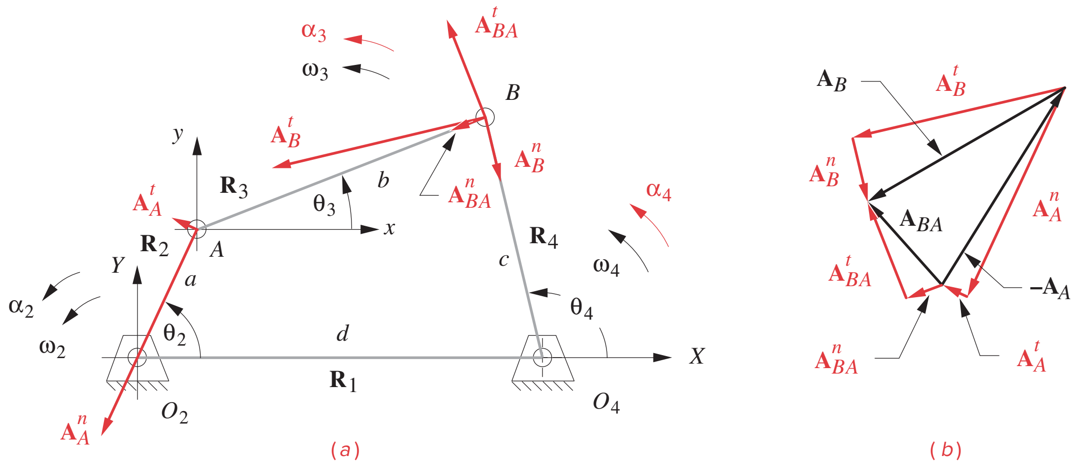

FIGURE 7-5

Position vector loop for a fourbar linkage showing acceleration vectors

Compare the terms grouped in parentheses with equations 7.2. Equation 7.7 contains the tangential and normal components of the accelerations of points A and B and of the acceleration difference of B to A. Note that these are the same relationships that we used to solve this problem graphically in Section 7.2. Equation 7.7 is, in fact, the **acceleration difference equation** 7.4 which, with the labels used here, is:

$$\mathbf{A}_A + \mathbf{A}_{BA} - \mathbf{A}_B = 0 \tag{7.8a}$$

where:

$$\begin{aligned} \mathbf{A}_A &= \left(\mathbf{A}_A^t + \mathbf{A}_A^n\right) = \left(a\alpha_2\, je^{j\theta_2} - a\omega_2^2\, e^{j\theta_2}\right) \\ \mathbf{A}_{BA} &= \left(\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n\right) = \left(b\alpha_3\, je^{j\theta_3} - b\omega_3^2\, e^{j\theta_3}\right) \\ \mathbf{A}_B &= \left(\mathbf{A}_B^t + \mathbf{A}_B^n\right) = \left(c\alpha_4\, je^{j\theta_4} - c\omega_4^2\, e^{j\theta_4}\right) \end{aligned} \tag{7.8b}$$

The vector diagram in Figure 7-5b shows these components and is a graphical solution to equation 7.8a. The vector components are also shown acting at their respective points on Figure 7-5a.

We now need to solve equation 7.7 for α_3 and α_4, knowing the input angular acceleration α_2, the link lengths, all link angles, and angular velocities. Thus, the position analysis derived in Section 4.5 and the velocity analysis from Section 6.7 must be done first to determine the link angles and angular velocities before this acceleration analysis can be completed. We wish to solve equations 7.8 to get expressions in this form:

$$\alpha_3 = f\left(a, b, c, d, \theta_2, \theta_3, \theta_4, \omega_2, \omega_3, \omega_4, \alpha_2\right) \tag{7.9a}$$

$$\alpha_4 = g\left(a, b, c, d, \theta_2, \theta_3, \theta_4, \omega_2, \omega_3, \omega_4, \alpha_2\right) \tag{7.9b}$$

The strategy of solution will be the same as was done for the position and velocity analysis. First, substitute the Euler identity from equation 4.4a in each term of equation 7.7:

$$\left[a\alpha_2\, j(\cos\theta_2 + j\sin\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2)\right] + \left[b\alpha_3\, j(\cos\theta_3 + j\sin\theta_3) - b\omega_3^2(\cos\theta_3 + j\sin\theta_3)\right] - \left[c\alpha_4\, j(\cos\theta_4 + j\sin\theta_4) - c\omega_4^2(\cos\theta_4 + j\sin\theta_4)\right] = 0 \tag{7.10a}$$

Multiply by the operator j and rearrange:

$$\left[a\alpha_2(-\sin\theta_2 + j\cos\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2)\right] + \left[b\alpha_3(-\sin\theta_3 + j\cos\theta_3) - b\omega_3^2(\cos\theta_3 + j\sin\theta_3)\right] - \left[c\alpha_4(-\sin\theta_4 + j\cos\theta_4) - c\omega_4^2(\cos\theta_4 + j\sin\theta_4)\right] = 0 \tag{7.10b}$$

We can now separate this vector equation into its two components by collecting all real and all imaginary terms separately:

real part (x component):

$$-a\alpha_2\sin\theta_2 - a\omega_2^2\cos\theta_2 - b\alpha_3\sin\theta_3 - b\omega_3^2\cos\theta_3 + c\alpha_4\sin\theta_4 + c\omega_4^2\cos\theta_4 = 0 \tag{7.11a}$$

imaginary part (y component):

$$a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 + b\alpha_3\cos\theta_3 - b\omega_3^2\sin\theta_3 - c\alpha_4\cos\theta_4 + c\omega_4^2\sin\theta_4 = 0 \tag{7.11b}$$

Note that the j's have canceled in equation 7.11b. We can solve equations 7.11a and 7.11b simultaneously to get:

$$\alpha_3 = \frac{CD - AF}{AE - BD} \tag{7.12a}$$

$$\alpha_4 = \frac{CE - BF}{AE - BD} \tag{7.12b}$$

where:

$$\begin{aligned}
A &= c\sin\theta_4 \\
B &= b\sin\theta_3 \\
C &= a\alpha_2\sin\theta_2 + a\omega_2^2\cos\theta_2 + b\omega_3^2\cos\theta_3 - c\omega_4^2\cos\theta_4 \\
D &= c\cos\theta_4 \\
E &= b\cos\theta_3 \\
F &= a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 - b\omega_3^2\sin\theta_3 + c\omega_4^2\sin\theta_4
\end{aligned} \tag{7.12c}$$

Once we have solved for α_3 and α_4, we can then solve for the linear accelerations by substituting the Euler identity into equations 7.8b,

$$\mathbf{A}_A = a\alpha_2(-\sin\theta_2 + j\cos\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2)$$
$$\mathbf{A}_{A_x} = -a\alpha_2\sin\theta_2 - a\omega_2^2\cos\theta_2 \qquad \mathbf{A}_{A_y} = a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 \tag{7.13a}$$

$$\mathbf{A}_{BA} = b\alpha_3(-\sin\theta_3 + j\cos\theta_3) - b\omega_3^2(\cos\theta_3 + j\sin\theta_3)$$
$$\mathbf{A}_{BA_x} = -b\alpha_3\sin\theta_3 - b\omega_3^2\cos\theta_3 \qquad \mathbf{A}_{BA_y} = b\alpha_3\cos\theta_3 - b\omega_3^2\sin\theta_3 \tag{7.13b}$$

$$\mathbf{A}_B = c\alpha_4(-\sin\theta_4 + j\cos\theta_4) - c\omega_4^2(\cos\theta_4 + j\sin\theta_4)$$
$$\mathbf{A}_{B_x} = -c\alpha_4\sin\theta_4 - c\omega_4^2\cos\theta_4 \qquad \mathbf{A}_{B_y} = c\alpha_4\cos\theta_4 - c\omega_4^2\sin\theta$$

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where the real and imaginary terms are the x and y components, respectively. Equations 7.12 and 7.13 provide a complete solution for the angular accelerations of the links and the linear accelerations of the joints in the pin-jointed fourbar linkage.

EXAMPLE 7-2

Acceleration Analysis of a Fourbar Linkage with the Vector Loop Method.

Problem: Given a fourbar linkage with the link lengths $L_1 = d = 100$ mm, $L_2 = a = 40$ mm, $L_3 = b = 120$ mm, $L_4 = c = 80$ mm. For $\theta_2 = 40°$, $\omega_2 = 25$ rad/sec, and $\alpha_2 = 15$ rad/sec^2 find the values of α_3 and α_4, A_A, A_{BA}, and A_B for the open circuit of the linkage. Use the angles and angular velocities found for the same linkage and position in Example 6-7.

Solution: (See Figure 7-5 for nomenclature.)

1 Example 4-1 found the link angles for the open circuit of this linkage in this position to be $\theta_3 = 20.298°$ and $\theta_4 = 57.325°$. Example 6-7 found the angular velocities at this position to be $\omega_3 = -4.121$ and $\omega_4 = 6.998$ rad/sec.

2 Use these angles, angular velocities, and equations 7.12 to find α_3 and α_4 for the open circuit. First find the parameters in equation 7.12c.

$$
\begin{aligned}
A &= c\sin\theta_4 = 80\sin 57.325° = 67.340\\
B &= b\sin\theta_3 = 120\sin 20.298° = 41.628\\
C &= a\alpha_2\sin\theta_2 + a\omega_2^2\cos\theta_2 + b\omega_3^2\cos\theta_3 - c\omega_4^2\cos\theta_4\\
&= 40(15)\sin 40° + 40(25)^2\cos 40° + 120(-4.121)^2\cos 20.298° - 80(6.998)^2\cos 57.325°\\
&= 19332.98\\
D &= c\cos\theta_4 = 80\cos 57.325° = 43.190 \qquad (a)\\
E &= b\cos\theta_3 = 120\cos 20.298° = 112.548\\
F &= a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 - b\omega_3^2\sin\theta_3 + c\omega_4^2\sin\theta_4\\
&= 40(15)\cos 40° - 40(25)^2\sin 40° - 120(-4.121)^2\sin 20.298° + 80(6.998)^2\sin 57.325°\\
&= -13\,019.25
\end{aligned}
$$

3 Then find α_3 and α_4 with equations 7.12a and b.

$$
\alpha_3 = \frac{CD - AF}{AE - BD} = \frac{19332.98(43.190) - 67.340(-13\,019.25)}{67.340(112.548) - 41.628(43.190)} = 296.089 \text{ rad/sec}^2 \qquad (b)
$$

$$
\alpha_4 = \frac{CE - BF}{AE - BD} = \frac{19332.98(112.548) - 41.628(-13\,019.25)}{67.340(112.548) - 41.628(43.190)} = 470.134 \text{ rad/sec}^2 \qquad (c)
$$

4 Use equations 7.13 to find the linear accelerations of points A and B.

$$
\begin{aligned}
\mathbf{A}_{A_x} &= -a\alpha_2\sin\theta_2 - a\omega_2^2\cos\theta_2 = -40(15)\sin 40° - 40(25)^2\cos 40° = -19.537 \text{ m/sec}^2\\
\mathbf{A}_{A_y} &= a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 = 40(15)\cos 40° - 40(25)^2\sin 40° = -15.617 \text{ m/sec}^2
\end{aligned} \qquad (d)
$$

$$\begin{aligned} \mathbf{A}_{BA_x} &= -b\alpha_3 \sin\theta_3 - b\omega_3^2 \cos\theta_3 \\ &= -120(269.089)\sin 20.298° - 120(-4.121)^2 \cos 20.298° = -14\,237 \text{ m/sec}^2 \\ \mathbf{A}_{BA_y} &= b\alpha_3 \cos\theta_3 - b\omega_3^2 \sin\theta_3 \\ &= 120(269.089)\cos 20.298° - 120\,(-4.121)^2 \sin 20.298° = 32.617 \text{ m/sec}^2 \end{aligned} \tag{e}$$

$$\begin{aligned} \mathbf{A}_{B_x} &= -c\alpha_4 \sin\theta_4 - c\omega_4^2 \cos\theta_4 \\ &= -80(470.134)\sin 57.325° - 80(6.998)^2 \cos 57.325° = -33.774 \text{ m/sec}^2 \\ \mathbf{A}_{B_y} &= c\alpha_4 \cos\theta_4 - c\omega_4^2 \sin\theta_4 \\ &= 80(470.134)\cos 57.325° - 80(6.998)^2 \sin 57.325° = 17.007 \text{ m/sec}^2 \end{aligned} \tag{f}$$

The Fourbar Crank-Slider

The first inversion of the offset crank-slider has its slider block sliding against the ground plane as shown in Figure 7-6a. Its accelerations can be solved for in similar manner as was done for the pin-jointed fourbar.

The position equations for the fourbar offset crank-slider linkage (inversion #1) were derived in Section 4.6. The linkage was shown in Figures 4-9 and 6-21 and is shown again in Figure 7-6a on which we also show an input angular acceleration α_2 applied to link 2. This α_2 can be a time-varying input acceleration. The vector loop equations 4.14 are repeated here for your convenience.

$$\mathbf{R}_2 - \mathbf{R}_3 - \mathbf{R}_4 - \mathbf{R}_1 = 0 \tag{4.14a}$$

$$a e^{j\theta_2} - b e^{j\theta_3} - c e^{j\theta_4} - d e^{j\theta_1} = 0 \tag{4.14b}$$

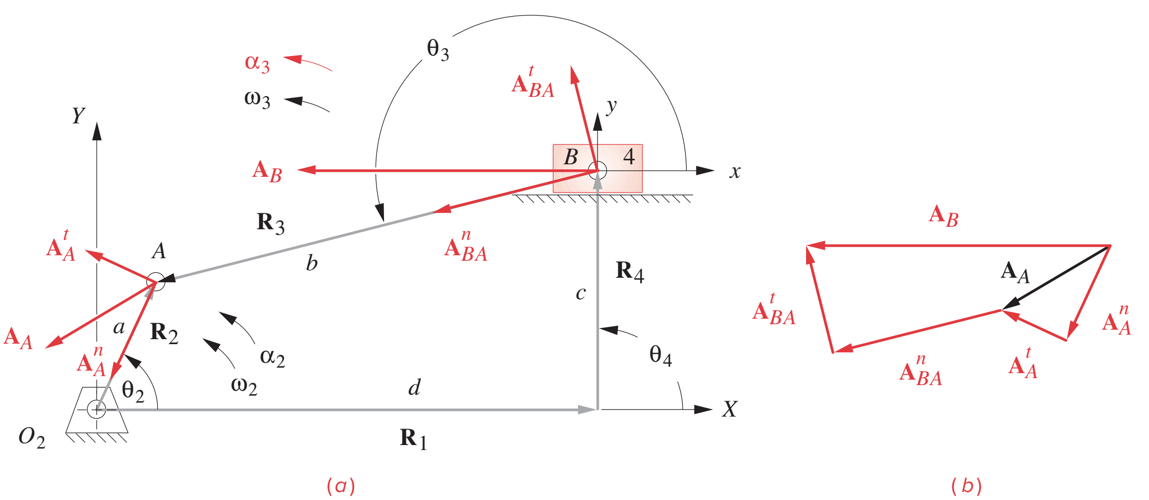

FIGURE 7-6

Position vector loop for a fourbar crank-slider linkage showing acceleration vectors

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In Section 6.7 we differentiated equation 4.14b with respect to time noting that a, b, c, θ_1, and θ_4 are constant but the length of link d varies with time in this inversion.

$$j a \omega_2 e^{j\theta_2} - j b \omega_3 e^{j\theta_3} - \dot{d} = 0 \tag{6.20a}$$

The term $\dot{d}$ is the linear velocity of the slider block. Equation 6.20a is the velocity difference equation.

We now will differentiate equation 6.20a with respect to time to get an expression for acceleration in this inversion of the crank-slider mechanism.

$$\left(j a \alpha_2 e^{j\theta_2} + j^2 a \omega_2^2 e^{j\theta_2}\right) - \left(j b \alpha_3 e^{j\theta_3} + j^2 b \omega_3^2 e^{j\theta_3}\right) - \ddot{d} = 0 \tag{7.14a}$$

Simplifying:

$$\left(a\alpha_2 j e^{j\theta_2} - a\omega_2^2 e^{j\theta_2}\right) - \left(b\alpha_3 j e^{j\theta_3} - b\omega_3^2 e^{j\theta_3}\right) - \ddot{d} = 0 \tag{7.14b}$$

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Note that equation 7.14 is again the acceleration difference equation:

$$\begin{aligned} \mathbf{A}_A - \mathbf{A}_{AB} - \mathbf{A}_B &= 0 \\ \mathbf{A}_{BA} &= -\mathbf{A}_{AB} \\ \mathbf{A}_B &= \mathbf{A}_A + \mathbf{A}_{BA} \end{aligned} \tag{7.15a}$$

$$\begin{aligned} \mathbf{A}_A &= \left(\mathbf{A}_A^t + \mathbf{A}_A^n\right) = \left(a\alpha_2 j e^{j\theta_2} - a\omega_2^2 e^{j\theta_2}\right) \\ \mathbf{A}_{BA} &= \left(\mathbf{A}_{BA}^t + \mathbf{A}_{BA}^n\right) = \left(b\alpha_3 j e^{j\theta_3} - b\omega_3^2 e^{j\theta_3}\right) \\ \mathbf{A}_B &= \mathbf{A}_B^t = \ddot{d} \end{aligned} \tag{7.15b}$$

In this mechanism, link 4 is in pure translation and so has zero ω_4 and zero α_4. The acceleration of link 4 has only a "tangential" component of acceleration along its path.

The two unknowns in the vector equation 7.14 are the angular acceleration of link 3, α_3, and the linear acceleration of link 4, $\ddot{d}$. To solve for them, substitute the Euler identity,

$$\begin{aligned} &a\alpha_2\left(-\sin\theta_2 + j\cos\theta_2\right) - a\omega_2^2\left(\cos\theta_2 + j\sin\theta_2\right) \\ &\quad - b\alpha_3\left(-\sin\theta_3 + j\cos\theta_3\right) + b\omega_3^2\left(\cos\theta_3 + j\sin\theta_3\right) - \ddot{d} = 0 \end{aligned} \tag{7.16a}$$

and separate the real (x) and imaginary (y) components:

real part (x component):

$$-a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 + b\alpha_3 \sin\theta_3 + b\omega_3^2 \cos\theta_3 - \ddot{d} = 0 \tag{7.16b}$$

imaginary part (y component):

$$a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 - b\alpha_3 \cos\theta_3 + b\omega_3^2 \sin\theta_3 = 0 \tag{7.16c}$$

Equation 7.16c can be solved directly for α_3 and the result substituted in equation 7.16b to find $\ddot{d}$.

$$\alpha_3 = \frac{a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 + b\omega_3^2 \sin\theta_3}{b\cos\theta_3} \tag{7.16d}$$

$$\ddot{d} = -a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 + b\alpha_3 \sin\theta_3 + b\omega_3^2 \cos\theta_3 \tag{7.16e}$$

The other linear accelerations can be found from equation 7.15b and are shown in the vector diagram of Figure 7-6b.

EXAMPLE 7-3

Acceleration Analysis of a Fourbar Crank-Slider Linkage with a Vector Loop Method.

Problem: Given a fourbar crank-slider linkage with the link lengths $L_2 = a = 40$ mm, $L_3 = b = 120$ mm, *offset* $= c = -20$ mm. For $\theta_2 = 60°$, $\omega_2 = -30$ rad/sec, and $\alpha_2 = 20$ rad/sec^2, find α_3 and linear acceleration of the slider for the open circuit. Use the angles, positions, and angular velocities found for the same linkage in Examples 4-2 and 6-8.

Solution: (See Figure 7-6 for nomenclature.)

1 Example 4-2 found angle $\theta_3 = 152.91°$ and slider position $d = 126.84$ mm for the open circuit. Example 6-8 found the the coupler angular velocity ω_3 to be 5.616 rad/sec.

2 Using equation 7.16d and the data from step 1, calculate the coupler angular acceleration α_3.

$$\alpha_3 = \frac{a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 + b\omega_3^2 \sin\theta_3}{b\cos\theta_3}$$

$$= \frac{40(20)\cos 60° - 40(-30)^2 \sin 60° + 120(5.616)^2 \sin 152.91°}{120\cos 152.91°} = 271.94 \text{ rad/sec}^2 \tag{a}$$

3 Using equation 7.16e and the data from steps 1 and 3, calculate the slider acceleration $\ddot{d}$.

$$\ddot{d} = -a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 + b\alpha_3 \sin\theta_3 + b\omega_3^2 \cos\theta_3$$

$$= -40(20)\sin 60° - 40(-30)^2 \cos 60° + 120(271.94)\sin 152.91° + 120(5.616)^2 \cos 152.91°$$

$$= -7.203 \text{ m/sec}^2 \tag{b}$$

The Fourbar Slider-Crank

The *fourbar* ***slider-crank*** *linkage* has the same geometry as the *fourbar* ***crank-slider*** *linkage* that was analyzed in the previous section. The name change indicates that it will be driven with the slider as input and the crank as output. This is sometimes referred to as a "back-driven" crank-slider. We will use the term **slider-crank** to define it as slider-driven. This is a very commonly used linkage configuration. Every internal-combustion, piston engine has as many of these as it has cylinders. The vector loop is as shown in Figure 7-6 and the vector loop equation is identical to that of the crank-slider (equation 4.14a). The derivation for θ_2 and ω_2 as a function of slider position d and slider velocity $\dot{d}$ were done, respectively, in Sections 4-7 and 6-7. Now we want to solve for α_2 and α_3 as a function of slider acceleration $\ddot{d}$ and the known lengths, angles, and angular velocities of the links.

We can start with equations 7.16b and c, which also apply to this linkage:

$$-a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 + b\alpha_3 \sin\theta_3 + b\omega_3^2 \cos\theta_3 - \ddot{d} = 0 \tag{7.16b}$$

$$a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 - b\alpha_3 \cos\theta_3 + b\omega_3^2 \sin\theta_3 = 0 \tag{7.16c}$$

Solve equation 7.16c for α_3 in terms of α_2.

$$\alpha_3 = \frac{a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 + b\omega_3^2 \sin\theta_3}{b\cos\theta_3} \tag{7.17a}$$

Substitute equation 7.17a for α_3 in equation 7.16b and solve for α_2.

$$\alpha_2 = \frac{a\omega_2^2\left(\cos\theta_2 \cos\theta_3 + \sin\theta_2 \sin\theta_3\right) - b\omega_3^2 + \ddot{d}\cos\theta_3}{a\left(\cos\theta_2 \sin\theta_3 - \sin\theta_2 \cos\theta_3\right)} \tag{7.17b}$$

The circuit of the linkage depends on the value of d chosen and the angular accelerations will be for the branch represented by the values of θ_2 and θ_3 used from equation 4.21.*

EXAMPLE 7-4

Acceleration Analysis of a Fourbar Slider-Crank Linkage with a Vector Loop Method.

Problem: Given a fourbar slider-crank linkage with the link lengths $L_2 = a = 40$ mm, $L_3 = b = 120$ mm, *offset* $= c = -20$ mm. For $d = 100$ mm and $\ddot{d} = 900$ mm/sec^2, find α_2 and α_3 for both branches of one circuit of the linkage. Use the angles and angular velocities found for the same linkage in Example 4-3 and Example 6-9, respectively.

Solution: (See Figure 7-6 for nomenclature.)

1 Example 4-3 found angles $\theta_{2_1} = 95.80°$, $\theta_{3_1} = 150.11°$ for branch 1 of this linkage. Example 6-9 found the the angular velocities to be $\omega_{2_1} = -32.023$ and $\omega_{3_1} = -1.244$ rad/sec for branch 1.

2 Using equation 7.17b and the data from step 1, calculate the crank angular acceleration α_{2_1}.

$$\alpha_{2_1} = \frac{a\omega_{2_1}^2\left(\cos\theta_{2_1} \cos\theta_{3_1} + \sin\theta_{2_1} \sin\theta_{3_1}\right) - b\omega_{3_1}^2 + \ddot{d}\cos\theta_{3_1}}{a\left(\cos\theta_{2_1} \sin\theta_{3_1} - \sin\theta_{2_1} \cos\theta_{3_1}\right)}$$

$$\alpha_{2_1} = \frac{40(-32.023)^2\left(\cos 95.80° \cos 150.11° + \sin 95.80° \sin 150.11°\right) - 120(-1.244)^2 + 900\cos 150.11°}{40\left(\cos 95.80° \sin 150.11° - \sin 95.80° \cos 150.11°\right)}$$

$$\alpha_{2_1} = 706.753 \text{ rad/sec}^2 \tag{a}$$

3 Using equation 7.17a and the data from steps 1 and 2, calculate the coupler angular acceleration α_{3_1}.

* The crank-slider and slider-crank linkage both have two circuits or configurations in which they can be independently assembled, sometimes called open and crossed. Because effective link 4 is always perpendicular to the slider axis, it is parallel to itself on both circuits. This results in the two circuits being mirror images of one another, mirrored about a line through the crank pivot and perpendicular to the slide axis. Thus, the choice of value of slider position *d* in the calculation of the slider-crank linkage determines which circuit is being analyzed. But, because of the change points at TDC and BDC, the slider-crank has two branches on each circuit and the two solutions obtained from equation 4.21 represent the two branches on the one circuit being analyzed. In contrast, the crank-slider has only one branch per circuit because when the crank is driven, it can make a full revolution and there are no change points to separate branches. See Section 4.13 for a more complete discussion of circuits and branches in linkages.

$$\alpha_{3_1} = \frac{a\alpha_{2_1}\cos\theta_{2_1} - a\omega_{2_1}^2\sin\theta_2 + b\omega_{3_1}^2\sin\theta_{3_1}}{b\cos\theta_{3_1}}$$

$$\alpha_{3_1} = \frac{40(-706.753)\cos 95.80° - 40(-32.023)^2\sin 95.80° + 120(-1.244)^2\sin 150.11°}{120\cos 150.11°}$$

$$\alpha_{3_1} = 418.804 \text{ rad/sec}^2 \qquad (b)$$

4 Example 4-3 found angles $\theta_{2_2} = -118.42°$, $\theta_{3_2} = 187.27°$ for branch 2 of this linkage. Example 6-9 found the the angular velocities to be $\omega_{2_2} = 36.64$ and $\omega_{3_2} = 5.86$ rad/sec for branch 2. Using equation 7.17b and the data from step 3, calculate the crank angular acceleration α_{2_2} for branch 2.

$$\alpha_{2_2} = \frac{a\omega_{2_2}^2\left(\cos\theta_{2_2}\cos\theta_{3_2} + \sin\theta_{2_2}\sin\theta_{3_2}\right) - b\omega_{3_2}^2 + \ddot{d}\cos\theta_{3_2}}{a\left(\cos\theta_{2_2}\sin\theta_{3_2} - \sin\theta_{2_2}\cos\theta_{3_2}\right)}$$

$$\alpha_{2_2} = \frac{40(36.64)^2(\cos -118.42°\cos 187.27° + \sin -118.42°\sin 187.27°) - 120(5.86)^2 + 900\cos 187.27°}{40\left[\cos(-118.42°)\sin 187.27° - \sin(-118.42°)\cos 187.27°\right]}$$

$$\alpha_{2_2} = -809.801 \text{ rad/sec}^2 \qquad (c)$$

5 Using equation 7.17a and the data from steps 3 and 4, calculate the coupler angular acceleration α_{3_2}.

$$\alpha_{3_2} = \frac{a\alpha_{2_2}\cos\theta_{2_2} - a\omega_{2_2}^2\sin\theta_2 + b\omega_{3_2}^2\sin\theta_{3_2}}{b\cos\theta_{3_2}}$$

$$\alpha_{3_2} = \frac{40(-809.801)\cos -118.42° - 40(36.64)^2\sin -118.42° + 120(5.859)^2\sin 187.27°}{120\cos 187.27°}$$

$$\alpha_{3_2} = -521.852 \text{ rad/sec}^2 \qquad (d)$$

Coriolis Acceleration

The examples used for acceleration analysis above have involved only pin-jointed linkages or the inversion of the crank-slider in which the slider block has no rotation. When a sliding joint is present on a rotating link, an additional component of acceleration will be present, called the **Coriolis component**, after its discoverer. Figure 7-7a shows a simple, two-link system consisting of a link with a radial slot and a slider block free to slip within that slot.

The instantaneous location of the block is defined by a position vector ($\mathbf{R}_P$) referenced to the global origin at the link center. *This vector is both rotating and changing length as the system moves.* As shown this is a two-degree-of-freedom system. The **two inputs to the system** are the angular acceleration (α) of the link and the relative linear slip velocity ($\mathbf{V}_{Pslip}$) of the block versus the disk. The angular velocity ω is a result of the time history of the angular acceleration. The situation shown, with a counterclockwise α and a clockwise ω, implies that earlier in time the link had been accelerated up to a clockwise angular velocity and is now being slowed down. The transmission component of velocity ($\mathbf{V}_{Ptrans}$) is a result of the ω of the link acting at the radius $\mathbf{R}_P$ whose magnitude is p.

We show the situation in Figure 7-7 at one instant of time. However, the equations to be derived will be valid for all time. We want to determine the acceleration at the center of the block (P) under this combined motion of rotation and sliding. To do so, we first write the expression for the position vector $\mathbf{R}_P$ that locates point P.

$$\mathbf{R}_P = pe^{j\theta_2} \tag{7.18a}$$

Note that there are two functions of time in equation 7.17, p and θ. When we differentiate versus time, we get two terms in the velocity expression:

$$\mathbf{V}_P = p\omega_2 je^{j\theta_2} + \dot{p}e^{j\theta_2} \tag{7.18b}$$

These are the transmission component and the slip component of velocity.

$$\mathbf{V}_P = \mathbf{V}_{P_{trans}} + \mathbf{V}_{P_{slip}} \tag{7.18c}$$

The $p\omega$ term is the transmission component and is directed at 90 degrees to the axis of slip that, in this example, is coincident with the position vector $\mathbf{R}_P$. The $\dot{p}$ term is the **slip component** and is directed along the **axis of slip** in the same direction as the position vector in this example. Their vector sum is $\mathbf{V}_P$ as shown in Figure 7-7a.

To get an expression for acceleration, we must differentiate equation 7.18 versus time. Note that the transmission component has **three** functions of time in it, p, ω, and θ. The chain rule will yield three terms for this one term. The slip component of velocity contains two functions of time, p and θ, yielding two terms in the derivative for a total of five terms, two of which turn out to be the same.

$$\mathbf{A}_P = \left(p\alpha_2 je^{j\theta_2} + p\omega_2^2 j^2 e^{j\theta_2} + \dot{p}\omega_2 je^{j\theta_2}\right) + \left(\dot{p}\omega_2 je^{j\theta_2} + \ddot{p}e^{j\theta_2}\right) \tag{7.19a}$$

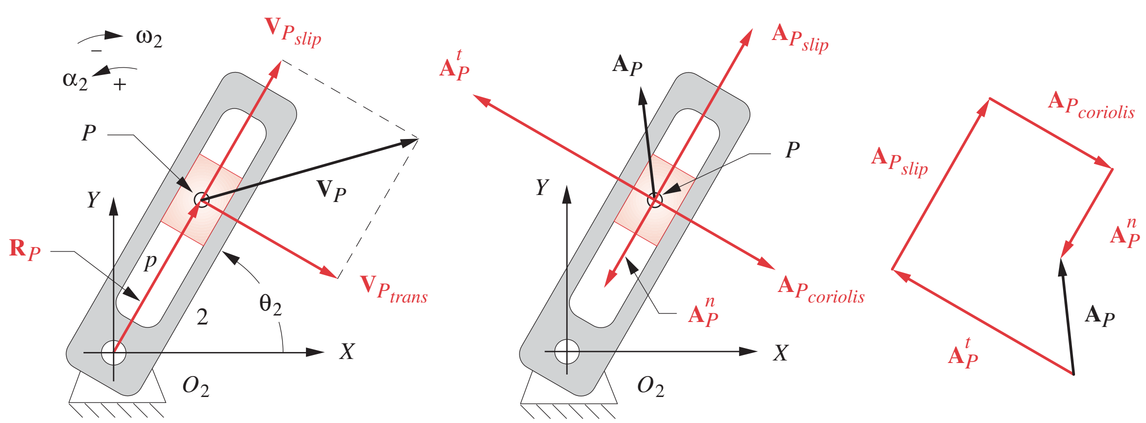

FIGURE 7-7

The Coriolis component of acceleration shown in a system with a positive (CCW) α and a negative (CW) ω

Simplifying and collecting terms:

$$\mathbf{A}_P = p\alpha_2 j e^{j\theta_2} - p\omega_2^2 e^{j\theta_2} + 2\dot{p}\omega_2 j e^{j\theta_2} + \ddot{p} e^{j\theta_2} \tag{7.19b}$$

These terms represent the following components:

$$\mathbf{A}_P = \mathbf{A}_{P_{\text{tangential}}} + \mathbf{A}_{P_{\text{normal}}} + \mathbf{A}_{P_{\text{coriolis}}} + \mathbf{A}_{P_{\text{slip}}} \tag{7.19c}$$

Note that the Coriolis term has appeared in the acceleration expression as a result of the differentiation simply because the length of the vector p is a function of time. The Coriolis component magnitude is twice the product of the velocity of slip (equation 7.18) and the angular velocity of the link containing the slider slot. Its direction is rotated 90 degrees from that of the original position vector $\mathbf{R}_P$ either clockwise or counterclockwise depending on the sense of ω.* (Note that we chose to align the position vector $\mathbf{R}_P$ with the axis of slip in Figure 7-7 which can always be done regardless of the location of the center of rotation—also see Figure 7-6 where $\mathbf{R}_1$ is aligned with the axis of slip.) All four components from equations 7.19 are shown acting on point P in Figure 7-7b. The total acceleration $\mathbf{A}_P$ is the vector sum of the four terms as shown in Figure 7-7c. Note that the normal acceleration term in equation 7.19b is negative in sign, so it becomes a subtraction when substituted in equation 7.19c.

This Coriolis component of acceleration will always be present when there is a velocity of slip associated with any member that also has an angular velocity. In the absence of either of those two factors the Coriolis component will be zero. You have probably experienced Coriolis acceleration if you have ever ridden on a carousel or merry-go-round. If you attempted to walk radially from the outside to the inside (or vice versa) while the carousel was turning, you were thrown sideways by the inertial force due to the Coriolis acceleration. You were the *slider block* in Figure 7-7, and your *slip velocity* combined with the rotation of the carousel created the Coriolis component. As you walked from a large radius to a smaller one, your tangential velocity had to change to match that of the new location of your foot on the spinning carousel. Any change in velocity requires an acceleration to accomplish. It was the *"ghost of Coriolis"* that pushed you sideways on that carousel.

Another example of the Coriolis component is its effect on weather systems. Large objects that exist in the earth's lower atmosphere, such as hurricanes, span enough area to be subject to significantly different velocities at their northern and southern extremities. The atmosphere turns with the earth. The earth's surface tangential velocity due to its angular velocity varies from zero at the poles to a maximum of about 1000 mph at the equator. The winds of a storm system are attracted toward the low pressure at its center. These winds have a slip velocity with respect to the surface, which in combination with the earth's ω creates a Coriolis component of acceleration on the moving air masses. This Coriolis acceleration causes the inrushing air to rotate about the center, or "eye" of the storm system. This rotation will be counterclockwise in the northern hemisphere and clockwise in the southern hemisphere. The movement of the entire storm system from south to north also creates a Coriolis component that will tend to deviate the storm's track eastward, though this effect is often overridden by the forces due to other large air masses such as high-pressure systems that can deflect a storm. These complicated factors make it difficult to predict a large storm's true track.

* This approach works in the 2-D case. Coriolis acceleration is the cross product of 2ω and the velocity of slip. The cross product operation will define its magnitude, direction, and sense in the 3-D case.

Note that in the analytical solution presented here, the Coriolis component will be accounted for automatically as long as the differentiations are correctly done. However, when doing a graphical acceleration analysis, one must be on the alert to recognize the presence of this component, calculate it, and include it in the vector diagrams when its two constituents $\mathbf{V}_{slip}$ and ω are both nonzero.

The Fourbar Inverted Crank-Slider

The position equations for the fourbar inverted crank-slider linkage were derived in Section 4.7. The linkage was shown in Figures 4-10 and 6-22 and is shown again in Figure 7-8a on which we also show an input angular acceleration α_2 applied to link 2. This α_2 can vary with time. The vector loop equations 4.14 are valid for this linkage as well.

All slider linkages will have at least one link whose effective length between joints varies as the linkage moves. In this inversion the length of link 3 between points A and B, designated as b, will change as it passes through the slider block on link 4. In Section 6.7 we got an expression for velocity by differentiating equation 4.14b with respect to time, noting that a, c, d, and θ_1 are constant and b, θ_3, and θ_4 vary with time.

$$ja\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{b}e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0 \tag{6.25a}$$

Differentiating this with respect to time will give an expression for accelerations in this inversion of the crank-slider mechanism.

$$\left(ja\alpha_2 e^{j\theta_2} + j^2 a\omega_2^2 e^{j\theta_2}\right) - \left(jb\alpha_3 e^{j\theta_3} + j^2 b\omega_3^2 e^{j\theta_3} + j\dot{b}\omega_3 e^{j\theta_3}\right)$$
$$-\left(\ddot{b}e^{j\theta_3} + j\dot{b}\omega_3 e^{j\theta_3}\right) - \left(jc\alpha_4 e^{j\theta_4} + j^2 c\omega_4^2 e^{j\theta_4}\right) = 0 \tag{7.20a}$$

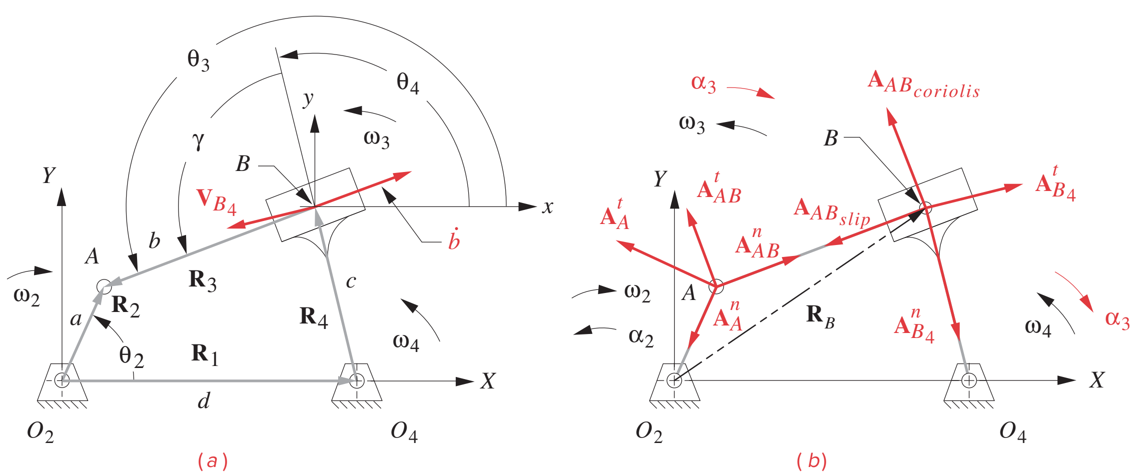

FIGURE 7-8

Acceleration analysis of a fourbar crank-slider-inversion #3 driven with positive (CCW) α_2 and negative (CW) ω_2

Simplifying and collecting terms:

$$\left(a\alpha_2\, je^{j\theta_2} - a\omega_2^2 e^{j\theta_2}\right) - \left(b\alpha_3\, je^{j\theta_3} - b\omega_3^2 e^{j\theta_3} + 2\dot{b}\omega_3\, je^{j\theta_3} + \ddot{b}e^{j\theta_3}\right)$$
$$-\left(c\alpha_4\, je^{j\theta_4} - c\omega_4^2 e^{j\theta_4}\right) = 0 \qquad (7.20b)$$

Equation 7.20 is in fact the acceleration difference equation (equation 7.4) and can be written in that notation as shown in equations 7.21.

$$\mathbf{A}_A - \mathbf{A}_{AB} - \mathbf{A}_B = 0$$

but: $$\mathbf{A}_{BA} = -\mathbf{A}_{AB} \qquad (7.21a)$$

and: $$\mathbf{A}_B = \mathbf{A}_A + \mathbf{A}_{BA}$$

$$\mathbf{A}_A = \mathbf{A}_{A_{tangential}} + \mathbf{A}_{A_{normal}}$$
$$\mathbf{A}_{AB} = \mathbf{A}_{AB_{tangential}} + \mathbf{A}_{AB_{normal}} + \mathbf{A}_{AB_{coriolis}} + \mathbf{A}_{AB_{slip}} \qquad (7.21b)$$
$$\mathbf{A}_B = \mathbf{A}_{B_{tangential}} + \mathbf{A}_{B_{normal}}$$

$$\mathbf{A}_{A_{tangential}} = a\alpha_2\, je^{j\theta_2} \qquad \mathbf{A}_{A_{normal}} = -a\omega_2^2 e^{j\theta_2}$$
$$\mathbf{A}_{B_{tangential}} = c\alpha_4\, je^{j\theta_4} \qquad \mathbf{A}_{B_{normal}} = -c\omega_4^2 e^{j\theta_4}$$
$$\mathbf{A}_{AB_{tangential}} = b\alpha_3\, je^{j\theta_3} \qquad \mathbf{A}_{AB_{normal}} = -b\omega_3^2 e^{j\theta_3} \qquad (7.21c)$$
$$\mathbf{A}_{AB_{coriolis}} = 2\dot{b}\omega_3\, je^{j\theta_3} \qquad \mathbf{A}_{AB_{slip}} = \ddot{b}e^{j\theta_3}$$

Because this sliding link also has an angular velocity, there will be a nonzero Coriolis component of acceleration at point B which is the $2\dot{b}$ term in equation 7.20. Since a complete velocity analysis was done before doing this acceleration analysis, the Coriolis component can be readily calculated at this point, knowing both ω and $\mathbf{V}_{slip}$ from the velocity analysis.

The $\ddot{b}$ term in equations 7.20b and 7.21c is the *slip component of acceleration*. This is one of the variables to be solved for in this acceleration analysis. Another variable to be solved for is α_4, the angular acceleration of link 4. Note, however, that we also have an unknown in α_3, the angular acceleration of link 3. This is a total of three unknowns. Equation 7.20 can only be solved for two unknowns. Thus we require another equation to solve the system. There is a fixed relationship between angles θ_3 and θ_4, shown as γ in Figure 7-8 and defined in equation 4.22, repeated here:

$$\text{open configuration: } \theta_3 = \theta_4 + \gamma; \qquad \text{crossed configuration: } \theta_3 = \theta_4 + \gamma - \pi \qquad (4.22)$$

Differentiate it twice with respect to time to obtain:

$$\omega_3 = \omega_4; \qquad \alpha_3 = \alpha_4 \qquad (7.22)$$

We wish to solve equation 7.20 to get expressions in this form:

$$\alpha_3 = \alpha_4 = f\left(a, b, \dot{b}, c, d, \theta_2, \theta_3, \theta_4, \omega_2, \omega_3, \omega_4, \alpha_2\right) \qquad (7.23a)$$

$$\frac{d^2 b}{dt^2} = \ddot{b} = g\left(a, b, \dot{b}, c, d, \theta_2, \theta_3, \theta_4, \omega_2, \omega_3, \omega_4, \alpha_2\right) \qquad (7.23b)$$

Substitution of the Euler identity (equation 4.4a) into equation 7.20 yields:

$$\begin{aligned} &a\alpha_2\, j(\cos\theta_2 + j\sin\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2) \\ &\quad - b\alpha_3\, j(\cos\theta_3 + j\sin\theta_3) + b\omega_3^2(\cos\theta_3 + j\sin\theta_3) \\ &\qquad - 2\dot{b}\omega_3\, j(\cos\theta_3 + j\sin\theta_3) - \ddot{b}(\cos\theta_3 + j\sin\theta_3) \\ &\qquad\quad - c\alpha_4\, j(\cos\theta_4 + j\sin\theta_4) + c\omega_4^2(\cos\theta_4 + j\sin\theta_4) = 0 \end{aligned} \tag{7.24a}$$

Multiply by the operator j and substitute α_4 for α_3 from equation 7.22:

$$\begin{aligned} &a\alpha_2(-\sin\theta_2 + j\cos\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2) \\ &\quad - b\alpha_4(-\sin\theta_3 + j\cos\theta_3) + b\omega_3^2(\cos\theta_3 + j\sin\theta_3) \\ &\qquad - 2\dot{b}\omega_3(-\sin\theta_3 + j\cos\theta_3) - \ddot{b}(\cos\theta_3 + j\sin\theta_3) \\ &\qquad\quad - c\alpha_4(-\sin\theta_4 + j\cos\theta_4) + c\omega_4^2(\cos\theta_4 + j\sin\theta_4) = 0 \end{aligned} \tag{7.24b}$$

We can now separate this vector equation 7.24b into its two components by collecting all real and all imaginary terms separately:

real part (x component):

$$\begin{aligned} &-a\alpha_2\sin\theta_2 - a\omega_2^2\cos\theta_2 + b\alpha_4\sin\theta_3 + b\omega_3^2\cos\theta_3 \\ &\quad + 2\dot{b}\omega_3\sin\theta_3 - \ddot{b}\cos\theta_3 + c\alpha_4\sin\theta_4 + c\omega_4^2\cos\theta_4 = 0 \end{aligned} \tag{7.25a}$$

imaginary part (y component):

$$\begin{aligned} &a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 - b\alpha_4\cos\theta_3 + b\omega_3^2\sin\theta_3 \\ &\quad - 2\dot{b}\omega_3\cos\theta_3 - \ddot{b}\sin\theta_3 - c\alpha_4\cos\theta_4 + c\omega_4^2\sin\theta_4 = 0 \end{aligned} \tag{7.25b}$$

Note that the j's have canceled in equation 7.25b. We can solve equations 7.25 simultaneously for the two unknowns, α_4 and $\ddot{b}$. The solution is:

$$\alpha_4 = \frac{a\left[\alpha_2\cos(\theta_3 - \theta_2) + \omega_2^2\sin(\theta_3 - \theta_2)\right] + c\omega_4^2\sin(\theta_4 - \theta_3) - 2\dot{b}\omega_3}{b + c\cos(\theta_3 - \theta_4)} \tag{7.26a}$$

$$\ddot{b} = -\frac{\left\{\begin{aligned} &a\omega_2^2\left[b\cos(\theta_3 - \theta_2) + c\cos(\theta_4 - \theta_2)\right] + a\alpha_2\left[b\sin(\theta_2 - \theta_3) - c\sin(\theta_4 - \theta_2)\right] \\ &\quad + 2\dot{b}c\omega_4\sin(\theta_4 - \theta_3) - \omega_4^2\left[b^2 + c^2 + 2bc\cos(\theta_4 - \theta_3)\right] \end{aligned}\right\}}{b + c\cos(\theta_3 - \theta_4)} \tag{7.26b}$$

Equation 7.26a provides the **angular acceleration** of link 4. Equation 7.26b provides the **acceleration of slip** at point B. Once these variables are solved for, the linear accelerations at points A and B in the linkage of Figure 7-8 can be found by substituting the Euler identity into equations 7.21.

$$\mathbf{A}_A = a\alpha_2\left(-\sin\theta_2 + j\cos\theta_2\right) - a\omega_2^2\left(\cos\theta_2 + j\sin\theta_2\right) \tag{7.27a}$$

$$\mathbf{A}_{BA} = b\alpha_3\left(\sin\theta_3 - j\cos\theta_3\right) + b\omega_3^2\left(\cos\theta_3 + j\sin\theta_3\right)$$
$$+ 2\dot{b}\omega_3\left(\sin\theta_3 - j\cos\theta_3\right) - \ddot{b}\left(\cos\theta_3 + j\sin\theta_3\right) \tag{7.27b}$$

$$\mathbf{A}_B = -c\alpha_4\left(\sin\theta_4 - j\cos\theta_4\right) - c\omega_4^2\left(\cos\theta_4 + j\sin\theta_4\right) \tag{7.27c}$$

These components of these vectors are shown in Figure 7-8b.

7.4 ACCELERATION ANALYSIS OF THE GEARED FIVEBAR LINKAGE

The velocity equation for the geared fivebar mechanism was derived in Section 6.8 and is repeated here. See Figure P7-4 for notation.

$$a\omega_2 je^{j\theta_2} + b\omega_3 je^{j\theta_3} - c\omega_4 je^{j\theta_4} - d\omega_5 je^{j\theta_5} = 0 \tag{6.32a}$$

Differentiate this with respect to time to get an expression for acceleration.

$$\left(a\alpha_2 je^{j\theta_2} - a\omega_2^2 e^{j\theta_2}\right) + \left(b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3}\right)$$
$$-\left(c\alpha_4 je^{j\theta_4} - c\omega_4^2 e^{j\theta_4}\right) - \left(d\alpha_5 je^{j\theta_5} - d\omega_5^2 e^{j\theta_5}\right) = 0 \tag{7.28a}$$

Substitute the Euler equivalents:

$$a\alpha_2\left(-\sin\theta_2 + j\cos\theta_2\right) - a\omega_2^2\left(\cos\theta_2 + j\sin\theta_2\right)$$
$$+ b\alpha_3\left(-\sin\theta_3 + j\cos\theta_3\right) - b\omega_3^2\left(\cos\theta_3 + j\sin\theta_3\right)$$
$$- c\alpha_4\left(-\sin\theta_4 + j\cos\theta_4\right) + c\omega_4^2\left(\cos\theta_4 + j\sin\theta_4\right)$$
$$- d\alpha_5\left(-\sin\theta_5 + j\cos\theta_5\right) + d\omega_5^2\left(\cos\theta_5 + j\sin\theta_5\right) = 0 \tag{7.28b}$$

Note that the angle θ_5 is defined in terms of θ_2, the gear ratio λ, and the phase angle ϕ. This relationship and its derivatives are:

$$\theta_5 = \lambda\theta_2 + \phi; \qquad \omega_5 = \lambda\omega_2; \qquad \alpha_5 = \lambda\alpha_2 \tag{7.28c}$$

Since a complete position and velocity analysis must be done before an acceleration analysis, we will assume that the values of θ_5 and ω_5 have been found and will leave these equations in terms of θ_5, ω_5, and α_5.

Separating the real and imaginary terms in equation 7.28b:

real:

$$-a\alpha_2\sin\theta_2 - a\omega_2^2\cos\theta_2 - b\alpha_3\sin\theta_3 - b\omega_3^2\cos\theta_3$$
$$+ c\alpha_4\sin\theta_4 + c\omega_4^2\cos\theta_4 + d\alpha_5\sin\theta_5 + d\omega_5^2\cos\theta_5 = 0 \tag{7.28d}$$

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imaginary:

$$a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 + b\alpha_3\cos\theta_3 - b\omega_3^2\sin\theta_3$$
$$- c\alpha_4\cos\theta_4 + c\omega_4^2\sin\theta_4 - d\alpha_5\cos\theta_5 + d\omega_5^2\sin\theta_5 = 0 \qquad (7.28e)$$

The only two unknowns are α_3 and α_4. Either equation 7.28d or 7.28e can be solved for one unknown and the result substituted in the other. The solution for α_3 is:

$$\alpha_3 = \frac{\begin{bmatrix} -a\alpha_2\sin(\theta_2-\theta_4) - a\omega_2^2\cos(\theta_2-\theta_4) \\ - b\omega_3^2\cos(\theta_3-\theta_4) + d\omega_5^2\cos(\theta_5-\theta_4) \\ + d\alpha_5\sin(\theta_5-\theta_4) + c\omega_4^2 \end{bmatrix}}{b\sin(\theta_3-\theta_4)} \qquad (7.29a)$$

and angular acceleration α_4 is:

$$\alpha_4 = \frac{\begin{bmatrix} a\alpha_2\sin(\theta_2-\theta_3) + a\omega_2^2\cos(\theta_2-\theta_3) \\ - c\omega_4^2\cos(\theta_3-\theta_4) - d\omega_5^2\cos(\theta_3-\theta_5) \\ + d\alpha_5\sin(\theta_3-\theta_5) + b\omega_3^2 \end{bmatrix}}{c\sin(\theta_4-\theta_3)} \qquad (7.29b)$$

With all link angles, angular velocities, and angular accelerations known, the linear accelerations for the pin joints can be found from:

$$\mathbf{A}_A = a\alpha_2(-\sin\theta_2 + j\cos\theta_2) - a\omega_2^2(\cos\theta_2 + j\sin\theta_2) \qquad (7.29c)$$
$$\mathbf{A}_{BA} = b\alpha_3(-\sin\theta_3 + j\cos\theta_3) - b\omega_3^2(\cos\theta_3 + j\sin\theta_3) \qquad (7.29d)$$
$$\mathbf{A}_C = c\alpha_5(-\sin\theta_5 + j\cos\theta_5) - c\omega_5^2(\cos\theta_5 + j\sin\theta_5) \qquad (7.29e)$$
$$\mathbf{A}_B = \mathbf{A}_A + \mathbf{A}_{BA} \qquad (7.29f)$$

7.5 ACCELERATION OF ANY POINT ON A LINKAGE

Once the angular accelerations of all the links are found, it is easy to define and calculate the acceleration of *any point on any link* for any input position of the linkage. Figure 7-9 shows the fourbar linkage with its coupler, link 3, enlarged to contain a coupler point *P*. The crank and rocker have also been enlarged to show points *S* and *U* which might represent the centers of gravity of those links. We want to develop algebraic expressions for the accelerations of these (or any) points on the links.

To find the acceleration of point *S*, draw the position vector from the fixed pivot O_2 to point *S*. This vector $\mathbf{R}_{SO_2}$ makes an angle δ_2 with the vector $\mathbf{R}_{AO_2}$. This angle δ_2 is completely defined by the geometry of link 2 and is constant. The position vector for point *S* is then:

$$\mathbf{R}_{SO_2} = \mathbf{R}_S = se^{j(\theta_2+\delta_2)} = s\left[\cos(\theta_2+\delta_2) + j\sin(\theta_2+\delta_2)\right] \qquad (4.29)$$

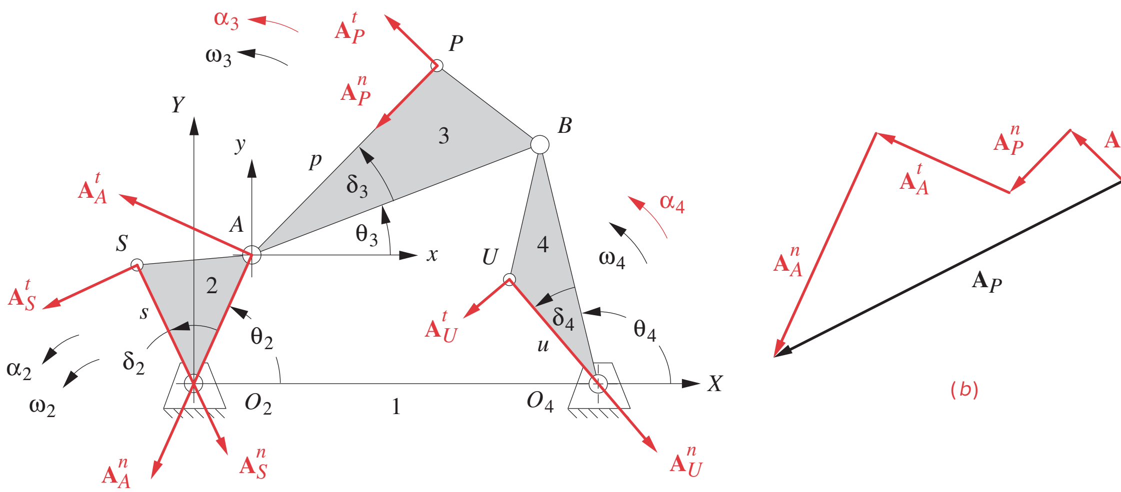

FIGURE 7-9

Finding the acceleration of any point on any link

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We differentiated this position vector in Section 6.9 to find the velocity of that point. The equation is repeated here for your convenience.

$$\mathbf{V}_S = jse^{j(\theta_2+\delta_2)}\omega_2 = s\omega_2\left[-\sin(\theta_2+\delta_2)+j\cos(\theta_2+\delta_2)\right] \tag{6.34}$$

We can differentiate again versus time to find the acceleration of point S.

$$\begin{aligned}\mathbf{A}_S &= s\alpha_2\, je^{j(\theta_2+\delta_2)} - s\omega_2^2\, e^{j(\theta_2+\delta_2)}\\ &= s\alpha_2\left[-\sin(\theta_2+\delta_2)+j\cos(\theta_2+\delta_2)\right]\\ &\quad - s\omega_2^2\left[\cos(\theta_2+\delta_2)+j\sin(\theta_2+\delta_2)\right]\end{aligned} \tag{7.30}$$

The position of point U on link 4 is found in the same way, using the angle δ_4 which is a constant angular offset within the link. The expression is:

$$\mathbf{R}_{UO_4} = ue^{j(\theta_4+\delta_4)} = u\left[\cos(\theta_4+\delta_4)+j\sin(\theta_4+\delta_4)\right] \tag{4.30}$$

We differentiated this position vector in Section 6.9 to find the velocity of that point. The equation is repeated here for your convenience.

$$\mathbf{V}_U = jue^{j(\theta_4+\delta_4)}\omega_4 = u\omega_4\left[-\sin(\theta_4+\delta_4)+j\cos(\theta_4+\delta_4)\right] \tag{6.35}$$

We can differentiate again versus time to find the acceleration of point U.

$$\begin{aligned}\mathbf{A}_U &= u\alpha_4\, je^{j(\theta_4+\delta_4)} - u\omega_4^2\, e^{j(\theta_4+\delta_4)}\\ &= u\alpha_4\left[-\sin(\theta_4+\delta_4)+j\cos(\theta_4+\delta_4)\right]\\ &\quad - u\omega_4^2\left[\cos(\theta_4+\delta_4)+j\sin(\theta_4+\delta_4)\right]\end{aligned} \tag{7.31}$$

The acceleration of point P on link 3 can be found from the addition of two acceleration vectors, such as $\mathbf{A}_A$ and $\mathbf{A}_{PA}$. Vector $\mathbf{A}_A$ is already defined from our analysis of the link accelerations. $\mathbf{A}_{PA}$ is the acceleration difference of point P with respect to point A. Point A is chosen as the reference point because angle θ_3 is defined at a local coordinate system whose origin is at A. Position vector $\mathbf{R}_{PA}$ is defined in the same way as $\mathbf{R}_U$ or $\mathbf{R}_S$, using the internal link offset angle δ_3 and the angle of link 3, θ_3. We previously analyzed this position vector and differentiated it in Section 6.9 to find the velocity difference of that point with respect to point A. Those equations are repeated here for your convenience.

$$\mathbf{R}_{PA} = pe^{j(\theta_3+\delta_3)} = p\left[\cos(\theta_3+\delta_3) + j\sin(\theta_3+\delta_3)\right] \tag{4.31a}$$

$$\mathbf{R}_P = \mathbf{R}_A + \mathbf{R}_{PA} \tag{4.31b}$$

$$\mathbf{V}_{PA} = jpe^{j(\theta_3+\delta_3)}\omega_3 = p\omega_3\left[-\sin(\theta_3+\delta_3) + j\cos(\theta_3+\delta_3)\right] \tag{6.36a}$$

$$\mathbf{V}_P = \mathbf{V}_A + \mathbf{V}_{PA} \tag{6.36b}$$

We can differentiate equation 6.36 again versus time to find $\mathbf{A}_{PA}$, the acceleration of point P versus A. This vector can then be added to the vector $\mathbf{A}_A$ already found to define the absolute acceleration $\mathbf{A}_P$ of point P.

$$\mathbf{A}_P = \mathbf{A}_A + \mathbf{A}_{PA} \tag{7.32a}$$

where:

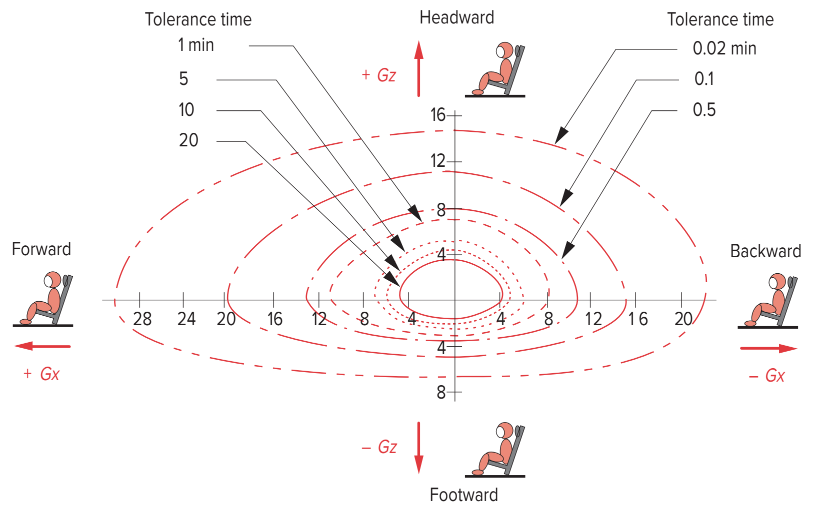
$$\begin{aligned}\mathbf{A}_{PA} &= p\alpha_3\, je^{j(\theta_3+\delta_3)} - p\omega_3^2\, e^{j(\theta_3+\delta_3)}\\ &= p\alpha_3\left[-\sin(\theta_3+\delta_3) + j\cos(\theta_3+\delta_3)\right]\\ &\quad - p\omega_3^2\left[\cos(\theta_3+\delta_3) + j\sin(\theta_3+\delta_3)\right]\end{aligned} \tag{7.32b}$$

Compare equation 7.32 with equation 7.4. It is again the acceleration difference equation. Note that this equation applies to **any point** on **any link** at any position for which the positions and velocities are defined. It is a general solution for any rigid body.*

* The video *Fourbar Linkage Virtual Laboratory* shows the measured acceleration of the coupler point on an actual linkage mechanism and also discusses the reasons for differences between the measured values and those calculated with equation 7.32. The measured data are also provided.

7.6 HUMAN TOLERANCE OF ACCELERATION

It is interesting to note that the human body does not sense velocity, except with the eyes, but is very sensitive to acceleration. Riding in an automobile, in the daylight, one can see the scenery passing by and have a sense of motion. But, traveling at night in a commercial airliner at a 500 mph constant velocity, we have no sensation of motion as long as the flight is smooth. What we will sense in this situation is any change in velocity due to atmospheric turbulence, takeoffs, or landings. The semicircular canals in the inner ear are sensitive accelerometers that report to us on any accelerations that we experience. You have no doubt also experienced the sensation of acceleration when riding in an elevator and starting, stopping, or turning in an automobile. Accelerations produce dynamic forces on physical systems, as expressed in Newton's second law, **F**=m**a**. Force is proportional to acceleration, for a constant mass. The dynamic forces produced within the human body in response to acceleration can be harmful if excessive. The human body is, after all, not rigid. It is a loosely coupled bag of water and tissue, most of which is quite internally mobile. Accelerations in the headward or footward directions will tend to either starve or flood the brain with blood as this liquid responds to Newton's law and effectively moves



Average levels of linear acceleration, in different directions that can be tolerated on a voluntary basis for specified periods. Each curve shows the average *G* load that can be tolerated for the time indicated. The data points obtained were actually those on the axes; the lines as such are extrapolated from the data points to form the concentric figures.

(*Source: Adapted from reference* [1], *Fig. 17-17, p. 505, reprinted with permission*)

FIGURE 7-10

Human tolerance of acceleration

within the body in a direction opposite to the imposed acceleration as it lags the motion of the skeleton. Lack of blood supply to the brain causes blackout; excess blood supply causes redout. Either results in death if sustained for a long enough period.

A great deal of research has been done, largely by the military and NASA, to determine the limits of human tolerance to sustained accelerations in various directions. Figure 7-10 shows data developed from such tests.[1] The units of linear acceleration were defined in Table 1-4 as in/sec^2, ft/sec^2, or m/sec^2. Another common unit for acceleration is the g, defined as the acceleration due to gravity, which on earth at sea level is approximately 386 in/sec^2, 32.2 ft/sec^2, or 9.8 m/sec^2. The g is a very convenient unit to use for accelerations involving the human as we live in a 1g environment. Our weight, felt on our feet or buttocks, is defined by our mass times the acceleration due to gravity or mg. Thus an imposed acceleration of 1g above the baseline of our gravity, or 2g's, will be felt as a doubling of our weight. At 6g's we would feel six times as heavy as normal and would have great difficulty even moving our arms against that acceleration. Figure 7-10 shows that the body's tolerance of acceleration is a function of its direction versus the body, its magnitude, and its duration. Note also that the data used for this chart were developed from tests on young, healthy military personnel in prime physical condition. The general population, children and elderly in particular, should not be expected to be able to withstand such high levels of acceleration. Since much machinery is designed for human use, these acceleration tolerance data should be of great interest and value to the machine designer. Several references dealing with these human factors data are provided in the bibliography to Chapter 1.

TABLE 7-1 Acceleration Levels Commonly Encountered in Human Activities

Gentle acceleration in an automobile	+0.1 *g*
Commercial jet aircraft on takeoff	+0.3 *g*
Hard acceleration in an automobile	+0.5 *g*
Panic stop in an automobile	–0.7 *g*
Fast cornering in a sports car (e.g., BMW, Corvette, Porsche, Ferrari)	+0.9 *g* to +1.0*g*
Formula 1 race car	+2.0 *g*, –4.0 *g*
Roller coasters (various)	±3.5 to ±6.5 *g**
NASA space shuttle on takeoff	+4.0 *g*
Top fuel dragster with drogue chute (>300 mph in 1/4 mile)	±4.5 *g*
Military jet fighter (e.g., F-15, F-16, F-22, F-35—note: pilot wears a G-suit)	±9.0 *g*

*Some U.S. state laws currently limit roller coaster accelerations to a maximum of 5.0 to 5.4 g.

7

Another useful benchmark when designing machinery for human occupation is to attempt to relate the magnitudes of accelerations that you commonly experience to the calculated values for your potential design. Table 7-1 lists some approximate levels of acceleration, in *g*'s, that humans can experience in everyday life. Your own experience of these will help you develop a "feel" for the values of acceleration that you encounter in designing machinery intended for human occupation.

Acceleration levels in machinery that does not carry humans is limited only by considerations of the stresses in its parts. These stresses are often generated in large part by the dynamic forces due to accelerations. The range of acceleration values in such machinery is so wide that it is not possible to comprehensively define any design guidelines for acceptable or unacceptable levels of acceleration. If the moving mass is small, then very large numerical values of acceleration are reasonable. If the mass is large, the dynamic stresses that the materials can sustain may limit the allowable accelerations to low values. Unfortunately, the designer usually does not know how much acceleration is too much in a design until completing it to the point of calculating stresses in the parts. This usually requires a fairly complete and detailed design. If the stresses turn out to be too high and are due to dynamic forces, then the only recourse is to iterate back through the design process and reduce the accelerations and/or masses in the design. This is one reason that the design process is a circular and not a linear one.

As one point of reference, the acceleration of the piston in a small, four-cylinder economy car engine (about 1.5-L displacement) at idle speed is about 40*g*'s. At highway speeds the piston acceleration can be as high as 700*g*'s. At the engine's top speed of 6000 rpm the peak piston acceleration is 2000*g*'s! As long as you're not riding on the piston, this is acceptable. These engines last a long time in spite of the high accelerations their components experience. One key factor is the choice of proper part geometry and use of low-mass, high-strength, high-stiffness materials for the moving parts to minimize dynamic forces at high acceleration and enable the parts to tolerate the applied stresses.

7.7 JERK

No, not you! The **time derivative of acceleration** is called *jerk*, *pulse,* or *shock*. The name is apt, as it conjures the proper image of this phenomenon. **Jerk** is *the time rate of change of acceleration.* Force is proportional to acceleration. Rapidly changing acceleration means a rapidly changing force. Rapidly changing forces tend to "jerk" the object about! You have probably experienced this phenomenon when riding in an automobile. If the driver is inclined to "jackrabbit" starts and accelerates violently away from the traffic light, you will suffer from large jerk because your acceleration will go from zero to a large value quite suddenly. But, when Jeeves, the chauffeur, is driving the Rolls, he always attempts to minimize jerk by accelerating gently and smoothly, so that Madame is entirely unaware of the change.

Controlling and minimizing jerk in machine design is often of interest, especially if low vibration is desired. Large magnitudes of jerk will tend to excite the natural frequencies of vibration of the machine or structure to which it is attached and cause increased vibration and noise levels. Jerk control is of greater interest in the design of cams than of linkages, and we will investigate it in greater detail in Chapter 8 on cam design.

7

The procedure for calculating the jerk in a linkage is a straightforward extension of the methods shown for acceleration analysis. Let angular jerk be represented by:

$$\varphi = \frac{d\alpha}{dt} \tag{7.33a}$$

and linear jerk by:

$$\mathbf{J} = \frac{d\mathbf{A}}{dt} \tag{7.33b}$$

To solve for jerk in a fourbar linkage, for example, the vector loop equation for acceleration (equation 7.7) is differentiated versus time. Refer to Figure 7-5 for notation.

$$\begin{aligned}
&-a\omega_2^3 je^{j\theta_2} - 2a\omega_2\alpha_2 e^{j\theta_2} + a\alpha_2\omega_2 j^2 e^{j\theta_2} + a\varphi_2 je^{j\theta_2} \\
&\quad - b\omega_3^3 je^{j\theta_3} - 2b\omega_3\alpha_3 e^{j\theta_3} + b\alpha_3\omega_3 j^2 e^{j\theta_3} + b\varphi_3 je^{j\theta_3} \\
&\quad\quad + c\omega_4^3 je^{j\theta_4} + 2c\omega_4\alpha_4 e^{j\theta_4} - c\alpha_4\omega_4 j^2 e^{j\theta_4} - c\varphi_4 je^{j\theta_4} = 0
\end{aligned} \tag{7.34a}$$

Collect terms and simplify:

$$\begin{aligned}
&-a\omega_2^3 je^{j\theta_2} - 3a\omega_2\alpha_2 e^{j\theta_2} + a\varphi_2 je^{j\theta_2} \\
&\quad - b\omega_3^3 je^{j\theta_3} - 3b\omega_3\alpha_3 e^{j\theta_3} + b\varphi_3 je^{j\theta_3} \\
&\quad\quad + c\omega_4^3 je^{j\theta_4} + 3c\omega_4\alpha_4 e^{j\theta_4} - c\varphi_4 je^{j\theta_4} = 0
\end{aligned} \tag{7.34b}$$

Substitute the Euler identity and separate into x and y components:

real part (x component):

$$\begin{aligned}
&a\omega_2^3 \sin\theta_2 - 3a\omega_2\alpha_2 \cos\theta_2 - a\varphi_2 \sin\theta_2 \\
&\quad + b\omega_3^3 \sin\theta_3 - 3b\omega_3\alpha_3 \cos\theta_3 - b\varphi_3 \sin\theta_3 \\
&\quad\quad - c\omega_4^3 \sin\theta_4 + 3c\omega_4\alpha_4 \cos\theta_4 + c\varphi_4 \sin\theta_4 = 0
\end{aligned} \tag{7.35a}$$

TABLE P7-0 Part 1
Topic/Problem Matrix

7.1 Definition of Acceleration
7-1, 7-2, 7-10, 7-56

7.2 Graphical Acceleration Analysis
Pin-Jointed Fourbar
7-3, 7-14a, 7-21, 7-24, 7-30, 7-33, 7-70a, 7-72a, 7-77
Fourbar Crank-Slider
7-5, 7-13a, 7-27, 7-36, 7-89, 7-91
Fourbar Slider-Crank
7-93
Other Fourbar 7-15a
Fivebar 7-79
Sixbar
7-52, 7-53, 7-61a, 7-63a, 7-65a, 7-75, 7-82
Eightbar 7-86

7.3 Analytic Solutions for Acceleration Analysis
Pin-Jointed Fourbar
7-22, 7-23, 7-25, 7-26, 7-34, 7-35, 7-41, 7-46, 7-51, 7-70b, 7-71, 7-72b
Fourbar Crank-Slider
7-6, 7-28, 7-29, 7-37, 7-38, 7-45, 7-50, 7-58, 7-90, 7-92
Fourbar Slider-Crank
7-94
Coriolis Acceleration
7-12, 7-20
Fourbar Inverted Crank-Slider
7-7, 7-8, 7-16, 7-59
Other Fourbar
7-15b, 7-74
Fivebar 7-80, 7-81
Sixbar
7-17, 7-18, 7-19, 7-48, 7-54, 7-61b, 7-62, 7-63b, 7-64, 7-65b, 7-66, 7-76, 7-83, 7-84, 7-85
Eightbar 7-67

imaginary part (*y* component):

$$\begin{aligned}
-a\omega_2^3\cos\theta_2 - 3a\omega_2\alpha_2\sin\theta_2 + a\varphi_2\cos\theta_2 \\
- b\omega_3^3\cos\theta_3 - 3b\omega_3\alpha_3\sin\theta_3 + b\varphi_3\cos\theta_3 \\
+ c\omega_4^3\cos\theta_4 + 3c\omega_4\alpha_4\sin\theta_4 - c\varphi_4\cos\theta_4 = 0
\end{aligned} \tag{7.35b}$$

These can be solved simultaneously for φ_3 and φ_4, which are the only unknowns. The driving angular jerk, φ_2, if nonzero, must be known in order to solve the system. All the other factors in equations 7.35 are defined or have been calculated from the position, velocity, and acceleration analyses. To simplify these expressions we will set the known terms to temporary constants.

In equation 7.35a, let:

$$\begin{aligned}
A &= a\omega_2^3\sin\theta_2 & D &= b\omega_3^3\sin\theta_3 & G &= 3c\omega_4\alpha_4\cos\theta_4 \\
B &= 3a\omega_2\alpha_2\cos\theta_2 & E &= 3b\omega_3\alpha_3\cos\theta_3 & H &= c\sin\theta_4 \\
C &= a\varphi_2\sin\theta_2 & F &= c\omega_4^3\sin\theta_4 & K &= b\sin\theta_3
\end{aligned} \tag{7.36a}$$

Equation 7.35a then reduces to:

$$\varphi_3 = \frac{A - B - C + D - E - F + G + H\varphi_4}{K} \tag{7.36b}$$

Note that equation 7.36b defines angle φ_3 in terms of angle φ_4. We will now simplify equation 7.35b and substitute equation 7.36b into it.

In equation 7.35b, let:

$$\begin{aligned}
L &= a\omega_2^3\cos\theta_2 & P &= b\omega_3^3\cos\theta_3 & S &= c\omega_4^3\cos\theta_4 \\
M &= 3a\omega_2\alpha_2\sin\theta_2 & Q &= 3b\omega_3\alpha_3\sin\theta_3 & T &= 3c\omega_4\alpha_4\sin\theta_4 \\
N &= a\varphi_2\cos\theta_2 & R &= b\cos\theta_3 & U &= c\cos\theta_4
\end{aligned} \tag{7.37a}$$

Equation 7.35b then reduces to:

$$R\varphi_3 - U\varphi_4 - L - M + N - P - Q + S + T = 0 \tag{7.37b}$$

Substituting equation 7.36b in equation 7.35b:

$$R\left(\frac{A - B - C + D - E - F + G + H\varphi_4}{K}\right) - U\varphi_4 - L - M + N - P - Q + S + T = 0 \tag{7.38}$$

The solution is:

$$\varphi_4 = \frac{KN - KL - KM - KP - KQ + AR - BR - CR + DR - ER - FR + GR + KS + KT}{KU - HR} \tag{7.39}$$

The result from equation 7.39 can be substituted into equation 7.36b to find φ_3. Once the angular jerk values are found, the linear jerk at the pin joints can be found from:

$$\begin{aligned}
\mathbf{J}_A &= -a\omega_2^3 je^{j\theta_2} - 3a\omega_2\alpha_2 e^{j\theta_2} + a\varphi_2 je^{j\theta_2} \\
\mathbf{J}_{BA} &= -b\omega_3^3 je^{j\theta_3} - 3b\omega_3\alpha_3 e^{j\theta_3} + b\varphi_3 je^{j\theta_3} \\
\mathbf{J}_B &= -c\omega_4^3 je^{j\theta_4} - 3c\omega_4\alpha_4 e^{j\theta_4} + c\varphi_4 je^{j\theta_4}
\end{aligned} \tag{7.40}$$

The same approach as used in Section 7.5 to find the acceleration of any point on any link can be used to find the linear jerk at any point.

$$\mathbf{J}_P = \mathbf{J}_A + \mathbf{J}_{PA} \tag{7.41}$$

The jerk difference equation 7.41 can be applied to any point on any link if we let P represent any arbitrary point on any link and A represent any reference point on the same link for which we know the value of the jerk vector. Note that if you substitute equations 7.40 into 7.41, you will get equation 7.34.

7.8 LINKAGES OF *N* BARS

The same analysis techniques presented here for position, velocity, acceleration, and jerk, using the fourbar and fivebar linkage as the examples, can be extended to more complex assemblies of links. Multiple vector loop equations can be written around a linkage of arbitrary complexity. The resulting vector equations can be differentiated and solved simultaneously for the variables of interest. In some cases, the solution will require simultaneous solution of a set of nonlinear equations. A root-finding algorithm such as the Newton-Raphson method will be needed to solve these more complicated cases. A computer is necessary. An equation solver software package such as *TKSolver* or *Mathcad* that will do an iterative root-finding solution will be a useful aid to the solution of any of these analysis problems, including the examples shown here.

TABLE P7-0 Part 2
Topic/Problem Matrix

7.5 Acceleration of Any Point on a Linkage

Pin-Jointed Fourbar
7-4, 7-13b, 7-14b, 7-31, 7-32, 7-39, 7-40, 7-42, 7-43, 7-44, 7-49, 7-55, 7-68, 7-70b, 7-71, 7-72b, 7-73, 7-78

Other Fourbar
7-15b, 7-47

Geared Fivebar
7-9, 7-60

Sixbar
7-69, 7-87, 7-88

7.7 Jerk
7-11, 7-57

7.9 REFERENCE

1 **Sanders, M. S., and E. J. McCormick**. (1987). *Human Factors in Engineering and Design*, 6th ed., McGraw-Hill Co., New York. p. 505.

7.10 PROBLEMS[§]

7-1 A point at a 6.5-in radius is on a body that is in pure rotation with $\omega = 100$ rad/sec and a constant $\alpha = -500$ rad/sec^2 at point A. The rotation center is at the origin of a coordinate system. When the point is at position A, its position vector makes a 45° angle with the X axis. It takes 0.01 sec to reach point B. Draw this system to some convenient scale, calculate the θ and ω of position B, and:

a. Write an expression for the particle's acceleration vector in position A using complex number notation, in both polar and cartesian forms.

b. Write an expression for the particle's acceleration vector in position B using complex number notation, in both polar and cartesian forms.

c. Write a vector equation for the acceleration difference between points B and A. Substitute the complex number notation for the vectors in this equation and solve for the acceleration difference numerically.

d. Check the result of part c with a graphical method.

7-2 In problem 7-1 let A and B represent points on separate, rotating bodies both having the given ω and α at $t = 0$, $\theta_A = 45°$, and $\theta_B = 120°$. Find their relative acceleration.

*7-3 The link lengths, coupler point location, and the values of θ_2, ω_2, and α_2 for the same fourbar linkages as used for position and velocity analysis in Chapters 4 and 6 are redefined in Table P7-1, which is basically the same as Table P6-1. The general link-

§ All problem figures are provided as PDF files, and some are also provided as animated Working Model files. PDF filenames are the same as the figure number. Run the file *Animations.html* to access and run the animations.

* Answers in Appendix F.

TABLE P7-1 Data for Problems 7-3, 7-4, and 7-11‡

Row	Link 1	Link 2	Link 3	Link 4	θ_2	ω_2	α_2	R_{pa}	δ_3
a	6	2	7	9	30	10	0	6	30
b	7	9	3	8	85	– 12	5	9	25
c	3	10	6	8	45	– 15	–10	10	80
d	8	5	7	6	25	24	–4	5	45
e	8	5	8	6	75	– 50	10	9	300
f	5	8	8	9	15	– 45	50	10	120
g	6	8	8	9	25	100	18	4	300
h	20	10	10	10	50	– 65	25	6	20
i	4	5	2	5	80	25	– 25	9	80
j	20	10	5	10	33	25	– 40	1	0
k	4	6	10	7	88	– 80	30	10	330
l	9	7	10	7	60	– 90	20	5	180
m	9	7	11	8	50	75	–5	10	90
n	9	7	11	6	120	15	– 65	15	60

‡Drawings of these linkages are in the *PDF Problem Workbook* folder.

7

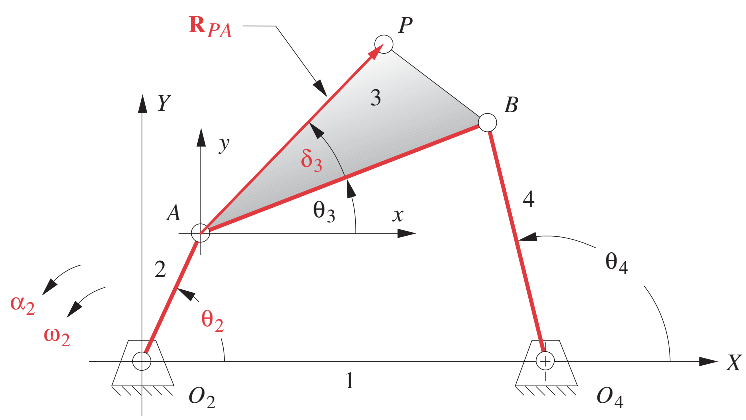


FIGURE P7-1

Configuration and terminology for Problems 7-3, 7-4, and 7-11

age configuration and terminology are shown in Figure P7-1. *For the row(s) assigned,* draw the linkage to scale and graphically find the accelerations of points *A* and *B*. Then calculate α_3 and α_4 and the acceleration of point *P*.

*†7-4 Repeat Problem 7-3, solving by the analytical vector loop method of Section 7.3.

*7-5 The link lengths and offset and the values of θ_2, ω_2, and α_2 for some noninverted, offset fourbar crank-slider linkages are defined in Table P7-2. The general linkage configuration and terminology are shown in Figure P7-2. *For the row(s) assigned*, draw the linkage to scale and graphically find the accelerations of the pin joints *A* and *B* and the acceleration of slip at the sliding joint.

*†7-6 Repeat Problem 7-5 using an analytical method.

* Answers in Appendix F.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

TABLE P7-2 Data for Problems 7-5 to 7-6 and 7-58‡

Row	Link 2	Link 3	Offset	θ_2	ω_2	α_2
a	1.4	4	1	45	10	0
b	2	6	–3	60	– 12	5
c	3	8	2	– 30	– 15	–10
d	3.5	10	1	120	24	-4
e	5	20	–5	225	– 50	10
f	3	13	0	100	– 45	50
g	7	25	10	330	100	18

‡Drawings of these linkages are in the *PDF Problem Workbook* folder.

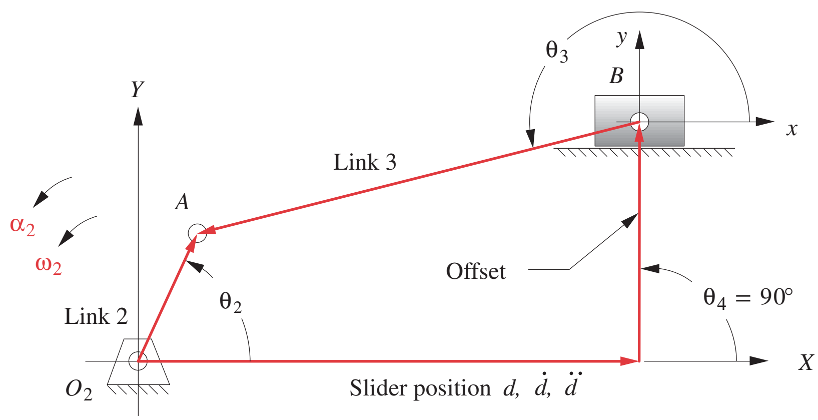


FIGURE P7-2

Configuration and terminology for Problems 7-5 to 7-6, 7-58, and 7-93 to 7-94

*†7-7 The link lengths and the values of θ_2, ω_2, and γ for some inverted fourbar crank-slider linkages are defined in Table P7-3. The general linkage configuration and terminology are shown in Figure P7-3. *For the row(s) assigned*, find accelerations of the pin joints *A* and the acceleration of slip at the sliding joint. Solve by the analytical vector loop method of Section 7.3 for the open configuration of the linkage.

*†7-8 Repeat Problem 7-7 for the crossed configuration of the linkage.

*7-9 The link lengths, gear ratio (λ), phase angle (ϕ), and the values of θ_2, ω_2, and α_2 for some geared fivebar linkages are defined in Table P7-4. The general linkage configuration and terminology are shown in Figure P7-4. *For the row(s) assigned*, find α_3 and α_4 and the linear acceleration of point *P*.

†7-10 An automobile driver took a curve too fast. The car spun out of control about its center of gravity (*CG*) and slid off the road in a northeasterly direction. The friction of the skidding tires provided a 0.25 *g* linear deceleration. The car rotated at 100 rpm. When the car hit the tree head-on at 30 mph, it took 0.1 sec to come to rest.

a. What was the acceleration experienced by the child seated on the middle of the rear seat, 2 ft behind the car's *CG*, just prior to impact?

b. What force did the 100-lb child exert on her seatbelt harness as a result of the

* Answers in Appendix F.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

TABLE P7-3 Data for Problems 7-7 to 7-8 and 7-59

Row	Link 1	Link 2	Link 4	γ	θ_2	ω_2	α_2
a	6	2	4	90	30	10	–25
b	7	9	3	75	85	–15	–40
c	3	10	6	45	45	24	30
d	8	5	3	60	25	–50	20
e	8	4	2	30	75	–45	–5
f	5	8	8	90	150	100	–65

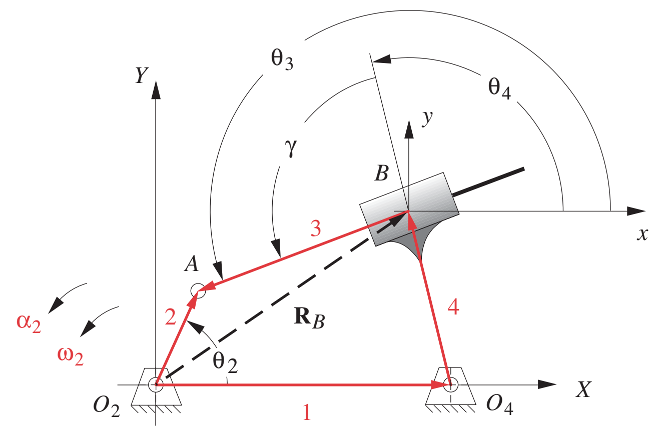


FIGURE P7-3

Configuration and terminology for Problems 7-7 to 7-8 and 7-59

acceleration, just prior to impact?

c. Assuming a constant deceleration during the 0.1 sec of impact, what was the magnitude of the average deceleration felt by the passengers in that interval?

†7-11 For the row(s) assigned in Table P7-1, find the angular jerk of links 3 and 4 and the linear jerk of the pin joint between links 3 and 4 (point *B*). Assume an angular jerk of zero on link 2. The linkage configuration and terminology are shown in Figure P7-1.

*†7-12 You are riding on a carousel that is rotating at a constant 12 rpm. It has an inside radius of 4 ft and an outside radius of 12 ft. You begin to run from the inside to the outside along a radius. Your peak velocity with respect to the carousel is 4 mph and occurs at a radius of 8 ft. What are your maximum Coriolis acceleration magnitude and its direction with respect to the carousel?

7-13 The linkage in Figure P7-5a has $O_2A = 0.8$, $AB = 1.93$, $AC = 1.33$, and *offset* = 0.38 in. The crank angle in the position shown is 34.3° and angle $BAC =$ 38.6°. Find α_3, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\omega_2 = 15$ rad/sec and $\alpha_2 = 10$ rad/sec^2 in directions shown:

a. Using the acceleration difference graphical method.

†b. Using an analytical method.

7-14 The linkage in Figure P7-5b has $I_{12}A = 0.75$, $AB = 1.5$, and $AC = 1.2$ in. The effective crank angle in the position shown is 77° and angle $BAC = 30°$. Find α , **A** , **A** , and

* Answers in Appendix F.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

TABLE P7-4 Data for Problem 7-9 and 7-60

Row	Link 1	Link 2	Link 3	Link 4	Link 5	λ	φ	θ_2	ω_2	α_2	R_{pa}	δ_3
a	6	1	7	9	4	2.0	30	60	10	0	6	30
b	6	5	7	8	4	– 2.5	60	30	– 12	5	9	25
c	3	5	7	8	4	– 0.5	0	45	– 15	– 10	10	80
d	4	5	7	8	4	– 1.0	120	75	24	– 4	5	45
e	5	9	11	8	8	3.2	– 50	– 39	– 50	10	9	300
f	10	2	7	5	3	1.5	30	120	– 45	50	10	120
g	15	7	9	11	4	2.5	– 90	75	100	18	4	300
h	12	8	7	9	4	– 2.5	60	55	– 65	25	6	20
i	9	7	8	9	4	– 4.0	120	100	25	– 25	9	80

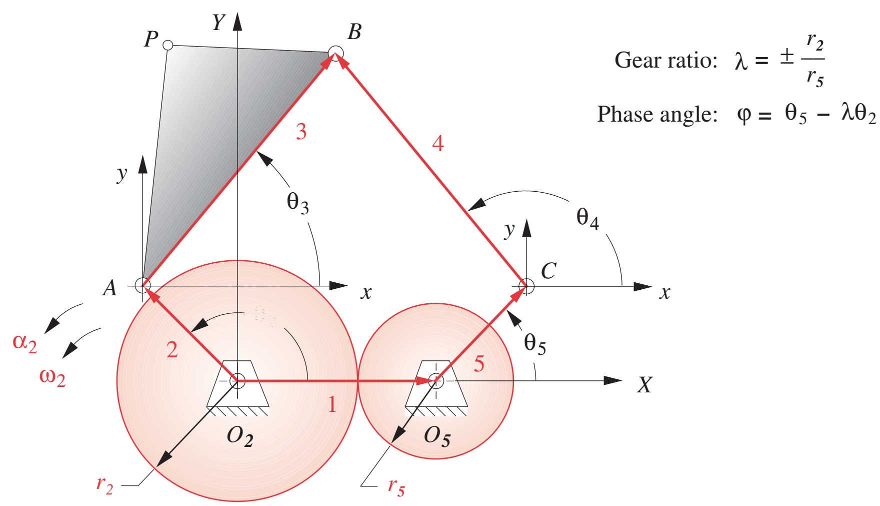


FIGURE P7-4

Configuration and terminology for Problems 7-9 and 7-60

7

$\mathbf{A}_C$ for the position shown for $\omega_2 = 15$ rad/sec and $\alpha_2 = 10$ rad/sec^2 in the directions shown:

a. Using the acceleration difference graphical method.

†b. Using an analytical method. (Hint: Create an effective linkage for the position shown and analyze it as a pin-jointed fourbar.)

7-15 The linkage in Figure P7-5c has $AB = 1.8$ and $AC = 1.44$ in. The angle of AB in the position shown is 128° and angle $BAC = 49°$. The slider at B is at an angle of 59°. Find α_3, $\mathbf{A}_B$, and $\mathbf{A}_C$ for the position shown for $\mathbf{V}_A = 10$ in/sec and $\mathbf{A}_A = 15$ in/sec^2 in the directions shown:

a. Using the acceleration difference graphical method.

†b. Using an analytical method.

†7-16 The linkage in Figure P7-6a has $O_2A = 5.6$, $AB = 9.5$, $O_4C = 9.5$, $L_1 = 38.8$ mm. θ_2 is 135° in the xy coordinate system. Write the vector loop equations; differentiate them, and do a complete position, velocity, and acceleration analysis of the linkage. Assume $\omega_2 = 10$ rad/sec and $\alpha_2 = 20$ rad/sec^2.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

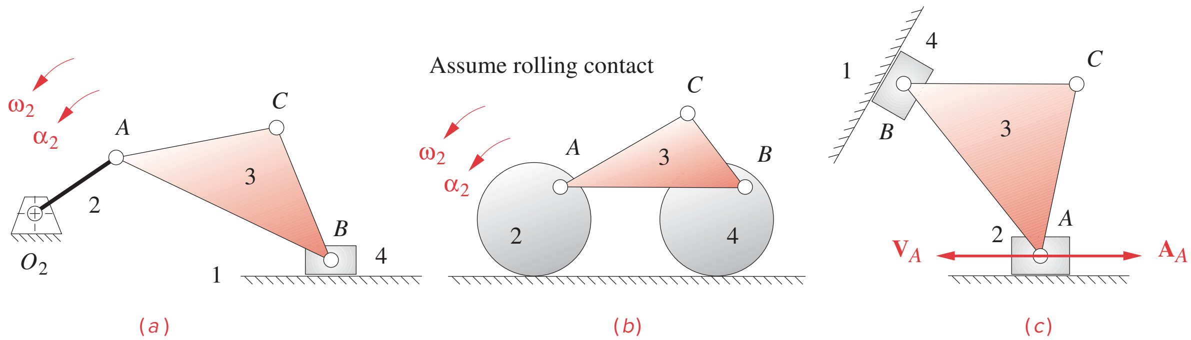


FIGURE P7-5

Problems 7-13 to 7-15

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†7-17 Repeat Problem 7-16 for the linkage shown in Figure P7-6b which has the dimensions: $L_1 = 61.9$, $L_2 = 15$, $L_3 = 45.8$, $L_4 = 18.1$, $L_5 = 23.1$ mm. θ_2 is 68.3° in the *xy* coordinate system, which is at –23.3° in the *XY* coordinate system. The *X* component of O_2C is 59.2 mm.

†7-18 Repeat Problem 7-16 for the linkage shown in Figure P7-6c which has the dimensions: $O_2A = 11.7$, $O_2C = 20$, $L_3 = 25$, $L_5 = 25.9$ mm. Point *B* is offset 3.7 mm from the x_1 axis and point *D* is offset 24.7 mm from the x_2 axis. θ_2 is at 13.3° in the x_2y_2 coordinate system.

†7-19 Repeat Problem 7-16 for the linkage shown in Figure P7-6d which has the dimensions: $L_2 = 15$, $L_3 = 40.9$, $L_5 = 44.7$ mm. θ_2 is 24.2° in the *XY* coordinate system.

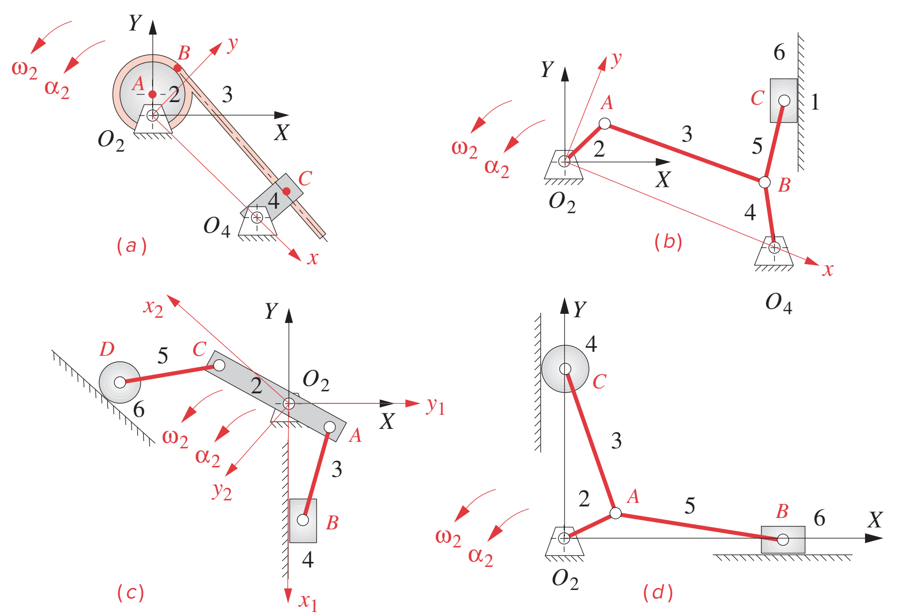


FIGURE P7-6

Problems 7-16 to 7-19

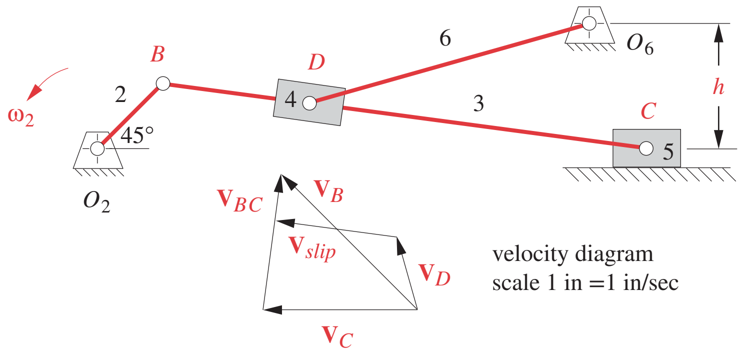


FIGURE P7-7

Problem 7-20

†7-20 Figure P7-7 shows a sixbar linkage with $O_2B = 1$, $BD = 1.5$, $DC = 3.5$, $DO_6 = 3$, and $h = 1.3$ in. Find the angular acceleration of link 6 if ω_2 is a constant 1 rad/sec.

*7-21 The linkage in Figure P7-8a has link 1 at –25° and link 2 at 37° in the global *XY* coordinate system. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if $\omega_2 = 15$ rad/sec CW and $\alpha_2 = 25$ rad/sec^2 CCW. Use the acceleration difference graphical method. (Print the figure from its PDF file and draw on it.)

†7-22 The linkage in Figure P7-8a has link 1 at –25° and link 2 at 37° in the global *XY* coordinate system. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if $\omega_2 = 15$ rad/sec CW and $\alpha_2 = 25$ rad/sec^2 CCW. Use an analytical method.

†7-23 At $t = 0$, the non-Grashof linkage in Figure P7-8a has link 1 at –25° and link 2 at 37° in the global *XY* coordinate system and $\omega_2 = 0$. Write a computer program or use an equation solver to find and plot ω_4, α_4, $\mathbf{V}_A$, $\mathbf{A}_A$, $\mathbf{V}_B$, and $\mathbf{A}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\alpha_2 = 15$ rad/sec CW constant.

*7-24 The linkage in Figure P7-8b has link 1 at –36° and link 2 at 57° in the global *XY* coordinate system. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if $\omega_2 = 20$ rad/sec CCW, constant. Use the acceleration difference graphical method. (Print the figure from its PDF file and draw on it.)

†7-25 The linkage in Figure P7-8b has link 1 at –36° and link 2 at 57° in the global *XY* coordinate system. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if $\omega_2 = 20$ rad/sec CCW, constant. Use an analytical method.

†7-26 For the linkage in Figure P7-8b, write a computer program or use an equation solver to find and plot α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if $\omega_2 = 20$ rad/sec CCW, constant.

7-27 The offset crank-slider linkage in Figure P7-8f has link 2 at 51° in the global *XY* coordinate system. Find $\mathbf{A}_A$ and $\mathbf{A}_B$ in the global coordinate system for the position shown if $\omega_2 = 25$ rad/sec CW, constant. Use the acceleration difference graphical method. (Print the figure from its PDF file and draw on it.)

*†7-28 The offset crank-slider linkage in Figure P7-8f has link 2 at 51° in the global *XY* coordinate system. Find $\mathbf{A}_A$ and $\mathbf{A}_B$ in the global coordinate system for the position shown if $\omega_2 = 25$ rad/sec CW, constant. Use an analytical method.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

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* Answers in Appendix F.

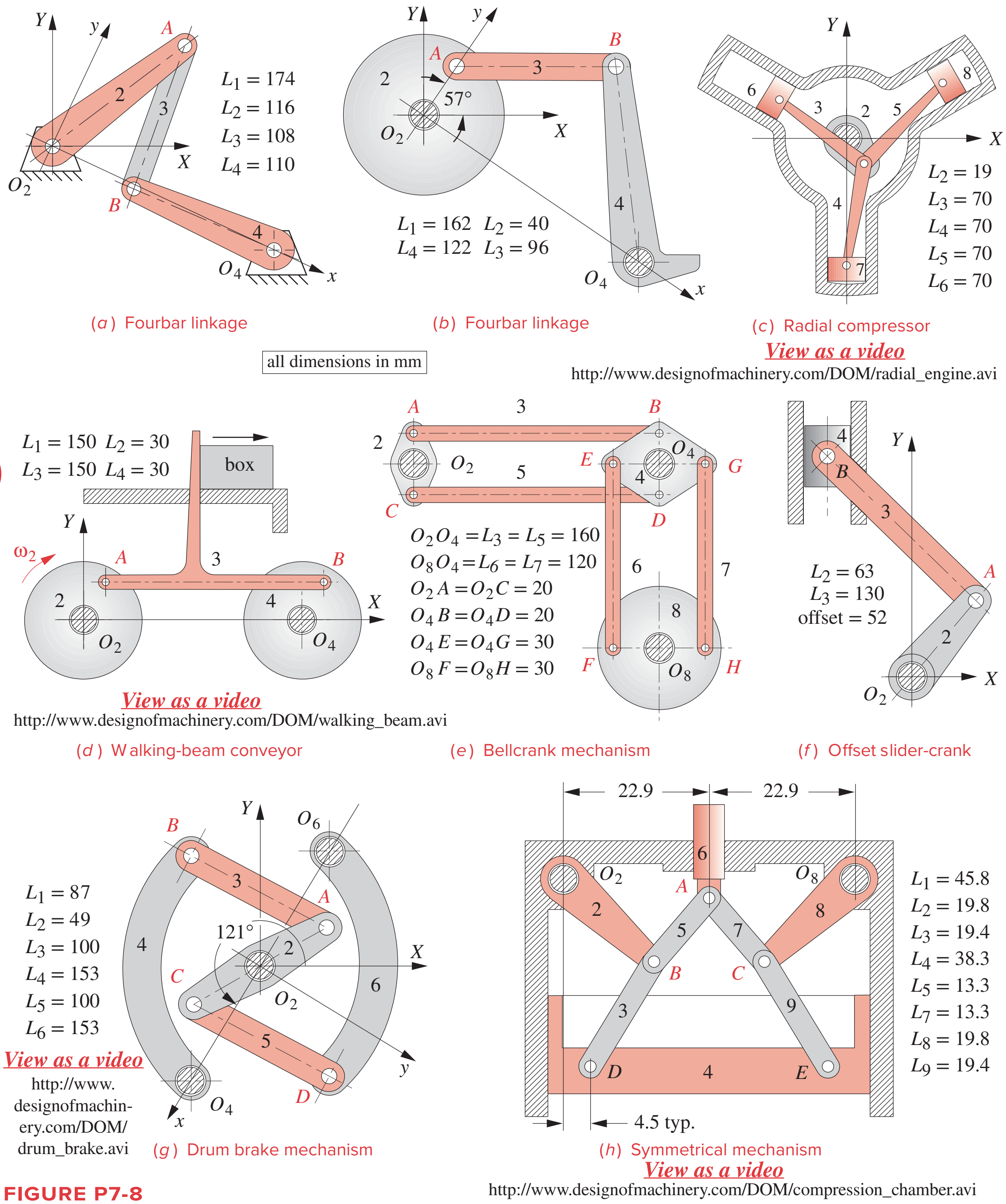


FIGURE P7-8

Problems 7-21 to 7-38

†7-29 For the offset crank-slider linkage in Figure P7-8f, write a computer program or use an equation solver to find and plot $\mathbf{A}_A$ and $\mathbf{A}_B$ in the global coordinate system for the maximum range of motion that this linkage allows if ω_2 = 25 rad/sec CW, constant.

7-30 The linkage in Figure P7-8d has link 2 at 58° in the global *XY* coordinate system. Find $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ (the acceleration of the box) in the global coordinate system for the position shown if ω_2 = 30 rad/sec CW, constant. Use the acceleration difference graphical method. (Print the figure from its PDF file and draw on it.)

†7-31 The linkage in Figure P7-8d has link 2 at 58° in the global *XY* coordinate system. Find $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ (the acceleration of the box) in the global coordinate system for the position shown if ω_2 = 30 rad/sec CW, constant. Use an analytical method.

†7-32 For the linkage in Figure P7-8d, write a computer program or use an equation solver to find and plot $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_{box}$ (the acceleration of the box) in the global coordinate system for the maximum range of motion that this linkage allows if ω_2 = 30 rad/sec CW, constant.

7-33 The linkage in Figure P7-8g has the local *xy* axis at –119° and O_2A at 29° in the global *XY* coordinate system. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW, constant. Use the acceleration difference graphical method. (Print the figure from its PDF file and draw on it.)

†7-34 The linkage in Figure P7-8g has the local *xy* axis at –119° and O_2A at 29° in the global *XY* coordinate system. Find α_4, $\mathbf{A}_A$, and $\mathbf{A}_B$ in the global coordinate system for the position shown if ω_2 = 15 rad/sec CW and α_2 = 10 rad/sec CCW, constant. Use an analytical method.

†7-35 At t = 0, the non-Grashof linkage in Figure P7-8g has the local *xy* axis at –119° and O_2A at 29° in the global *XY* coordinate system and ω_2 = 0. Write a computer program or use an equation solver to find and plot ω_4, α_4, $\mathbf{V}_A$, $\mathbf{A}_A$, $\mathbf{V}_B$, and $\mathbf{A}_B$ in the local coordinate system for the maximum range of motion that this linkage allows if α_2 = 15 rad/sec CCW, constant.

7-36 The 3-cylinder radial compressor in Figure P7-8c has its cylinders equispaced at 120°. Find the piston accelerations $\mathbf{A}_6$, $\mathbf{A}_7$, $\mathbf{A}_8$ with the crank at –53° using a graphical method if ω_2 = 15 rad/sec CW, constant. (Print the figure's PDF file and draw on it.)

†7-37 The 3-cylinder radial compressor in Figure P7-8c has its cylinders equispaced at 120°. Find the piston accelerations $\mathbf{A}_6$, $\mathbf{A}_7$, $\mathbf{A}_8$ with the crank at –53° using an analytical method if ω_2 = 15 rad/sec CW, constant.

†7-38 For the 3-cylinder radial compressor in Figure P7-8f, write a program or use an equation solver to find and plot the piston accelerations $\mathbf{A}_6$, $\mathbf{A}_7$, $\mathbf{A}_8$ for one revolution of the crank.

*†7-39 Figure P7-9 shows a linkage in one position. Find the instantaneous accelerations of points *A*, *B*, and *P* if link O_2A is rotating CW at 40 rad/sec.

*†7-40 Figure P7-10 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point *P* at 2° increments of crank angle for ω_2 = 100 rpm. Check your result with program LINKAGES.

*†7-41 Figure P7-11 shows a linkage that operates at 500 crank rpm. Write a computer program or use an equation solver to calculate and plot the magnitude and direction

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

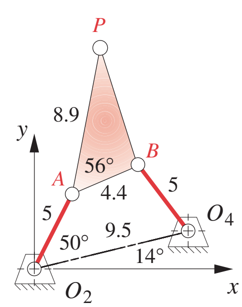


FIGURE P7-9
Problem 7-39

* Answers in Appendix F.

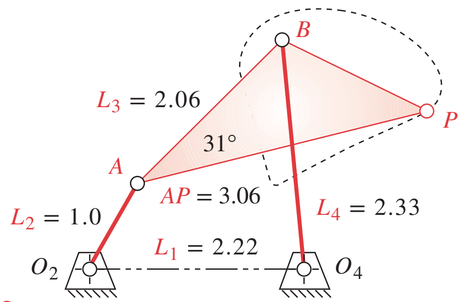
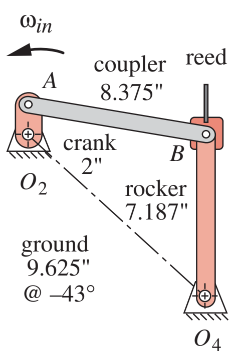


FIGURE P7-10

Problem 7-40 A fourbar linkage with a double straight-line coupler curve



View as a video

http://www.designof-machinery.com/DOM/loom_laybar_drive.avi

FIGURE P7-11

Problem 7-41 Loom laybar drive

* Answers in Appendix F.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

of the acceleration of point B at 2° increments of crank angle. Check your result with program LINKAGES.

*†7-42 Figure P7-12 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2° increments of crank angle for $\omega_2 = 20$ rpm over the maximum range of motion possible. Check your result with program LINKAGES.

†7-43 Figure P7-13 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2° increments of crank angle for $\omega_2 = 80$ rpm over the maximum range of motion possible. Check your result with program LINKAGES.

*†7-44 Figure P7-14 shows a linkage and its coupler curve. Write a computer program or use an equation solver to calculate and plot the magnitude and direction of the acceleration of the coupler point P at 2° increments of crank angle for $\omega_2 = 80$ rpm over the maximum range of motion possible. Check your result with program LINKAGES.

†7-45 Figure P7-15 shows a power hacksaw, used to cut metal. Link 5 pivots at O_5 and its weight forces the sawblade against the workpiece while the linkage moves the blade (link 4) back and forth on link 5 to cut the part. It is an offset crank-slider mechanism

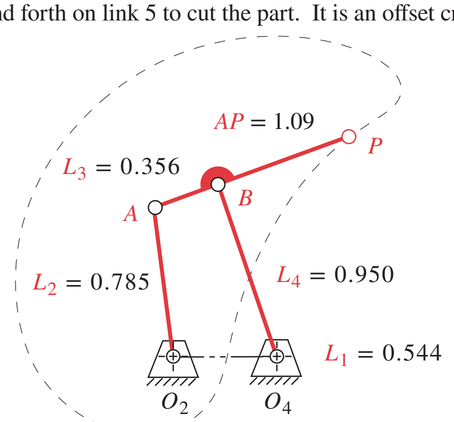


FIGURE P7-12

Problem 7-42

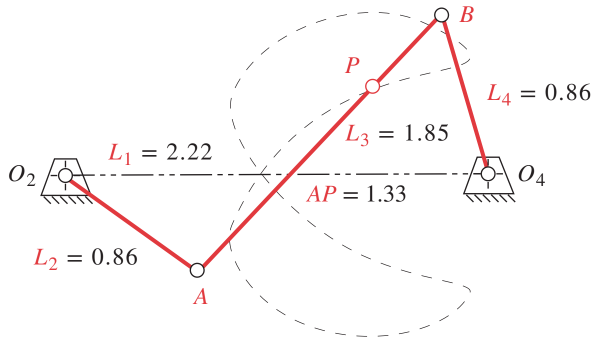


FIGURE P7-13

Problem 7-43

with the dimensions shown in the figure. Draw an equivalent linkage diagram, and then calculate and plot the acceleration of the sawblade with respect to the piece being cut over one revolution of the crank at 50 rpm.

†7-46 Figure P7-16 shows a walking-beam indexing and pick-and-place mechanism that can be analyzed as two fourbar linkages driven by a common crank. The link lengths are given in the figure. The phase angle between the two crankpins on links 4 and 5 is indicated. The product cylinders being pushed have 60-mm diameters. The point of contact between the left vertical finger and the leftmost cylinder in the position shown is 58 mm at 80° versus the left end of the parallelogram's coupler (point *D*). Calculate and plot the relative acceleration between points *E* and *P* for one revolution of gear 2.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

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†7-47 Figure P7-17 shows a paper roll off-loading mechanism driven by an air cylinder. In the position shown O_4A is 0.3 m at 226° and $O_2O_4 = 0.93$ m at 163.2°. The V-links are rigidly attached to O_4A. The paper roll center is 0.707 m from O_4 at –181° with respect to O_4A. The air cylinder is retracted at a constant acceleration of 0.1 m/sec^2. Draw a kinematic diagram of the mechanism, write the necessary equations, and calculate and plot the angular acceleration of the paper roll and the linear acceleration of its center as it rotates through 90° CCW from the position shown.

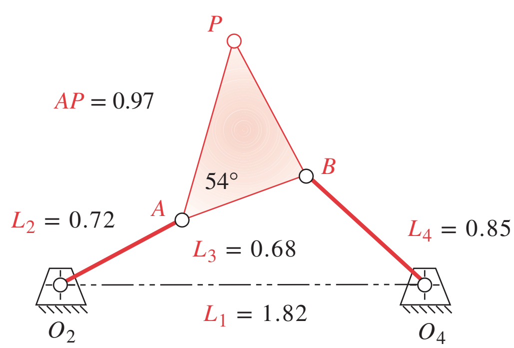


FIGURE P7-14

Problem 7-44

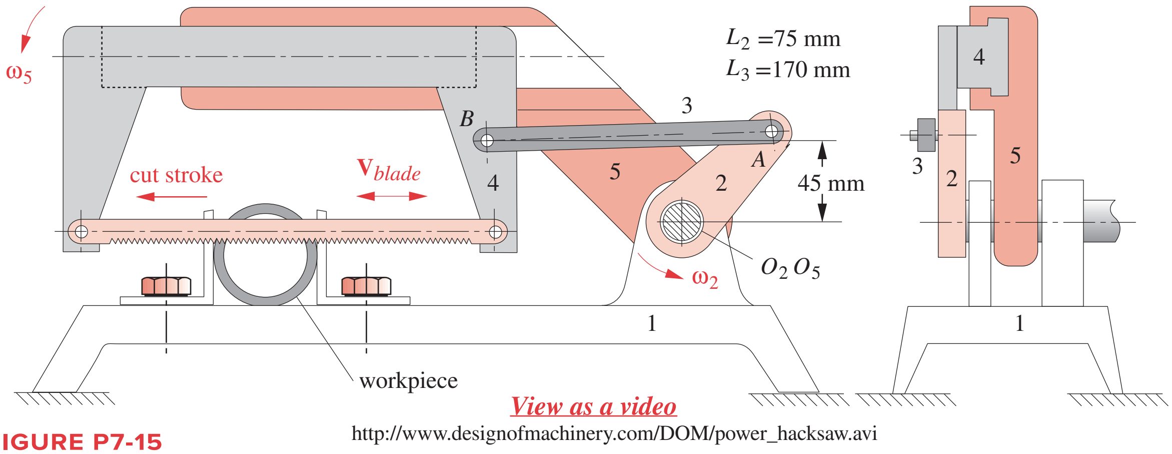


FIGURE P7-15

Problem 7-45 Power hacksaw

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†7-48 Figure P7-18 shows a mechanism and its dimensions. Find the accelerations of points *A*, *B*, and *C* for the position shown if $\omega_2 = 40$ rad/min and $\alpha_2 = -1500$ rad/min^2 as shown.

†7-49 Figure P7-19 shows a walking-beam mechanism. Calculate and plot the acceleration A_{out} for one revolution of the input crank 2 rotating at 100 rpm.

†7-50 Figure P7-20 shows a surface grinder. The workpiece is oscillated under the spinning 90-mm-diameter grinding wheel by the crank-slider linkage which has a 22-mm crank, a 157-mm connecting rod, and a 40-mm offset. The crank turns at 30 rpm, and the

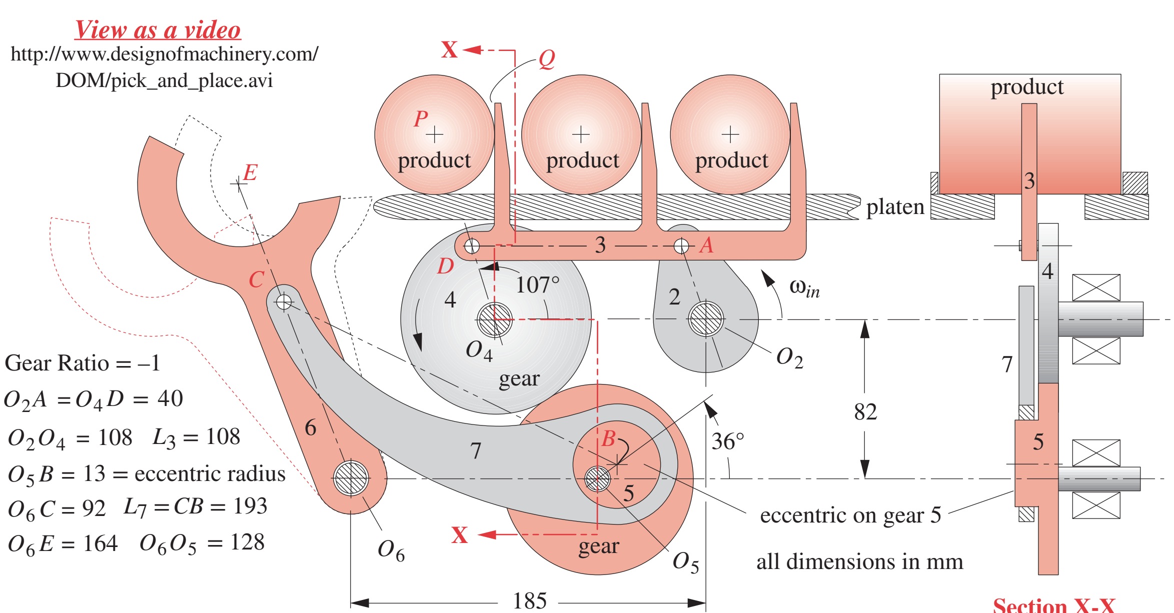


FIGURE P7-16

Problem 7-46 Walking-beam indexer with pick-and-place mechanism

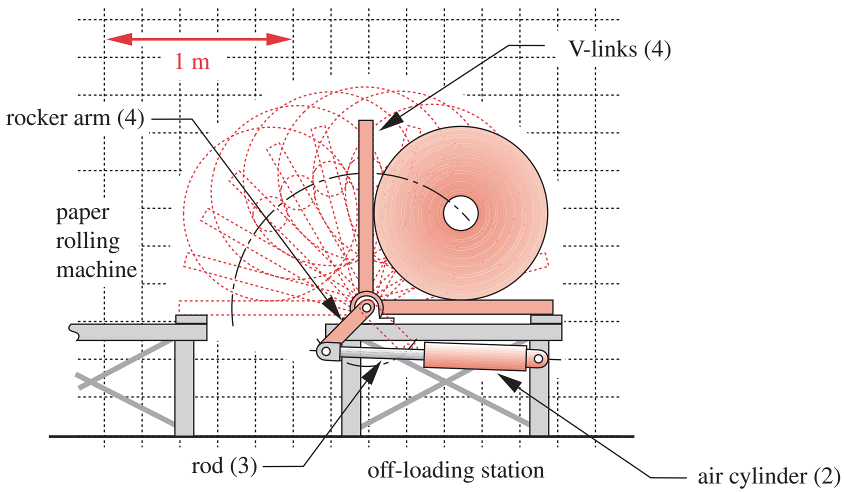


FIGURE P7-17

Problem 7-47

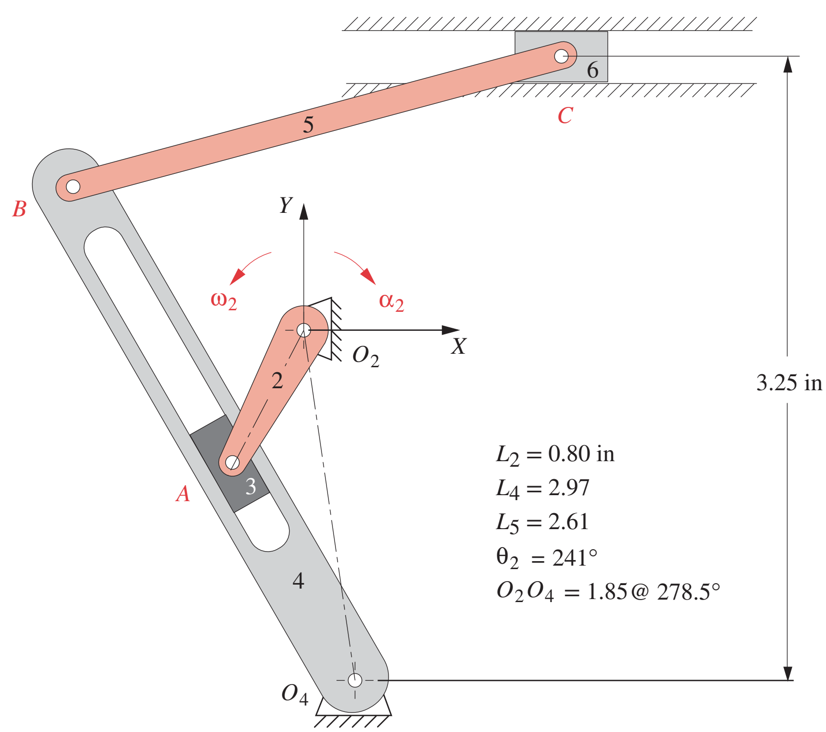


FIGURE P7-18

Problem 7-48

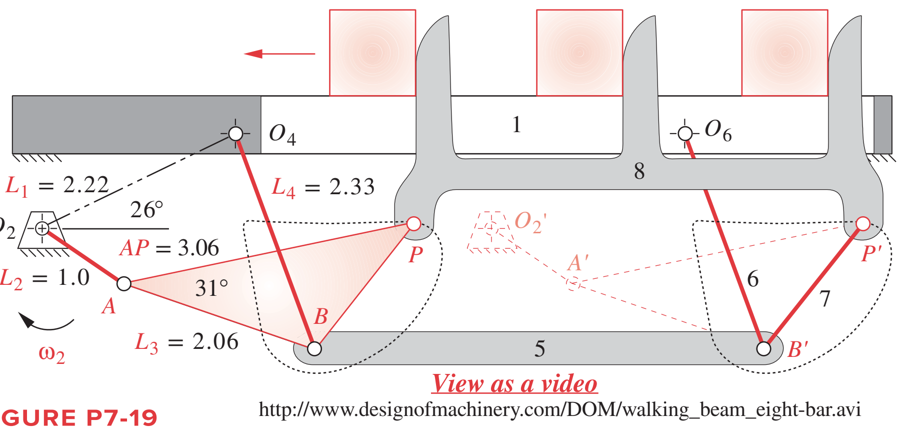


FIGURE P7-19

Problem 7-49 Straight-line walking-beam eightbar transport mechanism

grinding wheel turns at 3450 rpm. Calculate and plot the acceleration of the grinding wheel contact point relative to the workpiece over one revolution of the crank.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†7-51 Figure P7-21 shows a drag link mechanism with dimensions. Write the necessary equations and solve them to calculate the angular acceleration of link 4 for an input of $\omega_2 = 1$ rad/sec. Comment on uses for this mechanism.

7-52 Figure P7-22 shows a mechanism with dimensions. Use a graphical method to calculate the accelerations of points *A*, *B*, and *C* for the position shown. $\omega_2 = 20$ rad/sec.

7-53 Figure P7-23 shows a quick-return mechanism with dimensions. Use a graphical method to calculate the accelerations of points *A*, *B*, and *C* for the position shown. $\omega_2 = 10$ rad/sec.

†7-54 Figure P7-23 shows a quick-return mechanism with dimensions. Use an analytical method to calculate the accelerations of points *A*, *B*, and *C* for one revolution of the input link. $\omega_2 = 10$ rad/sec.

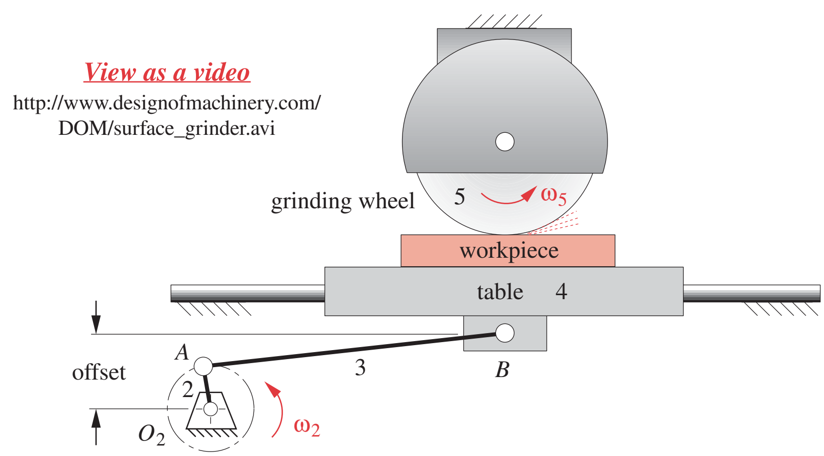


FIGURE P7-20

Problem 7-50 A surface grinder

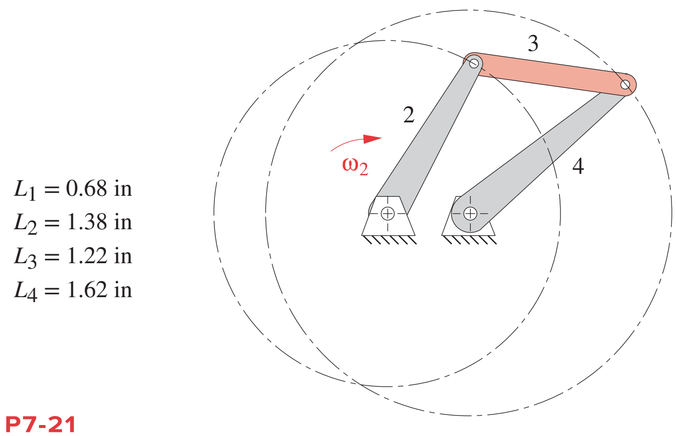


FIGURE P7-21

Problem 7-51

[†]7-55 Figure P7-24 shows a drum-pedal mechanism. $O_2A = 100$ mm at 162° and rotates to 171° at A'. $O_2O_4 = 56$ mm, $AB = 28$ mm, $AP = 124$ mm, and $O_4B = 64$ mm. The distance from O_4 to F_{in} is 48 mm. If the input velocity V_{in} is a constant magnitude of 3 m/sec, find the output acceleration over the range of motion.

[*†]7-56 A tractor-trailer tipped over while negotiating an on-ramp to the New York Thruway. The road has a 50-ft radius at that point and tilts 3° toward the outside of the curve. The 45-ft-long by 8-ft-wide by 8.5-ft-high trailer box (13 ft from ground to top) was loaded with 44 415 lb of paper rolls in two rows by two layers as shown in Figure P7-25. The rolls are 40 in diameter by 38 in long, and weigh about 900 lb each. They are wedged against backward rolling but not against sideward sliding. The empty

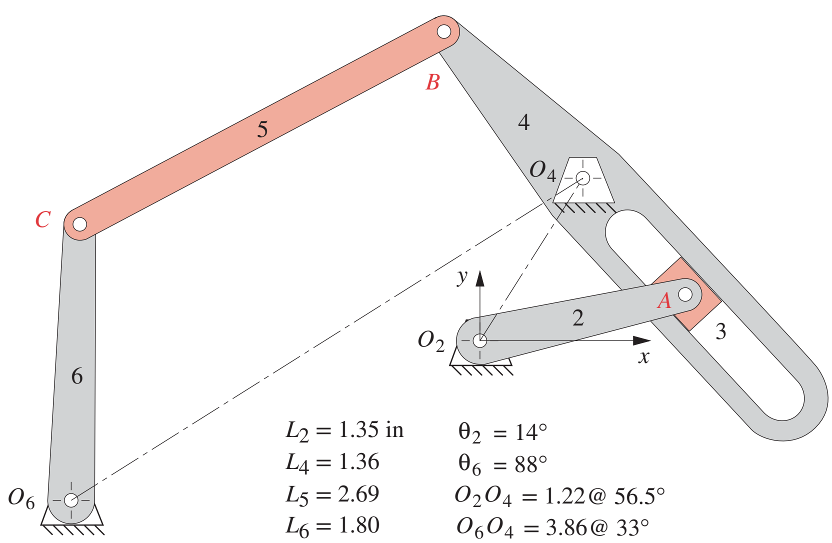
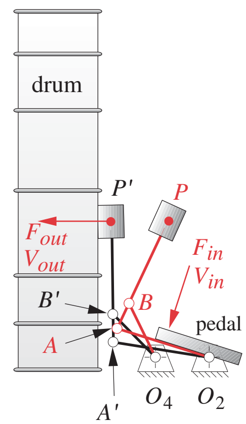


FIGURE P7-22

Problems 7-52 and 7-89 to 7-90



View as a video
http://www.designofmachinery.com/DOM/drum_pedal.avi

FIGURE P7-24

Problem 7-55

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[*] Answers in Appendix F.

[†] These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

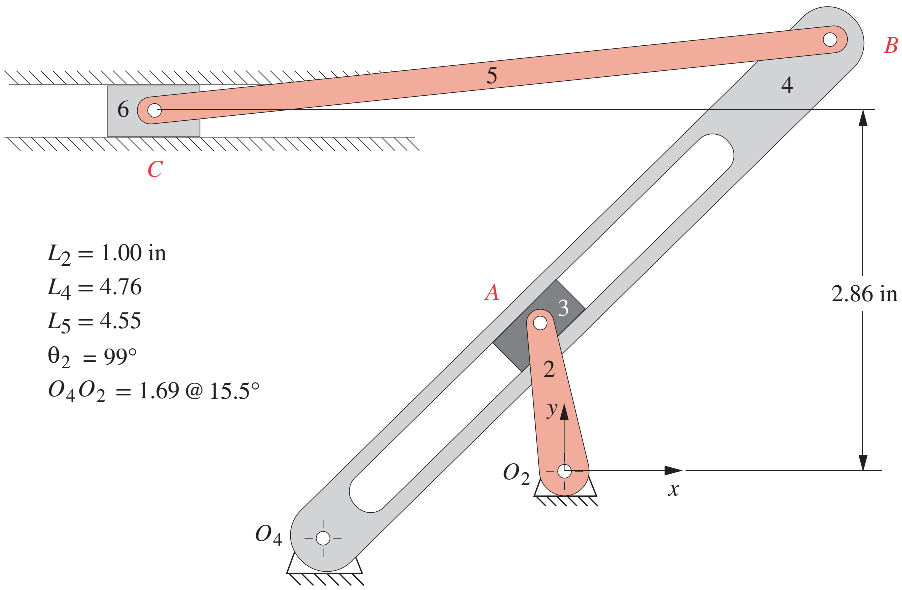

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FIGURE P7-23

Problems 7-53 to 7-54 and 7-91 to 7-92

trailer weighed 14 000 lb. The driver claims that he was traveling at less than 15 mph and that the load of paper shifted inside the trailer, struck the trailer sidewall, and tipped the truck. The paper company that loaded the truck claims the load was properly stowed and would not shift at that speed. Independent tests of the coefficient of friction between similar paper rolls and a similar trailer floor give a value of 0.43 ± 0.08. The composite center of gravity of the loaded trailer is estimated to be 7.5 ft above the road. Determine the truck speed that would cause the truck to just begin to tip and the speed at which the rolls will just begin to slide sideways. What do you think caused the accident?

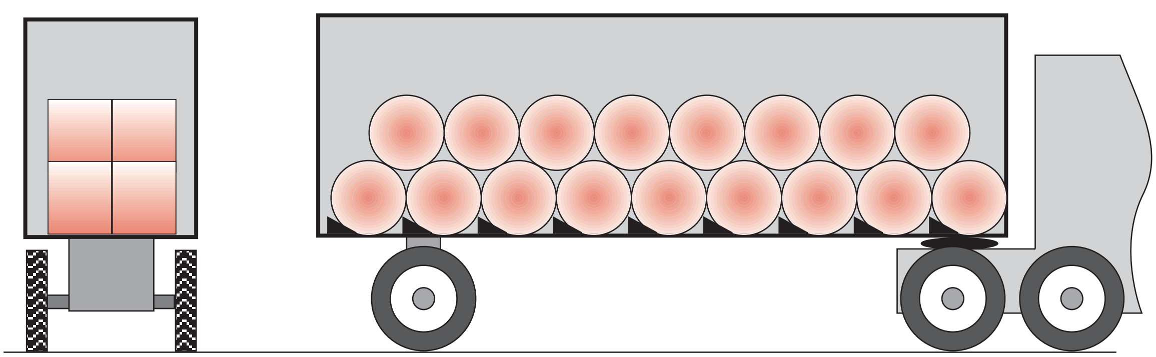

FIGURE P7-25

Problem 7-56

†7-57 Figure P7-26 shows a V-belt drive. The sheaves have pitch diameters of 150 and 300 mm, respectively. The smaller sheave is driven at a constant 1750 rpm. For a cross-sectional differential element of the belt, write the equations of its acceleration for one complete trip around both sheaves including its travel between the sheaves. Compute and plot the acceleration of the differential element versus time for one circuit around the belt path. What does your analysis tell about the dynamic behavior of the belt? Relate your findings to your personal observation of a belt of this type in operation. (Look in your school's machine shop or under the hood of an automobile—but mind your fingers!)

FIGURE P7-26

Problem 7-57
V-belt drive *Courtesy of T.B. Wood's Sons Co., Chambersburg, PA*

†7-58 Write a program using an equation solver or any computer language to solve for the displacements, velocities, and accelerations in an offset crank-slider linkage as shown in Figure P7-2. Plot the variation in all links' angular and all pins' linear positions, velocities, and accelerations with a constant angular velocity input to the crank over one revolution for both open and crossed configurations of the linkage. To test the program, use data from row *a* of Table P7-2. Check your results with program LINKAGES.

†7-59 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TK-Solver* to solve for the displacements, velocities, and accelerations in an inverted crank-slider linkage as shown in Figure P7-3. Plot the variation in all links' angular and all pins' linear positions, velocities, and accelerations with a constant angular velocity input to the crank over one revolution for both open and crossed configurations of the linkage. To test the program, use data from row *e* of Table P7-3 except for the value of α_2 which will be set to zero for this exercise.

†7-60 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TK-Solver* to solve for the displacements, velocities, and accelerations in a geared fivebar linkage as shown in Figure P7-4. Plot the variation in all links' angular and all pins' linear positions, velocities, and accelerations with a constant angular velocity input to the crank over one revolution for both open and crossed configurations of the linkage. To test the program, use data from row *a* of Table P7-4 . Check your results with program LINKAGES.

7-61 Find the acceleration of the slider in Figure 3-33 for the position shown if $\theta_2 = 110°$ with respect to the global *X* axis assuming a constant $\omega_2 = 1$ rad/sec CW:

a. Using a graphical method. †b. Using an analytical method.

†7-62 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TK-Solver* to calculate and plot the angular acceleration of link 4 and the linear acceleration of slider 6 in the sixbar crank-slider linkage of Figure 3-33 as a function of the angle of input link 2 for a constant $\omega_2 = 1$ rad/sec CW. Plot $\mathbf{A}_c$ both as a function of θ_2 and separately as a function of slider position as shown in the figure.

7-63 Find the angular acceleration of link 6 of the linkage in Figure 3-34 part (*b*) for the position shown ($\theta_6 = 90°$ with respect to the *x* axis) assuming constant $\omega_2 = 10$ rad/sec CW:

a. Using a graphical method. †b. Using an analytical method.

†7-64 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TKSolver* to calculate and plot the angular acceleration of link 6 in the sixbar linkage of Figure 3-34 as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CW.

7-65 Use a compass and straightedge (ruler) to draw the linkage in Figure 3-35 with link 2 at 90° and find the angular acceleration of link 6 of the linkage assuming constant $\omega_2 =$ 10 rad/sec CCW when $\theta_2 = 90°$:

a. Using a graphical method. †b. Using an analytical method.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

[†] These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

[†]7-66 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TKSolver* to calculate and plot the angular acceleration of link 6 in the sixbar linkage of Figure 3-35 as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CCW.

[†]7-67 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TKSolver* to calculate and plot the angular acceleration of link 8 in the linkage of Figure 3-36 as a function of θ_2 for a constant $\omega_2 = 1$ rad/sec CCW.

[†]7-68 Write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TKSolver* to calculate and plot magnitude and direction of the acceleration of point *P* in Figure 3-37a as a function of θ_2. Also calculate and plot the acceleration of point *P* versus point *A*.

[†]7-69 Repeat Problem 7-68 for the linkage in Figure 3-37b.

7-70 Find the angular accelerations of links 3 and 4 and the linear accelerations of points *A*, *B,* and P_1 in the *XY* coordinate system for the linkage in Figure P7-27 in the position shown. Assume that $\theta_2 = 45°$ in the *XY* coordinate system and $\omega_2 = 10$ rad/sec, constant. The coordinates of the point P_1 on link 4 are (114.68, 33.19) with respect to the *xy* coordinate system:

a. Using a graphical method. [†]b. Using an analytical method.

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[†]7-71 Using the data from Problem 7-70, write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TKSolver* to calculate and plot magnitude and direction of the absolute acceleration of point P_1 in Figure P7-27 as a function of θ_2.

7-72 Find the angular accelerations of links 3 and 4, and the linear acceleration of point *P* in the *XY* coordinate system for the linkage in Figure P7-28 in the position shown. Assume that $\theta_2 = -94.121°$ in the *XY* coordinate system, $\omega_2 = 1$ rad/sec, and $\alpha_2 = 10$ rad/sec^2. The position of the coupler point *P* on link 3 with respect to point *A* is: $p = 15.00$, $\delta_3 = 0°$:

a. Using a graphical method. [†]b Using an analytical method.

[†]7-73 For the linkage in Figure P7-28, write a computer program or use an equation solver such as *Mathcad, Matlab*, or *TKSolver* to calculate and plot the angular velocity and acceleration of links 2 and 4, and the magnitude and direction of the velocity and acceleration of point *P* as a function of θ_2 through its possible range of motion starting at the position shown. The position of the coupler point *P* on link 3 with respect to point *A* is: $p = 15.00$, $\delta_3 = 0°$. Assume that, @ $t = 0$, $\theta_2 = -94.121°$ in the *XY* coordinate system, $\omega_2 = 0$, and $\alpha_2 = 10$ rad/sec^2, constant.

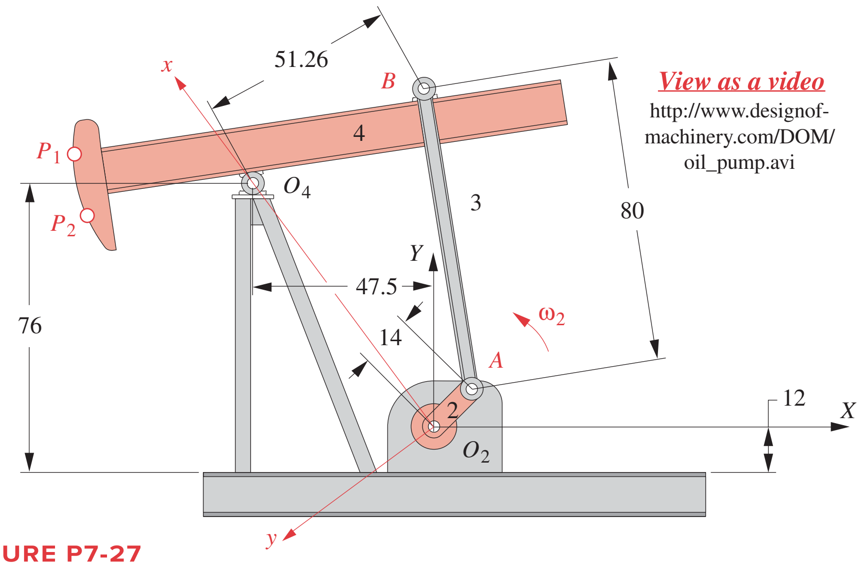
7-74 Derive analytical expressions for the accelerations of points *A* and *B* in Figure P7-29 as a function of θ_3 , ω_3, α_3, and the length *AB* of link 3. Use a vector loop equation. Code them in an equation solver or a programming language and plot them.

7-75 The linkage in Figure P7-30a has link 2 at 120° in the global *XY* coordinate system. Find α_6 and $\mathbf{A}_D$ in the global coordinate system for the position shown if $\omega_2 = 10$ rad/sec CCW and $\alpha_2 = 50$ rad/sec^2 CW. Use the acceleration difference graphical method. (Print the figure from its PDF file and draw on it.)

[*] Answers in Appendix F.

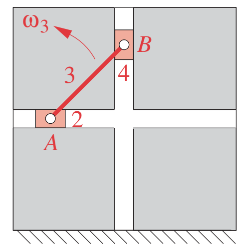
[*]7-76 The linkage in Figure P7-30a has link 2 at 120° in the global *XY* coordinate system. Find α_6 and $\mathbf{A}_D$ in the global coordinate system for the position shown if $\omega_2 = 10$ rad/sec CCW and $\alpha_2 = 50$ rad/sec^2 CW. Use an analytical method.

7-77 The linkage in Figure P7-30b has link 3 perpendicular to the *X* axis and links 2 and 4 are parallel to each other. Find α_4, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_P$ if $\omega_2 = 15$ rad/sec CW and $\alpha_2 =$



View as a video
http://www.designof-machinery.com/DOM/oil_pump.avi

FIGURE P7-27
Problems 7-70 to 7-71 An oil field pump—dimensions in inches



View as a video
http://www.designof-machinery.com/DOM/elliptic_trammel.avi

FIGURE P7-29
Elliptical trammel
Problem 7-74

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100 rad/ sec^2 CW. Use the acceleration difference graphical method. (Print the figure from its PDF file and draw on it.)

*7-78 The linkage in Figure P7-30b has link 3 perpendicular to the X axis and links 2 and 4 are parallel to each other. Find α_4, $\mathbf{A}_A$, $\mathbf{A}_B$, and $\mathbf{A}_P$ if ω_2 = 15 rad/sec CW and α_2 = 100 rad/sec^2 CW. Use an analytical method.

* Answers in Appendix F.

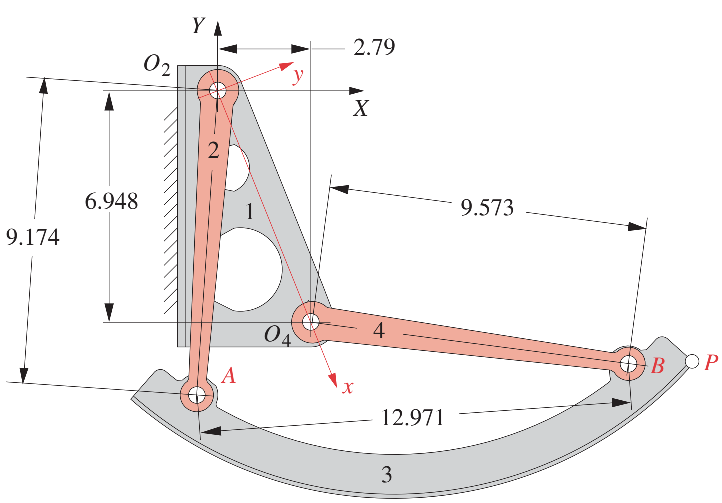

FIGURE P7-28
Problems 7-72 and 7-73 An aircraft overhead bin mechanism—dimensions in inches

7-79 The crosshead linkage shown in Figure P7-30c has 2 *DOF* with inputs at crossheads 2 and 5. Find $\mathbf{A}_B$, $\mathbf{A}_{P_3}$, and $\mathbf{A}_{P_4}$ if the crossheads are each moving toward the origin of the *XY* coordinate system with a speed of 20 in/sec and are decelerating at 75 in/sec^2. Use the acceleration difference method. (Print the figure from its PDF file and draw on it.)

†7-80 The crosshead linkage shown in Figure P7-30c has 2 *DOF* with inputs at crossheads 2 and 5. Find $\mathbf{A}_B$, $\mathbf{A}_{P_3}$, and $\mathbf{A}_{P_4}$ if the crossheads are each moving toward the origin of the *XY* coordinate system with a speed of 20 in/sec and are decelerating at 75 in/sec^2. Use an analytical method.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†§7-81 The crosshead linkage shown in Figure P7-30c has 2 *DOF* with inputs at crossheads 2 and 5. At $t = 0$, crosshead 2 is at rest at the origin of the global *XY* coordinate system and crosshead 5 is at rest at (70, 0). Write a computer program to find and plot $\mathbf{A}_{P_3}$ and $\mathbf{A}_{P_4}$ for the first 5 sec of motion if $\mathbf{A}_2 = 0.5$ in/sec^2 upward and $\mathbf{A}_5 = 0.5$ in/sec^2 to the left.

§ Note that these can be long problems to solve and may be more appropriate for a project assignment than an overnight problem. In most cases, the solution can be checked with program LINKAGES.

7-82 The linkage in Figure P7-30d has the path of slider 6 perpendicular to the global *X* axis and link 2 aligned with the global *X* axis. Find α_2 and $\mathbf{A}_A$ in the position shown if the velocity of the slider is constant at 20 in/sec downward. Use the acceleration difference graphical method. Print the figure's PDF file and draw on it.

†7-83 The linkage in Figure P7-30d has the path of slider 6 perpendicular to the global *X* axis and link 2 aligned with the global *X* axis. Find α_2 and $\mathbf{A}_A$ in the position shown if the velocity of the slider is constant at 20 in/sec downward. Use an analytical method.

†7-84 The linkage in Figure P7-30d has the path of slider 6 perpendicular to the global *X* axis and link 2 aligned with the global *X* axis at $t = 0$. Write a computer program or use an equation solver to find and plot $\mathbf{A}_D$ as a function of θ_2 over the possible range of motion of link 2 in the global *XY* coordinate system.

†§7-85 For the linkage of Figure P7-30e, write a computer program or use an equation solver to find and plot $\mathbf{A}_D$ in the global coordinate system for one revolution of link 2 if ω_2 is constant at 10 rad/sec CW.

7-86 The linkage of Figure P7-30f has link 2 at 130° in the global *XY* coordinate system. Find $\mathbf{A}_D$ in the global coordinate system for the position shown if $\omega_2 = 15$ rad/sec CW and $\alpha_2 = 50$ rad/sec^2 CW. Use the acceleration difference graphical method. (Print the figure from its PDF file and draw on it.)

* Answers in Appendix F.

*7-87 Figure 3-14 shows a crank-shaper quick-return mechanism with the dimensions: $L_2 =$ 4.80 in, $L_4 = 24.00$ in, $L_5 = 19.50$ in. The distance from link 4's pivot (O_4) to link 2's pivot (O_2) is 16.50 in. The vertical distance from O_2 to point *C* on link 6 is 6.465 in. Use a graphical method to find the acceleration of point *C* on link 6 when the linkage is near the rightmost position shown with $\theta_2 = 45°$ measured from an axis running from an origin at O_2 through O_4. Assume that link 2 has a constant angular velocity of 2 rad/sec CW.

§7-88 Use the data in Problem 7-87 and an analytical method to calculate and plot the acceleration of point *C* on link 6 of that mechanism for one revolution of input crank 2.

7-89 Figure P7-22 shows a mechanism with dimensions. Use a graphical method to determine the acceleration of points *A* and *B* for the position shown for $\omega_2 = 24$ rad/s CW. Ignore links 5 and 6.

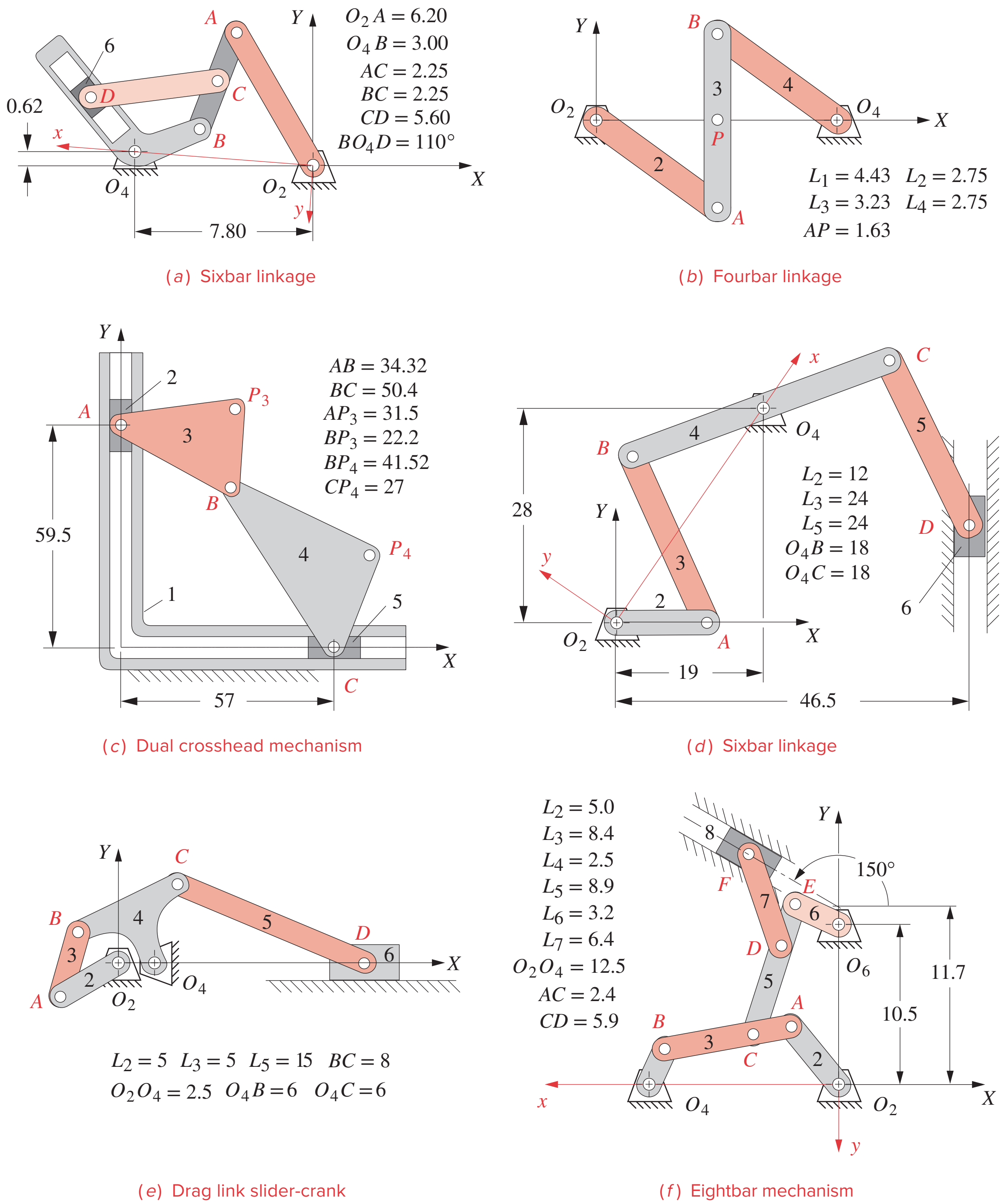

FIGURE P7-30

Problems 7-75 to 7-86

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TABLE P7-5 Data for Problems 7-93 to 7-94 ‡

Row	Link 2	Link 3	Offset	d	$\dot{d}$	$\ddot{d}$
a	1.4	4	1	2.5	10	0
b	2	6	–3	5	–12	5
c	3	8	2	8	–15	–10
d	3.5	10	1	–8	24	–4
e	5	20	–5	15	–50	10
f	3	13	0	–12	–45	50
g	7	25	10	25	100	18

‡ Drawings of these linkages are in the *PDF Problem Workbook* folder.

7-90 Figure P7-22 shows a mechanism with dimensions. Use an analytical method to calculate the accelerations of points *A* and *B* for the position shown for ω_2 = 24 rad/s CW. Ignore links 5 and 6.

7-91 Figure P7-23 shows a quick-return mechanism with dimensions. Use a graphical method to determine the accelerations of points *A* and *B* for the position shown for ω_2 = 16 rad/s CCW. Ignore links 5 and 6.

7-92 Figure P7-23 shows a quick-return mechanism with dimensions. Use an analytical method to calculate the accelerations of points *A* and *B* for the position shown for ω_2 = 16 rad/s CCW. Ignore links 5 and 6.

7-93 The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Figure P7-2. The link lengths and the values of d, $\dot{d}$, and $\ddot{d}$ are defined in Table P7-5. For the row(s) assigned, find the acceleration of the pin joint *A* and the angular acceleration of the crank using a graphical method.

7-94 The general linkage configuration and terminology for an offset fourbar slider-crank linkage are shown in Figure P7-2. The link lengths and the values of d, $\dot{d}$, and $\ddot{d}$ are defined in Table P7-5. For the rows assigned, find the acceleration of pin joint *A* and the angular acceleration of the crank using the analytic method. Draw the linkage to scale and label it before setting up the equations.

7.11 VIRTUAL LABORATORY *View the video* (35:38)† *View the lab* §

L7-1 View the video *Fourbar Linkage Virtual Laboratory*. Open the file *Virtual Fourbar Linkage Lab 7-1.doc* and follow the instructions as directed by your professor.

† http://www.designofmachinery.com/DOM/Fourbar_Machine_Virtual_laboratory.mp4

§ http://www.designofmachinery.com/DOM/Fourbar_Virtual_Lab.zip

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