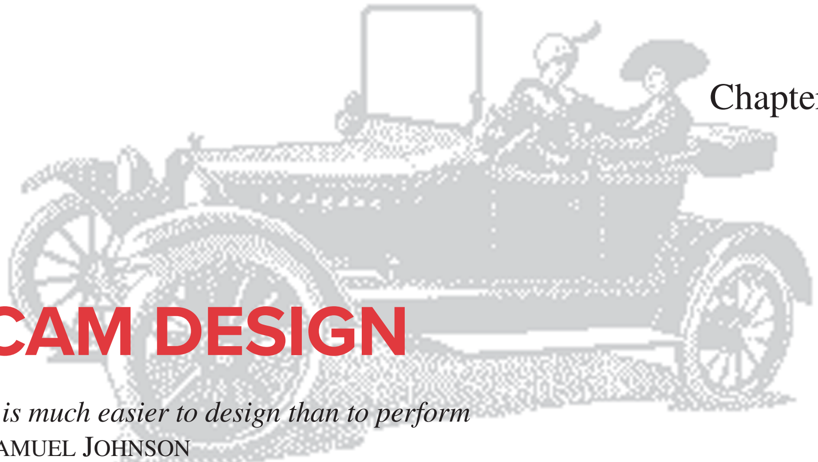

Chapter 8

CAM DESIGN

It is much easier to design than to perform
SAMUEL JOHNSON

8.0 INTRODUCTION *View the lecture video* (50:42)†

Cam-follower systems are frequently used in all kinds of machines. The valves in your automobile engine are opened by cams. Machines used in the manufacture of many consumer goods are full of cams.* Compared to linkages, cams are easier to design to give a specific output function, but they are much more difficult and expensive to make than a linkage. Cams are a form of degenerate fourbar linkage in which the coupler link has been replaced by a half joint as shown in Figure 8-1. This topic was discussed in Section 2.10 on linkage transformation (see also Figure 2-12). For any one instantaneous position of cam and follower, we can substitute an effective linkage that will, for that instantaneous position, have the same motion as the original. In effect, the cam-follower is a fourbar linkage with variable-length (effective) links. It is this conceptual difference that makes the cam-follower such a versatile and useful **function generator**. We can specify virtually any output function we desire and quite likely create a curved surface on the cam to generate that function in the motion of the follower. We are not limited to fixed-length links as we were in linkage synthesis. The cam-follower is an extremely useful mechanical device, without which the machine designer's tasks would be more difficult to accomplish. But, as with everything else in engineering, there are trade-offs. These will be discussed in later sections. A list of the variables used in this chapter is provided in Table 8-1.

This chapter will present the proper approach to designing a cam-follower system, and in the process also present some less than proper designs as examples of the problems that inexperienced cam designers often get into. Theoretical considerations of the mathematical functions commonly used for cam curves will be discussed. Methods for the derivation of custom polynomial functions, to suit any set of boundary conditions, will be presented. The task of sizing the cam with considerations of pressure angle and radius of curvature will be addressed, and manufacturing processes and their limitations discussed. The computer program DYNACAM will be used throughout the chapter as a tool

† http://www.designofmachinery.com/DOM/Cam_Design_I.mp4

* View the video http://www.designofmachinery.com/DOM/Pick_and_Place_Mechanism.mp4 to see an example of a cam driven mechanism from an actual production machine.

TABLE 8-1 Notation Used in This Chapter

t = time, seconds

θ = camshaft angle, degrees or radians (rad)

ω = camshaft angular velocity, rad/sec

β = total angle of any segment, rise, fall, or dwell, degrees or rad

h = total lift (rise or fall) of any one segment, length units

s or S = follower displacement, length units

$v = ds/d\theta$ = follower velocity, length/rad

$V = dS/dt$ = follower velocity, length/sec

$a = dv/d\theta$ = follower acceleration, length/rad^2

$A = dV/dt$ = follower acceleration, length/sec^2

$j = da/d\theta$ = follower jerk, length/rad^3

$J = dA/dt$ = follower jerk, length/sec^3

$s\ v\ a\ j$ refers to the group of diagrams, length units versus radians

$S\ V\ A\ J$ refers to the group of diagrams, length units versus time

R_b = base circle radius, length units

R_p = prime circle radius, length units

R_f = roller follower radius, length units

ε = eccentricity of cam-follower, length units

ϕ = pressure angle, degrees or radians

ρ = radius of curvature of cam surface, length units

ρ_{pitch} = radius of curvature of pitch curve, length units

ρ_{min} = minimum radius of curvature of pitch curve or cam surface, length units

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to present and illustrate design concepts and solutions. Information about this program is in Appendix A.

8.1 CAM TERMINOLOGY

Cam-follower systems can be classified in several ways: by *type of follower motion*, either **translating** or **rotating** (oscillating); by *type of cam*, radial, cylindrical, three-dimensional; by *type of joint closure*, either **force-** or **form**-closed; by *type of follower*, **curved** or **flat**, **rolling** or **sliding**; by *type of motion constraints*, **critical extreme position** (CEP), **critical path motion** (CPM); by *type of motion program*, **rise-fall** (RF), **rise-fall-dwell** (RFD), **rise-dwell-fall-dwell** (RDFD). We will now discuss each of these classification schemes in greater detail.

Type of Follower Motion

Figure 8-1a shows a system with an oscillating, or **rotating**, **follower**. Figure 8-1b shows a **translating follower**. These are analogous to the crank-rocker fourbar and the crank-slider fourbar linkages, respectively. An effective fourbar linkage can be substituted for the cam-follower system for any instantaneous position. The lengths of the effective links are determined by the instantaneous locations of the centers of curvature of cam and follower as shown in Figure 8-1. The velocities and accelerations of the cam-follower system can be found by analyzing the behavior of the effective linkage for any position. A proof of this can be found in reference [1]. Of course, the effective links change length as the cam-follower moves, giving it an advantage over a pure linkage as this allows greater flexibility in meeting the desired motion constraints.

The choice between these two forms of the cam-follower is usually dictated by the type of output motion desired. If true rectilinear translation is required, then the translating follower is dictated. If pure rotation output is needed, then the oscillator is the obvious choice. There are advantages to each of these approaches, separate from their motion characteristics, depending on the type of follower chosen. These will be discussed in a later section.

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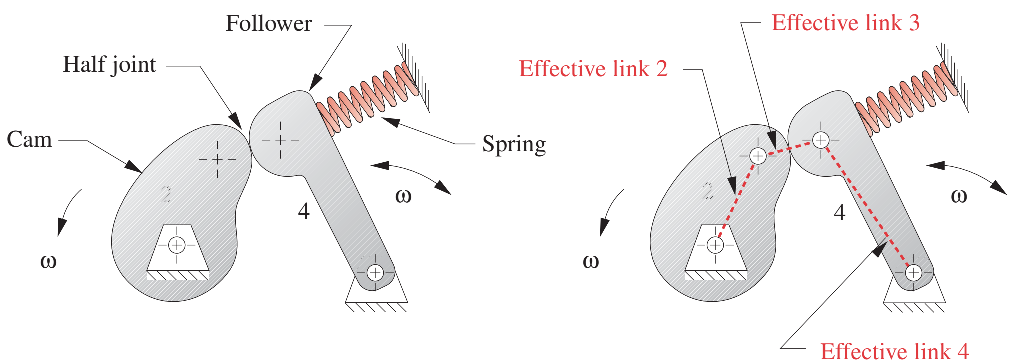
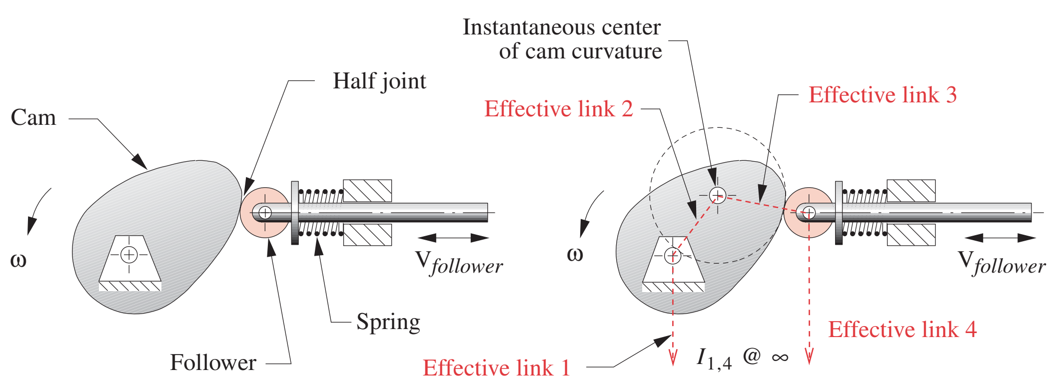


(*a*) An oscillating cam-follower has an effective pin-jointed fourbar equivalent



(*b*) A translating cam-follower has an effective fourbar slider-crank equivalent

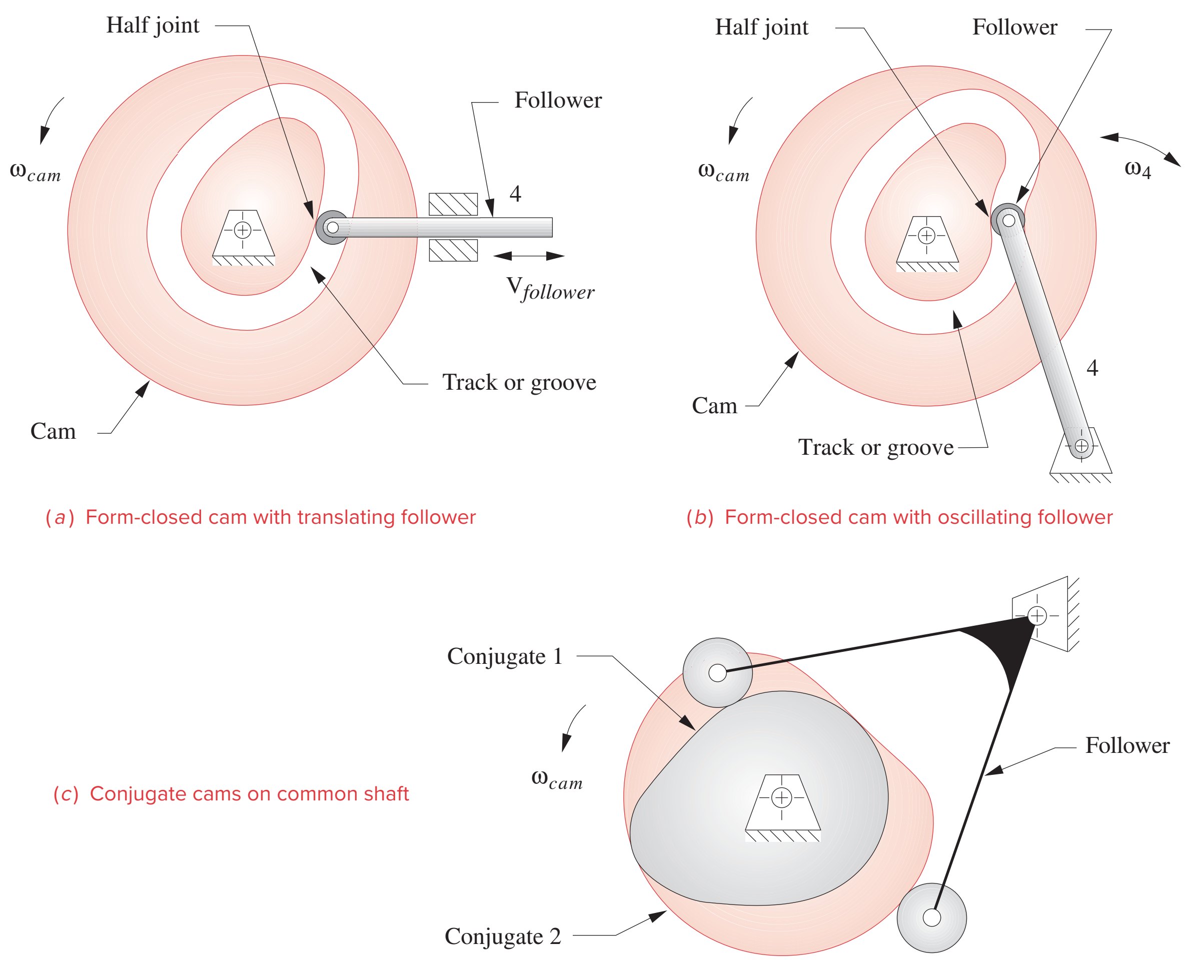
FIGURE 8-1

Effective linkages in the cam-follower mechanism

Type of Joint Closure

Force and form closure were discussed in Section 2.3 on the subject of joints and have the same meaning here. **Force closure**, as shown in Figure 8-1, *requires an external force be applied to the joint* in order to keep the two links, cam and follower, physically in contact. This force is usually provided by a spring. This force, defined as positive in a direction that closes the joint, cannot be allowed to become negative. If it does, the links have lost contact because a *force-closed joint can only push, not pull*. **Form closure**, as shown in Figure 8-2, *closes the joint by geometry*. No external force is required. There are really two cam surfaces in this arrangement, one surface on each side of the follower. Each surface pushes, in its turn, to drive the follower in both directions.

Figure 8-2a and b shows track or groove cams that capture a single follower in the groove and both push and pull on the follower. Figure 8-2c shows another variety of form-



(*a*) Form-closed cam with translating follower

(*b*) Form-closed cam with oscillating follower

(*c*) Conjugate cams on common shaft

FIGURE 8-2

Form-closed cam-follower systems

closed cam-follower arrangement, called **conjugate cams**. There are two cams fixed on a common shaft that are mathematical conjugates of one another. Two roller followers, attached to a common arm, are each pushed in opposite directions by the conjugate cams. When form-closed cams are used in automobile or motorcycle engine valve trains, they are called **desmodromic*** cams. There are advantages and disadvantages to both force- and form-closed arrangements that are discussed in Section 8-7.

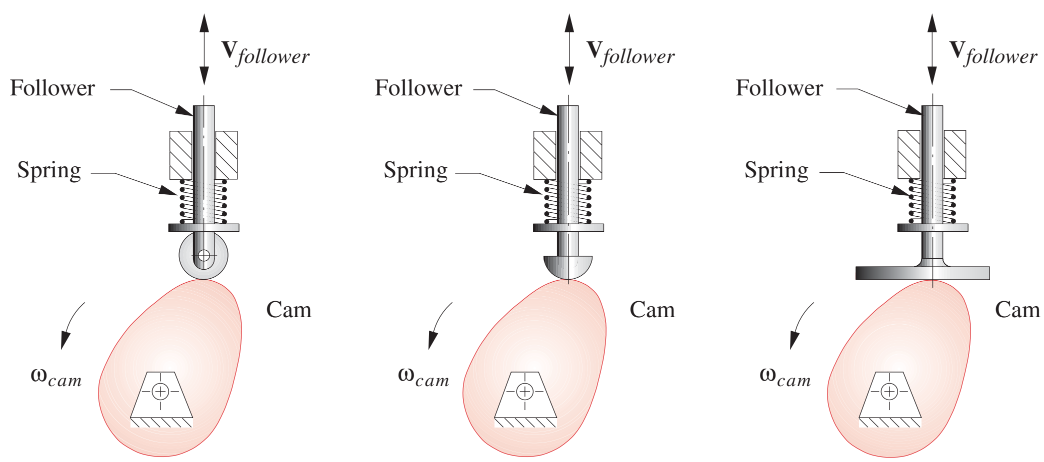
* More information on desmodromic cam-follower mechanisms can be found at http://members.chello.nl/~wgj.jansen/ where a number of models of their commercial implementations can be viewed in operation as movies.

Type of Follower

Follower, in this context, refers only to that part of the follower link that contacts the cam. Figure 8-3 shows three common arrangements, **flat-faced**, **mushroom** (curved), and **roller**. The roller follower has the advantage of lower (rolling) friction than the sliding contact of the other two but can be more expensive. **Flat-faced followers** can package smaller than roller followers for some cam designs and are often favored for that reason as well as cost for automotive valve trains. **Roller followers** are most frequently used in production machinery where their ease of replacement and availability from bearing manufacturers' stock in any quantities are advantages. Grooved or track cams require roller followers. Roller followers are essentially ball or roller bearings with customized mounting details. Figure 8-5a shows two common types of commercial roller followers. Flat-faced or **mushroom followers** are usually custom-designed and manufactured for each application. For high-volume applications such as automobile engines, the quantities are high enough to warrant a custom-designed follower.

Type of Cam

The direction of the follower's motion relative to the axis of rotation of the cam determines whether it is a **radial** or **axial** cam. All cams shown in Figures 8-1 to 8-3 are radial cams



(*a*) Roller follower (*b*) Mushroom follower (*c*) Flat-faced follower

FIGURE 8-3

Three common types of cam followers

because the follower motion is generally in a radial direction. Open **radial cams** are also called **plate cams**.

Figure 8-4 shows an **axial cam** whose follower moves parallel to the axis of cam rotation. This arrangement is also called a **face** cam if open (force-closed) and a **cylindrical** or **barrel** cam if grooved or ribbed (form-closed).

Figure 8-5b shows a selection of cams of various types.* Clockwise from the lower left, they are: an open (force-closed) axial or face cam; an axial grooved (track) cam (form-closed) with external gear; an open radial, or plate cam (force-closed); a ribbed axial cam (form-closed); an axial grooved (barrel) cam.

* View the video http://www.designofmachinery.com/DOM/Spring_Manufacturing.mp4 to see an example of spring manufacturing machinery that uses many cams.

Three-dimensional cams (Figure 8-5c) are a combination of radial and axial cams. The input rotation of the cam drives a follower train having both radial and axial motion. The follower motion has two coupled degrees of freedom.

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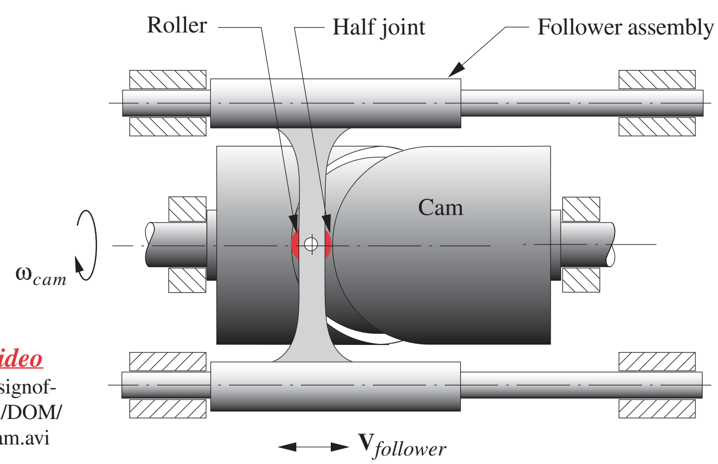
Type of Motion Constraints

There are two general categories of motion constraint, **critical extreme position** (CEP; also called endpoint specification) and **critical path motion** (CPM). **Critical extreme position** refers to the case in which the design specifications define the start and finish positions of the follower (i.e., extreme positions) but do not specify any constraints on the path motion between the extreme positions. This case is discussed in Sections 8.3 and 8.4 and is the easier of the two to design as the designer has great freedom to choose the cam functions that control the motion between extremes. **Critical path motion** is a more constrained problem than CEP because the path motion and/or one or more of its derivatives are defined over all or part of the interval of motion. This is analogous to **function generation** in the linkage design case except that with a cam we can achieve a continuous output function for the follower. Section 8.5 discusses this CPM case. It may only be possible to create an approximation of the specified function and still maintain suitable dynamic behavior.



View a video
http://www.designofmachinery.com/DOM/cylindrical_cam.avi

FIGURE 8-4

Axial, cylindrical, or barrel cam with form-closed, translating follower

(*a*) Commercial roller followers

*Courtesy of McGill Manufacturing Co.
South Bend, IN*

(*b*) Commercial cams and a motorcycle camshaft

(*c*) Three-dimensional cams

FIGURE 8-5

Cams and roller followers

Type of Motion Program

The motion programs **rise-fall** (RF), **rise-fall-dwell** (RFD), and **rise-dwell-fall-dwell** (RDFD) all refer mainly to the CEP case of motion constraint and in effect define how many dwells are present in the full cycle of motion, none (RF), one (RFD), or more than one (RDFD). **Dwells,** defined as *no output motion for a specified period of input motion*, are an important feature of cam-follower systems because it is very easy to create exact dwells in these mechanisms. The cam-follower is the design type of choice whenever a dwell is required. We saw in Section 3.9 how to design dwell linkages and found that at best we could obtain only an approximate dwell. The resulting single- or double-dwell linkages tend to be quite large for their output motion and are somewhat difficult to design. (See program LINKAGES for some built-in examples of these dwell linkages.) Cam-follower systems tend to be more compact than linkages for the same output motion.

If your need is for a **rise-fall** (RF) CEP motion, with no dwell, then you should really be considering a crank-rocker linkage rather than a cam-follower to obtain all the linkage's advantages over cams of reliability, ease of construction, and lower cost that were discussed in Section 2.18. If your needs for compactness outweigh those considerations, then the choice of a cam-follower in the RF case may be justified. Also, if you have a CPM design specification, and the motion or its derivatives are defined over the interval, then a cam-follower system is the logical choice in the RF case.

The **rise-fall-dwell** (RFD) and **rise-dwell-fall-dwell** (RDFD) cases are obvious choices for cam-followers for the reasons discussed above. However, each of these two cases has its own set of constraints on the behavior of the cam functions at the interfaces between the segments that control the rise, the fall, and the dwells. In general, we must match the **boundary conditions** (BCs) of the functions and their derivatives at all interfaces between the segments of the cam. This topic will be thoroughly discussed in the following sections.

8.2 *S V A J* DIAGRAMS

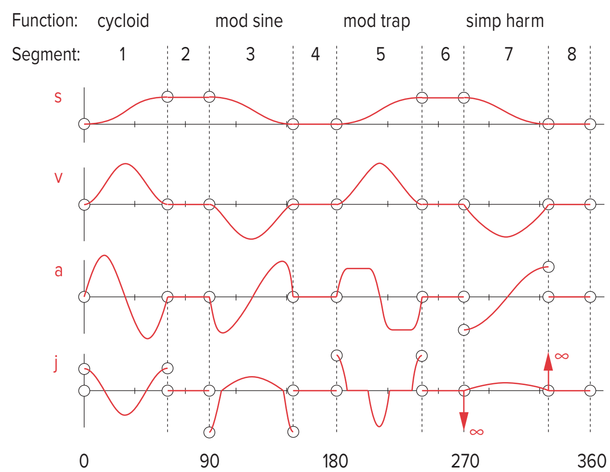
The first task faced by the cam designer is to select the mathematical functions to be used to define the motion of the follower. The easiest approach to this process is to "linearize" the cam, i.e., "unwrap it" from its circular shape and consider it as a function plotted on cartesian axes. We plot the displacement function s, its first derivative velocity v, its second derivative acceleration a, and its third derivative jerk j, all on aligned axes as a function of camshaft angle θ as shown in Figure 8-6. Note that we can consider the independent variable in these plots to be either time t or shaft angle θ, as we know the constant angular velocity ω of the camshaft and can easily convert from angle to time and vice versa.

$$\theta = \omega t \tag{8.1}$$

Figure 8-6a shows the specifications for a four-dwell cam that has eight segments, RDFDRDFDRDFD. Figure 8-6b shows the *s v a j* curves for the whole cam over 360 degrees of camshaft rotation. A cam design begins with a definition of the required cam functions and their *s v a j* diagrams. Functions for the nondwell cam segments should be chosen based on their velocity, acceleration, and jerk characteristics and the relationships at the interfaces between adjacent segments including the dwells. These function characteristics can be conveniently and quickly investigated with program DYNACAM which generated the data and plots shown in Figure 8-6.

Segment Number	Function Used	Start Angle	End Angle	Delta Angle
1	Cycloid rise	0	60	60
2	Dwell	60	90	30
3	ModSine fall	90	150	60
4	Dwell	150	180	30
5	ModTrap rise	180	240	60
6	Dwell	240	270	30
7	SimpHarm fall	270	330	60
8	Dwell	330	360	30

(*a*) Cam program specifications



(*b*) Plots of cam-follower's *s v a j* diagrams

FIGURE 8-6

Cycloidal, modified sine, modified trapezoid, and simple harmonic motion functions on a four-dwell cam

8.3 DOUBLE-DWELL CAM DESIGN—CHOOSING *S V A J* FUNCTIONS

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Many cam design applications require multiple dwells. The double-dwell case is quite common. Perhaps a **double-dwell** cam is driving a part feeding station on a production machine that makes toothpaste. This hypothetical cam's follower is fed an empty toothpaste tube (during the low dwell), then moves the empty tube into a loading station (during the rise), holds the tube absolutely still in a **critical extreme position** (CEP) while toothpaste is squirted into the open bottom of the tube (during the high dwell), and then retracts the filled tube back to the starting (zero) position and holds it in this other critical extreme position. At this point, another mechanism (during the low dwell) picks the tube up and carries it to the next operation, which might be to seal the bottom of the tube. A similar cam could be used to feed, align, and retract the tube at the bottom-sealing station as well.

Cam specifications such as this are often depicted on a timing diagram as shown in Figure 8-7 which is a graphical representation of the specified events in the machine cycle. A **machine's cycle** is defined as *one revolution of its master driveshaft*. In a complicated machine, such as our toothpaste maker, there will be a **timing diagram** for each subassembly in the machine. The time relationships among all subassemblies are defined by their timing diagrams which are all drawn on a common time axis. Obviously all these operations must be kept in precise synchrony and time phase for the machine to work.

This simple example in Figure 8-7 is a critical extreme position (CEP) case, because nothing is specified about the functions to be used to get from the low dwell position (one extreme) to the high dwell position (other extreme). The designer is free to choose any function that will do the job. Note that these specifications contain only information about the displacement function. The higher derivatives are not specifically constrained in this example. We will now use this problem to investigate several different ways to meet the specifications.

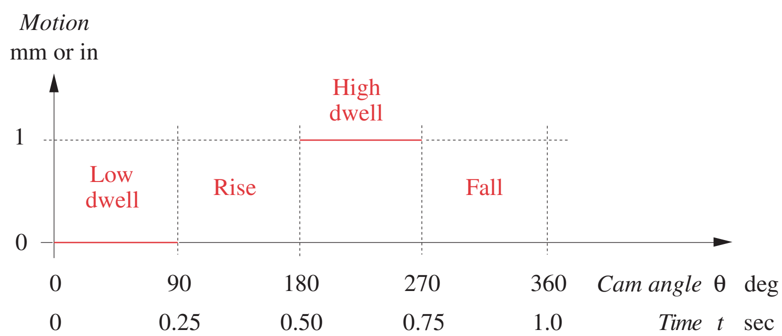

FIGURE 8-7

Cam timing diagram

EXAMPLE 8-1

Naive Cam Design—A Bad Cam.

Problem: Consider the following cam design CEP specification:

dwell	at zero displacement for 90 degrees (low dwell)
rise	1 in (25 mm) in 90 degrees
dwell	at 1 in (25 mm) for 90 degrees (high dwell)
fall	1 in (25 mm) in 90 degrees
cam ω	2π rad/sec = 1 rev/sec

Solution:

1 The naive or inexperienced cam designer might proceed with a design as shown in Figure 8-8a. Taking the given specifications literally, it is tempting to merely "connect the dots" on the timing diagram to create the displacement (s) diagram. (After all, when we wrap this s diagram around a circle to create the actual cam, it will look quite smooth despite the sharp corners on the s diagram.) The mistake our beginning designer is making here is to ignore the effect on the higher derivatives of the displacement function that results from this simplistic approach.

2 Figure 8-8b, c, and d shows the problem. Note that we have to treat each segment of the cam (rise, fall, dwell) as a separate entity in developing mathematical functions for the cam. Taking the rise segment (#2) first, the displacement function in Figure 8-8a during this portion is a straight line, or first-degree polynomial. The general equation for a straight line is:

$$y = mx + b \tag{8.2}$$

where m is the slope of the line and b is the y intercept. Substituting variables appropriate to this example in equation 8.2, angle θ replaces the independent variable x, and the displacement s replaces the dependent variable y. By definition, the constant slope m of the displacement is the velocity constant K_v.

3 For the rise segment, the y intercept b is zero because the low dwell position typically is taken as zero displacement by convention. Equation 8.2 then becomes:

$$s = K_v \theta \tag{8.3}$$

4 Differentiating with respect to θ gives a function for velocity during the rise.

$$v = K_v = \text{constant} \tag{8.4}$$

5 Differentiating again with respect to θ gives a function for acceleration during the rise.

$$a = 0 \tag{8.5}$$

This seems too good to be true (and it is). Zero acceleration means zero dynamic force. This cam appears to have no dynamic forces or stresses in it!

Figure 8-8 shows what is really happening here. If we return to the displacement function and graphically differentiate it twice, we will observe that, from the definition of the derivative as the instantaneous slope of the function, the acceleration is in fact zero **during the interval**. *But, at the boundaries of the interval*, where rise meets low dwell on one side and high dwell on the other, note that *the velocity function is multivalued*. *There are discontinuities at these boundaries*. The effect of these discontinuities is to create a portion of the velocity curve that has **infinite slope** and zero duration. This results in the *infinite spikes of acceleration* shown at those points.

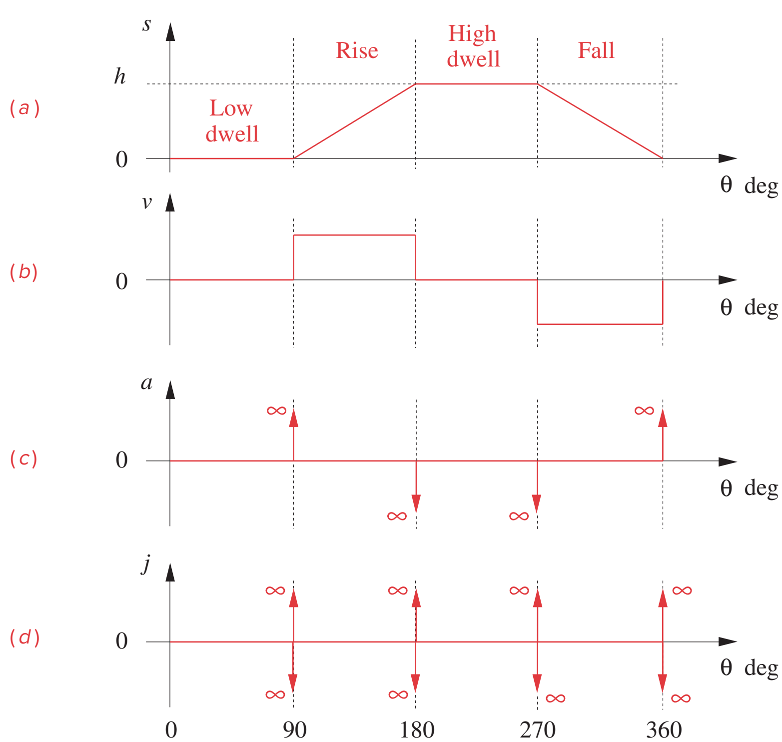

FIGURE 8-8

The *s v a j* diagrams of a "bad" cam design

These spikes are more properly called **Dirac delta functions**. Infinite acceleration cannot really be obtained, as it requires infinite force. Clearly the dynamic forces will be very large at these boundaries and will create high stresses and rapid wear. In fact, if this cam were built and run at any significant speeds, the sharp corners on the displacement diagram that are creating these theoretical infinite accelerations would be quickly worn to a smoother contour by the unsustainable stresses generated in the materials. *This is an unacceptable design.*

The unacceptability of this design is reinforced by the **jerk** diagram which shows theoretical values of **±infinity** at the discontinuities (the **doublet** function). The problem has been engendered by an inappropriate choice of displacement function. In fact, the cam designer should not be as concerned with the displacement function as with its higher derivatives.

The Fundamental Law of Cam Design

Any cam designed for operation at other than very low speeds must be designed with the following constraints:

The cam function must be continuous through the first and second derivatives of displacement across the entire interval (360 degrees).

Corollary:

The jerk function must be finite across the entire interval (360 degrees).

In any but the simplest of cams, the cam motion program cannot be defined by a single mathematical expression, but rather must be defined by several separate functions, each of which defines the follower behavior over one segment, or piece, of the cam. These expressions are sometimes called *piecewise functions*. These functions must have **third-order continuity** (the function plus two derivatives) at all boundaries. **The displacement, velocity, and acceleration functions must have no discontinuities in them.***

If any discontinuities exist in the acceleration function, then there will be infinite spikes, or Dirac delta functions, appearing in the derivative of acceleration, jerk. Thus the corollary merely restates the fundamental law of cam design. Our naive designer failed to recognize that by starting with a low-degree (linear) polynomial as the displacement function, discontinuities would appear in the upper derivatives.

Polynomial functions are one of the best choices for cams as we shall shortly see, but they do have one fault that can lead to trouble in this application. Each time they are differentiated, they reduce by one degree. Eventually, after enough differentiations, polynomials degenerate to zero degree (a constant value) as the velocity function in Figure 8-8b shows. Thus, by starting with a first-degree polynomial as a displacement function, it was inevitable that discontinuities would soon appear in its derivatives.

In order to obey the fundamental law of cam design, one must start with at least a fifth-degree polynomial (quintic) as the displacement function for a double-dwell cam. This will degenerate to a cubic function in the acceleration. The parabolic jerk function will have discontinuities, and the (unnamed) derivative of jerk will have infinite spikes in it. This is acceptable, as the jerk is still finite.

8

* This rule is stated by Neklutin[2] but is disputed by some other authors.[3],[4] Nevertheless, this author believes that it is a good (and simple) rule to follow in order to get acceptable dynamic results with high-speed cams. There are clear simulation data and experimental evidence that smooth jerk functions reduce residual vibrations in cam-follower systems.[10]

Simple Harmonic Motion (SHM)

Our naive cam designer recognized his mistake in choosing a straight-line function for the displacement. He also remembered a family of functions he had met in a calculus course that have the property of remaining continuous throughout any number of differentiations. These are the harmonic functions. On repeated differentiation, sine becomes cosine, which becomes negative sine, which becomes negative cosine, etc., ad infinitum. One never runs out of derivatives with the harmonic family of curves. In fact, differentiation of a harmonic function really only amounts to a 90° phase shift of the function. It is as though, when you differentiated it, you cut out, with a scissors, a different portion of the same continuous sine wave function, which is defined from minus infinity to plus infinity. The equations of simple harmonic motion (SHM) for a rise motion are:

$$s=\frac{h}{2}\left[1-\cos\left(\pi\frac{\theta}{\beta}\right)\right] \tag{8.6a}$$

$$v=\frac{\pi}{\beta}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right) \tag{8.6b}$$

$$a=\frac{\pi^2}{\beta^2}\frac{h}{2}\cos\left(\pi\frac{\theta}{\beta}\right) \tag{8.6c}$$

$$j=-\frac{\pi^3}{\beta^3}\frac{h}{2}\sin\left(\pi\frac{\theta}{\beta}\right) \tag{8.6d}$$

where h is the total rise, or lift, θ is the camshaft angle, and β is the total angle of the rise interval.

We have here introduced a notation to simplify the expressions. The independent variable in our cam functions is θ, the camshaft angle. The period of any one segment is defined as the angle β. Its value can, of course, be different for each segment. We normalize the independent variable θ by dividing it by the period of the segment β. Both θ and β are measured in radians (or both in degrees). The value of θ/β will then vary from 0 to 1 over any segment. It is a dimensionless ratio. Equations 8.6 define simple harmonic motion and its derivatives for this rise segment in terms of θ/β.

This family of harmonic functions appears, at first glance, to be well suited to the cam design problem of Figure 8-7. If we define the displacement function to be one of the harmonic functions, we should not "run out of derivatives" before reaching the acceleration function.

8

✍ EXAMPLE 8-2

Sophomoric* Cam Design—Simple Harmonic Motion—Still a Bad Cam.

Problem: Consider the same cam design CEP specification as in Example 8-1:

dwell	at zero displacement for 90 degrees (low dwell)
rise	1 in (25 mm) in 90 degrees
dwell	at 1 in (25 mm) for 90 degrees (high dwell)
fall	1 in (25 mm) in 90 degrees
cam ω	2π rad/sec = 1 rev/sec

* **Sophomoric**, from sophomore, *def. wise fool*, from the Greek, *sophos* = *wisdom*, *moros* = *fool.*

Solution:

1 Figure 8-9 shows a full-rise simple harmonic function* applied to the rise segment of our cam design problem.

2 Note that the velocity function is continuous, as it matches the zero velocity of the dwells at each end. The peak value is 6.28 in/sec (160 mm/sec) at the midpoint of the rise.

3 The acceleration function, however, is **not** continuous. It is a half-period cosine curve and has nonzero values at start and finish that are ±78.8 in/sec^2 (2.0 m/sec^2).

4 Unfortunately, the dwell functions, which adjoin this rise on each side, have zero acceleration as can be seen in Figure 8-6. Thus there are **discontinuities in the acceleration at each end of the interval** that uses this simple harmonic displacement function.

5 This violates the fundamental law of cam design and creates **infinite spikes of jerk** at the ends of this fall interval. **This is also an unacceptable design**.

---

* Though this is actually a half-period cosine wave, we will call it a *full-rise* (or *full-fall*) simple harmonic function to differentiate it from the *half-rise* (and *half-fall*) simple harmonic function which is actually a quarter-period cosine.

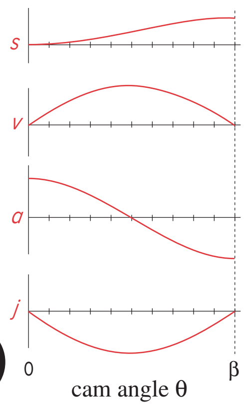

FIGURE 8-9

Simple harmonic motion with dwells has discontinuous acceleration.

What went wrong? While it is true that harmonic functions are differentiable ad infinitum, we are not dealing here with single harmonic functions. Our cam function over the entire interval is a **piecewise function** (Figure 8-6) made up of several segments, some of which may be dwell portions or other functions. A dwell will always have zero velocity and zero acceleration. Thus we must match the dwells' zero values at the ends of those derivatives of any nondwell segments that adjoin them. The simple harmonic displacement function, when used with dwells, does **not** satisfy the fundamental law of cam design. Its second derivative, acceleration, is nonzero at its ends and thus does not match the dwells required in this example.

The only case in which the simple harmonic displacement function will satisfy the fundamental law is the non-quick-return RF case, i.e., rise in 180° and fall in 180° with no dwells. Then the cam profile, if run against a flat-faced follower, becomes an eccentric as shown in Figure 8-10. As a single continuous (not piecewise) function, its derivatives are continuous also. Figure 8-11 shows the displacement (in inches) and acceleration functions (in *g*'s) of an eccentric cam as actually measured on the follower. The noise, or "ripple," on the acceleration curve is due to small, unavoidable, manufacturing errors. Manufacturing limitations will be discussed in a later section.

Cycloidal Displacement *View the lecture video* (51:17)†

The two bad examples of cam design described above should lead the cam designer to the conclusion that consideration only of the displacement function when designing a cam is erroneous. The better approach is to start with consideration of the higher derivatives, especially acceleration. The acceleration function, and to a lesser extent the jerk function, should be the principal concern of the designer. In some cases, especially when the mass of the follower train is large, or when there is a specification on velocity, that function must be carefully designed as well.

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† http://www.designof-machinery.com/DOM/Cam_Design_II.mp4

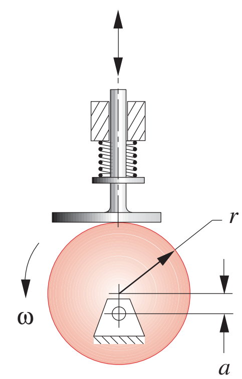

FIGURE 8-10

A flat-faced follower on an eccentric cam has simple harmonic motion.*

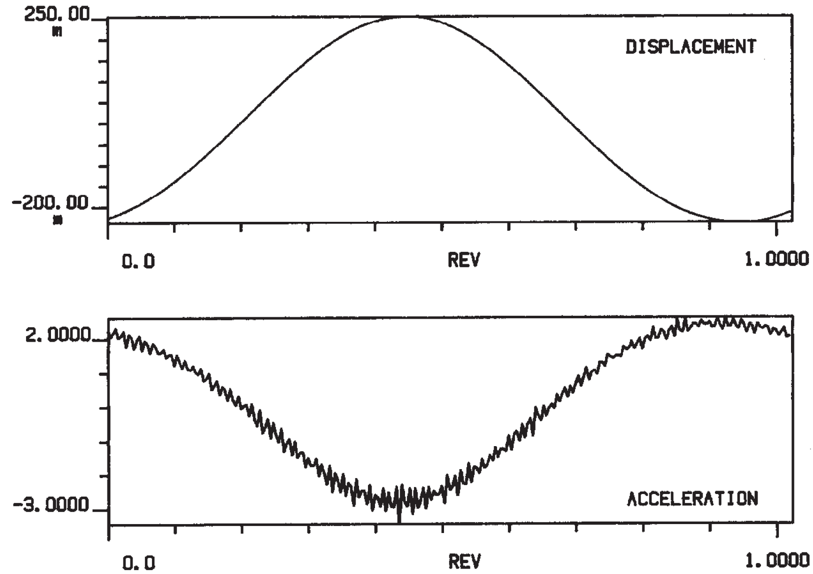

FIGURE 8-11

Displacement and acceleration as measured on the follower of an eccentric cam

With this in mind, we will redesign the cam for the same example specifications as above. This time we will start with the acceleration function. The harmonic family of functions still has advantages that make them attractive for these applications. Figure 8-12 shows a full-period sinusoid applied as the acceleration function. It meets the constraint of zero magnitude at each end to match the dwell segments that adjoin it. The equation for a sine wave is:

$$a = C\sin\left(2\pi\frac{\theta}{\beta}\right) \tag{8.7}$$

We have again normalized the independent variable θ by dividing it by the period of the segment β with both θ and β measured in radians. The value of θ/β ranges from 0 to 1 over any segment and is a dimensionless ratio. Since we want a full-cycle sine wave, we must multiply the argument by 2π. The argument of the sine function will then vary between 0 and 2π regardless of the value of β. The constant C defines the amplitude of the sine wave.

Integrate to obtain velocity,

$$\begin{aligned} a &= \frac{dv}{d\theta} = C\sin\left(2\pi\frac{\theta}{\beta}\right) \\ \int dv &= \int C\sin\left(2\pi\frac{\theta}{\beta}\right)d\theta \\ v &= -C\frac{\beta}{2\pi}\cos\left(2\pi\frac{\theta}{\beta}\right) + k_1 \end{aligned} \tag{8.8}$$

* If a roller follower is used instead of a flat-faced follower, then the trace of the roller follower center will still be a true eccentric, but the cam surface will not. This is due to the lead-lag error of the contact point of the roller with the cam surface. When going "uphill," the contact point leads the follower center and when going "downhill," it lags the center. This distorts the cam surface shape from that of a true eccentric circle. However, the motion of the follower will be simple harmonic motion as defined in Figure 8-10 regardless of follower type.

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where k_1 is the constant of integration. To evaluate k_1, substitute the boundary condition $v = 0$ at $\theta = 0$, since we must match the zero velocity of the dwell at that point. The constant of integration is then:

$$k_1 = C\frac{\beta}{2\pi}$$

and:

$$v = C\frac{\beta}{2\pi}\left[1 - \cos\left(2\pi\frac{\theta}{\beta}\right)\right] \tag{8.9}$$

Note that substituting the boundary values at the other end of the interval, $v = 0$, $\theta = \beta$, will give the same result for k_1. Integrate again to obtain displacement:

$$v = \frac{ds}{d\theta} = C\frac{\beta}{2\pi}\left[1 - \cos\left(2\pi\frac{\theta}{\beta}\right)\right]$$

$$\int ds = \int\left\{C\frac{\beta}{2\pi}\left[1 - \cos\left(2\pi\frac{\theta}{\beta}\right)\right]\right\}d\theta \tag{8.10}$$

$$s = C\frac{\beta}{2\pi}\theta - C\frac{\beta^2}{4\pi^2}\sin\left(2\pi\frac{\theta}{\beta}\right) + k_2$$

To evaluate k_2, substitute the boundary condition $s = 0$ at $\theta = 0$, since we must match the zero displacement of the dwell at that point. To evaluate the amplitude constant C, substitute the boundary condition $s = h$ at $\theta = \beta$, where h is the maximum follower rise (or lift) required over the interval and is a constant for any one cam specification.

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$$k_2 = 0$$
$$C = 2\pi\frac{h}{\beta^2} \tag{8.11}$$

Substituting the value of the constant C in equation 8.7 for acceleration gives:

$$a = 2\pi\frac{h}{\beta^2}\sin\left(2\pi\frac{\theta}{\beta}\right) \tag{8.12a}$$

Differentiating with respect to θ gives the expression for jerk.

$$j = 4\pi^2\frac{h}{\beta^3}\cos\left(2\pi\frac{\theta}{\beta}\right) \tag{8.12b}$$

Substituting the values of the constants C and k_1 in equation 8.9 for velocity gives:

$$v = \frac{h}{\beta}\left[1 - \cos\left(2\pi\frac{\theta}{\beta}\right)\right] \tag{8.12c}$$

This velocity function is the sum of a negative cosine term and a constant term. The coefficient of the cosine term is equal to the constant term. This results in a velocity curve that starts and ends at zero and reaches a maximum magnitude at $\beta/2$ as can be seen in Figure 8-12. Substituting the values of the constants C, k_1, and k_2 in equation 8.10 for displacement gives:

$$s = h\left[\frac{\theta}{\beta} - \frac{1}{2\pi}\sin\left(2\pi\frac{\theta}{\beta}\right)\right] \tag{8.12d}$$

Note that this displacement expression is the sum of a straight line of slope h and a negative sine wave. The sine wave is, in effect, "wrapped around" the straight line as can be seen in Figure 8-12. Equation 8.12d is the expression for a cycloid. This cam function is referred to either as **cycloidal displacement** or **sinusoidal acceleration**.

In the form presented, with θ (in radians) as the independent variable, the units of equation 8.12d are length, of equation 8.12c length/rad, of equation 8.12a length/rad^2, and of equation 8.12b length/rad^3. To convert these equations to a time base, multiply velocity v by the camshaft angular velocity ω (in rad/sec), multiply acceleration a by ω^2, and jerk j by ω^3.

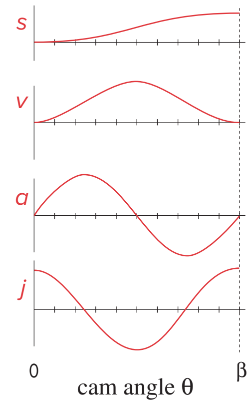

FIGURE 8-12
Sinusoidal acceleration gives cycloidal displacement.

EXAMPLE 8-3

Junior Cam Design—Cycloidal Displacement—An Acceptable Cam.

Problem: Consider the same cam design CEP specification as in Examples 8-1 and 8-2:

dwell	at zero displacement for 90 degrees (low dwell)
rise	1 in (25 mm) in 90 degrees
dwell	at 1 in (25 mm) for 90 degrees (high dwell)
fall	1 in (25 mm) in 90 degrees
cam ω	2π rad/sec = 1 rev/sec

Solution:

1 The cycloidal displacement function is an acceptable one for this double-dwell cam specification. Its derivatives are continuous through the acceleration function as seen in Figure 8-12. The peak acceleration is 100.4 in/sec^2 (2.55 m/sec^2).

2 The jerk curve in Figure 8-12 is discontinuous at its boundaries but is of finite magnitude, and this is acceptable. Its peak value is 2523 in/sec^3 (64 m/sec^3).

3 The velocity is smooth and matches the zeros of the dwell at each end. Its peak value is 8 in/sec (0.2 m/sec).

4 The only drawback to this function is that it has relatively large magnitudes of peak acceleration and peak velocity compared to some other possible functions for the double-dwell case.

The reader may open the file E08-03.cam in program DYNACAM to investigate this example in more detail.

Combined Functions

Dynamic force is proportional to acceleration. We generally would like to minimize dynamic forces, and thus should be looking to minimize the magnitude of the acceleration function as well as to keep it continuous. Kinetic energy is proportional to velocity

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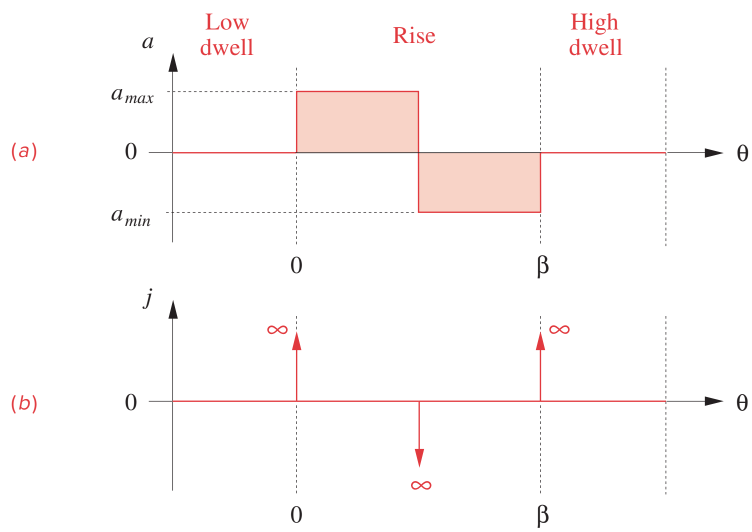

FIGURE 8-13

Constant acceleration gives infinite jerk.

squared. We also would like to minimize stored kinetic energy, especially with large mass follower trains, and so are concerned with the magnitude of the velocity function as well.

CONSTANT ACCELERATION If we wish to minimize the peak value of the magnitude of the acceleration function for a given problem, the function that would best satisfy this constraint is the square wave as shown in Figure 8-13. This function is also called **constant acceleration**. The square wave has the property of minimum peak value for a given area in a given interval. However, this function is not continuous. It has discontinuities at the beginning, middle, and end of the interval, so, by itself, **this is unacceptable as a cam acceleration function**.

TRAPEZOIDAL ACCELERATION The square wave's discontinuities can be removed by simply "knocking the corners off" the square wave function and creating the **trapezoidal acceleration** function shown in Figure 8-14a. The area lost from the "knocked off corners" must be replaced by increasing the peak magnitude above that of the original square wave in order to maintain the required specifications on lift and duration. But, this increase in peak magnitude is small, and the theoretical maximum acceleration can be significantly less than the theoretical peak value of the sinusoidal acceleration (cycloidal displacement) function. One disadvantage of this trapezoidal function is its discontinuous jerk function, as shown in Figure 8-14b. Ragged jerk functions such as this tend to excite vibratory behavior in the follower train due to their high harmonic content. The cycloidal's sinusoidal acceleration has a relatively smoother cosine jerk function with only two discontinuities in the interval and is preferable to the trapezoid's square waves of jerk. But the cycloidal's theoretical peak acceleration will be larger, which is not desirable. So, trade-offs must be made in selecting the cam functions.

MODIFIED TRAPEZOIDAL ACCELERATION An improvement can be made to the trapezoidal acceleration function by substituting pieces of sine waves for the sloped sides

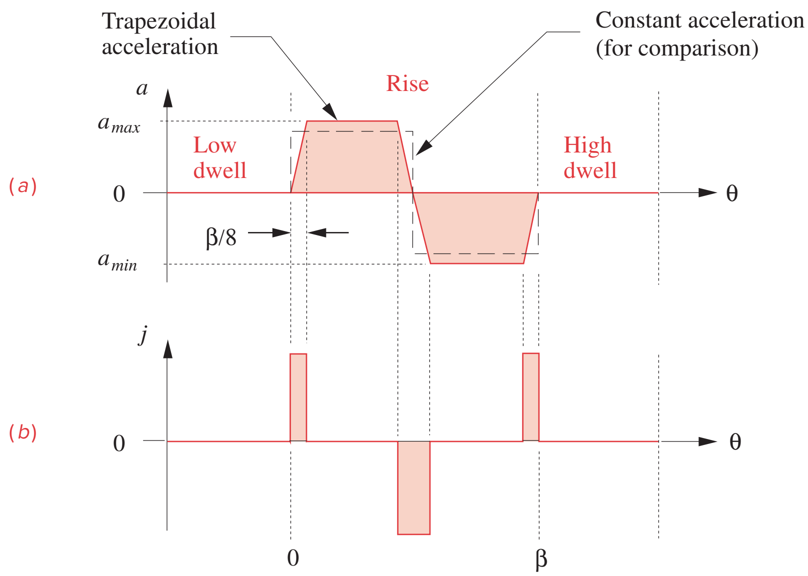

FIGURE 8-14

Trapezoidal acceleration gives finite jerk.

of the trapezoids as shown in Figure 8-15. This function is called the **modified trapezoidal acceleration** curve.* This function is a marriage of the sine acceleration and constant acceleration curves. Conceptually, a full period sine wave is cut into fourths and "pasted into" the square wave to provide a smooth transition from the zeros at the endpoints to the maximum and minimum peak values, and to make the transition from maximum to minimum in the center of the interval. The portions of the total segment period (β) used for the sinusoidal parts of the function can be varied. The most common arrangement is to cut the square wave at β/8, 3β/8, 5β/8, and 7β/8 to insert the pieces of sine wave as shown in Figure 8-15.

The modified trapezoidal function defined above is one of many combined functions created for cams by piecing together various functions, while being careful to match the values of the *s, v,* and *a* curves at all the interfaces between the joined functions. It has the advantage of relatively low theoretical peak acceleration, and reasonably rapid, smooth transitions at the beginning and end of the interval. The modified trapezoidal cam function has been a popular and often used program for double-dwell cams.

Modified Sinusoidal Acceleration† The sine acceleration curve (cycloidal displacement) has the advantage of smoothness (less ragged jerk curve) compared to the modified trapezoid but has higher theoretical peak acceleration. By combining two harmonic (sinusoid) curves of different frequencies, we can retain some of the smoothness characteristics of the cycloid and also reduce the peak acceleration compared to the cycloid. As an added bonus we will find that the peak velocity is also lower than in either the cycloidal or modified trapezoid. Figure 8-16 shows how the modified sine acceleration curve is made up of pieces of two sinusoid functions, one of higher frequency than the other. The first and last quarters of the high-frequency (short period, β/2) sine curve

* Developed by C. N. Neklutin of Universal Match Corp. See ref. [2].

† Developed by E. H. Schmidt of DuPont.

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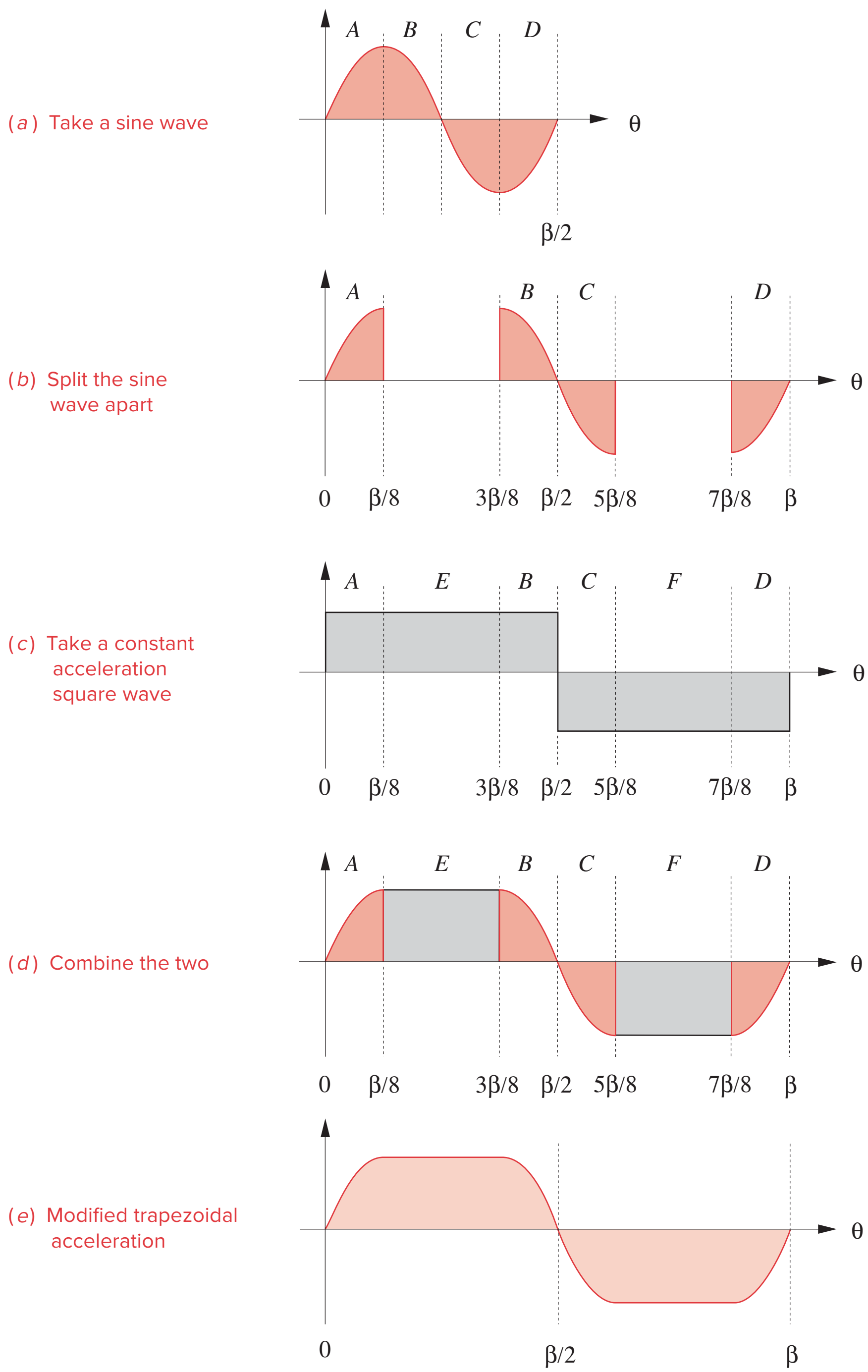

FIGURE 8-15

Creating the modified trapezoidal acceleration function are used for the first and last eighths of the combined function. The center half of the low-frequency (long period, $3\beta/2$) sine wave is used to fill in the center three-fourths of the combined curve. Obviously, the magnitudes of the two curves and their derivatives must be matched at their interfaces in order to avoid discontinuities.

The SCCA Family of Double-Dwell Functions

SCCA stands for *Sine-Constant-Cosine-Acceleration* and refers to a family of acceleration functions that includes constant acceleration, simple harmonic, modified trapezoid, modified sine, and cycloidal curves.[11] These very different looking curves can all be defined by the same equation with only a change of numeric parameters. In like fashion, the equations for displacement, velocity, and jerk for all these SCCA functions differ only by their parametric values.

To reveal this similitude, it is first necessary to normalize the variables in the equations. We have already normalized the independent variable, cam angle θ, dividing it by the interval period β. We will further simplify the notation by defining

$$x = \frac{\theta}{\beta} \tag{8.13a}$$

The normalized variable x then runs from 0 to 1 over any interval. The normalized follower displacement is

$$y = \frac{s}{h} \tag{8.13b}$$

where s is the instantaneous follower displacement and h is the total lift. The normalized variable y then runs from 0 to 1 over any follower displacement.

The general shapes of the $s\ v\ a\ j$ functions of the SCCA family are shown in Figure 8-17. The interval β is divided into five zones, numbered 1 through 5. Zones 0 and 6 represent the dwells on either side of the rise (or fall). The widths of zones 1 to 5 are defined in terms of β and one of three parameters, b, c, d. The values of these three parameters define the shape of the curve and define its identity within the family of functions. The normalized velocity, acceleration, and jerk are denoted, respectively, as:

$$y' = \frac{dy}{dx} \qquad y'' = \frac{d^2y}{dx^2} \qquad y''' = \frac{d^3y}{dx^3} \tag{8.14}$$

In zone 0, all functions are zero. The expressions for the functions within each other zone of Figure 8-17 are as follows:

Zone 1: $\quad 0 \le x \le \dfrac{b}{2}: \qquad b \ne 0$

$$y = C_a\left[\frac{b}{\pi}x - \left(\frac{b}{\pi}\right)^2 \sin\left(\frac{\pi}{b}x\right)\right] \tag{8.15a}$$

$$y' = C_a\left[\frac{b}{\pi} - \frac{b}{\pi}\cos\left(\frac{\pi}{b}x\right)\right] \tag{8.15b}$$

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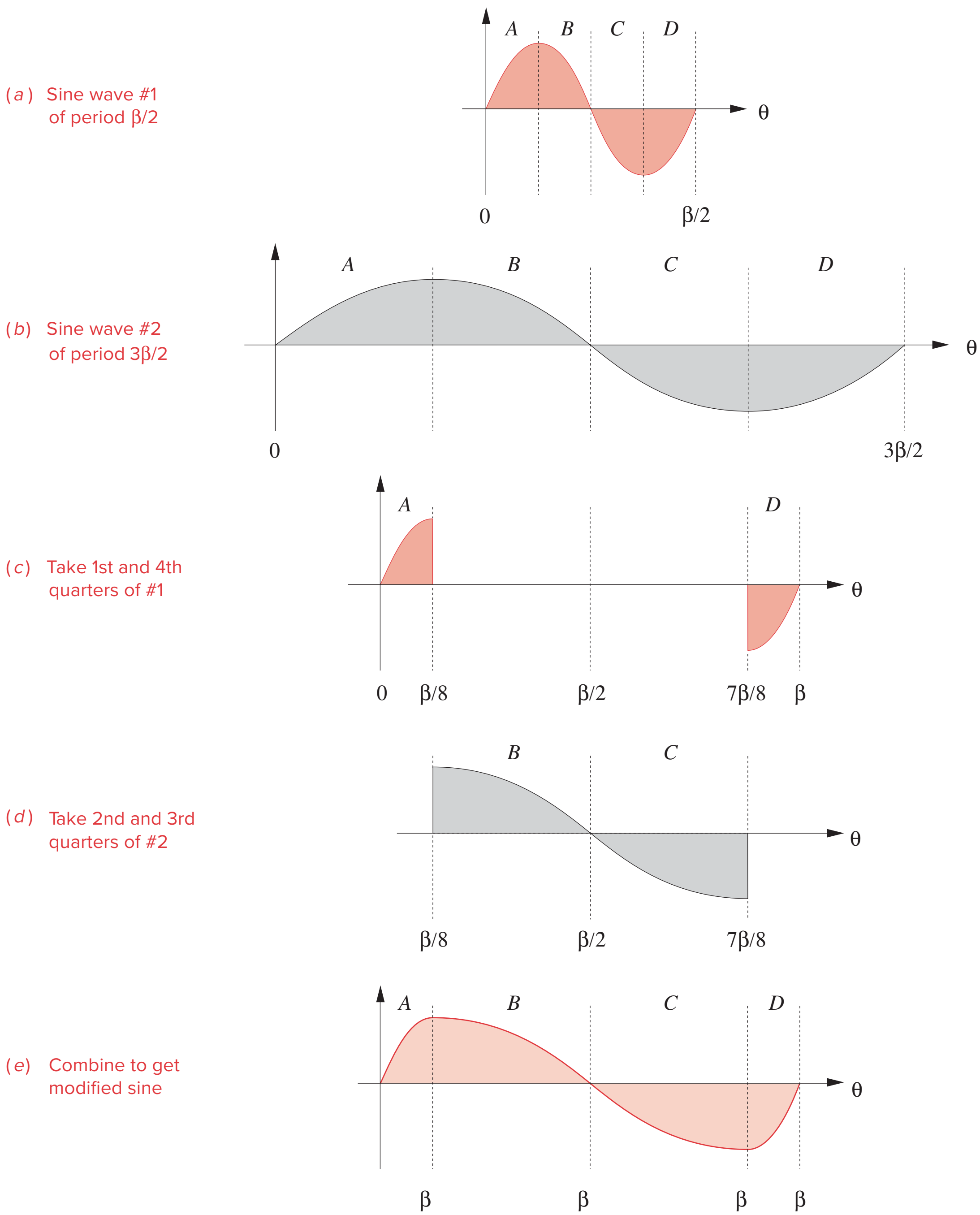

FIGURE 8-16

Creating the modified sine acceleration function

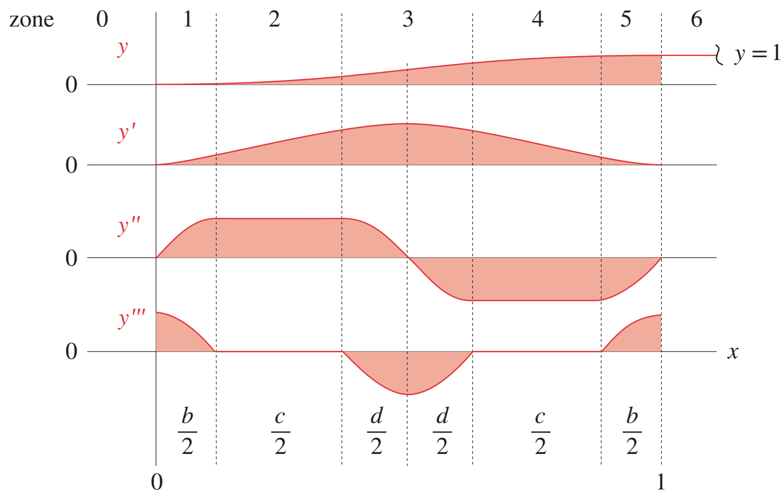

FIGURE 8-17

Parameters for the normalized SCCA family of curves

$$y'' = C_a \sin\left(\frac{\pi}{b}x\right) \tag{8.15c}$$

$$y''' = C_a \frac{\pi}{b}\cos\left(\frac{\pi}{b}x\right) \tag{8.15d}$$

Zone 2: $\frac{b}{2} \le x \le \frac{1-d}{2}$

$$y = C_a\left[\frac{x^2}{2} + b\left(\frac{1}{\pi} - \frac{1}{2}\right)x + b^2\left(\frac{1}{8} - \frac{1}{\pi^2}\right)\right] \tag{8.16a}$$

$$y' = C_a\left[x + b\left(\frac{1}{\pi} - \frac{1}{2}\right)\right] \tag{8.16b}$$

$$y'' = C_a \tag{8.16c}$$

$$y''' = 0 \tag{8.16d}$$

Zone 3: $\frac{1-d}{2} \le x \le \frac{1+d}{2}$: $d \ne 0$

$$y = C_a\left\{\left(\frac{b}{\pi} + \frac{c}{2}\right)x + \left(\frac{d}{\pi}\right)^2 + b^2\left(\frac{1}{8} - \frac{1}{\pi^2}\right) - \frac{(1-d)^2}{8} - \left(\frac{d}{\pi}\right)^2 \cos\left[\frac{\pi}{d}\left(x - \frac{1-d}{2}\right)\right]\right\} \tag{8.17a}$$

$$y' = C_a\left\{\frac{b}{\pi} + \frac{c}{2} + \frac{d}{\pi}\sin\left[\frac{\pi}{d}\left(x - \frac{1-d}{2}\right)\right]\right\} \tag{8.17b}$$

$$y'' = C_a \cos\left[\frac{\pi}{d}\left(x - \frac{1-d}{2}\right)\right] \tag{8.17c}$$

$$y''' = -C_a \frac{\pi}{d} \sin\left[\frac{\pi}{d}\left(x - \frac{1-d}{2}\right)\right] \tag{8.17d}$$

Zone 4: $\frac{1+d}{2} \le x \le 1 - \frac{b}{2}$

$$y = C_a\left[-\frac{x^2}{2} + \left(\frac{b}{\pi} + 1 - \frac{b}{2}\right)x + \left(2d^2 - b^2\right)\left(\frac{1}{\pi^2} - \frac{1}{8}\right) - \frac{1}{4}\right] \tag{8.18a}$$

$$y' = C_a\left(-x + \frac{b}{\pi} + 1 - \frac{b}{2}\right) \tag{8.18b}$$

$$y'' = -C_a \tag{8.18c}$$

$$y''' = 0 \tag{8.18d}$$

Zone 5: $1 - \frac{b}{2} \le x \le 1: \quad b \ne 0$

$$y = C_a\left\{\frac{b}{\pi}x + \frac{2\left(d^2 - b^2\right)}{\pi^2} + \frac{(1-b)^2 - d^2}{4} - \left(\frac{b}{\pi}\right)^2 \sin\left[\frac{\pi}{b}(x-1)\right]\right\} \tag{8.19a}$$

$$y' = C_a\left\{\frac{b}{\pi} - \frac{b}{\pi}\cos\left[\frac{\pi}{b}(x-1)\right]\right\} \tag{8.19b}$$

$$y'' = C_a \sin\left[\frac{\pi}{b}(x-1)\right] \tag{8.19c}$$

$$y''' = C_a \frac{\pi}{b}\cos\left[\frac{\pi}{b}(x-1)\right] \tag{8.19d}$$

Zone 6: $x > 1$

$$y = 1, \qquad y' = y'' = y''' = 0 \tag{8.20}$$

The coefficient C_a is a dimensionless peak acceleration factor. It can be evaluated from the fact that, at the end of the rise in zone 5 when $x = 1$, the expression for displacement (equation 8.19a) must have $y = 1$ to match the dwell in zone 6. Setting the right side of equation 8.19a equal to 1 gives:

$$C_a = \frac{4\pi^2}{\left(\pi^2 - 8\right)\left(b^2 - d^2\right) - 2\pi(\pi - 2)b + \pi^2} \tag{8.21a}$$

We can also define dimensionless peak factors (coefficients) for velocity (C_v) and jerk (C_j) in terms of C_a. The velocity is a maximum at $x = 0.5$. Thus C_v will equal the right side of equation 8.17b when $x = 0.5$.

TABLE 8-2 Parameters and Coefficients for the SCCA Family of Functions

Function	b	c	d	C_v	C_a	C_j
Constant acceleration	0.00	1.00	0.00	2.0000	4.0000	infinite
Modified trapezoid	0.25	0.50	0.25	2.0000	4.8881	61.426
Simple harmonic	0.00	0.00	1.00	1.5708	4.9348	infinite
Modified sine	0.25	0.00	0.75	1.7596	5.5280	69.466
Cycloidal displacement	0.50	0.00	0.50	2.0000	6.2832	39.478

$$C_v = C_a\left(\frac{b+d}{\pi} + \frac{c}{2}\right) \tag{8.21b}$$

The jerk is a maximum at $x = 0$. Setting the right side of equation 8.15d to zero gives:

$$C_j = C_a \frac{\pi}{b} \qquad b \neq 0 \tag{8.21c}$$

Table 8-2 shows the values of *b, c, d* and the resulting factors C_v, C_a, and C_j for the five standard members of the SCCA family. There is an infinity of related functions with values of these parameters between those shown. Figure 8-18 shows these five members of the "acceleration family" superposed with their design parameters noted. Note that all the functions shown in Figure 8-18 were generated with the same set of equations (8.15 through 8.21) with only changes to the values of the parameters *b, c*, and *d*. A *TKSolver* file (SCCA.tk) that is provided calculates and plots any of the SCCA family of normalized functions, along with their coefficients C_v, C_a, and C_j, in response to the input of values for *b, c*, and *d*. Note also that there is an infinity of family members as *b*, *c*, and *d* can take on any set of values that add to 1.

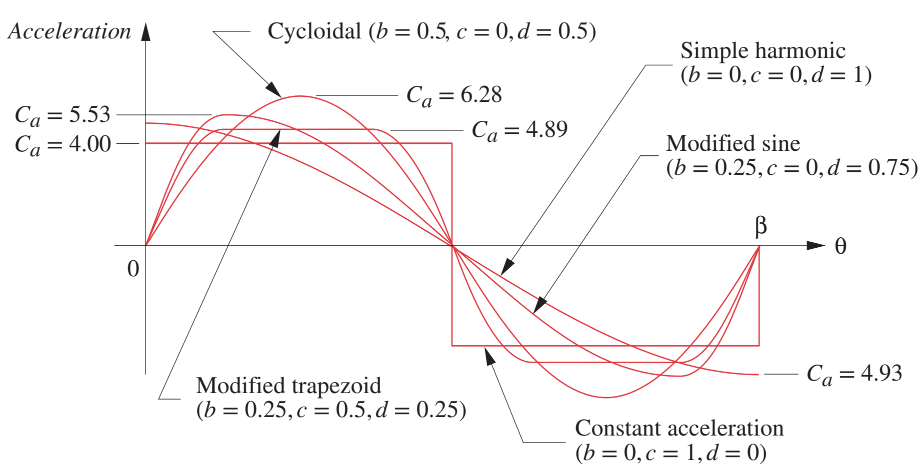

FIGURE 8-18

Comparison of five acceleration functions in the SCCA family

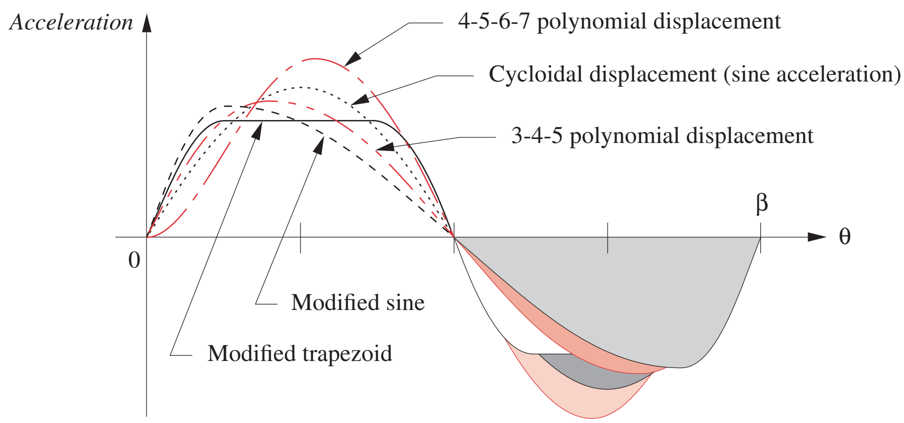

FIGURE 8-19

Comparison of five acceptable double-dwell cam acceleration functions

To apply the SCCA functions to an actual cam design problem only requires that they be multiplied or divided by factors appropriate to the particular problem, namely the actual rise h, the actual duration β (rad), and the cam velocity ω (rad/sec).

$$\begin{aligned} s &= hy \quad \text{length} & S &= s \quad \text{length} \\ v &= \frac{h}{\beta} y' \quad \text{length/rad} & V &= v\omega \quad \text{length/sec} \\ a &= \frac{h}{\beta^2} y'' \quad \text{length/rad}^2 & A &= a\omega^2 \quad \text{length/sec}^2 \\ j &= \frac{h}{\beta^3} y''' \quad \text{length/rad}^3 & J &= j\omega^3 \quad \text{length/sec}^3 \end{aligned} \tag{8.22}$$

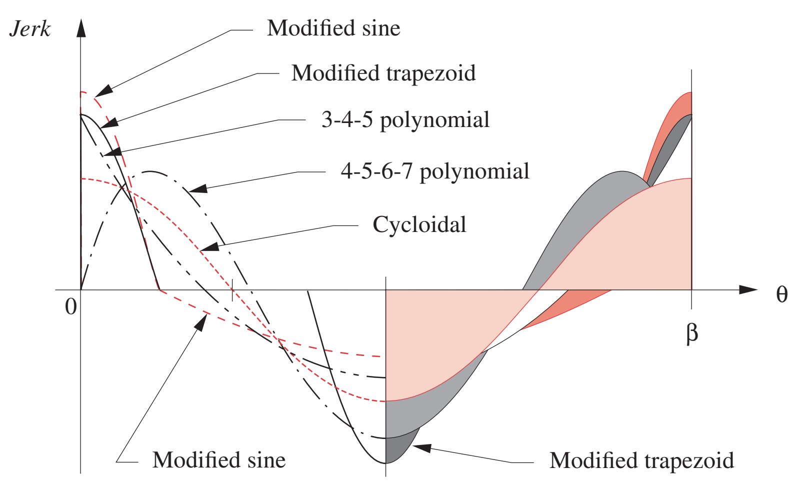

FIGURE 8-20

Comparison of five double-dwell cam jerk functions

TABLE 8-3 Factors for Peak Velocity and Acceleration of Some Cam Functions

Function	Max. Veloc.	Max. Accel.	Max. Jerk	Comments
Constant accel.	$2.000\,h/\beta$	$4.000\,h/\beta^2$	Infinite	∞ jerk—not acceptable
Harmonic disp.	$1.571\,h/\beta$	$4.945\,h/\beta^2$	Infinite	∞ jerk—not acceptable
Trapezoid accel.	$2.000\,h/\beta$	$5.300\,h/\beta^2$	$44\,h/\beta^3$	Not as good as mod. trap.
Mod. trap. accel.	$2.000\,h/\beta$	$4.888\,h/\beta^2$	$61\,h/\beta^3$	Low accel. but rough jerk
Mod. sine accel.	$1.760\,h/\beta$	$5.528\,h/\beta^2$	$69\,h/\beta^3$	Low veloc., good accel.
3-4-5 poly. disp.	$1.875\,h/\beta$	$5.777\,h/\beta^2$	$60\,h/\beta^3$	Good compromise
Cycloidal disp.	$2.000\,h/\beta$	$6.283\,h/\beta^2$	$40\,h/\beta^3$	Smooth accel. and jerk
4-5-6-7 poly. disp.	$2.188\,h/\beta$	$7.526\,h/\beta^2$	$52\,h/\beta^3$	Smooth jerk, high accel.

Figure 8-19 shows a comparison of the shapes and relative magnitudes of five acceptable cam acceleration programs including the cycloidal, modified trapezoid, and modified sine acceleration curves.* The cycloidal curve has a theoretical peak acceleration that is approximately 1.3 times that of the modified trapezoid's peak value for the same cam specification. The peak value of acceleration for the modified sine is between those of the cycloidal and modified trapezoids. Table 8-3 lists the peak values of acceleration, velocity, and jerk for these functions in terms of the total rise h and period β.

* The 3-4-5 and 4-5-6-7 polynomial functions also shown in the figure will be discussed in a later section.

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Figure 8-20 compares the jerk curves for the same functions. The modified sine jerk is somewhat less ragged than the modified trapezoid jerk but not as smooth as that of the cycloid, which is a full-period cosine. Figure 8-21 compares their velocity curves. The peak velocities of the cycloidal and modified trapezoid functions are the same, so each will store the same peak kinetic energy in the follower train. The peak velocity of the modified sine is the lowest of the five functions shown. This is the principal advantage

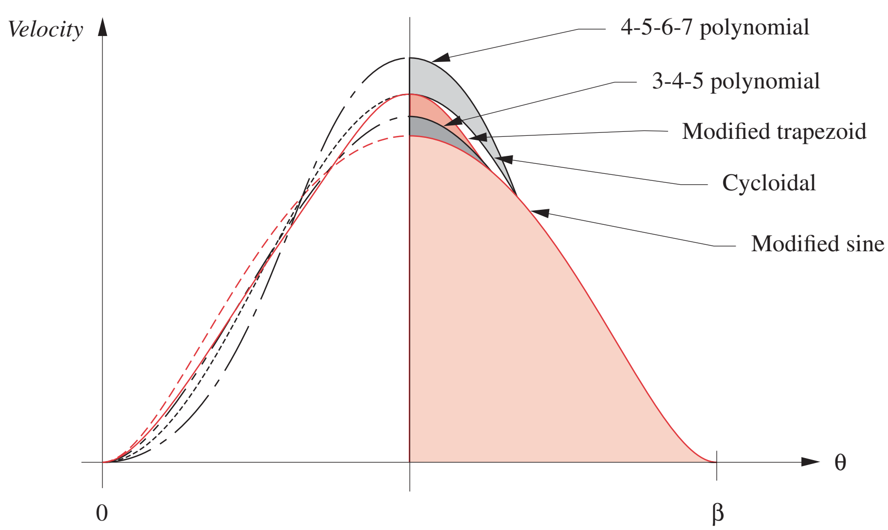

FIGURE 8-21
Comparison of five double-dwell cam velocity functions

of the modified sine acceleration curve and the reason it is often chosen for applications in which the follower mass or moment of inertia is very large.

An example of such an application is shown in Figure 8-22 which is an indexing table drive used for automated assembly lines. The round indexing table is mounted on a vertical spindle and driven as part of the rotary follower train by a form-closed barrel cam that moves it through some angular displacement, and then holds the table still in a dwell

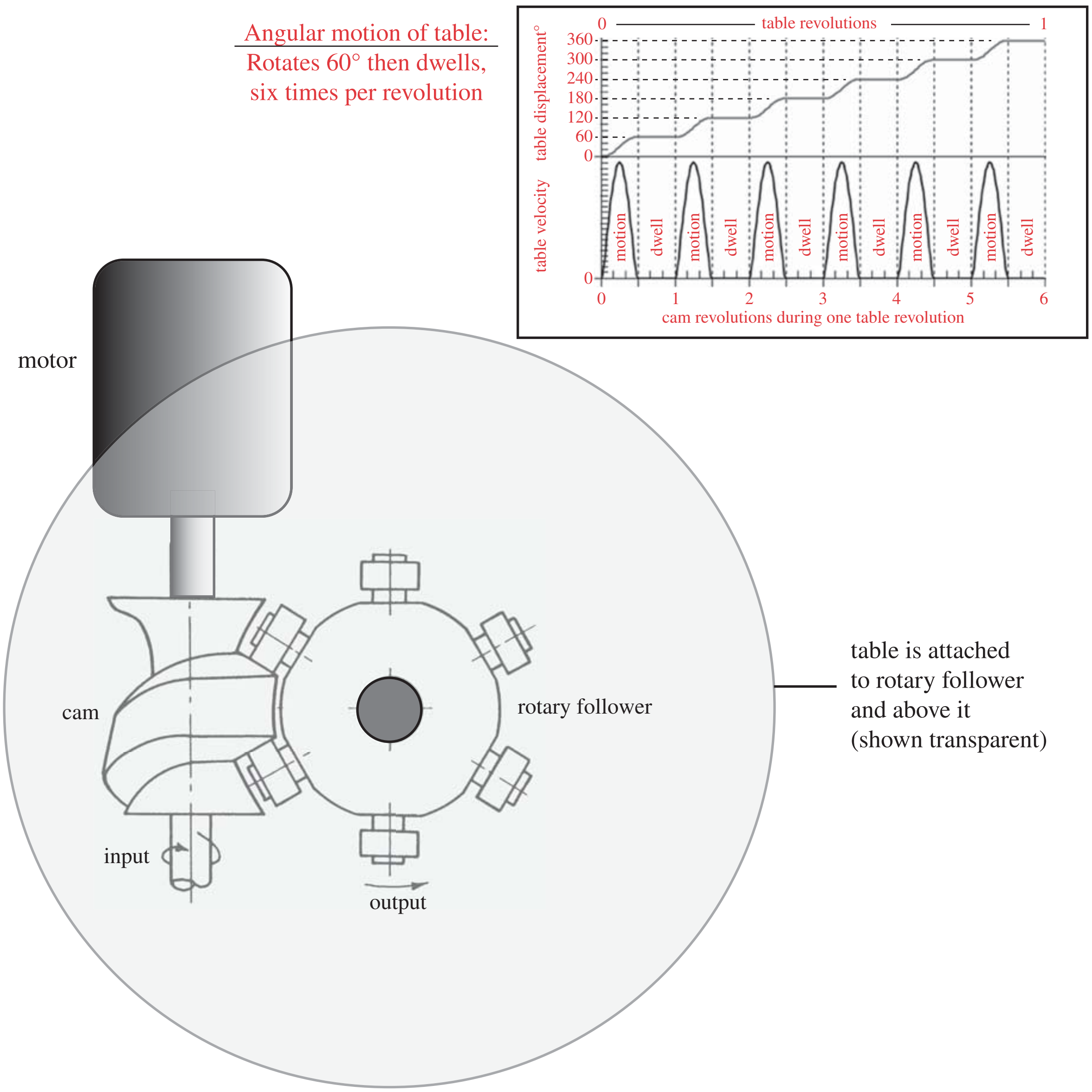

FIGURE 8-22

Six-stop rotary indexer. Table carries tooling to make a product during the dwells.

(called a "stop") while an assembly operation is performed on the workpiece carried on the table. These indexers may have three or more stops, each corresponding to an index position. The table is solid steel and may be several feet in diameter; thus its mass moment of inertia is large. To minimize the stored kinetic energy, which must be dissipated each time the table is brought to a stop, the manufacturers often use the modified sine program on these multidwell cams, because of its lower peak velocity.

Let us again try to improve the double-dwell cam example using the SCCA combined functions of modified trapezoid and modified sine acceleration.

EXAMPLE 8-4

Senior Cam Design—Combined Functions—Better Cams.

Problem: Consider the same cam design CEP specification as in Examples 8-1 to 8-3:

dwell	at zero displacement for 90 degrees (low dwell)
rise	1 in (25 mm) in 90 degrees
dwell	at 1 in (25 mm) for 90 degrees (high dwell)
fall	1 in (25 mm) in 90 degrees
cam ω	2π rad/sec = 1 rev/sec

Solution:

1 The modified trapezoidal function is an acceptable one for this double-dwell cam specification. Its derivatives are continuous through the acceleration function as shown in Figure 8-19. The peak acceleration is 78.1 in/sec^2 (1.98 m/sec^2).

2 The modified trapezoidal jerk curve in Figure 8-20 is discontinuous at its boundaries but has finite magnitude of 3925 in/sec^3 (100 m/sec^3), and this is acceptable.

3 The modified trapezoidal velocity in Figure 8-21 is smooth and matches the zeros of the dwell at each end. Its peak magnitude is 8 in/sec (0.2 m/sec).

4 The advantage of this modified trapezoidal function is that it has smaller theoretical peak acceleration than the cycloidal but its peak velocity is identical to that of the cycloidal.

5 The modified sinusoid function is also an acceptable one for this double-dwell cam specification. Its derivatives are also continuous through the acceleration function as shown in Figure 8-19. Its peak acceleration is 88.3 in/sec^2 (2.24 m/sec^2).

6 The modified sine jerk curve in Figure 8-20 is discontinuous at its boundaries but is of finite magnitude and is larger in magnitude at 4439 in/sec^3 (113 m/sec^3) but smoother than that of the modified trapezoid.

7 The modified sine velocity (Figure 8-21) is smooth, matches the zeros of the dwell at each end, and is lower in peak magnitude than either the cycloidal or modified trapezoidal at 7 in/sec (0.178 m/sec). This is an advantage for high-mass follower systems as it reduces stored kinetic energy. This, coupled with a peak acceleration lower than the cycloidal (but higher than the modified trapezoidal), is its chief advantage.

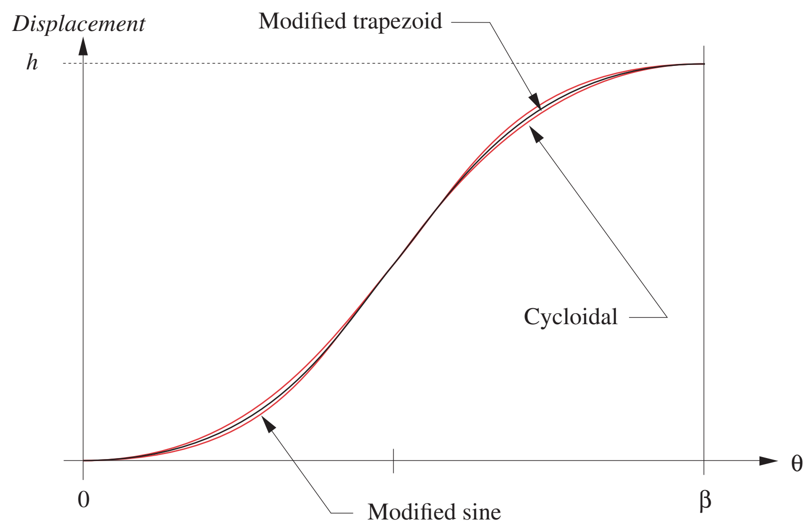

FIGURE 8-23

Comparison of three SCCA double-dwell cam displacement functions

Figure 8-23 shows the displacement curves for these three cam programs. (Open the file E08-04.cam in program DYNACAM to plot these also.) Note how little difference there is between the displacement curves despite the large differences in their acceleration waveforms in Figure 8-19. This is evidence of the smoothing effect of the integration process. Differentiating any two functions will exaggerate their differences. Integration tends to mask their differences. It is nearly impossible to recognize these very differently behaving cam functions by looking only at their displacement curves. This is further evidence of the folly of our earlier naive approach to cam design that dealt exclusively with the displacement function. The cam designer must be concerned with the higher derivatives of displacement. The displacement function is primarily of value to the manufacturer of the cam who needs its coordinate information in order to cut the cam.

FALL FUNCTIONS We have used only the rise portion of the cam for these examples. The fall is handled similarly. The rise functions presented here are applicable to the fall with slight modification. To convert rise equations to fall equations, it is only necessary to subtract the rise displacement function s from the maximum lift h and to negate the higher derivatives, v, a, and j.

Polynomial Functions

The class of polynomial functions is one of the more versatile types that can be used for cam design. They are not limited to single- or double-dwell applications and can be tailored to many design specifications. The general form of a polynomial function is:

$$s = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + \cdots + C_n x^n \tag{8.23}$$

where s is the follower displacement; x is the independent variable, which in our case will be replaced by either θ/β or time t. The constant coefficients C_n are the unknowns to be determined in our development of the particular polynomial equation to suit a design specification. The degree of a polynomial is defined as the highest power present in any term. Note that a polynomial of degree n will have $n + 1$ terms because there is an x^0 or constant term with coefficient C_0, as well as coefficients through and including C_n.

We structure a polynomial cam design problem by deciding how many boundary conditions (BCs) we want to specify on the *s v a j* diagrams. The number of BCs then determines the degree of the resulting polynomial. We can write an independent equation for each BC by substituting it into equation 8.16 or one of its derivatives. We will then have a system of linear equations that can be solved for the unknown coefficients C_0, . . ., C_n. If k represents the number of chosen boundary conditions, there will be k equations in k unknowns C_0, . . ., C_n and the **degree** of the polynomial will be $n = k - 1$. The **order** of the n-degree polynomial is equal to the number of terms, k.

Double-Dwell Applications of Polynomials

THE 3-4-5 POLYNOMIAL Reconsider the double-dwell problem of the previous three examples and solve it with polynomial functions. Many different polynomial solutions are possible. We will start with the simplest one possible for the double-dwell case.

EXAMPLE 8-5

The 3-4-5 Polynomial for the Double-Dwell Case.

Problem: Consider the same cam design CEP specification as in Examples 8-1 to 8-4:

dwell	at zero displacement for 90 degrees (low dwell)
rise	1 in (25 mm) in 90 degrees
dwell	at 1 in (25 mm) for 90 degrees (high dwell)
fall	1 in (25 mm) in 90 degrees
cam ω	2π rad/sec = 1 rev/sec

Solution:

1 To satisfy the fundamental law of cam design the values of the rise (and fall) functions at their boundaries with the dwells must match with no discontinuities in, at a minimum, *s*, *v*, and *a*.

2 Figure 8-24 shows the axes for the *s v a j* diagrams on which the known data have been drawn. The dwells are the only fully defined segments at this stage. The requirement for continuity through the acceleration defines a minimum of **six boundary conditions** for the rise segment and six more for the fall in this problem. They are shown as filled circles on the plots. For generality, we will let the specified total rise be represented by the variable h. The minimum set of required BCs for this example is then:

for the rise:

$$\begin{array}{llllll} \text{when} & \theta = 0; & \text{then} & s = 0, & v = 0, & a = 0 \\ \text{when} & \theta = \beta_1; & \text{then} & s = h, & v = 0, & a = 0 \end{array} \qquad (a)$$

for the fall:

$$\begin{array}{llllll} \text{when} & \theta = 0; & \text{then} & s = h, & v = 0, & a = 0 \\ \text{when} & \theta = \beta_2; & \text{then} & s = 0, & v = 0, & a = 0 \end{array} \tag{b}$$

3 We will use the rise for an example solution. (The fall is a similar derivation.) We have six BCs on the rise. This requires six terms in the equation. The highest term will be fifth degree. We will use the normalized angle θ/β as our independent variable, as before. Because our boundary conditions involve velocity and acceleration as well as displacement, we need to differentiate equation 8.23 versus θ to obtain expressions into which we can substitute those BCs. Rewriting equation 8.23 to fit these constraints and differentiating twice, we get:

$$s = C_0 + C_1\left(\frac{\theta}{\beta}\right) + C_2\left(\frac{\theta}{\beta}\right)^2 + C_3\left(\frac{\theta}{\beta}\right)^3 + C_4\left(\frac{\theta}{\beta}\right)^4 + C_5\left(\frac{\theta}{\beta}\right)^5 \tag{c}$$

$$v = \frac{1}{\beta}\left[C_1 + 2C_2\left(\frac{\theta}{\beta}\right) + 3C_3\left(\frac{\theta}{\beta}\right)^2 + 4C_4\left(\frac{\theta}{\beta}\right)^3 + 5C_5\left(\frac{\theta}{\beta}\right)^4\right] \tag{d}$$

$$a = \frac{1}{\beta^2}\left[2C_2 + 6C_3\left(\frac{\theta}{\beta}\right) + 12C_4\left(\frac{\theta}{\beta}\right)^2 + 20C_5\left(\frac{\theta}{\beta}\right)^3\right] \tag{e}$$

8

4 Substitute the boundary conditions $\theta = 0$, $s = 0$ into equation (c):

$$\begin{aligned} 0 &= C_0 + 0 + 0 + \cdots \\ C_0 &= 0 \end{aligned} \tag{f}$$

5 Substitute $\theta = 0$, $v = 0$ into equation (d):

$$\begin{aligned} 0 &= \frac{1}{\beta}\left(C_1 + 0 + 0 + \cdots\right) \\ C_1 &= 0 \end{aligned} \tag{g}$$

6 Substitute $\theta = 0$, $a = 0$ into equation (e):

$$\begin{aligned} 0 &= \frac{1}{\beta^2}\left(C_2 + 0 + 0 + \cdots\right) \\ C_2 &= 0 \end{aligned} \tag{h}$$

7 Substitute $\theta = \beta$, $s = h$ into equation (c):

$$h = C_3 + C_4 + C_5 \tag{i}$$

8 Substitute $\theta = \beta$, $v = 0$ into equation (d):

$$0 = \frac{1}{\beta}\left(3C_3 + 4C_4 + 5C_5\right) \tag{j}$$

9 Substitute $\theta = \beta$, $a = 0$ into equation (e):

$$0 = \frac{1}{\beta^2}\left(6C_3 + 12C_4 + 20C_5\right) \tag{k}$$

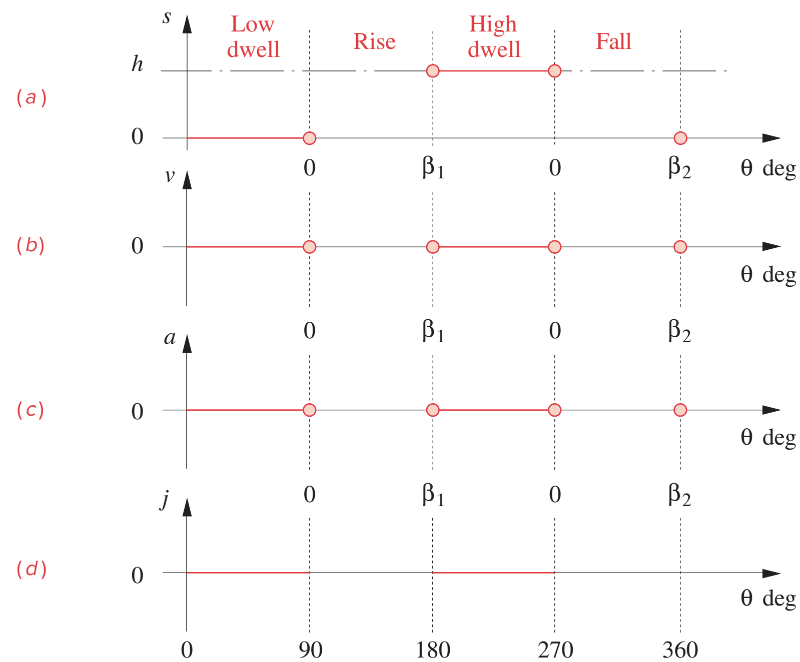

FIGURE 8-24

Minimum boundary conditions for the double-dwell case

10 Three of our unknowns are found to be zero, leaving three unknowns to be solved for, C_3, C_4, C_5. Equations (i), (j), and (k) can be solved simultaneously to get:

$$C_3 = 10h; \qquad C_4 = -15h; \qquad C_5 = 6h \tag{l}$$

11 The equation for this cam design's displacement is then:

$$s = h\left[10\left(\frac{\theta}{\beta}\right)^3 - 15\left(\frac{\theta}{\beta}\right)^4 + 6\left(\frac{\theta}{\beta}\right)^5\right] \tag{8.24}$$

12 The expressions for velocity and acceleration can be obtained by substituting the values of C_3, C_4, and C_5 into equations 8.18b and c. This function is referred to as the **3-4-5 polynomial**, after its exponents. Open the file E08-07.cam in program DYNACAM to investigate this example in more detail.

Figure 8-25 shows the resulting *s v a j* diagrams for a **3-4-5 polynomial rise** function. Note that the acceleration is continuous but the jerk is not, because we did not place any constraints on the boundary values of the jerk function. It is also interesting to note that the acceleration waveform looks very similar to the sinusoidal acceleration of the cycloidal function in Figure 8-12. Figure 8-19 shows the relative peak accelerations of this 3-4-5 polynomial compared to four other functions with the same h and β. Table 8-3 lists factors for the maximum velocity, acceleration, and jerk of these functions.

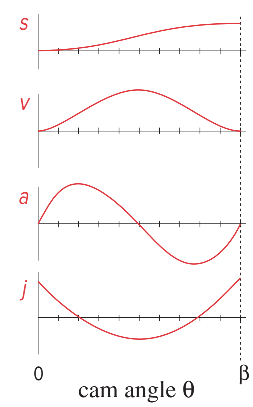

FIGURE 8-25

3-4-5 polynomial rise. Its acceleration is very similar to the sinusoid of cycloidal motion

THE 4-5-6-7 POLYNOMIAL We left the jerk unconstrained in the previous example. We will now redesign the cam for the same specifications but will also constrain the jerk function to be zero at both ends of the rise. It will then match the dwells in the jerk function with no discontinuities. This gives eight boundary conditions and yields a seventh-degree polynomial. The solution procedure to find the eight unknown coefficients is identical to that used in the previous example. Write the polynomial with the appropriate number of terms. Differentiate it to get expressions for all orders of boundary conditions. Substitute the boundary conditions and solve the resulting set of simultaneous equations.* This problem reduces to four equations in four unknowns, as the coefficients C_0, C_1, C_2, and C_3 turn out to be zero. For this set of boundary conditions the displacement equation for the rise is:

$$s = h\left[35\left(\frac{\theta}{\beta}\right)^4 - 84\left(\frac{\theta}{\beta}\right)^5 + 70\left(\frac{\theta}{\beta}\right)^6 - 20\left(\frac{\theta}{\beta}\right)^7\right] \tag{8.25}$$

This is known as the **4-5-6-7 polynomial**, after its exponents. Figure 8-26 shows the *s v a j* diagrams for this function. Compare these functions to the 3-4-5 polynomial functions shown in Figure 8-25. Note that the acceleration of the 4-5-6-7 starts off slowly, with zero slope (as we demanded with our zero jerk BC), and as a result goes to a larger peak value of acceleration in order to replace the missing area in the leading edge.

This **4-5-6-7 polynomial** function has the advantage of smoother jerk for better vibration control, compared to the **3-4-5 polynomial**, the **cycloidal**, and all other functions so far discussed, but it pays a price in the form of higher peak theoretical acceleration than all those functions. See also Table 8-3.

SUMMARY The previous two sections have attempted to present an approach to the selection of appropriate double-dwell cam functions, using the common rise-dwell-fall-dwell cam as the example, and to point out some of the pitfalls awaiting the cam designer. The particular functions described are only a few of the ones that have been developed for this double-dwell case over many years, by many designers, but they are probably the most used and most popular among cam designers. Most of them are also included in program DYNACAM. There are many trade-offs to be considered in selecting a cam program for any application, some of which have already been mentioned, such as function continuity, peak values of velocity and acceleration, and smoothness of jerk. There are many other trade-offs still to be discussed in later sections of this chapter, involving the sizing and the manufacturability of the cam.

8.4 SINGLE-DWELL CAM DESIGN—CHOOSING *S V A J* FUNCTIONS

Many applications in machinery require a **single-dwell** cam program, **rise-fall-dwell** (RFD). Perhaps a single-dwell cam is needed to lift and lower a roller that carries a moving paper web on a production machine that makes envelopes. This cam's follower lifts the paper up to one critical extreme position at the right time to contact a roller that applies a layer of glue to the envelope flap. Without dwelling in the up position, it immediately retracts the web back to the starting (zero) position and holds it in this other critical extreme position (low dwell) while the rest of the envelope passes by. It repeats the cycle for the

* Any matrix solving calculator, equation solver such as *Matlab*, *Mathcad*, or *TKSolver*, or programs MATRIX and DYNACAM (supplied with this text) will do the simultaneous equation solution for you. Programs MATRIX and DYNACAM are discussed in Appendix A. You need only to supply the desired boundary conditions to DYNACAM and the coefficients will be computed. The reader is encouraged to do so and examine the example problems presented here with the DYNACAM program.

8

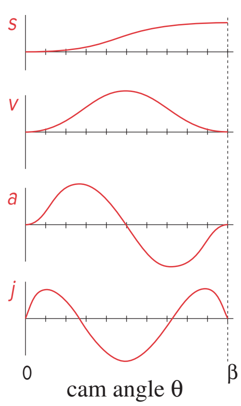

FIGURE 8-26

4-5-6-7 polynomial rise whose jerk is piecewise continuous with the dwells next envelope as it comes by. Another common example of a single-dwell application is the cam that opens the valves in your automobile engine. This lifts the valve open on the rise, immediately closes it on the fall, and then keeps the valve closed in a dwell while the compression and combustion take place.

If we attempt to use the same type of cam programs as were defined for the double-dwell case for a single-dwell application, we will achieve a solution that may work but is not optimal. We will nevertheless do so here as an example in order to point out the problems that result. Then we will redesign the cam to eliminate those problems.

EXAMPLE 8-6

Using Cycloidal Motion for a Symmetrical Rise-Fall Single-Dwell Case.

Problem: Consider the following single-dwell cam specification:

rise	1 in (25 mm) in 90 degrees
fall	1 in (25 mm) in 90 degrees
dwell	at zero displacement for 180 degrees (low dwell)
cam ω	15 rad/sec

Solution:

1 Figure 8-27 shows a cycloidal displacement rise and separate cycloidal displacement fall applied to this single-dwell example. Note that the displacement (s) diagram looks acceptable in that it moves the follower from the low to the high position and back in the required intervals.

2 The velocity (v) also looks acceptable in shape in that it takes the follower from zero velocity at the low dwell to a peak value of 19.1 in/sec (0.49 m/sec) to zero again at the maximum displacement, where the glue is applied.

3 Figure 8-27 also shows the acceleration function for this solution. Its maximum absolute value is about 573 in/sec^2.

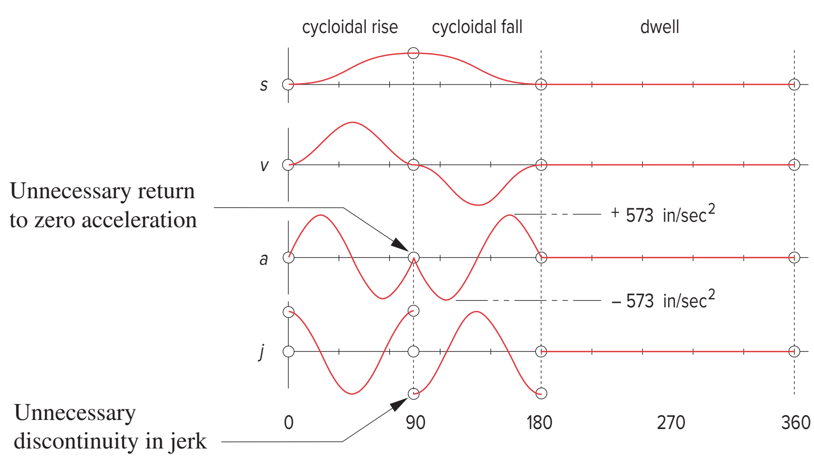

FIGURE 8-27

Cycloidal motion (or any double-dwell program) is a poor choice for the single-dwell case.

4 The problem is that this acceleration curve has an **unnecessary return to zero** at the end of the rise. It is unnecessary because the acceleration during the first part of the fall is also negative. It would be better to keep it in the negative region at the end of the rise.

5 This unnecessary oscillation to zero in the acceleration causes the jerk to have more abrupt changes and discontinuities. The only real justification for taking the acceleration to zero is the need to change its sign (as is the case halfway through the rise or fall) or to match an adjacent segment that has zero acceleration.

The reader may open the file E08-06.cam in program DYNACAM to investigate this example in more detail.

For the single-dwell case we would like a function for the rise that does not return its acceleration to zero at the end of the interval. The function for the fall should begin with the same nonzero acceleration value as ended the rise and then be zero at its terminus to match the dwell. One function that meets those criteria is the **double harmonic** which gets its name from its two cosine terms, one of which is a half-period harmonic and the other a full-period wave. The equations for the double harmonic functions are:

for the rise:

$$
\begin{aligned}
s &= \frac{h}{2}\left\{\left[1-\cos\left(\pi\frac{\theta}{\beta}\right)\right]-\frac{1}{4}\left[1-\cos\left(2\pi\frac{\theta}{\beta}\right)\right]\right\} \\
v &= \frac{\pi}{\beta}\frac{h}{2}\left[\sin\left(\pi\frac{\theta}{\beta}\right)-\frac{1}{2}\sin\left(2\pi\frac{\theta}{\beta}\right)\right] \\
a &= \frac{\pi^2}{\beta^2}\frac{h}{2}\left[\cos\left(\pi\frac{\theta}{\beta}\right)-\cos\left(2\pi\frac{\theta}{\beta}\right)\right] \qquad (8.26a) \\
j &= -\frac{\pi^3}{\beta^3}\frac{h}{2}\left[\sin\left(\pi\frac{\theta}{\beta}\right)-2\sin\left(2\pi\frac{\theta}{\beta}\right)\right]
\end{aligned}
$$

for the fall:

$$
\begin{aligned}
s &= \frac{h}{2}\left\{\left[1+\cos\left(\pi\frac{\theta}{\beta}\right)\right]-\frac{1}{4}\left[1-\cos\left(2\pi\frac{\theta}{\beta}\right)\right]\right\} \\
v &= -\frac{\pi}{\beta}\frac{h}{2}\left[\sin\left(\pi\frac{\theta}{\beta}\right)+\frac{1}{2}\sin\left(2\pi\frac{\theta}{\beta}\right)\right] \\
a &= -\frac{\pi^2}{\beta^2}\frac{h}{2}\left[\cos\left(\pi\frac{\theta}{\beta}\right)+\cos\left(2\pi\frac{\theta}{\beta}\right)\right] \qquad (8.26b) \\
j &= \frac{\pi^3}{\beta^3}\frac{h}{2}\left[\sin\left(\pi\frac{\theta}{\beta}\right)+2\sin\left(2\pi\frac{\theta}{\beta}\right)\right]
\end{aligned}
$$

Note that these double harmonic functions should **never** be used for the double-dwell case because their acceleration is nonzero at one end of the interval.

8

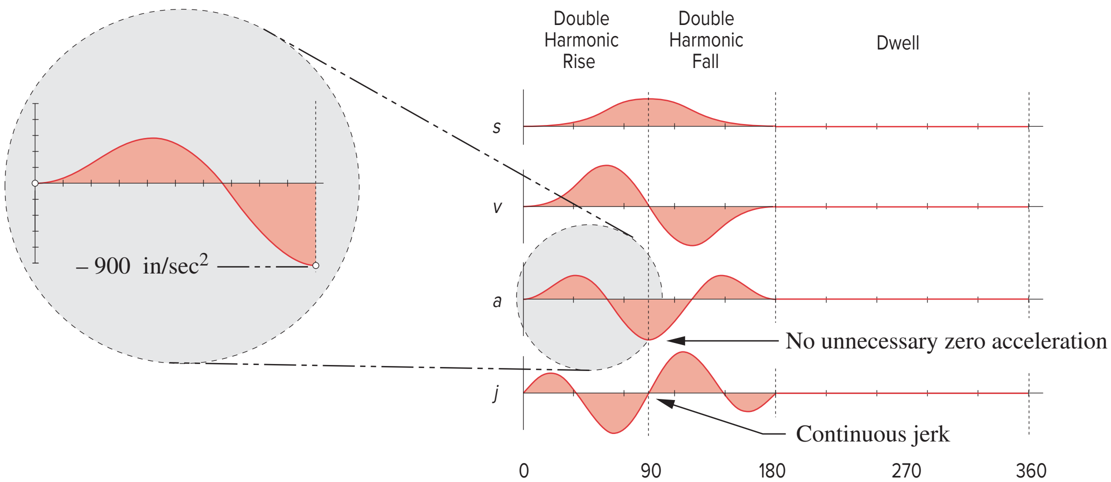

FIGURE 8-28

Double harmonic motion can be used for the single-dwell case if rise and fall durations are equal.

EXAMPLE 8-7

Double Harmonic Motion for Symmetrical Rise-Fall Single-Dwell Case.

Problem: Consider the same single-dwell cam specification as in Example 8-5:

rise	1 in (25 mm) in 90 degrees
fall	1 in (25 mm) in 90 degrees
dwell	at zero displacement for 180 degrees (low dwell)
cam ω	15 rad/sec

Solution:

1 Figure 8-28 shows a double harmonic rise and a double harmonic fall. The peak velocity is 19.5 in/sec (0.50 m/sec) which is similar to that of the cycloidal solution of Example 8-6.

2 Note that the acceleration of this double harmonic function does not return to zero at the end of the rise. This makes it more suitable for a single-dwell case in that respect.

3 The double harmonic jerk function peaks at 36 931 in/sec^3 (938 m/sec^3) and is quite smooth compared to the cycloidal solution.

4 Unfortunately, the peak negative acceleration is 900 in/sec^2, nearly twice that of the cycloidal solution. This is a smoother function but will develop higher dynamic forces. Open the file E08-07.cam in program DYNACAM to see this example in greater detail.

5 Another limitation of this function is that it may only be used for the case of an equal time (symmetrical) rise and fall. If the rise and fall times are different, the acceleration will be discontinuous at the juncture of rise and fall, violating the fundamental law of cam design.

Neither of the solutions in Examples 8-6 and 8-7 is optimal. We will now apply polynomial functions and redesign it to both improve its smoothness and reduce its peak acceleration.

Single-Dwell Applications of Polynomials

To solve the problem of Example 8-7 with a polynomial, we must decide on a suitable set of boundary conditions. But first, we must decide how many segments to divide the cam cycle into. The problem statement seems to imply three segments, a rise, a fall, and a dwell. We could use those three segments to create the functions as we did in the two previous examples, but a better approach is to use only **two segments**, one for the rise-fall combined and one for the dwell. *As a general rule we would like to minimize the number of segments in our polynomial cam function*s. Any dwell requires its own segment. So, the minimum number possible in this case is two segments.

Another rule of thumb is that *we would like to minimize the number of boundary conditions specified* because the degree of the polynomial is tied to the number of BCs. As the degree of the function increases, so will the number of its **inflection points** and its number of **minima and maxima**. The polynomial derivation process will guarantee that the function will pass through all specified BCs but says nothing about the function's behavior between the BCs. *A high-degree function may have undesirable oscillations between its BCs.*

With these assumptions we can select a set of boundary conditions for a trial solution. First we will restate the problem to reflect our two-segment configuration.

EXAMPLE 8-8

Designing a Polynomial for the Symmetrical Rise-Fall Single-Dwell Case.

Problem: Redefine the CEP specification from Examples 8-5 and 8-6.

rise-fall	1 in (25.4 mm) in 90° and fall 1 in (25.4 mm) in 90° over 180°
dwell	at zero displacement for 180° (low dwell)
cam ω	15 rad/sec

Solution:

1 Figure 8-29 shows the minimum set of seven BCs for this symmetrical problem, which will give a sixth-degree polynomial. The dwell on either side of the combined rise-fall segment has zero values of *s, v, a,* and *j*. The fundamental law of cam design requires that we match these zero values, through the acceleration function, at each end of the rise-fall segment.

2 These then account for six BCs; $s, v, a = 0$ at each end of the rise-fall segment.

3 We also must specify a value of displacement at the 1-in peak of the rise that occurs at $\theta = 90°$. This is the seventh BC. Note that due to symmetry, it is not necessary to specify the velocity to be zero at the peak. It will be anyway.

4 Figure 8-29 also shows the coefficients of the displacement polynomial that result from the simultaneous solution of the equations for the chosen BCs. For generality we have substituted

Segment number	Function used	Start angle	End angle	Delta angle
1	Poly 6	0	180	180

Boundary Conditions Imposed

Function	Theta	% Beta	Boundary Cond.
Displ	0	0	0
Veloc	0	0	0
Accel	0	0	0
Displ	180	1	0
Veloc	180	1	0
Accel	180	1	0
Displ	90	0.5	1

Equation Resulting

Exponent	Coefficient
0	0
1	0
2	0
3	64
4	– 192
5	192
6	– 64

FIGURE 8-29

Boundary conditions and coefficients for a single-dwell polynomial application

the variable *h* for the specified 1-in rise. The function turns out to be a 3-4-5-6 polynomial whose equation is:

$$s = h\left[64\left(\frac{\theta}{\beta}\right)^3 - 192\left(\frac{\theta}{\beta}\right)^4 + 192\left(\frac{\theta}{\beta}\right)^5 - 64\left(\frac{\theta}{\beta}\right)^6\right] \qquad (a)$$

Figure 8-30 shows the *s v a j* diagrams for this solution with its maximum values noted. Compare these acceleration and *s v a j* curves to the double harmonic and cycloidal solutions to the same problem in Figures 8-27 and 8-28. Note that this sixth-degree polynomial function is as smooth as the double harmonic functions (Figure 8-28) and does not unnecessarily return the acceleration to zero at the top of the rise as does the cycloidal (Figure 8-27). The polynomial has a peak acceleration of 547 in/sec^2, which is less than that of either the cycloidal or double harmonic solution. This 3-4-5-6 polynomial is a

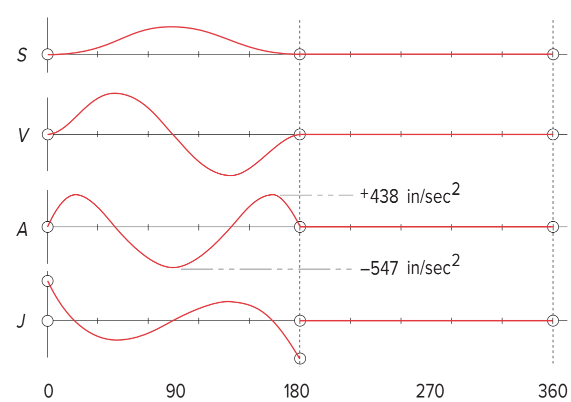

FIGURE 8-30

3-4-5-6 polynomial function for two-segment symmetrical rise-fall, single-dwell cam superior solution to either of those presented for the symmetrical rise-fall case and is an example of how polynomial functions can be easily tailored to particular design specifications. The reader may open the file E08-08.cam in program DYNACAM to investigate this example in greater detail.

Effect of Asymmetry on the Rise-Fall Polynomial Solution

The examples so far presented in this section all had equal time for rise and fall, referred to as a symmetrical rise-fall curve. What will happen if we need an asymmetric program and attempt to use a single polynomial as was done in the previous example?

EXAMPLE 8-9

Designing a Polynomial for an Asymmetrical Rise-Fall Single-Dwell Case.

Problem: Redefine the specification from Example 8-8 as:

rise-fall rise 1 in (25.4 mm) in 45° and fall 1 in (25.4 mm) in 135° over 180°
dwell at zero displacement for 180° (low dwell)
cam ω 15 rad/sec

Solution:

8

1 Figure 8-31 shows the minimum set of seven BCs for this problem that will give a sixth-degree polynomial. The dwell on either side of the combined rise-fall segment has zero values for *S, V, A,* and *J*. The fundamental law of cam design requires that we match these zero values, through the acceleration function, at each end of the rise-fall segment.

2 The endpoints account for six BCs; $S = V = A = 0$ at each end of the rise-fall segment.

3 We also must specify a value of displacement at the 1-in peak of the rise that occurs at $\theta = 45°$. This is the seventh BC.

4 Simultaneous solution of this equation set gives a 3-4-5-6 polynomial whose equation is:

$$s = h\left[151.704\left(\frac{\theta}{\beta}\right)^3 - 455.111\left(\frac{\theta}{\beta}\right)^4 + 455.111\left(\frac{\theta}{\beta}\right)^5 - 151.704\left(\frac{\theta}{\beta}\right)^6\right] \qquad (a)$$

For generality we have substituted the variable h for the specified 1-in rise.

5 Figure 8-31 shows the *S V A J* diagrams for this solution with its maximum values noted. Observe that the derived sixth-degree polynomial has obeyed the stated boundary conditions and does in fact pass through a displacement of 1 unit at 45°. But note also that it overshoots that point and reaches a height of 2.37 units at its peak. The acceleration peak is also 2.37 times that of the symmetrical case of Example 8-8. Without any additional boundary conditions applied, the function seeks symmetry. Note that the zero velocity point is still at 90° when we would like it to be at 45°. We can try to force the velocity to zero with an additional boundary condition of $V = 0$ when $\theta = 45°$.

6 Figure 8-32 shows the *S V A J* diagrams for a seventh-degree polynomial having 8 BCs, $S = V = A = 0$ at $\theta = 0°$, $S = V = A = 0$ at $\theta = 180°$, $S = 1$, $V = 0$ at $\theta = 45°$. Note that the resulting

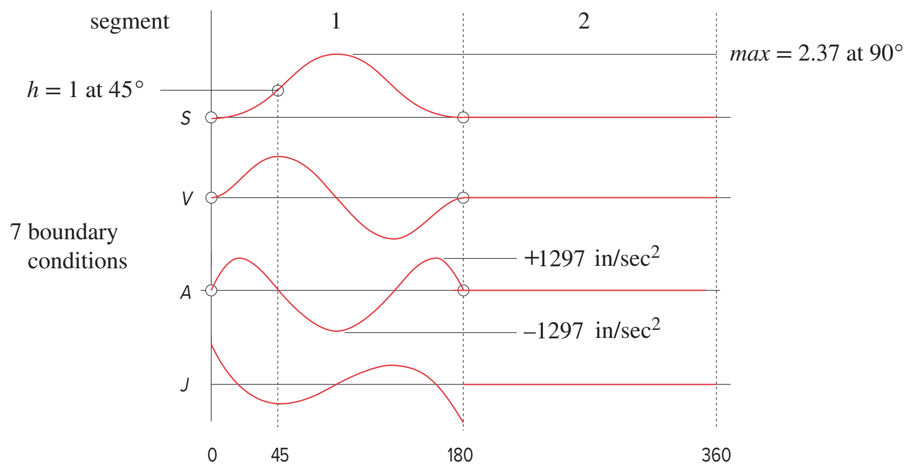

FIGURE 8-31

Unacceptable polynomial for a two-segment asymmetrical rise-fall, single-dwell cam

elsewhere. It now plunges to a negative displacement of –3.934, and the peak acceleration is much larger. This points out an inherent problem in polynomial functions, namely that their behavior between boundary conditions is not controllable and may create undesirable deviations in the follower motion. This problem is exacerbated as the degree of the function increases since it then has more roots and inflection points, thus allowing more oscillations between the boundary conditions.

7 Open the files Ex_08-09a and b in program DYNACAM to see this example in greater detail.

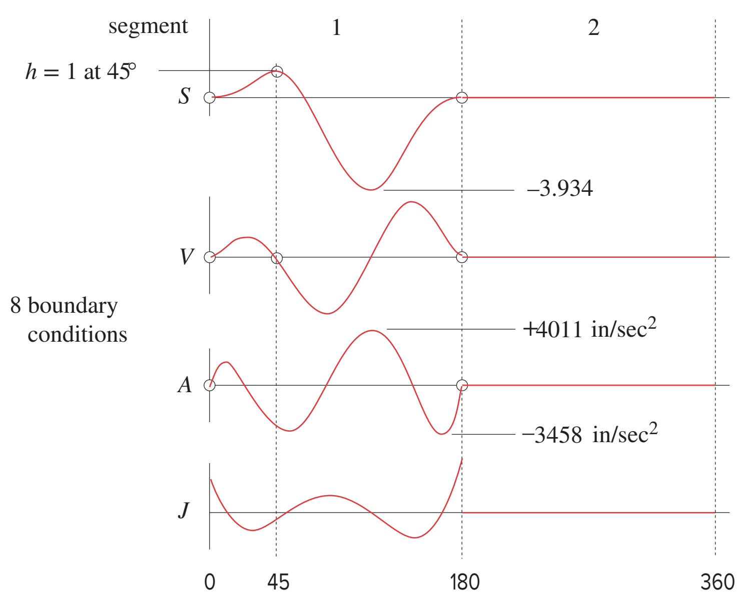

FIGURE 8-32

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In this case, the rule of thumb to minimize the number of segments is in conflict with the rule of thumb to minimize the degree of the polynomial. One alternative solution to this asymmetrical problem is to use three segments, one for the rise, one for the fall, and one for the dwell. Adding segments will reduce the order of the functions and bring them under control.

EXAMPLE 8-10

Designing a Three-Segment Polynomial for an Asymmetrical Rise-Fall Single-Dwell Case Using Minimum Boundary Conditions.

Problem: Redefine the specification from Example 8-9 as:

rise	1 in (25.4 mm) in 45°
fall	1 in (25.4 mm) in 135°
dwell	at zero displacement for 180° (low dwell)
cam ω	15 rad/sec

Solution:

1 The first attempt at this solution specifies 5 BCs; $S = V = A = 0$ at the start of the rise (to match the dwell), $S = 1$ and $V = 0$ at the end of the rise. Note that the rise segment BCs leave the acceleration at its end unspecified, but the fall segment BCs must include the value of the acceleration at the end of the rise that results from the calculation of its acceleration. Thus, the fall requires one more BC than the rise.

2 This results in the following fourth degree equation for the rise segment:

$$s = h\left[4\left(\frac{\theta}{\beta}\right)^3 - 3\left(\frac{\theta}{\beta}\right)^4\right] \tag{a}$$

3 Evaluating the acceleration at the end of rise gives –4377.11 in/sec^2. This value becomes a BC for the fall segment. The set of 6 BCs for the fall is then: $S = 1$, $V = 0$, $A = -4377.11$ at the start of the fall (to match the rise) and $S = V = A = 0$ at the end of the fall to match the dwell. The fifth-degree equation for the fall is then:

$$s = h\left[1 - 54\left(\frac{\theta}{\beta}\right)^2 + 152\left(\frac{\theta}{\beta}\right)^3 - 147\left(\frac{\theta}{\beta}\right)^4 + 48\left(\frac{\theta}{\beta}\right)^5\right] \tag{b}$$

4 Figure 8-33 shows the *S V A J* diagrams for this solution with its extreme values noted. Observe that this polynomial on the fall also has a problem—the displacement still goes negative.

5 The trick in this case (and in general) is to first calculate the segment with the smaller acceleration (here the second segment) because of its larger duration angle β. Then use its smaller acceleration value as a boundary condition on the first segment. The 5 BCs for segment 2 are then $S = 1$ and $V = 0$ at the start of the fall and $S = V = A = 0$ at the end of the fall (to match the dwell). These give the following fourth-degree polynomial for the fall.

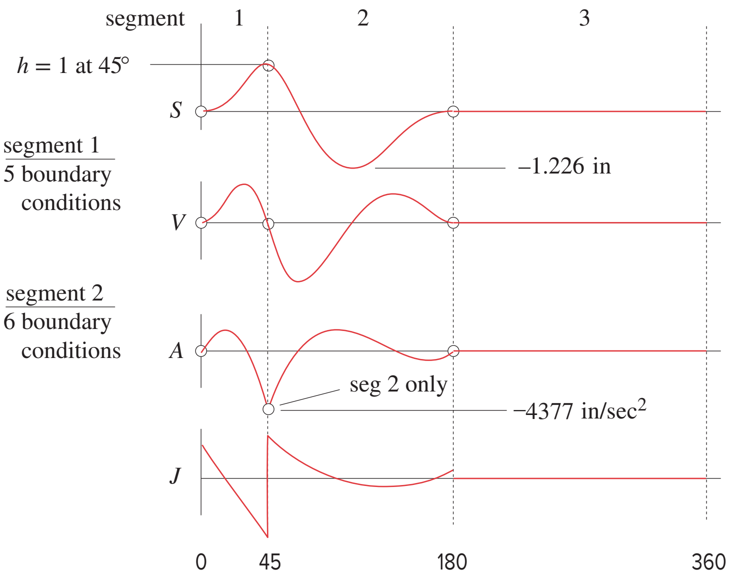

FIGURE 8-33

Uacceptable polynomials for a three-segment asymmetrical rise-fall, single-dwell cam

$$s = h\left[1 - 6\left(\frac{\theta}{\beta}\right)^2 + 8\left(\frac{\theta}{\beta}\right)^3 - 3\left(\frac{\theta}{\beta}\right)^4\right] \tag{c}$$

6 Evaluating the acceleration at the start of the fall gives –486.4 in/sec^2. This value becomes a BC for the rise segment. The set of 6 BCs for the rise is then: $S = V = A = 0$ at the start of the rise to match the dwell*s*, and $S = 1$, $V = 0$, $A = –486.4$ at the end of the rise (to match the fall). The fifth-degree equation for the rise is then:

$$s = h\left[9.333\left(\frac{\theta}{\beta}\right)^3 - 13.667\left(\frac{\theta}{\beta}\right)^4 + 5.333\left(\frac{\theta}{\beta}\right)^5\right] \tag{d}$$

7 The resulting cam design is shown in Figure 8-34. The displacement is now under control and the peak acceleration is much less than the previous design at about 2024 in/sec^2.

8 The design of Figure 8-34 is acceptable (though not optimum)* for this example. Open the files Ex_08-10a and b in program DYNACAM to see this example in greater detail.

* An optimum solution to this generic problem can be found in reference [5].

8.5 CRITICAL PATH MOTION (CPM)

Probably the most common application of **critical path motion** (CPM) specifications in production machinery design is the need for **constant velocity motion**. There are two

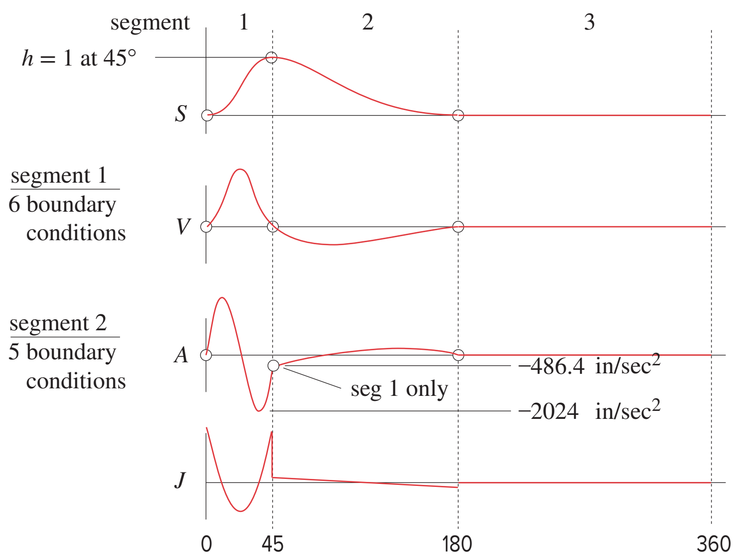

FIGURE 8-34

Acceptable polynomials for a three-segment asymmetrical rise-fall, single-dwell cam

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general types of automated production machinery in common use, **intermittent motion** assembly machines and **continuous motion** assembly machines.

Intermittent motion assembly machines carry the manufactured goods from workstation to workstation, stopping the workpiece or subassembly at each station while another operation is performed upon it. The throughput speed of this type of automated production machine is typically limited by the dynamic forces that are due to accelerations and decelerations of the mass of the moving parts of the machine and its workpieces. The workpiece motion may be either in a straight line as on a conveyor or in a circle as on a rotary table as shown in Figure 8-22.

Continuous motion assembly machines never allow the workpiece to stop and thus are capable of higher throughput speeds. All operations are performed on a moving target. Any tools that operate on the product have to "chase" the moving assembly line to do their job. Since the assembly line (often a conveyor belt or chain, or a rotary table) is moving at some constant velocity, there is a need for mechanisms to provide constant velocity motion, matched exactly to the conveyor, in order to carry the tools alongside for a long enough time to do their job. These cam driven "chaser" mechanisms must then return the tool quickly to its start position in time to meet the next part or subassembly on the conveyor (quick-return). There is a motivation in manufacturing to convert from intermittent motion machines to continuous motion in order to increase production rates. Thus there is some demand for this type of constant velocity mechanism. Though we met some linkages in Chapter 6 that give approximate constant velocity output, the cam-follower system is well suited to this problem, allowing theoretically exact constant follower velocity, and the polynomial cam function is particularly adaptable to the task.

Polynomials Used for Critical Path Motion

EXAMPLE 8-11

Designing a Polynomial for Constant Velocity Critical Path Motion.

Problem: Consider the following statement of a critical path motion (CPM) problem:

Accelerate	the follower from zero to 10 in/sec
Maintain	a constant velocity of 10 in/sec for 0.5 sec
Decelerate	the follower to zero velocity
Return	the follower to start position
Cycle time	exactly 1 sec

Solution:

1 This unstructured problem statement is typical of real design problems as was discussed in Chapter 1. No information is given as to the means to be used to accelerate or decelerate the follower or even as to the portions of the available time to be used for those tasks. A little reflection will cause the engineer to recognize that the specification on total cycle time in effect defines the camshaft velocity to be its reciprocal or **one revolution per second**. Converted to appropriate units, this is an angular velocity of 2π rad/sec.

2 The constant velocity portion uses half of the total period of 1 sec in this example. The designer must next decide how much of the remaining 0.5 sec to devote to each other phase of the required motion.

3 The problem statement seems to imply that four segments are needed. Note that the designer has to somewhat arbitrarily select the lengths of the individual segments (except the constant velocity one). Some iteration may be required to optimize the result. Program DYNACAM makes the iteration process quick and easy, however.

4 Assuming four segments, the timing diagram in Figure 8-35 shows an acceleration phase, a constant velocity phase, a deceleration phase, and a return phase, labeled as segments 1 through 4.

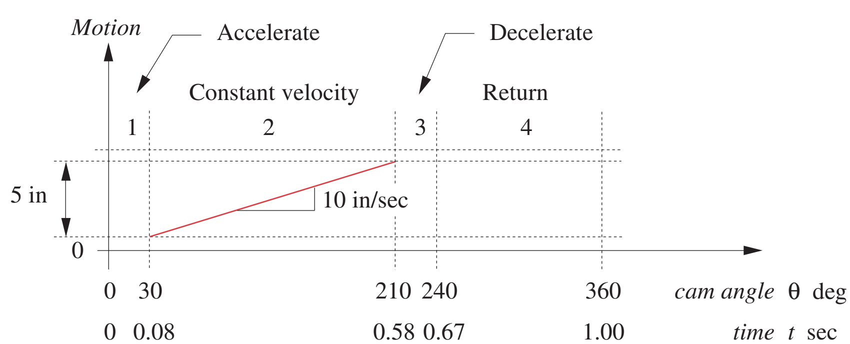

FIGURE 8-35

Constant velocity cam timing diagram

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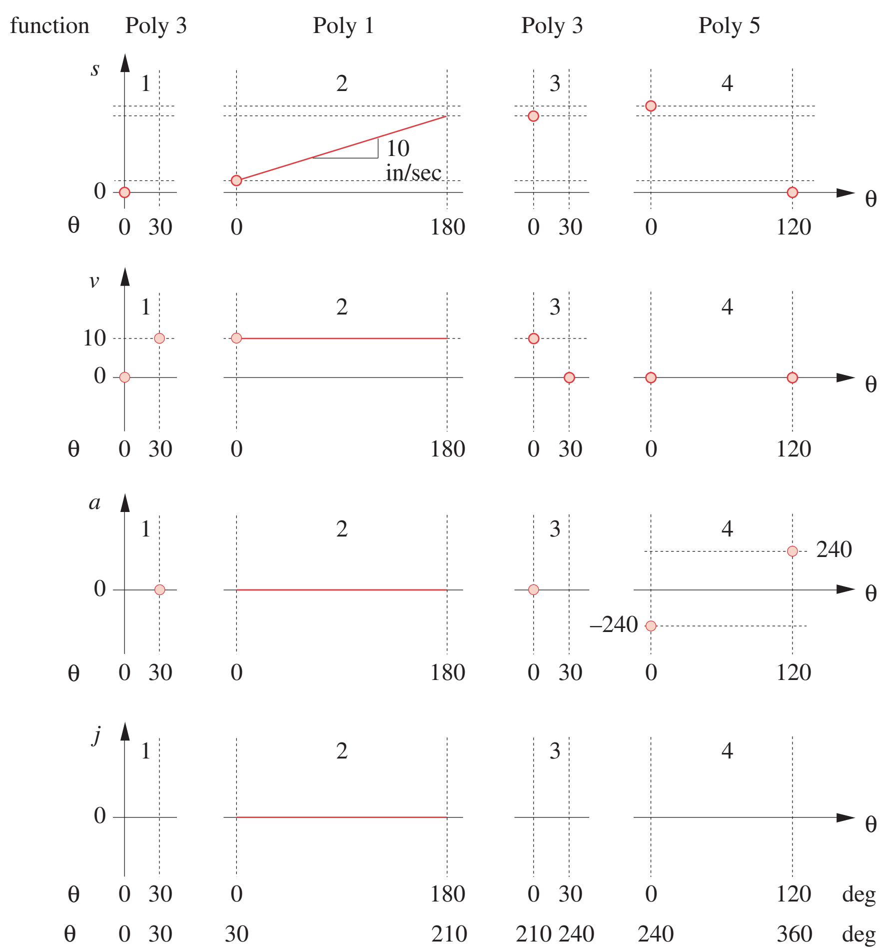

FIGURE 8-36

A possible set of boundary conditions for the four-segment constant velocity solution

5 The segment angles (β's) are assumed, for a first approximation, to be 30° for segment 1, 180° for segment 2, 30° for segment 3, and 120° for segment 4 as shown in Figure 8-36. These angles may need to be adjusted in later iterations, except for segment 2 which is rigidly constrained in the specifications.

6 Figure 8-36 shows a tentative set of boundary conditions for the *s v a j* diagram. The solid circles indicate a set of boundary conditions that will constrain the continuous function to these specifications. These are for segment 1:

$$\begin{array}{lllll} \text{when } \theta = 0°; & s = 0, & v = 0, & \textit{none} \\ \text{when } \theta = 30°; & \textit{none}, & v = 10, & a = 0 \end{array} \qquad (a)$$

7 Note that the displacement at θ = 30° is left unspecified. The resulting polynomial function will provide us with the values of displacement at that point, which can then be used as a

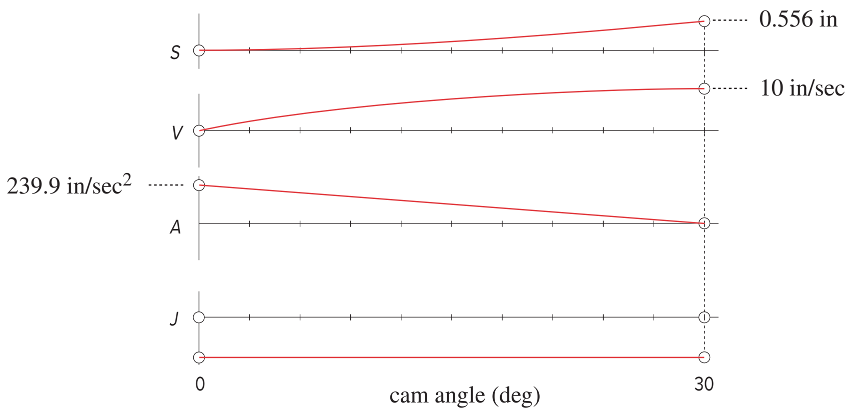

FIGURE 8-37

Segment one for the four-segment solution to the constant velocity problem (Example 8-11)

boundary condition for the next segment, in order to make the overall functions continuous as required. The acceleration at $\theta = 30°$ must be zero in order to match that of the constant velocity segment 2. The acceleration at $\theta = 0$ is left unspecified. The resulting value will be used later to match the end of the last segment's acceleration.

8 Putting these four BCs for segment 1 into program DYNACAM yields a cubic function whose *s v a j* plots are shown in Figure 8-37. Its equation is:

$$s = 0.83376\left(\frac{\theta}{\beta}\right)^2 - 0.27792\left(\frac{\theta}{\beta}\right)^3 \qquad (8.27a)$$

The maximum displacement occurs at $\theta = 30°$. This will be used as one BC for segment 2. The entire set for segment 2 is:

$$\begin{array}{llll} \text{when} & \theta = 30°; & s = 0.556, & v = 10 \\ \text{when} & \theta = 210°; & \textit{none}, & \textit{none} \end{array} \qquad (b)$$

9 Note that in the derivations and in the DYNACAM program each segment's local angles run from zero to the β for that segment. Thus, segment 2's local angles are 0° to 180°, which correspond to 30° to 210° globally in this example. We have left the displacement, velocity, and acceleration at the end of segment 2 unspecified. They will be determined by the computation.

10 Since this is a constant velocity segment, its integral, the displacement function, must be a polynomial of degree one, i.e., a straight line. If we specify more than two BCs we will get a function of higher degree than one that will pass through the specified endpoints but may also oscillate between them and deviate from the desired constant velocity. Thus we can *only* provide two BCs, a slope and an intercept, as defined in equation 8.2. But, we must provide at least one displacement boundary condition in order to compute the coefficient C_0 from equation 8.23. Specifying the two BCs at only one end of the interval is perfectly acceptable. The equation for segment 2 is:

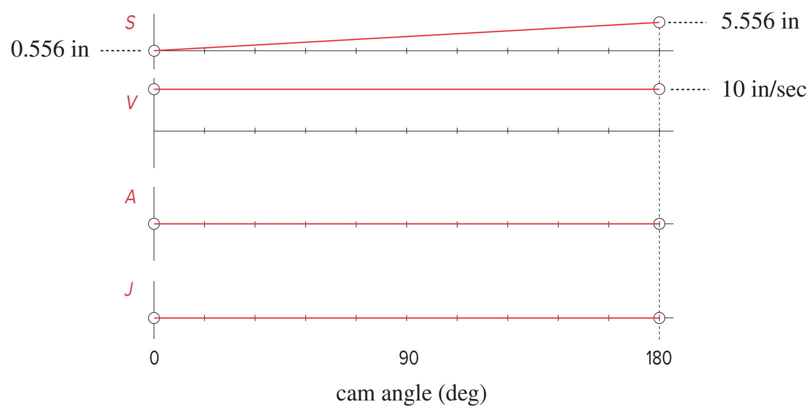

FIGURE 8-38

Segment two for the four-segment solution to the constant velocity problem (Example 8-11)

$$s = 5\left(\frac{\theta}{\beta}\right) + 0.556 \tag{8.27b}$$

11 Figure 8-38 shows the displacement and velocity plots of segment 2. The acceleration and jerk are both zero. The resulting displacement at $\theta = 210°$ is 5.556.

12 The displacement at the end of segment 2 is now known from its equation. The four boundary conditions for segment 3 are then:

$$\begin{aligned} &\text{when} \quad \theta = 210°; \quad s = 5.556, \quad v = 10, \quad a = 0 \\ &\text{when} \quad \theta = 240°; \quad \textit{none}, \quad v = 0, \quad \textit{none} \end{aligned} \tag{c}$$

13 This generates a cubic displacement function for segment 3 as in Figure 8-39. Its equation is:

$$s = -0.27792\left(\frac{\theta}{\beta}\right)^3 + 0.83376\left(\frac{\theta}{\beta}\right) + 5.556 \tag{8.27c}$$

14 The boundary conditions for the last segment 4 are now defined, as they must match those of the end of segment 3 and the beginning of segment 1. The displacement at the end of segment 3 is found from the computation in DYNACAM to be $s = 6.112$ at $\theta = 240°$ and the acceleration at that point is –239.9. We left the acceleration at the beginning of segment 1 unspecified. From the second derivative of the equation for displacement in that segment we find that the acceleration is 239.9 at $\theta = 0°$. The BCs for segment 4 are then:

$$\begin{aligned} &\text{when} \quad \theta = 240°; \quad s = 6.112, \quad v = 0, \quad a = -239.9 \\ &\text{when} \quad \theta = 360°; \quad s = 0, \quad v = 0, \quad a = 239.9 \end{aligned} \tag{d}$$

15 The equation for segment 4 is then:

$$s = -9.9894\left(\frac{\theta}{\beta}\right)^5 + 24.9735\left(\frac{\theta}{\beta}\right)^4 - 7.7548\left(\frac{\theta}{\beta}\right)^3 - 13.3413\left(\frac{\theta}{\beta}\right)^2 + 6.112 \tag{8.27d}$$

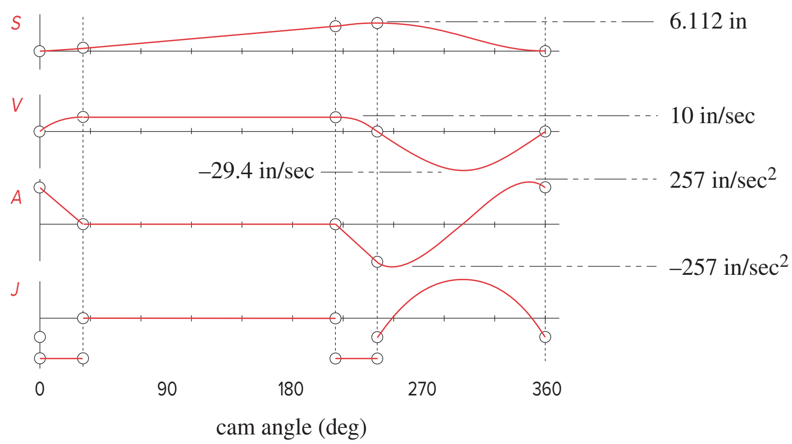

FIGURE 8-39

Four-segment solution to the constant velocity problem of Example 8-11

16 Figure 8-39 shows the *s v a j* plots for the complete cam. It obeys the fundamental law of cam design because the piecewise functions are continuous through the acceleration. The maximum value of acceleration is 257 in/sec^2. The maximum negative velocity is –29.4 in/sec. We now have four piecewise-continuous functions, equations 8.27, which will meet the performance specifications for this problem.

The reader may open the file E08-11.cam in program DYNACAM to investigate this example in greater detail.

While this design is acceptable, it can be improved. One useful strategy in designing polynomial cams is to minimize the number of segments, provided that this does not result in functions of such high degree that they misbehave between boundary conditions. Another strategy is to always start with the segment for which you have the most information. In this example, the constant velocity portion is the most constrained and must be a separate segment, just as a dwell must be a separate segment. The rest of the cam motion exists only to return the follower to the constant velocity segment for the next cycle. If we start by designing the constant velocity segment, it may be possible to complete the cam with only one additional segment. We will now redesign this cam, to the same specifications but with only two segments as shown in Figure 8-40.

EXAMPLE 8-12

Designing an Optimum Polynomial for Constant Velocity Critical Path Motion.

Problem: Redefine the problem statement of Example 8-11 to have only two segments.

Maintain a constant velocity of 10 in/sec for 0.5 sec
Decelerate and **accelerate** follower to constant velocity
Cycle time exactly 1 sec

Solution: See Figures 8-40 and 8-41.

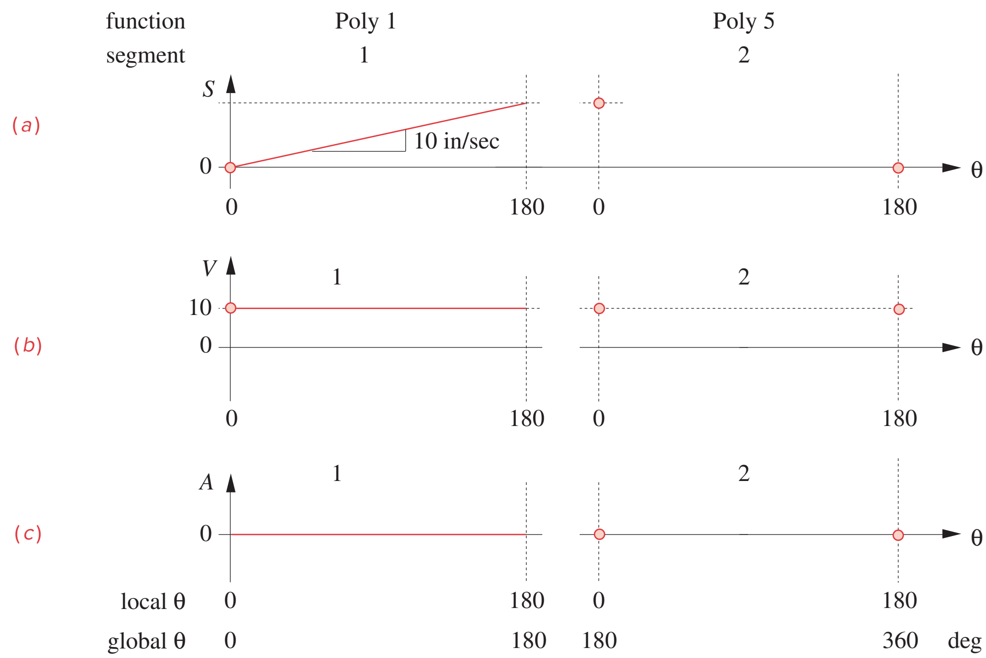

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FIGURE 8-40

Boundary conditions for the two-segment constant velocity solution

1 The BCs for the first, constant velocity, segment will be similar to our previous solution except for the global values of its angles and the fact that we will start at zero displacement rather than at 0.556 in. They are:

$$\begin{aligned} &\text{when} \quad \theta = 0°; \qquad s = 0, \qquad v = 10 \\ &\text{when} \quad \theta = 180°; \qquad \textit{none}, \qquad \textit{none} \end{aligned} \tag{a}$$

2 The displacement and velocity plots for this segment are identical to those in Figure 8-38 except that the displacement starts at zero. The equation for segment 1 is:

$$s = 5\left(\frac{\theta}{\beta}\right) \tag{8.28a}$$

3 The program calculates the displacement at the end of segment 1 to be 5.00 in. This defines that BC for segment 2. The set of BCs for segment 2 is then:

$$\begin{aligned} &\text{when} \quad \theta = 180°; \qquad s = 5.00, \qquad v = 10, \qquad a = 0 \\ &\text{when} \quad \theta = 360°; \qquad s = 0, \qquad v = 10, \qquad a = 0 \end{aligned} \tag{b}$$

The equation for segment 2 is:

$$s = -60\left(\frac{\theta}{\beta}\right)^5 + 150\left(\frac{\theta}{\beta}\right)^4 - 100\left(\frac{\theta}{\beta}\right)^3 + 5\left(\frac{\theta}{\beta}\right)^1 + 5 \tag{8.28b}$$

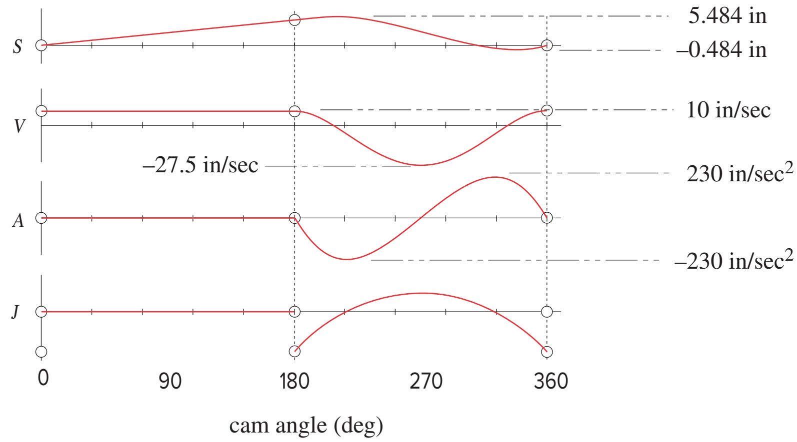

FIGURE 8-41

Two-segment solution to the constant velocity problem of Example 8-12

4 The *s v a j* diagrams for this design are shown in Figure 8-41. Note that they are much smoother than the four-segment design. The maximum acceleration in this example is now 230 in/sec^2, and the maximum negative velocity is –27.5 in/sec. These are both less than in the previous design of Example 8-11.

5 The fact that our displacement in this design contains negative values as shown in the *s* diagram of Figure 8-41 is of no concern. This is due to our starting with the beginning of the constant velocity portion as zero displacement. The follower has to go to a negative position in order to have distance to accelerate up to speed again. We will simply shift the displacement coordinates by that negative amount to make the cam. To do this, simply calculate the displacement coordinates for the cam. Note the value of the largest negative displacement. Add this value to the displacement boundary conditions for all segments and recalculate the cam functions with DYNACAM. (Do not change the BCs for the higher derivatives.) The finished cam's displacement profile will be shifted up such that its minimum value will now be zero.

So, not only do we now have a smoother cam but the dynamic forces and stored kinetic energy are both lower. Note that we did not have to make any assumptions about the portions of the available nonconstant velocity time to be devoted to speeding up or slowing down. This all happened automatically from our choice of only two segments and the specification of the minimum set of necessary boundary conditions. This is clearly a superior design to the previous attempt and is in fact an optimal polynomial solution to the given specifications. The reader is encouraged to open the file E08-12.cam in program DYNACAM to investigate this example in more detail.

SUMMARY These sections have presented polynomial functions as the most versatile approach (of those shown here) to virtually any cam design problem. It is only since the development and general availability of computers that polynomial functions have become practical to use, as the computation to solve the simultaneous equations is often beyond hand calculation abilities. With the availability of a design aid to solve the equations such as program DYNACAM, polynomials have become a practical and preferable way to solve many, but not all, cam design problems. **Spline functions**, of which polynomials are a subset, offer even more flexibility in meeting boundary constraints and other cam performance criteria. Space does not permit a detailed exposition of spline functions as applied to cam systems here. See reference [6] for more information.

8.6 SIZING THE CAM—PRESSURE ANGLE AND RADIUS OF CURVATURE *View the lecture video (48:55)*[‡]

[†] http://www.designof-machinery.com/DOM/Cam_Design_III.mp4

Once the *s v a j* functions have been defined, the next step is to size the cam. There are two major factors that affect cam size, the **pressure angle** and the **radius of curvature**. Both of these involve either the **base circle radius** on the cam (R_b) when using flat-faced followers, or the **prime circle radius** on the cam (R_p) when using roller or curved followers.

The base circle's and prime circle's centers are at the center of rotation of the cam. The base circle is defined as *the smallest circle that can be drawn tangent to the physical cam surface* as shown in Figure 8-42. All radial cams will have a base circle, regardless of the follower type used.

The prime circle is only applicable to cams with roller followers or radiused (mushroom) followers and is measured to the center of the follower. The **prime circle** is defined as *the smallest circle that can be drawn tangent to the locus of the centerline of the follower* as shown in Figure 8-42. *The locus of the centerline of the follower* is called the **pitch curve**. Cams with roller followers are in fact defined for manufacture with respect to the pitch curve rather than with respect to the cam's physical surface. Cams with flat-faced followers must be defined for manufacture with respect to their physical surface, as there is no pitch curve.

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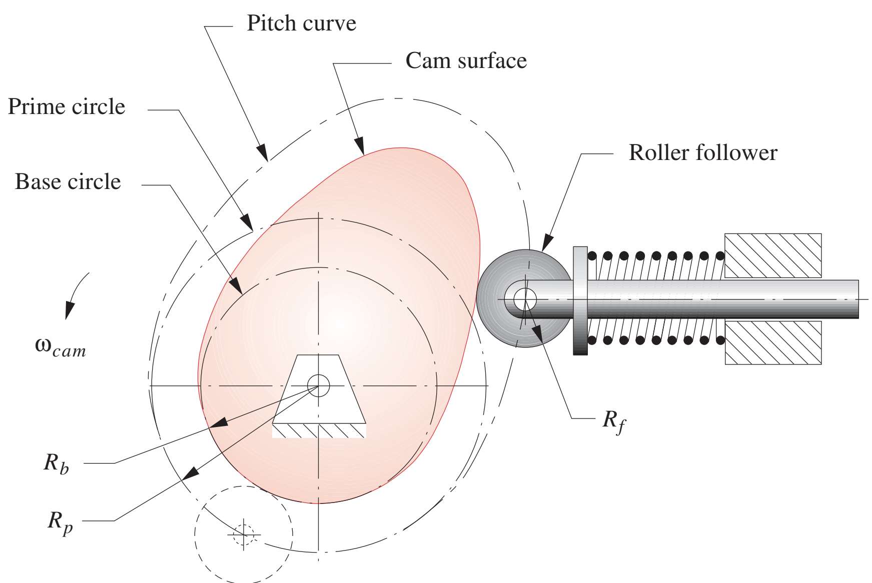

FIGURE 8-42

Base circle R_b, prime circle R_p, and pitch curve of a radial cam with roller follower

The process of creating the physical cam from the *s* diagram can be visualized conceptually by imagining the *s* diagram to be cut out of a flexible material such as rubber. The *x* axis of the *s* diagram represents the circumference of a circle, which could be either the **base circle**, or the **prime circle**, around which we will "wrap" our "rubber" *s* diagram. We are free to choose the initial length of our *s* diagram's *x* axis, though the height of the displacement curve is fixed by the cam displacement function we have chosen. In effect we will choose the base or prime circle radius as a design parameter and stretch the length of the *s* diagram's axis to fit the circumference of the chosen circle.

We will present equations for pressure angle and radius of curvature only for radial cams with translating followers here. For related information on oscillating followers and axial (barrel) cams, see Chapter 7 of reference [5].

Pressure Angle—Translating Roller Followers

The **pressure angle** is defined as shown in Figure 8-43. It is the complement of the transmission angle that was defined for linkages in previous chapters and has a similar meaning with respect to cam-follower operation. By convention, the pressure angle is used for cams, rather than the transmission angle. Force can only be transmitted from cam to follower or vice versa along the **axis of transmission** which is perpendicular to the **axis of slip**, or common tangent.

Pressure Angle The **pressure angle** ϕ is *the angle between the direction of motion (velocity) of the follower and the direction of the axis of transmission.** When $\phi = 0$, all the transmitted force goes into motion of the follower and none into slip velocity. When ϕ becomes 90° there will be no motion of the follower. As a rule of thumb, we would like the pressure angle to be between zero and about 30° for translating followers to avoid excessive side load on the sliding follower. If the follower is oscillating on a pivoted arm, a pressure angle up to about 35° is acceptable. Values of ϕ greater than this can increase the follower sliding or pivot friction to undesirable levels and may tend to jam a translating follower in its guides.

* Dresner and Buffington[7] point out that this definition is only valid for single-degree-of-freedom systems. For multi-input systems, a more complicated definition and calculation of pressure angle (or transmission angle) are needed.

Eccentricity Figure 8-44 shows the geometry of a cam and translating roller follower in an arbitrary position. This shows the general case in that the axis of motion of the follower does not intersect the center of the cam. There is an **eccentricity** ε defined as *the perpendicular distance between the follower's axis of motion and the center of the cam*. Often this eccentricity ε will be zero, making it an **aligned follower**, which is the special case.

In Figure 8-44, the axis of transmission is extended to intersect effective link 1, which is the ground link. (See Section 8.0 and Figure 8-1 for a discussion of effective links in cam systems.) This intersection is instant center $I_{2,4}$ (labeled *B*) which, by definition, has the same velocity in link 2 (the cam) and in link 4 (the follower). Because link 4 is in pure translation, all points on it have identical velocities $V_{follower}$, which are equal to the velocity of $I_{2,4}$ in link 2. We can write an expression for the velocity of $I_{2,4}$ in terms of cam angular velocity and the radius *b* from cam center to $I_{2,4}$,

$$V_{I_{2,4}} = b\omega = \dot{S} \tag{8.29}$$

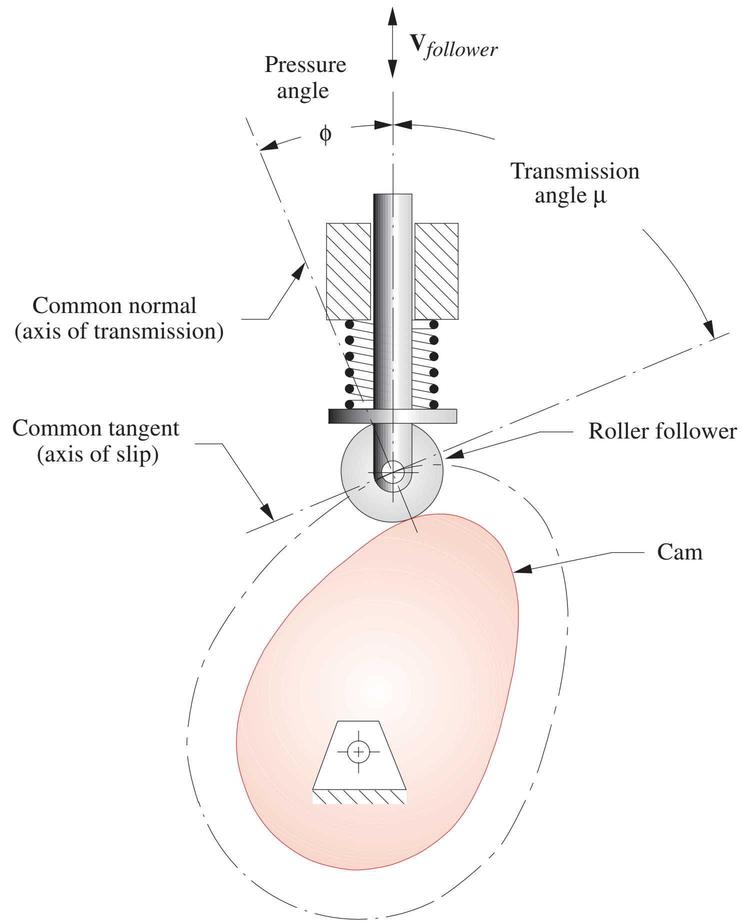

FIGURE 8-43

Cam pressure angle

where s or S is the instantaneous displacement of the follower from the S diagram and $\dot{S}$ is its time derivative in units of length/sec. (Note that capital V, A, J denote time-based variables and v, a, j are functions of cam angle—length/rad, length/rad^2, length/rad^3.)

But
$$\dot{S} = \frac{dS}{dt}$$

and
$$\frac{dS}{dt}\frac{d\theta}{d\theta} = \frac{dS}{d\theta}\frac{d\theta}{dt} = \frac{dS}{d\theta}\omega = v\omega$$

so
$$b\omega = v\omega$$

then
$$b = v \tag{8.30}$$

This is an interesting relationship which says that the **distance** b **to the instant center** $\boldsymbol{I_{2,4}}$ **is equal to the velocity of the follower** v in units of length per radian as derived in previous sections. We have reduced this expression to pure geometry, independent of the angular velocity ω of the cam.

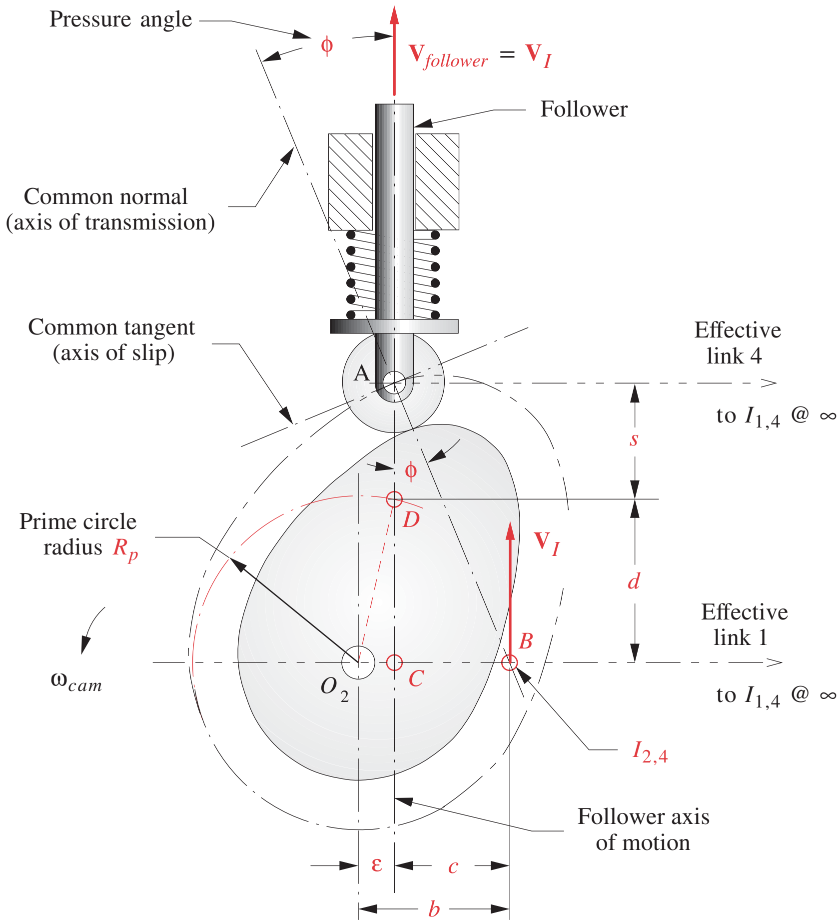

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FIGURE 8-44

Geometry for the derivation of the equation for pressure angle

Note that we can express the distance b in terms of the prime circle radius R_p and the eccentricity ε, by the construction shown in Figure 8-44. Swing the arc of radius R_p until it intersects the axis of motion of the follower at point D. This defines the length of line d from effective link 1 to this intersection. This is constant for any chosen prime circle radius R_p. Points A, C, and $I_{2,4}$ form a right triangle whose upper angle is the pressure angle ϕ and whose vertical leg is $(s + d)$, where s is the instantaneous displacement of the follower. From this triangle:

$$c = b - \varepsilon = (s + d)\tan\phi$$

and

$$b = (s + d)\tan\phi + \varepsilon \tag{8.31a}$$

Then from equation 8.30,

$$v = (s + d)\tan\phi + \varepsilon \tag{8.31b}$$

and from triangle CDO_2,

$$d = \sqrt{R_P^2 - \varepsilon^2} \tag{8.31c}$$

Substituting equation 8.31c into equation 8.31b and solving for ϕ give an expression for pressure angle in terms of displacement s, velocity v, eccentricity ε, and the prime circle radius R_p.

$$\phi = \arctan \frac{v - \varepsilon}{s + \sqrt{R_P^2 - \varepsilon^2}} \tag{8.31d}$$

The velocity v in this expression is in units of length/rad, and all other quantities are in compatible length units. We have typically defined s and v by this stage of the cam design process and wish to manipulate R_p and ε to get an acceptable maximum pressure angle ϕ. As R_p is increased, ϕ will be reduced. The only constraints against large values of R_p are the practical ones of package size and cost. Often there will be some upper limit on the size of the cam-follower package dictated by its surroundings. There will always be a cost constraint and bigger = heavier = more expensive.

Choosing a Prime Circle Radius

8

Both R_p and ε are within a transcendental expression in equation 8.31d, so they cannot be conveniently solved for directly. The simplest approach is to assume a trial value for R_p and an initial eccentricity of zero, and use program DYNACAM, your own program, or an equation solver such as *Matlab*, *TKSolver* or *Mathcad* to quickly calculate the values of ϕ for the entire cam, and then adjust R_p and repeat the calculation until an acceptable arrangement is found. Figure 8-45 shows the calculated pressure angles for a four-dwell cam. Note the similarity in shape to the velocity functions for the same cam in Figure 8-6, as that term is dominant in equation 8.31d.

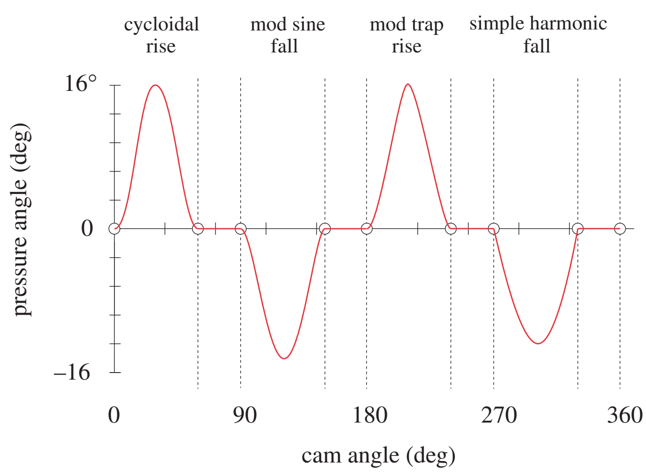


FIGURE 8-45

Pressure angle functions are similar in shape to velocity functions.

Using Eccentricity If a suitably small cam cannot be obtained with acceptable pressure angle, then eccentricity can be introduced to change the pressure angle. Using eccentricity to control the pressure angle has its limitations. For a positive ω, a positive value of eccentricity will *decrease the pressure angle on the rise* but will *increase it on the fall*. Negative eccentricity does the reverse.

This is of little value with a form-closed (groove or track) cam, as it is driving the follower in both directions. For a force-closed cam with spring return, you can sometimes afford to have a larger pressure angle on the fall than on the rise because the stored energy in the spring is attempting to speed up the camshaft on the fall, whereas the cam is storing that energy in the spring on the rise. The limit of this technique can be the degree of overspeed attained with a larger pressure angle on the fall. The resulting variations in cam angular velocity may be unacceptable.

The most value gained from adding eccentricity to a follower comes in situations where the cam program is asymmetrical and significant differences exist (with no eccentricity) between maximum pressure angles on rise and fall. Introducing eccentricity can balance the pressure angles in this situation and create a smoother running cam.

If adjustments to R_p or ε do not yield acceptable pressure angles, the only recourse is to return to an earlier stage in the design process and redefine the problem. Less lift or more time to rise or fall will reduce the causes of the large pressure angle. Design is, after all, an iterative process.

Overturning Moment—Translating Flat-Faced Follower

Figure 8-46 shows a translating, flat-faced follower running against a radial cam. The pressure angle can be seen to be zero for all positions of cam and follower. This seems to be giving us something for nothing, which can't be true. As the contact point moves left and right, the point of application of the force between cam and follower moves with it. There is an overturning moment on the follower associated with this off-center force which tends to jam the follower in its guides, just as did too large a pressure angle in the roller follower case. In this case, we would like to keep the cam as small as possible in order to minimize the moment arm of the force. Eccentricity will affect the average value of the moment, but the peak-to-peak variation of the moment about that average is unaffected by eccentricity. Considerations of too-large pressure angle do not limit the size of this cam, but other factors do. The minimum radius of curvature (see below) of the cam surface must be kept large enough to avoid undercutting. This is true regardless of the type of follower used.

Radius of Curvature—Translating Roller Follower

The **radius of curvature** is a *mathematical property of a function*. Its value and use is not limited to cams but has great significance in their design. The concept is simple. No matter how complicated a curve's shape may be, nor how high the degree of the describing function, it will have an instantaneous radius of curvature at every point on the curve. These radii of curvature will have instantaneous centers (which may be at infinity), and the radius of curvature of any function is itself a function that can be computed and plotted. For example, the radius of curvature of a straight line is infinity everywhere; that of a circle is a constant value. A parabola has a constantly changing radius of curvature that

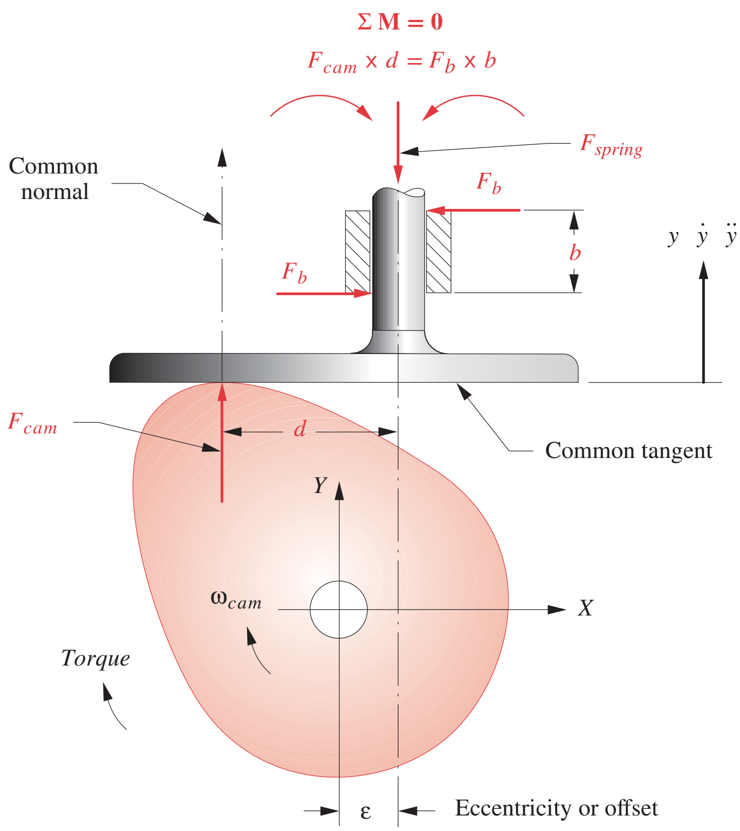


FIGURE 8-46

Overturning moment on a flat-faced follower

approaches infinity. A cubic curve will have radii of curvature that are sometimes positive (convex) and sometimes negative (concave). The higher the degree of a function, in general, the more potential variety in its radius of curvature.

Cam contours are usually functions of high degree. When they are wrapped around their base or prime circles, they may have portions that are concave, convex, or flat. Infinitesimally short flats of infinite radius will occur at all inflection points on the cam surface where it changes from concave to convex or vice versa.

The radius of curvature of the finished cam is of concern regardless of the follower type, but the concerns are different for different followers. Figure 8-47 shows an obvious (and exaggerated) problem with a roller follower whose own (constant) radius of curvature R_f is too large to follow the locally smaller concave (negative) radius $-\rho$ on the cam. (Note that, normally, one would not use that large a roller compared to the cam.)

A more subtle problem occurs when the roller follower radius R_f is larger than the smallest positive (convex) local radius $+\rho$ on the cam. This problem is called **undercutting** and is depicted in Figure 8-48. Recall that for a roller follower cam, the cam contour is actually defined as the locus of the center of the roller follower, or the **pitch curve**. The

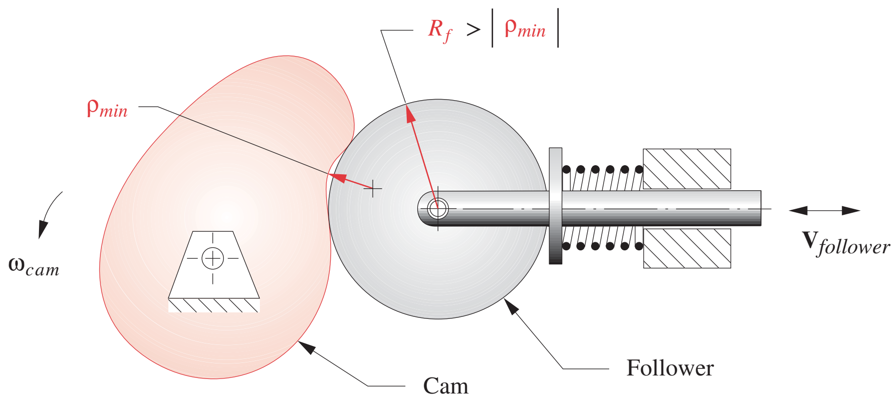


FIGURE 8-47

The result of using a roller follower larger than the one for which the cam was designed

machinist is given these *x,y* coordinate data (on computer tape or disk) and also told the radius of the follower R_f. The machinist will then cut the cam with a cutter of the same effective radius as the follower, following the pitch curve coordinates with the center of the cutter.

Figure 8-48a shows the situation in which the follower (cutter) radius R_f is at one point exactly equal to the minimum convex radius of curvature of the cam ($+\rho_{min}$). The cutter creates a perfect sharp point, or **cusp**, on the cam surface. This cam will not run very well at speed! Figure 8-48b shows the situation in which the follower (cutter) radius is greater than the minimum convex radius of curvature of the cam. The cutter now undercuts or removes material needed for cam contours in different locations and also creates a sharp point or cusp on the cam surface. This cam no longer has the same displacement function you so carefully designed.

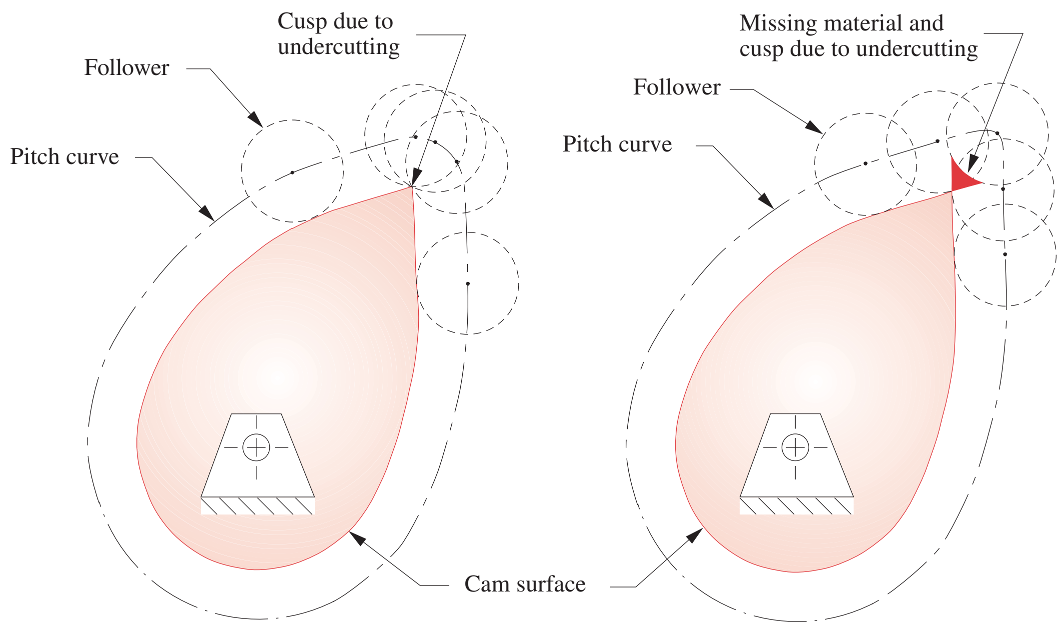
The rule of thumb is to keep the absolute value of the minimum radius of curvature ρ_{min} of the cam pitch curve preferably at least 2 to 3 times as large as the radius of the roller follower R_f.

$$|\rho_{\min}| >> R_f \tag{8.32}$$

A derivation for radius of curvature can be found in any calculus text. For our case of a roller follower, we can write the equation for the radius of curvature of the pitch curve of the cam as:

$$\rho_{pitch} = \frac{\left[(R_P + s)^2 + v^2\right]^{3/2}}{(R_P + s)^2 + 2v^2 - a(R_P + s)} \tag{8.33}$$

In this expression, *s*, *v*, and *a* are the displacement, velocity, and acceleration of the cam program as defined in a previous section. Their units are length, length/rad, and length/rad^2, respectively. R_p is the prime circle radius. **Do not confuse** this *prime circle radius* R_p with the *radius of curvature*, ρ_{pitch}. R_p is a **constant value** which you choose as a design parameter and ρ_{pitch} is the constantly changing radius of curvature that results from your design choices.



(*a*) Radius of curvature of pitch curve equals the radius of the roller follower

(*b*) Radius of curvature of pitch curve is less than the radius of the roller follower

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FIGURE 8-48

Small positive radius of curvature can cause undercutting.

Also do not confuse R_p, the *prime circle radius,* with R_f, the *radius of the roller follower*. See Figure 8-43 for definitions. You can choose the value of R_f to suit the problem, so you might think that it is simple to satisfy equation 8.32 by just selecting a roller follower with a small value of R_f. Unfortunately it is more complicated than that, as a small roller follower may not be strong enough to withstand the dynamic forces from the cam. The radius of the pin on which the roller follower pivots is substantially smaller than R_f because of the space needed for roller or ball bearings within the follower. Dynamic forces will be addressed in later chapters where we will revisit this problem.

We can solve equation 8.33 for ρ_{pitch} since we know *s*, *v,* and *a* for all values of θ and can choose a trial R_p. If the pressure angle has already been calculated, the R_p found for its acceptable values should be used to calculate ρ_{pitch} as well. If a suitable follower radius cannot be found which satisfies equation 8.32 for the minimum values of ρ_{pitch} calculated from equation 8.33, then further iteration will be needed, possibly including a redefinition of the cam specifications.

Program DYNACAM calculates ρ_{pitch} for all values of θ for a user supplied prime circle radius R_p. Figure 8-49 shows ρ_{pitch} for the four-dwell cam of Figure 8-6. Note that this cam has both positive and negative radii of curvature. The large values of radius of curvature are truncated at arbitrary levels on the plot as they are heading to infinity at the inflection points between convex and concave portions. Note that the radii of curvature

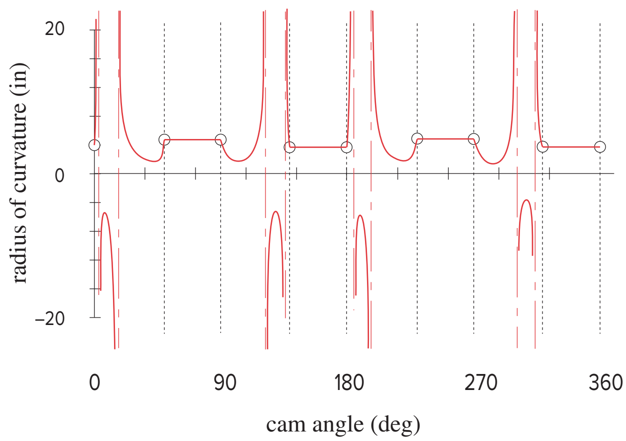

FIGURE 8-49

Radius of curvature of a four-dwell cam

go out to positive infinity and return from negative infinity or vice versa at these inflection points (perhaps after a round trip through the universe?).

Once an acceptable prime circle radius and roller follower radius are determined based on pressure angle and radius of curvature considerations, the cam can be drawn in finished form and subsequently manufactured. Figure 8-50 shows the profile of the four-dwell cam from Figure 8-6. The cam surface contour is swept out by the envelope of follower positions just as the cutter will create the cam in metal. The sidebar shows the parameters for the design, which is an acceptable one. The ρ_{min} is 1.7 times R_f and the pressure angles are less than 30°. The contours on the cam surface appear smooth, with no sharp corners. Figure 8-51 shows the same cam with only one change. The radius of follower R_f has been made the same as the minimum radius of curvature, ρ_{min}. The sharp corners or cusps in several places indicate that undercutting has occurred. This has now become an **unacceptable cam**, *simply because of a roller follower that is too large*.

The coordinates for the cam contour, measured to the locus of the center of the roller follower, or the **pitch curve** as shown in Figure 8-50, are defined by the following expressions, referenced to the center of rotation of the cam. See Figure 8-44 for nomenclature. The subtraction of the cam input angle θ from 2π is necessary because the relative motion of the follower versus the cam is opposite to that of the cam versus the follower. In other words, to define the contour of the centerline of the follower's path around a stationary cam, we must move the follower (and also the cutter to make the cam) in the opposite direction of cam rotation.

$$x = \cos\lambda\sqrt{(d+s)^2 + \varepsilon^2}$$
$$y = \sin\lambda\sqrt{(d+s)^2 + \varepsilon^2} \tag{8.34}$$

where:

$$\lambda = (2\pi - \theta) - \arctan\left(\frac{\varepsilon}{d+s}\right)$$

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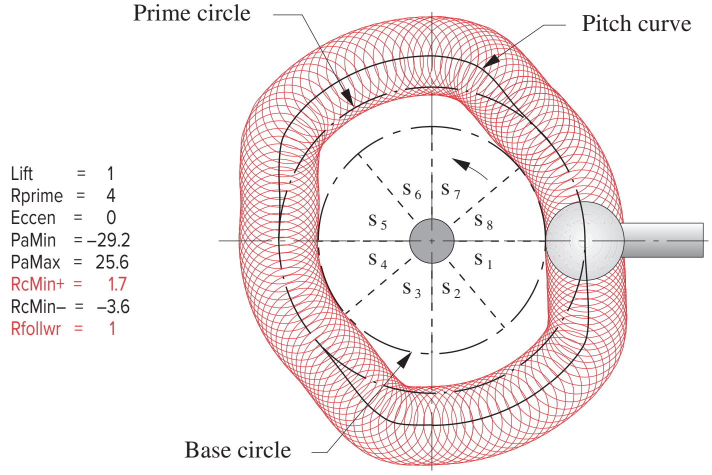

FIGURE 8-50

Radial plate cam profile is generated by the locus of the roller follower (or cutter)

Radius of Curvature—Translating Flat-Faced Follower

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The situation with a flat-faced follower is different from that of a roller follower. A negative radius of curvature on the cam cannot be accommodated with a flat-faced follower. The flat follower obviously cannot follow a concave cam. Undercutting will occur when the radius of curvature becomes negative if a cam with that condition is made.

Figure 8-52 shows a cam and translating flat-faced follower in an arbitrary position. The origin of the global *XY* coordinate system is placed at the cam's center of rotation, and the *X* axis is defined parallel to the common tangent, which is the surface of the flat

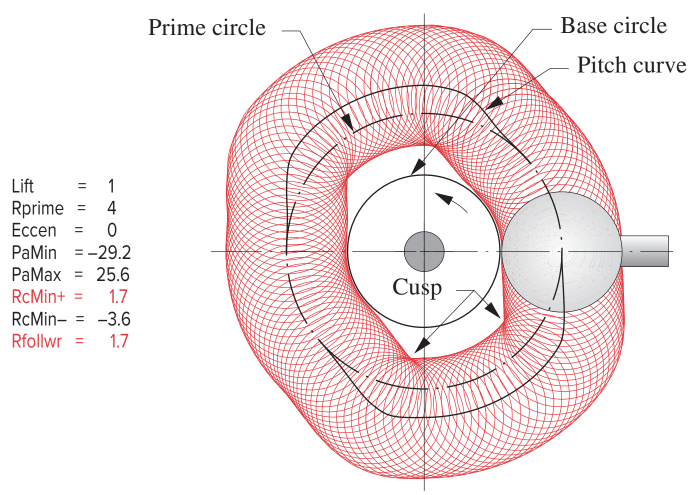

FIGURE 8-51

Cusps formed by undercutting due to radius of follower $R \geq$ cam radius of curvature ρ

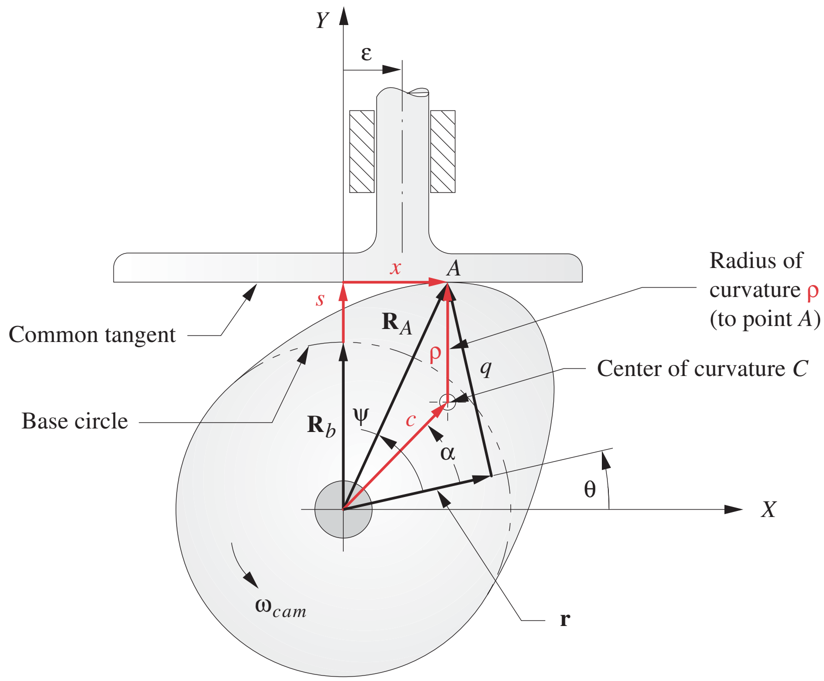

FIGURE 8-52

Geometry for derivation of radius of curvature and cam contour with flat-faced follower

follower. The vector **r** is attached to the cam, rotates with it, and serves as the reference line to which the cam angle θ is measured from the *X* axis. The point of contact *A* is defined by the position vector $\mathbf{R}_A$. The instantaneous center of curvature is at *C* and the radius of curvature is ρ. R_b is the radius of the base circle and *s* is the displacement of the follower for angle θ. The eccentricity is ε.

We can define the location of contact point *A* from two vector loops (in complex notation).

$$\mathbf{R}_A = x + j\left(R_b + s\right)$$

and

$$\mathbf{R}_A = ce^{j(\theta+\alpha)} + j\rho$$

so:

$$ce^{j(\theta+\alpha)} + j\rho = x + j\left(R_b + s\right) \tag{8.35a}$$

Substitute the Euler equivalent (equation 4.4a) in equation 8.35a and separate the real and imaginary parts.

real:

$$c\cos\left(\theta+\alpha\right) = x \tag{8.35b}$$

imaginary:

$$c\sin(\theta+\alpha)+\rho = R_b + s \tag{8.35c}$$

The center of curvature C is **stationary** on the cam, meaning that the magnitudes of c and ρ, and angle α do not change for small changes in cam angle θ. (These values are not constant but are at stationary values. Their first derivatives with respect to θ are zero, but their higher derivatives are not zero.)

Differentiating equation 8.35a with respect to θ then gives:

$$jce^{j(\theta+\alpha)} = \frac{dx}{d\theta} + j\frac{ds}{d\theta} \tag{8.36}$$

Substitute the Euler equivalent (equation 4.4a) in equation 8.36 and separate the real and imaginary parts.

real:

$$-c\sin(\theta+\alpha) = \frac{dx}{d\theta} \tag{8.37}$$

imaginary:

$$c\cos(\theta+\alpha) = \frac{ds}{d\theta} = v \tag{8.38}$$

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Inspection of equations 8.35b and 8.36 shows that:

$$x = v \tag{8.39}$$

This is an interesting relationship that says the x position of the contact point between cam and follower is equal to the velocity of the follower in length/rad. This means that the v diagram gives a direct measure of the necessary minimum face width of the flat follower.

$$facewidth > v_{max} - v_{min} \tag{8.40}$$

If the velocity function is asymmetric, then a minimum-width follower will have to be asymmetric also, in order not to fall off the cam.

Differentiating equation 8.39 with respect to θ gives:

$$\frac{dx}{d\theta} = \frac{dv}{d\theta} = a \tag{8.41}$$

Equations 8.35c and 8.37 can be solved simultaneously and equation 8.41 substituted in the result to yield:

$$\rho = R_b + s + a \tag{8.42a}$$

and the minimum value of radius of curvature is

$$\rho_{min} = R_b + (s+a)_{min} \tag{8.42b}$$

Base Circle Note that equations 8.42 define the radius of curvature in terms of the base circle radius and the displacement and acceleration functions from the *s v a j* diagrams only. Because ρ cannot be allowed to become negative with a flat-faced follower, we can formulate a relationship from equation 8.42b that will predict the minimum base circle radius R_b needed to avoid undercutting. The only factor on the right side of equations 8.42 that can be negative is the acceleration, a. We have defined s to be always positive, as is R_b. Therefore, the worst case for undercutting will occur when a is near its **largest negative value**, a_{min}, whose value we know from the a diagram. The minimum base circle radius can then be defined as:

$$R_{b_{\min}} > \rho_{\min} - (s + a)_{\min} \tag{8.43}$$

Because the value of a_{min} is negative and it is also negated in equation 8.43, it dominates the expression. To use this relationship, we must choose some minimum radius of curvature ρ_{min} for the cam surface as a design parameter. Since the hertzian contact stresses at the contact point are a function of local radius of curvature, that criterion can be used to select ρ_{min}. That topic is beyond the scope of this text and will not be further explored here. See reference [1] for further information on contact stresses.

Cam Contour For a flat-faced follower cam, the coordinates of the physical cam surface must be provided to the machinist as there is no pitch curve to work to. Figure 8-52 shows two orthogonal vectors, **r** and **q**, which define the cartesian coordinates of contact point A between cam and follower with respect to a rotating axis coordinate system embedded in the cam. Vector **r** is the rotating "x" axis of this embedded coordinate system. Angle ψ defines the position of vector $\mathbf{R}_A$ in this system. Two vector loop equations can be written and equated to define the coordinates of all points on the cam surface as a function of cam angle θ.

$$\mathbf{R}_A = x + j(R_b + s)$$

and

$$\mathbf{R}_A = re^{j\theta} + qe^{j\left(\theta + \frac{\pi}{2}\right)}$$

so:

$$re^{j\theta} + qe^{j\left(\theta + \frac{\pi}{2}\right)} = x + j(R_b + s) \tag{8.44}$$

Divide both sides by $e^{j\theta}$:

$$r + jq = xe^{-j\theta} + j(R_b + s)e^{-j\theta} \tag{8.45}$$

Separate into real and imaginary components and substitute v for x from equation 8.39:

real (x component):

$$r = (R_b + s)\sin\theta + v\cos\theta \tag{8.46a}$$

imaginary (y component):

$$q = (R_b + s)\cos\theta - v\sin\theta \tag{8.46b}$$

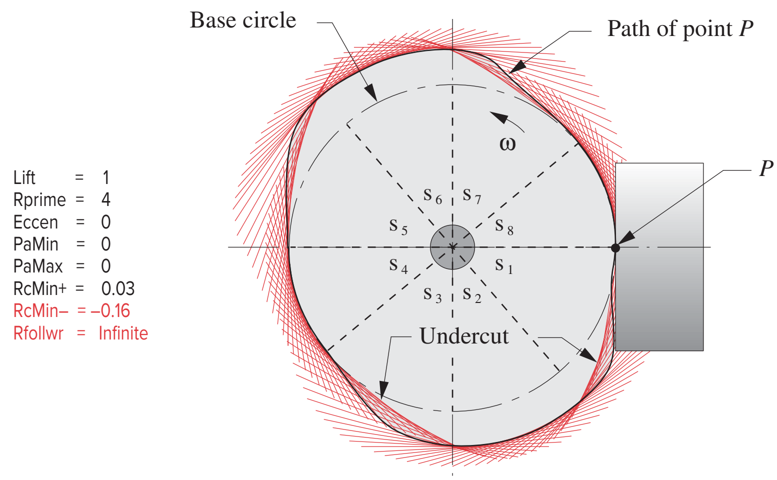

FIGURE 8-53

Undercutting due to negative radius of curvature used with flat-faced follower

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Equations 8.46 can be used to machine the cam for a flat-faced follower. These *x, y* components are in the rotating coordinate system that is embedded in the cam.

Note that none of the equations developed above for this case involve the **eccentricity**, ε. It is only a factor in cam size when a roller follower is used. It does not affect the geometry of a flat follower cam.

Figure 8-53 shows the result of trying to use a flat-faced follower on a cam whose theoretical path of follower point *P* has negative radius of curvature due to a base circle radius that is too small. If the follower tracked the path of *P* as is required to create the motion function defined in the *s* diagram, the cam surface would actually be as developed by the envelope of straight lines shown. But, these loci of the follower face are cutting into cam contours that are needed for other cam angles. The line running through the forest of follower loci is the path of point *P* needed for this design. The undercutting can be clearly seen as the crescent-shaped missing pieces at four places between the path of *P* and the follower face loci. Note that if the follower were zero width (at point *P*), it would work kinematically, but the stress at the knife edge would be infinite.

SUMMARY The task of sizing a cam is an excellent example of the need for and value of iteration in design. Rapid recalculation of the relevant equations with a tool such as program DYNACAM makes it possible to quickly and painlessly arrive at an acceptable solution while balancing the often conflicting requirements of pressure angle and radius of curvature constraints. In any cam, either the pressure angle or radius of curvature considerations will dictate the minimum size of the cam. Both factors must be checked. The choice of follower type, either roller or flat-faced, makes a big difference in the cam geometry. Cam programs that generate negative radii of curvature are unsuited to the flat-faced type of follower unless very large base circles are used to force ρ to be positive everywhere.

8.7 PRACTICAL DESIGN CONSIDERATIONS

The cam designer is often faced with many confusing decisions, especially at an early stage of the design process. Many early decisions, often made somewhat arbitrarily and without much thought, can have significant and costly consequences later in the design. The following is a discussion of some of the trade-offs involved with such decisions in the hope that it will provide the cam designer with some guidance in making these decisions.

Translating or Oscillating Follower?

There are many cases, especially early in a design, when either translating or rotating motion could be accommodated as output from the cam, though in other situations, the follower motion and geometry is dictated to the designer. If some design freedom is allowed, and straight-line motion is specified, the designer should consider the possibility of using an approximate straight-line motion, which is often adequate and can be obtained from a large-radius rocker follower. The rocker or oscillating follower has advantages over the translating follower when a roller is used. A round-cross-section translating follower slide is free to rotate about its axis of translation and needs to have some antirotation guiding provided (such as a keyway or second slide) to prevent z axis misalignment of the roller follower with the cam. Many commercial, nonrotating slide assemblies are now available, often fitted with ball bearings, and these provide a good way to deal with this issue. However, an oscillating follower arm will keep the roller follower aligned in the same plane as the cam with no guiding other than its own pivot.

Also, the pivot friction in an oscillating follower typically has a small moment arm compared to the moment of the force from the cam on the follower arm. But, the friction force on a translating follower has a one-to-one geometric relationship with the cam force. This can have a larger parasitic effect on the system.

Translating flat-faced followers are often deliberately arranged with their axis slightly out of the plane of the cam in order to create a rotation about their own axis due to the frictional moment resulting from the offset. The flat follower will then *precess* around its own axis and distribute the wear over its entire face surface. This is common practice with automotive valve cams that use flat-faced followers or "tappets."

Force- or Form-Closed?

A form-closed (track or groove) cam or conjugate cams are more expensive to make than a force-closed (open) cam simply because there are two surfaces to machine and grind. Also, heat treating will often distort the track of a form-closed cam, narrowing or widening it such that the roller follower will not fit properly. This virtually requires post heat-treat grinding for track cams in order to resize the slot. An open (force-closed) cam will also distort on heat-treating, but can still be usable without grinding.

FOLLOWER JUMP The principal advantage of a form-closed (track) or conjugate-pair cam is that it does not need a return spring, and thus can be run at higher speeds than a force-closed cam whose spring and follower mass will go into resonance at some speed, causing potentially destructive follower jump. This phenomenon will be investigated in Chapter 15 on cam dynamics. High-speed automobile and motorcycle racing engines often use form-closed (desmodromic)* valve cam trains to allow higher engine rpm without incurring valve "float," or **follower jump**.

* More information on desmodromic cam-follower mechanisms can be found at http://members.chello.nl/~wgj.jansen/ where a number of models of their commercial implementations can be viewed in operation as movies.

CROSSOVER SHOCK Though the lack of a return spring can be an advantage, it comes, as usual, with a trade-off. In a form-closed (track) cam there will be **crossover shock** each time the acceleration changes sign. Crossover shock describes the impact force that occurs when the follower suddenly jumps from one side of the track to the other as the dynamic force (*ma*) reverses sign. There is no flexible spring in this system to absorb the force reversal as in the force-closed case. The high impact forces at crossover cause noise, high stresses, and local wear. Also, the roller follower has to reverse direction at each crossover, which causes sliding and accelerates follower wear. Studies have shown that roller followers running against a well-lubricated open radial cam have slip rates of less than 1%.[9]

Radial or Axial Cam?

This choice is largely dictated by the overall geometry of the machine for which the cam is being designed. If the follower must move parallel to the camshaft axis, then an axial cam is dictated. If there is no such constraint, a radial cam is probably a better choice simply because it is a less complicated, thus less expensive, cam to manufacture.

Roller or Flat-Faced Follower?

The roller follower is a better choice from a cam design standpoint simply because it accepts negative radius of curvature on the cam. This allows more variety in the cam program. Also, for any production quantity, the roller follower has the advantage of being available from several manufacturers in any quantity from one to a million. For low quantities it is not usually economical to design and build your own custom follower. In addition, replacement roller followers can be obtained from suppliers on short notice when repairs are needed. Also, they are not particularly expensive even in small quantities.

Perhaps the largest users of flat-faced followers are automobile engine makers. Their quantities are high enough to allow any custom design they desire. It can be made or purchased economically in large quantity and can be less expensive than a roller follower in that case. Also with engine valve cams, a flat follower can save space over a roller. Nevertheless, many manufacturers have switched to roller followers in automobile engine valve trains to reduce friction and improve fuel economy. Most new automotive internal combustion engines designed in the United States in recent years have used roller followers for those reasons. Diesel engines have long used roller followers (tappets) as have racers who "hop-up" engines for high performance.

Cams used in automated production line machinery use stock roller followers almost exclusively. The ability to quickly change a worn follower for a new one taken from the stockroom without losing much production time on the "line" is a strong argument in this environment. Roller followers come in several varieties (see Figure 8-5a). They are based on roller or ball bearings. Plain bearing versions are also available for low-noise requirements. The outer surface, which rolls against the cam, can be either cylindrical or spherical in shape. The "crown" on the spherical follower is slight, but it guarantees that the follower will ride near the center of a flat cam even with some inaccuracy of alignment of the axes of rotation of cam and follower. If a cylindrical follower is chosen and care is not taken to align the axes of cam and roller follower, or if it deflects under load, the follower will ride on one edge and wear rapidly.

Commercial roller followers are typically made of high carbon alloy steel such as AISI 52100 and hardened to Rockwell HRC 60–62. The 52100 alloy is well suited to thin sections that must be heat-treated to a uniform hardness. Because the roller makes many revolutions for each cam rotation, its wear rate will typically be higher than that of the cam. Chrome plating the follower can markedly improve its life. Chrome is harder than steel at about HRC 70. Steel cams are typically hardened to a range of HRC 50–55.

To Dwell or Not to Dwell?

The need for a dwell is usually clear from the problem specifications. If the follower must be held stationary for any time, then a dwell is required. Some cam designers tend to insert dwells in situations where they are not specifically needed for follower stasis, in a mistaken belief that this is preferable to providing a rise-return motion when that is what is really needed. If the designer is attempting to use a double-dwell program in what really needs only to be a single-dwell case, with the motivation to "let the vibrations settle out" by providing a "short dwell" at the end of the motion, he or she is misguided. Instead, the designer probably should be using a different cam program, perhaps a polynomial or a B-spline tailored to the specifications. Taking the follower acceleration to zero, whether for an instant or for a "short dwell," is generally undesirable unless absolutely required for machine function. (See Examples 8-6 , 8-7, and 8-8.) A dwell should be used only when the follower is required to be stationary for some measurable time. Moreover, if you do not need any dwell at all, consider using a linkage instead. They are a lot easier and less expensive to manufacture.

To Grind or Not to Grind?

Some production machinery cams are used as-milled, and not ground. Automotive valve cams are ground. The reasons are largely due to cost and quantity considerations as well as the high speeds of automotive cams. There is no question that a ground cam is superior to a milled cam, but a hard-machined* cam can perform nearly as well as a well-ground cam. The question in each case is whether the grinding advantage gained is worth the cost. In small quantities, as are typical of production machinery, grinding about doubles the cost of a cam. The advantages in terms of smoothness and quietness of operation, and of wear, are not in the same ratio as the cost difference.[9, 10] Automotive cams are made in large quantity, run at very high speed, and are expected to last for a very long time with minimal maintenance. This is a very challenging specification. It is a great credit to the engineering of these cams that they very seldom fail in 150 000 miles or more of operation. These cams are made on specialized equipment which keeps the cost of their grinding to a minimum.

Industrial production machine cams also see very long lives, often 10 to 20 years, running into billions of cycles at typical machine speeds. Unlike the typical automotive application, industrial cams often run around the clock, 7 days a week, 50+ weeks a year.

* "Hard machining" is a relatively recent addition to the machinist's toolbox. Modern boron-nitride cutting tools are able to machine pre-hardened steel at up to about HRC 50 hardness. This allows the cam blank to be pre-hardened and then machined (rather than ground) to final contour in a CNC machining center. This technique has allowed cam manufacturers to reduce the cost of finished cams significantly. Instead of machining the cam blank from soft steel, then hardening it, followed by a grinding operation to generate the final contour and remove the distortion from hardening, they can now directly machine the hardened blank and get finishes close to those from grinding. This has greatly reduced the cost and tunaround time for cam manufacturing. Cams that formerly took multiple days to manufacture are now made in hours from a stock of pre-hardened cam blanks.

TABLE P8-0
Topic/Problem Matrix

8.1 Cam Terminology
8-1, 8-3, 8-5

8.3 Double-Dwell Cam Design
Simple Harmonic Motion (SHM)
8-26
Cycloidal Displacement
8-23, 8-70
Modified Trapezoidal
8-7, 8-11, 8-21, 8-44, 8-64
Modified Sinusoidal
8-8. 8-10, 8-22, 8-45, 8-66
Polynomial
8-24, 8-25, 8-33, 8-46, 8-59, 8-60, 8-68

8.4 Single-Dwell Cam Design
8-9, 8-41, 8-42, 8-47, 8-53, 8-61

8.5 Critical Path Motion
8-17, 8-43, 8-48, 8-54, 8-63

8.6 Sizing the Cam
Pressure Angle
8-2, 8-4, 8-6, 8-34, 8-56, 8-57, 8-58, 8-71, 8-72, 8-73
Radius of Curvature - Roller Followers
8-18, 8-19, 8-20, 8-27, 8-28, 8-29, 8-30, 8-31, 8-32, 8-35, 8-36, 8-37, 8-38, 8-39, 8-40
Radius of Curvature - Flat-Faced Followers
8-49, 8-50, 8-51, 8-52, 8-62, 8-65, 8-67, 8-69,8-74, 8-75
Roller & Flat-Faced Followers
8-12, 8-13, 8-14, 8-15

To Lubricate or Not to Lubricate?

Cams like lots of lubrication. Automotive cams are literally drowned in a flow of filtered and sometimes cooled engine oil. Many production machine cams run immersed in an oil bath. These are reasonably happy cams. Others are not so fortunate. Cams that operate in close proximity to the product on an assembly machine in which oil would cause contamination of the product (food products, personal products) often are run dry. Camera mechanisms, which are full of linkages and cams, are often run dry. Lubricant would eventually find its way to the film or sensors.

Unless there is some good reason to eschew lubrication, a cam and follower should be provided with a generous supply of clean lubricant, preferably a hypoid-type oil containing additives for boundary lubrication conditions. The geometry of a cam-follower joint (half-joint) is among the worst possible from a lubrication standpoint. Unlike a journal bearing, which tends to trap a film of lubricant within the annulus of the joint, the half-joint is continually trying to squeeze the lubricant out of itself. This can result in a boundary, or mixed boundary / elasto-hydrodynamic lubrication state in which some metal-to-metal contact will occur. Lubricant must be continually resupplied to the joint. Another purpose of the liquid lubricant is to remove the heat of friction from the joint. If run dry, significantly higher material temperatures will result, with accelerated wear and possible early failure.

8.8 REFERENCES

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8.9 PROBLEMS‡

Programs DYNACAM *and* MATRIX *may be used to solve these problems or to check your solution where appropriate.*

*8-1 Figure P8-1 shows the cam and follower from Problem 6-65. Using graphical methods, find and sketch the equivalent fourbar linkage for this position of the cam and follower.

*8-2 Figure P8-1 shows the cam and follower from Problem 6-65. Using graphical methods, find the pressure angle at the position shown.

8-3 Figure P8-2 shows a cam and follower. Using graphical methods, find and sketch the equivalent fourbar linkage for this position of the cam and follower.

*8-4 Figure P8-2 shows a cam and follower. Using graphical methods, find the pressure angle at the position shown.

8-5 Figure P8-3 shows a cam and follower. Using graphical methods, find and sketch the equivalent fourbar linkage for this position of the cam and follower.

*8-6 Figure P8-3 shows a cam and follower. Using graphical methods, find the pressure angle at the position shown.

‡8-7 Design a double-dwell cam to move a follower from 0 to 2.5″ in 60°, dwell for 120°, fall 2.5" in 30°, and dwell for the remainder. The total cycle must take 4 sec. Choose suitable functions for rise and fall to minimize accelerations. Plot the *s v a j* diagrams.

‡8-8 Design a double-dwell cam to move a follower from 0 to 1.5” in 45°, dwell for 150°, fall 1.5” in 90°, and dwell for the remainder. The total cycle must take 6 sec. Choose suitable functions for rise and fall to minimize velocities. Plot the *s v a j* diagrams.

‡8-9 Design a single-dwell cam to move a follower from 0 to 2” in 60°, fall 2” in 90°, and dwell for the remainder. The total cycle must take 2 sec. Choose suitable functions for rise and fall to minimize accelerations. Plot the *s v a j* diagrams.

‡8-10 Design a three-dwell cam to move a follower from 0 to 2.5” in 40°, dwell for 100°, fall 1.5” in 90°, dwell for 20°, fall 1” in 30°, and dwell for the remainder. The total cycle must take 10 sec. Choose suitable functions for rise and fall to minimize velocities. Plot the *s v a j* diagrams.

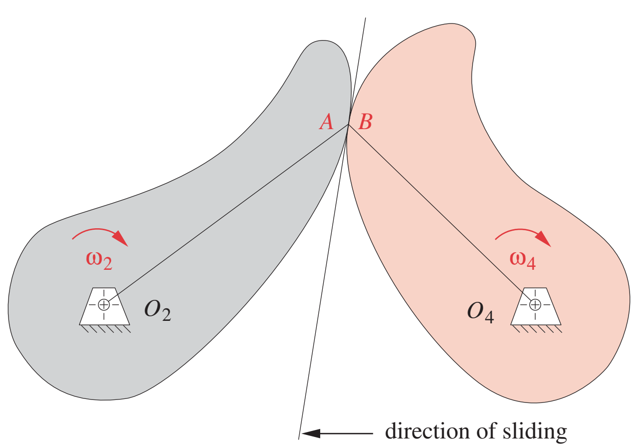


FIGURE P8-1
Problems 8-1 to 8-2

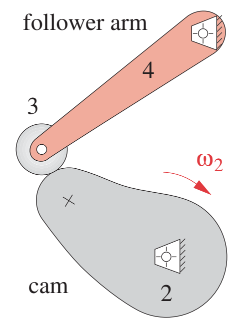


FIGURE P8-2
Problems 8-3 to 8-4

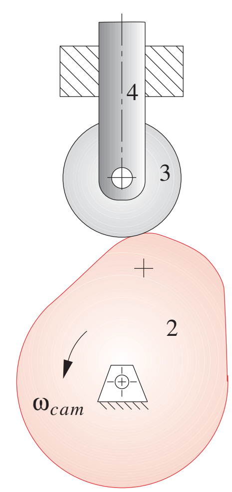


FIGURE P8-3
Problems 8-5 to 8-6

* Answers in Appendix F.

‡ Problem figures are provided as downloadable PDF files with same names as the

‡ These problems are suited to solution using program DYNACAM.

‡8-11 Design a four-dwell cam to move a follower from 0 to 2.5” in 40°, dwell for 100°, fall 1.5” in 90°, dwell for 20°, fall 0.5” in 30°, dwell for 40°, fall 0.5” in 30°, and dwell for the remainder. The total cycle must take 15 sec. Choose suitable functions for rise and fall to minimize accelerations. Plot the *s v a j* diagrams.

‡8-12 Size the cam from Problem 8-7 for a 1” radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Plot both those functions. Draw the cam profile. Repeat for a flat-faced follower. Which would you use?

‡8-13 Size the cam from Problem 8-8 for a 1.5” radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Plot both those functions. Draw the cam profile. Repeat for a flat-faced follower. Which would you use?

‡8-14 Size the cam from Problem 8-9 for a 0.5” radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Plot both those functions. Draw the cam profile. Repeat for a flat-faced follower. Which would you use?

‡8-15 Size the cam from Problem 8-10 for a 2” radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Plot both those functions. Draw the cam profile. Repeat for a flat-faced follower. Which would you use?

‡8-16 Size the cam from Problem 8-11 for a 0.5” radius roller follower considering pressure angle and radius of curvature. Use eccentricity only if necessary to balance those functions. Plot both those functions. Draw the cam profile. Repeat for a flat-faced follower. Which would you use?

‡8-17 A high friction, high inertia load is to be driven. We wish to keep peak velocity low. Combine segments of polynomial displacements with a constant velocity segment on both rise and fall to reduce the maximum velocity below that obtainable with a full period modified sine acceleration alone (i.e., one with no constant velocity portion). Rise 1” in 90°, dwell for 60°, fall in 50°, dwell for remainder. Compare the two designs and comment. Use an ω of one for comparison.

‡8-18 A constant velocity of 0.4 in/sec is to be matched for 1.5 sec. The follower must return to your choice of start point and dwell for 2 sec. Total cycle is 6 sec. Design a cam for a follower radius of 0.75” and a maximum pressure angle of 30° absolute value.

‡8-19 A constant velocity of 0.25 in/sec must be matched for 3 sec. Then the follower must return to your choice of start point and dwell for 3 sec. The total cycle time is 12 sec. Design a cam for a follower radius of 1.25” and a maximum pressure angle of 35° absolute value.

‡8-20 A constant velocity of 2 in/sec must be matched for 1 sec. Then the follower must return to your choice of start point. The total cycle time is 2.75 sec. Design a cam for a follower radius of 0.5” and a maximum pressure angle of 25° absolute value.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†8-21 Write a computer program or use an equation solver to calculate and plot the *s v a j* diagrams for a modified trapezoidal acceleration cam function for any specified values of lift and duration. Test it using a lift of 20 mm over 60° at 1 rad/sec.

†8-22 Write a computer program or use an equation solver to calculate and plot the *s v a j* diagrams for a modified sine acceleration cam function for any specified values of lift

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†8-23 Write a computer program or use an equation solver to calculate and plot the *s v a j* diagrams for a cycloidal displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over 60° at 1 rad/sec.

†8-24 Write a computer program or use an equation solver to calculate and plot the *s v a j* diagrams for a 3-4-5 polynomial displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over 60° at 1 rad/sec.

†8-25 Write a computer program or use an equation solver to calculate and plot the *s v a j* diagrams for a 4-5-6-7 polynomial displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over 60° at 1 rad/sec.

†8-26 Write a computer program or use an equation solver to calculate and plot the *s v a j* diagrams for a simple harmonic displacement cam function for any specified values of lift and duration. Test it using a lift of 20 mm over 60° at 1 rad/sec.

†8-27 Write a computer program or use an equation solver to calculate and plot the pressure angle and radius of curvature for a modified trapezoidal acceleration cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over 60° at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20°. What is the minimum diameter of roller follower needed to avoid undercutting with these data?

†8-28 Write a computer program or use an equation solver to calculate and plot the pressure angle and radius of curvature for a modified sine acceleration cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over 60° at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20°. What is the minimum diameter of roller follower needed to avoid undercutting with these data?

†8-29 Write a computer program or use an equation solver to calculate and plot the pressure angle and radius of curvature for a cycloidal displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over 60° at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20°. What is the minimum diameter of roller follower needed to avoid undercutting with these data?

†8-30 Write a computer program or use an equation solver to calculate and plot the pressure angle and radius of curvature for a 3-4-5 polynomial displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over 60° at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20°. What is the minimum diameter of roller follower needed to avoid undercutting with these data?

†8-31 Write a computer program or use an equation solver to calculate and plot the pressure angle and radius of curvature for a 4-5-6-7 polynomial displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over 60° at 1 rad/sec, and determine the prime circle radius needed to obtain a maximum pressure angle of 20°. What is the minimum diameter of roller follower needed to avoid undercutting with these data?

†8-32 Write a computer program or use an equation solver to calculate and plot the pressure angle and radius of curvature for a simple harmonic displacement cam function for any specified values of lift, duration, eccentricity, and prime circle radius. Test it using a lift of 20 mm over 60° at 1 rad/sec, and determine the prime circle radius needed to

obtain a maximum pressure angle of 20°. What is the minimum diameter of roller follower needed to avoid undercutting with these data?

8-33 Derive equation 8.25 for the 4-5-6-7 polynomial function.

8-34 Derive an expression for the pressure angle of a barrel cam with zero eccentricity.

‡ These problems are suited to solution using program DYNACAM.

‡8-35 Design a radial plate cam to move a translating roller follower through 30 mm in 30°, dwell for 100°, fall 10 mm in 10°, dwell for 20°, fall 20 mm in 20°, and dwell for the remainder. Camshaft ω = 200 rpm. Minimize the follower's peak velocity and determine the minimum prime circle radius that will give a maximum 25° pressure angle. Determine the minimum radii of curvature on the pitch curve.

‡8-36 Repeat Problem 8-35, but minimize the follower's peak acceleration instead.

‡8-37 Repeat Problem 8-35, but minimize the follower's peak jerk instead.

‡8-38 Design a radial plate cam to lift a translating roller follower through 10 mm in 65°, return to 0 in 65° and dwell for the remainder. Camshaft ω = 3500 rpm. Minimize the cam size while not exceeding a 25° pressure angle. What size roller follower is needed?

‡8-39 Design a cam-driven quick-return mechanism for a 3:1 time ratio. The translating roller follower should move forward and back 50 mm and dwell in the back position for 80°. It should take one-third the time to return as to move forward. Camshaft ω = 100 rpm. Minimize the package size while maintaining a 25° maximum pressure angle. Draw a sketch of your design and provide *s v a j*, ϕ, and ρ diagrams.

8

‡8-40 Design a cam-follower system to drive a linear translating piston at constant velocity for 200° through a stroke of 100 mm at 60 rpm. Minimize the package size while maintaining a 25° maximum pressure angle. Draw a sketch of your design and provide *s v a j*, ϕ, and ρ diagrams.

‡8-41 Design a cam-follower system to rise 20 mm in 80°, fall 10 mm in 100°, dwell at 10 mm for 100°, fall 10 mm in 50°, and dwell at 0 for 30°. Total cycle time is 4 sec. Avoid unnecessary returns to zero acceleration. Minimize the package size and maximize the roller follower diameter while maintaining a 25° maximum pressure angle. Draw a sketch of your design and provide *s v a j*, ϕ, and ρ diagrams.

‡8-42 Design a single-dwell cam to move a follower from 0 to 35 mm in 75°, fall 35 mm in 120°, and dwell for the remainder. The total cycle time is 3 sec. Choose suitable functions to minimize acceleration and plot the *s v a j* diagrams for the rise/fall.

‡8-43 Design a cam to move a follower at a constant velocity of 100 mm/sec for 2 sec then return to its starting position with a total cycle time of 3 sec.

‡8-44 Design a double-dwell cam to move a follower from 0 to 50 mm in 75°, dwell for 75°, fall 50 mm in 75°, and dwell for the remainder. The total cycle must take 5 sec. Use a modified trapezoidal function for rise and fall and plot the *s v a j* diagrams.

‡8-45 Design a double-dwell cam to move a follower from 0 to 50 mm in 75°, dwell for 75°, fall 50 mm in 75°, and dwell for the remainder. The total cycle must take 5 sec. Use a modified sinusoidal function for rise and fall and plot the *s v a j* diagrams.

‡8-46 Design a double-dwell cam to move a follower from 0 to 50 mm in 75°, dwell for 75°, fall 50 mm in 75°, and dwell for the remainder. The total cycle must take 5 sec. Use a 4-5-6-7 polynomial function for rise and fall and plot the *s v a j* diagrams.

‡8-47 Design a single-dwell cam to move a follower from 0 to 65 mm in 90°, fall 65 mm in 180°, and dwell for the remainder. The total cycle time is 2 sec. Choose suitable functions to minimize acceleration and plot the *s v a j* diagrams for the rise/fall.

‡8-48 Design a cam to move a follower at a constant velocity of 200 mm/sec for 3 sec then return to its starting position with a total cycle time of 6 sec.

‡8-49 Size the cam from Problem 8-42 for a translating flat-faced follower considering follower face width and radius of curvature. Plot radius of curvature and cam profile.

‡8-50 Size the cam from Problem 8-44 for a translating flat-faced follower considering follower face width and radius of curvature. Plot radius of curvature and cam profile.

‡8-51 Size the cam from Problem 8-45 for a translating flat-faced follower considering follower face width and radius of curvature. Plot radius of curvature and cam profile.

‡8-52 Size the cam from Problem 8-46 for a translating flat-faced follower considering follower face width and radius of curvature. Plot radius of curvature and cam profile.

‡8-53 Design a single-dwell cam to move a follower from 0 to 50 mm in 100°, fall 50 mm in 120°, and dwell for the remainder. The total cycle time is 1 sec. Choose suitable functions to minimize acceleration and plot the *s v a j* diagrams for the rise/fall.

‡8-54 Design a cam to move a follower at a constant velocity of 300 mm/sec for 2 sec then return to its starting position with a total cycle time of 4 sec.

†8-55 Write a computer program or use an equation solver to calculate and plot the *s v a j* diagrams for the family of SCCA cam functions for any specified values of lift and duration. It should allow changing values of the parameters b, c, d, and C_a to plot any member of the family. Test all functions with 100 mm rise in 100°, dwell 80°, fall in 120°, dwell for remainder. Shaft turns at 1 rad/sec.

†8-56 Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot the pressure angle for the cam of Problem 8-42 for any given prime circle radius and follower eccentricity. Test it using $R_p = 45$ mm and $e = 10$ mm.

†8-57 Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot the pressure angle for the cam of Problem 8-43 for any given prime circle radius and follower eccentricity. Test it using $R_p = 100$ mm and $e = -15$ mm.

†8-58 Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot the pressure angle for the rise segment of the cam of Problem 8-46 for any given prime circle radius and follower eccentricity. Test it using $R_p = 75$ mm and $e = 20$ mm.

‡8-59 Design a cam to move a follower from 20.5 to 15 mm in 60°, fall an additional 15 mm in 90°, rise 20.5 mm in 110°, and dwell for the remainder. Use polynomial functions for the rise and falls. Some of the boundary conditions are given in Table P8-1; however, in order to make the polynomials piecewise continuous, other boundary conditions will have to be determined. The shaft speed is 250 rpm. Plot the *s v a j* diagrams.

‡8-60 Design a cam to move a follower from 32 to 12 mm in 60°, fall an additional 12 mm in 50°, dwell 35°, rise 12 mm in 45°, rise an additional 20 mm in 65°, and dwell for the remainder. Use polynomial functions for the rises and falls. Velocity and acceleration are zero at the beginning and end of each event and jerk is zero at $\theta = 0°$, 110°, 145°, and 255°. The shaft speed is 37.5 rpm. Plot the *s v a j* diagrams.

‡ These problems are suited to solution using program DYNACAM.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

TABLE P8-1 Data for Problem 8-59

Event	S	V	A	J
First fall (60°)				
Beginning	20.5	0	0	0
Ending	15.0	0	0	
Second fall (90°)				
Beginning	15.0	0	0	
Ending	0.0	0	A_1	0
Rise (110°)				
Beginning	0.0	0	match A_1	0
Ending	20.5	0	0	0

‡ These problems are suited to solution using program DYNACAM.

8

‡8-61 Design a single-dwell cam to move a follower from 0 to 0.6” in 0.8 sec, fall 0.6” in 1.2 sec and dwell for the remainder of the cycle. The total cycle must take 4 sec. Choose suitable programs for rise and fall to minimize velocities. Plot the *s v a j* diagrams.

‡8-62 Size the cam from Problem 8-61 for a flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

‡8-63 Design a cam to move a follower at a constant velocity of 4 in/sec for 2 sec then return to its starting position with a total cycle time of 4 sec.

‡8-64 Design a double-dwell cam to move a follower from 0 to 2” in 4/3 sec, dwell for 1 sec, fall 2” in 4/3 sec and dwell for the remainder of the cycle. The total cycle must take 6 sec. Use a modified trapezoidal function for rise and fall and plot the *s v a j* diagrams.

‡8-65 Size the cam from Problem 8-64 for a flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

‡8-66 Design a double-dwell cam to move a follower from 0 to 2” in 4/3 sec, dwell for 1 sec, fall 2” in 4/3 sec and dwell for the remainder of the cycle. The total cycle must take 6 sec. Use a modified sinusoidal function for rise and fall and plot the *s v a j* diagrams.

‡8-67 Size the cam from Problem 8-66 for a flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

‡8-68 Design a double-dwell cam to move a follower from 0 to 2” in 4/3 sec, dwell for 1 sec, fall 2” in 4/3 sec and dwell for the remainder of the cycle. The total cycle must take 6 sec. Use a 4-5-6-7 polynomial function for rise and fall and plot the *s v a j* diagrams.

‡8-69 Size the cam from Problem 8-68 for a flat-faced follower considering follower face width and radius of curvature. Plot the radius of curvature and draw the cam profile.

‡8-70 Design a double-dwell cam to move a follower from 0 to 1.5” in 1 sec, dwell for 2 sec, fall 1.5” in 1 sec and dwell for the remainder of the cycle. The total cycle must take 8 sec. Use a cycloidal displacement function for rise and fall and plot the *s v a j* diagrams.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

†8-71 Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot the pressure angle for the cam of Problem 8-61 for any given prime circle radius and follower eccentricity. Test it using $R_p = 1.500$ in and $\varepsilon = 0.250$ in.

†8-72 Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot the pressure angle for the cam of Problem 8-63 for any given prime circle radius and follower eccentricity. Test it using $R_p = 5.000$ in and $\varepsilon = -1.250$ in.

†8-73 Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to calculate and plot the pressure angle for the rise segment of the cam of Problem 8-68 for any given prime circle radius and follower eccentricity. Test it using $R_p = 3$ in and $\varepsilon = 0.750$ in.

†8-74 Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to draw the cam profile for the cam of Problem 8-61 with a translating flat-faced follower for any given base circle radius. Test it using $R_b = 1.500$ in.

†8-75 Write a computer program or use an equation solver such as *Mathcad* or *TKSolver* to draw the cam profile for the cam of Problem 8-63 with a translating flat-faced follower for any given base circle radius. Test it using $R_b = 2.000$ in.

† These problems are suited to solution using *Mathcad, Matlab,* or *TKSolver* equation solver programs.

8.10 VIRTUAL LABORATORY *View the video* (21:28)† *View the lab handout*§

L8-1 View the video *Cam Machine Virtual Laboratory* that is downloadable. Open the file *Virtual Cam Machine Lab.doc* and follow the instructions as directed by your professor.

† http://www.designofmachinery.com/DOM/Cam_machine_virtual_laboratory.mp4

§ http://www.designofmachinery.com/DOM/Cam_Virtual_Lab.zip

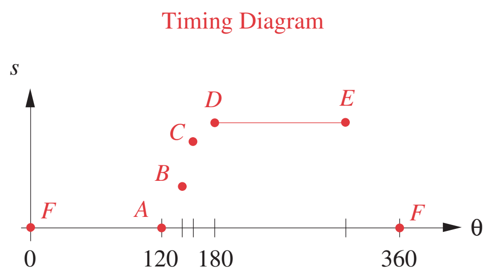
8.11 PROJECTS

These larger-scale project statements deliberately lack detail and structure and are loosely defined. Thus, they are similar to the kind of "identification of need" or problem statement commonly encountered in engineering practice. It is left to the student to structure the problem through background research and to create a clear goal statement and set of task specifications before attempting to design a solution. This design process is spelled out in Chapter 1 and should be followed in all of these examples. Document all results in a professional engineering report. (See Section 1.9 and the Chap. 1 bibliography for information on report writing.)

‡P8-1 A timing diagram for a halogen headlight filament insertion device is shown in Figure P8-4. Four points are specified. Point *A* is the start of rise. At *B* the grippers close to grab the filament from its holder. The filament enters its socket at *C* and is fully inserted at *D*. The high dwell from *D* to *E* holds the filament stationary while it is soldered in place. The follower returns to its start position from *E* to *F*. From *F* to *A* the follower is stationary while the next bulb is indexed into position. It is desirable to have low to zero velocity at point *B* where the grippers close on the fragile filament. The velocity at *C* should not be so high as to "bend the filament in the breeze." Design and size a complete cam-follower system to do this job.

‡P8-2 A cam-driven pump to simulate human aortic pressure is needed to serve as a consistent, repeatable pseudo-human input to a hospital's operating room computer monitoring equipment, in order to test it daily. Figure P8-5 shows a typical aortic pressure curve and a pump pressure-volume characteristic. Design a cam to drive the piston and give as close an approximation to the aortic pressure curve shown as can be obtained without violating the fundamental law of cam design. Simulate the dicrotic notch as best you can.

‡ These problems are suited to solution using program DYNACAM.



Displacement Table

Cam angle,°	Point	s
120	*A*	0
140	*B*	2
150	*C*	3
180	*D*	3.5
300	*E*	3.5
360	*F*	0

FIGURE P8-4

Data for cam design Project P8-1

‡ These problems are suited to solution using program DYNACAM.

‡P8-3 A fluorescent light bulb production machine moves 5500 lamps per hour through a 550°C oven on a chain conveyor which is in constant motion. The lamps are on 2-in centerlines. The bulbs must be sprayed internally with a tin oxide coating as they leave the oven, still hot. This requires a cam-driven device to track the bulbs at constant velocity for the 0.5 sec required to spray them. The spray guns will fit on a 6 x 10 in table. The spray creates hydrochloric acid, so all exposed parts must be resistant to that environment. The spray head transport device will be driven from the conveyor chain by a shaft having a 28-tooth sprocket in mesh with the chain. Design a complete spray gun transport assembly to these specifications.

‡P8-4 A 30-ft-tall drop tower is being used to study the shape of water droplets as they fall through air. A camera is to be carried by a cam-operated linkage which will track the droplet's motion from the 8-ft to the 10-ft point in its fall (measured from release

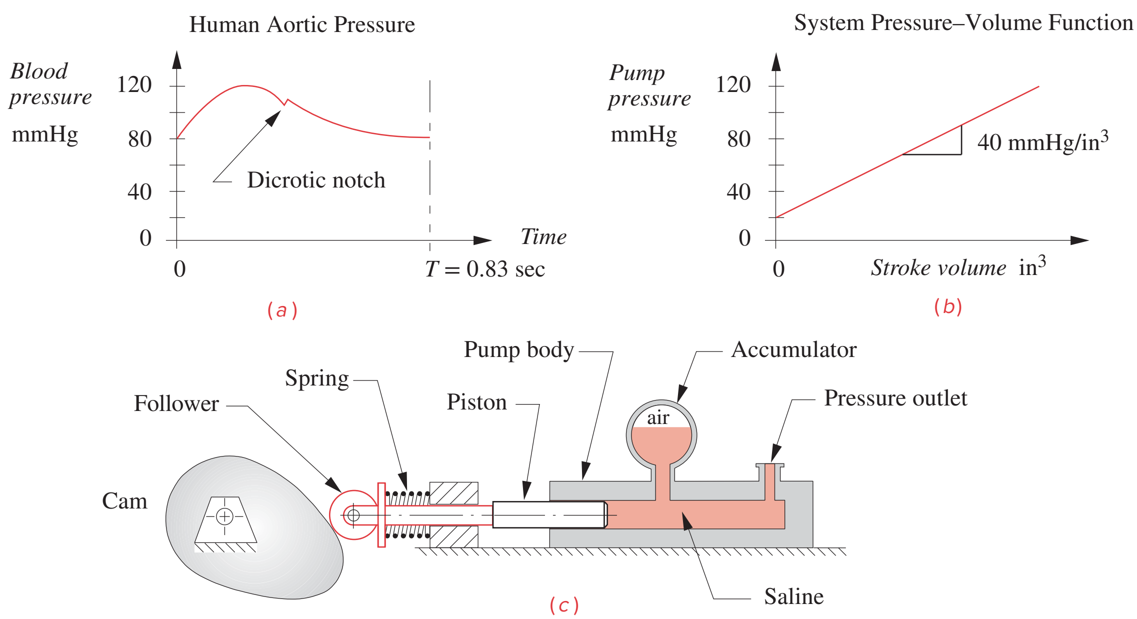


FIGURE P8-5

Data for cam design Project P8-2

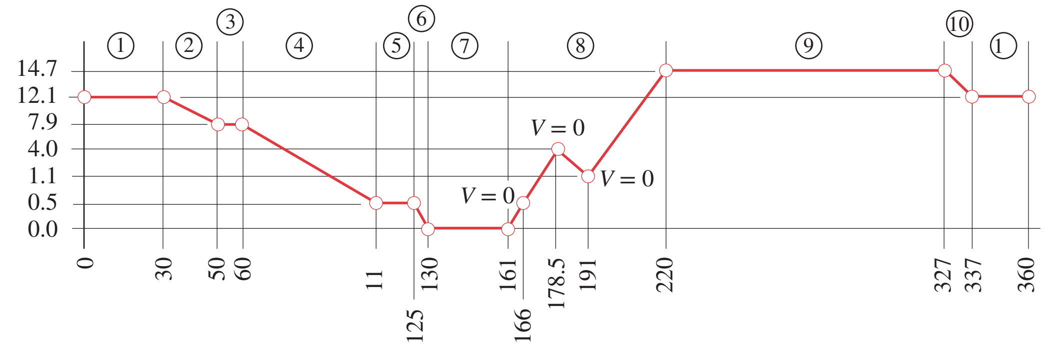


FIGURE P8-6

Timing diagram for Project P8-6. Displacements in mm (not to scale)

point at the top of the tower). The drops are released every 1/2 sec. Every drop is to be filmed. Design a cam and linkage which will track these droplets, matching their velocities and accelerations in the 1-ft filming window.

‡P8-5 A device is needed to accelerate a 3000-lb vehicle into a barrier with constant velocity, to test its 5 mph bumpers. The vehicle will start at rest, move forward, and have constant velocity for the last part of its motion before striking the barrier with the specified velocity. Design a cam-follower system to do this. The vehicle will leave contact with your follower just prior to the crash.

‡P8-6 Figure P8-6 shows a timing diagram for a machine cam to drive a translating roller follower. Design suitable functions for all motions and size the cam for acceptable pressure angles and roller follower diameter. Note points of required zero velocity at particular displacements. Cam speed is 30 rpm. Hint: Segment 8 should be solved with polynomial functions, the fewer the better.

‡P8-7 An athletic footwear manufacturer wants a device to test rubber heels for their ability to withstand millions of cycles of force similar to that which a walking human's foot applies to the ground. Figure P8-7 shows a typical walker's force-time function and a pressure-volume curve for a piston-accumulator. Design a cam-follower system to drive the piston in a way that will create a force-time function on the heel similar to the one shown. Choose suitable piston diameters at each end.

‡P8-8 Design an engine exhaust-valve cam with 10-mm lift over 132 camshaft deg. The rest of the cycle is a dwell. The valve-open duration is measured between cam-follower displacements of 0.5 mm above the dwell position, where valve clearance is taken up and the valve begins to move as shown in Figure P8-8. Engine crankshaft speed ranges from 1000 to 10 000 rpm. The cam should take up the clearance with minimal impact, then continue to lift to 10-mm at 66° as rapidly as possible, close to the 0.5 mm point by 132° and then return it to zero at a controlled velocity. See Figure 8-3a. Select a spring from the Appendix to prevent valve float (follower jump) assuming an effective follower train mass of 200 grams. The camshaft turns at half the crank speed.

‡P8-9 Design a cam-driven peanut-butter (PB) pump for a 600/min cookie assembly line. The cookies are spaced at 40-mm centers on a constant-velocity conveyor. A square, 1-mm thick patch containing 0.4 cc of peanut butter is applied to the cookie as it passes by a nozzle. Entrained air in the PB makes it compressible. Figure P8-5 shows a similar setup with a cam driving a follower attached to a piston pump. The peanut

‡ These problems are suited to solution using program DYNACAM.

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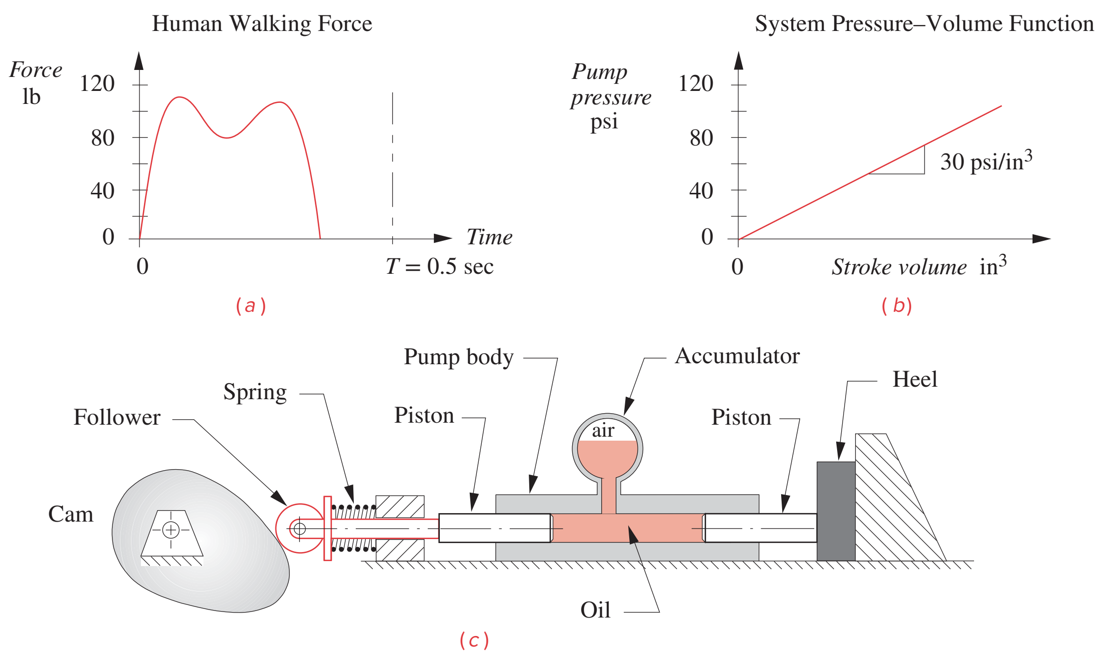

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FIGURE P8-7

Data for cam design Project P8-7

butter flows from the "pressure outlet." The accumulator represents entrained air in the PB. If pumped at constant rate with a piston pump, there is a lag at the start as the entrained air is compressed. Once compressed, it flows uniformly when the piston moves at constant velocity. At the end of the stroke, the stored energy in the entrained air causes "peanut-butter drool," making a messy cookie. To get a sharp-edged start to the "patch" of peanut butter, we need an extra "kick" at the beginning of the pumping cycle to wind up the "air spring," followed by a period of constant velocity motion to lay down a uniform thickness of PB. At the end of the patch, we need a "sniff" to rapidly retract the piston slightly and prevent drool. The piston then returns to the start point at constant velocity to refill the pump and repeat the cycle. The velocity of the "kick" should be about 3 times the steady-state velocity and of as short a duration as practical. The velocity of the "sniff" is optimal at about –4 times the steady-state veloc-

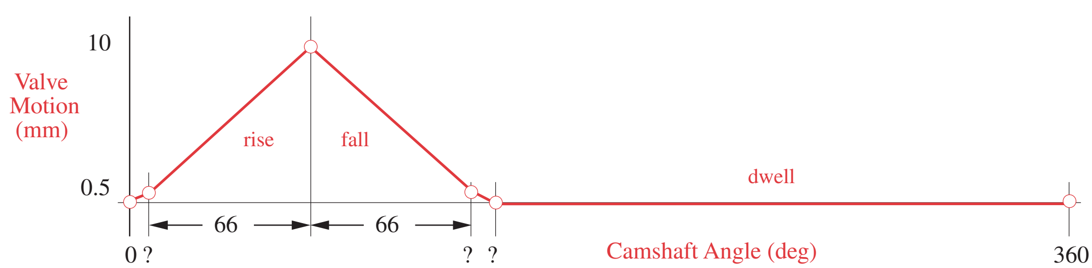

FIGURE P8-8

Timing diagram for Project P8-8—exhaust-valve cam. Determine suitable values for ? from problem statement.

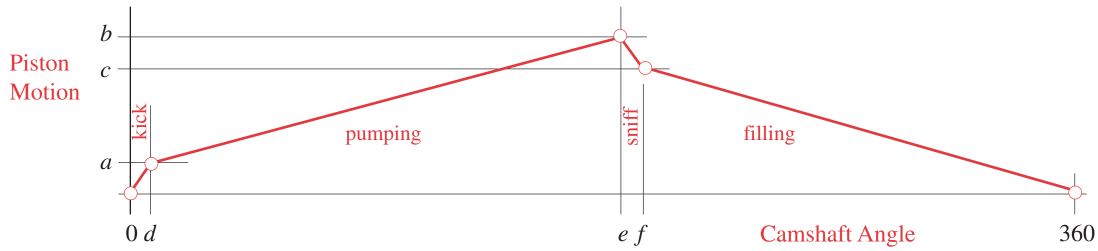

FIGURE P8-9

Timing diagram for Project P8-9—peanut butter pump. Determine suitable values for a–f from problem statement

ity with as short a duration as practical. Figure P8-9 shows the displacement timing diagram. Size the piston and design the piston-driver cam for good dynamic operation with reasonable accelerations and size it in a reasonable package. Select a return spring for a moving follower mass of 0.5 kg.

‡P8-10 Figure P8-10 shows timing diagrams for 3 cams used in a production machine. Design suitable *SVAJ* functions to run at 250 rpm with 10-kg effective mass on each follower. Size the cams for suitable pressure angles and radii of curvature using a 20-mm diameter roller follower. Select a suitable spring for each follower from the Appendix, specify its preload and sketch the assembly, showing all three cams on a common camshaft driving the three follower trains along the *X* axis.

‡ These problems are suited to solution using program DYNACAM.

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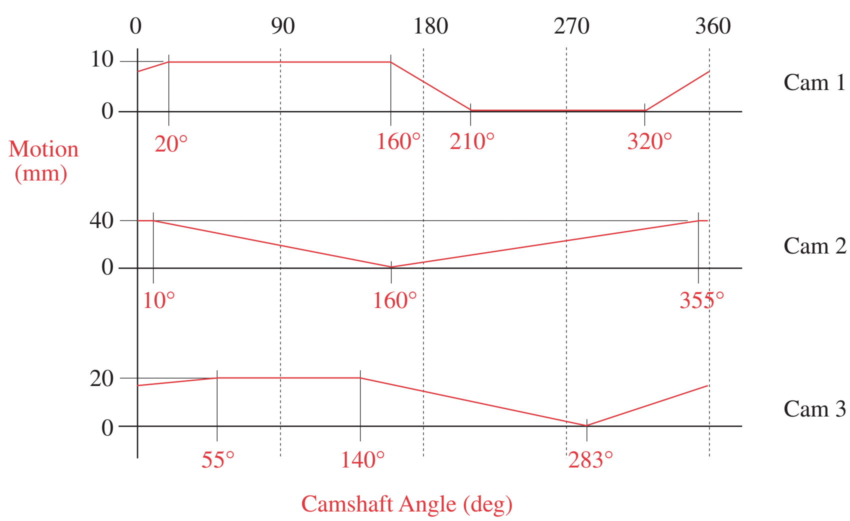

FIGURE P8-10

Timing diagram for Project P8-10