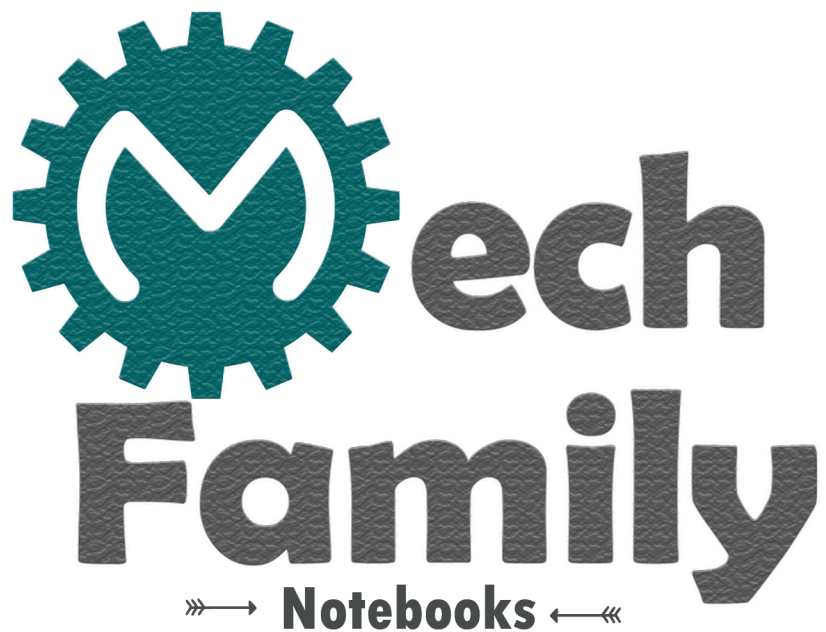


THERMO 2

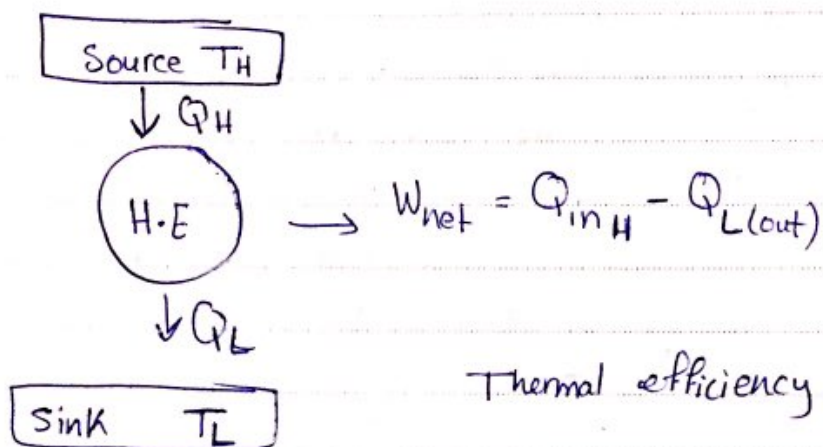
DR. JIHAD YAMEEN

2ND SEMESTER 2017



Chapter 9: Air standard cycles (power plants)

- Objective is to develop power by consuming heat
- Heat engines follow Kelvin-Planck statements



Thermal efficiency $\eta_{th} = \frac{W}{Q_H}$

$$= \frac{Q_H - Q_L}{Q_H}$$

Air standard cycles

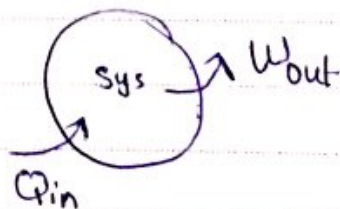
- Closed (Otto, Diesel, Dual)
- open gas turbine
- Jet engines

Revise Ch 4

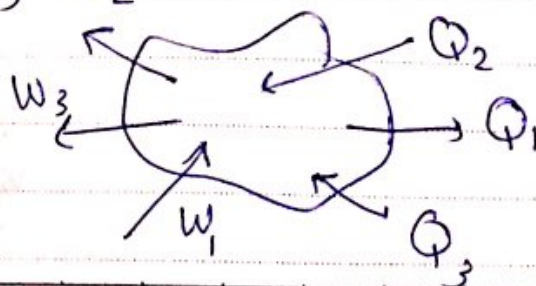
First law of Thermodynamics

$$\sum E_{in} - \sum E_{out} = \Delta E_{sys}$$

$$Q_{in} - W_{out} = \Delta E_{sys}$$



$$(Q_3 + Q_2 - Q_1) - (W_2 + W_3 - W_1) = \Delta E_{sys}$$



$$(Q_{in} - Q_{out}) + (W_{in} - W_{out}) = \Delta E_{sys} = \Delta U$$

$$\Delta u = \int c_v dT, \quad c_v = \frac{du}{dT}$$

* For closed system

$$W_b = \int_{V_1}^{V_2} p \, dV$$

(In closed system, at const 'p', as T ↓ the system gets compressed liquid)

$pV = mRT$ (because we will deal with air, and

air is an ideal gas)

① P = const ② V = const ③ T = const

④ Adiabatic

⑤ polytropic

▢ Const Volume ($V = \text{const}$, $dV = 0$)

$$Q_{in} - \overset{\text{zero}}{W_{out}} = \Delta U$$

$Q_{in} = \Delta U \Rightarrow$ (All heat added is stored as internal energy)

$$= m \int c_v dT$$

if C_v is const

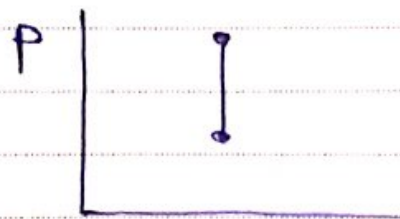
$$Q_{in} = m C_v (T_f - T_i)$$

$$= m \frac{R}{k-1} (T_f - T_i)$$

In ideal gas $C_p - C_v = R$

$$C_v \left(\frac{C_p}{C_v} - 1 \right) = R$$

$$C_v = \frac{R}{k-1}$$



Area = 0

$W_b = 0$

② Constant pressure ($P = \text{const}$)

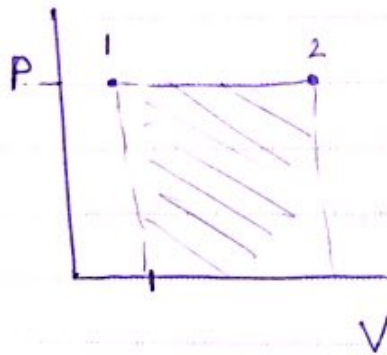
$$P_1 = P_2 = P$$

$$W_b = \text{Area} = P(V_2 - V_1)$$

$$Q_{in} - W_{out} = \Delta U$$

$$Q_{in} - P(V_2 - V_1) = \Delta U$$

$$Q_{in} = \Delta H = m C_P (T_2 - T_1)$$



In a process ~~too~~ All heat $\rightarrow$ may be converted into work

③ Isothermal process ($T = c$)

$$\Delta U = \int C_V dT$$

$$dT = 0 \therefore \Delta U = 0$$

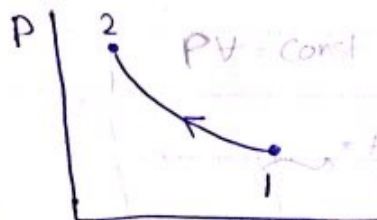
$$Q - W = \Delta U \rightarrow \text{zero}$$

$$Q = W = \int P dV$$

$$Q = \int_{V_1}^{V_2} \frac{C}{V} dV$$

$$= C \ln\left(\frac{V_2}{V_1}\right)$$

$$C = P_1 V_1 = P_2 V_2, \quad \frac{V_2}{V_1} = \frac{P_1}{P_2}$$



In a cycle All heat ~~will~~ be converted into work

there is area under the PV curve: this is boundary work

All heat is converted to work (Carnot cycle / Otto cycle / Diesel cycle) contains isothermal processes

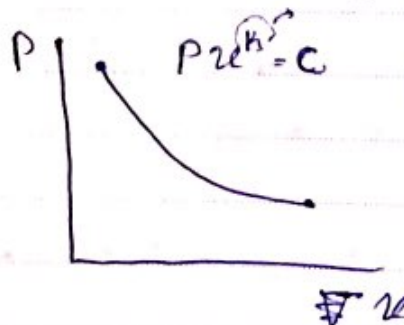
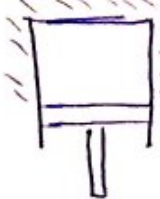
④ Isothermal processes examples: 1) Boiling 2) Condensation

We have isothermal processes in real life

but we don't have isothermal processes that convert heat into work

④ Adiabatic process

Isentropic, with constant C_P, C_V



$$P = \frac{C}{V^{\gamma}}$$

In an adiabatic process, no heat crosses the boundary

$$\textcircled{4} q - w = \Delta u \quad (\text{kJ/kg})$$

Adiabatic: $q = 0$

$$-W = \Delta u$$

$$= \int C_v dT$$

$$-W = C_v(T_2 - T_1) \quad \text{kJ/kg}$$

$$\left\{ \begin{aligned} W &= \int p dv \\ &= \int \frac{C}{v^k} dv \\ &= C \frac{v^{1-k} - v_1^{1-k}}{1-k} \end{aligned} \right.$$

$$C = P_1 v_1^k = P_2 v_2^k$$

$$= \frac{P_2 v_2 - P_1 v_1}{1-k}$$

$$-W = \frac{R(T_2 - T_1)}{k-1}$$

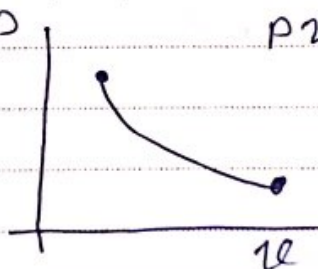
$$-W = \frac{C_p - C_v}{\frac{C_p}{C_v} - 1} \times (T_2 - T_1) = C_v(T_2 - T_1) \quad \text{kJ/kg}$$

⑤ Polytropic process $1 < n < k$

there is heat crossing the boundary

$$W_b = \frac{P_2 v_2 - P_1 v_1}{1-n}$$

$$= \frac{R(T_2 - T_1)}{1-n}$$



$$P v^n = C$$

$$q_{in} - w_{out} = \Delta u, \quad q_{in} = \frac{P_2 v_2 - P_1 v_1}{n-1} = C_v(T_2 - T_1)$$

$$P v = RT \Rightarrow \frac{P}{T} = \frac{R}{v}$$

* first law $\delta q = P dv + du$

$$\text{divide by } T \quad \frac{\delta q}{T} = \frac{P}{T} dv + \frac{du}{T}$$

$$\text{divide by } m \quad \frac{\delta q}{T} = \frac{P}{T} dv + \frac{du}{T} \Rightarrow$$

at const pressure

$$\frac{\delta q}{T} = \frac{dh}{T} = C_p \frac{dT}{T} \Rightarrow \frac{\delta q}{T} = ds = C_p \frac{dT}{T} \Rightarrow \Delta s = C_p \ln\left(\frac{T_2}{T_1}\right)$$

at const volume, $dv=0$

$$\frac{\delta q}{T} = \frac{du}{T} \Rightarrow ds = C_v \frac{dT}{T} \Rightarrow \Delta s = C_v \ln\left(\frac{T_2}{T_1}\right)$$

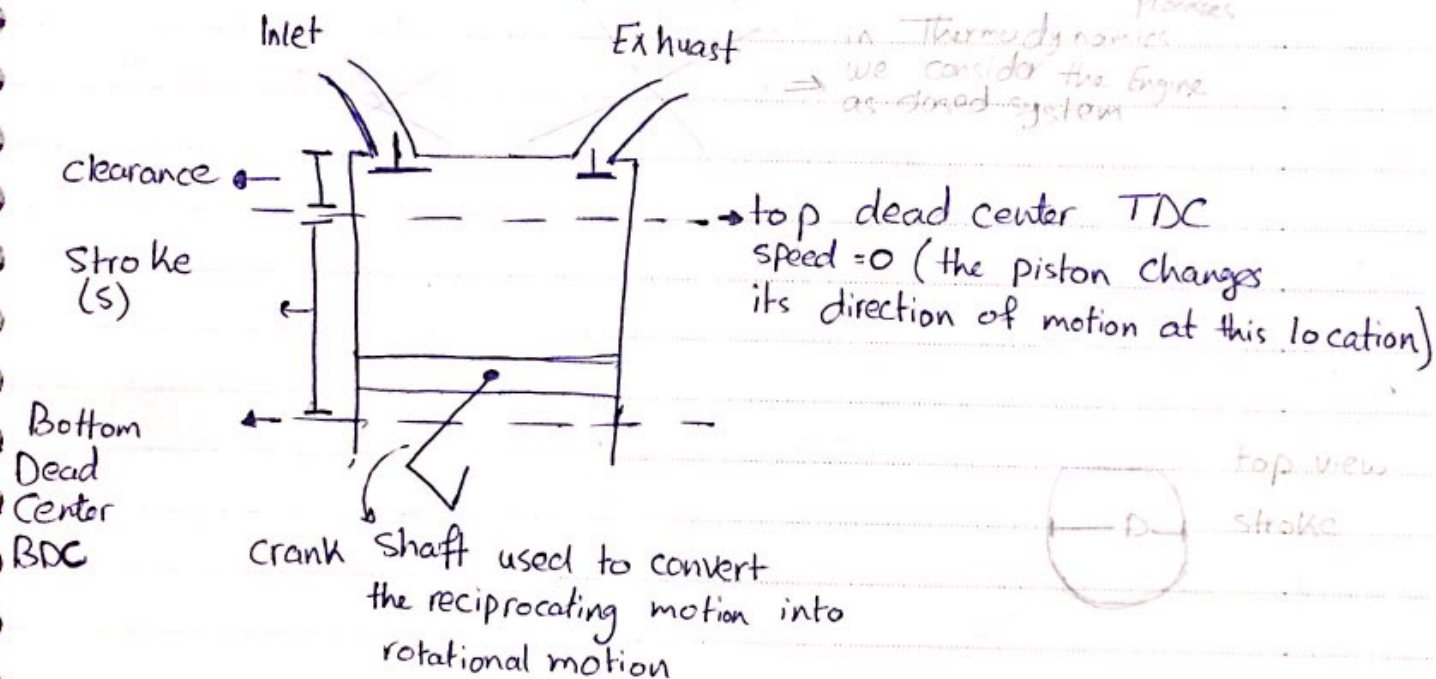
$$\begin{aligned} * \frac{\delta q}{T} &= \frac{R}{v} dv + \frac{du}{T} \\ \int \frac{\delta q}{T} &= R \ln\left(\frac{v_2}{v_1}\right) + C_v \ln\left(\frac{T_2}{T_1}\right) \end{aligned}$$

Air standard cycles

* Assumptions

- Air is the working medium
- Air behaves as an ideal gas $PV = mRT$
- C_p, C_v are const, @ $T = 20^\circ\text{C}$ $C_p = 1.005$, $C_v = 0.712$
- heat addition/rejection are represented by Q_{in}, Q_{out}
- Q_{in}/Q_{out} are instantaneous
- Compression/expansion are adiabatic

→ In Thermodynamics
we consider the engine
as closed system
isothermal processes
correct will



$$V_{\text{stroke}} = \frac{\pi}{4} d^2 s$$

$$\text{Area piston} = \frac{\pi}{4} d^2$$

$$V_{\text{clearance}} = \frac{\pi}{4} d^2 C = V_{\text{min}} = V_{\text{TDC}}$$

$$V_{\text{BDC}} = V_{\text{clearance}} + V_{\text{stroke}} = V_{\text{max}}$$

centimeter³

$$V_{\text{Engine}} = V_s \times \text{number of cylinders}$$

one cylinder

$$V_{\text{toyota 4x4 Engine}} = 3600 \text{ cc}$$

6 cylinders

$$V_{\text{stroke}} = 600 \text{ cc}$$

* The closed system will undergo a cyclic process

Engines

four strokes

1 cycle is done by 2 rotations
720°

6000 rotation/min rpm

100 rotations/s

50 cycles/s

2 strokes

1 cycle is done by 1 rotation
360°

6000 rotation/min

100 rotation/s

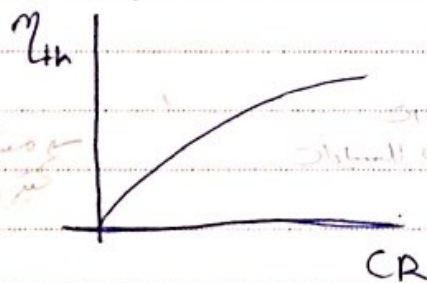
100 cycles/s

$$\text{Cycle/s} = \frac{N}{60} \times \frac{1}{n_r}$$

$N \rightarrow$ number of rotations per minute

$n_r \rightarrow$ number of rotations needed for one cycle

* Compression ratio = $\frac{V_{\max}}{V_{\min}} = \frac{V_{\text{BDC}}}{V_{\text{TDC}}}$
(CR)

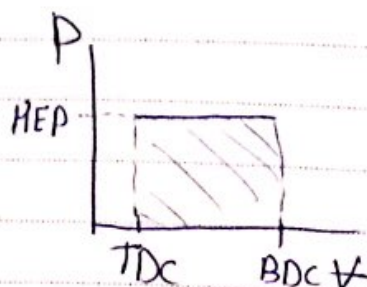
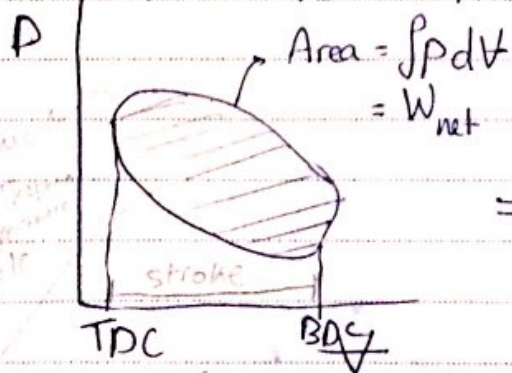


* CR:

spark engines 8-5-11

Diesel engines 14-25

* Mean effective pressure



* HEP = mean effective pressure

Work for cycle = $MEP \times V_{\text{stroke}} = HEP \times (V_{\text{BDC}} - V_{\text{TDC}})$

one cylinder

$$W_{net} = MEP \times V_s \quad J/cycle$$

torque
W
Power
الطاقة الميكانيكية
التي تنتجها المحرك
في كل دورة

$$\dot{W}_{net} = \text{Power} = MEP \times V_s \times \left(\frac{N}{60} \right) \times \frac{\# \text{ cylinders}}{nr} \rightarrow \text{cycle/s}$$

power → is calculated for the engine (All cylinders)

W_{net} → is calculated for one cylinder

$$\dot{W}_{net} = W \times \omega = \frac{2\pi N}{60} \times \tau$$

السرعة الزاوية
clock shaft
الزاوية
في الثانية
clock shaft
الزاوية
في الثانية
clock shaft

ω → omega (angular velocity)

τ → torque

$$\text{Work} = F \cdot d = F \times r \theta$$

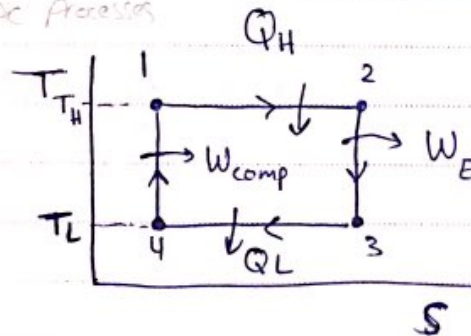
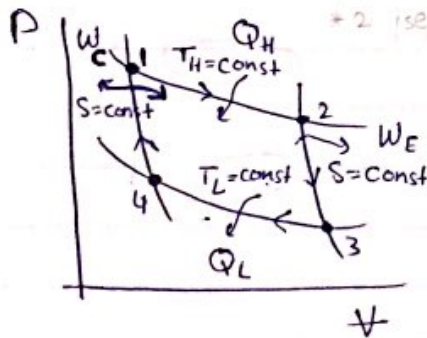
$$\text{Work} = \tau \times \theta$$

$$\text{Power} = \tau \times \frac{\theta}{t} = \tau \times \omega$$

Gas Carnot cycle power cycle

+ 2 isothermal processes
+ 2 isentropic processes

$$q_{in} - q_{out} + w_{in} - w_{out} = \Delta u$$



+ 1→2 Expansion
أنتفاخ

+ 3→4 Compression
ضغط

(1→2) Isothermal, $q_{out} = 0$, $w_{in} = 0$, $\Delta u = 0$

$$q_{in} - w_{out} = 0 \Rightarrow q_{in} = w_{out}$$

$$q_H = q_{in} = T_H \times \Delta S = RT_H \ln\left(\frac{V_2}{V_1}\right)$$

* 1→2 Expansion
P & T ↓
T = -ve
* 2→3 Expansion
P & T ↓
u → l compression, P & T ↑

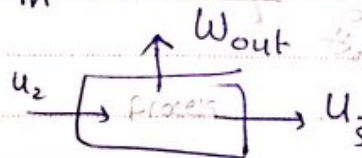
* for reversible processes $q = Tds$

(2→3) Adiabatic, Isentropic, $q = 0$, $w_{in} = 0$, expansion

reversible

$$-W_{out} = \Delta u = u_3 - u_2$$

$$W_{out} = u_2 - u_3$$



(3 → 4) Isothermal, $q_{in} = 0$, $W_{out} = 0$, $\Delta U = 0$

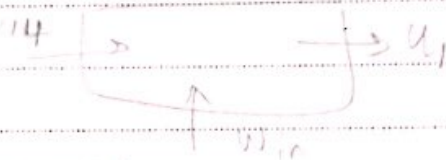
Compression → work done on the system
 $W_{in} \neq 0$

$$W_{in} - q_{out} = 0$$

$$q_{out} = W_{in} \Rightarrow q_{out} = q_L = W_{in} = RT_L \ln\left(\frac{v_2}{v_1}\right) = T_L (s_3 - s_1) \text{ or } s_2 - s_1$$

(4 → 1) Isentropic, Compression, $q = 0$, $W_{out} = 0$

$$W_{in} = \Delta U = u_1 - u_4$$



Review

for reversible isothermal process
specific entropy $\Delta s = \frac{Q}{T}$

Carnot cycles

(1 → 2) $Q_H = m T_H (s_2 - s_1)$ kJ

Isothermal = $W_{out\ 1 \rightarrow 2} = m R T_H \ln\left(\frac{v_2}{v_1}\right)$

* $P v = \text{const}$ ✓

* Remember that we are dealing with Carnot cycle, closed

(2 → 3) $S = \text{const} \rightarrow$ reversible + adiabatic = isentropic system, and the working fluid is an ideal gas

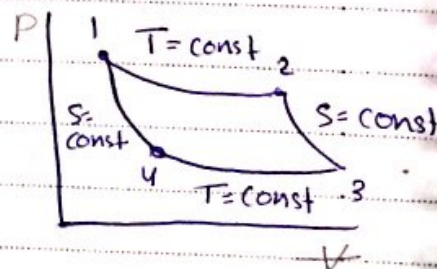
$$W_{\text{Expansion}} = u_2 - u_3$$

$$\left(\frac{T_2}{T_3}\right)_{S=\text{const}} = \left(\frac{P_2}{P_3}\right)^{\frac{k-1}{k}} = \left(\frac{v_3}{v_2}\right)^{k-1}$$

(3 → 4)

$$Q_L = m T_L (s_4 - s_3)$$

$$= W_{\text{compression}} = m R T_L \ln\left(\frac{v_4}{v_3}\right)$$



(4→1) $S = \text{const}$ Compression
 Work done → internal energy
 $W_C = u_1 - u_4$

for processes 1→2 and 3→4

$$\begin{aligned} s_f - s_i &= \overset{\text{zero}}{c_p \ln\left(\frac{T_f}{T_i}\right)} - R \ln\left(\frac{P_f}{P_i}\right) \\ s_f - s_i &= \overset{\text{zero}}{c_v \ln\left(\frac{T_f}{T_i}\right)} + R \ln\left(\frac{v_f}{v_i}\right) \end{aligned} \quad \left. \begin{array}{l} \text{because it is an} \\ \text{isothermal process} \end{array} \right\} \text{Change in entropy for ideal gases}$$

* for Carnot H.E.B

$$\begin{aligned} W_{\text{net}} = Q_H - Q_L &= m(T_H - T_L) \overset{\text{or } S_3 - S_4}{(S_2 - S_1)} \\ &= m \cdot \underbrace{w_{\text{net}}}_{\text{work per unit mass [kJ/kg]}} \end{aligned}$$

maximum work

Substitute

$$\eta_{\text{Carnot}} = \frac{W_{\text{net}}}{Q_H} = 1 - \frac{T_L}{T_H}$$

$Q_H = m T_H (S_2 - S_1)$

Question 9-18

Air, Carnot cycle, closed system, $T_L = 350 \text{ K}$, $T_H = 1200 \text{ K}$, $P_3 = 150 \text{ kPa}$, $P_4 = 300 \text{ kPa}$

$W_{\text{net}} = 0.5 \text{ kJ}$, Find P_1 , m_{air} , η_{th} , Q_H

Solution (constant specific heats)

must be in kelvin

$$\boxed{1} \quad \eta_{\text{th}} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{1200} = 0.7083 \approx 0.71$$

$$\boxed{2} \quad \eta_{\text{th}} = \frac{W_{\text{net}}}{Q_H} \Rightarrow Q_H = \frac{W_{\text{net}}}{\eta_{\text{th}}} = \frac{0.5}{0.7083} = 0.7059 \text{ kJ}$$

$$[3] \quad Q_H = m_{\text{air}} T_H (S_2 - S_1)$$

$$\text{but } |S_2 - S_1| = |S_4 - S_3|$$

$$\text{specific entropy } S_4 - S_3 = \underbrace{C_p \ln \left(\frac{T_4}{T_3} \right)}_{\text{zero}} - R \ln \left(\frac{P_4}{P_3} \right), \text{ isothermal}$$

$$= -0.287 \ln \left(\frac{300}{150} \right)$$

$$= -0.1989$$

$$0.7059$$

$$Q_H = m_{\text{air}} \times 1200 \times 0.1989$$

$$m_{\text{air}} = 2.95 \times 10^{-3} \text{ kg}$$

[4] Since the process 4→1 is isentropic

$$\left(\frac{P_1}{P_4} \right)^{\frac{k-1}{k}}_{S=\text{const}} = \left(\frac{T_1}{T_4} \right) \rightarrow \text{constant specific heats}$$

$$\left(\frac{P_1}{300} \right)^{\frac{k-1}{k}} = \left(\frac{1200}{350} \right)$$

for variable specific heat

$$\left(\frac{P_1}{P_4} \right)_{S=\text{const}} = \left(\frac{P_{r1}}{P_{r4}} \right) \quad \begin{array}{l} \text{at } T_1 = 1200 \rightarrow P_{r1} = 238 \\ T_4 = 350 \rightarrow P_{r4} = 2.379 \end{array}$$

$$\therefore P_1 = 300 \times 2.6 \text{ kPa} = 30 \text{ MPa}$$

* To find P_2 zero (isothermal)

$$S_2 - S_1 = C_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

$$+0.1989 = -R \ln \left(\frac{P_2}{300 \times 2.6} \right)$$

$$\downarrow 0.287$$

9-18

Q_H and Q_{H1} are the same $R = 2.0769$

$$Q_H = m_{\text{helium}} * T_H * (S_2 - S_1)$$

$$\begin{aligned} (S_2 - S_1) &= (S_4 - S_3) \\ (S_4 - S_3) &= C_p \ln\left(\frac{T_4}{T_3}\right) - R \ln\left(\frac{P_4}{P_3}\right) \\ &= -2.0769 \ln\left(\frac{300}{150}\right) = -1.439 \end{aligned}$$

$$Q_H = m_{\text{helium}} * T_H * (S_2 - S_1) \Rightarrow m_{\text{helium}} = 4.087 \times 10^{-4} \text{ kg}$$

$$0.7059 = m_{\text{helium}} * 1200 * 1.439$$

for isentropic process:

$$\left(\frac{P_1}{P_4}\right)^{\frac{k-1}{k}} = \left(\frac{T_1}{T_4}\right) \Rightarrow \left(\frac{P_1}{300}\right)^{\frac{1.667-1}{1.667}} = \left(\frac{1200}{350}\right)$$

$$P_1 = 6523.83 \text{ kPa}$$

9-19 Carnot cycle, closed system

$m_{\text{Air}} = 0.6 \text{ kg}$ $T_L = 300 \text{ K}$ $T_H = 1100 \text{ K}$ $P_1 = 3000 \text{ kPa}$ $P_3 = 20 \text{ kPa}$
assume constant specific heats, W_{net}

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1100} = 0.7272$$

$$\eta_{\text{Carnot}} = \frac{W_{\text{net}}}{Q_H}, \quad Q_H = m * T_H * (S_2 - S_1)$$

zero (isothermal)

$$S_2 - S_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

process $2 \rightarrow 3$ is isentropic

$$\left(\frac{P_2}{P_3}\right)^{\frac{k-1}{k}} = \left(\frac{T_2}{T_3}\right)$$

$$\left(\frac{P_2}{20}\right)^{\frac{1.4-1}{1.4}} = \left(\frac{1100}{300}\right) \Rightarrow P_2 = 12888 \text{ kPa}$$

So

$$S_2 - S_1 = -R \ln\left(\frac{P_2}{P_1}\right) = -0.287 \ln\left(\frac{12888}{3000}\right) = 0.132 \text{ kJ/k.Kg}$$

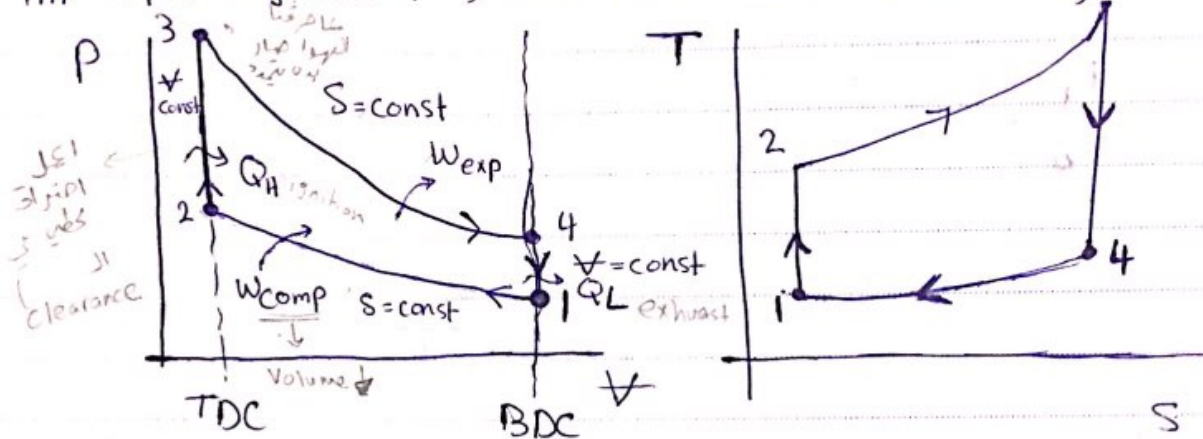
then

$$Q_H = 0.6 * 1100 * 0.132 = 87.12 \text{ kJ}$$

$$\rightarrow W_{\text{net}} = Q_H * \eta_{\text{Carnot}} = 0.727 * 87.12 = 63.33 \text{ kJ}$$

Otto cycle = Constant volume cycle (q_{in} , q_{out} at constant volume)

All spark ignition (SI) engines are based on this cycle



1 compression, 2 expansion while in Carnot. 2 expansion, 2 compression
Instead of 2 isothermal (As in Carnot), there are ~~isochoric~~ ^{isochoric}

Isentropic process

Isenthalpic process

تغير الإنتروبي مع الإنتروبي
تغير الإنتروبي مع الإنتروبي
Polytropic

تغير الإنتروبي مع الإنتروبي
تغير الإنتروبي مع الإنتروبي
Polytropic

* Heat addition and rejection at constant volume is more realistic and more practical than at constant Temp

1) (1 → 2) isentropic $S=C$, $Q=0$, $W_{in} = u_2 - u_1 \Rightarrow kJ/kg$

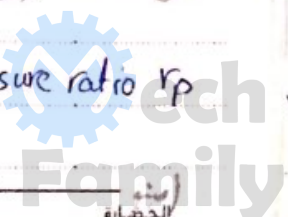
if $C_v = \text{const}$ $\Delta u = C_v(T_2 - T_1)$ specific internal energy

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{v_1}{v_2}\right)^{k-1} \quad \begin{matrix} v_1 = \text{BDC} \\ v_2 = \text{TDC} \end{matrix} \quad \frac{v_1}{v_2} = CR$$

$$S = \text{const} \quad = CR^{k-1}$$

2) (2 → 3) Isochoric $v = \text{const}$, $W=0$, $q_{in} = u_3 - u_2 = C_v(T_3 - T_2) \Rightarrow kJ/kg$

$$Pv = mRT \Rightarrow \frac{P}{T} = \text{const} \Rightarrow \frac{P_2}{T_2} = \frac{P_3}{T_3} \Rightarrow \frac{T_3}{T_2} = \frac{P_3}{P_2} = \text{Pressure ratio } r_p$$



3) (3 → 4) Isentropic $S = \text{Const}$, $Q = 0$

$$W_{\text{out}} = u_3 - u_4 = C_v(T_3 - T_4) \quad \text{kJ/kg}$$

$$\left(\frac{T_3}{T_4}\right) = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = \left(\frac{v_4}{v_3}\right)^{k-1} = CR^{k-1}$$

$S = \text{Const}$ $v_4 = v_1$
 $v_3 = v_2$

4) (4 → 1) $v = \text{const}$, $W = 0$, $q_{\text{out}} = u_4 - u_1 = C_v(T_4 - T_1)$

$$\frac{P_4}{T_4} = \frac{P_1}{T_1} \Rightarrow \frac{T_4}{T_1} = \frac{P_4}{P_1}$$

$$\frac{T_3}{T_4} = \frac{T_2}{T_1} = CR^{k-1}$$

$$\downarrow$$

$$\frac{T_3}{T_2} = \frac{T_4}{T_1}$$

$$\frac{T_4}{T_1} = \frac{T_3}{T_2} = r_p$$

$$\frac{P_3}{P_2} = \frac{P_4}{P_1} = r_p$$

Since $\frac{T_2}{T_1} = \frac{P_2}{P_1}$ & $\frac{T_4}{T_1} = \frac{P_4}{P_1}$

Then $\frac{P_4}{P_1} = \frac{P_2}{P_1} = r_p$, $\frac{P_2}{P_1} = \frac{P_2}{P_1}$

1) (1 → 2) $W_{\text{in}} = u_2 - u_1 = C_v(T_2 - T_1) = C_v T_1 \left(\frac{T_2}{T_1} - 1\right) = C_v T_1 (CR^{k-1} - 1)$

2) (2 → 3) $q_{\text{in}} = u_3 - u_2 = C_v(T_3 - T_2) = C_v T_2 (r_p - 1) = C_v \underbrace{T_1 CR^{k-1}}_{T_2} (r_p - 1)$ $\frac{T_2}{T_1} = CR^{k-1}$

3) (3 → 4) $W_{\text{out}} = u_3 - u_4 = C_v(T_3 - T_4) = C_v T_4 (CR^{k-1} - 1) = C_v T_1 r_p (CR^{k-1} - 1)$

$$4) (4 \rightarrow 1) \quad q_{out} = u_4 - u_1 = C_v(T_4 - T_1) \\ = C_v T_1 (r_p - 1)$$

$$\bullet W_{net} = q_{in} - q_{out} \quad \text{kJ/kg} \\ = C_v T_1 C_R^{k-1} (r_p - 1) - C_v T_1 (r_p - 1) \\ = C_v T_1 (r_p - 1) (C_R^{k-1} - 1)$$

الحرارة النوعية
في الضغط
P, ثابت R

$$\bullet \eta_{otto} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{C_v T_1 (r_p - 1)}{C_v T_1 C_R^{k-1} (r_p - 1)} = 1 - \frac{1}{C_R^{k-1}}$$

$$= 1 - \frac{C_v (T_4 - T_1)}{C_v (T_3 - T_2)} = 1 - \frac{T_1 (r_p - 1)}{T_2 (r_p - 1)} = 1 - \frac{T_1}{T_2} \rightarrow \text{درجات الحرارة التي جاز من (الحجم)}$$

$$= 1 - \frac{T_4}{T_3} \rightarrow \text{بالنسبة } T_1 \text{ بالنتيجة } \eta_{otto} \uparrow$$

to increase η_{otto} $\left\{ \begin{array}{l} T_3 > T_2 \uparrow / T_4 \downarrow \\ CR \uparrow \Rightarrow \eta_{otto} \uparrow \end{array} \right.$ is a loss
كل ما زاد Q_{in} heat rejected $\downarrow$
كل ما زاد Q_{in} $W_{net} \uparrow$

* التي يمكن ان تكون متغيرة

Variable specific heat

متغير

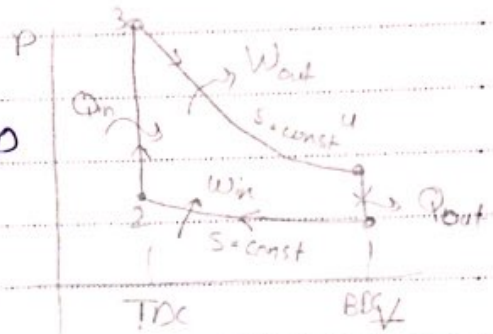
$$\eta_{otto} = 1 - \frac{q_{out}}{q_{in}} = \frac{W_{net}}{q_{in}}$$

Example 9-2

Otto cycle $CR=8$, Air, $P_1 = 100 \text{ kPa}$, $T_1 = 17 + 273 \text{ K}$

$$q_{in} = 800 \text{ kJ/kg}$$

- a) P_3, T_3 b) W_{net} c) η_{th} d) MEP



Case 1: constant specific heat

$$\left(\frac{T_2}{T_1}\right) = CR^{k-1} \Rightarrow T_2 = T_1 CR^{k-1} = 666.25 \text{ K}$$

$$k = 1.4$$

$$c_v = 0.718$$

$$R = 0.287$$

$$q_{in} = c_v(T_3 - T_2) \Rightarrow 800 = 0.718(T_3 - 666.25)$$

$$T_3 = 1780.46 \text{ K}$$

$$P_1 v_1 = mRT_1 \Rightarrow v_1 = \frac{1 \times 0.287 \times 290}{100} = 0.8323 \text{ m}^3/\text{kg}$$

$$\frac{v_1}{v_2} = CR \Rightarrow v_2 = \frac{v_1}{CR} = 0.104 \text{ m}^3/\text{kg}$$

$$P_3 v_3 = mRT_3 \Rightarrow P_3 = \frac{mRT_3}{v_3} = \frac{1 \times 0.287 \times 1780.46}{0.104} = 4913 \text{ kPa}$$

$$\eta_{th} = 1 - \frac{1}{CR^{k-1}} = 1 - \frac{1}{8^{1.4-1}} = 0.564$$

$$\eta_{th} = \frac{W_{net}}{q_{in}} \Rightarrow W_{net} = 0.564 \times 800 = 451.2 \text{ kJ/kg}$$

$$MEP = \frac{W_{net}}{v_1 - v_2} = \frac{451.2}{0.8323 - 0.104} = 619.5 \text{ kPa}$$

another solution

$$W_{\text{comp}} = C_v (T_2 - T_1) = 0.718 (666.25 - 290) \\ = 270.1475 \text{ kJ/kg}$$

$$\left(\frac{T_3}{T_4} \right) = CR^{k-1} \Rightarrow T_4 = \frac{T_3}{CR^{k-1}} = \frac{1780.46}{8^{1.4-1}} = 775 \text{ K}$$

$$W_E = C_v (T_3 - T_4) = 0.718 (1780.46 - 775) = 722 \text{ kJ/kg}$$

$$q_{\text{out}} = C_v (T_4 - T_1) = 0.718 (775 - 290) = 348.23 \text{ kJ/kg}$$

$$W_{\text{net}} = 722 - 270.1475 = 451.85 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{q_{\text{in}}} = \frac{451.85}{800} = 0.564$$

Case 2: Variable specific heat (Isentropic)

1) $Q_w = \Delta U$

2) use P_r , v_r and don't use c_p , c_v , k

$$\frac{T_1}{T_2} = CR = \frac{v_{r1}}{v_{r2}} \Rightarrow 8 = \frac{676.1}{v_{r2}} \Rightarrow v_{r2} = 84.51$$

T	v_r
650	85.34
660	81.89

$$T = 650 + \frac{660 - 650}{81.89 - 85.34} (84.51 - 85.34) = 652.4 \text{ K}$$

$$T_2 = 652.4 \text{ K}$$

at $T_1 = 290 \text{ K}$ $u_1 = 206.91 \text{ kJ/kg}$

at $T_2 = 652.4 \text{ K}$ $u_2 = 475.11 \text{ kJ/kg}$

$$W_{\text{comp}} = u_2 - u_1 = 268.2 \text{ kJ/kg}$$

$$q_{\text{in}} = u_3 - u_2 \Rightarrow 800 = u_3 - 475.11$$

$$u_3 = 1275.11 \text{ kJ/kg}$$

$$u_3 = 1275.11 \text{ kJ/kg}$$

$$T_3 = 1575.13 \text{ K}$$

$$v_{r3} = 6.108$$

$$P_3 v_3 = R T_3 \Rightarrow P_3 = \frac{0.287 \times 1575.13}{0.104} = 4346 \text{ kPa}$$

$$\frac{v_4}{v_3} = \frac{v_{r4}}{v_{r3}} \Rightarrow 8 = \frac{v_{r4}}{6.108} \Rightarrow v_{r4} = 48.864$$

$$T_4 = 795.6 \text{ K}$$

$$u_4 = 588.74 \text{ kJ/kg}$$

$$W_{\text{exp}} = u_3 - u_4 = 686.37 \text{ kJ/kg}$$

$$q_{\text{out}} = u_4 - u_1 = 588.74 - 206.91 = 381.83 \text{ kJ/kg}$$

$$W_{\text{net}} = W_{\text{exp}} - W_{\text{comp}} = 418.17 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{q_{\text{in}}} = \frac{418.17}{800} = 0.5227$$

$$= 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 0.5227$$

We cannot use $1 - \frac{1}{r^{\gamma}}$ because we are dealing with variable specific heat problems.

$$\text{MEP} = \frac{W_{\text{net}}}{v_3 - v_1} = \frac{418.17}{0.8323 - 0.104} = 574.2 \text{ kPa}$$

Otto cycle (Polytropic with constant specific heats)

* There is heat transfer during $1 \rightarrow 2$
 $3 \rightarrow 4$

* $PV^n = C$
 $\Downarrow$

$$W_b = \int P dv = \int \frac{C}{v^n} dv = C \int v^{-n} dv = \frac{C}{1-n} \left[v^{1-n} \right]_{v_1}^{v_2} = \frac{C}{1-n} \left(\frac{v_2^{1-n}}{2} - \frac{v_1^{1-n}}{1} \right)$$

since $C = P_1 v_1^n = P_2 v_2^n$

* $W_{comp} = \frac{R}{n-1} (T_2 - T_1)$

* $W_{Exp} = \frac{R}{n-1} (T_3 - T_4)$

then $W_b = \frac{C v_2^{1-n}}{1-n} - \frac{C v_1^{1-n}}{1-n}$
 $= \frac{P_2 v_2^n v_2^{1-n}}{1-n} - \frac{P_1 v_1^n v_1^{1-n}}{1-n}$
 $= \frac{P_2 v_2}{1-n} - \frac{P_1 v_1}{1-n} = \frac{R T_2}{1-n} - \frac{R T_1}{1-n}$

$W_b = \frac{R}{1-n} (T_2 - T_1)$

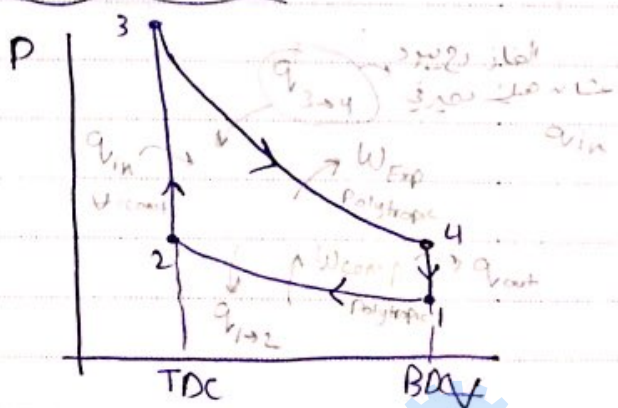
$W_b = \frac{R}{n-1} (T_2 - T_1)$ $\Rightarrow$ $\frac{R}{n-1} (T_2 - T_1)$

process $1 \rightarrow 2$ polytropic

$W_c - q_{1 \rightarrow 2} = \Delta u = u_2 - u_1$
 $\frac{R}{n-1} (T_2 - T_1) - q_{1 \rightarrow 2} = C_v (T_2 - T_1)$

$q_{1 \rightarrow 2} = \left(\frac{R}{n-1} - C_v \right) (T_2 - T_1)$

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = \left(\frac{v_1}{v_2} \right)^{n-1} = CR^{n-1} \Rightarrow$ For ideal gas polytropic process



Process 2→3 (constant volume)

$$q_{in} = u_3 - u_2 = C_v(T_3 - T_2) \quad (\text{No change})$$

Process 3→4 isentropic polytropic

$$q_{3 \rightarrow 4} - W_{Exp} = u_4 - u_3$$

$$q_{3 \rightarrow 4} - \left(\frac{R}{n-1} (T_3 - T_4) \right) = C_v(T_4 - T_3)$$

= magnitude of work exp

$$q_{3 \rightarrow 4} = C_v(T_4 - T_3) + \frac{R}{n-1} (T_3 - T_4)$$

Process 4→1 Constant volume

$$q_{out} = u_4 - u_1 = C_v(T_4 - T_1) \quad \text{No change}$$

$$W_{net} = W_{Exp} - W_{comp}$$

$$= (q_{in} + q_{3 \rightarrow 4})_{in\,tot} - (q_{out} + q_{1 \rightarrow 2})_{out\,tot}$$

$$\eta_{th} = \frac{W_{net}}{q_{in\,tot}} = 1 - \frac{q_{out\,tot}}{q_{in\,tot}}$$

Never over use

$$\eta_{th} = 1 - \frac{1}{CR^{n-1}}$$

Question Otto cycle, $CR=8$, $PV^{1.3}=C$, $P_1=95 \text{ kPa}$
 $T_1=15+273 \text{ K}$
 $T_3=1473 \text{ K}$

Find q_{intot} , q_{outtot} , W_{net} , η_{th}

$$n=1.3$$

Process 1→2 Polytropic

$$\left(\frac{T_2}{T_1}\right) = CR^{n-1} \Rightarrow T_2 = T_1 \times CR^{n-1} = 288 \times 8^{1.3-1} = 537.42 \text{ K}$$

$$\left(\frac{T_3}{T_4}\right) = CR^{n-1} \Rightarrow T_4 = \frac{T_3}{CR^{n-1}} = \frac{1473}{8^{1.3-1}} = 789.36 \text{ K}$$

$$q_{1 \rightarrow 2} = \left(\frac{R}{n-1} - C_v\right)(T_2 - T_1) =$$

$$= \left(\frac{0.287}{1.3-1} - 0.718\right)(537.42 - 288) = 59.5 \text{ kJ/kg}$$

$$q_{\text{in}} = u_3 - u_2 = C_v(T_3 - T_2) = 0.718(1473 - 537.42) = 671.74 \text{ kJ/kg}$$

$$q_{3 \rightarrow 4} = \left(\frac{R}{n-1} - C_v\right)(T_3 - T_4) = \left(\frac{0.287}{1.3-1} - 0.718\right)(1473 - 789.36) = 163.16 \text{ kJ/kg}$$

$$q_{\text{out}} = u_4 - u_1 = C_v(T_4 - T_1) = 0.718(789.36 - 288) = 359.97 \text{ kJ/kg}$$

$$W_c = \frac{R}{n-1}(T_2 - T_1) = \frac{0.287}{0.3}(537.42 - 288) = 238.6 \text{ kJ/kg}$$

$$W_e = \frac{R}{n-1}(T_3 - T_4) = \frac{0.287}{0.3}(1473 - 789.36) = 654 \text{ kJ/kg}$$

$$q_{\text{intot}} = q_{\text{in}} + q_{3 \rightarrow 4} = 671.74 + 163.16 = 834.9 \text{ kJ/kg}$$

$$q_{\text{outtot}} = q_{\text{out}} + q_{1 \rightarrow 2} = 59.5 + 359.97 = 419.47 \text{ kJ/kg}$$

$$\eta_{\text{th}} = 1 - \frac{q_{\text{outtot}}}{q_{\text{intot}}} = 1 - \frac{419.5}{834.9} = 0.4975 \times 100 = 49.75\%$$

Diesel cycle

Isothermal processes in Carnot are exchanged by isobaric process & isochoric process

how is it done at constant pressure

* q_h (heat addition) $\Rightarrow$ at constant pressure (i.e. $P_2 = P_3$) $\left(\begin{matrix} P = \text{const} \\ T \uparrow \\ V \uparrow \end{matrix} \right)$

* q_L (heat rejection) $\Rightarrow$ at constant volume (like Otto)

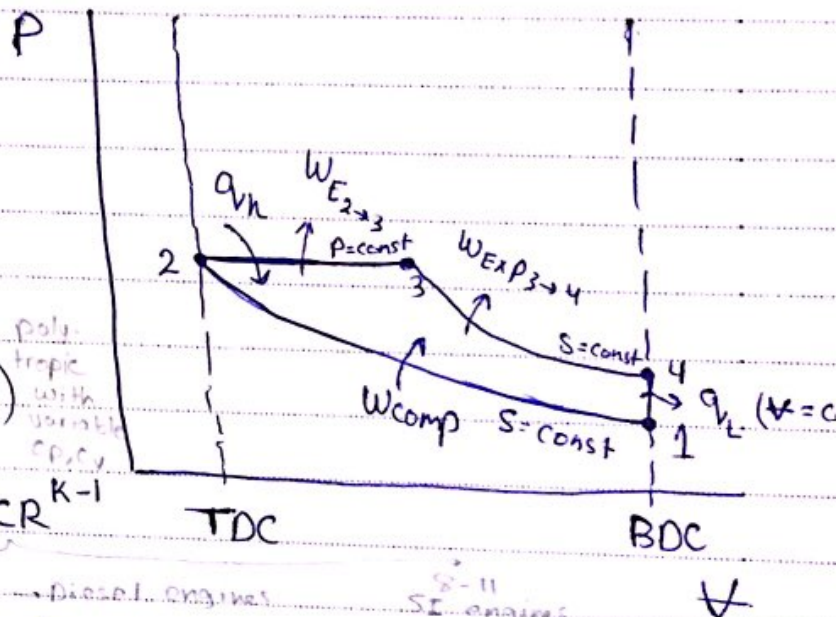
$\uparrow \downarrow P - \text{MRTN}$
 $\uparrow \downarrow V$
const

process 1 $\rightarrow$ 2

Comp / S = const / $q_v = 0$

$$W_{\text{comp}} = u_2 - u_1 = C_v(T_2 - T_1)$$

$$\left(\frac{T_2}{T_1} \right) = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = \left(\frac{V_1}{V_2} \right)^{k-1} = CR^{k-1}$$



process 2 $\rightarrow$ 3 (heat addition at const pressure)

$$q_h - W_{E2 \rightarrow 3} = u_3 - u_2$$

there is area under the curve (there is work)

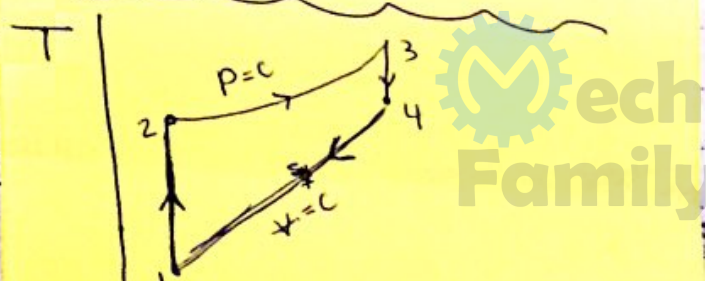
$$q_h - P(V_3 - V_2) = u_3 - u_2 \Rightarrow q_h = (u_3 + PV_3) - (u_2 + PV_2) = \Delta h = C_p(T_3 - T_2) = h_3 - h_2$$

* Cut-off ratio " r_c " = $\frac{V_3}{V_2} = \frac{T_3}{T_2}$

$$\frac{P_3 V_3}{T_3} = \frac{P_2 V_2}{T_2}, \text{ since } P_2 = P_3 \Rightarrow \frac{V_3}{T_3} = \frac{V_2}{T_2}$$

r_c for Otto cycle = 1

T-S Diagram for Diesel cycle



In diesel engines
CR is large in order to
make T_2 ~~very~~ huge
 $T_2 = T_1 CR^{k-1}$
* There are no spark plugs in
diesel engines, because diesel
can't be ignited using sparks
* High pressure and Temperature
cause diesel to ignite, so
CR must be large to attain
high pressure and Temperature
at ~~the~~ states

We increase the CR by
decreasing θ_{min} (clearance) (such
that the valves can move) and
increasing θ_{max}

off ratio

بشكل اديت ديزل دخل عاكس 6 ادا
كانت كمية كبيرة ديزل عند احتراق
غير كامل

Fuels react with oxygen to
release energy. Complete combustion
happens in a plentiful supply of
air. Incomplete combustion occurs
when the supply of air is
limited. Complete combustion releases
more energy than incomplete
combustion

Fuels are substances that react
with O_2 to release useful energy (heat)
Fuels contain hydrocarbons
These are compounds of hydrogen
and carbon only. When they
burn completely
Carbon oxidises to carbon dioxide
hydrogen oxidises to water
when they burn incompletely water,
carbon monoxide and carbon
are produced
poisonous gas soot

process 1 → 2
 $p/S = \text{const} / q$

شاي الاحتراق →

مثل injection ع مرحلتين
المرحلة الاولى بينج جزء من الوقود وجزءه
at const volume
المرحلة الثانية والدفعة الثانية تستقرق
at const pressure

محركات ال Honda تستعمل هذه

process 1 → 2
 $p/S = \text{const} / q = c$

بشكل اديت ضغط
الهوا قبل ما ادخل الوقود

بشكل كمية الوقود
الي دخلت ال cylinder
وبالتالي بضغطي فكرة عن
كمية الديزل الي احتترقت
(كل ما كانت كمية الديزل اكبر
كان الاحتراق غير تام)

في الشتاء يكون بارد الهواء
يادخل ال cylinder يكون بارد (T_1 صغيرة)
فما يعني
 $T_2 = T_1 CR^{k-1}$
 T_2 كما ان ربح يكون صغير 6 اذا لم تكن
 T_2 مرتفعة لدرجة كافية لحرق الديزل 6 فالباقي
ما ربح يشتغل 6 فيفضل القوي يشتغل الباص على
وتنصر T_2 كافية لحرق الديزل

بالاسكيو كيف الناس يتعفش ← في اسي اسه
دفاية 6 يشتغل الباص مرة اول 6 فيفضل الهواء
هو ال cylinder 6 يشتغل مرة ثانية يشتغل
الباص بتدوير T_2 كافية لحرق الديزل

بعضه من
اشي اعلى injection ع مرحلتين
أصغر CR 6 في محركات الديزل انما يصغر
CR عالية ع 14 أضغط الهواء وأرفع درجة
حرارة لدرجة كافية لحرق الديزل 6 لكن
في ال dual جزء من احتراق ربح ديزل
من ضغط الهواء وجزء من مرحلة الاحتراق
الدول 6 فيقلل من CR

محركات الديزل المتزجج 6 جزء منه ناجح
ال CR العالية 6 فن خلال
ال CR انما يقلل من CR

(3→4) Expansion / $S = \text{const}$ / $q = 0$

$$W_{\text{EXP}3 \rightarrow 4} = u_3 - u_4 = C_v(T_3 - T_4)$$

$$\left(\frac{T_3}{T_4}\right) = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = \left(\frac{v_4}{v_3}\right)^{k-1} = \left(\frac{CR}{r_c}\right)^{k-1}$$

$v_4 = v_{\text{BDC}}$
 $v_3 \neq v_{\text{BDC}}$
 $\frac{v_4}{v_3} \neq CR$

$$\frac{v_4}{v_3} = \frac{v_4}{v_1} \cdot \frac{v_1}{v_2} \cdot \frac{v_2}{v_3} \quad (v_1 = v_4 = \text{BDC } v_{\text{BDC}})$$

$$= CR \cdot \frac{1}{r_c}$$

process (4-1) heat rejection / $v = \text{const}$

$$q_L = u_4 - u_1 = C_v(T_4 - T_1)$$

$$W_{\text{net}} = q_h - q_L = W_{E2 \rightarrow 3} + W_{E3 \rightarrow 4} - W_{\text{comp}}$$

$$\eta_{th} = 1 - \frac{q_L}{q_h} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{C_v}{C_p} \cdot \frac{T_1}{T_2} \cdot \frac{(T_4/T_1 - 1)}{(T_3/T_2 - 1)}$$

$$= 1 - \frac{1}{k} \cdot \frac{1}{CR^{k-1}} \left(\frac{T_4/T_1 - 1}{r_c - 1} \right)$$

$$= 1 - \frac{1}{CR^{k-1}} \left(\frac{r_c^k - 1}{k(r_c - 1)} \right)$$

$$\begin{aligned} * \frac{T_4}{T_1} &= \frac{T_4}{T_3} \cdot \frac{T_3}{T_2} \cdot \frac{T_2}{T_1} \\ &= \left(\frac{r_c}{CR} \right)^{k-1} \cdot r_c \cdot CR^k \\ &= r_c^k \end{aligned}$$

$$\lim_{r_c \rightarrow 1} \frac{r_c^k - 1}{k(r_c - 1)} = 1$$

في diesel η_{th} يتغير
 او في Otto η_{th}

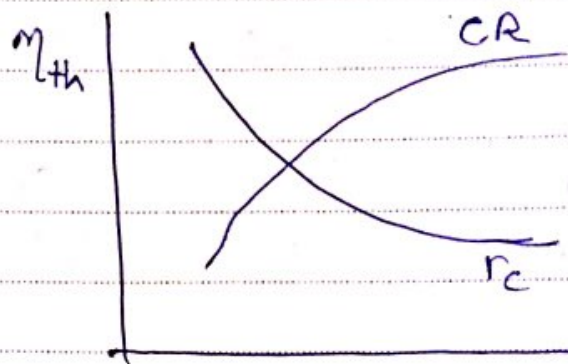
η_{th} depends on $\begin{cases} k \\ CR \\ r_c \end{cases}$

$$\eta_{HEP} = \frac{W_{net}}{V_{stroke}}$$

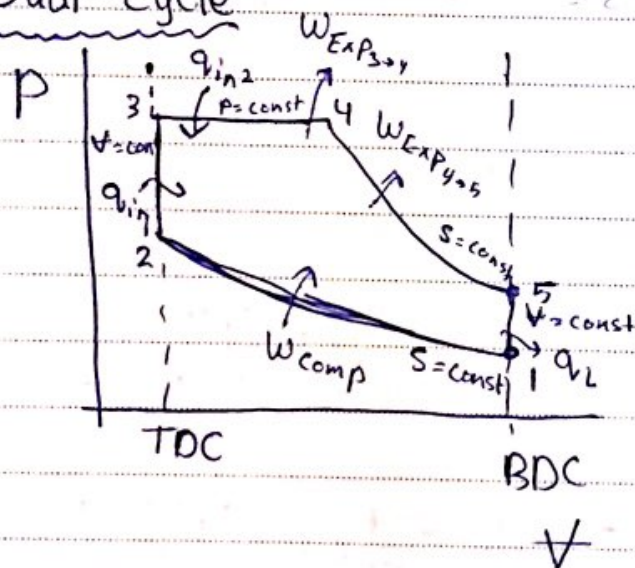
$$\eta_{th} = 1 - \frac{1}{CR^{k-1}} \cdot \frac{1}{k} \left(\frac{r_c^k - 1}{r_c - 1} \right)$$

When $r_c \uparrow$ $\left(\frac{r_c^k - 1}{r_c - 1} \right) \uparrow$

$\eta_{th} \downarrow$



Dual cycle diesel, Otto cycle



$$* W_{comp} = u_2 - u_1 = C_v(T_2 - T_1)$$

$$* q_{in1} = u_3 - u_2 = C_v(T_3 - T_2)$$

$$* W_{Exp3 \rightarrow 4} = P(v_4 - v_3)$$

$$* q_{in2} = h_4 - h_3 = C_p(T_4 - T_3)$$

$$* W_{Exp4 \rightarrow 5} = u_4 - u_5 = C_v(T_4 - T_5)$$

$$* q_{out} = u_5 - u_1 = C_v(T_5 - T_1)$$

Example 9-3

Ideal diesel cycle, air, $CR = 18$, $\gamma_c = 2$

$$P_1 = 100 \text{ kPa}$$

$$T_1 = 27 + 273 \text{ K}$$

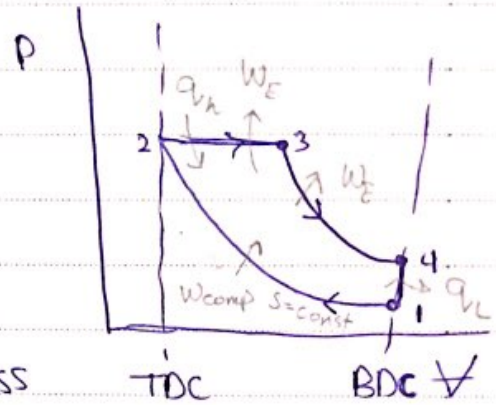
$$V_1 = 1917 \text{ cm}^3 = 1917 \times 10^{-6} \text{ m}^3$$

Cold air standard assumption (i.e. use constant specific heats)

a) Find T and p at the end of each process

b) W_{net} , M_{th}

c) MEP



(a) Solution:

1 $\rightarrow$ 2 $S = \text{const}$ + constant specific heats

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = CR^{\gamma-1}$$

$$T_2 = T_1 \times CR^{\gamma-1} = 300 \times 18^{1.4-1}$$

$$T_2 = 953 \text{ K}$$

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = CR^{\gamma-1} \Rightarrow P_2 = P_1 \times CR^{\gamma} = 5720 \text{ kPa}$$

$$\gamma = 1.4$$

$$R = 0.287$$

$$C_v = 0.718$$

$$C_p = 1.005$$

2 $\rightarrow$ 3 $P = \text{const}$

$$P_3 = P_2 = 5720 \text{ kPa}$$

$$\gamma_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} \Rightarrow T_3 = T_2 \times \gamma_c$$

$$T_3 = 953 \times 2 = 1906 \text{ K}$$

3 $\rightarrow$ 4 $S = \text{const}$, const specific heats

$$\left(\frac{T_3}{T_4}\right) = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$$

$$\frac{T_3}{T_4} = \left(\frac{CR}{\gamma_c}\right)^{\gamma-1} \Rightarrow T_4 = \frac{T_3}{\left(\frac{CR}{\gamma_c}\right)^{\gamma-1}}$$

$$T_4 = 791 \text{ K}$$

$$\frac{V_4}{V_3} = \frac{V_4}{V_1} \cdot \frac{V_1}{V_2} \cdot \frac{V_2}{V_3} = \frac{CR}{\gamma_c}$$

$$\left(\frac{P_3}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{CR}{rc}\right)^{k-1} \Rightarrow P_1 = \frac{P_3}{\left(\frac{CR}{rc}\right)^k} = 264 \text{ kPa}$$

b) $W_{net} = Q_H - Q_L$

$$Q_H = P \Delta V = \Delta U \Rightarrow Q_H = \Delta U + P \Delta V = \Delta H$$

$$Q_H = C_p(T_2 - T_1) \cdot m$$

$$P_1 V_1 = m R T_1 \Rightarrow m = \frac{P_1 V_1}{R T_1} = \frac{100 \times 1917 \times 10^{-6}}{0.287 \times 300} = 0.00223 \text{ kg}$$

$$Q_H = 1.005 \times 0.00223 \times (1906 - 953) = 2.136 \text{ kJ}$$

$$-Q_L = U_1 - U_4$$

$$\begin{aligned} Q_L = U_4 - U_1 &= C_v(T_4 - T_1) \cdot m \\ &= 0.718 \times 0.00223 \times (791 - 300) \\ &= 0.786 \text{ kJ} \end{aligned}$$

$$\therefore W_{net} = Q_H - Q_L = 2.136 - 0.786 = 1.35 \text{ kJ}$$

$$\eta_{th} = \frac{W_{net}}{Q_H} = \frac{1.35}{2.136} \times 100 = 63.2\%$$

c) $HEP = \frac{W_{net}}{V_{stroke}} = \frac{1.35}{(1917 - 18.5) \times 10^{-6}} = 746 \text{ kPa}$

$$CR = \frac{V_{max}}{V_{min}} \Rightarrow V_{min} = \frac{V_{max}}{CR} = \frac{1917 \times 10^{-6}}{18} = 106.5 \times 10^{-6} \text{ m}^3$$

Repeat Example 9-3 assuming polytropic processes with $n=1.3$

(a)

$$P_1 = 100 \text{ kPa} \quad T_1 = 300 \text{ KPa} \quad V_1 = 1917 \times 10^{-6} \text{ m}^3$$

$$m = 0.00223 \text{ kg}$$

1 → 2 polytropic

$$\left(\frac{T_2}{T_1} \right) = \left(\frac{V_1}{V_2} \right)^{n-1}$$

$$T_2 = T_1 * CR^{n-1} = 300 * 18^{0.3}$$

$$T_2 = 714 \text{ K}$$

$$P_2 = P_1 * CR^n = 100 * 18^{1.3} = 4284 \text{ kPa} \Rightarrow P_2 = 4284 \text{ kPa}$$

2 → 3 constant pressure

$$P_3 = P_2 = 4284 \text{ kPa}$$

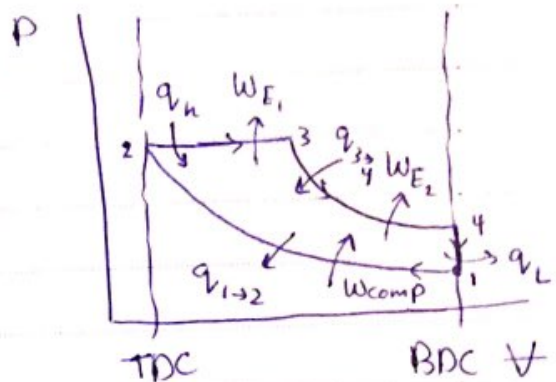
$$r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2}$$

$$T_3 = T_2 * r_c = 2 * 714 = 1428 \text{ K}$$

3 → 4 polytropic

$$\left(\frac{T_3}{T_4} \right) = \left(\frac{CR}{r_c} \right)^{n-1} \Rightarrow T_4 = \frac{T_3}{\left(\frac{CR}{r_c} \right)^{n-1}} = \frac{1428}{9^{0.3}} = 738.7 \text{ K}$$

$$\left(\frac{P_3}{P_4} \right) = \left(\frac{CR}{r_c} \right)^n \Rightarrow P_4 = \frac{P_3}{\left(\frac{CR}{r_c} \right)^n} = \frac{4284}{9^{1.3}} = 246.2 \text{ kPa}$$



(b) process 1→2

$$W_{\text{comp}} - Q_{1 \rightarrow 2} = U_2 - U_1$$

$$\frac{mR(T_2 - T_1)}{n-1} - Q_{1 \rightarrow 2} = mC_v(T_2 - T_1)$$

$$Q_{1 \rightarrow 2} = (T_2 - T_1) \left(\frac{mR}{n-1} - mC_v \right)$$

$$= (714 - 300) \left(\frac{0.00223 \times 0.287}{0.3} - 0.718 \times 0.00223 \right)$$

$$= 0.22 \text{ kJ}$$

process 2→3

$$Q_h - W_{E_{2 \rightarrow 3}} = U_3 - U_2$$

$$Q_h = U_3 - U_2 + P(V_3 - V_2)$$

$$= m C_p (T_3 - T_2) = 0.00223 \times 1.005 \times (1428 - 714)$$

$$Q_h = 1.6001 \text{ kJ}$$

process 3→4

$$Q_{3 \rightarrow 4} - W_E = U_4 - U_3$$

$$Q_{3 \rightarrow 4} - \frac{mR(T_3 - T_4)}{n-1} = mC_v(T_4 - T_3)$$

$$Q_{3 \rightarrow 4} = (T_4 - T_3) \left(mC_v - \frac{mR}{n-1} \right)$$

$$= (738.7 - 1428) \left(0.00223 \times 0.718 - \frac{0.00223 \times 0.287}{0.3} \right)$$

$$= 0.3668 \text{ kJ}$$

process 4 → 1

$$Q_L = m C_v (T_4 - T_1) = 0.00223 * 0.718 * (738.7 - 300) \\ = 0.7024 \text{ kJ}$$

$$W_{net} = Q_{in tot} - Q_{out tot} \\ = (Q_{12} + Q_{34}) - (Q_L + Q_{12}) \\ = (1.6001 + 0.3668) - (0.7024 + 0.22) \\ = 1.0445 \text{ kJ}$$

$$\eta_{th} = \frac{W_{net}}{Q_{in tot}} = \frac{1.0445}{1.9669} = 0.53$$

$$(C) \text{ MEP} = \frac{W_{net}}{V_s} = \frac{1.0445}{(1917 - 106.5) \times 10^{-6}} = 576 \text{ kPa}$$



Repeat example 9-3 assuming isentropic processes with variable specific heats. Assume $q_{in} = 958 \text{ kJ/kg}$

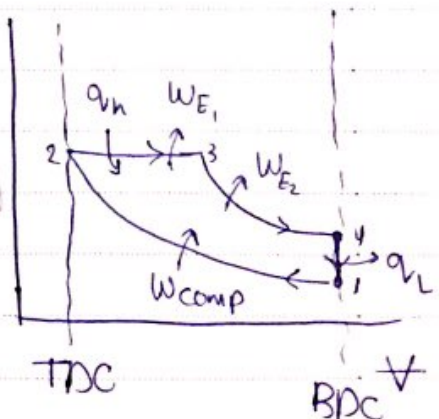
$$P_1 = 100 \text{ kPa} \quad T_1 = 300 \text{ K} \quad V_1 = 1917 \times 10^{-6} \text{ m}^3$$

$$V_2 = \frac{V_1}{CR} = 106.5 \times 10^{-6} \quad , \quad u_1 = 214.07 \text{ kJ/kg}$$

at state 2

$$\frac{V_1}{V_2} = \frac{V_{1r}}{V_{2r}} \Rightarrow 18 = \frac{621.2}{V_{2r}}$$

$$V_{2r} = \frac{621.2}{18} = 34.51$$



T	v_f	h
900	34.31	932.93
880	36.61	910.56

$$T = 900 = \frac{900 - 880}{34.31 - 36.61} (v_f - 34.31)$$

$$T_2 = 898 \text{ K}$$

$$h = 932.93 = 932.93 - 910.56 \times \frac{(T - 900)}{900 - 880}$$

$$h_2 = 931 \text{ kJ/kg}$$

$$P_2 V_2 = m R T_2 \Rightarrow P_2 = \frac{m R T_2}{V_2} = \frac{2.2264 \times 10^{-3} \times 0.287 \times 898}{106.5 \times 10^{-6}} = 5388 \text{ kPa}$$

$$P_3 = P_2 = 5388 \text{ kPa}, \quad r_c = \frac{V_2}{V_3} \Rightarrow V_3 = V_2 \times r_c = 106.5 \times 10^{-6} \times 2 = 213 \times 10^{-6} \text{ m}^3$$

$$P_3 V_3 = m R T_3 \Rightarrow T_3 = \frac{P_3 \times V_3}{m \times R} = \frac{5388 \times 213 \times 10^{-6}}{2.2264 \times 10^{-3} \times 0.287}$$

$$T_3 = 1796 \text{ K}$$

T	h	v_f
1750	1941.6	4.328
1800	2003.3	3.994

$$h = 2003.3 = 1941.6 \times \frac{(T - 1800)}{1800 - 1750}$$

$$h_3 = 1998 \text{ kJ/kg}$$

$$v_f = 4.328 = \frac{4.328 - 3.994}{1750 - 1800} (T - 1750)$$

$$v_{f3} = 4.02072$$

$$V_4 = V_1 = 1917 \times 10^{-6} \text{ m}^3$$

process 3→4 isentropic

$$V_3 = 213 \times 10^{-6} \text{ m}^3$$

$$T_3 = 1796 \text{ K}$$

$$v_{r3} = 4.02072$$

$$\frac{V_4}{V_3} = \frac{v_{r4}}{v_{r3}} \Rightarrow v_{r4} = \frac{1917}{213} \times 4.02072$$

$$= 36.18648$$

T	v_r	u
880	36.61	657.95
900	34.31	674.58

$$T - 880 = \frac{880 - 900}{36.61 - 34.31} * (v_r - 36.61)$$

$$T_4 = 884 \text{ K}$$

$$u - 674.58 = \frac{674.58 - 657.95}{900 - 880} * (T - 900)$$

$$u_4 = 661.276 \text{ kJ/kg}$$

$$P_4 V_4 = m R T_4 \Rightarrow P_4 = \frac{2.2264 \times 10^{-3} \times 0.287 \times 884}{1917 \times 10^{-6}} = 294.65 \text{ kPa}$$

$$Q_H = m(h_3 - h_2) = 2.2264 \times 10^{-3} \times (1998 - 931) = 2.3756 \text{ kJ}$$

$$Q_L = m(u_4 - u_1) = 2.2264 \times 10^{-3} \times (661.276 - 214.07) = 0.99566 \text{ kJ}$$

$$W_{\text{net}} = Q_H - Q_L = 1.37994 \text{ kJ}$$

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_H} = 0.5808$$

$$MEP = \frac{W_{net}}{V_1 - V_2} = \frac{1.37994}{(1917 - 106.5) \times 10^{-6}} = 762.18 \text{ kPa}$$

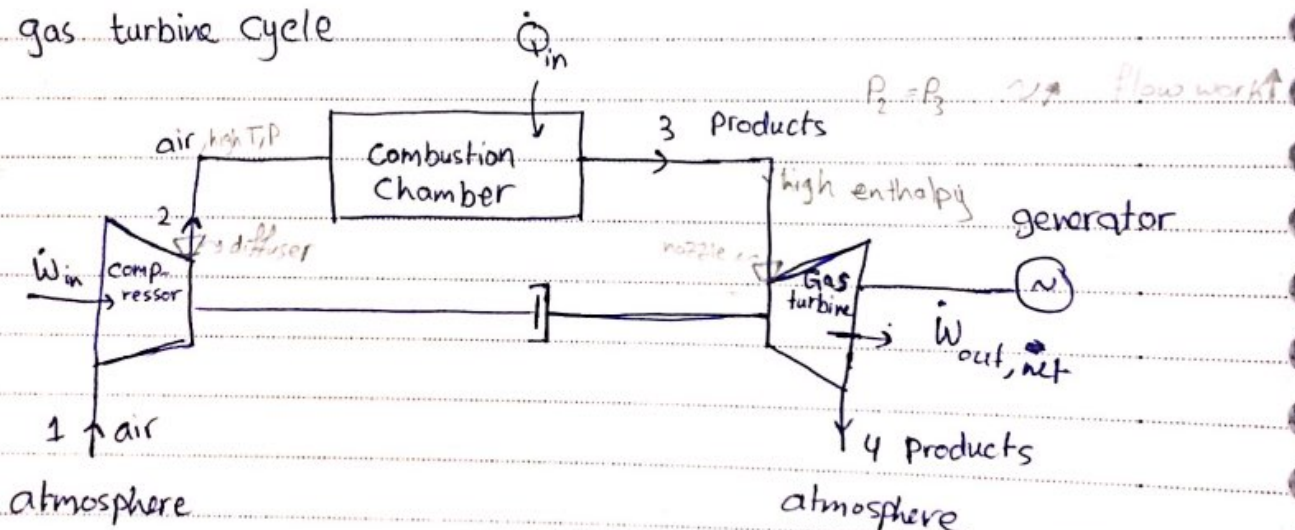
Brayton Cycle (Gas turbine cycle / Joule cycle)

1st law of Thermodynamics for open systems (steady-flow) Control volume (volume is const.)

$$(\dot{Q}_{in} - \dot{Q}_{out}) + (\dot{W}_{in} - \dot{W}_{out}) = \dot{m}(\Delta h + \Delta ke + \Delta pe)$$

$\frac{d(\text{Property})}{dt} = 0$

- The cycle consists of steady flow devices (not piston cylinder device)
- Assume constant specific heats
- Assume that the compressor and the turbine are adiabatic.
reversible (i.e. isentropic) ↗
irreversible ↗
- neglect $\Delta ke, \Delta pe$, unless given in the question
- Open gas turbine cycle



$$\dot{W}_{in, comp} = \dot{m}_a (h_2 - h_1) \rightarrow \text{isentropic } Q=0 \quad * P_2 = P_3$$

$$\dot{Q}_{in} = (\dot{m}_a + \dot{m}_f) (h_3 - h_2) \quad * \dot{W}_{net} = \dot{W}_{out, Tur} - \dot{W}_{in, comp}$$

$$\dot{W}_{out, Tur} = (\dot{m}_a + \dot{m}_f) (h_3 - h_4) \rightarrow \text{isentropic } Q=0$$

Products

Brayton cycle, consists of 4 processes
 1) compression 2) heat addition
 3) heat expansion 4) heat rejection

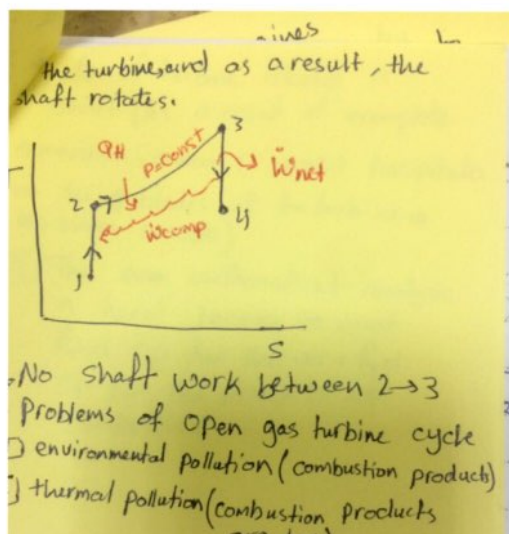
* Power plants in Jordan are based on Brayton cycle

* Joule proposed this cycle in order to burn medical waste, But Nowadays this cycle is used in power plants and jet engines

Open gas turbine cycle:
 → Mixing air with fuel is allowed in this cycle
 → It is called "Open", because it is open to the atmosphere
 → Actually, it is not a complete cycle (because the air that enters the compressor is not the same as the air leaves the turbine, but we can consider it as a cycle)
 دورة الغازات المفتوحة
 الهواء الداخل إلى الضاغط ليس هو الذي يخرج من التوربين، لكن يمكننا اعتباره دورة

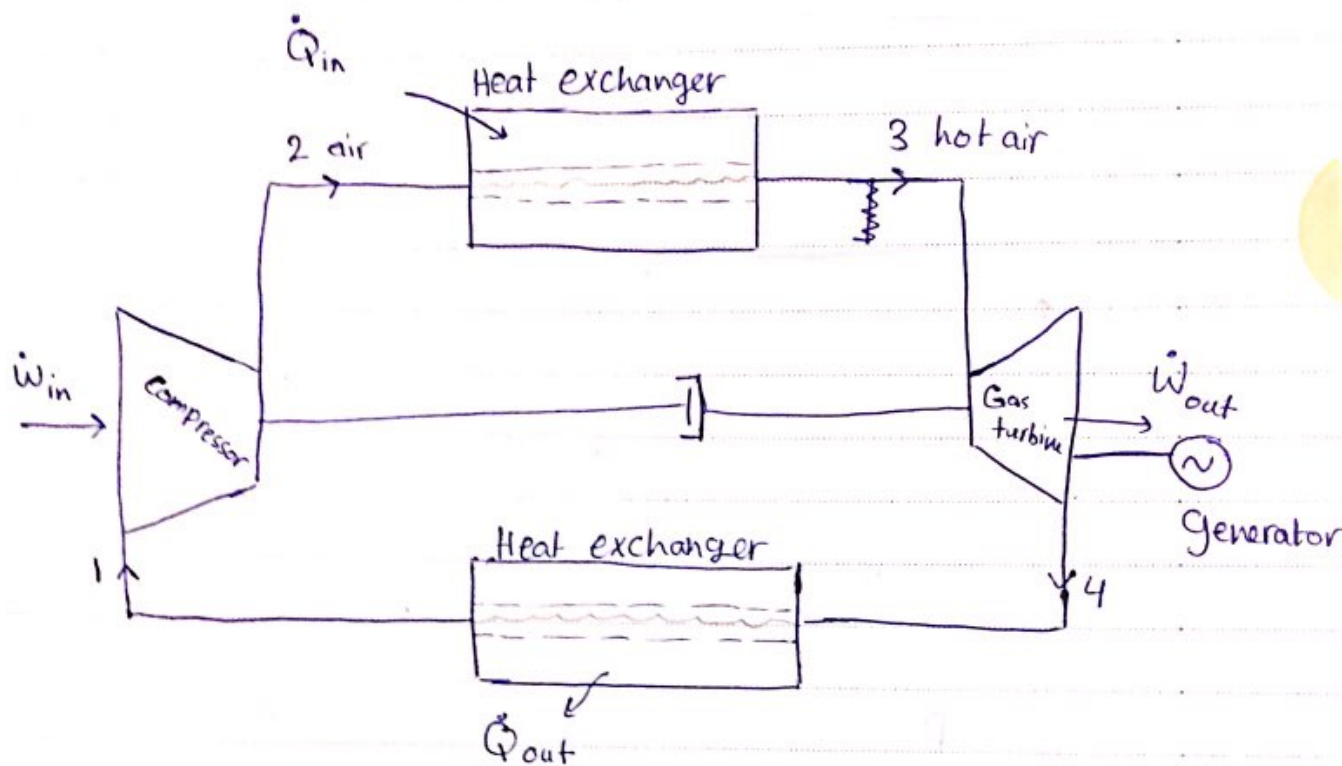
Compressor:
 → at the beginning of the operation, a diesel engine is connected to the compressor to operate the compressor, when the cycle becomes steady, we disconnect the diesel engine and connect the compressor with the turbine (ie $\dot{W}_{turb} \neq \dot{W}_{net, cycle}$)
 some of it goes to operate the turbine
 → the compressor increases the speed of the air. at 2 there is a diffuser that decreases the speed and increases the pressure

as P_1, T_1
 * Combustion Chamber
 → $P_2 = P_3$ (actually P_2 is slightly greater than P_3)
 if $P_3 > P_2$ the working fluid will move in the opposite direction
 if $P_2 > P_3$, the air will move fast, without gaining the sufficient heat
 Gas turbine:
 there is a nozzle before the turbine this nozzle increases the K.E of the air, so the air hits the blades



life of turbine blades is short. (As a result of incomplete combustion, carbon () precipitates on the blades of the turbine → the blades corrode)
 4) The mathematical analysis is hard because we must find C_p for the air + fuel C_p for the products of combustion

• Closed gas turbine cycle

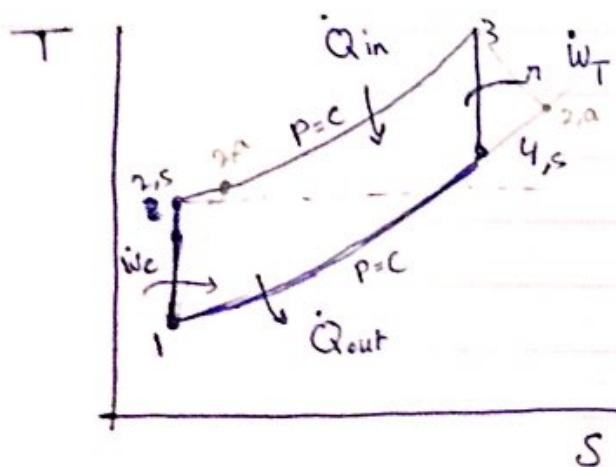


$$* \dot{W}_{incomp} = \dot{m}_a (h_2 - h_1)$$

$$* \dot{W}_{outtur} = \dot{m}_a (h_3 - h_4)$$

$$* \dot{Q}_{in} = \dot{m}_a (h_3 - h_2)$$

$$* \dot{Q}_{out} = \dot{m}_a (h_4 - h_1)$$



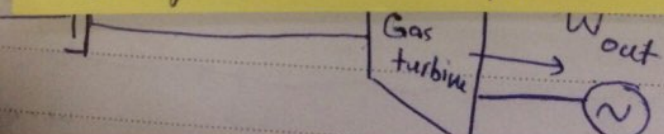
$$* \dot{W}_{net} = \dot{W}_{outtur} - \dot{W}_{incomp}$$

Closed gas turbine cycle

- mixing air with fuel is not allowed in this cycle
- The air that is entering the cycle at 1 is filtered in order to protect the blade of the turbine

→ Gas turbine, steam turbine
↓ ↓
for gases for steam

→ compressor, pumps
↓ ↓
for gases for liquids



- At 4, since the air is filtered and the enthalpy of the air is relatively high. (high volume), we must reuse it

P_2 is constant

the compressor is

- In order to compress the air using the compressor. Greater work is needed W_{comp} because the air has a great volume. ~~the order is~~

W_{comp} is taken from the turbine

W_{net} will decrease

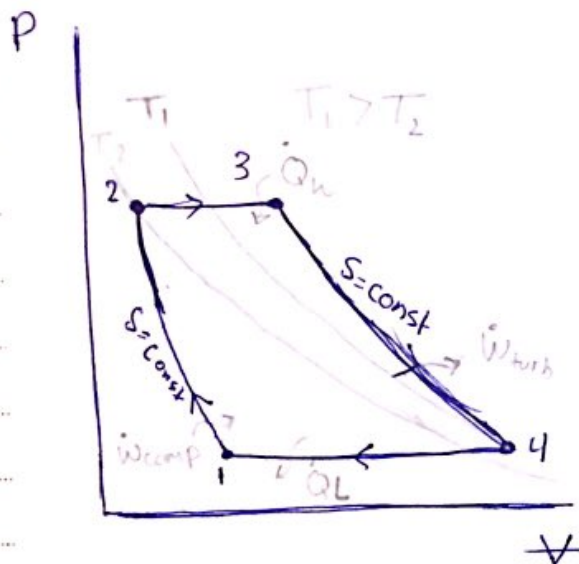
In order to reduce w_{comp}

We use a HEX to cool the air

∴ lower its volume

$$\frac{\dot{W}_s}{\dot{m}} = \int v dp$$

$\downarrow$
 $\downarrow$
 W



1→2 Compression $v_2 < v_1$
3→4 Expansion $v_4 > v_3$

* لو سخت رقم 2 قبل ما ادخله

حالة HEX و مع أقل كمية الوقود المبرح

استهلاكها لرفع درجة الحرارة إلى T_3

* إذا كانت $T_2 < T_4$ بقدر استخدامها

في سخين في فنيك يكون قلت استهلاك

الوقود + قلت التلوث اكراري

$$P_2 = P_3, P_4 = P_1$$

$$\frac{P_2}{P_1} = \frac{P_3}{P_4} = r_p$$

process 1→2 (isentropic)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}$$

process 3→4 (isentropic)

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4} \right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

* Compressor (1→2) ($s = \text{const}$, $\dot{Q} = 0$, $\dot{W}_{\text{out}} = 0$, $\Delta KE = 0$, $\Delta PE = 0$)

$$\dot{W}_{\text{in}} = \dot{m}_{\text{air}} (h_2 - h_1)$$

$$= \dot{m}_{\text{air}} C_{p,\text{air}} (T_2 - T_1) = \dot{m}_{\text{air}} C_{p,\text{air}} T_2 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}} \right)$$

We know $\frac{T_2}{T_1} = r_p^{\frac{k-1}{k}}$

$$T_2 = T_1 r_p^{\frac{k-1}{k}}, \text{ if } T_1 \downarrow \Rightarrow T_2 \downarrow$$

one way to decrease $\dot{W}_{\text{comp}}$ is to cool the air before it enters the comp.

* Combustion Chamber (2→3) ($P = \text{const}$, $\dot{W}_s = 0$, $\Delta KE = 0$, $\Delta PE = 0$)

if the air enters the combustion chamber with high speed, it will not suck enough heat

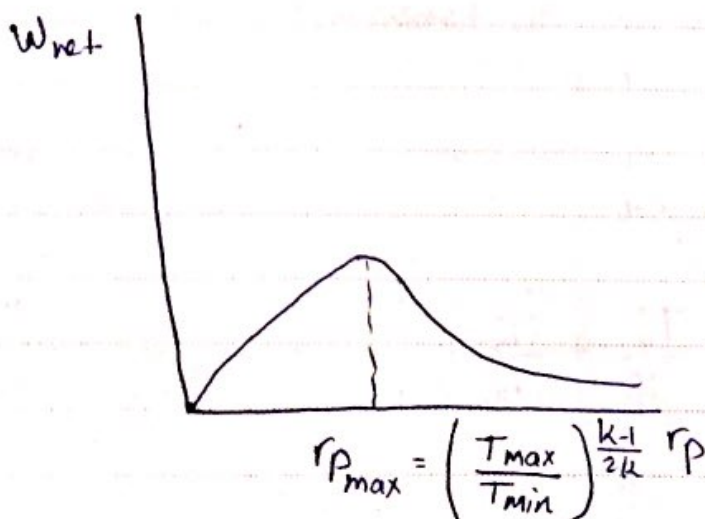
$$\dot{Q}_H = \dot{m}_{\text{air}} (h_3 - h_2) = \dot{m}_{\text{air}} C_{p,\text{air}} (T_3 - T_2)$$

* Heat exchanger (4→1) ($P = \text{const}$, $\dot{W}_s = 0$, $\Delta KE = 0$, $\Delta PE = 0$)

$$\dot{Q}_L = \dot{m}_{\text{air}} (h_4 - h_1) = \dot{m}_{\text{air}} C_{p,\text{air}} (T_4 - T_1)$$

* Gas turbine ($s = \text{const}$, $\dot{Q} = 0$, $\dot{W}_{\text{in}} = 0$, $\Delta KE = 0$, $\Delta PE = 0$) (3→4)

$$\dot{W}_{\text{out}} = \dot{m}_a (h_3 - h_2) = \dot{m}_a C_p (T_3 - T_2) = \dot{m}_{\text{air}} C_p T_3 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}} \right)$$



In order to increase $\dot{W}_{\text{Turb}}$
 $T_3 \uparrow \Rightarrow$ consuming more fuel

* Compressor $\rightarrow$ I compress ~~the~~ ^{P $\uparrow$} the air + heat it in the HEX, so it will arrive the turbine with high flow energy $\Rightarrow$ So it will hit the blades and ~~make~~ makes the shaft rotate.

* If I leave it with compressing it and only heat it $\Rightarrow$ it will arrive the turbine with low flow energy $\Rightarrow$ It will not be able to rotate the shaft

It reacts with O_2 , form NO_x (Nitrogen oxides). When NO_x reacts with hydrocarbons, under the light of the sun, it forms gases that causes cancer and blindness

لا يوجد شيء في الطبيعة

In order to increase $\dot{W}_{out}$, we can increase r_p , but if $r_p \uparrow \Rightarrow \dot{W}_{comp}$ will also $\uparrow$ until

$$\dot{W}_{turb} = \dot{W}_{comp} \quad \text{i.e. } \dot{W}_{net} = 0$$

$$\dot{Q} = \dot{m}_{air} c_{p,air} (T_3 - T_2) \quad \text{suck enough heat}$$

$$\Delta KE = 0, \Delta PE = 0$$

In order to increase $\dot{W}_{out}$, we can increase T_3 (T_3 is the maximum temperature in the cycle) but increasing T_3 has limitations:

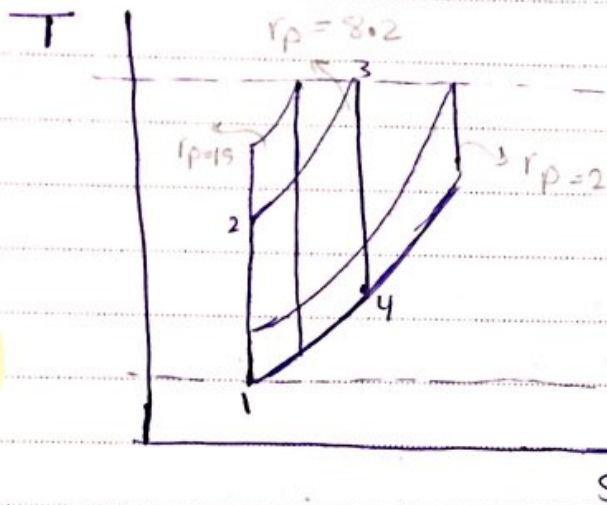
[1] Thermal strains occur in the pipes
The may melt. ~~to~~
Blades of the turbine may melt also

[2] Thermal Pollution $\Rightarrow$

[3] Consuming more fuel

N_2 ~~is~~ ^{atmosphere} in air is not active. But at high temperatures ~~is~~ (1100-1200°C), it becomes active

$11 \leq r_p \leq 16$ (for civil uses)



$T_{\max} = T_3$ (it has limitations)

$$\dot{W}_{\text{net}} = \dot{m} c_p (T_3 - T_2) \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}} \right)$$

$T_{\min} = T_1$ ($T_1 = T_{\text{atmospheric}}$)

⇒ Notice that the area ↑ when r_p ↑, but then the area ↓

$$\text{Area} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \dot{W}_{\text{net}}$$

~~As r_p ↑, T_2 ↑ because $T_2 = T_1 r_p^{\frac{k-1}{k}}$~~
 ~~$(T_3 - T_2)$ ↓ $(1 - \frac{1}{r_p^{\frac{k-1}{k}}})$ ↑~~

* For the isentropic compressor process in the compressor

$$r_p = \left(\frac{P_2}{P_1} \right) = \left(\frac{T_2}{T_1} \right)^{\frac{k}{k-1}}$$

* For the isentropic process in the turbine

$$r_p = \left(\frac{P_3}{P_4} \right) = \left(\frac{T_3}{T_4} \right)^{\frac{k}{k-1}}$$

$$\Rightarrow \left(\frac{T_2}{T_1} \right) = \left(\frac{T_3}{T_4} \right) \Rightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\eta_{th} = 1 - \frac{q_L}{q_H} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1}{T_2} \times \left(\frac{T_4/T_1 - 1}{T_3/T_2 - 1} \right)$$

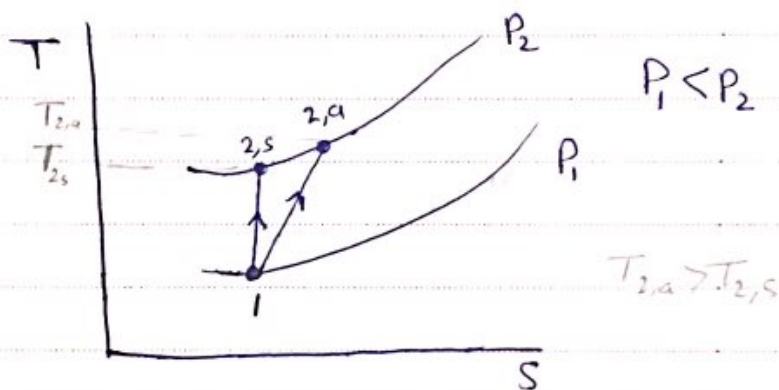
$$= 1 - \frac{1}{r_p^{\frac{k-1}{k}}}$$

W_{BR} = Back work ratio = $\frac{W_C}{W_T}$ 50 %
المحرك المثالي
الحرارة

* Compressor

$W_{C,s}$ = Ideal comp work
 $= h_{2,s} - h_1$

$W_{C,a}$ = actual comp work
 $= h_{2,a} - h_1$

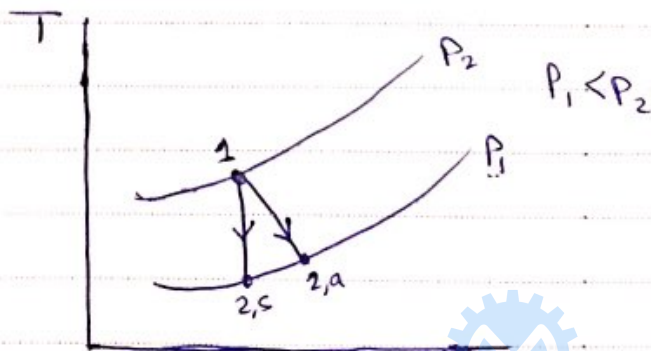


$$\eta_{fs,comp} = \frac{W_{C,s}}{W_{C,a}} = \frac{h_{2,s} - h_1}{h_{2,a} - h_1} = \frac{T_{2,s} - T_1}{T_{2,a} - T_1}$$

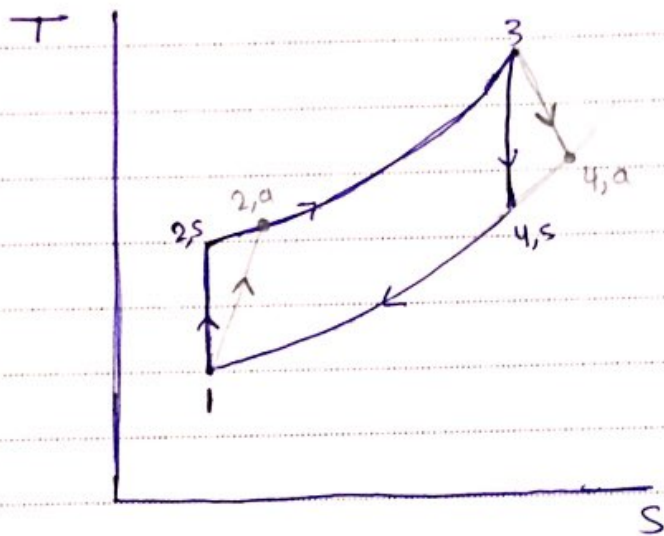
* Turbine

$W_{T,s} = h_1 - h_{2,s}$

$W_{T,a} = h_1 - h_{2,a}$



$$\eta_{s,T} = \frac{W_{T,a}}{W_{T,s}} = \frac{h_1 - h_{2,a}}{h_1 - h_{2,s}} = \frac{T_1 - T_{2,a}}{T_1 - T_{2,s}}$$



In actual cycles

$W_T \downarrow$

$W_C \uparrow$

$W_{net} \downarrow$

$\eta_{th} \downarrow$

8.3 - 8.4

Question: simple brayton power cycle, using Air as the working fluid.
 $r_p = 10$, $T_{min,i} = 295 K$, $T_{max,3} = 1240 K$, $\eta_{s,comp} = 83\%$, $\eta_{s,turbine} = 87\%$
 (a) Find $T_{4,a}$ (b) W_{net} (c) η_{th}

Solution:

$$\left(\frac{T_{2,s}}{T_1}\right) = r_p^{\frac{k-1}{k}} \Rightarrow T_{2,s} = 10^{\frac{0.4}{1.4}} \times 295 = 569.55 K$$

$$\eta_{s,comp} = \frac{T_{2,s} - T_1}{T_{2,a} - T_1} \Rightarrow 0.83 = \frac{569.55 - 295}{T_{2,a} - 295} \Rightarrow T_{2,a} = 625.8 K$$

$$\left(\frac{T_3}{T_{4,s}}\right) = r_p^{\frac{k-1}{k}} \Rightarrow T_{4,s} = \frac{1240}{10^{\frac{0.4}{1.4}}} = 642.25 K$$

$$\eta_{s,tur} = \frac{T_3 - T_{4,a}}{T_3 - T_{4,s}} \Rightarrow 0.87 = \frac{1240 - T_{4,a}}{1240 - 642.25} \Rightarrow T_{4,a} = 720 K$$

$$w_{comp,a} = c_p(T_{2,a} - T_1) = 1.005 * (625.8 - 295) \\ = 332.45 \text{ kJ/kg}$$

$$q_{in} = h_3 - h_{2,a} = c_p(T_3 - T_{2,a}) = 1.005(1240 - 625.8) \\ = 617.27 \text{ kJ/kg}$$

$$w_{T,a} = h_3 - h_{4,a} = c_p(T_3 - T_{4,a}) = 1.005(1240 - 720) \\ = 522.6 \text{ kJ/kg}$$

$$q_{out} = h_{4,a} - h_1 = c_p(T_{4,a} - T_1) = 1.005 * (720 - 295) \\ = 427.1 \text{ kJ/kg}$$

$$w_{net} = w_{T,a} - w_{comp} \\ = q_{in} - q_{out} = 190.15 \text{ kJ/kg}$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{190.15}{617.27} = 0.308 = 30.8\%$$

$$w_{BR} = \frac{w_{c,a}}{w_{T,a}} = \frac{332.45}{522.6} = 0.636$$

* Repeat the problem assuming $\eta_{s, comp} = \eta_{s, tur} = 100\%$

$$w_{comp,s} = h_{2,s} - h_1 = c_p(T_{2,s} - T_1) = 1.005 * (569.55 - 295) \\ = 275.92 \text{ kJ/kg}$$

$$q_{in} = h_3 - h_{2,s} = c_p(T_3 - T_{2,s}) = 1.005 * (1240 - 569.55) = 673.8 \text{ kJ/kg}$$

$$w_{T,s} = h_3 - h_{4,s} = c_p(T_3 - T_{4,s}) = 1.005 * (1240 - 642.25) \\ = 600.74 \text{ kJ/kg}$$

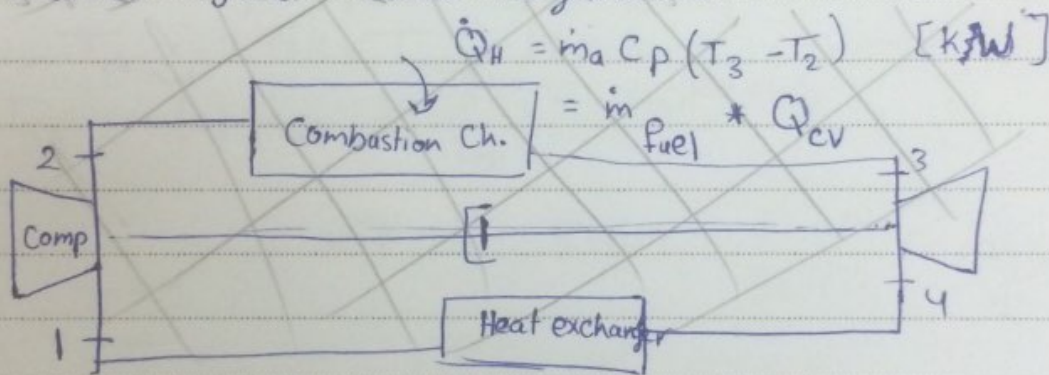
$$q_{out} = h_{4,s} - h_1 = c_p (T_{4,s} - T_1) = 1.005 * (642.5 - 295) = 348.99 \text{ kJ/kg}$$

$$W_{net} = W_{T,s} - W_{C,s} = q_{in} - q_{out} = 324.82 \text{ kJ/kg}$$

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{324.82}{673.8} = 0.482 = 48.2\%$$

$$W_{BR} = \frac{W_{comp,s}}{W_{Turb,s}} = 0.46$$

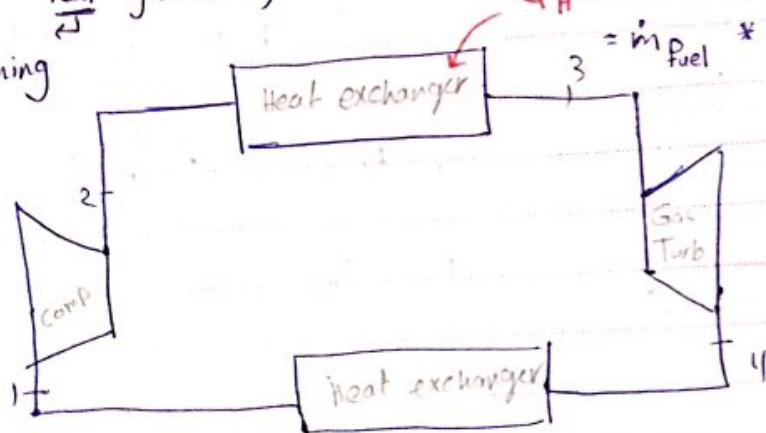
Gas turbine cycle with regenerator



* How to improve the gasturbine cycle

Gas turbine cycle with regenerator

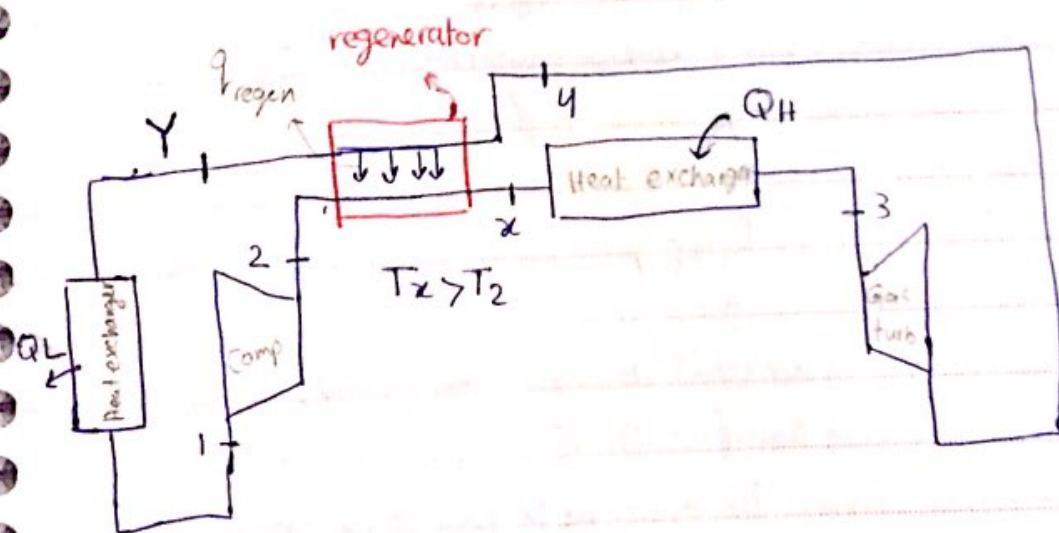
heat gained by the air $\leftarrow \dot{Q}_H = \dot{m}_{air} C_p (T_3 - T_2) \text{ [Kwatt]}$
 it comes from burning the fuel $= \dot{m}_{fuel} * Q_{cv} * \eta_c$



$\Rightarrow Q_{cv}$: Calorific value [KJ/kg] . it represents the amount of energy (heat) in 1 kg of fuel.

$\Rightarrow \eta_c$: efficiency of combustion

$$\dot{m}_{fuel} * Q_{cv} * \eta_c = \dot{m}_a C_p (T_3 - T_2)$$



* Regenerator: device that is used to heat the air at 2. before it enters the heat exchanger. This operation is called regeneration.

(تجديد الهواء)

* How to decrease the consumption of fuel? (decrease Φ_H) *

1) Increase the efficiency of combustion (η_c may reach 90% nowadays)

2] If I decrease $T_3 \Rightarrow$ This is a problem because the enthalpy of the air will decrease $\therefore$ the turbine will not produce the sufficient work.

بديل ما أدخل السوا على HEX
مقدرة حرارة 600 مئلا على 2000
لو استطعت اني أدخله ع 1000 على اظلم
ع 2000 بقله على
Fuel

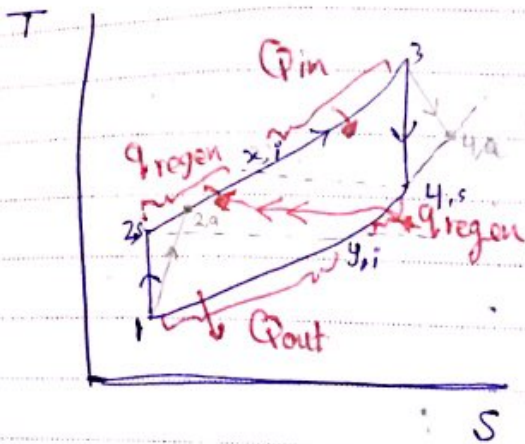
(1) أخذ المول قبل ما أضافه مع H₂ بطريقة

if $T_4 > T_2$, then I can use the air leaving the turbine to heat the air at 2, instead of rejecting the heat into the atmosphere (decrease thermal pollution)

جاء في مرقوم بخطوة رقم (3) +
مستورد الغاز الطبيعي لأنه الـ

Calorific value

50000 kJ/kg ~~is the~~ the needed fuel



Assuming no losses in heat exchanger

* Ideally $T_{x,i} = T_4$
 $T_{y,i} = T_2$ if the effectiveness is 100%

$$\dot{m} C_p (T_4 - T_{y,i}) = \dot{m} C_p (T_{x,i} - T_2)$$

heat gained by 2 = heat lost by 4

$$T_4 - T_{y,i} = T_{x,i} - T_2$$

*

* Effectiveness of regenerator = $\frac{Q_{\text{regen actual}}}{Q_{\text{regen ideal}}} = \frac{h_{x,a} - h_2}{h_{x,i} - h_2}$

effectiveness of real HEX is 80-89%
 i.e. 80% of heat will be transferred to 2, which means $T_{x,a}$ will be less than $T_{x,i}$ and $T_{y,a} > T_{y,i}$

$$* Q_{\text{in}} = h_3 - h_{x,a} = C_p (T_3 - T_{x,a})$$

$$* Q_{\text{out}} = h_{y,a} - h_1 = C_p (T_{y,a} - T_1)$$

■ $Q_{\text{out} \rightarrow 1}$ can be rejected to atmosphere, or I can give this heat to another thermodynamic cycle

■ Notice that $\dot{W}_{\text{net}}$ didn't change, but I ~~reduce~~ have reduced $\dot{Q}_{\text{in}}$, $\eta_{\text{th}} \uparrow$

■ Without regenerator Area under 2-3 curve represents $\dot{Q}_{\text{in}}$
 with regenerator the Area becomes smaller (under x-3 curve)

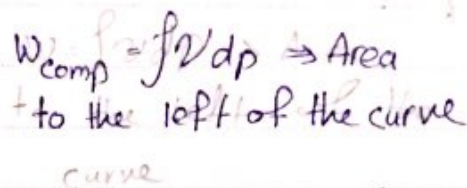
■ T_x may be greater than T_4 , if I use the solar energy, if this occurs, regenerator is not needed

■ Efficiency: is a word that is used to describe the efficiency of transforming energy from one form to another (تحويل)

■ In regenerator, I didn't transform the energy, ~~it is not~~ Instead, energy is transferred from one place to another. This is called effectiveness (فعالية), no efficiency

1 T

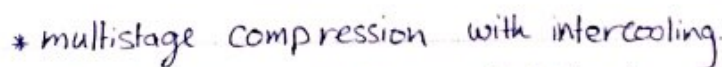
$P_{2n} = C$



$$n = k, \quad W = \frac{k}{k-1} * R (T_2 - T_1) = \frac{k}{k-1} * T_1 \left(r_p^{\frac{k-1}{k}} - 1 \right) \Rightarrow \text{isentropic with const. } C_p, C_v \text{ for an ideal gas}$$

$$n=1, W = RT \ln\left(\frac{P_2}{P_1}\right) \Rightarrow \text{isothermal process for an ideal gas}$$

Notice, We can minimize W_{comp} by doing compression at constant temperature

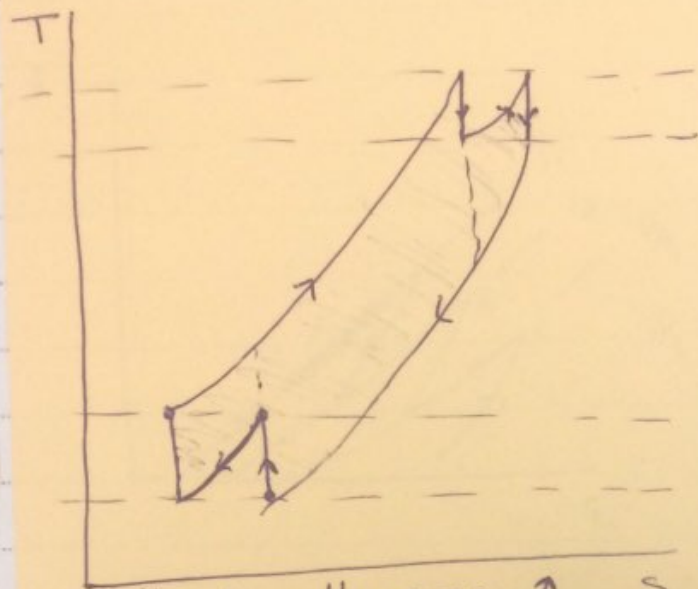


(اضبط برّاد اضبط برّاد

turbine

If the compression occurs at constant temperature $\Rightarrow$ no resistance to compression

هكذا ينخفض التوترين $\Rightarrow W_{comp} \downarrow$



Notice how the area $\uparrow$
by intercooling and reheating

* How to increase $\dot{W}_{Turb}$?

In turbines $\Rightarrow$ expansion occurs

$\Rightarrow$ So, ~~the~~ Temperature drop occurs

if I reheat the air for every temperature drop, the air will

~~not~~ expand, enthalpy $\uparrow$, $\dot{W}_{Turb} \uparrow$

(This is called multistage expansion with reheating)

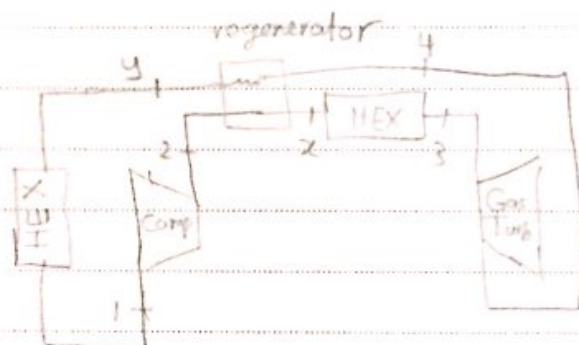
* رج زعفران لا طعماً كيف نسخن من غير

ما نسخن ولا ذرة بنزين

Question: A gas turbine for an automobile is designed with a regenerator. Air enters the compressor of this engine at 100 kPa and 20°C. The compressor has a pressure ratio of 8; the maximum cycle temperature is 800°C and the cold air stream leaves the regenerator 10°C cooler than the hot airstream at the inlet of the regenerator. Assuming both the compressor and the turbine to be isentropic, determine the rates of heat addition and rejection of the cycle ~~that~~ when it produces 150 kW. Use constant specific heats at room temperature.

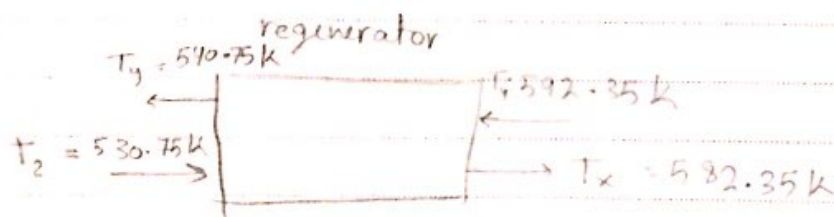
Solution $\dot{Q}_H = \dot{Q}_L$

- * $P_1 = 100 \text{ kPa}$ * Comp + Gas turb
- * $T_1 = 293 \text{ K}$ isentropic
- * $r_p = 8$ * $\dot{W}_{net} = 150 \text{ kW}$
- * $T_3 = 1073 \text{ K}$
- * $T_4 - T_x = 10^\circ\text{C}$



$$\left(\frac{T_2}{T_1}\right) = r_p^{\frac{k-1}{k}} \Rightarrow T_2 = T_1 r_p^{\frac{k-1}{k}} = (293) \times 8^{\frac{0.4}{1.4}} = 530.75 \text{ K}$$

$$\left(\frac{T_3}{T_4}\right) = r_p^{\frac{k-1}{k}} \Rightarrow T_4 = \frac{T_3}{r_p^{\frac{k-1}{k}}} = \frac{1073}{8^{\frac{0.4}{1.4}}} = 592.35 \text{ K}$$



Heat gained by the cold stream = Heat lost by the hot stream

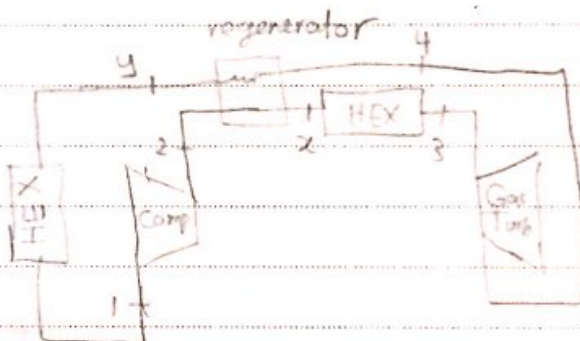
$$\dot{m}_c p (T_x - T_2) = \dot{m}_h p (T_4 - T_3)$$

$$T_x - T_4 = T_2 - T_3 = -10$$

Question: A gas turbine for an automobile is designed with a regenerator. Air enters the compressor of this engine at 100 kPa and 20°C. The compressor has a pressure ratio of 8; the maximum cycle temperature is 800°C and the cold air stream leaves the regenerator 10°C cooler than the hot air stream at the inlet of the regenerator. Assuming both the compressor and the turbine to be isentropic, determine the rates of heat addition and rejection of the cycle ~~that~~ when it produces 150 kW. Use constant specific heats at room temperature.

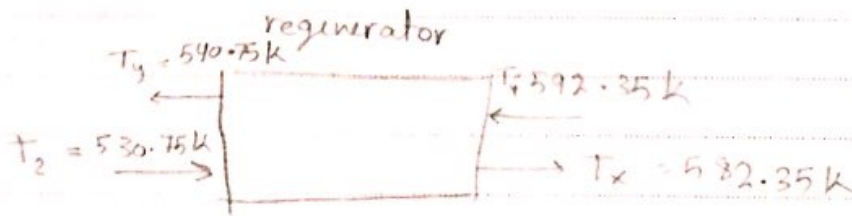
Solution $\dot{Q}_H = \dot{Q}_L$

- * $P_1 = 100 \text{ kPa}$
- * $T_1 = 293 \text{ K}$
- * $r_p = 8$
- * $T_3 = 1073 \text{ K}$
- * $T_4 - T_x = 10^\circ\text{C}$
- * Comp + Gas turb isentropic
- * $\dot{W}_{\text{net}} = 150 \text{ kW}$



$$\left(\frac{T_2}{T_1}\right) = r_p^{\frac{k-1}{k}} \Rightarrow T_2 = T_1 r_p^{\frac{k-1}{k}} = (293) \times 8^{\frac{0.4}{1.4}} = 530.75 \text{ K}$$

$$\left(\frac{T_3}{T_4}\right) = r_p^{\frac{k-1}{k}} \Rightarrow T_4 = \frac{T_3}{r_p^{\frac{k-1}{k}}} = 592.35 \text{ K}$$



heat gained by the cold stream = heat lost by the hot stream

$$\dot{m} c_p (T_x - T_2) = \dot{m} c_p (T_4 - T_3)$$

$$T_x - T_4 = T_2 - T_3 = -10$$

$$\frac{\dot{W}_{\text{comp}}}{\dot{m}} = c_p(T_2 - T_1) = 238.93 \text{ kJ/kg}$$

$$\frac{\dot{W}_{\text{turb}}}{\dot{m}} = c_p(T_3 - T_4) = 483.06 \text{ kJ/kg}$$

$$\dot{W}_{\text{net}} = \dot{m} (\dot{W}_{\text{turb}} - \dot{W}_{\text{comp}})$$

$$\dot{m} = \frac{150}{483.06 - 238.93} = 0.615 \text{ kg/s}$$

$$\dot{Q}_H = \dot{m} c_p(T_3 - T_{x1}) = 0.615 \times 1.005 \times (1073 - 582.35) = 303.2 \text{ kW}$$

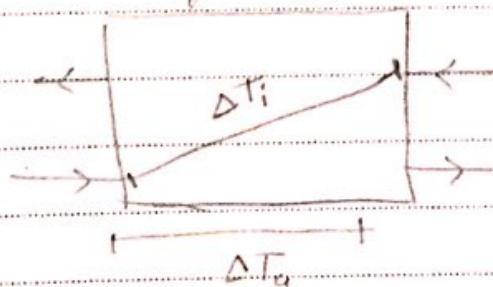
$$\dot{Q}_L = \dot{m} c_p(T_4 - T_1) = 0.615 \times 1.005 \times (540.75 - 293) = 153.12 \text{ kW}$$

$$W_{\text{PR}} = \frac{W_c}{W_T} = \frac{238.93}{483.06} = 0.4946$$

$$\text{Effectiveness of the regenerator} = \frac{T_{x2} - T_2}{T_{x1} - T_2} = \frac{582.35 - 530.75}{592.35 - 530.75}$$

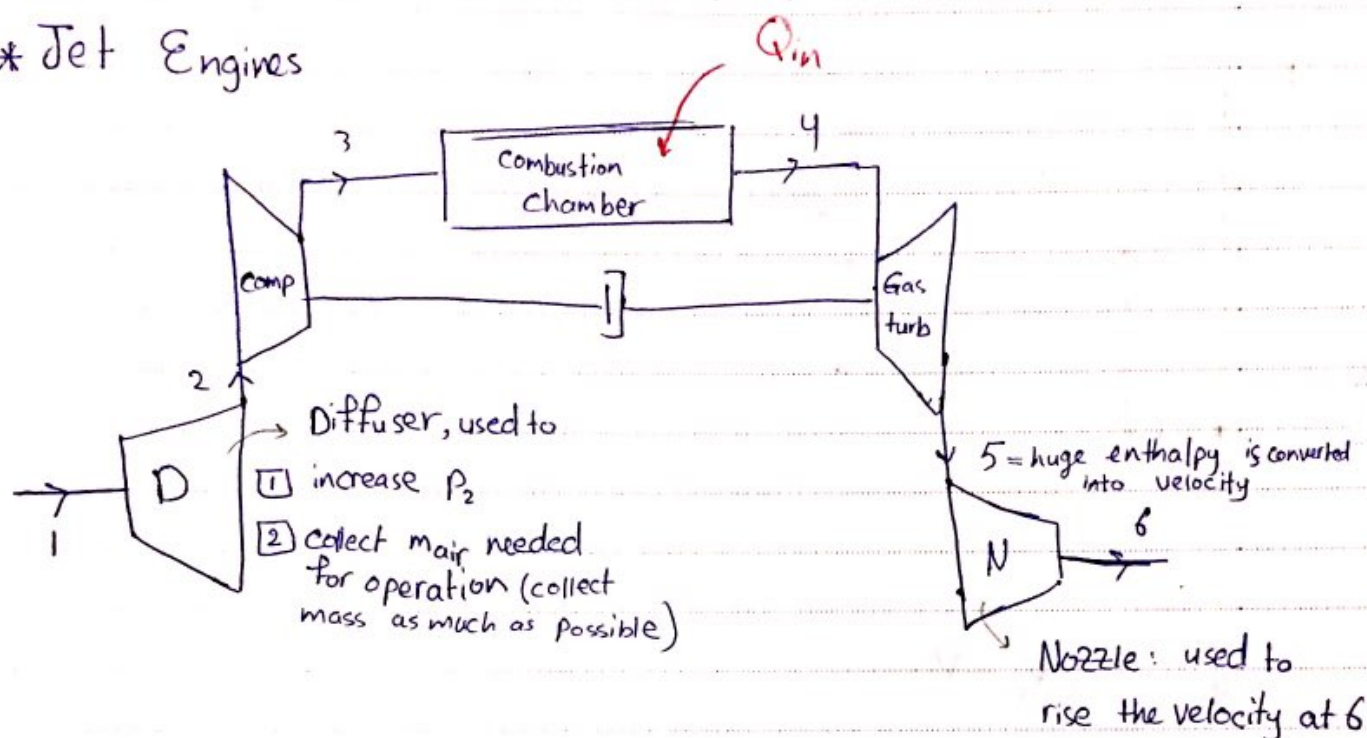
regenerator

$$= 0.8376$$



* Repeat the problem when the compressor isentropic efficiency is 87% and the turbine isentropic efficiency is 93%.

* Jet Engines

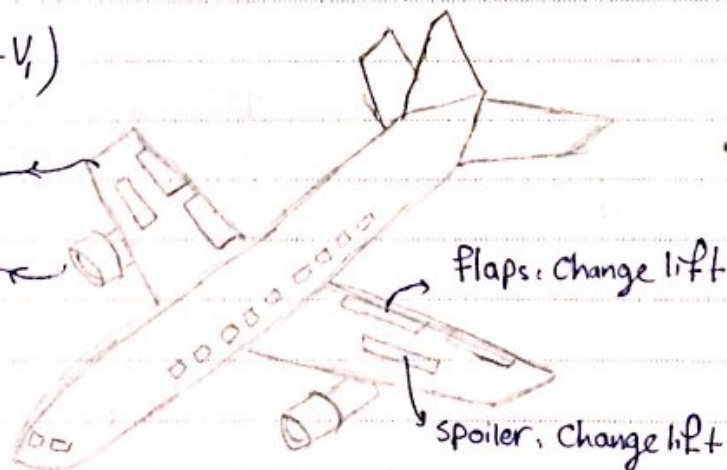


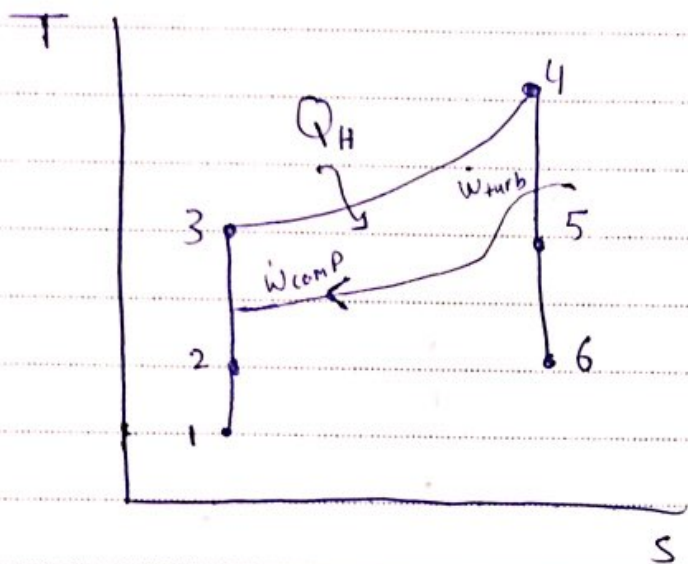
- Jet engines are used to produce net work $\dot{W}_{net}$
- They are used to produce thrust force
- $r_p = 10-20$
- ~~Theoretically~~ Theoretically $\dot{W}_{Turb} = \dot{W}_{comp}$ actually $\dot{W}_{Turb}$ is slightly greater than $\dot{W}_{comp}$, in order to operate AC, TV's, screens, in the plane
- The air expands in a nozzle to produce V_6

$$F_{thrust} = \dot{m}_a (V_6 - V_1)$$

Wing:
(Generate lift)

Jet engine:
(Generate thrust)





* $\dot{W}_{net} = 0$

* $Q_L \equiv$ the hot air that leaves the engine

$$F_{th} = \dot{m} (v_6 - v_1)$$

* الطائرات التي لا تتحرك في الجو

تسمى هذه المحركات

$\rightarrow v_1$ when the air is static

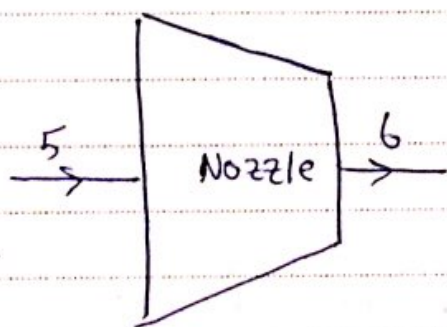
* $\dot{W}_p = F_{th} * V_{aircraft}$ ($\dot{W}_{propulsion} \equiv \dot{W}_p$)

* Propulsion efficiency (كفاءة الدفع) = $\frac{\dot{W}_p}{\dot{Q}_{in}}$

القوة الدافعة هي

القوة الدافعة هي

إذا كانت الطائرة تتحرك في الجو



$$h_5 + \frac{v_5^2}{2} \overset{\text{zero}}{=} h_6 + \frac{v_6^2}{2}$$

$$v_5 \ll v_6$$

$$v_6 = \sqrt{2 * C_p (T_5 - T_6)}$$

$\hookrightarrow 1005 \text{ J/kg} \cdot \text{K}$ For air

* When the air in the atmosphere is not moving $v_1 = V_{aircraft}$

* The back work ratio = 1

1 → 2) Diffuser, $Q=0$, $W=0$, $S=\text{const}$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad \text{zero} \quad V_2 \ll V_1$$

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_2$$

$$T_2 = T_1 + \frac{V_1^2}{2 C_p}$$

$$\left(\frac{T_2}{T_1} \right) = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$$

2 → 3) Compressor, $S=\text{const}$, $Q=0$

$$r_{\text{comp}} = \frac{P_3}{P_2}$$

$$W = C_p (T_3 - T_2)$$

$$\left(\frac{T_3}{T_2} \right) = r_p^{\frac{k-1}{k}}$$

3 → 4) Combustion chamber, $W=0$, $Q_{\text{out}}=0$, $P=\text{const}$

$$q_{\text{in}} = C_p (T_4 - T_3)$$

$$P_3 = P_4$$

4 → 5) Turbine, $S = \text{const}$, $Q = 0$

$$W_{\text{out}} = h_4 - h_5 \\ = c_p(T_4 - T_5)$$

$$\left(\frac{T_5}{T_4}\right)_{S=\text{const}} = \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}}, \quad r_{P_{\text{Turb}}} = \frac{P_4}{P_5} \Rightarrow \text{different from } r_{P_{\text{comp}}}$$

5 → 6) Nozzle, $S = \text{const}$, $W = 0$, $Q = 0$

$$h_5 + \overset{\text{zero}}{\frac{V_5^2}{2}} = h_6 + \frac{V_6^2}{2}, \quad V_5 \ll V_6$$

$$c_p T_5 = c_p T_6 + \frac{V_6^2}{2}$$

$$T_5 = T_6 + \frac{V_6^2}{2 \cdot c_p}, \quad V_6 = \sqrt{2 \cdot c_p (T_5 - T_6)}$$

$$\left(\frac{T_5}{T_6}\right) = \left(\frac{P_5}{P_6}\right)^{\frac{k-1}{k}}$$

Since 1 → 2 → 3 $S = \text{const}$ with const c_p, c_v

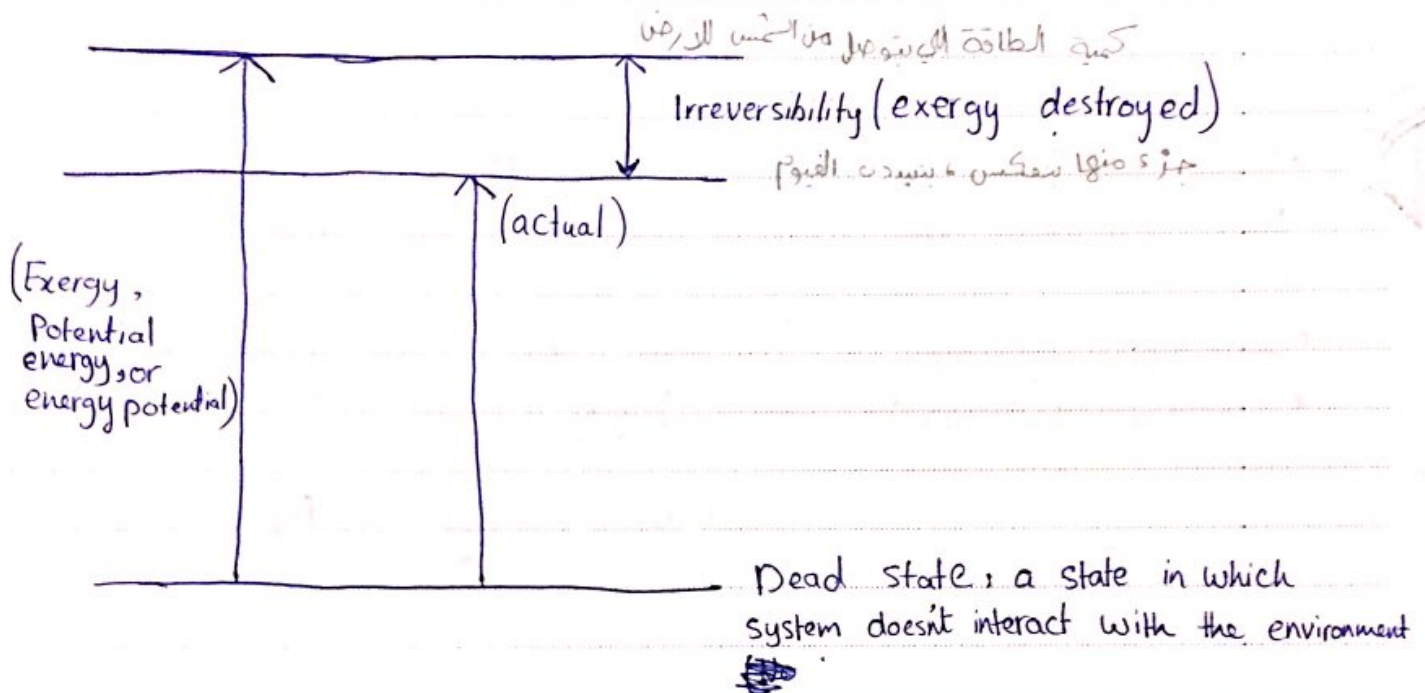
$$\left(\frac{T_3}{T_1}\right) = \left(\frac{P_3}{P_1}\right)^{\frac{k-1}{k}}$$

Since 4 → 5 → 6 $S = \text{const}$ with const c_p, c_v

$$\left(\frac{T_6}{T_4}\right) = \left(\frac{P_4}{P_6}\right)^{\frac{k-1}{k}}$$

* Exergy analysis (Availability)

Exergy: It is the maximum useful energy



$$* W_{\max} = W_{\text{rev}} \quad (\text{output})$$

$$* W_{\min} = W_{\text{rev}} \quad (\text{input})$$

$$* I \equiv \text{Irreversibility} = W_{\text{rev}} - W_{\text{act}} \quad (\text{output})$$

$$* I \equiv \text{Irreversibility} = W_{\text{act}} - W_{\text{rev}} \quad (\text{input})$$

Irreversibility $\equiv$ Exergy destroyed

$$* W_{\text{rev}} = Q_{\text{in}} \left(1 - \frac{T_L}{T_H} \right)$$

$\eta_{\text{th, Carnot}}$

/ /

* Exergy $\equiv$ work potential $\equiv$ availability $\equiv$ available work (state function
ie it depends on the initial and final states only)

At the dead state $\rightarrow$ properties are denoted by T_0, P_0, V_0, u_0, S_0
 $\rightarrow$ Exergy = 0

* Work and heat are maximum when ~~the~~ reversible (output)
Work and heat are minimum when reversible (input)

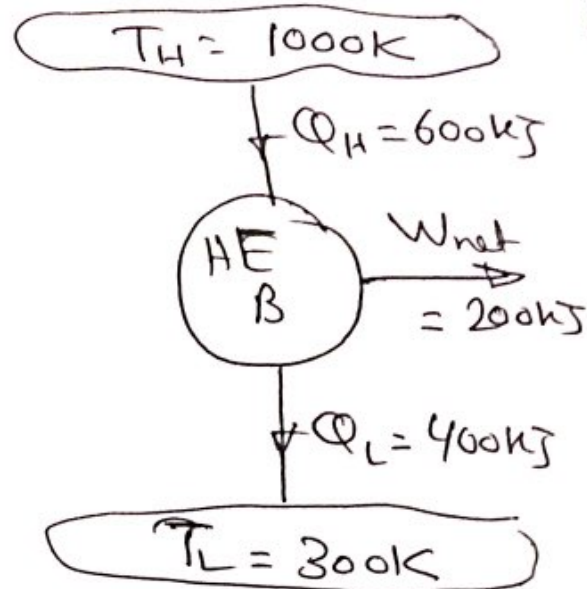
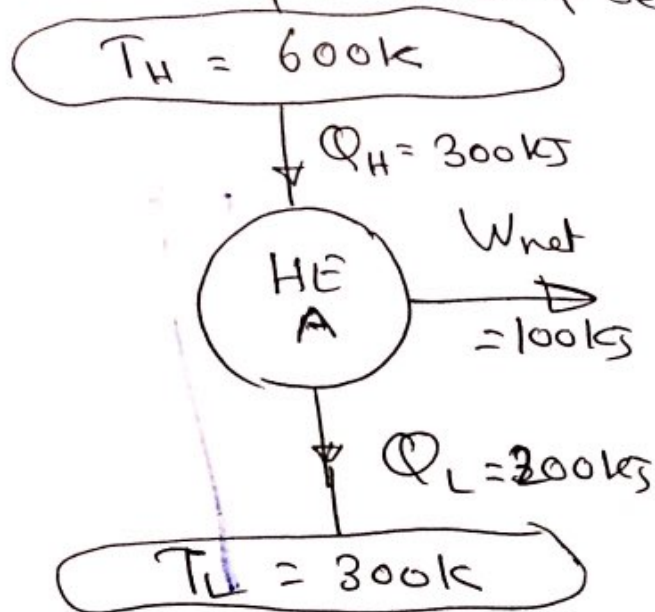
* $W_{\text{reversible}} = \text{Exergy}$ if the final state is the dead state

* Irreversibility (Exergy destroyed) = $W_{\text{max (Exergy)}} - W_{\text{act (output)}}$

$$= W_{\text{act}} - W_{\text{max (Exergy)}} \quad (\text{Input})$$

$$* W_{\text{rev}} = Q_{\text{in}} * \left(1 - \frac{T_L}{T_H} \right)$$

* Importance of Second Law Analysis:



Q: Which engine is better?

HE(A) is better than HE(B), because

$\eta_{th, HE(A)} = \frac{1}{3}$ and the maximum thermal efficiency of A = $\frac{1}{2}$ while

$\eta_{th, HE(B)} = \frac{1}{3}$ and the maximum thermal efficiency of B = 0.7

* For Closed System (Control mass)

$$\text{Exergy } (\phi) \text{ at any point} = (U - U_0) + P_0(V - V_0) - T_0(S - S_0) + m\left(\frac{V^2}{2} + gz\right)$$

$$\Delta\phi_{1-2} = \phi_f - \phi_i = U_2 - U_1 + P_0(V_2 - V_1) - T_0(S_2 - S_1) + m\left(\frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

For Open System (Control Volume)

$$\text{Exergy } (\psi) \text{ at any point} = (H - H_0) - T_0 (S - S_0) + m \left(\frac{V^2}{2} + g z \right)$$

$$\Delta \psi = (H_2 - H_1) - T_0 (S_2 - S_1) + m \left(\frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right)$$

* Exergy Destroyed (X_{dest}) 2 $\rightarrow$ final
1 $\rightarrow$ Initial

$$= T_0 S_{\text{gen}}$$

$$= T_0 [\Delta S_{\text{system}} - S_{\text{in}} + S_{\text{out}}]$$

$$= T_0 \left[(S_f - S_i)_{\text{system}} - \frac{Q_{\text{in}}}{T_{\text{in}}} + \frac{Q_{\text{out}}}{T_{\text{out}}} \right] \quad \frac{\text{kJ}}{\text{kg}}$$

$$X_{\text{dest}} = T_0 \left[(S_f - S_i) - \frac{q_{\text{in}}}{T_{\text{in}}} + \frac{q_{\text{out}}}{T_{\text{out}}} \right] \quad \frac{\text{kJ}}{\text{kg}}$$

$$T_{\text{in}} = T_{\text{source}}, \quad T_{\text{out}} = T_{\text{sink}}$$

* For Gases, With Constant Sp. Heat:

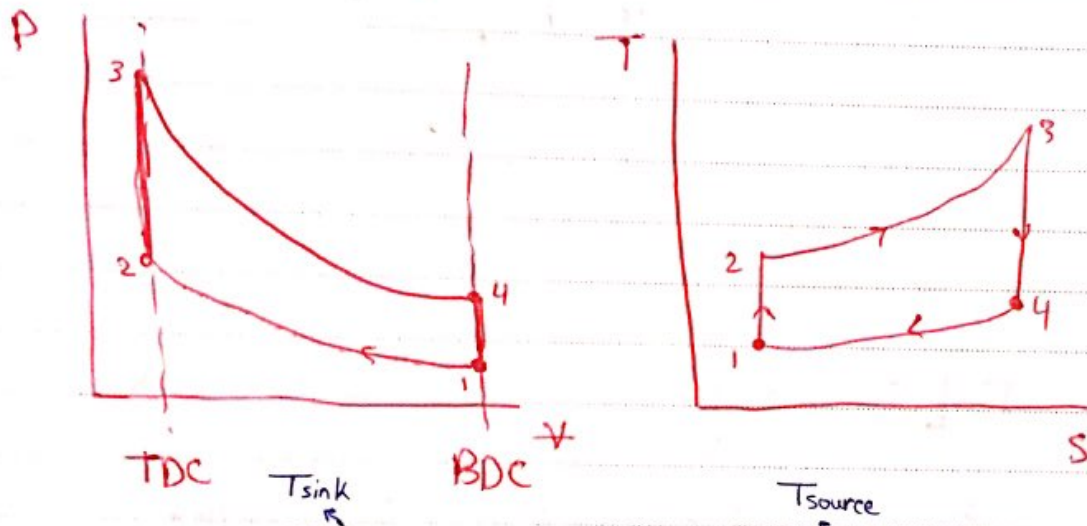
$$S_2 - S_1 = C_{v, \text{avg}} \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{V_2}{V_1} \right) \quad v = c$$

$$= C_{p, \text{avg}} \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{P_2}{P_1} \right) \quad p = c$$

* For Variable Sp. heat

$$S_2 - S_1 = S_2^\circ - S_1^\circ - R \ln(P_2/P_1)$$

Problem 148 (7th edition) Which of the Otto cycle processes loses the greatest amount of work potential. The energy supply reservoirs temperature is the same as the maximum cycle temperature and the energy-sink reservoir's temperature is the same as the minimum cycle



Given: $T_1 = 288 \text{ K}$, $P_1 = 95 \text{ kPa}$, $T_3 = 1473 \text{ K}$, $CR = 8$
 $q_{in} = 582.55 \text{ kJ/kg}$, $q_{out} = 253.58 \text{ kJ/kg}$, $\eta_{th} = 0.5647$
 $w_{net} = 328.965$

Solution

$$X_{dest} = T_0 \left[S_f - S_i - \frac{q_{in}}{T_H} + \frac{q_{out}}{T_L} \right]$$

- For processes $1 \rightarrow 2$, $3 \rightarrow 4$, ($\Delta S = 0$, $Q = 0$)
 So $X_{dest} = 0$

- For process $2 \rightarrow 3$ ($v = \text{const}$)

$$X_{dest} = T_0 \left[S_f - S_i - \frac{q_{in}}{T_H} \right] \Rightarrow$$

$$S_3 - S_2 = C_{v,avg} \ln\left(\frac{T_3}{T_2}\right) + R \ln\left(\frac{v_3}{v_2}\right) \rightarrow \text{zero}$$

$$\Delta S = 0.718 \ln\left(\frac{147.3}{661.6}\right) = 0.5746 \text{ kJ/kg}\cdot\text{K}$$

$$X_{\text{dest } 2 \rightarrow 3} = 288 \left[0.5746 - \frac{582.55}{1473} \right] = 51.6 \frac{\text{kJ}}{\text{kg}}$$

• for process 4 → 1 $v = \text{const}$

$$X_{\text{dest } 4 \rightarrow 1} = T_0 \left[S_1 - S_4 + \frac{q_L}{T_H} \right]$$

$$S_1 - S_4 = -(S_2 - S_2)$$

$$= 288 \left[-0.5746 + \frac{253.58}{288} \right] = 88.12 \text{ kJ/kg}$$

$$\begin{aligned} * X_{\text{dest tot}} &= X_{\text{dest } 1 \rightarrow 2} + X_{\text{dest } 2 \rightarrow 3} + X_{\text{dest } 3 \rightarrow 4} + X_{\text{dest } 4 \rightarrow 1} \\ &= 88.12 + 51.6 = 139.72 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} * \eta_{II} &= \frac{\eta_{th}}{\eta_{carnot}} = \frac{X_{\text{recovered}}}{X_{\text{expended}}} = \frac{W_{\text{net actual}}}{W_{\text{net, Carnot}}} = 1 - \frac{X_{\text{destroyed}}}{X_{\text{expended}}} \\ &= \frac{56.47\%}{80.44\%} = 70.2\% \end{aligned}$$

$$W_{\text{net, Carnot}} = X_{\text{expended}} = \left(1 - \frac{T_L}{T_H} \right) \dot{q}_h = 468.65 \text{ kJ/kg}$$

Example 9-10

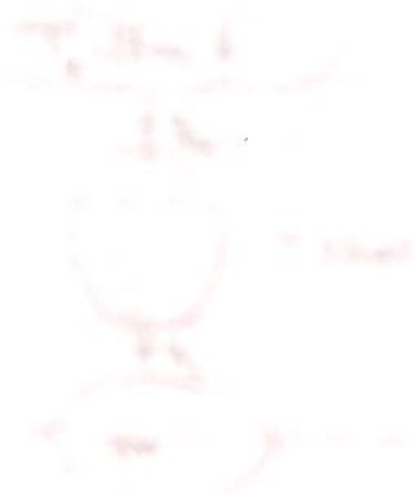
Ch. 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100

Thrust = 1000 lb, Thrust = 1000 lb

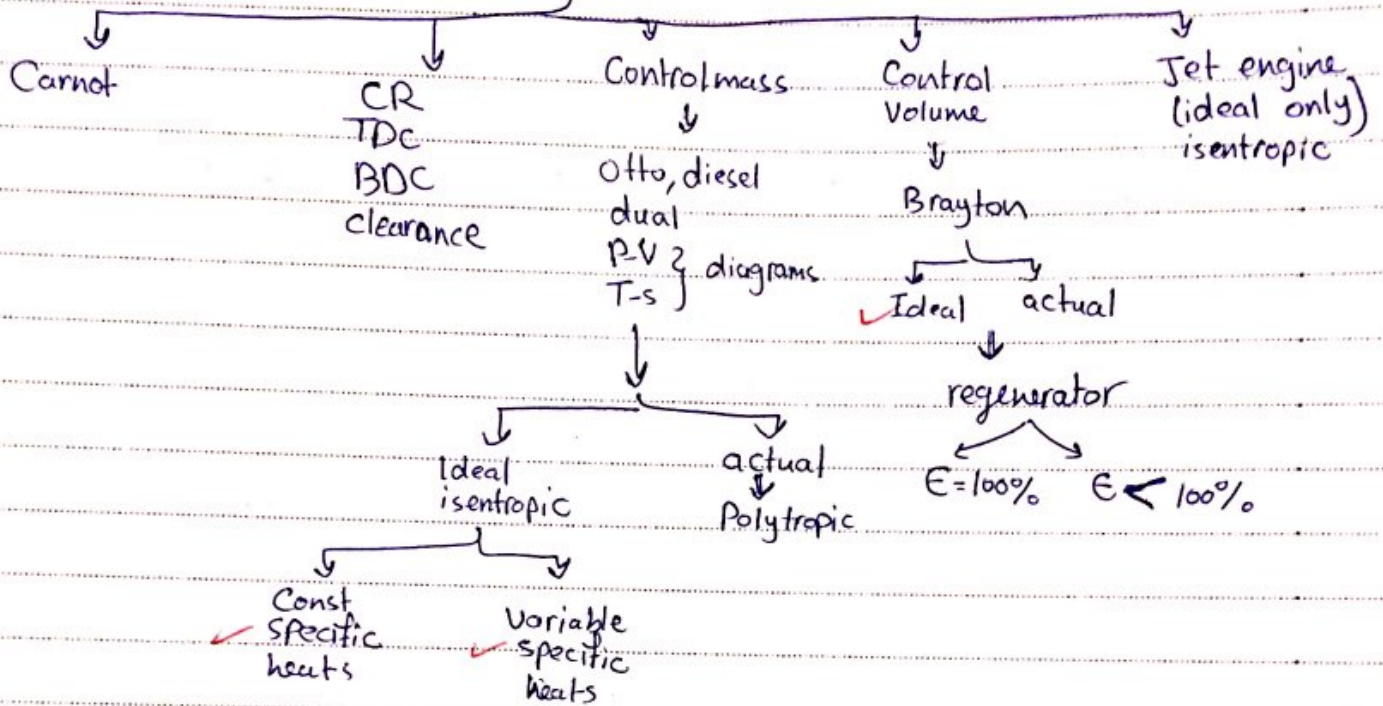
$\dot{m} = 200 \text{ lb/s}$ velocity, $\dot{m} = 200 \text{ lb/s}$ velocity, $\dot{m} = 200 \text{ lb/s}$

• The velocity of the jet is

$$v = \frac{F}{\dot{m}} = \frac{1000 \text{ lb}}{200 \text{ lb/s}} = 5 \text{ ft/s}$$



Ch 9



✓ ⇒ study exergy for these cycles

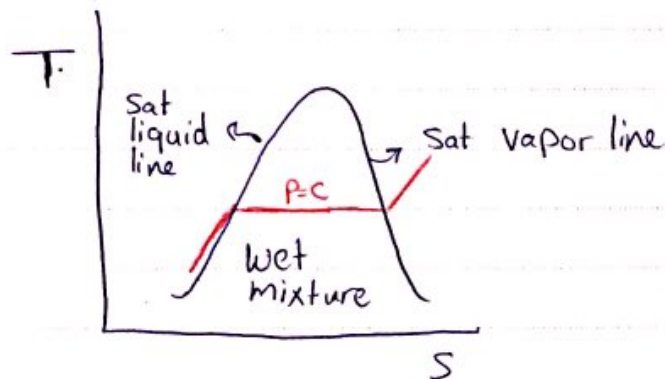
Chapter 10: Vapor power cycle

We are no more dealing with gas

The working fluid is liquid that changes its phase

- ① Carnot Vapor power cycle
- ② Rankine cycle (أبسط أبسط)

Recall

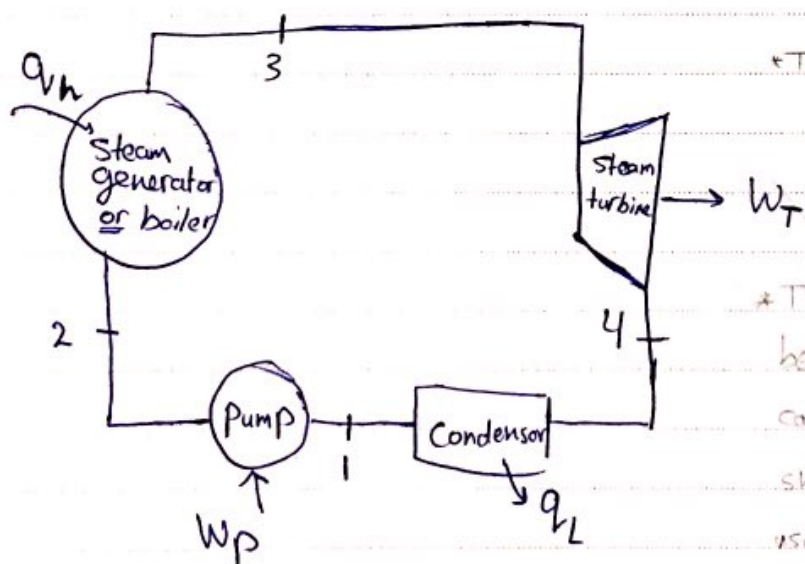


* In order to find the properties of wet mixture, we have identified the quality $x = \frac{\text{mass of vapor}}{\text{mass of the vapor + liquid}}$

$$h_{avg} = h_f + x h_{fg}$$
$$v_{avg} = v_f + x v_{fg}$$

* At constant P as $T \uparrow$, $v \uparrow$

* Saturated temperature $\approx$ depends on the pressure



* The objective of the generator is to produce steam.

* The working fluid (water) must be filtered otherwise it will cause erosion in the power plant may shut down. Water will be recycled using the condenser which will condense the steam coming from the turbine, so that the pump can work properly.

T-S Diagram for Carnot vapor power cycle

It will produce the highest possible net work.

$$\eta_{th, \text{Carnot}} = 1 - \frac{T_L}{T_H}$$

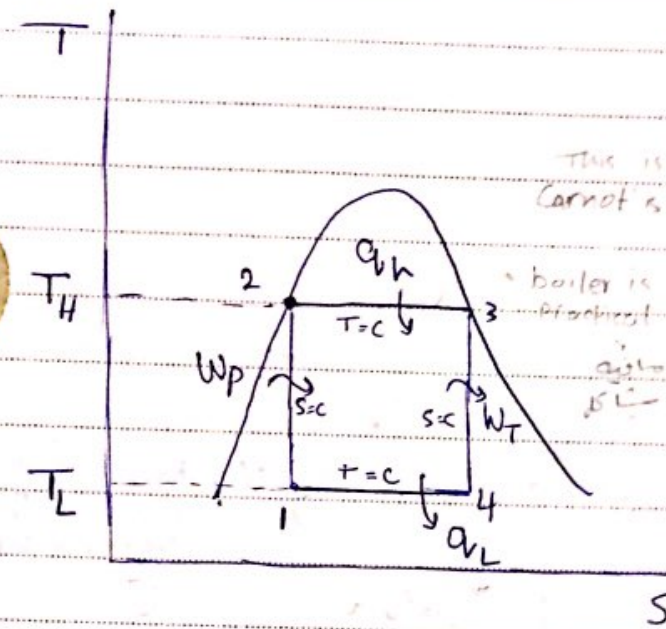
This is why - problems of Carnot cycle - Carnot is not practical

① 1 → a wet mixture is entering the pump
there is liquid + vapor

pump cannot deal with upper

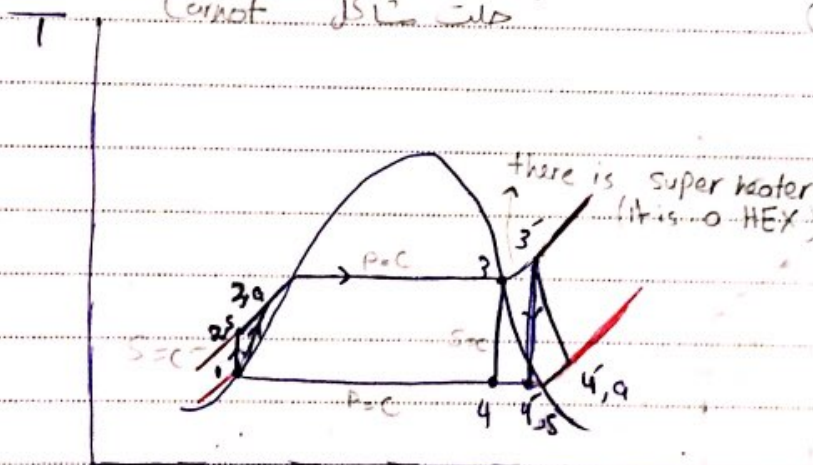
If 1 is saturated liquid a wet
cannot go from 1 → 2 isothermally
So 1 must be under 2 in order
to make $\eta = s_2$

② Turbine cannot endure quality less
than 0.25, the amount of liquid
water will be more drops of water
can destroy the blades of the
turbine. (i.e. x at 4 is unacceptable)



T-S Diagram for Rankine cycle

Carnot is not practical



③ operating the condenser at
low effectiveness (4 is wet mixture
1 is also a wet mixture)

* 1 → can be saturated liquid & subcooled

* Instead of adding heat at constant temperature, heat addition
is made at constant pressure

* q_H is the heat added in the boiler
 q_L is the heat rejected in the condenser

Boiler is a HEX

1 → 2, 2 → 3, 3 → 4, 4 → 1

Condenser is a HEX

al turbine + w, q, u, v

turbine life



$$(\dot{Q}_{in} - \dot{Q}_{out}) + (\dot{W}_{in} - \dot{W}_{out}) = \dot{m} (\Delta h + \underbrace{\Delta ke}_{\text{zero}} + \underbrace{\Delta pe}_{\text{zero}})$$

* Steam turbine, $S = \text{const}$, $\dot{Q} = 0$, $\dot{W}_{in} = 0$

$$-\dot{W}_{out} = \dot{m}(h_4 - h_3) \Rightarrow \dot{W}_{out} = \dot{m}(h_3 - h_4) + \dot{Q}_{out}$$

$cp(\Delta T)$ tables $h_3 \rightarrow$ Super heated
 $X_{dest} = 0$ $h_4 \rightarrow$ wet mixture
 if there is heat loss

* Steam generator, $\dot{W} = 0$, $\dot{Q}_{out} = 0$

$$-\dot{Q}_{out} + \dot{Q}_{in} = \dot{m}(h_3 - h_2)$$

if there is heat loss

* Condensor, $\dot{W} = 0$, $\dot{Q}_{in} = 0$

$$\dot{Q}_{out} = \dot{m}(h_4 - h_1)$$

يعطى في الجدول
 أو
 cycle

* Pump, $S = \text{const}$, $\dot{Q} = 0$, $\dot{W}_{out} = 0$

$$\dot{W}_p = \dot{m}(h_2 - h_1) = \dot{m} v_f (P_2 - P_1)$$

$$h_{2,s} - h_1 = v_f (P_2 - P_1)$$

only valid for reversible processes

v_f is very very small
 $\therefore \dot{W}_p$ isn't very large!

$$\dot{W}_{pump} \ll \dot{W}_{comp}$$

* Notes:

1. There is a very slight difference between v_f and v_g and $v_f \approx v_g$ (since liquids are incompressible)

$$\dot{W}_p = \dot{m} \int v dp = m v_f \int dp = m v_f (P_2 - P_1)$$

this is true for

□ ss process

□ int, rev process

$$\eta_{s, \text{pump}} = \frac{h_{2,s} - h_1}{h_{2a} - h_1} \Rightarrow \text{find } h_{2a}$$

$$\eta_{th} = 1 - \frac{q_L}{q_H}$$

Notes:

* The stream leaving the turbine may be superheated, so the condenser may not be able to convert it into a sat. liquid (i.e. the stream leaving the stream may be wet mixture). So we put a device between the condenser and the pump that is called 'Flash chamber'. This device separates the liquid from the ~~gas~~ vapor, allows the liquid to ~~continue~~ to enter the pump and returns the vapor to the condenser.

* State 3 is a saturated vapor. There are HEX's that are called 'superheaters'. They heat the saturated vapor to form superheated water (3 → 3').

* (The stream that is leaving the boiler is always saturated vapor.

Exergy Analysis

(*) Turbine

1st law

$$\dot{m}h_3 = \dot{m}h_4 + \dot{Q}_{out} + \dot{W}_T$$

$$\dot{W}_{T, actual} = \dot{m}(h_3 - h_4) = \dot{Q}_{out}$$

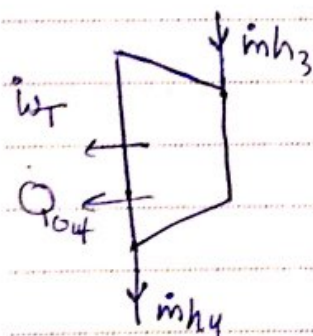
$$\dot{\Psi}_3 = \dot{m}(h_3 - h_0 - T_0(s_3 - s_0))$$

$$\dot{\Psi}_4 = \dot{m}(h_4 - h_0 - T_0(s_4 - s_0))$$

$$\dot{W}_{T, rev} = \dot{\Psi}_3 - \dot{\Psi}_4$$

$$= \dot{m}(h_3 - h_4 - T_0(s_3 - s_4)) = \dot{W}_{act} + I$$

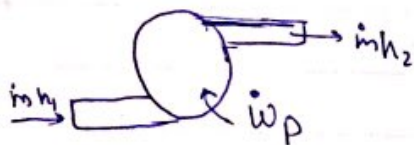
$$I = \dot{W}_{rev} - \dot{W}_{act}$$



$$\dot{X}_{dest, 3 \rightarrow 4} = \dot{m} T_0 \left[S_4 - S_3 + \frac{\dot{Q}_{out}}{T_0} \right]$$

* if the process is isentropic $\dot{X}_{dest, 3 \rightarrow 4} = 0$

2) Pump



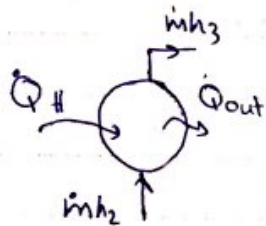
$$\begin{aligned} \dot{m} h_1 + \dot{w}_p &= \dot{m} h_2 \\ \dot{w}_p &= \dot{m} (h_2 - h_1) \\ &= \dot{m} v_f (P_2 - P_1)_{s=c} \end{aligned}$$

$$\dot{w}_{rev, in} = \dot{\psi}_2 - \dot{\psi}_1 = \dot{m} [h_2 - h_1 - T_0 (S_2 - S_1)] = \dot{w}_{p, act} - \dot{I}$$

$$\dot{X}_{dest, 1 \rightarrow 2} = \dot{m} T_0 [S_2 - S_1]$$

$$\dot{I} = \dot{w}_p - \dot{w}_{rev, in} \quad , \text{ if the process is isentropic } \dot{X}_{dest, 1 \rightarrow 2} = 0$$

3) Steam generator



$$\dot{m} h_2 + \dot{Q}_H = \dot{m} h_3 + \dot{Q}_{out}$$

$$\dot{Q}_H = \dot{m} (h_3 - h_2) + \dot{Q}_{out}$$

$$\dot{X}_{dest, 2 \rightarrow 3} = \dot{m} T_0 \left[S_3 - S_2 - \frac{\dot{Q}_{in}}{T_H} + \frac{\dot{Q}_{out}}{T_H} \right]$$

$$\dot{X}_{heat, in} = \left(1 - \frac{T_0}{T_H} \right) \dot{Q}_H$$

$$\begin{aligned} \dot{X}_{heat, in} &= \dot{\psi}_3 - \dot{\psi}_2 = \dot{m} [h_3 - h_2 - T_0 (S_3 - S_2)] \\ &= \dot{X}_{heat, in} - T_0 \dot{S}_{gen} \end{aligned}$$

Exergy = Energy - Energy lost to System

the difference between them is the $\dot{X}_{dest}$

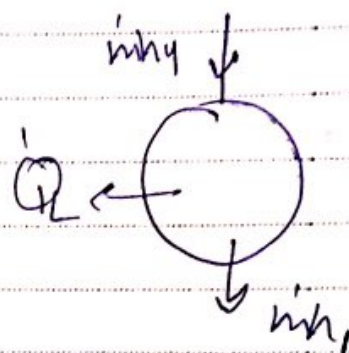
$$\dot{X}_{heat, in} = \dot{\psi}_3 - \dot{\psi}_2 + T_0 \dot{S}_{gen}$$

(4) Condenser

$$\dot{Q}_L = \dot{m}(h_4 - h_1)$$

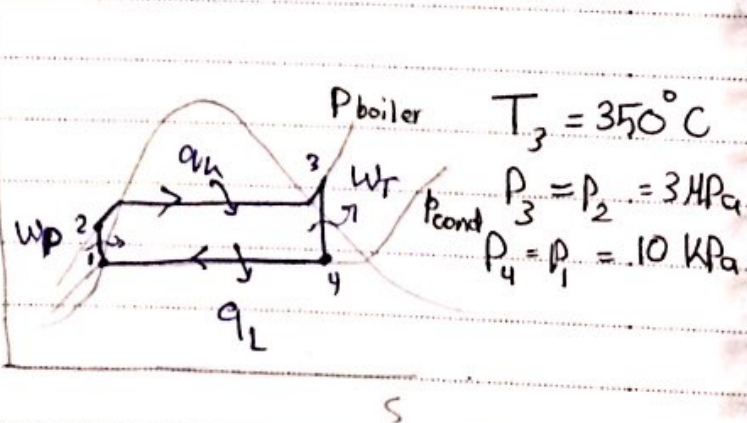
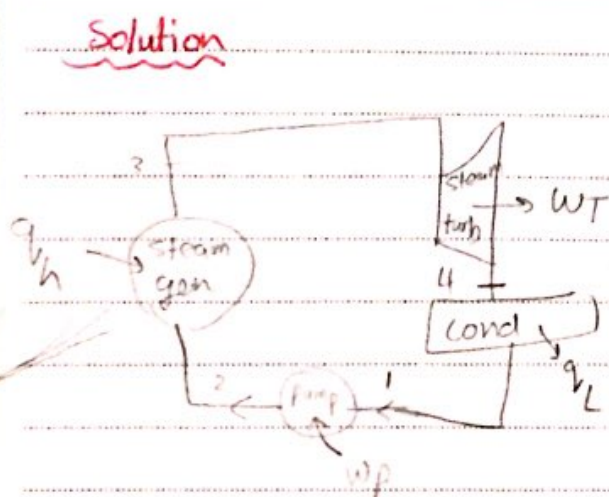
the difference between them is the exergy destroyed

$$\begin{aligned} \dot{X}_{\text{heat out}} &= \left(1 - \frac{T_0}{T_L}\right) \dot{Q}_L \\ \dot{X}_{\text{heat}} &= \dot{\Psi}_4 - \dot{\Psi}_1 = \dot{m}(h_4 - h_1 - T_0(s_4 - s_1)) \\ &= \dot{X}_{\text{heat out}} + T_0 \dot{S}_{\text{gen}} \\ \dot{X}_{\text{dest } 4 \rightarrow 1} &= \dot{m} T_0 \left(s_4 - s_1 + \frac{q_{\text{out}}}{T_L} \right) \end{aligned}$$



Question: A steam power plant operates on a simple ideal Rankine cycle. The steam enters the turbine at 3 MPa and 350°C. It is then condensed in the condenser at 10 kPa. Calculate pump work, Heat input, X_4 , Turbine work, heat output, W_{net} , η_{th} , $\eta_{\text{th, Carnot}}$, η_{II}

Solution



state 1 is Sat. liquid $T_{sat@10 \text{ kPa}} = 45.81^\circ\text{C}$

$$v_{f1} = 0.001010 \text{ m}^3/\text{kg}$$

$$h_{f1} = 191.81 \text{ kJ/kg}$$

$$h_{fg} = 2392.1 \text{ kJ/kg}$$

$$s_{f1} = 0.6492 \text{ kJ/k}\cdot\text{kg}$$

$$s_{fg} = 7.4996 \text{ kJ/k}\cdot\text{kg}$$

state 2

$$P_2 = 3 \text{ MPa} \quad s_2 = s_1 = 0.6492 \text{ kJ/kg}\cdot\text{K}$$

Since $s_2 < s_{fg}@P=3000 \text{ kPa}$: state 2 is subcooled liquid

We don't have data for compressed liquid at $P=3 \text{ MPa}$
but we know that

$$w_p = h_2 - h_1 = v_{f1} (P_2 - P_1)_{s=c}$$

$$h_2 = h_1 + v_{f1} (P_2 - P_1) = 191.81 + 0.001010 (3000 - 10)$$
$$h_2 = 194.8299 \text{ kJ/kg}$$

state 3

$$T_3 = 350^\circ\text{C} \quad P_3 = 3 \text{ MPa} \quad T_{sat@P=3 \text{ MPa}} = 233.85^\circ\text{C}$$

Since $T_3 > T_{sat@P=3 \text{ MPa}}$: state 3 is superheated

$$h_3 = 3116.1 \text{ kJ/kg}$$

$$s_3 = 6.74150 \text{ kJ/kg}\cdot\text{K}$$

State 4

$$S_4 = S_3 = 6.7450 \text{ kJ/kg} \cdot \text{K}, P_4 = 10 \text{ kPa}$$

Since $S_f < S_4 < S_g$ at $P = 10 \text{ kPa}$ then S_4 is wet mixture

$$S_f = 0.6492 \text{ kJ/kg} \cdot \text{K}, S_g = 8.1482 \text{ kJ/kg} \cdot \text{K}$$

$$S_g = 7.4796 \text{ kJ/kg} \cdot \text{K}$$

$$S_4 = S_f + x S_{fg}$$

$$x = \frac{6.7450 - 0.6492}{7.4796} = 0.812 \quad (\text{this value is not acceptable in practical case})$$

$$h_4 = h_f + x h_{fg} \quad h_f = 191.81 \text{ kJ/kg}, h_{fg} = 2392.1 \text{ kJ/kg}$$

$$= 191.81 + 0.812 \times 2392.1$$

$$h_4 = 2134.1952 \text{ kJ/kg}$$

$$1) W_p = h_2 - h_1 = 194.8299 - 191.81 = 3.0199 \text{ kJ/kg}$$

$$\text{or } W_p = v_f (P_2 - P_1) = 0.001010 (3000 - 10) = 3.0199 \text{ kJ/kg}$$

$$2) q_{in} = h_3 - h_2 = 3116.1 - 194.8299 = 2921.2701 \text{ kJ/kg}$$

$$3) x_4 = 0.812$$

$$4) W_T = h_3 - h_4 = 3116.1 - 2134.1952 = 981.9048 \text{ kJ/kg}$$

$$5) q_{out} = h_4 - h_1 = 2134.1952 - 191.81 = 1942.3852 \text{ kJ/kg}$$

$$6) W_{net} = W_T - W_p = 978.8849 \text{ kJ/kg}$$

$$7) M_{th} = \frac{W_{net}}{q_{in}} = 0.335$$

$$8) M_{Carnot} = 1 - \frac{45.81 + 273}{350 + 273}$$

$$9) M_{II} = \frac{M_{th}}{M_{Carnot}} = 0.686 = 0.4582$$

My Job as a Mechanical engineer in power plants

1) service ((خدمة))

2) Control unit

3) R&D = Research and development

Comparison between different methods to improve cycle efficiency

Point 3

A steam power plant operates on a simple ideal Rankine cycle. The steam enters the turbine at 3 MPa and 350 °C. It is then condensed in the condenser at 10 kPa. Calculate:

- 1) The engine performance parameters for the above power plant.
- 2) The performance parameters if the steam is superheated to 600 °C instead of 350 °C keeping rest of conditions the same.
- 3) The performance parameters if the boiler pressure is raised to 15 MPa keeping turbine inlet at 350 °C.
- 4) The performance parameters if the boiler pressure is raised to 15 MPa keeping turbine inlet at 600 °C.
- 5) The performance parameters if for case No. (1) the condenser pressure is lowered to 7.5 kPa.

❖ Results comparison for the above cases.

	Ideal	Increase $T_{max} = T_3$	Increase P_{boiler}	Increase $P_{boiler} \& T_{max}$	Decrease $P_{condenser}$
• Pump Work (kJ/Kg)	3.0199	3.0199	15.14	15.1399	3.016
• Heat Input (kJ/Kg)	2920.48	3487.48	2485.47	3375.36	2943.51
• Steam Quality x_4	0.8123	0.9144	0.6388	0.803	0.8035
• Turbine Work (kJ/Kg)	979.6	1302.375	971.87	1467.4	1013.18
• Heat Output (kJ/Kg)	1943.89	2188.12	1528.74	1923.1	1933.349
• Net Work out (kJ/Kg)	976.58	1299.355	956.73	1452.26	1010.164
• Thermal Efficiency (%)	33.4	37.25	38.49	43.025	34.31
• Carnot Efficiency (%)	48.82	63.48	48.82	63.48	49.71
• Relative Efficiency (%)	68.41	58.67	78.84	67.77	69.02

بالنسبة لـ
increase
P_{boiler}

الـ Pump
تضيق بشكل
أكبر
W_p ↑

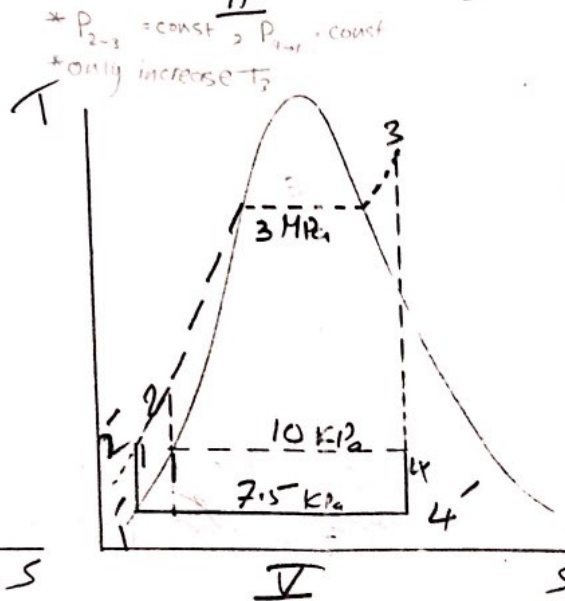
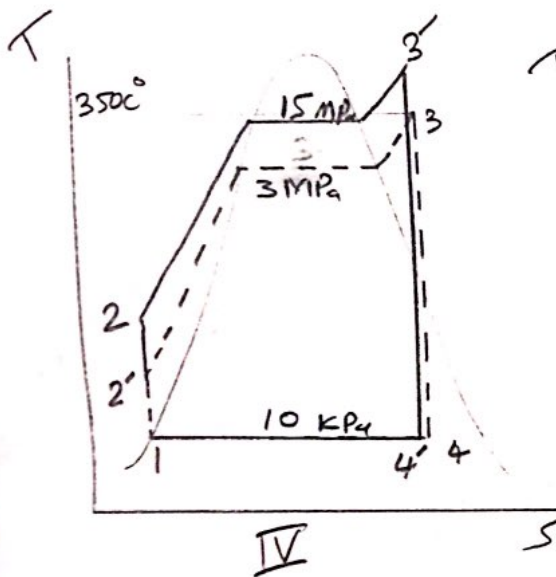
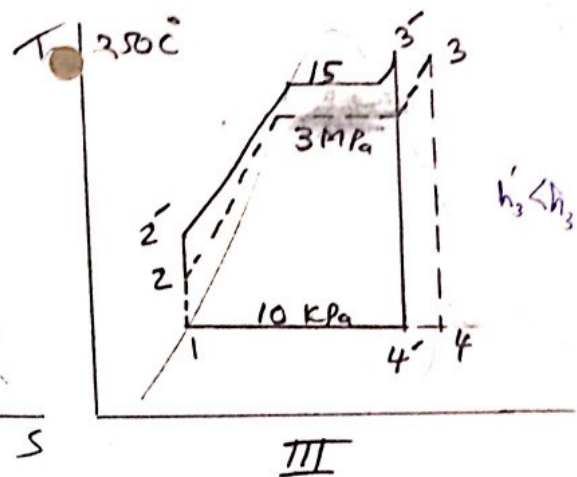
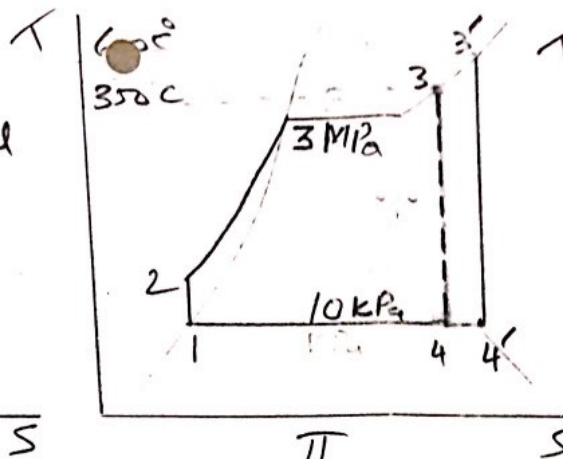
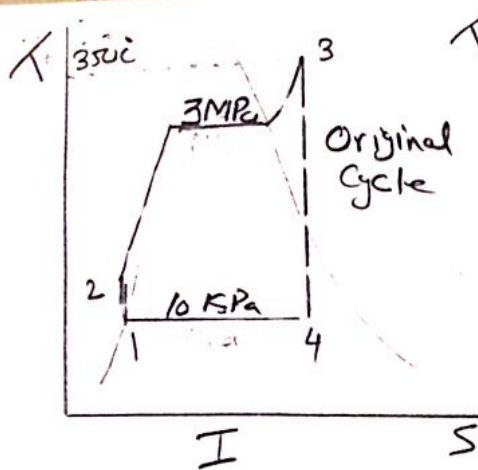
طاقة البخار
بجهد الذي عالما طلاق
قلت استهلاك الغاز (W_g)

- x_4 is acceptable now
- heat input is greater
- $W_T \uparrow$, W_P const, $W_{net} \uparrow$
- x_4 is unacceptable
- Area under 4-1 curve ↓ (i.e. $Q_{loss} \downarrow$)
- $W_T \downarrow$
- $Q_{in} \downarrow$

(not acceptable)

* In order to transfer heat from the steam to cool the sea, T_1 between them must be greater than 10°C i.e. if $T_{sea} = 20^\circ C$

$T_{steam} = 30^\circ C$ at 10 bar



* increase $P_{\text{boiler}} = \times 4$
 1-2-3-4 Original Cycle
 1'-2'-3'-4' Modified Cycle



* decrease Q_L (heat loss)
 i.e. decrease the pressure of the Condenser
 * Q_H will increase slightly in order to maintain state 2 as sat liquid

* Decrease $P_{\text{condenser}}$

(heat transfer) (تحويل الحرارة) P (الضغط) (T) (درجة الحرارة)

In order to make heat transfer more effective ΔT between the 2 objects must be greater than 10°C . If $T_{\text{sea}} = 20^\circ\text{C}$ and T_{steam} at the condenser $= 35^\circ\text{C}$ $P_{\text{sat}@T=35^\circ\text{C}} = 5.6291 \text{ kPa}$

and $P_{\text{atmosphere}} = 101.3 \text{ kPa}$ $P_{\text{atmospheric}} \gg P_{\text{condenser}}$, mechanical

stresses will develop. So we must design a condenser that endures this high pressure. + air leakage into the condenser, if air enters + unacceptable quality X_4 for the condenser to any system, the heat transfer effectiveness $\downarrow$ (air is insulator) + increase in efficiency is $\approx 1\%$. This method is not really effective.

* Increase T_{max}

- $X_4 \uparrow \rightarrow$ thermal stresses will develop in the blades of the turbine
- increasing T_{max} is accomplished by increasing the consumption of the fuel $\Rightarrow$ air pollution
- heat in $\uparrow$ (state 3 is shifted to the right $\therefore$ state 4 is shifted to the right also i.e. heat loss $\uparrow$) $\Rightarrow$ sea pollution, O_2 in water will $\downarrow$, fish will die / Turbine work $\uparrow$ (because $h_3 \uparrow$ i.e. energy given to the turbine $\uparrow$)
- We can think of it as a method to increase η
- * Increase ~~boiler~~ P_{boiler}

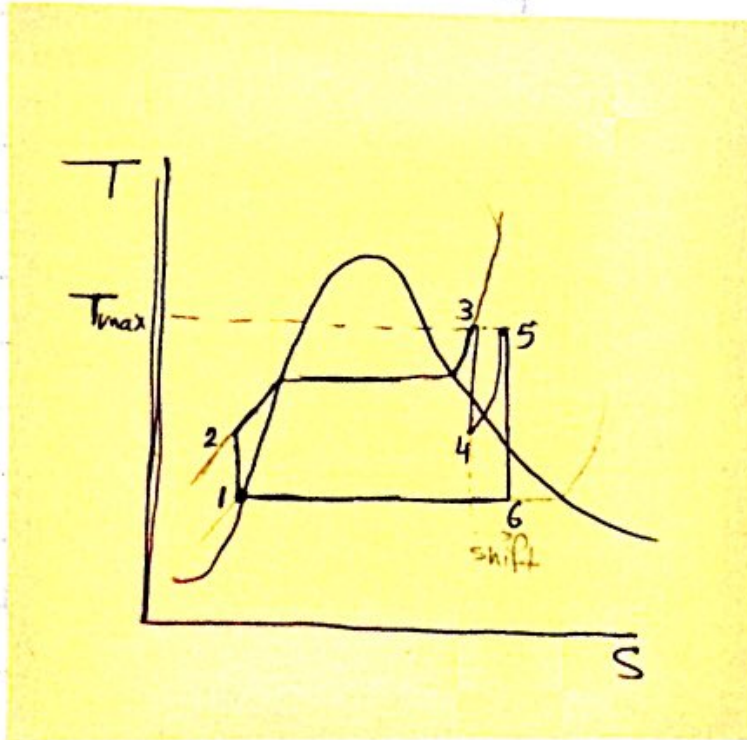
- $W_T \downarrow$ why? as we go to the left of the T-s diagram, enthalpy $\downarrow$ $\therefore$ energy given to the turbine $\downarrow$
- But $\eta_{\text{th}} \uparrow$, why? $q_L \downarrow$, $q_H \downarrow$ $\eta_{\text{th}} = 1 - \frac{q_L}{q_H} \downarrow \therefore \eta_{\text{th}} \uparrow$ (q_H decreased more than q_L)
- Mechanical stresses will develop in boilers. Any crack or failure in design will cause explosion

■ $Q_{in} \downarrow$ (Fuel consumption $\downarrow$ and $\eta_{LH} \uparrow$ which is good according to me)

But the only problem is x_4 is unacceptable (ما يقبله)

* Increase $T_{max} + P_{boiler}$

■ We can shift x_4 by doing expansion using 2 stages instead of one stage (this is called reheat cycle)



* This cycle is used in power plants

3 $\rightarrow$ 4 W_{Turb1}

4 $\rightarrow$ 5 heat addition at const P
ما صرنا بنزين

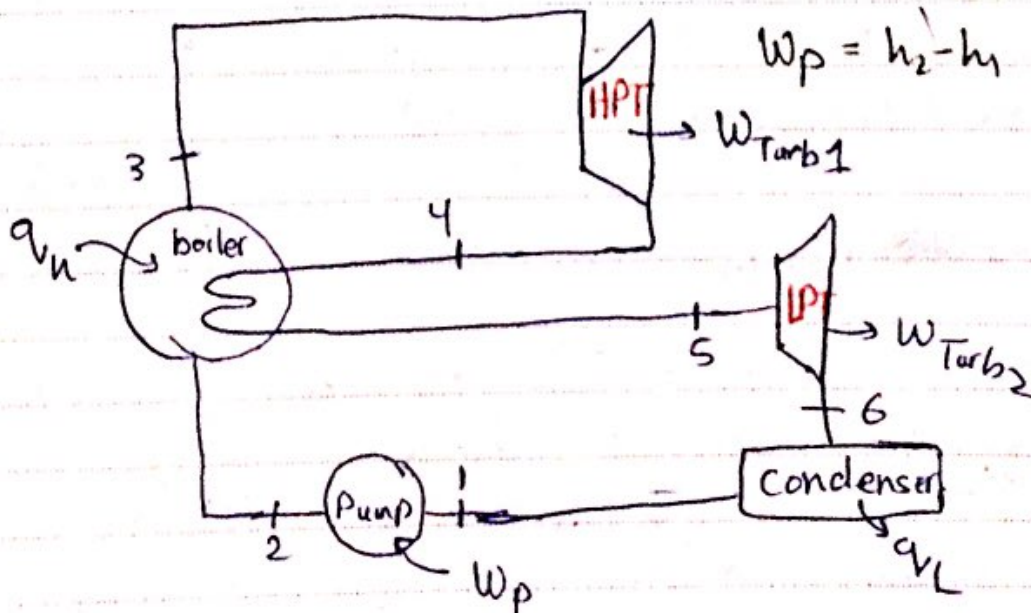
5 $\rightarrow$ 6 W_{Turb2}

$$W_T = W_{T1} + W_{T2} = (h_3 - h_4) + (h_5 - h_6)$$

$$q_h = (h_3 - h_2) + (h_5 - h_4)$$

$$q_l = h_6 - h_1$$

$$W_p = h_2 - h_1$$

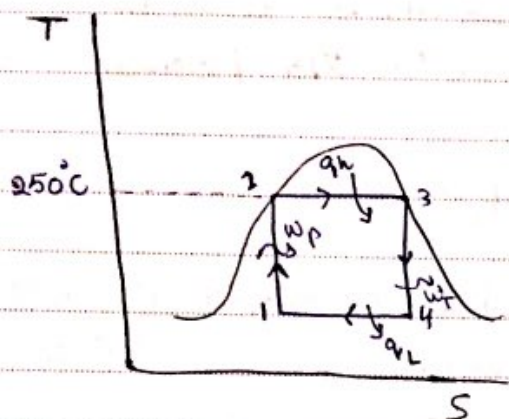


HPT $\rightarrow$ high pressure turbine

LPT $\rightarrow$ low pressure turbine

Problem

10-2 Carnot cycle uses water as working fluid



* Heat rejection takes place at a pressure of 20 kPa find $\eta_{th, \text{Carnot}}$, q_L , w_{net}

Solution:

$$T_H = 250^\circ\text{C}$$

$$T_L = T_{\text{sat}} @ P = 20 \text{ kPa} = 66.06^\circ\text{C}$$

$$\eta_{th} = 1 - \frac{T_L}{T_H} = 1 - \frac{66.06 + 273}{250 + 273} = 0.362$$

$$q_h = h_3 - h_2 = h_{fg} @ 250^\circ\text{C} = 1715.3 \text{ kJ/kg}$$

$$\eta_{th} = \frac{w_{\text{net}}}{q_h} \Rightarrow w_{\text{net}} = \eta_{th} * q_h = 623 \text{ kJ/kg}$$

$$w_{\text{net}} = q_h - q_L \Rightarrow q_L = q_h - w_{\text{net}} = 1715.3 - 623 = 1092.3 \text{ kJ/kg}$$

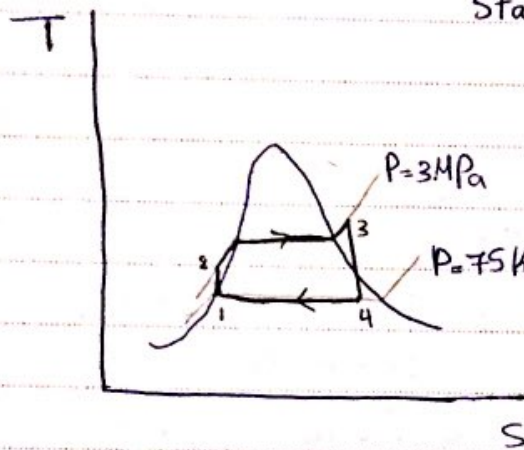
or

$$q_h = T_H s_{fg} @ T = 250^\circ\text{C} = (250 + 273)(3.2788) = 1714.8124 \text{ kJ/kg}$$

$$q_L = T_L s_{fg} @ T = 250^\circ\text{C} = (66.06 + 273)(3.2788) = 1092.03 \text{ kJ/kg}$$

Example 10-1 Consider a steam power plant operating on the simple ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed at a pressure of 75 kPa. Determine the thermal efficiency of the cycle.

State 3: $T_3 = 350^\circ\text{C}$, $P_3 = 3 \text{ MPa}$



State 1: Sat liquid at $P = 75 \text{ kPa}$, $T_{\text{sat}} = 91.76^\circ\text{C}$ @ $P = 75 \text{ kPa}$

$$v_f = 6.001037 \text{ m}^3/\text{kg}$$

$$h_f = 384.44 \text{ kJ/kg}$$

$$h_{fg} = 2278 \text{ kJ/kg}$$

$$s_f = 1.2132 \text{ kJ/k}\cdot\text{kg}$$

$$s_{fg} = 6.2426 \text{ kJ/k}\cdot\text{kg}$$

State 2: $s_2 = s_1 = 1.2132 \text{ kJ/k}\cdot\text{kg}$, $P_2 = 3000 \text{ kPa}$

since $s_2 < s_{fg} @ P = 3000$

$\therefore$ state 2 is subcooled liquid

$$w_p = h_2 - h_1 = v_f (P_2 - P_1)$$

$$w_p = 6.001037 (3000 - 75) = 3.033225 \text{ kJ/kg}$$

$$h_2 - 384.44 = 3.033225 \Rightarrow h_2 = 387.473225 \text{ kJ/kg}$$

State 3: $T_3 = 350^\circ\text{C}$, $P_3 = 3 \text{ MPa}$

$$T_{\text{sat}} @ P = 3000 \text{ kPa} = 233.85^\circ\text{C}$$

Since $T_3 > T_{\text{sat}} @ P = 3000 \text{ kPa}$

$\therefore$ State 3 is superheated

$$s_3 = 6.7450 \text{ kJ/k}\cdot\text{kg}$$

$$h_3 = 3116.1 \text{ kJ/kg}$$

State 4:

$$s_4 = s_3 = 6.745 \text{ kJ/k.kg} \quad p_4 = 75 \text{ kPa}$$

$$s_p < s_4 < s_g \quad \text{at } p = 75 \text{ kPa} \quad \therefore \text{State 4 is wet mixture}$$

$$\begin{array}{c} 1.2132 \\ \text{kJ/k.kg} \end{array} \quad \begin{array}{c} 7.4558 \\ \text{kJ/k.kg} \end{array}$$

$$s_4 = s_p + x s_{fg} \Rightarrow x = \frac{6.745 - 1.2132}{6.2426} = 0.8861 \quad \text{this quality is acceptable in practical case}$$

$$h_4 = h_p + x h_{fg} = 384.44 + 0.8861 (2278) = 2402.9758 \text{ kJ/kg}$$

$$q_h = h_3 - h_2 = 3116.1 - 387.473225 = 2728.626775 \text{ kJ/kg}$$

$$q_L = h_4 - h_1 = 2402.9758 - 384.44 = 2018.5358 \text{ kJ/kg}$$

$$\eta_{th} = 1 - \frac{q_L}{q_h} = 1 - \frac{2018.5358}{2728.626775} = 0.2602$$

Example 10-7 In example 10-1 Suppose that Heat is supplied to the stream in a Furnace maintained at 800K, and waste heat is rejected to the surroundings at 300K Find (a) exergy destroyed in each process (b) $\dot{M}$ (c) $\dot{X}$ expended (d) $\dot{X}$ recovered

$$\text{process } 1 \rightarrow 2 \text{ (pump)} \Delta S = 0, Q = 0 \quad \therefore \dot{X}_{dest, 1 \rightarrow 2} = 0$$

$$\text{process } 3 \rightarrow 4 \text{ (Turbine)} \Delta S = 0, Q = 0 \quad \therefore \dot{X}_{dest, 3 \rightarrow 4} = 0$$

$$\text{Process } 2 \rightarrow 3 \quad \dot{X}_{dest, 2 \rightarrow 3} = T_0 \left[s_3 - s_2 - \frac{q_{in}}{T_H} \right] = 300 (6.745 - 1.2132 - \frac{2728.626775}{800})$$

$$= 636.3049 \text{ kJ/kg}$$

process 4 → 1

$$\begin{aligned} X_{\text{dest}}^{4 \rightarrow 1} &= T_0 \left[S_1 - S_4 + \frac{q_{\text{out}}}{T_0} \right] \\ &= 300 \left(1.2132 - 6.745 + \frac{2018.5358}{300} \right) \\ &= 358.9958 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} X_{\text{dest, cycle}} &= X_{\text{dest}, 1 \rightarrow 2} + X_{\text{dest}, 2 \rightarrow 3} + X_{\text{dest}, 3 \rightarrow 4} + X_{\text{dest}, 4 \rightarrow 1} \\ &= 0 + 636.305 + 0 + 358.9958 \\ &= 995.308 \text{ kJ/kg} \end{aligned}$$

$$\eta_{\text{th, Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{800} = 0.625$$

$$\eta_{\text{II}} = \frac{\eta_{\text{th, Rankine}}}{\eta_{\text{th, Carnot}}} = \frac{0.2602}{0.625} = 0.41632$$

$$x_{\text{recovered}} = W_T = h_3 - h_4 = 3116.1 - 2402.9758 = 713.1242 \text{ kJ/kg}$$

$$x_{\text{expended}} = \left(1 - \frac{T_L}{T_H} \right) q_h + W_p = 1708.43 \text{ kJ/kg}$$

$$\text{or } x_{\text{expended}} = \frac{x_{\text{recovered}}}{\eta_{\text{II}}}$$

$$x_{\text{recovered}} = x_{\text{expended}} - x_{\text{dest, cycle}}$$

How to find $\dot{m}_6$?

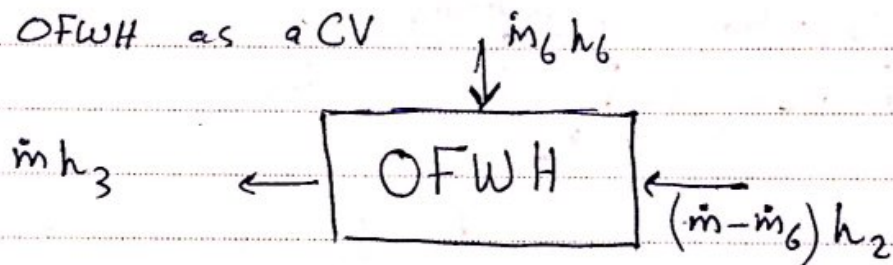
$$\begin{aligned}\dot{W}_T &= \dot{m}(h_5 - h_6) + (\dot{m} - \dot{m}_6)(h_6 - h_7) \\ \dot{W}_p &= (\dot{m} - \dot{m}_6)(h_2 - h_1) + \dot{m}(h_4 - h_3) \\ \dot{Q}_H &= \dot{m}(h_5 - h_4) \\ \dot{Q}_L &= (\dot{m} - \dot{m}_6)(h_7 - h_1)\end{aligned}$$

} divide by $\dot{m}$
and let $y = \frac{\dot{m}_6}{\dot{m}}$

↓

$$\begin{aligned}W_T &= (h_5 - h_6) + (1 - y)(h_6 - h_7) \\ W_p &= (1 - y)(h_2 - h_1) + (h_4 - h_3) \\ q_H &= h_5 - h_4 \\ q_L &= (1 - y)(h_7 - h_1)\end{aligned}$$

* take the OFWH as a CV



$$(\dot{m} - \dot{m}_6) h_2 + \dot{m}_6 h_6 = \dot{m} h_3$$

$$(1 - y) h_2 + y h_6 = h_3 \Rightarrow h_2 - y h_2 + y h_6 = h_3$$

↓

using y I can find $\dot{m}_6$

$$y = \frac{h_3 - h_2}{h_6 - h_2}$$

Rankine cycle with regeneration

①

We will reheat the working fluid before it enters the steam generator. So, for the same W_{net} , the amount of q_{in} will $\downarrow$ (i.e. the consumption of the fuel $\downarrow$). Then $\eta_{th} \uparrow$ (actually W_{net} will decrease very slightly because ^{we} will add a pump).
+ some of the steam in the turbine will be extracted

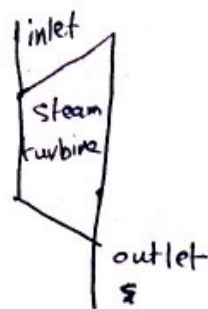
How can I reheat the working fluid? Many solutions are proposed.

- ① Building pipes around the turbine so the working fluid will suck the heat from the turbine (illogic, because the steam will condensate inside the turbine i.e. x may become below 0.85) turbine ΔT ΔT pipes will
- ② Using the solar energy (This solution is not useful in winter)
- ③ The working fluid leaving the turbine has heat, so instead of rejecting the heat in the condenser to the river for example, try to reuse it. Studies found that there is a location (within the turbine between the inlet and the outlet) that has steam this steam has heat but it doesn't have any potential to do work or to produce effective power. If we leave this steam inside the turbine, it will cause back pressure (it will not let the working fluid to enter the turbine) coming from the steam generator

→ So we take this steam bleeding at a certain location. We mix this hot steam with the relatively cold water that is coming from the first pump. Mixing occurs at a room that is called open feed water heater. Steam will condensate / the water will be hot / (i.e. the working fluid at point 3 is hot ~~water~~)

* Why does this steam have weak potential to do work?

In turbines as we go out to the outlet, the cross-sectional area $\uparrow$, the speed of the flow $\downarrow$. We need large ~~velocity~~ speed to hit the blades of the turbine and rotate the shaft, so when the speed of the flow $\downarrow$, the flow will not be able to rotate the shaft effectively, but it has ~~some~~ amount of heat. So we take



that steam at certain location. there is a certain location between the inlet and the outlet, if ~~it~~ ^{we} take the steam from that location $\eta_{th} \uparrow$ greatly

mixing between the hot steam and the cold water is direct
(this is why it is called open feed water heater)

We can use a flash chamber between the OFWH and the 2nd pump.

* The working fluid that is going to enter the 2nd pump will be hot water. So when the working fluid enters the steam generator less q_{in} will be needed to raise the temperature of the working fluid to T_5 , for the same W_{net} .

* Condenser $\rightarrow$ Will relax because they will deal with less amount + the first Pump of mass

* $P_2 = P_3 = P_6$ (because they are connected to the same HEX)

* $P_{SG} > P_{OFWH} > P_{condenser}$

$\rightarrow$ If $P_3 > P_6, P_2$ both 6 and 2 will go back to the turbine and pump

* If $P_3 < P_6, P_2 \Rightarrow$ mixing will occur very fast (مختلج سريع)

* Point 6 $\rightarrow$ wet mixture
 $\rightarrow$ super heated
 $\rightarrow$ Sat. vapor



* The first Pump will increase the pressure of $P_1 \rightarrow P_2$

cold water $\leftarrow$ (2) + (6) $\rightarrow$ steam

We mix them $\Rightarrow$ (3) must be sat liquid at least.

because (3) will ~~not~~ enter the pump (the 2nd pump)

* The 2nd pump will raise the pressure $P_3 \rightarrow P_4$ / $P_4 = P_5$

$q_{in} = h_5 - h_4$ which will be less than before (notice the T-s diagram)

* I don't take the steam at the inlet because this steam has high potential to do work.

* I don't take the steam at the outlet because this steam doesn't have any potential to do work or to heat

* I can find the best location for the steam bleeding by knowing $P_7 + P_5$ (best location means the location that gives the highest thermal efficiency)

* $h_3 \rightarrow$ Sat liquid * $h_2 \rightarrow$ can find it using $h_2 - h_1 = v_{f,1} (P_2 - P_1)_{s-c}$

* $h_6 \rightarrow$ must be given

* don't worry about the 2nd pump because $v_{f,3} (P_4 - P_3)$ is very small. • the difference between P_4 and P_3 is not large

* The disadvantages of OFWH:

$\rightarrow$ Mixing is direct So P_2 must equal $P_3 = P_6$, this requires selection of pumps that work at these pressures

$\rightarrow$ Friction $\rightarrow$ causes changes in pressure (which is not desirable)

$\rightarrow$ pumps may be damaged, leakage in the OFWH $\Rightarrow$ the cycle fails

Steam + water leakage causes the ~~boiler~~ explosion of the boiler

(if the amount of water entering the boiler is small and q_h is large, the boiler will melt + explosion in the boiler will occur)

So we use the CFWH.

* In closed gas turbine cycle, there is no mix between the fuel and the air. $\rightarrow$ Brayton cycle/Joule cycle

* In CFWH, there is no mix between the steam bleeding and the water coming from the first pump.

* Water is heated $2 \rightarrow 9$ } mixing them in the mixing chamber
* Steam is condensed $7 \rightarrow 3$ } gathering them in the mixing chamber

Revision

Rankine cycle with regeneration

* In Gas turbine cycle, we have used the regenerator, to heat the air before it enters the combustion chamber $\Rightarrow q_{in} \downarrow \Rightarrow$ Fuel consumption $\downarrow$ for the same w_{net} , $\eta_{th} = \frac{w_{net} \rightarrow const}{q_{in} \downarrow}$, $\eta_{th} \uparrow$

* The objective of the regeneration is to decrease q_{in} for the same w_{net} or slightly less.

* In the OFWH:

- $\rightarrow$ the pump (without the FWH) increases the pressure from $P_{cond} \Rightarrow P_{boiler}$
- $\rightarrow$ but in the OFWH, the 2 pumps work with small difference of pressure + the first pump deals with (in-mb) not the total mass.
- $\rightarrow$ We can use the flash chamber between the OFWH and the 2nd pump
- $\rightarrow$ Instead of raising the temperature from $T_2 \rightarrow T_5$, now we are raising the temperature from $T_4 \rightarrow T_5$
- $\rightarrow$ if state 3 is compressed liquid, $q_{in} \uparrow$ (check the T-s diagram)
- $\rightarrow y = 0.10 - 0.15 \Rightarrow$ a small amount is needed to supply 2 with heat to increase its temperature but not to evaporate it. (because it will enter the pump after that)

$\rightarrow$ We can put pumps in parallel (~~same~~ ^{spare} pumps)

$\rightarrow$ In the OFWH: once I choose the 2 pumps point 6 is fixed

$\rightarrow$ In the CFWH: I can take the steam bleeding from any location I want because there is no mixing
Check the T-s diagram for location I want because there is no mixing
no need for the $P_{steam\ bleeding}$ to be equal to P_2 .
 P_2 (the outlet of the 1st pump) (2nd + 1st pumps) 1 bar + steam bleeding

is fixed

فاد في الكون
ما أقص
بس بوجبة نظري
المسألة في الـ OFWH
أني لازم أرفع الضغط (ضغط الـ 1st pump)
وهذا الشيء مش موجود بالـ CFWH

$\rightarrow$ We put a mixing chamber in order to gather the ~~working~~ total mass of the working fluid and to be sure that the pressure is fixed (SG can't endure any fluctuations in pressure)

How does 'the freedom of taking the steam bleeding from any location' benefit me?
 If I want to decrease the consumption of the fuel, I can ~~change~~ choose a location that is slightly above, without sacrifice the objective of this cycle (i.e. producing power).

For example, In summer, ^{many} citizens travel, so the consumption of electricity decreases. I can choose a location that is slightly above $\rightarrow$ $\downarrow$ the consumption of the fuel

+ $\downarrow$ W_{net} (less power is required because ~~people~~ the citizens are abroad.

- * there is a location that increases the thermal efficiency (decreases q_h for the same W_{net})
- * there is a location that increases W_{net} (maybe at the expense of increasing the heat addition (i.e. the fuel consumption))

في 8 (pump) نسير في السائل + نسير في البخار إلى 8
 . also جولة الـ pump من $\leftarrow$ Vapor إلى

$$\textcircled{1} - \frac{\textcircled{y} = h_g - h_2}{(h_7 - h_3) + (h_g - h_2)}$$

$h_2 \rightarrow$ is known using the equation of the pump

$h_3 \rightarrow$ is known because it's saturated liquid

$h_7 \rightarrow$ must be given because I must know the location of the steam bleeding

$$\textcircled{2} - (1-y)h_g + yh_g = h_s$$

Equation of the Pump

h_s must be given so that 1+2 are 2 ~~equations~~ equations with 2 unknowns
 if h_s is not given
 $T_3 = T_g$ so
 assume $h_g = h_3$

notes on steam bleeding

- there is a certain location between the inlet and the outlet + there is a certain 'y' (~~amount~~ certain amount of steam extracted from the turbine) that gives the best η_{th}
- If I choose a location that is slightly above, $y \downarrow$, but I have lost turbine work to heat the working fluid
- However, if I choose a location that is below, $y \uparrow$, but $W_{Turbine}$ will not be lost

Cogeneration

- In all the previous cycles that we have studied, the purpose of the cycles ~~was~~ ^{was} to produce power
- In Cogeneration cycles, Steam is utilized for more than one purpose (the main purpose is still producing power)

→ Some industries that require steam

- * biscuits (Food industry)
- * Ironing
- * Sterilization in hospitals
- * Drying textiles
- * Heating buildings

→ any industry that is based on steam is called Process heater (PH)

Case

→ Which ~~state~~ is more economical for the manufacturer?

- * To build his own Power plant
- * To contract an agreement with a power plant to supply him with the required steam

→ In this case, the ~~steam~~ manufacturer will be supplied with steam, and return this steam as a liquid. Only the pump that will be used to return the liquid to the power plant will cost him

→ Rankine cycles that are used for more than one purpose is called Cogeneration cycle

→ the goal of the cogeneration cycle is investment. (In order to provide the repairing costs, workers' salary and ^{the costs required to} develop the power plant.

→ Sometimes, Sailing electricity is not sufficient for the powerplant's outlay, (2)
So you can sail steam ~~when~~ only when you don't need this steam. (Because the main purpose of this steam is to produce power)

→ In Summer vacations, citizens travel for tourism, so the consumption of electricity ↓, and you want to make an investment, there are 2 ways for investment.

- Sailing electricity → (not effective, because the citizens are abroad the consumption of electricity ↓)

- Sailing steam for factories that require steam (factories or hospitals or for building heatings)

→ If there is a great demand on the power plant, steam can be extracted from the turbine.

→ A valve is installed at point (5) (it is a gate valve that is used only to regulate the flow of the fluid. It doesn't cause any changes in the properties of the flowing fluid)

- * the valve is closed when the demand on electricity ↑

- * the valve is opened when the demand on electricity ↓ (In summer vacations, so it becomes a way for investment)

→ Steam must be supplied to the factory according to the required features:

- Drying textiles: Superheated steam is required to dry the textile. This steam is not required to be at high pressure, so the pressure of the steam is decreased using a throttling valve (without changing the amount of heat in the steam i.e. const enthalpy).

- Biscuits: Superheated steam at lower pressure is required for biscuits. (wet mixture will ~~moor~~ moisturize the biscuits, which is not desired, so we don't use wet mixture)

- Heating buildings: Superheated steam is not required for heating buildings. Wet mixture / Sat. vapor steam is used to heat buildings. (this steam will be supplied from the turbine at point (5))

□ P_{II} → it is not like a mixing chamber, think of it as factories, houses, hospitals etc.

■ $P_q = P_{10} = P_{II} = P_1 = P_2 = P_3$

P_{PH} is determined by the manufacturer $\Rightarrow$ according to the features of the steam (3)

- The steam at 4 may be used for sterilization in hospitals
- the steam at 5 may be used for heating building (because it doesn't require ~~high~~ ^{very} large amount of heat)

هذا ما يحدث عندما يمر البخار المشبع (الخارج من الدوم) عبر صمام التقييد
this is what happens when hot steam (outside the dome) passes through a throttling valve

- The throttling valve has a small diameter, so when steam enters the throttling valve $V \uparrow$, $P \downarrow$, $T \downarrow$ Joule-Thomson effect (For real gases: $T \downarrow$ when V (specific volume) $\uparrow$ i.e. when the gas expands)

- When the gas expands, work is done by the gas on the surroundings ($U \downarrow$, $T \downarrow$) - the opposite occurs when the gas is compressed

~~•~~ $V \uparrow$, $P \downarrow$ why?

(الغازات الغازية تزداد سرعتها مع زيادة حجمها
استطاعت بذلك عندها دون أن يتغير ضغطها بالجزئية
التي حوالها فيقبل الضغط)

- The manufacturer must agree to the terms and conditions of the owner of the power plant. these terms are

- 1) Not to change in the properties of the working fluid (because water is chemically processed)
- 2) Steam is supplied to the manufacturer and it must be returned as ~~wet mixture~~ saturated liquid (so the owner of the power plant doesn't have to use a condenser)
- 3) The working fluid must be returned free from impurities, as it was supplied free from impurities

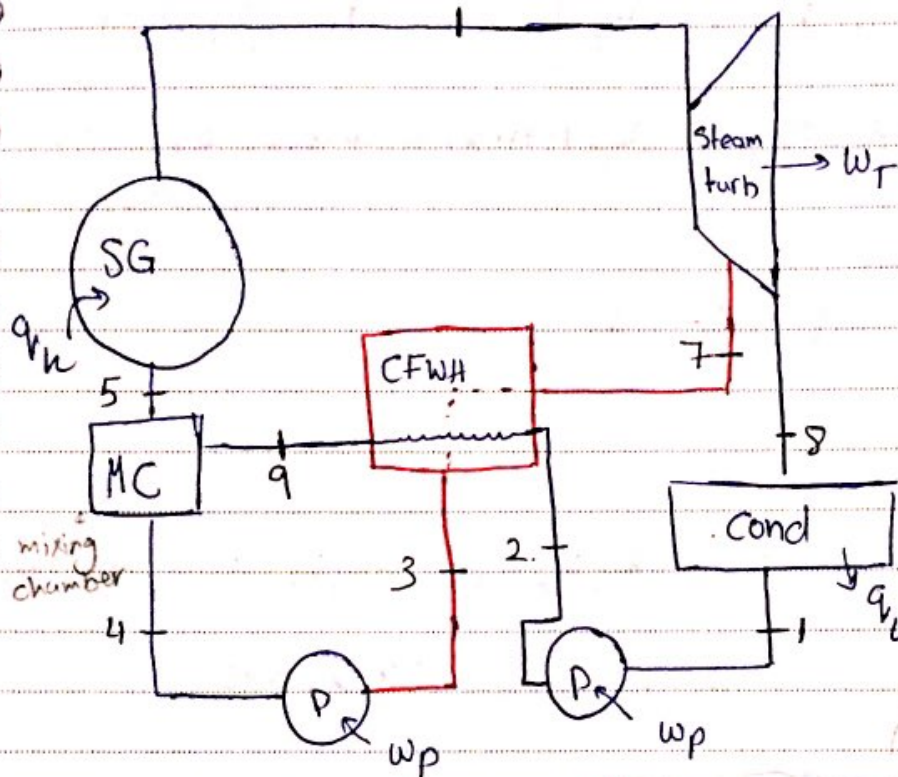
- Remember that the main purpose of the cycle is to produce power, so don't sell steam unless the demand on electricity is weak.

- If the demand on electricity $\uparrow$, steam bleeding is disconnected from the ~~process~~ process water and connected to the feed water heater

- As an owner of a power plant, you mustn't stop supplying the steam suddenly. You must give the manufacturer a time out, so he can look for another ~~power~~ power plant or steam supplier

- This requires having knowledge about how does the demand on electricity vary over the year.
- The value is closed gradually because the demand on electricity ↑ gradually.
- The purpose of the PH is investment
- In the 2nd design, steam is supplied with certain features
(In the 2nd design the owner of the power plant determines the features of the steam, according to the power plant's needs, unlike the first design, the manufacturer chooses the features of the steam)
- So, Any manufacturer who is interested to buy this steam with ^{these} ~~the~~ certain features, he is welcome, otherwise the power plant cannot supply him with his own features.

*Closed Feed Water Heaters

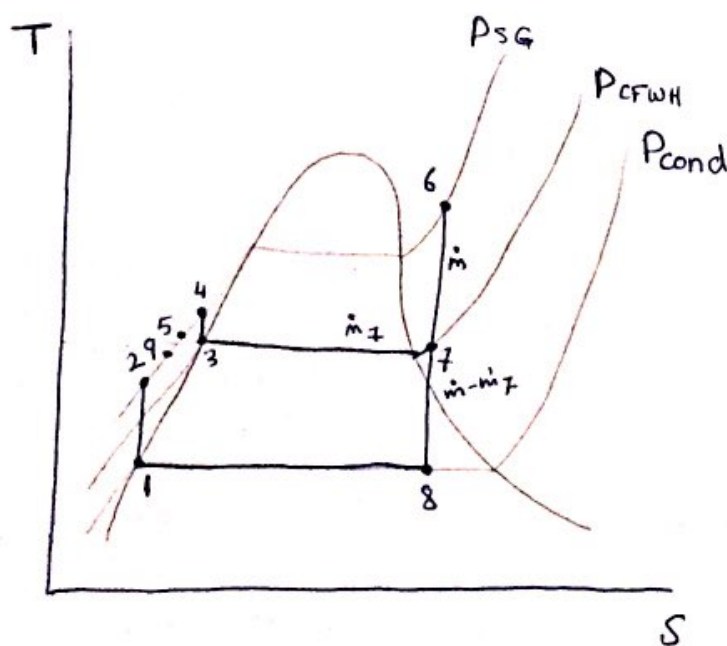


$$P_6 = P_5 = P_9 = P_4 = P_2$$

$$P_3 = P_7$$

It's impossible for T_6 to be greater than T_7 (T_6 may be equal to T_7 in the limiting ideal case)

It's impossible for T_3 to be less than T_2 (T_3 may be equal to T_2 in the limiting ideal case)



$T_4 > T_9$ why?
the steam at 7 is going to heat the liquid at 2. So $T_7 > T_2$. T_3 will be greater than T_9 or equal to T_9 (if thermal equilibrium occurs). The pump will raise the temp. of P3 to T_4 (T_4 is > than T_3) which means that $T_4 > T_9$

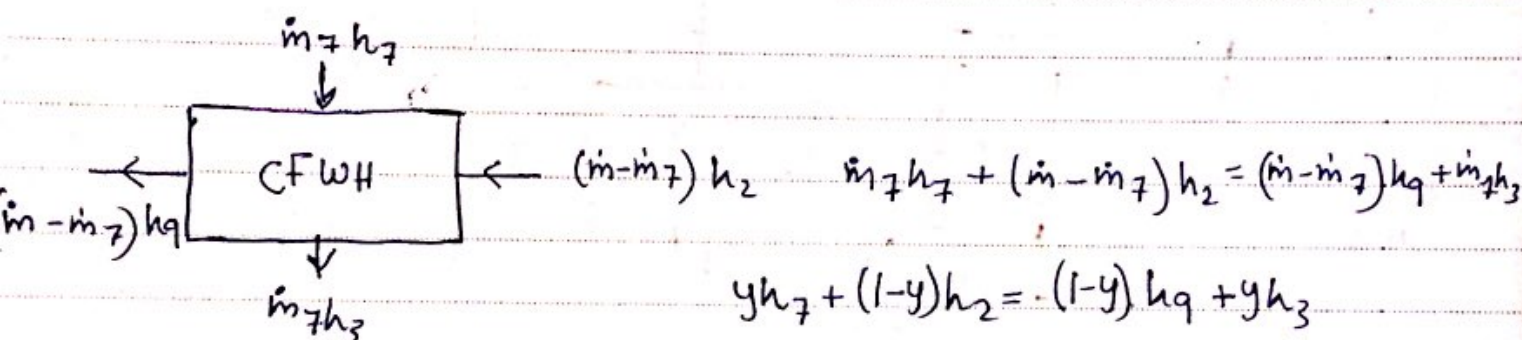
$$\dot{W}_T = \dot{m}(h_6 - h_7) + (\dot{m} - \dot{m}_7)(h_7 - h_8), w_T = (h_6 - h_7) + (1-y)(h_7 - h_8)$$

$$\dot{W}_P = (\dot{m} - \dot{m}_7)(h_2 - h_1) + \dot{m}_7(h_4 - h_3), w_P = (1-y)(h_2 - h_1) + y(h_4 - h_3)$$

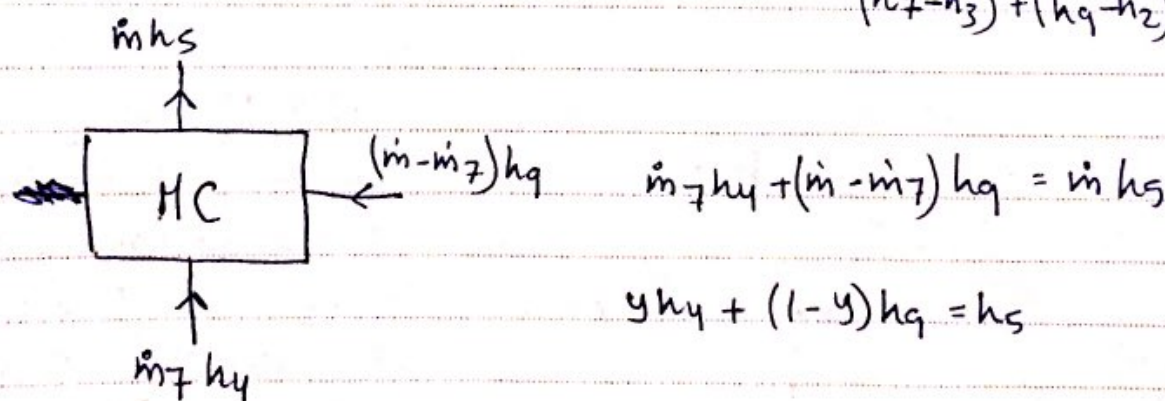
$$\dot{Q}_H = \dot{m}(h_6 - h_5), q_H = h_6 - h_5$$

$$\dot{Q}_L = (\dot{m} - \dot{m}_7)(h_8 - h_1) \Rightarrow q_L = (1-y)(h_8 - h_1)$$

divide by $\dot{m}$
and let $y = \frac{\dot{m}_7}{\dot{m}}$



$$y = \frac{h_q - h_2}{(h_7 - h_3) + (h_q - h_2)}$$

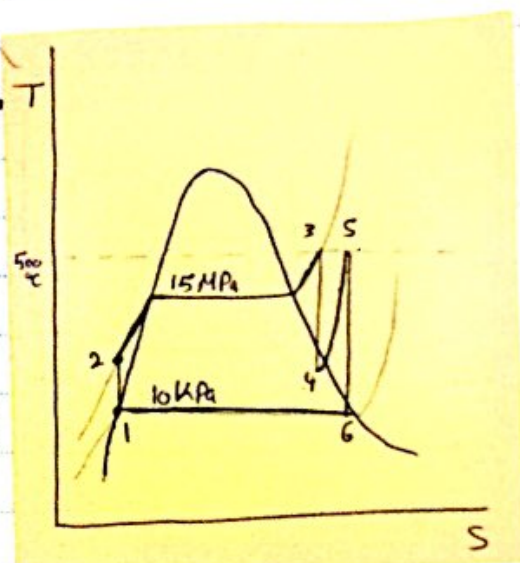
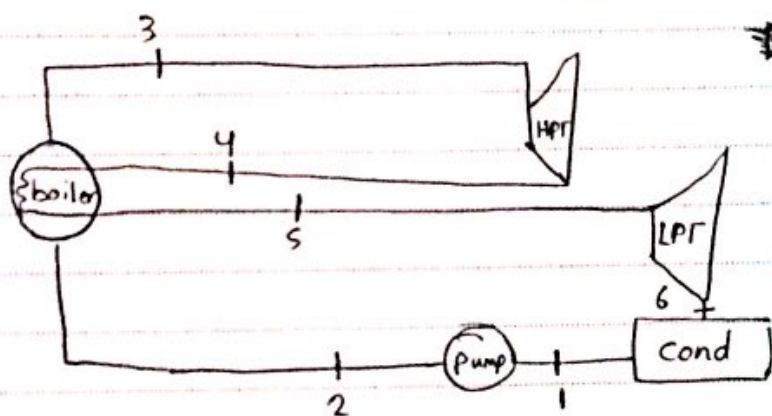


* If no information is given about either of the 3 (y, h_5, h_q), then assume $T_3 = T_q$ (effectiveness = 100%) and hence $h_3 = h_q$

h is a weak function of pressure

so although $P_3 \neq P_q$, we have assumed that $T_3 = T_q$

- 10-407med. A steam power plant operates on an ideal reheat Rankine cycle between the pressure limits of 15 MPa and 10 kPa. The mass flow rate is 12 kg/s. Steam ~~enter~~ enters both stages of the turbine at 500°C. If the moisture content is not to exceed 10 percent, determine
- the pressure at which reheating takes place
 - the total rate of heat input in the boiler
 - $\dot{M}_{th}$



* $x_6 = 0.9$, $\dot{m} = 12 \text{ kg/s}$

1) Sat liquid

$\nu_f = 0.001010$

$h_f = 191.81 \text{ kJ/kg}$

$h_{fg} = 2392.1 \text{ kJ/kg}$

$s_f = 0.6492 \text{ kJ/k}\cdot\text{kg}$

$s_{fg} = 7.4996 \text{ kJ/k}\cdot\text{kg}$

2) $h_2 - h_1 = \nu_f (P_2 - P_1)$ $h_2 = 191.81 + 0.001010(15000 - 10)$
 $= 206.9499 \text{ kJ/kg}$

3) $T_{sat @ 15 \text{ MPa}} = 342.16^\circ\text{C} < 500^\circ\text{C}$: State 3 is SH

$h_3 = 3310.8 \text{ kJ/kg}$

$s_3 = 6.3480 \text{ kJ/k}\cdot\text{kg}$

5) $h_6 = h_f + x h_{fg} = 191.81 + 0.9 \times 2392.1 = 2344.7 \text{ kJ/kg}$

$s_6 = s_f + x s_{fg} = 0.6492 + 0.9 \times 7.4996 = 7.29884 \text{ kJ/k}\cdot\text{kg}$

$$At T = 500^{\circ}C$$

5) $S_5 = S_6 = 7.39884 \text{ kJ/K} \cdot \text{kg}$	$S.H$	$P = 2 \text{ MPa}$	$P = 2.5 \text{ MPa}$
$T_5 = 500^{\circ}C$		$S = 7.4337$	7.3254
$P_5 = 2.161 \text{ MPa}$		$h = 3468.3$	3462.8
$h_5 = 3466.529 \text{ kJ/kg}$			

$$P-2 = \frac{2.5 - 2}{7.3254 - 7.4337} (7.39884 - 7.4337)$$

$$h - 3468.3 = \frac{3468.3 - 3462.8}{2 - 2.5} (2.161 - 2)$$

$$4) P_4 = P_5 = 2.161 \text{ MPa}, S_4 = S_3' = 6.3480 \text{ kJ/K} \cdot \text{kg}$$

$$\text{state 4 is superheated} \quad h_4 = 2817.6 \text{ kJ/kg}$$

$$\begin{aligned} \dot{Q}_H &= \dot{Q}_{3-2} + \dot{Q}_{RH} = \dot{m} (h_3 - h_2) + \dot{m} (h_5 - h_4) \\ &= 12 (3310.8 - 206.9499) + 12 (3466.529 - 2817.6) \\ &= 45039 \text{ kW} \end{aligned}$$

$$\eta_{th} = \frac{W_{net}}{\dot{Q}_H} = 42.5$$

$P \text{ (kPa)}$	$T_{\text{sat}} \text{ (}^\circ\text{C)}$	$S_g \text{ (kJ/k}\cdot\text{kg)}$	$h_g \text{ (kJ/kg)}$
2000	212.38	6.3390	2798.3
2161	<u>223.45</u>	<u>6.3109</u>	<u>2799.7</u>
2250	223.95	6.2951	2800.5

at $P = 2 \text{ MPa}$

$T = 212.38^\circ\text{C}$

$T = 223.95^\circ\text{C}$

$T = 225^\circ\text{C}$

h
2798.3
2832.9
2836.1

S
6.3390
6.4096
6.4160

at $T = 223.95^\circ\text{C}$

$P = 2.5 \text{ MPa}$

$P = 2.161 \text{ MPa}$

$P = 2 \text{ MPa}$

h
2801.9
2822.9
2832.9

S
6.2558
6.3600
6.4096

at 2.161 MPa

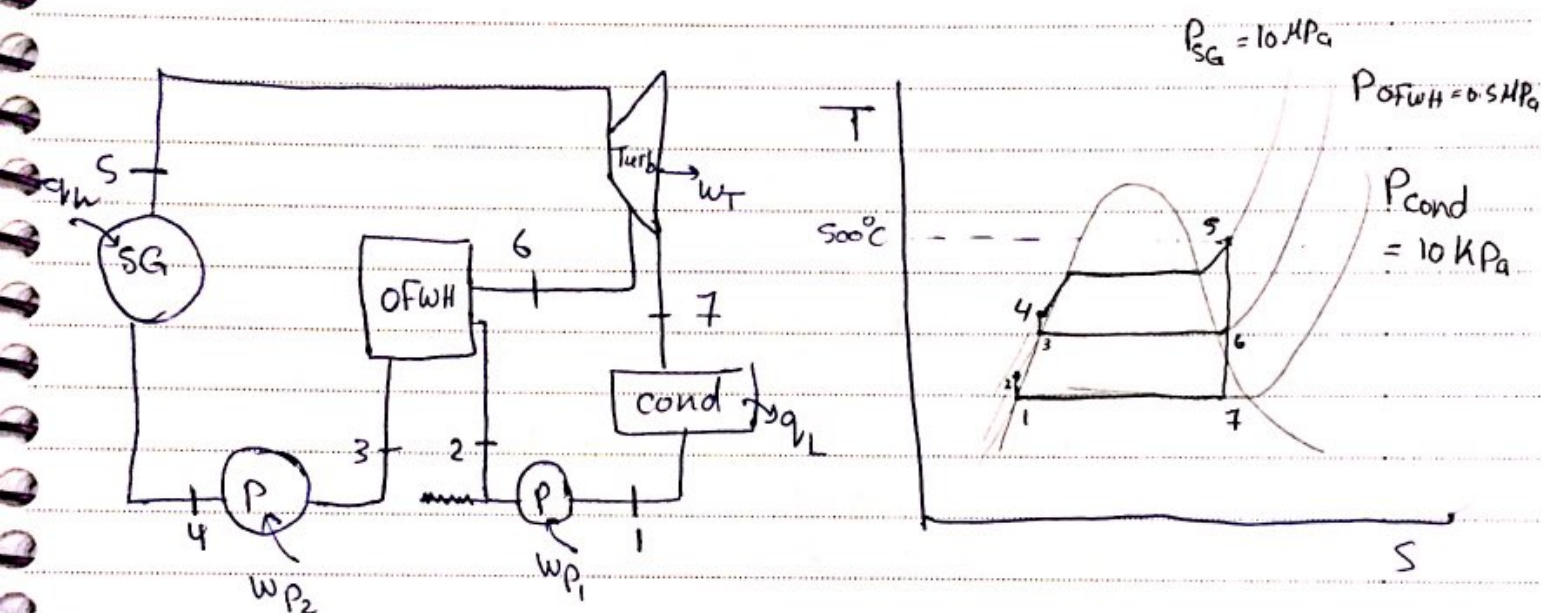
$T = 223.95^\circ\text{C}$

S
6.3600
6.3480

h
2822.9
2817.2

$T = 223.45^\circ\text{C}$ 6.3109 2799.7

10-90 8th ed. Consider a steam power plant that operates on a regenerative Rankine cycle and has a net power output of 150 MW. Steam enters the turbine at 10 MPa and 500°C and the condenser at 10 kPa. Steam is extracted from the turbine at 0.5 MPa. Water leaves the open feedwater heater as saturated liquid a) in b) η_{th}



1) Sat liquid $P = 10 \text{ kPa}$

$$v_f = 0.001010 \text{ m}^3/\text{kg} \quad h_f = 191.81 \text{ kJ/kg} \quad h_{fg} = 2392.1 \text{ kJ/kg}$$

$$s_f = 0.6492 \frac{\text{kJ}}{\text{K} \cdot \text{kg}} \quad s_{fg} = 7.4996 \frac{\text{kJ}}{\text{K} \cdot \text{kg}}$$

$$2) h_2 = h_1 + v_f (P_2 - P_1) = 191.81 + 0.001010 (500 - 10)$$

$$= 192.3 \text{ kJ/kg}$$

3) Sat liquid $P = 500 \text{ kPa}$

$$v_f = 0.001093 \text{ m}^3/\text{kg} \quad h_f = 640.09 \text{ kJ/kg} \quad h_{fg} = 2108 \text{ kJ/kg}$$

$$s_f = 1.8604 \frac{\text{kJ}}{\text{K} \cdot \text{kg}} \quad s_{fg} = 4.9603 \frac{\text{kJ}}{\text{K} \cdot \text{kg}}$$

$$4) h_4 = h_3 + v_f (P_4 - P_3) = 640.09 + 0.001093 (10,000 - 500)$$

$$= 650.47 \text{ kJ/kg}$$

5) $T_{\text{sat}@10 \text{ MPa}} = 311^\circ\text{C} < 500^\circ\text{C} \therefore$ state 5 is superheated

$$h_5 = 3375.1 \text{ kJ/kg}$$

$$s_5 = 6.5995 \text{ kJ/k}\cdot\text{kg}$$

$$6) s_6 = s_5 = 6.5995 \text{ kJ/k}\cdot\text{kg}$$

$$p_6 = 500 \text{ kPa}$$

$$s_f < s_6 < s_g \text{ at } 500 \text{ kPa}$$

$\therefore$ State 6 is wet mixture

$$s_6 = s_f + x s_{fg} \Rightarrow x = \frac{6.5995 - 1.8604}{4.9603} = 0.955$$

$$h_6 = h_f + x h_{fg} = 640.09 + 0.955 \times 2108 = 2654.1 \text{ kJ/kg}$$

$$7) s_7 = s_5 = 6.5995 \text{ kJ/k}\cdot\text{kg} \quad p = 10 \text{ kPa}$$

$$s_f < s_7 < s_g \text{ at } 10 \text{ kPa} \therefore \text{State 7 is wet mixture}$$

$$s_7 = s_f + x s_{fg} \Rightarrow x = \frac{6.5995 - 0.6492}{7.4996} = 0.8$$

$$h_7 = h_f + x h_{fg} = 191.81 + x (2392.1) = 2089.7 \text{ kJ/kg}$$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{640.09 - 192.3}{2654.1 - 192.3} = 0.182$$

$$w_T = (h_5 - h_6) + (1-y)(h_6 - h_7) = (3375.1 - 2654.1) + (1-0.182) \times (2654.1 - 2089.7) = 1182.6792 \text{ kJ/kg}$$

$$w_p = (1-y)(h_2 - h_1) + (h_4 - h_3) = (1-0.182)(192.3 - 191.81) + (650.47 - 640.09) = 10.78082 \text{ kJ/kg}$$

$$a) \dot{W}_{net} = 150 \times 10^3 = \dot{m}_s (\dot{W}_T - \dot{W}_P)$$

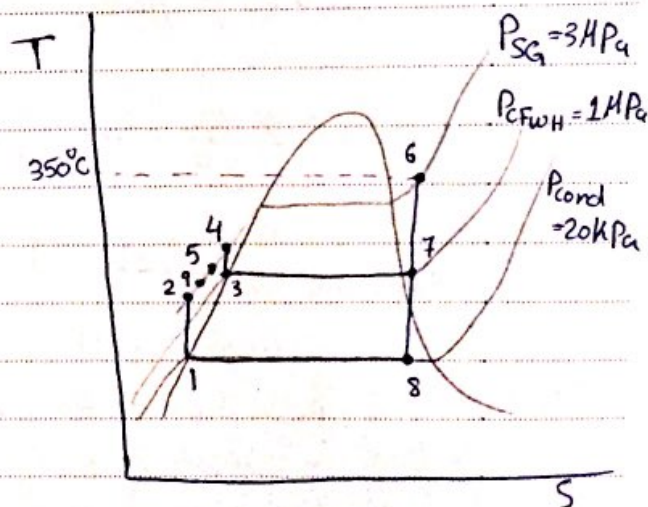
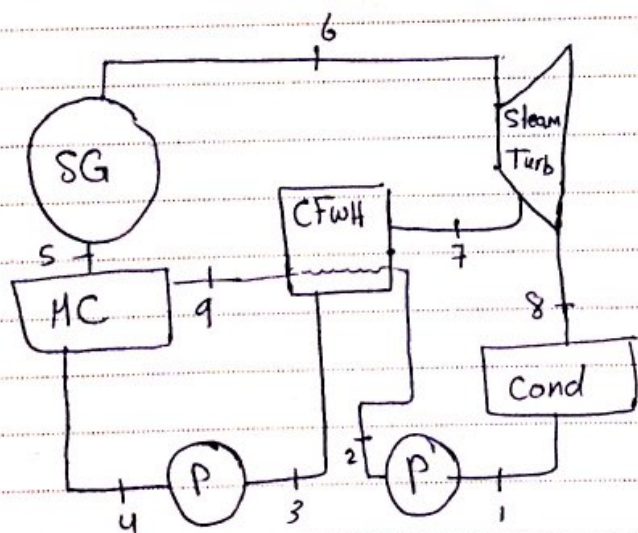
$$\dot{m}_s = \frac{150 \times 10^3}{1182.6792 - 10.78082}$$

$$= 127.99 \text{ kg/s}$$

$$b) \eta_{th} = \frac{\dot{W}_{net}}{\dot{q}_{in}} = \frac{1182.6792 - 10.78082}{2724.63} = 0.43011$$

$$\dot{q}_{in} = h_5 - h_4 = 3375.1 - 650.47 = 2724.63 \text{ kJ/kg}$$

10-48 (tutoring) Consider a steam power plant that operates on ideal regenerative Rankine cycle with a closed feedwater heater. The plant maintains the turbine inlet at 3 MPa and 350°C; and operates the condenser at 20 kPa. Steam is extracted at 1 MPa to serve the closed feedwater heater, which discharges into the condenser after being throttled to condenser pressure. Find $\dot{W}_P$, $\dot{W}_T$, $\dot{q}_w$.



st) * Assume $T_3 = T_g \therefore h_3 = h_g$

or

1) Sat liquid at 20 kPa

$$v_f = 0.001017 \text{ m}^3/\text{kg} \quad h_f = 251.42 \text{ kJ/kg} \quad h_{fg} = 2357.5 \text{ kJ/kg}$$
$$s_f = 0.8320 \text{ kJ/kg.K} \quad s_{fg} = 7.0752 \text{ kJ/kg.K}$$

2) $s_2 - s_1 = 0.8320$ $P_2 = 3000 \text{ kPa}$

$$s_2 < s_{p@3000} = 2.6454 \text{ kJ/kg.K}$$

$$(1-y)(h_2 - h_1) = (1-y)v_f(P_2 - P_1)$$

$$h_2 = 251.42 + 0.001017(3000 - 20)$$
$$= 254.45066 \text{ kJ/kg}$$

3) Sat liquid at 1 MPa

$$T_3 = 179.88^\circ\text{C}, \quad v_{f3} = 0.001127 \text{ m}^3/\text{kg}, \quad h_{f3} = 762.51 \text{ kJ/kg}$$

$$h_{fg} = 2014.6 \text{ kJ/kg} \quad s_{f3} = 2.1381 \text{ kJ/kg.K}$$

$$s_{fg} = 4.4470 \text{ kJ/kg.K}$$

$$4) y(h_4 - h_3) = yv_{f3}(P_4 - P_3)$$

$$h_4 = 762.51 + 0.001127(3000 - 1000)$$
$$= 764.764 \text{ kJ/kg}$$

6) $P_6 = 3000 \text{ kPa}$ $T_6 = 350^\circ\text{C} > T_{sat@3000} = 233.85^\circ\text{C}$

$\therefore$ State 6 is superheated

$$h_6 = 3116.1 \text{ kJ/kg}$$

$$s_6 = 6.7450 \text{ kJ/kg.K}$$

② $P_7 = 1 \text{ MPa}$ $S_7 = S_6 = 6.7450 \text{ kJ/K} \cdot \text{kg}$ $S_g = 6.9850 \text{ kJ/K} \cdot \text{kg}$ @ 1 MPa

∴ State 7 is super heated

$h_7 = 2852.86 \text{ kJ/kg}$

T	h	S
200	2828.3	6.6956
250	2943.1	6.9265

$$h = 2828.3 = \frac{2828.3 - 2943.1}{6.6956 - 6.9265} (6.7450 - 6.6956)$$

③ $P_8 = 20 \text{ kPa}$ $S_8 = S_6 = 6.7450 \text{ kJ/K} \cdot \text{kg}$

$S_p < S_8 < S_g$ @ 20 kPa
 $0.8320 \text{ kJ/K} \cdot \text{kg}$ $7.073 \text{ kJ/K} \cdot \text{kg}$

∴ State 8 is wet mixture

$S_8 = S_p + x S_{fg}$

$x = \frac{6.7450 - 0.8320}{7.0732}$

$= 0.8357$

to increase x → reheat
 → increase the pressure of the condenser

$h_8 = h_f + x h_{fg}$
 $= 251.42 + x \times 2357.5$
 $= 2221.67 \text{ kJ/kg}$

4) $T_9 = T_3$ ∴ $h_9 = h_3 = 762.51 \text{ kJ/kg}$

$y = \frac{h_9 - h_2}{h_7 - h_2} = \frac{762.51 - 254.45066}{2852.86 - 254.45066} = 0.155$

$h_5 = (1-y)h_3 + y h_4 = 762.950 \text{ kJ/kg}$

$q_h = h_6 - h_5$
 $= 2353.15 \text{ kJ/kg}$

$w_p = (1-y)(h_2 - h_1) + y(h_4 - h_3)$
 $= 2.878 \text{ kJ/kg}$

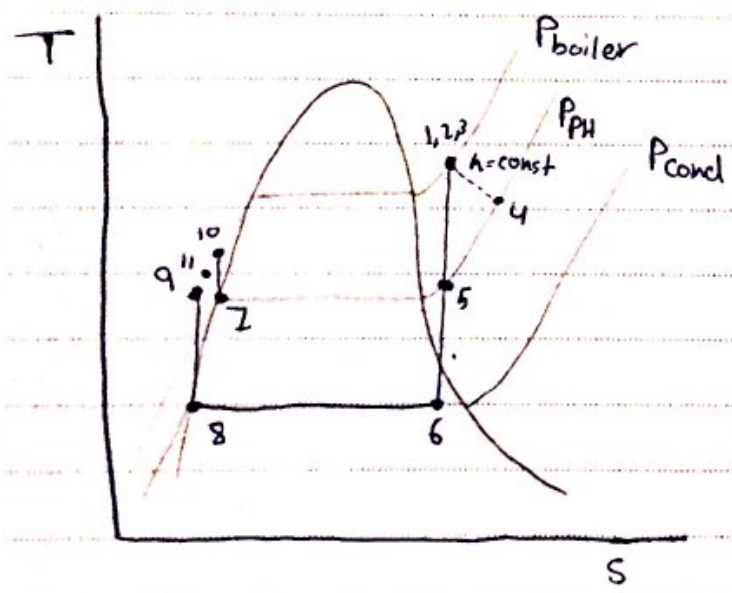
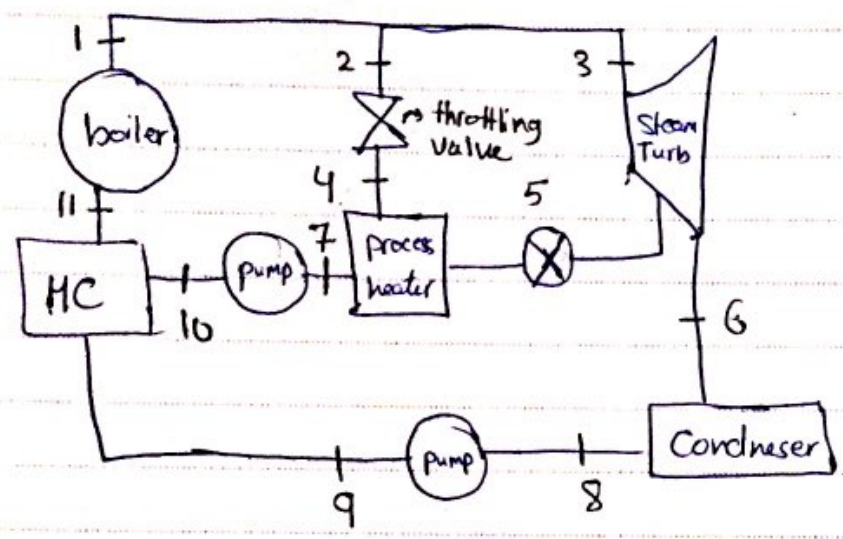
$w_T = (h_6 - h_7) + (1-y)(h_7 - h_5)$
 $= 771.03 \text{ kJ/kg}$

ost
ou

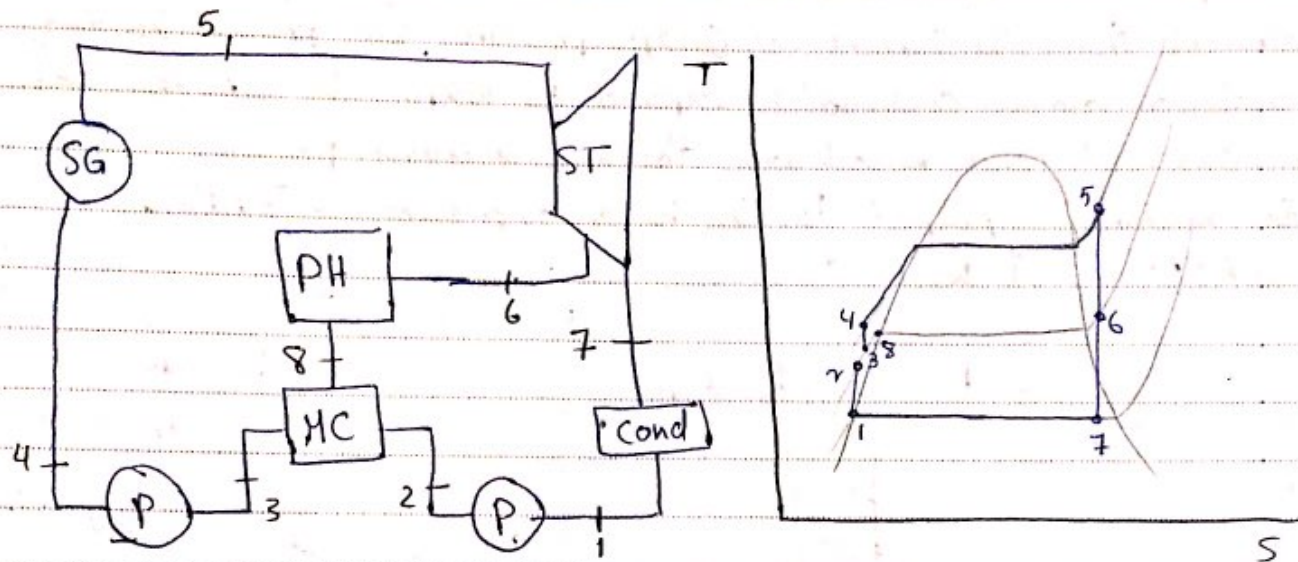
Cogeneration cycle

Steam will be utilized for ① electricity
② process heater (PH)

* Cycle I (1st design)



* Cycle II (2nd design)



$$\dot{Q}_H = \dot{m}(h_5 - h_4)$$

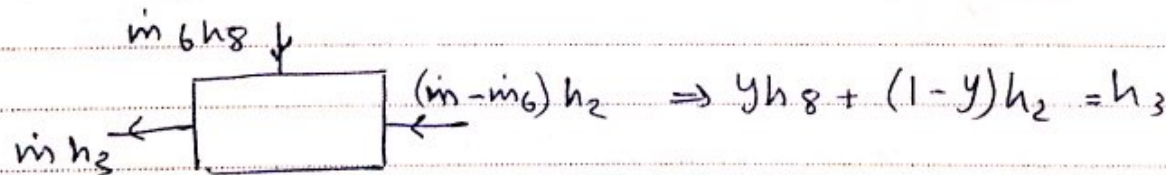
$$\dot{Q}_L = (\dot{m} - \dot{m}_6)(h_7 - h_1)$$

$$\dot{Q}_{PH} = \dot{m}_6(h_6 - h_8)$$

$$\dot{W}_T = \dot{m}(h_5 - h_6) + (\dot{m} - \dot{m}_6)(h_6 - h_7)$$

$$\dot{W}_P = (\dot{m} - \dot{m}_6)(h_2 - h_1) + \dot{m}(h_4 - h_3)$$

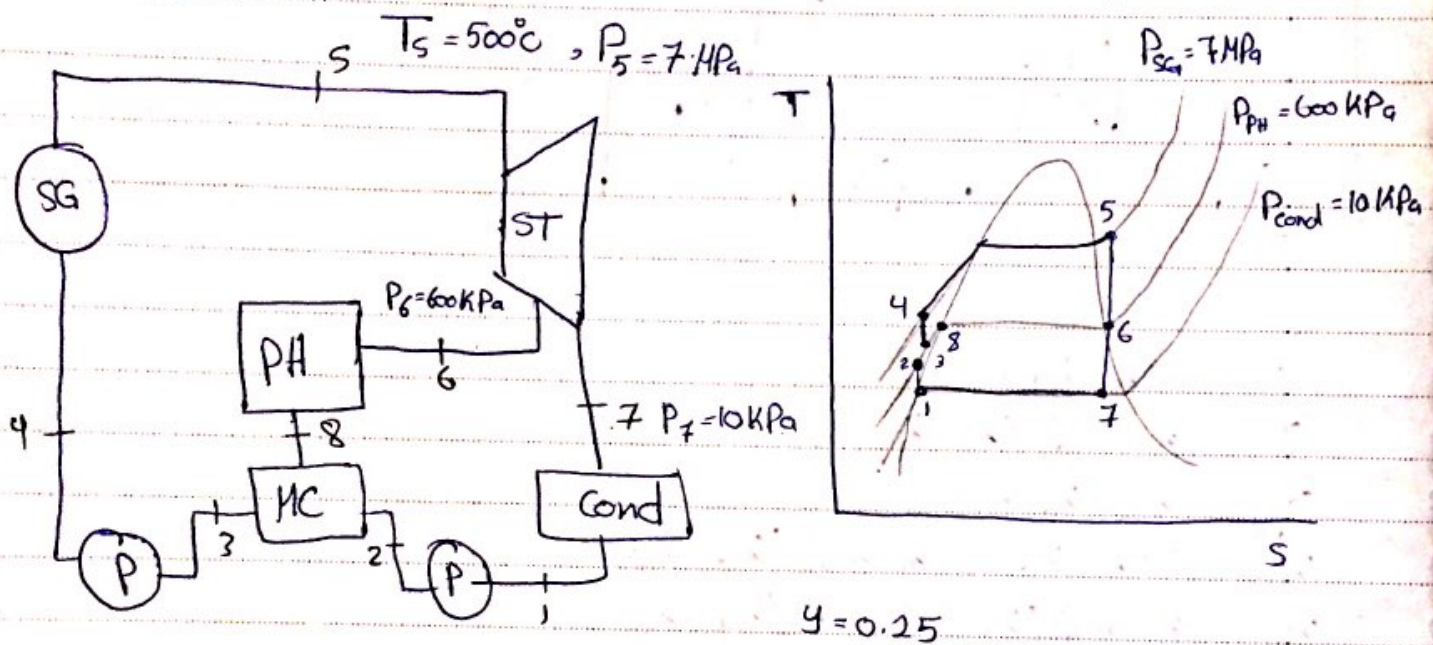
Take HC as CV



$$y = \frac{\dot{m}_6}{\dot{m}} = \frac{h_3 - h_2}{h_8 - h_2}$$

$$\epsilon_u = \text{utilization factor} = \frac{\dot{Q}_{PH} + \dot{W}_{net}}{\dot{Q}_H}$$

ost (tutoring question) Steam enters the turbine of a cogeneration power plant at 7 MPa and 500°C. One-fourth of this steam is extracted from the turbine at 600 kPa pressure for process heating. The remaining steam continues to expand to 10 kPa. The extracted steam is then condensed and mixed with feedwater at constant pressure and the mixture is pumped to the boiler of pressure of 7 MPa. $\dot{m} = 30 \text{ kg/s}$, Find $\dot{W}_{net}$, ϵ_u .



* Check the tutoring sheet for the solution

Ch 11: Refrigeration cycles

* Basic definitions

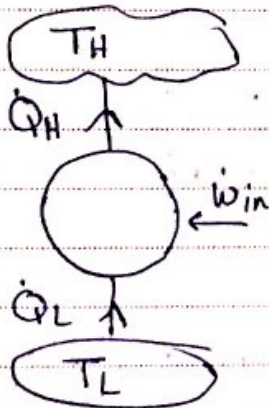
Refrigerant
(the working fluid)

R_{AB} $\left\{ \begin{array}{l} R_{12} \\ R_{22} \end{array} \right\}$ they cause ozone layer depletion
 R_{1AB} $\left\{ \begin{array}{l} R_{134a} \\ R_{170} \\ R_{140} \end{array} \right\}$

refrigeration cycle

Refrigeration process (Heat transfer $\dot{Q}_L \rightarrow \dot{Q}_H$)

* Clausius Statement



$$COP_{R,HP} = \frac{\text{desired output}}{\text{required input } (w_{in})}$$

In refrigeration: desired output is $\dot{Q}_L$
In heat pumps: the desired output is $\dot{Q}_H$

* Cooling capacity (tonne) قدرة التبريد

h_{fs} = latent heat of fusion = 303.85 kJ/kg for water
liquid $\leftarrow$ solid

$$\text{tonne} = \frac{1000 \text{ kg} \times 303.85 \text{ kJ/kg}}{24 \text{ hrs}} = 12660 \text{ kJ/hr}$$

$$= 211 \text{ kJ/min}$$

$$= 200 \text{ Btu/min}$$

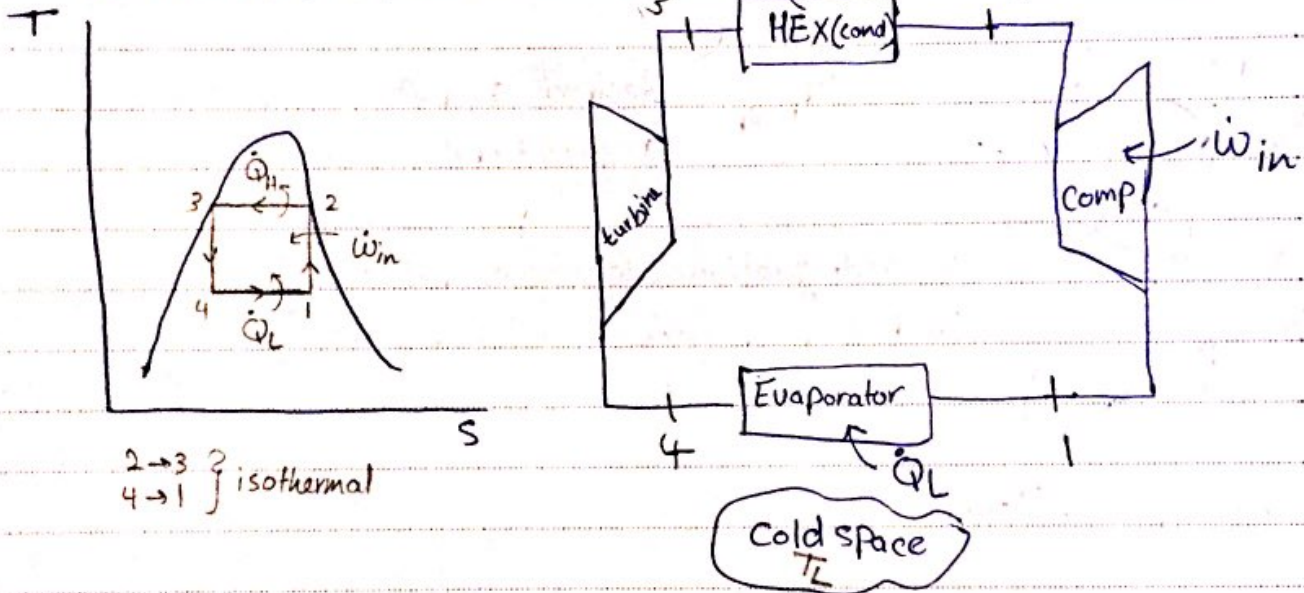
British Thermal unit

$$1 \text{ btu} = 1.055 \text{ kJ}$$

$$\text{Cooling Capacity} = \frac{\dot{m}_R * \dot{Q}_L * 3600}{\dot{m}_w * h_{fs} * \frac{1}{24}}$$

Carnot refrigeration cycle

R134a Table A-11



* Compressor inlet is wet mixture, compressor cannot handle with wet mixture.

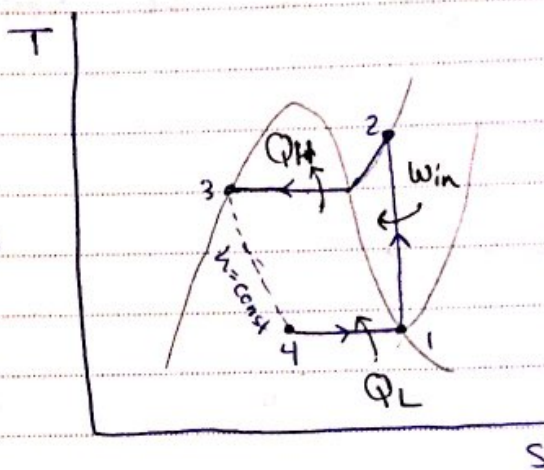
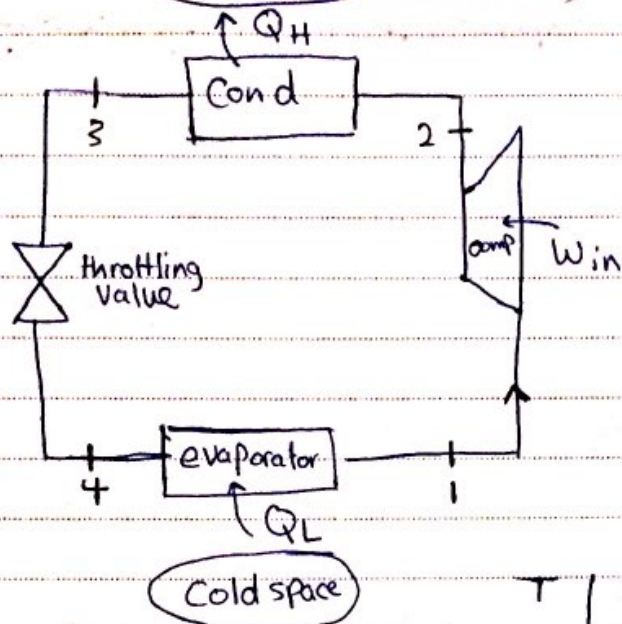
$$COP_{R_{max}} = \frac{T_L}{T_H - T_L}$$

* Turbine inlet is sat. liquid. Turbines can't handle with liquids
+ liquids can't expand largely

$$COP_{HP_{max}} = \frac{T_H}{T_H - T_L}$$

Ideal Vapor-Compression refrigeration cycle

Hot space (environment)

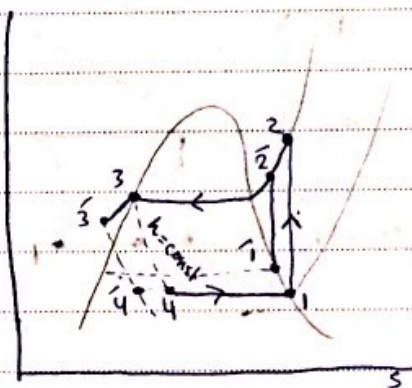


$$- \dot{W}_c = \dot{m}_R (h_2 - h_1)$$

refrigerant

$$- \dot{Q}_L = \dot{m}_R (h_1 - h_4)$$

$$- \text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_c} \Rightarrow \text{in order to increase } \text{COP}_R : \dot{Q}_L \uparrow, \dot{W}_c \downarrow$$



Increase the

Area under 4-1

$\uparrow \dot{Q}_L$

Subcool Point 3

raise $T_{\text{evaporator}}$ (Area under 4-1)

i.e. increase the pressure of the evaporator

$\downarrow \dot{W}_{in}$

raise $T_{\text{evaporator}}$ ($P_1 \uparrow$)

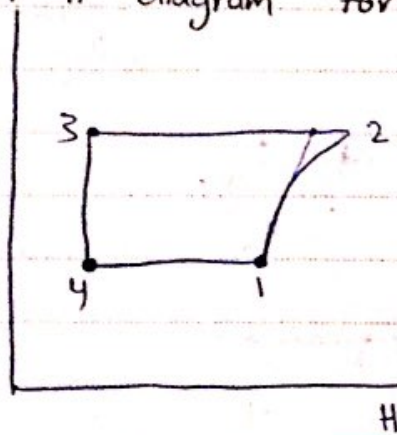
lower T_{cond} ($P_2 \downarrow$)

$$W_{\text{comp}} = \gamma_g (P_2 - P_1)$$

* Notice that (raising $T_{\text{evaporator}}$ causes $\dot{W}_{in} \downarrow, \dot{Q}_L \uparrow$)

* 1 bar = 100 kPa = 0.1 MPa the $\text{COP} \uparrow$ increases by 2-4% for raising $T_{\text{evaporator}}$ by 1°C

* P-h diagram for the ideal vapor-compression refrigeration cycle.



Example 11.1

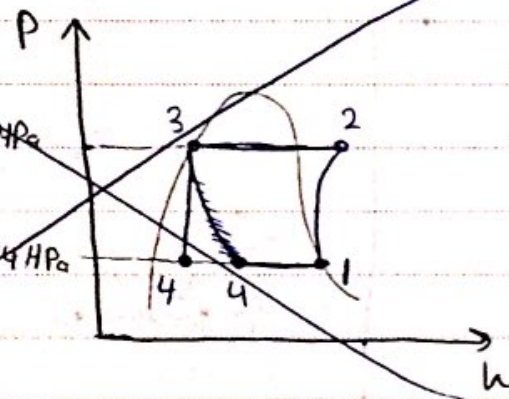
$$\dot{m}_R = 0.05 \text{ kg/s}$$

$$8 \text{ bar} = 0.8 \text{ MPa}$$

$$1.4 \text{ bar} = 0.14 \text{ MPa}$$

$$* P_{\text{evaporator}} = 0.14 \text{ MPa}$$

$$* P_{\text{cond}} = 0.8 \text{ MPa}$$

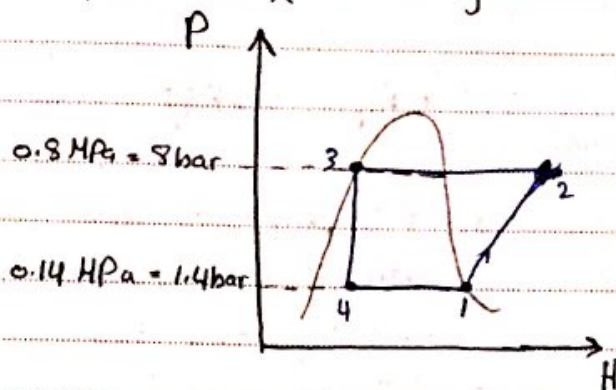


From table A-11

* Some notes about the Ph chart

- BAR(A) ~ Absolute
- pressure measurements of devices are gauge pressure
- The enthalpy in the Ph diagram differs from the values of enthalpy in the tables. why? because the references (i.e. the state where $h=0$) are different, but the change in enthalpy must be the same
- Tables are more accurate than charts, but getting information from charts is faster (because no need for interpolation)

Example 11.1, $\dot{m}_R = 0.05 \text{ kg/s}$



From table A-11 | From charts

① $h_1 = 239.19$

$h_1 = 290$

* $\dot{W}_C = \dot{m}_R (h_2 - h_1)$

$= 1.81 \text{ kW (From table A-11)}$

② $h_2 = 275.4$

$h_2 = 325$

* $\dot{W}_C = \dot{m}_R (h_2 - h_1) = 1.75 \text{ kW (From charts)}$

③ $h_3 = 95.48$

$h_3 = 145$

④ $h_4 = 95.48$

$h_4 = 145$

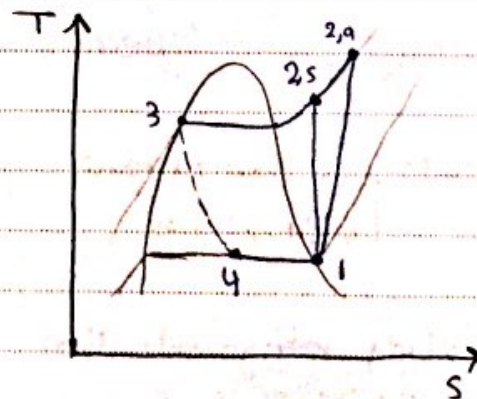
* $\text{COP}_{\text{table}} = 3.97$

$\text{COP}_{\text{charts}} = 4.14$

Actual cycle

$$\eta_{C,S} = \frac{h_{2,S} - h_1}{h_{2,A} - h_1}$$

⇒ If information is given at every point
or if the efficiency of the compressor
is given, then it's an actual cycle

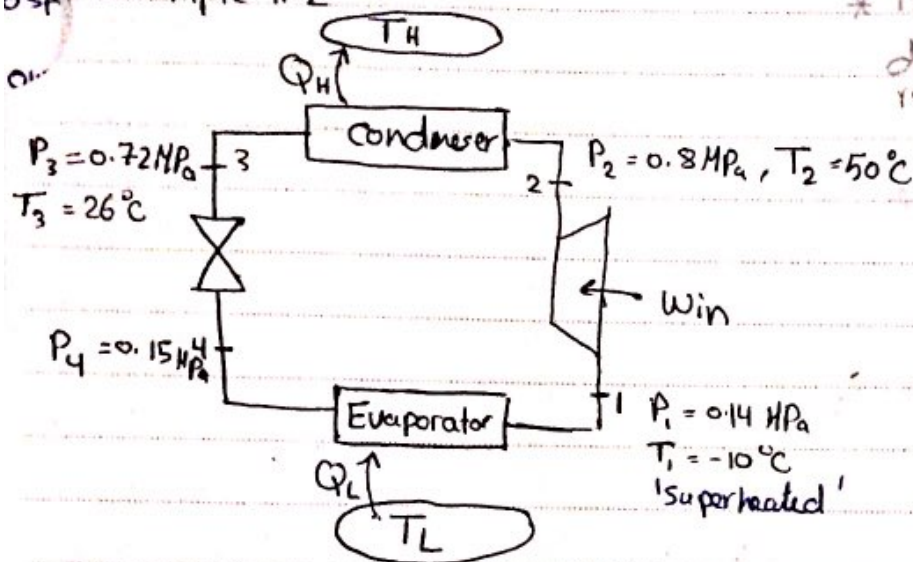


* In ideal cycle 2 pieces of information are given:

① P_{cond} and $P_{\text{evaporator}}$
or

② T_{cond} or $T_{\text{evaporator}}$

Example 11-2



* There is pressure drop due to friction between the refrigerant and the pipes of the fluid
* $\dot{m}_R = 0.05 \text{ kg/s}$

* If 2 properties are given at state 4, then find h_4 using tables
* If 1 property is given at ~~state 4~~ state 4, then assume $h_4 = h_3$

* using charts

$$h_1 = 295$$

$$h_2 = 338$$

$$h_3 = 136$$

$$h_4 = 136$$

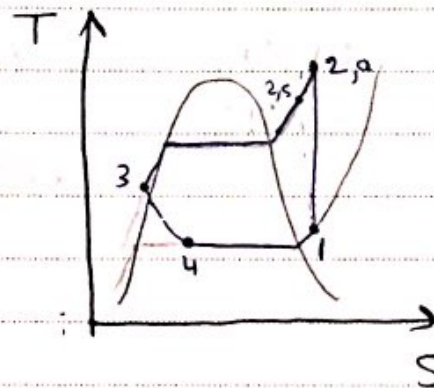
* using tables

$$h_1 = 246.37$$

$$h_2 = 286.71$$

$$h_3 = 87.83$$

$$h_4 = 87.83$$



(This sketch is not accurate)

Note: $h_3 = h_f @ T = 26^\circ\text{C}$, because it is subcooled liquid (we don't have subcooled liquid tables)

* Refrigerants → How to read properties

⇒ We will study refrigerants that consist of (C, F, Cl). They are called CFC (Chloro-Fluoro-Carbon). "Halogenes"

The origin of refrigerants

CH_4 "methane"

R_{ab}

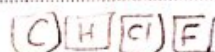
C_2H_6 "ethane"

R_{1ab}

$a-1 = \text{H}$ (the number of hydrogen atoms that are not replaced)

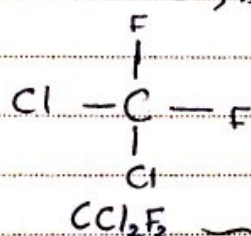
$b = \text{F}$ (the number of Fluorine atoms that are available)

e.g



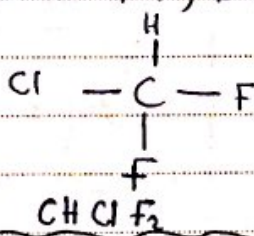
R_{12} (مركب 12)

$a-1 = 1-1 = 0, b = 2$



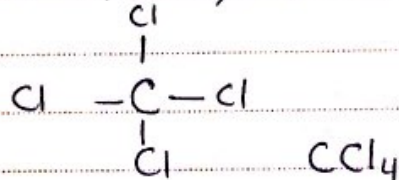
R_{22}

$a-1 = 2-1 = 1, b = 2$



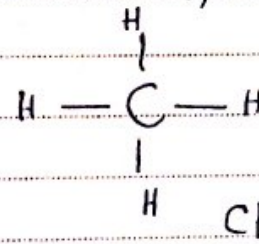
R_{10}

$a-1 = 1-1 = 0, b = 0$



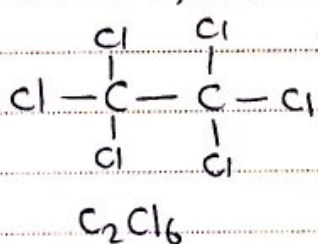
R_{50}

$a-1 = 5-1 = 4, b = 0$



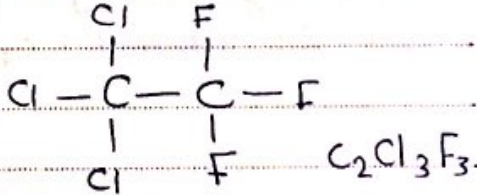
R_{110}

$a-1 = 1-1 = 0, b = 0$



R_{113}

$a-1 = 1-1 = 0, b = 3$



* Note. R_{134-a}

③ it indicates how atoms are ordered (or arranged) in the compound

In developed countries

Constructing malls

that use NH_3 as

a refrigerant in chillers

is not allowed

ليس مسموح باستخدامه

NH_3 → blindness
→ death
→ skin burns

ost * Innovative Ref. cycle

→ Cascade ref. cycle
Stages

* Gas Ref. cycle

→ Ammonia Absorption cycle

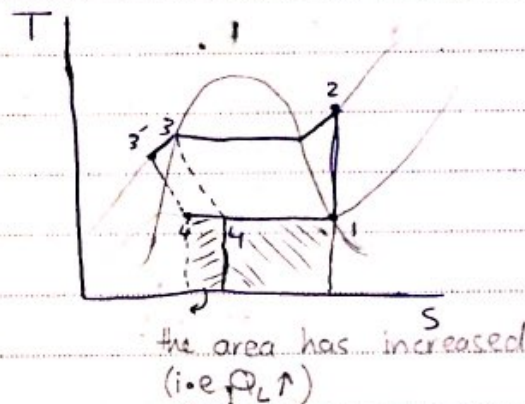
$$COP_R = \frac{Q_L \uparrow}{W_{cl} \downarrow}$$

① Subcool point 3

$$Q_L = m(h_1 - h_4)$$

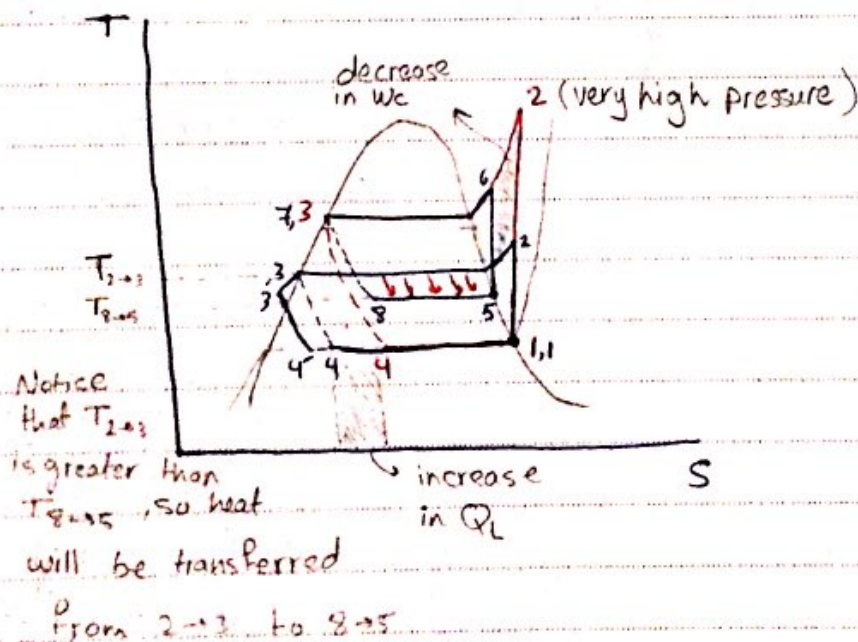
$$h_4 = h_f + x h_{fg}$$

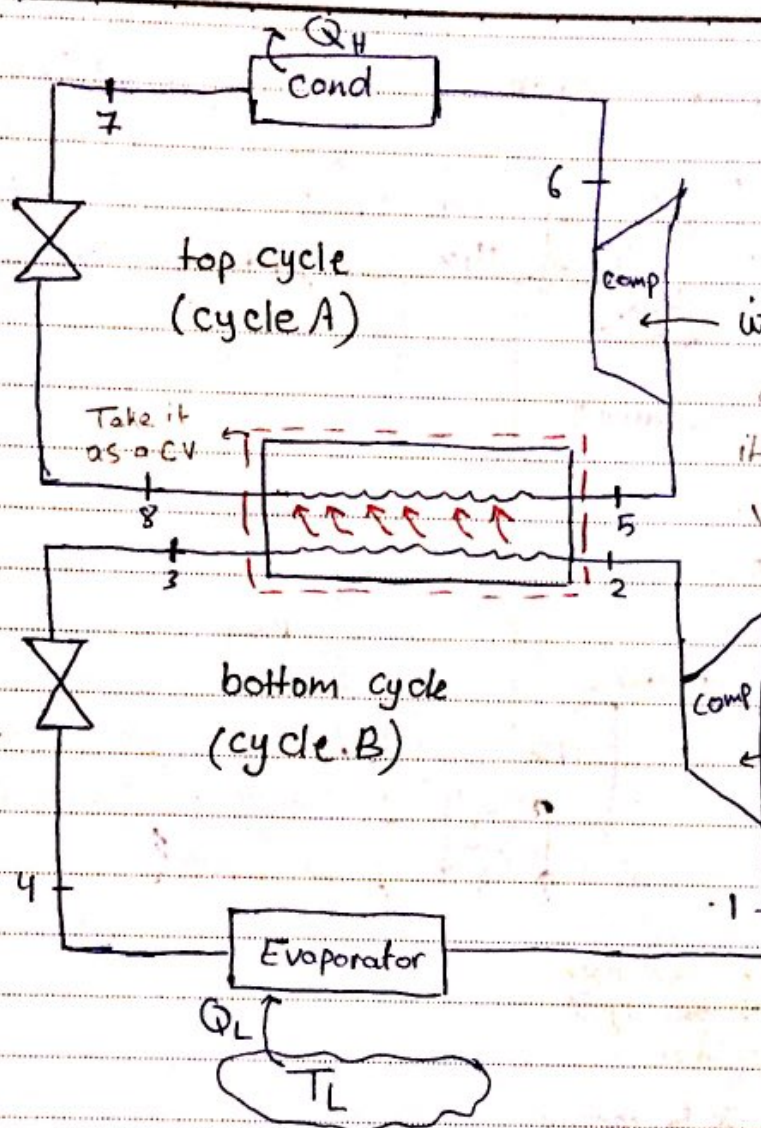
$$(as \ x \downarrow \Rightarrow h_4 \downarrow \Rightarrow (h_1 - h_4) \uparrow \Rightarrow Q_L \uparrow)$$



* Cascade refrigeration cycle

(it is used for industrial applications)

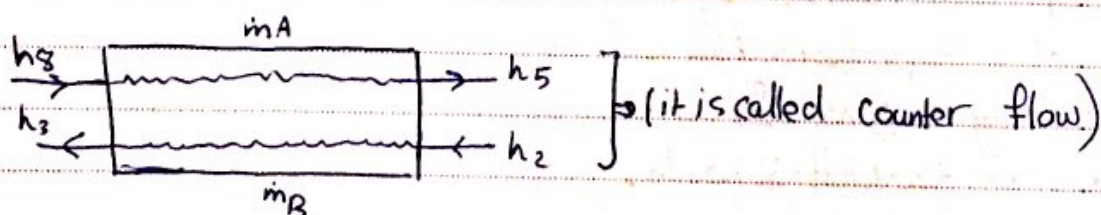




* c_p and h_{fg} for the working fluid in cycle A must be high. (So, w_c it will gain a great amount of heat before it is converted to sat. vapor) e.g. Water

* P_{8-5} is less than P_{2-3} (because the working fluid in the evaporator of cycle A is at lower temperature)

* c_p and h_{fg} for the working fluid in cycle B must be less. (This will cause state 3 to be subcooled as the working fluid is losing heat in the condenser) e.g. Refrigerant



(it is called counter flow)

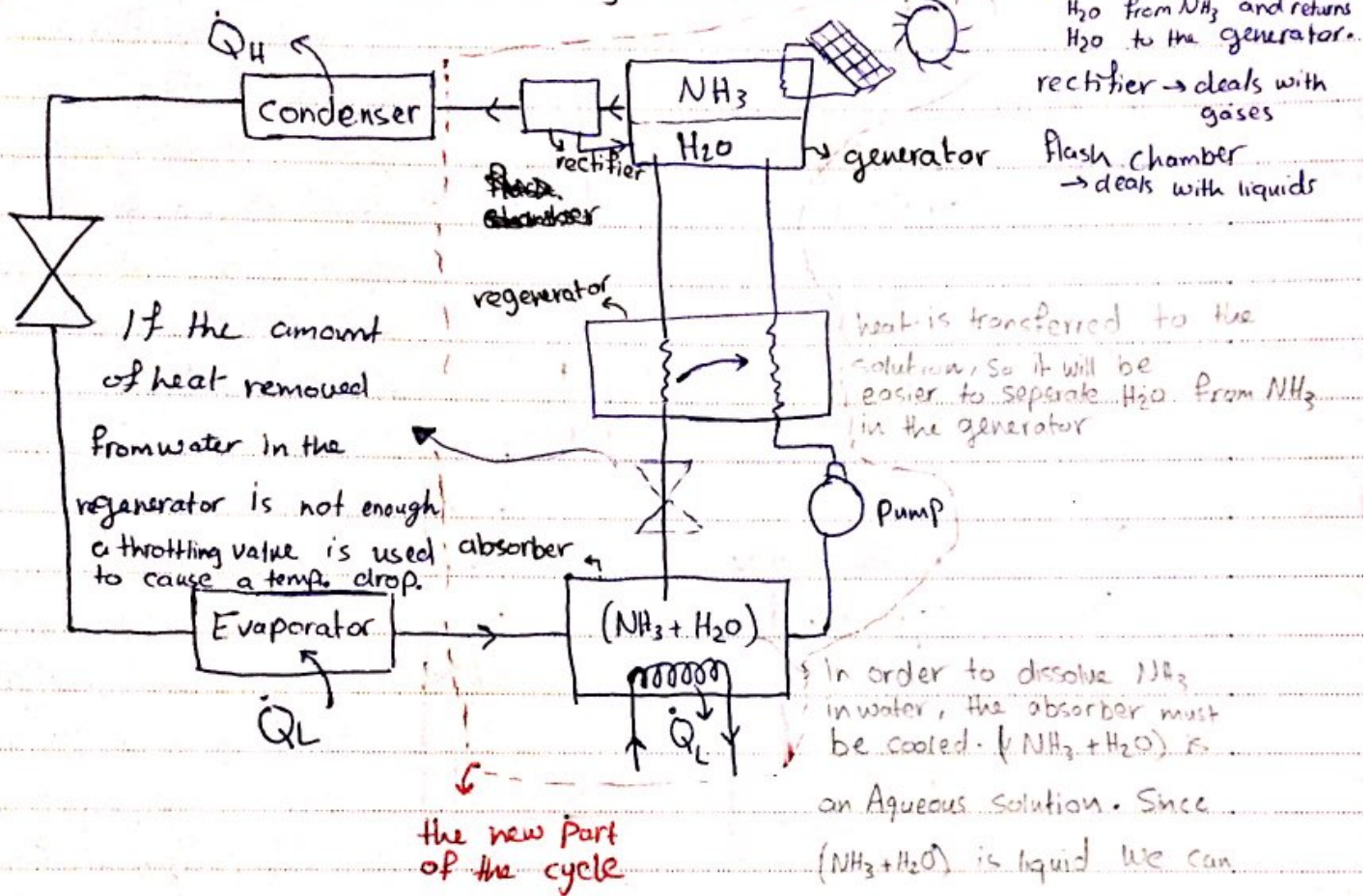
* Assume effectiveness = 100%

$$\text{Heat lost by (B)} = \text{Heat gained by (A)}$$

$$\dot{m}_B (h_2 - h_3) = \dot{m}_A (h_5 - h_8)$$

$$\therefore \frac{\dot{m}_B}{\dot{m}_A} = \frac{h_5 - h_8}{h_2 - h_3}$$

* Ammonia absorption cycle



* The goal of this cycle is to reduce the compression work by using a pump instead of a compressor. This can be achieved by looking for a refrigerant that can be dissolved in water, so pump can be used. (e.g. NH_3 is a refrigerant that can be dissolved in water)

* The absorber is used to dissolve NH_3 in water. the absorber must be cooled, so that NH_3 can be dissolved in water (dissolving NH_3 in water is an exothermic reaction)

* In the regenerator, heat is transferred from water to the aqueous solution. So water is cooled (it is ready now to enter the absorber and react with NH_3) and the aqueous solution is heated (it is ready to be separated in the generator using a small amount of solar energy)

$$COP = \frac{Q_L}{W_{pump}}$$

(This will increase very small the COP greatly)

$$\uparrow \text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_C} \uparrow$$

multistage comp
Cascade Ref. cycle

$$\dot{Q}_L = \dot{m}_R (h_1 - h_4)$$

$$h_4 = h_f + x_4 h_{fg}$$

$$\dot{W}_C = \dot{m}_R (h_2 - h_1)$$

$$h_1 = h_g$$

* If I want to increase $\dot{Q}_L$, I can increase $\dot{m}_R$.
Increasing $\dot{m}_R$ causes $\dot{W}_{\text{comp}}$ to increase. (this choice is excluded).
* I can increase h_1 , increasing h_1 causes h_2 to increase and $\dot{W}_{\text{comp}}$ will increase. (choose any 2 arbitrary pressures and notice how does h_2 change when h_1 changes. + η_c will increase $\dot{W}_{\text{comp}}$ will increase) this choice is also excluded)

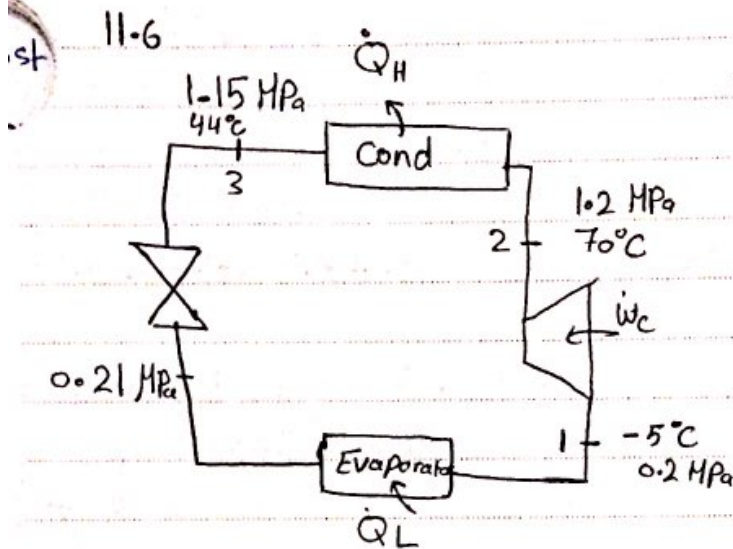
~~Decrease h_4 by either reducing x_4 or decreasing T_4~~

* Decrease h_4 by either reducing x_4 or decreasing T_4

* To decrease h_2 , one can compress the refrigerant by 2 stages instead of one stage (i.e. cascade refrigeration cycle)

* Increasing $\dot{m}_R$ causes $\dot{W}_{\text{comp}}$ to increase

+ ice formation inside the refrigerator which is not desirable (ice prevents or decreases the rate of heat transferred from between the evaporator and the stuff inside the refrigerator).



$$\dot{m}_R = 0.07 \text{ kg/s}$$

① at $P = 0.2 \text{ MPa}$

$$T = -10^\circ\text{C} \quad h = 244.56 \text{ kJ/kg}$$

$$T = 0^\circ\text{C} \quad h = 253.07 \text{ kJ/kg}$$

$$h_1 = 244.56 = \frac{253.07 - 244.56}{0 + 10} (-5 + 10)$$

$$h_1 = 248.8 \text{ kJ/kg}$$

$$S = 0.9381 \text{ kJ/kg} \cdot \text{K} \quad @ -10^\circ\text{C}$$

$$S = 0.9699 \text{ kJ/kg} \cdot \text{K} \quad @ 0^\circ\text{C}$$

$$S_1 = 0.9381 = \frac{0.9699 - 0.9381}{10} (-5 + 10)$$

$$S_1 = 0.954 \text{ kJ/kg} \cdot \text{K}$$

Find $\dot{Q}_L$, $\dot{W}_{\text{comp}}$, $\dot{m}_{\text{C,s}}$, COP

② State 2 is super heated

$$h_{2,a} = 300.63 \text{ kJ/kg} \quad S_{2,a} = 0.9939 \text{ kJ/kg} \cdot \text{K}$$

(notice that $S_2 > S_1$, which indicates that it is not an isentropic process)

$$\text{At } S_{2,s} = 0.954 \text{ kJ/kg} \cdot \text{K} \text{ and } P = 1.2 \text{ MPa} \quad (T_{2,s} \neq T_{2,a})$$

$$T = 50^\circ\text{C} \quad h = 278.28 \text{ kJ/kg} \quad S = 0.9268 \text{ kJ/kg} \cdot \text{K}$$

$$T = 60^\circ\text{C} \quad h = 289.66 \text{ kJ/kg} \quad S = 0.9615 \text{ kJ/kg} \cdot \text{K}$$

$$h = 289.66 = \frac{289.66 - 278.28}{0.9615 - 0.9268} * (0.954 - 0.9615)$$

$$h_{2,s} = 287.2 \text{ kJ/kg}$$

$$\dot{m}_{\text{C,s}} = \frac{h_{2,s} - h_1}{h_{2,a} - h_1} = \frac{287.2 - 248.8}{300.63 - 248.8} = 74.1\%$$

③

$$P = 1000 \text{ kPa}, T_{\text{sat}} = 39.37^\circ\text{C}$$

$$P = 1200 \text{ kPa}, T_{\text{sat}} = 46.26^\circ\text{C}$$

$$T = 39.37 = \frac{46.26 - 39.37}{1200 - 1000} (1150 - 1000)$$

$$T_{\text{sat}} = 44.5375^\circ\text{C} > 44.8^\circ\text{C}$$

$\therefore$ State 3 is subcooled

$$h_{f,3} = 114.3 \text{ kJ/kg} \quad S_3 = 0.41371 \text{ kJ/kg} \cdot \text{K}$$

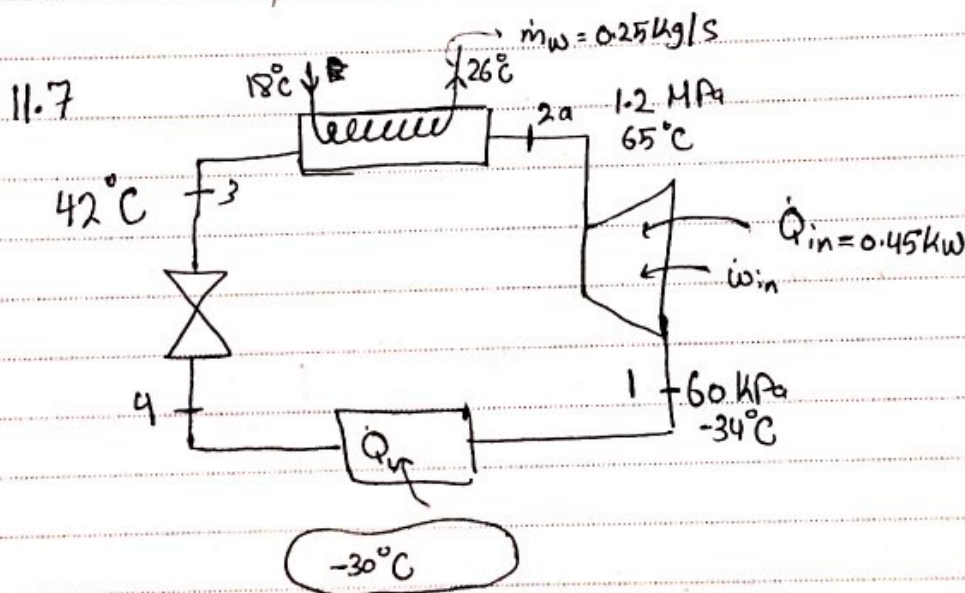
$$(4) h_4 = h_3 = 114.3 \text{ kJ/kg}$$

$$\dot{Q}_L = \dot{m}_R (h_1 - h_4) = 0.07 (248.8 - 114.3) = 9.415 \text{ kW}$$

$$\dot{W}_{\text{comp}} = \dot{m}_R (h_{2,s} - h_1) = 0.07 (300.63 - 248.8) = 3.6281 \text{ kW}$$

$$\dot{Q}_H = \dot{m}_R (h_{2,s} - h_3) = 0.07 (300.63 - 114.3) = 13.0431 \text{ kW} = \dot{Q}_L + \dot{W}_{\text{comp}}$$

$$\text{COP}_{R} = \frac{\dot{Q}_L}{\dot{W}_{\text{comp}}} = \frac{9.415}{3.6281} = 2.6$$



Find x_4 , $\dot{Q}_L$
COP, $\dot{Q}_{L,\text{Carnot}}$

① State 1 is superheated $T_{\text{sat}} @ 60 \text{ kPa} = -36.95^\circ\text{C} < -34^\circ\text{C}$

$$T = -36.95^\circ\text{C} \quad h = 227.8 \text{ kJ/kg} \quad s = 0.9645 \text{ kJ/kg}\cdot\text{K}$$

$$T = -20^\circ\text{C} \quad h = 240.78 \text{ kJ/kg} \quad s = 1.0175 \text{ kJ/kg}\cdot\text{K}$$

$$h_1 = 230.059 \text{ kJ/kg}$$

$$s_1 = 0.9737 \text{ kJ/kg}\cdot\text{K}$$

② State 2 is superheated $T_{\text{sat}} = 46.29^\circ\text{C} < 65^\circ\text{C}$

$$T = 60^\circ\text{C} \quad h = 289.66 \text{ kJ/kg} \quad s = 0.9615 \text{ kJ/kg}\cdot\text{K}$$

$$T = 70^\circ\text{C} \quad h = 300.63 \text{ kJ/kg} \quad s = 0.9939 \text{ kJ/kg}\cdot\text{K}$$

$$h_2 = 295.145 \text{ kJ/kg} \quad s_2 = 0.9777 \text{ kJ/kg}\cdot\text{K}$$

3) Subcooled

$$h_3 = 111.28 \text{ kJ/kg} \quad s_3 = 0.40432 \text{ kJ/K.kg}$$

$$4) h_4 = h_3 = 111.28 \text{ kJ/kg}$$

$$P = 60 \text{ kPa}$$

$$h_f = 3.837 \text{ kJ/kg} \quad h_{fg} = 223.96 \text{ kJ/kg}$$

$$h_4 = h_f + x_4 h_{fg}$$

$$x_4 = \frac{111.28 - 3.837}{223.96} = 0.4795$$

* In the condenser

the heat gained by water = the heat lost by the refrigerant

$$\dot{m}_w (h_{f, 30^\circ\text{C}} - h_{f, 15^\circ\text{C}}) = \dot{m}_R (h_{2,s} - h_3) \Rightarrow \dot{m}_R = \frac{0.25 (109.012 - 75.5418)}{295.145 - 111.28} = 0.04551 \text{ kg/s}$$

$T(^{\circ}\text{C})$ $h_f \text{ (kJ/kg)}$

15 62.982

18 75.5418

20 83.915

25 104.83

26 109.012

30 125.74

$$\dot{Q}_H = 8.36755 \text{ kW}$$

$$* \dot{Q}_L = \dot{m}_R (h_1 - h_4) = 0.04551 (230.059 - 114.38) = 5.4557 \text{ kW}$$

$$* \dot{Q}_L + \dot{W}_{\text{comp}} + \dot{Q}_{\text{in}} = \dot{Q}_H$$

$$\dot{W}_{\text{comp}} = 8.36755 - 5.4557 - 0.45 = 2.51185 \text{ kW}$$

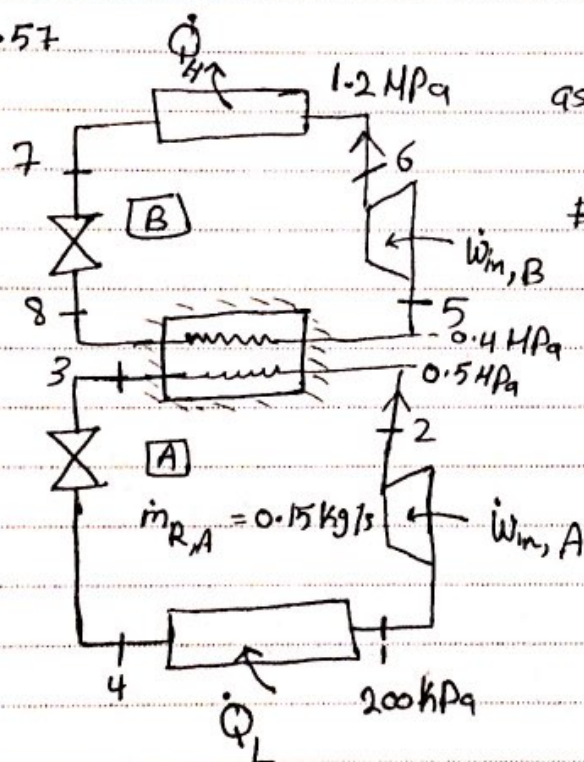
$$* COP = \frac{\dot{Q}_L}{\dot{W}_{comp}} = \frac{5.4857}{2.5185} = 2.2195$$

$$T_L = -30 + 273 \text{ K}, T_H = 18 + 273 \text{ K} = 291 \text{ K}$$

$$COP = \frac{T_L}{T_H - T_L} = 5.0625$$

$$\dot{Q}_L = COP \times \dot{W}_{comp} = 5.0625 \times 2.5185 = 12.7462 \text{ kW}$$

11.57



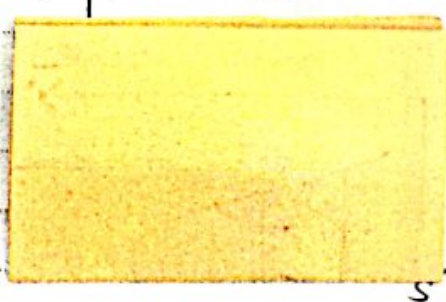
assume $\eta_{s,c} = 80\%$

* Heat rejection from the lower cycle to the upper cycle takes place in an adiabatic counter flow HEX

state 8 is very cold. I want to use the

coldness to condense the refrigerant

at state 2. if it is not insulated heat will be transferred to state 8 from the atmosphere (i.e. the effectiveness will ↓)



$$① h_1 = 244.50 \text{ kJ/kg}, s_1 = 0.93788 \text{ kJ/kg} \cdot \text{K}$$

$$② P_2 = 0.5 \text{ MPa}, s_{2,s} = 0.93788 \text{ kJ/kg} \cdot \text{K}$$

$$0.80 = \frac{263.3 - 244.50}{h_{2,a} - 244.50}, h_{2,a} = 268 \text{ kJ/kg}$$

s (kJ/kg·K)	h (kJ/kg)
0.9242	259.36

0.93788	263.3
---------	-------

0.9384	263.48
--------	--------

ost ③ $h_3 = 73.32 \text{ kJ/kg}$ ④ $h_4 = h_3 = 73.32 \text{ kJ/kg}$

⑤ $h_5 = 255.61 \text{ kJ/kg}$ $s_5 = 0.9271 \text{ kJ/kg}$

⑥ $P_6 = 1.2 \text{ MPa}$ $s_{6,s} = s_5$ (superheated) because $s_{6,s} > s_g @ 1.2 \text{ MPa}$

$$m_{c,s} = \frac{h_{6,s} - h_s}{h_{6,a} - h_s}$$

$s \text{ kJ/kg}$ $w \text{ kJ/kg}$

0.9268 278.28

0.9271 278.38

$0.8 = \frac{278.38 - 255.61}{h_{6,a} - 255.61}$ $h_{6,a} = 284.07 \text{ kJ/kg}$ $\left\{ \begin{array}{l} 0.9615 \\ 289.66 \end{array} \right.$

⑦ $h_7 = 117.79 \text{ kJ/kg}$ ⑧ $h_8 = h_7 = 117.79 \text{ kJ/kg}$

Take the HEX as a CV

$$\dot{m}_{R,A} (-h_3 + h_2) = \dot{m}_{R,B} (h_5 - h_8)$$

$$\dot{m}_{R,B} = \frac{0.15 (-73.32 + 268)}{255.61 - 117.79} = 0.212 \text{ kg/s}$$

$$\dot{Q}_L = \dot{m}_{R,A} (h_1 - h_4) = 0.15 (244.50 - 73.32) = 25.7 \text{ kW}$$

$$\dot{W}_{c,tot} = \dot{m}_{R,A} (h_{2,a} - h_1) + \dot{m}_{R,B} (h_{6,a} - h_5)$$

$$= 0.15 (268 - 244.50) + 0.212 (284.07 - 255.61) = 9.56 \text{ kW}$$

$$COP = \frac{\dot{Q}_L}{\dot{W}_c} = \frac{25.7}{9.56} = 2.68$$

$$\dot{X}_{\text{dest}} = \dot{m}_R T_0 \left[s_f - s_i - \frac{q_L}{T_L} + \frac{q_H}{T_H} \right]$$

know how
to use them

$T_0 = T_H$ for the ref
 $T_0 > T_L$ for a heat pump

$$\dot{X}_{\text{dest cycle}} = \overset{\text{expended}}{\dot{W}_{\text{in,act}}} - \overset{\text{recovered}}{\dot{X}_{Q_L}}$$

$$\dot{X}_{Q_L} = \dot{Q}_L \left(\frac{T_0 - T_L}{T_L} \right)$$

$$\text{COP} = \frac{\dot{Q}_L}{\dot{W}_{\text{in}}} = \frac{T_L}{T_H - T_L}$$

Chapter 13: Gas mixtures

• How to find mixtures properties

→ What are the components

→ concentration for each component

→ Individual properties

• To specify the concentration of each component

moles (N)

mole fraction (y)

→ This analysis is called
molar analysis

(on volume basis)

mass (m)

mass fraction (w)

→ This analysis is called
Gravimetric analysis

on weight basis

gases in a chamber

A	m_A	N_A
B	m_B	N_B
C	m_C	N_C
D	m_D	N_D

• Total mass = $m_{mix} = m_A + m_B + m_C + m_D$
 or m_{total}

$$= \sum_{i=1}^n m_i$$

• Total moles = $N_{mix} = N_A + N_B + N_C + N_D$
 or N_{total}

$$= \sum_{i=1}^n N_i$$

• Mass fraction for the gas A (m_{fA}) = $\frac{m_A}{m_{mix}}$

In General

$$m_{f_i} = \frac{m_i}{m_{mix}}, \quad \sum m_{f_i} = 1$$

• Mole fraction for the gas A (y_A) = $\frac{N_A}{N_{mix}}$

In General

$$y_i = \frac{N_i}{N_{mix}}, \quad \sum y_i = 1$$

• Relationship between m_{f_i} , y_i

$$m_{f_i} = \frac{m_i}{m_{mix}} = \frac{N_i \cdot M_i}{\sum N_i M_i} \Rightarrow \text{divide by } N_{mix}$$

Recall $m = N \cdot M$

$$= \frac{y_i M_i}{\sum y_i M_i}$$

$\downarrow$
 M_{mix}

$$y_i = \frac{N_i}{N_{\text{mix}}} = \frac{m_i/M_i}{\sum \left(\frac{m_i}{M_i} \right)} \Rightarrow \text{divide by } m_{\text{mix}}$$

$$= \frac{m_{f_i}}{M_i} \cdot \frac{1}{\sum \frac{m_{f_i}}{M_i}}$$

$$M_{\text{mix}} = \frac{m_{\text{mix}}}{N_{\text{mix}}} = \frac{\sum N_i H_i}{N_{\text{mix}}} = \sum y_i M_i$$

$$R_{\text{mixture}} = \frac{R_u}{H_{\text{mix}}} \rightarrow 8.314 \text{ kJ/kmol}\cdot\text{K}$$

Extensive properties

H

U $X_{\text{mix}} = \sum X_i = \sum (m_i x_i) = \sum (N_i \bar{x}_i)$

S e.g.

m $H_{\text{mix}} = \sum H_i = \sum \left(\underset{\substack{\downarrow \\ \text{kg}}}{m_i} \underset{\substack{\downarrow \\ \frac{\text{kJ}}{\text{kg}}}}{h_i} \right) = \sum \left(\underset{\substack{\downarrow \\ \text{kmol}}}{N_i} \underset{\substack{\downarrow \\ \frac{\text{kJ}}{\text{kmol}}}}{\bar{h}_i} \right)$

$$H_{\text{mix}} = m_{\text{mix}} h_{\text{mix}} = \sum m_i h_i$$

$$\frac{H_{\text{mix}}}{m_{\text{mix}}} = h_{\text{mix}} = \sum m_{f_i} h_i$$

Intensive properties

$$x_{\text{mix}} = \sum m_{f_i} x_i$$

$$\bar{x}_{\text{mix}} = \sum y_i \bar{x}_i$$

Examples

$$\begin{aligned}
 \bullet C_{p,mix} &= \sum m_{f,i} C_{p,i} & \bullet M_{mix} &= \sum y_i M_i \\
 \bullet u_{mix} &= \sum m_{f,i} u_i & \bullet k_{mix} &= \frac{C_{p,mix}}{C_{v,mix}} = \frac{\sum m_{f,i} C_{p,i}}{\sum m_{f,i} C_{v,i}}
 \end{aligned}$$

Example 13-1

O ₂	3 kg
N ₂	5 kg
CH ₄	12 kg

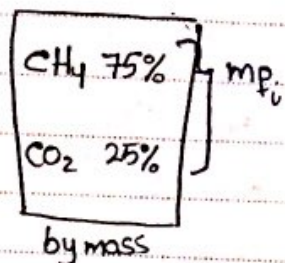
Components	Given m (kg)	M _i (kg/kmol)	N _i (kmol)	m _{f,i} = $\frac{m_i}{m_{mix}}$	y _i = $\frac{N_i}{N_{mix}}$
O ₂	3	32	$\frac{3}{32} = 0.094$	0.15	0.092
N ₂	5	28	$\frac{5}{28} = 0.179$	0.25	0.175
CH ₄	12	16	$\frac{12}{16} = 0.75$	0.6	0.733
	20 kg = m _{mix}		N _{mix} = 1.023		

$$\begin{aligned}
 R_{mix} &= \frac{R_u}{M_{mix}} & M_{mix} &= \frac{m_{mix}}{N_{mix}} = 19.6 \text{ kJ/kmol} \\
 &= \frac{8.314}{19.6} = 0.424 \text{ kJ/kg} \cdot \text{K}
 \end{aligned}$$

* Check questions 10-11-28-40-50-52

13-10 Find y_i

* Assume $m_{mix} = 100 \text{ kg}$



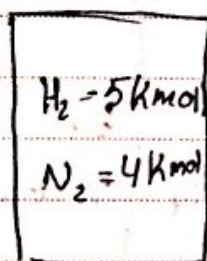
Component	m_i	H_i	N_i	$y_i = \frac{N_i}{N_{mix}}$
CH ₄	75	16	$\frac{75}{16} = 4.68$	0.892
CO ₂	25	44	$\frac{25}{44} = 0.568$	0.108
	$\Sigma m_{mix} = 100 \text{ kg}$		$N_{mix} = 5.256 \text{ kmol}$	$\Sigma y_i = 1$

$$M_{mix} = \Sigma y_i H_i$$

$$= \frac{m_{mix}}{N_{mix}} = 19.03 \text{ kJ/kmol}$$

$$R_{mix} = \frac{R_u}{M_{mix}} = \frac{8.314}{19.03} = 0.437 \text{ kJ/kg} \cdot \text{K}$$

13-11 Find M_{mix} , R_{mix}



Component	$N_i \text{ (kmol)}$	H_i	$m_i \text{ (kg)}$	$mf_i = \frac{m_i}{m_{mix}}$
H ₂	5	2	10	0.082
N ₂	4	28	112	0.918
	$N_{mix} = 9 \text{ kmol}$		$m_{mix} = 122 \text{ kg}$	$\Sigma mf_i = 1$

$$M_{mix} = \frac{m_{mix}}{N_{mix}} = \frac{122}{9} = 13.56 \text{ kJ/kmol}$$

$$R = \frac{R_u}{M_{mix}} = 0.613 \text{ kJ/kg} \cdot \text{K}$$

13-28 Find H_{mix} , R_{mix}

* Assume $N_{mix} = 1 \text{ kmole}$

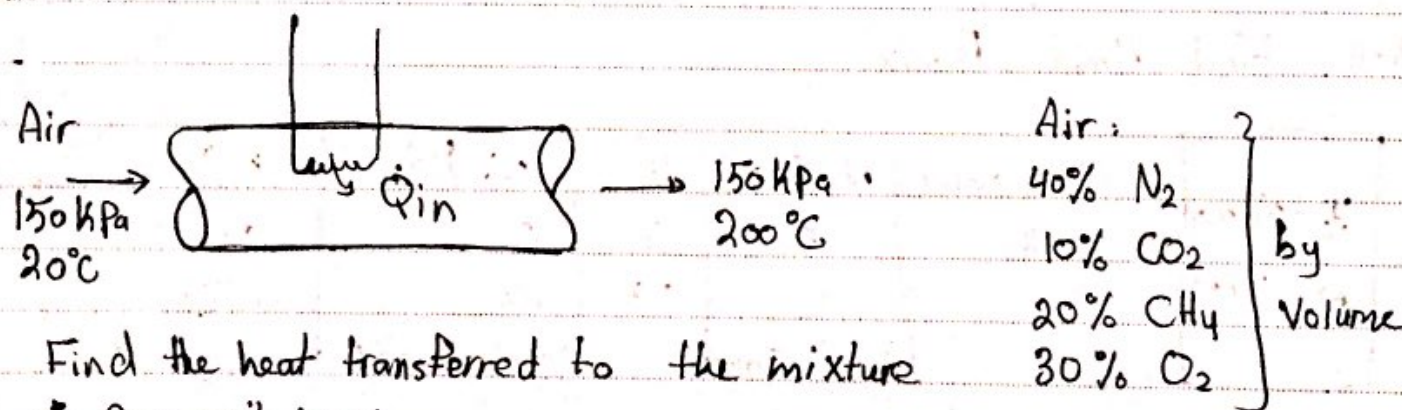
H_2 30%
He 40%
 N_2 30%
by Volume

Component	N_i	H_i	$m_i = N_i H_i \text{ (kg)}$	$m f_i = \frac{m_i}{m_{mix}}$
H_2	0.3	2	0.6	0.0566
He	0.4	4	1.6	0.1509
N_2	0.3	28	8.4	0.7924
$N_{mix} = 1 \text{ kmole}$			$m_{mix} = 10.6 \text{ kg}$	

$$H_{mix} = \frac{10.6}{1} = 10.6 \text{ kg/kmol}$$

$$R = \frac{R_u}{H_{mix}} = \frac{8.314}{10.6} = 0.7843 \text{ kJ/kg}\cdot\text{K}$$

13-50



Find the heat transferred to the mixture

Per unit mass

$$q_{in} = h_2 - h_1 = C_{p,mix} (T_2 - T_1), \quad C_{p,mix} = \sum m f_i C_{p,i}$$

Assume $N_{mix} = 100 \text{ kmol}$

Component	$N_i \text{ (kmol)}$	$M_i \text{ (kg/kmol)}$	$m_i = N_i M_i \text{ (kg)}$	m_f	C_{p_i}	$m_f C_{p_i}$
N_2	40	28	1120	0.3943	1.039	0.4696
CO_2	10	44	440	0.155	0.8469	0.1315
CH_4	20	16	320	0.1126	2.2537	0.2539
O_2	30	32	960	0.338	0.918	0.313
$N_{mix} = 100 \text{ kmol}$			$m_{mix} = 2840$			

$$C_{p_{mix}} = 1.1051 \text{ kJ/kg.K}$$

$$q_{in} = 1.1051 \times (200 - 20) = 200 \text{ kJ/kg}$$

$$\Delta T (^{\circ}\text{C}) = \Delta T (\text{K})$$

13.52*

H_2 30%
 He 40%
 N_2 30%

by Volume

* The gas mixture ~~and~~ undergoes in an isentropic expansion

5 MPa $600^{\circ}\text{C} \rightarrow 200 \text{ kPa}$

Find the workdone per unit mass of the mixture

Assume it is a closed system

$$W_{exp} = C_{v_{mix}} (T_1 - T_2)$$

$$C_{v_{mix}} = \sum m_{f_i} C_{v_i}$$

* Assume $N_{mix} = 100 \text{ kmol}$

Component	$N_i \text{ (kmol)}$	$M_i \text{ (kg/kmol)}$	$m_i = N_i M_i \text{ (kg)}$	m_{f_i}	$C_{v_i} \text{ (kJ/kg.K)}$
H_2	30	2	60	0.0566	10.183
He	40	4	160	0.1509	3.1156
N_2	30	28	840	0.7925	0.743

$$m_{mix} = 1060 \text{ kg}$$

C_{p_i}	$m_{p_i} C_{v_i}$	$C_{p_i} \times m_{p_i}$
14.307	0.5764	0.8098
5.1926	0.4701	0.7836
1.039	0.5888	0.8234
$C_{v_{mix}} = 1.6353 \text{ kJ/k}\cdot\text{kg}$		$C_{p_{mix}} = 2.4168 \text{ kJ/k}\cdot\text{kg}$

$$R_{mix} = C_{p_{mix}} - C_{v_{mix}} = 0.7815 \text{ kJ/k}\cdot\text{kg}$$

$$k_{mix} = \frac{C_{p_{mix}}}{C_{v_{mix}}} = 1.48$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k_{mix}-1}{k_{mix}}} \quad T_2 = (600 + 273) \times \left(\frac{200}{5 \times 10^3}\right)^{\frac{1.48-1}{1.48}} = 307 \text{ K}$$

$$W_{exp} = C_{v_{mix}} (T_1 - T_2) = 1.6353 \times (873 - 307) = 925 \text{ kJ/kg}$$

13-56

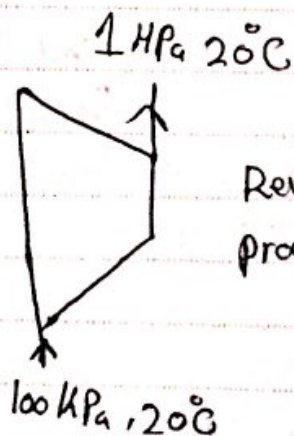
Gas mixture:

60% CH_4

25% C_3H_8

15% C_4H_{10}

by weight =
by mass



$PV = \text{Const}$

Reversible isothermal process, Find W_{in}

isothermal $\Rightarrow \Delta h = 0$

$$W_{in} = q_{out} = R_{mix} T \ln\left(\frac{P_2}{P_1}\right)$$

$\Rightarrow$ follow

Assume $m_{mix} = 100 \text{ kg}$

Component	$m_i \text{ (kg)}$	$M_i \text{ (kg/kmol)}$	$N_i = \frac{m_i}{M_i} \text{ (kmol)}$
CH_4	60	16	3.75
C_3H_8	25	44	0.5681
C_4H_{10}	15	58	0.2586
$m_{mix} = 100 \text{ kg}$			$N_{mix} = 4.5767 \text{ kmol}$

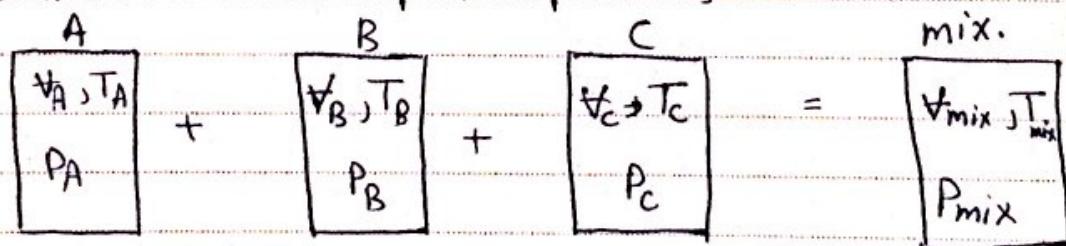
$$H_{mix} = \frac{m_{mix}}{N_{mix}} = \frac{100}{4.5767} = 21.85 \text{ kg/kmol}$$

$$R_{mix} = \frac{R_u}{H_{mix}} = \frac{8.314}{21.85} = 0.3805 \text{ kJ/k.kg}$$

$$W_{in} = q_{out} = R_{mix} T \ln \left(\frac{P_2}{P_1} \right) = 0.3805 * (20+273) * \ln \left(\frac{1 \times 10^3}{100} \right) = 257 \text{ kJ/kg}$$

P-V-T behavior of gas mixtures

(I) Dalton's law of partial pressures



$$V_A = V_B = V_C = V_{mix}, T_A = T_B = T_C = T_{mix}$$

$$\text{then } P_{mix} = P_A + P_B + P_C$$

$$P_{\text{mix}} V_{\text{mix}} = n_{\text{mix}} T_{\text{mix}} + n_{\text{ext}} T_{\text{ext}}$$

$$P_{\text{mix}} = \frac{P_1 V_1}{V_{\text{mix}}}$$

$$P_{\text{mix}} V_{\text{mix}} = n_{\text{mix}} \cdot \frac{R T_{\text{mix}}}{P_{\text{mix}}} + T_{\text{ext}} + n_{\text{ext}} \cdot \frac{R T_{\text{ext}}}{P_{\text{mix}}}$$

we notice that if $T \neq T_{\text{ext}}$ we must have the pressure is a function of the number of moles n

$$P_1 = P_2 = P_3 = P_4 = T_1$$

$$\frac{P_1}{P_{\text{mix}}} = \frac{n_1 \cdot R \cdot T_{\text{mix}}}{V_{\text{mix}}} \Rightarrow \frac{P_1}{P_{\text{mix}}} = \frac{n_1}{n_{\text{mix}}} = y_1$$

Equivalent pressure at partial pressure

$$P_{\text{mix}} = \sum_{i=1}^n P_i \text{ when } V \text{ and } T \text{ are const}$$

① Dalton's law of partial pressure



$$P_1 = P_2 = P_3 = P_4 = T_1 = T_2 = T_3 = T_4 = T_{\text{mix}}$$

$$P_{\text{mix}} V_{\text{mix}} = P_1 V_1 + P_2 V_2 + P_3 V_3$$

$$V_A = \frac{N_A * R_u * T_{mix}}{P_{mix}}$$

$$V_{mix} = \frac{N_{mix} * R_u * T_{mix}}{P_{mix}}$$

$$\frac{V_A}{V_{mix}} = \frac{N_A}{N_{mix}} = y_i \quad \text{When } T \text{ and } P \text{ are const}$$

→ This is why "on Volume basis" means molar analysis

In General (only for ideal gases)

$$\frac{P_i}{P_{mix}} = \frac{N_i}{N_{mix}} = y_i, \quad \frac{V_i}{V_{mix}} = \frac{N_i}{N_{mix}} = y_i$$

"For const V, T "

"For const P, T "

General ideal gas law

$$PV = ZNR_uT, \quad \text{for ideal gases } Z=1$$

low P high T

* In ideal gases, the molecules are far from each other, and they are not affected by each other

$$* P_R = \frac{P}{P_{cr}} \rightarrow \text{Table A-1}$$

* Compressibility Charts can be used for any gas

* Every division in the x-axis is 0.05 and every division in the y-axis is 0.01 in the chart

How to find Z_{mix} ?

First method: Amagat's rule

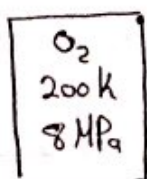
- * Find y_i for each gas
- * Find P_R, T_R for each gas
- * Find z for each gas
- * $Z_{mix} = \sum y_i z_i$
- * Then you can use $PV = ZNR_uT$

2nd method: Kay's rule

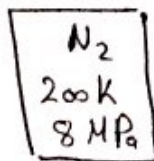
- * Find y_i for each gas
- * Find Pseudo (P'_{CR}, T'_{CR})
 - $T'_{CR} = \sum y_i T_{CR}$
 - $P'_{CR} = \sum y_i P_{CR}$

$$\begin{aligned} * T_{Rmix} &= \frac{T_{mix}}{T'_{CR}} \\ P_{Rmix} &= \frac{P_{mix}}{P'_{CR}} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Find } Z_{mix}$$

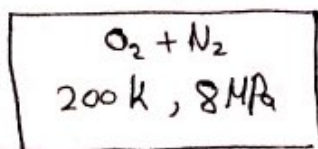
13-37



$V = 0.3 \text{ m}^3$



$V = 0.5 \text{ m}^3$



* Find V_{mix} if we are consider ideal gases? real gases?

1) Ideal gases; by Amagat's law

$$V_{mix} = \sum V_i = 0.8 \text{ m}^3$$

* Note: Amagat's law is only applicable for ideal gases

1) using the ideal gas
since $(T_i, P_i) = (T_{mix}, P_{mix})$

using Amagat's law

$$V_{mix} = \sum V_i \quad (P_i, T_i) = 0.8 \text{ m}^3$$

or

$$V_{mix} = \frac{N_{mix} * R_u * T_{mix}}{P_{mix}}$$

$$= \frac{3.849 * 8.314 * 200}{8000}$$

$$= 0.8 \text{ m}^3$$

$$* N_{N_2} = \frac{V_{N_2} * P_{N_2}}{R_u * T_{N_2}}$$

$$= \frac{0.35 * 8000}{8.314 * 200}$$

$$= 2.405 \text{ kmol}$$

$$* N_{O_2} = \frac{0.3 * 8000}{8.314 * 200}$$

$$= 1.443 \text{ kmol}$$

$$N_{mix} = \sum N_i = 3.849 \text{ kmol}$$

2) Real gases

I) Amagat's rule:

Find y_i

$$y_{O_2} = \frac{1.443}{3.849} = 0.375$$

$$y_{N_2} = \frac{2.406}{3.849} = 0.625$$

[2] Find T_R and P_R for each gas

* for O_2 (from table A-1)

$$T_{CR} = 154.8 \text{ K} \quad P_{CR} = 5.08 \text{ MPa}$$

$$T_R = \frac{T}{T_{CR}} = \frac{200}{154.8} = 1.3, \quad P_R = \frac{P}{P_{CR}} = \frac{8}{5.08} = 1.6$$

* for N_2

$$T_{CR} = 126.2 \text{ K} \quad P_{CR} = 3.39 \text{ MPa}$$

$$T_R = \frac{200}{126.2} = 1.6 \quad P_R = \frac{8}{3.39} = 2.35$$

[3] Find z for each gas

$$z_{O_2} = 0.77, \quad z_{N_2} = 0.86$$

[4] Find z_{mix}

$$z_{mix} = \sum y_i z_i = 0.375 * 0.77 + 0.625 * 0.86 \\ = 0.826$$

$$P_{mix} v_{mix} = z_{mix} N_{mix} R_u T_{mix}$$

$$v_{mix} = z_{mix} * \underbrace{(N_{mix} R_u T_{mix})}_{P_{mix}}^{0.8} = 0.826 * 0.8 = 0.661 \text{ m}^3$$

II) Kay's rule

[1] Find y_i for each gas $y_{O_2} = 0.375, \quad y_{N_2} = 0.625$

[2] Find T_{cr}, P_{cr} for each gas

for O_2 $T_{cr} = 154.8 \text{ K}, \quad P_{cr} = 5.08 \text{ MPa}$

for N_2 $T_{cr} = 126.2 \text{ K}, \quad P_{cr} = 3.39 \text{ MPa}$

[3] Find T_{cr}, P_{cr}

$$T_{cr, mix} = \sum y_i T_{cr} = 0.375 * 154.8 + 0.625 * 126.2 \\ = 137 \text{ K}$$

$$P_{cr, mix} = \sum y_i P_{cr} = 0.375 * 5.08 + 0.625 * 3.39 = 4.02 \text{ MPa}$$

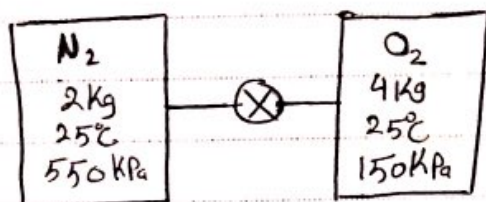
[4] Find T_R, P_R of the mixture

$$T_R = \frac{200}{137} = 1.45$$

$$P_R = \frac{8}{4.02} = 1.99 \approx 2 \rightarrow z_{mix} = 0.82$$

$$v_{mix} = z_{mix} * 0.8 = 0.656 \text{ m}^3$$

13-32



determine the volume of each tank and the final mixture pressure

* Before mixing:

$$M_{O_2} = 32 \text{ kg/kmol}$$

$$N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{4}{32} = 0.125 \text{ kmol}$$

$$V_{O_2} = \frac{N_{O_2} * R_u * T_{O_2}}{P_{O_2}} = \frac{0.125 * 8.314 * (25 + 273)}{150} = 2.07 \text{ m}^3$$

OR

$$R_{O_2} = \frac{R_u}{M_{O_2}} = \frac{8.314}{32} = 0.2598 \text{ kJ/k.Kg or (from table A-1)}$$

$$V_{O_2} = \frac{m_{O_2} * R_{O_2} * T_{O_2}}{P_{O_2}} = \frac{4 * 0.2598 * (25 + 273)}{150} = 2.07 \text{ m}^3$$

For N_2

$$M_{N_2} = 28 \text{ kg/kmol}, N_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{2}{28} = 0.07143 \text{ kmol}$$

$$V_{N_2} = \frac{N_{N_2} * R_u * T_{N_2}}{P_{N_2}} = \frac{0.07143 * 8.314 * (25 + 273)}{550} = 0.322 \text{ m}^3$$

OR

$$R_{N_2} = 0.2968 \text{ kJ/k.Kg}$$

$$V_{N_2} = \frac{m_{N_2} * R_{N_2} * T_{N_2}}{P_{N_2}} = \frac{2 * 0.2968 * (25 + 273)}{550} = 0.322 \text{ m}^3$$

* After mixing:

$$V_{\text{mix}} = V_{O_2} + V_{N_2} = 2.392 \text{ m}^3$$

$$N_{\text{mix}} = N_{O_2} + N_{N_2} = 0.125 + 0.07143 = 0.19643 \text{ kmol}$$

$$P_{\text{mix}} = \frac{N_{\text{mix}} * R_u * T_{\text{mix}}}{V_{\text{mix}}} = \frac{0.19643 * 8.314 * (25 + 273)}{2.392} = 203.45 \text{ kPa}$$

Chapter 14: Gas-vapor mixture:

Atmospheric air = dry air + Vapor

⇒ The temperature of air in air-conditioning applications ranges from about -10°C to about 50°C .

• For dry air

$T, (^{\circ}\text{C})$	$c_p (\text{kJ/Kg} \cdot ^{\circ}\text{C})$
-10	1.0038
20	1.0049
50	1.0065

⇒ In this range of temp, the dry air can be treated as an ideal gas.
⇒ with a constant c_p value of $1.005 \text{ kJ/Kg} \cdot ^{\circ}\text{C}$ with error under 0.2%

$$\Rightarrow h_{\text{dry air}} = c_p T, \quad \Delta h_{\text{dry air}} = c_p \Delta T, \quad \text{where } T \text{ is in } ^{\circ}\text{C}$$

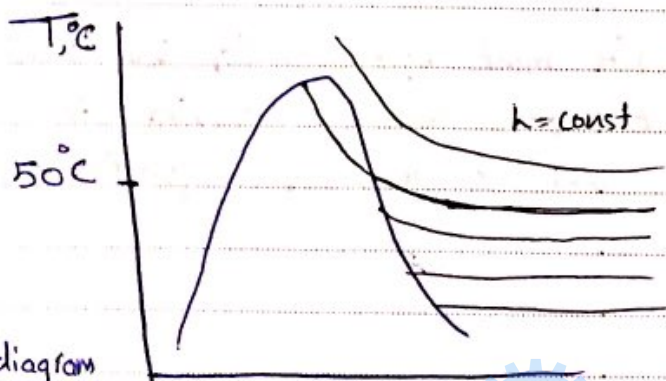
• For water vapor

⇒ At $T = 50^{\circ}\text{C}$ $P_{\text{sat}} = 12.3 \text{ kPa}$, At pressures below this value, water vapor can be treated as ideal gas with error under 0.2% even when it is a saturated vapor (Try to find $T_R = \frac{50+273}{T_{\text{CR}}}$, $P_R = \frac{12.3 \times 10^{-3}}{P_{\text{CR}}}$,

and notice how $z \rightarrow 1$)

⇒ ~~So water~~ Since water vapor ~~is not~~ behaves as an ideal gas, the enthalpy of water vapor is a function of temperature only (i.e $h = h(T)$)

This can be observed from the T-s diagram where the constant enthalpy lines coincide with const temp lines at temperatures below 50°C .



⇒ Since we can treat the dry air and the water vapor as ~~an~~ ideal gases, then the atmospheric air can be treated as an ideal gas

$$P_{\text{atmospheric air}} = P_{\text{dry air}} + P_{\text{water vapor}} \quad (\text{according to Dalton's law})$$

T and T are constants for air

* For dry air $h_a = c_p T = 1.005 * T$ → °C
 $\Delta h_a = c_p \Delta T = 1.005 * \Delta T$

* For ~~the~~ vapor $h_v(T, \text{low } P) = h_g(T)$ → °C
 $= 2500.9 + 1.82 T$
 Curve Fitting equation
 " such that ~~the~~ the range of T is +50 °C → -10 °C "

* $P = P_a + P_v$ (T is const, according to Dalton's law)
 T is const

* For totally dry air $P_v = 0$ (because $\frac{P_i}{P_{\text{mix}}} = \frac{N_i}{N_{\text{mix}}}$, $N_v = 0 \Rightarrow P_{\text{vapor}} = 0$)

* The most important step that must be done by the mechanical engineers who are working in air conditioning field is to determine the vapor quantity of air

How to define the vapor quantity in air? There are 4 parameters
الرطوبة النوعية

Specific (absolute) humidity or humidity ratio $w = \frac{m_v}{m_a}$ kg water vapor per kg of dry air

$$w = \frac{\frac{P_v}{R_v T_v}}{\frac{P_a}{R_a T_a}} = \frac{0.622 P_v}{P_a} = \frac{0.622 P_v}{P - P_v}$$

kg of water vapor per kg of dry air

* if $P_v = 0 \Rightarrow m_v = 0 \Rightarrow w = 0$ dry air

* if $P_v = P_g = (P_{sat})_{@T} \Rightarrow$ the air is saturated $\Rightarrow w = 0.622 \frac{P_g}{P - P_g}$

* if $0 < P_v < (P_{sat})_{@T} \Rightarrow$ the air is unsaturated

at 100 kPa, 25°C $\Rightarrow P_{sat @ 25^\circ C} = 3.169 \text{ kPa}$
(Total pressure)

Can be used to find the amount of saturated vapor in saturated air

الرطوبة النسبية

Relative humidity $\phi = \frac{m_v}{m_g} = \frac{P_v}{P_g}$ $0 < \phi < 1$

relative to the maximum amount of moisture the air can hold at the same temperature.

Dry bulb temperature: the temperature of the atmospheric air.

Example

• if $T = 30^\circ\text{C}$, $\phi = 80\%$

$$* P_g = P_{\text{sat}} @ 30^\circ\text{C} = 4.246 \text{ kPa}, \quad \phi = \frac{P_v}{P_g} \Rightarrow P_v = 0.8 * 4.246 = 3.4 \text{ kPa}$$

$$P = P_a + P_v \quad (\text{assuming } P = 100 \text{ kPa})$$
$$\Rightarrow P_a = 100 - 3.4 = 96.6 \text{ kPa}$$

• as $T \downarrow$, $\phi = \frac{P_v}{P_g} \rightarrow$ is a function of temperature

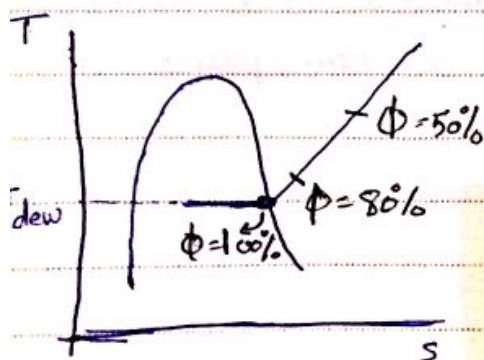
$P_g \downarrow \therefore \phi \uparrow$ (P_v is const, because it depends on the amount of vapor in the air, and the cooling process doesn't affect m_v)

$\phi = 1$ when P_g reaches P_v

in our previous example T must ~~become 26°C~~ decrease to 26°C so that $P_g = P_v$ (i.e. $\phi = 1$)

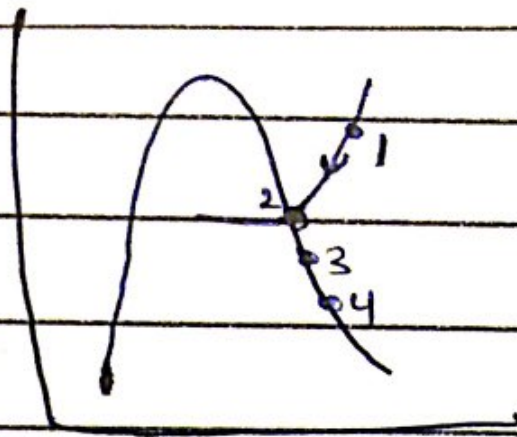
$T = 26^\circ\text{C} \Rightarrow$ is called "dew point" $\Rightarrow$ the temperature at which condensation begins when air is cooled at constant pressure. $T_{\text{dp}} = T_{\text{sat}} @ P_v$

• if ϕ is low, more cooling is required to reach the dew point.



• as $\phi \uparrow$, we are approaching the dew point

10 The air remains saturated during the condensation process and thus follows a path of 100% relative humidity



When the air is saturated, the temperature of the air is equal to the dew point temperature

* The dew-point temperature of room air can be determined easily by cooling some water in a metal ~~cup~~ cup by adding small amounts of ice and stirring. The temperature of the outer surface of the cup when dew starts to form on the surface is the dew point temperature of the air.

* Knowing the T_{dew} , we can find P_v and thus the relative humidity ϕ

$$\phi = \frac{P_v}{P_g}, \quad W = 0.622 \frac{P_v}{P - P_v} \Rightarrow \text{Combining these two equations gives}$$

$$\phi = \frac{WP}{(0.622 + W)P_g}, \quad W = \frac{0.622 \phi P_g}{P - \phi P_g}$$

• In the previous example, if T decreases to a temperature that is less than 26°C (dew point), some of the vapor will condense such that ϕ stays equal to 1

$$\text{mass (condensed)} = m_{v, 30^\circ\text{C}} - m_{v, 25^\circ\text{C}}$$

Some applications:

~~Bedouins~~

1. Tents in deserts are made of burlap. Bedouins moisturizes this burlap with water in order to cool the tents. When the hot dry air ^{evaporates by sucking heat from itself} passes through the burlap, ~~it~~ ^{the water} (the hot dry air can hold a great amount of vapor); So, ~~the~~ ^{the} air enters the tent and the burlap becomes cool.

2. Why does water drop out from the ~~AC~~ ^{AC} when it is on?

In the ~~AC~~ ^{AC} unit, there is a refrigerant that is circulated in pipes. The AC takes the hot air from your room, passes it over the pipes in which refrigerant flows. Heat is taken from the air by the refrigerant and the air is ~~is~~ cooled. This air ~~is~~ contains vapor. So when the air passes over the pipes, the vapor condenses over the surface of the pipes. The water ~~is~~ ^{condensed} on the pipes is then sent out using drain tube and that's the water you see.

Recall:

- If the air is completely saturated, then

$$w = 0.622 \frac{P_g}{P - P_g}$$

$$P_g = P_{\text{sat}@T} \quad \text{as } T \uparrow \quad P_g \uparrow$$

- For simple heating or simple cooling (i.e. changing the temperature of air, without changing the following:

$$\left. \begin{array}{l} \text{const } \leftarrow \left(\frac{m_v}{m_a} \right) \\ \text{(i.e. no} \\ \text{condensation)} \end{array} \right\} \begin{array}{l} P_v \\ P_a \end{array} \quad \left. \begin{array}{l} w \text{ is const} \\ P_g \text{ varies, so } \phi \text{ varies} \end{array} \right\}$$

$$\bullet \text{ } h_{\text{atmospheric}} = h_a + \left(\frac{m_v}{m_a} \right) m_v h_v$$

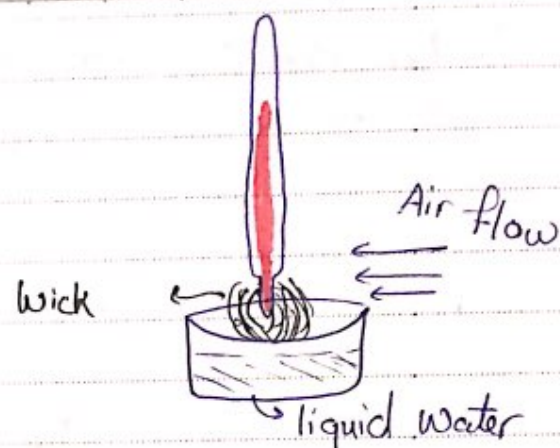
divide by m_a , to get the enthalpy of the atmospheric air per unit mass of dry air

$$h = \underbrace{(h_a)}_{c_p T} + w \underbrace{(h_v)}_{h_g \text{ at } T}$$

note: in most practical applications the amount of dry air in the air-vapor mixture remains const but the amount of vapor water changes. So the enthalpy of atmospheric air is expressed per unit mass of dry air instead of per unit mass of air-vapor mixture

Wet bulb temperature

When unsaturated ~~water~~ air passes over the wet wick, some of the water in the wick evaporates. (Note: the latent heat of vaporization is taken from the water itself). As a result the temperature of the water drops, creating a temperature difference between the air and water. So heat is transferred to the wick (because it has less temperature) from the air. Also, at the same time, there is heat loss ~~from~~ from the water, due to evaporation. After a while, the rate of heat loss from the water by evaporation equals the rate of heat gain from the air, so the temperature of water stabilizes. The temperature reading is called wet bulb temperature.



Note: Wet bulb temperature can also be measured by placing the wet-wicked thermometer in a holder attached to a handle and rotating the holder rapidly, that is moving the thermometer ~~to~~ instead of the air. A device that works on this principle is called sling psychrometer.

The psychrometric chart

$$\square \phi = 50\% , W = 0.01 \Rightarrow T_{db} = 25^\circ\text{C} \quad T_{dp} = 14^\circ\text{C} \quad T_{wb} = 18^\circ\text{C}$$
$$h = 50.75 \frac{\text{kJ}}{\text{kg dry air}} \quad v = 0.858 \frac{\text{m}^3}{\text{kg dry air}}$$

$$\boxed{2} \quad T_{db} = 25^{\circ}\text{C}$$

$$\phi = 80\%$$

$$\Rightarrow W = 0.016$$

$$T_{dp} = 21.3^{\circ}\text{C}$$

$$h = 66 \text{ kJ/kg dry air}$$

$$T_{wb} = 22.4^{\circ}\text{C}$$

$$v = 0.866 \frac{\text{m}^3}{\text{kg dry air}}$$

$$\boxed{3} \quad T_{db} = 20^{\circ}\text{C}$$

$$T_{dp} = 20^{\circ}\text{C}$$

$$\Rightarrow h = 57.6 \text{ kJ/kg dry air}$$

$$W = 0.0147$$

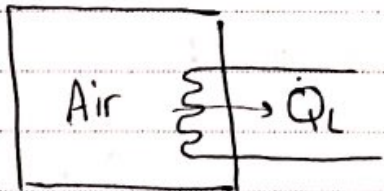
$$T_{wb} = 20^{\circ}\text{C}$$

$$, v = 0.85 \text{ m}^3/\text{kg dry air}$$

Note:

www.cytsoft.com $\Rightarrow$ use it to check you readings from the psychrometric chart

Question Rigid tank contains dry air + vapor. the volume of the rigid tank is 2 m^3



$$\phi = 20\% \quad T = 160^{\circ}\text{C} \quad P = 400 \text{ kPa}$$

After cooling

$$T_{\text{final}} = 20^{\circ}\text{C}$$

Find $\dot{Q}_L$, Will ~~the water~~ some of the vapor condense?

*Before cooling (state 1)

$$P_{g@160^{\circ}\text{C}} = P_{\text{sat}@160^{\circ}\text{C}} = 618.23 \text{ kPa} \quad \phi = \frac{P_v}{P_g} \Rightarrow P_{v_1} = 0.2 * 618.23 = 123.646 \text{ kPa}$$

$$P_{\text{dry air}} = P - P_{v_1} = 400 - 123.646 = 276.354 \text{ kPa}$$

$$m_{\text{dry air}} = \frac{PV}{RT} = \frac{276.354 * 2}{(0.287) * (160 + 273)} = 4.448 \text{ kg}$$

*After cooling

$$P_{g@20^{\circ}\text{C}} = P_{\text{sat}@20^{\circ}\text{C}} = 2.3398 \text{ kPa}$$

since $P_{g@20^{\circ}\text{C}} < P_{v_1}$ $\therefore$ there will be some condensation

as $T \downarrow$ $P_g \downarrow$ $P_g = 123.646 \text{ kPa}$ when $T = 105.612^{\circ}\text{C}$

$T(^{\circ}\text{C})$	$P_{\text{sat}}(\text{kPa})$
105	120.90
110	143.38

$$T_{\text{dew}} = 105.612^{\circ}\text{C}$$

since $T_2 < T_{\text{dew}} \therefore$ there will be condensation

after cooling $T_2 = 20^{\circ}\text{C}$ $\phi = 1$ $P_{v_2} = P_{g@20^{\circ}\text{C}} = 2.3398 \text{ kPa}$

~~$m_{\text{air}} = \frac{P_{v_1}}{P - P_{v_1}} \times 2$~~
before cooling

$$m_f = m_{\text{dry air}} (w_1 - w_2) = 4.448 (0.278 - 0.00778) = 1.2 \text{ kg}$$

$$w_1 = 0.622 \frac{P_{v_1}}{P - P_{v_1}} = 0.622 \times \frac{123.646}{400 - 123.646} = 0.278 \text{ kg moist / kg dry air}$$

$$w_2 = 0.622 \frac{P_{v_2}}{P_{\text{dry air}_2}} = 0.622 \times \frac{2.3398}{187} = 0.00778 \text{ kg moist / kg dry air}$$

For dry air

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{276.354}{(160+273)} = \frac{P_2}{(20+273)} \Rightarrow P_2 = 187 \text{ kPa}$$

$$\begin{aligned} Q_L &= m_a \Delta u \\ &= m_a (u_{a2} - u_{a1}) + (m_{v_2} u_{v_2} - m_{v_1} u_{v_1}) + m_f h_{fg} \\ &= 4.45 (C_v(T_2 - T_1) + (w_2 u_{v_2} - w_1 u_{v_1})) + m_f h_{fg} \end{aligned}$$

$$= 4.45 (0.718(20-160) + (0.00778 \times 2402.3 - 0.278 \times 2567.8)) + 1.2 h_{fg} = -6290$$

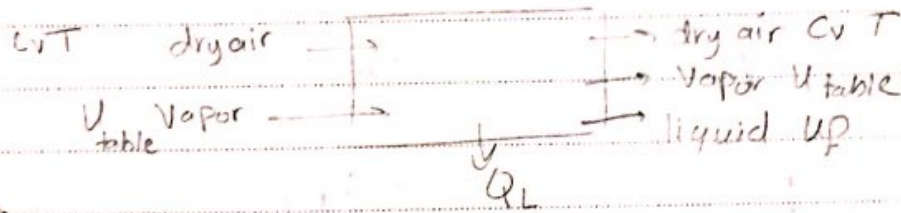
$P = 0.1 \text{ MPa}$

T(°C)	u kJ/kg
150	2582.9
160°C	2641.42
200	2875.5

$P = 0.2 \text{ MPa}$

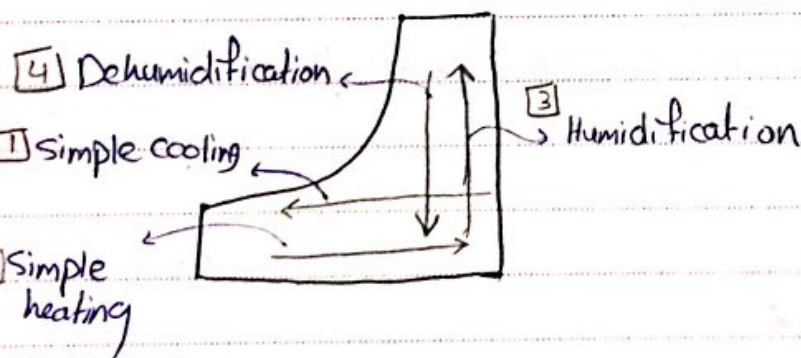
T(°C)	u kJ/kg
150	2577.1
160°C	2592.6
200	2654.6

$P = 0.12365 \text{ MPa}$
at 160°C
 $u = 2629.87 \text{ kJ/kg}$

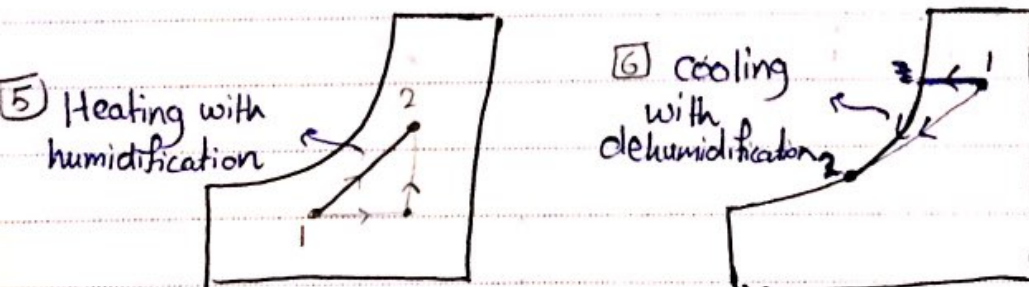


$$m_a C_v T_1 + m_{v_1} u_{g_1} = Q_L + m_a C_v T_2 + m_{v_2} u_{g_2} + m_f u_f$$

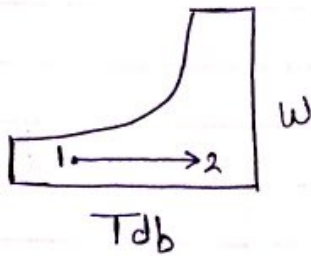
* Air conditioning processes



simple heating or cooling means w remains constant



I) Simple (sensible) Heating

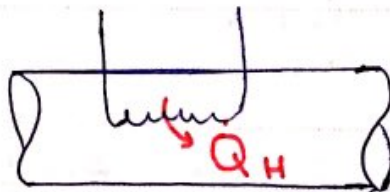


$$w_1 = w_2 ; m_a = \text{const}$$

$$\therefore m_{v1} = m_{v2}$$

$$P_{v1} = P_{v2} \rightarrow \text{vapor}$$

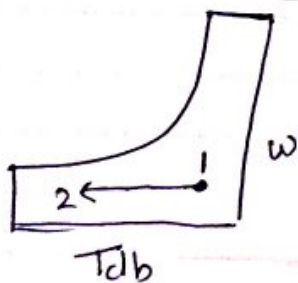
$$\phi_2 < \phi_1 \text{ since } P_{g2} > P_{g1}$$



$$\dot{Q}_H = \dot{m}_a (h_2 - h_1)$$

enthalpy of air
Per ~~unit~~ kg of dry air

II) Simple cooling

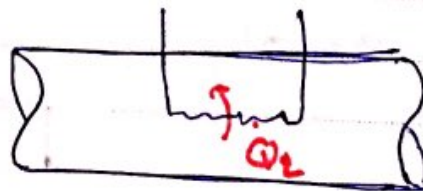


* the process is considered as simple cooling only if $T_2 \gg T_{dp}$

$$w = c, m_a = \text{const}, m_v = \text{const}$$

$$P_v = \text{const}$$

$$\phi_2 > \phi_1 \text{ since } P_{g2} < P_{g1}$$



$$\dot{Q}_L = \dot{m}_a (h_1 - h_2)$$

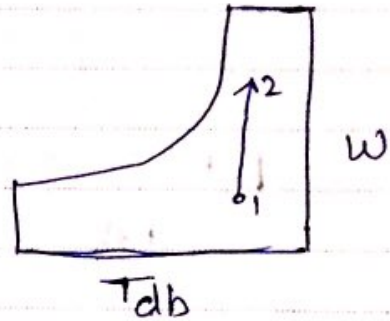
III) Humidification

$$T_{db} = \text{const} ; m_a = \text{const}$$

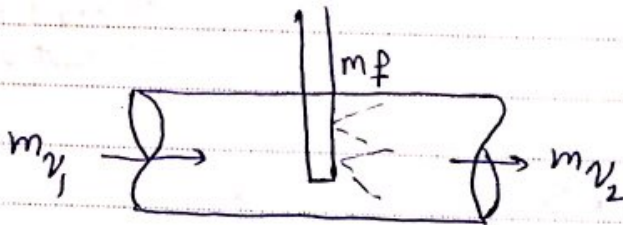
$$w_2 > w_1$$

$$\therefore m_{v_2} > m_{v_1} \Rightarrow P_{v_2} > P_{v_1}$$

$$\phi_2 > \phi_1 \quad \phi = \frac{P_v}{P_g} \begin{matrix} \nearrow \text{increasing} \\ \searrow \text{is const, it is a function of } T \end{matrix}$$



$$m_f = m_a(w_2 - w_1) \Rightarrow \text{mass balance equation}$$



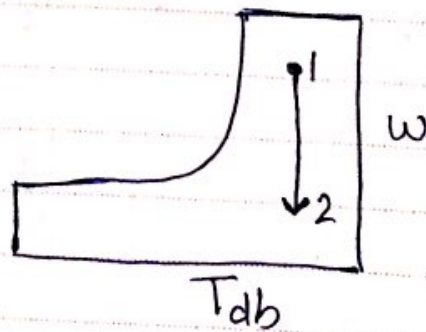
$$m_f = (m_{v_2} - m_{v_1}) \times \frac{m_a}{m_v} = m_a(w_2 - w_1)$$

IV) Dehumidification

$$T_{db} = \text{const} ; m_a = \text{const}$$

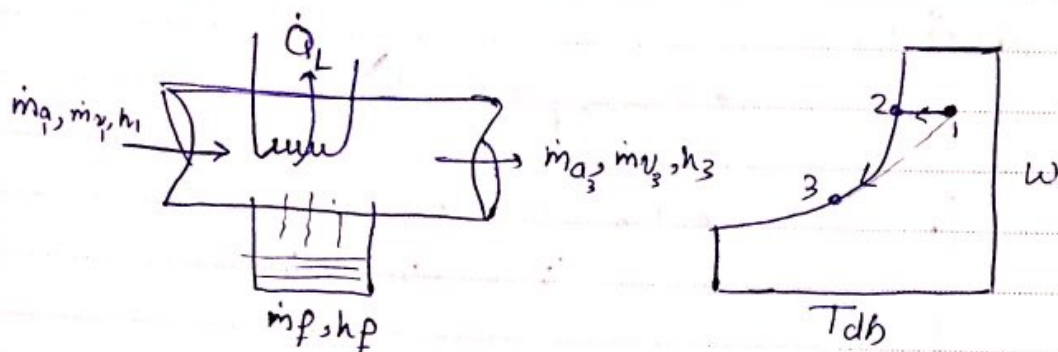
$$w_2 < w_1 \quad \therefore m_{v_2} < m_{v_1} \Rightarrow P_{v_2} < P_{v_1}$$

$$\phi_2 < \phi_1$$



$$m_f = m_a(w_1 - w_2)$$

V) Cooling with dehumidification



1-2) Simple cooling

$$W_3 < W_2 ; \phi_3 = \phi_2 = 100\% ; \dot{m}_a = \dot{m}_a$$

Energy balance equation

$$\dot{m}_a h_1 = \dot{m}_a h_3 + \dot{m}_p h_p + \dot{Q}_L \Rightarrow \dot{Q}_L = \dot{m}_a (h_1 - h_3) - \dot{m}_p h_p$$

for a refrigeration cycle $\dot{Q}_L$ also = $\dot{m}_{ROR} (h_1 - h_4)$

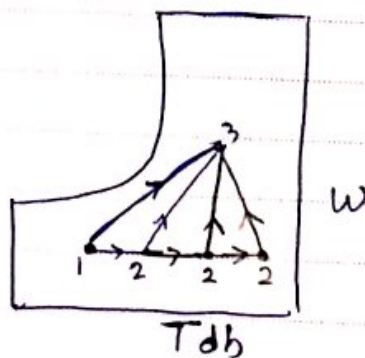
VI) Heating with humidification

1-2) simple Heating

$$\dot{Q}_H = \dot{m}_a (h_2 - h_1)$$

$$W_3 > W_1 ; \dot{m}_{v_3} > \dot{m}_{v_1}$$

$$T_3 > T_1 ; P_{g_3} > P_{g_2}$$



2-3) Humidification

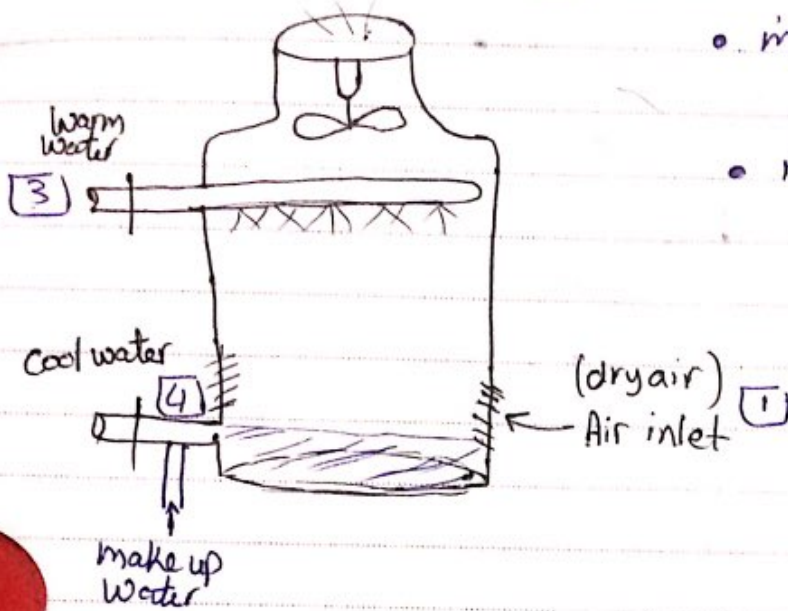
$$\dot{m}_{water} = \dot{m}_a (W_3 - W_2)$$

Cooling Tower

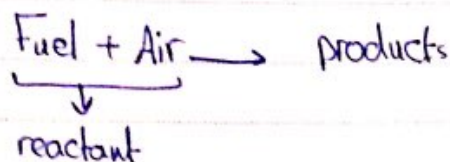
Air exit [2]

$$\dot{m}_{\text{makeup}} = \dot{m}_a (w_2 - w_1)$$

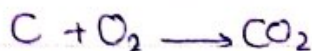
$$\dot{m}_a = \frac{\dot{m}_3 (h_3 - h_4)}{(h_2 - h_1) - (w_2 - w_1) h_{f1}}$$



Chapter 15: Chemical Thermodynamics

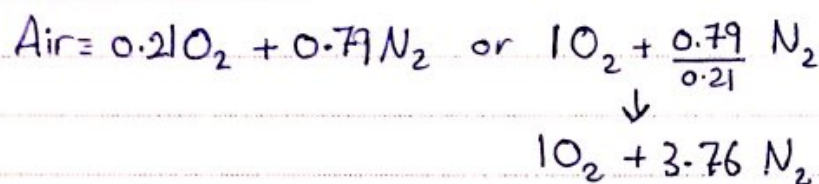


e.g

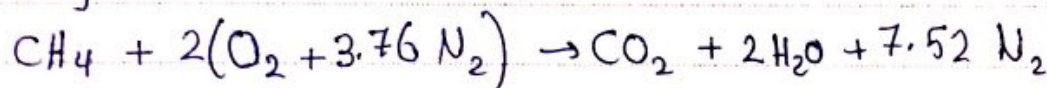


Note: don't forget
to balance the chemical
equation.

~~also~~ • Atmospheric air consists of 79% N_2 + 21% O_2 by volume



e.g



* If all $\text{C} \rightarrow \text{CO}_2$

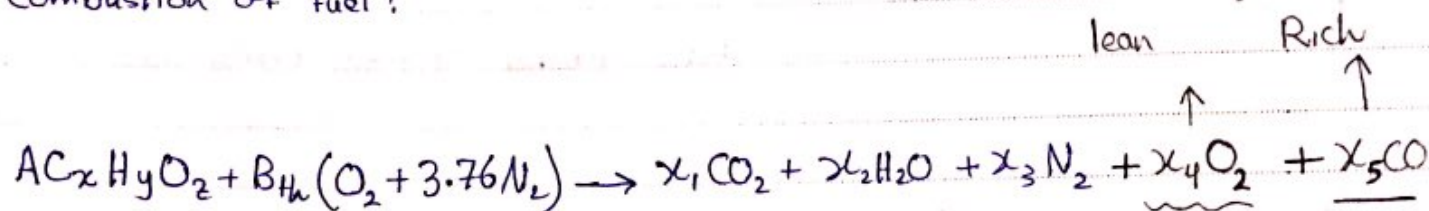
$\text{H} \rightarrow \text{H}_2\text{O}$

$\text{S} \rightarrow \text{SO}_2$

No O_2 in the products

} The reaction is called
Stoichiometric
theoretical
100% theoretical

How to find the minimum amount of air needed for complete combustion of fuel?



* if there is "S" in the reactants, we must add x_6SO_2 in the products

Mass Balance:

$$C: A x * 12 = x_1 * 12 \Rightarrow x_1 = Ax$$

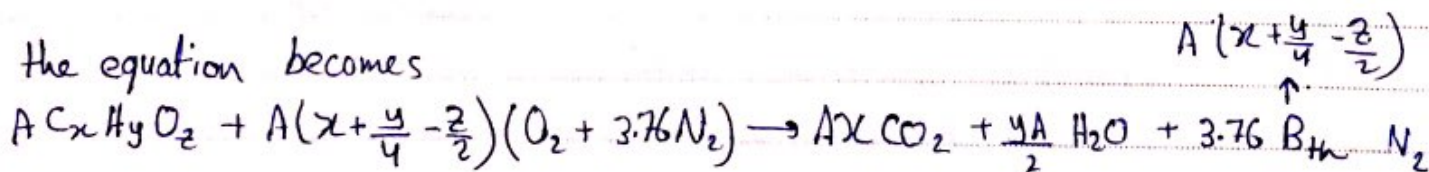
$$H: y * A * 1 = x_2 * 2 * 1 \Rightarrow x_2 = \frac{yA}{2}$$

$$O: A * z * 16 + B_{th} * 2 * 16 = x_1 * 2 * 16 + x_2 * 1 * 16$$

$$\begin{aligned} B_{th} &= \frac{2x_1 + x_2 - Az}{2} = \frac{x_1 + \frac{x_2}{2} - \frac{Az}{2}}{1} \\ &= Ax + \frac{yA}{4} - \frac{Az}{2} \\ &= A \left(x + \frac{y}{4} - \frac{z}{2} \right) \end{aligned}$$

$$N: B_{th} * 3.76 * 2 * 14 = x_3 * 2 * 14 \Rightarrow x_3 = 3.76 B_{th}$$

the equation becomes



* Try to use the equation with C_8H_{12} . $x=8$, $y=12$, $z=0$

$$\left(\frac{A}{F}\right)_s = \frac{\text{mass of air}}{\text{mass of fuel}} = 15 \text{ for } C_8H_{12}$$

mass = number of moles * Molar mass

$$\frac{A_{\text{theoretical}}}{A_{\text{actual}}}$$

ϕ = equivalence ratio
 $= \frac{(A/F)_s}{(A/F)_{\text{actual}}}$

$\phi < 1$ lean (excess O_2)
 $\phi = 1$ stoichiometric
 $\phi > 1$ Rich

the amount of heat ~~and~~ resulting from the reaction

$$Q_H = \left(\sum N_p (\bar{h}_{T,p} - \bar{h}_{25^\circ C}^\circ) - \bar{h}_f^\circ \right)_{\text{Products}} - \left(\sum N_R (\bar{h}_{T,R} - \bar{h}_{25^\circ C}^\circ) - \bar{h}_f^\circ \right)_{\text{Reactants}}$$

* Example 15-1, 15-2

* For the same amount of fuel if $air_{\text{actual}} < air_{\text{theoretical}}$ then it is called rich

كمية ال fuel أكبر مقارنة بكمية ال fuel التي يمكن تحرق من خلال كمية الهواء (الحقيقية)

* if $air_{\text{actual}} > air_{\text{theoretical}}$ "lean"

كمية ال fuel أقل مقارنة بكمية ال fuel التي يمكن تحرق من خلال كمية الهواء