

# Thermo 2

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Full Notes

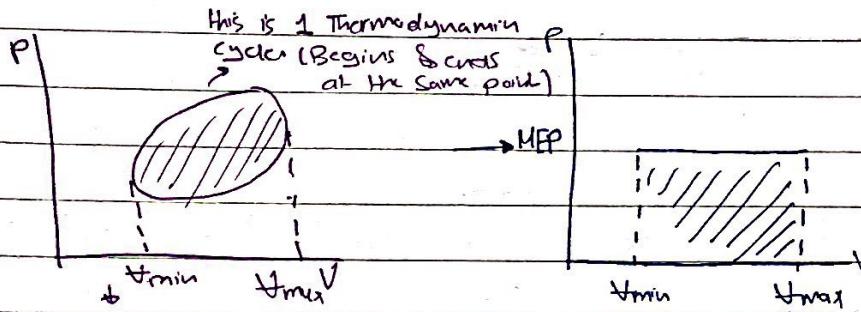






\* Mean effective pressure :-

\* note → it's like



$$\text{Area} = \int P dV = W_{\text{net}}$$

$$\Rightarrow W_{\text{net}} = \text{MEP} \times (H_{\text{max}} - H_{\text{min}})$$

I want to convert the Irregular shape into a Regular shape that is easier to deal with (regularity)

→ assume you go to a car dealer, you want to know the work (have the power)

that a certain car could do, you can't tell him give me a p-v diagram

to find it - it is not practical (no use), but that's why we deal with (MEP), which is the value of pressure that if multiplied by ( $H_s$ ) you'd get the same value of work obtained from the P-V diagram.

$$\text{So} \rightarrow W_{\text{net}} = \text{MEP} \times H_s = \text{MEP} (H_{\text{max}} - H_{\text{min}})$$

But what we need is (POWER), to obtain power: (from work)

→ we usually calculate work for 1 cycle. (1 cylinder)

$$W_{\text{net}} = \text{MEP} \times H_s \rightarrow \text{J/cycle}$$

## \* Gas Carnot Power Cycle \*:

→ The main objective when we study any of the Thermodynamic cycles is to find the max. performance (slip,  $\eta_{max}$ )

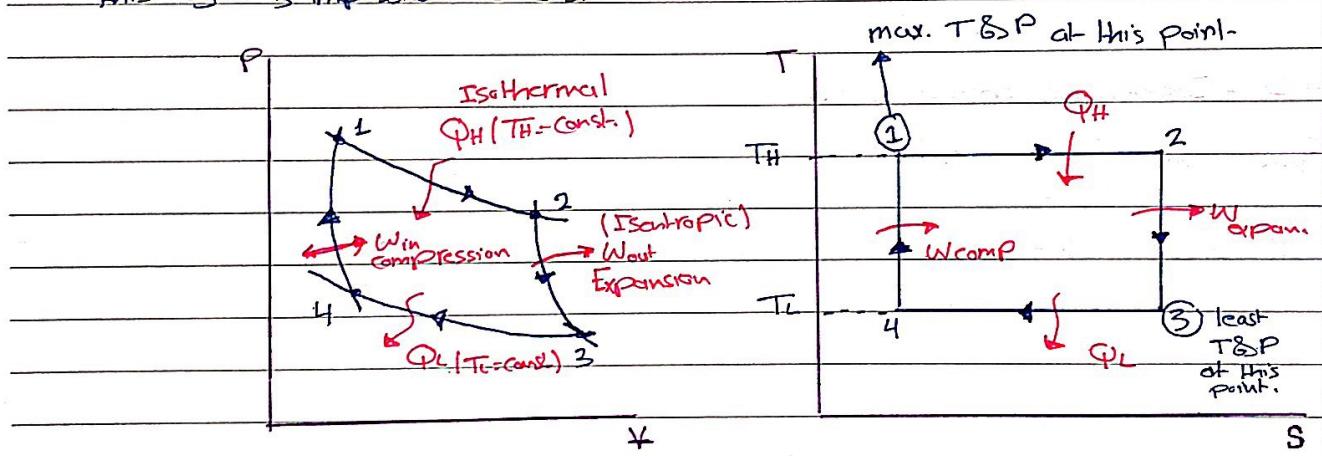
→ among all the cycles Carnot Cycles gives the max efficiency possible so we care to study this cycle to compare its performance with the performance of the other cycles, hence to know how far is the cycle from the max. efficiency possible. ( $\text{if this cycle is possible}$ )

→ Carnot proposed this cycle & it is neither realistic nor practical. But he wanted to propose the cycle that would achieve the max. efficiency between  $T_H$  &  $T_L$ .

→ same previous assumptions are made.

→ This cycle consists of → 2 Isothermal processes,  
2 Isentropic processes.

→ so, the two processes that makes the cycle are hypothetical that's why this cycle is impossible to exist.



•  $Q_H$  is supplied by burning fuel since  $Q_H = \eta_{max} \cdot Q_{in}$   $\leftarrow (1-1\right)$

Wout is given to the surroundings  $\leftarrow (2-2\right)$

compression at  $Q_L$  is given by the surroundings  $\leftarrow (3-3\right)$

constant  $T_L$  we should continuously cool it down.

•  $W_{in} \leftarrow \text{Work done by fuel} \leftarrow (4-4\right)$

look into every process by it self

Recall → 1st law:  $(Q_{in} - Q_{out} + W_{in} - W_{out} = \Delta U)$

(1 → 2) - Isothermal

$$Q_{out} = 0 \quad W_{in} = 0 \quad \Delta U = 0$$

constant temperature,

thermal energy ( $U$ )  
is a function of  $T$ .

Work done  $\downarrow$

size piston  $\downarrow$

$$Q_{in} - Q_{out} + W_{in} - W_{out} = \Delta U$$

(Work  $\uparrow$ )

size of the work

$$Q_{in} - W_{out} = 0 \rightarrow Q_{in} = W_{out}$$

$$\rightarrow = T_H \Delta S$$

$$Q_{in} = Q_{H} = R T_H \ln \left( \frac{V_2}{V_1} \right)$$

same thing  
you choose  
either one.

area under  $T-S$

Diagram gives

us heat.

### $(2 \rightarrow 3)$ - Isentropic, adiabatic - Expansion.

Volume increases

$$q_{in} = q_{out} = 0 \quad w_{in} = 0$$

$$q_{in}^0 - q_{out}^0 + w_{in} - w_{out}^0 = \Delta U$$

$$-w_{out} = \Delta U = u_3 - u_2$$

$$w_{out} = u_2 - u_3$$

so air pushes

the piston

(Wout).

### $(3 \rightarrow 4)$ - Isothermal

$$q_{in} = 0, w_{out} = 0, \Delta U = 0$$

$$w_{in} - q_{out} = 0$$

$$q_{out} = w_{in} \rightarrow q_{out} = q_L = w_{in} = RT_L \ln\left(\frac{V_4}{V_3}\right)$$

$$\rightarrow \text{or } (S_2 - S_1) \\ = T_L (S_3 - S_4)$$

### $(4 \rightarrow 1)$ - Isentropic, compression.

$$q_{in} = q_{out} = 0, w_{out} = 0$$

$$w_{in} = \Delta U = u_1 - u_4$$

## From last lecture (Revision) :-

• Carnot- Cycle has two isothermal, two isentropic processes

First, isothermal expansion with heat addition then an isentropic expansion, isothermal compression with heat loss then another compression but isentropic.

→ we derived the equations related to every process :-

$$(1 \rightarrow 2) \rightarrow \text{could be } (S_1 - S_2) - \text{we take absolute value}$$

$$Q_H = m T_H (S_2 - S_1)$$

$$Q_H = W_{1 \rightarrow 2} = m R T_H \ln\left(\frac{V_2}{V_1}\right) ; \frac{V_2}{V_1} = \frac{P_1}{P_2} \rightarrow \text{we derived this earlier.}$$

(2 → 3) Reversible + adiabatic = isentropic

$S = \text{const.} \rightarrow$  Here there is no change in heat since entropy is constant

∴ So, the expansion is at the cost of Internal energy (U)

$W_{\text{exp}} = U_2 - U_3 \rightarrow$  we have no other equation except

one we will use in calculations:

$$\left(\frac{T_2}{T_3}\right)_{S=\text{const.}} = \left(\frac{P_2}{P_3}\right)^{\frac{1}{k-1}} = \left(\frac{V_3}{V_2}\right)^{k-1} \rightarrow \text{If polytropic (k) becomes an} \\ \text{int.} \rightarrow \text{But in Carnot there is} \\ \text{no polytropic because } \text{d- means} \\ \text{loss \& Carnot means no loss.}$$

↳ from CH#7

(3 → 4) - Isothermal  $T_L = \text{const.}$

$$Q_L = m T_L (S_4 - S_3) - \text{again, Don't look at the sign take the} \\ \text{absolute}$$

$$Q_L = W_{\text{exp}}_{\text{compression}} = m R T_L \ln\left(\frac{V_4}{V_3}\right) \rightarrow \text{always final} \rightarrow \text{Entropy (f - i)}$$

(4 → 1) - Isentropic compression  $S = \text{const.}$

→ compression is at the cost of Internal energy.

$$W_{\text{compression}} = U_1 - U_4$$

now, we care a lot to know  $\Delta S$  :- (change in Entropy for ideal gases)

sec (7-9)

$$** S_f - S_i = C_p \ln\left(\frac{T_f}{T_i}\right) - R \ln\left(\frac{P_f}{P_i}\right)$$

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$$** S_f - S_i = C_v \ln\left(\frac{T_f}{T_i}\right) + R \ln\left(\frac{V_f}{V_i}\right)$$

Find  $\Delta S$  for processes  $(1 \rightarrow 2)$  &  $(3 \rightarrow 4)$

Remember:

$$S_2 - S_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

For processes

$\rightarrow$  BUT! :  $T_2 = T_1$  (isothermal)  $\rightarrow \ln 1 = 0$   $\rightarrow (2 \rightarrow 3) \& (4 \rightarrow 1)$

$$S_2 - S_1 = -R \ln\left(\frac{P_2}{P_1}\right)$$

$\Delta S = 0$  (Isentropic)

also:

$$S_2 - S_1 = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)$$

$$S_2 - S_1 = R \ln \frac{V_2}{V_1}$$

$\rightarrow$  same thing goes for  $(S_4 - S_3)$  or  $(S_3 - S_4)$  if you want. (only the sign differs but the same result).

$$W_{net} = Q_H - Q_L$$

$$= mT_H(S_2 - S_1) - mT_L(S_4 - S_3) \quad \text{But: } S_2 - S_1 = S_4 - S_3$$

$$= m(S_2 - S_1)(T_H - T_L) \quad \rightarrow \text{since we used two isothermal}$$

$$\rightarrow \text{or } (S_4 - S_3)$$

processes to obtain  $W_{net}$ , the

two are reversible, hence this is the MAX. work

Efficiency:

$$\eta_{Carnot} = \frac{W_{net}}{Q_H} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{1200} \quad \rightarrow \text{must be in Kelvin!}$$

Question (9-18)  $\rightarrow$  working medium is air

(An air standard) Carnot cycle is executed in a closed system between temperature limits between  $(350\text{ K}) \& (1200\text{ K})$ .

The pressures before and after isothermal compression are  $(150) \& (300)$  kPa. If the net work output per cycle is  $(0.5 \text{ kJ}) \rightarrow W_{\text{net}} = mW_{\text{net}}$

Determine  $\rightarrow$  (a) the max. pressure in the cycle  $\rightarrow P_1$   
 (b) the heat transfer to air.  
 (c) the mass of air.

Solution:-

$\rightarrow$  Assume constant specific heats

$$1 - \eta_{\text{th}} = 1 - \frac{T_L}{T_H} = 1 - \frac{350}{1200} = 0.7083$$

$\hookrightarrow$  must be in Kelvin.

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_H} \rightarrow Q_H = W_{\text{net}} = \frac{0.5}{0.7083} = 0.7059 \text{ kJ}$$

$$Q_H = m_{\text{air}} T_H (S_2 - S_1)$$

But, we know that  $|S_2 - S_1| = |S_4 - S_3| \rightarrow$  Because I have  $P_3 \& P_4$ , this will help.

$$S_4 - S_3 = C_p \ln \left( \frac{T_4}{T_3} \right) - R \ln \left( \frac{P_4}{P_3} \right)$$

$\hookrightarrow (3 \rightarrow 4)$  is an isothermal processes

$$= -0.287 \ln \left( \frac{300}{150} \right) = -0.1989 \rightarrow$$
 Take the absolute value.

$$m_{\text{air}} = \frac{0.7059}{(1200)(0.1989)} = 1.95 \cdot 10^{-3} \text{ kg}$$

Since the process  $4 \rightarrow 1$  is isentropic

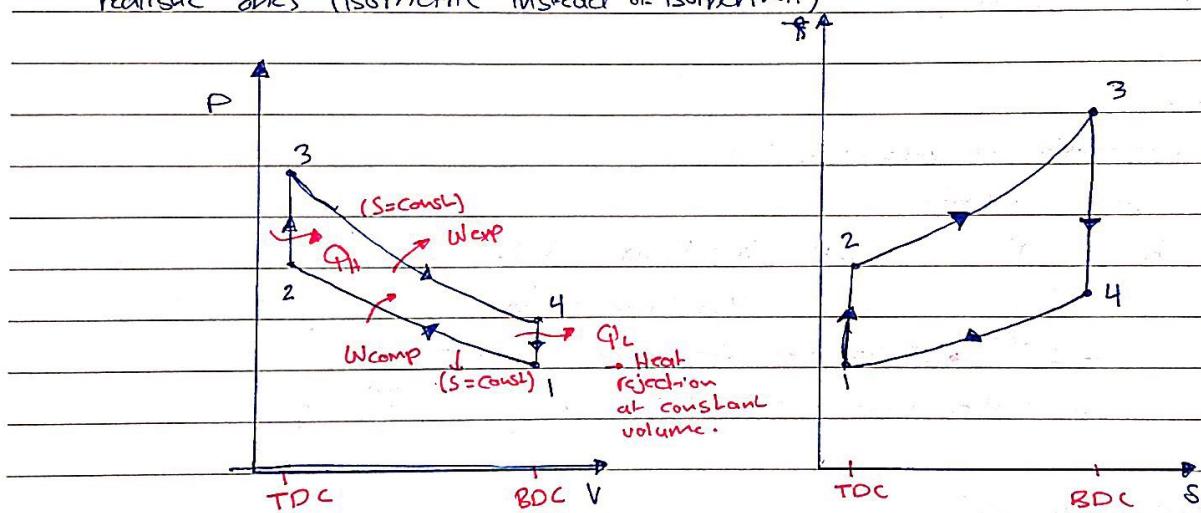
$$\left( \frac{P_1}{P_4} \right)^{\frac{k-1}{k}} = \frac{T_1}{T_4}$$

$k = 1.4 \rightarrow$  This is only used when you assume specific heat.

$$\left( \frac{P_1}{300} \right)^{\frac{1.4-1}{1.4}} = \frac{1200}{350} \rightarrow P_1 = 22 \text{ MPa}$$

# OTTO CYCLE

- This cycle is like a realistic version of carnot cycle
- we will find the max. work obtained from this cycle & divide it by the max. work from carnot cycle, this is called Exergy → CH #8
- All spark ignition engines are based on otto cycle → important
- The problem in carnot cycle and what makes it unrealistic is the isothermal processes, they are only achieved in the cases of condensation and boiling, otherwise it is impossible to achieve.
- If I want to achieve this, I have to connect it with a continuous heat-sink which is impossible (sun is a cont. heat sink for example).
- so what Nicolas Otto did is to replace these two processes with more realistic ones (isometric instead of isothermal)



∴ const. volume is heat add. & rejection

∴ compression & expansion is at const. volume. At TDC, piston is at max. volume. At BDC, piston is at min. volume.

→ you can see that Otto cycle has 1 compression & 1 expansion while Carnot had 2 comp. & 2 expn.

→ Note: even isentropic processes are hard to achieve but we can convert them to polytropic which is very close enough.

→ This cycle is also called constant volume cycle

→ even heat addition and rejection at constant volume are not real but we assume that to get the max. possible performance.

Q → What is the goal (main objective) of what we are doing in Thermo II?

If you want to know how good is the performance of any device working on Thermodynamic cycles, you need a good reference, here Otto cycle is the ideal cycle that we use as a reference, it is ideal because we assume Isentropic, Isometric, Frictionless processes which is not really the case, and we use Otto instead of Carnot because it is closer to reality while Carnot is unrealistic.

Equations:

$$\text{Recall} \rightarrow Q_{in} - Q_{out} + W_{in} - W_{out} = \Delta U$$

(1 → 2) Isentropic ( $S = \text{const.}$ )

$$Q_{in} = Q_{out} = 0$$

$$W_{in} = U_2 - U_1$$

$$\text{assuming } C_V = \text{const.} \rightarrow \Delta U = C_V(T_2 - T_1)$$

\* For isentropic we know that:

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{k-1}{k}} = \left( \frac{V_1}{V_2} \right)^{\frac{k-1}{k}} \quad \text{But} \quad V_1 = V_{\max} = V_{B.D.C}$$

$$V_2 = V_{\min} = V_{T.P.C}$$

$$= CR^{k-1} \rightarrow \text{(Important) Remember } CR = \frac{V_{\min}}{V_{\max}}$$

This work is into the system meaning that it is done by the piston on air

(2 → 3) Isometric ( $V = \text{const.}$ )

$$W_{in} = W_{out} = 0 \rightarrow \text{for constant volume there is no boundary work}$$

$$Q_{in} = U_3 - U_2$$

$$= C_V(T_3 - T_2) \text{ kJ/kg}$$

To find the relation between  $T_1, P_1, V_1$  i-

$$pV = mRT \rightarrow \frac{P}{T} = mR = \text{const.}$$

$$\text{so} \rightarrow \frac{P_2}{T_2} = \frac{P_3}{T_3} \rightarrow \frac{T_3}{T_2} = \frac{P_3}{P_2} = r_p \quad (\text{Pressure Ratio})$$

(3 → 4) Isentropic ( $S = \text{const.}$ )

$$Q_{in} = Q_{out} = 0 \rightarrow \text{From air on the piston.}$$

$$-W_{out} = U_4 - U_3 \rightarrow W_{out} = U_3 - U_4$$

$$\left( \frac{T_3}{T_4} \right) = \left( \frac{P_3}{P_4} \right)^{\frac{k-1}{k}} = \left( \frac{V_4}{V_3} \right)^{\frac{k-1}{k}} = CR^{k-1} \rightarrow \text{notice that } V_4 = V_1 \text{ & } V_2 = V_3$$

$S = \text{const.}$

(1  $\rightarrow$  1) Isometric (Isochoric)  $T = \text{const.}$

$$W_{\text{in}} = W_{\text{out}} = 0$$

$$q_{\text{out}} = u_4 - u_1$$

$$= C_v (T_4 - T_1)$$

$$\frac{P_1}{T_4} = \frac{P_1}{T_1} \rightarrow \frac{P_4}{P_1} = \frac{T_4}{T_1}$$

another way to express the relations:

(1  $\rightarrow$  2)

$$W_{\text{in}} = u_2 - u_1$$

$$= C_v (T_2 - T_1)$$

$$= C_v T_1 \left( \frac{T_2}{T_1} - 1 \right) \quad \text{But } \frac{T_2}{T_1} = CR^{k-1}$$

$$= C_v T_1 (CR^{k-1} - 1)$$

(2  $\rightarrow$  3)

$$q_{\text{in}} = u_3 - u_2$$

$$= C_v (T_3 - T_2)$$

$$= C_v (T_2) \left( \frac{T_3}{T_2} - 1 \right)$$

$$BUL \quad \frac{T_3}{T_2} = \frac{P_3}{P_2} = r_p$$

$$= C_v T_2 (r_p - 1)$$

$$\text{But } \frac{T_3}{T_2} = T_1 (CR^{k-1})$$

$$= C_v T_1 CR^{k-1} (r_p - 1)$$

(3  $\rightarrow$  4)

$$W_{\text{out}} = u_3 - u_4$$

$$= C_v (T_3 - T_4) = C_v T_4 \left( \frac{T_3}{T_4} - 1 \right)$$

$$= C_v T_4 (CR^{k-1} - 1)$$

$$= C_v T_4 r_p (CR^{k-1} - 1)$$

$$T_3 = T_2 r_p$$

$$T_2 = T_1 (CR^{k-1}) \rightarrow T_3 = T_1 (CR^{k-1}) r_p$$

$$\frac{T_3}{T_4} = CR^{k-1}$$

$$\frac{T_3}{T_4} = \frac{T_3}{CR^{k-1}} = \frac{T_1 (CR^{k-1}) r_p}{CR^{k-1}} = T_1 r_p$$

(4  $\rightarrow$  1)

$$q_{\text{out}} = u_4 - u_1 = C_v (T_4 - T_1)$$

$$= C_v T_1 \left( \frac{T_4}{T_1} - 1 \right)$$

$$\text{But } \frac{T_4}{T_1} \times \frac{T_2}{T_1} = \frac{T_3}{T_4} = CR^{k-1}$$

$$= C_v T_1 (r_p - 1)$$

$$\frac{T_4}{T_1} = \frac{T_3}{T_2} \rightarrow \frac{T_3}{T_2} = r_p$$

$$\text{So } \rightarrow \frac{T_1}{T_1} = r_p$$

## For The Cycle:

\* Total work:

$$\begin{aligned} W_{net} &= q_{in} - q_{out} \\ &= C_V T_1 (CR^{\frac{1}{k}}(p-1)) - C_V T_1 (r_p-1) \\ &= C_V T_1 (r_p-1) (CR^{\frac{1}{k}}-1) \end{aligned}$$

$$\begin{aligned} \eta_{Otto} &= \frac{W_{net}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} \\ &= 1 - \frac{C_V T_1 (r_p-1)}{C_V T_1 (CR^{\frac{1}{k}}(p-1))} = 1 - \frac{1}{CR^{\frac{1}{k}}} \end{aligned}$$

$$\text{and also } \eta_{Otto} = 1 - \frac{C_V (T_4 - T_1)}{C_V (T_3 - T_2)} = 1 - \frac{T_1 (r_p-1)}{T_2 (r_p-1)} = \frac{1}{CR^{\frac{1}{k}}}$$

$$\textcircled{2} \text{ MEP} = \frac{W_{net}}{q_s}$$

Q → what can we do to increase the efficiency of Otto cycle ( $\eta_{Otto} \uparrow$ )?

Increase (CR), But we can only do that to a certain limit (knocking thing)

$$CR = \frac{T_3}{T_4} = \frac{T_2}{T_1}, \text{ so } T_4 T_3 \uparrow, T_4 \downarrow$$

In general, what exactly happens in Otto cycle?

The Otto cycle starts with the piston at B.D.C, then it moves to T.D.C with no heat in or out (process 1 → 2), the pressure will increase and volume decrease.

Then a sudden explosion happens and heat is added under a constant volume (the piston doesn't move) (process 2 → 3), then air expands until the piston goes back to B.D.C (here work is done by air on the piston so work (process 3 → 4)), then for the last process heat rejection occurs as the piston stays at B.D.C (constant volume  $q_{out}$ ) which is process (4 → 1)

note → If the specific heat is variable you can't take  $C_V$  out, neither  $k$  you have to use Table (A-17)

### PROBLEM (9-19)

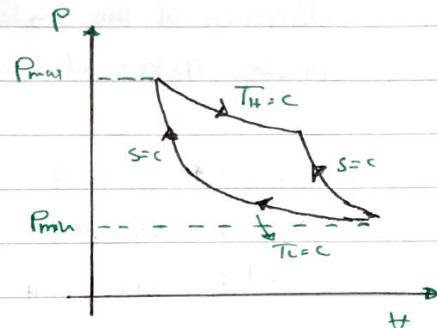
Consider a Carnot Cycle executed in a closed system with 0.6 kg of air. The temperature limits of the cycle are 300 and 1100 K, and the minimum and maximum pressures that occur during the cycle are 20 and 3000 kPa. Assuming constant specific heat.

Find ( $W_{net}$ )

→ Solution ←

Given :-  
 $T_1 = T_3$        $T_2 = T_4$   
 $m_a = 0.619$ ,  $T_L = 300\text{ K}$ ,  $T_H = 1100\text{ K}$

$$P_{min} = P_1 = 3\text{ MPa} \quad P_{max} = P_3 = 20\text{ kPa}$$



$$\eta_{Carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1100} = 0.7272$$

$$\eta_{Carnot} = \frac{W_{net}}{Q_H} \rightarrow \text{So to find } W_{net} \text{ we need } Q_H$$

$$\rightarrow Q_H = m_a T_H \Delta S$$

To find  $\Delta S$ , you have two relations :-

$$\Delta S = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \quad \left. \right\} \text{ change in Entropy for Ideal Gases}$$

$$\Delta S = C_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{P_2}{P_1}\right)$$

For process (1 → 2) : Isothermal  $\rightarrow T_1 = T_2 \rightarrow \frac{T_2}{T_1} = 1 \rightarrow \ln 1 = 0$

$$\text{So } \rightarrow \Delta S = -R \ln \frac{P_2}{P_1}$$

To find  $P_2$  :- (2 → 3) - Isentropic and you have  $P_3$  so use Isentropic Relations.

$$\left(\frac{P_2}{P_3}\right)_{S=c} = \left(\frac{T_2}{T_3}\right)^{\frac{K}{K-1}} \rightarrow P_2 = P_3 \left(\frac{T_2}{T_3}\right)^K = 20 \left(\frac{1100}{300}\right) \rightarrow P_2 = 1888\text{ kPa}$$

$$\rightarrow \Delta S = -R \ln \frac{P_2}{P_1} = -0.287 \ln \left(\frac{1888}{3000}\right) = -0.133 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$$

$$\rightarrow Q_H = m_a T_H \Delta S = (0.619)(1100)(0.133) \rightarrow Q_H = 87.78 \text{ kJ}$$

$$\rightarrow \eta_{Carnot} = \frac{W_{net}}{Q_H} \rightarrow W_{net} = (87.78)(0.7272) = 63.8 \text{ kJ}$$

## Problems for Otto Cycle :-

### [PROBLEM (9-30)]

An ideal Otto cycle has a compression ratio of 10.5, takes in air at 90 kPa and 40°C. Using constant specific heats at room temperatures, determine the thermal efficiency of this cycle and the rate of heat input if the cycle is to produce 90 kW of power.

→ Solution ←

Given :-  $CR = 10.5$ ,  $P_1 = 90 \text{ kPa}$ ,  $T_1 = 313 \text{ K}$ ,  $W_{net} = 90 \text{ kW}$

Find :-  $\eta_{otto}$ ,  $\dot{Q}_H$

$$\eta_{otto} = 1 - \frac{1}{CR^{k-1}} = 1 - \frac{1}{(10.5)^{1.4-1}} = 0.61$$

$$= \frac{W_{net}}{\dot{Q}_H} \rightarrow \dot{Q}_H = 148 \text{ kW}$$

### [PROBLEM (9-31)]

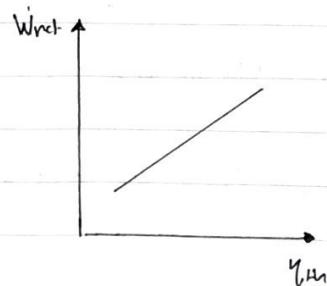
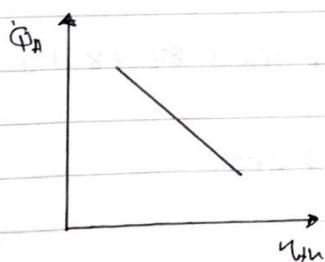
Repeat (9-30) with ( $CR = 8.5$ ) instead of (10.5)

→ Solution ←

$$\eta_{otto} = 1 - \frac{1}{CR^{k-1}} = 1 - \frac{1}{8.5^{0.4}} = 0.575$$

$$= \frac{W_{net}}{\dot{Q}_H} \rightarrow \dot{Q}_H = 157 \text{ kW}$$

From the Results of the two previous problems, you can conclude the Relation between the Thermal Efficiency with both  $W_{net}$  &  $\dot{Q}_H$



### PROBLEM (9-34)

An ideal Otto cycle has a CR of 8. At the beginning of the compression process air is at 95 kPa and 27°C, and 750 kJ/kg of heat is transferred to air during the constant volume heat-addition process. Assuming constant specific heat at room temperature. Find :- a.  $T_3$  &  $P_3$  c.  $\eta_{th}$   
b.  $W_{net}$  d. MEP

→ Solution ←

Given :-  $P_1 = 95 \text{ kPa}$ ,  $T_1 = 300 \text{ K}$ ,  $q_{in} = 750 \text{ kJ/kg}$ ,  $CR = 8$

$$\text{Point } [1] \rightarrow V_1 = \frac{RT_1}{P_1} = \frac{(0.287)(300)}{95} = 0.9063 \text{ m}^3 \text{ kg}$$

$$\text{Point } [2] \rightarrow T_2 = T_1 \text{ CR}^{k-1} = (300)(8)^{1.4-1} = 689.22 \text{ K}$$

$$P_2 = P_1 \text{ CR}^k = (95)(8)^{1.4} = 1746.02 \text{ kPa}$$

$$V_2 = \frac{V_1}{\text{CR}} = \frac{0.9063}{8} = 0.1133 \text{ m}^3/\text{kg}$$

$$\text{Point } [3] \rightarrow V_3 = V_2 = 0.1133$$

$$q_{in} = C_v(T_3 - T_2) \rightarrow T_3 = \frac{750}{0.718} + 689.22 = 1733.79 \text{ K}$$

From Ideal Gas law:

$$\frac{P_2 V_3}{m R T_3} = \frac{P_2 V_2}{m R T_2} \rightarrow \frac{P_3}{T_3} = \frac{P_2}{T_2} \rightarrow P_3 = \frac{(1733.79)}{(1746.02)} \cdot \frac{1}{(689.22)} = 4392.26 \text{ kPa}$$

$$\text{Point } [4] \rightarrow V_4 = V_1$$

$$\frac{T_3}{T_4} = \text{CR}^{k-1} \rightarrow T_4 = \frac{1733.79}{(8)^{1.4-1}} = 754.68 \text{ K}$$

$$\frac{P_3}{P_4} = \text{CR}^k \rightarrow P_4 = \frac{4392.26}{8^{1.4}} = 238.98 \text{ kPa}$$

$$W_c = C_v(T_2 - T_1) = 0.718(689.22 - 300) = 279.46$$

$$W_E = C_v(T_3 - T_4) = 0.718(1733.79 - 754.68) = 703$$

$$\eta_{th} = 1 - \frac{1}{\text{CR}^{k-1}} = 1 - \frac{1}{8^{0.4}} = 0.5647$$

To find  $W_{net}$ , two methods :  $\rightarrow W_{net} = \eta_{th} * q_{in} = 0.5647 * 750 = 423.48$

$$\rightarrow W_{net} = W_E - W_c = 703 - 279.46 = 423.54 \quad ] \text{ same.}$$

$$q_{out} = C_v(T_4 - T_1) = 0.718(754.68 - 300) = 326.46 \text{ kJ/kg}$$

$$\text{MEP} = \frac{W_{net}}{V_1 - V_2} = \frac{423.48}{(0.9063 - 0.1133)} = 534 \text{ kPa}$$

### PROBLEM (9-32)

Repeat the previous problem using VARIABLE specific Heat.

- Solution -

→ you CAN'T use  $C_v$  &  $C_p$  K, you have to find the Internal Energy (u)

for each point using Table (A-17) - Page 924

note → From the table you'll be using ( $V_r$ ) which is the volume at the Critical point

Point [1] →  $T_1 = 300\text{K}$

(A-17) →  $u_1 = 214.07 \text{ kJ/kg}$

$$V_r = 621.2 \text{ (from A-17)}$$

Point [2] → DON'T use isentropic relations that includes (k) in variable specific heat problems, use Eqn (7.49) & (7.50) for isentropic process:

$$\left(\frac{V_r_1}{V_r_2}\right)_{s=c} \xrightarrow{CR} = \frac{V_1}{V_2} \rightarrow \left(\frac{P_r_1}{P_r_2}\right)_{s=c} = \frac{P_1}{P_2}$$

↳ you can find  $V_r_2$  to find  $u_2$  and  $T_2$  from (A-17).

$$V_r_2 = \frac{V_r_1}{CR} = \frac{621.2}{8} = 77.65$$

using interpolation:  $u_2 = 491.2 \text{ kJ/kg}$

$$T_2 = 673.1 \text{ K}$$

Point [3] →  $q_{in} = u_3 - u_2$

$$u_3 = q_{in} + u_2 = 750 + 491.2 = 1241.2 \frac{\text{kJ}}{\text{kg}}$$

(A-17) → Interpolation →  $V_r_3 = 6.588$

$$T_3 = 1539 \text{ K}$$

Point [4] →  $\frac{V_r_4}{V_r_3} = \frac{V_4}{V_3} = CR$

$$V_r_4 = V_r_3 \cdot CR = (6.588) (8) = 52.7$$

↳ (A-17) →  $u_4 = 571.69 \text{ kJ/kg}$

$$T_4 = 774.5 \text{ K}$$

$$W_c = u_2 - u_1 = 491.2 - 214.07 = 277.18 \text{ kJ/kg}$$

$$W_E = u_3 - u_4 = 1241.2 - 571.69 = 669.51 \text{ kJ/kg}$$

$$q_{\text{out}} = u_4 - u_1 = 571.69 - 214.07 = 357.6 \text{ kJ/kg}$$

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{357.6}{750} = 0.523$$

→ note that we did not use the relations that included K

$$\text{MEP} = \frac{W_{\text{net}}}{v_1 - v_2} = \frac{W_E - W_c}{v_1 - v_2} = \frac{392.4}{0.9063 - 0.1133} = 492.8 \text{ kPa}$$

Q 8- compare the results of this problem with the previous one

→  $T_2 \& T_3$  Decreased

→  $T_4$  increased → This is a disadvantage because  $T_4$  is the temperature of the exhaust, which represents the  $T$  that we couldn't convert to work

∴ the thermal efficiency ( $\eta_{\text{th}}$ ) using variable specific heat is less than ( $\eta_{\text{th}}$ ) using constant specific heat

→ using variable specific heat is more accurate and closer to reality while assuming constant specific heat is considered an idealization.

$$W_c = u_2 - u_1 = 491.2 - 214.07 = 277.18 \text{ kJ/kg}$$

$$WE = u_3 - u_4 = 1241.2 - 571.69 = 669.51 \text{ kJ/kg}$$

$$q_{\text{out}} = u_4 - u_1 = 571.69 - 214.07 = 357.6 \text{ kJ/kg}$$

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{357.6}{750} = 0.523$$

→ note that we did not use the relations

$$MEP = \frac{W_{\text{net}}}{V_1 - V_2} = \frac{WE - W_c}{V_1 - V_2} = \frac{392.4}{0.9063 - 0.1133} = 495.8 \text{ kPa}$$

that included K

Q :- compare the results of this problem with the previous one

→  $T_2$  &  $T_3$  Decreased

→  $T_4$  increased → This is a disadvantage because  $T_4$  is the temperature of the exhaust, which represents the  $T$  that we couldn't convert to work

∴ the thermal efficiency ( $\eta_{\text{th}}$ ) using variable specific heat is less than ( $\eta_{\text{th}}$ ) using constant specific heat

→ using variable specific heat is more accurate and closer to Reality while assuming constant specific heat is considered an idealization.

## DIESEL CYCLE

→ Diesel proposed the heat addition to be at constant pressure and that can be done easily by letting the piston free to move while adding heat, so it is also more realistic than Carnot Cycle.

→ To understand :-

looking at the ideal Gas law :  $PV = mRT \rightarrow P = mR \left( \frac{T}{V} \right) \rightarrow$  you can see that to keep (P) constant, the Ratio  $\left( \frac{T}{V} \right)$  must remain constant, which is achieved when the piston is free to move.

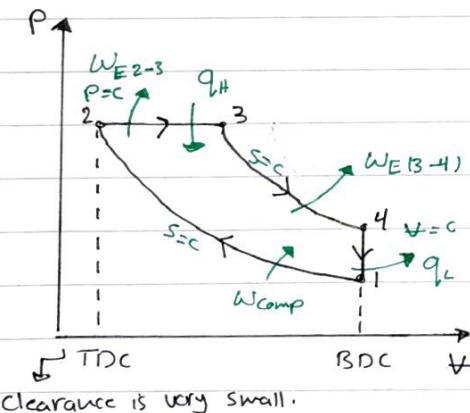
→ So, what happens in Diesel Cycle is :-

$q_H \rightarrow$  Constant pressure.

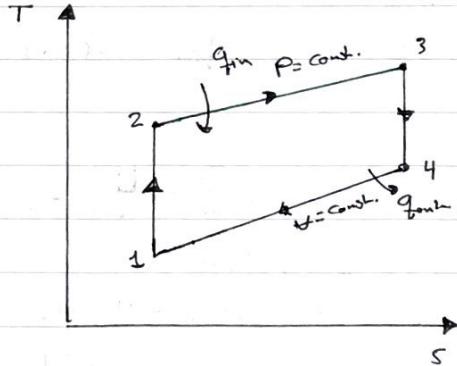
$q_L \rightarrow$  Constant volume.

$W_{in}, W_{out} \rightarrow$  Constant Entropy. (Isentropic)

→ DIAGRAMS :-



Clearance is very small.



→ Note that- There are two Expansion works,  $W_E(2-3)$  which results from the movement of the piston while the heat is being added, BUT the real work we are looking for is  $W_E(3-4)$

→ NOTE !!

$$\frac{H_1}{H_2} = \frac{W_1}{W_2} = CR$$

↳ This Does NOT work in Diesel cycle

$$\text{So} \rightarrow CR = \frac{H_1}{H_2} \text{ only.}$$

## EQUATIONS

process  $(1 \rightarrow 2)$  - Isentropic

$$q_f = 0, \omega_{out} = 0$$

$$\omega_{in} = \omega_c = u_2 - u_1$$

$$= Cu(T_2 - T_1)$$

Recall: For isentropic, you can use the relations:

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{K-1}{K}} = \left( \frac{V_1}{V_2} \right)^{K-1} = CR^{K-1} \rightarrow T_2 = T_1 CR^{K-1}$$

$$P_2 = P_1 CR^K$$

Process  $(2 \rightarrow 3)$  - constant pressure.

note  $\rightarrow$  since the process is under constant pressure ( $C_p$ ) not ( $Cv$ ) is used.

$$q_{out} = 0, \omega_{in} = 0$$

$$q_{in} - \omega_b(2 \rightarrow 3) = u_3 - u_2$$

but we know that:  $\omega_b(2 \rightarrow 3) = P(V_3 - V_2)$  - area under the curve -

$$\text{so } \rightarrow q_{in} - P(V_3 - V_2) = u_3 - u_2$$

$$q_{in} = (u_3 + PV_3) - (u_2 + PV_2) \quad \text{But } \rightarrow (u + PV = h)$$

$$= h_3 - h_2 = \Delta h$$

$$q_{in} = C_p(T_3 - T_2)$$

Process  $(3 \rightarrow 4)$  - Isentropic

$$q_f = 0, \omega_{in} = 0$$

$$- \omega_E(3 \rightarrow 4) = u_4 - u_3$$

$$\omega_E(3 \rightarrow 4) = Cu(T_3 - T_4)$$

$$\text{also! Isentropic Relations } \rightarrow \frac{T_3}{T_4} = \left( \frac{P_3}{P_4} \right)^{\frac{K-1}{K}} = \left( \frac{V_1}{V_3} \right)^{K-1} \quad \text{again } \rightarrow \frac{V_4}{V_3} \neq CR$$

Process  $(4 \rightarrow 1)$  - constant volume.

$$\omega = 0, q_{in} = 0$$

$$- q_{out} = u_1 - u_4$$

$$q_{out} = u_4 - u_1$$

$$= Cu(T_4 - T_1)$$

## CUT-OFF RATIO - $r_c$

→ It is the ratio of the cylinder volumes after & before the combustion process  
 So → From process (2 → 3) :-

$$\left[ r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} \right] \text{ note: This is Mathematically true but as a concept } r_c \text{ is volume to volume ratio, not temp. to temp.}$$

note :- for Otto cycle ( $r_c$ ) would = 1 because process (2 → 3) is at constant volume.  
 -  $r_c$  gives you an idea about how much heat was added.

summary - Diesel cycle relations :-

- \*  $W_c = Cv(T_2 - T_1)$
- \*  $q_{in} = h_3 - h_2 = Cp(T_3 - T_2)$
- \*  $W_E(2-3) = P_2(V_3 - V_2) = R(T_3 - T_2)$
- \* also  $\rightarrow q_{in} = W_E(2-3) + (u_3 - u_2) = W_E(2-3) + Cv(T_3 - T_2)$
- \*  $W_E(3 \rightarrow 4) = Cv(T_4 - T_1)$
- \*  $q_{out} = Cv(T_4 - T_1)$

Thermal Efficiency of Diesel Cycle :-

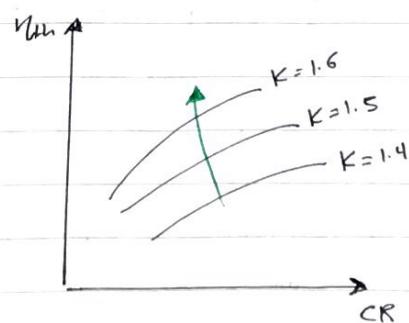
$$\begin{aligned} \eta_{th} &= 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{Cv(T_4 - T_1)}{Cp(T_3 - T_2)} \quad \text{But } \rightarrow \frac{Cv}{Cp} = \frac{1}{k} \\ &= 1 - \frac{1}{k} \frac{T_1}{T_2} \left[ \frac{\frac{T_4}{T_1} - 1}{\frac{T_3}{T_2} - 1} \right] \quad \text{But } \rightarrow \left( \frac{T_1}{T_2} = \frac{1}{CR^{k-1}} \right) \quad \& \quad \left( \frac{T_3}{T_2} = r_c \right) \end{aligned}$$

→ To relate  $\frac{T_4}{T_1}$  with CR &  $r_c$

$$\frac{T_4}{T_1} = \frac{T_4}{T_3} \times \frac{T_3}{T_2} \times \frac{T_2}{T_1}$$

$$= \left( \frac{r_c}{CR} \right)^{k-1} \times r_c \times CR^{k-1} = r_c^k$$

$$\text{So } \rightarrow \eta_{th} = 1 - \frac{1}{CR^{k-1}} \left[ \frac{r_c^k - 1}{k(r_c - 1)} \right]$$



NOTE → as  $k$  increases,  $r_c^k$  increases in a rate higher than the other terms so  $\eta_{th}$  will DECREASE, also increasing ( $r_c$ ) lowers ( $\eta_{th}$ ).

## PROBLEMS - Diesel Cycle

### PROBLEM (9-44+45)

An air standard Diesel cycle has a compression ratio of (16) and a cutoff ratio of (2). air is at (95 kPa) and (27°C), Find  $T_3, \eta_{th}, \text{MEP} \rightarrow$  (everything) using Both constant and variable specific heat

→ Solution ←

#### CASE - 1- Constant specific heat

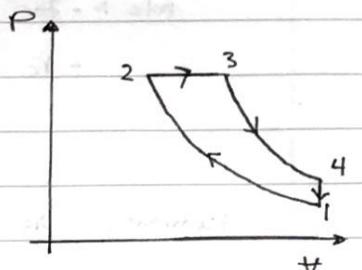
$$\text{Point } [1] \rightarrow P_1 = 95 \text{ kPa}, T_1 = 300 \text{ K}$$

$$V_1 = \frac{RT_1}{P_1} = 0.9063 \text{ m}^3/\text{kg}$$

$$\text{Point } [2] \rightarrow P_2 = P_1 (CR)^k = (95)(16)^{1/4} = 4607.77 \text{ kPa}$$

$$T_2 = T_1 (CR)^{k-1} = 909.43 \text{ K}$$

$$V_2 = \frac{V_1}{CR} = \frac{0.9063}{16} = 0.05664 \text{ m}^3/\text{kg}$$



$$\text{Point } [3] \rightarrow P_3 = P_2 = 4607.77$$

$$r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} \rightarrow V_3 = 2(0.05664) = 0.1132 \text{ m}^3/\text{kg}$$

$$T_3 = 2(909.43) = 1818.810 \text{ K}$$

$$\text{Point } [4] \rightarrow V_4 = V_1$$

$$\frac{P_4}{P_3} = \left( \frac{r_c}{CR} \right)^k \rightarrow P_4 = 250.7 \text{ kPa}$$

$$\frac{T_4}{T_3} = \left( \frac{r_c}{CR} \right)^{k-1} \rightarrow T_4 = 791.7 \text{ K}$$

$$\eta_{th} = 1 - \frac{1}{CR^{k-1}} \left( \frac{r_c^k - 1}{k(r_c - 1)} \right) = 1 - \frac{1}{16^{0.4}} \left( \frac{2^{1.4} - 1}{1.4(2-1)} \right) = 0.613$$

$$q_{in} = C_p(T_3 - T_2) = 1.005 (1818.86 - 909.43) = 914 \text{ kJ/kg}$$

$$w_{net} = \eta_{th} q_{in} = (0.613)(914) = 561 \text{ kJ/kg}$$

$$\text{MEP} = \frac{w_{net}}{V_1 - V_2} = \frac{561}{0.9063 - 0.05664} = 660.3 \text{ kPa}$$

For practice, Find the following:

$$q_{out} = 353.05$$

$$w_c = 432.6$$

$$w_E(2-3) = 261$$

$$w_E(3-4) = 737.5$$

Note:

you can find  $w_{net}$  using:

$$w_{net} = w_E_{23} + w_E_{34} - w_c$$

$$= q_{in} - q_{out}$$

CASE -2- Variable specific Heat.

Point 1  $\rightarrow T_1 = 300\text{ K}$

(A-17) :-  $u_1 = 214.027 \text{ kJ/kg}$

$\sqrt{r_1} = 621.2$

Point 2  $\rightarrow \left( \frac{\sqrt{r_2}}{\sqrt{r_1}} \right)_{s=c} = \frac{\sqrt{r_2}}{\sqrt{r_1}} = \frac{1}{CR} \rightarrow \sqrt{r_2} = \frac{621.2}{16} = 38.825$

(A-17) :-  $u_2 = 642.945 \text{ kJ/kg}$

$T_2 = 862.25 \text{ K}$

$h_2 = 890.9 \text{ kJ/kg}$

Point 3  $\rightarrow \frac{T_3}{T_2} = r_c \rightarrow T_3 = (862.25)(2) = 1724.2 \text{ K}$

(A-17) :-  $h_3 = 1910.5 \text{ kJ/kg}$

$u_3 = 1415.9 \text{ kJ/kg}$

$\sqrt{r_3} = 4.547$

Point 4  $\rightarrow \frac{\sqrt{r_4}}{\sqrt{r_3}} = \frac{\sqrt{r_4}}{\sqrt{r_3}} = \frac{CR}{r_c} = \frac{16}{2} = 8 \rightarrow \sqrt{r_4} = (4.547)(8) = 36.37$

(A-17) :-  $u_4 = 659.68$

$T_4 = 882.08$

### PROBLEM (9-46)

An ideal Diesel cycle has a compression Ratio of (17) and a cutoff Ratio of (1.3). Determine the max. Temp of the air and the rate of heat addition to this cycle when it produces (140 kW) of Power and the state of air at the beginning of the compression is (90 kPa) and (57°C), use constant specific heats at room temp.

→ Solution ↗

Given :  $r_c = 1.3$ ,  $(R = 17)$ ,  $P_1 = 90 \text{ kPa}$ ,  $T_1 = 57^\circ\text{C} = 330 \text{ K}$ ,  $W_{net} = 140 \text{ kW}$

$$\eta_{th} = 1 - \frac{1}{(R^{k-1})} \left( \frac{r_c^k - 1}{k(r_c - 1)} \right)$$

$$= 1 - \frac{1}{17^{0.4}} \left( \frac{1.3^{1.4} - 1}{1.4(1.3 - 1)} \right) = 0.66$$

$$\eta_{th} = 0.66 = \frac{W_{net}}{Q_{in}} \rightarrow Q_{in} = \frac{140 \text{ kW}}{0.66} = 212.2 \text{ kW}$$

$$T_2 = T_1 (R^{k-1})$$

$$= (330) (17)^{0.4} = 1024.9$$

$$r_c = \frac{T_3}{T_2} \rightarrow T_3 = 1332.4 \text{ K}$$

## DUAL CYCLE

→ Combustion in this cycle happens at two stages:-

First, a part of the fuel is injected and combustion happens at constant volume, as a result the temperature inside the cylinder increases to a value enough to burn the rest of the fuel but now at constant pressure.

→ This cycle combines both Otto & Diesel (it contains both spark ignition and auto ignition).

→ So, Heat is added ( $q_{in}$ ) in two stages 1- at constant volume.

2- at constant pressure.

→ Dual Cycle has 5 processes in total :  $q_{in1} \rightarrow$  constant  $T$

$q_{out1} \rightarrow$  constant  $T$

$q_{in2} \rightarrow$  constant  $P$

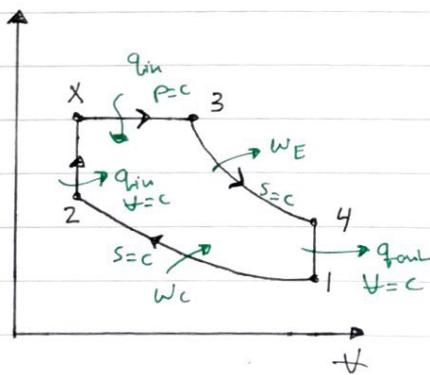
$W_{out} \rightarrow$  constant  $S$

$W_{in} \rightarrow$  constant  $S$

→ CR of Dual Cycle :

↳ note that it is less than Diesel because there is no need to compress the air as much as Diesel since it's not all self-ignition

→ Diagrams:



## EQUATIONS

$$q_{in} = u_x - u_2 = Cv(T_x - T_2)$$

$$q_{out} = h_3 - h_x = Cp(T_3 - T_x)$$

$$w_c = u_2 - u_1 = Cv(T_2 - T_1)$$

$$w_{E_{34}} = u_3 - u_4 = Cv(T_3 - T_4)$$

$$q_{out} = u_4 - u_1 = Cv(T_4 - T_1)$$

## PRESSURE RATIO ( $r_p$ ):-

$$r_p = \frac{P_x}{P_2} = \frac{T_x}{T_2}$$

## CUT OFF RATIO

$$r_c = \frac{T_3}{T_x} = \frac{\sqrt{3}}{\sqrt{x}}$$

now, let's the following

$$\frac{T_x}{T_1} = \frac{T_x}{T_2} \times \frac{T_2}{T_1} = r_p C R^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=c} = \left(\frac{\sqrt{1}}{\sqrt{2}}\right)^{k-1} = C R^{k-1}$$

$$\frac{T_2}{T_1} = \frac{T_3}{T_1} \cdot \frac{T_x}{T_2} = r_c r_p C R^{k-1}$$

$$\frac{T_3}{T_2} = \frac{\cancel{T_3}}{\cancel{T_x}} \cdot \frac{\cancel{T_x}}{\cancel{T_2}} \rightarrow r_c \rightarrow r_p$$

$$\frac{T_4}{T_1} = r_c^k r_p$$

### PROBLEM (9.47)

An air standard dual cycle has a compression ratio of (14) and a cutoff ratio of (1.2). The pressure ratio during the constant volume heat addition process is (1.5). Determine the thermal efficiency, amount of heat added, the max. gas pressure and T when the cycle is operated at (80 kPa) and (20°C) at the beginning of the compression. Use constant specific heats at room T.

→ Solution ←

Given:  $CR = 14$ ,  $r_c = 1.2$ ,  $r_p = 1.5$ ,  $P_1 = 80 \text{ kPa}$ ,  $T_1 = 20^\circ\text{C} = 293 \text{ K}$

Required:  $\eta_{th}$ ,  $q_{in}$ ,  $(P_1 T)_m = (P_1 T)_3$

Point  $\boxed{1}$   $\rightarrow P_1 = 80 \text{ kPa}$ ,  $T_1 = 20^\circ\text{C} = 293 \text{ K}$

Point  $\boxed{2}$   $\rightarrow P_2 = P_1 CR^k$

$$= (80)(14)^{1.4} = 842.013 \text{ K}$$

Point  $\boxed{3}$   $\rightarrow \frac{T_3}{T_2} = \frac{P_3}{P_2} = r_p = 1.5 \rightarrow T_3 = 1263.02 \text{ K}$

$$P_3 = 4827.9 \text{ kPa} = P_3 = P_{max}$$

$$q_{in}(H) = C_v (T_3 - T_2) = 302.28 \text{ kJ/kg}$$

$$q_{in}(P) = C_p (T_3 - T_1) = 253.86 \text{ kJ/kg}$$

$$q_{in} = q_{in}(H) + q_{in}(P) = 556.14 \text{ kJ/kg}$$

Point  $\boxed{4}$   $\rightarrow r_c = \frac{T_4}{T_1} = \frac{V_4}{V_1} \rightarrow T_4 = 1515.62 \text{ K}$

$$\eta_{th} = 1 - \frac{1}{CR^k} \left( \frac{r_p r_c^k - 1}{(1) + k(1)} \right) = 0.646$$

For you:

$$W_{net} = 359.2$$

$$T_4 = 567.3 \text{ K}$$

$$W_c = 394.2$$

$$P_4 = 154.89 \text{ kPa}$$

$$W_{E1-4} = 680.89$$

$$W_{E4-3} = 72.5$$

$$q_{out} = 196.95$$

## BRAYTON CYCLE

→ It is also called "Gas Turbine Cycle"

→ This cycle is CONTROL VOLUME

↳ Note that all the cycles we considered until now were CONTROL VOLUME

"Piston Cylinder" and we applied 1st Law for "CLOSED SYSTEMS"

→ The cycle consists of [STEADY FLOW DEVICES]

Recall → steady flow devices are like (Turbine, Compressor, Heat Exchanger, Pump, Nozzle, Diffuser, ... etc)

↳ They were considered in CH#5 "Revise them"

→ for steady flow devices, we'll be using:

\* 1st law for open system

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} m(h + K.E + P.E) = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} m(h + K.E + P.E)$$

\* Continuity Equation

$$\sum_{in} m_i = \sum_{out} m_i$$

↳ since we'll only deal with steady

flow devices (no Transient)  $\rightarrow \sum m_{system} = 0$

so for steady state  $\rightarrow \sum_{in} m_i = \sum_{out} m_i$

→ This cycle is used to generate electricity

↳ cycles NOT devices

→ In general, Cycles are two Types: Open and Closed

① Open

Final state is not the same as the initial state.

② Closed

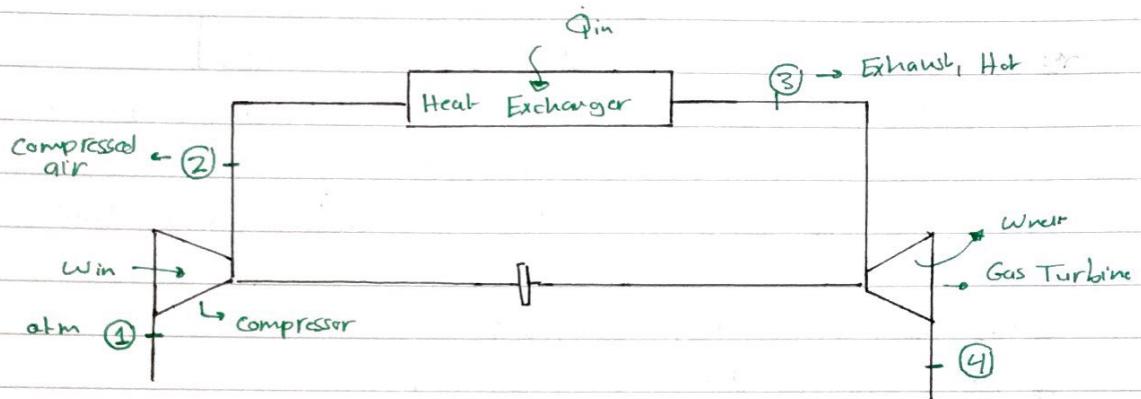
Final state is the same as the initial state.

note → The term "Cycle" means that the initial state has to be same as final state

BUT we approximate the (Open Cycle) to be a Cycle in Thermo.

→ There is an Open Brayton Cycle and a Closed Brayton Cycle, we will understand both, But the important one to us is the Closed cycle and all calculations will be based on it.

### OPEN CYCLE



What happens in this cycle? How does it work?

→ The compressor is a device used to raise the pressure of a Gas, to compress it! (note: it only deals with Gases so there is no need to say Gas Compressor).

→ This Compressor takes air from the atm (at point 1), increase its pressure until point (2)

↳ To do that it consumes energy that's why ( $W_{in}$ )  
so point 2 → High pressure and Temperature (compressed air)

→ This Compressed air (from 2) enters the Heat exchanger which we call here (a Combustion Chamber CC), Fuel is injected into the CC (which represents  $Q_{in}$ ) and mixed with air

→ at point (3), (Gases) are out of the CC with a very high Temperature ( $T_3$  is  $T_{max}$  in the cycle)

↳ Notice we said GASES NOT AIR, these gases are called Exhaust gases and they have different properties than air since they got mixed with the fuel in the CC

→ That's why we said this is an Open Cycle, air came in BUT Exhaust Gases with totally different properties come out (initial state is not the same as the final state)

→ Then, Hot exhaust Gases from point (3) enters the Gas Turbine with a very High Enthalpy, the Turbine converts this energy into (work) to generate electricity which is the objective of the cycle.

↳ we will learn how the Turbine does that later in (Power Plants)

Q → from where does the compressor takes Energy to work ?

→ The work produced by the Turbine will be used for two things:

1. a part will go to an Electric generator to generate electricity.
2. a part will be used to run the Compressor.

→ That's why the Turbine and the compressor are connected with a shaft.

→ One of the problems of Brayton Cycle, is that the compressor takes more than 50% of the Turbine's produced work, so what is left for electricity is not that much.

Q → why does the compressor consumes that much work?

we know that  $w = v S dp$ , so for the same ( $\Delta P$ ), as the volume of air increases, work needed will increase as well and since air comes into the compressor with a relatively High ( $T$ ), it has a big ( $v$ ) so it needs alot of work to be compressed, which will be taken from the Gas Turbine.

→ Exhaust Gases Finally comes out at point (4) with a lower  $T$  and  $P$  but still considered Hot, they are out to the atmosphere.

## PROBLEMS OF THE OPEN CYCLE !!

→ There are (3) important problems related to open Gas-Turbine Cycle:-

### 1. Air Pollution & Damage

- Since the exhaust gases at point (4) are rejected to the atmosphere they will cause air pollution (Because of their high T and their content).
- When exhaust gases at point (3) enters the Gas Turbine, they will cause damage of its internal parts (The mixture contains  $H_2SO_4$  (Gibson, 1969))  
(The chemical reaction produces carbon dioxide (1969))  
. (Carbon dioxide Turbine 1969)

### 2. Complication of Analysis.

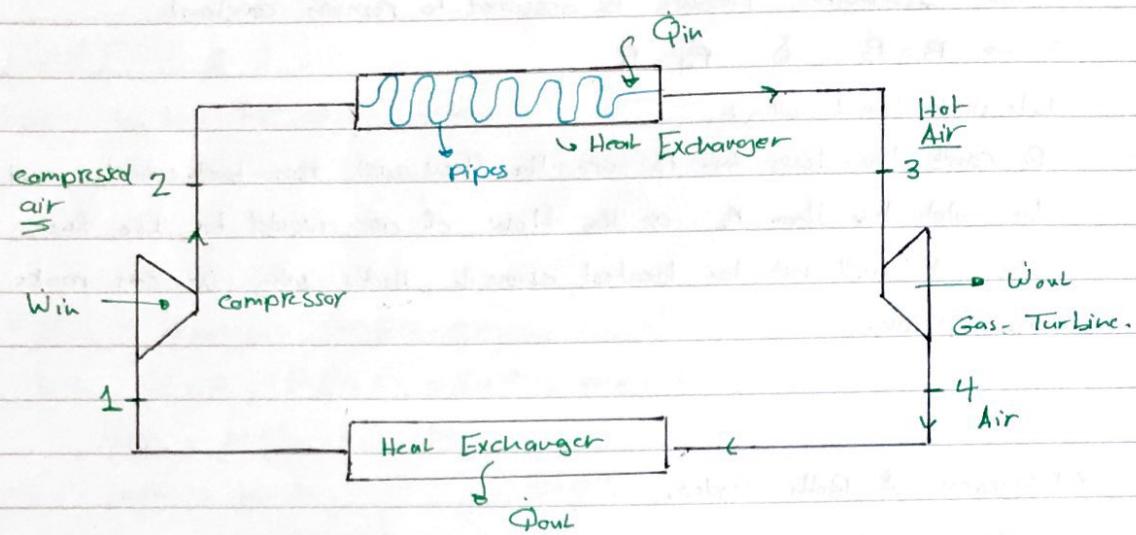
- The gas coming out of C.C (Point 3), have different properties than air, which means it has different ( $\rho$ ,  $C_p$ ,  $C_v$ , etc) and it is hard to find the values of these properties for exhaust gases because they contain

### 3. The need of a filter at the compressor's inlet.

- Since air will be constantly pulled from the atmosphere, it needs filterization, adding filters decreases the pressure of air coming into the compressor, hence it'll need more work to compress it, which will be taken from the Turbine, as a result the efficiency of the cycle will decrease.

→ Open Cycle is only used in very special cases, what is better and more efficient to be used is the (closed cycle), which we will consider next.

## CLOSED CYCLE



→ Closed - Turbine Cycle has the same concept of Open Cycle with two main differences :-

1] In Heat exchanger (from 2→3), There is NO DIRECT MIX between the Fuel and air (air is run through pipes), so at point (3) there is now HOT AIR not Exhaust Gases (same  $C_p$ ,  $C_v$ ,  $k$ , ... etc) which is better for calculations.

2] a Heat exchanger is added after point (4), this Heat exchanger cools down the air coming from the Turbine and sends it back to the compressor.

↳ it is better to cool down the air before entering the compressor to lower its specific volume hence, Reduce the work needed by the compressor to increase its pressure.

\* note that the problems introduced in the open cycle are solved when using the closed cycle.

↳ There will still be air pollution BUT add less than the one resulting from the open Turbine Cycle.

Important :-

In Heat exchangers, pressure is assumed to remain constant

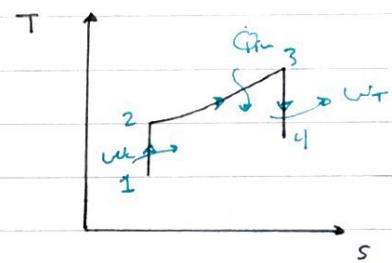
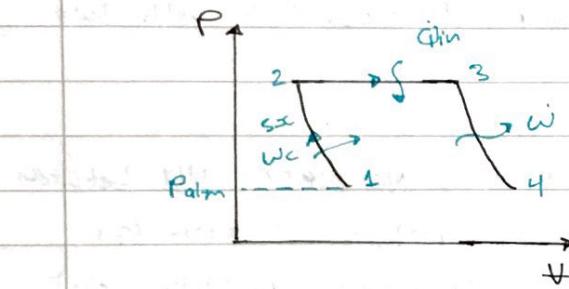
$$\text{So } \rightarrow P_2 = P_3 \quad \& \quad P_4 = P_1$$

Let's understand why :-

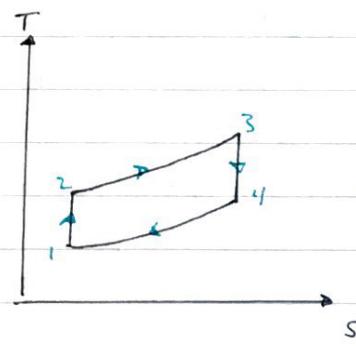
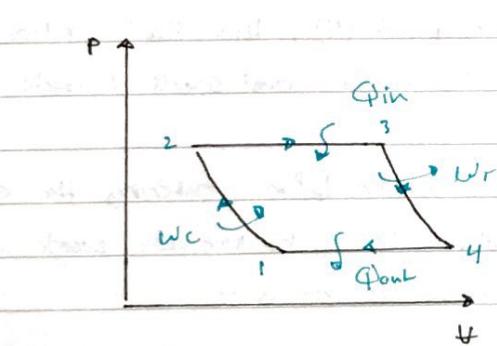
$P_2$  can't be less than  $P_3$  or the flow will run backwards, and  $P_3$  can't be a lot less than  $P_2$  or the flow of air would be too fast that the air will not be heated enough, that's why we can make this assumption.

\* Diagrams of Both Cycles:

→ Open Cycle :-



→ Closed Cycle :-



Notes :-

→ since we are dealing with (Shaft work) not (Boundary work), we don't have compression ratio now (only pressure Ratio)

→ in the open cycle diagram, you can see there is  $Wc$ ,  $Qout$ ,  $PnL$   
in reality there is ( $CPDT$ )

→ Remember: we are only required to solve problems for (closed cycle)

## EQUATIONS

Recall: 1st law for open system:

$$\dot{Q}_{in} + \dot{W}_{in} + \sum m(h + K.E + P.E) = \dot{Q}_{out} + \dot{W}_{out} + \sum m(h + K.E + P.E)$$

### (1 → 2) - isentropic, compression

$$s=c, \dot{Q}=0, P.E=0, K.E=0, \dot{W}_{out}=0$$

$$\dot{W}_{in} = m(h_2 - h_1) \text{ kW}$$

$$\dot{W}_{in} = h_2 - h_1$$

$$= Cp(T_2 - T_1)$$

note → we assume  $P.E = 0$  as: in CH4 we learned that every 100m would give 1kJ of energy ( $P.E = mgZ$ ), the height of a device is so much smaller than 100m, so we can neglect it.

•  $K.E \approx 0$  because the difference in the inlet and outlet velocity of a compressor is very small  $\Delta K.E = \frac{1}{2}m(V_2^2 - V_1^2) \approx 0$

### (2 → 3) - Isobaric, heat addition

$$p=c, \dot{Q}_{out}=0, \dot{W}=0$$

$$\dot{Q}_{in} = m(h_3 - h_2)$$

$$\dot{q}_{in} = h_3 - h_2$$

$$= Cp(T_3 - T_2)$$

### (3 → 4) - Isentropic, Expansion. → same reasons as compressor (process 1 → 2)

$$s=c, \dot{Q}=0, \dot{W}_{in}=0, \Delta P.E=0, \Delta K.E=0$$

$$\dot{W}_{out} = m(h_3 - h_4)$$

$$\dot{W}_{out} = h_3 - h_4 = Cp(T_3 - T_4)$$

### (4 → 1) - Isobaric,

$$p=c, \dot{Q}_{in}=0, \Delta P.E = \Delta K.E = 0$$

$$\dot{Q}_{out} = m(h_4 - h_1)$$

$$\dot{q} = Cp(T_4 - T_1) = h_4 - h_1$$

also, Isentropic Relations will be needed

for  $(1 \rightarrow 2)$  : Isentropic

$$\left(\frac{T_2}{T_1}\right)_{S=c} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}$$

for  $(3 \rightarrow 4)$  : Isentropic

$$\left(\frac{T_3}{T_4}\right)_{S=c} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}$$

BUT  $\rightarrow (P_2 = P_3), (P_4 = P_1)$

so, you can see that the Ratios are equal.

Thermal efficiency of the CYCLE :-

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$$

$$= 1 - \frac{C_p(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)} \quad \xrightarrow{1} \text{Because} \quad \text{But} : \frac{T_2}{T_1} = \frac{T_3}{T_1}$$

$$= 1 - \frac{T_1}{T_2} \quad \xrightarrow{\text{But}} \left(\frac{T_2}{T_1} = r_p^{\frac{k-1}{k}}\right), \text{ so:} \quad \xrightarrow{2} \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

$$\boxed{\eta_{th} = 1 - \frac{1}{r_p^{\frac{k-1}{k}}} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3}}$$

work calculations in terms of  $r_p$ :

$$w_c = C_p T_1 (r_p^{\frac{k-1}{k}} - 1)$$

$$w_T = C_p T_4 (r_p^{\frac{k-1}{k}} - 1)$$

\* BACK WORK RATIO :-  $w_{Br}$

$$w_{Br} = \frac{w_c}{w_T} \approx 50-60\%$$

→ This Ratio gives an idea of how much the compressor consumes of the Turbine's work.

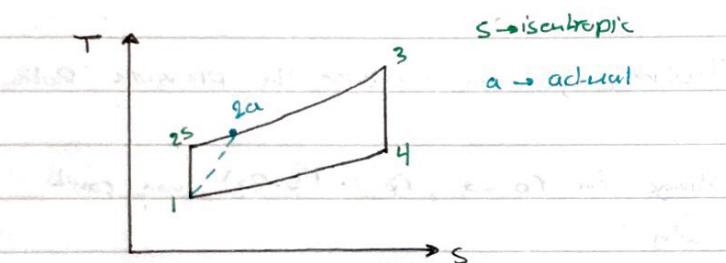
## [Adiabatic efficiencies] $\eta_s$

→ adiabatic efficiency was discussed in CH#7.

→ we want to find the adiabatic efficiency of the Turbine and the compressor.

→ In reality, the work needed for the compressor is more than the work we calculate, because we assumed the process to be isentropic which is not true in reality.

\* For the compressor: (process 1→2)



→ since process (1→2) is not really isentropic, the entropy will increase and line (1→2) will not be a straight line as you can see.

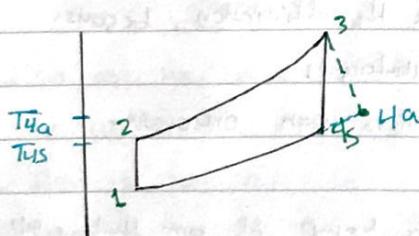
→ so, compressor needs more work in reality to do its job.

→ Recall: ( $W_c = h_2 - h_1$ )

$$\eta_{sc} = \frac{\text{ideal work}}{\text{actual work}} = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

( $W_{ideal} < W_{actual}$ )

\* For the Turbine (process 3→4)



→ we also assumed 3→4 to be isentropic while in reality the entropy will increase until point 4a.

→ Recall:  $W_t = h_3 - h_4$

→ Turbine will produce work

$$\eta_{st} = \frac{\text{actual work}}{\text{ideal work}} = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

less than it should.

Because  $W_{actual} < W_{ideal}$

∴ Thermal Efficiency of actual cycles is always less than ideal cycles.

note →  $T_{4a}$  is higher than  $T_{4s}$  which means that the amount of heat that the turbine didn't convert to work is higher, hence more loss.

## HOW CAN WE IMPROVE THE EFFICIENCY OF THE CYCLE ?

To study the choices of improving the efficiency, look at the equation:

$$\eta = 1 - \frac{1}{r_p \frac{k-1}{k+1}} = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3}$$

→ we will study every choice :-

note → ( $T_1$ ) is not a choice since you can't control it (it's the temperature of the atm).

### • Increasing ( $r_p$ ) :-

↳ to increase the efficiency you can increase the pressure Ratio  
BUT !!

there is a certain Range for  $r_p \rightarrow r_p = (5-20)$ , you can't exceed this number, why?

IF you keep on increasing ( $r_p$ ), eventually the work of the compressor will exceed the turbine work.

### \* Increasing ( $T_3$ ) :-

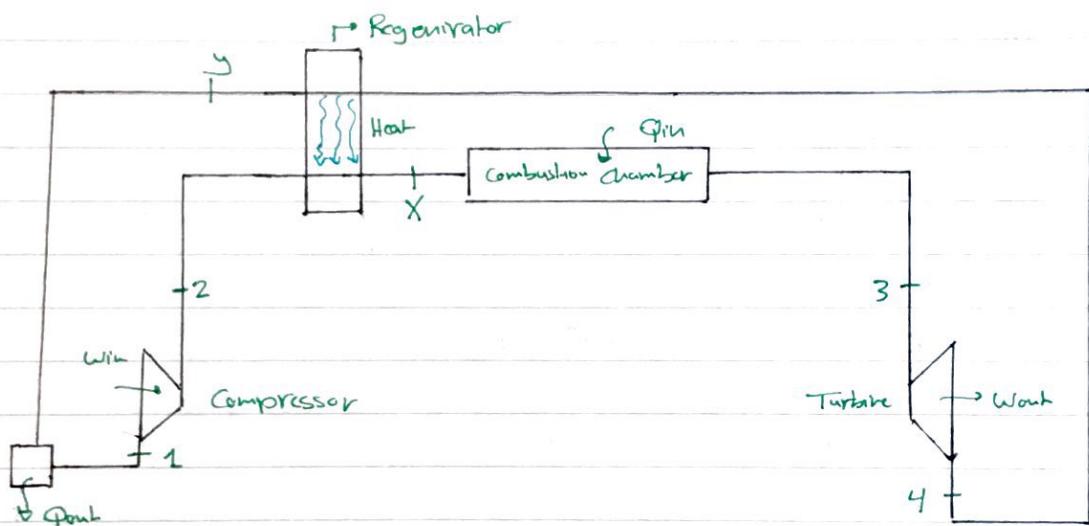
↳ this is not a good way to improve the efficiency, because increasing ( $T_3$ ) has too many disadvantages:  
→ it needs burning more fuel which is more expensive.  
→ more pollution  
→ material limit → since  $T_3$  is the temp of air that will enter the turbine, the materials in the turbine might not be able to take very high values of ( $T_3$ ).

### \* Increasing ( $T_1$ ) :-

↳ increasing ( $T_1$ ) is considered the best method for improving efficiency, how can we do it?  
→ it is done by what we call Regeneration, which will be discussed next.

## Brayton Cycle with Regeneration

- we said that increasing  $T_2$  is the best method to improve the efficiency of the cycle.
- increasing  $T_2$  means that less fuel ( $q_{in}$ ) will be needed to get the air into point (3) -  $T_2$
- we do that using Regeneration
  - \* Regeneration in general means heating the fluid before entering the C.C.



→ We use the temperature of air at  $T_4$ , so instead of  $T_4$  extra heat is wasted we use it to increase the Temperature at  $T_2$ .

→ Heat is done by adding a Regenerator as shown, so heat is transferred from air at  $T_1$  to air at  $T_2$  (heat exchanger)

→ what's the result?

now, instead of air entering the C.C. with  $T_2$ , it'll enter the C.C. with  $T_3$  which is higher (less  $q_{in}$  is needed  $\rightarrow v_l \uparrow$ )

also, instead of  $q_{out}$  coming out at point (4) it comes out at point (5) which has less Temp  $\rightarrow$  less  $q_{out}$   $\rightarrow$  Better.

→ this remains unchanged.

$$\eta = 1 - \frac{q_{\text{out}} \downarrow}{q_{\text{in}} \downarrow} = \frac{W_{\text{out}}}{q_{\text{in}} \downarrow} \quad (\text{Increase in efficiency !})$$

نحو ← اكي طالع هو (1) Turbin درجة حرارته اكلاه هو (2) ، بينما درجات بيتخانة هو نعمة (2) .  
ويعانه صماره 4 < 2 (كرارة تستعمل هو 4 ) 2 ، ناكارة بدل ما تروح لكبو يستعمل هو (2) .

Notes :-

→  $T_4$  is still high, so we can use a heat exchanger to cool it down before entering the compressor again.  $T_1$  remains  $\rightarrow$

→ The objective of decreasing  $q_{in}$  and increasing  $\eta$  was achieved (less fuel, less pollution, ...)

### Problem (9-87)

Simple Brayton Cycle, Air,  $r_p = 10$

$$T_{min} = T_1 = 295 \text{ K}$$

$$T_{max} = T_3 = 1240 \text{ K}$$

$$\eta_{SC} = 83\%, \quad \eta_{ST} = 87\%$$

→ Solution  $\leftarrow$

$$1) T_1 = 295 \text{ K}, \quad r_p = 10$$

$$2) \frac{T_1}{T_{2s}} = r_p^{k-1/k} \rightarrow T_{2s} = 569 \text{ K}$$

$$3) T_3 = 1240 \text{ K}$$

$$4) \frac{T_3}{T_{4s}} = r_p^{(k-1)/k} \rightarrow T_{4s} = 642.3 \text{ K}$$

$$\eta_{CS} = \frac{T_{2s} - T_1}{T_{2s} - T_1} = 0.83 \rightarrow T_{2a} =$$

$$\eta_{TS} = \frac{T_3 - T_{4a}}{T_3 - T_{4s}} = 0.87 \rightarrow T_{4a} =$$

$$q_{in} = Cp(T_3 - T_2) = 1.005(1240 - 569) =$$

$$q_{out} = Cp(T_4 - T_1)$$

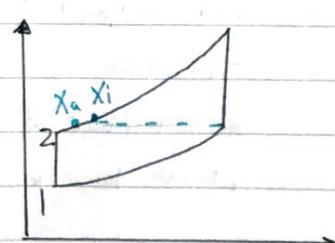
### Effectiveness ( $\epsilon$ )

$$\epsilon = \frac{h_{xa} - h_x}{h_{xi} - h_x} \rightarrow \text{actual}$$

$$\rightarrow \text{ideal}$$

→ It gives an idea of how much

a regenerator approaches an ideal regenerator.



[Brayton Cycle with Intercooling and Reheat] - Only Theory is Required.

Intercooling :-

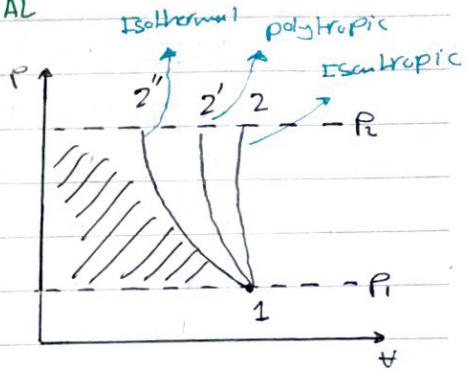
→ intercooling is used to minimize the compressor's work.

→ what is it, and how does it work?

↳ in CH#7 we learned that the compressor's work is minimum when the process is ISOTHERMAL

as you can see, the area under the curve is min for the isothermal process

$$W_c = \int v \, dP = \text{area}$$



→ so to make  $W_c$  minimum, we need to make the process isothermal. How?

→ practically, this is impossible to achieve at 100%.

But we can get close to it by using continuous intercooling while compressing the fluid.

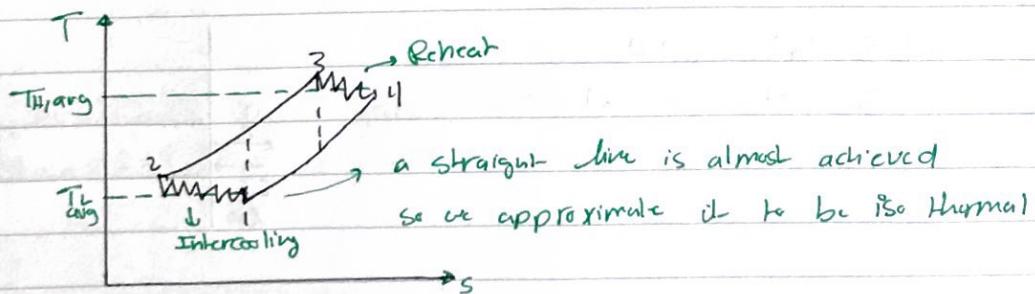
→ instead of compressing it at one stage, it is done in Multi-Stage

1 stage  $\rightarrow$  2 stage  $\rightarrow$  3 stage  $\rightarrow$  4 stage  $\rightarrow$  5 stage  $\rightarrow$

↳ until you reach point (2) -  $P_2$

→ usually, 3 compressors are used to achieve this.

(you can't use the same compressor because each compressor has a certain pressure level to work at)



Reheat :- the same concept is used in the Turbine - Multi stage Expansion - instead of the expansion happening at one stage, the air is expanded  $\rightarrow$  heated  $\rightarrow$  expanded  $\rightarrow$  heated  $\rightarrow$   $T_4$

↳ specific volume should be as high as possible during an expansion process.

note: why does the work increase when the process is isothermal?

→ If the Temp. increases while compressing the fluid, its volume will increase as well, which will resist the compressing process meaning that it would require more work.

### PROBLEM (9-100)

Solve problem 98 using constant specific heat

Ideal Brayton Cycle, Regeneration, Air

$r_p = 10$ ,  $T_1 = 300\text{ K}$ ,  $T_3 = 1200\text{ K}$

$\eta_{reg} = 100\%$

Find  $W_{net}$ ,  $q_{in}$ ,  $q_{out}$  (we will find every thing).

→ Solution ←

Assume  $\eta_{b,ts} = \eta_{b,cs} = 100\%$ .

→ you can make this assumption if it was not mentioned in the Q, you can choose any value you want but this is the easier.

1)  $T_1 = 300\text{ K}$

$$2) \left( \frac{T_2}{T_1} \right)_{s=c} = r_p \frac{k-1}{k} \rightarrow T_{2s} = 579.21\text{ K}$$

3)  $T_3 = 1200\text{ K}$

$$4) \left( \frac{T_3}{T_{4s}} \right)_{s=c} = r_p \frac{k-1}{k} \rightarrow T_{4s} = 621.5\text{ K}$$

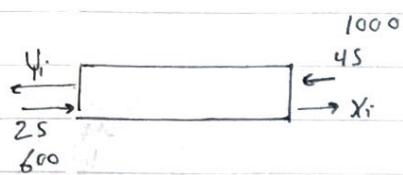
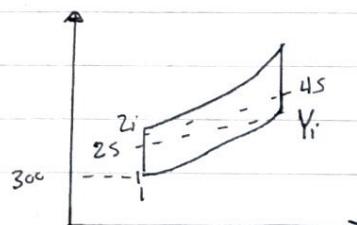
$$w_{cs} = C_p (T_{2s} - T_1) = 280.606$$

$$w_{ts} = C_p (T_3 - T_{4s}) = 581.4$$

$$w_{net} = 300.8$$

$$q_{in} = C_p (T_5 - T_{4s}) = 581.4$$

$$q_{out} = C_p (T_{5i} - T_1) = 280.6$$



## Problem (9-101) - IMPORTANT

Brayton Cycle, Regeneration, Air

$r_p = 10$ ,  $E_{reg} = 65\%$ ,  $T_1 = 310\text{ K}$ ,  $T_3 = 1150\text{ K}$

Find what,  $\eta_{th}$

assuming that  $y_{ts} = 82\%$ ,  $y_c = 75\%$

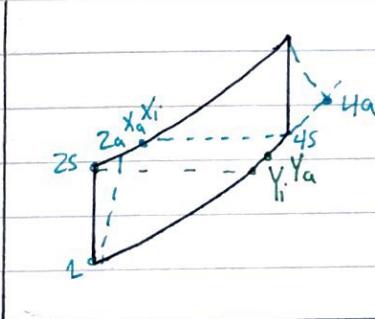
→ Solution ←

$$1) T_1 = 310\text{ K} \quad r_p = 10$$

$$2) \frac{T_{2s}}{T_1} = r_p^{\frac{K}{K-1}} \rightarrow T_{2s} = 540.53\text{ K}$$

$$3) \frac{T_3}{T_{4s}} = r_p^{\frac{K}{K-1}} \rightarrow T_{4s} = 659.51\text{ K}$$

$$2a) \eta_{sc} = \frac{T_{2s} - T_1}{T_{2u} - T_1} \rightarrow T_{2u} = 617.37\text{ K}$$



$$4a) \eta_{ts} = \frac{T_3 - T_{4a}}{T_3 - T_{4s}} \rightarrow T_{4a} = 747.82\text{ K}$$

$$E_{reg} = \frac{T_{4a} - T_{2u}}{T_{4a} - T_{2a}} \rightarrow T_{2u} = 702.16\text{ K}$$

$$T_{4a} = 663.02\text{ K}$$

$$w_{ca} = Cp(T_{2u} - T_1) = 309$$

$$w_{cs} = Cp(T_{2s} - T_1) = 231.7$$

$$w_{ta} = Cp(T_3 - T_{4a}) = 404.2$$

$$w_{ts} = Cp(T_3 - T_{4s}) = 492.91$$

$$w_{net-a} = 95.2$$

$$w_{net,s} = 261.2$$

$$q_{in} = Cp(T_3 - T_{4a}) = 447.83$$

$$q_{in,s} = Cp(T_3 - T_{4s}) = 492.92$$

$$q_{out} = Cp(T_{4a} - T_1) = 353.02$$

$$q_{out,s} = Cp(T_{4s} - T_1) = 231.7$$

$$\eta_{th,a} = 0.212$$

$$\eta_{th,s} = 0.53$$

$$WBR_{a,s} = 0.261$$

$$WBR_{s,s} = 0.17$$

→ Compressor, Heat exchanger, Turbine are the same as in Gas-Turbine Cycle, with one difference:

Turbine is only used to produce the work needed for the compressor to do its job and nothing more

$$W_T = W_C \quad (W_{Net} = 0)$$

Remember → the objective of this cycle is not power, But Thrust Force.

→ air comes out of the Turbine with a high  $T$  and  $P$  since not all the energy that was in it to produce work ( $T_5$  &  $P_5$  have high values)

→ air at  $T_5$  and  $P_5$  now enters the nozzle which will turn the energy into velocity → velocity means momentum ( $m \times v$ ) → thrust force will occur and the plane would move).

So, the Thrust Force depends on the velocity coming out of the nozzle ( $V_6$ )

→ even though there is direct mixing we assume all the cycle having the working fluid as AIR (assumption).

### IMPORTANT :-

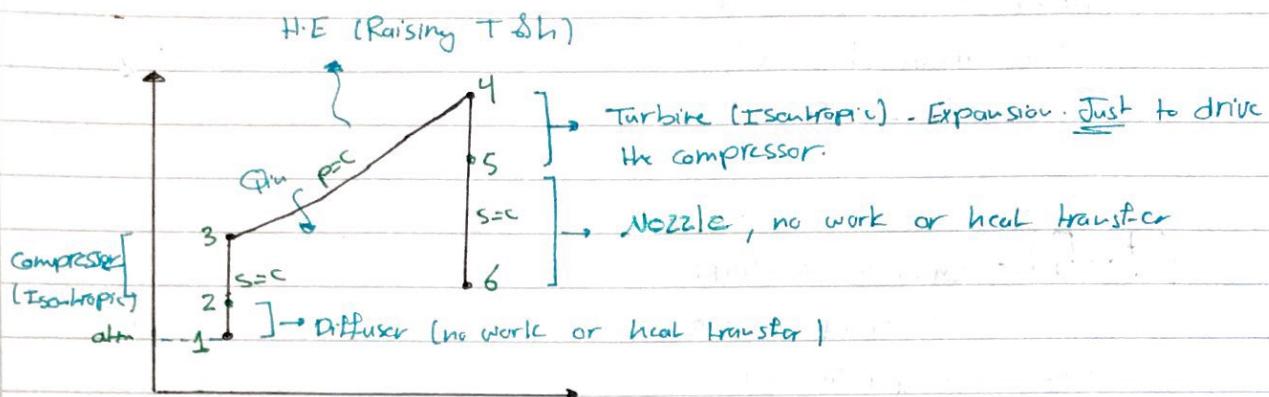
In Thermodynamics, we assume that  $(m_1)$  and  $(m_2)$  are the same even though we know that the mass of fuel was added, why?

→ we can make that assumption because mass-fuel Ratio is about 100 meaning that every 100 kg of air → 1 kg of fuel, which is nothing, so we can neglect it.

so → as you can see, Jet engine is similar to Gas-Turbine Cycle with adding a diffuser and nozzle.

Note → nozzles and diffusers don't need work neither do they produce work.

## T-S Diagram



Note  $\rightarrow T_6$  is always higher than  $T_1$ , why?

- we know that there must be  $Q_{out}$ ,  $Q_{out} = Cp(T_6 - T_1)$ , so if  $T_6 \leq T_1$  it would mean there is no  $Q_{out}$  which is IMPOSSIBLE

## EQUATIONS :-

### (1 $\rightarrow$ 2) - isentropic diffuser.

$s=c$ ,  $Q=0$ ,  $PE=0$   $\rightarrow$  DON'T assume  $KE=0$ , there is a big change in velocity.

↳ Horizontal flow so we can assume this.

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}$$

↳ we deal with ideal gas so  $h = CpT$

$$CpT_1 + \frac{V_1^2}{2} = CpT_2 + \frac{V_2^2}{2}$$

BUT  $\rightarrow V_2 \ll V_1$  so we assume  $V_2 \approx 0$

• usually  $T_2$  is unknown

$$T_2 = \frac{CpT_1 + \frac{V_1^2}{2}}{Cp} = T_1 + \frac{V_1^2}{2Cp}$$

where!  $Cp = 1005 \text{ J/kg}\cdot\text{K}$  not  $1.005$

↳  $\alpha_{120} \approx Cp$  (with  $\approx$  because

$\text{J/kg}\cdot\text{K}$  not  $\text{KJ/kg}\cdot\text{K}$

also, isentropic Relation:

$$\left(\frac{T_2}{T_1}\right)_{s=c} = \left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}$$

### (2 $\rightarrow$ 3) - compressor, isentropic

$$s=c, Q=0, KE=PE=0$$

$$W_{in} = m_{air} (h_3 - h_2) = m_{air} Cp(T_3 - T_2)$$

$$\left(\frac{T_3}{T_2}\right)_{s=c} = \left(\frac{P_3}{P_2}\right)^{\frac{K-1}{K}} = r_p^{\frac{K-1}{K}}$$

IMPORTANT!

$$(r_p)_{\text{comp.}} \neq (r_p)_{\text{Turbine}}$$

↳ not like Brayton

(3 → 4) - Head addition, isobaric

$$P=c, W=0, KE=PE=0$$

$$Q_{in} = m_{air} (h_4 - h_3)$$

$$= m_{air} C_p (T_4 - T_3)$$

(4 → 5) - Expansion, isentropic

$$S=c, Q=0, KE=PE=0$$

$$W_T = m_{air} C_p (T_4 - T_5)$$

$$Bul \rightarrow W_T = W_c$$

$$m_{air} C_p (T_4 - T_5) = m_{air} C_p (T_3 - T_2) \rightarrow T_4 - T_5 = T_3 - T_2$$

↳ usually needed to find  $T_5$

$$\left(\frac{T_4}{T_5}\right)_{S=c} = \left(\frac{P_4}{P_5}\right)^{\frac{K-1}{K}} \rightarrow \text{usually needed to find } (P_5)$$

(5 → 6) - nozzle,  $S=c, W=0, Q=0, PE=0$

$$h_5 + \frac{V_5^2}{2} = h_6 + \frac{V_6^2}{2}$$

$$Bul \rightarrow V_5 \ll V_6 \quad \text{so} \quad V_5 \approx 0$$

I need  $V_6$ , so:

$$V_6 = \sqrt{2 C_p (T_5 - T_6)}$$

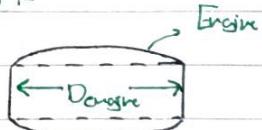
$$\rightarrow C_p = 1005$$

now, we care to know the MASS FLOW RATE ( $m$ ):-

$$\text{so} \rightarrow m_{air} = \rho_{air} A_{engine} V_1$$

$$A_{engine} = \frac{\pi}{4} D_{engine}^2 \quad (\text{approximately}).$$

$$\text{or: } m_{air} = \frac{A_{engine} V_1}{V_1 \text{ (specific volume)}} \rightarrow \text{velocity at } 1$$



↳ engines don't

have a uniform

diameter. But

we assume a constant

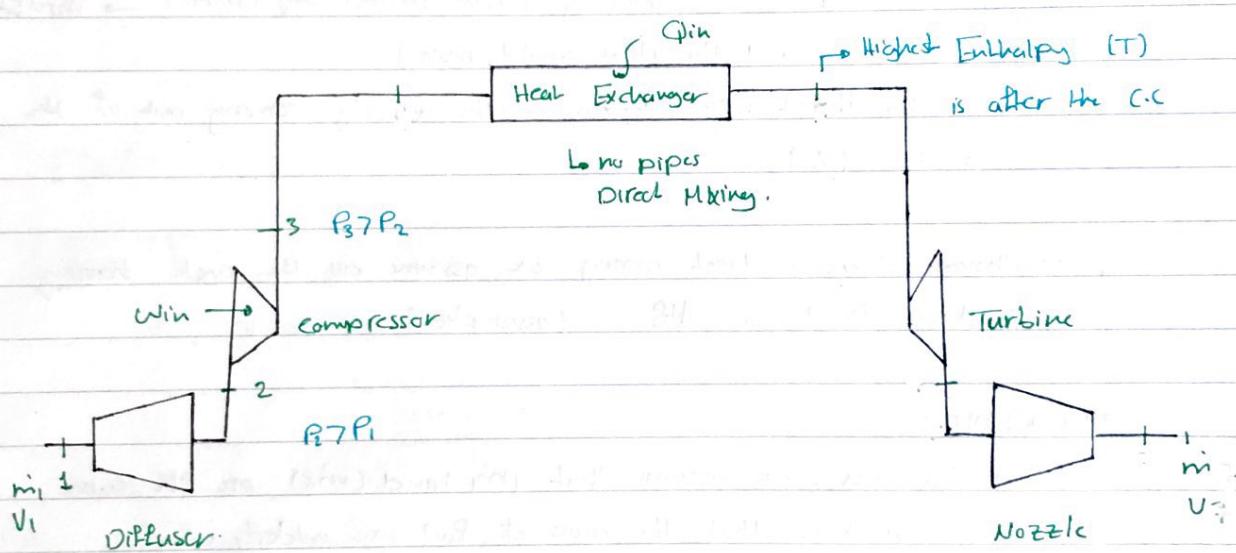
avg diameter:

$D_{engine}$ .

## JET ENGINE

- Jet engines are neither open or closed cycle, they don't follow any of the two categories.
- They are not designed to produce power ( $W_{net} = 0$ ), they are designed to give the Thrust Force needed to drive the plane.
  - ↳ also called Propulsion Force
- It is similar to the Gas-Turbine, but with some modifications.

### \* Basic Configuration of a Jet Engine :-



### How Does the Cycle work ?

- Knowing that planes fly at High elevations where  $\rho$  of air would be very low (very low density), the engine should be designed to adjust with that.
- That's why a diffuser is added to the cycle.
  - ↳ diffuser increases the pressure of air before entering the compressor because if the air entered with its original pressure which is very low at high elevations, compressor will need more work to compress it which is taken from the turbine so  $W_c$  would decrease.
- so, the objective of the diffuser in this cycle is:
  - 1- Raise Pressure ( $P_{inlet} \uparrow$ ), hence decrease  $W_c$
  - 2- Collect Max. possible air mass for the work of compressor.

## (THRUST FORCE) :-

→ Thrust force is the difference between the momentum coming in and out of the cycle.

$$F_{th} = m_i V_0 - m_i V_1 = m_i (V_0 - V_1)$$

$\frac{kg}{s}$        $\frac{m}{s}$

$$\text{THRUST POWER} \rightarrow W_p = F_{th} \times V_1 \text{ (WATT)}$$

$$\dot{Q}_{in} = m_i C_p (T_4 - T_3)$$

$$\dot{Q}_{out} = m_i C_p (T_0 - T_1)$$

## EFFICIENCY :-

→ in Jet Engines, there is no thermal ~~efficiency~~ efficiency because the obj is not work so we define →

### PROPELLION efficiency $\eta_{prop}$ .

$$\eta_{prop} = \frac{\text{output}}{\text{Input}} = \frac{\text{Thrust power}}{\text{Heat in}} = \frac{W_p}{\dot{Q}_{in}}$$

\* Calorific value :- (KJ/kg)

The standard fuel used globally is JP8

$$\therefore \dot{Q}_{cv} = 42800 \text{ KJ/kg}$$

∴ 1kg of JP8 is equivalent to 42800 KJ

Problem (9-125)

Jet engine, Vair craft =  $V_1 = 240 \text{ m/s}$

$$P_1 = 45 \text{ kPa} \quad D = 1.6 \text{ m}$$

$$T_1 = -13^\circ\text{C} \quad r_p = 13$$

$$= 260 \text{ K} \quad T_4 = 557^\circ\text{C} = 830 \text{ K}$$

Find everything.

→ Solution ←

$$1) P_1 = 45 \text{ kPa}, T_1 = 260 \text{ K}, f_1 = \frac{P_1}{RT_1}$$

$$v_1 = \frac{RT_1}{P_1} = 1.658 \frac{\text{m}^3}{\text{kg}}$$

$$m_{air} = \frac{\frac{1}{4} (1.6)^2 * 240}{1.658} = 291.05 \text{ kg/s}$$

$$2) T_2 = T_1 + \frac{V_1^2}{2C_p}$$

$$= 260 + \frac{(240)^2}{2 * 1005} = 288.7 \text{ K}$$

$$\left(\frac{P_2}{P_1}\right)_{scc} = \left(\frac{T_2}{T_1}\right)^{\frac{K}{K-1}} \rightarrow P_2 = 64.87 \text{ kPa}$$

$$3) r_p = \frac{P_3}{P_2} \rightarrow P_3 = 843.5 \text{ kPa}$$

$$\left(\frac{T_3}{T_2}\right)_{scc} = r_p^{\frac{K-1}{K}} \rightarrow T_3 = 600.7 \text{ K}$$

$$4) P_1 = P_3, T_4 = 830 \text{ K}$$

$$5) w_T = w_c$$

$$C_p(T_4 - T_5) = C_p(T_3 - T_2)$$

$$T_5 = 518 \text{ K}$$

$$P_5 =$$

$$6) P_6 = P_1$$

$$\frac{T_5}{T_6} = \left(\frac{P_1}{P_6}\right)^{\frac{K-1}{K}} \rightarrow T_6 = 359.3 \text{ K}$$

$$V_6 = \sqrt{2 * 1005 (518 - 359.3)} = 564.8 \text{ m/s}$$

$$F_{th} = m_a (V_6 - V_1)$$

$$= 94530.6 \text{ N}$$

$$W_P = F_{th} \times V_1$$

$$= 22687344 \text{ watt}$$

$$\dot{Q}_{in} = m_a C_p (T_4 - T_3) = 67071.45 \text{ kW}$$

$$\eta_P = \frac{22687.344}{67071.45} = 0.338$$

$$\dot{Q}_{out} = m_a C_p (T_6 - T_1) = 29045 = 77 \text{ kW}$$

To find  $m_f$   $\rightarrow$   $\dot{Q}_{in} = m_f \dot{Q}_{cv} \xrightarrow{42800}$

$$\therefore m_f = 1.6 \text{ kg/s}$$

## SECOND LAW ANALYSIS

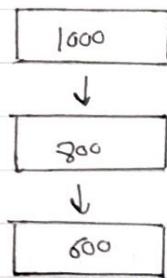
Objective → to maximize the output and minimize the input.

**EXERGY :-**

→ Exergy is the MAX available energy.

\* to understand the concept of exergy:-

نحو 800 جو ایجاد کرنا یا 800 جو ایجاد کرنا 1000 جو ایجاد کرنا  
600 جو ایجاد کرنا اسباب ایجاد کرنا



→ we call this TOTAL ENERGY (we don't deal with it, it is out of hand).

→ This is the MAX AVAILABLE energy which we call EXERGY

→ This is what I was capable to reach which we call useful or Actual energy.

→ we call the difference between the useful energy and the exergy "EXERGY DESTROYED" or Irreversibility.

↳ in reality this is never zero.

→ The exergy of a Turbine for example is the MAX work it should produce

→ for a compressor, it is the MIN work it should consume.

But → How do we know what is the Max or Min work?

to do that, we first Define the (DEAD STATE).

### DEAD STATE

→ Dead state of a system is the point where the system or working fluid can no more interact with the surroundings. (it has no energy to produce or to consume).

properties measured at the dead state are denoted with an (0)

$T_0, P_0, h_0, \dots$  etc

To understand :-

→ assume you heated a pen then you let it to cool down, when that pen's Temp. gets the same as the surroundings, it can no longer pr give or take energy (thermal equilibrium), so we say it reached its dead state.

Now, when can we say that the Turbine (for example) gives max. work?

Two conditions:

1) Reversible process

2) it have to reach its dead state. ( $T_0, P_0, \dots$ )

↳ Dead state will be mentioned in the Q or assumed  
(we will learn how later)

### IMPORTANT :-

→ We don't always want the system to reach the dead state, it depends on the application

for example, in STEAM Turbines, assume steam entered the turbine with ( $500^\circ\text{C}$ ), if we allow the steam to reach  $25^\circ\text{C}$  (assuming that the dead state  $T_0 = 25^\circ\text{C}$ ), steam will turn into liquid, and the Turbine would be ruined (since it doesn't deal with liquids).

so → Exergy is the Max Energy available, work = Exergy in two conditions:

1) the process is Reversible.

↳ so even if work is Rev it doesn't mean it = exergy.

2) the system reached the dead state

the work we used to calculate is the → (useful or actual work)

Max work - useful work = losses

↳ EXERGY DESTROYED (I)

To find Exergy Destroyed (I) :-

- In case the Device produces work:

$$I = W_{rev} - W_{useful}$$

$$I = Exergy - W_{useful}$$

↳ Both equations are used, depending whether you want the system to reach the dead state or not:

If you do  $\rightarrow$  use exergy

If you don't  $\rightarrow$  use  $W_{rev}$

- In case the Device takes work not produce it:

$$I = W_u - W_{rev}$$

$$I = W_u - Exergy .$$

### 2nd law Efficiency $\eta_{II}$

$$\eta_{II} = \eta_{II} = \frac{W_a}{W_{carrot}} = \frac{W_a}{W_{rev}} = \frac{W_u}{Exergy}$$

$$= \frac{X_{recovered}}{X_{expanded}} = 1 - \frac{X_{dest}}{X_{expanded}}$$

↳ exergy.

$$X_{expanded} = \left(1 - \frac{T_0}{T_{true}}\right) q_{in}$$

↓  
Same as  
Max. work

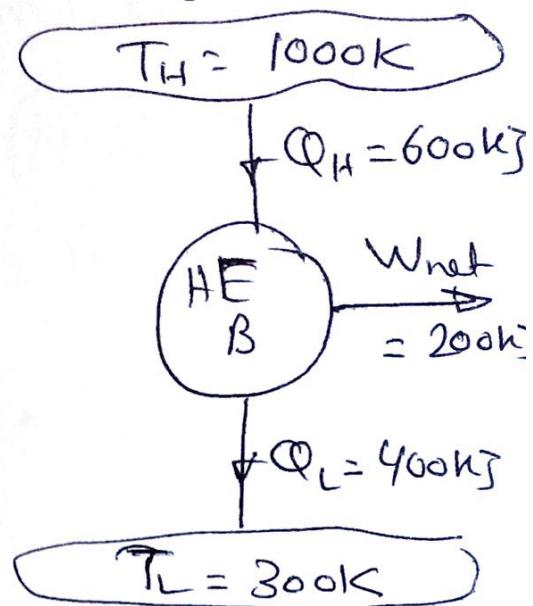
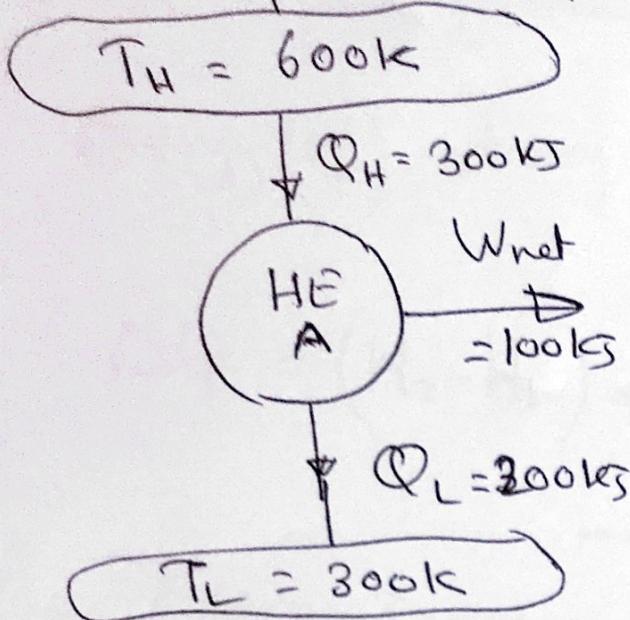
↳ carnot efficiency  $\times$  Heat in = Max. work

note  $\rightarrow$  For equations Related to exergy, look at the next two pages

it shows how to calculate the exergy (denoted by  $\$$ ) at any point for both closed and open systems

+ Exergy Destroyed calculations.

## \* Importance of Second Law Analysis:



Q: Which engine is better?

→ To know why do we discuss 2nd law analysis.

so 1st law gives me a no.  
But it has no engineering  
sense until you use 2nd law.

$$\eta_A = 1 - \frac{200}{300} = 33\%$$

$$\eta_B = 1 - \frac{400}{600} = 33\%$$

Lord Kelvin and Carnot both have the same efficiency, so using the 1st law here  
can't give me which cycle is better

So 2nd law analysis is needed to find out which cycle is better

$$\eta_{A,\text{Carnot}} = 1 - \frac{200}{500} = 50\%$$

2nd law efficiency

$$\eta_{B,\text{Carnot}} = 1 - \frac{400}{1000} = 70\%$$

$$\eta_{B,\text{Carnot}} = 1 - \frac{400}{1000} = 70\%$$

Carnot 81%  
Carnot is more  
efficient than  
Lord Kelvin

Lord Kelvin is less efficient than Carnot

Carnot is more efficient than Kelvin

Carnot is more efficient than Kelvin

\* For Closed System (Control Mass)

$$\text{Exergy } (\phi) \text{ at any point} = (U - U_0) + P_0(\mathcal{H} - \mathcal{H}_0)$$

$$-T_0(S - S_0) + m\left(\frac{V^2}{2} + gz\right)$$

$$\Delta\phi_{1-2} = \phi_f - \phi_i = U_2 - U_1 + P_0(\mathcal{H}_2 - \mathcal{H}_1) - T_0(S_2 - S_1) \\ + m\left(\frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right)$$

Dead state

دَمَدُوك

\* For Open System (Control Volume)

$$\text{Exergy } (\Psi) \text{ at any point} = (H - H_0) - T_0 (S - S_0) + m \left( \frac{V^2}{2} + gz \right)$$

$$\Delta \Psi = (H_2 - H_1) - T_0 (S_2 - S_1) + m \left( \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right)$$

\* Exergy Destroyed ( $\dot{X}_{dest}$ ) 2 → final  
1 → Initial

$$= T_0 S_{gen}$$

$$= T_0 [ \Delta S_{\text{system}} - S_{in} + S_{out} ]$$

$$= T_0 \left[ (\delta_f - \delta_i)_{\text{system}} - \frac{Q_{in}}{T_{in}} + \frac{Q_{out}}{T_{out}} \right] \text{  $\text{kJ}$  } \text{  $\text{kg}$  }$$

$$\dot{X}_{dest} = T_0 \left[ (\delta_f - \delta_i) - \frac{Q_{in}}{T_{in}} + \frac{Q_{out}}{T_{out}} \right] \text{  $\text{kJ}$  } \text{  $\text{kg}$  }$$

$$T_{in} = T_{\text{source}} \quad , \quad T_{out} = T_{\text{sink}}$$

\* For Gases, with Constant Sp. Heat:

$$S_2 - S_1 = C_v \text{avg} \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{P_2}{P_1} \right) \quad v = c$$

$$= C_p \text{avg} \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right) \quad p = c$$

\* For Variable Sp. heat

$$S_2 - S_1 = S_2^\circ - S_1^\circ - R \ln \left( \frac{P_2}{P_1} \right)$$

## OTTO Cycle Revision + Comparison

An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 95 kPa and 15°C, and the maximum cycle temperature is 1200 °C.

Determine:

The maximum pressure that occurs during the cycle, the net work output, the thermal efficiency, and the mean effective pressure for the cycle.

Perform second law analysis for this cycle and find the second law efficiency, exergy destroyed during each process.

$$CR = 8, P_1 = 95 \text{ kPa}, T_1 = 15^\circ\text{C}, T_3 = 1200^\circ\text{C}$$

$$\text{Point 1} \rightarrow P_1 = 95 \text{ kPa}$$

$$T_1 = 288 \text{ K}$$

$$V_1 = \frac{RT_1}{P_1} = 0.87$$

$$3) T_3 = T_{\max} = 1200^\circ\text{C} = 1473 \text{ K}$$

$$2) T_2 = T_1 CR^{k-1} = 661.65 \text{ K}$$

$$P_2 = P_1 CR^k = 1746.0215 \text{ kPa}$$

$$V_2 = \frac{V_1}{CR} = \frac{RT_2}{P_2} = 0.10875 \text{ m}^3/\text{kg}$$

$$3) T_3 = 1473$$

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} \rightarrow P_3 = 3887.085 \text{ kPa}$$

$$V_3 = V_2 = 0.10875$$

$$4) \frac{T_4}{T_3} \Big|_{\text{dead}} = \left( \frac{P_4}{P_3} \right)^{\frac{k-1}{k}} = \left( \frac{1}{CR} \right)^{\frac{k-1}{k}}$$

$T_4 = 641.16 \text{ K} \rightarrow$  notice this is NOT dead state ( $T_4 \neq T_0$ )

$$P_4 = 211.5 \text{ kPa}$$

$$q_{in} = C_v(T_3 - T_1) = 582.56$$

$$q_{out} = C_v(T_4 - T_1) = 253.568$$

$$W_{net} = 329 \text{ kJ/KJ}$$

$$W_c = C_v(T_2 - T_1) = 268.28$$

$$W_E = C_v(T_3 - T_4) = 597.26$$

$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}} = 0.5647$$

since  $T_0$  not given  $T_0 = T_1 = 285 \text{ K}$

$$T_{th} \quad \text{II} \quad T_{th} = T_3 = 1473 \text{ K}$$

Important note:  
in 2nd law, you  
are only required  
to solve ISENTROPIC  
OTTO, Diesel, Gas-Turbine

السؤال معنده صرف في داد  
النفخة انه يحول في صورة من  
الحرارة الى طاقة حركة  
السائلات  
as Dead State  
 $T_1 = T_0 = 285 \text{ K}$

IF Dead state values are not  
given then:

1. Dead state values are  
lowest values
2. If Heat source  $T$  is  
not given, Max  $T$  in  
the cycle is  $T_{th}$  which  
is  $T_3$  here.

Second law analysis:

$$X_{\text{desl}}(1-2) = X_{\text{desl}}(2-3) = 0 \quad \text{, since Isentropic } s = c$$

↑  
omp.  
↓  
Exp

$$X_{\text{desl}}(2-3) = T_0 \left[ (s_3 - s_2) - \frac{q_{\text{in}}}{T_0} \right]$$

$$s_3 - s_2 = \text{constant } u \quad \text{constant } u$$
$$= C_{\text{p,sg}} \ln \left( \frac{T_3}{T_2} \right) + R \ln \left( \frac{V_3}{V_2} \right) \quad V_3 = V_1$$

$$= 0.718 \ln \left( \frac{1473}{661.65} \right) = 0.5746 \text{ kJ/kg.K}$$

$$q_{\text{out}} = C_p (T_4 - T_1) = 253.568$$

## CHAPTER (10)

### « VAPOR POWER CYCLE »

→ We will study two parts:

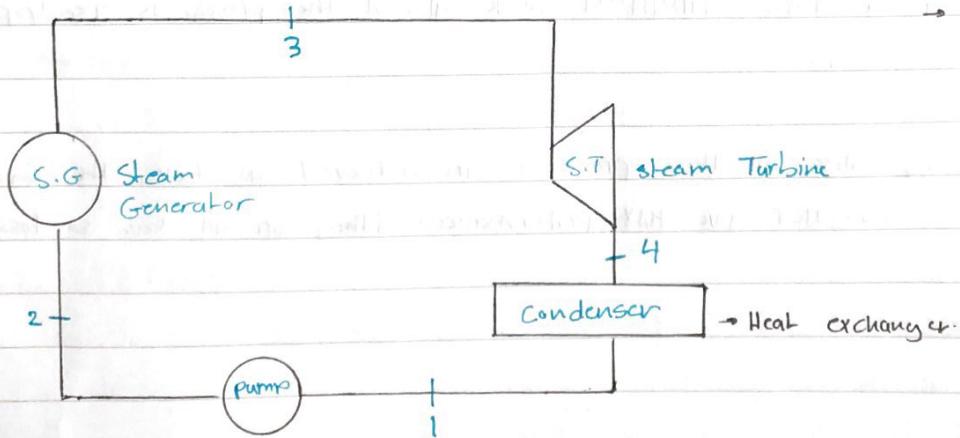
- Carnot Vapor. Power Cycle → Impractical, only studied to find Max. Performance
- Rankine Cycle → More Practical

→ about Vapor power Cycles in general :-

- Working Fluid is liquid
  - ↳ not exactly water
- During heat addition, it changes phase
  - when we heat the liquid, it will convert from compressed liquid
  - saturated liquid → wet mixture → saturated vapor → super heated.
- Power is produced by (HOT STEAM)
  - ↳ so what gives me power in these cycles is NOT the liquid, it's the steam after it is heated. (after it changes phase)
  - That's why it is called "Vapor power Cycle"

- Any Vapor power Cycles consists of (4) main Devices, all of them are open systems, they are:
  1. Steam generator
  2. Steam Turbine
  3. Condenser
  4. Pump.

- The following represents the Basic configuration of Any Vapor Power Cycle:



→ This is the Thermo cycle of "Vapor or Steam Power Cycles"

## CARNOT VAPOR POWER CYCLE

→ First we will study Carnot Vapor power Cycle, to see the Max. performance that this cycle would give then compare the practical cycle with it.

→ Carnot suggests that for this cycle to give Max. performance, all processes in it should be Reversible.

How?

- Heat Addition :-  $(2 \rightarrow 3)$

↳ To make the Heat addition process (in Steam Generator) Reversible it should be [Isothermal]

$$\text{So } T_H = \text{Constant}$$

Also:

- Condensation  $(4 \rightarrow 1)$

↳ To be Rev. it has to be ISOTHERMAL (like Heat addition)

$$\text{So } T_L = \text{constant}$$

↳ we will understand how it happens later

- Expansion  $(3 \rightarrow 4)$  in Steam Turbine.

↳ we already know that Turbines gives Max. Work when the process is ISENTROPIC

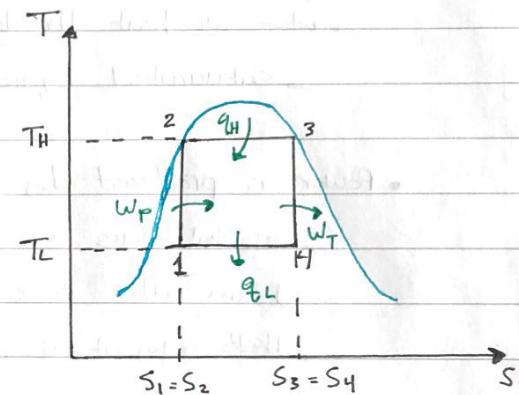
$$\text{So: } S_3 = S_4$$

- Pump  $(1 \rightarrow 2)$  (note → pumps only deals with LIQUID)

↳ Pump takes MINIMUM work also if the process is ISENTROPIC

$$\text{So: } S_1 = S_2$$

→ Unless all these processes are achieved as above, the cycle can NOT give MAX performance. (they are all Rev. so losses = 0)



→ To find The efficiency of Carnot Vapor power Cycle

$$\eta_{\text{Carnot}} = 1 - \frac{q_H}{q_L} = \frac{W_{\text{net}}}{q_H}$$

But → From TS Diagram, we know that  $q = \text{Area under the curve}$

$$\text{So } \rightarrow q_H = T_H (S_2 - S_1)$$

$$q_L = T_L (S_1 - S_2)$$

$$\text{But } \rightarrow S_2 - S_1 = S_1 - S_0 = \Delta S$$

$$\text{So } \rightarrow q_H = T_H \Delta S$$

$$q_L = T_L \Delta S$$

$$\hookrightarrow \frac{q_H}{q_L} = \frac{T_H}{T_L}$$

$$\therefore \eta_{\text{Carnot}} = 1 - \frac{T_H}{T_L}$$

→ To find  $W_{\text{net}} \text{ Max} \rightarrow W_{\text{net}} = \eta_{\text{Carnot}} \times q_H$

BUT !! Carnot Cycle has many problems that makes it Impractical ..

#### • PROBLEMS OF CARNOT CYCLE :-

[1] Pump inlet is Wet Mixture.

↳ we mentioned that pumps only deals with liquids, But as you can see in the TS Diagram point (1) is wet mixture and that's what will enter the pump. PROBLEM !!

[2] Steam Turbine outlet has very low  $x$  (steam quality).

↳ steam turbines can NOT deal with a steam quality ( $x$ ) less than (85%), Meaning that the percentage of liquid in the mixture can't exceed (15%), But as you can see, during the expansion process  $x$  keeps on decreasing until it reaches point (4), which is too low for the turbine.

[3] Condenser is not working effectively.

↳ The objective of using the Condenser (from its name) is to convert steam into liquid. But in Carnot cycle (look at TS Diagram)

The outlet of the condenser (Point 1) is wet mixture not liquid, this will enter the pump as mentioned and cause problems

↳ effectiveness of the condenser is low ( $\epsilon$ )

→ All these problems were to keep the pump and Turbine working at constant Entropy ( $S_1 = S_4$ ) & ( $S_2 = S_3$ )

→ The 3 problems mentioned previously plus some other problems makes Carnot Cycle Totally Impractical "Hypothetical Cycle"

→ From here, we introduce "Rankine Cycle" which is more practical

## RANKINE CYCLE

→ Rankine made some modifications on the cycle in order to make it practical

→ Rankine Modifications:

[1] Steam Generator works at constant pressure

↳ when Steam Generator works at constant pressure instead of constant temperature, the outlet would be "Super Heated Vapor" instead of saturated vapor, which is better.

so liquid enters the S.G at point (2) → comes out as Steam at point (3)

↳ note → it is almost impossible to make the S.G works Isothermally

so it's better to make the process Isobaric (P.c)

[2] Steam Generator output is super heated.

↳ this means that point (3) has very high steam quality 'x'

this is not exactly an advantage because it means that the condenser will have to deal with a higher T, which means  $q_u$  will be higher, BUT this has to be done to keep the Turbine safe (working with steam).

[3] Condenser works at constant pressure

↳ it is also impossible to make the condenser works at constant (T)

↳ when it works under constant (P), the outlet at point (4) would be saturated liquid (or compressed liquid) → (high  $\epsilon$ )

↳ that means that it worked more efficiently (not like Carnot)

so:

[4] Condenser output is Min Saturated liquid.

which means:

[5] Pump inlet is Min Saturated liquid.

↳ this means that another problem of Carnot is solved, now the pump only deals with liquids which is what it should work with

↳

→ Now we will take each device in Rankine Cycle in details!

## 1. STEAM GENERATOR :-

→ It is a Heat Exchanger that converts (Feed Water) from compressed liquid to sat. vapor then to super heated.

→ There are Two Types of Steam Generators :-

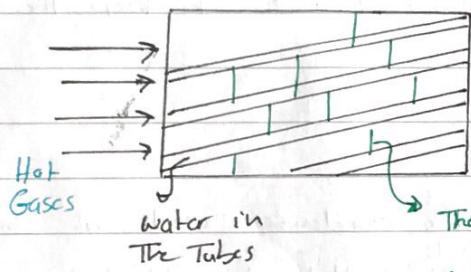
1. Water Tube.
2. Fire Tube.

### • Water Tube Steam Generator :-

↳ Imagine a Cylinder (Big Cylinder) with tubes inside of it, these tubes are filled with water, Hot gases are passed through these tubes, Heat is transferred from these Hot Gases into the water then the water turns from liquid to steam.

↳ It is called "Water Tube" Because there is water inside the tubes.

→ note that the tubes are inclined, Because the steam will go up due to its low density.



→ These things are called (Baffles), they are added to make the Hot Gases take a longer path, hence, they stay in contact with the water tubes for longer time and more heat is transferred (enough to turn all water into steam).

### • Fire Tube Steam Generator:

↳ In this Type, there is flames (Hot Gases) inside the tubes while the water is outside, so Heat is transferred from the Hot Tubes to the water (liquid) so it turns into steam.

#### IMPORTANT:

Water Tube Steam Generator is safer and consumes less fuel to heat the water, so fire tube is only used in special cases where less safety is needed.

→ Fire Tube is less expensive.

→ applying 1st law of open system:-

$$\text{Recall: } (\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}) + (\dot{W}_{\text{in}} - \dot{W}_{\text{out}}) = m (h_o - h_i) + \Delta K.E + \Delta P.E$$

for steam Generator:  $\dot{Q}_{\text{out}} = 0$   $\dot{W} = 0$   $\Delta K.E = 0$   $\Delta P.E = 0$

so →

↑ NOT AIR!

$$\dot{Q}_{\text{in}} = m_{\text{steam}} (h_o - h_i)$$

note →  $(h)$  is found from (STEAM TABLES)

## 2. STEAM TURBINE :-

↳ Converts energy (at point (3)) to power out  $P_{\text{ul}}$

↳ High (Inthalpy)

What Happens?

steam enters the Turbine with very High Inthalpy, first it goes through a nozzle (inside the Turbine), this nozzle converts the Inthalpy into High Velocity, then the steam with high velocity hits the blades of the Turbine which are connected to a shaft → power is produced.

↳ That's why the water should be super heated (as  $T$  increases, Inthalpy increases Velocity increases → More power).

↳ The process is Ideally (ISENTROPIC)

↳ Applying 1st law:  $\dot{Q}_{\text{in}} = \dot{W}_{\text{T}} + m_{\text{steam}} (h_i - h_o)$  Remember → out 1st law

condition is,  $\text{Quality (X) should be } \geq 85\%$ .

$\dot{W}_{\text{T}} = m_{\text{steam}} (h_i - h_o)$  the power output will be less if  $X$  is less is NOT acceptable.

↳ The power output will be less if  $X$  is less is NOT acceptable.

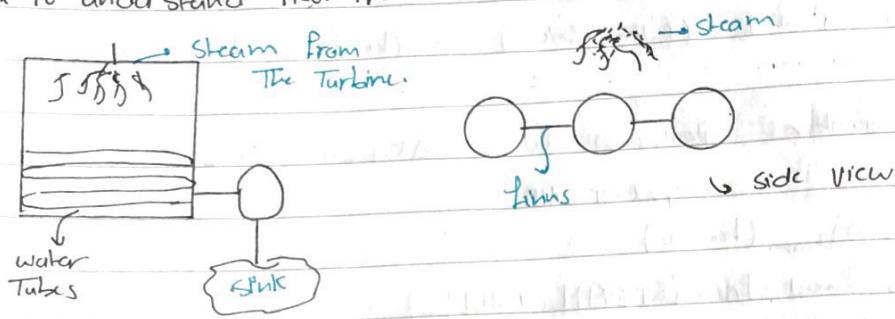
## 3. CONDENSER :-

↳ It is a Regular Heat Exchanger

so, it is used to: 1. Condense steam from wet. mix to Saturated liquid.

2. Creates vacuum needed to maintain circulation.

\* To understand How the condenser works:



- To condense the steam coming from the Turbine (process  $1 \rightarrow 2$ ) we need to use a cold surface, so pipes of water are used as the cold surface where steam will condense
- Water runs through the pipes (cold water), then Heat is transferred from the steam to the water, so water gets heated, that's why it should go back to the sink where it'll cool down, then back to the tubes again.
  - They have the same ( $T$ ) as the water tubes.
- Fins are added between the tubes, they are surfaces used to guarantee that the steam gets in touch of with a cold surface (without them it might just pass between the tubes without condensing).
- when steam is condensed, its volume decreases and a drop in pressure occurs, that creates (VACUUM) in the condenser, this vacuum creates "Back pressure", which helps maintaining the circulation
  - ↳ when steam enters the Turbine with High Inthalpy, this Inthalpy is turned into K.E that hits the blades, then it's velocity (the steam) will decrease, which affects the velocity of the new steam coming into the Turbine, so Back pressure prevents that from happening. (as air of steam is lower)

#### • Applying 1st law ( $4 \rightarrow 1$ )

$$\dot{Q}_L = m_s (h_4 - h_0)$$

## 4. PUMP :-

1. It creates pressure difference needed for circulation. (condenser) (condenser) suction
2. It increases feed water pressure to supply steam Generator.

→ notice that the pump is between the condenser and the steam Generator, we mentioned that vacuum occurs in the condenser due to the condensation process (which means water at point 1 will have very low pressure) while steam Generator works at high pressures (MPa), so the pump increases its pressure to achieve that.

• applying 1st law:

$$W_p = m s (h_o - h_i)$$

**IMPORTANT !**

Recall: shaft work for the pump:

$$W_p = m \frac{v_f}{in} (P_o - P_i)$$

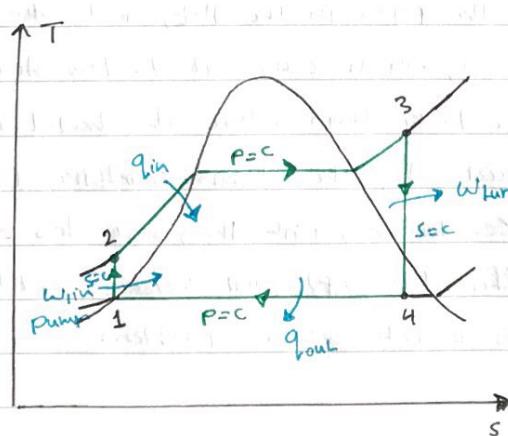
→ This must be in KPa

Both equations are used to find the work, they are both used because the water at point 2 (pump outlet) is compressed liquid

جاذبیتیں very pressurized      Compressed       $\rightarrow$   $W_p = m \frac{v_f}{in} (P_o - P_i)$   $\rightarrow$   $W_p = m s (h_o - h_i)$

$$m s (h_o - h_i) = m \frac{v_f}{in} (P_o - P_i) \rightarrow \text{Extremely Important.}$$

(T-S) Diagram of Rankine Cycle:



### \* Quick summary :-

Starting from point (1), water enters the pump as (sat. liquid) at point (2) it is compressed liquid (we said there is not enough tables for compressed liquid so we use shaft work eqn (VAP)), then liquid enters the steam generator and converts to super heated steam (point 3), it enters the steam turbine which is connected to a power generator and electric power is generated, then it enters the condenser which converts it to liquid.

### ADIABATIC EFFICIENCIES

↳ we said that the pump and the steam turbine works under an isentropic process

↳ In Reality this is not true (if you put your hand on a steam turbine it'll be hot).

↳ So I have  $h_3, h_4a, 2s, 2a$

$$\eta_T = \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

$$\eta_p = \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

### \* Other losses in Rankine Cycle :-

Losses in Steam Generator :- losses occur in heat exchangers, due to fluid friction between the fluid and the pipes in the H.E, and due to the long distance between points (2 & 3), which causes it to lose heat  $\delta T$ , this means that point 3 will be lower than where it should be. (less enthalpy)

↳ this affects the work produced by the steam turbine (problem)!

↳ these losses also occur in condensers, But they have less effect on the cycle since point 1<sub>s</sub> is affected  $(\Delta P)$  will increase and the pump will need more work. But this is not a big problem

### STEAM GENERATOR efficiency:

$$\eta_{S.G} = \frac{m_s (h_3 - h_2)}{m_p \eta_p h_{2s}}$$

not idealised

•  $m_p \eta_p h_{2s} \rightarrow$  نیازمندی کم پیوست  
 $m_s \Delta h \rightarrow$  نیازمندی کم پیوست  
 $\rightarrow$  کم نیازمندی

## Efficiency of Rankine Cycle.

$$\eta_{\text{lin}} = \frac{W_{\text{net}}}{Q_{\text{in}}} = 1 - \frac{q_L \downarrow}{q_H \uparrow} = \frac{W_T - W_P}{q_H \uparrow}$$

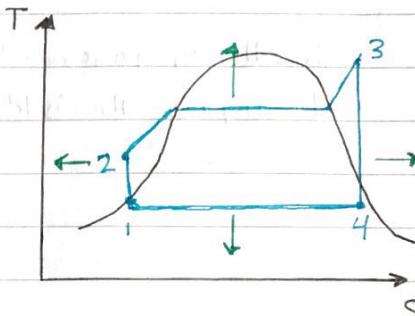
$$\text{Remember} \rightarrow q_L = h_4 - h_1 = c_p (T_4 - T_1)$$

$$q_H = h_3 - h_2 = c\rho(T_3 - T_2)$$

## • HOW TO IMPROVE THERMAL EFFICIENCY •

→ we will study each choice with its effects

→ The main idea is to increase the area under T-S Diagram, when the area increases  $\rightarrow$  Work  $\uparrow$   $\rightarrow$   $q$   $\uparrow$



→ To do that you have many choices, BUT not all of them ~~of~~ works, as follows:

## 10 Decreasing the Pressure of the condenser

Pcond. ↓

decreasing Prod. means decreasing (P<sub>L</sub>)  $\rightarrow \eta \uparrow$

problems of this class;

→ Steam Quality ( $X_4$ ) will decrease (problem in Turbine).

→ decreasing the pressure makes greater pressure difference between the atm and inside the condenser which could cause air leakage into the condenser, that prevents effects the heat transfer between the steam and surface of condenser

So if you want to increase  $P_{cond.}$  you must have a special Design that could tolerate the pressure difference.

→ Increase the  $P_u$

2. add more pumps, pumps don't consume a big amount of energy compared to the power produced by the turbine (it consumes about 10 kJ/kg while turbine produces 3 MW) so it's very small and it's fine. If you use more than one but this choice is not very helpful

- Unrealistic suggestion

→ we have two remaining solutions which we mostly care about, which are:

3. Increasing  $T_{max}$  ( $T_3$ ) → it affects the G.T (High Mechanical and Thermal stresses)

4. Increasing  $P_{boiler}$

The following page, contains a comparison between the different methods used to improve the cycle's efficiency.

## Comparison between different methods to improve cycle efficiency

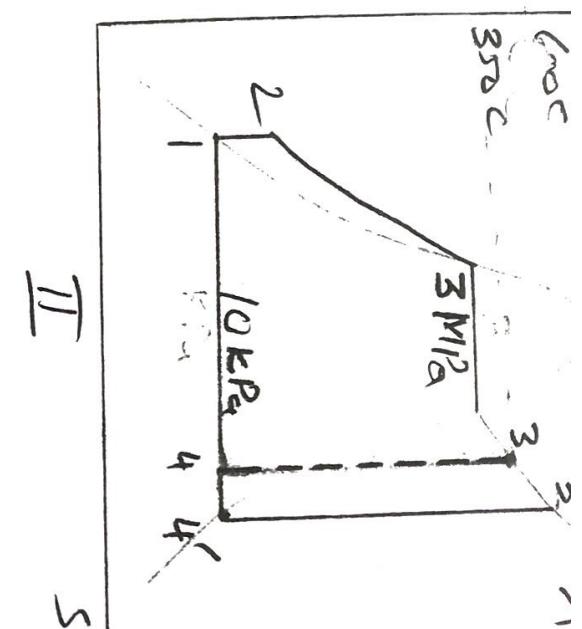
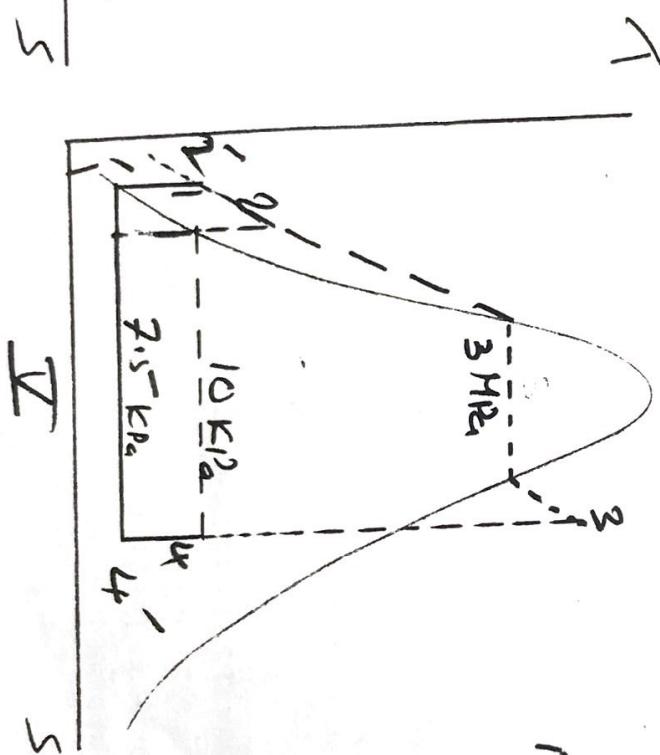
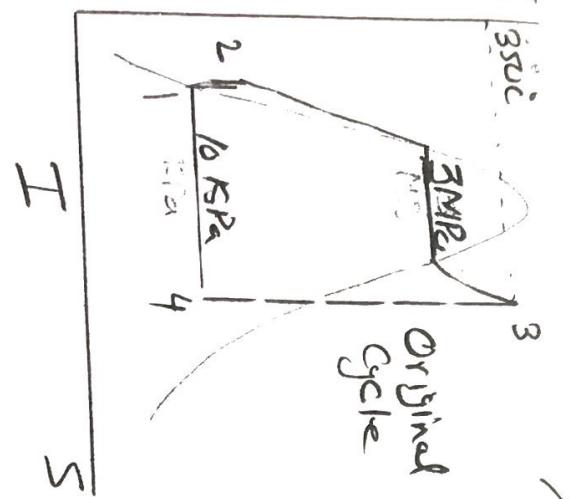
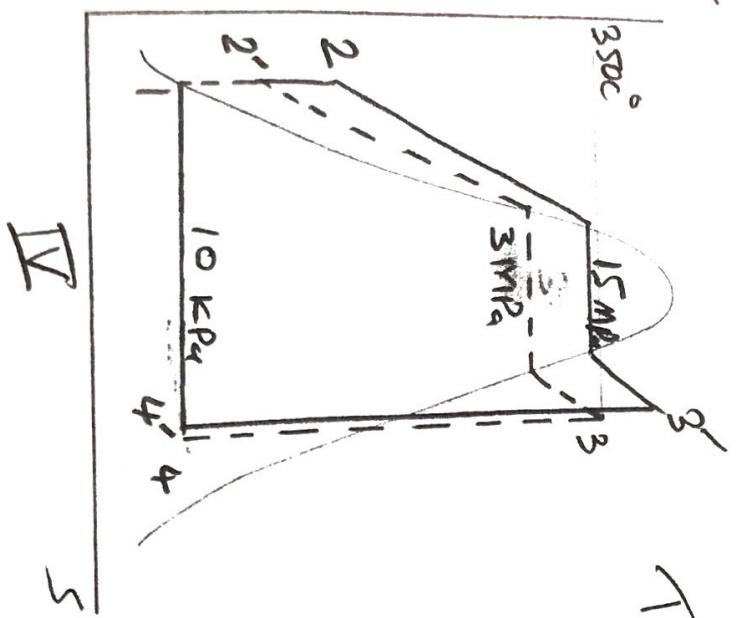
A steam power plant operates on a simple ideal Rankine cycle. The steam enters the turbine at 3 MPa and 350 °C. It is then condensed in the condenser at 10 kPa. Calculate:

- 1) The engine performance parameters for the above power plant.
- 2) The performance parameters if the steam is superheated to 600 °C instead of 350 °C keeping rest of conditions the same.
- 3) The performance parameters if the boiler pressure is raised to 15 MPa keeping turbine inlet at 350 °C.
- 4) The performance parameters if the boiler pressure is raised to 15 MPa keeping turbine inlet at 600 °C.
- 5) The performance parameters if for case No. (1) the condenser pressure is lowered to 7.5 kPa.

### Results comparison for the above cases. ① ② ③

↓ ④ ⑤

Ideal	Increase $T_{max}$	Increase $P_{boiler}$	Increase $P_{boiler} \& T_{max}$	Decrease $P_{condenser}$
Pump Work (kJ/Kg)	3.0199	3.0199	15.14	15.1399
Heat Input (kJ/Kg)	2920.48	3487.48	2485.47	3375.36
Steam Quality X4	0.8123	0.9144	0.6388	0.803
Turbine Work (kJ/Kg)	979.6	1302.375	971.87	1467.4
Heat Output (kJ/Kg)	1943.89	2188.12	1528.74	1923.1
Net Work out (kJ/Kg)	976.58	1299.355	956.73	1452.26
Thermal Efficiency (%)	33.4	37.25	38.49	43.025
Carnot Efficiency (%)	48.82	63.48	48.82	63.48
Relative Efficiency (%)	68.41	58.67	78.84	67.77
				69.02



1-2-3-4  
Original Cycle  
1'-2'-3'-4'  
Modified Cycle

## NOTES about the Results of the previous Question:

↳ The Table shows the output and different parameters in the ideal case then the results in case of using the different methods for improving the efficiency to make a comparison and choose the Best Method.

→ looking at the last column (Decreasing  $P_{cond}$ )

↳ comparing with the ideal :

$X_4 \downarrow$  which is not good.

Heat output  $\downarrow$  which is the only advantage

→ note that  $q_1$  increased by only (1), so this Method doesn't make a big difference.

→ looking at column 2, (Increasing  $T_{max}$  ( $T_3$ ))

→ This solution causes a problem on the Gas Turbine, its components might not take the high thermal and mechanical stresses resulting from increasing ( $T_3$ ). But we'll see its effects

→ Heat Input  $\uparrow \rightarrow$  Disadvantage.

$X_4 \uparrow \rightarrow$  Advantage to the Turbine

Heat out  $\uparrow$

→ when  $X_4$  increases, it means that the condenser will have to deal with a (T) higher than it is designed to take, which could effect its job (the mixture might not convert fully into saturated liquid) - Point 1 (Pump inlet might contain gas which is not good), this problem could be avoided if the condenser was designed to deal with it (tolerate extra heat).  
↳ we say that the condenser is (OVERLOADED)

→ looking at column (3) - Increasing ( $P_{boiler}$ )

→ pump work  $\uparrow$  Because the outlet pressure of S.G increases.  $\xrightarrow{\text{S.G}}$

Heat Input  $\downarrow$  advantage.

looking at the diagram III, point 3' has higher Enthalpy

∴ Heat out put  $\downarrow$  (Area decreased),  $W_{net} \downarrow$

$\eta_{th} \uparrow, \eta_{II} \uparrow$

$X_4 \downarrow$  a lot!

Turbine work  $\downarrow$

→ Looking at column [4] - Increase Boiler & Turbine

→ Area increased  $\rightarrow$  Wnet  $\uparrow$  - look at diagram

$X_4$  almost as it is

$q_{th}$  increased by almost 10%

$q_{II}$  " " 0.7

Turbine work  $\uparrow$  (Advantage)

Heat in  $\uparrow$  (Disadvantage)

Heat out  $\downarrow$  (Just a little)

↳ you can see that this is the best choice. But! it has a problem:

→ since P & T were increased, Mechanical & Thermal stresses increases, so safety is effected!

I can get the same results without increasing P & T using REHEAT

10-15)  $W_{net} = 210 \text{ MW}$ , Steam power plant

$$W_{net} = \dot{m}_s \left( \frac{W_T - W_p}{\text{kw}} \right)$$

1) Saturated Liquid at 10 kPa.

$$h_1 = h_f = 191.81 \text{ ,}$$

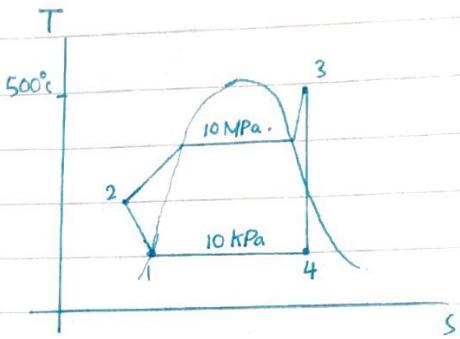
$$h_{fg} = 2392.1$$

$$v_1 = v_f = 0.00101$$

$$h_g = 2583.9 \text{ .}$$

$$s_g = 8.1488$$

$$s_f = 0.629 \text{ .}$$



$$2) (h_{2s} - h_1) = v_{f1} (P_2 - P_1)$$

$$\therefore (h_{2s} - 191.81) = 0.00101 (10000 - 10)$$

$$h_{2s} = 201.9 \text{ .}$$

$$3) T_{sat@10 \text{ MPa}} = 311^\circ \text{C}$$

$T_3 > T_{sat}$   $\rightarrow$  so, it's superheated.

$$h_3 = 3375.1$$

$$s_3 = 6.5995$$

$$4) s_3 = s_4, \text{ since } s_4 < s_g @ 10 \text{ kPa} \rightarrow \text{wet mixture.}$$

$$s_4 = s_f + x_4 s_{fg}$$

$$x_4 = 0.794 \quad \bullet \text{Not acceptable.}$$

$$h_4 = h_f + x_4 h_{fg}$$

$$h_4 = 2089.7$$

$$\bullet q_{in} = h_3 - h_2 = 3173.2$$

$$\bullet q_{out} = h_4 - h_1 = 1285.4$$

$$\bullet \eta_{th} = 40\%$$

$$\bullet W_p = h_{2s} - h_1 = 10 \text{ kJ}$$

$$\bullet W_T = h_3 - h_4 = 1285.4$$

$$\bullet \dot{m}_s = 9.5$$

$$10-16) \eta_{ps} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad h_{2a} = 203.68$$

$$\eta_{ps} = \frac{h_3 - h_{2a}}{h_3 - h_{2s}} \quad h_{4a} = 2282.5$$

$$\bullet q_H = h_3 - h_{2a} = 3171.42 \text{ .}$$

$$\bullet q_L = h_{4a} - h_1 = 2090.7$$

$$\bullet \eta_{th} = 0.34$$

→ CH#8 is not included, only the following two examples to clarify the idea of Exergy and 2nd law analysis:

## EXAMPLE (8-15)

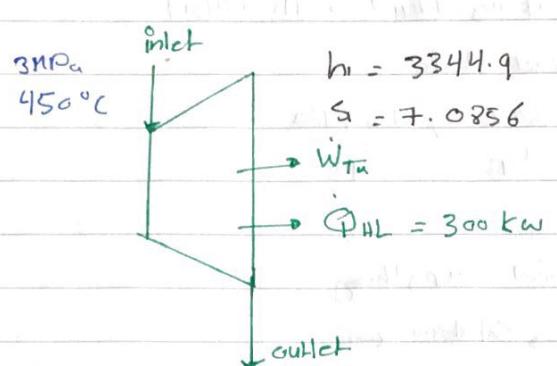
$$m_s = 8 \text{ kg/s}$$

Dead State:

$$100 \text{ kPa}, 25^\circ\text{C} =$$

$P_0$        $T_0$

Find everything.



→ Solution

from Tables, at Dead State:

$$h_0 = h_f|_{25^\circ\text{C}} = 104.83$$

$$s_0 = s_f|_{25^\circ\text{C}} = 0.3672$$

$$h_2 = 2769.1, s_2 = 7.281$$

$$m_s h_1 = W_{Tu} + Q_{HL} + m_s h_2$$

$$W_{Tu} = m_s (h_1 - h_2) - Q_{HL} = 43064 \text{ kW}$$

→ denotes exergy for open system ( $\phi$  is for closed).

$$W_{Tmax} = m_s (\psi_1 - \psi_2) = 5072.2 \text{ kPa}$$

$$\psi_1 = (h_1 - h_0) - T_0 (s_1 - s_0) = 123.8 \text{ kJ/kg}$$

$$\psi_2 = ( ) - T_0 ( ) = 609$$

$$I = W_{Tmax} - W_{Tu} = 5072.2 - 43064 = 765.83$$

$$\dot{X}_{dest} = m_s T_0 [s_2 - s_1 + \frac{q_{HL}}{T_0}]$$

$$= 767.4 \text{ kW}$$

note that (I) and ( $\dot{X}_{dest}$ ) should be equal or very close to each other

$$\eta_{II} = \frac{W_u}{W_{max}} = 0.849$$

## EXAMPLE (8-8)

Compressor

working Fluid  $\rightarrow$  (R-134a)

Dead State:

$20^\circ\text{C}$ ,  $95 \text{ kPa}$

$\downarrow T_0$

$\downarrow p_0$

Find everything.

→ Solution ←

note that  $m$  is not given so answers are per kg.

$h_2 = 286.71$

$0.8 \text{ MPa}$

$50^\circ\text{C}$

out let

$C$

$s_2 = 0.9803$

inlet

$0.14 \text{ MPa}$

$-10^\circ\text{C}$

$h_1 = 246.37$

$s_1 = 0.9724$

$$w_{\text{actual}} = h_2 - h_1$$

$$= 40.34 \text{ kJ/kg}$$

$$w_{\text{min}} = \psi_2 - \psi_1$$

$$= (h_2 - h_1) - T_0(s_2 - s_1) = 38.19$$

$$\rightarrow I = 2.3 \text{ kJ/kg}$$

$$x_{\text{dest}} = T_0 [s_2 - s_1] =$$

$$= 1000 [0.9803 - 0.9724] = 79 \text{ kJ/kg}$$

water and air

Chiller Works

and design

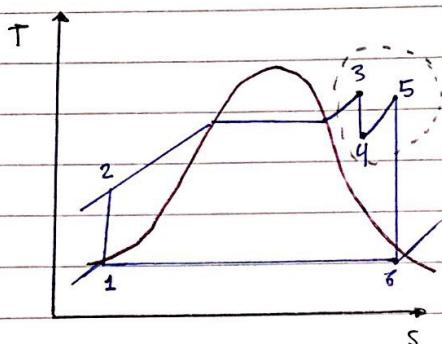
boiler and tank

## \* The Ideal Reheat Rankine Cycle :-

- ↳ it is one of the methods to increase the efficiency of the cycle
- ↳ we want to get the same results when raising ( $P_{cond}$ ) & ( $T_3$ ) as discussed before but without actually doing that, so we use  $\rightarrow$  REHEAT

so: Reheat is the first cycle of the improvement cycles.

T-S Diagram of The Ideal Rankine Cycle Cycle



What happens in a Reheat Rankine Cycle?

- ↳ Expansion process happens at two stages, which means there are two turbines now
- ↳ at point (3) steam enters the 1st turbine (High pressure turbine) and expands until it reaches point (4) which would cause a low  $X_1$  if it continued expanding
- ↳ so at point (4) steam is reheated mostly back to the Temp. of point (3), then expanded again in the second turbine (Low pressure turbine) until it reaches point (6) which has a very good quality ( $X_6$ )  $\rightarrow$  so Reheat solved the problem of extra moisture in steam (low  $X$ )

Q  $\rightarrow$  How is the steam reheated from point (4) to point (5)?

1. will be taken back to the steam generator, note that  $Q_{re}$  will increase but only a little because the steam at (4) already has a high Temperature

note  $\rightarrow$  no change on the pump

# Rankine cycle with regeneration

①

We will reheat the working fluid before it enters the steam generator. So, for the same  $W_{net}$ , the amount of  $q_{th}$  will  $\downarrow$  (i.e. the consumption of the fuel  $\downarrow$ ). Then  $n_{th} \uparrow$  (actually  $W_{net}$  will decrease very slightly because  $\uparrow$  will add a pump).   
 + some of the steam in the turbine will be

How can I reheat the working fluid? Many solutions are proposed.

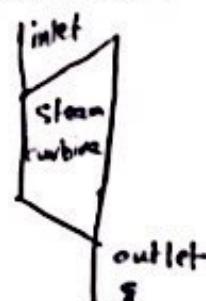
- 1) Building pipes around the turbine so the working fluid will suck the heat from the turbine (illogic, because the steam will condensate inside the turbine i.e.  $x$  may become below 0.85) turbine will go pipes will
- 2) Using the solar energy (This solution is not useful in winter)
- 3) The working fluid leaving the turbine has heat. So instead of rejecting the heat in the condenser to the river for example, try to reuse it. Studies found that there is a location (within the turbine between the inlet and the outlet) that has steam, this steam has effective power. If we leave this steam inside the turbine, it will cause back pressure (it will not let the working fluid to enter the turbine) coming from the steam generator

→ So we take this steam at a certain location. We mix this hot steam with the relatively cold water that is coming from the first pump. Mixing occurs at a room that is called Open feed water heater.   
 Steam will condensate / the water will be hot / (i.e. the working fluid at point 3 is hot  $\Rightarrow$  water)

\* Why does this steam have weak potential to do work?

In turbines as we go out to the outlet, the cross-sectional area  $\uparrow$ , the speed of the flow  $\uparrow$ . We need huge water speed to hit the blades of the turbine and rotate the shaft. So when the speed of the flow  $\downarrow$ , the flow will not be able to rotate the shaft effectively, but it has  $\approx$  some amount of heat. So we take

that steam at certain location. There is a certain location between the inlet and the outlet, if we take the steam from that location  $n_{th} \uparrow$  greatly



mixing between the hot steam and the cold water is direct  
(this is why it is called open feed water heater)

We can use a flash chamber between the OFWI and the 2nd pump.

The working fluid that is going to enter the 2nd pump will be hot water. So when the working fluid enters the steam generator less  $q_{VH}$  will be need to raise the temperature of the working fluid to  $T_5$ , for the same  $W_{net}$ .

\* Condenser  $\rightarrow$  Will relax because they will deal with less amount of mass

\*  $P_2 = P_3 = P_6$  (because they are connected to the same H-E-X)

\*  $P_{SG} > P_{OFWI} > P_{\text{condenser}}$

\* If  $P_3 > P_6, P_2$  both 6. and 2 will go back to the turbine and pump

\* If  $P_3 < P_6, P_2 \rightarrow$  mixing will occur very fast (mixing zone diagram)

\* Point 6 -  $\begin{cases} \text{Wet mixture} \\ \text{Superheated} \\ \text{Sat. Vapor} \end{cases}$

\* The first pump will increase the pressure of  $P_1 \rightarrow P_2$   
cold water  $\underbrace{(2)}_{\text{+ (6)}} \rightarrow$  steam

we mix them  $\rightarrow$  3 must be sat liquid at least.

because 3 will enter the pump (the 2nd pump)

\* The 2nd pump will raise the pressure  $P_3 \rightarrow P_4 / P_4 = P_5$

$q_{VH} = h_5 - h_4$  which will be less than before (Notice the T-s diagram)

\* I don't take the steam at the inlet because this steam has high potential to do work.

\* I don't take the steam at the outlet because this steam doesn't have any potential to do work or to heat

\* I can find the best location for the steam bleeding by knowing  $P_7 + P_5$  (best location means the location that gives the highest thermal efficiency)

\*  $h_3 \rightarrow$  Sat liquid &  $h_2 \rightarrow$  can find it using  $h_2 - h_1 = V_p (P_2 - P_1)_{sc}$

\*  $h_6 \rightarrow$  must be given

\* don't worry about the 2nd pump because  $V_p (P_4 - P_3)$  • the difference between  $P_4$  and  $P_3$  is not large  
( $P_3$  is very small)

\* The disadvantages of OFWA:

→ Mixing is direct So  $P_2$  must equal  $P_3 = P_6$ , this requires selection of pumps that work at these pressures

→ Friction → causes changes in pressure (which is not desirable)

→ Pumps may be damaged, leakage in the OFWH  $\Rightarrow$  the cycle fails

Steam + water leakage causes the ~~water~~ explosion of the boiler

(if the amount of water entering the boiler is small and  $q_{th}$  is large, the boiler will melt + explosion in the boiler will occur)

So we use the CFWH.

\* In Closed gas turbine cycle, there is no mix between the fuel and the air.  $\rightarrow$  Brayton cycle / Joule cycle

\* In CFWH, there is no mix between the steam bleeding and the water coming from the first pump.

\* Water is heated  $2 \rightarrow 9$  } mixing them in the mixing chamber  
\* Steam is condensed  $7 \rightarrow 3$  } gathering them in the mixing chamber

## Revision

### Rankine cycle with regeneration

In Gas turbine cycle, we have used the regenerator, to heat the air before it enters the combustion chamber  $\Rightarrow q_{th} \downarrow \Rightarrow$  Fuel consumption for the same  $W_{net}$ ,  $M_{th} = \frac{W_{net} + cond}{q_{th} \downarrow}$ ,  $M_{th} \uparrow$

\* The objective of the regeneration is to decrease  $q_{th}$  for the same  $W_{net}$  or slightly less.

\* In the OFWH:

- the pump (without the FWH) increases the pressure from  $P_{cond}$   $\rightarrow P_{boiler}$
- but in the OFWH, the 2 pumps work with small difference of pressure + the first pump deals with  $(m-m_b)$  not the total mass.
- We can use the flash chamber between the OFWH and the 2nd pump
- Instead of raising the temperature from  $T_2 \rightarrow T_5$ , now we are raising the temperature from  $T_4 \rightarrow T_5$
- if state 3 is compressed liquid,  $q_{th} \uparrow$  (check the T-S diagram)
- $y = 0.10 - 0.15 \Rightarrow$  a small amount is needed to supply 2 with heat to increase its temperature but not to evaporate it. (because it will enter the pump after that)

→ We can put pumps in parallel (~~spare~~ pumps)

is fixed

the pressure of the steam bleeding:  $P_1$   $\rightarrow$   $P_2$   $\rightarrow$   $P_3$   $\rightarrow$   $P_4$   $\rightarrow$   $P_5$

- In the OFWH: once I choose the 2 pumps point 6 is fixed
- In the CFWH: I can take the steam bleeding from any location I want because there is no mixing. Check the T-S diagram for the location needed for the  $P_{steam}$  bleeding to be equal to  $P_2$ .  $P_2$  (the outlet of the 1st pump)  $\rightarrow$  (2nd + 1st pumps)  $\rightarrow$   $P_1$   $\rightarrow$   $P_2$   $\rightarrow$   $P_3$   $\rightarrow$   $P_4$   $\rightarrow$   $P_5$

→ We put a mixing chamber in order to gather the ~~working~~ total mass of the working fluid and to be sure that the pressure is fixed (SG can't endure any fluctuations in pressure)

How does 'the freedom of taking the steam bleeding from any location' benefit me?  
 If I want to decrease the consumption of the fuel, I can choose a location that is slightly above, without sacrifice the objective of this cycle (i.e. producing power).

For example, In summer, many citizens travel, so the consumption of electricity decreases. I can choose a location that is slightly above  $\rightarrow \downarrow$  the consumption of the fuel

+  $W_{net}$  (less power is required because people  $\rightarrow$  the citizens are abroad.)

- \* There is a location that increases the thermal efficiency (decreases  $q_m$  for the  $\downarrow$ )
- \* There is a location that increases  $W_{net}$  (maybe at the expense of increasing the heat addition (i.e. the fuel consumption))

at 8% (pump) raises pressure +  $\downarrow$  heat addition to oil  $\rightarrow$   $h_2$   $\rightarrow$   $h_3$   $\rightarrow$   $h_7$   $\rightarrow$   $h_8$   
 also  $\downarrow$  heat addition  $\rightarrow$  pump  $\rightarrow$   $\leftarrow$  Vapor  $\rightarrow$

$$① - (1-y) = \frac{h_8 - h_2}{(h_7 - h_3) + (h_9 - h_2)}$$

$h_2$   $\rightarrow$  is known of the pump using the equation

$h_3$   $\rightarrow$  is known because it's saturated liquid

$h_7$   $\rightarrow$  must be given because I must know the location of the steam bleeding

$$② - (1-y) h_9 + y h_8, \text{ also } h_9 \text{ must be given so that } ①, ②$$

$\rightarrow$  2 equations with 2 unknowns  
 if  $h_9$  is not given assume  $h_9 = h_3$  so

- there is a certain location between the inlet and the outlet + there is a certain "y" (amount of steam extracted from the turbine) that gives the best  $\eta_{th}$
- If I choose a location that is slightly above,  $y$ , but I have lost turbine work to heat the working fluid
- However, if I choose a location that is below,  $y$ , but  $W_{Turbine}$  will not be lost

~~16.2~~

## Cogeneration

- In all the previous cycles that we have studied, the purpose of the cycles ~~was~~ to produce power
- In Cogeneration Cycles, Steam is utilized for more than one purpose (the main purpose is still producing power)
- Some industries that require steam
  - \* biscuits (food industry)
  - \* ironing
  - \* sterilization in hospitals
  - \* Drying textiles
  - \* Heating buildings

any industry that is based on steam is called process heater (PH)

case

- Which ~~state~~ is more economical for the manufacturer?
  - \* To build his own power plant
  - \* To contract an agreement with a power plant to supply him with the required steam
- In this case, the steam manufacturer will be supplied with steam, and return this steam as a liquid. Only the pump that will be used to return the liquid to the power plant will cost him

→ Rankine Cycles that are used for more than one purpose is called, Cogeneration cycle

→ the goal of the cogeneration cycle is investment. (In order to provide the repairing costs, workers' salary and the costs required to develop the power plant)

## Notes on Steam bleeding

- there is a certain location between the inlet and the outlet + there is a certain  $y^*$  (amount certain amount of steam extracted from the turbine) that gives the best  $M_{th}$
- If I choose a location that is slightly above,  $y^*$ , but I have lost turbine work to heat the working fluid
- However, if I choose a location that is below,  $y^*$ , but  $W_{Turbine}$  will not be lost

## Notes on Cogeneration

- In all the previous cycles that we have studied, the purpose of the cycles ~~was~~ to produce power
- In Cogeneration cycles, steam is utilized for more than one purpose (the main purpose is still producing power)
- Some industries that require steam
  - \* biscuits (Food industry)
  - \* Ironing
  - \* Sterilization in hospitals
  - \* Drying textiles
  - \* Heating buildings

} any industry that is based on steam is called process heater (PH)

### Case

- Which ~~stage~~ is more economical for the manufacturer?
  - \* To build his own power plant
  - \* To contract an agreement with a power plant to supply him with the required steam
- In this case, the ~~stage~~ manufacturer will be supplied with steam, and return this steam as a liquid. Only the pump that will be used to return the liquid to the power plant will cost him
- Rankine cycles that are used for more than one purpose is called Cogeneration cycle
- the goal of the cogeneration cycle is investment. (In order to provide the repairing costs, workers' salary and <sup>the costs required to</sup> develop the power plant)

- Sometimes, Sailing electricity is not sufficient for the powerplant's outlay, So you can sail steam ~~when only~~ when you don't need this steam. (Because the main purpose of this steam is to produce power) (2)
- In Summer Vacations, citizens travel for tourism, So the consumption of electricity  $\downarrow$ , and you want to make an investment, there are 2 ways for investment.
  - Sailing electricity  $\rightarrow$  (not effective, because the citizens are abroad the consumption of electricity  $\downarrow$ )
  - Sailing steam for factories that require steam (factories or hospitals) or for building heatings
- If there is a great demand on the power plant, steam can be extracted from the turbine.
- A Valve is installed at point (5) (it is a gate valve that is used only to regulate the flow of the fluid. It doesn't cause any changes in the properties of the flowing fluid)
  - \* the valve is closed  $\Rightarrow$  when the demand on electricity  $\uparrow$
  - \* the valve is opened  $\Rightarrow$  the demand on electricity  $\downarrow$  (In summer vacation, so it becomes a way for investment)
- Steam must be supplied to the factory according to the required features.
- Drying textiles: Super-heated steam is required to dry the textile. This steam is not required to be at high pressure, so the pressure of the steam is decreased using a throttling valve (without changing the amount of heat in the steam i.e. const enthalpy).
- Biscuits: Superheated steam at lower pressure is required for biscuits. (Wet mixture will not moisturize the biscuits, which is not desired, So we don't use wet mixture)
- Heating buildings: Superheated steam is not required for heating buildings. Wet mixture / Sat. vapor steam is used to heat buildings. (This steam will be supplied from the turbine at point (5))
- P11  $\rightarrow$  ii. is not like a mixing chamber, think of it as factories, houses, hospitals etc.
- $P_q = P_{10} = P_1 = P_2 = P_3$

$P_{ph}$  is determined by the manufacturer according to the features of the steam

(3)

→ the steam at 4 may be used for sterilization in hospitals

the steam at 5 may be used for heating building (because it doesn't require large amount of heat)

جذب سطح الماء، لـ ١٥٠، لـ ١٦٠ سـ١٠٠ جـ١٠٠ \*  
this is what happens when hot steam (outside the tube) passes through a throttling valve

→ the throttling valve has a small diameter, so when steam enters

the throttling valve  $V \uparrow, P \downarrow, T \downarrow$  Joule-Thomson effect (For real gases)  
( $T \downarrow$  when  $V$  (specific volume) (i.e. when the gas expands))

- When the gas expands, work is done by the gas on the surroundings ( $U_f, T_f$ ) - the opposite occurs when the gas is compressed

•  $V \uparrow, P \downarrow$  Why?  
جذب سطح الماء، لـ ١٥٠، لـ ١٦٠ سـ١٠٠ جـ١٠٠  
استكمال الماء، لـ ١٥٠، لـ ١٦٠ سـ١٠٠ جـ١٠٠  
(بـ ١٥٠ فـ ١٦٠)

→ The manufacturer must agree to the terms and conditions of the owner of the power plant. these terms are

1) Not to change in the properties of the working fluid (because water is chemically processed)

2) Steam is supplied to the manufacturer and it is must be returned as ~~saturated~~ Saturated liquid (So the owner of the power plant doesn't have to use a condenser)

3) The working fluid must be returned free from impurities, as it was supplied free from impurities

→ Remember that the main purpose of the cycle is to produce power,  
So don't sail steam unless the demand on electricity is weak.

→ If the demand on electricity  $\uparrow$ , steam bleeding is disconnected from the process water and connected to the feed water heater

→ As an owner of a power plant, you mustn't stop supplying the steam suddenly. You must give the manufacturer a time out, so he can look for another ~~power plant~~ power plant or steam supplier

→ This requires having knowledge about how does the demand on electricity vary over the year.

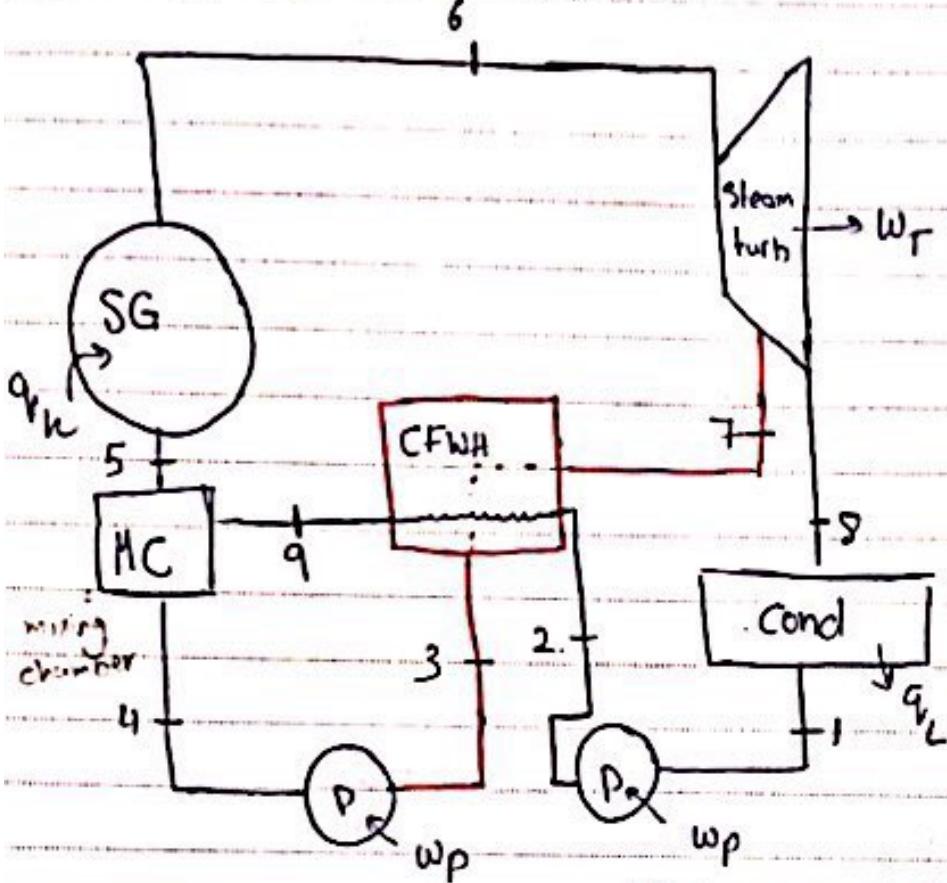
→ The value is closed gradually because the demand on electricity ↑ gradually.

→ The purpose of the PH is investment

→ In the 2nd design, steam is supplied with certain features  
(In the 2nd design the owner of the power plant determines the features of the steam, according to the power plant's needs, unlike the first design, the manufacturer chooses the features of the steam)

→ So, Any manufacturer who is interested to buy this steam with ~~the~~ certain features, he is welcome, otherwise the power plant cannot supply him with his own features.

# \*Closed Feed Water Heater,



$$P_6 = P_5 = P_9 = P_4 = P_2$$

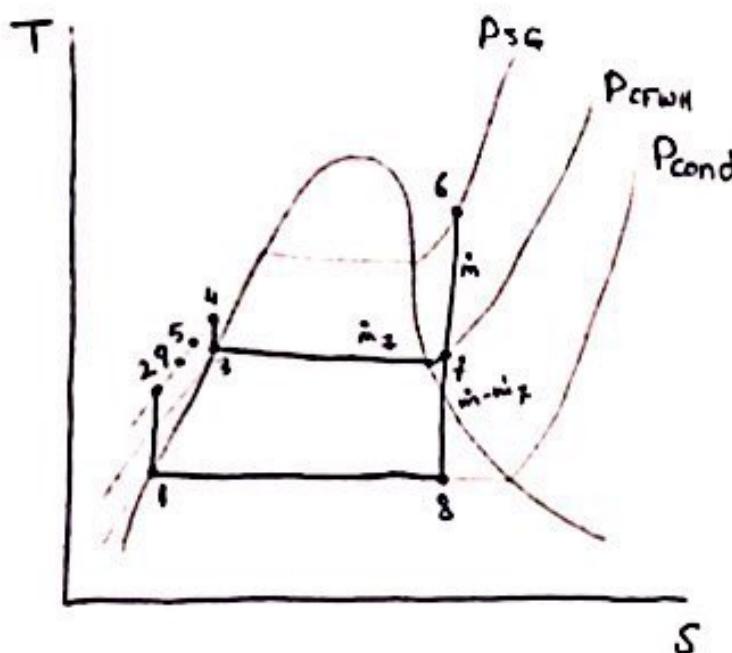
$$P_3 = P_7$$

It is impossible for  $T_4$  to be greater than  $T_2$  ( $T_2$  may be equal to  $T_3$  in the limiting ideal case).  
 $T_3$  to be less than  $T_2$  ( $T_3$  may be equal to  $T_2$  in the limiting ideal case).

$T_4 > T_9$  why?

The steam at 7 is going to heat the liquid at 2.

So  $T_7 > T_2$ .  $T_3$  will be greater than  $T_9$  or equal to  $T_9$  (if thermal equilibrium occurs). The pump will raise the temp. of  $P_3$  to  $T_4$  ( $T_4$  is  $>$  than  $T_3$ ) which means that  $T_4 > T_9$ .



$$W_T = \dot{m}(h_6 - h_7) + (\dot{m} - \dot{m}_7)(h_7 - h_8), W_T = (h_6 - h_7) + (1-y)(h_7 - h_8)$$

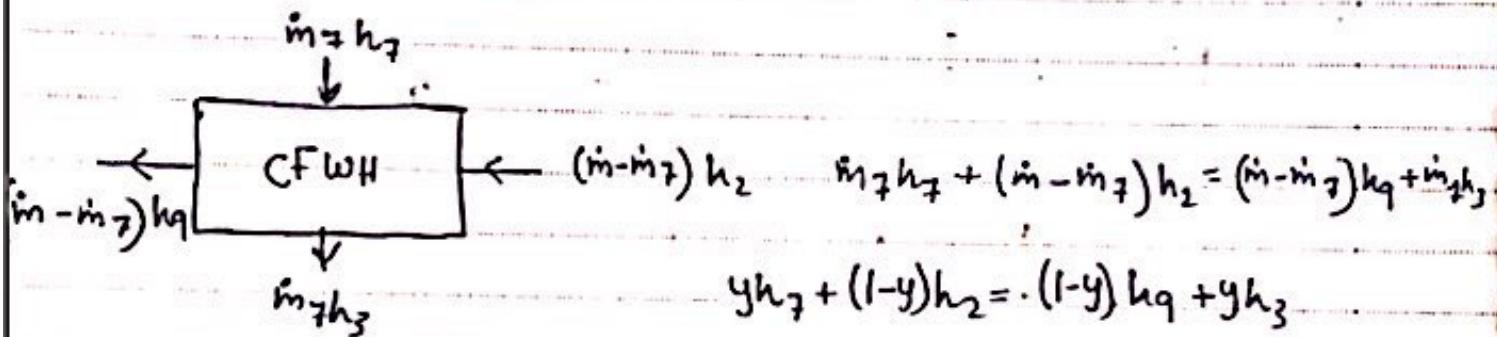
$$W_P = (\dot{m} - \dot{m}_7)(h_2 - h_1) + \dot{m}_7(h_4 - h_3), W_P = (1-y)(h_2 - h_1) + y(h_4 - h_3)$$

$$Q_H = \dot{m}(h_6 - h_5), q_v h_6 - h_5$$

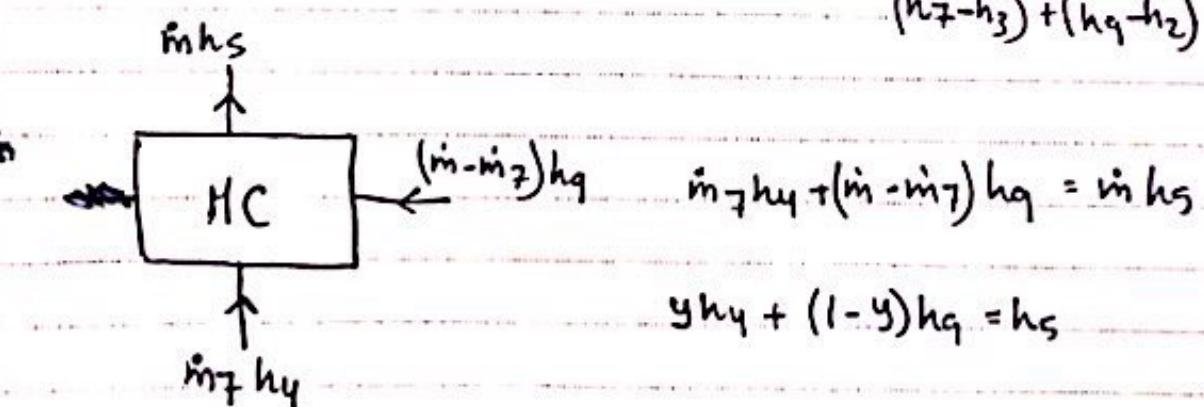
$$Q_L \cdot (\dot{m} - \dot{m}_7)(h_8 - h_1) \Rightarrow q_v = (1-y)(h_8 - h_1)$$

divide by  $\dot{m}$

and let  $y = \frac{\dot{m}_7}{\dot{m}}$



$$y = \frac{h_4 - h_2}{(h_7 - h_3) + (h_4 - h_2)}$$



\* If no information is given about either of the 3 ( $y, h_5, h_4$ ), then assume  $T_3 = T_4$  (effectiveness = 100%) and hence  $h_3 = h_4$ .

$h$  is a weak function of pressure

so although  $p_3 \neq p_4$  we have assumed

that  $T_3 = T_4$

## CHAPTER (II)

### REFRIGERATION CYCLES

- So far we used to deal with Heat Engines  $\rightarrow$  Kelvin Planck statement which states that it's impossible to have an engine that receives heat from heat source  $Q_H$  and produces only power (there must be  $Q_L$ )
- In this chapter we'll be dealing with 6th different "Refrigeration" or "Heat pumps" :-  
Their objective is to [Transfer heat from LOWER level to HIGHER level]
- we'll deal with "Clausius Statement" instead of Kelvin, which states that: "It's impossible to Transfer Heat from lower level to higher level without the help of external support"  
↳ To understand:  
Imagine you want to Raise a pen, you can't do that without adding energy while if you drop the pen it doesn't need any help  
(In English) ~~لأنك لا تقدر على رفعه إلا بجهد~~ ~~لأنك لا تقدر على رفعه إلا بجهد~~

#### → IMPORTANT :-

- ↳ In previous chapters, Heat was physically converted to work that's why we defined or used "Thermal Efficiency" BUT in this cycle work is NOT physically converted to heat, it is only used to Transfer Heat from level to another, that's why there isn't Thermal efficiency for the cycle. instead of Thermal Efficiency there is (COP)

↳ Coefficient of performance

#### → (COP) $\leftarrow$ coefficient of performance:

↳ It is used to measure the performance of a Refrigerator or a Heat pump.

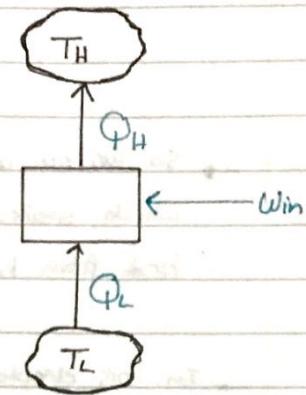
↳ We don't call it "Thermal efficiency" because work is NOT converted to heat or the opposite.

$(COP = \frac{\text{Desired output}}{\text{Required input}})$	<b>Refrigerator</b> $(COP_R = \frac{Q_L}{W_{in}})$	$Q_L \leftarrow$ $Q_L \rightarrow$
<b>Heat Pump</b> $(COP_{HP} = \frac{Q_H}{W_{in}})$		$Q_H \leftarrow$ $Q_H \rightarrow$

→ There are important Terms in which we need to know in this chapter:

#### \* REFRIGERATOR :-

Thermodynamic device that transfers heat from lower level to Higher level



#### \* REFRIGERANT :-

The working medium that helps achieving the Target of the Cycle ( Just like water or Gas in previous chapters).

it is denoted by  $\rightarrow (R134a)$  ; this is the Refrigerant Type that is used

→ There are other types like (R12) or (R22) But they are not used any more.

#### \* REFRIGERATION :

The Thermodynamic process itself ( Transferring Heat from lower level to Higher level ).

#### \* HEAT PUMP :

#### \* TONNE OF REF. :-

it is used to describe cooling/Heating capacity.

it is equal to  $(211 \frac{KJ}{min})$

↳ this number represents the amount of energy needed per min to melt an ice cube or the amount of energy that you must take away to keep the ice cube an ice cube without melting.

"Latent Heat of Fusion"

(So 1 tonne of heat is one  $211 KJ$  per degree celcius)

→ There are 3 cycles at which we'll consider:

1. Carnot Refrigeration Cycle.

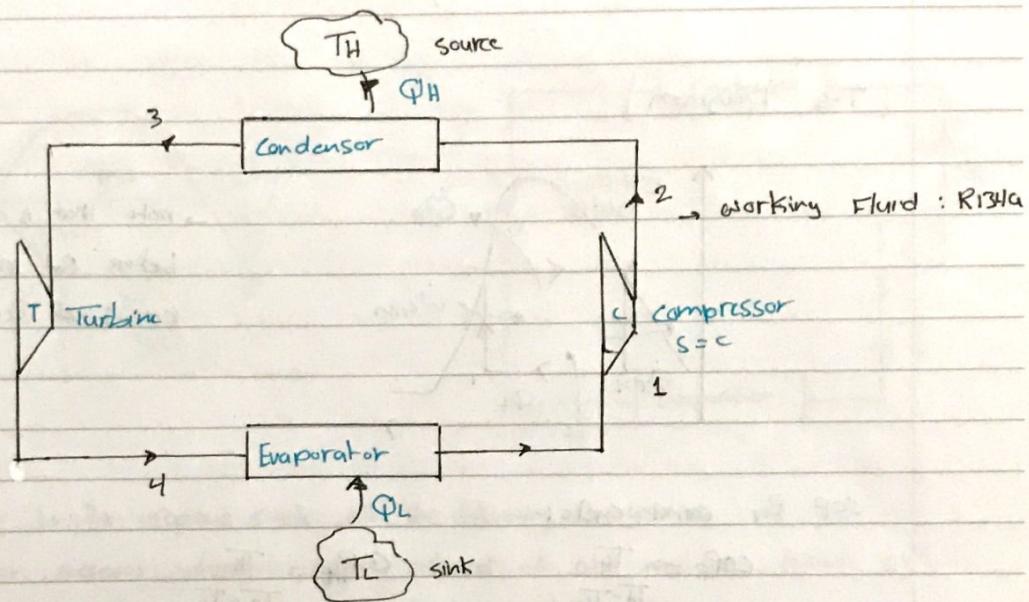
2. Vapor Comp. Ref. Cycle.

3.

## 1. CARNOT REFRIGERATION CYCLE

→ Carnot proposed the following Cycle:

(it's the exact Reverse of Rankine Cycle).



→ as you can see, it consists of 4 devices:

1. Condenser

2. Compressor

3. Turbine

4. EVAPORATOR → This is the only device that is new to us.

an Evaporator is used to Evaporate the liquid, so it takes in Heat in low quality ( $Q_L$ )

(it is the opposite of a condenser)

→ Recall in Carnot Cycle: 2 isothermal, 2 isentropic processes.

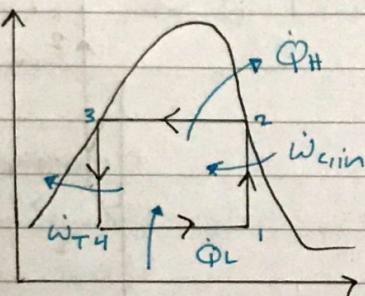
and it is a Hypothetical Cycle.

## \* How Does the Cycle work ?

→ The working fluid gets into the compressor at 1 and gets compressed isentropically, it goes out as super heated at (2) then goes into the condenser → so at (3) it is saturated liquid → then Expansion in the turbine also isentropically, at the end it goes into the evaporator at point (4) as very cold liquid to be evaporated.

↳ This cold liquid takes heat from the objects in the sink, so objects will get cold and it will evaporate.

(T-s Diagram)



→ note that is the same as before but now it is (c.w) not (c.i.w)

COP for Carnot cycle:-

$$COP_R = \frac{T_C}{T_H - T_C}$$

$$COP_{HP} = \frac{T_H}{T_H - T_C}$$

→ Carnot's Ref. Cycle is -as usual- not practical, why?

1. at point (3) The working fluid is (liquid) and since Turbines should deal with a quality not less than 90%, it'll NOT work efficiently (Turbine doesn't deal with (liquids) (for liquid  $X=0$ ) (2 → 3))

2. at point (1) The working fluid is a wet mixture while compressor deals with gases so this is another problem (1 → 2)

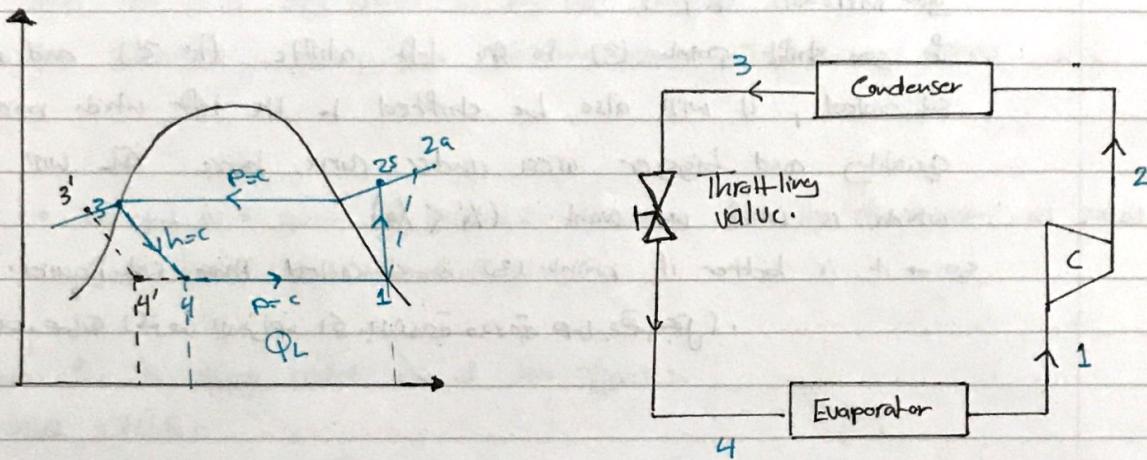
so → Carnot chose to use a Turbine and a compressor only to make the processes (isentropic) so Reversible

3. Evaporator receives wet mixture & gives wet mixture so it's working under low efficiency. (4 → 1)

∴ Carnot Cycle is NOT practical BUT it gives the Highest COP between two Temperature limits  $(T_L) \& (T_H)$

→ to make the cycle more practical they proposed "Vapor Compression Refrigeration Cycle" (similar to what we did in Rankine Cycle)

### VAPOR COMP. REF. CYCLE



→ It is similar to the previous cycle with the following changes:

1. compressor receives saturated vapor instead of wet mixture. (Point 1)  
↳ or super heated.  $(1 \rightarrow 2)$

2. Condenser works under constant PRESSURE not Temperature, so point (3) will be sat. liquid (or sub-cooled which is better)  $(2 \rightarrow 3)$

3. Turbine is replaced with a Throttling valve which works under constant Enthalpy (Isenthalpic)  $(3 \rightarrow 4)$  cycle JL (Subsidiary) will

4. Evaporator will work under constant pressure, it will receive wet mixture or what so ever and give (sat. vapor) or (superheated) which is the Target-  
cycle JL (Subsidiary) will

→ This represents the concept of vapor comp. Ref. Cycle

Q → where does the evaporator take heat to evaporate the liquid?

liquid at point (4) will have low quality and low T (about  $-20$  to  $-30^\circ\text{C}$ )

so assuming the sink having objects at about  $(5^\circ\text{C})$ ,  $5^\circ\text{C}$  is considered High Relative to  $-20$  or  $-30^\circ\text{C}$ , so heat will be transferred from the objects to the liquid and as a result → liquid will evaporate

→ objects will cool down

Q → What's the point of using a condenser in the cycle?

good Throttling happens at high pressures, so the condenser is responsible for keeping point (3) at a high pressure to get into the Throttling valve.

Note  $\rightarrow$  The area under  $(4 \rightarrow 1)$  represents  $Q_2$ .

Q → How can the Heat taken from the objects increase (How can you increase  $Q_1$ )?

If you shift point (3) to the left a little (to  $3'$ ) and make it subcritical, it will also be shifted to the left which means lower quality and bigger area under curve, hence  $Q_L$  will increase which is what we want. ( $x_3' < x_3$ )

so  $\rightarrow$  it is better if point (3) is subcooled than sat. liquid.

أهم سمات حركة الاتجاهات أو بالكلمة درجة حرارة بقلم .

Q → is it good to have part (1) as superheated ?

it is better for the condenser to deal with superheated (no liquid) BUT superheated means higher specific volume which also means that more work will be required from the compressor.

## - EQUATIONS -

$$W_C = m_2(h_2 - h_1)$$

$$\hat{Q}_L = \min_{\mathbf{h}_L} (h_1 - h_2)$$

$$\dot{Q}_A = m_p (h_2 - h_3)$$

note that  $mg$  is constant (steady flow devices).

$$\frac{COP_R}{HP} = \frac{Q_L}{W_{c, in}}$$

Recall  $\rightarrow$  Thermodynamic works under constant Entropy ( $\Delta S = 0$ )  
so it has no Heat or work.

Q → How does Enthalpy in process (3 → 4) remain constant?

We know that  $h = u + Pv$

$(u)$   $\downarrow$   $\downarrow$   $(v \uparrow)$ : ( $v$  will increase because at 3 it was liquid only then at 4 it becomes liquid and vapor  $\therefore v \uparrow$ )

Because  $u$  is a function of pressure  $\downarrow$   $T \downarrow$   $T$  will drop  $\therefore u$  will decrease)

$(P)$   $\downarrow$   $v$  will drop.

∴ Both Terms will change But  $(h)$  will remain constant.

Note → if you look at TS Diagram you'll notice that there is a small area under (3 → 4) which means there is  $Q_{3 \rightarrow 4}$  BUT compared to  $Q_C$  and  $Q_H$  it is very small so we can neglect it.

just give heat exchange just before Throttling valve in the diagram process  $\rightarrow$

NOTE → If you were given the isentropic efficiency for the compressor, as usual you'll have (2s) instead of (2).

\* Questions for this cycle could be of two types :-

### 1) IDEAL CYCLE :-

If you were only given Max. & Min pressures or Max T & P for example with no extra information this means you are dealing with an ideal cycle :- you can do the following assumptions:

- point (3) is SAT. LIQUID
- point (1) is SAT. VAPOR
- point (2) as the ideal case (2s) - isentropic
- $h_3 = h_4$

### 2) ACTUAL CYCLE:

- you'll be given the information of each point by itself (T & P)
- none of the previous assumptions is applicable if you know that it is an actual cycle
- you can't assume  $h_3 = h_4$  if you are given enough info. about point (4)
- you can only do that assumption if there is no info given about the point.

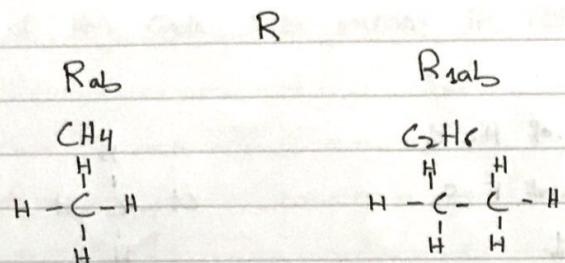
IMPORTANT :-

P-h Diagram will be used to obtain info instead of T-s.

See examples (II-1) & (II-2) IMPORTANT

## SELECTION OF REFRIGERANT

- All Refrigerants are noted with -R-
- They are divided into two types depending on the chemical compound they come from, as follows:



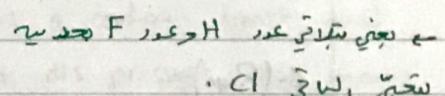
→ what they do is to replace Hydrogen atoms totally or partially with either (F) or (Cl) to create different Refrigerants.

→ you can know what atoms the Refrigerant consists of from its name as follows:

$$\text{number of (H) atoms} = a-1$$

$$\text{number of (F) atoms} = b$$

$$\text{number of (Cl) atoms} = \text{what is left other than H \& F}$$



### EXAMPLES :-

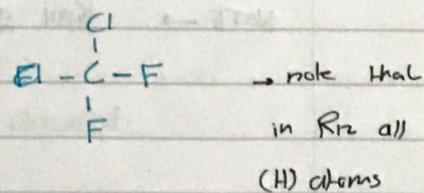
#### 1. (R<sub>12</sub>)

→ it is an (R<sub>ab</sub>) Refrigerant so from → CH<sub>4</sub>

$$a=1 \rightarrow a-1 = 1-1 = 0 \rightarrow \text{No. of (H)} = 0$$

$$b=2 \rightarrow \text{No. of (F)} = 2$$

$$\text{No. of (Cl) atoms} = 2$$



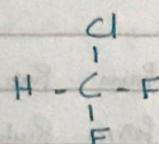
#### 2. (R<sub>22</sub>)

→ R<sub>ab</sub> : CH<sub>4</sub>

$$a=2 \rightarrow a-2 = 2-1 = 1 \rightarrow \text{No. of (H)} = 1$$

$$b=2 \rightarrow \text{No. of (F)} = 2$$

$$\text{No. of (Cl) atoms} = 1$$

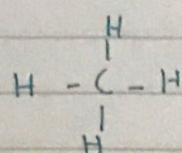


#### 3. (R<sub>50</sub>)

→ R<sub>ab</sub> : CH<sub>4</sub>

$$a=5 \rightarrow a-1 = 5-1 = 4 \rightarrow \text{No. of H} = 4$$

$$b=0 \rightarrow \text{No. of F} = 0 \text{ also Cl} = 0$$



→ note that (Methane) "CH<sub>4</sub>" is also considered a refrigerant BUT it is NOT practical to use it because it is Flammable

→ note: Having Both F & Cl in the Ref. is not good for environment.

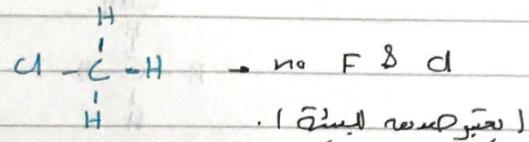
#### 4. (R<sub>40</sub>)

→ R<sub>40</sub> : CH<sub>4</sub>

→ a-1 = 3 → No. of H = 3

→ b = 0 → No. of F = 0

→ No. of Cl = 1



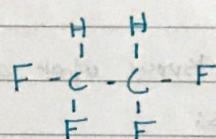
#### 5. (R<sub>134a</sub>)

→ R<sub>134a</sub> → C<sub>2</sub>H<sub>6</sub>

→ a = 3 → a-1 = 3-1 = 2 → No. of H = 2

→ b = 4 → No. of F = 4

→ No. of Cl = 0



#### 6. (R<sub>140</sub>)

→ R<sub>140</sub> : C<sub>2</sub>H<sub>6</sub>

a = 4 → a-1 = 4-1 = 3 → No. of H = 3

b = 0 → No. of F = 0

No. of Cl = 3

NOTE → R<sub>134a</sub>

↳ This symbol represents which of the (H) atoms were replaced (Related to chemical engineering) (we don't care that much about it but just to know).

7. (R<sub>110</sub>) → No. of H = 0, F = 0, Cl = 6

8. (R<sub>113</sub>) → H = 0, F = 3, Cl = 3

NOTE → From R<sub>40</sub> : R<sub>12</sub> & R<sub>22</sub> are most common and practical

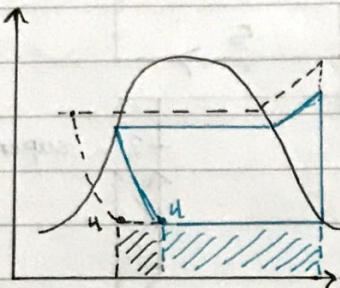
From R<sub>134a</sub> : R<sub>134a</sub>

## INNOVATIVE CYCLES

- in the Book there are 4 innovative cycles, we are only required to know one of them → (Cascade Ref. Cycles)

### CASCADE REF. CYCLE :-

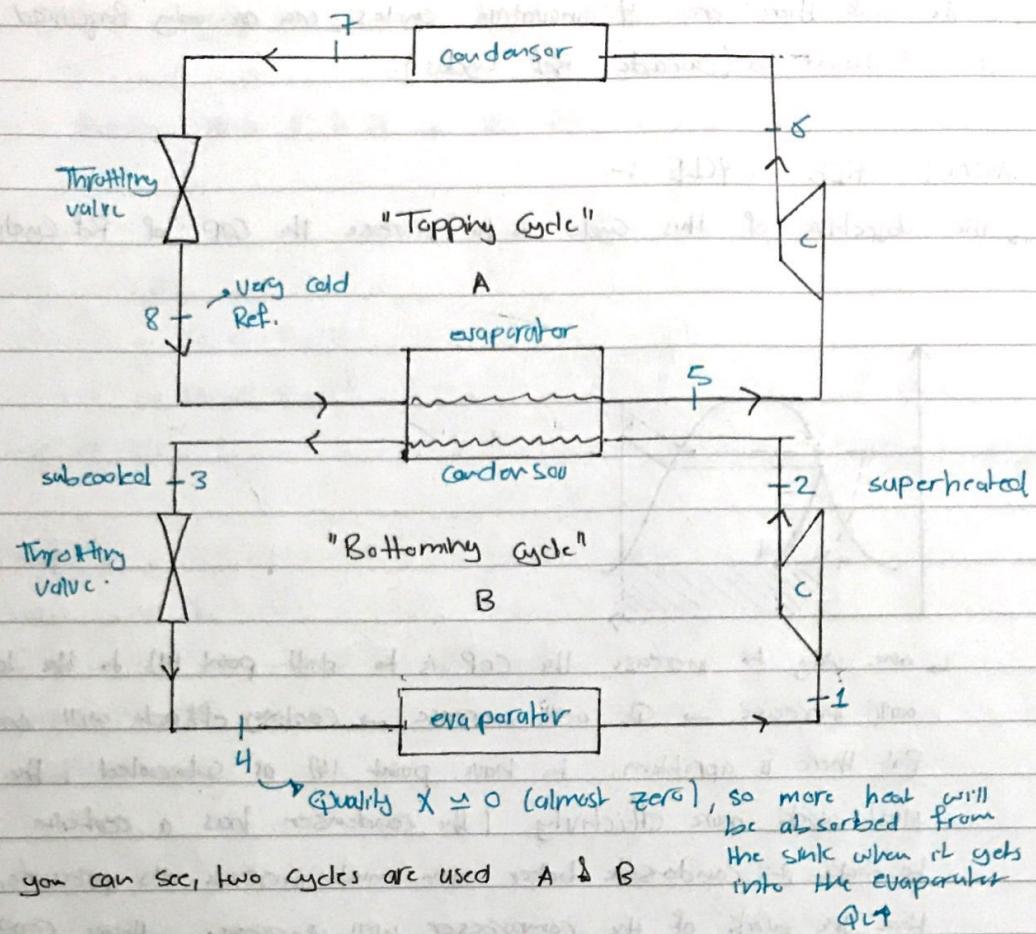
- The objective of this cycle is to increase the COP of Ref. Cycle.



→ one way to increase the COP is to shift point (4) to the left so area will increase →  $Q_L$  will increase → Cooling effect will increase  
But there is a problem: to have point (4) as sub-cooled, the condenser must work more efficiently (the condenser has a certain limit and to make it condensate better you must increase its pressure, that means that the work of the compressor will increase, hence COP will decrease which is NOT what we want).

→ so, to make the condenser work better without increasing its work Cascade Ref. Cycle was used, two cycles are used to achieve that target. look at the following page.

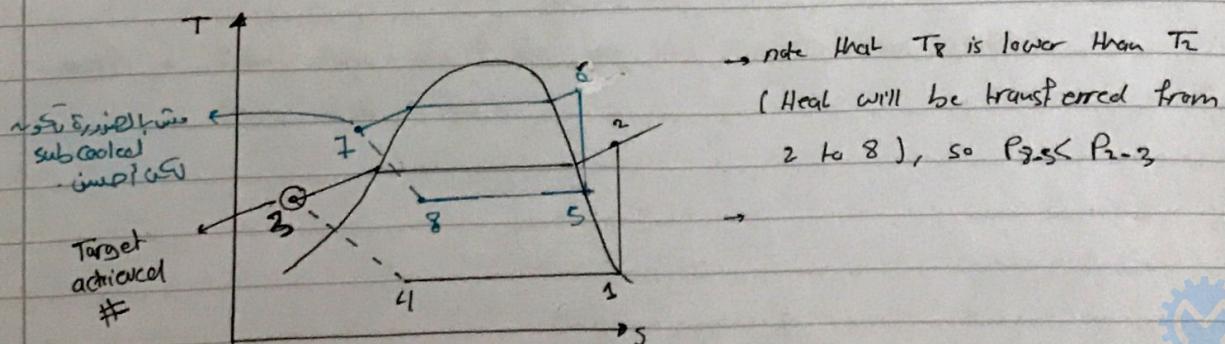
→ This cycle is used where high cooling loads are required.



what happens is that the condenser of the first cycle (B) is the evaporator of the upper one (A), and since the evaporator is supplied by a very cold Ref coming out of the throttling valve at "8", so this cold Ref will be used to condensate the superheated Ref coming at (2) which will make the condensation process better and at point (3) the liquid will be sub-cooled as we want without using any extra work

so  $\rightarrow Q_L$  increased  $\rightarrow$  COP increased  $\rightarrow$  Target is achieved. ( $COP = \frac{Q_L}{W_C}$ )

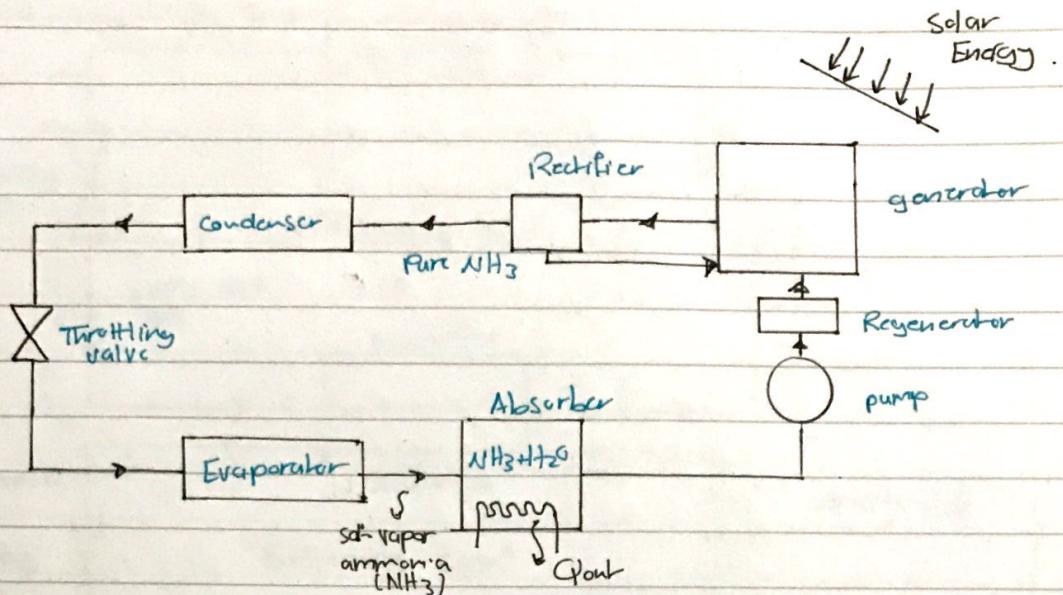
T-S Diagram :-



### Example (11-4) - notes

- There is a mistake in the T-s Diagram in this example, you can notice that the pressure lines for both (5,8) & (2,3) are assumed to be the same. But  $P_{2,5} < P_{2,3}$  This is just to simplify things.
- 3: sat. liquid (no info. so assume ideal case)
- 5: sat. vapor
- 6:  $h_5 = h_6$

## AMMONIA ABSORPTION CYCLE



- from its name, this cycle uses Ammonia ( $\text{NH}_3$ ) as the Ref.
- $\text{NH}_3$  is very harmful to use BUT its advantage is that it is soluble in water so the compressor could be replaced with a PUMP which Reduces the work needed.

→ what happens?

$\text{NH}_3$  leaves the evaporator as saturated vapor and goes into the "ABSORBER" where  $\text{NH}_3$  &  $\text{H}_2\text{O}$  are mixed and to minimize the amount of  $\text{NH}_3$  dissolved in water the absorber must be cooled (QonL) then it goes into the pump.

the Regenerator & generator are used to separate  $\text{NH}_3$  from water again (the solution is vaporized then that vapor goes to what's called a "Racifier" where  $\text{NH}_3$  is totally separated from any water and pure  $\text{NH}_3$  is obtained again).

→ So, the benefit of this cycle is that the compressor was replaced by a pump and a pump does not need a lot of work to do the job.

note → we are only required to know how this cycle works (possibly)

## SECOND LAW ANALYSIS OF REF. CYCLES

$$\dot{X}_{dest} = \dot{m}_R T_0 S_{generated}$$

$$S_{gen} = \sum S_{out} - \sum S_{in}$$

rate Heat in Ref. Cycle:

$Q_C \rightarrow$  into system

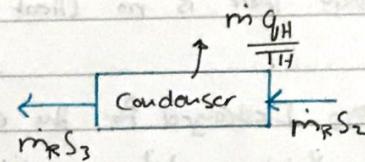
$Q_H \rightarrow$  out of system

For Condenser :-

By Mass Balance:

$$\dot{S}_{gen} = \dot{m}_R S_3 + \dot{m}_R \frac{q_H}{T_H} - \dot{m}_R S_2$$

$$= \dot{m}_R \left[ (S_3 - S_2) + \frac{q_H}{T_H} \right]$$



$$\text{So } \dot{X}_{dest} = \dot{m}_R T_0 S_{gen} = \dot{m}_R T_0 \left[ (S_3 - S_2) + \frac{q_H}{T_H} \right]$$

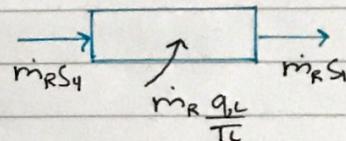
For Evaporator :-

By mass Balance:

$$\dot{S}_{gen} = \dot{m}_R S_4 + \dot{m}_R \frac{q_L}{T_C} - \dot{m}_R S_1$$

So:

$$\dot{X}_{dest} = \dot{m}_R T_0 \left[ S_1 - S_4 - \frac{q_L}{T_C} \right]$$



For Compressor:

$$\dot{X}_{dest} = \dot{m}_R T_0 [S_2 - S_1]$$

IMPORTANT :-

what is  $T_0$  in this cycle?

If Refrigeration  $\rightarrow T_0 = T_H$

If Heat Pump  $\rightarrow T_0 = T_C$

Recall  $\rightarrow$  previously we used

to take  $T_0$  as  $T_H$

Dead state now we

take it as  $T$  of

the surroundings which

is either  $(T_H)$  or  $(T_C)$

$$\dot{X}_{\text{recovered}} (\dot{X}_{\text{QL}}) = \left( \frac{T_0 - T_L}{T_C} \right) \dot{Q}_C = \dot{W}_{\text{in, min}}$$

$$\text{COP}_{\text{max}} = \frac{T_C}{T_H - T_C} = \frac{\dot{Q}_C \text{ max}}{\dot{W}_{\text{in, min}}} \uparrow$$

$$\dot{W}_{\text{in, min}} = \frac{T_0}{T_C} \left( \frac{T_H - T_C}{T_C} \right) \dot{Q}_C \quad \#$$

\* objective is to :

Maximize cooling effect  
at Min work input

Note that there is no (Heat in) But power to transfer Heat

\* Exergy Destroyed for the cycle:

$$\dot{X}_{\text{des, cycle}} = \dot{W}_{C, \text{in}} - \dot{W}_{\text{min, in}}$$

note → don't calculate

\* Second law efficiency

$$\eta_{II} = \frac{\text{COP}_R}{\text{COP}_{\text{max}} \text{ (Carnot)}} = \frac{\dot{W}_C \text{ min}}{\dot{W}_C \text{ actual}}$$

$\dot{X}_{\text{des}}$  for each  
process it's time  
consuming.

## CHAPTER (13)

### → GAS MIXTURES ←

- in this chapter we will only deal with Gas Mixtures → No liquids.
- we will learn how to find the properties of gas Mixtures
- to understand → we used to say that  $C_p = 1.005$  for air because N<sub>2</sub> is the dominant gas in it, now we will find the exact value and compare the two.

- assuming you have a Mixture A, B & C;

A	$m_A$	$n_A$
B	$m_B$	$n_B$
C	$m_C$ mass	$n_C$ mole

there are (3) Questions you need to answer:-

- \* what are the Gases ?
- \* what are their Respective Concentrations.
- \* what are their properties

- Concentrations could be given in two ways:

1) on Mass Basis (Gravimetric)

2) on Mole Basis (Molar)

↳ we will use Both

- o Total Mass of mixture is the sum of masses of individual components;

$$m_{\text{tot}} = \sum_{i=1}^N m_i$$

$$= m_A + m_B + m_C$$

we use subscript (tot) or (mix), it's the same thing.

- o The Mole number of the Mixture is the sum of the mole numbers of individual components;

$$n_{\text{tot}} = \sum_{i=1}^N n_i$$

$$= n_A + n_B + n_C$$

→ we will be dealing with what's called "Mass Fraction" & "Mole Fraction"

\* Mass Fraction: -  $(m_f)$

$$m_{fi} = \frac{m_i}{m_{tot}}$$

$$(\text{for A} \rightarrow m_{fA} = \frac{M_A}{M_{tot}})$$

$$\sum m_{fi} = 1$$

$$(m_{fA} + m_{fB} + m_{fC} = 1)$$

\* Mole Fraction: -  $(y_i)$

$$y_i = \frac{n_i}{n_{tot}}$$

$$(\text{for A} \rightarrow y_A = \frac{n_A}{n_{tot}})$$

$$\sum y_i = 100\%$$

(we want to relate  $y_i$  with  $m_f$ ) → Because sometimes you'll be given

$y_i$  But you need  $m_f$  and vice versa

→ from previous knowledge:

$$m = n M \rightarrow \text{Molecular weight } \left( \frac{\text{kg}}{\text{mole}} \right)$$

$\underbrace{\text{kg}}_{\text{mole}}$

$$\text{so} \rightarrow n_i = m_i / M_i$$

→ this is Mass Fraction  $m_{fi}$

$$\left[ y_i = \frac{n_i}{n_{tot}} = \frac{m_i / M_i}{n_{tot} / M_{tot}} = \left( \frac{m_i}{m_{tot}} \right) * \frac{M_{tot}}{M_i} \right] \rightarrow \begin{array}{l} \text{Mole Fraction in terms of} \\ \text{Mass Fraction} \end{array}$$
$$= m_{fi} \frac{M_{tot}}{M_i}$$

$$\text{so} \rightarrow y_i = m_{fi} \frac{M_{tot}}{M_i} \rightarrow m_{fi} = \frac{y_i M_i}{M_{tot}}$$

$$\left[ m_{fi} = \frac{m_i}{m_{tot}} = \frac{n_i M_i}{n_{tot} M_{tot}} = \frac{y_i M_i}{M_{tot}} \right] \rightarrow \begin{array}{l} \text{Mass Fraction in terms} \\ \text{of Mole Fraction} \end{array}$$

$$y_i = m_{fi} \left( \frac{\sum (M_i / n_i)}{\sum (m_i / n_i)} \right)$$

$$M_{tot} = \frac{m_{tot}}{n_{tot}} = \sum \frac{(n_i M_i)}{n_{tot}}$$

$$\left[ M_{tot} = \sum y_i M_i \right]$$

$$R_{mix} = \frac{R_u}{M_{mix}} \rightarrow 8.314 \frac{\text{KJ}}{\text{kmole} \cdot \text{K}}$$

After you answered the first two questions, what are the gases and what are their concentrations, you can now find their properties, as follows:

\* Properties are of two types:

Extensive

→ doesn't depend on mass or mole

→ noted with capital letters usually  
(H, U, S, m)

Intensive

→ depends on mass or mole

(it's unit is J/kg or kmole)  
(h, u, s, Cp, Cv, v, ...)

mass

assuming you want to find the Enthalpy H of a mixture:

"for Extensive properties, Add them up"

$$H_{\text{tot}} = \sum H_i = \sum (m_i h_i) \quad \begin{matrix} \text{This line} \\ \text{means per kmole} \end{matrix}$$

If you decide over  $M_{\text{tot}}$ :

$$\frac{H_{\text{tot}}}{M_{\text{tot}}} = \sum \frac{m_i h_i}{M_{\text{tot}}} \rightarrow \left( \frac{H_{\text{tot}}}{M_{\text{tot}}} = \sum x_i \bar{h}_i \right) \quad \text{Intensive (per kg)}$$

If you decide over  $n_{\text{tot}}$ :

$$\frac{H_{\text{tot}}}{n_{\text{tot}}} = \sum \frac{n_i h_i}{n_{\text{tot}}} \rightarrow \left( \frac{H_{\text{tot}}}{n_{\text{tot}}} = \sum x_i \bar{h}_i \right) \quad \text{Intensive (per kmol)}$$

→ same thing goes for all other properties

for example: ( $\bar{U}_{\text{tot}} = \sum x_i \bar{U}_i$ )

GENERAL RULE :-

$$\begin{pmatrix} H \\ U \\ S \end{pmatrix}_{\text{tot}} = \sum \begin{pmatrix} H \\ U \\ S \end{pmatrix}_i = \sum \begin{pmatrix} m_i * \frac{h_i}{u_i} \\ \text{or} \\ x_i * \frac{h_i}{\bar{u}_i} \end{pmatrix} \rightarrow \text{For Extensive.}$$

$$\begin{pmatrix} h \\ u \\ s \\ cp \end{pmatrix}_i = \sum \begin{pmatrix} m_i * \frac{h_i}{u_i} \\ \text{or} \\ x_i * \frac{h_i}{\bar{u}_i} \end{pmatrix} \rightarrow \text{For Intensive}$$

### EXAMPLE (13.1)

Given:

$$\begin{pmatrix} 3 \text{ Kg O}_2 \\ 5 \text{ Kg N}_2 \\ 12 \text{ Kg C}_H \end{pmatrix} \rightarrow \text{Gas Mixture}$$

Find  $\rightarrow m_i, x_i, M_{mix}, R_{mix}$

Solution:

↳ you can see that you are given (Mass) not (Mole)

↳ use the following Table to solve such problems:

$(i)$	$(m_i)$	$(M_i)$	$n_i = m_i/M_i$	$x_i = n_i/n_{total}$
$O_2$	3	32	$3/32 = 0.09375$	0.09375
$N_2$	5	28	$5/28 = 0.17857$	0.17857
$C_2H_6$	12	30	$12/30 = 0.4$	0.4
	$m_{mix} = 20$			
			$\downarrow 1$	

(1)  $\sum x_i = 1$

## EXAMPLE (B.1)

Given:

3 Kg O <sub>2</sub>	Gas Mixture
5 Kg N <sub>2</sub>	
12 Kg CH <sub>4</sub>	

Find  $\rightarrow m_i, y_i, M_{mix}, \rho_{mix}$

Solution:

→ you can see you are given Mass not Moles

→ use the following Table to solve such problems (first two columns are the given info)

i	m <sub>i</sub>	M <sub>i</sub>	m <sub>pi</sub>	from (A-1)	
				n <sub>i</sub> = m <sub>i</sub> /M <sub>i</sub>	y <sub>i</sub> = n <sub>i</sub> /n <sub>mix</sub>
O <sub>2</sub>	3	32	3/20 = 0.15	3/32 = 0.094	0.094/1.023 = 0.092
N <sub>2</sub>	5	28	5/20 = 0.25	5/28 = 0.179	0.175
CH <sub>4</sub>	12	16	12/20 = 0.6	12/16 = 0.75	0.732
	m <sub>mix</sub> = 20		(1)	n <sub>mix</sub> = 1.023	
				Kmol	
	This $\downarrow$ MUST Equal (1)				

→ you can use this But it is Time consuming.

$$\bar{M}_{mix} = \sum y_i M_i = \frac{m_{mix}}{n_{mix}} = 19.6$$

$$R_{mix} = \frac{R_u}{M_{mix}} = \frac{8.314}{19.6} = 0.424 \frac{\text{Kg}}{\text{Kg.K}}$$

### Problem (13-13)

Given

Gas Mix

6 kmol	H <sub>2</sub>
2 kmol	N <sub>2</sub>

Find M<sub>mix</sub> & R<sub>mix</sub>

Solution:

→ you are given moles not mass

i	n <sub>i</sub>	M <sub>i</sub>	m <sub>i</sub>	x <sub>i</sub>	m <sub>fi</sub>
H <sub>2</sub>	6	2	12	6/8	12/68
N <sub>2</sub>	2	28	56	2/8	56/68
Σ				(1)	(1)

$$M_{\text{mix}} = \frac{m_{\text{mix}}}{n_{\text{mix}}} = \frac{68}{8} = 8.5 \text{ kg/kmol}$$

$$M_{\text{mix}} = \sum x_i n_i = \frac{m_{\text{mix}}}{n_{\text{mix}}} = \frac{68}{8} = 8.5 \text{ kg/kmol}$$

$$R_{\text{mix}} = \frac{R_u}{M_{\text{mix}}} = 0.978 \text{ kJ/kg.K}$$

### Problem (13-8)

Moist air, on Molar Basis

78 %	N <sub>2</sub>
20 %	O <sub>2</sub>
2 %	H <sub>2</sub> O vapor

note → molar Basis is the same as Volume Basis.

Find M<sub>mix</sub>, R<sub>mix</sub>

i	y <sub>i</sub>	n <sub>i</sub>	M <sub>i</sub>	m <sub>i</sub>	m <sub>fi</sub>
N <sub>2</sub>	0.78	78	28	218.4	0.7634
O <sub>2</sub>	0.2	20	32	640	0.2237
H <sub>2</sub> O	0.02	2	18	36	0.012

$$n_{\text{mix}} = 100$$

$$m_{\text{mix}} = 2860 \text{ kg}$$

$$M_{\text{mix}} = \frac{2860}{100} = 28.6 \text{ kg/kmol}$$

$$R_{\text{mix}} = \frac{R_u}{M_{\text{mix}}} = 0.2907 \text{ kJ/kg.K} \rightarrow \text{compare with } R_{\text{air}}$$

### Problem (13.12)

By Mass:

$$\begin{cases} 75\% \text{ CH}_4 \\ 25\% \text{ CO}_2 \end{cases} \rightarrow M_{\text{mix}} = 100$$

Find  $M_{\text{mix}}$ ,  $R_{\text{mix}}$

$i$	$m_{fi}$	$m_i$	$M_i$	$n_i$	$x_i$
CH <sub>4</sub>	0.75	75	16	4.687	0.89
CO <sub>2</sub>	0.25	25	44	0.5	0.10812
(1)		(100)			

$$M_{\text{mix}} = 19.03 \text{ kg/Kmol}$$

$$R_{\text{mix}} = 0.437 \text{ kJ/kg.K}$$

### Problem (13.48) - application

"Volumetric Analysis" → mole or mole fraction

$y_i$	30%	O <sub>2</sub>			
	40%	N <sub>2</sub>			
	10%	CO <sub>2</sub>			
	20%	CH <sub>4</sub>			

→  $20^\circ\text{C}$   $\xrightarrow{\text{heat}} 200^\circ\text{C}$   $q_{\text{air}}$

The mix. is heated from  $20^\circ\text{C}$  to  $200^\circ\text{C}$  while flowing through a tube in which the pressure is constant (150 kPa). Find Heat transferred to the mix. per unit mass of the mixture.

Solution:

$$w, K, E, P, F = 0 \rightarrow q_{\text{in}} = \frac{h_0 - h_i}{m_{\text{mix}}} = C_p (T_0 - T_i)$$

$$C_{p,\text{mix}} = \sum C_p \quad \text{↳ we need to find this}$$

$i$	$x_i$	$n_i$	$M_i$	$m_{fi}$	$C_p$
O <sub>2</sub>	0.3	30	32	960	0.338
N <sub>2</sub>	0.4	40	28	1120	0.394
CO <sub>2</sub>	0.1	10	44	440	0.816
CH <sub>4</sub>	0.2	20	16	320	2.2537

$\rightarrow m_{\text{mix}} = 284 \text{ g}$

$$C_{p,\text{mix}} = \sum m_{fi} C_p = 1.1051 \text{ kJ/kg.K}$$

$$q_{\text{in}} = 1.1051 (200 - 20) = 199.92 \text{ kJ/kg}$$

(notice the difference)

we used  $h$  to find

$$C_p = 1.005$$

Problem (13-57)

60%	CH <sub>4</sub>
25%	C <sub>3</sub> H <sub>8</sub>
15%	C <sub>2</sub> H <sub>6</sub>

"By weight"  $\rightarrow$  Mass

The mixture is compressed from

100 kPa and 20°C to 1000 kPa in rev.

Isothermal, steady flow compressor, calculate

The work and Heat Transfer per unit mass.

Solution:

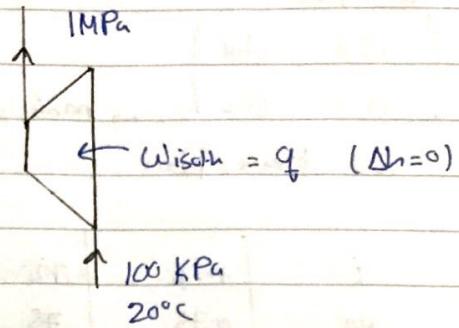
$$= R_{\text{mix}} T_{\text{mix}} \ln \left( \frac{P_2}{P_1} \right)$$

$$R_{\text{mix}} = R_u = \frac{\sum m_i R_i}{M_{\text{mix}}}$$

$$= 0.3805 \text{ kJ/kg.K}$$

$$W_{\text{isoth}} = 256.7 \text{ J}$$

$$W \Big|_{R=0.3805} = 193.6 \text{ (to compare)}$$



Problem (13-58)

Gas Turbine

He

Ar

Determine the work out put of the turbine per unit mass of the mixture

the mixture

$$W_T = h_1 - h_2 = C_p (T_1 - T_2)$$

$$i \quad n_i \quad M_i$$

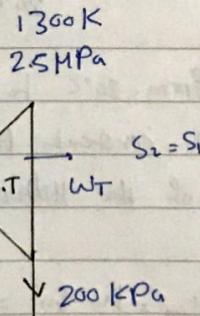
$$\text{He} \quad \text{Ar}$$

$$\text{Ar} \quad \text{40} \quad \text{0.5, 0.5}$$

answers:

$$C_p - C_v = R, \quad K-1 = \frac{R}{C_v}$$

$$K = \frac{R_{\text{mix}}}{C_{v\text{mix}}} + 1$$



$$\left( \frac{T_2}{T_1} \right)_{\text{mix}} = \left( \frac{P_2}{P_1} \right) \frac{K-1}{K}$$

$$C_{p\text{,mix}} = 0.948 \text{ kJ/kg.K}$$

$$M_{\text{mix}} = 22$$

$$C_{v\text{,mix}} = 0.56421$$

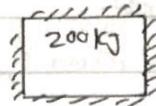
$$K_{\text{mix}} = 1.666 \quad (\text{it was 1.4})$$

$$T_2 = 483.1 \text{ K} \quad W_T = 781.16 \text{ kJ/kg}$$

Problem (B-54)

mass Basis, Rigid Body, Insulated

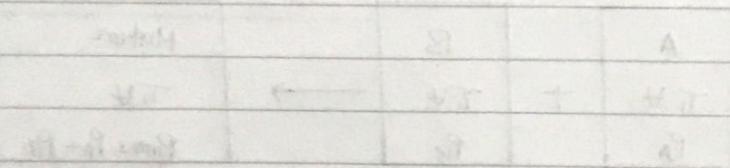
$$W = m \text{ mix } C_{\text{mix}} (T_2 - T_1)$$



200 kJ

200 kJ

200 kJ



# P-H-T BEHAVIOUR OF an Ideal / non ideal Gas Mixture

Ideal Gas law:

$$P_{\text{mix}} V_{\text{mix}} = n_{\text{mix}} R_{\text{u}} T_{\text{mix}}$$

$$\text{But } \rightarrow R_{\text{mix}} = R_{\text{u}}$$

$$\text{So } \rightarrow P_{\text{mix}} V_{\text{mix}} = n_{\text{mix}} R_{\text{u}} T_{\text{mix}}$$

↳ we want to know how to find  $P_{\text{mix}}$ ,  $V_{\text{mix}}$ ,  $T_{\text{mix}}$

## DALTON'S LAW FOR PARTIAL PRESSURES:

↳ The pressure of a gas mixture is equal to the sum of the pressures of each gas would exert if it exists alone at the mixture temperature and volume

A	+	B	→	Mixture
$T_{\text{f}}$		$T_{\text{f}}$		$T_{\text{f}}$
$P_A$		$P_B$		$P_{\text{mix}} = P_A + P_B$

↳ note that  $T$  &  $V$  MUST be the same for both (A) & (B) so that Dalton's law is applicable.

$$\frac{P_A}{P_{\text{mix}}} = \frac{n_A R_{\text{u}} T_{\text{mix}}}{n_{\text{mix}} R_{\text{u}} T_{\text{mix}}} = \frac{n_A}{n_{\text{mix}}} = x_A$$

So  $\rightarrow \frac{P_i}{P_{\text{mix}}} = x_i$  → you can see that pressure depends on the no. of moles which makes since

(~~जल्दी से से लाई सत्र की तरफ आया तो यह लाई की तरफ आया~~)  
(~~जल्दी से से लाई सत्र की तरफ आया तो यह लाई की तरफ आया~~)

∴ you can find partial pressures using  $x_i$

## AMAGAT'S LAW FOR PARTIAL VOLUMES

The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the same Temperature & Pressure.

A	+	B	=	Mixture
$T, P$		$T, P$		$T, P$
$V_A$		$V_B$		$V_{\text{mix}} = V_A + V_B$

$$\frac{V_A}{V_{\text{mix}}} = \frac{n_A R u T_{\text{mix}}}{P_{\text{mix}}} = \frac{n_A}{n_{\text{mix}}} = \chi_A$$

$$\frac{n_{\text{mix}} R u T_{\text{mix}}}{P_{\text{mix}}}$$

$$\text{so } \frac{V_i}{V_{\text{mix}}} = \chi_i \quad \text{so volume is also a function of moles.}$$

→ that's why we said that

Volume Basis is the same as mole Basis.

For non-ideal Gases:-

$$PV = Z_{\text{mix}} mRT$$

↳ compressibility Factor

We want to learn how to find  $Z_{\text{mix}}$ , But first we need to know how to find  $Z$  of one Gas

$Z$  is found using **FIGURE (A-15)**, to use it;

x-axis → Reduced Pressure.  $P_R$

y-axis →  $Z$

what is Reduced pressure ??

$$P_R = \frac{P}{P_{\text{cr}}} \quad (\text{the goal is to make the pressure dimensionless})$$

so if  $P_R = 5$  that means that the pressure of the gas is 5 times its critical pressure.

Note that for all gases there is one comp. chart because a certain mole of any gas will behave the same

\* Gas is considered ideal under very low pressures.

To find ( $Z_{mix}$ ) :-

We have two methods  $\rightarrow$  Kay Rule, Amagat's Rule, you can use Both.

### 1) AMAGAT'S RULE :-

1. Find  $y_i$

2. for each component ( $i$ ) find :

$$T_{Ri} = \frac{T_{mix}}{T_{ci}}$$
$$P_{Ri} = \frac{P_i}{P_{ci}}$$

$y_i$

↑ mole fraction

$$3. Z_{mix} = \sum (y_i z_i)$$

### 2) KAY RULE :-

1. Find  $y_i$

2. Pseudo Parameter  $\left[ \begin{array}{l} T_{cr}' = \sum y_i T_{ci} \\ P_{cr}' = \sum y_i P_{ci} \end{array} \right]$

$$3. T_R = \frac{T_{mix}}{T_{cr}'}$$

$$P_R = \frac{P_{mix}}{P_{cr}'}$$

This Rule treats a gas mixture as a pure substance with pseudo critical properties.

Problem (13 - 31)

Right Tank  $\rightarrow$   $H = \text{constant}$

$N_2$	$O_2$
2 kg	4 kg
25°C	25°C
520 kPa	150 kPa

If the final mixture  $T$  is 25°C, determine the volume of each tank and the final mix. pressure.

Solution:

→ note that  $H$  is not the same for both (you have to find  $n_B$  &  $n_A$ )

AFTER equil. you can apply Amagat's Rule.

$$n_2 = \frac{2}{28} = 0.0713$$

$$\text{for } H: \quad PH = nRT$$

$$H_{N_2} = \frac{n_{N_2} R_u T}{P_{N_2}} \downarrow \text{Kelvin}$$

$$PH = nR_u T$$

→ you can use any method

$$= 0.07143 \times 8.314 \times 298 \div 520 = 0.316$$

$$n_{O_2} = \frac{4}{32} = 0.125$$

$$H_{O_2} = 2.06412$$

After Equil, Temperatures & pressures are unified

$$H_{\text{mix}} = H_{O_2} + H_{N_2}$$

$$= 2.3861$$

$$P_{\text{mix}} = \frac{n_{\text{mix}} R_u T_{\text{mix}}}{H_{\text{mix}}} = 203.95$$

Problem (13-37)

Two Ideal Gases  $O_2$  &  $N_2$

$O_2$	
$0.3 \text{ m}^3$	
$200 \text{ K}$	
$8 \text{ MPa}$	
	$N_2 + O_2$
	$200 \text{ K}$
	$8 \text{ MPa}$
$N_2$	
$0.5 \text{ m}^3$	
$200 \text{ K}$	
$8 \text{ MPa}$	

calculate  $f$  using Ideal & non Ideal

$$n_{O_2} = \frac{8000 \times 0.3}{8.314 \times 200} = 1.443 \quad \left( \frac{PV}{RT} \right)$$

$$n_{N_2} = 2.406$$

case (I) - Ideal

$$V_{\text{mix}} = 0.3 + 0.5 = 0.8 \text{ m}^3 \quad (T, P \text{ are constants})$$

case (II) - non Ideal

using Amagat's Rule

$$x_{O_2} = 0.375 \quad y_{N_2} = 0.625$$

$$O_2 \rightarrow T_{R O_2} = \frac{200}{154.8} = 1.3$$

$$P_{R O_2} = \frac{8}{5.08} = 1.57$$

$$\left. \right\} z_{O_2} = 0.77$$

$$N_2 \rightarrow T_{R N_2} = \frac{200}{126.2} = 1.6$$

$$P_{R N_2} = \frac{8}{3.39} = 2.3$$

$$\left. \right\} z_{N_2} = 0.86$$

$$z_{\text{mix}} = \sum x_i z_i \\ = 0.826$$

$$V_{\text{mix}} = z_{\text{mix}} \frac{N_{\text{mix}} R_{\text{mix}}}{P_{\text{mix}}} = 0.601 \quad (\text{less than } 0.8)$$

### EXAMPLE (13-2)

rigid tank

Find  $T_{mix}$  Ideally & non-Ideally

Solution:

case I  $\rightarrow$  Ideal

$$n_{mix} = 2 + 6 = 8 \text{ kmol}$$

$$P_{mix} T_{mix} = n_{mix} R T_{mix}$$

$$V = 1.35 \text{ m}^3$$

$$\begin{bmatrix} 2 \text{ kmol } N_2 \\ 6 \text{ kmol } CO_2 \end{bmatrix}$$

$$300 \text{ K, } 15 \text{ MPa}$$

case II - Non Ideal

$i$	$n_i$	$x_i$	$T_{cr}$	$P_{cr}$	$T_{cr}'$	$P_{cr}'$
$N_2$	2	0.25	126.2	3.39	31.55	0.8475
$CO_2$	6	0.75	304.2	7.39	228.15	5.54
						$(259.7) \quad (6.39)$

using Kay's Rule

$$T_{Rmix} = \frac{T_{mix}}{T_{cr'mix}} = \frac{300}{259.7} = 1.15 \quad \left. \right\} \quad z_{mix} = 0.47$$

$$P_{Rmix} = \frac{P_{mix}}{P_{cr'mix}} = \frac{1500}{6390} = 2.35$$

$$V_{actual} = 0.47 \times 1.33 =$$

using Amagat's Rule

$T_{Ri} = T_{mix}/T_{cr}$	$P_{Ri}$	$z_i$
$300/126.2 = 2.38$	$15/3.39 = 4.42$	1.02
0.99	2.03	0.31

$$z_{mix} = \sum x_i z_i = 0.48$$

Problem (13-33)

60%  $\text{CH}_4$

25%  $\text{C}_3\text{H}_8$

15%  $\text{C}_4\text{H}_{10}$

By weight

calculate & Ideally & non-Ideally.

$i$	$m_i$	$m_i$	$M_i$	$n_i$	$y_i$	$T_{\text{cri}}$	$P_{\text{cri}}$	$Z_i$
$\text{CH}_4$	60	60	16	3.75	0.82			0.64
$\text{C}_3\text{H}_8$	25	25	44	0.5682	0.1241			0.95
$\text{C}_4\text{H}_{10}$	10	10	58	0.2582	0.0555			0.68
			$n_{\text{mix}} = 4.5768$			226.65		4.55

$$\text{If } T_{\text{R}} = 1.36 \quad \rightarrow \quad Z = 0.91$$

$$V = 3.93$$

# CHAPTER (14)

## GAS VAPOR MIXTURES & AIR CONDITIONING

→ Chapters 14 & 15 are applications on the previous chapters.

→ In this chapter we will be dealing with GAS - VAPOR MIXTURES

↳ NOT dry air

→ Atmospheric air consists of two parts:

ATM air = Dry air + water vapor

↳ until now all we dealt with was DRY AIR

→ in Air conditioning & Ref. processes the Temperature Range is as follows:

$$-10^\circ\text{C} \rightarrow C_p|_{10^\circ\text{C}} \approx 1.0037$$

$$+50^\circ\text{C} \rightarrow C_p|_{50^\circ\text{C}} \approx 1.0065$$

↳ we used to take  $C_p$  at  $20^\circ\text{C} \rightarrow C_p = 1.005$

if you calculate the error you'll get  $\rightarrow$  error  $\approx 0.24\%$  which is acceptable

so  $\rightarrow C_p$  for air could be taken as 1.005 in the Range  $(-10^\circ\text{C} - 50^\circ\text{C})$

∴ For DRY AIR:

$$\Delta h_{air} = C_p T = 1.005 \times T \rightarrow \text{IMPORTANT:}$$

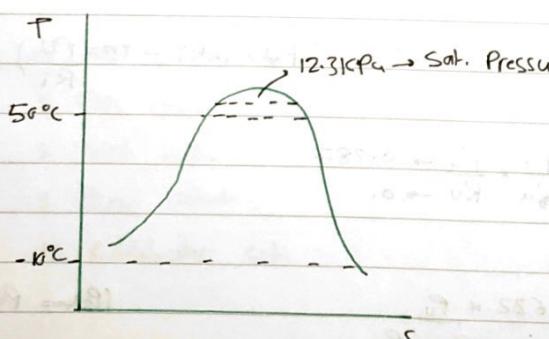
$$\Delta h_{air} = 1.005 \times \Delta T$$

(T) MUST BE IN  $^\circ\text{C}$

So  $\rightarrow$  Vapor is NOT Gas, But when can we say that vapor behaves as a gas?

look at Fig (3-45) p137

Vapor behaves as an ideal Gas at pressures lower than (10kPa)



when  $P < 10\text{ kPa}$ , we deal with Vapor as an ideal Gas, why?

① Pcr of water is  $22.06\text{ MPa}$

$$\text{if you find } P_r = \frac{10\text{ kPa}}{22.06\text{ MPa}} \approx 0.0005$$

now looking at the compressibility

chart  $\rightarrow$   $T$  in that Range is almost

(1) which means that it behaves as an ideal gas.

IF neither T or pressure is effecting  
(h) it means it's an ideal gas.

← ② the Enthalpy in that Range remains constant

again → if you look at steam tables, you'll notice that in the Range  $(-10^\circ\text{C} \rightarrow 50^\circ\text{C})$  variation in  $h$  does not exceed 0.2%. (only in that Range)  
so water vapor behaves as ideal gas, so we can say that:

$$h_v(T, \text{low } P) = h_g(T)$$

↑  
vapor      any T

$$(h_g(T) = 2500.9 + 1.82T) \quad \begin{array}{l} \text{you can also use this eqn} \\ \text{in } ^\circ\text{C} \end{array}$$

↓  
Entropy of Vapor  
at that Range ( $h_v = h_g$ )

(curve fitting) of Tables.

→ since vapor behaves as an ideal gas in that Range, if air is an ideal gas, if volume & T of the Room to be conditioned are constant you can apply Dalton's law

$$P_{\text{atm}} = P_{\text{dry air}} + P_{\text{vapor}}$$

summary → we proved that vapor behaves as an ideal gas in  $(-10^\circ\text{C} \rightarrow 50^\circ\text{C})$

which is the Range we usually deal with

to find  $h_{\text{vapor}}$  →  $C_p T$

$$h_{\text{vapor}} = h_g(T) \text{ either using Tables or the Equations.}$$

Dalton's law can be applied.

→ air conditioning processes depends on the amount of vapor that exists in air (Humidity), so we care to know that to find Humidity we have two ways:

→ HUMIDITY RATIO ( $w$ )

$$w = \frac{m_v}{m_a} \rightarrow \text{using Ideal gas law: } (P_{\text{atm}} = mRT \rightarrow m = \frac{P_{\text{atm}}}{RT})$$

$$w = \frac{\frac{P_v}{R_v}}{\frac{P_{\text{atm}}}{R_a}} = \frac{P_v}{P_{\text{atm}}} \cdot \frac{R_a}{R_v} = \frac{P_v}{P_{\text{atm}}} \cdot \frac{287}{460} \rightarrow 0.287$$

$$= 0.622 \cdot \frac{P_v}{P_{\text{atm}}} = 0.622 \cdot \frac{P_v}{P_{\text{atm}} - P_v} \quad (P_{\text{atm}} = P_a + P_v)$$

→ dry air has a humidity of zero, as vapor increases  $w$  will increase because  $P_v$  will increase, until it gets a point where air can't take any more vapor and becomes SATURATED

→ at that point  $P_v = P_g$

$$\text{so } \rightarrow w = 0.622 \frac{P_g}{P_a} \quad (\text{at saturation})$$

→ Any more moisture added after this point will condense.

To understand:

assume you have Air at  $25^\circ\text{C}$  &  $100\text{kPa}$

$$\text{from Tables} \rightarrow P_{\text{sat H}_2\text{O}}|_{25^\circ\text{C}} = 3.1698 \text{ kPa}$$

Now you can know the state of air according to the value of ( $P_v$ ) as follows:

$P_v = 0 \rightarrow \text{dry air}$

$P_v < 3.1698 \rightarrow \text{unsaturated air}$

$P_v = 3.1698 \rightarrow \text{Saturated air}$

## \* RELATIVE HUMIDITY:- ( $\phi$ )

→ It is the amount of moisture the air can hold (m<sub>u</sub>) Relative to the max.

amount of moisture the air can hold at the same Temperature (m<sub>g</sub>)

$$\phi = \frac{m_u}{m_g} = \frac{P_v \text{H}/R_v T}{P_g \text{H}/R_v T} = \frac{P_v}{P_g}$$

$$\therefore P_v = \phi P_g$$

→ Substitute in (w)

$$w = \frac{0.622 P_v}{P_{\text{atm}} - P_v} = \frac{0.622 \phi P_g}{P - \phi P_g}$$

at saturation  $\rightarrow P_v = P_g \Rightarrow \phi = 100\%$ .

→ we have (4) Temperatures we need to know to calculate w &  $\phi$  :-

1. Dry bulb
2. Wet bulb
3. Dew point
4. Adiabatic sat.

### \* Dew point Temperature (Tdp)

→ The temperature at which condensation begins when the air is cooled at constant pressure

$$Tdp = T_{sat} \Big|_{Pv}$$

you can find Tdp using the chart or using steam Tables.

### \* Dry bulb Temperature :- (Tdb)

Temp. is measured using a Thermometer, at the end of this Thermometer there exists a dry bulb, so the T measured using it is called a dry bulb temperature.

(it is on the x-axis of the chart)

### EXAMPLE (14-1)

Room  $\rightarrow$  5m  $\times$  5m  $\times$  3m, contains air at  $25^\circ\text{C}$  & 100kPa,  $\phi = 75\%$ .

Find  $\rightarrow$  w,  $P_v$ ,  $P_u$ ,  $h$ ,  $T_{dp}$ ,  $m_a$ ,  $m_v$

Solution:

$\rightarrow$  The Temp of the room is the dry bulb Temp., so

$$T_{db} = 25^\circ\text{C}$$

$\rightarrow$  first step you should do is to find:  $P_g$ ,  $h_g$  using steam tables

$$\text{at } T = 25^\circ\text{C} : P_g = 3.1698 \text{ kPa}$$

$$h_g = 2546.5$$

note: you can also find  $h_v$  using the equation:

$$h_g = h_v = 2500.9 + 1.82T = 2500.9 + 1.82(25) = 2546.4 \rightarrow \text{note that } h_g \text{ and } h_v \text{ are almost the same}$$

$$\rightarrow \phi = \frac{P_v}{P_g} \rightarrow P_v = \phi P_g = 0.75(3.1698) = 2.38 \text{ kPa}$$

$$\rightarrow P_{atm} = P_{atm} + P_v \rightarrow P_u = P_{atm} - P_v = 100 - 2.38 = 97.62 \text{ kPa}$$

$$\rightarrow w = 0.622 \frac{P_v}{P_u} = 0.622 \left( \frac{2.38}{97.62} \right) = 0.0152$$

$$\rightarrow h = C_p T + w h_v$$

$$= (1.005)(25) + (0.0152)(2546.5) = 63.8 \text{ kJ/kg dry air}$$

$$\rightarrow T_{dp} = T_{sat} @ P_v = T_{sat} \Big|_{2.38 \text{ kPa}} = 21^\circ\text{C} \quad (\text{L needs Interpolation})$$

$\rightarrow$  To find the Mass, use ideal gas law  $PV = mRT$

$$V = 5 \times 5 \times 3 = 75 \text{ m}^3 \quad T = 25^\circ\text{C} = 303 \text{ K}$$

$$m_v = \frac{P_v V}{R_u T} = \frac{(2.38)(75)}{(0.4615)(298)} = 1.3 \text{ kg}$$

$$m_a = \frac{P_u V}{R_u T} = \frac{(97.62)(75)}{(0.287)(298)} = 85.61$$

### EXAMPLE (14-2)

Consider a house contains air at  $20^\circ\text{C}$  & 75% relative humidity. At what window  $T$  will the moisture in the air start condensing on the inner surfaces of the windows.

Solution.

$$\rightarrow T = 20^\circ\text{C}, \phi = 75\%$$

→ condensation starts when  $T_{dp}$  is reached so you should find  $T_{dp}$ .

→ to find  $T_{dp}$  you need  $P_v$

$$\text{at } 20^\circ\text{C} : P_g = 2.3392 \text{ kPa}$$

$$P_v = \phi P_g = (0.75)(2.3392) = 1.754 \text{ kPa}$$

$$T_{dp} = T_{sat} \mid_{1.754}$$

$$= 15.4^\circ\text{C} \text{ (interpolation)}$$

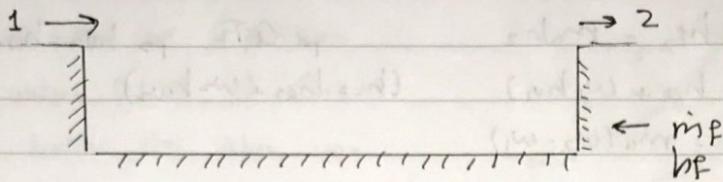
so Temp. of the inner surface must remain above  $15.4^\circ\text{C}$  to avoid condensation

(if the Temp. of the window is  $10^\circ\text{C}$  for example condensation will happen on the surface of the window).

\* you can find  $T_{dp}$  &  $h$  using the chart

## \* ADIABATIC SATURATION & WET BULB TEMPERATURES.

### 1. Adiabatic Saturation Temp.:



assume you have a long tunnel of water and you make a stream of air UNSATURATED air pass over it at  $(T_1, \phi_1, w_1)$  " $\phi_1 < 100\%$ .", since this air is unsaturated it will take moisture from water until it's humidity  $(\phi = 100\%)$  which means it'll give heat to water, hence it's temperature will decrease so at point 2:  $w_2 > w_1$   $\phi_2 = 100\%$ .

$$T_2 < T_1$$

Q → what is the relation between  $T_2$  &  $\phi$ ?

as the Humidity of air passing over water decreases,  $T_2$  will increase because less humidity means it is more capable of taking vapor to understand: if  $\phi = 100\%$  nothing will happen

if  $\phi = 90\%$  for example it can absorb 10% vapor out of water which means it's Temp. will decrease.

if  $\phi = 10\%$  it'll lose even more heat &  $T_2$  will be a lot less than  $T_1$ .

→ what we want to do is to find a relation between  $T_2$  &  $\phi$  or  $w$  :-

note: since a part of the water will vaporize the level of water in the tunnel

will decrease, so water will be added to compensate that loss ( $m_f$ )

By Mass Balance:

for dry air (it's mass will NOT change)

$$m_{a1} = m_{a2} = m_a$$

for vapor:

$$m_{v1} + m_f = m_{v2}$$

$$\therefore m_f = m_{v2} - m_{v1} \quad \text{But } m_v = w m_a \quad (\text{we want to relate } w)$$

$$\text{so } m_f = m_a (w_2 - w_1)$$

By Energy Balance:

(The process involves no heat or work interactions  $\therefore K.E. = P.E. \approx 0$ )

so:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}a h_1 + \dot{m}f h_{f2} = \dot{m}a h_2$$

$$\text{But } \rightarrow (h_1 = h_a + w_1 h_{v1})$$

$$(\dot{m}f = \dot{m}a (w_2 - w_1))$$

$$\xrightarrow{CPT_2} h_{v2} = h_{g2} \quad (\phi = 100\%)$$

$$(h_2 = h_{a2} + w_2 h_{v2})$$

so:

$$\dot{m}a (h_a + w_1 h_{v1}) + \dot{m}a (w_2 - w_1) h_{f2} = \dot{m}a (h_{a2} + w_2 h_{g2})$$

$$\therefore w_1 = C_p (T_2 - T_1) + w_2 h_{fg2} \rightarrow (h_{fg2} = h_{f2} - h_{g2})$$

$$h_{v1} - h_{f2}$$

$$w_2 = \frac{0.622 P_{v1}}{P_2 - P_{g2}}$$

$T_2$  is called adiabatic saturation Temperature & it is used as a method to find  $(w)$

### \* WET BULB TEMPERATURE

↳ It is a more practical approach to find

the humidity and more practical than

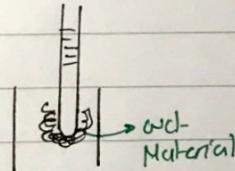
adiabatic sat. temp. (no need for a tunnel).

↳ The bulb is covered with a material (like cotton)

saturated with water) and air is passed over it

The Temp. measured in this case is called wet bulb Temp ( $T_{wb}$ )

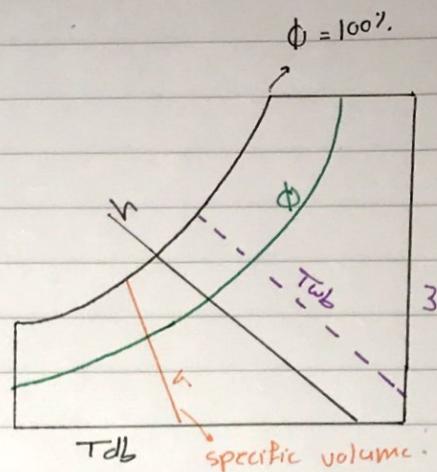
$\therefore T_2$  is  $T_{wb}$



See Example (14-3)

## Psychometric chart :-

↳ you can use this chart to find info. needed BUT it is used for  $P = 101.325 \text{ kPa}$  or a pressure close to value ( $\pm 2$ ), if  $P$  is above or below this value you have to calculate everything by hand as we did in previous examples.



↳ note: at  $\phi = 100^\circ \text{C}$   $T_{db} = T_{wb} = T_{dp}$

### Problem (14.39)

Atmospheric air at  $P = 1 \text{ atm}$  and dry bulb  $T$  of  $28^\circ \text{C}$  has a wet bulb temp of  $20^\circ \text{C}$ , using the chart find:

$\phi$ ,  $w$ ,  $h$ ,  $T_{dp}$ ,  $P_v$

Solution:

$P_{atm} = 1 \text{ atm}$ ,  $T_{db} = 28^\circ \text{C}$ ,  $T_{wb} = 20^\circ \text{C}$

using the chart:

$$w = 11.4$$

$$v = 0.868$$

$$T_{dp} = 16$$

$$h = 58 \text{ kJ/kg}_{\text{dry air}}$$

To practice:-

$$\begin{aligned} \text{1) } T_{db} &= 40^\circ \text{C} & h &= 101 & w &= 23.5 \\ \phi &= 50\% & T_{dp} &= 27.5^\circ \text{C} & v &= 0.922 \\ T_{wb} &= 30.25 \end{aligned}$$

$$\begin{aligned} \text{2) } w &= 12 & T_{wb} &= 23^\circ \text{C} & h &= 68.6 \\ \phi &= 30\% & T_{db} &= 37.2^\circ \text{C} & v &= 0.896 \\ T_{dp} &= 16.9^\circ \text{C} \end{aligned}$$

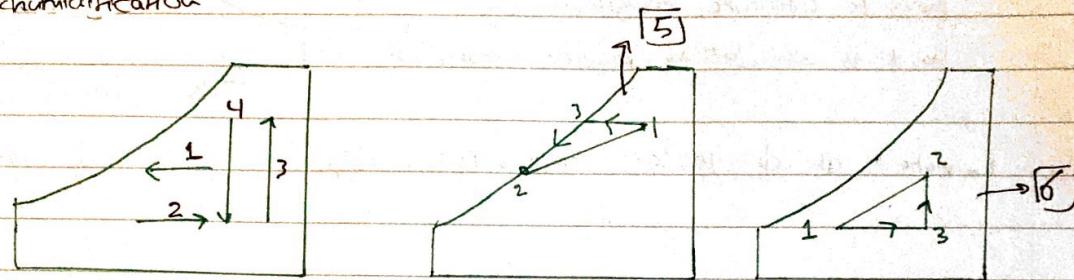
Try to solve example (14-1) using the chart, you should get the same numbers (approximately).

## AIR CONDITIONING PROCESSES :-

- air conditioning processes are used to maintain a certain space at a desired T & Humidity
- They are :

- 1 → simple cooling
- 2 → simple Heating
- 3 → Humidification
- 4 → Dehumidification

- 5 → Heating with Humidification
- 6 → cooling with dehumidification



### I - Simple (Sensible) Heating

$$w_1 = w_2 ; m_a = \text{Const.}$$

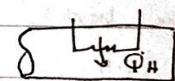
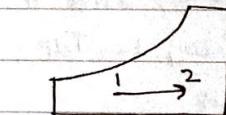
↳ Constant ( $\therefore m_{v1} = m_{v2}$ )

$\phi$  will decrease  $\rightarrow \phi_2 < \phi_1$

↳ Because  $P_{g2} > P_{g1}$

$$\dot{Q}_H = \text{min}(h_2 - h_1)$$

↳ per kg dry air



### II - simple

The process is considered as simple cooling if  $T_2 > T_{d, \text{op}}$

$$w = c, m_a = \text{Const.}, m_v = \text{Const.}, P_v = \text{Const}$$

$\phi_2 > \phi_1$  Because  $P_{g2} < P_{g1}$

$$\dot{Q}_C = \text{min}(h_1 - h_2)$$

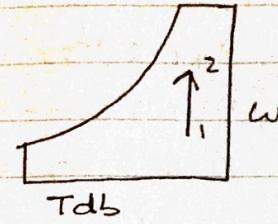
### III- Humidification (Adding Moisture)

$$T_{db} = \text{const.}; m_a = \text{const.}$$

$$w_2 > w_1$$

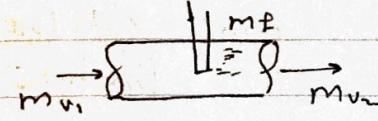
$$m_{v2} > m_{v1} \rightarrow P_{v2} > P_{v1}$$

$$\phi_2 > \phi_1, \quad \phi = \frac{P_v}{P_s} \rightarrow \uparrow$$



$$m_f = (m_{v2} - m_{v1}) * \frac{m_a}{m_a}$$

$$> m_a (w_2 - w_1)$$



### IV- Dehumidification (Removing Moisture)

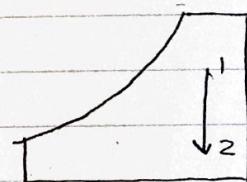
$$T_{db} = \text{const.}, m_a = \text{const.}$$

$$w_2 < w_1$$

$$\rightarrow m_{v2} < m_{v1} \rightarrow P_{v2} < P_{v1}$$

$$\phi_2 < \phi_1$$

$$m_f = m_a (w_1 - w_2)$$



### V- cooling with dehumidification

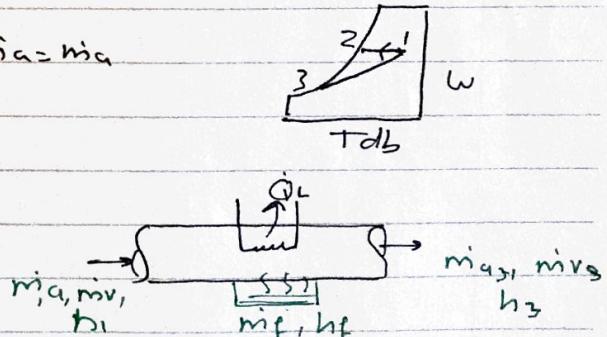
(1 → 2) - simple cooling

$$w_3 < w_2; \phi_3 = \phi_2 = 100\%; m_a = m_a$$

Energy Balance:

$$m_a h_1 = m_a h_3 + m_f h_f + \dot{Q}_L$$

$$\dot{Q}_L = m_a (h_1 - h_3) - m_f h_f$$



### VI Heating with Humidification

(1-2) simple Heating

$$\dot{Q}_H = m_a (h_2 - h_1)$$

$$w_3 > w_2; m_{v3} > m_{v2}$$

$$T_3 > T_1; P_{v3} > P_{v2}$$

(2-3) Humidification

$$m_{water} = m_a (w_3 - w_2)$$

## Cooling Tower

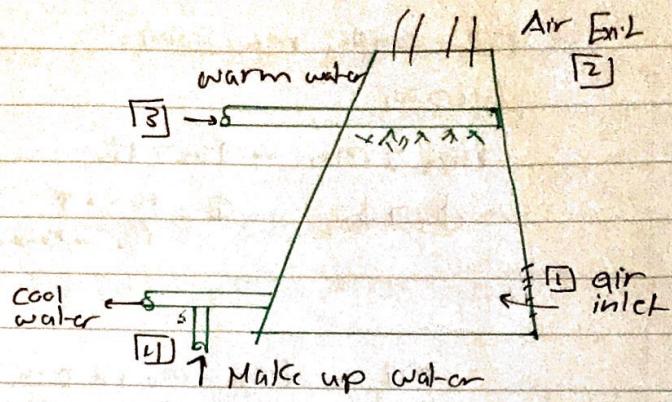
$$m_{F4} < m_{F3}$$

$$m_{ai} = m_{aq2} = m_a$$

$$m_{W1} + m_{F3} = m_{F11} + m_{W2}$$

;

$$m_a = \frac{m_{F3} (h_{F3} - h_{F4})}{(h_{F2} - h_{F1}) + (W_2 - W_1) h_{F4}}$$



problems:

(14.39)

Humidified using water

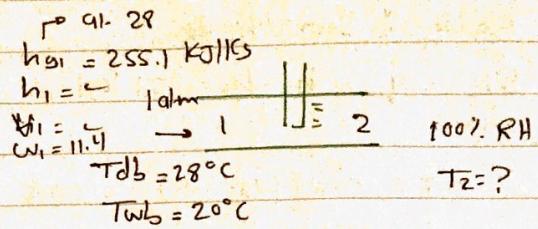
$$w_1 = \frac{1.005 (T_2 - T_1) + w_2 h_{fg2}}{h_{fg1} - h_{fg2}}$$

assume  $T_2 = 20^\circ\text{C}$

$$w_2 = 0.0147$$

$$h_{fg2} = 83.915$$

$$h_{fg1} = 2453.5$$



(14.62)

note that  $P < 100 \text{ kPa}$

↳ you can't use charts  
(manual)

$$\dot{Q}_{in} = m_a (h_2 - h_1) = 91.2 \text{ kJ/min} \quad 30\% \text{ RH}$$

$$m_a = \frac{\dot{V}}{V}$$

$$12^\circ\text{C} \rightarrow P_{a1} = 1.403$$

$$h_{g1} = 2522.9$$

$$P_{V1} = \Phi P_{g1} \rightarrow P_{V1} = 0.421 \text{ kPa}$$

$$P_a = 95 \text{ kPa} - 0.421 = 94.58 \text{ kPa}$$

$$w_1 = 0.622 \frac{P_{V1}}{P_{a1}} = 0.002768$$

$$h_1 = C_p T_1 + w_1 h_{fg1} = 19.04$$

$$w_2 = w_1 = 0.002768, \quad h_2 = C_p T_2 + w_2 h_{fg2} = 32.17$$

$$h_{fg2} = 2546.5$$

simplifying  
Heating

$$V_{a1} = \frac{R_a T_{a1}}{P_a} = 0.8648 \frac{\text{m}^3}{\text{kg}}$$

$$m_a = \frac{\dot{V}}{V} = \frac{6}{0.864} = 6.94 \rightarrow \text{find } \dot{Q}_{in}$$

(14.68)

### Heating & Humidification

$$\dot{Q}_{in} = q_{in} = h_2 - h_1 = 51107 \text{ kJ/kg.dry air}$$

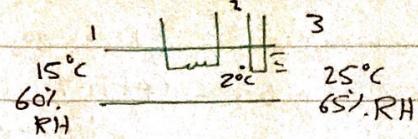
$$m_f = m_a (w_3 - w_2) = 0.0065$$

$$h_1 = 31.1$$

$$w_1 = w_2 = 0.0064$$

$$w_3 = 0.0129$$

$$h_3 = 58.1 \quad h_2 = 36.2$$



Sat. steam is

useful

(14.69) - Important

Properties at  $\frac{2}{3}$

$$m_a = \frac{V}{V}$$

$$m_f = m_a (w_3 - w_2)$$

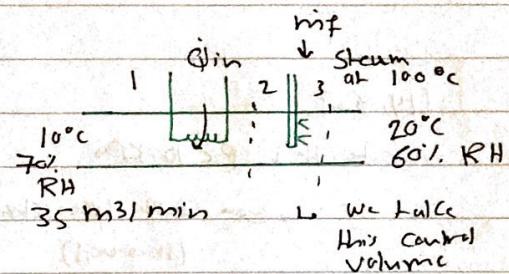
$$m_a h_2 + m_f h_f = m_a h_3$$

$$h_2 = 33.2$$

$$w_2 = w_1 = 0.0053$$

$$T_2 \text{ db} = 19.5 \quad \phi_2 = 37.8$$

$$m_f = 0.15 \text{ kg}$$



(14.72)

### Cooling & Dehumidification

Find the rates of Heat and Moisture Removal from air.

Solution:

$$m_a = \frac{V}{V} = 2.138 \text{ kg/min}$$

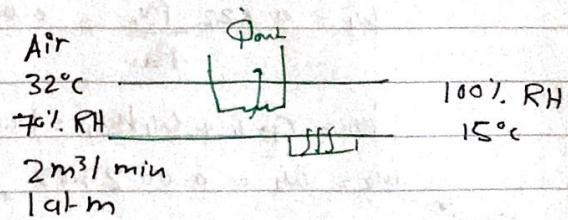
$$h_1 = 36.3$$

$$w_1 = 0.0211$$

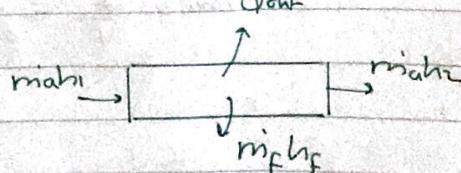
$$V_1 = 0.894$$

$$h_f = 62.982$$

$$h_2 = 42, \quad w_2 = 0.0167$$



Take control volume



$$\dot{Q}_{out} = m_a (h_1 - h_2) - m_f h_f = 97.7 \text{ kJ/min}$$

$$m_f = m_a (w_1 - w_2)$$

(14.9-8) cooling Tower

Find:

volume flow rate into the cooling tower and mass flow rate of the required make up water.

use charts  $\rightarrow$  atm

$$m_a = \frac{m_{\text{in}} (h_{f3} - h_{f1})}{(h_2 - h_1) - (w_2 - w_1) h_{f1}} \text{ kJ/s}$$

$$h_{f3} = 167.53$$

$$h_1 = 44.7$$

$$h_4 = 138.28$$

$$w_1 = 0.00875$$

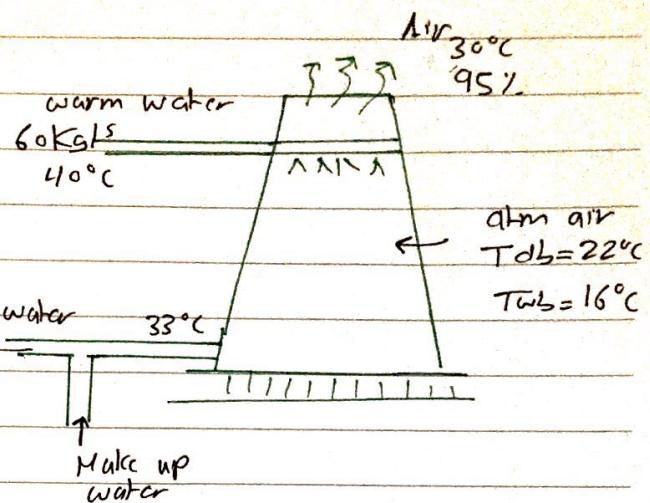
$$v_t = 0.848$$

$$h_2 = 96.13$$

$$w_2 = 0.02579$$

$$V_a = m_a V_a = 30.3 \text{ m}^3/\text{s}$$

$$m_{\text{makeup}} = m_{\text{in}} - m_{\text{out}} = m_a (w_2 - w_1) = 0.605 \text{ kg/s}$$



## CHAPTER (15)

### (CHEMICAL THERMODYNAMICS)

→ This chapter is also an application of the previous chapters

الطلاب، والذين يدرسونه، يجدونه ممتعًا، ويسهل فهمه.

#### \* FUELS:-

Fuels are materials that produce thermal energy when burned

الذين يحرقون، في موارد مختلفة، في مركبات مختلفة.

Fuels are of 3 Types:-

#### (1) liquids)

examples: Gasoline

Diesel (C<sub>12</sub>H<sub>26</sub>)

Octane (C<sub>8</sub>H<sub>18</sub>)

Alcohol (CH<sub>3</sub>OH)

Isooctane

Biodiesel

Also, Alcohol is considered a liquid fuel.

#### (2) Solids)

examples → coal, wood

#### (3) Gases)

examples → H<sub>2</sub>, CH<sub>4</sub>, C<sub>3</sub>H<sub>8</sub>

note: Gases are the best to use as fuels because they produce less amount of (CO<sub>2</sub>) than liquids BUT they have a problem when it comes to safety.

#### NOTE:-

##### \* For SI-Engines :- (Otto)

we care about a property of the fuel used called (Octane Number)

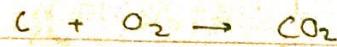
This number measures the fuel's ability to resist knocking (knocking is something we'll learn later but it's not good for engines.)

its Range is (0 → 100) and as it increases the Resistance increases.

##### \* For CI-Engines (Diesel) :-

we care about (Cetane number), it's the same concept as

Octane No. for SI but we want this number to be low not high unlike the Octane no.



Fuel + Air  $\rightarrow$  Products  $\rightarrow$  combustion process  
we call these "Reactants"

as you can see, the oxidizer used in combustion processes is AIR, But atmospheric air is not pure  $O_2$  as we know, it consists of ( $O_2, N_2, Ar, He, \dots$ ) we assume that air consists of:

21%  $O_2$   
79%  $N_2$   $\rightarrow$  Volume (mole) basis

so Air:  $(O_2 + 3.76 N_2)$

note  $\rightarrow$   
 $\frac{79}{21} = 3.76$

IMPORTANT  $\rightarrow$  in chemical Reactions, we care about Masses not Moles

example:



$\hookrightarrow$  what we want is to calculate the amount of thermal energy resulting from such a Reaction But first there are important terms we need to learn.

(\* AIR FUEL RATIO) :-

$\hookrightarrow$  it is the ratio of the mass of air to the mass of fuel for a combustion process

$$AF = \frac{m_{air}}{m_{fuel}} = \frac{\sum m_i M_i}{\sum m_i M_i}$$

\* STOICHIOMETRIC air fuel Ratio

it is also called: 100% Theoretical air

chemically correct air

$\hookrightarrow$  it is the minimum amount of air needed for the (complete combustion of a fuel)

and it is constant no. for

each type of fuel (it will be given)

it is a "theoretical" value.

for example :

for: Iso - Octane :-

its stoichiometric Ratio is 15.1

$$(\frac{A}{F})_s = 15.1 \text{ (from tables)}$$

what does that number mean?

it means that each 1kg of fuel will need 15.1 kg of air to achieve complete combustion.

EQUIVALENCE RATIO :- ( $\phi$ )

↳ The ratio of the actual fuel-air ratio to the stoichiometric fuel-air Ratio

$$\phi = \frac{(\frac{A}{F})_a}{(\frac{A}{F})_s}$$

There are three cases for  $\phi$  :-

[1]  $\phi = 1$  → This is the perfect case which we want to achieve.

[2]  $\phi > 1$  →  $\frac{(A/F)_a}{(A/F)_s} > 1$  → This is called

↳ To have  $\phi > 1$  it means that either the air is less than required or fuel is more than required for complete combustion

in this case it is called a (RICH MIXTURE)

[3]  $\phi < 1$  → This means that the amount of air is more than required or the amount of fuel is less than required and in both cases mol O<sub>2</sub> is used

in this case it is called a (LEAN MIXTURE)

summary :-

\* air fuel Ratio

$$\frac{m_f}{m_f} = \frac{(\sum m_i M_i)}{(\sum m_i M_i') a}$$

\* Stoichiometric Ratio (S)  $\rightarrow$  constant

100% Theoretical air

chemically correct

\* Equivalence Ratio  $\phi$

$$\phi = \frac{(A/F)s}{(A/F)a}$$

$\phi < 1 \rightarrow$  lean mixture

$\phi = 1 \rightarrow$  stoichiometric or perfect

$\phi > 1 \rightarrow$  rich mixture

- There are (3) conditions for a reaction to be stoichiometric:

1. perfect combustion.

2. all oxygen is consumed (no  $O_2$  in products).

3.  $C \rightarrow CO_2$   $S \rightarrow SO_2$   $H_2 \rightarrow H_2O$   $\rightarrow$  not  $CO$  or  $SO$ , the products must be in that form.

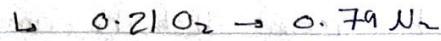
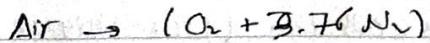
air in a Rich mixture  $\rightarrow$  "Deficient Air"

• less oxygen available

Air in a Lean mixture  $\rightarrow$  "Excess Air"

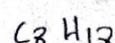
• more oxygen available

We agreed that air will be assumed to consist of 79%  $N_2$ , 21%  $O_2$  and since our calculations are per one mole of oxygen:



General Formula of Fuels  $\rightarrow [C_x H_y O_z S_l]$

examples:

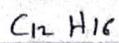


$$x = 8$$

$$y = 18$$

$$z = 0$$

$$l = 0$$

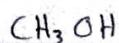


$$x = 2$$

$$y = 6$$

$$z = 0$$

$$l = 0$$



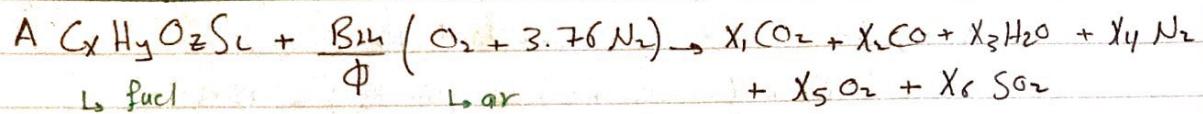
$$x = 1$$

$$y = 4$$

$$z = 1$$

$$l = 0$$

General Chemical Reaction is represented by the following chemical equation:



↳ Combustion process

↳ The products depends on the type of mixture (Stoich, lean or Rich).

A → No. of Moles

B<sub>2h</sub> → No. of Moles Required "Theoretically" to achieve full combustion.

X<sub>1</sub>, X<sub>2</sub>, L → no. of atoms

(CASE (I) - Stoichiometric Reaction  $\phi = 1$ )

→ we know that for a reaction to be stoichiometric, there must be no (O<sub>2</sub>, CO) in the products so:

$$X_2 = 0, X_5 = 0$$

→ so, we need to find the values of X<sub>1</sub>, X<sub>3</sub>, X<sub>4</sub>, X<sub>6</sub>, to do that we apply (Mass Balance) for each element, as follows:

$$C: (\text{mass})_{\text{in}} = (\text{mass})_{\text{out}}$$

$$M_{\text{carbon}} = 12$$

$$(nH)_m = (nH)_{\text{out}}$$

$$(A \times X) * 12 = X_1 * 12 \rightarrow (X_1 = A \times X)$$

$$H: (A \times Y) * 1 = X_3 * 2 * 1 \rightarrow (X_3 = \frac{AY}{2})$$

$$O: (A \times Z) * 16 + (\frac{B_{2h}}{2} * 2) * 16 = X_1 * 2 * 16 + X_3 * 1 * 16$$

$$\hookrightarrow AX \quad \hookrightarrow \frac{AY}{2}$$

$$2B_{2h} = 2AX + \frac{AY}{2} - AZ$$

$$(B_{2h} = A \left[ X + \frac{Y}{4} - \frac{Z}{2} + L \right])$$

→ as you can see, B<sub>2h</sub> depends only on the formula of the fuel (X<sub>1</sub>, Y, Z, L)

$$S: A \times L * 16 = X_6 * 16 \rightarrow (X_6 = AL)$$

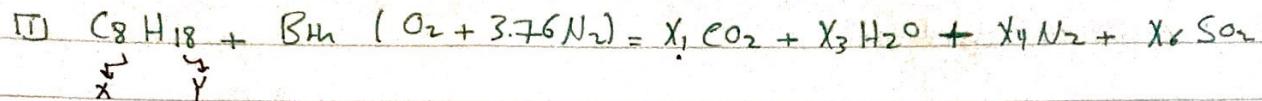
so you can find it by only knowing its type

$$N: 3.76 * B_{2h} * 2 * 14 = X_4 * 2 * 14$$

$$(X_4 = 3.76 B_{2h})$$

↳ you can see that no. of moles of N<sub>2</sub> is not changed, because it will not React.

Examples on Case I :-



$\downarrow x$        $\downarrow y$

$A=1, Z=0, L=0$ , you can find  $B_{Hn}$

$$B_{Hn} = \left( x + \frac{y}{4} - \frac{z}{2} + L \right) = \left( 8 + \frac{18}{4} \right) = 12.5$$

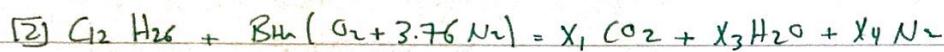
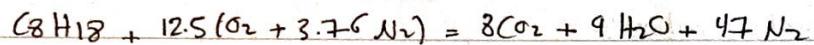
$$X_1 = A \cdot x = 1 \cdot 8 = 8$$

$$X_3 = A \cdot \frac{y}{2} = 1 \left( \frac{18}{2} \right) = 9$$

$$X_4 = 3.76 B_{Hn} = (3.76)(12.5) = 47$$

$$X_6 = 0$$

So, Reaction is:

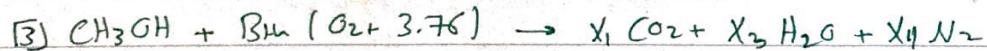
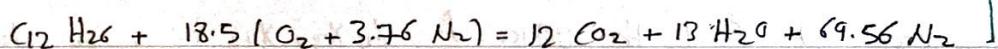


$$x = 12, y = 26, z = 0, L = 0, N = 1$$

$$B_{Hn} = \left( x + \frac{y}{4} - \frac{z}{2} + L \right) = 12 + \frac{26}{4} = 18.5$$

$$X_1 = Ax = 1(12) = 12, X_3 = A \cdot \frac{y}{2} = 1 \left( \frac{26}{2} \right) = 13, X_4 = 3.76 B_{Hn} = 69.56$$

So:



$$A=1, x=1, y=4, z=1$$

$$B_{Hn} = x + \frac{y}{4} - \frac{z}{2} + L = 1 + \frac{4}{4} - \frac{1}{2} = 1.5$$

$$X_1 = Ax = 1, X_3 = A \cdot \frac{y}{2} = 1 \left( \frac{4}{2} \right) = 2, X_4 = 3.76 B_{Hn} = 5.64$$

So:



To find  $\left( \frac{A}{S} \right)_s$ :

$$\left( \frac{A}{S} \right)_s = \left( \frac{m_a}{m_f} \right)_s = \left( \frac{\sum n_i M_i}{\sum n_f M_f} \right)_s = \frac{B_{Hn}}{\phi} \left( 32 * + 3.76 * 28 \right)$$

$$A * (x * 12 + y * 1 + z * 16 + L * 16) \quad \text{for stoichiometric } \phi = 1$$

for  $C_8H_{18}$ :

$$MW = 8 * 12 + 18 * 1 = 114$$

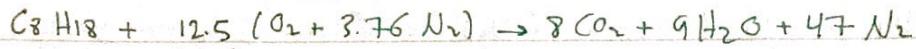
the fact

$$\left( \frac{A}{F} \right)_s = \frac{12.5 (32 + 3.76 * 28)}{114} = 15.05$$

for  $C_{12}H_{26}$ :

$$\left( \frac{A}{F} \right)_s = \frac{18.5 (32 + 3.76 * 28)}{12 * 12 + 26 * 1} = 14.9$$

To understand Case II & Case III :- assume you have the following Reaction



↳ This is Btu (Stoichiometric case)

That means 100% theo. air  $\& \phi = 1$

now assume you give it 130% th. air  $\rightarrow$  30% extra air

$$\% \text{ theo air} = \left( \frac{1}{\phi} + 100\% \right)$$

$$\hookrightarrow \phi = \frac{1}{1.3} < 1$$

extra air

lean  $\& O_2$  fine mols

$$\% \text{ excess air} = \left( \frac{1}{\phi} - 1 \right) * 100\%$$

$$\hookrightarrow \text{excess air} = \frac{1}{1.3} - 1 = 1.3 - 1 = \boxed{0.3} \rightarrow 30\% \text{ excess air}$$

air

so  $\rightarrow$  since  $O_2$  will be in the products it is not stoichiometric any more  
also:

If you give it 90% Dilute air

instead of having 125 it will have  $0.9 * 12.5$  which is less than the  
Required value  $\rightarrow$  Rich Mixture  
as a result  $(CO)$  will be found in the products.

so:

(CASE II :  $\phi < 1$ , Lean Mixture.)

$x_2$  is still zero (no  $CO$ )

(look at the general eqn)

But  $x_5 \neq 0$  Because there is  $O_2$ , to find  $x_5$

assume  $\rightarrow 120\% \text{ th. air}$

$$1.2 * 12.5 = 15$$

which is less than 15 also air is less

$$x_5 = 15 - 12.5 = 2.5$$

which is  $12.5 - 10.5 = 2$  less

$$\text{so } \rightarrow x_5 = \text{Bactual} - B\text{tho.}$$

increasing

also  $x_4$  (for  $N_2$ ) will be effected:

$$x_4 = 15 * 3.76 = 56.4$$

(CASE III :  $\phi > 1$ , Rich Mixture)

$x_5 = 0$ ,  $x_2 \neq 0 \rightarrow$  there is  $CO$

assume  $80\% \text{ th. air} \rightarrow \phi = \frac{1}{0.8} = 1.25$

$$0.8 * 12.5 = \frac{10}{\phi}$$

$$\text{Bactual} = \frac{B\text{tho}}{\phi} = \frac{12.5}{1.25} \rightarrow \text{so you are}$$

giving less  $O_2$

that what

the Reaction

needs!

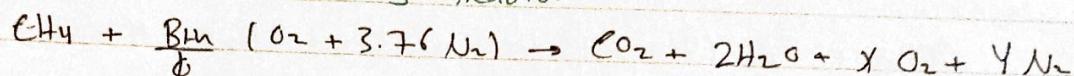
to find  $\& x_2$

$$\rightarrow x_1 CO_2 + x_2 CO + \dots$$

$\hookrightarrow$  you'll Balance the eqn again

Example:

assume you have the following reaction:



$$B_{th} = \frac{X + Y}{4} \leftarrow \frac{2}{2} + L = 1+1 = 2 \rightarrow B_{th} = 2$$

\* if you give 12% th. air, find X & Y :-

$$1.1 * 2 = 2.2$$

$$X = B_{actual} - B_{th} = 2.2 - 2 = 0.2$$

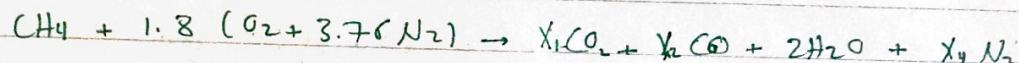
$$Y = (3.76)(2.2) = 8.272$$

\* find air-fuel Ratio :-

$$\frac{A}{F} = \frac{2.2(32 + 3.76 + 28)}{12 + 4} = 18.87$$

\* if you have 90% th air :-

$$\frac{B_{th}}{\phi} = 0.9 * 2 = 1.8$$



$$X_1 = 3.76 * 1.8 = 6.768$$

Mass Balance:

$$O: 1.8 * 2 * 16 = X_1 * 2 * 16 + X_2 * 1 * 16 + 2 * 1 * 16$$

$$2X_1 + X_2 = 1.6$$

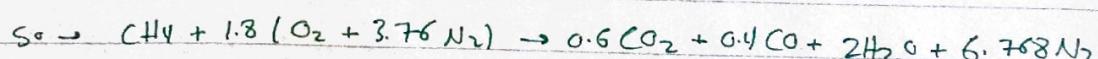
$$C: 1 * 1 * 12 = X_1 * 1 * 12 + X_2 * 1 * 12$$

$$1 = X_1 + X_2$$

$$.6 = 2X_1 + X_2$$

$$.6 = -X_1 \rightarrow X_1 = 0.6$$

$$X_1 + X_2 = 1 \rightarrow 0.6 + X_2 = 1 \rightarrow X_2 = 0.4$$

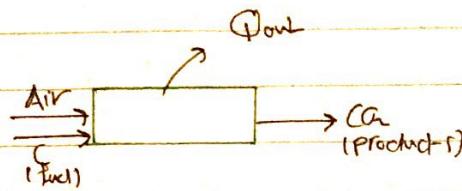


IMPORTANT:

See Example (14.5)

do self

## FIRST LAW ANALYSIS



- for the above chemical Reaction, we have to find  $Q_{out}$  and we know that  $Q_{out}$  is related to the change in Enthalpy
- in chemical Reactions, the change in Energy could happen from two things → change in state (at inlet & outlet) or the change in  $\Delta E_{sys} = \Delta E_{chem} + \Delta E_{state}$  like  $T, P$  the chemical composition

→ since the composition of the reactants differs from the products it is better to use a common reference state for all substances

standard ref. state  $\rightarrow 25^\circ\text{C}, 1 \text{ atm}$

→ property values at that state are denoted by  $(^\circ)$   $h, U, S$   
 $h^\circ \rightarrow$  Enthalpy at  $25^\circ\text{C}$  &  $1 \text{ atm}$ .

### ENTHALPY OF COMBUSTION:

enthalpy of combustion is: the amount of heat released during a steady flow combustion process when 1kmol (or 1kg) of fuel is burned at (a specified  $T \& P$ )

→ no change in state so  $\Delta E_{sys} = \Delta E_{chem}$

$$Q_{out} = \text{Enthalpy of comb.} = H_R - H_P = \sum_R h_i^\circ - \sum_P h_i^\circ$$

→ this is  $Q_{out}$  in case there is

no change in state

### ENTHALPY OF FORMATION: $\text{H}_f$

→ the Enthalpy of a substance at a specified state due to its chemical composition

at Ref. state  $\rightarrow \text{H}_f^\circ$ : this value can be found for each substance in Table (A-26)

To find the Enthalpy of a substance at any state:

$$\text{Enthalpy } h(T, P) = \bar{h}_f + \Delta h_T$$

intake mass specific enthalpy  $\downarrow \rightarrow (\bar{h} - \bar{h}_0)$   
outlet  $\downarrow \rightarrow$  Enthalpy at Ref state  
Enthalpy at that state

Note  $\rightarrow \bar{h}_f$  for stable elements ( $O_2, N_2, H_2$  and  $C$ ) is zero.

\* OPEN SYSTEMS:

In steady flow devices the following is used:

$$Q = \underbrace{H_p - H_{\text{Reactants}}}_{(Q_{\text{in}} - Q_{\text{out}})} \quad \text{assuming } W = 0$$

$$= \sum_i n_i (\bar{h}_f + \Delta \bar{h}_T) - \sum_i n_i (\bar{h}_f + \Delta \bar{h}_T)$$

If the process includes  $W_{\text{in}}$  or  $W_{\text{out}}$ :

$$Q - W = \sum_i n_i (\bar{h}_f + \Delta \bar{h}_T) - \sum_i n_i (\bar{h}_f + \Delta \bar{h}_T)$$

Note  $\rightarrow$  usually, for combustion in steady flow devices there is only  $Q_{\text{out}}$

So the eqn becomes  $\rightarrow Q_{\text{out}} = H_p - H_f$

### EXAMPLE (15-85)

Find Enthalpy of combustion of  $C_8H_{18}$

Solution:

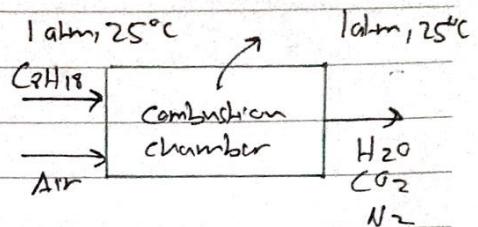
From Table (A-26):

$$CO_2: \bar{h}_f^\circ = -393.520 \text{ kJ/Kmol}$$

$$H_2O: \bar{h}_f^\circ = -285.83 \text{ kJ/Kmol}$$

$N_2$ : 0 (stable)

$$C_8H_{18}: -249.950 \text{ kJ/Kmol}$$



$$Q_{out} = H_{p, \text{out}} - H_{p, \text{in}} \quad \text{→ no change in } T$$

$$= \sum_r n_r (\bar{h}_f^\circ + \Delta H_f^\circ) - \sum_r n_r (\bar{h}_f^\circ + \Delta H_f^\circ)$$

$$= (8 \times -393.520) + 9 + (-285.83) + 0 - (1 \times -249.950)$$

$$= -5741000 \text{ kJ/Kmol fuel}$$

### IMPORTANT

↳ See Example (15-18)