

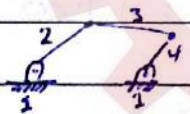
Machinery 33

• machine: A Device that transforms power and motion
It consists of mechanisms (linkages).

• mechanisms: A machine component that transmits motion.
It consists of Two or more bodies (links)
connected by joints (pairs).

Relative Motion: $\rightarrow$ $\leftarrow$ link goes link $\leftarrow$

• Link: A body that has at least two points (nodes) for attachment to other links.



machine $\rightarrow$ link $\rightarrow$ Ground link

• Based on the number of links connected a link may be classified as
Binary link $\leftarrow$

• Ternary link (3 point)



$\rightarrow$ $\leftarrow$

Quaternary (4 point)



• Based on the type of motion links may be classified as:
Ground of fixed (stationary)

Crank: $\rightarrow$ full Rotation

$\rightarrow$ pivoted to Ground

Crank

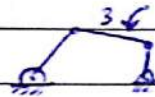


Rockers: $\rightarrow$ Oscillation

$\rightarrow$ pivoted to Ground



Coupler or connecting rod: 1 General plane motion



2 Connected to other moving links.

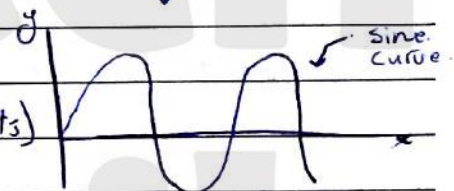
slider: 1 sliding motion (translation)



** Degree of Freedom (DOF)

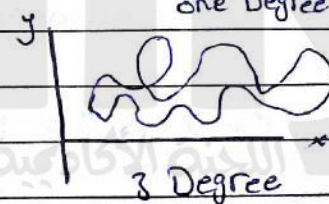
The number of independent coordinates (parameters) needs to describe the position of an object.

** Joints (Pairs, kinematic pairs, pairing elements)



Elements by which two or more links are joined together in such a way that motion is allowed.

one Degree.



- Joint classification: (see fig 2.3)

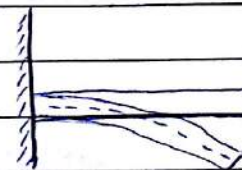
[I] Based on the Type of Contact

→ Lower pair: Joints with surface contact.

E.g. Revolute joint (R)

Prismatic (sliding) (P)

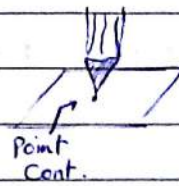
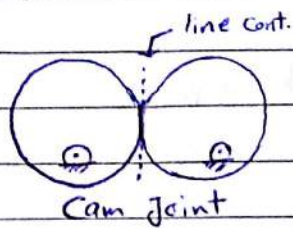
Rotation 3 Degree



→ Higher pair: Joints with point or line contact.

∞ Degree.

Ex



$\uparrow R$: Translation
 $\downarrow P$: Rotation

II Based on the number of (D.o.f) allowed by joint:

→ 1 (Dof) joint : R, P joint (Full joint)

→ 2 (Dof) joint : Roll slide joint (Half joint) Ex Cam. (RP)

III Based on the number of links joint: [Joint order]:

1st order joint : 2 links joined.

2nd order joint : 3 links joined.

kth order joint : (k+1) links joined.

IV Based on the type of physical closure

Form closed



Force closed



قوة
 مغلقة
 قفل

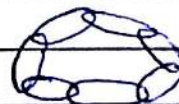
**** Kinematic Chain :** A system consisting of a number of links connected together by joints.

closed

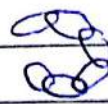
open

Two link ب
 Two link ب

one link ب
 one link ب



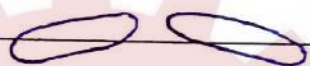
closed



open

No. _____

** Mobility (D.o.F) of mechanism...



6 D.o.F



4 D.o.F



2 D.o.F



Zero D.o.F ← structure



structure
Deflection



mechanism
Rotation

$$M = 3(L-1) - 2J_1 - J_2$$

L →

M : Mobility (D.o.F) of the mechanism.

L : number of link.

J_1 : No. of Joints with 1 d.o.f.

J_2 : No. of Joints with 2 d.o.f.

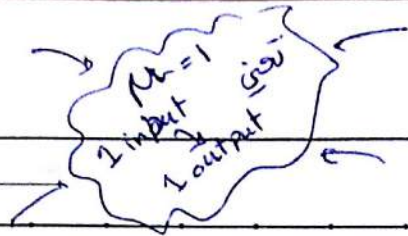
→ $M \geq 1$: mechanism (linkage)

$M \leq 0$: structure

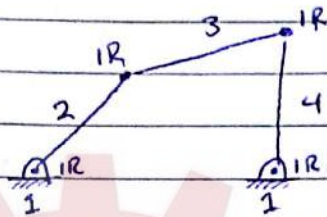
→ 0 stat. def.

→ -ve stat. Indef.

No.



E.x 4-Bar



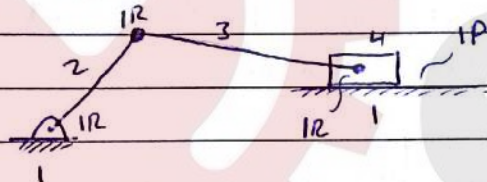
$$L = 4$$

$$J_1 = 4 (4R)$$

$$J_2 = 0$$

$$M = 3(4-1) - 2(4) - 0 = 1$$

E.x slider - crank



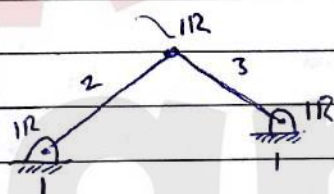
$$L = 4$$

$$J_1 = 4 (3R + 1P)$$

$$J_2 = 0$$

$$M = 1$$

E.x



$$L = 3$$

$$J_1 = 3$$

$$J_2 = 0$$

$$M = 3(3-1) - 2(3) - 0$$

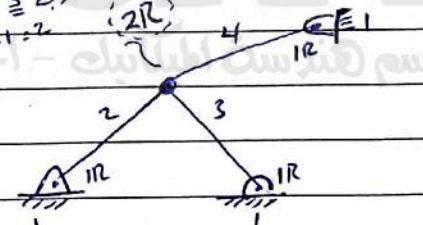
$$= 0 \rightarrow \text{structure.}$$

$$\rightarrow \text{statically det.}$$

$$\rightarrow \text{Truss}$$

E.x

Joint no.
link $\equiv$ 2R
 $3-1=2$



$$L = 4$$

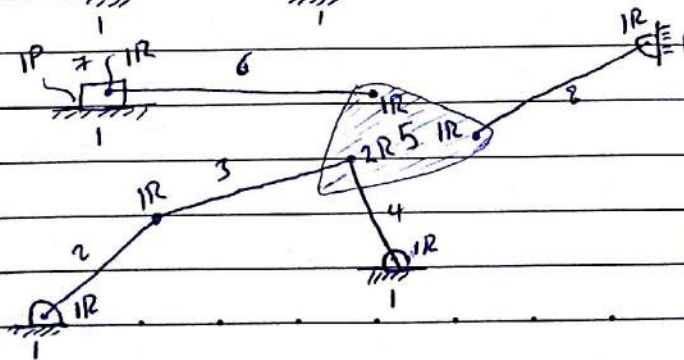
$$J_1 = 5 (5R)$$

$$J_2 = 0$$

$$M = -1$$

$$\text{stat. indet. structure.}$$

E.x



$$L = 8$$

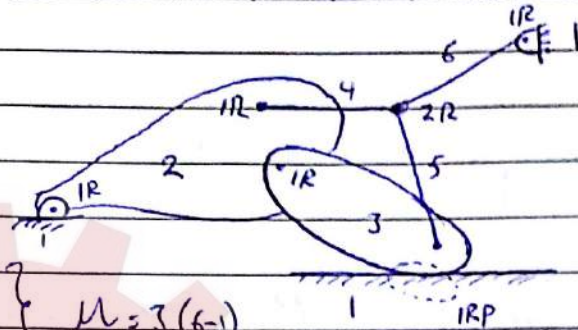
$$J_1 = 10 (9R + 1P)$$

$$J_2 = 0$$

$$M = 1$$

No. _____

Ex



link 1 is
spring
fixed link
link 6
link

$$L = 6$$

$$J_1 = 7 (7R)$$

$$J_2 = 1 (1RP)$$

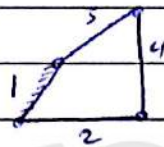
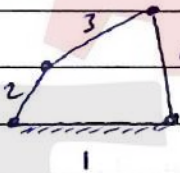
Cam
Joint

$$\mu = 3(6-1)$$

$$-2(7) - 1 = 0$$

** Inversion of a mechanism :

A new mechanism that results when the fixed link is allowed to move and another link is fixed.



4-Bar

Grashof

Non-Grashof

Self
Reading

L_1, L_2, L_3, L_4

S, L, P, Q

Grashof

$\Rightarrow$

Grashof

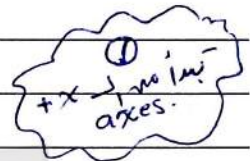
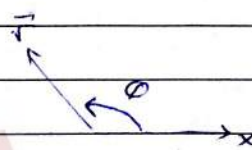
$$(S+L) \leq (P+Q)$$

** Position analysis of mechanism ---

Analytical
Approach
(Vector)

Numerical.

Graphical.



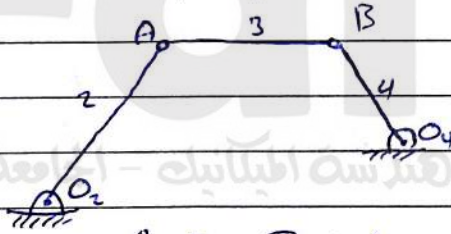
$$r_x = r \cos \theta$$

$$r_y = r \sin \theta$$

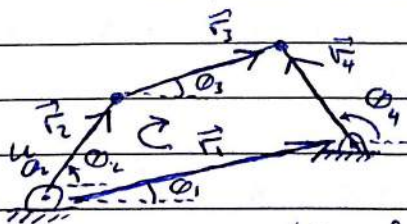
$$\vec{r} = r \cos \theta \hat{i} + r \sin \theta \hat{j} \rightarrow \vec{r} = r (\cos \theta \hat{i} + \sin \theta \hat{j})$$

$\hat{u}_\theta$

* Vector Loop Equation [loop closure Equ.] :-



loop : $O_2 \rightarrow A \rightarrow B \rightarrow O_4 \rightarrow O_2$



$$\vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_1 = \vec{0}$$

$$r_2 \vec{u}_{\theta_2} + r_3 \vec{u}_{\theta_3} - r_4 \vec{u}_{\theta_4} - r_1 \vec{u}_{\theta_1} = \vec{0}$$

$$r_2 (\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j}) + r_3 (\cos \theta_3 \hat{i} + \sin \theta_3 \hat{j}) - r_4 (\cos \theta_4 \hat{i} + \sin \theta_4 \hat{j}) - r_1 (\cos \theta_1 \hat{i} + \sin \theta_1 \hat{j}) = 0 \hat{i} + 0 \hat{j}$$

moving link
link two
link three
link four
link ground

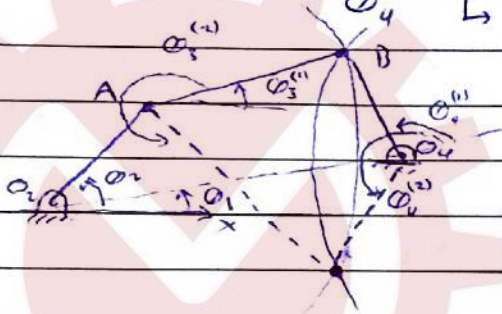
No. _____

x-Comp: $r_2 \cos \theta_2 + r_3 \cos \theta_3 - r_4 \cos \theta_4 - r_1 \cos \theta_1 = 0$

y-Comp: $r_2 \sin \theta_2 + r_3 \sin \theta_3 - r_4 \sin \theta_4 - r_1 \sin \theta_1 = 0$

4-Bar: Given: $\theta_1, r_1, r_2, r_3, r_4$ + Input: θ_2

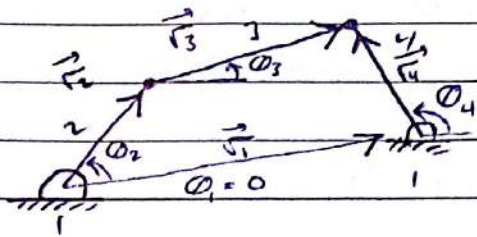
Find θ_3, θ_4
 Analytical
 Numerical
 Graph.



KK

ech
Family

الأجنحة الأكاديمية في قسم هندسة الميكانيك - الجامعة الأردنية



$\odot_5, \odot_4$
 مجاهيل
 ۱۰۰٪ اقل
 $\odot_5$
 ۱۰۰٪ بي
 $\odot_4$

$$\sqrt{2} \hat{U}_{02} + \sqrt{3} \hat{U}_{03} - \sqrt{4} \hat{U}_{04} - \sqrt{5} \hat{U}_0 = 0$$

→ y-comp. : $\sqrt{2} \sin \theta_2 + \sqrt{5} \sin \theta_3 - \sqrt{4} \sin \theta_4 = 0$

$$\rightarrow \vec{r}^2 = (\vec{r}_1 \hat{u}_0 - \vec{r}_2 \hat{u}_2 + \vec{r}_4 \hat{u}_4) \cdot (\vec{r}_1 \hat{u}_0 - \vec{r}_2 \hat{u}_2 + \vec{r}_4 \hat{u}_4)$$

$$= r_1^2 + r_2^2 + r_4^2 - 2r_1r_2 \cos \theta_2 + 2r_1r_4 \cos \theta_4 - 2r_2r_4 (\cos \theta_4 \cos \theta_2 + \sin \theta_4 \sin \theta_2)$$

$$\rightarrow A \cos \theta_4 + B \sin \theta_4 + C = 0$$

$$A = 4\sqrt{5} - 2\sqrt{2}\sqrt{5}\cos 60^\circ$$

$$R = -2\sqrt{2}\sqrt{y} \sin \theta_2$$

$$C = r_1^2 + r_2^2 - r_3^2 + r_4^2 - 2r_1 r_2 \cos \theta_2$$

$\cos \theta_4 = \frac{1 - \tan^2\left(\frac{\theta_4}{2}\right)}{1 + \tan^2\left(\frac{\theta_4}{2}\right)}, \quad \sin \theta_4 = \frac{2 \tan\left(\frac{\theta_4}{2}\right)}{1 + \tan^2\left(\frac{\theta_4}{2}\right)}$

$$\frac{0.4}{1} \rightarrow \text{let } x = \tan\left(\frac{0.4}{2}\right)$$

$$A \left(\frac{1-x^2}{1+x^2} \right) + \frac{2Bx}{1+x^2} + C = 0$$

المقام $(1+x^2)$
العدد 1
المقام $(1+x^2)$

$$(C-A)x^2 + (2B)x + (A+C) = 0$$

same of

$$ax^2 + bx + c = 0$$

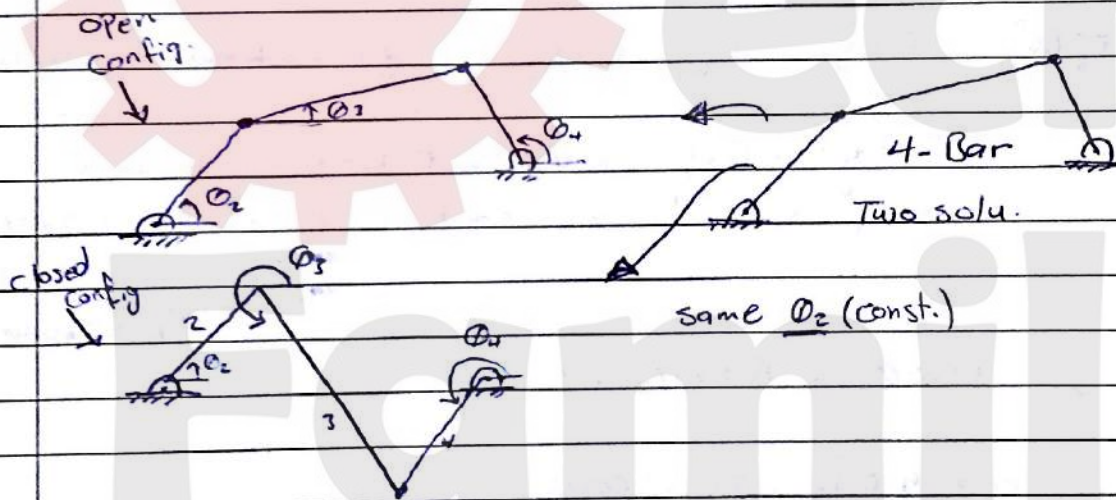
$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Two solution.

$$\tan \left[\frac{(\theta_4)_2}{2} \right] = \frac{-2B \pm \sqrt{(2B)^2 - 4(C-A)(C+A)}}{2(C-A)}$$

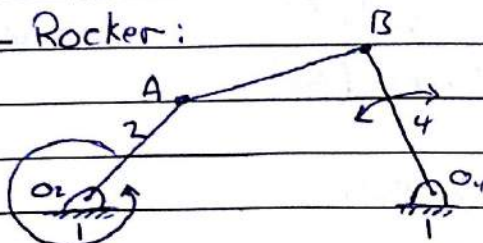
$$(\theta_4)_{1,2} = 2 \tan^{-1} x_{1,2}$$

Then find $(\theta_3)_{1,2}$ from the component equ.

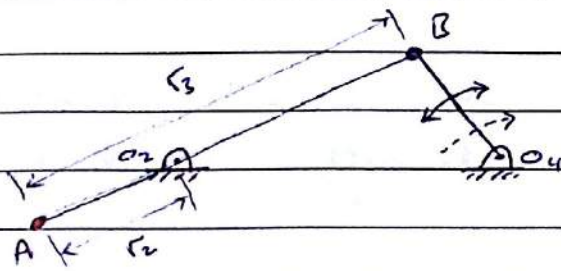


** Limiting position (toggle position):

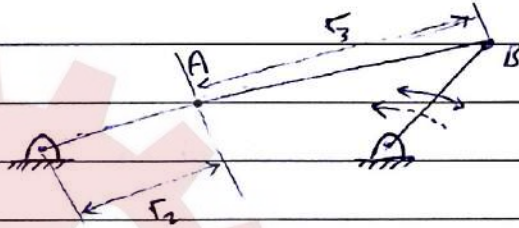
- Crank - Rocker:



Limit Pos. 1



Limit Pos. 2



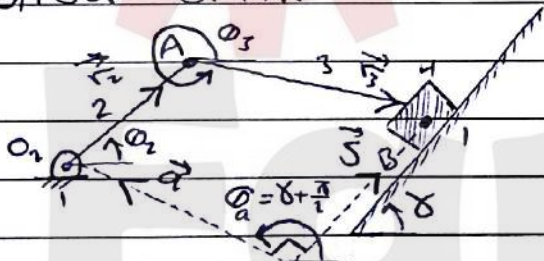
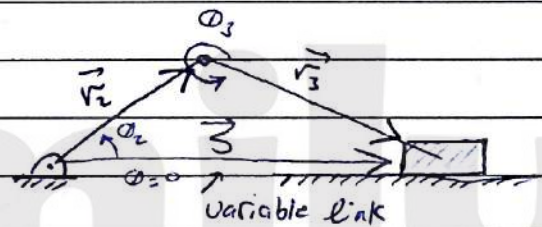
* H.W. #1 : Due Wed 8/10

4-6(a) + find θ_2 in limit position.4-7(a) + find θ_2 " " " "

Cover Page.

Course title
Sec. 2
H.W. #1
Date
Name
Seat No.
ID No.

** slider-crank :-

Given: r_2, r_3, δ input: θ_2 Find: θ_3, s 

$$\vec{r}_2 \hat{u}_{\theta_2} + \vec{r}_3 \hat{u}_{\theta_3} - s \hat{u}_s = 0$$

To eliminate θ_3 :

$$\vec{r}_3 \hat{u}_{\theta_3} = s \hat{u}_s - \vec{r}_2 \hat{u}_{\theta_2} \quad \left| \begin{array}{l} \text{dot each} \\ \text{side by it} \\ \text{self} \end{array} \right.$$

cosine law

$$r_3^2 = s^2 + r_2^2 - 2sr_2 \cos \theta_2$$

$$As^2 + Bs + C = 0$$

$$s_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$A=1, B=-2r_2 \cos \theta_2, C=r_2^2 - r_3^2$$

No. _____

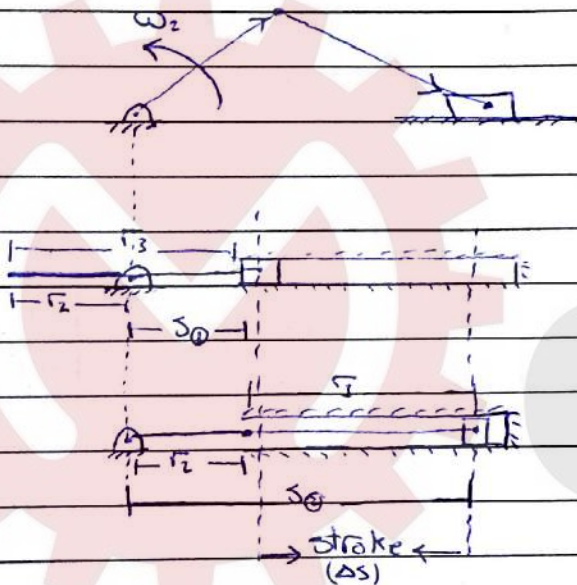
OR

x-comp. : $r_2 \cos \theta_2 + r_3 \cos \theta_3 - S = 0$

y-comp. : $r_2 \sin \theta_2 + r_3 \sin \theta_3 = 0 \rightarrow \text{find } \theta_3$

find S

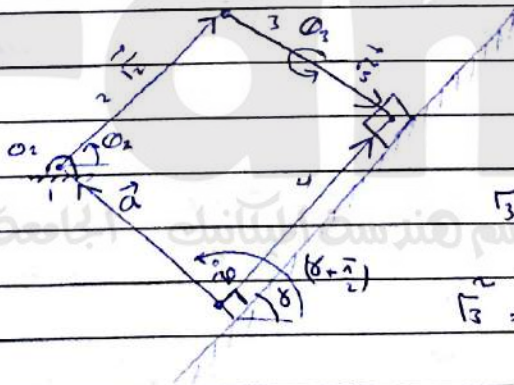
→ Limit position:



$$S_0 = r_2 - r_3$$

$$S_0 = r_2 + r_3$$

$$\Delta S = S_0 - S_0 = 2r_2$$



L.C.E:

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - S \hat{u}_{\theta} + a \hat{u}_{\theta+\pi} = 0$$

$$r_3 \hat{u}_{\theta_3} = S \hat{u}_{\theta} - a \hat{u}_{\theta+\pi} - r_2 \hat{u}_{\theta_2} \quad \text{Dot}$$

$$r_3^2 = S^2 + a^2 + r_2^2 - 2aS \cos(\theta_3 - \theta) + 2ar_2 \cos(\theta_2 - \theta + \pi)$$

$\sin(\theta_3 - \theta)$

$$AS^2 + BS + C = 0$$

$$A = 1$$

$$B = -2r_2 \cos(\theta_2 - \theta)$$

$$C = a^2 + r_2^2 - r_3^2 + 2ar_2 \sin(\theta_2 - \theta)$$

} →

$$S_{1,2} = \dots$$

go Back and find θ_3

No. _____

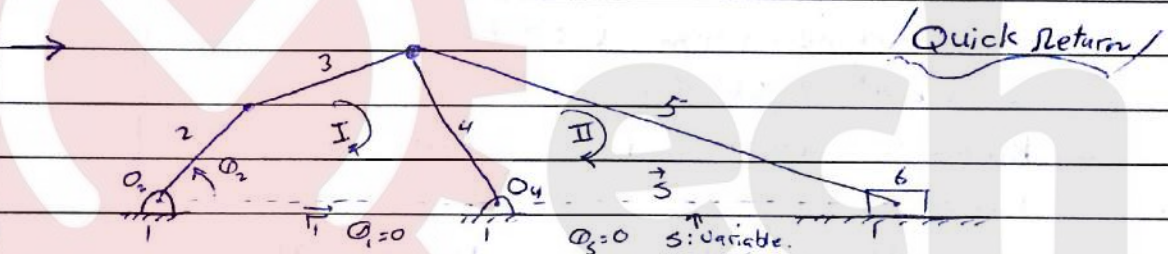
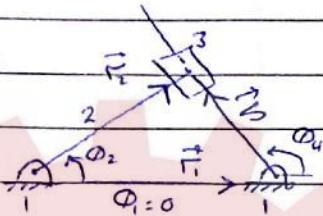
→ Inverted slider-crank (sliding-contact mech):-

L.C.E :

$$\sqrt{2} \hat{u}_x - 5 \hat{u}_y - 7 \hat{u}_z = \vec{0}$$

$$S\hat{U}_{04} = \sqrt{2}\hat{U}_{03} - \pi\hat{U}_0$$

$$S^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos \theta_2$$



Given : $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \phi_1, \phi_5$

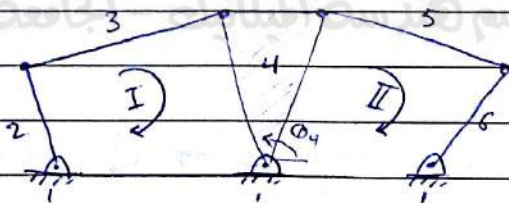
input : O_2

find : 5

$(0_3, 0_4, 0_5, 5)$ Two 3! loops.

5) Two loops.

Watt's six-bar mechanism:

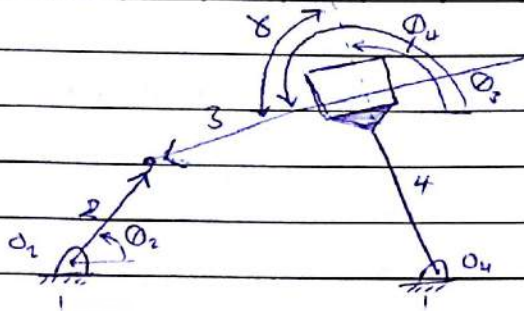


Give: O_2

find: $\odot_6$

??
*

No. _____



$\vec{r}_2, \vec{r}_3, \vec{r}_4$

Given: ω_2

find: S, θ_3, ω_3

$$\omega_3 = \omega_4 + \delta$$

** Project → Due Mon. 27/11/2014

- Choose a mechanism (≥ 5 bar's)
- Explain its Application.
- Kinematic Analysis (Pos., Velocity, acc.).
- Dynamic Analysis
- Animation. [Software, working model 2D].

** Velocity Analysis:-

$$\vec{U} = \frac{d\vec{r}}{dt}, \quad \frac{d\hat{i}}{dt} = 0 = \frac{d\hat{j}}{dt} = \frac{d\hat{k}}{dt}$$

$$\frac{d\vec{U}_0}{dt} = \frac{d}{dt} (\cos\theta \hat{i} + \sin\theta \hat{j})$$

$$\frac{d\vec{U}_0}{dt} = -(\sin\theta)\dot{\theta}\hat{i} + (\cos\theta)\dot{\theta}\hat{j}$$

$$\vec{U}_0 = \dot{\theta} (-\sin\theta \hat{i} + \cos\theta \hat{j})$$

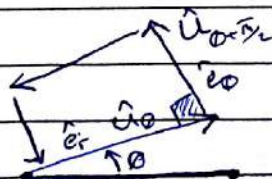
Note

$$\frac{d(\cos\theta)}{dt} = \frac{d(\cos\theta)}{d\theta} \frac{d\theta}{dt} = -\sin\theta \dot{\theta}$$

$$\frac{d(\sin\theta)}{dt} = \frac{d(\sin\theta)}{d\theta} \frac{d\theta}{dt} = \cos\theta \dot{\theta}$$

$$\vec{U}_{\theta+\pi/2} = \cos(\theta+\pi/2)\hat{i} + \sin(\theta+\pi/2)\hat{j}$$

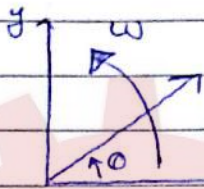
$$= -\sin\theta \hat{i} + \cos\theta \hat{j}$$



$$d\vec{r} = \vec{r} d\theta + (\vec{r} \times \vec{\omega}) \times d\vec{r}$$

$$\vec{r} = r \hat{u}_\theta \rightarrow \frac{d\vec{r}}{dt} = \frac{dr}{dt} \hat{u}_\theta + r \frac{d\hat{u}_\theta}{dt}$$

$$= \dot{r} \hat{u}_\theta + \cancel{r \hat{u}_\theta} + r \dot{\theta} \hat{u}_{\theta+\frac{\pi}{2}}$$



(ω) is a vector

$$\vec{\omega} = \begin{matrix} \text{ccw} \\ \oplus \\ \odot \\ \text{cw} \end{matrix} \hat{k}$$

* 4-Bar :-

Given: $r_1, r_2, r_3, r_4, \theta_1$

input: θ_2, ω_2

↓
solve: θ_3, θ_4

Find $\vec{\omega}_3, \vec{\omega}_4$ ← السرعة الزاوية

دالة بنية
في المفاصل
المعروفة
المكانة
Position

$$\text{L.C.E} : r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = 0$$

$$\text{V.E} : \frac{d}{dt} (\text{L.C.E})$$

$$\rightarrow (\dot{r}_2 \hat{u}_{\theta_2} + r_2 \omega_2 \hat{u}_{\theta_2+\frac{\pi}{2}}) + (\dot{r}_3 \hat{u}_{\theta_3} + r_3 \omega_3 \hat{u}_{\theta_3+\frac{\pi}{2}})$$

$$- (\dot{r}_4 \hat{u}_{\theta_4} + r_4 \omega_4 \hat{u}_{\theta_4+\frac{\pi}{2}}) - (\dot{r}_1 \hat{u}_{\theta_1} + r_1 \omega_1 \hat{u}_{\theta_1+\frac{\pi}{2}}) = 0$$

$$\rightarrow r_2 \omega_2 \hat{u}_{\theta_2+\frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta_3+\frac{\pi}{2}} - r_4 \omega_4 \hat{u}_{\theta_4+\frac{\pi}{2}} = 0$$

Sol 1: X-Vel: $-r_2 \omega_2 \sin \theta_2 - r_3 \omega_3 \sin \theta_3 + r_4 \omega_4 \sin \theta_4 = 0 \dots (1)$

Y-Vel: $r_2 \omega_2 \cos \theta_2 + r_3 \omega_3 \cos \theta_3 - r_4 \omega_4 \cos \theta_4 = 0 \dots (2)$

$\Rightarrow$ find ω_3, ω_4

Sol 2: To eliminate ω_3 :

dot $\cdot \hat{u}_{\theta_3}$

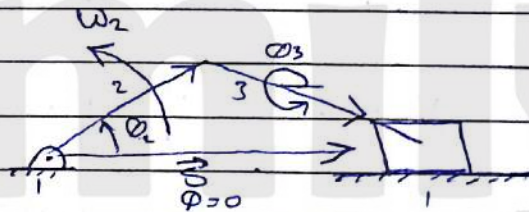
$$\begin{aligned} & r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} - r_4 \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} = 0 \\ & r_2 \omega_2 \sin(\theta_3 - \theta_2) - r_4 \omega_4 \sin(\theta_3 - \theta_4) = 0 \end{aligned} \quad \left\{ \begin{aligned} & \hat{u}_{\theta_2 + \frac{\pi}{2}} \cdot \hat{u}_{\theta_3} \\ & (-\sin \theta_2 \hat{i} + \cos \theta_2 \hat{j}) \cdot (\cos \theta_3 \hat{i} + \sin \theta_3 \hat{j}) \\ & = -\sin \theta_2 \cos \theta_3 + \sin \theta_3 \cos \theta_2 \\ & = \sin \theta_3 \cos \theta_2 - \cos \theta_3 \sin \theta_2 \\ & = \sin(\theta_3 - \theta_2) \end{aligned} \right.$$

$$\Rightarrow \omega_4 = \left[\frac{r_2 \sin(\theta_3 - \theta_2)}{r_4 \sin(\theta_3 - \theta_4)} \right] \omega_2$$

$\rightarrow$ then find $\omega_3 \checkmark$

* slider-crank:-

find $\dot{s}, \omega_3$



L.C.E :-

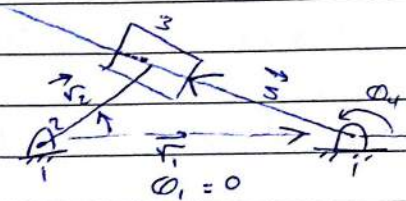
$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - s \hat{u}_0 = 0$$

V.E :- $r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} - \dot{s} \hat{u}_0 = 0 \quad (\cdot \hat{u}_{\theta_3} \text{ to find } \dot{s})$

$$\rightarrow \dot{s} = \frac{r_2 \omega_2 \sin(\theta_3 - \theta_2)}{\cos \theta_3}$$

L.C.E:

$$r_2 \hat{u}_{\theta_2} - s \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = 0$$

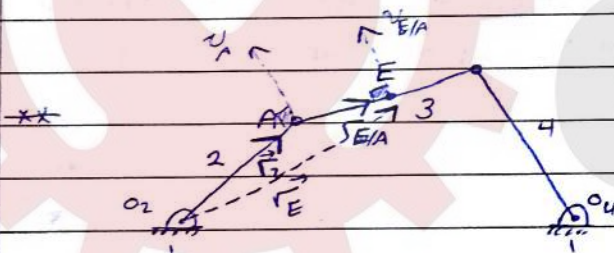


V.E:

$$r_2 \omega_2 \hat{u}_{\theta_2 + \pi/2} - s \hat{u}_{\theta_4} - s \omega_4 \hat{u}_{\theta_4 + \pi/2} = 0 \quad (\cdot \hat{u}_{\theta_4})$$

$$r_2 \omega_2 \sin(\theta_4 - \theta_2) = \dot{s}$$

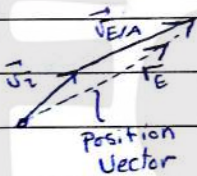
Self Reading :- $\rightarrow$ Angular Velocity Ratio (m_v)
 $\rightarrow$ Torque Ratio (m_T)
 $\rightarrow$ Mechanical Advantage (m_A)

Find $\vec{v}_E$?

$$\vec{v}_E = \vec{v}_A + \vec{v}_{E/A}$$

$$|\vec{v}_A| = \omega_2 r_2$$

$$|\vec{v}_{E/A}| = \omega_3 (\bar{AE})$$



$$\vec{r}_E = \vec{r}_2 + \vec{r}_{E/A} = r_2 \hat{u}_{\theta_2} + (\bar{AE}) \hat{u}_{\theta_3}$$

$$\vec{v}_E = \frac{d\vec{r}_2}{dt} + \frac{d\vec{r}_{E/A}}{dt}$$

$$= r_2 \omega_2 \hat{u}_{\theta_2 + \pi/2} + (\bar{AE}) \omega_3 \hat{u}_{\theta_3 + \pi/2}$$

* Instantaneous Center of Rotation :-

$\rightarrow$ A point common to two links and has the same instant velocity in each link.

$\rightarrow$ some (I.C's) can be found by inspection (ex pin joints).

other can be found by Kennedy's Rule).

→ No. of instant No I.C's :-

$$N_I = \frac{L(L-1)}{2} \quad ; \quad L: \text{No. of link's}$$

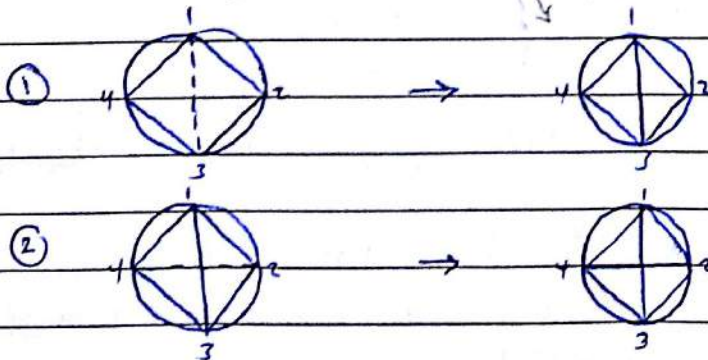
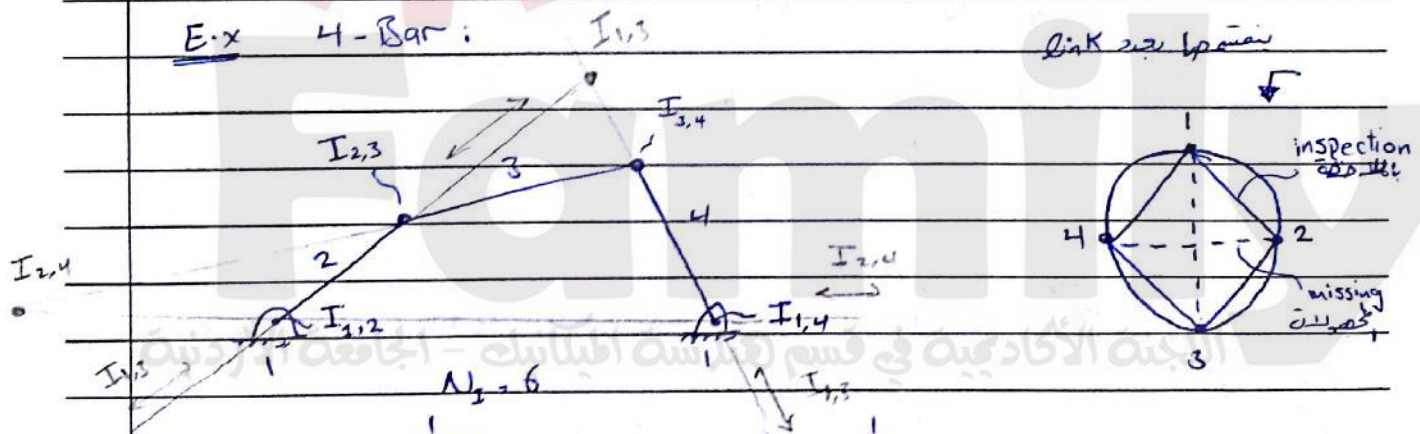
Ex 4-Bar : $N_I = \frac{4(4-1)}{2} = 6$

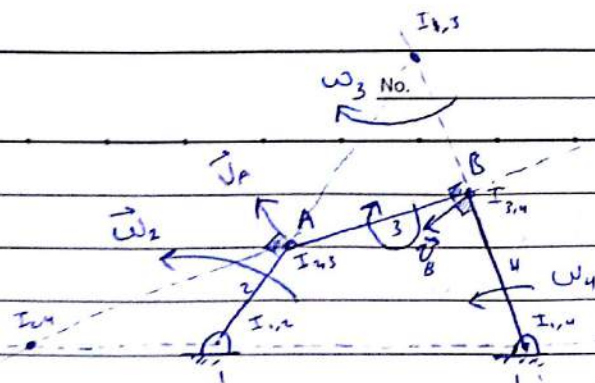
→ Kennedy's Rule :-

→ Link

Any three bodies in plane motion will have exactly three instant centers, and they will lie on the same straight line.

Ex 4-Bar :





Given: Pos. + input $\vec{\omega}_2$
find $\vec{\omega}_3, \vec{\omega}_4$

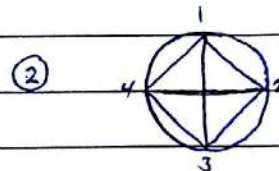
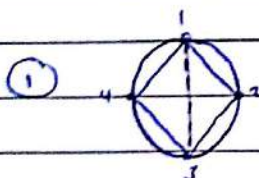
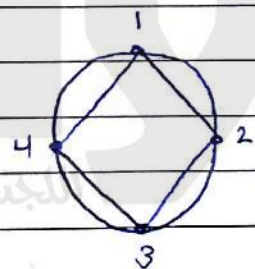
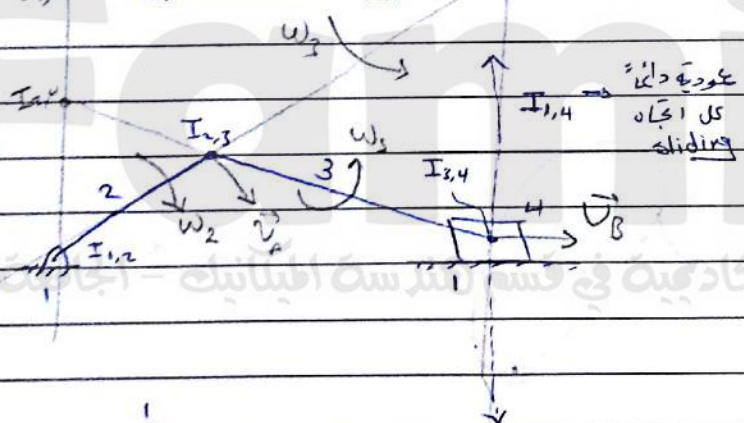
1) $V_A = \omega_2 r_2$, dir: $\perp$ Link 2

2) $V_A = \omega_3 (\overline{AI_{1,3}}) \rightarrow \omega_3 = \frac{V_A}{\overline{AI_{1,3}}}$; dir: cw

3) $V_B = \omega_3 (\overline{I_{1,3}B})$, dir $\perp$ Link 4

4) $V_B = \omega_4 r_4 \rightarrow \omega_4 = \frac{V_B}{r_4}$; dir: ccw

$V_B = r_4 \omega_4 \rightarrow \omega_4 = \frac{V_B}{r_4}$



Given: Pos + ω_2 , find $\omega_3, S = V_B = V_4$

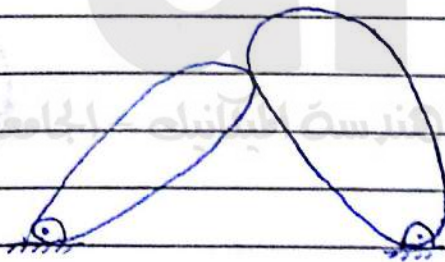
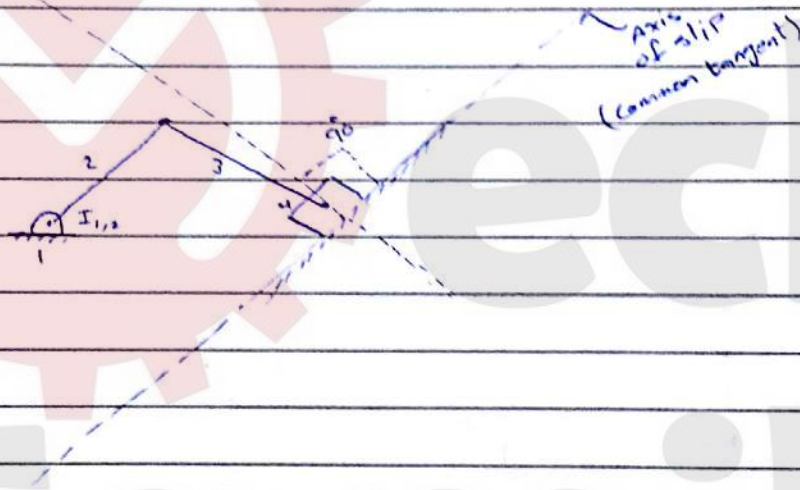
$$1) V_A = \omega_2 r_2, \text{ dir } \perp \text{ Link } 2$$

$$2) \omega_3 = \frac{V_A}{AI_{1,3}}, \text{ dir: CCW}$$

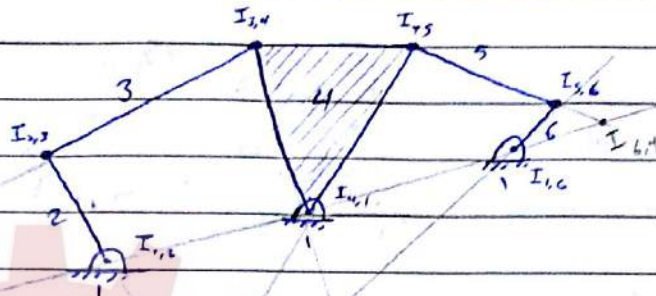
$$3) V_B = \omega_3 (\overline{BI_{1,3}})$$

→ H.W #2 Due Mon 27/10 ; 6-18 (b.c) , 7-15 (b)
suggested 6-13, 21 (b) , 44, 61

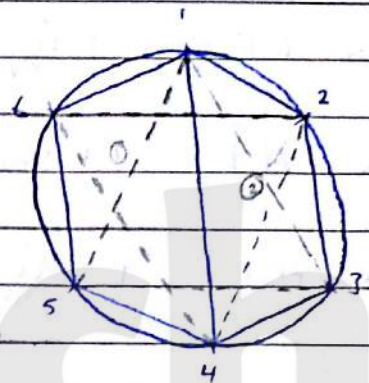
Axis of
Transmission
(Common normal)



No. _____



$$N_I = \frac{6(6-1)}{2} = 15$$



** Acceleration Analysis :-

$$\vec{r} = r \hat{u}_\theta$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{r} \hat{u}_\theta + r \dot{\theta} \hat{u}_{\theta+\frac{\pi}{2}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\ddot{r} \hat{u}_\theta + \dot{r} \dot{\theta} \hat{u}_{\theta+\frac{\pi}{2}} \right) + \dot{r} \dot{\theta} \hat{u}_{\theta+\frac{\pi}{2}} + r \ddot{\theta} \hat{u}_{\theta+\frac{\pi}{2}} + r \dot{\theta} \dot{\theta} \hat{u}_{\theta+\frac{\pi}{2}}$$

α : angular acc. = rad/s^2

$$\vec{a} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{\text{Radial acc.}} \hat{u}_\theta + \underbrace{(\dot{r}\dot{\theta} + 2\dot{r}\dot{\theta})}_{\text{Trans. acc.}} \hat{u}_{\theta+\frac{\pi}{2}}$$

a_{slip} a_{normal} a_{tang} a_{Coriolis}

Note

$$\hat{u}_\theta = \cos\theta \hat{i} + \sin\theta \hat{j}$$

$$\hat{u}_{\theta+\frac{\pi}{2}} = -\sin\theta \hat{i} + \cos\theta \hat{j}$$

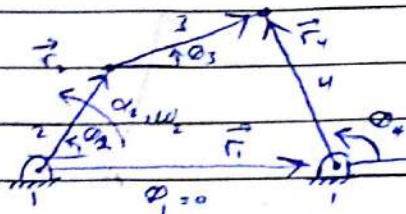
$$\dot{\hat{u}}_{\theta+\frac{\pi}{2}} = -\dot{\theta} \cos\theta \hat{i} - \dot{\theta} \sin\theta \hat{j}$$

$$= -\dot{\theta} \hat{u}_\theta$$

4-Bar

Given: Position, Velocity

Find: α_2, α_4 input α_2



→ L.C.E: $r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = 0$

→ V.E: $r_2 \omega_2 \hat{u}_{\theta_2 + \pi/2} + r_3 \omega_3 \hat{u}_{\theta_3 + \pi/2} - r_4 \omega_4 \hat{u}_{\theta_4 + \pi/2} = 0$

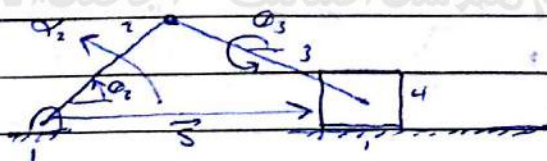
→ A.E: $r_2 \alpha_2 \hat{u}_{\theta_2 + \pi/2} - r_2 \omega_2^2 \hat{u}_{\theta_2} + r_3 \alpha_3 \hat{u}_{\theta_3 + \pi/2} - r_3 \omega_3^2 \hat{u}_{\theta_3} - r_4 \alpha_4 \hat{u}_{\theta_4 + \pi/2} + r_4 \omega_4^2 \hat{u}_{\theta_4} = 0$

✓ Sol.1 → x comp. → y comp.

✓ Sol.2 To eliminate α_3 :- we dot $\cdot \hat{u}_{\theta_3}$

$$r_2 \alpha_2 \sin(\theta_3 - \theta_2) - r_2 \omega_2^2 \cos(\theta_3 - \theta_2) - r_3 \omega_3^2 - r_4 \alpha_4 \sin(\theta_3 - \theta_4) + r_4 \omega_4^2 \cos(\theta_3 - \theta_4) = 0 \quad \text{then find } \alpha_3$$

slider-crank



Given: Pos. + Vel. - input α_2

Find: $\alpha_3, \ddot{s}(\alpha_4)$

→ L.C.E: $r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - s \hat{u}_{\theta_0} = \vec{0}$

→ V.E: $r_2 \omega_2 \hat{u}_{\theta_2 + \pi/2} + r_3 \omega_3 \hat{u}_{\theta_3 + \pi/2} - \dot{s} \hat{u}_{\theta_0} = 0$

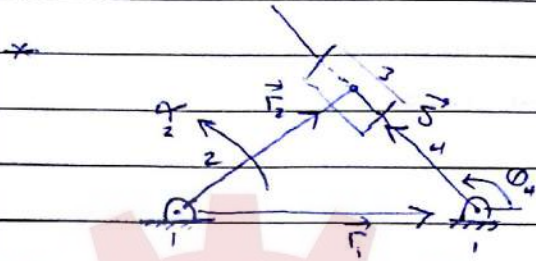
→ A.E: $r_2 \alpha_2 \hat{u}_{\theta_2 + \pi/2} - r_2 \omega_2^2 \hat{u}_{\theta_2} + r_3 \alpha_3 \hat{u}_{\theta_3 + \pi/2} - r_3 \omega_3^2 \hat{u}_{\theta_3} - \ddot{s} \hat{u}_{\theta_0} = 0$

ccw
 $\vec{\alpha} = \alpha \hat{k}$
 cw

نصف دائرة
 acc. نصف
 نصف دائرة
 Dec. نصف

No. _____

$$\text{dot}(\cdot \hat{u}_{\theta_2}) \rightarrow \ddot{S} = v$$



Given: Pos + Vel - input α_2

find: $\ddot{S} = \alpha_4$

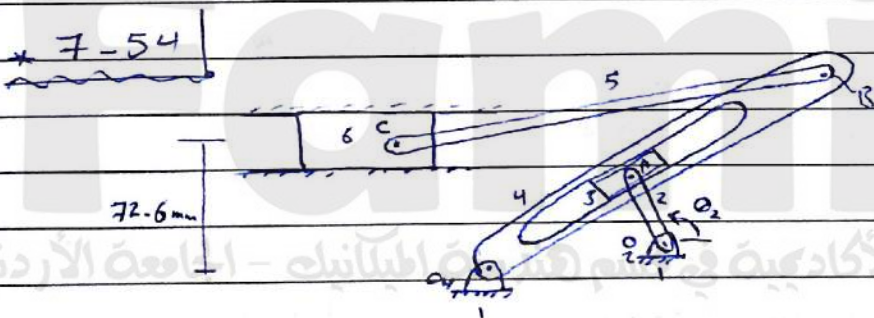
$$\rightarrow \text{L.C.E: } r_2 \hat{u}_{\theta_2} - \dot{S} \hat{u}_{\theta_4} - r_3 \hat{u}_{\theta_3} = 0$$

$$\rightarrow \text{V.E: } r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - \dot{S} \hat{u}_{\theta_4} - \dot{S} \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} = 0$$

$$\rightarrow \text{A.E: } [r_2 \alpha_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - r_2 \omega_2^2 \hat{u}_{\theta_2} - \ddot{S} \hat{u}_{\theta_4} - \dot{S} \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} - \dot{S} \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} - \dot{S} \alpha_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} + \dot{S} \omega_4^2 \hat{u}_{\theta_4} = \vec{0}] \cdot \hat{u}_{\theta_4}$$

$$\rightarrow \ddot{S} = r_2 \alpha_2 \sin(\theta_4 - \theta_2) - r_2 \omega_2^2 \cos(\theta_4 - \theta_2) + \dot{S} \omega_4^2$$

then $\alpha_4 = v$



$$L_2 = 25.4 \text{ mm}$$

$$L_3 = 120.9 \text{ mm}$$

$$L_4 = 115.9 \text{ mm}$$

$$\theta_4, \theta_2 = 42.9^\circ @ 15.5^\circ$$

$$\text{For } \theta_2 = 99^\circ$$

$$\omega_2 = 10 \text{ rad/s (CCW) = const.}$$

find:

$$\vec{a}_A, \vec{a}_B, \vec{a}_C$$

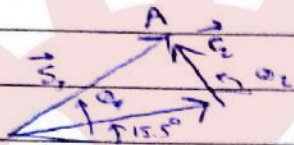
Given: $\alpha_2 = 0$
 $\omega_2 = \text{const.}$

Soln:- Loop I: $O_4 \rightarrow O_2 \rightarrow A \rightarrow O_4$

Loop II: $O_4 \rightarrow B \rightarrow C \rightarrow D \rightarrow O_4$

or $O_2 \rightarrow A \rightarrow B \rightarrow C \rightarrow D \rightarrow O_4 \rightarrow O_2$

→ Loop I:



L.C.E.I:

$$S_3 \hat{u}_{O_4} - 25.4 \hat{u}_{O_2} - 42.9 \hat{u}_{15.5} = 0$$

Rewrite $\Rightarrow S_3 \hat{u}_{O_4} + 25.4 \hat{u}_{O_2} + 42.9 \hat{u}_{15.5}$

[cat each self side by
it self]
for $\phi_1 = 99^\circ$

$$S_3^2 = 25.4^2 + 42.9^2 + 2(25.4)(42.9) \cos(99 - 15.5)$$

$$\Rightarrow S_3 = 52.27 \text{ mm}$$

$$\phi_4 = \tan^{-1} \left[\frac{25.4 \sin 79 + 42.9 \sin 15.5}{25.4 \cos 79 + 42.9 \cos 15.5} \right]$$

y = Comp.
x = Comp.

V.E.I:

$$\dot{S}_3 \hat{u}_{O_4} + S_3 \omega_4 \hat{u}_{\phi_4} - 25.4 \omega_2 \hat{u}_{\phi_2} = 0$$

cat each
side by
 $\hat{u}_{\phi_4}$

$$\dot{S}_3 = 25.4(+10) \sin(44.87 - 99)$$

$$\dot{S}_3 = -207.12 \left(\frac{\text{mm}}{\text{s}} \right)$$

Position: $\omega_2 \hat{u}_{\phi_2} \hat{u}_{\phi_4} \hat{u}_{\phi_1}$

$$\omega_4 = \frac{25.4(+10) \cos(44.87 - 99)}{52.27} = 2.81 \text{ (rad/s)}$$

CCW

$$\tilde{D}_3 = \tilde{S}_3 \omega_1^2 = 15.4 \omega_1^2 \cos(\theta_4 - \theta_2) \rightarrow \tilde{D}_3 = -1057.56$$

A.E.I :-

$$\vec{S}_3 \hat{U}_{\phi_4} + 2 \vec{S}_3 \omega_4 \hat{U}_{\phi_4 + \frac{\pi}{2}} + \vec{S}_3 \alpha_4 \hat{U}_{\phi_4 + \frac{\pi}{2}} - \vec{S}_3 \omega_4^2 \hat{U}_{\phi_4} - 25.4 \alpha_2 \hat{U}_{\phi_2 + \frac{\pi}{2}} + 25.4 \omega_2^2 \hat{U}_{\phi_2} = \vec{0}$$

$$(\cdot \hat{U}_{\theta_q}) \rightarrow \ddot{z}_3 =$$

بشكل عام $\vec{a}_A$ عند مركزه اعتباراً لنقطة $A =$ نقطة على ρ_{rod} $\rightarrow$

$$\vec{L}_A = \vec{L}_2 = 25.4 \hat{L}_{02}$$

$$\vec{v}_n = \frac{d\vec{r}_A}{dt} = 25.4 \omega_2 \hat{u}_{\phi + \pi/2}$$

$$\vec{a}_A = \frac{d\vec{v}_A}{dt} = 25.4 \cancel{\omega_2} \hat{u}_{\theta_2 + \gamma_2} - 25.4 \omega_2 \hat{u}_{\theta_2}$$

$$\vec{a}_A = -25.4 (10)^2 \hat{u}_{qg} = 2540 \hat{u}_{qg} \rightarrow |\vec{a}| = 2540 \text{ m/s}^2$$

عالم $\vec{a}$ على $\vec{b}$ ، لنقطة B نقطة على Rod 4 $\rightarrow$

$$\vec{v}_R = 120.9 \hat{u}_{0.4} \rightarrow \vec{v}_R = 120.9 \omega_A \hat{u}_{0.4 + \pi/2}$$

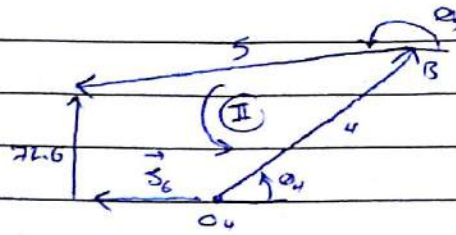
$$\vec{a}_B = 120.9 \alpha_4 \hat{u}_{\phi_4 + \frac{\pi}{2}} - 120.9 \omega_4^2 \hat{u}_{\phi_4}$$

$$= (120.9)(-17.36) \hat{u}_{\theta + \frac{\pi}{2}} = 120.9(2.81)^2 \hat{u}_{\theta}$$

$$= -2098.82 \hat{u}_{\theta_4 + \pi/2} - 954.64 \hat{u}_{\theta_4}$$

L.C.E II:

$$120.9 \hat{u}_{\theta_4} + 115.9 \hat{u}_{\theta_5} - 72.6 \hat{u}_{\theta_6} - \sum \hat{u}_{180^\circ} = 0$$



$$\rightarrow y\text{-comp: } \theta_5 = \sin^{-1} \frac{72.6 - 120.9 \sin 44.37^\circ}{115.9}$$

$$\theta_5 = 185.91^\circ$$

$$\sum_6 = 28.86 \text{ mm}$$

لأثر $\vec{\sum}_6$ Horizontal

نستخدم $\times \cdot \hat{u}$

Comp

اسهل

ملاحظة: في حالة
الاجابة في اقل من
تكونه بالنظر في

V.E. II:

$$120.9 \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} + 115.9 \omega_5 \hat{u}_{\theta_5 + \frac{\pi}{2}} - \sum \hat{u}_{180^\circ} = 0$$

$$\rightarrow y\text{-Comp: } \omega_5 = 2.107 \text{ rad/s}$$

ما يحتاج $\sum_6$

الاجابة في اقل من
الاجابة في اقل من
acc. $\hat{u}$
dec. $\hat{u}$

A.E. II:

$$120.9 \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} + 115.9 \omega_5 \hat{u}_{\theta_5 + \frac{\pi}{2}} - \sum \hat{u}_{180^\circ} = 0$$

$$120.9 \alpha_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} - 120.4 \omega_4^2 \hat{u}_{\theta_4} + 115.9 \alpha_5 \hat{u}_{\theta_5 + \frac{\pi}{2}} - 115.9 \omega_5^2 \hat{u}_{\theta_5} = \sum \hat{u}_{180^\circ}$$

$$\sum_6 = -107.78 \text{ mm/s}^2 = \alpha_c = -1077.78 \hat{u}_{180^\circ} = 1077.78 \hat{i}$$

No.

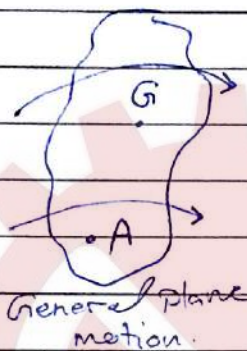
suggested

7, 16, 20, 48, 50, 52

(Kinetic)

** Dynamic Analysis: ch 11

- Knowns: Kinematic + masses



Equation of motion:

$$\sum \vec{F} = m \vec{a}$$

$$\sum \vec{M}_G = \sum (M_G)_{\text{eff}} = I_G \vec{\alpha}$$

$$\sum \vec{M}_A = \sum (M_A)_{\text{eff}}$$

Note:
Centroid $\rightarrow$ geometric center
center of mass
center of gravity
if homogeneous body, all three are the same

- Rotation about fixed axis

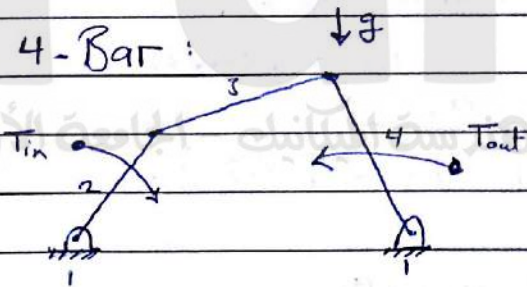
Centroid (G)
 $\sum M_G = I_G \alpha$

Non-Centroid (A)
 $\sum M_A = I_A \alpha$
 $I_A = I_G + m d^2$

Note:
Parallel axis theorem
if you know I_G and d , you can find I_A



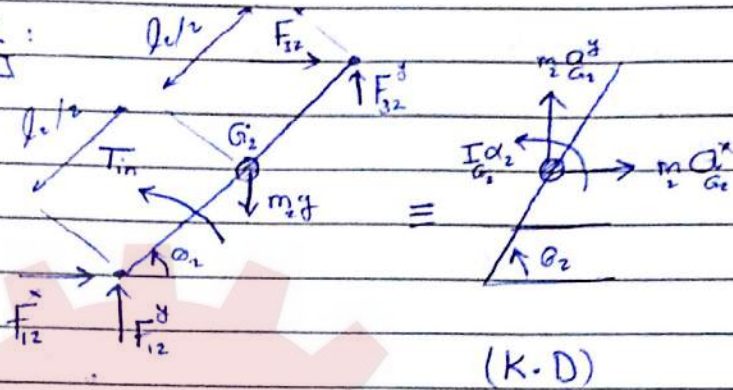
* 4-Bar:



Given: Kinematic + masses.

Find: For a certain output torque on Link 4 (ex) find the necessary input torque on Link 2 (ex) and the pin forces.

Link 2:



$$\rightarrow \sum F_x = \sum (F_x)_{\text{eff.}}$$

$$F_{12} + F_{32} = m_2 a_{G_2} \quad \dots \dots (1)$$

$$+\uparrow \sum M_y = \sum (F_y)_{\text{eff}}$$

$$F_{12}^y + F_{21}^y - m_2 g = m_2 a_2^y \quad \text{----- (2)}$$

$$J \leq \mu_0 = \Sigma(\mu_0)_{\text{eff}}$$

$$T_0 - m_1 g \frac{l_1}{2} \cos \theta_2 + F_{32}^y l_1 \cos \theta_2 - F_{32}^x l_1 \sin \theta_2 = I_{G_1} \alpha_2$$

$$+ m_2 a_{G_2}^x \frac{l_1}{2} \sin \theta_2 + m_2 a_{G_2}^y \frac{l_1}{2} \cos \theta_2$$

$$(-\ddot{\theta}_2) = \ddot{\theta}_2 \quad (3)$$

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2) \hat{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \hat{u}_\theta$$

$$= [\ddot{r} \cos\theta - \ddot{\theta} r \sin\theta] \hat{i} + [\ddot{r} \sin\theta + \ddot{\theta} r \cos\theta] \hat{j}$$

$$\rightarrow a_{\theta_2}^* = -\frac{l_2}{2} \omega_1^2 \cos \theta_2 - \frac{l_2}{2} \alpha_1 \sin \theta_2$$

$$-r\omega^2 = -\frac{r_2}{2}\omega_2^2 \} a_n$$

$$\gamma x = \frac{b_2}{2} x_2 \quad \gamma a_f$$

$$2\dot{\Gamma}\omega = 0$$

$$\rightarrow \frac{a_y}{G_2} = -\frac{l_2}{2} \omega^2 \sin \theta + \frac{l_2}{2} \alpha \cos \theta$$

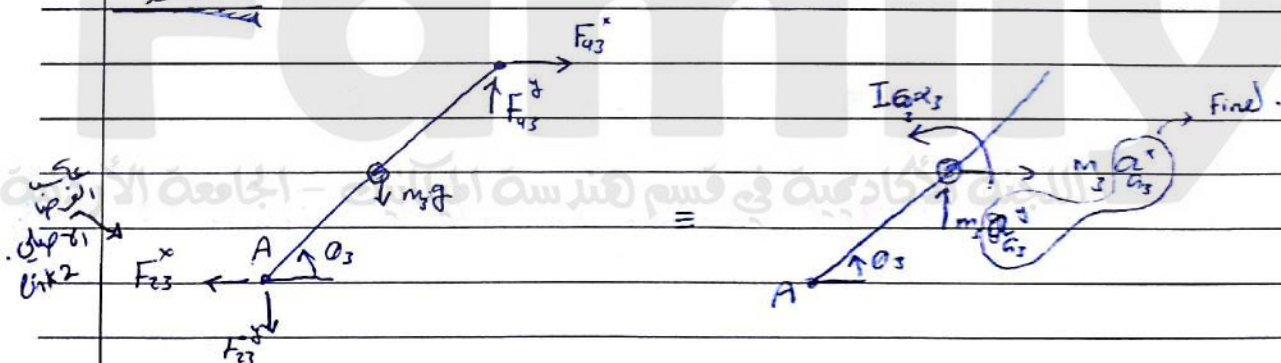
No. _____

$$\begin{aligned} \xrightarrow{\text{دالة}} (x) \quad m_2 a_{G_2}^x \left(\frac{l_2}{2} \sin \theta_2 \right) &= m_2 \frac{l_2}{2} \sin \theta_2 \left(-\frac{l_2}{2} \omega_2^2 \cos \theta_2 - \frac{l_2}{2} \ddot{\theta}_2 \sin \theta_2 \right) \\ &= -m_2 \left(\frac{l_2}{2} \right)^2 \omega_2^2 \sin \theta_2 \cos \theta_2 - m_2 \left(\frac{l_2}{2} \right)^2 \ddot{\theta}_2 \sin^2 \theta_2 \end{aligned}$$

$$\begin{aligned} (y) \quad m_2 a_{G_2}^y \left(\frac{l_2}{2} \right) \cos \theta_2 &= m_2 \frac{l_2}{2} \cos \theta_2 \left(-\frac{l_2}{2} \omega_2^2 \sin \theta_2 - \frac{l_2}{2} \ddot{\theta}_2 \cos \theta_2 \right) \\ &= -m_2 \left(\frac{l_2}{2} \right)^2 \omega_2^2 \sin \theta_2 \cos \theta_2 + m_2 \left(\frac{l_2}{2} \right)^2 \ddot{\theta}_2 \cos^2 \theta_2 \end{aligned}$$

$$\begin{aligned} \text{R.S (3)} : \quad I_{G_2} \alpha_2 - m_2 a_{G_2}^x \frac{l_2}{2} \sin \theta_2 - m_2 a_{G_2}^y \frac{l_2}{2} \cos \theta_2 \\ &= I_{A_2} \alpha_2 + 0 - m \left(\frac{l_2}{2} \right)^2 \ddot{\theta}_2 (\sin^2 \theta_2 - \cos^2 \theta_2) \\ &= I_{G_2} \alpha_2 + m \left(\frac{l_2}{2} \right)^2 \ddot{\theta}_2 = \underbrace{\left[I_{G_2} + m \left(\frac{l_2}{2} \right)^2 \right]}_{I_{A_2}} \ddot{\theta}_2 \\ &\Rightarrow \boxed{I_{A_2} \alpha_2} \neq \end{aligned}$$

Link 3:



$$(x) \quad -F_{23}^x + F_{43}^x = m_3 a_{G_3}^x \quad \dots (4)$$

$$(y) \quad -F_{23}^y - m_3 g + F_{43}^y = m_3 a_{G_3}^y \quad \dots (5)$$

$$+\circlearrowleft \sum M_A = \sum (M)_{eff}$$

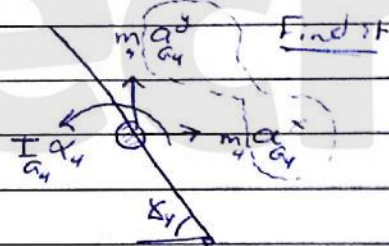
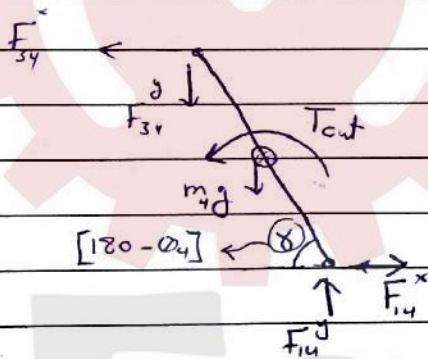
لا بد من إيجاد M_A و M_B من أجل حل المسألة
لأننا نحتاج إلى M_A و M_B من أجل حل المسألة
كل واحد على حدة.

$$-m_3 g \left(\frac{l_3}{2} \right) \cos \theta_3 + F_{43}^y l_3 \cos \theta_3 - F_{43}^x l_3 \sin \theta_3 = I_{G_3} \alpha_3 + m_3 a_{G_3}^y \left(\frac{l_3}{2} \right)$$

$$\cos \theta_3 - \frac{m_3 a_{G_3}^y \left(\frac{l_3}{2} \right) \sin \theta_3}{I_{G_3} \alpha_3} \quad (6)$$

ليس
مطلوب

Link 4:



$$\rightarrow x: -F_{34}^x + F_{4x} = m_4 a_{G_4}^x \quad (7)$$

$$\uparrow y: -F_{34}^y - m_4 g + F_{4y} = m_4 a_{G_4}^y \quad (8)$$

$$+\circlearrowleft M_{G_4}: T_{cut} + m_4 g \left(\frac{l_4}{2} \right) \cos \theta_4 + F_{34}^y l_4 \cos \theta_4 + F_{34}^x l_4 \sin \theta_4 = I_{G_4} \alpha_4 \quad (9)$$

* Solution:

- ① solve (6) and (4) simultaneously to find F_{43}^x and F_{43}^y
- ② solve (4) for F_{32}^x
- ③ solve (5) for F_{32}^y
- ④ solve (3) for T_{in}

هناك حاجة فقط لـ (4) لمعادلات
من أجل إيجاد T_{in} و F_{32}
التي هي النتيجة التي إذا طلب
مطلوب آخر.

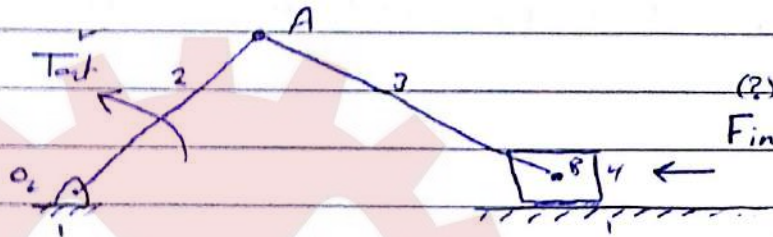
(5) " (1) " F_{12}^x

(7) " (7) " F_{14}^x

(6) " (1) " F_{12}^y

(8) " (8) " F_{14}^y

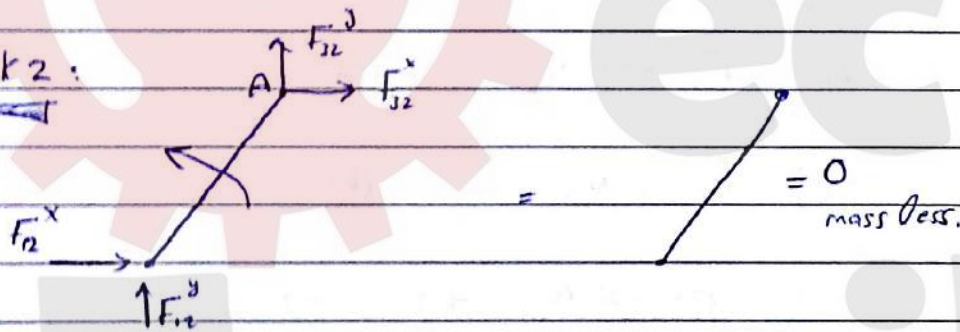
** slider-crank:



for certine T_{out} , find F_{in} ?

Assume links 2 and 3 are massless.

Link 2:



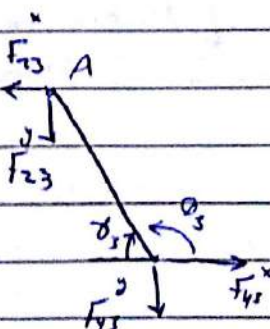
$\rightarrow \sum F_x: F_{12}^x + F_{32}^x = 0 \quad \dots (1)$

$\uparrow \sum F_y: F_{12}^y + F_{32}^y = 0 \quad \dots (2)$

$\sum M_{O_1}: T_{out} + F_{32}^y l_2 \cos \theta_2 - F_{32}^x l_2 \sin \theta_2 = 0 \quad \dots (3)$

sun
known
input
moment
is 12

Link 3:



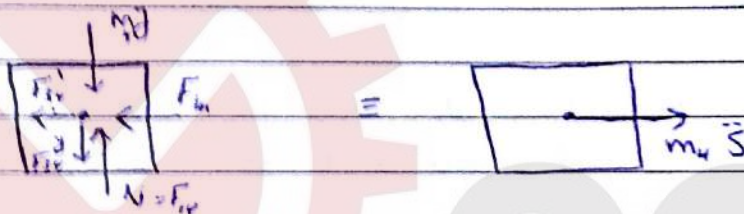
= 0 (massless)

$$\sum F_x = -F_{12}^x - F_{43}^x = 0 \quad \text{--- (4)}$$

$$\sum F_y = -F_{12}^y + F_{43}^y = 0 \quad \text{--- (5)}$$

$$\sum M_A = F_{12}^x l_3 \sin \theta_3 + F_{12}^y l_3 \cos \theta_3 = 0 \quad \text{--- (6)}$$

Block 4:



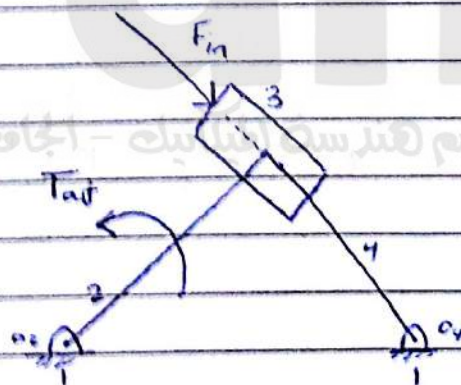
$$\sum F_x = -F_{12}^x + F_{43}^x = m_4 \ddot{s} \quad \text{--- (7)}$$

$$\sum F_y = -Mg - F_{12}^y + N = 0 \quad \text{--- (8)}$$

Solu: solve (3) and (6) $\rightarrow F_{12}^x$ and F_{12}^y

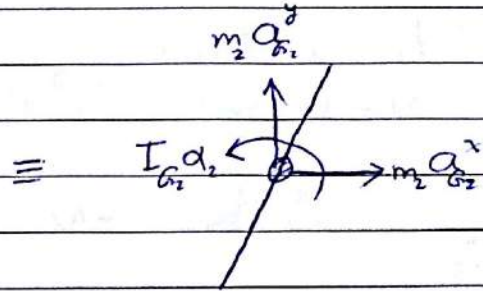
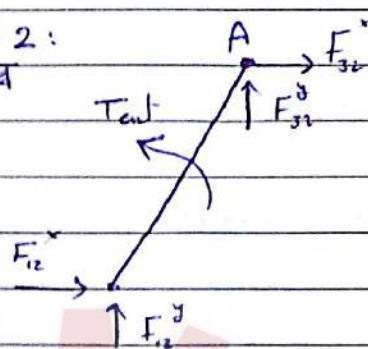
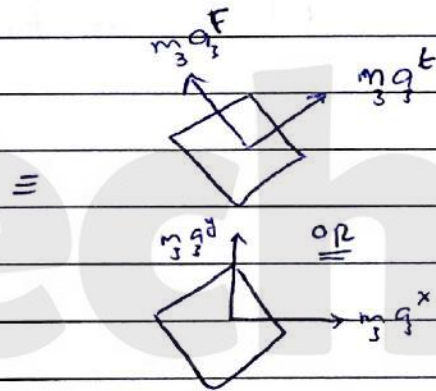
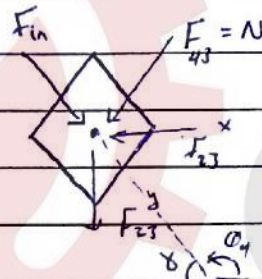
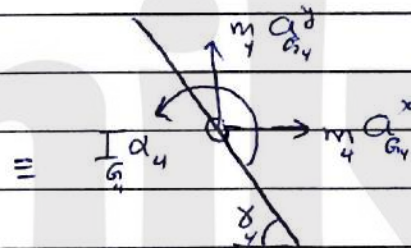
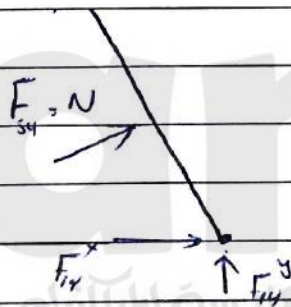
" (4) $\rightarrow F_{43}^x$

" (7) $\rightarrow F_{in}$



Find a relation between T_{air} and F_{in} .

[Motion in horizontal plane] ?!

Link 2:Link 3:Link 4:

equ.

Link 2: $\sum M_{O_2} = \sum (M)_{eff}$

$$T_{out} + F_{32}^y l_2 \cos \theta_2 - F_{32}^x l_2 \sin \theta_2 = I_{O_2} \alpha_2 \quad \dots (1)$$

equ.

Link 3: $\uparrow \sum F_r = \sum (F_r)_{eff}$

$$-F_{13} + F_{23}^x \cos \theta_4 - F_{23}^y \sin \theta_4 = m_3 a_3^x \quad \dots (2)$$

No. _____

$$\uparrow \sum F_k = \sum (F_k)_{eff.}$$

$$-N - F_{23}^x \sin \theta_4 - F_{23}^y \cos \theta_4 = m_2 \ddot{x}_2 \quad \dots (3)$$

equ. link 4 : $\rightarrow \sum M_{A_4} : -N r = I_{O_4} \ddot{\alpha}_4 \quad \dots (4)$

$\rightarrow$ solve (4) $\rightarrow N$

solve (3) and (1) $\rightarrow F_{23}^x, F_{23}^y$

solve (2) $\rightarrow F_{12}$