

* Machinery.

* Mechanism: "linkage"

→ A machine component that is concerned mainly with motion transmission, it consists of a number of bodies "Links" connected by joints "pairs".

• links أجسام الجسمين

* Links: A body that has at least two points (nodes) for attachment to other links.

• كائناً (holes) موجودة بال (Bodies) أجسام الجسمين (Pins) مملوءة

* Classifications

1. Based on the number of (nodes), links are classified as:

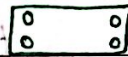
1. Binary link



2. Ternary link



3. Quaternary link

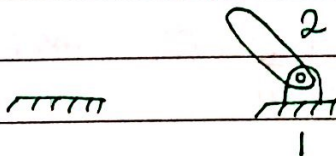


• كائناً ثنائياً (two links only) كائناً ثلاثياً (three links only) كائناً رباعياً (four links only)

2. Based on the motion type, links are classified as:

1. Ground [fixed or frame] link, (Fixed, stationary)

→ zero motion. "Exist in all mechanisms".



* Note! "Ground link" if it was existed many times in the mechanism, we count it as one link only (موجود في مكان واحد).

2. Crank: A link pivoted to the ground, and makes full Rotation.

~> Pivoted $\equiv$ pinned "fixed"

~> Full Rotation $\equiv$ Complete Revolution $= (360^\circ)$.

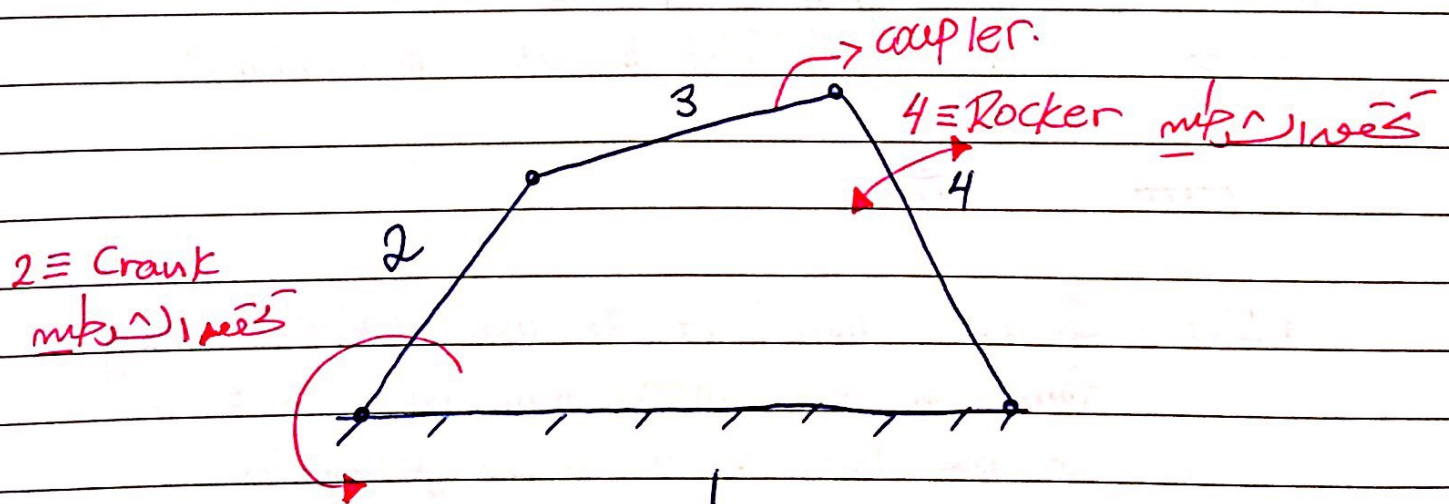
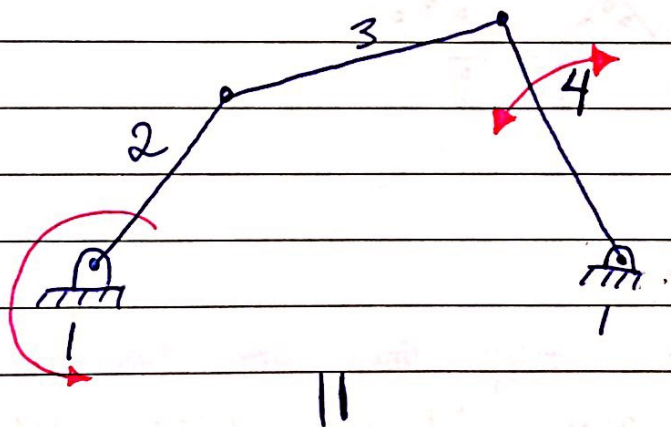
3. Rocker: A link pivoted to the ground and makes oscillation.

(حركة تذبذبية دائرية)

~> Resprocation: (حركة تذبذبية خطية)

~> Full Rotation "Crank"

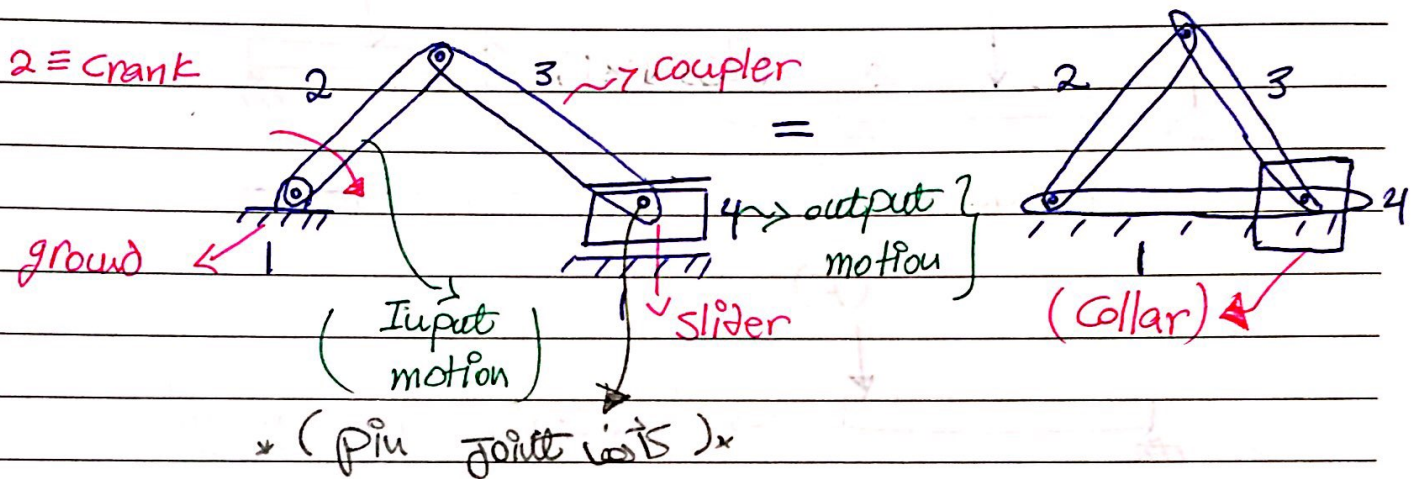
~> Oscillation "Rocker"



4. Coupler (connecting Rod): A link not pivoted to the ground [but connected to other links] and makes complex motion "general plane motion".

~> complex motion $\equiv$ G.P.M $\equiv$ translation + Rotation.

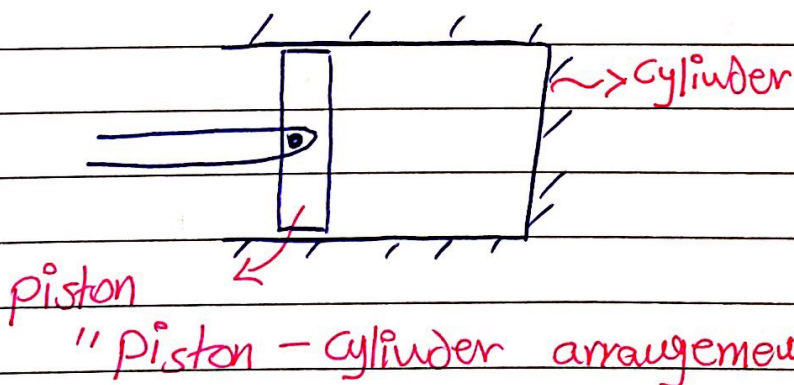
* $\vec{v} = \vec{v}_0 + \vec{\omega} \times \vec{r}$ *



5. Slider: A sliding (translating) link.

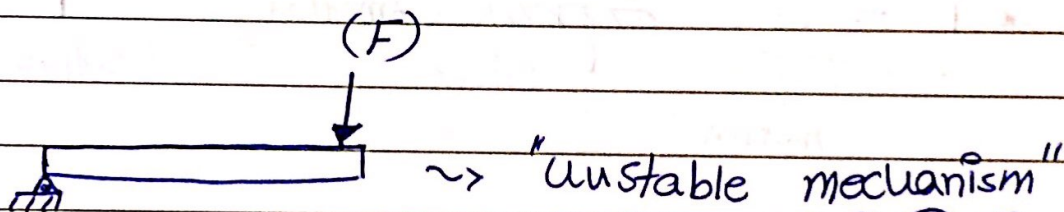
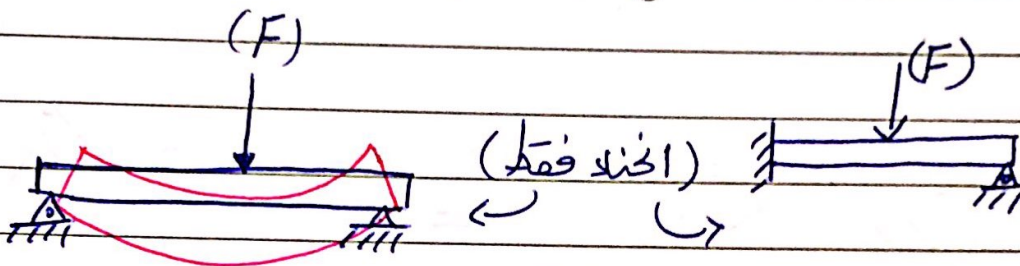
~> Example (piston).

$\vec{v} = \vec{v}_0$
 $\vec{\omega} = 0$



Note! Input motion must gives output motion.
If (output = 0), it becomes "structure" not a "mechanism".

~> Structures (loads) $\rightarrow$ $\bar{\delta}$ $\rightarrow$ δ



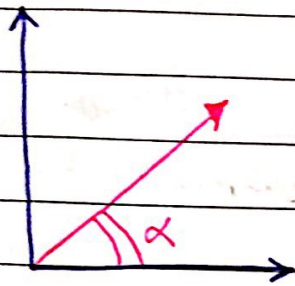
* $\bar{\delta}$ (load) $\rightarrow$ δ $\rightarrow$ δ

* In mechanisms, Number of unknown Reactions less than number of equilibrium equations.



Degree of freedom: "DOF"

درجۂ حرکت الحركه



~ (2-Independent axes) (One DOF) ~

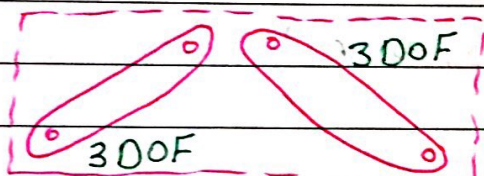
محاور متعامدين (x & y) بالاقوس (alpha)

√ Rigid bodies

1. Planar motion → maximum DOF's ~ (3) . "x, y, Rotation".
2. Space → maximum DOF's ~ (6).

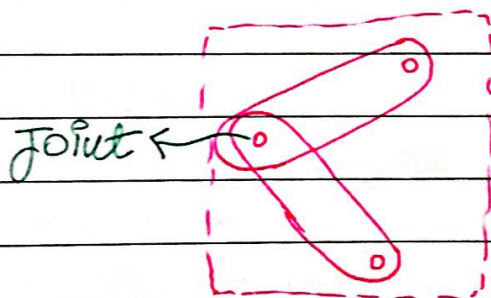
→ For particles there is no Rotation.

Ex 8

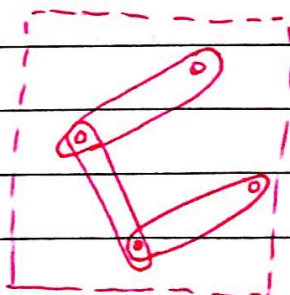


~ System.

* total for the system = 6 DOF.
→ (3) (2) = 6
max ~ [no. of DOF] [the same type]



* 4 DOF "2 translation + 2 Rotation"



* 5 DOF "3 Rotation + 2 translation"



* 1 DOF.

→ [Ground link]

Parameters $\equiv (x, y, z)$ / $(\theta_x, \theta_y, \theta_z)$.
 + translation Rotation.

Joints classification "Fig 2-3"

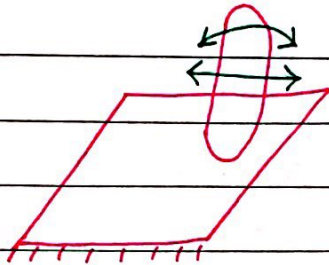
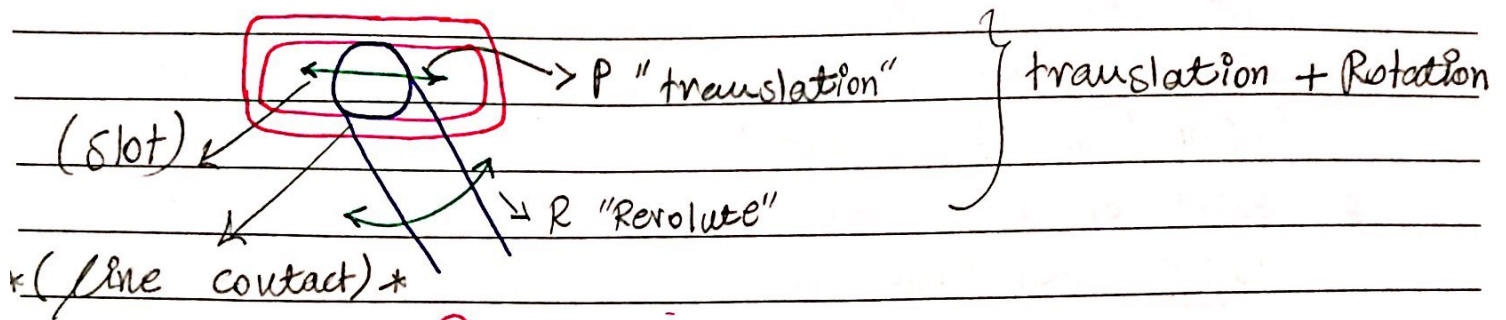
(I) Based on the number of DOF allowed: $\left. \begin{array}{l} \rightarrow 1 \text{ DOF} \\ \rightarrow 2 \text{ DOF} \end{array} \right\}$

$\hookrightarrow$ only for planar mechanisms.

Lower Joint - 1 DOF

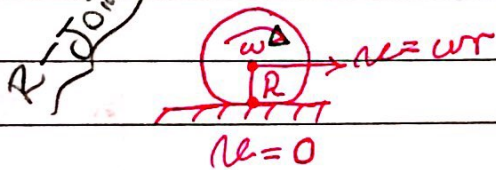
2. Prismatic or sliding joint (P-joint) $\Rightarrow$ 1 DOF
 $\rightarrow$ "translation".

2 # Ex: RP-Joint "Revolute + Prismatic". $\Rightarrow$ 2 DOF

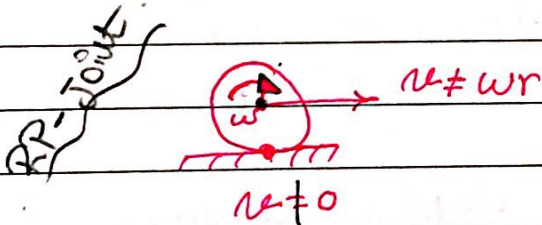


Notes:

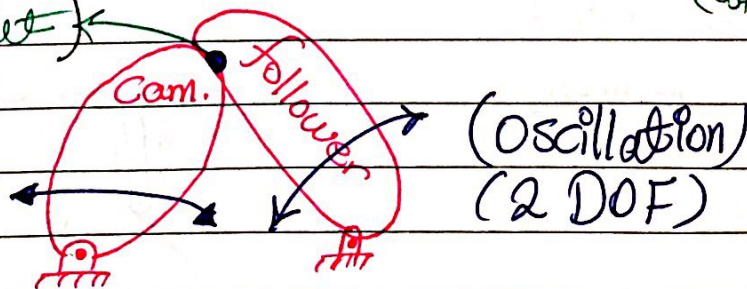
1. Rolling without slipping: (1 DOF) "Rotation only."
R-Joint



2. Rolling with slipping: (2 DOF) "trans. + Rotation."



Cam-follower: Radius of Rotation (is fixed)
(Cam-Joint) (suppl. plane)



(II) Based on the type of contact between bodies:
"Point, Surface, line".

1. Surface contact $\leadsto$ "lower pair" $\leadsto$ "R-joint, P-joint" $\leadsto$ "1 DoF" $\leadsto$ "Full joint".

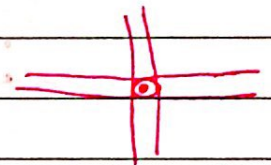
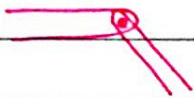
2. (Point or line) contact $\leadsto$ "Higher pair" $\leadsto$ "RP joint" $\leadsto$ "2 DOF" $\leadsto$ "Half joint".
(Point or line) / (Surface) / (Point or line)

(III) Based on the number of links joined:

1st-order joint
(2 links connected)

2nd-order joint
(3 links connected)

3rd-order joint
(4 links connected)



Equation 8

Joint order (k) = (No. of links joined (L) - 1) $\leadsto$ "Joint order"

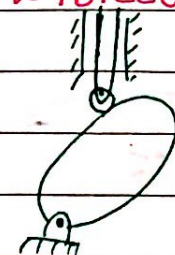
(IV) Based on the type of physical closures

1. Form closed, Ex: (R or P)

$\leadsto$ Depends on geometry.

(Form closed)

2. Forced closed, Ex: (RP)



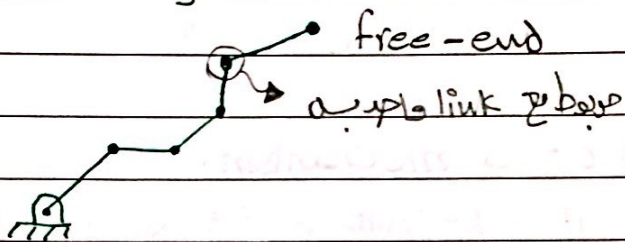
Forced closed joint
links

$\leadsto$ Depends on the external forces.

* Kinematic chain: A system of a number of links connected by Joints.

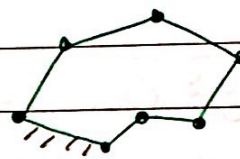
Open loop chain

(At least one link is connected to one only other link).



closed loop chain

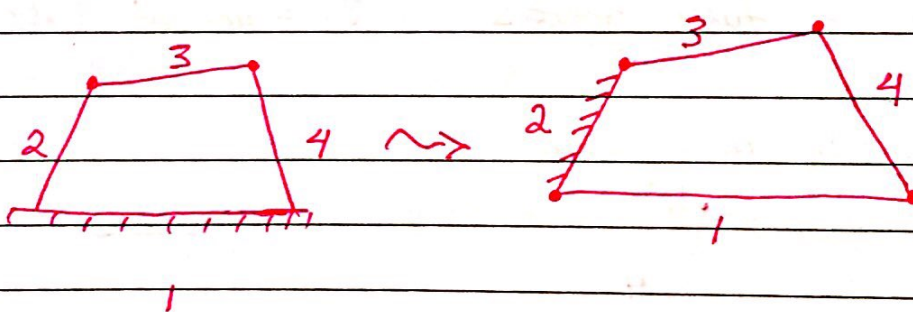
(Each link is connected to two or more other links).



~> Not a mechanism since there is no ground link.

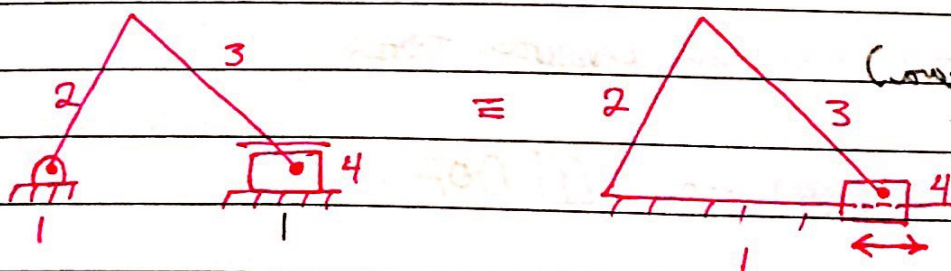
* A mechanism "linkage" is a closed loop chain with one link fixed.

* Inversion of mechanisms: "Inverse $\equiv$ عكس"



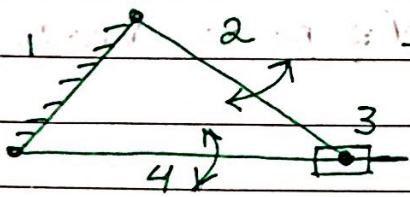
* خلية ار كذا تتحرك وانقلاب

→ no. of Inversions. = no. of links



* (لحركة ثمانية الدروس)

→ Inversion 8



Inverted Slider Crank mech.

OR

Sliding - Contact mech.

* 2, 4 ~> Rotation

* 3 ~> General plane motion [$\omega_3 = \omega_4$]
trans. + Rot. "Coriolis acc."

* Mobility (No. of DOF) of a mechanism.

* Full * إذا درجته تمام (R-Joint) تبين مقدار (DOF) مقدار (2)
(P-Joint)

* half * إذا درجته نصف تمام (RP-Joint) تبين مقدار (DOF) مقدار (1)

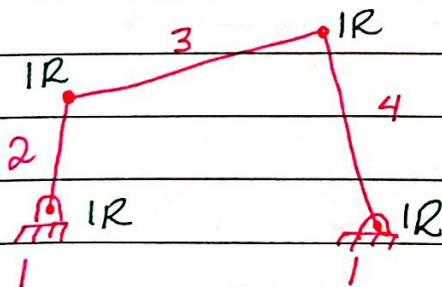
→ For planar mechanisms

$$M = 3(L-1) - 2J_1 - J_2$$

M = mobility / L = no. of links

J_1 = no. of full joints , J_2 = no. of half joints.

* Ex. 1: 4 Bar mechanism.



$L = 4$, $J_1 = 4$ (4R), full Revolute joints

$J_2 = 0$

$$\rightarrow M = 3(4-1) - 2(4) - 0 = \boxed{1} \text{ DOF.}$$

A k^{th} -order joint is counted (k) joints.

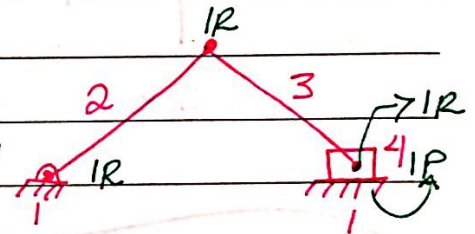
$\sim M \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \left. \begin{array}{l} \text{Mechanism with } (M) \text{ DOF.} \\ \text{"Structures"} \\ \text{Statically Indeterminate.} \end{array} \right\}$

Ex. 2: Slider-Crank

$$L = 4, \quad \bar{J}_1 = 4 (3R + 1P), \quad \bar{J}_2 = 0$$

$$\sim M = 3(4-1) - 2(4) = 1 \text{ DOF}$$

1 input $\rightarrow$ 1 output "Motion"



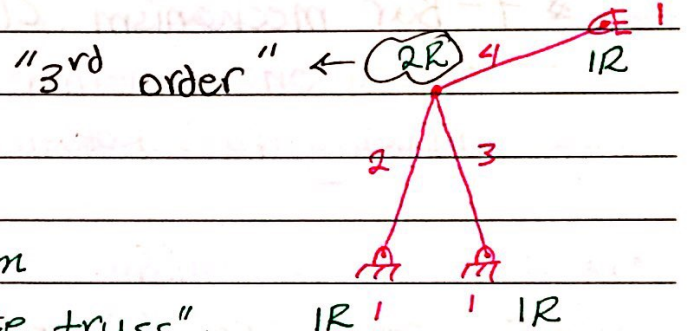
Ex. 3: 8

$$L = 4, \quad \bar{J}_1 = 5 (5R), \quad \bar{J}_2 = 0$$

$$\sim M = 3(4-1) - 2(5) = -1$$

$\hookrightarrow$ This is not a mechanism

"Statically Indeterminate truss".

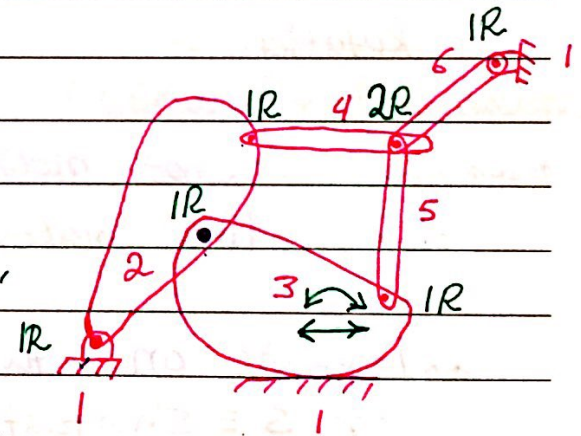


Ex. 4: 8

$$L = 6, \quad \bar{J}_1 = 7 (7R), \quad \bar{J}_2 = 1 (1RP)$$

$$\sim M = 3(6-1) - 2(7) - 1 = 0 \text{ "structure"}$$

(mech.) $\leftarrow$ مآخذ الحركة

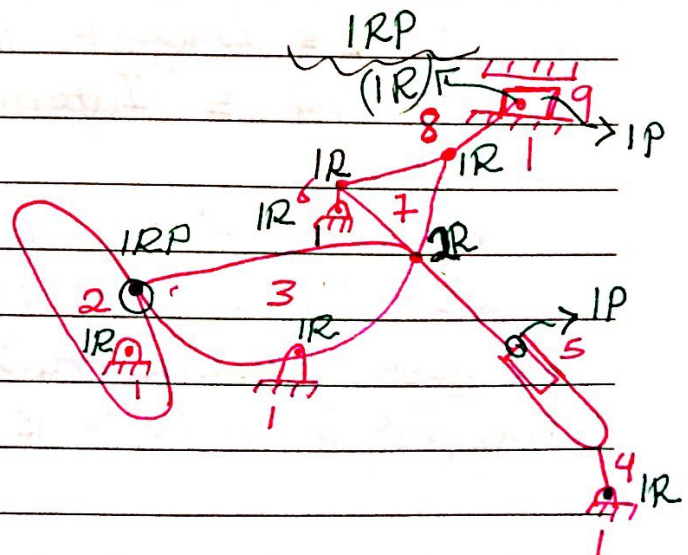


Ex. 5: 8

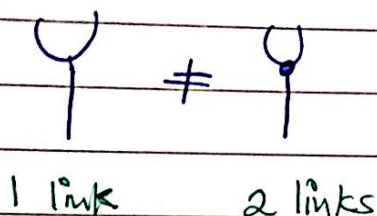
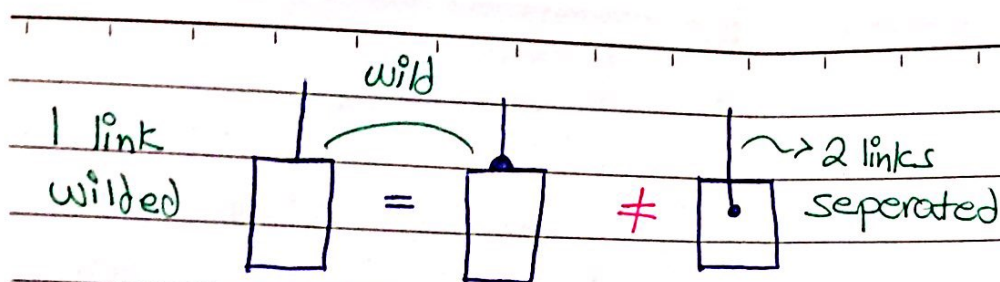
$$L = 9, \quad \bar{J}_1 = 11 (9R + 2P), \quad \bar{J}_2 = 1$$

$$\sim M = 3(9-1) - 2(11) - 1 = 1$$

(1 Input $\rightarrow$ 1 output).



+ using spring $\rightarrow$ force Flow. (cont.)

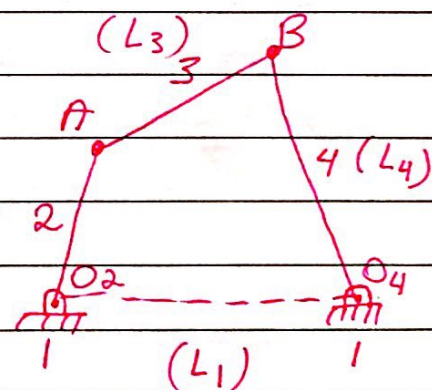


* 4-Bar mechanism classification:

$\rightarrow$ Based on whether there is a link capable of making full Rotation or not!!

* 1* Grashof mechanism:

$\rightarrow$ At least one link makes full Rotation.



* 2* Non-Grashof mechanism:

$\rightarrow$ No link makes full Rotation.

* Depends on the links lengths:

$\rightarrow S \equiv$ Shortest link

$\rightarrow L \equiv$ Longest link

$\rightarrow P, Q \equiv$ Intermediate lengths.

* If $(S+L) \leq (P+Q) \Rightarrow$ Grashof

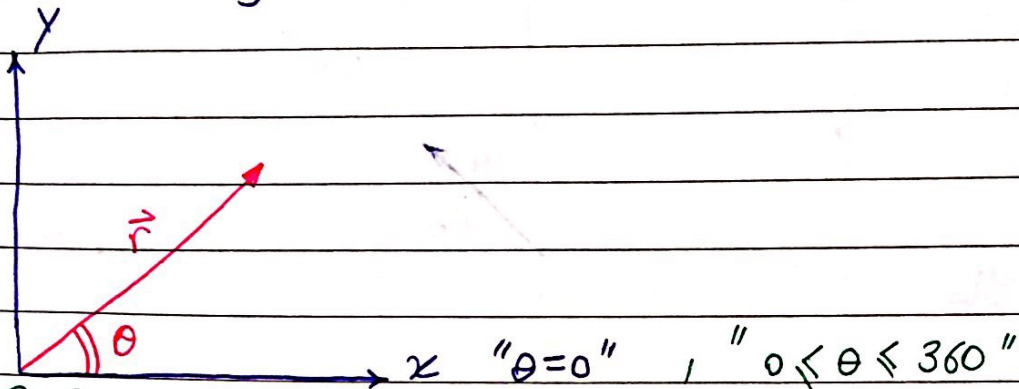
* If $(S+L) > (P+Q) \Rightarrow$ Non-Grashof.

* Suggested problems "ch. 2"

1(a, d, f, g, i, j, n), 4, 7, 15, 21, 22, 26, 32, 42, 44

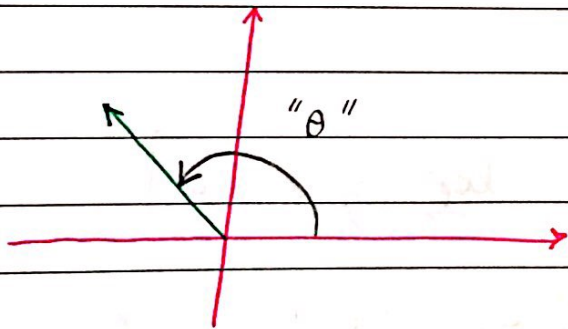
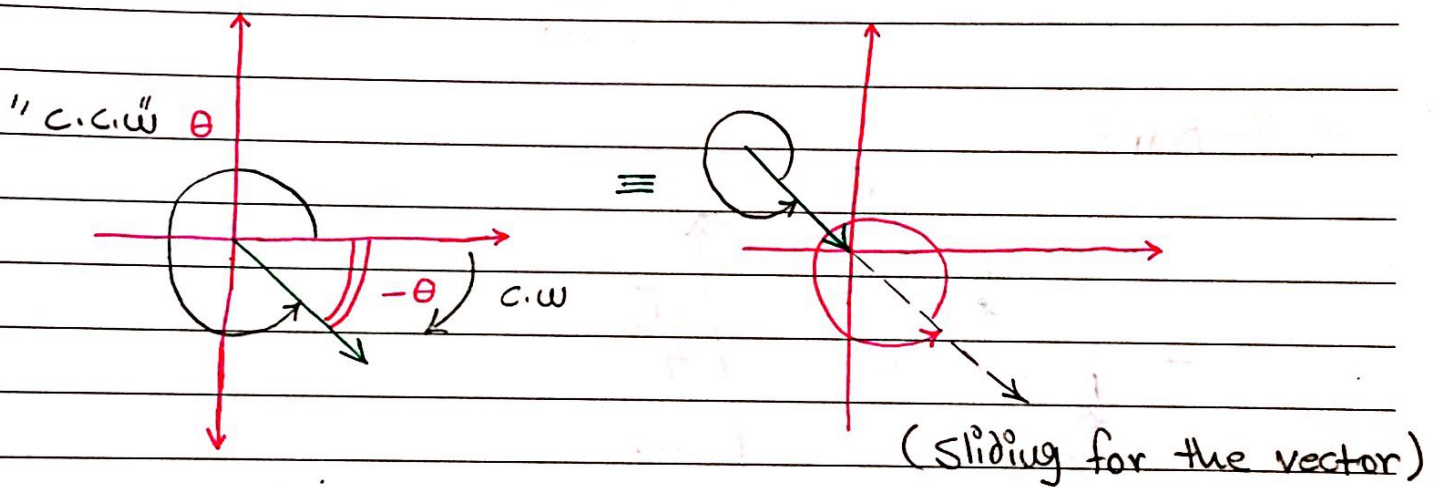
Kinematic Analysis

Position analysis of mechanisms: "For planar linkages".



~> Reference of θ is +ve x-axis " +ve c.c.w from +x "

~> tail to head Vector.



1x Cartesian Coordinates:

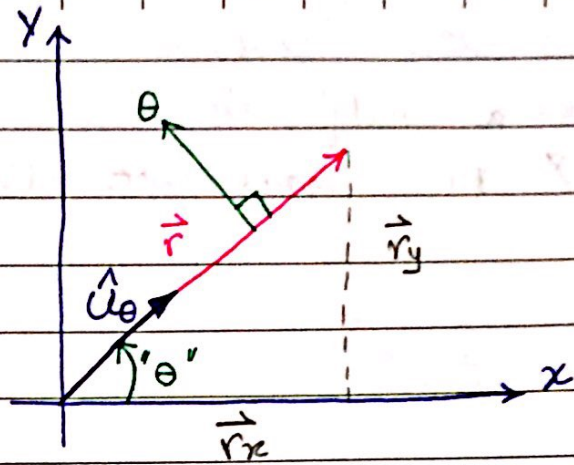
$$\vec{r} = \underbrace{r_x}_{r \cos \theta} \hat{i} + \underbrace{r_y}_{r \sin \theta} \hat{j}$$

2* Polar coordinates

$$(x, y) \rightarrow (r, \theta)$$

$$\rightarrow r = \sqrt{r_x^2 + r_y^2}$$

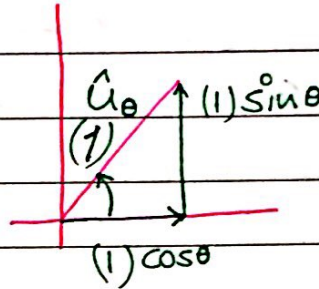
$$\rightarrow \theta = \tan^{-1} \left(\frac{r_y}{r_x} \right)$$



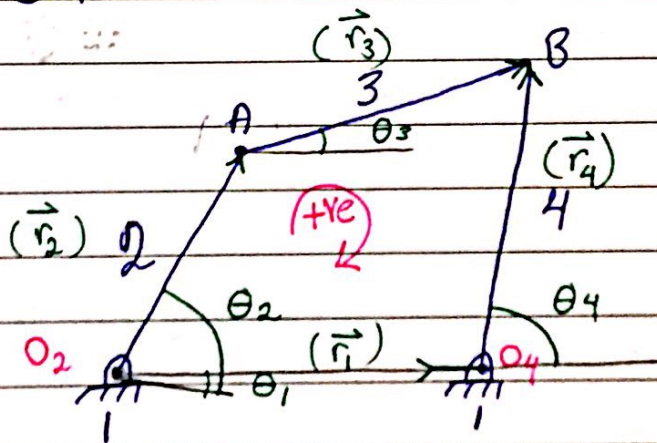
$$\rightarrow \vec{r} = r \hat{u}_\theta$$

$$\rightarrow \hat{u}_\theta = \cos \theta \hat{i} + \sin \theta \hat{j}$$

$$\vec{r} = r (\cos \theta \hat{i} + \sin \theta \hat{j})$$



4-Bar:



* loop closure equations: [vector loop equations]

$$O_2 \rightarrow A \rightarrow B \rightarrow O_4 \rightarrow O_2$$

* Note! "0" $\rightarrow$ $\vec{O_2A} + \vec{AB} + \vec{BO_4} + \vec{O_4O_2} = \vec{0}$

$$\rightarrow \vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_1 = 0$$

-ve "clockwise"

$$\rightarrow r_2 \hat{a}_{\theta_2} + r_3 \hat{a}_{\theta_3} - r_4 \hat{a}_{\theta_4} - r_1 \hat{a}_{\theta_1} = \vec{0}$$

$$\left(\quad \right) \hat{i} + \left(\quad \right) \hat{j} = 0\hat{i} + 0\hat{j}$$

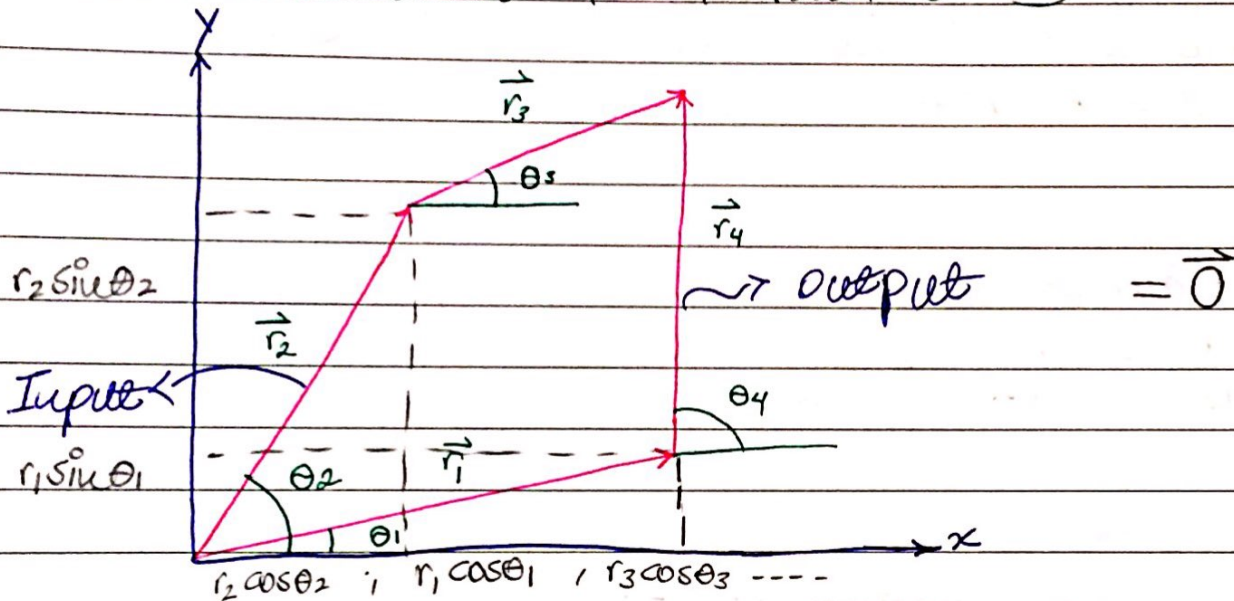
two Independent equations.

* X-Position:

$$r_2 \cos \theta_2 + r_3 \cos \theta_3 - r_4 \cos \theta_4 - r_1 \cos \theta_1 = 0$$

* Y-Position:

$$r_2 \sin \theta_2 + r_3 \sin \theta_3 - r_4 \sin \theta_4 - r_1 \sin \theta_1 = 0$$



* الثوابت، الأطوال (r_1, r_2, r_3, r_4) و θ_1

$\rightarrow \theta_2$: Input
 $\rightarrow \theta_4$: output

* المتغيرات: $\theta_2, \theta_3, \theta_4$

* Trigonometric Identities: "Relations"

$$* \cos(-\theta) = \cos(\theta) \quad \text{"Even odd"}$$

$$* \sin(-\theta) = -\sin(\theta)$$

$$* \cos(\theta_1 \pm \theta_2) = \cos \theta_1 \cos \theta_2 \mp \sin \theta_1 \sin \theta_2$$

$$* \sin(\theta_1 \pm \theta_2) = \sin \theta_1 \cos \theta_2 \pm \sin \theta_2 \cos \theta_1$$

$$* \cos(\theta + \frac{\pi}{2}) = -\sin\theta \quad \text{"shifting"}$$

$$* \sin(\theta + \frac{\pi}{2}) = \cos\theta$$

$$* \cos\theta = \frac{1 - \tan^2(\theta/2)}{1 + \tan^2(\theta/2)}$$

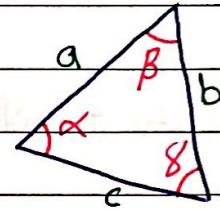
$$* \sin\theta = \frac{2\tan(\theta/2)}{1 + \tan^2(\theta/2)}$$

*** Law of cosines:**

$$1 * a^2 = b^2 + c^2 - 2bc \cos\gamma$$

$$2 * b^2 = a^2 + c^2 - 2ac \cos\alpha$$

$$3 * c^2 = a^2 + b^2 - 2ab \cos\beta$$



*** Law of Sines:**

$$\frac{a}{\sin\gamma} = \frac{b}{\sin\alpha} = \frac{c}{\sin\beta}$$

*** Ex: Loop**

→ loop closure equation (L.C.E)

$$\vec{r}_3 - \vec{r}_2 - \vec{r}_1 = \vec{0}$$

$$\vec{r}_3 = \vec{r}_2 + \vec{r}_1$$

$$r_3 \hat{u}_{\theta_3} = r_1 \hat{u}_{\theta_1} + r_2 \hat{u}_{\theta_2}$$

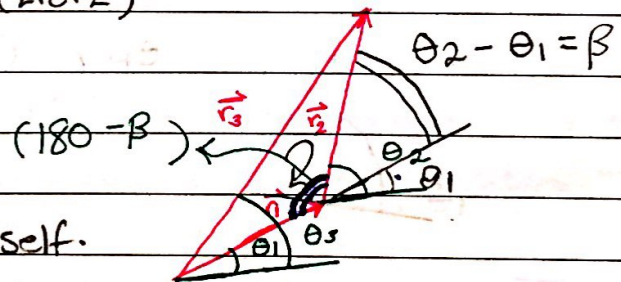
→ Dot each side by itself.

$$\vec{r}_3 \cdot \vec{r}_3 = (\vec{r}_1 + \vec{r}_2) \cdot (\vec{r}_1 + \vec{r}_2)$$

$$\vec{r}_3^2 (\hat{u}_{\theta_3} \cdot \hat{u}_{\theta_3}) = r_1^2 + r_2^2 + 2r_1 r_2 \hat{u}_{\theta_1} \cdot \hat{u}_{\theta_2}$$

$$(1) =$$

$$(\vec{r}_3^2)$$



$$(\hat{i} \cdot \hat{j} = 0 \perp)$$

$$\rightarrow \hat{u}_{\theta_1} \cdot \hat{u}_{\theta_2} = (\cos\theta_1 \hat{i} + \sin\theta_1 \hat{j}) \cdot (\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j})$$

$$= \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2$$

$$= \cos(\theta_1 - \theta_2) = \cos(\theta_2 - \theta_1) \quad \text{even fu.}$$

$$r_3^2 = r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_2 - \theta_1)$$

$$\rightarrow \cos(180 - \beta) = \cos 180 \cos \beta + \sin 180 \sin \beta$$

$$= -\cos \beta$$

$$\cos(180 - \beta) = -\cos(\theta_2 - \theta_1)$$

$$r_3^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos \alpha$$

4-Bar mechanism

(L.C.E)

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = \vec{0}$$

* x-component:

$$r_2 \cos \theta_2 + r_3 \cos \theta_3 - r_4 \cos \theta_4 - r_1 \cos \theta_1 = 0$$

* y-component:

$$r_2 \sin \theta_2 + r_3 \sin \theta_3 - r_4 \sin \theta_4 - r_1 \sin \theta_1 = 0$$

$\theta_2 \rightarrow$ Input
 $\theta_4 \rightarrow$ output

$\rightarrow$ System of equations:

$$r_3 \cos \theta_3 - r_4 \cos \theta_4 = r_1 \cos \theta_1 - r_2 \cos \theta_2$$

$$r_3 \sin \theta_3 - r_4 \sin \theta_4 = r_1 \sin \theta_1 - r_2 \sin \theta_2$$

(* Non-linear equations)
How to solve?

* Solution of equation (**):

* To eliminate θ_3 : Analytical solution.

$$r_3 \hat{u}_{\theta_3} = r_1 \hat{u}_{\theta_1} - r_2 \hat{u}_{\theta_2} + r_4 \hat{u}_{\theta_4}$$

$\rightarrow$ (Dot each side by itself.)

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 - 2r_1 r_2 (\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2)$$

$$+ 2r_1 r_4 (\cos \theta_1 \cos \theta_4 + \sin \theta_1 \sin \theta_4)$$

$$- 2r_2 r_4 (\cos \theta_2 \cos \theta_4 + \sin \theta_2 \sin \theta_4)$$

$$(0 = -r_3^2)$$

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases} \text{ Linear}$$

$$\leadsto \text{OR: } A \cos \theta_4 + B \sin \theta_4 + C = 0 \quad (***)$$

$$\leadsto A = 2r_1r_4 \cos \theta_1 - 2r_2r_4 \cos \theta_2$$

$$\leadsto B = 2r_1r_4 \sin \theta_1 - 2r_2r_4 \sin \theta_2$$

$$\leadsto C = r_1^2 + r_2^2 - r_3^2 + r_4^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)$$

"given" } means known
"given" } (disposable)
"given"

$$* (\cos \theta_4, \sin \theta_4) \rightarrow x \rightarrow \text{fix}$$

$$* \cos \theta = \frac{1 - \tan^2(\theta/2)}{1 + \tan^2(\theta/2)} \quad | \quad x = \tan\left(\frac{\theta}{2}\right) = \left(\frac{1 - x^2}{1 + x^2}\right)$$

$$* \sin \theta = \frac{2x}{1 + x^2}$$

$\leadsto$ Rewrite (***) :

$$\left(A \left(\frac{1 - x^2}{1 + x^2} \right) + B \left(\frac{2x}{1 + x^2} \right) + C = 0 \right) * (1 + x^2) \rightarrow A(1 - x^2) + B(2x) + C(1 + x^2) = 0$$

$$\leadsto A(1 - x^2) + B(2x) + C(1 + x^2) = 0$$

$\leadsto$ Re-write:

$$ax^2 + bx + c = 0$$

$$\rightarrow a = C - A$$

$$\rightarrow b = 2B$$

$$\rightarrow c = A + C$$

* Solution:

$$* x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\tan(\theta_4)_{1,2}}{2}$$

$$\rightarrow (\theta_4)_{1,2} = 2 \tan^{-1}(x_{1,2})$$

$\hookrightarrow$ this means for each $\theta_2 \rightarrow (\theta_4)_1, (\theta_4)_2$

~> To find θ_3 !

x-equ. : $r_3 \cos \theta_3 = r_1 \cos \theta_1 - r_2 \cos \theta_2 + r_4 \cos \theta_4$

y-equ. : $r_3 \sin \theta_3 = r_1 \sin \theta_1 - r_2 \sin \theta_2 + r_4 \sin \theta_4$

أضرب المعادلة الأولى في $(\tan^{-1})$ ونحصل على (θ_3) من (y) و (x) ونقسمها على المعادلة الثانية.

~> Y-equation. : $\tan(\theta_3) = \frac{r_1 \sin \theta_1 - r_2 \sin \theta_2 + r_4 \sin \theta_4}{r_1 \cos \theta_1 - r_2 \cos \theta_2 + r_4 \cos \theta_4}$ (1,2)

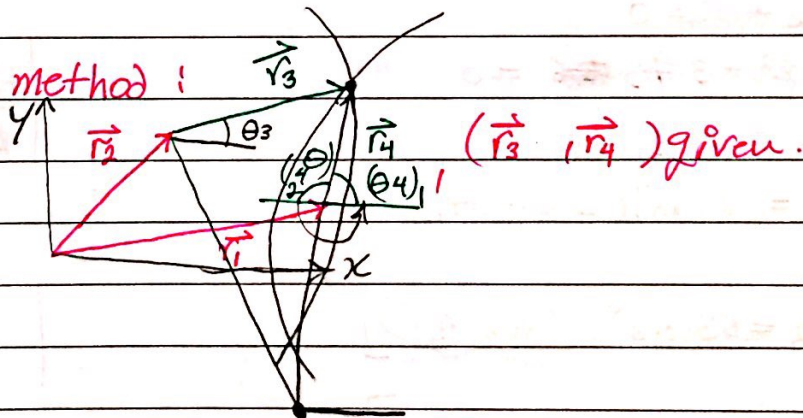
X-equation. : (1,2)

نلاحظ أن الإشارة للمتغير (θ_3) في أي ربع تقع.

$$= \frac{\pm ()}{\pm ()}$$

$(\theta_3)_1 = \checkmark$, $(\theta_3)_2 = \checkmark$
 $\rightarrow [(\theta_3)_1, (\theta_4)_1]$, $[(\theta_3)_2, (\theta_4)_2]$ for the same θ_2 .

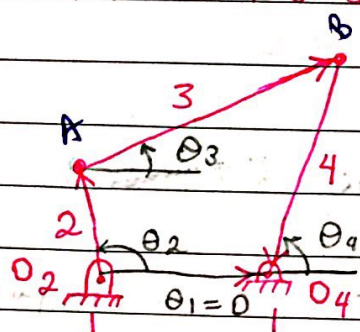
* Graphical method :



1. نرسم المتجهات $(\vec{r}_2)$ ونقيس لمتجه $(\vec{r}_3)$ ونقسم.
2. نرسم المتجهات $(\vec{r}_4)$ ونقسم.
3. متقاطعتا المنحنيين يكونان الحل (2-solutions).

* Ex. : 4-Bar.

$L_1 = 1''$, $L_2 = 2''$, $L_3 = 3.5''$, $L_4 = 4''$



For $\theta_1 = 0$, $\theta_2 = 0$

Find θ_3 & $\theta_4 = ?!$

this is given at instant moment.

* $\theta_2 \rightarrow$ Input.

* Solution:

$$2\hat{u}_{\theta_2} + 3.5\hat{u}_{\theta_3} - 4\hat{u}_{\theta_4} - \hat{u}_{\theta_1} = \vec{0}$$

$$\begin{matrix} \hat{a}_0 = \hat{i} \\ \hat{a}_{90} = \hat{j} \end{matrix}$$

$$A \cos \theta_4 + B \sin \theta_4 + C = 0$$

$$\sim A = -8$$

$$\sim B = 0$$

$$\sim C = 4.75$$



$$ax^2 + bx + c = 0$$

$$\sim 12.75x^2 - 3.75 = 0 \quad "b=0"$$

$$\sim (x)_{1,2} = \pm 0.5049$$

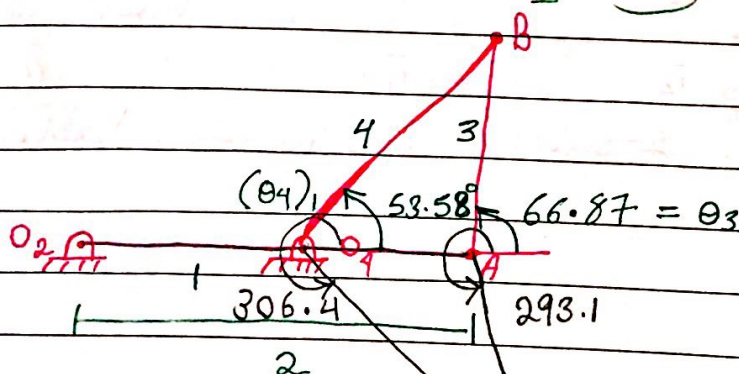
$$(\theta_4)_{1,2} = 2 \tan^{-1}(\pm 0.5049)$$

$$[(\theta_4)_{1,2} = 53.58^\circ, 306.42^\circ]$$

$$\sim [(\theta_3)_{1,2} = 66.87^\circ, 293.13^\circ]$$

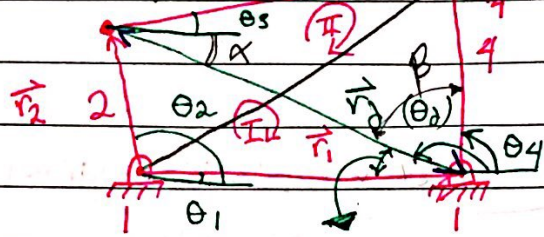
H.W!

Re-Do [graphical]



* Alternative analysis Given (r_1, r_2, r_3, r_4) & $(\theta_1), (\theta_2)$ Input

$(\alpha = \pi - \theta_0)$ $\vec{r}_3$ $\vec{r}_4$ $\sim \text{Find } (\theta_3, \theta_4) = !!$



* I can choose one triangle of the 2-Diagonals

* I choose the diagonal which I have some info about θ .

$\sim \text{L.C.E. I:}$

$$r_2 \hat{u}_{\theta_2} - r_3 \hat{u}_{\theta_3} - r_1 \hat{u}_{\theta_1} = \vec{0} \sim \text{find } (\theta_0, r_0)$$

1. $x-y$ components.

$$2. \frac{y}{x} = \tan \theta_0 \quad 3. \tan^{-1}$$

$\sim \text{L.C.E. II:}$

$$r_0 \hat{u}_{\theta_0} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} = \vec{0} \sim \text{find } (\theta_3, \theta_4)$$

$x-y$ components

** law of cosines:

$$r_0^2 = r_1^2 + r_2^2 - 2(r_1)(r_2) \cos(\theta_2 - \theta_1) \quad \left(\begin{array}{l} \text{also give given} \\ \text{Right side} \end{array} \right)$$

** law of sines:

$$\frac{r_0}{\sin(\theta_2 - \theta_1)} = \frac{r_2}{\sin(\pi - \theta_0)} \sim (\theta_0 = \checkmark)$$

** law of cosines:

$$r_3^2 = r_0^2 + r_4^2 - 2r_0r_4 \cos(\beta) \quad (\beta = \checkmark)$$

$$\sim \theta_4 + \beta + 180 - \theta_0 = 180$$

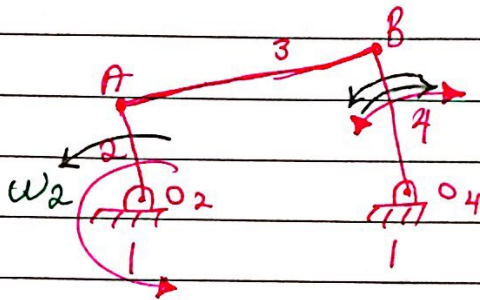
$$\Rightarrow \theta_4 = \theta_0 - \beta$$

** law of sines

$$\theta_3 = \checkmark \quad \frac{r_3}{\sin(\theta_0 - \theta_4)} = \frac{r_4}{\sin(\alpha + \theta_3)}$$

limiting Positions (Toggle positions) + (توقف الميكانيكية) +
 ~> Instant (stationary positions):

($\omega = 0$) ~~no given is $\dot{\theta}_2$ & $\dot{\theta}_4$~~ +
 velocity.

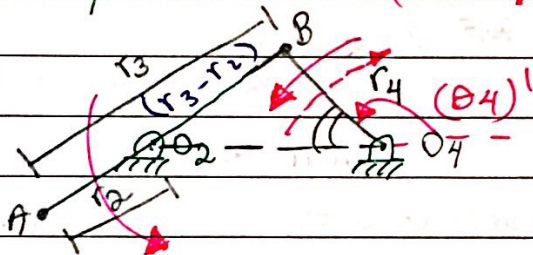


~> Crank - Rocker grashof mechanism.

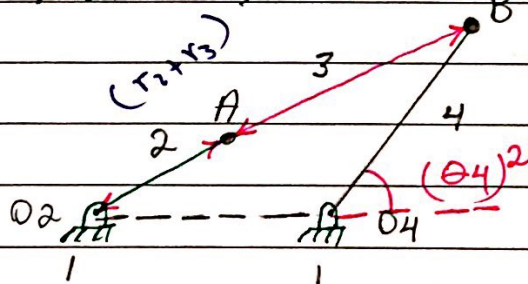
~> 2-Positions (توقف الميكانيكية في 2-مواقع).

(2-3) ~> (توقف الميكانيكية)

(1) limit position (1) + (Over-lap) above each other +

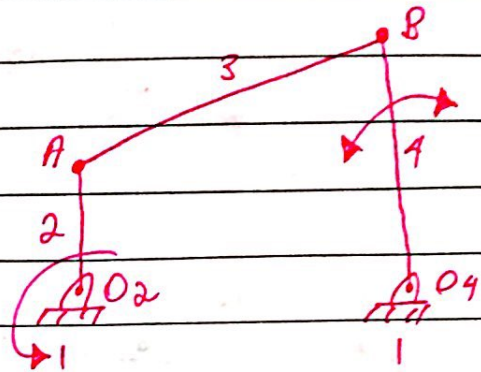


(2) limit Position (2) * Extended *

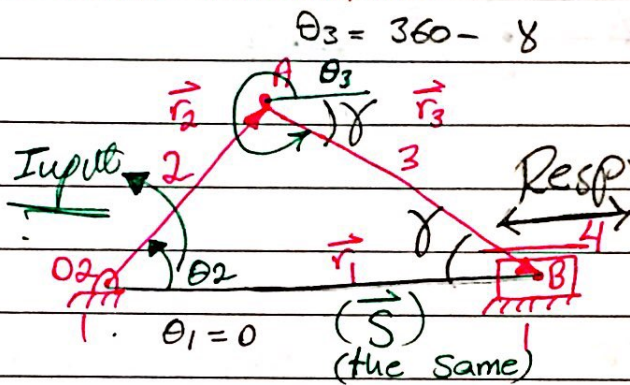


* Ex: $r_1 = 2 \text{ cm}$, $r_2 = 1 \text{ cm}$ } * Find limit positions
 $r_3 = 2.5 \text{ cm}$, $r_4 = 4 \text{ cm}$ } $(\theta_4)'$, $(\theta_4)'' = !!$
 (3) [Analytical & Graphical.]

~> Input at Link (2)



* Slider - Crank Mechanism:



~> Given: $\theta_1 = 0$, r_2 , r_3 and input θ_2
 ~> Find (S, θ_3)

** L.C.E: ?
 $r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - S \hat{u}_0 = 0$

[or * using Law of cosines and sines.]

~> x-Position!

$$r_2 \cos \theta_2 + r_3 \cos \theta_3 - S = 0$$

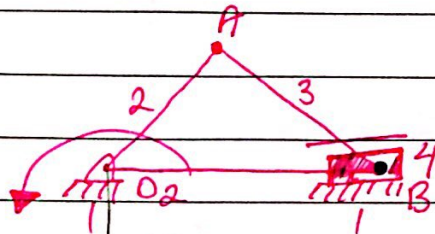
$$\frac{r_2}{\sin \gamma} = \frac{r_3}{\sin \theta_2}$$

Law of cosine to find $S = ?!$
 $\theta_3 = 360 - \gamma$

~> y-Position!

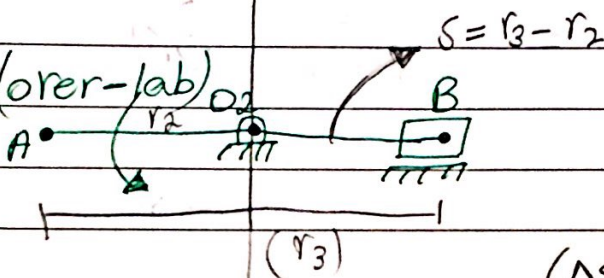
$$r_2 \sin \theta_2 + r_3 \sin \theta_3 = 0 \sim \text{Find } \theta_3$$

* limit position 1

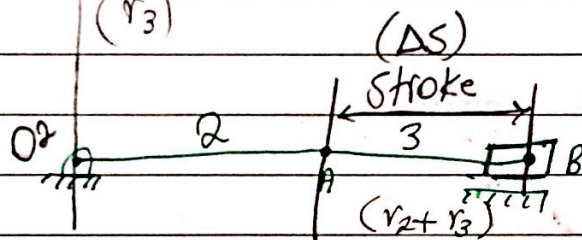


* (2-3) (over-lap)

Position (1)



Position (2)



~> Show that $(\Delta S = 2r_2)$

* Suggested ch.4:

4-6 (a)

4-11 (b)

~> Analytical + Graphical.

4-7 (a)

4-12 (b)

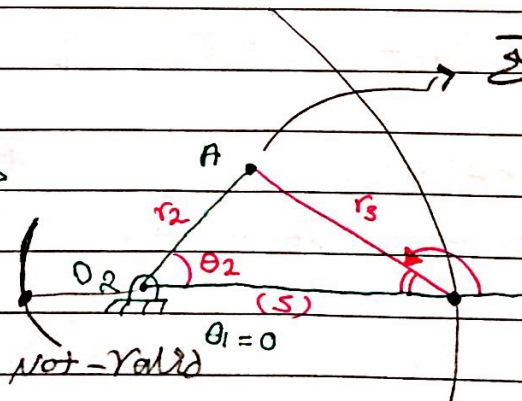
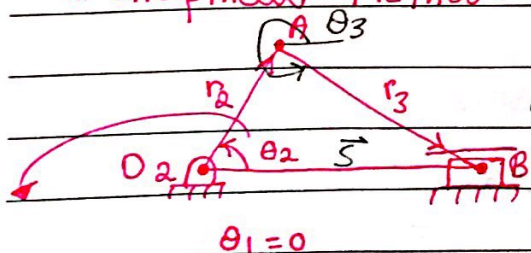
-9 (c)

-27

-10 (c)

-48 (No program)

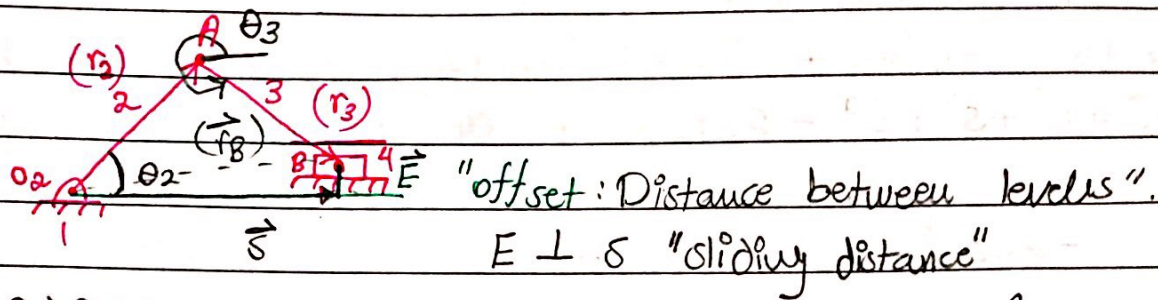
* Graphical Method 8



نسبة الفرجار ونقطة
عقدار (3)

** offset slider - crank Mechanism:

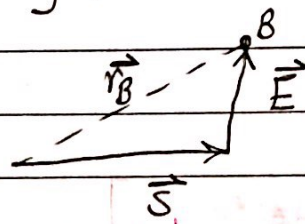
~> sliding "دفع وسحب"
(Horizontal)



~> $\theta_1 = 0$

* Given: r_2, r_3, E + input(θ_2)

* Find: S, θ_3



~> L.C.E

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - E \hat{u}_{\frac{\pi}{2}} - S \hat{u}_0 = \vec{0}$$

(f) (v)

Sliding distance

→ x-Position:

$$r_2 \cos \theta_2 + r_3 \cos \theta_3 - S = 0$$

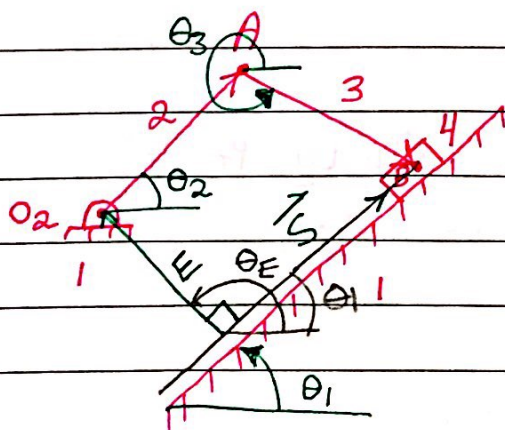
$$S = L$$

→ y-Position

$$r_2 \sin \theta_2 + r_3 \sin \theta_3 - E = 0, \theta_3 = L$$

"Inverse" حل المسألة العكس

* Most General Case: "slider - crank"



$$\theta_E = \theta_1 + \frac{\pi}{2}$$

* L.C.E^o

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - s \hat{u}_{\theta_1} + E \hat{u}_{(\theta_1 + \frac{\pi}{2})} = \vec{0}$$

$\leadsto$ To eliminate Θ_{38}

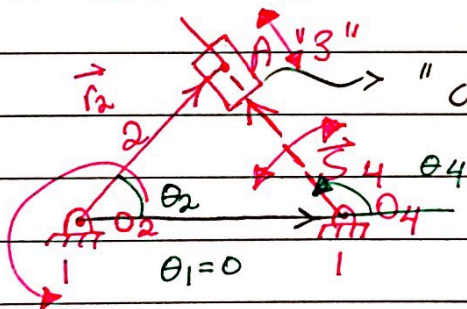
$$r_3^2 = r_2^2 + s^2 + E^2 - 2sr_2 \cos(\theta_2 - \theta_1) + 2r_2 E (\cos \theta_2 \cos(\theta_1 + \frac{\pi}{2}) + \sin \theta_2 \sin(\theta_1 + \frac{\pi}{2}))$$

$$\begin{aligned} &\leadsto \cos(\theta_1 + \frac{\pi}{2}) = -\sin\theta_1 \\ &\leadsto \sin(\theta_1 + \frac{\pi}{2}) = \cos\theta_1 \\ &\Rightarrow \begin{bmatrix} \sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1 \\ \cos\theta_2 \cos\theta_1 + \sin\theta_2 \sin\theta_1 \end{bmatrix} \\ &\hookrightarrow \begin{bmatrix} \sin(\theta_2 - \theta_1) \\ \cos(\theta_2 - \theta_1) \end{bmatrix} \end{aligned}$$

→ find ["S", 1, 2]

$$(\theta_3)_{1,2} = \tan^{-1} \left[\frac{-r_2 \sin \theta_2 + S_{1,2} \sin \theta_1 - E \cos \theta_1}{-r_2 \cos \theta_2 + S_{1,2} \cos \theta_1 + E \sin \theta_1} \right], \text{ all Right Side known}$$

Inverted slider - crank mechanism



$$\begin{aligned} (04 \rightarrow 0_2) &\equiv \theta = 180 \\ (0_2 \rightarrow 04) &\equiv \theta = 0 \end{aligned}$$

* L.C.E^o (?) (?)

$$n_2 \hat{a}_2 - 5 \hat{a}_4 - n \hat{a}_0 = \vec{0}$$

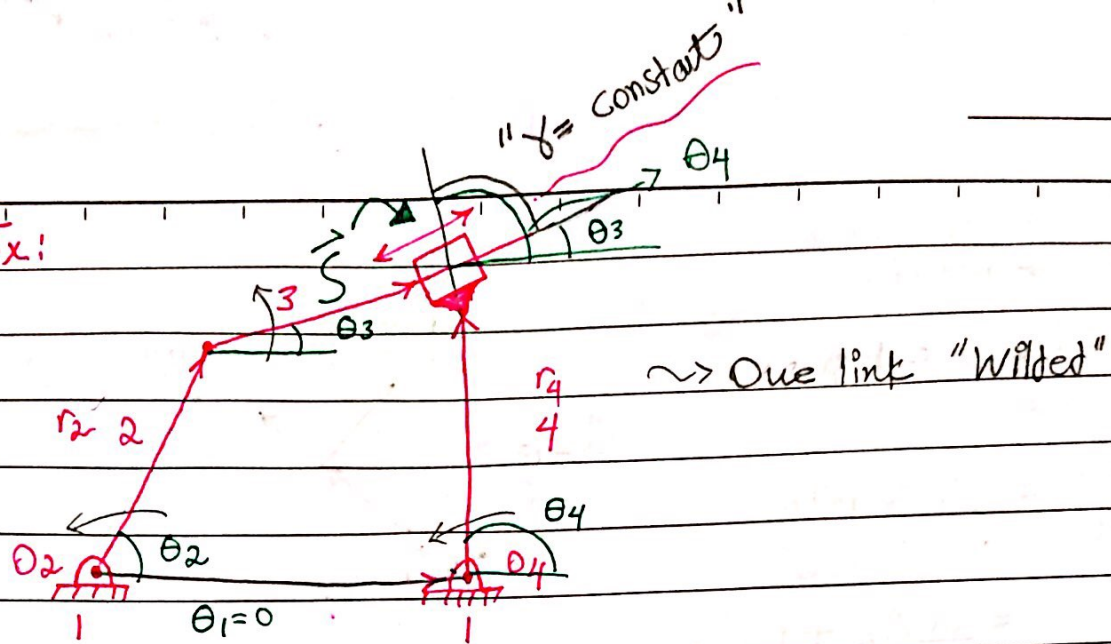
$\downarrow$
 Input

- * Re-writes

$\hat{S} \hat{U} \theta_1 = r_2 \hat{U} \theta_2 - r_1 \hat{U} \theta_0$ "Dot each side by Itself"
 $\rightarrow S^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta_2 \leadsto (S)_{1,2}$

$$(\theta_4)_{1,2} = \tan^{-1} \left[\frac{r_2 \sin \theta_2}{r_2 \cos \theta_2 - r_1} \right]$$

* Ex:

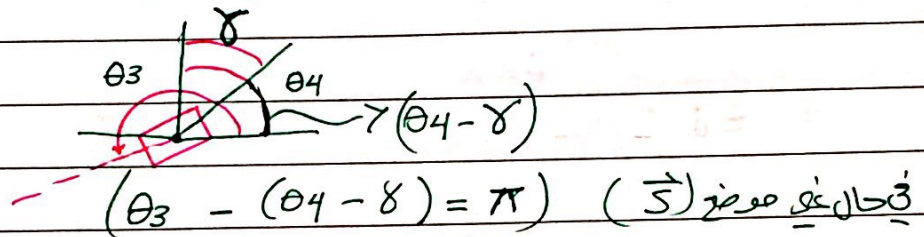


$\sim \rightarrow$ Given: $r_1, r_2, r_4, \gamma, \theta_1$ + Input θ_2 .

$\sim \rightarrow$ Find: θ_3, θ_4, s "3-unknowns = ?!"

oo $\gamma = \theta_4 - \theta_3$ "using diagonal"

* Other case:



$$r_2 \dot{\theta}_2 + s \dot{\theta}_3 - r_4 \dot{\theta}_4 - r_1 \dot{\theta}_1 = 0$$

$\sim \rightarrow$ Find θ_3 / θ_4 , $\gamma = \theta_4 - \theta_3$

(zero) = $\pm$ بالسرعة $(\hat{i}, \hat{j}, \hat{k})$ direction

Velocity analysis

$\vec{v} = \frac{d\vec{r}}{dt}$
 magnitude
 Direction

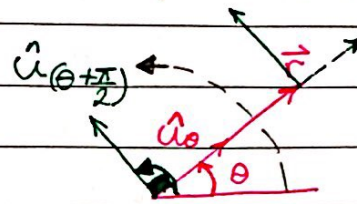
$\vec{r} = r\hat{u}_\theta$

$\omega \times \vec{r} = \vec{v} \equiv \text{velocity (tangential)}$

$$\vec{v} = \frac{d\vec{r}}{dt} = \left(\frac{d}{dt} \right) (r \hat{u}_\theta)$$

$$= \frac{dr}{dt} \hat{u}_\theta + r \frac{d\hat{u}_\theta}{dt}$$

 $(\dot{r}) = \frac{dr}{dt}$



$\hat{u}_\theta = \cos\theta \hat{i} + \sin\theta \hat{j}$

$\frac{d\hat{u}_\theta}{dt} = \left(\frac{d\hat{u}_\theta}{d\theta} \right) \left(\frac{d\theta}{dt} \right)$

Implicitly $\left(\frac{d\theta}{dt} \right) = \dot{\theta}$

$\dot{\theta} = \omega$

$= \dot{\theta} (-\sin\theta \hat{i} + \cos\theta \hat{j}) \rightarrow \hat{u}_{(\theta + \frac{\pi}{2})}$

$\rightarrow \hat{u}_{(\theta + \frac{\pi}{2})} = \underbrace{\cos(\theta + \frac{\pi}{2})}_{-\sin\theta} \hat{i} + \underbrace{\sin(\theta + \frac{\pi}{2})}_{\cos\theta} \hat{j}$

$$\vec{v} = \dot{r} \hat{u}_\theta + r \dot{\theta} \hat{u}_{(\theta + \frac{\pi}{2})}$$

 radial velocity | tangential velocity

$\dot{\theta} = \omega \rightarrow \text{c.c. } \omega (+ve)$
 $\vec{\omega} = \pm \dot{\theta} \hat{k}$
 $\hookrightarrow \text{cur } (-ve)$

2D-motion, so ω in $(z\text{-Direction})$.

$$\vec{\omega} \times \vec{r} = (\dot{\theta} \hat{k}) \times \vec{r} (\cos\theta \hat{i} + \sin\theta \hat{j})$$

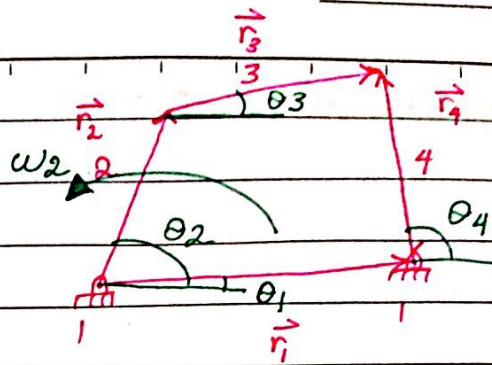
$$= \dot{\theta} (\cos\theta \hat{j} - \sin\theta \hat{i})$$

$$= \dot{\theta} \hat{u}_{(\theta + \frac{\pi}{2})}$$

* 4 Bar mechanism:

~> Knowns:

1. Complete position.
2. Input: ω_2
3. Find: ω_3 / ω_4



* L.C.E! Positions لا تعرف الزاوية في البداية

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = \vec{0}$$

* V.E! $\frac{d}{dt}(\text{L.C.E}) = 0$ } $r_{2,3,4,1} = \text{constants}$ $\dot{r} = 0$

$$** \quad r_2 \dot{\hat{u}}_{\theta_2} + r_2 \dot{\theta}_2 \hat{u}_{\theta_2} + \frac{\pi}{2} + 0 + r_3 \dot{\hat{u}}_{\theta_3} + \frac{\pi}{2} - 0 - r_4 \dot{\hat{u}}_{\theta_4} + \frac{\pi}{2} - 0 - 0 = 0$$

($r_i = \text{no motion}$)
ground link vel. always (zero).

* Solution. 1:

→ x-velocity:

$$-r_2 \omega_2 \sin \theta_2 - r_3 \omega_3 \sin \theta_3 + r_4 \omega_4 \sin \theta_4 = 0$$

it does more
→ constant
mag. Dir.

→ y-velocity:

$$r_2 \omega_2 \cos \theta_2 + r_3 \omega_3 \cos \theta_3 - r_4 \omega_4 \cos \theta_4 = 0$$

~> Solve simultaneously ~> Find (ω_3, ω_4)

* Solution. 2: Eliminate (ω_3)

~> Dot product by ($\hat{u}_{\theta_3}$): ** (dot product $\hat{u}_{\theta_3}$)
 "V.E"

$$r_2 \omega_2 (\hat{u}_{\theta_2 + \frac{\pi}{2}} \cdot \hat{u}_{\theta_3}) - r_4 \omega_4 (\hat{u}_{\theta_4 + \frac{\pi}{2}} \cdot \hat{u}_{\theta_3}) = 0$$

$$\rightarrow (\hat{u}_{\theta_2 + \frac{\pi}{2}} \cdot \hat{u}_{\theta_3}) = -\sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3$$

$$= \sin(\theta_3 - \theta_2)$$

$$\rightarrow (\hat{u}_{\theta_4 + \frac{\pi}{2}} \cdot \hat{u}_{\theta_3}) = \sin(\theta_3 - \theta_4)$$

$$\rightarrow \omega_4 = \frac{r_2 \sin(\theta_3 - \theta_2)}{r_4 \sin(\theta_3 - \theta_4)} \cdot \omega_2$$

To find (ω_3) $\left\{ \begin{array}{l} \text{X-Y Position (OR)} \\ \text{Eliminate } \omega_4 \end{array} \right.$

$$\rightarrow \omega_3 = \frac{-r_2 \sin(\theta_4 - \theta_2)}{r_3 \sin(\theta_4 - \theta_3)} \cdot \omega_2$$

* Self-Reading:

1. Angular Velocity Ratio (m_v), output (ω) / Input (ω)
2. Torque Ratio (m_T)
3. Mechanical advantage (m_A)

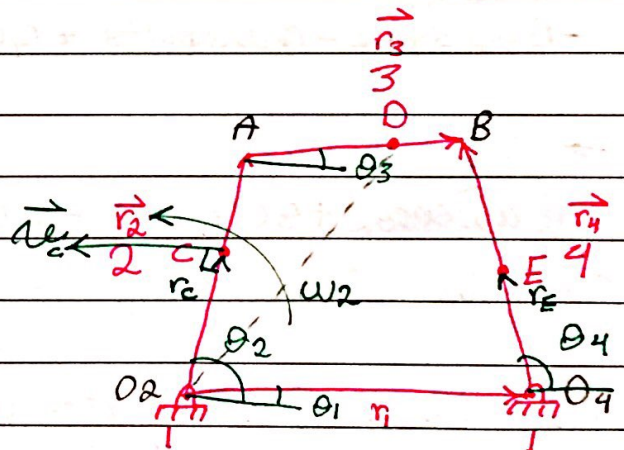
* Velocity:

$$\vec{\tau}_C = \vec{\omega}_2 \times \vec{r}_C \quad \rightarrow \text{Dynamics}$$

$$r_C \hat{u}_{\theta_2} \quad \text{Analysis}$$

$$\vec{\tau}_E = \vec{\omega}_4 \times \vec{r}_E$$

$$r_E \hat{u}_{\theta_4}$$



$$\left\{ \begin{array}{l} \vec{\tau}_D = \vec{\tau}_A + \vec{\tau}_{D/A} \text{ [G.P.M.]} \\ = \vec{\omega}_2 \times \vec{r}_2 + \vec{\omega}_3 \times \vec{r}_{D/A} \end{array} \right.$$

$\rightarrow$ (OR)

$$\rightarrow \vec{r}_C = r_C \hat{u}_{\theta_2} \quad \text{"0" "machinery Analysis"}$$

$$\vec{\tau}_C = \frac{d\vec{r}_C}{dt} = r_C \dot{\theta}_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} + r_C \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}}$$

$$\vec{r}_E = r_E \hat{u}_{\theta_4}$$

$$\vec{v}_E = r_E \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}}$$

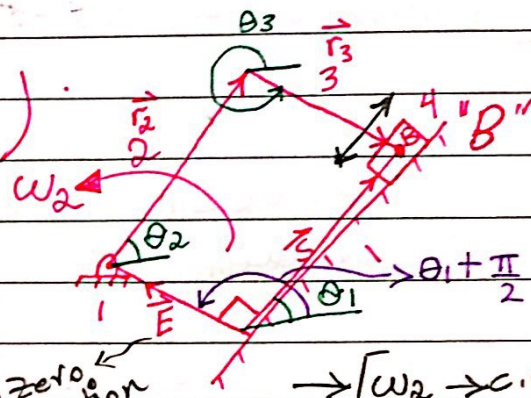
$$\vec{r}_D = \vec{r}_2 + \vec{r}_{D/A} = r_2 \hat{u}_{\theta_2} + r_{D/A} \hat{u}_{\theta_3}$$

$$\vec{v}_D = r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} + r_{D/A} \omega_3 \hat{u}_{\theta_3 + \frac{\pi}{2}}$$

* Ex: Slider-crank: "Most general case"

~ Known: Position + inputs ω_2

~ Find: $\dot{S}$ (Velocity of slider(4)
or Vel. of point B)



~ L.C.E

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - S \hat{u}_{\theta_1} + E \hat{u}_{\theta_1 + \frac{\pi}{2}} = \vec{0}$$

~ V.E

$$r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} - \dot{S} \hat{u}_{\theta_1} = \vec{0}$$

zero motion

$\rightarrow [\omega_2 \rightarrow \text{c.w} \rightarrow -ve]$

$\rightarrow$ Sliding $\hat{u}$

$\theta_1 = \text{const.}$

$S = \text{not "}$

$\hookrightarrow$ variable "not const."

slider $\rightarrow$ $\hat{u}$ $\rightarrow$ $\hat{u}$

$\hookrightarrow$ makes translation, no angular velocity.

* x-velocity

* y-velocity

** Eliminate $\omega_3 \Rightarrow (\cdot \hat{u}_{\theta_3}) : \hat{u}_{\theta_3 + \frac{\pi}{2}} \cdot \hat{u}_{\theta_3} = \text{zero}$

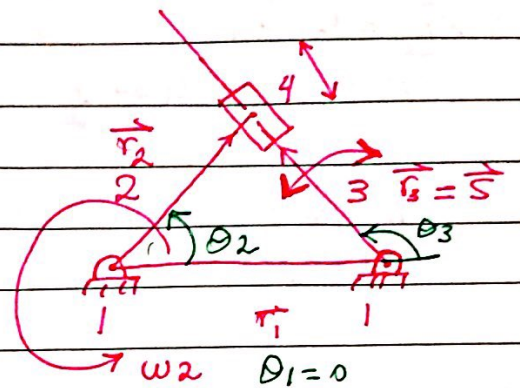
$$\dot{S} = \frac{r_2 \sin(\theta_3 - \theta_2)}{\cos(\theta_3 - \theta_1)} \cdot \omega_2$$

~ Then find (ω_3)

* Ex. : Inverted slider crank :

→ Knowns : Position + ω_2

→ Find : ω_3 , $\dot{s}$ (slip)



* L.C.E :

$$r_2 \hat{u}_{\theta_2} - s \hat{u}_{\theta_3} - r_1 \hat{u}_0 = \vec{0}$$

* V.E :

$$\left[r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - \dot{s} \hat{u}_{\theta_3} - s \omega_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} = \vec{0} \right]$$

→ 4 : translation + Rotation.

→ $\theta_3 + \frac{\pi}{2} = \text{const.}$

→ To find $\dot{s}$:

$$\dot{s} = r_2 \omega_2 \sin(\theta_3 - \theta_2)$$

$$= v_{\text{slip}}$$

ω_3

→ To find ω_3 : $\cdot \hat{u}_{\theta_3 + \frac{\pi}{2}}$

$$\omega_3 = \frac{r_2 \omega_2 \cos(\theta_3 - \theta_2)}{s}$$

+

* Multi-loop Mechanisms : loop mechanism

* Ex. : quick-Return mechanisms :

$$\rightarrow M = 3(6-1) - 2(7) - 0 = (1 \text{ DOF})$$

* L.C.E (I) :

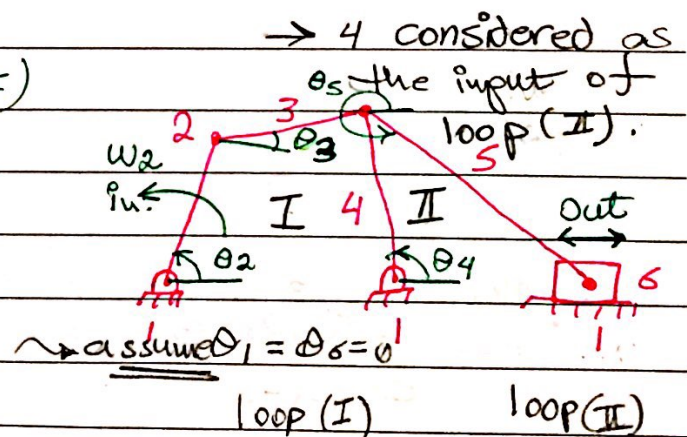
$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = \vec{0}$$

→ to find θ_4, θ_3

* L.C.E (II) :

$$r_4 \hat{u}_{\theta_4} + r_5 \hat{u}_{\theta_5} - s_6 \hat{u}_{\theta_6} = \vec{0}$$

→ to find s_6, θ_5



→ assumed $\theta_1 = \theta_6 = 0$

loop (I)

loop (II)

$$\sim \rightarrow CD: \theta = 270^\circ (+ve)$$

$$\sim \rightarrow DC: \theta = 90^\circ (-ve)$$

Problem 6-66:

$\sim \rightarrow 1, 2, 3, 4$: Inverted slider crank.

$\sim \rightarrow 4, 5, 6$: Slider - crank.

$$* L_2 = 25.4 \text{ mm } (O_2A)$$

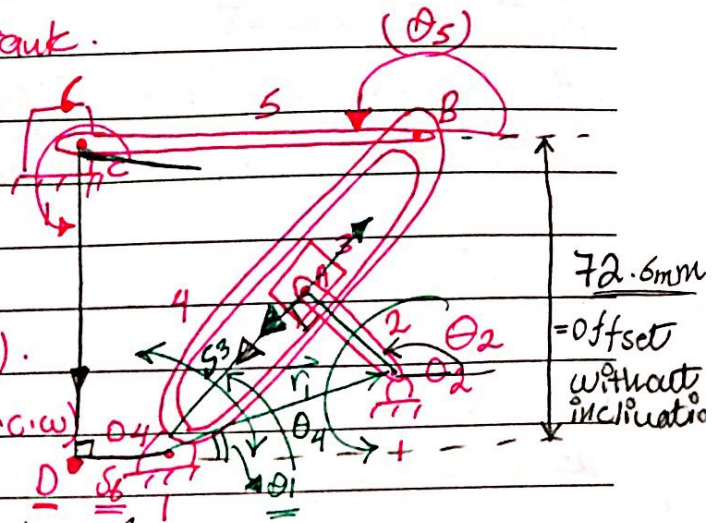
$$* L_4 = 120.9 \text{ mm } (O_4B)$$

$$* L_5 = 115.9 \text{ mm } (BC)$$

$$* O_4O_2 = 42.9 \text{ mm } (r_1) \text{ \& } 15.5^\circ (\theta_1)$$

$$\rightarrow \text{For } \theta_2 = 99^\circ \text{ \& } \omega_2 = 10 \text{ rad/s (C.C.W.)}$$

$$\rightarrow \text{Find } \vec{v}_A, \vec{v}_B, \vec{v}_C, \vec{v}_{\text{slip}} = ?$$



Sliding

1. Rotation + trans.
2. Rotation.

$$\sim \rightarrow (\vec{v}, \vec{\omega})$$

$$\rightarrow \vec{v}_A = \vec{\omega}_2 \times \vec{r}_{A/O_2}$$

since $\omega_2 =$ known

$$* \vec{v}_A:$$

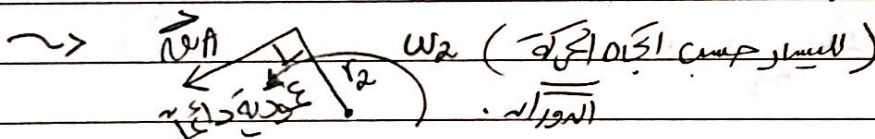
$$\vec{r}_A = \vec{r}_2 = 25.4 \hat{u}_{\theta_2}$$

$$\vec{v}_A = \frac{d\vec{r}_2}{dt} = 25.4 \omega_2 \hat{u}_{\theta_2 + \pi/2}$$

$$= 25.4 (+10) \hat{u}_{99^\circ + \pi/2} = (25.4)(+10)(-\sin 99^\circ \hat{i} + \cos 99^\circ \hat{j})$$

$\rightarrow$ don't forget the

$$= -250.87 \hat{i} - 39.73 \hat{j} \text{ (mm/s)} \leftarrow \text{units.}$$



* Loop (I) : $O_4 \rightarrow A \rightarrow O_2 \rightarrow O_4 \sim$ Inverted slider-crank.

$\sim \rightarrow$ L.C.E. I:

$$? = 53 \hat{u}_{\theta_4} - 25.4 \hat{u}_{\theta_2} - 42.9 \hat{u}_{15.5} = \vec{0}$$

* Loop (II) : $O_4 \rightarrow B \rightarrow C \rightarrow D \rightarrow O_4$ offset (CD) / slider - crank.

$\sim \rightarrow$ L.C.E. II:

$$120.9 \hat{u}_{\theta_4} + 115.9 \hat{u}_{\theta_5} + 72.6 \hat{u}_{3\pi} - 56 \hat{u}_{\pi} = \vec{0}$$

* L.C.E (I): $\vec{r}_{O_4O_2} + \vec{r}_2 - \vec{S}_3 = \vec{0}$

$$42.9\hat{u}_{15.5} + 25.4\hat{u}_{\theta_2} - S_3\hat{u}_{\theta_4} = \vec{0}$$

$$25.4\hat{u}_{\theta_2} = S_3\hat{u}_{\theta_4} - 42.9\hat{u}_{15.5}$$

$$25.4\hat{u}_{\theta_2} = S_3\hat{u}_{\theta_4} - 42.9\hat{u}_{15.5}$$

$$\vec{r}_A = 25.4\omega_2\hat{u}_{\theta_2} + \frac{\pi}{2} \quad \text{or} \quad \vec{r}_A = S_3\hat{u}_{\theta_4} + S_3\omega_4\hat{u}_{\theta_4} + \frac{\pi}{2} = \vec{0}$$

* V.E.I:

$$25.4\omega_2\hat{u}_{\theta_2} + \frac{\pi}{2} - S_3\hat{u}_{\theta_4} - S_3\omega_4\hat{u}_{\theta_4} + \frac{\pi}{2} = \vec{0} \quad \cdot \hat{u}_{\theta_4}$$

$$\rightarrow 42.9\hat{u}_{15.5} + 25.4\hat{u}_{\theta_2} = S_3\hat{u}_{\theta_4} \quad \text{Dot each side by itself.}$$

$$S_3^2 = 42.9^2 + 25.4^2 + (2)(42.9)(25.4)\cos(\theta_2 - 15.3) \quad \text{law of cosines}$$

$$\rightarrow S_3 = 52.27 \text{ mm}$$

$$\rightarrow \theta_4 = \tan^{-1} \left(\frac{42.9\sin 15.5 + 25.4\sin 99}{42.9\cos 15.5 + 25.4\cos 99} \right) = (44.37^\circ)$$

$$\rightarrow \dot{S}_3 = 25.4\omega_2\sin(\theta_4 - \theta_2)$$

$$\text{For } \theta_2 = 99^\circ, \theta_4 = 44.37^\circ, \omega_2 = +10 \text{ ;}$$

$$\dot{S}_3 = -207.12 \text{ mm/s} = v_{\text{slip}}$$

$$\rightarrow \vec{v}_{\text{slip}} = -207.12\hat{u}_{44.37} = ()\hat{i} + ()\hat{j}$$

$$\rightarrow \text{To find } \omega_4: (\cdot \hat{u}_{\theta_4} + \frac{\pi}{2})$$

$$\omega_4 = \frac{25.4(+10)\cos(\theta_2 - \theta_4)}{S_3} = +2.81 \text{ rad/s (c.c.w)}$$

$$\rightarrow \vec{v}_B$$

$$\vec{r}_B = \vec{r}_{B/O_4}$$

$$= 120.9\hat{u}_{\theta_4}$$

$$\vec{v}_B = 120.9\omega_4\hat{u}_{\theta_4} + \frac{\pi}{2}$$

$$= 120.9(+2.81)(-\sin 44.37\hat{i} + \cos 44.37\hat{j})$$

$$\vec{v}_B = -239.25\hat{i} + 242.85\hat{j} \text{ (mm/s)}$$

$O_2 \rightarrow A \rightarrow B$ another complicated way.

* L.C.E. (II) :

$$\vec{OB} + \vec{BC} - \vec{E} - \vec{S}_6 = \vec{0} \quad \begin{matrix} (\hat{j}) & (-\hat{i}) \end{matrix}$$

$$\underbrace{120.9 \hat{u}_{\theta_4} + 115.9 \hat{u}_{\theta_5}}_{r_c} = \underbrace{72.6 \hat{u}_{\frac{\pi}{2}} + 56 \hat{u}_{\pi}}_{r_c}$$

بسم الله الرحمن الرحيم

$\star V \cdot E \cdot (\pi) :$

$$120.9 \omega_4 \hat{u}_{04} + \frac{\pi}{2} + 115.9 \omega_5 \hat{u}_{05} + \frac{\pi}{2} = \dot{\phi}_c \hat{u}_\pi$$

(c) and (d) are not in the same class.

→ From position analysis: (θ_s) (only)

* Y-Component of the L.C.F (II):

$$120.9 \sin(44.37) + 115.9 \sin \theta_5 = 72.6$$

$$\theta_5 = 185.9^\circ \text{ (CW)}$$

$\leadsto$ To find $\dot{x}_i(\cdot, u_0)$

$$120.9 \omega_4 \sin(\theta_5 - \theta_4) = \dot{S}_6 \cos(\pi - \theta_5)$$

$$(120.9)(2.81) \sin(185.4 - 44.37) = 56 \cos(-5.9^\circ)$$

→ $\dot{S}_6 = 212.47 \text{ mm/s} = v_c \leftarrow \underline{S_6}$ مع ایام خضه

$$N_z = -212.47 \hat{i} \text{ (mm/s)} \cdot \hat{u}_{88.5}$$

$$\rightarrow \omega_5 = 2.107 \text{ rad/s (ccw)}$$

* Method of Instant. Centers: (I.C) "Pure graphical".

→ An Instant center is a point ~~common~~ to two (or more) links and has the same velocity in each link.
 (Ex. all joints (O_2/O_4) , (A/B)).

For planner linkages: the no. of (I.c's) is:

$$N_{I.C} = \frac{L(L-1)}{2}, \quad L \equiv \text{no. of links.}$$

→ Ex: (4-Bar mech.)

$$N_{I.C} = \frac{4(4-1)}{2} = 6 \text{ I.C's}$$

* Some of the I.C's are found by inspection.
→ (Ex. R-Joint).

* The remaining I.C's can be found using (Kennedy's Rule):

→ Kennedy's Rule: links

Any three bodies in plane motion will have exactly (3) I.C's and they will lie on the same straight line.

$$(Ex: I_{1,2}, I_{2,3}, I_{1,3}). \quad (\text{non zero})$$

$(I_{2,1}, I_{3,2}, I_{3,1})$ give

← 31/8/2018 اكل على فطور الصباح / لا فطور

* Ex. 4: 4-Bar mechanism:

$$\rightarrow \text{I.C's} = \frac{4(4-1)}{2} = 6 \text{ I.C's}$$

→ Kennedy's Rule!

→ links (1,2,3) : $I_{1,2}$
 $I_{2,4}$ → $I_{1,3}$ (circled)
 $I_{3,2}$
 * unknown

→ links(1, 3, 4)!

* Note 8 $I_{(1,2)} / I_{(1,14)}$
~~ground link~~
 (ground link)

← رابطہ (links) اور شناخت (I.C's)

* Suggested ch. 6 :

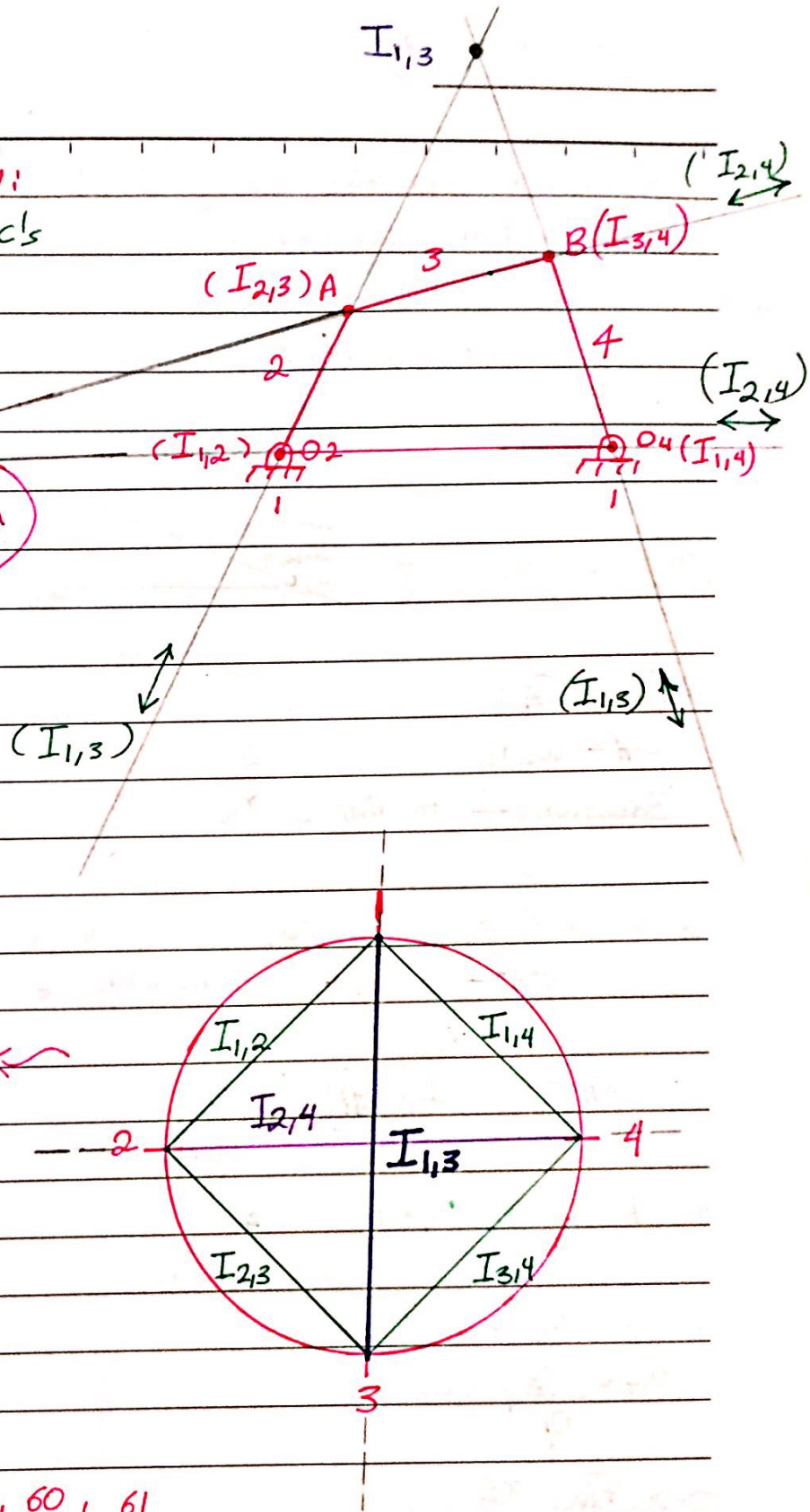
$$6 - 5(a)$$

7(c)

9(b)

12, 13, 14, 15, 21 (b), 60, 61

64, 88, 90, 93, 97.



→ Scale 1:12
1:10

اضرب بـ 2
اضرب بـ 10

→ link 3
Rotates about
($I_{1,3}$)

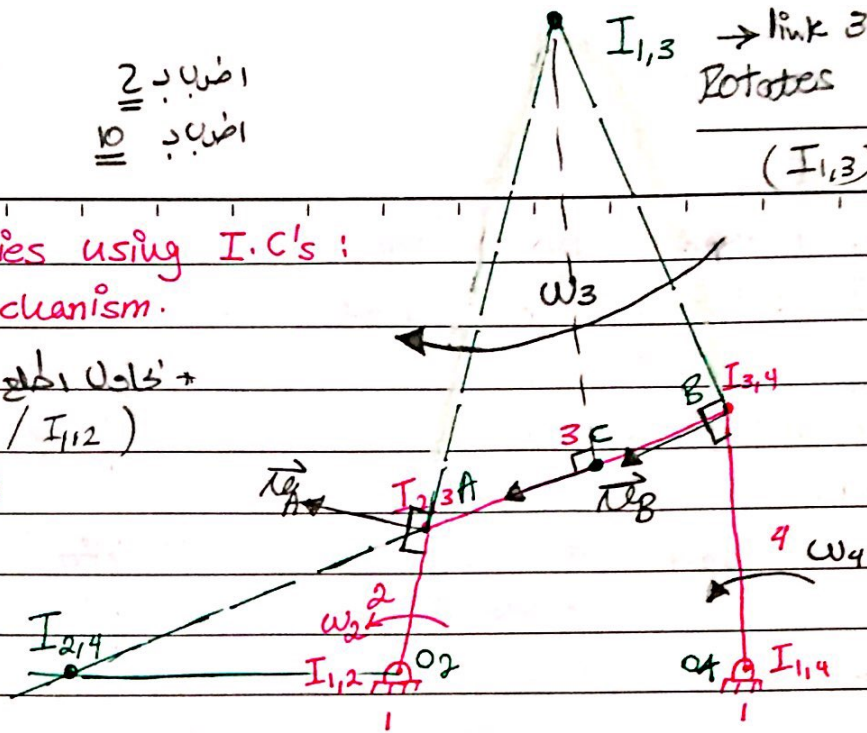
* Finding velocities using I.C.'s :

* Ex: 4-Bar mechanism.

no instantaneous velocity of any links +

if any link ($I_{1,4}$ / $I_{1,2}$)

→ Find (ω_3 / ω_4) :



1. Find $\vec{v}_A$:

$$v_A = \omega_2 r_2$$

Direction: $\perp$ to link 2 according to direction of ω_2 "tangential"

2. $\omega_3 = \frac{v_A}{I_{1,3} I_{2,3}}$; (CW) according to direction of v_A .

→ we measure it. (Scale) $\frac{1}{10}$

$$\rightarrow v_A = \omega_3 (I_{3,1}) (A)$$

→ measure it

3. $v_B = \omega_3 (I_{1,3} B)$, ✓

$$4. v_B = \omega_4 r_4$$

$$\omega_4 = \frac{v_B}{r_4} , (CC \cdot \omega)$$

→ Find $\vec{v}_C$:

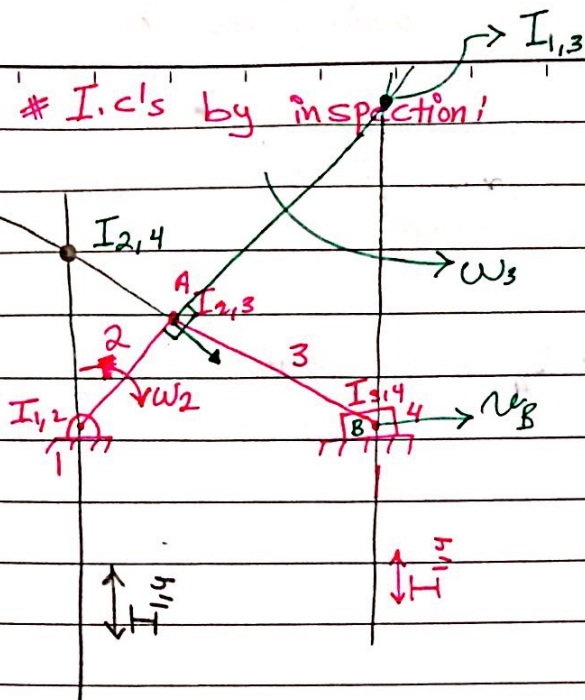
$$v_C = \omega_3 (I_{1,3} C)$$

→ measured

* Reminder (mv) → I.C "4-Bar" (self-Reading).

Note* ($I_{1,3}$ for slider $\rightarrow \pm\infty$)

* I.c's by inspection!



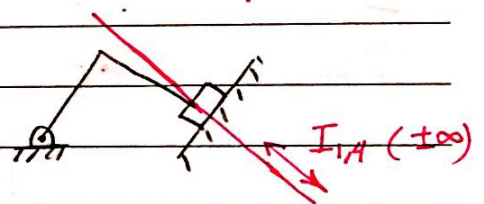
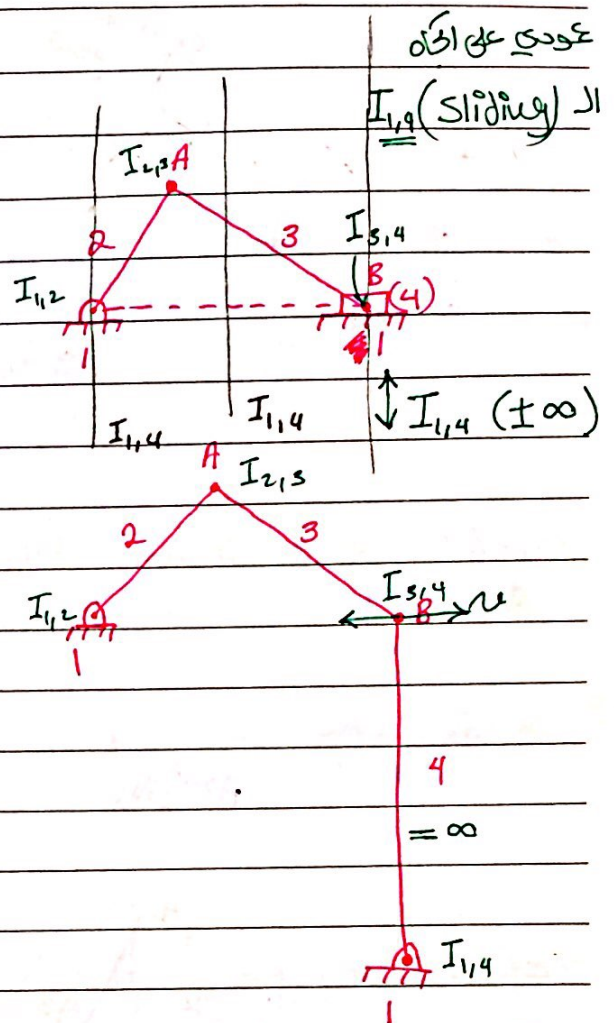
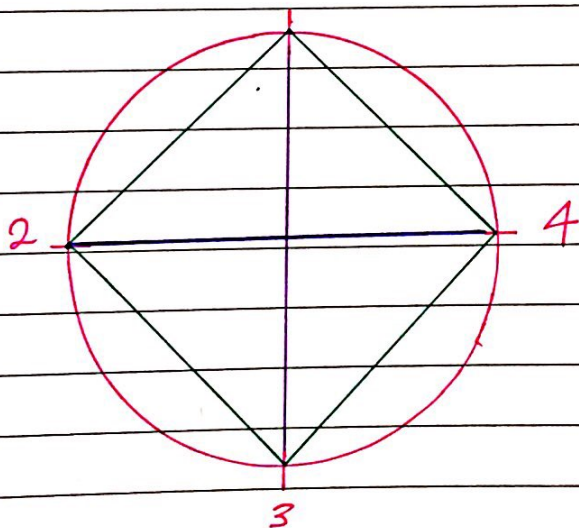
$\rightarrow$ Find ω_3 , $\vec{v}_4 = ?$

1. $\vec{v}_A = \omega_2 \vec{r}_2$

2. $\omega_3 = \omega_A$ (CCW)

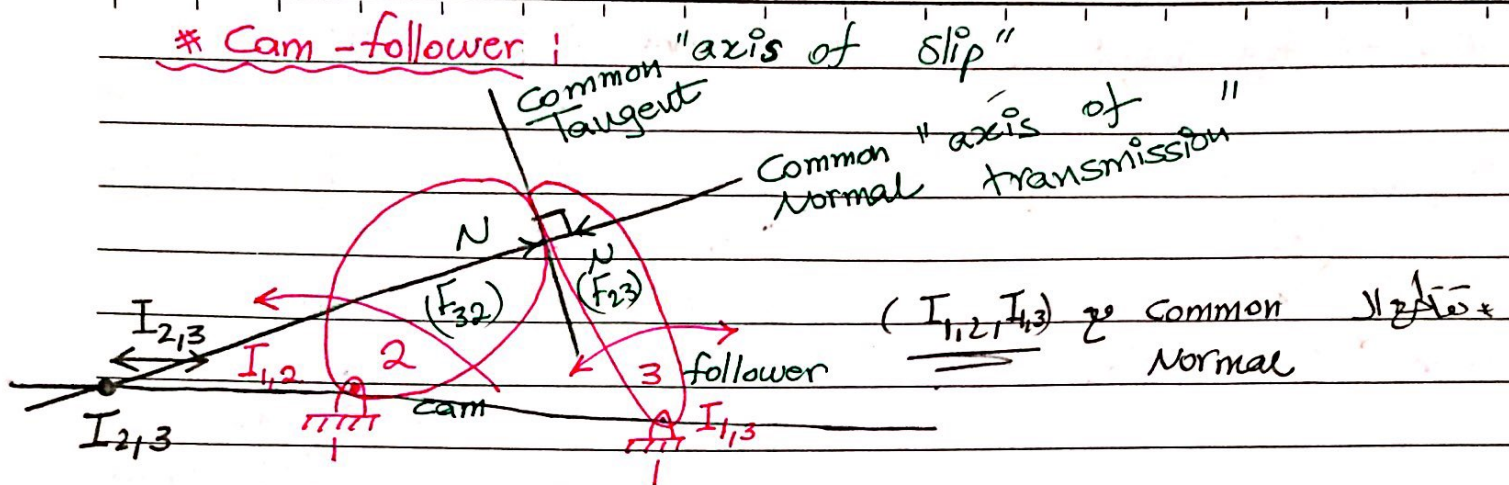
$I_{1,3}A$

3. $\vec{v}_B = \omega_3 (I_{1,3}B) \rightarrow$



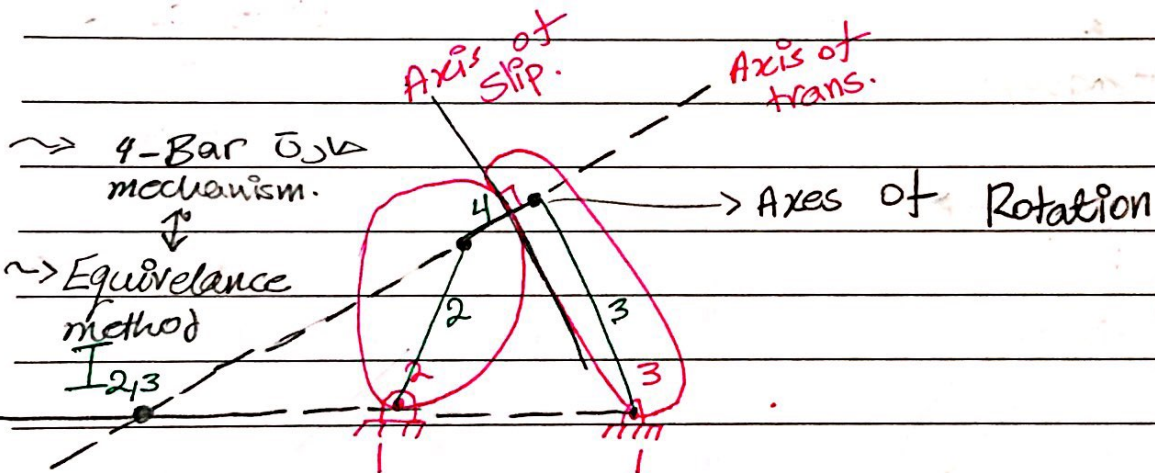
$(F_{23} = F_{32})$ action & Reaction.

* Cam-follower :



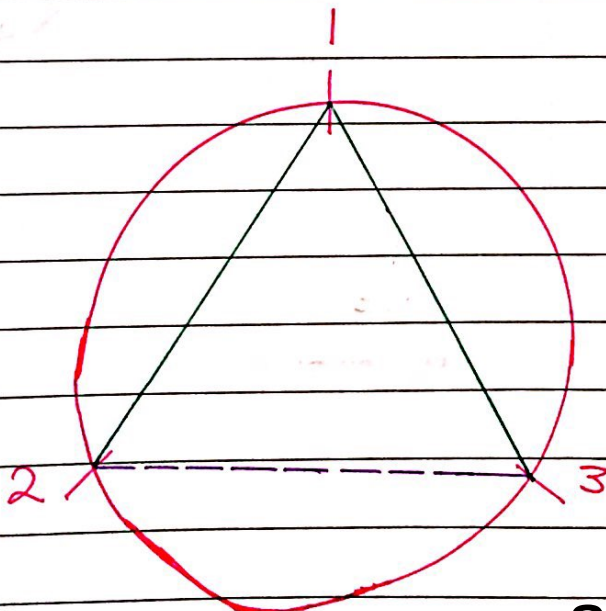
~> Axis of Transmission:

~> "follower" $\rightarrow$ $I_{1,2}, I_{2,3}, I_{3,1}$ are the axes of rotation.



~> 4-Bar mechanism.

~> Equivalence method



(a) Not included.

* 6-18: (b, c) (2, 4) $\rightarrow$ translation / 3 $\rightarrow$ GPM

$AB = 43.7 \text{ mm} \rightarrow \vec{r}_3 = 45.7 \hat{u}_{\theta_3}$

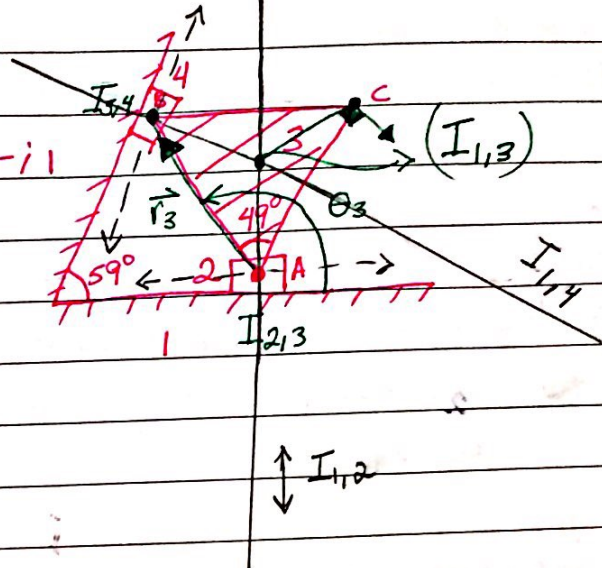
$AC = 36.6 \text{ mm}$

* For $\theta_3 = 128^\circ$ & $\vec{v}_A = 254 \text{ mm/s} \leftarrow i$

* Find: $\vec{v}_B, \vec{v}_C$

b) I.C.

c) Analytic.



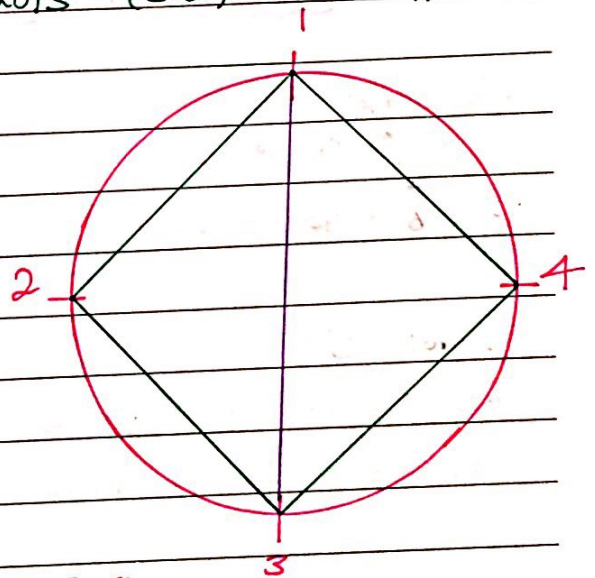
$\rightarrow I.C.'s = \frac{4(4-1)}{2} = 6$

* (b)

$\rightarrow v_A = \omega_3 (I_{1,3}A) \Rightarrow \omega_3 = 13.28 \text{ rad/s (c.w) since } \vec{v}_A \leftarrow$
(19 mm)

$\rightarrow v_B = \omega_3 (I_{1,3}B) = 436 \text{ mm/s}$
13.28" 33 mm

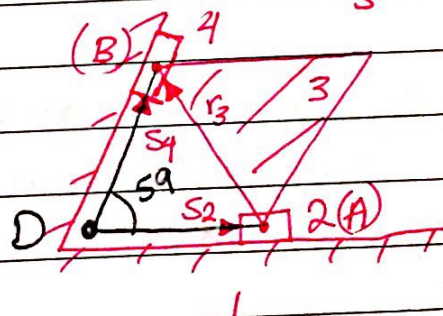
$\rightarrow v_C = \omega_3 (I_{1,3}C) = 241.5 \text{ mm/s}$



* (c)

$\rightarrow$ L.C.E!

$\vec{DA} + \vec{AB} + \vec{BD} = \vec{0}$
 $\rightarrow S_2 \hat{u}_0 + 45.7 \hat{u}_{\theta_3} - S_4 \hat{u}_{59^\circ} = \vec{0}$



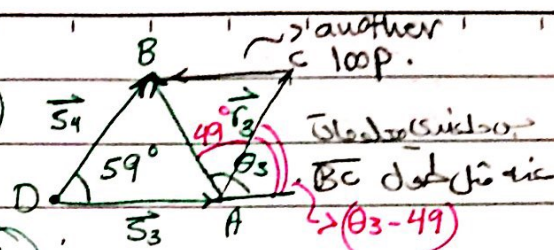
$$\rightarrow S_2 \hat{u}_0 + 45.7 \hat{u}_{\theta_3} - S_4 \hat{u}_{59} = \vec{0}$$

* Position analysis (For $\theta_3 = 128^\circ$ "Input")

$\rightarrow$ law of sines

$$\frac{45.7}{\sin 59^\circ} = \frac{S_4}{\sin (180 - 128)} \Rightarrow S_4 = 42.013 \text{ mm}$$

then find $S_2 \approx 49$.



* V.E 8

$$\rightarrow \dot{S}_2 \hat{u}_0 + 45.7 \dot{\omega}_3 \hat{u}_{(\theta_3 + 90^\circ)} - \dot{S}_4 \hat{u}_{59} = \vec{0}$$

$$\rightarrow \text{For } \dot{S}_2 = -254 \text{ mm/s} \Rightarrow \vec{v}_A = \dot{S}_2 \hat{u}_0$$

$$\theta_3 = 128^\circ$$

$$\rightarrow \dot{S}_2 \cos \theta_3 - \dot{S}_4 \cos (59 - \theta_3) = 0$$

$$\rightarrow \dot{S}_4 = 436.36 \text{ mm/s} \left[(\cos 59^\circ \hat{i} + \sin 59^\circ \hat{j}) = \vec{r}_B \right]$$

$$* \vec{r}_B = \vec{r}_{B/D}$$

$$= \dot{S}_4 \hat{u}_{59^\circ}$$

$$\rightarrow \vec{v}_B = 436.36 \hat{u}_{59^\circ}$$

* Y-velocity 8

$$45.7 \dot{\omega}_3 \cos 128^\circ = 436.36 \sin 59^\circ$$

$$\Rightarrow \dot{\omega}_3 = -13.29 \text{ rad/s}$$

$$= 13.29 \text{ rad/s (CW)}$$

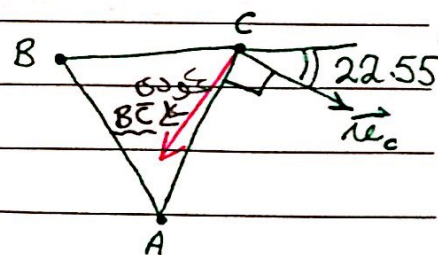
$$* \vec{r}_C = \vec{r}_{C/D} = S_2 \hat{u}_0 + 36.6 \hat{u}_{\theta_3 - 49^\circ}$$

$$\rightarrow \vec{v}_C = \dot{S}_2 \hat{u}_0 + 36.6 \dot{\omega}_3 \hat{u}_{\theta_3 - 49^\circ + 90^\circ}$$

(-254) -13.29

$$\vec{v}_C = 223.477 \hat{i} - 92.812 \hat{j} \text{ mm/s}$$

$$v_C = \sqrt{223.477^2 + 92.812^2} = 242 \text{ mm/s}$$



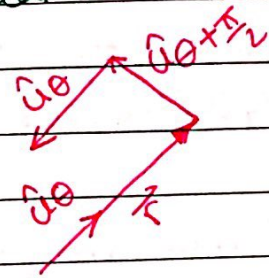
* Acceleration analysis

$$\vec{a} = \frac{\partial \vec{v}}{\partial t} = \frac{\partial}{\partial t} (r \hat{u}_\theta + r \dot{\theta} \hat{u}_{\theta+\pi/2})$$

$$= \ddot{r} \hat{u}_\theta + \underbrace{\dot{r} \dot{\theta} \hat{u}_{\theta+\pi/2}}_{\omega} + \underbrace{\dot{r} \ddot{\theta} \hat{u}_{\theta+\pi/2}}_{\alpha \text{ angular acc.}} + r \ddot{\theta} \hat{u}_{\theta+\pi/2} + r \dot{\theta} \dot{\theta} \hat{u}_\theta$$

(+/-), (+/-) (لازم نشاء)

$$\begin{aligned} \hat{u}_{\theta+\pi/2} &= -\sin\theta \hat{i} + \cos\theta \hat{j} \\ \dot{\theta} \hat{u}_{\theta+\pi/2} &= \dot{\theta} (-\cos\theta \hat{i} - \sin\theta \hat{j}) \\ &= -\dot{\theta} \hat{u}_\theta \end{aligned}$$



$$\vec{a} = (\ddot{r} - r\dot{\theta}^2) \hat{u}_\theta + (r\alpha + 2\dot{r}\dot{\theta}) \hat{u}_{\theta+\pi/2}$$

$\vec{a}_{slip} = \underbrace{\ddot{r} - r\dot{\theta}^2}_{\text{rad. acc.}} \hat{u}_\theta + \underbrace{(r\alpha + 2\dot{r}\dot{\theta})}_{\text{tangential acc.}} \hat{u}_{\theta+\pi/2}$

* Normal acc.

حاشا وجود جاك الكلاسيكي

$$\rightarrow \ddot{r} = \vec{a}_{slip} \equiv \text{Slider acc.}$$

$$\rightarrow -r\dot{\theta}^2 = \text{Normal acc.} \text{ باقىاى}$$

$$\rightarrow r\alpha = \vec{a}_{\text{tangential}}$$

$$\rightarrow 2\dot{r}\dot{\theta} = \vec{a}_{\text{Coriolis acc.}} \text{ "Sliding + Rot."}$$

$$\rightarrow \text{Note 8 } (\alpha \text{ with } \omega) \text{ acc.} \quad | \quad (\alpha \text{ with } \omega) \text{ deceleration}$$

* 4-Bar mechanism

knowns complete positions & velocities.
+ input α_2

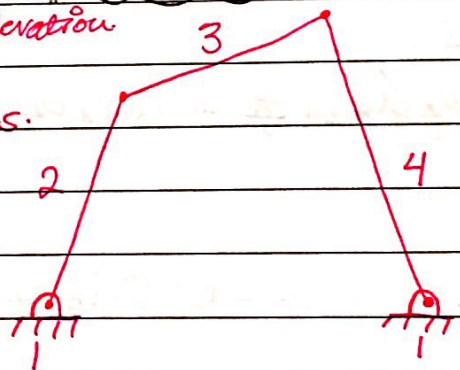
Find: α_3, α_4

* L.C.E:

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = \vec{0}$$

* V.E:

$$r_2 \omega_2 \hat{u}_{\theta_2+\pi/2} + r_3 \omega_3 \hat{u}_{\theta_3+\pi/2} - r_4 \omega_4 \hat{u}_{\theta_4+\pi/2} = \vec{0}$$



* A.E: No $\vec{a}_{slip}$ or $\vec{a}_{cor}$?

$$\left[\overset{\text{input}}{r_2 \alpha_2 \hat{u}_{\theta_2 + \frac{\pi}{2}}} - r_2 \omega_2^2 \hat{u}_{\theta_2} + r_3 \alpha_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} - r_3 \omega_3^2 \hat{u}_{\theta_3} \right] \cdot \hat{u}_{\theta_3} \\ - r_4 \alpha_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} + r_4 \omega_4^2 \hat{u}_{\theta_4} = 0$$

$\leadsto$ To find $\alpha_4 \rightarrow$ Dot by $\hat{u}_{\theta_3}$

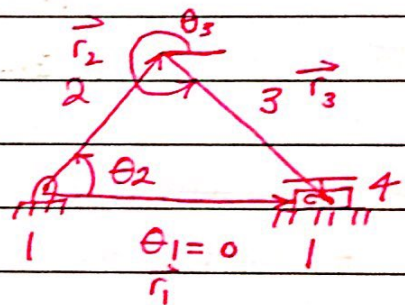
$$r_2 \alpha_2 \sin(\theta_3 - \theta_2) - r_2 \omega_2^2 \cos(\theta_3 - \theta_2) - r_3 \omega_3^2 - r_4 \alpha_4 \sin(\theta_3 - \theta_4) \\ + r_4 \omega_4^2 \cos(\theta_3 - \theta_4) = 0 \leadsto \alpha_4 = \checkmark$$

$\leadsto$ Then find α_3 $\begin{cases} \rightarrow \text{Dot by } \hat{u}_{\theta_4} \\ \rightarrow x-y \text{ components.} \end{cases}$ or

* Slider - Crank mechanism:

$\leadsto$ knows: Positions + vel. + input α_2 .

$\leadsto$ Find: $\vec{\alpha}_3$, ($\vec{a}_c = \vec{a}_{sliding}$)



* L.C.E:

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_0 = 0$$

* V.E:

$$r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} - \dot{S} \hat{u}_0 = 0$$

* A.E:

$$\left[r_2 \alpha_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - r_2 \omega_2^2 \hat{u}_{\theta_2} + r_3 \alpha_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} - r_3 \omega_3^2 \hat{u}_{\theta_3} - \ddot{S} \hat{u}_0 = 0 \right] \cdot \hat{u}_{\theta_3}$$

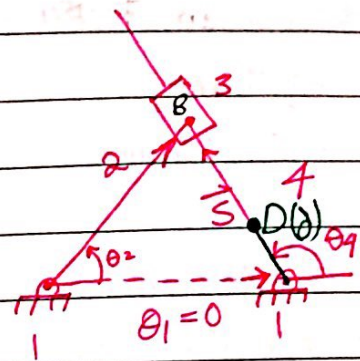
$\rightarrow \vec{a}_c = \ddot{S}$; To find $\ddot{S}$, dot with $\hat{u}_{\theta_3}$.

$$\ddot{S} = \frac{r_2 \alpha_2 \sin(\theta_3 - \theta_2) - r_2 \omega_2^2 \cos(\theta_3 - \theta_2) - r_3 \omega_3^2}{\cos \theta_3}$$

* Inverted slider - crank mech.

* L.C.E!

$$\vec{r}_2 \hat{u}_{\theta_2} - \vec{s} \hat{u}_{\theta_4} - r_1 \hat{u}_0 = \vec{0}$$



* V.E!

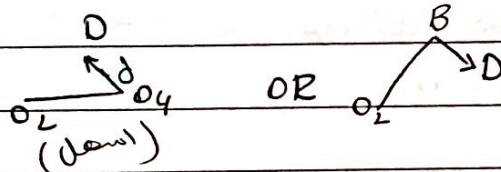
$$r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - \dot{s} \hat{u}_{\theta_4} - s \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} = \vec{0}$$

* A.E!

$$\left[r_2 \alpha_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - r_2 \omega_2^2 \hat{u}_{\theta_2} - \ddot{s} \hat{u}_{\theta_4} - \left(\dot{s} \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} - \dot{s} \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} - 2 \dot{s} \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} \right) \cdot \hat{u}_{\theta_4} - s \alpha_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} + s \omega_4^2 \hat{u}_{\theta_4} \right] = \vec{0}$$

$$\rightarrow \ddot{s} = r_2 \alpha_2 \sin(\theta_4 - \theta_2) - r_2 \omega_2^2 \cos(\theta_4 - \theta_2) + s \omega_4^2; \text{ Then find } \alpha_4!$$

* Find $\vec{a}_D = ?!$



$$\begin{aligned} \vec{r}_D &= r_1 \hat{u}_0 + d \hat{u}_{\theta_4} \\ \vec{v}_D &= \vec{0} + d \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} \\ \vec{a}_D &= d \alpha_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} - d \omega_4^2 \hat{u}_{\theta_4} \\ &= (\quad) \hat{r} + (\quad) \hat{s} \end{aligned}$$

* Suggested (CH. 7):

7 - 4, 6, 8, 15(b), 16, 18, 20, 48, 50, 52.

* Problem 7-54 !

$$L_2 = 25.4 \text{ mm}$$

$$L_4 = 120.9 \text{ mm}$$

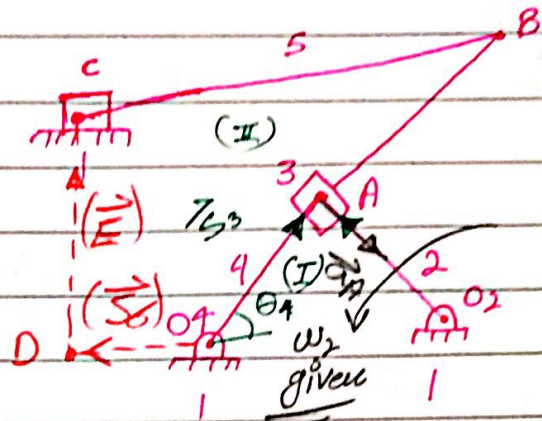
$$L_5 = 115.9$$

$$O_4 O_2 = 42.9 \text{ mm @ } 15.5^\circ$$

~> For $\theta_2 = 99^\circ$

$$\omega_2 = 10 \text{ rad/s} = \text{const.}$$

$$\alpha_2 = 0$$



~> find $\vec{a}_A, \vec{a}_B, \vec{a}_C$!!

$$\vec{r}_A = 25.4 \hat{u}_{\theta_2}$$

$$\vec{\omega}_A = 25.4 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}}$$

$$\vec{a}_A = 25.4 \alpha_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - 25.4 \omega_2^2 \hat{u}_{\theta_2} \rightarrow \text{"mag. of } \vec{a}_A \text{"}$$

$$= -2540 (\cos 99^\circ \hat{i} + \sin 99^\circ \hat{j})$$

$$= 397.34 \hat{i} - 2508.7 \hat{j} \text{ mm/s}^2$$

* L.C.E (I) :

$$S_3 \hat{u}_{\theta_4} - 25.4 \hat{u}_{\theta_2} - 42.9 \hat{u}_{15.5} = \vec{0}$$

* V.E (I) :

$$S_3 \hat{u}_{\theta_4} + S_3 \omega_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} - 25.4 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} = \vec{0}$$

* A.E (I) :

$$\left[(\ddot{S}_3 - S_3 \omega_4^2) \hat{u}_{\theta_4} + (S_3 \alpha_4 + 2\dot{S}_3 \omega_4) \hat{u}_{\theta_4 + \frac{\pi}{2}} - 25.4 \ddot{\omega}_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} + 25.4 \omega_2^2 \hat{u}_{\theta_2} = \vec{0} \right] \cdot \hat{u}_{\theta_4 + \frac{\pi}{2}}$$

* To find (α_4) : $(\hat{u}_{\theta_4 + \frac{\pi}{2}})$

$$\alpha_4 = \frac{(-2\dot{S}_3 \omega_4 - 25.4 \omega_2^2 \sin(\theta_2 - \theta_4))}{S_3}$$

$$= \frac{-2(-207.12)(2.81) - (25.4)(10)^2 \sin(99 - 44.37)}{52.27}$$

$$= -17.36 \text{ rad/s}^2$$

~> $\alpha_4 = 17.36 \text{ rad/s}^2$ (c.w) deceleration (-) (ω_4) clockwise

$$\rightarrow \vec{r}_B = r_4 \hat{u}_{\theta 4} = 120.9 \hat{u}_{\theta 4}$$

$$\vec{v}_B = 120.9 \omega_4 \hat{u}_{\theta 4 + \frac{\pi}{2}}$$

$$\vec{a}_B = 120.9 \alpha_4 \hat{u}_{\theta 4 + \frac{\pi}{2}} - 120.9 \omega_4^2 \hat{u}_{\theta 4}$$

(-17.36)

$$= (120.9)(-17.36)(-\sin 44.37^\circ \hat{i} + \cos 44.37^\circ \hat{j})$$

$$- (120.9)(2.81)^2 (\cos 44.37^\circ \hat{i} + \sin 44.37^\circ \hat{j})$$

$$= 785.3 \hat{i} - 2167.9 \hat{j} \text{ (mm/s}^2\text{)}$$

* L.C.E (II) :

$$\rightarrow 120.9 \hat{u}_{\theta 4} + 115.9 \hat{u}_{\theta 5} - 72.6 \hat{u}_{\theta 0} - \vec{s}_6 \hat{u}_\pi = \vec{0}$$

* V.E (II)

$$\rightarrow 120.9 \omega_4 \hat{u}_{\theta 4 + \frac{\pi}{2}} + 115.9 \omega_5 \hat{u}_{\theta 5 + \frac{\pi}{2}} - \dot{s}_6 \hat{u}_\pi = \vec{0}$$

* A.E (II) :

$$\rightarrow 120.9 \alpha_4 \hat{u}_{\theta 4 + \frac{\pi}{2}} - 120.9 \omega_4^2 \hat{u}_{\theta 4} + 115.9 \alpha_5 \hat{u}_{\theta 5 + \frac{\pi}{2}} - 115.9 \omega_5^2 \hat{u}_{\theta 5} - \ddot{s}_6 \hat{u}_{180} = \vec{0}$$

• $\hat{u}_{\theta 5}$

$$\Rightarrow \vec{a}_c = \ddot{s}_6 = \frac{120.9(-17.36) \sin(185.9 - 44.37) - 120.9(2.81)^2 \cos(185.9 - 44.37) - 115.9(2.017)^2 (1)}{\cos(185.9 - 180)}$$

$$\vec{a}_c = -1077.78 \text{ mm/s}^2 = -1077.78 \hat{u}_\pi$$

$$= 1077.78 \hat{i} \text{ (mm/s}^2\text{)}$$

* Sliders: we treat them as particles regardless to their dimensions.

* Force Analysis:

Dynamic
Static

(translation)

$$\sim \sum F_x = \sum (F_x)_{\text{eff.}} = m a_x$$

$$\sim \sum F_y = \sum (F_y)_{\text{eff.}} = m a_y$$

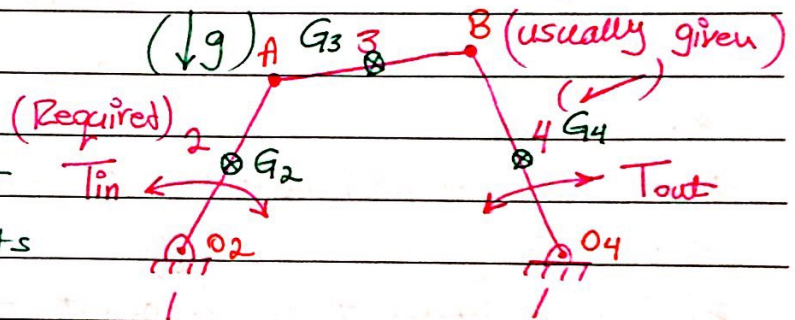
$$\sim \sum M_z = \sum (M_z)_{\text{eff.}} = I_z \alpha$$

$G \equiv (\text{center of gravity}) = (\text{mass center})$ * eff = effective. "Inertia forces"

* 4-Bar mechanism:

$T \equiv \text{Torque}$

$(\downarrow g)$ means the weight is included and it affects on the motion ($\perp$).
vertical plane

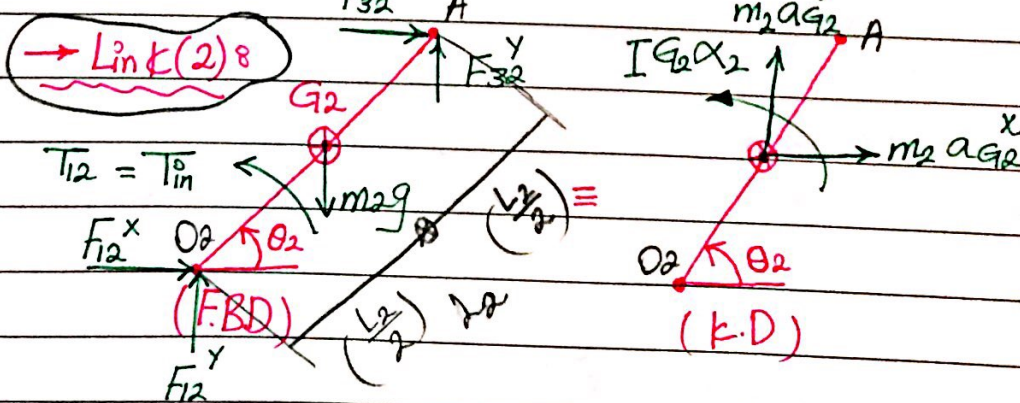


* knowns: kinematics "position / velocity / acc." + Masses + T_{out} .
* Find: $(T_{\text{in}}) = ?!$

* No. of unknowns:

$$(2)(4) = 8 \text{ (Joints Forces)} ; 8 + 1 = 9 \text{ For } (T_{\text{in}})$$

* 3 equ.s with 3 analysis for each link (2, 3, 4) = 9 equ.s



0 direction.
Source $\leftarrow$ $\rightarrow$ Target

* FBD & free body diagram

* KD & kinematic diagram

(FBD)

(KD)

$$\rightarrow \sum F_x = m_2 a_{G_2}^x \rightarrow \text{no unknowns here.}$$

$$F_{12}^x + F_{32}^x = m_2 a_{G_2}^x \quad \dots (1)$$

?"

?"

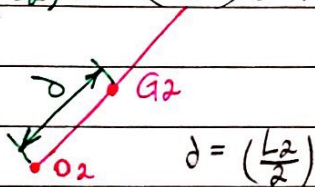
$$\uparrow \sum F_y = m_2 a_{G_2}^y$$

$$F_{12}^y + F_{32}^y - m_2 g = m_2 a_{G_2}^y \quad \dots (2)$$

$$\rightarrow \sum M_{O_2} = \sum (M_{O_2})_{\text{eff.}} \quad (\text{chap 19, 19.1c})$$

$$T_{in} - m_2 g \left(\frac{L_2}{2} \right) \cos \theta_2 + F_{32}^y (L_2) \cos \theta_2 - F_{32}^x (L_2) \sin \theta_2 = \left(I_{G_2} \alpha_2 + m_2 a_{G_2}^y \left(\frac{L_2}{2} \right) \cos \theta_2 - m_2 a_{G_2}^x \left(\frac{L_2}{2} \right) \sin \theta_2 \right)$$

$$\rightarrow (I_{O_2})_z = (I_{G_2})_z + m d^2 \quad \text{"parallel axis thm."}$$



$$d = \left(\frac{L_2}{2} \right)$$

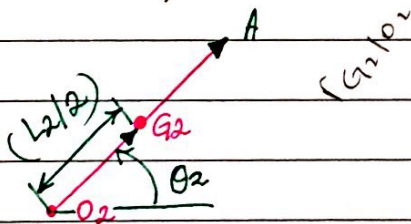
(OR) $\dots (3)$

$$= I_{O_2} \alpha_2$$

$\rightarrow$ valid only for links

Rotating about fixed axis $\left\{ \begin{array}{l} \text{link 3} \\ \text{GPM} \end{array} \right.$

* Consider kinematics of link 2



$$\rightarrow \vec{r}_{G_2} = \vec{r}_{G_2/O_2} = \frac{L_2}{2} \hat{u}_{\theta_2}$$

$$\rightarrow \vec{v}_{G_2} = \frac{L_2}{2} \omega_2 \hat{u}_{\theta_2 + \pi/2}$$

$$\rightarrow \vec{a}_{G_2} = \frac{L_2}{2} \alpha_2 \hat{u}_{\theta_2 + \pi/2} - \frac{L_2}{2} \omega_2^2 \hat{u}_{\theta_2}$$

$$\rightarrow a_{G_2}^x = \frac{L_2}{2} \left[-\alpha_2 \sin \theta_2 - \omega_2^2 \cos \theta_2 \right]$$

$$\rightarrow a_{G_2}^y = \frac{L_2}{2} \left[\alpha_2 \cos \theta_2 - \omega_2^2 \sin \theta_2 \right]$$

$$1 \rightarrow m_2 a_{G_2}^y \frac{L_2}{2} \cos \theta_2 = \left(\frac{m_2 L_2}{2} \right)^2 [\alpha_2 \cos^2 \theta_2 - \omega_2^2 \cos \theta_2 \sin \theta_2]$$

$$2 \rightarrow m_2 a_{G_2}^x \frac{L_2}{2} \sin \theta_2 = m_2 \left(\frac{L_2}{2} \right)^2 [-\alpha_2 \sin \theta_2 \cos \theta_2 - \omega_2^2 \sin^2 \theta_2]$$

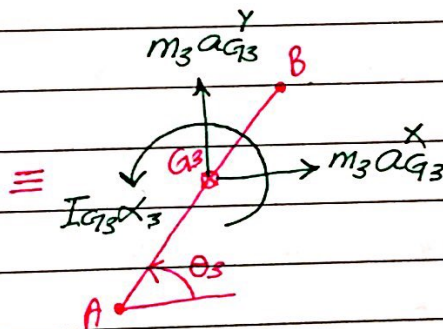
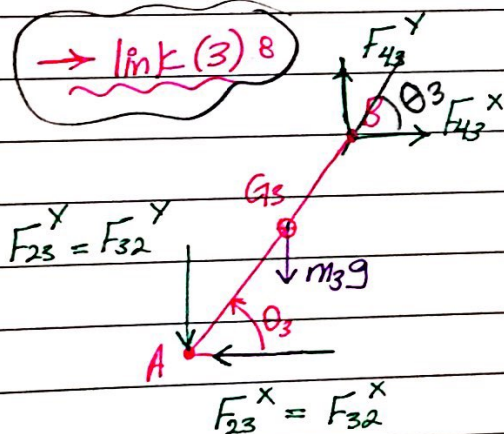
* Subtracting 1 of 2 :

$$\begin{aligned} \rightarrow m_2 \left(\frac{L_2}{2} \right)^2 [\alpha_2 (\underbrace{\cos^2 \theta_2 + \sin^2 \theta_2}_{(1)}) - \omega_2^2 (\underbrace{\sin \theta_2 \cos \theta_2 - \sin \theta_2 \cos \theta_2}_{(0)})] \\ = m_2 \left(\frac{L_2}{2} \right)^2 \alpha_2 \end{aligned}$$

** Right-hand side :

$$\begin{aligned} &= (I_{G_2} + m_2 \left(\frac{L_2}{2} \right)^2) \alpha_2 \\ &= I_{O_2} \alpha_2 \end{aligned}$$

Link (3) :



$$x: -F_{32}^x + F_{43}^x = m_3 a_{G_3}^x \quad \text{--- (4)}$$

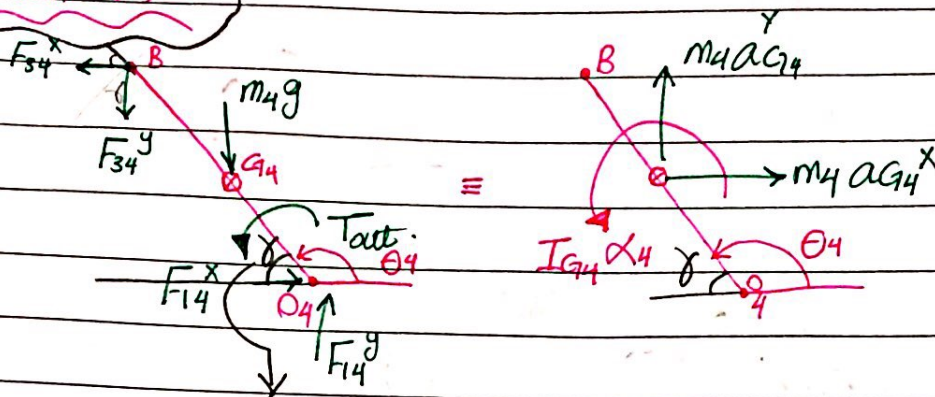
$$y: -F_{32}^y + F_{43}^y - m_3 g = m_3 a_{G_3}^y \quad \text{--- (5)}$$

Prove it

$$\begin{aligned} \text{+ve) } M_A: F_{43}^y L_3 \cos \theta_3 - F_{43}^x L_3 \sin \theta_3 - m_3 g \frac{L_3}{2} \cos \theta_3 &\neq I_A \alpha_3 \\ &= I_{G_3} \alpha_3 + m_3 a_{G_3}^y \frac{L_3}{2} \cos \theta_3 \\ &\quad - m_3 a_{G_3}^x \frac{L_3}{2} \sin \theta_3 \quad \text{--- (6)} \end{aligned}$$

about A

→ Link (4) *



Link 4 dard olgan qisqir (x) *

$$\sum F_{14}^x - F_{43}^x = m_4 a_{G4}^x \quad \text{--- (7)}$$

$$\sum F_{14}^y - F_{43}^y - m_4 g = m_4 a_{G4}^y \quad \text{--- (8)}$$

→ Mo4 *

$$T_{act} + F_{43}^x L_4 \sin \gamma + F_{43}^y L_4 \cos \gamma + m_4 g \frac{L_4}{2} \cos \gamma = I_{O4} \alpha_4 \quad \text{--- (9)}$$

$\rightarrow \frac{1}{12} m L^2$

→ (6) & (9): (qisqir olgan)

about (A) qisqir (6) dard olgan

* Solution *

1. Solve (6) & (9) same. → $(F_{43}^x), (F_{43}^y)$

2. Solve (4) → F_{32}^x

3. Solve (5) → F_{32}^y

4. Solve (3) → T_{in} "our main target"

5. " (1) → $F_{12}^x = R_{02}^x$

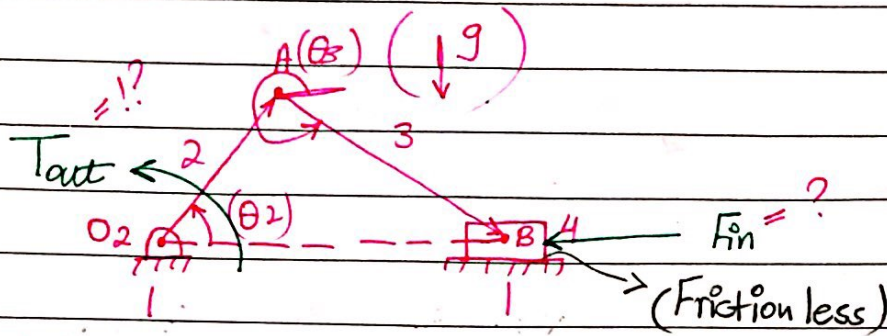
6. " (2) → $F_{12}^y = R_{02}^y$

7. " (7) → $F_{14}^x = R_{04}^x$

8. " (8) → $F_{14}^y = R_{04}^y$

all of them depends on moment equation in each link.

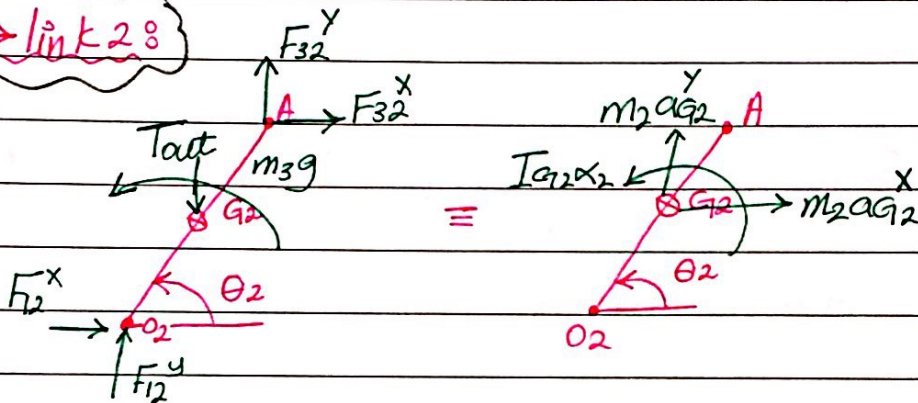
* Slider - Crank mechanism:



~> knowns: kinematics + masses + T_{act}

~> Find: F_{in} & Joint forces.

→ Link 2:

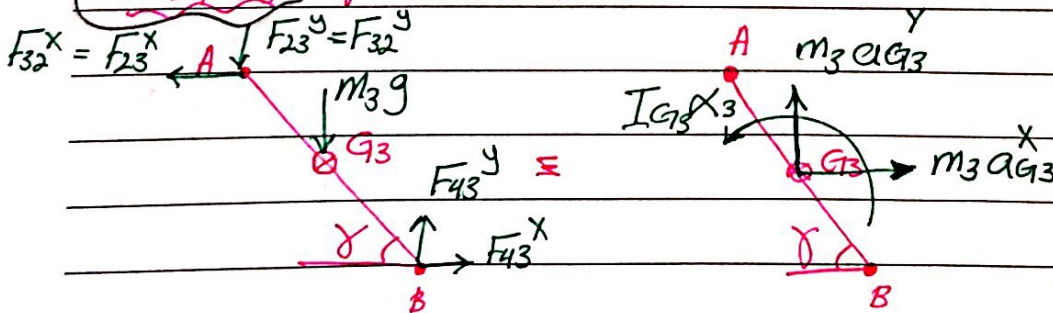


$$x: F_{12}^x + F_{32}^x = m_2 a_{G_2}^x \quad \text{--- (1)}$$

$$y: F_{12}^y + F_{32}^y - m_2 g = m_2 a_{G_2}^y \quad \text{--- (2)}$$

$$M_2: T_{act} - F_{32}^x l_2 \sin \theta_2 + F_{32}^y l_2 \cos \theta_2 - m_2 g \frac{l_2}{2} \cos \theta_2 = I_{O_2} \alpha_2 \quad \text{--- (3)}$$

→ Link 3: $\gamma = 360 - \theta_3$

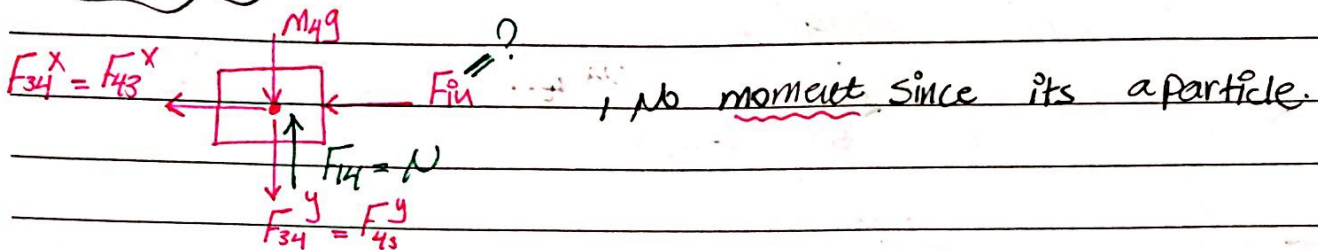


$$x: -F_{32}^x + F_{43}^x = m_3 a_{G3}^x \quad \text{--- (4)}$$

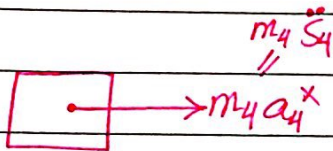
$$y: -F_{32}^y + F_{43}^y + m_3 g = m_3 a_{G3}^y \quad \text{--- (5)}$$

$$\begin{aligned} \text{+)} M_B: F_{32}^x L_3 \sin \gamma + F_{32}^y L_3 \cos \gamma + m_3 g \frac{L_3}{2} \cos \gamma &= I_{G3} \alpha_3 - m_3 a_{G3}^x L_3 \sin \gamma \\ &\quad - m_3 a_{G3}^y \frac{L_3}{2} \cos \gamma \quad \text{--- (6)} \end{aligned}$$

→ Link 4



|||



$$x: -F_{14} - F_{43}^x = m \ddot{s} \quad \text{--- (7)}$$

$$y: N - m_4 g - F_{43}^y = 0 \quad \text{--- (8)}$$

* Solution:

1. Solve (3) & (6) → F_{32}^x, F_{32}^y

2. // (4) → F_{43}^x

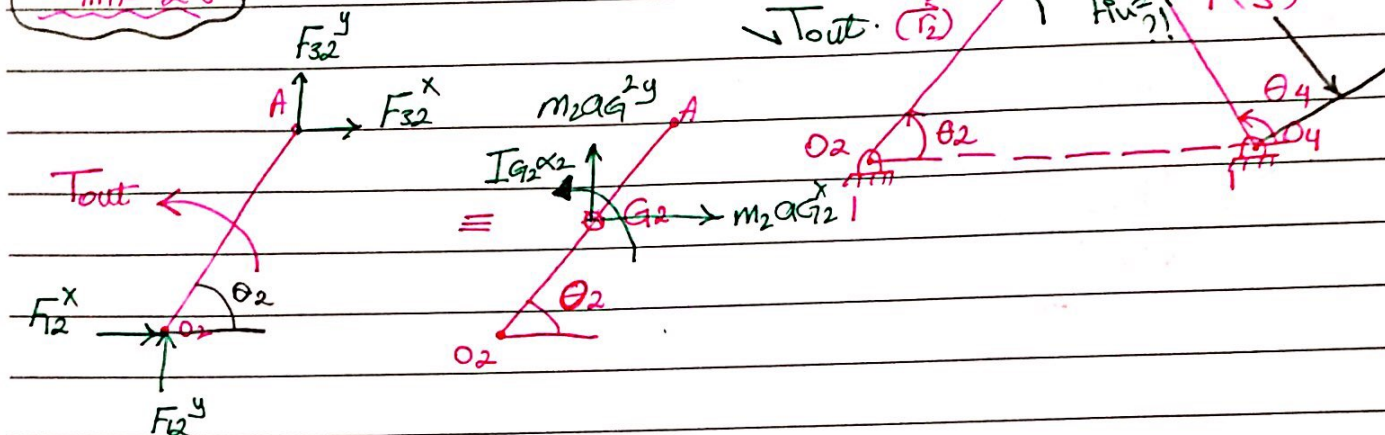
3. // (5) → F_{43}^y

4. // (7) → F_{14}

* Inverted slider-crank mechanism

→ Horizontal plane weight doesn't affect.

→ link 2

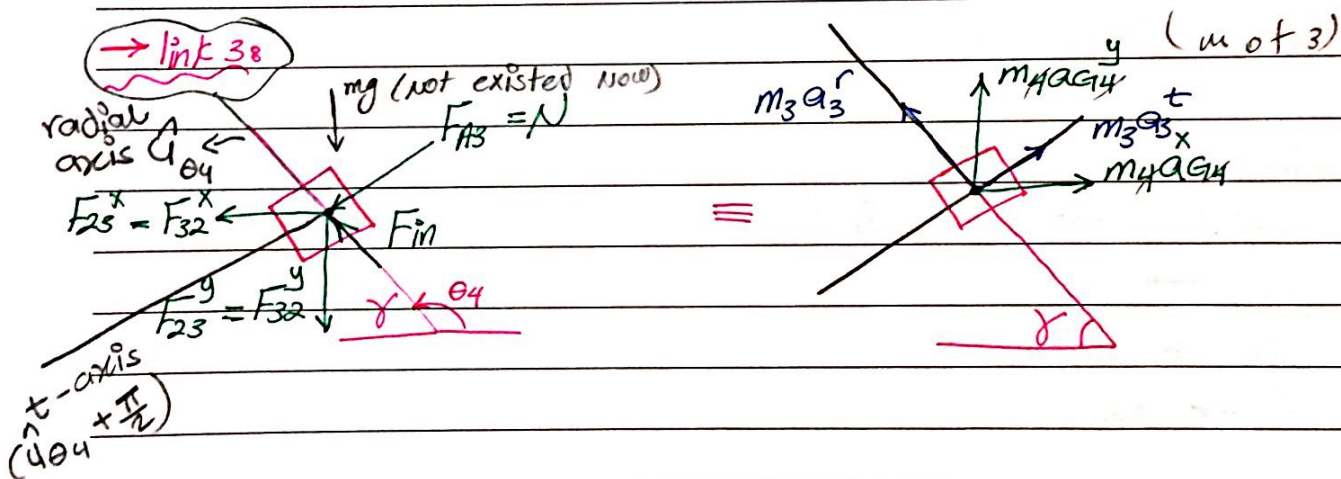


$$x: F_{12}^x + F_{32}^x = m_2 a_{G2}^x \quad \text{--- (1)}$$

$$y: F_{12}^y + F_{32}^y = m_2 a_{G2}^y \quad \text{--- (2)}$$

$$M_{O2}: T_{out} + F_{32}^y L_2 \cos \theta_2 - F_{32}^x L_2 \sin \theta_2 = I_{O2} \alpha_2 \quad \text{--- (3)}$$

→ link 3



$$\sum F_{rad.} = (\sum F_{rad.})_{eff.}$$

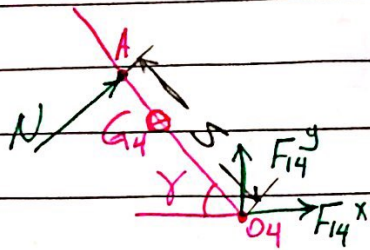
$$F_{13} + F_{32}^x \cos \gamma - F_{32}^y \sin \gamma = m_3 (\ddot{s} - \dot{\omega}_4^2 r_{G3}) \quad \text{--- (4)}$$

$$\sum F_t = (\sum F_t)_{eff.}$$

$$N + F_{32}^x \sin \gamma + F_{32}^y \cos \gamma = m_3 (s \alpha_4 + 2 \dot{s} \omega_4) \quad \text{--- (5)}$$

→ Link 4:

دھڑکی (L4) کی ابتدا (O4) ←
موجہ (3)



$$\equiv I_{G4} \alpha_4$$

$$x: N \sin \gamma + F_{14}^x = m_4 a_{G4}^x \quad \text{--- (6)}$$

$$y: N \cos \gamma + F_{14}^y = m_4 a_{G4}^y \quad \text{--- (7)}$$

$$M_{O4} - Ns = I_{O4} \alpha_4 \quad \text{--- (8)}$$

* Solution: our target ($\vec{F}_{in}$)

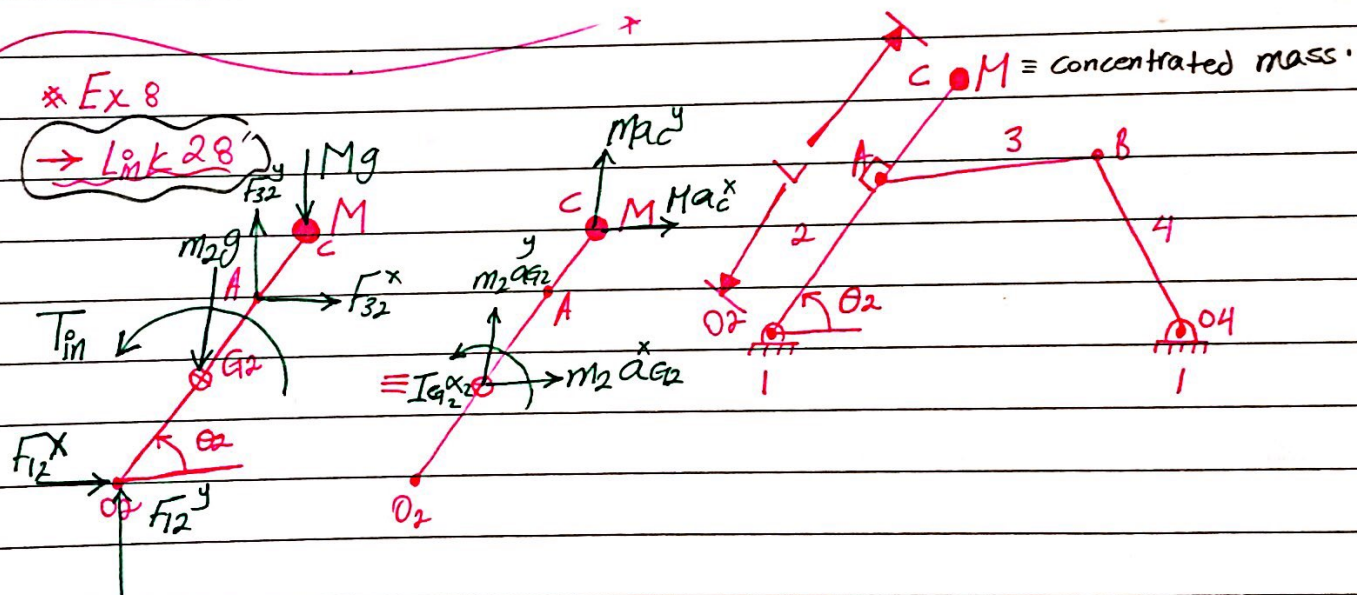
1. Find (8) → N

2. Solve (3) & (5) → F_{32}^x, F_{32}^y

3. Solve (4) → ($\vec{F}_{in}$)

* Ex 8

→ Link 2:



$$\vec{r}_c = L \hat{u}_{\theta_2}, \quad \vec{v}_c, \quad \vec{a}_c$$

$$x: F_{12}^x + F_{32}^x = m_2 a_{G2}^x + M a_c^x \quad \text{--- (1)}$$

$$y: F_{12}^y + F_{32}^y - m_2 g - Mg = m_2 a_{G2}^y + M a_c^y \quad \text{--- (2)}$$

$$M_2 \left(T_{in} - m_2 g \left(\frac{L_2}{2} \right) \cos \theta_2 - MgL \cos \theta_2 + F_{32}^y L_2 \cos \theta_2 - F_{32}^x L_2 \sin \theta_2 \right) =$$

$$\left(I_{G_2} \alpha_2 + m_2 a_{G_2}^y \left(\frac{L_2}{2} \right) \cos \theta_2 - m_2 a_{G_2}^x \left(\frac{L_2}{2} \right) \sin \theta_2 + \right.$$

$$\left. M a_c^y L \cos \theta_2 - M a_c^x L \sin \theta_2 \right) \rightarrow (I_G + m \frac{L^2}{2}) \alpha_2$$

Show that $(ML^2 \alpha_2) = ?!$

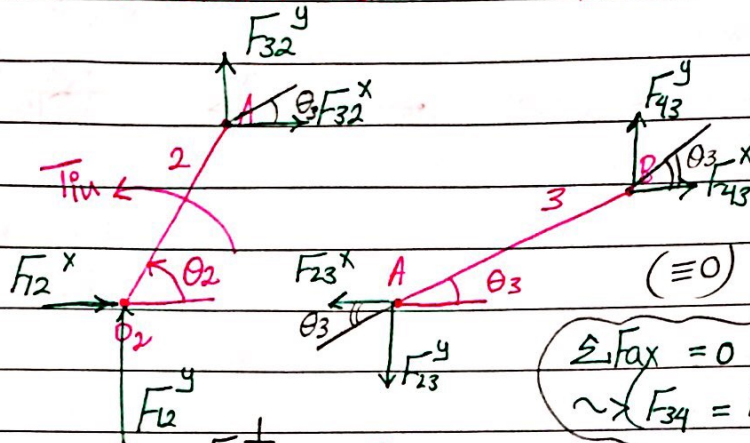
$$= \left[\underbrace{I_G + m \frac{L^2}{2}}_{(I_{O_2} \alpha_2)} + ML^2 \right] \alpha_2$$

~> Beam 8 can hold axial loading.
(transversie).

* Static force Analysis 8 "mass-less links"

* Ex 8 4-Bar mechanism (Assume all links are mass less)

~> Find a relation between T_{in} & T_{out} = !



* 2 force member:

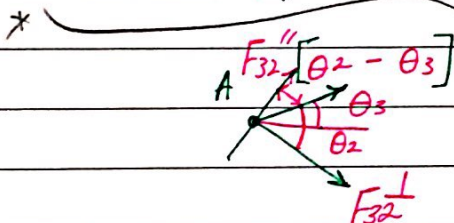
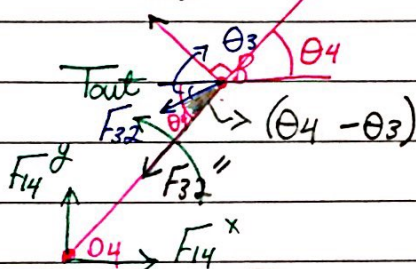
1. No axial loading
2. No moment
3. mass less ($R \cdot S = 0$).
(Right side)

$$\sum F_{ax} = 0$$

$$\sim F_{34} = F_{23}$$

$$F_{32}^{\perp} = F_{32} \sin(\theta_4 - \theta_3)$$

$$F_{32}'' = F_{32} \cos(\theta_4 - \theta_3)$$



$$F_{32}'' = F_{32} \cos(\theta_2 - \theta_3)$$

$$F_{32}^{\perp} = F_{32} \sin(\theta_2 - \theta_3)$$

$$\sim \sum M_{O2} = 0 (+)$$

$$T_{in} = F_{32} L_2 \sin(\theta_2 - \theta_3)$$

$$\sim \sum M_{O4} = 0 (+) \rightarrow = T_{in} / L_2 \sin(\theta_2 - \theta_3)$$

$$T_{out} + F_{32} L_4 \sin(\theta_4 - \theta_3) = 0$$

$$\Rightarrow T_{in} = \frac{-L_2 \sin(\theta_2 - \theta_3)}{L_4 \sin(\theta_4 - \theta_3)} T_{out}$$

* Suggested ch. 11: 11-12, 14, 16

Ex 8

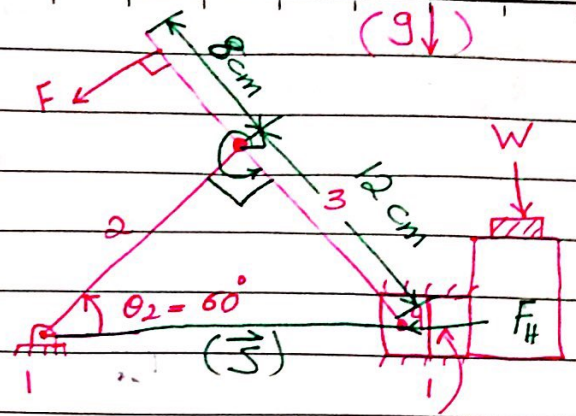
$L_2 = 6.93 \text{ cm}$, $m_4 = 7 \text{ kg}$

$m_2 = m_3 = 0$

→ For $\theta_2 = 60^\circ \Rightarrow F = 50 \text{ N}$

$\omega_3 = 5 \text{ rad/s}$, (ccw)

$\omega_2 = 15 \text{ rad/s}$ (cw)
→ const. / $\alpha_2 = 0$



→ $F_H = ?!$

Hydraulic Fluid

* Kinematic analysis:

* $\hat{S}\hat{U}_0 = 6.93 \hat{U}_{\theta_2} + 12 \hat{U}_{\theta_3}$

→ For $\theta_2 = 60^\circ \Rightarrow \theta_3 = 35^\circ$

* $\dot{\hat{S}}\hat{U}_0 = 6.93 \omega_2 \hat{U}_{\theta_2 + \frac{\pi}{2}} + 12 \omega_3 \hat{U}_{\theta_3 + \frac{\pi}{2}}$

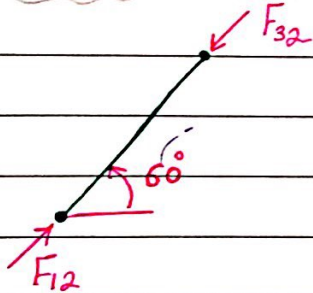
→ $\ddot{\hat{S}}\hat{U}_0 = -6.93 \omega_2^2 \hat{U}_{\theta_2} + 12 \alpha_3 \hat{U}_{\theta_3 + \frac{\pi}{2}} - 12 \omega_3^2 \hat{U}_{\theta_3}$

→ for ---

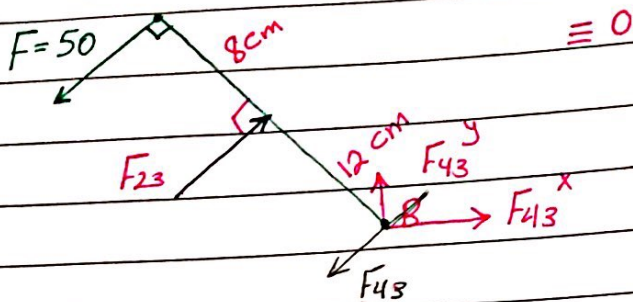
$\Rightarrow \ddot{S} = -3.464 \text{ m/s}^2$

* Dynamics:

→ link 2 is 2-force member. " $F_{12} = F_{32}$ "



→ link 38 not 2-force member.

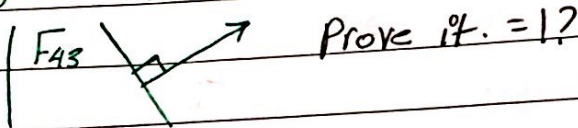


$$\sum M_B = 0 \Rightarrow$$

$$(50)(20) = F_{32}(12) \Rightarrow F_{32} = 83.33 \text{ N} = F_{32} = F_{12}$$

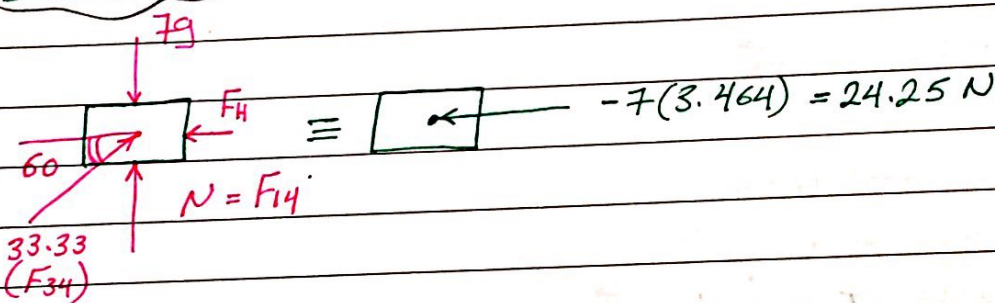
$$\sum F_x = 0$$

$$\sum F_y = 0$$



$$\rightarrow F_{43} = 33.33 \text{ N}$$

→ link 48



$$\sum F = 0$$

$$F_H - 33.33 \cos 60 = 24.25$$

$$\Rightarrow F_H = 40.9 \text{ N}$$

$$\sum F_y = 0$$

$$\Rightarrow F_{14} + 33.33 \sin 60 = (7)(9.81)$$

$$F_{14} = 39.806 \text{ (N)}$$

* Self-Reading:

- Transmission Angle (4Bar, slider-crank) (ch. 4) (position).
- Joint Force Index (JFI) (ch. 11)

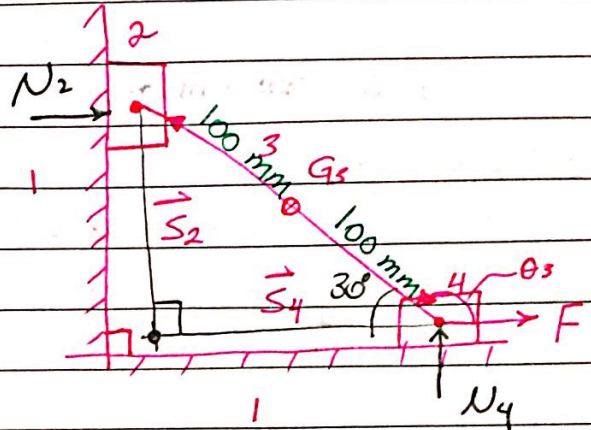
* Ex 8

$$L_3 = 200 \text{ mm}, m_3 = 5 \text{ kg}.$$

$$I_{G_3} = 0.2 \text{ (kg.m}^2\text{)}$$

→ For $\theta_3 = 150^\circ$, find $(\vec{F})$

needed so that $V_A = 700 \text{ mm/s} \rightarrow$
= const.



$$* m_2 = m_4 = 0.$$

* Horizontal plane.

$$\vec{a}_{G_3} = \underset{x''}{0} \hat{i} - \underset{y''}{9.8} \hat{j} \text{ (m/s}^2\text{)}$$

* Kinematics:

* L.C.F:

$$* \dot{s}_4 \hat{u}_0 + 0.2 \dot{\omega}_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} - \dot{s}_2 \hat{u}_{\frac{\pi}{2}} = \vec{0}$$

→ For $\theta_3 = 150^\circ$;

$$s_2 = 0.1 \text{ m}, s_4 = 0.173 \text{ m}$$

* V.E:

$$* \dot{s}_4 \hat{u}_0 + 0.2 \dot{\omega}_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} - \dot{s}_2 \hat{u}_{\frac{\pi}{2}} = \vec{0}$$

(0.7 m/s), for ---

⇒ $\omega_3 = 7 \text{ rad/s}$ (ccw), it must be (ccw) since the motion is to the right.

* A.E:

$$\ddot{s}_4 \hat{u}_0 + 0.2 \alpha_3 \hat{u}_{\theta_3 + \frac{\pi}{2}} - 0.2 \omega_3^2 \hat{u}_{\theta_3} - \ddot{s}_2 \hat{u}_{\frac{\pi}{2}} = \vec{0}$$

For $\theta_3 = 150^\circ$, $\dot{s}_4 = 0.7$, $\omega_3 = 7$;

$$\Rightarrow \alpha_3 = 84.87 \text{ rad/s}^2 \text{ (ccw)}$$

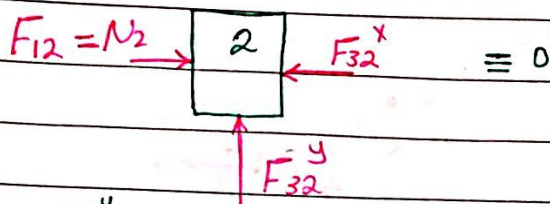
$$* \vec{r}_{G3} = 54 \hat{u}_0 + 0.1 \hat{u}_{\theta_3}$$

$$* \vec{v}_{G3} = \dot{54} \hat{u}_0 + 0.1 \omega_3 \hat{u}_{\theta_3} + \frac{\pi}{2}$$

$$* \vec{a}_{G3} = 0 + 0.1 \alpha_3 \hat{u}_{\theta_3} - 0.1 \omega_3^2 \hat{u}_{\theta_3} + \frac{\pi}{2}$$

* Dynamics :

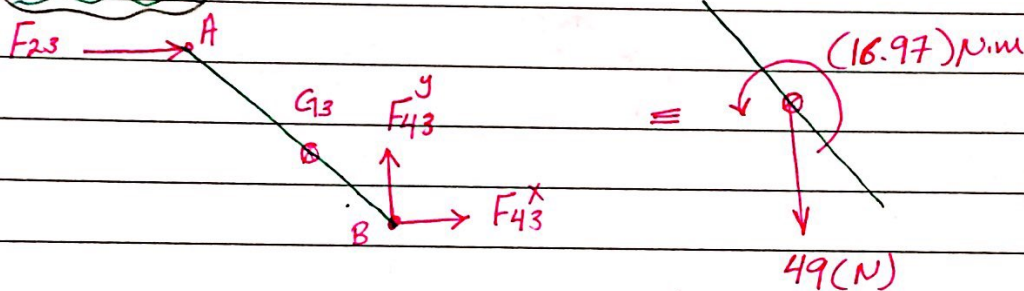
→ link 2: 2 force-member but its a particle.



$$\rightarrow F_{32}^y = 0$$

$$F_{32}^x = F_{12} = N_2$$

→ link 3:



$$\sum F_y = m_3 a_{G3}^y$$

$$F_{43}^y = -49 = 49(N) \downarrow$$

$$\sum M_A = (\sum M_A)_{eff.} \neq I_A \alpha_3 \quad (GPM)$$

$$\Rightarrow F_{43}^x (0.2) \sin 30 = 16.97 - 49(0.1) \cos 30$$

$$\Rightarrow F_{43}^x = 2.122(N) \rightarrow$$

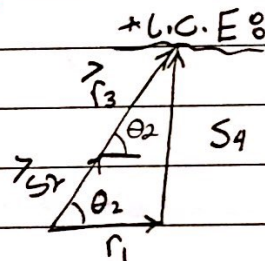


②

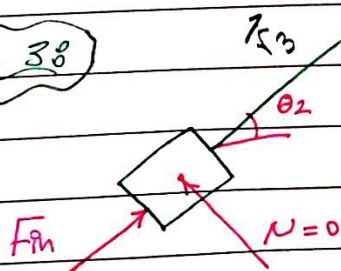
A free body diagram of a mechanical system. At the top, a horizontal bar is supported by a pin joint on the right and has a red dot on the left. A red line (cable) runs from this red dot, passes over a pulley (labeled G_s), and then goes down to a slider block (labeled 4). The slider block is on a horizontal surface and has a horizontal force F applied to its right side. A vertical force F_{out} is applied downwards to the slider block. A red line (cable) also runs from the slider block, goes up and to the left, and then down to a slider block (labeled 3) on an inclined plane. The inclined plane makes an angle θ_2 with the horizontal. A force F_{in} is applied to slider block 3 along the incline. At the bottom, there is a horizontal line with an upward arrow labeled $+L.C.E \otimes$ and a force F_c pointing upwards.

link 28

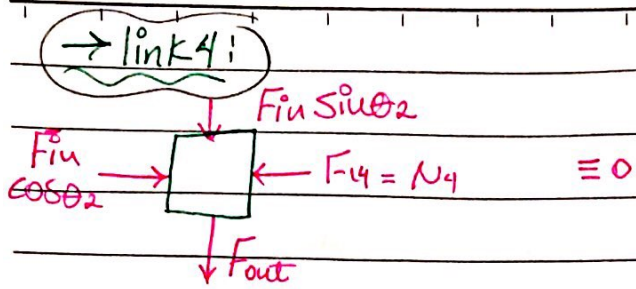
三〇



→ link 30



$$\Rightarrow F_{43}^x = f_{in} \cos \theta_2 \leftarrow / F_{43}^y = f_{in} \sin \theta_2 \downarrow$$

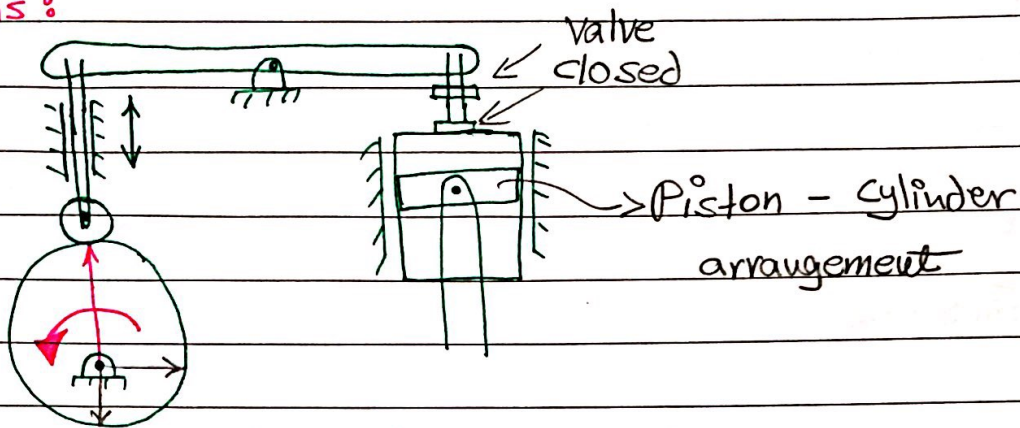


$$F_{out} = F_{in} \sin \theta_2.$$

*

* Cams :

* Ex:



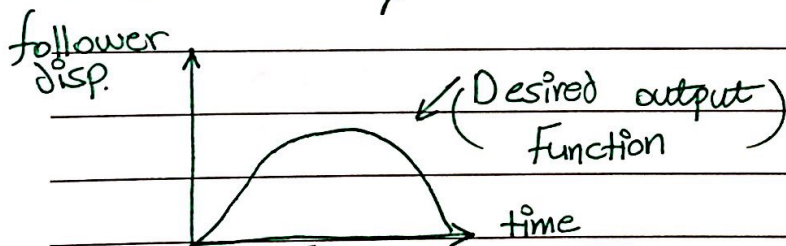
→ A cam with a translating roller follower.

** A cam is a general shaped mechanism component that rotates about a fixed axis (not usually the centroid) and causes a driven link "follower" to conform to a definite path of motion.

(centroid) ← (cam-shaft) ← (desired output function) ← (time)

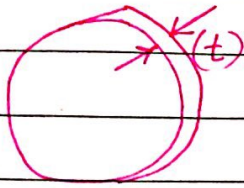
** A cam-follower mechanism is considered a "Function generator" 8

→ We can specify a desired output and design the cam surface required to achieve that output.



[cam-shape] = ?!

* planar cams $\rightarrow$ "Disk cams".
(2-D) cams



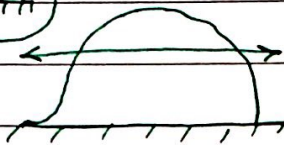
* Types of (2-D) cam mechanism "cam-follower classification".

** Based on cam

✓ 1. Rotating



2. Translating

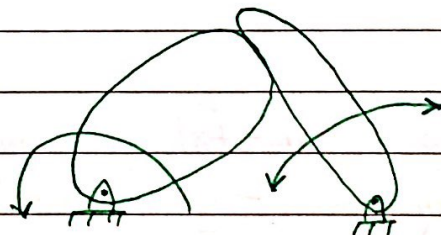


** Based on follower type: * all of them make "line contact" between cam & follower

1. Translating follower: ✓



2. Rotating or Oscillating

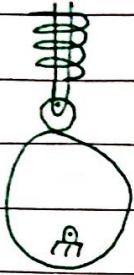


* Types of translating followers

1. Flat - forced



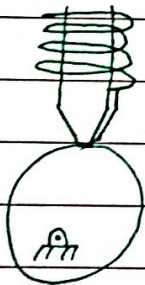
2. Roller-follower :



3. Mushroom-follower :

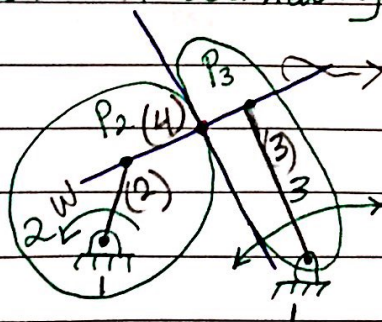


4. Edge-follower :

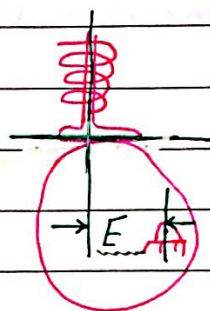
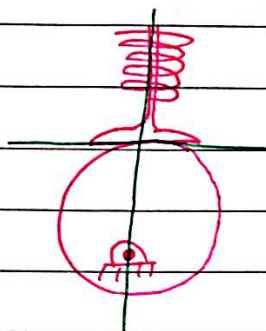
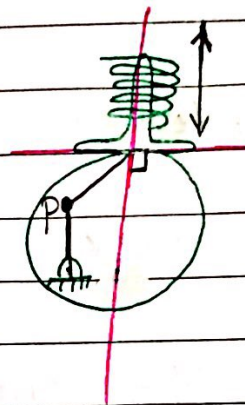
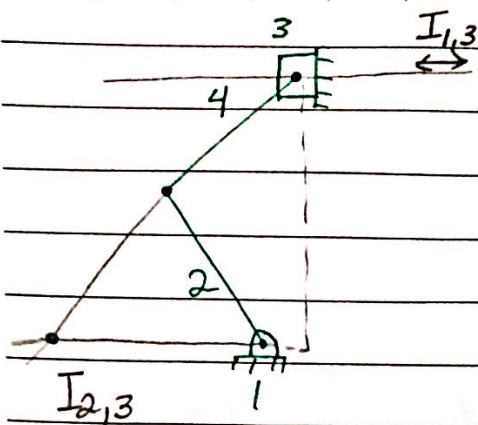


* Equivalency of cam-follower mechanism :

"Cam with an oscillating follower" :



As "4-Bar mechanism. "Equivalent".

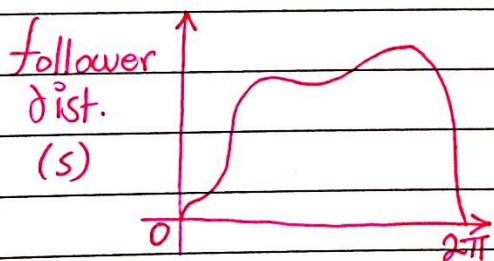


~> In-line "without offset" } ~> with offset.

* Motion program:

Follower motion in one cycle of cam rotation.

~> One revolution.



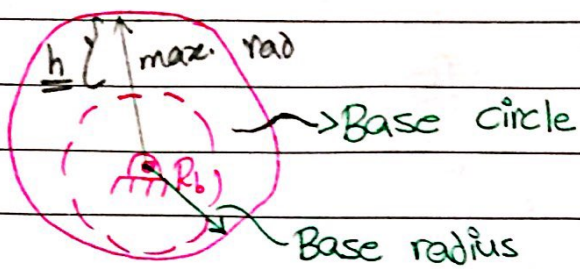
"s- θ " or "s-t"
Diagram.

θ : cam-rotation (Input).
OR

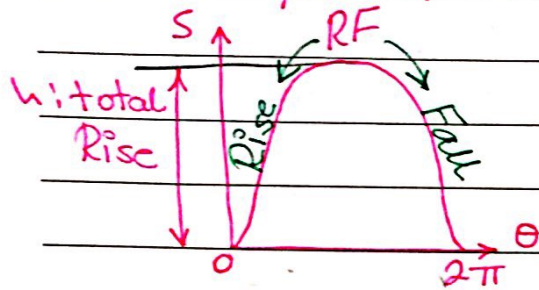
t: time of cam rotation.

~> $\omega = \frac{\partial \theta}{\partial t}$ (const.) $\Rightarrow \theta = \omega t$ (const.)

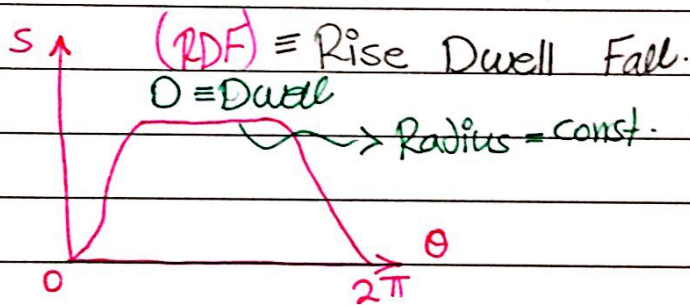
"the same diagram"



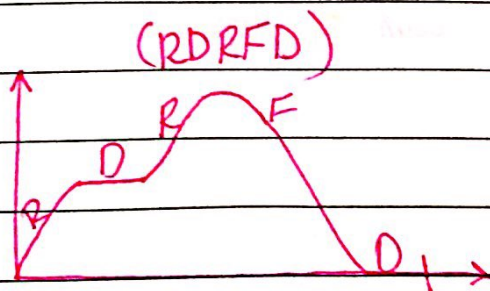
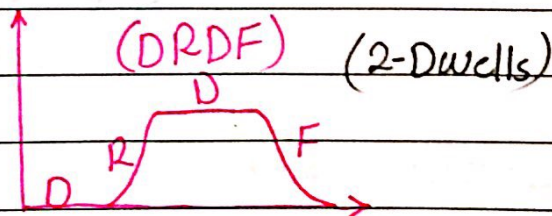
* Examples of Motion programs all of them have (R & F)



$h = \text{maximum rad.} - \text{Base Rad.}$



* Dwell: No output (follower) motion for a specified period of input (cam Rotation) $\rightarrow$ it doesn't Rise or fall since the radius is const.



S-V-a-j diagrams

$s \equiv$ displacement.

$v \equiv$ Velocity

$a \equiv$ acc.

$\dot{a} \equiv$ Jerk.

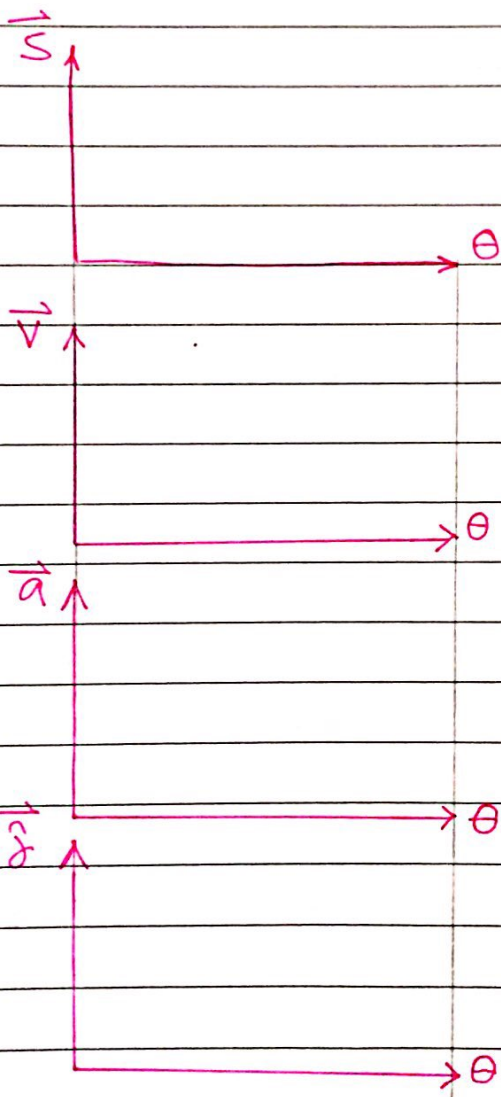
$$\leadsto \text{Jerk } (\dot{a}) = \frac{\partial a}{\partial t}$$

$$F = ma$$

$$\frac{\partial F}{\partial t} = m \frac{\partial a}{\partial t} = (\dot{a})$$

* The time derivative of acceleration.

* It is a measure of the time rate of change of inertia forces and hence is an indication of the impact characteristics of loading. "Ex. Elevators".



$$\rightarrow \vec{v} = \frac{\partial \vec{s}}{\partial t} = \frac{\partial s}{\partial \theta} \cdot \frac{\partial \theta}{\partial t} = (\omega)$$

$$v(\theta) = \omega \frac{\partial s}{\partial \theta}$$

$$\rightarrow \vec{a} = \frac{\partial \vec{v}}{\partial t} = \frac{\partial v}{\partial \theta} \cdot \frac{\partial \theta}{\partial t}$$

$$a(\theta) = \dot{\theta} \frac{\partial v}{\partial \theta} = \omega \frac{\partial v}{\partial \theta}$$

$$= \omega^2 \frac{\partial^2 s}{\partial \theta^2}$$

$$\rightarrow \vec{j} = \frac{\partial \vec{a}}{\partial t} = \frac{\partial a}{\partial \theta} \cdot \frac{\partial \theta}{\partial t}$$

$$j(\theta) = \omega \frac{\partial a}{\partial \theta}$$

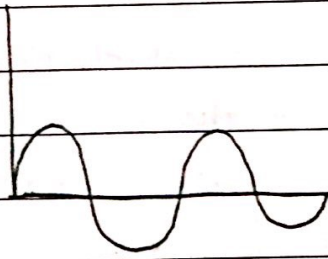
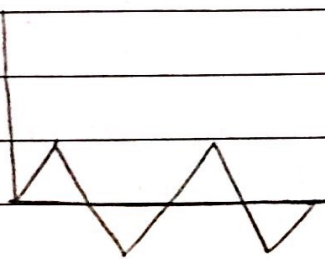
* Examples of some common types of follower motion function:

- Uniform motion' $\vec{v} = \text{const.}$, $\vec{a} = 0$

- Simple Harmonic motion.

~ Periodic $\rightarrow$ Harmonic $\rightarrow$ Simple Har.

(Sine + cosine) functions.

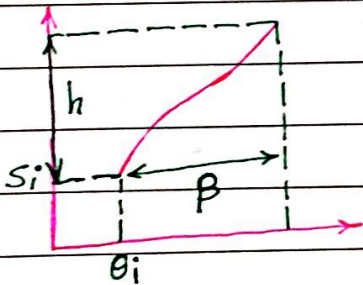


(Periodic)

- Parabolic

- cycloidal

- Polynomial.



* Domain: $(\theta_i \leq \theta \leq \theta_i + \beta)$

$$s = s_i + f(\theta - \theta_i)$$

S.H.M Cy. Polyn.

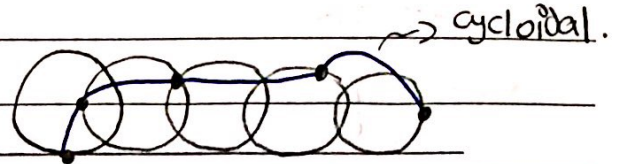
* S.H.M: (sine or cosine) OR (sine + cosine with the same frequency)

$$f(\theta) = \frac{h}{2} \left[1 - \cos \frac{\pi \theta}{\beta} \right] \rightarrow \text{var.}$$

const. = β

* Cycl:

$$f(\theta) = h \left[\frac{\theta}{\beta} - \frac{1}{2\pi} \sin \left(\frac{2\pi \theta}{\beta} \right) \right]$$



* Polyn.

$$f(\theta) = \sum_{i=0}^n (c_i \theta^i)$$

* Ex. 8 A cam with a flat faced follower is to have the following motion for one cycle:

* $0 \rightarrow 150^\circ$: Rise of 10 cm, S.H.M

* $150^\circ \rightarrow 240^\circ$: Dwell

* $240^\circ \rightarrow 360^\circ$: Fall, S.H.M

$\rightarrow$ Plot & label the "SVA" diagram.

* Solving Vertically.

* Rise 8

$$S = S_i + \frac{h}{2} \left[1 - \cos \left(\frac{\pi}{\beta} (\theta - \theta_i) \right) \right]$$

$$\rightarrow S_i = 0, \theta_i = 0, h = 10 \text{ cm}, \beta = 150^\circ = \frac{5\pi}{6} \text{ In rad.}$$

$$\rightarrow S = 5 \left[1 - \cos (1.2\theta) \right], 0 \leq \theta \leq 150^\circ$$

* check 8

$$S(\theta=0) = 0, S(\theta=150^\circ) = 10 \quad \checkmark$$

$$v = \omega \frac{ds}{d\theta} = 6\omega \sin (1.2\theta), 0 \leq \theta \leq 150^\circ$$

$$\rightarrow v(\theta=0) = 0, v(\theta=150^\circ) = 0, v(\theta=75^\circ) = 6\omega = v_{\max}.$$

$$*\vec{a} = \omega \frac{\partial v}{\partial \theta} = 7.2 \omega^2 \cos(1.2\theta), \quad 0 \leq \theta \leq 150^\circ$$

$$\sim \vec{a}(\theta=0) = 7.2 \omega^2 = \vec{a}_{\max} \quad | \quad \vec{a}(\theta=150^\circ) = -7.2 \omega^2 = \vec{a}_{\min}$$

$$\vec{a}(\theta=75^\circ) = \vec{0}$$

$$*\dot{j} = \omega \frac{\partial a}{\partial \theta}$$

$$\dot{j} = \begin{cases} -8.64 \omega^3 \sin(1.2\theta), & 0 < \theta < 150^\circ \\ \pm \infty & \begin{cases} \theta=0 \\ \theta=150^\circ \end{cases} \end{cases} \quad \text{"lim from Right} \neq \text{"lim from left"}$$

→ Discout.

$$\sim \dot{j}(\theta=0^+) = 0, \quad \dot{j}(\theta=75^\circ) = -8.64 \omega^3, \quad \dot{j}(\theta=150^\circ-) = 0$$

*Note: Symmetry "only" if ($\beta_{\text{rise}} = \beta_{\text{fall}}$)

$$\hookrightarrow (150^\circ - 0) \neq (360^\circ - 240^\circ)$$

$$S = S_i + \frac{h}{2} \left[1 - \cos\left(\frac{\pi}{\beta} (\theta - \theta_i)\right) \right]$$

Fall 8 $S_i = 10$, $\beta = 120^\circ = \frac{2\pi}{3}$, $\theta_i = 240^\circ$, $h_i = -10 \text{ cm}$

$$\rightarrow S = 10 - 5 \left[1 - \cos(1.5(\theta - 240^\circ)) \right], \quad (240^\circ \leq \theta \leq 360^\circ)_{\text{fall}}$$

*check:

$$\hookrightarrow S(\theta=240^\circ) = 10 \text{ cm}, \quad S(\theta=360^\circ) = 0$$

$$*\vec{S} = 5 + 5 \cos[1.5(\theta - 240^\circ)]$$

$$\sim v = \omega \frac{\partial S}{\partial \theta} = -7.5 \omega \sin[1.5(\theta - 240^\circ)]$$

$$v(\theta=240^\circ) = 0, \quad v(\theta=360^\circ) = 0, \quad v(\theta=300^\circ) = -7.5 \omega$$

$$\sim \vec{a} = \omega \frac{\partial v}{\partial \theta} = -11.25 \omega^2 \cos[1.5(\theta - 240^\circ)]$$

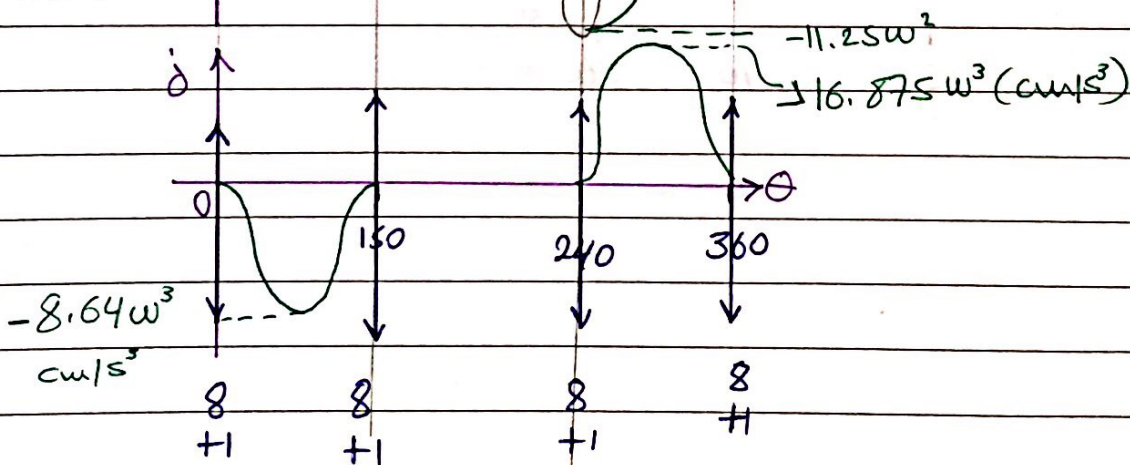
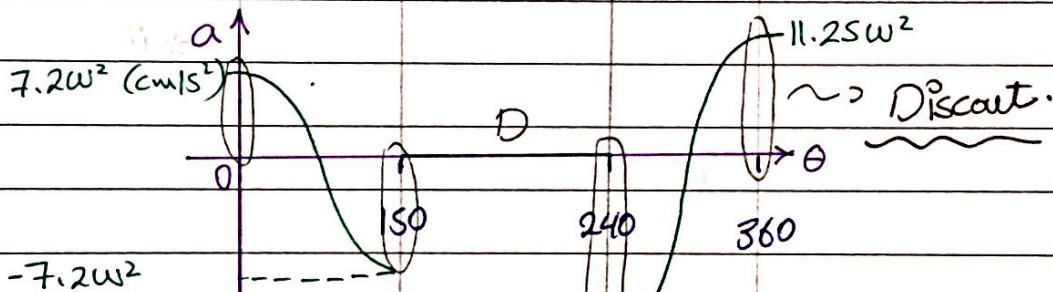
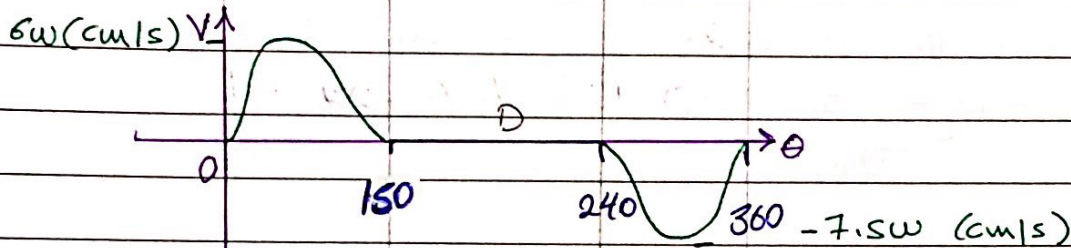
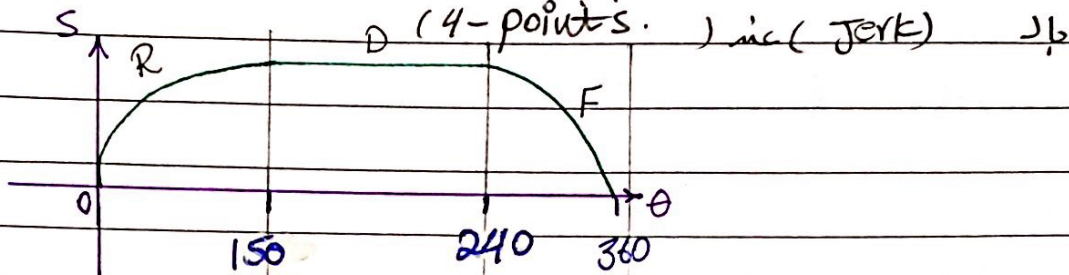
$$\vec{a}(\theta=240^\circ) = -11.25 \omega^2, \quad \vec{a}(\theta=300^\circ) = 0, \quad \vec{a}(\theta=360^\circ) = 11.25 \omega^2$$

$$\sim \ddot{\theta} = \begin{cases} 16.875\omega^3 \sin[1.5(\theta - 240)] & , 240^\circ < \theta < 360^\circ \\ \pm \infty & , \theta = 240^\circ \\ & \theta = 360^\circ \end{cases} \quad \text{without equal.}$$

$$\sim \ddot{\theta}(240^+) = 0, \ddot{\theta}(300^\circ) = 16.875\omega^3 \equiv \text{max. finite value}$$

$$\ddot{\theta}(360^-) = 0$$

د (Program) د (Jerk) د (4-points) د (Jerk) د (Jerk)



* Ex. 28 the same (Ex. 1), but using "cycloidal"

* $S = S_i + h \left[\frac{\theta - \theta_i}{\beta} - \frac{1}{2\pi} \sin \left(\frac{2\pi}{\beta} (\theta - \theta_i) \right) \right]$, solving Horizontally
Rise $\rightarrow$ Dwell $\rightarrow$ Fall

* Rise:

$S_i = 0$, $\theta_i = 0$, $h = 10 \text{ cm}$, $\beta = 150^\circ = \frac{5\pi}{6}$

$\rightarrow S = 10 \left[\frac{6\theta}{5\pi} - \frac{1}{2\pi} \sin(2.4\theta) \right]$, $(0 \leq \theta \leq 150^\circ)$

$= \frac{12\theta}{\pi} - \frac{5}{\pi} \sin(2.4\theta)$

* Check:

$\rightarrow S(\theta=0) = 0$, $S(\theta=150^\circ) = 10$ ✓

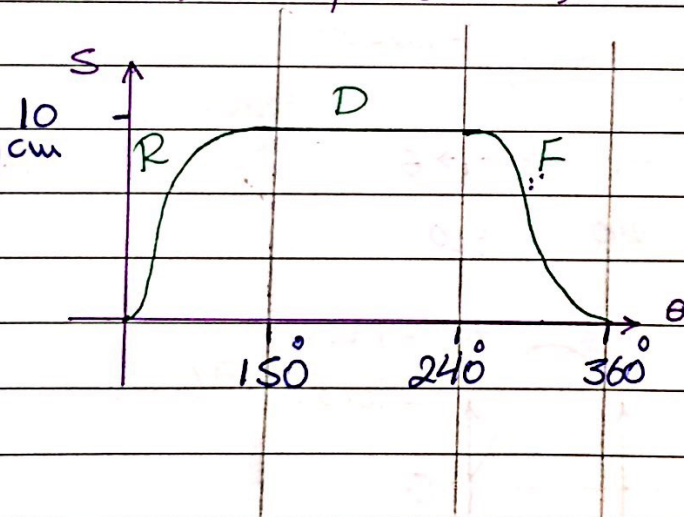
* Dwell: $S = 10 \text{ cm}$, $(150 \leq \theta \leq 240)$

* Fall: $S_i = 10$, $\theta_i = 240$, $h = -10 \text{ cm}$, $\beta = 120 = \frac{2\pi}{3}$

$\rightarrow S = 10 - 10 \left[\frac{\theta - \frac{4\pi}{3}}{\frac{2\pi}{3}} - \frac{1}{2\pi} \sin \left[3(\theta - 240) \right] \right]$

* Check:

$S(\theta=240) = 10$, $S(\theta=360) = 0$



(+ for rise, - for fall)
S.H.M & cyc.

$$* s = \begin{cases} \frac{12\theta}{\pi} - \frac{5}{\pi} \sin(2.4\theta) & ; 0 \leq \theta \leq 150^\circ \\ 10 \text{ cm} & ; 150^\circ \leq \theta \leq 240^\circ \\ 10 - 10 \left[\frac{\theta - \frac{4}{3}\pi}{\frac{2\pi}{3}} - \frac{1}{2\pi} \sin[3(\theta - 240)] \right] & ; 240^\circ \leq \theta \leq 360^\circ \end{cases}$$

Rise * $v = \omega \frac{ds}{d\theta} = \frac{12\omega}{\pi} - \frac{12\omega}{\pi} \cos(2.4\theta)$

$$\sim v(\theta=0^\circ) = 0, v(\theta=150^\circ) = 0, v(\theta=75^\circ) = \frac{24\omega}{\pi} = 7.64\omega$$

Dwell * $v = 0$

Fall * $v = \omega \frac{ds}{d\theta} = -10\omega \left[\frac{3}{2\pi} - \frac{3}{2\pi} \cos[3(\theta - 240)] \right]$

$$= -\frac{15\omega}{\pi} [1 - \cos[3(\theta - 240)]] ; 240 \leq \theta \leq 360$$

$$\sim v(240) = 0, v(360) = 0, v(300) = -\frac{30\omega}{\pi} = -9.54\omega \text{ (cm/s}^2\text{)}$$

Rise * $\vec{a} = \omega \frac{dv}{d\theta} = \frac{28.8\omega^2}{\pi} \sin(2.4\theta)$

$$\sim \vec{a}(0) = 0, \vec{a}(150) = 0, \vec{a}(75) = 0 \quad \left(\vec{a}(37.5) = \frac{28.8}{\pi} \omega^2 = 9.16\omega^2 = \vec{a}_{\max} \right)$$

$$\vec{a}(112.5) = -9.16\omega^2 = \vec{a}_{\min} \quad \left(\text{At Energy } \frac{\pi}{2} \text{ (at Force } 2\omega^2) \right)$$

Dwell * $\vec{a} = 0$

Fall * $\vec{a} = \omega \frac{dv}{d\theta} = -\frac{45\omega^2}{\pi} \sin[3(\theta - 240)]$

$$\sim \vec{a}(240) = 0, \vec{a}(360) = 0, \vec{a}(300) = 0, \vec{a}(270) = -\frac{45\omega^2}{\pi} = -14.32\omega^2$$

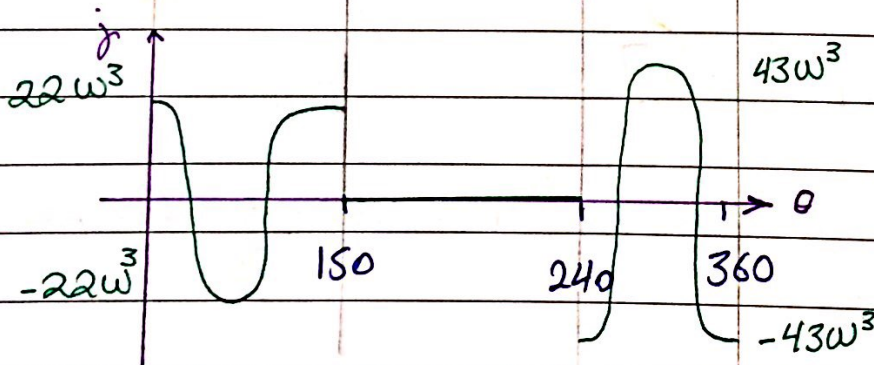
$$\vec{a}(330) = \frac{45\omega^2}{\pi} = 14.32\omega^2$$

$$\dot{r} = 22\omega^3 \cos(2.4\theta) \quad ; \quad 0 \leq \theta \leq 150^\circ$$

Dwell: $\dot{\gamma} = 0$

$$\delta = -43\omega^3 \cos [3(\theta - 240)] \quad ; 240^\circ \leq \theta \leq 360^\circ$$

• S.H.M is inf. to all other cycloidal M.



* Polynomial $N=3$

... No. of terms is 4

$$\leadsto S = S_i + \sum_{k=0}^N C_k (\theta - \theta_i)^k$$

* Ex: 4 conditions (4 Requirements), $N=0,1,2,3$

4-cond.

$$1. S(\theta_i) = S_i$$

$$2. V(\theta_i) = 0$$

$$3. S(\theta_i + \beta) = S_i + h$$

$$4. V(\theta_i + \beta) = 0$$

$$\leadsto S = S_i + C_0 + C_1(\theta - \theta_i) + C_2(\theta - \theta_i)^2 + C_3(\theta - \theta_i)^3$$

$$\leadsto V = \frac{dS}{d\theta} = C_1 + 2C_2(\theta - \theta_i) + 3C_3(\theta - \theta_i)^2$$

$$1. S(\theta_i) = S_i$$

$$= S_i + C_0 + 0 + 0 + 0 \Rightarrow C_0 = 0$$

$$2. V(\theta_i) = 0$$

$$= C_1 + 0 + 0 \Rightarrow C_1 = 0$$

$$3. S(\theta_i + \beta) = S_i + h$$

$$= S_i + C_2(\beta)^2 + C_3(\beta)^3$$

$$C_2\beta^2 + C_3\beta^3 = h \quad \text{--- (1)}$$

$$4. V(\theta_i + \beta) = 0$$

$$= 2C_2\beta + 3C_3\beta^2$$

$$C_2(2\beta) + C_3(3\beta^2) = 0 \quad \text{--- (2)}$$

$$\Rightarrow C_2 = \frac{3h}{\beta^2}$$

$$\Rightarrow C_3 = -\frac{2h}{\beta^3}$$

$$** S = S_i + \frac{3h}{\beta^2}(\theta - \theta_i)^2 - \frac{2h}{\beta^3}(\theta - \theta_i)^3 ; (\theta_i \leq \theta \leq \theta_i + \beta)$$

(3rd - order polynomial).

+ Suggested :

* Same Ex. !

→ must be in rad.

"Solving Vertically"

* Rise: $s_i = 0$, $\theta_i = 0$, $\beta = \frac{5\pi}{6}$, $h = 10$

$$s = \frac{30}{\left(\frac{5\pi}{6}\right)^2} \theta^2 - \frac{20}{\left(\frac{5\pi}{6}\right)^2} \theta^3$$

$$= 4.37\theta^2 - 1.11\theta^3, \quad 0 \leq \theta \leq \frac{5\pi}{6} = 150^\circ$$

* check !

$$s(\theta=0) = 0 \quad \checkmark, \quad s(\theta = \frac{5\pi}{6}) = 10 \text{ cm}$$

$$* v = \omega \frac{\partial s}{\partial \theta} = 8.74\omega\theta - 3.33\omega\theta^2$$

$$\sim v(\theta=0) = 0, \quad v(\frac{5\pi}{6}) = 0, \quad v(\frac{5\pi}{12}) = 5.73\omega$$

$$* \vec{a} = \omega \frac{\partial v}{\partial \theta} = 8.74\omega^2 - 6.66\omega^2\theta$$

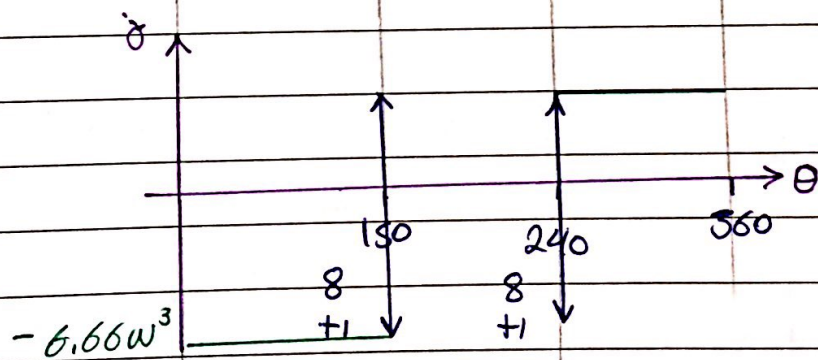
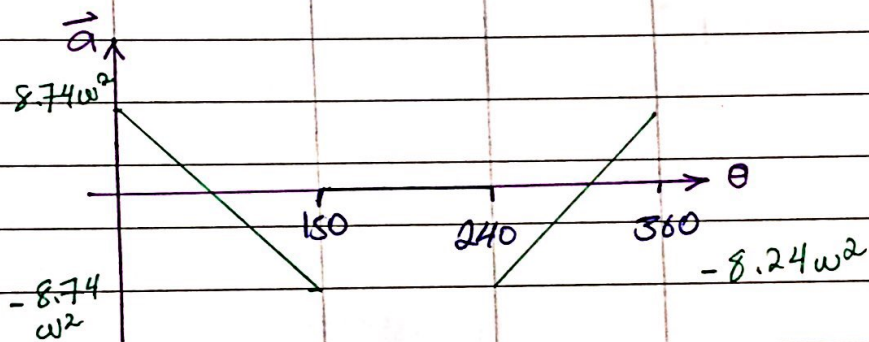
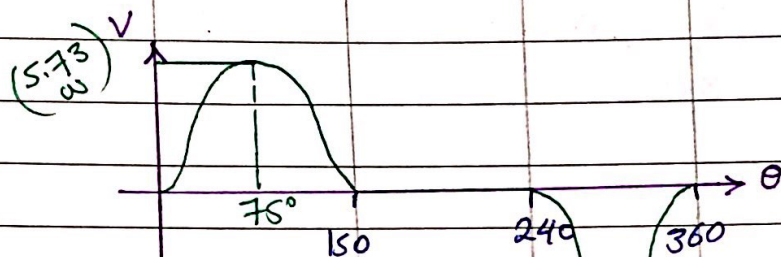
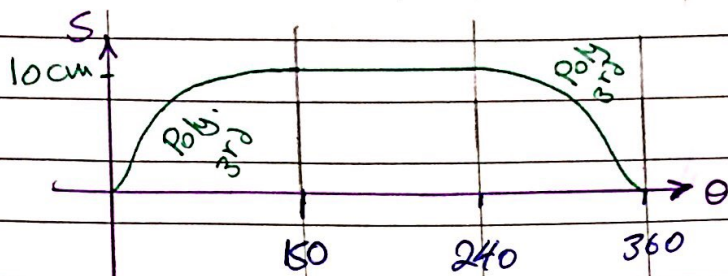
$$\sim \vec{a}(0) = 8.74\omega^2, \quad \vec{a}(\frac{5\pi}{6}) = -8.74\omega^2, \quad \vec{a}(\frac{5\pi}{12}) = 0$$

$$* j = \omega \frac{\partial \vec{a}}{\partial \theta} = -6.66\omega^3$$

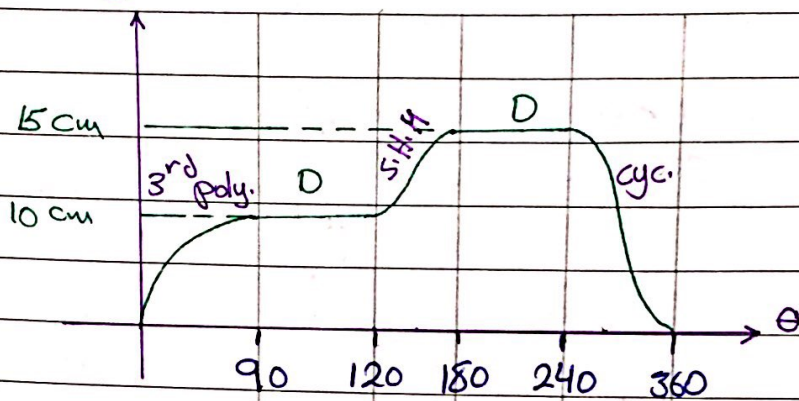
$$j = \begin{cases} -6.66\omega^3 & ; 0 < \theta < 150^\circ \\ \pm \infty & ; \theta = 0, \theta = 150^\circ \end{cases}$$

* Fall : $s = h - \frac{3h}{\beta^2} \left(\theta - \frac{4\pi}{3}\right)^2 + \frac{2h}{(\beta)^3} \left(\theta - \frac{4\pi}{3}\right)^3$

$$s = h - \frac{30}{(2\pi\beta)^2} \left(\theta - \frac{4\pi}{3}\right)^2 + \frac{30}{\left(\frac{2\pi}{3}\right)^3} \left(\theta - \frac{4\pi}{3}\right)^3$$

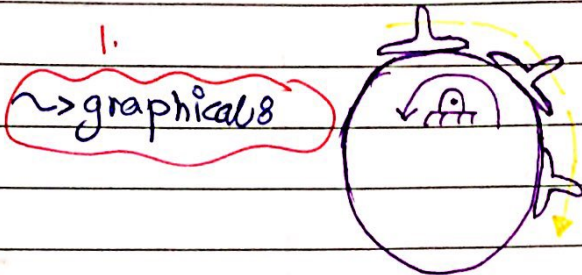
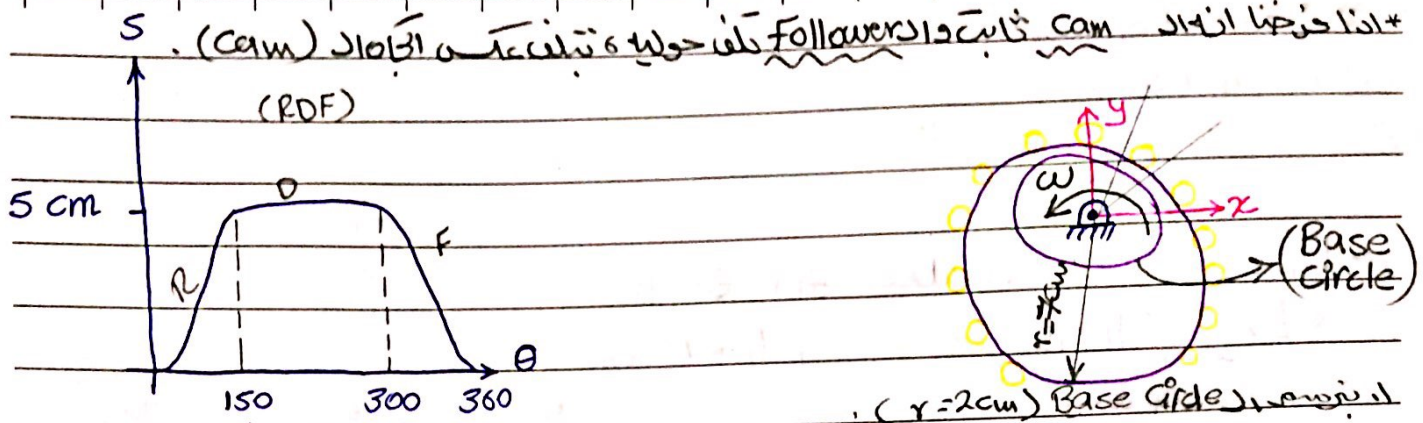


Suggested 8



* Cam design8

* Ex8



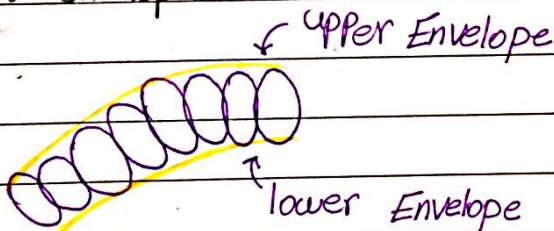
1. $(0 \rightarrow 150^\circ)$ Range of cam
 2. Dwell
 3. Fall (Base circle)
 # Remember: $h = \text{max. rad.} - \text{Base rad.}$

2. Analytical cam design8 "Theorey of Envelopes".

$\sim$ Family of similar curves8.

* Family of circles8

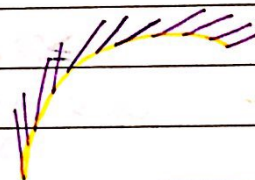
2-envelopes



$\sim$ Roller-follower

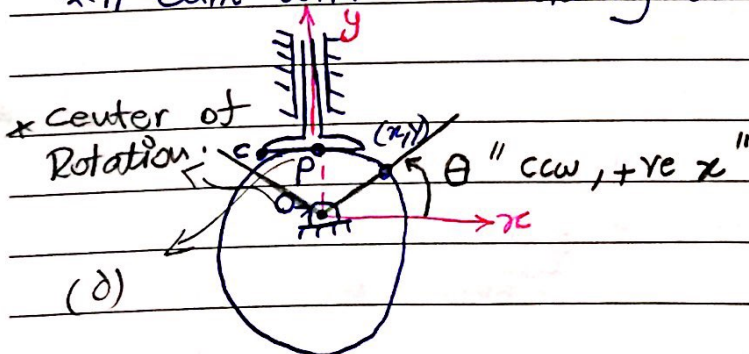
* Family of straight lines8

1-envelope



$\sim$ Flat-Faced follower

* A cam with a translating flat-faced follower8



$\left[\begin{matrix} (d) \\ c \end{matrix} \right]$

$$x(\theta) \quad s(\theta) \quad \frac{ds}{d\theta}(\theta)$$

$$\sim x = -(r_b + s) \sin \theta - \frac{ds}{d\theta} \cos \theta$$

$$\sim y = (r_b + s) \cos \theta - \frac{ds}{d\theta} \sin \theta$$

$y(\theta)$

→ Find (x, y) coordinates for $(\theta = 90^\circ)$:

* Ex: S.H.M "as Ex.1", $r_b = 15 \text{ cm}$.

→ Rise s

$$s(\theta) = 5 - 5 \cos(1.2\theta) \quad (0 \leq \theta \leq 150^\circ)$$

$$\frac{ds}{d\theta} = \frac{v}{\omega} = 6 \sin(1.2\theta) \quad ; \quad "v = \omega \frac{ds}{d\theta}"$$

$$\sim s(90^\circ) = (6.54) \text{ cm}$$

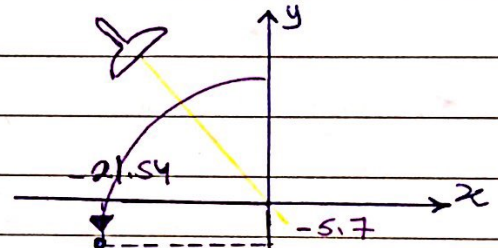
$$\left. \frac{ds}{d\theta} \right|_{\theta=90^\circ} = (5.7) \text{ cm/rad}$$

θ	$s(\theta)$	$\frac{ds}{d\theta}(\theta)$	$x(\theta)$	$y(\theta)$
0				
360				

$$\sim x(90^\circ) = -(15 + 6.54) \sin 90^\circ - 5.7 \cos 90^\circ = -21.54 \text{ cm}$$

$$\sim y(90^\circ) = -5.7 \text{ cm}$$

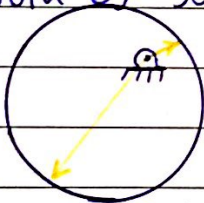
* at $\theta = 0^\circ$, it will give me "Base rad."



$$\rightarrow \delta = \frac{ds}{d\theta}$$

→ width of follower $\geq 2\delta_{\max}$. "to keep in contact with Cam Surface".

$$OR \geq 2 \left. \frac{ds}{d\theta} \right|_{\max}$$



→ Cam Surface curvature $(k = \frac{1}{\rho})$



→ sharp edges (α) → radius of curvature.
"cusp" i "sharp angle"

$(-ve \text{ i } out), (\alpha)$

1. No sharp Edges $\wedge$

2. $\rho > 0$

→ Radius of curvature.

* conclusion $(\rho > 0)$

$$\sim \rho = \frac{[1 + (\frac{\partial y}{\partial x})^2]^{3/2}}{(\frac{\partial^2 y}{\partial x^2})}$$

$$\rightarrow x(\theta) = -(r_b + s) \sin \theta - \frac{\partial s}{\partial \theta} \cos \theta$$

$$\rightarrow y(\theta) = (r_b + s) \cos \theta - \frac{\partial s}{\partial \theta} \sin \theta$$

$$\sim \frac{\partial y}{\partial x} = \frac{\partial y / \partial \theta}{\partial x / \partial \theta} \rightarrow f(s)$$

* Exo 8

$$\begin{aligned} * \frac{\partial x}{\partial \theta} &= -r_b \cos \theta - \frac{\partial s}{\partial \theta} \sin \theta - s \cos \theta - \frac{\partial^2 s}{\partial \theta^2} \cos \theta + \frac{\partial s}{\partial \theta} \sin \theta \\ &= -(r_b + s) \cos \theta - \frac{\partial^2 s}{\partial \theta^2} \cos \theta \end{aligned}$$

$$* \frac{\partial y}{\partial \theta} = \dots$$

$$\begin{aligned} \rightarrow \frac{\partial^2 y}{\partial x^2} &= \frac{\partial}{\partial x} \left(\frac{\partial y}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{\partial y / \partial \theta}{\partial x / \partial \theta} \right) = \frac{\partial}{\partial \theta} \left(\frac{\partial y / \partial \theta}{\partial x / \partial \theta} \right) * \frac{\partial \theta}{\partial x} \\ &= \frac{\frac{\partial}{\partial \theta} \left(\frac{\partial y / \partial \theta}{\partial x / \partial \theta} \right)}{(\partial x / \partial \theta)} \end{aligned}$$

$$\sim \rho(\theta) = r_b + s + \frac{\partial^2 s}{\partial \theta^2}$$

β جى ۋى ۋى ۋى ۋى ۋى Symmetry ۋى ۋى ۋى ۋى ۋى
S.H.M. cyc. Poly. Function ۋى ۋى

* Exo: Find $\rho(\theta = 90^\circ)$; $r_b = 15 \text{ cm}$, $s(90) = 6.54 \text{ cm}$.

$$\sim s = 5 - 5 \cos(1.2\theta), \quad \frac{\partial s}{\partial \theta} = 6 \sin(1.2\theta)$$

$$\frac{\partial^2 s}{\partial \theta^2} \Big|_{\theta=90} = 7.2 \cos(1.2\theta) = -2.22 \text{ cm/rad}^2$$

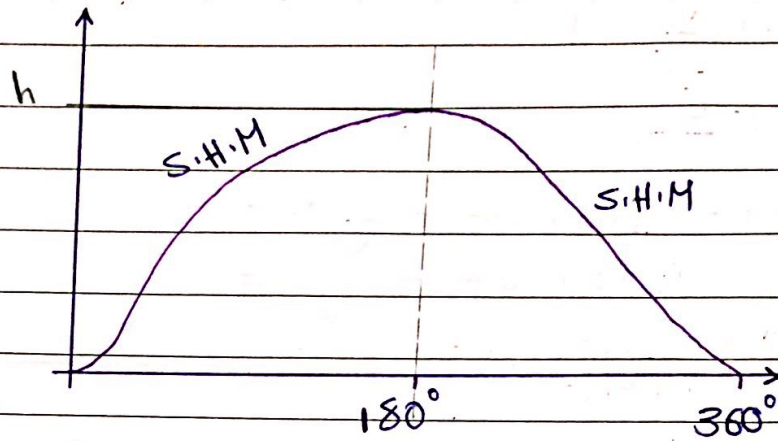
$$\sim \rho = 15 + 6.54 - 2.22 = 19.3 \text{ cm}$$

* Exo 8 RF

$\leadsto (0 \rightarrow 180^\circ)$

$$s = \frac{h}{2} - \frac{h}{2} \cos(\theta)$$

$$\frac{ds}{d\theta} = \frac{h}{2} \sin \theta$$



$$\rightarrow x = -(r_b + s) \sin \theta - \frac{ds}{d\theta} \cos \theta$$

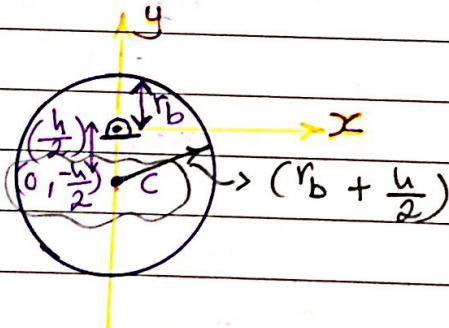
$$= -\left(r_b + \frac{h}{2} - \frac{h}{2} \cos \theta\right) \sin \theta - \frac{h}{2} \sin \theta \cos \theta$$

$$= -\left(r_b + \frac{h}{2}\right) \sin \theta \rightarrow \sin \theta \Rightarrow$$

$$\rightarrow y = \left(r_b + \frac{h}{2}\right) \cos \theta - \frac{h}{2} \rightarrow \cos \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

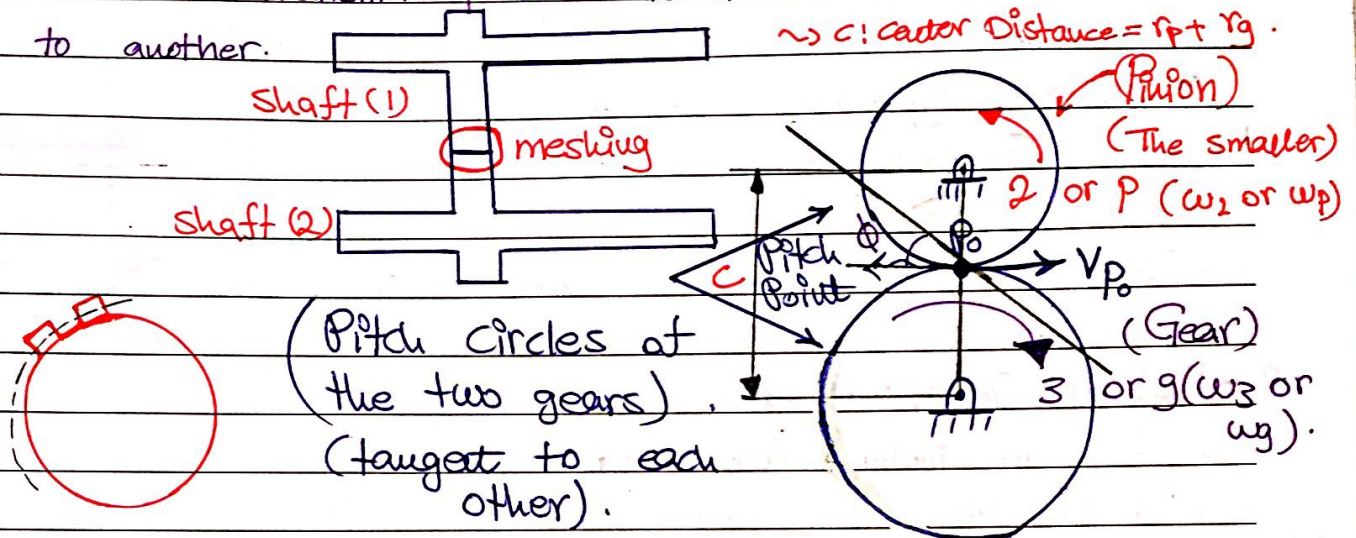
$$x^2 + \left(y + \frac{h}{2}\right)^2 = \left(r_b + \frac{h}{2}\right)^2 \quad , \quad x^2 + y^2 = r^2 \quad \text{"Circle formula"}$$



* Gears

CH. 9

- Toothed members.
- They mesh with each other.
- Used to transmit power and motion from one shaft to another.



(it Divide them $\div 2$) P_o = along the two centers

* pitch circles: theoretical circles tangent to each other at the pitch point, " P_o ".

$\leadsto$ Pinion: the smallest gear.

$\leadsto$ Gear: the largest gear.

V_p : pitch line velocity.

$$\rightarrow (V_p)_p = \omega_p r_p = (V_p)_g \quad ; \quad r_p \text{ is pitch radius of pinion.}$$

(pitch line velocity) $= \omega_g r_g$

$$\frac{\omega_g}{\omega_p} = -\frac{r_p}{r_g} \leadsto \left| \frac{\omega_g}{\omega_p} \right| = \frac{r_p}{r_g} \quad \leadsto \text{abs. value.}$$

(Gear) Pinion $\rightarrow$ $\frac{\omega_g}{\omega_p}$ is the ratio of angular velocities.
 gear $\rightarrow$ $\frac{\omega_g}{\omega_p}$ is the ratio of angular velocities.
 Torque $\rightarrow$ $\frac{T_g}{T_p}$ is the ratio of torques.

* Assuming 100% efficiency:

$$(Power)_{in} = (Power)_{out} \quad ; \quad mv = \text{angular velocity ratio}$$

$$(Power)_p = (Power)_g \quad mv = \omega_g / \omega_p = \text{output / Input}$$

$$\leadsto T_p \omega_p = T_g \omega_g$$

$$\Rightarrow \omega_g = \frac{T_p}{T_g} = \frac{r_p}{r_g}$$

$$m_v \leftarrow \omega_p \quad m_T \leftarrow \frac{T_g}{T_p}$$

$m_T \equiv$ Torque ratio.

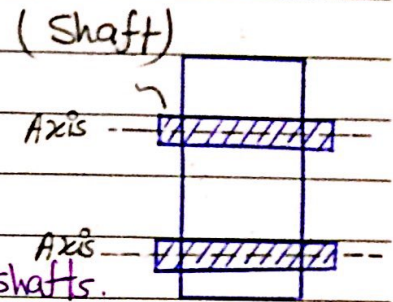
$$m_T = \frac{T_g}{T_p} = \frac{r_g}{r_p} > 1$$

* Gears Types:

1 - Spur Gears: "Basic type"

→ Parallel shafts. (Parallel teeth)

→ Teeth are straight and parallel to the shafts.

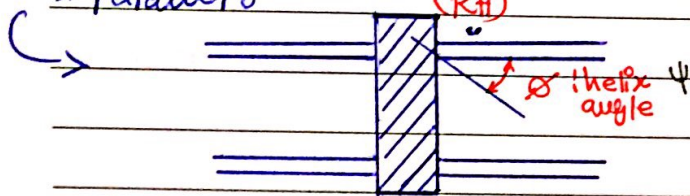


2 - Helical Gears: (Inclined teeth).

→ Teeth are inclined w.r.t shafts.

→ Shafts are either Parallel or crossed. → In diff. planes.

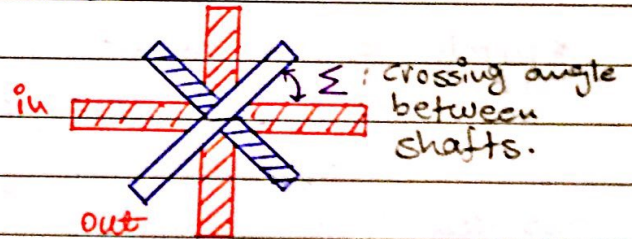
* Parallel



* Parallel Helical *

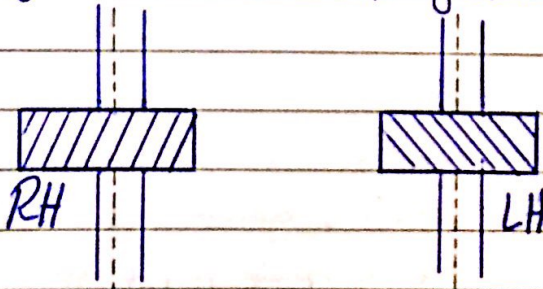
$|\phi_p| = |\phi_g|$; opposite hand

* crossed



Σ: crossing angle between shafts.

→ Hand of the helical gears →



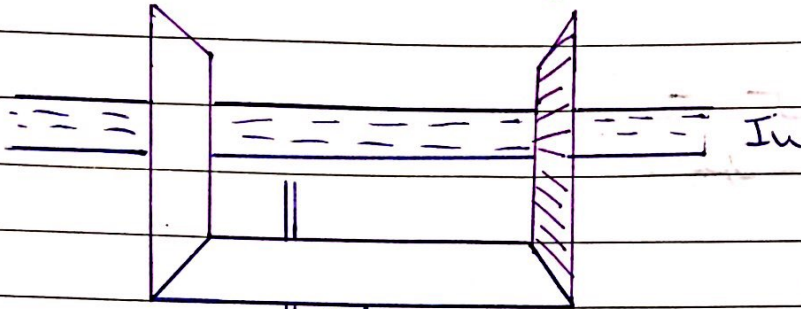
→ Parallel meshing gears (opposite handed) will work like spur gears.

→ unlike Sudden impact between teeth of the spur gear which causes vibrations.

3- Bevel gears at the same plane

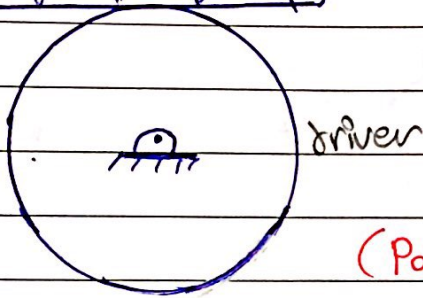
→ Teeth are cut on conical surfaces.

→ Shafts are intersecting. crossing at 90° or at any compatible angle.

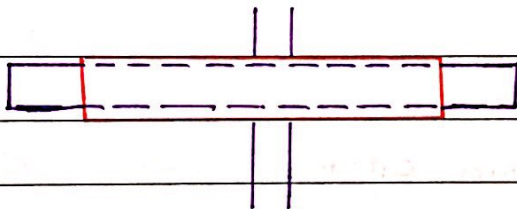


4- Worm gears like crossed gears.

→ Worm (screw)



(Power $\neq$ Power)
 I_{in} out



Top View

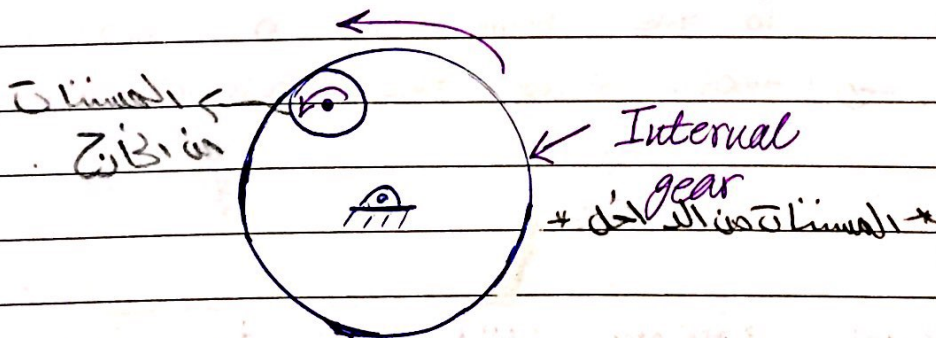
*we can increase surface area of contact between

worm & worm gear increase of capacity of carrying very high load. and cost.

*it can't be backdriven

$$\text{gearset Ratio} = \frac{1}{\left(\frac{\text{no. of teeth of worm gear}}{\text{no. of teeth of gear}} \right)}$$

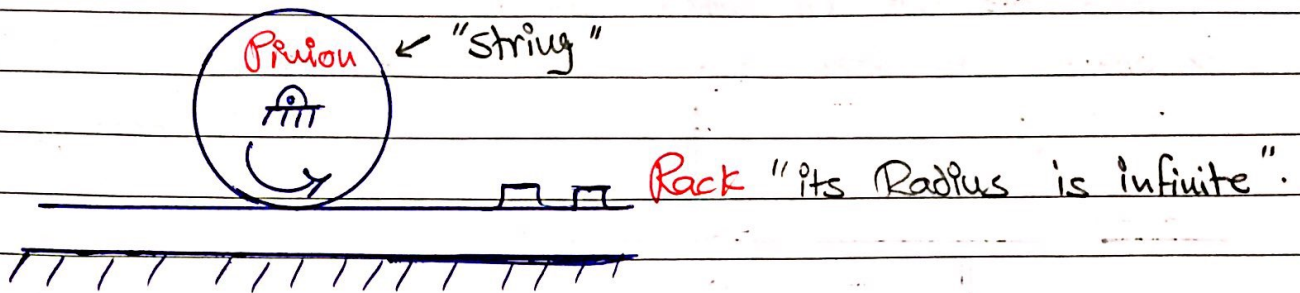
* Internal meshing gears



→ efficiency 40-85%

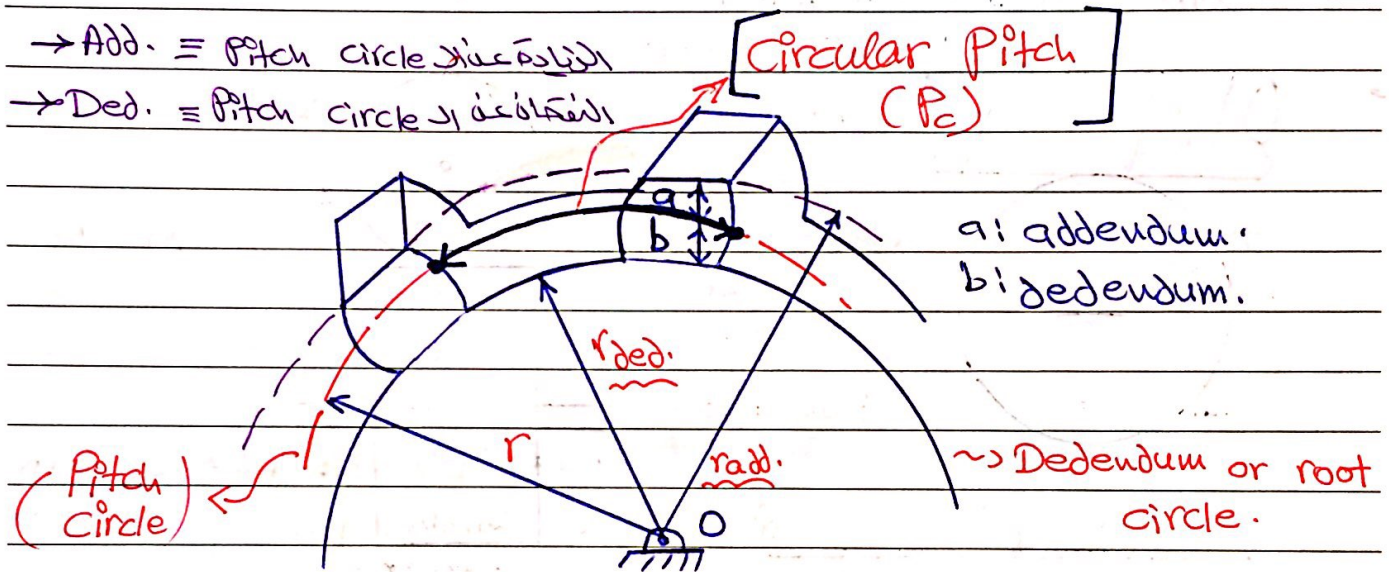
* The worm-gear can be driven only from the worm.

* Rack and pinion gears "teeth are trapezoides"



→ Add. $\equiv$ Pitch circle distance

→ Ded. $\equiv$ Pitch circle distance



a: Radial distance between addendum circle and Pitch circle.

b: Radial distance between dedendum circle and Pitch circle.

→ $a+b$ whole depth of tooth $= [h_t = a+b]$.

→ circular pitch (P_c) & Arc length from one point to the same point on another tooth along the pitch circle.

$$\left. \begin{aligned} \sim r_{add.} &= r + a \\ \sim r_{ded.} &= r - b \end{aligned} \right\}$$

No. of circular pitch (P_c) = No. of teeth.

No. of teeth.

$$\sim P_c = \frac{\pi d}{N} = 2r, \quad d: \text{pitch diameter.}$$

$$\sim \frac{d}{N} = m: \text{module [mm] "SI"}, \quad P_c = \pi m$$

$$\sim \frac{N}{d} = P: \text{diametral pitch} \left[\frac{\text{teeth}}{\text{Inch}} \right] \text{ "US" .}$$

$\sim$ Tooth thickness + width space = Pitch circle.
which lies on pitch circle.

$\sim$ Clearance circle.

$\sim$ Backlash: point of contact ; 'دور' (clearance)

$$\sim a = 1 \text{ (m)} \rightarrow \text{module not (1) meter.}, \quad \text{clearance} = 0.25$$

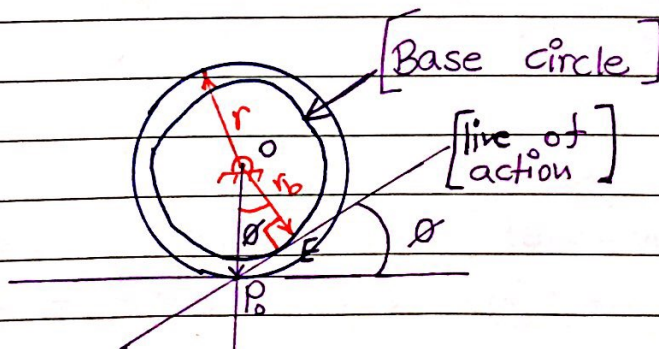
* common Normal $\equiv$ Axis of transmission $\equiv$ line of action
Pressure line.

(2-point) of contact

* Pressure angle $\equiv$ $\angle$ between line of action (line of action) and line of centers (line of centers).

tooth thickness $\rightarrow$ Pressure angle

* ϕ (fine mesh) $\rightarrow$ ϕ (coarse mesh)



$$\sim \overline{OP_b} = r$$

$$= \frac{(OE)}{\cos \phi} \rightarrow r_b =$$

$$\Rightarrow r_b = r \cos \phi$$



* standard pressure angle $\equiv \phi = 20^\circ$.

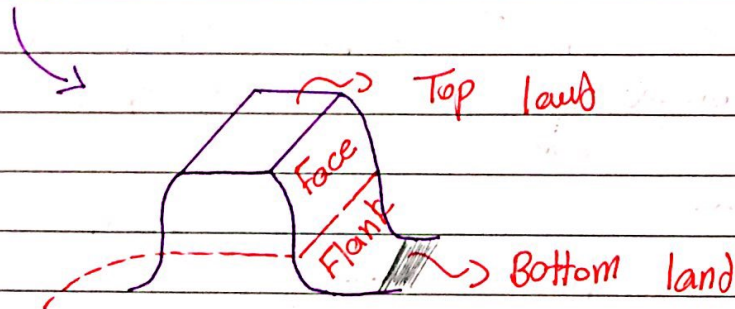
$\leadsto$ Base pitch: $P_b = \frac{\pi d_b}{N}$; $d_b = d \cos \phi$
 $= \frac{\pi d \cos \phi}{N}$

$\leadsto$ Face width: thickness of each tooth "the same".

$\leadsto$ Tip of tooth & Edge $\uparrow$ related.

$\leadsto$ Face

$\leadsto$ Flank



$\ast \frac{W_{out}}{W_{in}} = \frac{r_{out}}{r_{in}} = \frac{N_{in}}{N_{out}} = \text{const.}$

1. module m (دائرة التماس); 2-gears in mesh d_1 & d_2

2. diametral pitch

3. Circular pitch.

$\ast m_p = m_g \rightarrow \frac{d_p}{N_p} = \frac{d_g}{N_g}$

$\left. \begin{matrix} (P_c)_p = (P_c)_g \\ P_p = P_g \end{matrix} \right\} \Rightarrow \frac{d_p}{d_g} = \frac{N_p}{N_g}$

$\ast$ Fundamental law of gears

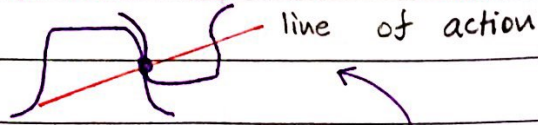
$\hookrightarrow$ Conjugate action. [$m_v = \text{const.}$] during meshing. as a ratio.

$\rightarrow$ Angular velocity ratio " m_v " remains constant during the meshing.

$$m_A = m_T$$

→ mechanical Advantage. "Reduce the velocity & increase Torque"

* Profile of the tooth:



Conjugate action (mv = const.)

→ "Involute profile".

Involute Curve:

→ "mathematical curve".

curve of a gear is a conjugate action.

Conjugate Action.

Base circle of Involute

Involute (Dedendum circle)

Non-Involute ← Circle
Non-conjugate action, $mv \neq \text{const.}$

→ Interference problem: (Non-Involute portion)
[Contact at these points]

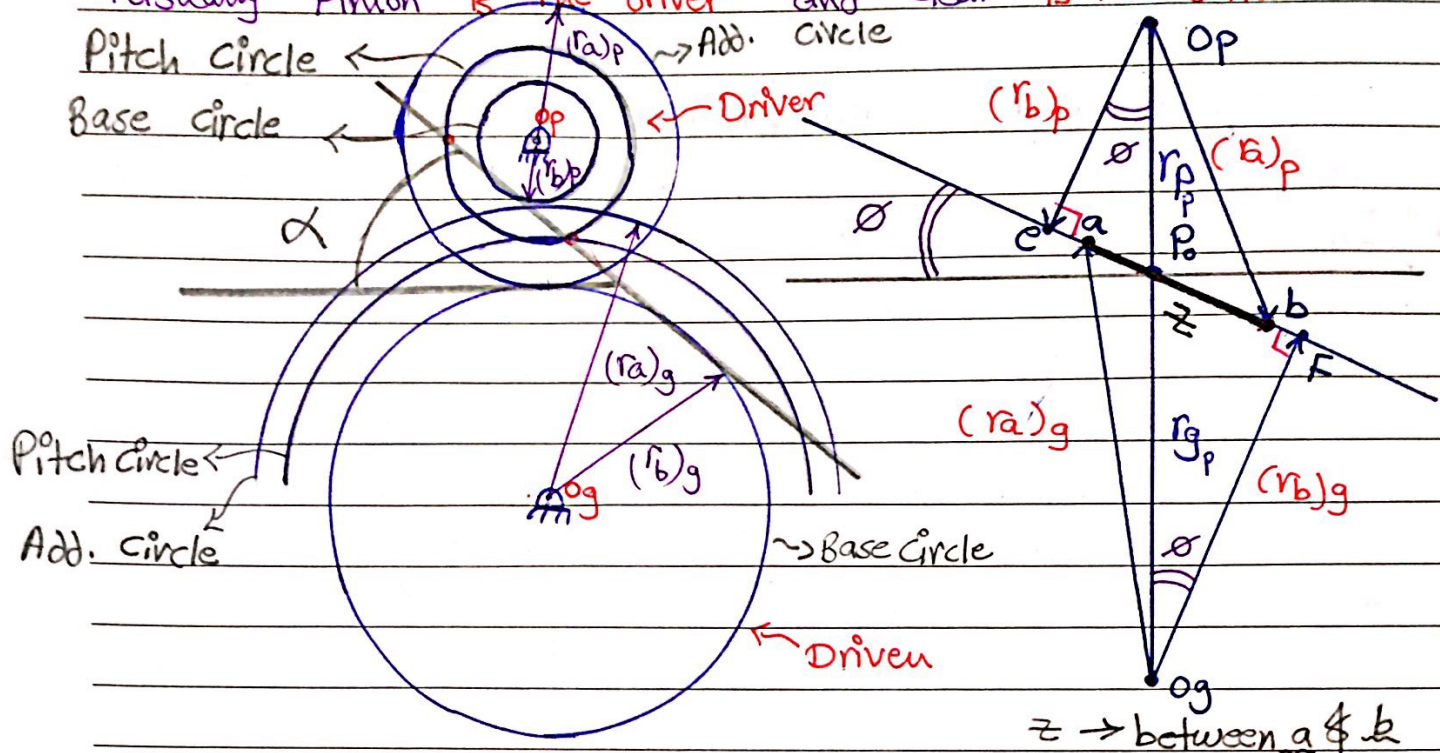
below (base circle) non-Involute (base circle)

* Beginning of contact:

* End of contact:

(length of line of action) : contact

* usually "Pinion" is the driver and "Gear" is the driven.



$\sim z = \overline{ab}$: length of line of action.

$$z = \overline{ap_0} + \overline{p_0b}$$

$\rightarrow$ Consider triangle $(Op_e b)$

$$\left[\begin{array}{l} \overline{ap_0} = 1m \\ \overline{ap_0} = 1m \end{array} \right] \begin{array}{l} r_p^2 = r_{bp}^2 + (eb)^2 = [r_p + \overline{ap_0}] \\ (eb) = \sqrt{(r_p + \overline{ap_0})^2 - (r_p \cos \phi)^2} = (ep_0 + p_0b) \end{array}$$

$\rightarrow$ Consider triangle $(Og f a)$

$$(fa) = \sqrt{(r_g + \overline{ap_0})^2 - (r_g \cos \phi)^2} = \overline{fp_0} + \overline{ap_0}$$

$\rightarrow \Delta (Op_e p_0)$

$$ep_0 = r_p \sin \phi$$

$\rightarrow \Delta (Og f p_0)$

$$fp_0 = r_g \sin \phi$$

$$\Rightarrow \overline{p_0b} = z_1 - r_p \sin \phi$$

$$\Rightarrow \overline{ap_0} = z_2 - r_g \sin \phi$$

$$; \quad z = z_1 + z_2 - z_3 \quad \left\{ \begin{array}{l} z_3 = (r_p + r_g) \sin \phi \\ z_3 = c \sin \phi \end{array} \right.$$

* Note: If (a, b) out of (e, f) there will be an Interference; since it will be larger than base circle at (e, f)

* No-Inter. u Tol, g

1 * **Angle of approach** : The angle of intersects between tooth profile "دائري" with the pitch circle of gear "Driven".
 (line of centers) $O_1 O_2$ and pitch circle

[Initial point of contact @] : intersection of pitch circles

2 * **Arc of approach** : (q_a) (angle of approach) $\rightarrow$ (Arc) $\rightarrow$ pitch circle

1 * **Angle of recess** : Final point of contact @

2 * **Arc of recess** : the same as arc of approach. (q_r)

* **Arc of action** (q_t) :

$$q_t = q_a + q_r$$

arc of approach $\rightarrow$ arc of Recess.

* **Contact Ratio** $\equiv m_c = \frac{q_t}{P_c} \rightarrow$ Arc of action . $\left\{ \begin{array}{l} \text{minimum} \approx 1 \\ \text{referred} \approx 1.3 \end{array} \right.$
 $P_c \rightarrow$ circular pitch .

$\rightarrow m_c = \frac{z}{P_b} \rightarrow$ length of line of action.

$P_b \rightarrow$ Base pitch.

* **Center Distance** : $\left\{ \begin{array}{l} \text{Actual} \\ \text{Theoretical} \end{array} \right.$ (CD)

* (دائري (CD) دائري

Changed	Un-changed
Pitch circle	m_T
Pitch radius	add. circle
m_c	ded. circle
circular pitch	z_1, z_2 "physically"
diametral pitch	Base circle
module	
Pitch point	
ϕ "pressure angle"	
z_3, z	

Add. + ded.

$$\begin{aligned} & a \neq a' \\ & b \neq b' \\ & a + b = a' + b' \end{aligned}$$

~> Some proofs of "changed" and "unchanged":

$$* m_T = \frac{T_{out}}{T_{in}} = \frac{\omega_{out}}{\omega_{in}} \leftarrow \text{constant}$$

$$= \frac{N_{out}}{N_{in}} \Rightarrow \text{No. of teeth didn't change.}$$

$$= \frac{\partial G'}{\partial p'} \text{ "new situation"}$$

$$= \frac{\partial G}{\partial p} \text{ "old situation".}$$

$$* m_G' = \frac{z'}{P_b} \quad ; \quad d_b = d \cos \phi$$

$$; \quad d_b' = d' \cos \phi' = d_b$$

$$\rightarrow [d' \cos \phi' = d \cos \phi]$$

=> Assignment 8 Ex (9.4 or 9.3) -> changing center distance.
(as self Reading)

~~~~~ \*

$$* r_b = r_b' \Rightarrow r_p \cos \phi = r_p' \cos \phi'$$

$$\Rightarrow \phi' = \cos^{-1} \left[ \frac{r_p \cos \phi}{r_p'} \right] \quad \text{"shift in center distance".}$$

$$\frac{r_p}{r_g} = \frac{r_p'}{r_g'} \quad \text{---- } c'$$

\* Suggested ch.9 "part 1" 8

(1->4), 52, 59, (67->70) + Table (9-1)

+ Ex (9.1) }  
SI  
as

\* Gear Ratio 8 (mG)

$$m_G = \frac{N_g}{N_p}$$

; whole depth  $\equiv h_t = a + b$

working depth  $\equiv h_w = h_t - c$   
clearance.



## \* Gear Trains

"meshing" stage  $\rightarrow$   $\omega_1$   $\rightarrow$   $\omega_2$   
 $\omega_1$  (Torque magnification)  $\omega_2$

## Gear Trains

Fixed Axis (1DOF)

Moving axis (2DOF)

(Planetary / epicyclic)

1. Simple

$\rightarrow$  shaft  $\omega_1$   $\omega_2$

Ex.: 5 shafts  $\rightarrow$  4 stages (4-meshes)

Intermediate shafts act as driving & driven gears at same time

2. Compound.

3. Reverted compound.

### \* Simple

Ex.: Figure 9.28

$\rightarrow$  Stage 1:  $m_v = \frac{\omega_3}{\omega_2} = -\frac{N_2}{N_3}$

$\rightarrow$  Stage 2:  $m_v = \frac{\omega_4}{\omega_3} = -\frac{N_3}{N_4}$

$\rightarrow$  Stage 3:  $m_v = \frac{\omega_5}{\omega_4} = -\frac{N_4}{N_5}$

$\rightarrow$  Stage 4:  $m_v = \frac{\omega_6}{\omega_5} = -\frac{N_5}{N_6}$

$m_v = \frac{\omega_6}{\omega_2} = \frac{\omega_{out}}{\omega_{in}} = \left( \frac{N_2}{N_3} \right) \left( \frac{N_3}{N_4} \right) \left( \frac{N_4}{N_5} \right) \left( \frac{N_5}{N_6} \right)$

\* no. of gears (odd)  $\rightarrow +ve (e)$   
 \* " " " (even)  $\rightarrow -ve (e)$  } only for simple

### \* Train value $e$ (e)

$$e = \frac{\omega_L}{\omega_f} \quad \text{last gear ; first gear}$$

$\rightarrow$  Same direction of rotation (f & L)

$$\sim \rightarrow e = \frac{\omega_L}{\omega_f} = (\pm)$$

$\rightarrow$  opposite direction of rotation (f & L)

$$= \frac{\text{Product of teeth of driving gears}}{\text{Product of teeth of driven gears}}$$

القيمة (driving & driven gears)  $\rightarrow$  gears 1 و 2  
 (في المحرك والمحرك)

### \* Intermediate gears

Torque  $\rightarrow$  القوة الدافعة

shafts.  $\rightarrow$  محاور

$$\rightarrow F = 2, L = 6$$

$$e = \frac{\omega_6}{\omega_2} = (+) \frac{N_2 N_3 N_4 N_5}{N_3 N_4 N_5 N_6} = + \frac{N_2}{N_6}$$

القيمة  $\omega_{out}$  و  $\omega_{in}$  في

### \* Compound gear Trains

\* at least one shaft having 2-gears\*

$\rightarrow$  also fixed axis.

\* Advantage : I need less spacing for gears in the box.

\* Ex. : Figure 9.29:

$\rightarrow$  No. of stages = 2 "meshing points"

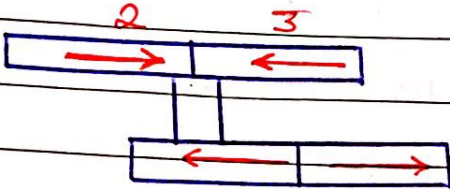
$\rightarrow$  2 Ideal shafts.

$\rightarrow$  gears (3,4)  $\rightarrow$  Not in mesh. ("Same  $\omega$ ")  
 $\omega_3 = \omega_4$



→  $F = 2$  ;  $L = 5$  ,  $e = \frac{\omega_5}{\omega_2}$

→  $e = \frac{\omega_5}{\omega_2} = (+) \left( \frac{N_2}{N_3} \right) \times \left( \frac{N_4}{N_5} \right)$   
 (2 stages) , (2 mv)  
 (win, Wout) (0.581, 1)



\* Ex. 9.28 mv =  $\frac{1}{180}$  (Figure 9-30)

$e = \frac{\omega_7}{\omega_2} = (-) \left( \frac{N_2}{N_3} \right) \left( \frac{N_4}{N_5} \right) \left( \frac{N_6}{N_7} \right)$  : 3 stages.  
 (2, 7) (0.581, 1)

### \* # Reverted compound gear train

\* Input and output shafts ( $\omega_{in}$  &  $\omega_{out}$ ) at the same line (axis). (Compound)  $\rightarrow$  the same calculations.

\* Ex. 8 Figure (9/31).



# # Planetary or epicyclic gear train

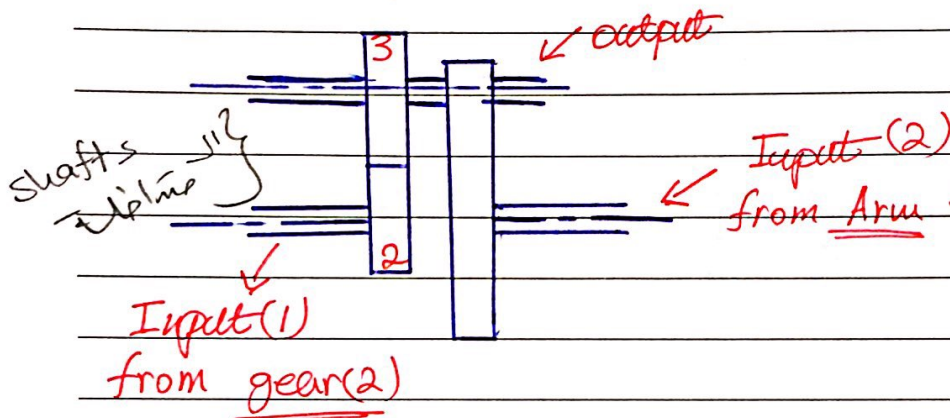
# moving axis gear train.

(connecting arm) ← الفرق بين دو محور

# Ex. (Figure) 8

→ meshing between gears (2 and 3)°

→ Gear (3) rotates about its axis & rotates about gear (2) by the arm.



\* Independent  $\Rightarrow$  2 input ; 1 output

\* Figure 9.34 8

(define shaft ك)

→ Shaft of arm  $\rightarrow$  shaft of gear (2)  $\rightarrow$  shaft of gear (4) "above each other"

→ Arm & Input  $\rightarrow$  gears  $\rightarrow$  output

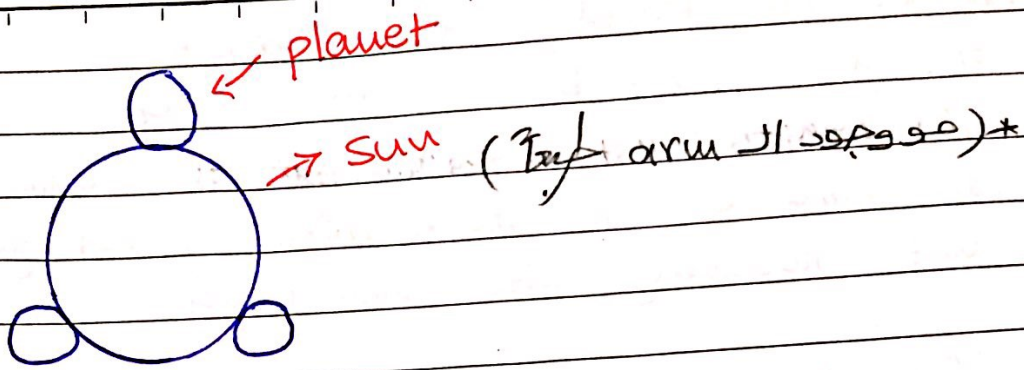
with clearance  $\leftarrow$  (constraint)  $\leftarrow$  Ring gear (4)  $\leftarrow$  Sun gear (2)  $\leftarrow$  Arm

→ Sun gear meshes with planet gear.

\* Sun gear & Rotates about its own axis. like (SUN)

\* Planet gear & Rotates about its own axis and around the sun gear. ; like (EARTH) and other Planets





\* For planetary gears train  
 $\rightarrow (e) = \frac{\omega_L - \omega_A}{\omega_f - \omega_A}$   $\rightarrow$  arm ; (Relative motion)  $\rightarrow$  1 gear  $\rightarrow$  2 gear

$$= \frac{(\pm) \text{Product of teeth of driving gears}}{\text{Product of teeth of driven gears}}$$

## \* Condition for planetary gear trains

1.  $(f, L)$  gears must be at fixed axes.
2.  $(f, L)$  shafts must be parallel at the same plane.
3.  $(f, L)$  must be in mesh with a planet gear but not necessary with same planet.

#  $(F, L) \rightarrow (2, 4)$  for ex. (figure 9, 34)  
 Sum  $\leftarrow$  Ring  $\rightarrow$

\* Internal gear meshing rotate with the same direction (gears 4, 3).

$$(e) = (-) \frac{N_2 N_3}{N_3 N_4} = \frac{\omega_4 - \omega_A}{\omega_2 - \omega_A} \quad ; \quad \left. \begin{array}{l} \omega_2 \text{ (ccw)} \\ \omega_3 \text{ (cw)} \\ \omega_4 \text{ (cw)} \end{array} \right\} \begin{array}{l} \text{fixed} \\ \text{وجود ال} \\ \text{arm.} \end{array}$$

\*  $\omega_2 \text{ (ccw)} \Rightarrow \omega_3 \text{ (cw)} \Rightarrow \omega_4 \text{ (cw)}$  (theoretical or imaginary way)  $\omega_2 \text{ (ccw)} \Rightarrow \omega_3 \text{ (cw)} \Rightarrow \omega_4 \text{ (cw)}$  (arm)  $\omega_2 \text{ (ccw)} \Rightarrow \omega_3 \text{ (cw)} \Rightarrow \omega_4 \text{ (cw)}$

## \* Important 8 No. of teeth for $\omega_2 \text{ (ccw)}$

$N(2, 3)$ ; then I have to find  $(N_4)$

$$r_4 = r_2 + 2r_3 \quad ; \text{ radius gear (4)}$$

$$d_4 = d_2 + 2d_3$$

$$N_4 = N_2 + 2N_3$$

$\rightarrow$  For  $N_2 = 40$  ;  $N_3 = 20$  ,  $N_4 = ?!$

$$N_4 = 40 + (2)(20) = 80$$



→  $\omega$  (rad/s)

$$* (e) = \frac{n_L - n_A}{n_f - n_A} = \left( \frac{+}{-} \right) \frac{\text{Product of teeth of driving gears}}{\text{Product of teeth of driven gears.}}$$

\*  $n \rightarrow$  (Rotation Speed in Rev./min (rpm))  
 → (ccw  $\Rightarrow$  +ve) ; (cw  $\Rightarrow$  -ve).

\* Figure (9-34); Ex (9-5) 8

→  $n_A = -200$  rpm ;  $n_f = n_2 = -100$  rpm ;  $[n_1 = n_4 = ?!]$

→ (F/L) of 81 on 1c

$$* \frac{n_4 + 200}{-100 + 200} = \left( - \right) \frac{n_2 n_3}{n_3 n_4} = - \frac{n_2}{n_4} = \frac{-40}{80} = -0.5$$

$$n_4 + 200 = -50 \rightarrow n_4 = -250 = 250 \text{ rpm (cw)}$$

\* Figure (9-35): 1 → Arm (0/1/3/5/7) \*

(a) → 1: arm , 2: Sun gear , 3: Planet gear.

(c) → (2, 7) → Sun gears "Fixed axes"  
 (3, 4, 5, 6) : planet gears

(h) → (2, 5) : Sun gears , (3, 4) : planet gears.

(j) → (2) : Sun gear , (7) → Internal , (3, 4) planet gears.

\* Figure P(9-1), page (505) 8

2 → Sun gear

3, 4, 5 → Planet gears

6 → Ring

\* 2 in mesh with 4.

\* 3 in mesh with 5.

\* (3 and 4) → the same axis of Rotation "shaft".

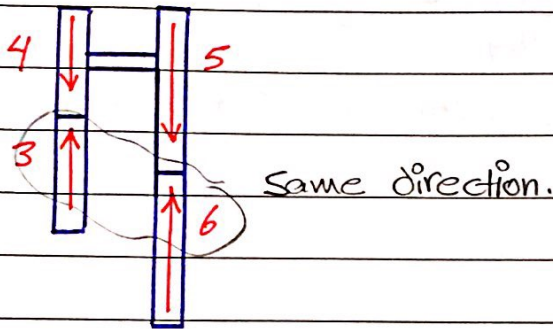


\* Figure P(9-2), Page (506) 8 (prob. 9-26)

\* 5 in mesh with 6. }  $\times(4+5)$  with the same shaft.  
 \* 4 " " " 3. }  $\times(3+6)$  with the same shaft.  
 \* 2 " " " 3. }  $\times(3) \rightarrow$  Sun gear.

$\leadsto$  (2) not in mesh with a planet gear. (one condition violated).

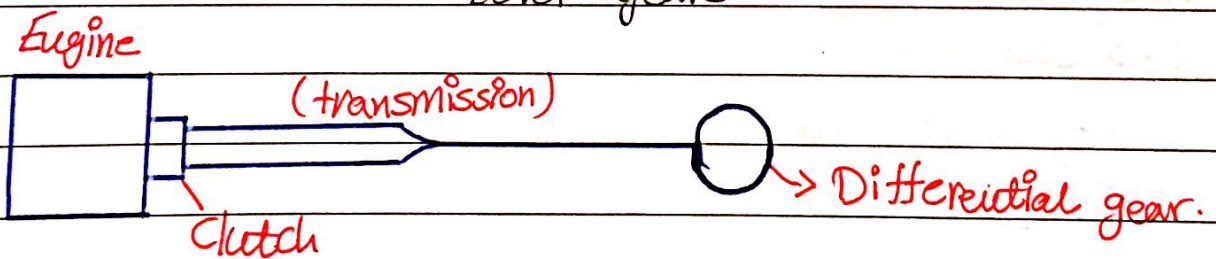
$\leadsto$  Solve (2+3) as an ordinary gears the (3) will act like a sun gear.



$$e = (+ve)$$

# Differential gear a gear that makes difference in the velocity in the two wheels (to avoid slipping).

\* Gear Box made of a group of Bevel gears.



\* Differential gear

1. Rear - wheel drive.
2. Front - wheel drive.
3. All - wheel drive. 3 - diff. gears.



\* Figure P(9-3)8 Page (507).

[train value (e) = -1]

\* when the planet gears are not rotating; the car would be on a straight road (moving forward).

\* when the planet gears are rotating; the car would be rotating.

\* the pinion gear rotates at a different speeds.

\* Types of differential gears:

1. Open (Normal)

2. Lst (amount of power is small).

3. locking

4. Auto (Brake activated).

\* Suggested (Ch. 9); (Part 2).

9-25, 26, 27, (37 → 41).

\* Problem 9.678

$r_{bp} = 46 \text{ mm}$ ,  $r_{bg} = 92 \text{ mm}$ ,  $\phi = 25^\circ$  "correct, theoretical, standard, design"

→ Find  $\phi'$  (new, Actual, Pressure angle) = ? | Pressure angle"

if  $\Delta c = 1.5 \text{ mm}$ .

\*  $r_{bp} = r_p \cos \phi \Rightarrow r_p = 50.755 \text{ mm}$  } Pitch, standard, theoretical.

\*  $r_{bg} = r_g \cos \phi \Rightarrow r_g = 101.511 \text{ mm}$

\*  $C = r_p + r_g = 152.266 \text{ mm}$

\*  $\Delta c = \frac{C'}{C} \Rightarrow 1.5 = \frac{C' - 152.266}{152.266} \Rightarrow 1.5 = \frac{C' - 152.266}{152.266}$   
(قبل - بعد)

→  $C' = 153.766 = r_p' + r_g'$  --- (1)

\*  $m_g = \frac{N_g}{N_p} = \frac{r_p}{r_g} \Rightarrow 2 = \frac{r_g'}{r_p'} \Rightarrow r_g' = 2r_p'$  --- (2)

→ Substitute (2) in (1)

$3r_p' = 153.766$ ,  $r_g' = 2r_p' = 102.511 \text{ mm}$ .  
→  $r_p' = 51.255 \text{ mm}$

⇒  $r_{bp} = r_p \cos \phi = r_p' \cos \phi'$  (or)  $r_{bg} = r_g \cos \phi = r_g' \cos \phi'$   
 $\phi = \cos^{-1} \left[ \frac{r_p \cos \phi}{r_p'} \right] = \cos^{-1} \left[ \frac{(50.755)(25)}{(51.255)} \right] = 26.173^\circ$



\* Problem 9-37, figure (a) 8 B-27 means no. of teeth.

C-100

→ Fixed means  $\Rightarrow \omega = 0$  , speed } magnitude  
→ direction.

→  $F = A$  ,  $L = D$

\* A in mesh with B } (B & C) at the same shaft  
\* D in mesh with C } so the same ( $\omega$ ) and direction of rotation.

$$(e) = (+) (100)(108)$$

$$A \& D \curvearrowright (27)(35)$$

in same Direction

# some notes 8 figure (b)

\* (D & A) are not in mesh with any gear (ما بينهما لا يتصل).

\* Arm & shaft of gear (D) and perpendicular.

\* C → planet gear.

\* A, B → F, L

\* Bevel gears اذا السطحين (الأسطح) المتصلين (التي تتصل بها)

تتحركوا معاً



$$(e) = (-ve)$$

### \* Problem (9-39)8 Figure (a).

\* Motor moves (arm & gear (A)).

1. train (A, B, C, D)

2. train (E, F, F<sub>2</sub>, G, arm) / planetary gears  $\rightarrow (e) = (-)$

$$\left. \begin{array}{l} [g=h] \\ [D=E] \end{array} \right\} \text{ (دنيا السرعة)}$$

3. Simple (h, j) } ordinary gear trains.

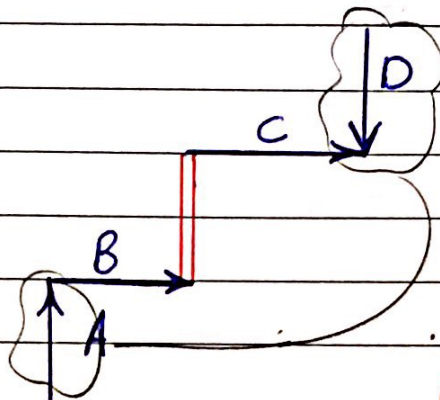
4. Simple (h, k)

A في دنيا D في دنيا

### \* problem (9-41)8

\* Speed of  $\left[ \begin{array}{l} A=2 \\ F=1 \\ \text{arm}=3 \end{array} \right]$

\* (D, A, F)  $\Rightarrow$  Fixed  
 $F \rightarrow A$  ,  $L \rightarrow D$



على وجه  
 $(e) = (-ve)$

1. D, C, B, A  $\rightarrow (e) = (-)$

2. D, C, E, f  $\rightarrow (e) = (-)$

### \* problem (9-42)8 the same

\* Arm (1)  $\equiv$  Arm (2) = Arm

1. A  $\rightarrow$  F , D  $\rightarrow$  L , planetary gears  $\rightarrow$  (B, C)  $e = (+)$

2. D, E, F, G , (E, G)  $\rightarrow$  planetary gears.



## \*Problems (9.43) 8

\*G  $\rightarrow$  Internal gear.

1. (A, B)  $\rightarrow$  the Same Velocity
2. (C, Arm)  $\rightarrow$  " " "
3. velocity D.
4. planetary gear (E, f)  $\rightarrow$  last  $\rightarrow$  G  
 $\rightarrow$  first  $\rightarrow$  D

$\sim (D)$  External and Internal at the same time.

$$(e) = (+) \frac{(54)(40)}{(24)(38)}$$

## \*Problem (9-27) 8

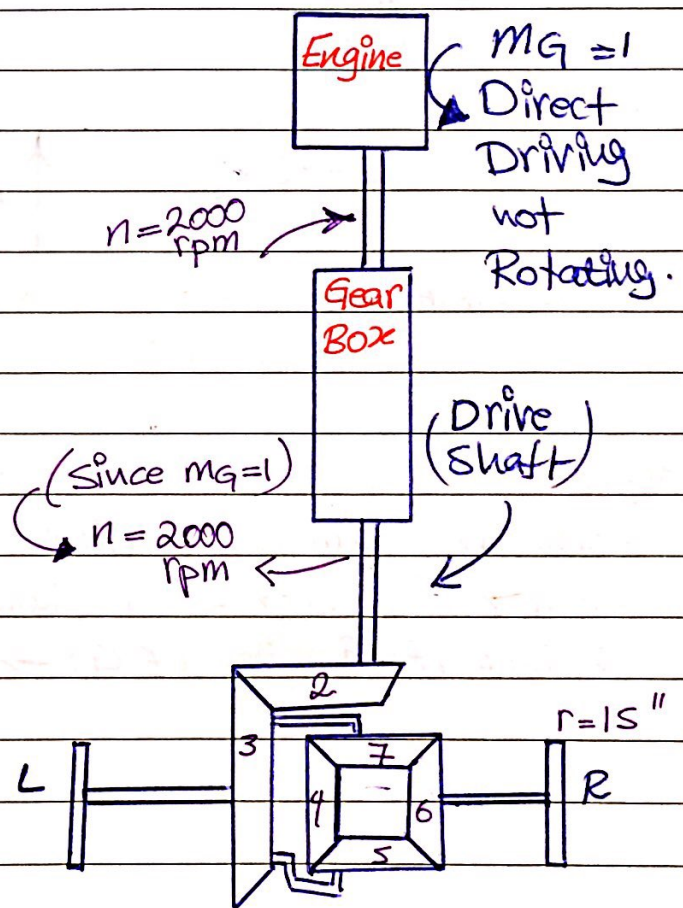
\*  $n_2 = 2000$  rpm

$$n_3 = n_2 \left( \frac{N_2}{N_3} \right)$$

given

$$n_3 = n_{arm}$$

\* axle Ratio.



$R \equiv L =$  the same velocity.

Direct Driving

$\omega_6 = \omega_4$  [  $\omega_{arm} = \omega_3$  ]

$V_{car} = 50$  mile / hour