MACHINERY

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FULL NOTES

Mech Family

Notebooks

Lec (1)
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MECHANICS OF MACHINES

Introduction :-

→ The name of this course consists of two parts : [1] Mechanics
[2] Machines.

[I] Mechanics :

- The word "mechanics" include : (objects + Interactions + loads)
- An "object" could be :

1. Particle → - a single point in space with zero dimentions
   - I can assume a body to be a particle when it's dimentions doesn't affect calculations.
2. Rigid Body → The shape and size of the body doesn't change under load.
3. Deformable Body → When the shape or size of the body change under load.

* Courses that (mechanics) include :

1- Statics
2. Dynamics ↓ particle + Rigid body
3. Mechanics of materials ↓ Deformable bodies.

* According to (Dynamics) motion types are :

1. Translation
   → Rectilinear → motion along a straight path
   → Curvilinear → motion along a curved path

   → It DOES NOT rotate
2. Rotation
3. General motion (rotation + Translation)

Dynamics
- → Kinematics → motion parameters only without regards to the loads (position, velocity, acceleration)
- → Kinetics → motion parameters + Mechanical loads
  - Force (Translation)
  - Moment (Rotation)

Note :- moment is a vector that has 3 components

$$\vec{M} = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$

↳ $M_x = M_{yz}$ (you can use both to express rotation about x-axis)

Twisting the body about it's axis (in this case the x-axis) is (Torque) ←

* Stiffness or (Elasticity) → The property that defines whether the body is deformable or not (it's capability to resist deformation)

* If stiffness goes to ∞, the body is considered (Rigid)
  If stiffness is a small value the body is more elastic & easy to deform.

Notes: → In this course we will only deal with "Rigid Bodies"
→ We will have have (springs) in some problems which are deformable BUT we don't consider them as a significant component (not a link)

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[2] Machines :-

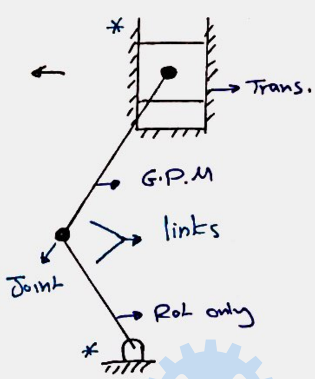
→ A device that transforms power & motion and consists of "Mechanisms" or (linkage)

↳ $P = \vec{F} \cdot \vec{v}$ (Transational system)

$P = \vec{T} \cdot \vec{\omega}$ (Rotational system)

Mechanism → - a machine component that is mainly concerned with (motion) transmissions

"Slider- Crank mechanism" ←



- It consists of Bodies (Links) connected by Joints (Pairing elements or kinematic parts)

  note : welding is not considered as a kinematic Joint, it's permenant.

→ We consider the two links with (*) as one link called (Ground or frame link) - Fixed.

→ Joints keep the links separate but connects them with a relative motion.

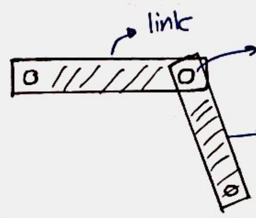
LINKS

→ A link is a body that is connected to at least two other links through nodes.

↳ The Body could be
a Rigid body or
sometimes a particle (ex: Slider)

nodes :- points for attachment to other links

note :- since we said that each point is connected to at least other two, this means the system must be a (closed loop)

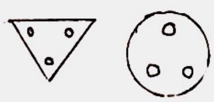


* Links could be classified based on:

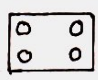
I → Number of nodes :-

1. Binary link
(two nodes)

2. Ternary link
(Three nodes)



3. Quaternary link
(Four links)



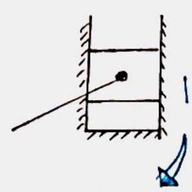
note :- The Slider is a Binary link, the 1st node connects it to the ground link & the other connects it to the rod.

II → Motion Type :-

I. Ground link (Fixed link, Frame)

* no motion

* note → The ground link is the only type of links that must exist at least two locations in every mechanism. (if it didn't that would mean it's an open system!)
↳ (open loop)



we usually give ground links number (1)

[2] Crank

→ A link Pivoted (connected by a pin) to the ground.

→ It makes full rotation about a fixed axis.

* full rotation means (360°) which also means that it doesn't change the direction of rotation (c.w or c.c.w)

[3] Rocker

→ A link pivoted to the ground

→ It makes (oscillation)
(حركة تذبذبية متكررة)

* oscillation is also rotation about a fixed axis, but the difference is that it changes the direction of rotation after a certain angle.

[4] Coupler or Connecting Rod

→ Not connected to the ground.

→ It makes General Plane Motion.

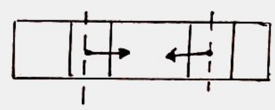
general motion is also rotation BUT about a moving axis.

[5] Slider (Collar)

→ Sliding (Translating link)

Translation could be Rectilinear or curvilinear.

→ It usually makes Reciprocation
(حركة خطية ترددية)



piston in an internal combustion engine moves between two positions. (example of reciprocation)

EXAMPLES:

Classify links in the following Machnisms:

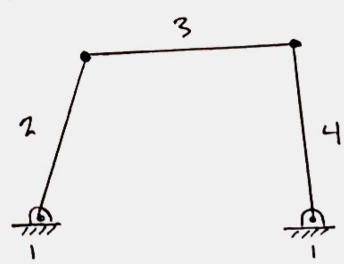
[1] 4-Bar Machnism

link (1) → Ground - Binary

link (2) → Crank or Rocker - Binary

link (3) → Coupler - Binary

link (4) → Crank or Rocker - Binary



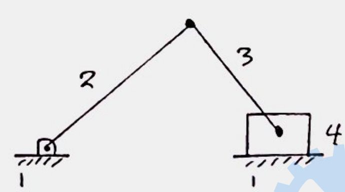
[2] Slider-Crank Mechanism

link (1) → Ground - Binary

link (2) → Crank - Binary

link (3) → Coupler - Binary

link (4) → Slider - Binary



DEGREE OF FREEDOM

(DOF) is the number of independent coordinates needed to fully describe the motion of an object or a system of objects.

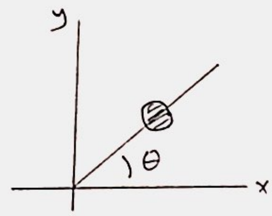
example → * If you throw a rigid body in space (↓ 3D), the body would have
6 DOF → 3 Translation $(x, y \,\&\, z)$
3 Rotation about $(x, y \,\&\, z)$

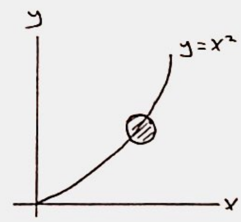
* If an object is free to move in 2D plane, it would have
3DOF : 2 Translation (x, y)
1 Rotation, about z

$$\boxed{(DOF)_{sys} = (\Sigma\, DOF)_{objects}}$$

, so DOF increases as number of objects increase.

For such systems, there is only 1 DOF, Because it is "dependent motion"
_ if you have the x-coordinate you can obtain the (y).





* Deformable Bodies:
→ They have an infinite number of degrees of freedom, Because they consist of an infinite number of particles and each particle moves on it's own path.
↳ we call it a (continuous system).

note → If you assume the object you throw in space to be a particle, DOF will reduce to only 3 (translation) - no rotation could happen -

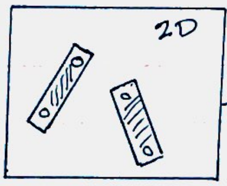
JOINTS

Joints are also called (kinematic parts) or (pairing elements)

→ Joints are elements by which two or more links are connected (Joined) together in such a way that certain motion is still allowed.

→ Joints limits down the number of DOF of any system.

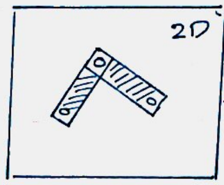
→ Example :



→ no Joint

6-DOF

x,y & Rotation For each link



with Joint

→ 4-DOF

x & Rotation For each link

* Classifications of Joints :-

I → Based on DOF :-

[1] one DOF : R-Joint → Revolute or rotational (Rotation only)

P-Joint → Prismatic (Translation only)

] "Full Joint"

[2] Two DOF : RP-Joint → Rotation & Translation "Half Joint"

II → Type of Contact :-

[1] lower Pairs (surface contact)

ex → R & P Joints

[2] Higher Pairs (line contact)

ex → Roll - Slide Joint

Cam Joint

Gear Joint

Types of Contact :

1. Point $\xrightarrow{ex}$ sphere on surface.
2. line $\xrightarrow{ex}$ Rigid cylinder on surface.
3. surface $\xrightarrow{ex}$ cylinder inside cylinder

↳ In ideal cases where the body is perfectly Rigid

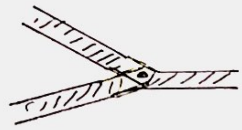
III → Based on the number of links Joined by the Joint

Joint Order = number of links Joined - 1

Ex → 1st order Joint (Two links)

2nd order Joint (Three links)

⋮ and so on



→ order = no. of Joints counted, so we count this Joint as two (we will need this later on).

IV → Based on the type of physical closure:

[1] Form closed

How are they closed together?

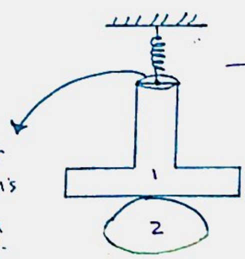
→ If you don't need any thing like a force to keep the links connected

Ex → R & P Joints (The Design of the Joint allows the links to stay connected without any Forces)

[2] Force closed

→ you need sth (a Force) to keep them connected

note: the spring is NOT a link and this point is NOT a kinematic joint.



→ I add a spring under compression to keep the two links connected. (a Force)

Kinematic chain

→ A system consisting of a no. of links connected by Joints.

* Closed loop :- each link is connected to at least two others
* Open loop :- At least one link is connected to only one other.

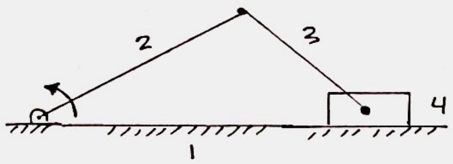
A mechanism is a closed loop kinematic chain with one ground link

→ another definition of mechanism from the "kinematic chain" prespective.

Inversion

Inversion of a machnism :- a new machnism that results when the fixed link in the original machnism is allowed to move & another link is fixed.

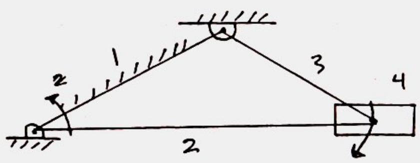
EX → Slider - Crank Mechanism.



→ Here 1. Fixed
2. Free

For each machnism I have a number of Inversions equal to the number of links

make 1 Free and 2 Fixed



→ This mechanism is called :
"Inverted - Slider Crank mechanism"
or "Sliding Contact mechanism"

* note → All Inversions will have the same relative motion. (If you assume an observer to be on link (2) in both cases he'll see the slider coming Back & Forth (Relative - Motion).

Notes From The Slides

→ We have sth called (Limiting positions) which are the positions where the mechanism stops the motion and the velocity = 0.

→ After the limiting positions the Mechanism keeps on moving because of the Inertia of the bodies.

Rolling without slipping	Rolling with slipping
→ Static Friction	→ Rolling with slipping.
→ Rotation and Translation are dependent so it's considered to have (1DOF)	→ Translation & Rotation are independent of each other so (2DOF).

$V = wr$

$V \neq wr$!

* The Default is with slipping (If not mentioned)

MOBILITY

→ Mobility is the Degrees of Freedom of planar machnisms.

$\boxed{M = 3(L-1) - 2J_1^* - J_2}$ → M: mobility

L: no. of links

J_1: no. of full Joints (1 D.O.F)

J_2: no. of Half Joints (2 D.O.F)

According to the value of M you get:

M > 0 → A mechanism with (M) - DOF

M = 0 → statically Determinant structure

M < 0 → statically In-determinant structure.

a structure is NOT a mechnism we have no intrest in structures in this course

notes : → Mobility is an indication of the number of degrees of freedom.

→ Joints decrease the no. of DOF.

Examples :

Find the Mobility of each of the following :

[1] Slider - Crank Mechanism

solution:

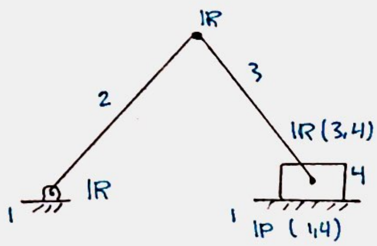
L = 4 $\quad J_1 = 4$ (3R + 1P)

$J_2 = 0$

$M = 3(L-1) - 2J_1 - J_2$

$= 3(4-1) - 2(4) = \boxed{+1}$

↳ So, it's a machnism with a mobility of 1.



steps:

1. number the links
2. Identify Joints
3. Find L, J_1, J_2
4. Solve.

note → what does M=1 or M=2 ... mean ?

If M = 1 , I need 1 input to define the output

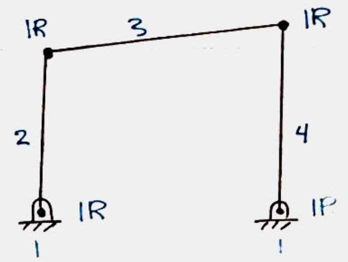
If M = 2 , I need 2 inputs . ... and so on

[2] Four - Bar Machnism

Solution :-

$L = 4 \quad J_1 = 4(R) \quad J_2 = 0$

$M = 3(L-1) - 2J_1 - J_2 = 3(4-1) - 2(4) = \boxed{1}$



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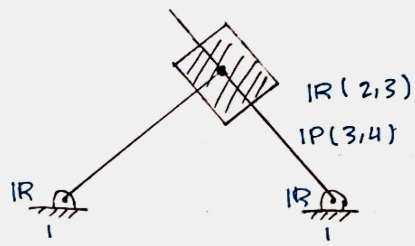
[3]

$L = 3 \quad J_1 = 4 \ (3R + 1P) \quad J_2 = 0$

$M = 3(4-1) - 2(4) - 0$

$= 9 - 8 = \boxed{1} \rightarrow$ Machnism



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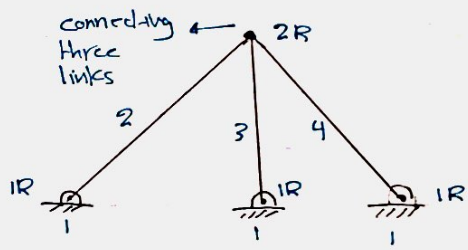
[4]

$L = 4 \quad J_1 = 5 \ (5R) \quad J_2 = 0$

$M = 3(4-1) - 2(5) = 9 - 10 = \boxed{-1}$

Statically indeterminant structure.



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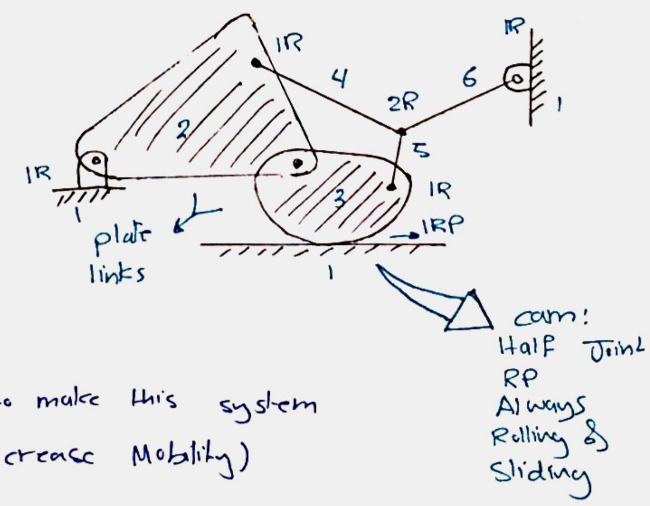
[5]

$L = 6 \quad J_1 = 7 \ (7R)$

$J_2 = 1 \ (RP)$

$M = 3(5) - (7)(2) - 1$

$= \boxed{0} \rightarrow$ Statically Determinant structure.



THINK $\longrightarrow$ What can you do to make this system a mechanism (increase Mobility)

* Ideas from class:

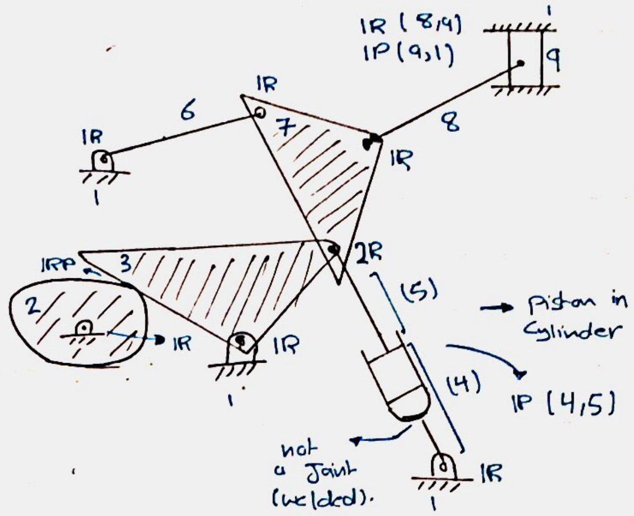
1. I can add a spring instead of link (4) $\rightarrow L=5, J_1 = 5, J_2 = 1 \rightarrow \boxed{M = 1}$
2. you can convert the R-Joint between (2,3) to RP-Joint by adding a slot
   ↳ $L = 6 \quad J_1 = 6 \quad J_2 = 2 \rightarrow M = 1$ # Mechanism
3. Remove the contact between the cam and the ground
   ↳ $L = 6 \quad J_1 = 7 \quad J_2 = 0 \rightarrow M = 1$ # Mechanism

[6]

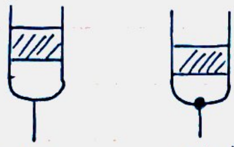
$L = 9 \quad J_1 = 11 \quad J_2 = 1$

$M = 3(L-1) - 2J_1 - J_2$

$= \boxed{1} \rightarrow$ Mechanism ✓



Note :-



1 link (welded) 2 links (kinematic Joints).

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* Multi-Loop Mechanisms :-

[1] Quick Return Mechanism :

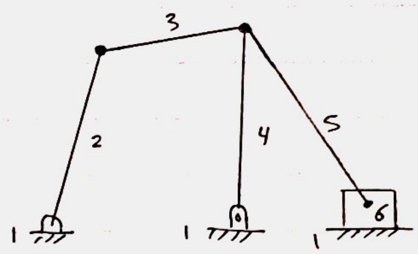
it consists of → 1. 4-Bar Machnism

2. Slider Crank Machnism

* link (4) is called "(Intermediate link)

this link turns the output of one machnism to the input of another

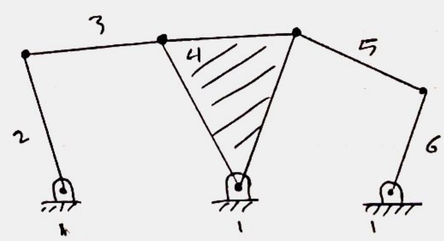
↳ In this example the output of the 4-Bar Machnism turns into the input of slider-crank mechanism.



[2] Watt's 6-Bar Machnism :

→ it consists of two 4-Bar Mechanism

→ link (4) is the Intermediate link



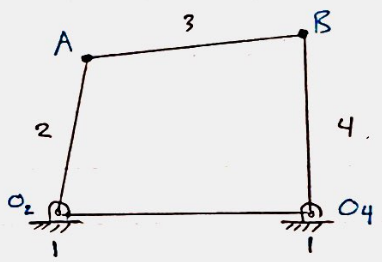
Four-Bar Mechanism

We have two main Types of 4-Bar Mechanism:

1. Grashof → At least one link can do full Rotation.
2. Non-Grashof → no link can do full rotation.

Naming links and their lengths:

→ you give ground links the symbol (O) followed by the number of link after it. ex → (O_2, O_4)



→ lengths:

L_1 : $\overline{O_2 O_4}$ L_3 : $\overline{AB}$

L_2 : $\overline{O_2 A}$ L_4 : $\overline{O_4 B}$

↳ once I have these lengths, I can (sort) them as follows:

S → The length of the shortest length.

in this example → $S = L_2$

L → The length of the longest length

ex → $L = L_1$

P, Q → Intermediate lengths

ex → $P = L_3$, $Q = L_4$

→ note → It doesn't matter which link is the larger one (Random).

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* Things you have to do by yourself:

1. Grashof & non Grashof → Self-Reading
2. Find the proper combination between (S, L, P, Q) to form a closed-loop 4-Bar machnism.
3. What are Grashof's classifications or (inversions)

* Suggested Problems for CH2:

1 (a, d, f, g, J,) - 2 - 3 - 4 - 5

7 (Part(1)) - 7 (Part 2, F) - 15 - 21

22 - 26 - 27 - 32 - 42 - 44

POSITION ANALYSIS OF MECHANISMS

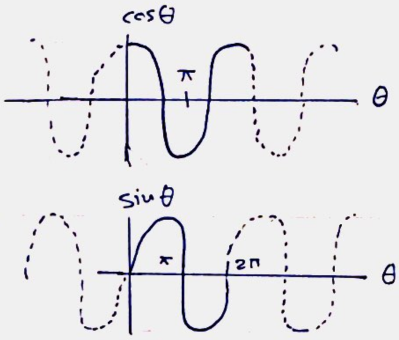
* Revision:-

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\cos(-\theta) = \cos\theta$ → Even Function

$\sin(-\theta) = -\sin\theta$ → Odd Function



Angle Shift:

$\sin(\theta + \frac{\pi}{2}) = \cos\theta$ → we say " sin is lagging by 90° "

$\sin(\theta - \frac{\pi}{2}) = -\cos\theta$

$\cos(\theta + \frac{\pi}{2}) = -\sin\theta$

$\cos(\theta - \frac{\pi}{2}) = \sin\theta$

note → Orthogonal functions are considered to be independent.

(x, y)
(sin, cos)] → orthogonal

$\sin(\theta + \pi) = -\sin\theta$

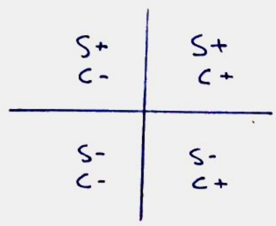
$\sin(\pi - \theta) = \sin\theta$

$\cos(\pi - \theta) = -\cos\theta$

$\cos(\pi + \theta) = -\cos\theta$] → Because it's an even function it gives the same answer.

$\cos(2\pi - \theta) = \cos\theta$

↳ (θ & $(2\pi - \theta)$ have the same cosine)



$\tan\theta = \frac{\sin\theta}{\cos\theta}$

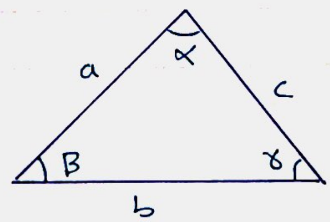
$(\pm, \pm, \mp, \mp)$ → Pay attention to the signs to specify the quadratic of the angle.

$$\cos\theta = \frac{1 - \tan^2(\frac{\theta}{2})}{1 + \tan^2(\frac{\theta}{2})}$$

$$\sin\theta = \frac{2\tan(\frac{\theta}{2})}{1 + \tan^2(\frac{\theta}{2})}$$

* Law of Sines:

$$\frac{a}{\sin \gamma} = \frac{B}{\sin \alpha} = \frac{c}{\sin B}$$



* Law of Cosines:

$$c^2 = a^2 + b^2 - 2(a)(b)\cos B$$

$$a^2 = c^2 + b^2 - 2(c)(b)\cos \gamma$$

$$b^2 = a^2 + c^2 - 2(a)(c)\cos \alpha$$

---

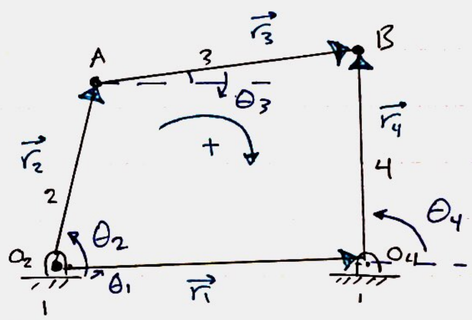
* In Position Analysis Of Mechanisms, we mainly depend on:

"Loop Closure Equations" → (LCE)

Example:

(The angular position of each vector is (the angle it makes with the x-axis).)

To find the loop closure equation:



→ you can choose to go C.W or C.C.W

↳ "choose your path"

→ $O_2 \rightarrow A \rightarrow B \rightarrow O_4 \rightarrow O_2 = 0$

↳ This means that the net displacement $= 0$

→ you can randomly choose the directions of your vectors (only signs will be affected).

→ So, The L.C.E:

$$\vec{r_2} + \vec{r_3} - \vec{r_4} - \vec{r_1} = \vec{0}$$

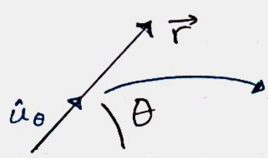
→ I start from a point and finish at the same point.

→ now, write the equation with unit vectors.

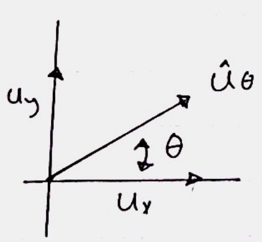
↓

* Unit Vectors :-

$$\vec{r} = |r|\,\hat{u}_\theta$$



This is a unit vector along the vector ($\vec{r}$) named ($\hat{u}_\theta$) and has a mag. of (1)



$$\rightarrow \hat{u}_\theta = \underbrace{(\cos\theta)}_{u_x}\hat{\imath} + \underbrace{(\sin\theta)}_{u_y}\hat{\jmath}$$

"Resolution of $\hat{u}_\theta$ along x & y"

* The L.C.E we found can solve 2 unknowns because it represents 2 Eqns in 2D - eqn for the magnitude (r)
- eqn for the Direction (angle). $(\hat{u}_\theta)$

$$\vec{r_2} + \vec{r_3} - \vec{r_4} - \vec{r_1} = 0$$

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = 0$$

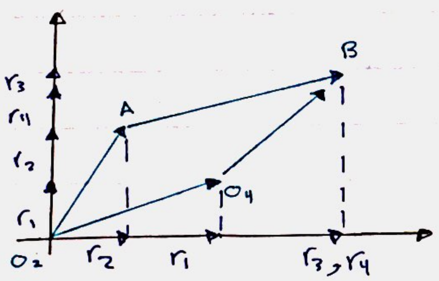
x- Position: (scalar eqn)

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 \cos\theta_1 = 0$$

y- Position: (scalar eqn)

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 - r_1 \sin\theta_1 = 0$$

* Graphical clarification:



→ If you apply the L.C.E $(\vec{r_2} + \vec{r_3} - \vec{r_4} - \vec{r_1})$ to the x-components and y-components in the graph you'll get a zero #

*Solution of the (LCE) :-

→ In any problem you are usually Given : - Fixed quantities
- Input position → The default Mobility =1 so we only need 1 input, If it was 2 I would need 2.

Example (4-Bar Machnism)

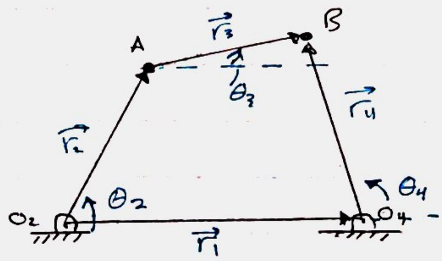
Given → fixed quantities :

quantities NOT directions ← $r_1, r_2, r_3, r_4, \theta_1$

Input position :

one of $(\theta_2, \theta_3, \theta_4)$ is must be given. let's assum it's (θ_2)

Find → θ_3, θ_4



continued →

(lec 6)

Solution:

We have 3 methods to find the solution →
- Analytical (exact)
- Numerical (Approximate)
- Graphical

Using the analytical method (which is the most used):-

Steps:

*1. write the L.C.E :

$$r_2\hat{u}_{\theta_2} + r_3\hat{u}_{\theta_3} - r_4\dot{\theta}_4 - r_1\hat{u}_{\theta_1} = \vec{0}$$

*2. Rearrange the eqn : (Put $r_3\hat{u}_{\theta_3}$ on one side)

$$r_3\hat{u}_{\theta_3} = \vec{r}_1\hat{u}_{\theta_1} - \vec{r}_2\hat{u}_{\theta_2} + \vec{r}_4\hat{u}_{\theta_4}$$

*3. Dot each side by it's self :- → To eliminate θ_3

$$(r_3\hat{u}_{\theta_3})\cdot(r_3\hat{u}_{\theta_3}) = (\vec{r}_1\hat{u}_{\theta_1} - \vec{r}_2\hat{u}_{\theta_2} + \vec{r}_4\hat{u}_{\theta_4})\cdot(\vec{r}_1\hat{u}_{\theta_1} - \vec{r}_2\hat{u}_{\theta_2} + \vec{r}_4\hat{u}_{\theta_4})$$

$$(r_3)^2 = r_1^2 + r_2^2 + r_3^2 - 2r_2r_4\,\hat{u}_{\theta_2}\cdot\hat{u}_{\theta_4} - 2r_1r_2\,\hat{u}_{\theta_1}\hat{u}_{\theta_2} + 2r_1r_4\,\hat{u}_{\theta_1}\hat{u}_{\theta_4} \quad — *$$

→ Find $\hat{u}_{\theta_1}\cdot\hat{u}_{\theta_2} = (\cos\theta_1\hat{i} + \sin\theta_1\hat{j})\cdot(\cos\theta_2\hat{i} + \sin\theta_2\hat{j})$

$= \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$ ↗ from Identities

$= \cos(\theta_2 - \theta_1) = \cos(\theta_1 - \theta_2)$ → even function.

$\hat{u}_{\theta_2}\cdot\hat{u}_{\theta_4} = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4$

$\hat{u}_{\theta_1}\cdot\hat{u}_{\theta_4} = \cos\theta_1\cos\theta_4 + \sin\theta_1\sin\theta_4$

* Rewrite equation (*) as follows:

$$A\cos\theta_4 + B\sin\theta_4 + C = 0$$

where → $A = 2r_1r_4\cos\theta_1 - 2r_2r_4\cos\theta_4$

$B = 2r_1r_4\sin\theta_1 - 2r_1r_4\sin\theta_1$

$C = r_1^2 + r_2^2 - r_3^2 + r_4^2 - 2r_1r_4\cos(\theta_2 - \theta_1)$

* note → This is for a certain Input of θ_2, if θ_2 changes θ_3 & θ_4 will change too.

* use the identities : $\cos\theta_4 = \dfrac{1 - \tan^2(\frac{\theta_4}{2})}{1 + \tan^2(\frac{\theta_4}{2})}$

$$\sin\theta_4 = \frac{2\tan(\frac{\theta_4}{2})}{1 + \tan^2(\frac{\theta_4}{2})}$$

→ This step is to make the unknown Interms of ($\tan\theta_4$) only instead of $\sin\theta_4$, $\cos\theta_3$.

* let $X = \tan\left(\frac{\theta_4}{2}\right)$ (Just to simplify)

* now, you have :

$$A\left(\frac{1-X^2}{1+X^2}\right) + B\left(\frac{2X}{1+X^2}\right) + C = 0$$

* Multiply Both Sides By $(1+X^2)$:

$$A(1-X^2) + B(2X) + C(1+X^2) = 0$$

$$(C-A)X^2 + 2BX + A+C$$ (quadratic equation)

$$aX^2 + bX + c = 0$$ where : $a = C-A$

$b = 2B$

$c = A+C$

* Solve the quadratic equation:

$$X_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$X_{1,2} = \tan\left(\frac{(\theta_4)_{1,2}}{2}\right) \longrightarrow (\theta_4)_{1,2} = 2\tan^{-1} X_{1,2}$$

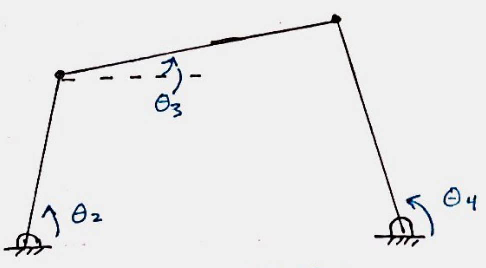
* Find (θ_3) By Dividing the y-eqn by the x-eqn:

$$\frac{y\text{-eqn}}{x\text{-eqn}} = \tan(\theta_3)_{1,2} = \frac{r_1\sin\theta_1 - r_2\sin\theta_2 + r_4\sin(\theta_4)_{1,2}}{r_1\cos\theta_1 - r_2\cos\theta_2 + r_4\cos(\theta_4)_{1,2}}$$

$$(\theta_3)_{1,2} = \tan^{-1}(\qquad)$$

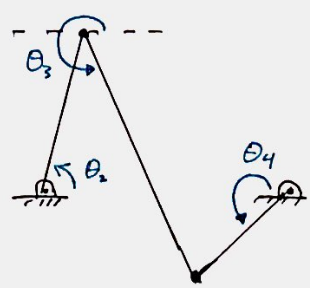
note → If you use the X or Y equations directly you'll lose the intermediate step of finding what quadratic is the angle in.

* Showing the positions in the Graph :



First case $(\theta_3, \theta_4)_1$

OPEN CONFIGURATION

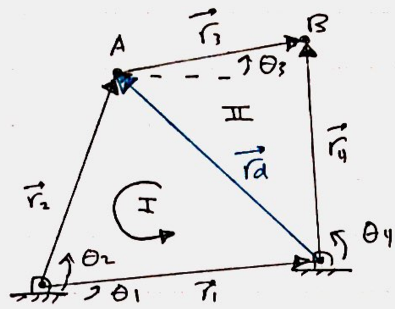


second case (θ_3, θ_4)

CROSSED - CONFIGURATION

Alternative Solution (Analytical Also) :-

→ Adding a radial vector $\vec{r_d}$



* For loop (1) :

$r_1 \hat{u}_{\theta_1} + r_d \hat{u}_{\theta_d} - r_2 \hat{u}_{\theta_2} = \vec{0}$

$r_d \hat{u}_{\theta_d} = r_2 \hat{u}_{\theta_2} - r_1 \hat{u}_{\theta_1}$

(Dot each side by it's self to eliminate θ_d)

$(r_d)^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta_2 - \theta_1)$

$r_d = \leftarrow$ (magnitude)

→ note that this is the law of cosines, you can use it directly from the start.

To Find The Direction:

$$\frac{y\text{-eqn}}{x\text{-eqn}} = \tan\theta_d = \frac{r_2 \sin\theta_2 - r_1 \sin\theta_1}{r_2 \cos\theta_2 - r_1 \cos\theta_1}$$

* Now you can use loop 2 to find θ_3 & θ_4

L.C.E :

$\vec{r_d} + \vec{r_3} - \vec{r_4} = 0$

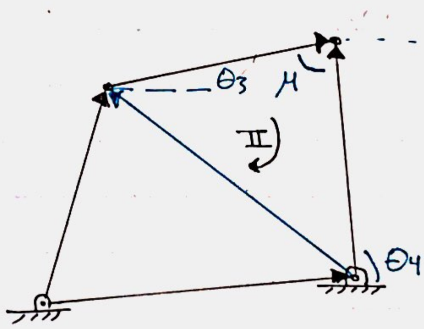
$r_d \hat{u}_{\theta_d} = r_4 \hat{u}_{\theta_4} - r_3 \hat{u}_{\theta_3}$

→ Dot each side by it's self

$(r_d)^2 = r_4^2 + r_3^2 - 2 r_3 r_4 \; \hat{u}_{\theta_3} \cdot \hat{u}_{\theta_4}$

$= r_4^2 + r_3^2 - 2 r_3 r_4 \cos(\theta_3 - \theta_4)$

$(r_d)^2 = r_4^2 + r_3^2 - 2 r_3 r_4 \cos(\mu)$



* continue to find θ_3 & θ_4

Hint: use law of sines.

μ → Transmission angle.

$\mu = \theta_3 - \theta_4$

If $\mu > \frac{\pi}{2} \rightarrow \mu = 180 - \mu$

(خ الكاملة)

* Graphical Solution :-

4-Bar Mechanism

Given: $r_1, r_2, r_3, r_4, \theta_1$ + Input θ_2

Find θ_3 & θ_4

Steps:

→ Start from O_2

→ Draw vector $\vec{r_1}$ (you have it's length & angle)

→ Draw $\vec{r_2}$ (you also have r_2 & θ_2)

→ Draw an arc an a distance (r_3) from point (A). (فرجار)

→ Draw another arc from the end of ($\vec{r_1}$) - (point O_4) - after a distance (r_4)

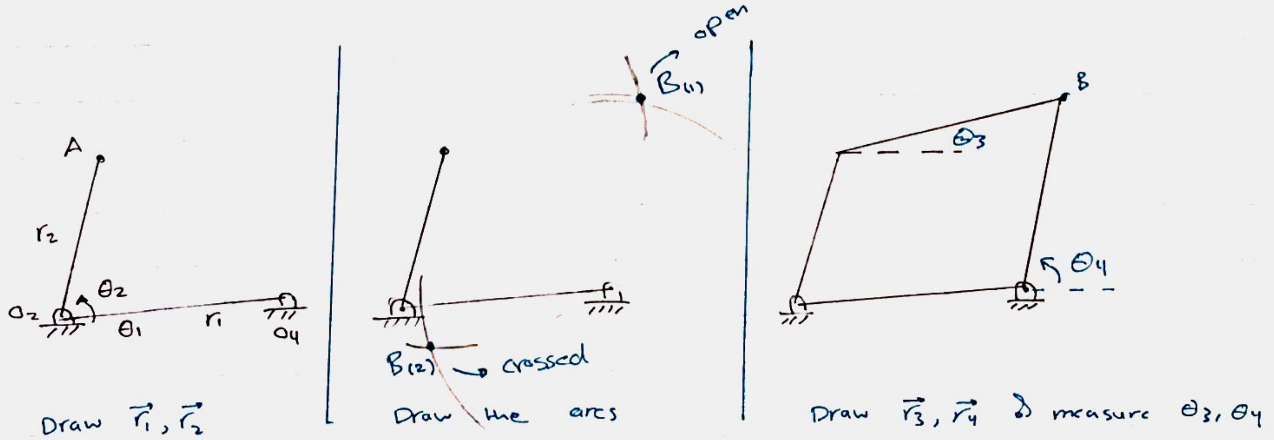
→ The Two arcs will (CROSS) at a point (B).

→ Draw $\overline{AB}$ & $\overline{O_4B}$

→ Measure the angles θ_3 & θ_4 (منقلة)

note → This will give me the first values of (θ_3, θ_4), if you want to find the second case, continue the arc to a full circle and you'll have another point where the two circles will cross, then Measure $(\theta_3)_2$ & $(\theta_4)_2$ #

note → The disadvantage of the Graphical method is that you have to Draw a new sketch for each (θ_2) you have, but in the Analyitical Method you obtain a general formula. that you can use



Draw $\vec{r_1}, \vec{r_2}$ | Draw the arcs | Draw $\vec{r_3}, \vec{r_4}$ & measure θ_3, θ_4

note → sketches are not accurate, only for clarification.

LIMIT POSITIONS (Toggle Positions)

→ limiting positions are the positions where the Mechanism changes the direction of Motion.

→ When the limiting positions are reached (V=0)

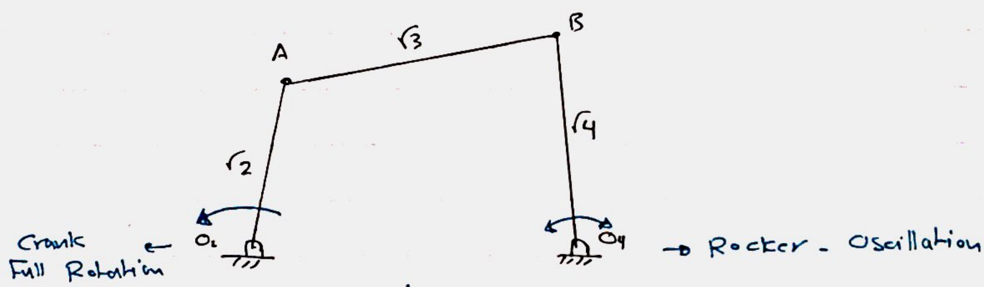
→ The Mechanism continuous the motion due to the bodies Inertia.

Example :-

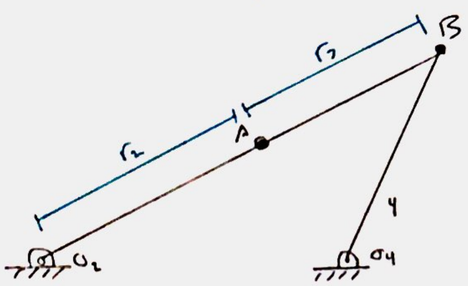
limiting positions of (Crank-Rocker 4-Bar Mechanism) :-

→ There is two limiting positions

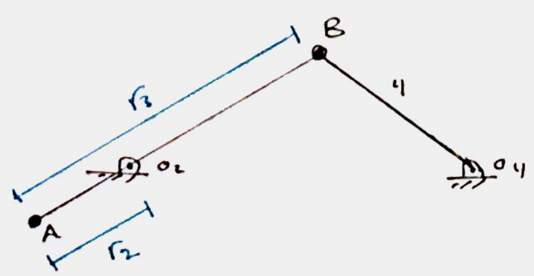
→ They occure when links 2,3 are on the SAME level.



limit position (1)



limit position (2)



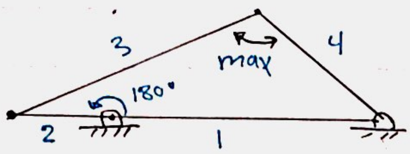
→ at this position link (4) will change the direction of rotation & link (2) will continue rotating.

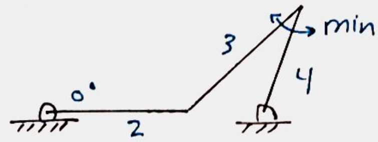
Extreme Values of Transmission Angle (μ)

In 4-Bar Mechanism there is two extreme positions for the transmission angle:

$\mu_{max} \rightarrow$ when the position angle of crank $= 180°$ $(\theta_2 = 180°)$

$\mu_{min} \rightarrow$ when the position angle of crank $= 0°$ $(\theta_2 = 0°)$



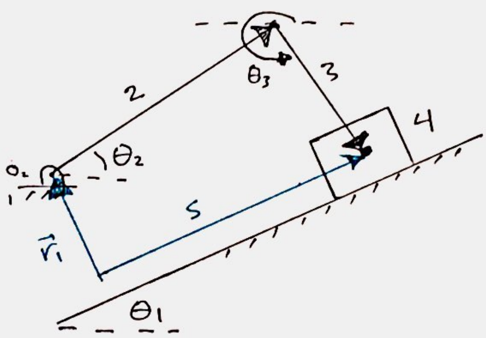


Position Analysis Of Slider-Crank Mechanism

I General case:

Given r_2, r_3, r_1, θ_1 + Input θ_2

Find S, θ_3



L.C.E:

$$\vec{r_2} + \vec{r_3} - \vec{S} + \vec{r_1} = 0$$

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - S\hat{u}_{\theta_1} + r_1 u_{\theta_1 + \frac{\pi}{2}} = 0$$

↳ now I can use any of the methods to solve this equation (Just like the 4-Bar)

Analytical:

To find $S \rightarrow$ Eliminate θ_3

$$r_3 \hat{u}_{\theta_3} = S\hat{u}_{\theta_1} - r_1 u_{\theta_1 + \frac{\pi}{2}} - r_2 \hat{u}_{\theta_2}$$

$$r_3^2 = S^2 + r_1^2 + r_2^2 - 2r_2 S \cos(\theta_2 - \theta_1) + 2r_1 r_2 \underbrace{\cos\left((\theta_2 - \theta_1) - \frac{\pi}{2}\right)}_{\sin(\theta_2 - \theta_1)}$$

$$\underline{S^2} - (2r_2 \cos(\theta_2 - \theta_1))\underline{S} + r_1^2 + r_2^2 - r_3^2 + 2r_1 r_2 \sin(\theta_2 - \theta_1)$$ "quadratic eqn" in (S)

$S_{1,2} =$ —

To find θ_3:

$$\frac{y\text{-eqn}}{x\text{-eqn}} = \tan\theta_3 = \frac{S\sin\theta_1 - r_1 \overbrace{\sin(\theta_1 + \frac{\pi}{2})}^{\cos\theta_1} - r_2 \sin\theta_2}{S\cos\theta_1 - r_1 \underbrace{\cos(\theta_1 + \frac{\pi}{2})}_{-\sin\theta_1} - r_2\cos\theta_2}$$

$$\theta_3 = \tan^{-1}\left[\frac{S\sin\theta_1 - r_1\cos\theta_1 - r_2\sin\theta_2}{S\cos\theta_1 + r_1\sin\theta_1 - r_2\cos\theta_2}\right]$$

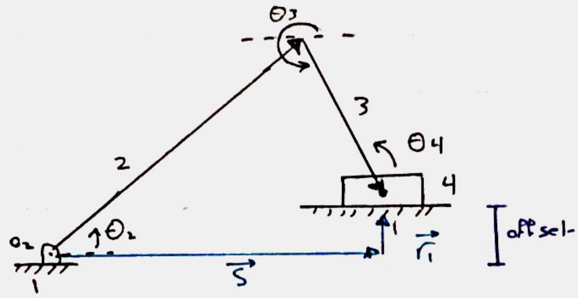
[2] Position Analysis of Offset Slider-Crank Mechanism:

↳ (special case of the general case where $\theta_1 = 0$)

Given → r_1, r_2, r_3, θ_1 + Input θ_2

Find → s, θ_3

L.C.E:



$\vec{r_2} + \vec{r_3} - \vec{r_1} - \vec{s} = \vec{0}$

$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_1 \hat{u}_{\frac{\pi}{2}} - s \hat{u}_0 = 0$

↳ you can use elimination of (θ_3) BUT in this particular case it's easier to use x & y equations:

x - Position

$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_1 \cos\frac{\pi}{2} - s \cos 0 = 0$

$r_2 \cos\theta_2 + r_3 \cos\theta_3 - s = 0$

$\vec{r_1}$ → sliding vector along s, constant mag & direction.

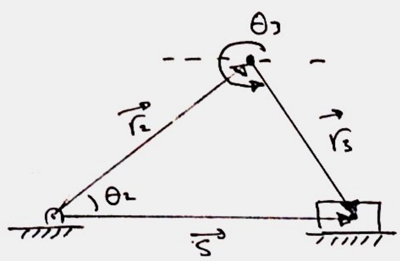
y - Position

$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_1 \sin\frac{\pi}{2} - s \sin 0 = 0$

$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_1 = 0$ (one unknown θ_3)

[3] Basic Slider-Crank Mechanism:

L.C.E:



$\vec{r_2} + \vec{r_3} - \vec{s} = \vec{0}$

$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - s \hat{u}_0 = 0$

$r_3 \hat{u}_{\theta_3} = s \hat{u}_0 - r_2 \hat{u}_{\theta_2}$

eliminate θ_3

$r_3^2 = s^2 - r_2^2 + 2 s r_2 \cos\theta_2$

$s^2 - 2 r_2 s - r_2^2 - r_3^2 = 0$

$s_{1,2} = \leftarrow$

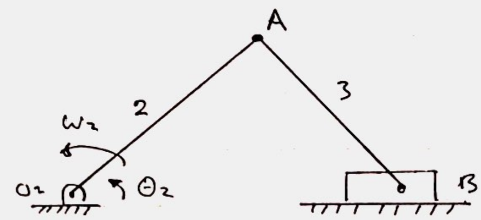
↳ you can also use law of signs but you'll need more steps.

x-Pos → $r_2 \cos\theta_2 + r_3 \cos\theta_3 - s = 0$

y-Pos → $r_2 \sin\theta_2 + r_3 \sin\theta_3 = 0$

Limiting Positions of Slider-Crank Mechanism:

→ when link 3 & link 2 are on the same level

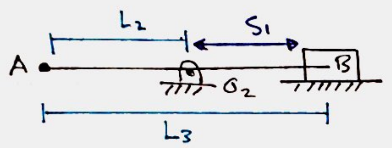


limit position (1)

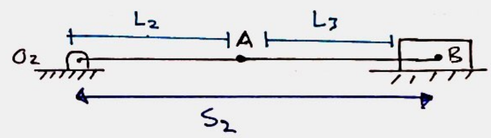
$\theta_2 = 180°$

limit position (2)

$\theta_2 = 0$



S_1: position of slider at limit position (1)



S_2: position of slider at limit position (2).

STROKE:-

→ The stroke is the displacement of the slider. (distance between the two limit positions).

stroke $= \Delta S = S_2 - S_1$ (O_2 is the reference)

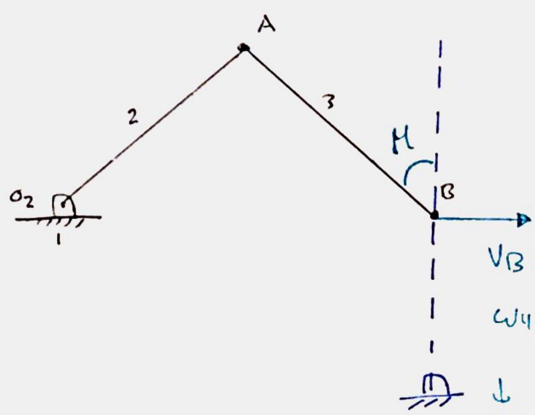
$= (L_2 + L_3) - (L_3 - L_2) = L_2 + L_3 - L_3 + L_2$

$= 2L_2$ # Rule → In slider-crank mechanism, the stroke is double the length of link 2.

Transmission Angle In Slider-Crank Mechanism:

→ There is an equiv between slider-crank mechanism & 4-bar mechanism.

↳ Slider-crank mechanism is a 4-bar mechanism with link 4 rotating about a radius of (∞).



$V_B = \omega_4 r_4$

$\omega_4 = \frac{V_B}{r_4}$

↓

when $r_4 = \infty$

$\omega_4 = 0$

point B will be sliding only. (Slider-Crank

So:

Transmission angle (μ) is the angle between link (3) & the line ⊥ to the sliding axis.

* If $\mu > \frac{\pi}{2}$ → $\mu = 180 - \mu$

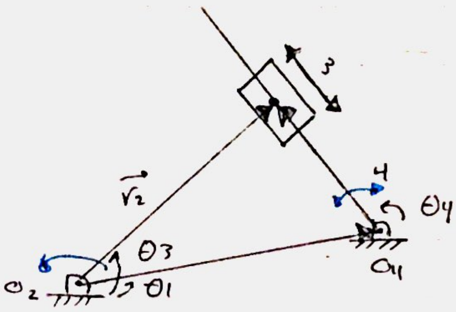
لو بناخد الفعلية .

Example (Inverted Slider-Crank mechanism)

Given : r_1, θ_1, r_2 + Input θ_2

Find θ , S

↳ position of slider on link (4)



Solution :

loop closure equation: $O_2 \to 3 \to O_4 \to O_2$

$r_2 \hat{U}_{\theta_2} - S\hat{U}_{\theta_4} - r_1 U_{\theta_1} = \vec{0}$ → 2 unknowns in the same term (S, θ_4)

$S\hat{U}_{\theta_4} = r_2 \hat{U}_{\theta_2} - r_1 \hat{U}_{\theta_1}$

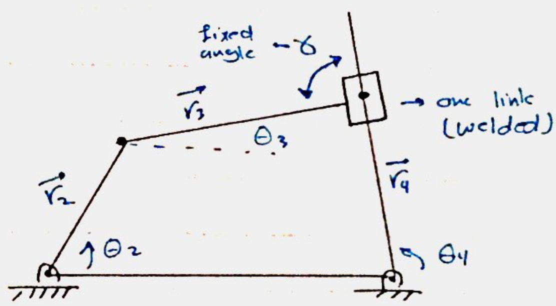
$S^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)$

$\theta_4 = \tan^{-1}\left[\dfrac{r_2 \sin\theta_2 - r_1 \sin\theta_1}{r_2 \cos\theta_2 - r_1 \cos\theta_1}\right]$

Problem (4-11)

Given → $\vec{r_1}$, θ_1, $\vec{r_2}$, $\vec{r_4}$, γ + Input θ_2

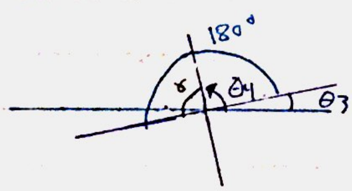
Find θ_3, θ_4, S_1

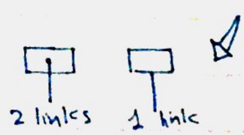


Solution;

→ There is a relation between θ_3 & θ_4

$180° = \gamma + \theta_4 - \theta_3$ → $\theta_4 = \theta_3 - \gamma + \pi$





Suggested CH4:

6(n) . 7(n) , 9(b) , 10(b) , 11 (d) , 12 (d)

27 + Example 4 (1, 2, 3)

Self - Reading :

1. Velocity Ratio (m_v)
2. Torque Ratio (m_T)
3. Mechanical advantage (m_a)

VELOCITY ANALYSIS

Position vector ($\vec{r}$) is a function of time (implicitly):

$\vec{r}(t) = \vec{r}(\theta(t))$

→ Take the derivative with respect to time to find the velocity:

$\vec{V} = \frac{d\vec{r}}{dt}$ (it has Both mag. & direction).

$\vec{r} = r\hat{u}_\theta$

$\frac{d}{dt}(r\hat{u}_\theta) = \dot{r}\hat{u}_\theta + r\dot{\hat{u}}_\theta$; where $\dot{\hat{u}}_\theta = \frac{du_\theta}{dt}$

$\hat{u}_\theta = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

- $\dot{}$ → with respect to time.
- $'$ → with respect to position

$$\dot{\hat{u}}_\theta = \frac{d\hat{u}_\theta}{dt} = \frac{d}{dt}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$= \frac{d}{d\theta}(\cos\theta)\cdot\frac{d\theta}{dt}\,\hat{i} + \frac{d}{d\theta}(\sin\theta)\cdot\frac{d\theta}{dt}\,\hat{j}$$

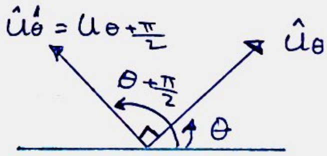
$$= -\sin\theta\,\dot{\theta}\,\hat{i} + \cos\theta\,\dot{\theta}\,\hat{j}$$

$$= (-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})\,\dot{\theta}$$

$$= (\hat{u}'_\theta)(\dot{\theta})$$

$$= (\hat{u}_{\theta+\frac{\pi}{2}})\,\dot{\theta}$$

BUT $\hat{u}'_\theta = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$



so → $\dot{\hat{u}}_\theta = (\hat{u}_{\theta+\frac{\pi}{2}})(\dot{\theta}) = \omega\,\hat{u}_{\theta+\frac{\pi}{2}}$

→ when you take the derivative of a rotating unit vector ($\hat{u}_\theta$) you'll get a unit vector ⊥ to it.

so → $\hat{u}'_\theta = u_{\theta+\frac{\pi}{2}}$

note → For planar motion (x-y plane)

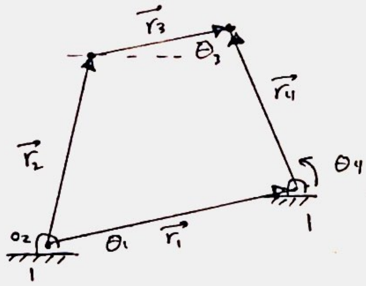
$\vec{\omega} = \dot{\theta}\hat{k}$ (about z-axis only).

4-Bar Mechanism velocity Analysis:

Knowns $\rightarrow$ All positions (from position Analysis)

Input $\dot{\theta}_2$

Find ω_3, ω_4



Solution:

L.C.E:

$$\vec{r}_2 + \vec{r}_3 - \vec{r}_4 - \vec{r}_1 = 0$$

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = 0$$

$$V.E = \frac{d}{dt}(L.C.E) \qquad \Rightarrow \frac{d}{dt}(\underbrace{r_1 \hat{u}_{\theta_1}}_{\text{constant.}}) = 0$$

$$\frac{d}{dt}(L.C.E) = r_2 \omega_2 \hat{u}_{\theta_2+\frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta_3+\frac{\pi}{2}} - r_4 \omega_4 \hat{u}_{\theta_4+\frac{\pi}{2}} = 0$$

x-velocity:

$$-r_2 \omega_2 \sin\theta_2 - r_3 \overset{?}{\omega_3} \sin\theta_3 + r_4 \overset{?}{\omega_4} \sin\theta_4 = 0$$

y-velocity:

$$r_2 \omega_2 \cos\theta_2 + r_3 \overset{?}{\omega_3} \cos\theta_3 - r_4 \overset{?}{\omega_4} \cos\theta_4 = 0$$

$\rightarrow$ Two equations, Two unknowns (ω_3, ω_4)

* linear $\rightarrow$ can be solved

$\hookrightarrow$ you can either use x & y equations or elimination method (easier):

Elimination:

To eliminate ω_3, Dot the V.E By $\hat{u}_{\theta_3}$ Because $(\hat{u}_{\theta_3} \cdot \hat{u}_{\theta_3+\frac{\pi}{2}} = 0)$

$$\left(r_2 \omega_2 \hat{u}_{\theta_2+\frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta_3+\frac{\pi}{2}} - r_4 \omega_4 \hat{u}_{\theta_4+\frac{\pi}{2}} = 0\right) \cdot \hat{u}_{\theta_3}$$

$$-r_2 \omega_2 \sin(\theta_2 - \theta_3) + 0 + r_4 \omega_4 \sin(\theta_4 - \theta_3) = 0$$

$$\omega_4 = \frac{r_2 \omega_2 \sin(\theta_2 - \theta_3)}{r_4 \omega_4 \sin(\theta_4 - \theta_3)}$$

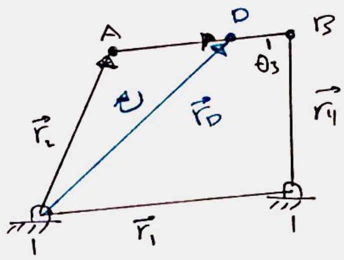
Using the same approach (eliminating ω_4), you can get:

$$\omega_3 = \left(\frac{r_2 \sin(\theta_4 - \theta_2)}{r_3 \sin(\theta_4 - \theta_3)}\right) \omega_2$$

continued $\rightarrow$

Find V_d (from the previous example).

I can find V_d with two methods



1) Relative motion analysis:

$\vec{V}_D = \vec{V}_A + \vec{V}_{D/A}$; V_A is known

2) Loops:

$\vec{r}_2 + \vec{AD} - \vec{r}_D = 0$

$\vec{r}_D = r_2 \hat{u}_{\theta 2} + \bar{AD}\, \hat{u}_{\theta 3}$

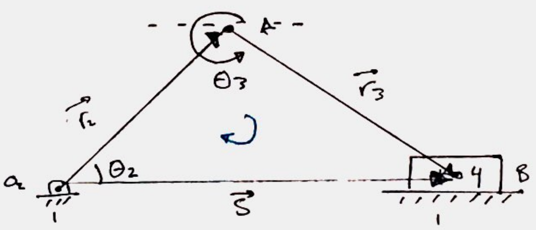
$\vec{V}_D = \frac{d}{dt}(\vec{r}_D)$

$= \underbrace{r_2 \omega_2 u_{\theta 2 + \frac{\pi}{2}}}_{\vec{V}_A} + \underbrace{\bar{AD}\, \omega_3\, u_{\theta 3 + \frac{\pi}{2}}}_{\vec{V}_{D/A}}$ → notice that it gives you the same result as the relative motion analysis.

Ex → Velocity Analysis of Slider-Crank Mechanism

knowns → All positions

Find → ω_3, $\vec{V}_B$



Solution:

L.C.E:

$\vec{r}_2 + \vec{r}_3 - \vec{S} = \vec{0}$

$r_2 \hat{u}_{\theta 2} + r_3 \hat{u}_{\theta 3} - \vec{S} u_0 = \vec{0}$

V.E:

$r_2 \omega_2 \hat{u}_{\theta 2 + \frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta 3 + \frac{\pi}{2}} - \dot{S} u_0 = \vec{0}$ → you can solve using $\begin{cases} \text{x-y velocity} \\ \text{Vector elimination.} \end{cases}$

y-velocity → $r_2 \omega_2 \cos\theta_2 + r_3 \omega_3 \cos\theta_3 = 0$ → $\omega_3 = \leftarrow$

x-velocity → $-r_2 \omega_2 \sin\theta_2 - r_3 \omega_3 \sin\theta_3 - \dot{S} = 0$ → $\dot{S} = \leftarrow$

OR → Dot the (V.E) By $u_{\theta 3}$ (elimination)

$r_2 \omega_2 \sin(\theta_3 - \theta_2) - \dot{S} \cos\theta_3 = 0$

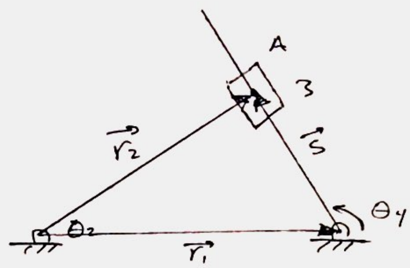
$\dot{S} = \frac{r_2 \omega_2 \sin(\theta_3 - \theta_2)}{\cos\theta_3} = \leftarrow$ → $\omega_3 = \leftarrow$

$\vec{V}_B = \vec{V}_4 = \dot{S}\hat{i}$ → $\dot{S}$: $V_{sliding}$ or V_{slip}

Example:

Knowns → All positions + Input ω_2

Find $\omega_4, \dot{s}$



Solution:

loop closure eqn:

$\vec{r_2} - \vec{s} - \vec{r_1} = 0$ → Both variables

$r_2 \hat{u}_{\theta_2} - s\hat{u}_{\theta_4} - r_1 \hat{u}_0 = 0$

↳ Both constants.

velocity eqn:

$\frac{d}{dt}(L.C.E) = r_2\omega_2 \hat{u}_{\theta_2+\frac{\pi}{2}} - \dot{s}\hat{u}_{\theta_4} - s\hat{u}_{\theta_4+\frac{\pi}{2}}\omega_4 - 0 = 0$

you can also use x & y equations.

elimination:

1- Dot By u_{θ_4}

$$\dot{s} = r_2\omega_2 \sin(\theta_4 - \theta_2)$$

2- Dot By $u_{\theta_4+\frac{\pi}{2}}$

$$r_2\omega_2 \hat{u}_{\theta_2+\frac{\pi}{2}} \cdot \hat{u}_{\theta_4+\frac{\pi}{2}} - s\omega_4 = 0$$

$$\omega_4 = \frac{r_2\omega_2 \cos(\theta_4 - \theta_1)}{s}$$

To find V_A:

you can use 3 or 2 to find V_A Because the point links 2 with 3

3 → Rotation & Translation

2 → Rotation only.

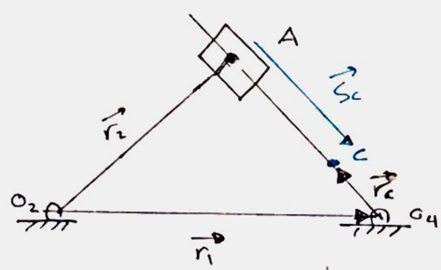
using L.C.E:

$r_2 u_{\theta_2} = s\hat{u}_{\theta_4} + r_1\hat{u}_0$

$$r_2\omega_2 \hat{u}_{\theta_2+\frac{\pi}{2}} = \underbrace{\dot{s}\hat{u}_{\theta_4}}_{\text{sliding}} + \underbrace{s\omega_4 \hat{u}_{\theta_4+\frac{\pi}{2}}}_{\text{Rotation}}$$

$\vec{V_A} = \vec{V_A}$

$= (\quad)\hat{i} + (\quad)\hat{j}$



Find $\vec{V_c}$:

→ you should choose the best Path

where minimum unknowns exist. (2 choices:-

[1] $\vec{r_c} = O_4C\,\hat{u}_{\theta_4}$

$\vec{V_c} = O_4C\,\omega_4\,\hat{u}_{\theta_4+\frac{\pi}{2}}$

$O_4 \to C$

[2] $\vec{r_c} = r_2 u_{\theta_2} - S_c u_{\theta_4}$

$\vec{V_c} = r_1\omega_1 \hat{u}_{\theta_2+\frac{\pi}{2}} - \dot{S_c}\,u_{\theta_4} - S_c\omega_4 \hat{u}_{\theta_4+\frac{\pi}{2}}$

$O_2 \to A \to C$

notice that this path has $\vec{s}$ which has a variable length (more complicated).

↳ This path is easier since you pass through two fixed points.

P(6-66)

Given that:

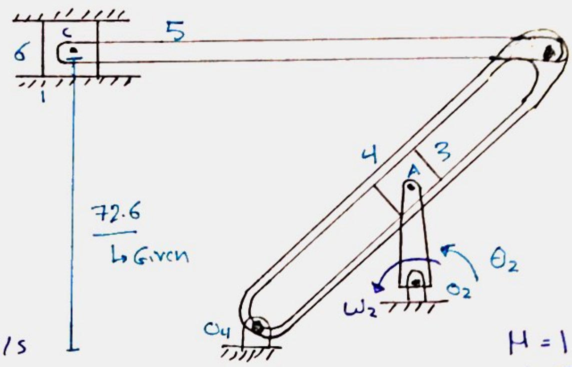
$L_2 = 25.4$ mm (O_2A)

$L_4 = 120.9$ mm (O_4B)

$L_5 = 115.9$ mm (BC)

$O_4O_2 = 42.9$ at $15.5°$

Inputs $\rightarrow$ $\theta_2 = 99°$, $\omega_2 = 10$ rad/s c.c.w



$M = 1$
↳ I need one Input only.

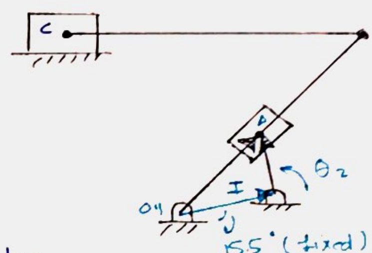
Find :- $\vec{V}_{slip}$ (link 3) , $\vec{V}_C$, $\vec{V}_A$

Solution:

This is multi loop Mechanism with:

① Slider Crank ② Inverted Slider-Crank

Kinematic Diagram



Loop (1):

Path : $O_2 \rightarrow A \rightarrow O_4 \rightarrow O_2$

L.C.E:

$$\left[25.4\,\hat{u}_{\theta_2} - \underline{\underline{S_3}}\,\hat{u}_{\underline{\underline{\theta_4}}} + \overset{\text{constants}}{42.9\,\hat{u}_{15.5}} = \vec{0} \right]$$ vector equation with two unknowns (θ_4, S_3)

* Position Analysis:

$S_3\hat{u}_{\theta_4} = 25.4\,\hat{u}_{\theta_2} + 42.9\,\hat{u}_{15.5}$ (Dot each side by it's self).

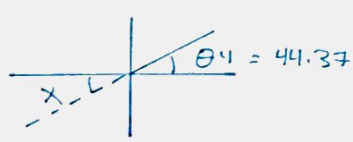
$S_3^2 = (25.4)^2 + (42.9)^2 + 2(25.4)(42.9)\cos(99-15.5)$

$S_3 = 52.27$ mm

$$\tan\theta_4 = \frac{y\text{-position}}{x\text{-position}} = \frac{25.4\sin 99 + 42.9\sin 15.5}{25.4\cos 99 + 42.9\cos 15.5}$$

$$\theta_4 = \tan^{-1}\frac{36.55}{37.37} = 44.37°$$

note:



↳ you can't choose the other angle (look at the Graph of your mechanism).

Note $\rightarrow$ Always keep variables such as θ_2 here as they are till the last step, Because you'll need to derive the equ later to find the velocity

* loop I - Velocity Analysis:

L.C.E → $25.4\hat{u}_{\theta_2} - S_3\hat{u}_{\theta_4} + 42.9\,\hat{u}_{15.5} = \vec{0}$

V.E = $\frac{d}{dt}$ (L.C.E)

$\left(25.4\,\omega_2\, u_{\theta_2+\frac{\pi}{2}} - \dot{S}_3\hat{u}_{\theta_4} - S_3\omega_4\, u_{\theta_4+\frac{\pi}{2}} = \vec{0}\right) \cdot u_{\theta_4}$ (to find $\dot{S}_3$)

$\dot{S}_3 = 254\,(\omega_2)\,\sin(\theta_4 - \theta_2)$

$= 254\,(+10)\,\sin(44.37 - 99) = -207.12$ mm/s

$\vec{V}_{slip} = -207.12\,(\cos 44.37\,\hat{i} + \sin 44.37\,\hat{j})$

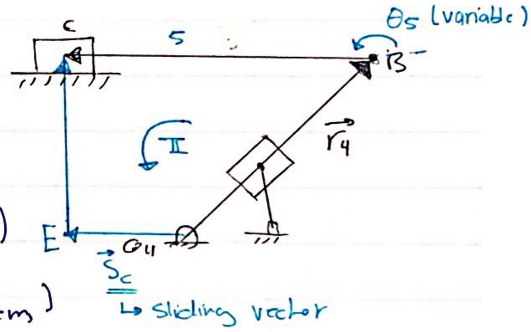
To find ω_4 → (V.E) . $u_{\theta_4+\frac{\pi}{2}}$:-

$\omega_4 = \dfrac{24.4\,(10)\cos(44.37 - 99)}{52.27} = 2.84$ rad/s c.c.w

Loop II :

Path: $O_4 \to B \to C \to E \to O_4$

* L.C.E :



$\left(120.9\,\hat{u}_{\theta_4} + 115.9\,\hat{u}_{\theta_5} - 72.6\,\hat{u}_{90} - S_6\hat{u}_{180} = 0\right)$

I can use (y-position) only to find θ_5 :

$115.9\sin\theta_5 = 72.6 - 120.9\sin\theta_4$ (θ_4 is known)

$\theta_5 = \sin^{-1}(0.1031)$ → 185.9° ✓ (Based on my system)
→ 354.

X-Position :

$S_6 = -120.9\cos\theta_4 - 115.9\cos\theta_5$

$= 28.86$ mm

$\vec{r}_B = 120.9\,\hat{u}_{\theta_4}$

$\vec{V}_B = 120.9\,\omega_4\, u_{\theta_4+\frac{\pi}{2}}$

$= 120.9\,(2.81)\,(-\sin 44.37\,\hat{i} + \cos 44.37\,\hat{j})$

$= (\quad)\hat{i} + (\quad)\hat{j}$ mm/s

* V.E loop II :

$\left(120.9\,\omega_4\, u_{\theta_4+90} + 115.9\,\omega_5\, u_{\theta_5+90} - 0 - \dot{S}_6\hat{u}_{180} = \vec{0}\right) \cdot u_{\theta_5}$ (eliminate ω_5)

$\dot{S}_6\cos(\theta_5 - 180) = 120.9\,(2.81)\sin(\theta_5 - \theta_4)$

$\dot{S}_6 = \dfrac{120.9\,(2.81)\sin(185.9 - 44.37)}{\cos(185.9 - 180)} = -212.47\,\hat{i}$ mm/s

$\vec{V}_c = -212.47\,\hat{i}$

↳ $\omega_5 = 2.107$ rad/s c.c.w (Do it yourself).

note → you could have chosen the path $O_2 \to B \to C \to O_2$ BUT you have no information about $\overline{CO_2}$, so this path is the best choice.

suggested problems CH6:

9(d), $\underbrace{12, 13, 14}_{I.C}$, 15, 18 (B, C), 60, 61, 64, 88, 90

I.C

INSTANT CENTER OF ROTATION

I.C → A point common to two links that has the same instant velocity in each link.

$I_{\boxed{i,j}}$ → links linked to that I.C

note → you can look at G.P.M as Rotation about an instant center

ex → $I_{1,2}$ (The instant center that is common to links 1,2).

↳ Notice : we said common to two links not on so I.C's can be located away from the links.

* For a certain linkage :

$NO_{I.C's} = \frac{L(L-1)}{2}$; L: no. of links

Finding I.C's
- → some IC are found by Inspection (ex → R-Joints)
- → Remaining IC's are found using kennedy's Rule.

note → any I.C common with link 1 ($I_{1,sth}$) has a velocity = 0 , because link (1) is fixed (v=0)

Kennedys Rule

Any three bodies in plane Motion will have exactly three Instant centers and they will lie on the same straight line

* How is it used to find I.C's :

In general I need 2 points to draw a straight line , so If I have two IC's , I can know where the 3rd one is located.

Example (1)

Find all Instant centers for 4-Bar Mechanism.

1. Find No of I.C's:

$$No = \frac{L(L-1)}{2} = \frac{4(4-1)}{2} = \frac{12}{2} = 6 \text{ I.C's}$$

2. Find I.C's By Inspection:

4 R-Joints ($I_{1,2}$, $I_{2,3}$, $I_{3,4}$, $I_{1,4}$)

3. Remaining 2 are found by Kennedy's Rule.

First Method → use combinations.

→ choose 3 bodies and write their I.C's

* Bodies 1,2,3 → $I_{1,2}$, $\boxed{I_{1,3}}$, $I_{2,3}$ → I know that it will lie on the same line as ($I_{1,2}$, $I_{2,3}$) But I don't know on what Point, so I need another combination with ($I_{1,3}$).

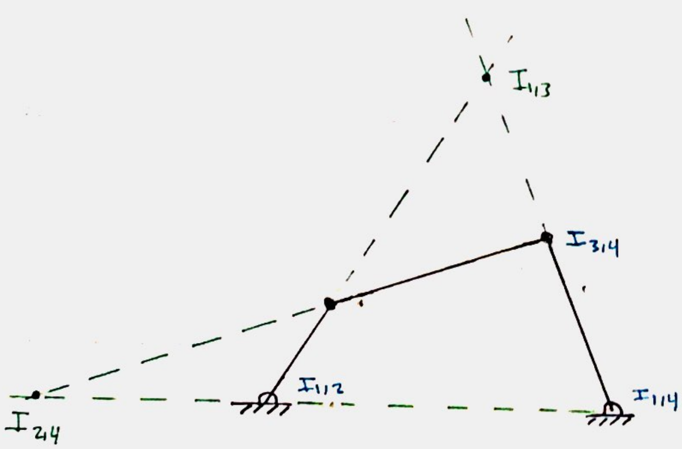
* Bodies 1,4,3 → $I_{1,4}$, $\boxed{I_{1,3}}$, $I_{4,3}$

→ Draw a line connecting $I_{1,2}$ & $I_{2,3}$ and another connecting $I_{4,3}$ & $I_{1,4}$ (the Intersection is $I_{1,3}$)

→ To find $I_{2,4}$ (choose two combinations that has 2 & 4), Repeat the steps ↑

2,4,1 → $I_{1,2}$, $I_{1,4}$, $\boxed{I_{2,4}}$

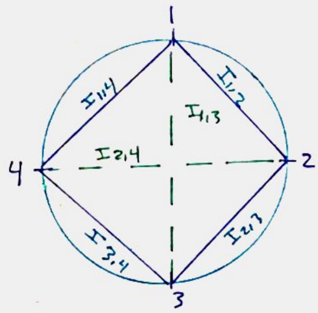
2,4,3 → $\boxed{I_{2,4}}$, $I_{2,3}$, $I_{4,3}$



Method II : Draw a circle

Points → links

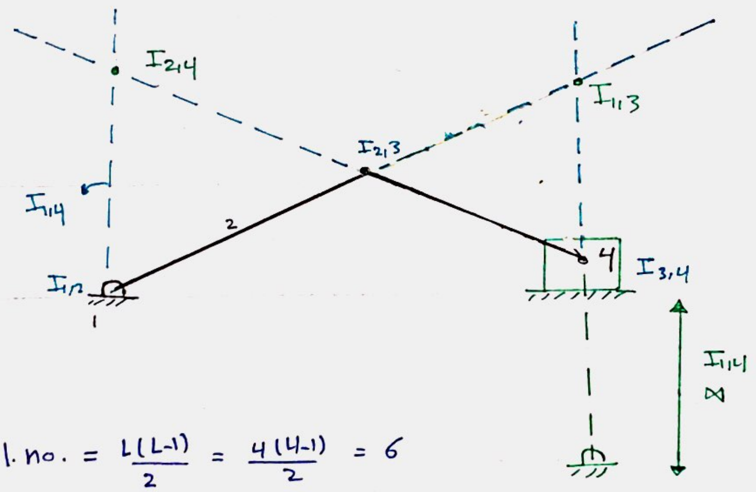
lines → Instant centers.

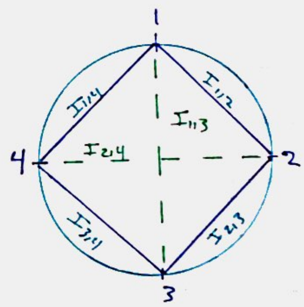


* since each line represents an I.c you should have 6 lines in the circle.

The Dashed line represents the I.c that you Don't know, you can make it solid after you find it.

Example (2) - Slider Crank Mechanism





1. no. = $\frac{L(L-1)}{2} = \frac{4(4-1)}{2} = 6$

2. Draw the circle.

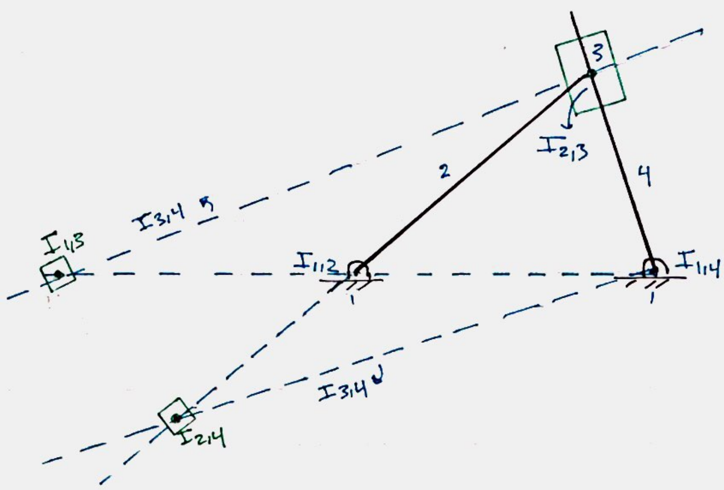
3. R-Joints I have → $I_{1,2}$, $I_{2,3}$, $I_{3,4}$

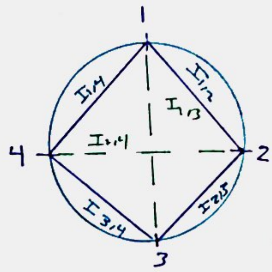
→ I still need one More By inspection ($I_{1,4}$) → By using equivuvalency between slider-crank & 4-Bar, $I_{1,4}$ is located in ∞ and it's direction is $\perp$ on the direction of sliding.

* since ($I_{1,4}$) represents only a direction, we can move the line to the location of point ($I_{1,2}$).

* $I_{1,4}$ Represents a direction not apoint since it is in ∞.

Example (3) - Inverted Slider-Crank Mechanism.





no I.C's $= \frac{4(4-1)}{2} = 6$

* $I_{3,4}$ → Perpendicular to the Direction of sliding & located in ∞.
* note → The Difference between the inverted & standard is that the direction of $I_{3,4}$ here is not constant it changes with the rotation of link (4)

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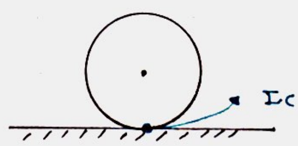
* Rolling without slipping:

The I.C is the point of contact between the ground & the Disk.



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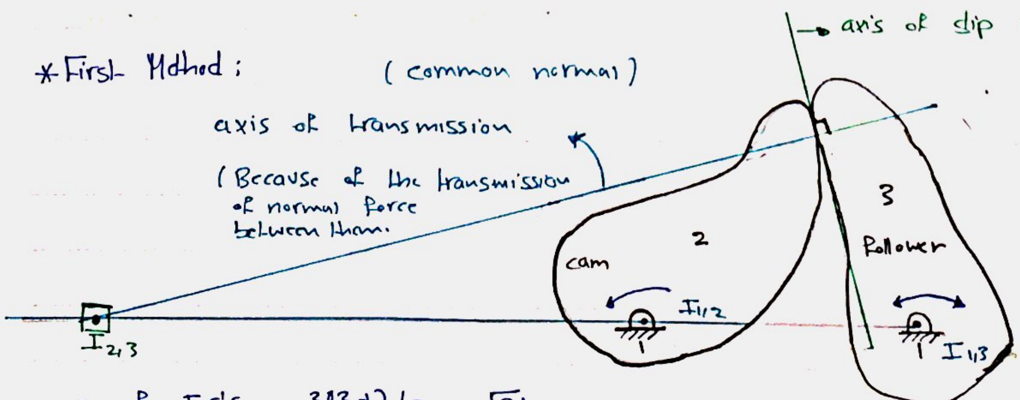
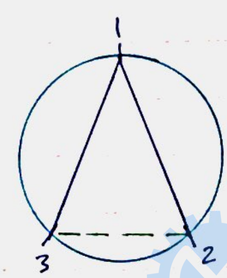
*First Method:



↳ slipping occurs tangentially to both surfaces.

no of I.C's $= 3(3-1)/2 = \boxed{3}$



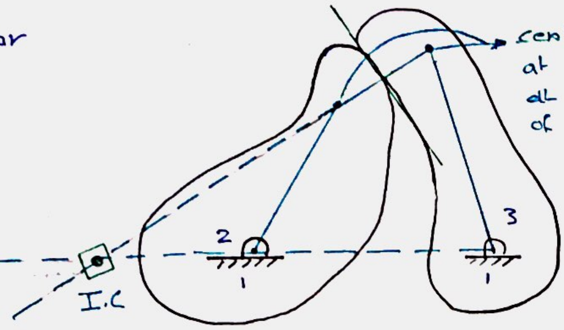
→ $I_{2,3}$ is the intersection between the (common normal) and the line connecting $I_{1,3}$ & $I_{1,2}$

→ $I_{1,2}$ & $I_{1,3}$ are by Inspection.

Method II :

→ using equivalent 4-Bar Mechanism.

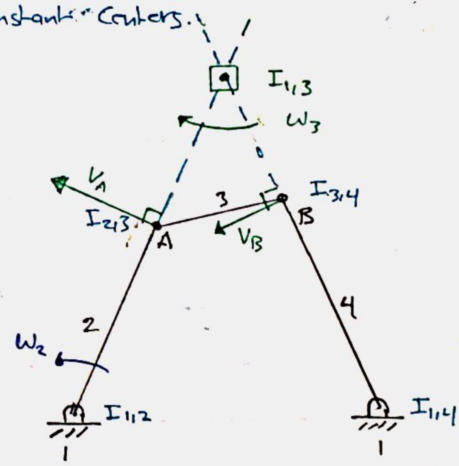
→ I.C is the Intersection of the coupler line with the line connecting $I_{1,2}$ & $I_{1,3}$



→ center of curvature at the point of contact at this certain position of the Mechanism.

↳ They can be found analytically using certain formulas or Graphically.

Velocity Analysis using Instant Centers.



1 4-Bar Mechanism.

Given → Fixed lengths + Input (ω_2)

usually given by the drawing with a scale (1:1) - (you have to measure each length by your self).

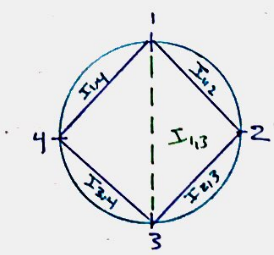
Find → ω_3 & ω_4

Solution:

1. Find V_A as a point on link (2)

$V_A = \omega_2 (O_2 A) =$ ⟶ → Rot only

↳ Direction ⊥ to link 2



→ I only need $I_{1,3}$ for velocity analysis

2. use V_A as a point on link 3

$V_A = \omega_3 (I_{1,3} A)$

$\rightarrow \omega_3 = \dfrac{V_A}{(I_{1,3} A)}$ ↻ (C.W)

↳ Find the direction according to ω_3.

link (3) is rotating about $I_{1,3}$ at that Instant, A is a point on link (3) so we can use $V_A = \omega_3 (I_{1,3} A)$

3. use (ω_3) to find V_B (B as a point on link (3)).

$V_B = (I_{1,3} B)\, \omega_3 =$ ⟵

4. use V_B to find ω_4 (B as a point on link (4)).

$V_B = \omega_4 (I_{3,4} I_{1,4})$

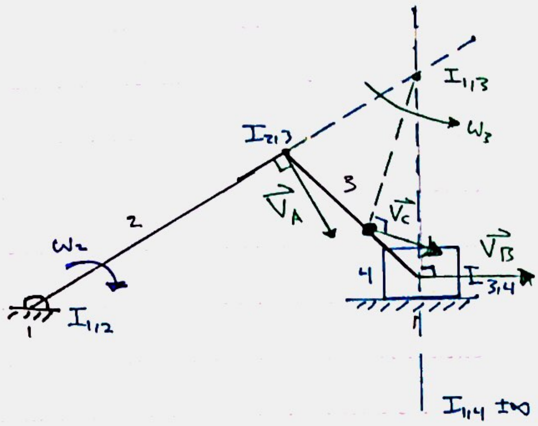
$\omega_4 = \dfrac{V_B}{I_{3,4} I_{1,4}}$ (C.C.W)

you have to go step by step $V_A \rightarrow \omega_3 \rightarrow V_B \rightarrow \omega_4$

[2] Slider-Crank Mechanism.

Given → All lengths + Input (ω_2)

Find ω_3, V_B



Solution:

1. $V_A = (O_2A)\,\omega_2$ (A as a point on 2)
2. $V_A = (I_{1,3}A)\,\omega_3$ (A as a point on 3)

$\omega_3 = \frac{V_A}{I_{1,3}A}$ (c.c.w)

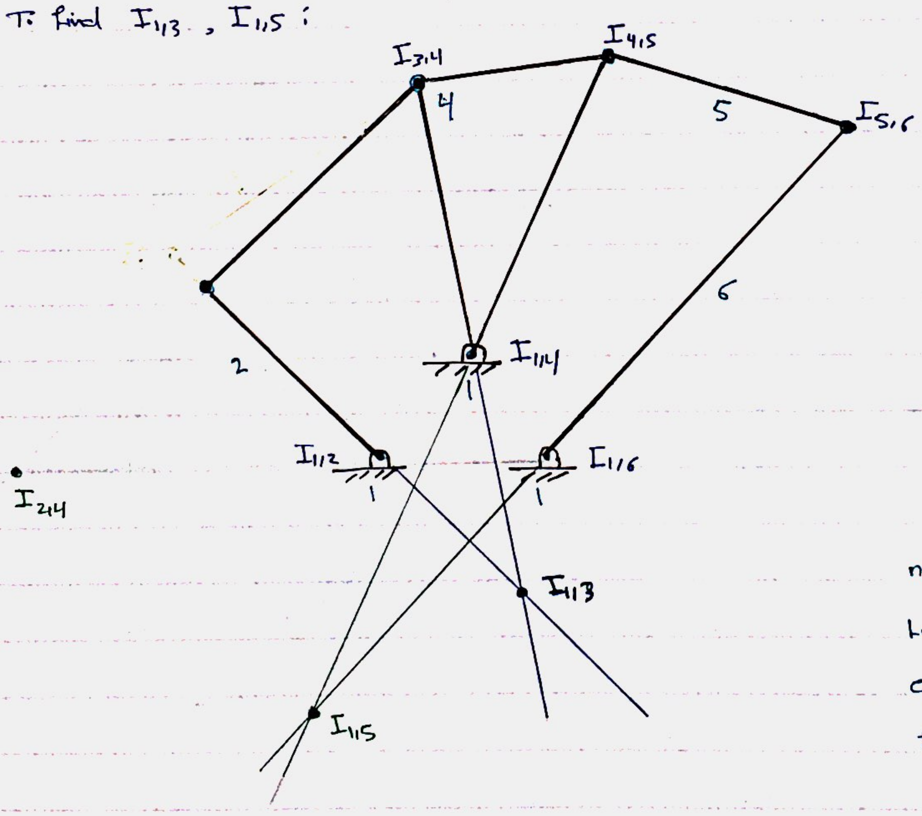
3. $V_B = (I_{1,3}B)\,\omega_3$

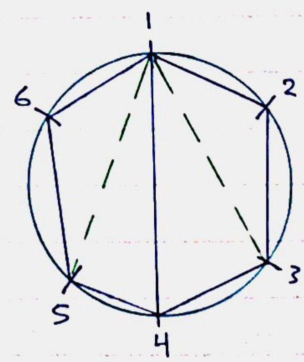
Also → Find V_C

$V_C = (I_{1,3}C)\,\omega_3 = c$

Example → I.C for Watt's 6-Bar Mechanism.

To find $I_{1,3}$, $I_{1,5}$:





note → If you want to find $I_{2,5}$ for example, you need to find $I_{2,4}$ first.

no I.C's $= \frac{6(6-1)}{2} = \boxed{15}$

---

* self-Reading: using I.C To find Angular Velocity Ratio

P(6-18)

2,4 → sliding on a fixed direction.

3 → G.P.M (Imagine)

AB = 45.7 mm

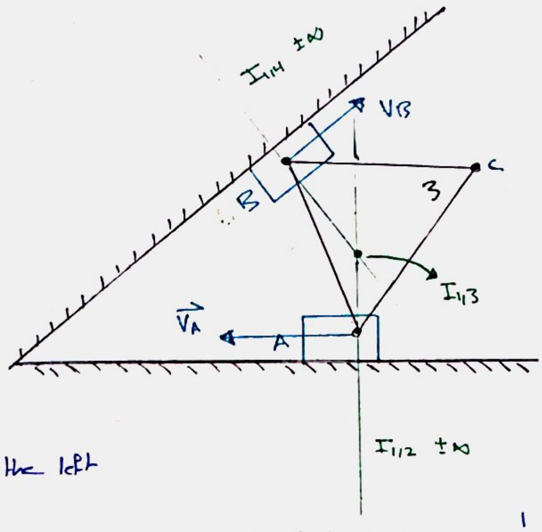
AC = 3

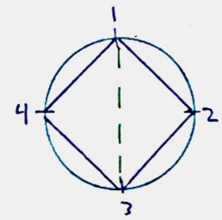
∢BAC = 49° (constant)

B at 59° from A

For $\theta_3 = 128°$ & $V_A = 254$ mm/s to the left

Find V_B & V_C





Solution:-

$V_A = 254$ m/s

$= \omega_3 (I_{13} A)$

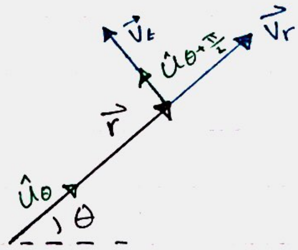
$\omega_3 = \frac{254}{19} = 13.28$ rad/s c.w

$V_B = \omega_3 (I_{13} B)$

$= 436$ mm/s

$V_C = (I_{13} C) \omega_3 = 241.5$ mm/s

ACCELERATION ANALYSIS



* $\vec{r} = r\hat{u}_\theta$
* $\vec{v} = \frac{dr}{dt} = \underbrace{\dot{r}\hat{u}_\theta}_{\substack{V_r \\ \text{(rotation)}}} + \underbrace{r\dot{\theta}\, u_{\theta+\frac{\pi}{2}}}_{\substack{V_t \\ \text{(translation)}}}$
* $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\dot{r}\hat{u}_\theta) + \frac{d}{dt}(r\dot{\theta}\, u_{\theta+\frac{\pi}{2}})$

$$= (\ddot{r}\hat{u}_\theta + \dot{r}\dot{\hat{u}}_\theta) + (\dot{r}\dot{\theta}\hat{u}_{\theta+\frac{\pi}{2}} + r\ddot{\theta}\hat{u}_{\theta+\frac{\pi}{2}} + r\dot{\theta}\dot{\hat{u}}_{\theta+\frac{\pi}{2}})$$

$$\hookrightarrow \dot{\hat{u}}_{\theta+\frac{\pi}{2}} = \frac{d}{dt}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$$
$$= -\dot{\theta}(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$
$$= -\dot{\theta}\hat{u}_\theta$$

$$\hookrightarrow = \ddot{r}\hat{u}_\theta + \dot{r}\dot{\hat{u}}_\theta + \dot{r}\dot{\theta}\hat{u}_{\theta+\frac{\pi}{2}} + r\ddot{\theta}\hat{u}_{\theta+\frac{\pi}{2}} - r(\dot{\theta})^2\hat{u}_\theta$$

$$= \overbrace{(\ddot{r} - r\omega^2)}^{\vec{a}_r\ \text{(radial)}}\hat{u}_\theta + \overbrace{(r\ddot{\theta} + 2\dot{r}\dot{\theta})}^{\vec{a}_t\ \text{(transverse)}}\hat{u}_{\theta+\frac{\pi}{2}}$$

$$= (\underbrace{\ddot{r}}_{\vec{a}_{slip}} - \underbrace{r\omega^2}_{a_{normal}})\hat{u}_\theta + (\underbrace{r\alpha}_{a_{tangential}} + \underbrace{2\dot{r}\omega}_{a_{coriolis}})\hat{u}_{\theta+\frac{\pi}{2}}$$

$a_{coriolis}$ (Rotation & sliding at the same time)

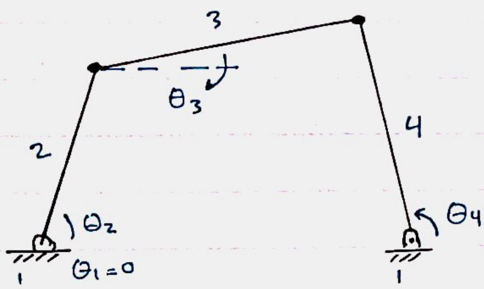
so $\rightarrow$ $$\boxed{\vec{a} = (\ddot{r} - r\omega^2)\hat{u}_\theta + (r\alpha + 2\dot{r}\omega)\hat{u}_{\theta+\frac{\pi}{2}}}$$

Example (4-Bar Mechanism)

Given → Positions + Velocities

Input α_2

Find → α_3, α_4



Solution:

* L.C.E:

$$r_2 \hat{u}_{\theta 2} + r_3 \hat{u}_{\theta 3} - r_4 \hat{u}_{\theta 4} - r_1 \hat{u}_0 = 0$$

* V.E:

$$r_2 \omega_2 \hat{u}_{\theta 2+\frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta 3+\frac{\pi}{2}} - r_4 \omega_4 \hat{u}_{\theta 4+\frac{\pi}{2}} = 0$$

* A.E:

$$(r_2 \alpha_2 \hat{u}_{\theta 2+\frac{\pi}{2}} - r_2 \omega_2^2 \hat{u}_{\theta 2}) + (r_3 \alpha_3 \hat{u}_{\theta 3+\frac{\pi}{2}} - r_3 \omega_3^2 \hat{u}_{\theta 3}) - (r_4 \alpha_4 \hat{u}_{\theta 4+\frac{\pi}{2}} - r_4 \omega_4^2 \hat{u}_{\theta 4}) = 0$$

* Method I → x,y eqns:

→ X - Acceleration

$$-r_2 \alpha_2 \sin\theta_2 - r_2 \omega_2^2 \cos\theta_2 - r_3 \underline{\underline{\alpha_3}} \sin\theta_3 - r_3 \omega_3^2 \cos\theta_3 + r_4 \underline{\underline{\alpha_4}} \sin\theta_4 + r_4 \omega_4^2 \cos\theta_4 = 0$$

→ y - Acceleration:

$$r_2 \alpha_2 \cos\theta_2 - r_2 \omega_2^2 \sin\theta_2 + r_3 \alpha_3 \cos\theta_3 - r_3 \omega_3^2 \sin\theta_3 - r_4 \alpha_4 \cos\theta_4 + r_4 \omega_4^2 \sin\theta_4 = 0$$

↳ 2 eqns - 2 unknowns → solve!

* Method II → vector elimination:

To find (α_4) → eliminate (α_3) → $(A.E) \cdot \hat{u}_{\theta 3}$

↳ $r_2 \alpha_2 \sin(\theta_3 - \theta_2) - r_2 \omega_2^2 \cos(\theta_3 - \theta_2) - r_3 \omega_3^2 - r_4 \alpha_4 \sin(\theta_3 - \theta_4) + r_4 \omega_4^2 \cos(\theta_3 - \theta_4) = 0$

$\alpha_4 =$ ↩

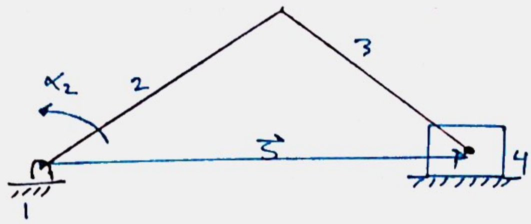
To find (α_3) → eliminate (α_4) → $(A.E) \cdot \hat{u}_{\theta 4}$

note → all terms with $\dot{r}$ and $\ddot{r}$ are omitted because all the links have constant lengths in 4-Bar Mechanism.

Example (Slider - Crank Mechanisms)

Given → positions + velocities

Input α_2



Find → α_3, a_4 ($\ddot{S}$)

Solution:

L.C.E:

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - S\hat{u}_0 = 0$$

V.E:

$$r_2 \omega_2 \hat{u}_{\theta_2+\frac{\pi}{2}} + r_3 \omega_3 \hat{u}_{\theta_3+\frac{\pi}{2}} - \dot{S}\hat{u}_0 = 0$$

A.E:

$$r_2 \alpha_2 \hat{u}_{\theta_2+\frac{\pi}{2}} - r_2 \omega_2^2 \hat{u}_{\theta_2} + r_3 \underline{\alpha_3} \hat{u}_{\theta_3+\frac{\pi}{2}} - r_3 \omega_3^2 \hat{u}_{\theta_3} - \underline{\underline{\ddot{S}}}\hat{u}_0 = \vec{0}$$

Method (I) → use y-eqn

y- Acceleration:

$$r_2 \alpha_2 \cos\theta_2 - r_2 \omega_2^2 \sin\theta_2 + r_3 \underline{\underline{\alpha_3}} \cos\theta_3 - r_3 \omega_3^2 \sin\theta_3 = 0 \qquad \text{1 unknown } (\alpha_3)$$

$\alpha_3 = \checkmark$ #

Method (II) → Vector elimination

$(A.E) \cdot \hat{u}_{\theta_3}$

$$r_2 \alpha_2 \sin(\theta_3 - \theta_2) - r_2 \omega_2^2 \cos(\theta_3 - \theta_2) - r_3 \omega_3^2 - \underline{\underline{\ddot{S}}} = 0 \qquad \text{1 unknown } (\ddot{S})$$

$\ddot{S} = \checkmark$ #

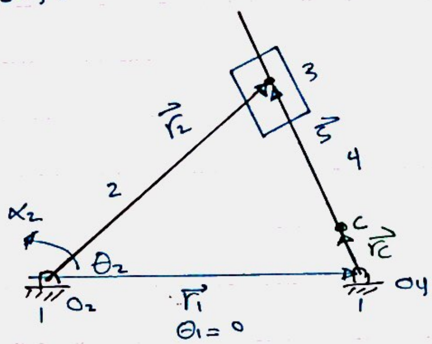
$\vec{a}_4 = \ddot{S}\hat{u}_0 = \ddot{S}\,\hat{i}$

→ note ←

to know if the body is accelerating or deaccelerating, compare the signs of ω & α → similar (acceleration)

different (deacceleration)

Example → (Inverted Slider-Crank Mechanism).



Given : positions + velocities

Input α_2

Find → α_4 , $\ddot{s}$

Solution:

L.C.E:

$$r_2 \hat{u}_{\theta_2} - \underbrace{s \hat{u}_{\theta_4}}_{\text{Both variable}} - \overbrace{r_1 \hat{u}_0}^{\text{constants}} = 0$$

V.E:

$$r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - \dot{s} u_{\theta_4} - s \omega_4 u_{\theta_4 + \frac{\pi}{2}} = 0$$

A.E:

$$r_2 \alpha_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - r_2 \omega_2^2 \hat{u}_{\theta_2} - \ddot{s} u_{\theta_4} - \dot{s} \omega_4 u_{\theta_4 + \frac{\pi}{2}} - \dot{s} \omega_4 u_{\theta_4 + \frac{\pi}{2}} - s \alpha_4 u_{\theta_4 + \frac{\pi}{2}} + s \omega_4^2 \hat{u}_{\theta_4} = 0$$

$$\rightarrow r_2 \alpha_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} - r_2 \omega_2^2 \hat{u}_{\theta_2} - (\ddot{s} - s\omega_4^2) \hat{u}_{\theta_4} - (2\dot{s}\omega_4 + s\alpha_4) u_{\theta_4 + \frac{\pi}{2}} = 0$$

Elimination → (A.E) . $\hat{u}_{\theta_4}$ (to find $\ddot{s}$)

$\ddot{s}$ → acc of slider (3) w.r.t link (4)

$$r_2 \alpha_2 \sin(\theta_4 - \theta_2) - r_2 \omega_2^2 \cos(\theta_4 - \theta_2) - \ddot{s} + s\omega_4^2 = 0$$

$$\ddot{s} = r_2 \alpha_2 \sin(\theta_4 - \theta_2) - r_2 \omega_2^2 \cos(\theta_4 - \theta_2) + s\omega_4^2 = \checkmark \ \#$$

(A.E) . $u_{\theta_4 + \frac{\pi}{2}}$ (to find α_4)

$$r_2 \alpha_2 \cos(\theta_4 - \theta_2) - r_2 \omega_2^2 \sin(\theta_2 - \theta_4) - 2\dot{s}\omega_4 - s\alpha_4 = 0$$

$$\alpha_4 = \frac{r_2 \alpha_2 \cos(\theta_4 - \theta_2) - r_2 \omega_2^2 \sin(\theta_2 - \theta_4) - 2\dot{s}\omega_4}{s} = \checkmark \ \#$$

Also → Find $\vec{a_c}$:

$$\vec{r_c} = \overline{O_4C} \, \hat{u}_{\theta_4}$$

$$\vec{V_c} = O_4C \, \omega_4 \, \hat{u}_{\theta_4 + \frac{\pi}{2}}$$

$$\vec{a_c} = (O_4C) \alpha_4 \hat{u}_{\theta_4 + \frac{\pi}{2}} - (O_4C) \omega_4^2 \hat{u}_{\theta_4} \quad \text{(all knowns)}$$

$$= (\quad) \hat{i} + (\quad) \hat{j}$$

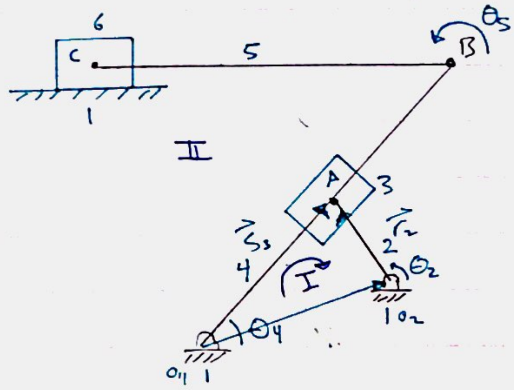
↳ you can try to take the path ($O_2 \rightarrow 3 \rightarrow c$) , you'll find that ($O_4C$) is MUCH easier because it doesn't have (S).

→ (6-66) in velocity Analysis.

P(7-54)

Given → $\theta_2 = 99°$

$\omega_2 = 10$ rad/s = constant ($\alpha_2 = 0$)



Solution:

Positions & velocities have been found previously.

* L.C.E (I):

$$S_3\hat{u}_{\theta_4} - 25.4\,\hat{u}_{\theta_2} - 42.9\,\hat{u}_{15.5} = \vec{0}$$

V.E (I):

$$\dot{S}_3\hat{u}_{\theta_4} + S_3\hat{u}_{\theta_4+\frac{\pi}{2}}\,\omega_4 - 25.4\,\omega_2\,\hat{u}_{\theta_2+\frac{\pi}{2}} = \vec{0}$$

A.E (I):

$$\ddot{S}_3\hat{u}_{\theta_4} + \dot{S}_3\omega_4\hat{u}_{\theta_4+\frac{\pi}{2}} + \dot{S}_3\hat{u}_{\theta_4+\frac{\pi}{2}}\omega_4 + S_3\alpha_4\hat{u}_{\theta_4+\frac{\pi}{2}} - S_3\omega_4^2 u_{\theta_4} - 25.4\alpha_2\hat{u}_{\theta_2+\frac{\pi}{2}} + 25.4\omega_2^2 u_{\theta_2} = 0$$

Rearrange:

$$\to (\ddot{S}_3 - S_3\omega_4^2)\hat{u}_{\theta_4} + (S_3\alpha_4 + 2\omega_4\dot{S}_3)u_{\theta_4+\frac{\pi}{2}} - 25.4\alpha_2\hat{u}_{\theta_2+\frac{\pi}{2}} + 25.4\,\omega_2^2\,\hat{u}_{\theta_2} = \vec{0}$$

To find $\ddot{S}_3$ (eliminate α_4)

(A.E) $\cdot\,\hat{u}_{\theta_4}$

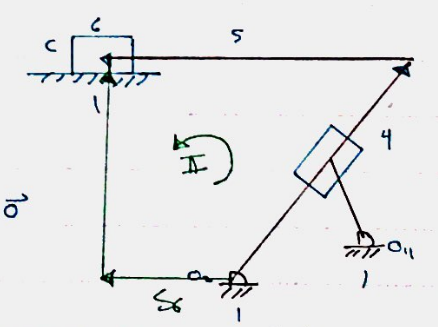
$$\ddot{S}_3 = S_3\omega_4^2 - 25.4\,\omega_2^2\cos(\theta_4-\theta_2) + 25.4\alpha_2\sin(\theta_4-\theta_2)$$

$$= (52.27)(2.81)^2 - 25.4\,(10)^2\cos(44.37-99) + 0 \quad \alpha_2 = 0$$

\# $\ddot{S}_3 = -1057.56$ (mm/s²)

To find α_4 (A.E)$\cdot\hat{u}_{\theta_4+\frac{\pi}{2}}$

\# $\alpha_4 = -17.36$ rad/s → α_4 (−), ω_4 (+)] → Deacceleration



*loop (II)

L.C.E:

$$120.9\,\hat{u}_{\theta_4} + 115.9\,\hat{u}_{\theta_5} - 72.6\,\hat{u}_{90} - S_6\,\hat{u}_{180} = \vec{0}$$

V.E:

$$120.9\,\omega_4\hat{u}_{\theta_4+\frac{\pi}{2}} + 115.9\,\omega_5 u_{\theta_5+\frac{\pi}{2}} - \dot{S}_6\,\hat{u}_{180} = \vec{0}$$

A.E:

$$120.9\,\alpha_4 u_{\theta_4+\frac{\pi}{2}} - 120.9\,\omega_4^2\hat{u}_{\theta_4} + 115.9\,\alpha_5\hat{u}_{\theta_5+\frac{\pi}{2}} - 115.9\,\omega_5^2 u_{\theta_5} - \ddot{S}_6\,\hat{u}_{180} = \vec{0}$$

To find $\ddot{s}_6$ (A.E) . $\hat{u}_s$

$$\ddot{s}_6 = \frac{(120.9)(-17.36)\sin(185.9-44.37) - 120(2.81)^2\cos(185.9-44.37) - 115.9(2.167)^2}{\cos(185.9-180)}$$

$\ddot{s}_6 = -1077.78 \; mm/s^2$

$\vec{a}_c = -1077.78\,\hat{u}_{180} = 1077.78\,\hat{i}$ (mm/s^2)

*note : you can use two paths to find $\vec{r}_c$

1) $\vec{r}_c = S_6\,\hat{u}_{180} + 72.6\,\hat{u}_{90}$

$\vec{V}_c = \dot{S}_6\,\hat{u}_{180}$

$\vec{a}_c = \ddot{S}_6\,\hat{u}_{180}$

2) $\vec{r}_c = 120.9\,\hat{u}_{\theta 4} + 115.9\,\hat{u}_{\theta 5}$

$\vec{V}_c = \cdots$

$\vec{a}_c = \cdots$

↳ we used that one BUT both should give you the same answer.

*To find a_A:

$\vec{r}_A = 25.4\,\hat{u}_{\theta 2}$

$\vec{V}_A = 25.4\,W_2\,\hat{u}_{\theta 2+\frac{\pi}{2}}$

$\vec{a}_A = 25.4\,(10)^2\,\hat{u}_{\theta 2}$ $\quad(\theta_2 = 99^\circ)$

$= -2540(\cos 99\,\hat{i} + \sin 99\,\hat{j})$

$\vec{a}_A = 397.34\,\hat{i} - 2508.7\,\hat{j}$

$|a_A| = 2540 \; mm/s^2 = \sqrt{(\;)^2 + (\;)^2}$

* note → you can choose any path you want, but it's better to choose the one that would give you less terms so it doesn't take too much time in exams.

* To find $\vec{a}_B$:

$\vec{r}_B = 120.9\,\hat{u}_{\theta 4}$

$\vec{V}_B = 120.9\,W_4\,\hat{u}_{\theta 4+\frac{\pi}{2}}$

$\vec{a}_B = 120.9\,\alpha_4\,\hat{u}_{\theta 4+\frac{\pi}{2}} - 120.9\,W_4^2\,\hat{u}_{\theta 4}$

$= 120.9\,(17.36)\,\hat{u}_{\theta 4+\frac{\pi}{2}} - 120.9\,(2.81)^2\,\hat{u}_{\theta 4}$

$= -2098.82\,(-\sin 44.37\,\hat{i} + \cos 44.37\,\hat{j}) - 954.64\,(\cos 44.37\,\hat{i} + \sin 44.37\,\hat{j})$

$\vec{a}_B = -2150\,\hat{i} - 2167.8\,\hat{j}$ $\quad mm/s^2$

$|\vec{a}_B| = 2305.7 \; mm/s^2$

---

*suggested Problems CH#7 :

7(a) - 8(a) - 15(b) - 19 - 20 - 48 - 50 - 52 - 80

DYNAMIC ANALYSIS OF MECHANISMS

→ Now, we will start to consider the causes of Motion.

* Translation:

$$\sum \vec{F} = \sum (\vec{F})_{eff} = m\vec{a}$$

↳ Inertia forces

* Rotation:

$$\sum \vec{M} = \sum (\vec{M})_{eff} = I_z \alpha$$

; α about the z-axis

↳ Inertia Moment (not moment of Inertia).

I_z → Mass Moment of Inertia to resist rotation about z-axis.

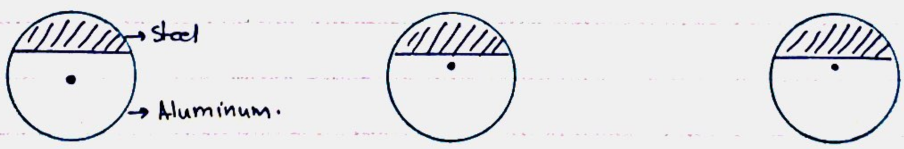
---

* Moment of Inertia :-

Two Types:

- → Mass Moment of Inertia → Rigid body's Inertia to resist Rotation (Geometry & mass are considered). - Dynamics
- → (Area) or (second) Moment of Inertia → The body's Inertia to resist Deformation. (only Geometry) - Strength.

→ In this course, we only deal with Mass Moment of Inertia



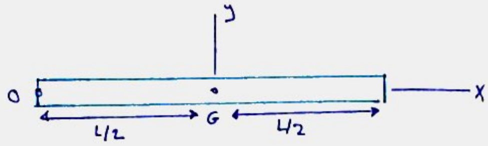
Centroid "Geometric center"

center of Mass (MC) Because steel is heavier it is closer to it.

center of gravity (almost always the same as MC.)

→ For homogeneous bodies Mass center & centroid are on the same location so in this course since all bodies are assumed homogeneous, it doesn't make a difference if you use any of them.

* Mass Moment of Inertia for a Rod:



About centroid:

$$I_G = \frac{1}{12} mL^2$$

About point O:

$$I_O = \frac{1}{12} mL^2 + m\left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12} mL^2 + \frac{1}{4} mL^2 = \frac{1}{3} mL^2$$

↳ using PAT

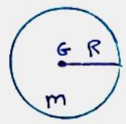
Parallel Axis Theorem

$$I_O = I_G + m\,(d_{O \to G})^2$$

→ used to find I about any point other than G BUT MUST be about the same axis (x or y or z)

* Mass Moment of Inertia of a Disk:

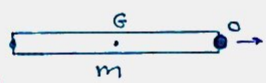
$$I_G = \frac{1}{2} m R^2$$



* For a particles

since the particle is not a Body (no Dimentions), it's Mass Moment of Inertia is zero, Unless it was about another point (use PAT)

Example:



→ particle with mass (M)

Find I_G

centroid of the rod

for the particle → $I_G = I_O + M\,d_{G \to O}^2 = 0 + Md^2 = Md^2$

$$I_G = \frac{1}{12} mL^2 + M\left(\frac{L}{2}\right)^2$$

Example (4-Bar Mechanism):

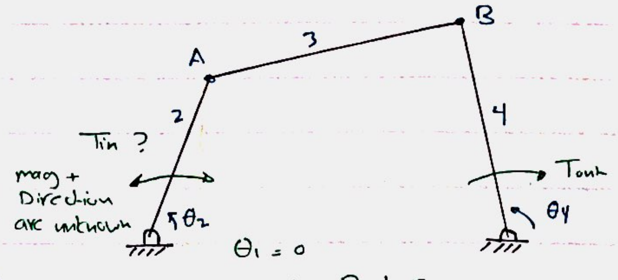
Given $\rightarrow$ kinematics + Masses or Inertias

(m_2, m_3, m_4)

\+ Tout

Find $\rightarrow$ $\underbrace{\text{Tin}}$ & Joint Forces

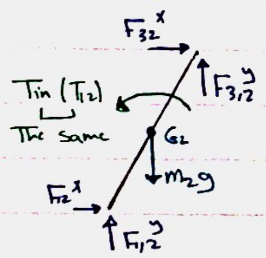
$\hookrightarrow$ For that certain Tout find the necessary Tin



Solution:

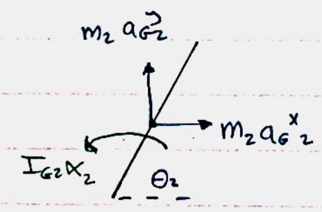
$\rightarrow$ Because each Joint is affected by two forces, each link will have 3-eqns we have 3 links, so 9 equations - 9 unknowns !

Link (2)



F.B.D
applied & Joint loads

$\equiv$



K.D
effective loads

$F_{i,j}$

مصدر الـ Force

تأثير

$G_2 \rightarrow$ center of gravity of link (2)

Directions are assumed.

$\overset{+ve}{\rightarrow} \Sigma F_x = \Sigma (F_x)_{eff} = m a_x$

$F_{12}^x + F_{32}^x = m_2 a_{G_2}^x$ — $\boxed{1}$ 2unknowns

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$

$F_{12}^y + F_{32}^y - m_2 g = m_2 a_{G_2}^y$ — $\boxed{2}$ 2unknowns

$+\circlearrowleft \Sigma (M)_{G_2} = \Sigma (M_{G_2})_{eff}$

$$\text{Tin} - m_2 g \left(\frac{L_2}{2}\right) \cos\theta_2 + F_{32}^y L_2 \cos\theta_2 - F_{32}^x L_2 \sin\theta_2 = I_{G_2} \alpha_2 - m_2 a_{G_2}^x \frac{L_2}{2} \sin\theta_2 + m_2 a_{G_2}^y \left(\frac{L_2}{2}\right) \cos\theta_2 \quad — \boxed{3}$$

① → Prove that:

$$I_{G2}\alpha_2 - m_2 a_{G2}^x \left(\frac{L_2}{2}\right)\sin\theta_2 + m_2 a_{G2}^y \left(\frac{L_2}{2}\right)\cos\theta_2 = I_{O2}\alpha_2$$

$$\vec{r}_{G2} = \frac{L_2}{2}\hat{u}_{\theta 2}$$

$$\vec{V}_{G2} = \frac{L_2}{2}\omega_2 \hat{u}_{\theta 2 + \frac{\pi}{2}}$$

$$\vec{a}_{G2} = \frac{L_2}{2}\alpha_2 \hat{u}_{\theta 2+\frac{\pi}{2}} - \frac{L_2}{2}\omega_2^2 \hat{u}_{\theta 2}$$

$$= \frac{L_2}{2}\alpha_2 \left(-\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j}\right) - \frac{L_2}{2}\omega_2^2 \left(\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}\right)$$

$$= \underbrace{-\frac{L_2}{2}\left(\alpha_2 \sin\theta_2 + \omega_2^2\cos\theta_2\right)}_{a_{G2}^x}\hat{i} + \underbrace{\frac{L_2}{2}\left(\alpha_2\cos\theta_2 - \omega_2^2\sin\theta_2\right)}_{a_{G2}^y}\hat{j}$$

→ substitute the values of a_G^x & a_G^y in the Right hand side of eqn (3)

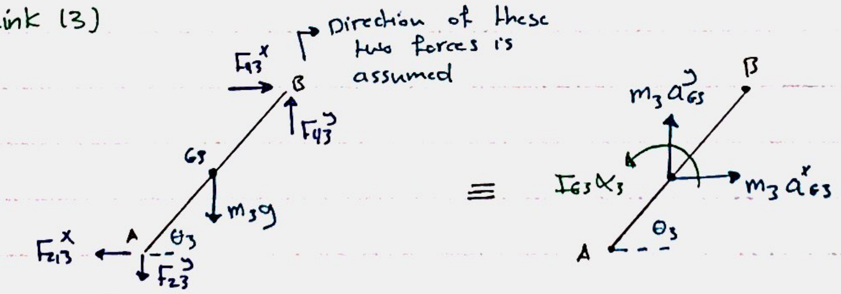
$$I_G\alpha_2 + m_2\frac{L_2^2}{4}\left(\alpha_2\sin^2\theta_2 + \omega_2^2\sin\theta_2\cos\theta_2\right) + m_2\frac{L_2^2}{4}\left(\alpha_2\cos^2\theta_2 - \omega_2^2\sin\theta_2\cos\theta_2\right)$$

$$= I_{G2}\alpha_2 + m_2\frac{L_2^2}{4}\left(\alpha_2\underbrace{\left(\sin^2\theta_2 + \cos^2\theta_2\right)}_{1} + \omega_2^2\underbrace{\left(\sin\theta_2\cos\theta_2 - \sin\theta_2\cos\theta_2\right)}_{zero}\right)$$

$$= I_{G2}\alpha_2 + m_2\alpha_2\frac{L_2^2}{4} = \left(I_{G2} + m_2\left(\frac{L_2}{2}\right)^2\right)\alpha_2$$

$$= I_{O2}\alpha_2 \quad \#$$

Link (3)



Direction of these two forces is determined by newton's 3rd law (Based on link (2)).

$$\begin{bmatrix} F_{23}^x = F_{32}^x \\ F_{23}^y = F_{32}^y \end{bmatrix}$$

$\overset{+ve}{\rightarrow} \Sigma F_x = \Sigma (F_x)_{eff}$

$$-F_{32}^x + F_{43}^x = m_3 a_{G3}^x \quad — \boxed{4}$$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$

$$-F_{32}^y - m_3 g + F_{43}^y = m_3 a_{G3}^y \quad — \boxed{5}$$

$\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$

$$-m_3 g\left(\frac{L_3}{2}\right)\cos\theta_3 + F_{43}^y (L_3)\cos\theta_3 - F_{43}^x (L_3)\sin\theta_3 = I_G\alpha_3 + m_3 a_{G3}^y\left(\frac{L_3}{2}\right)\cos\theta_3 - m_3 a_{G3}^x \frac{L_3}{2}\sin\theta_3 \quad \boxed{6}$$

note → choose point A so that eqn 6 can be solved with eqn (4)

Q → Does the Right hand Side of eqn (6) = $I_A \alpha_3$?

no ; proof:

$$\vec{r}_{G3} = L_2 \hat{U}_{\theta 2} + \frac{L_3}{2} \hat{U}_{\theta 3}$$

$$\vec{V}_{G3} = L_2 \omega_2 \hat{U}_{\theta 2+\frac{\pi}{2}} + \frac{L_3}{2} \omega_3 \hat{U}_{\theta 3+\frac{\pi}{2}}$$

$$\vec{a}_{G3} = L_2 \alpha_2 \hat{U}_{\theta 2+\frac{\pi}{2}} - L_2 \omega_2^2 \hat{U}_{\theta 2} + \frac{L_3}{2} \alpha_3 \hat{U}_{\theta 3+\frac{\pi}{2}} - \frac{L_3}{2} \omega_3^2 \hat{U}_{\theta 3}$$

$$= L_2 \alpha_2 (-\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j}) - L_2 \omega_2^2 (\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}) + \left(\frac{L_3}{2}\right) \alpha_3 (-\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j})$$

$$\hookrightarrow \quad -\left(\frac{L_3}{2}\right)\omega_3^2 (\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j})$$

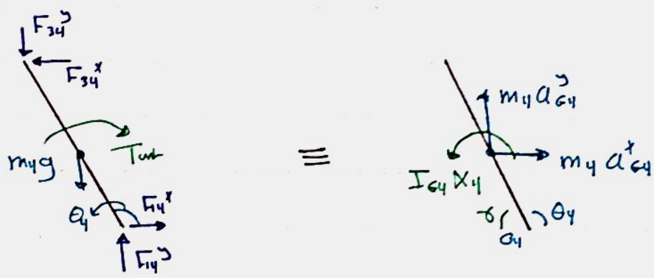
$$= \left(-L_2 (\sin\theta_2 \alpha_2 + \omega_2^2 \cos\theta_2) - \frac{L_3}{2}(+\alpha_3 \sin\theta_3 + \omega_3^2 \cos\theta_3)\right) \hat{i} \rightarrow a_{G3}^x$$

$$+ L_2 (\alpha_2 \cos\theta_2 - \omega_2^2 \sin\theta_2) + \frac{L_3}{2}(\alpha_3 \cos\theta_3 - \omega_3^2 \sin\theta_3)) \hat{j} \rightarrow a_{G3}^y$$

→ substitute the values of a_{G3}^x & a_{G3}^y in the Right hand side of eq (6) and notice that it doesn't equal ($I_A \alpha_3$)

Link (4)

$\gamma = 180 - \theta_4$



$\overset{+}{\rightarrow} \Sigma F_x = \Sigma (F_x)_{eff}$; $F_{14}^x - F_{34}^x = m_4 a_{G4}^x$ — $\boxed{7}$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$; $F_{14}^y - F_{34}^y = m_4 a_{G4}^y$ — $\boxed{8}$

$+\circlearrowright \Sigma M_{O4} = \Sigma (M_{O4})_{eff}$; $-T_{out} + M_4 g \frac{L_4}{2} \cos\gamma + F_{43}^y L_4 \cos\gamma + F_{43}^x L_4 \sin\gamma = I_{O4} \alpha_4$ — $\boxed{9}$

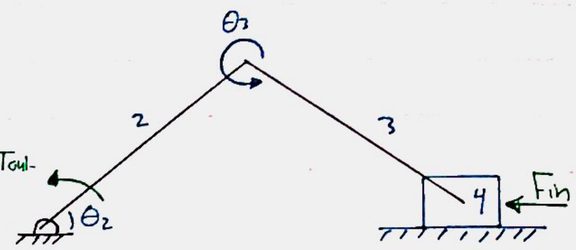
* Solution Procedure :

1. Solve (6) & (9) simultaneously → F_{43}^x , F_{43}^y
2. Solve eqn (4) → F_{32}^x
3. Solve eqn (5) → F_{32}^y
4. Solve eqn (3) → (T_{in}) (we reached our goal)
5. Solve eqn (1) → F_{12}^x
6. Solve eqn (2) → F_{12}^y
7. Solve eqn (7) → F_{14}^x
8. Solve eqn (8) → F_{14}^y

Example (Slider-Crank Mechanism)

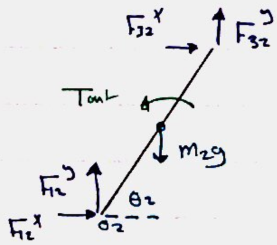
Given → Kinematics + Masses + T_{out}

Find → T_{in}

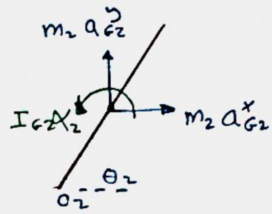


Solution:

link (2)



$\equiv$



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$

$$F_{12}^x + F_{32}^y = m_2 a_{G2}^x \quad \text{—} \boxed{1}$$

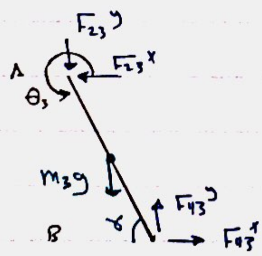
$+\uparrow \ \Sigma F_y = \Sigma (F_y)_{eff}$

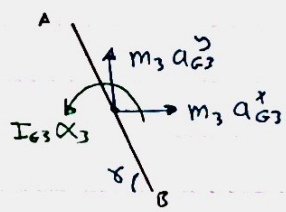
$$F_{12}^y + F_{32}^y - m_2 g = m_2 a_{G2}^y \quad \text{—} \boxed{2}$$

$+\circlearrowleft \ \Sigma (M)_{O2} = \Sigma (M_{O2})_{eff}$

$$T_{out} - m_2 g \left(\frac{L_2}{2}\right) \cos\theta_2 - F_{32}^x L_2 \sin\theta_2 + F_{32}^y L_2 \cos\theta_2 = I_{O2} \alpha_2 \quad \text{—} \boxed{3}$$

link (3)





$\gamma = 360 - \theta_3$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$

$$-F_{23}^x + F_{43}^x = m_3 a_{G3}^x \quad \text{—} \boxed{4}$$

$+\uparrow \ \Sigma F_y = \Sigma (F_y)_{eff}$

$$-F_{23}^y + F_{43}^y - m_3 g = m_3 a_{G3}^y \quad \text{—} \boxed{5}$$

$+\circlearrowleft \ \Sigma M_B = \Sigma (M_B)_{eff}$

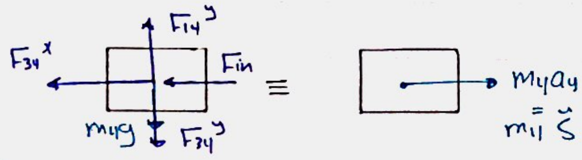
$$F_{32}^x (L_3) \sin\gamma + F_{32}^y L_3 \cos\gamma + m_3 g \left(\frac{L_3}{2}\right) \cos\gamma = I_{G3}\alpha_3 - m_3 a_{G3}^x \left(\frac{L_3}{2}\right) \sin\gamma - m_3 a_{G3}^y \left(\frac{L_3}{2}\right) \cos\gamma$$

↳ $\boxed{6}$

note → choose point (B) so that eq (6) can be solved with eq (3)

continued →

link (4)



$F_{14}^y = N$ (normal force)

$F_{14}^x = \vec{F}$ (friction force $= \mu_k N$; not considered in this example) .

notes :

* Always treat sliders as Particles (i.e Moment eqn will not be applied to Slider 4).
* Sliding occurs on smooth surfaces unless stated otherwise.
* F_{in} is the force resulting from combustion of the fuel.

$$\Sigma F_x = \Sigma (F_x)_{eff}$$

$$F_{34}^x - F_{in} = m_4 \ddot{S} \quad — \boxed{7}$$

$$\Sigma F_y = \Sigma (F_y)_{eff} = 0 \quad \text{"Static eqn"}$$

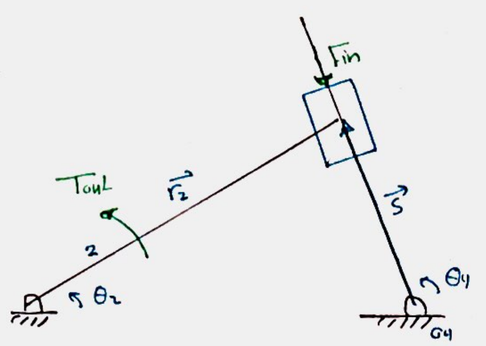
$$F_{14}^y - F_{43}^y - m_4 g = 0 \quad — \boxed{8}$$

Solution Procedure :

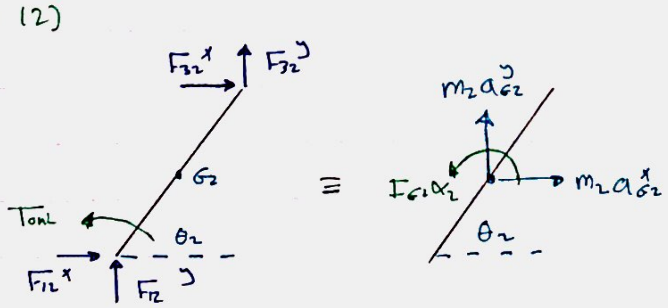
1. Solve 3 & 6 $\longrightarrow F_{32}^x , F_{32}^y$
2. Solve (4) $\longrightarrow F_{43}^x$
3. Solve (5) $\longrightarrow F_{43}^y$
4. Solve (7) $\longrightarrow F_{in}$ #
5. Solve (1) $\longrightarrow F_{12}^x$
6. Solve (2) $\longrightarrow F_{12}^y$
7. Solve (3) $\longrightarrow F_{14}^y$

Example (Inverted Slider - Crank Mechanism)

Assume the Motion to be in the Horizontal plane.



link (2)

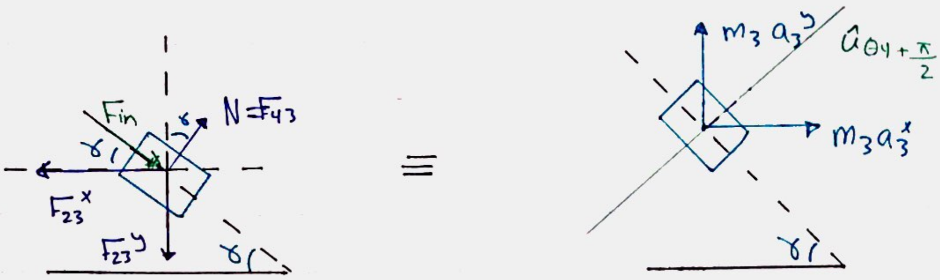


$$\Sigma M_{O2} = \Sigma (M_{O2})_{eff}$$

$$T_{out} - F_{32}^x L_2 \sin\theta_2 + F_{32}^y L_2 \cos\theta_2 = I_{G2}\alpha_2 \quad — \boxed{3}$$

continued

link (3)



$\Sigma F_x = \Sigma (F_x)_{eff}$

$-F_{32}^x + F_{in}\cos\gamma + F_{43}\sin\gamma = m_3 a_3^x$ — $\boxed{4}$

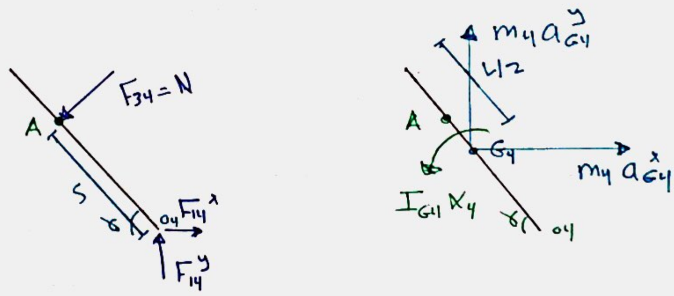
$\Sigma F_y = \Sigma (F_y)_{eff}$

$-F_{32}^y - F_{in}\sin\gamma + F_{43}\cos\gamma = m_3 a_3^y$ — $\boxed{5}$

$\boxed{OR}$ $\Sigma F_{radial} = -F_{in} + \cdots$ (Do it your self).

$\Sigma F_{trans} = F_{34} + \cdots$

link (4)



$\overset{+}{\rightarrow} \Sigma F_x = \Sigma (F_x)_{eff}$ ... $\boxed{6}$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$ ... $\boxed{7}$

$+\circlearrowleft \Sigma M_{O_4} = \Sigma (M_{O_4})_{eff}$

$F_{34} S = I_{O_4}\alpha_4$ — $\boxed{8}$

→ Start the solution with eqn 8

notice that (mg) wasn't included since the Motion is on the Horizontal plane.

STATIC ANALYSIS

→ Assume that the links are Massless ($m = 0$, $I = 0$)

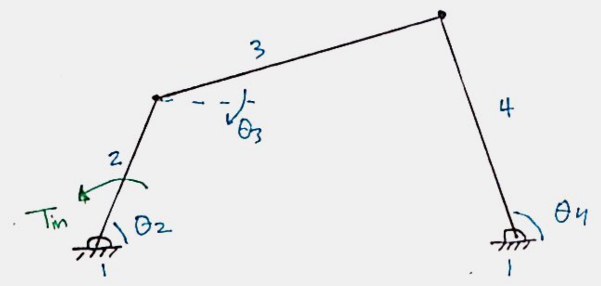
→ $\sum F_x = 0$ $\sum F_y = 0$ $\sum M = 0$ (static equations)

→

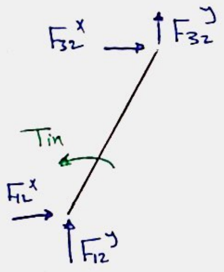
Example (4-Bar Mechanism)

Assuming Massless links:

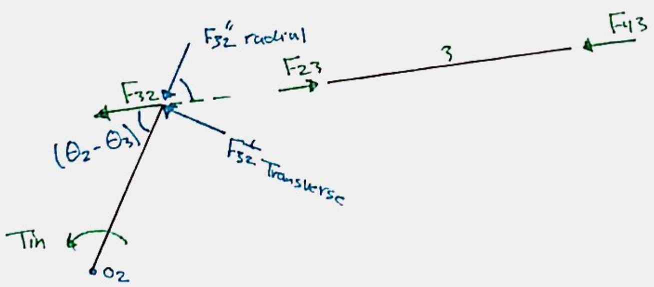
notice that link (3) will become a two force member, it satisfies the 3 conditions:



1. Static Conditions.
2. no Forces along the link, only on Joints.
3. no applied Moment on the link.



→ usually (not in static Analysis) this is how we would analyze the forces on link (2) BUT since link (3) is a two Force Member, we will use the following analysis:-



$F_{32}^{\parallel} = F_{32} \sin(\theta_2 - \theta_3)$

$F_{32}^{\perp} = F_{32} \cos(\theta_2 - \theta_3)$

From link (3):

$\boxed{F_{23} = F_{43}}$

link (2):

$\sum M_{O_2} = 0$

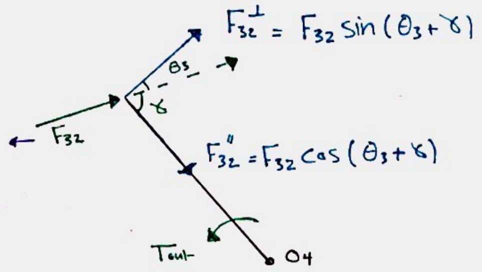
$- F_{32}^{\perp} = T_{in}$

$T_{in} = - F_{32} L_2 \sin(\theta_2 - \theta_3)$ — *

continued →

link (4)

From the axial force member $F_{32} = F_{34}$



$+\circlearrowleft \sum M_{O_4} = 0$

$T_{out} = F_{32} L_4 \sin(\theta_3+\gamma)$ — $**$

$\hookrightarrow \gamma = 180 - \theta_4 \rightarrow \sin(\theta_3 + 180 - \theta_4) = \sin(\theta_4 - \theta_3)$

$F_{32} = \dfrac{T_{out}}{L_4 * \sin(\theta_4-\theta_3)}$ $\rightarrow$ substitute this in $*$:-

$T_{in} = \dfrac{L_2}{L_4} * \dfrac{\sin(\theta_2-\theta_3)}{\sin(\theta_4-\theta_3)} T_{out}$ $\rightarrow$ Relationship between T_{in} & T_{out}

P(11-16)

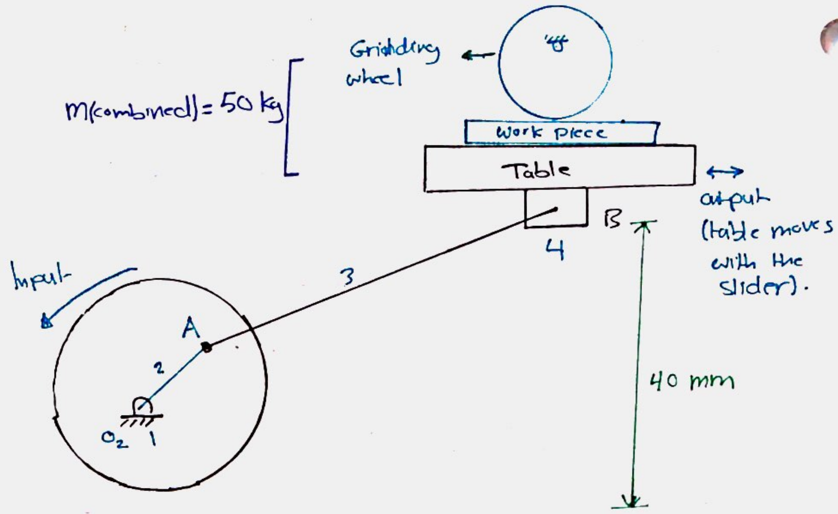
$O_2A = 22$ mm

$AB = 150$ mm

links are Massless

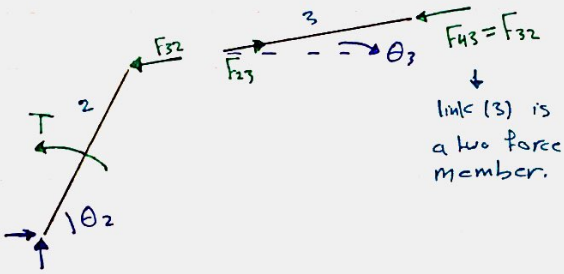
Find driving Torque over one Revolution

↳ $T(\theta_2) = ??$



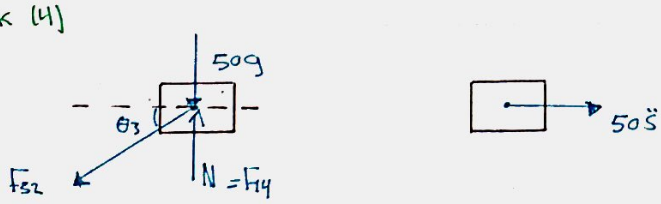
Solution :-

link (2)



$$+\circlearrowleft \sum M_{O_2} = 0 \; ; \quad T + F_{32} L_2 \sin(\theta_2 - \theta_3) = 0 \quad - \boxed{1}$$

link (4)



$$\sum F_x = m a_x \; ; \quad F_{32}\cos\theta_3 = 50\ddot{S} \rightarrow F_{32} = \frac{50\ddot{S}}{\cos\theta_3} \quad - \boxed{2}$$

substitute $\boxed{2}$ in $\boxed{1}$

$$T = 1.1 \frac{\sin(\theta_3 - \theta_2)}{\cos\theta_3} \ddot{S}$$ → kinematics

* For each θ_2:

1. Find θ_3
2. Find $\ddot{S}$
3. Find T

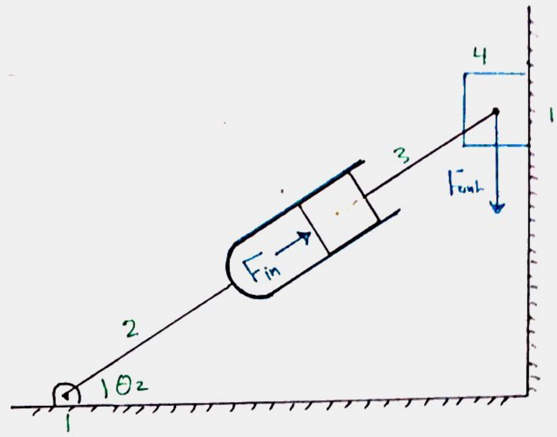
→ note : Because the Question said for 1 Rev, that means for any θ_2

Example (from past papers)

Assuming Massless links

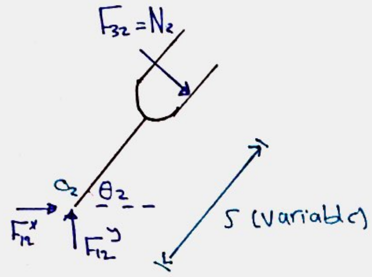
Find a relation between F_{in} & F_{out}

interms of θ_2



Solution:

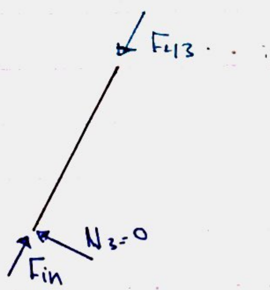
link (2)



$+\circlearrowleft \Sigma M_{O_2} = 0$

$F_{32} S = 0 \rightarrow F_{32} = 0 = N_2 \rightarrow \therefore$ link (2) & link (3) are force members.

link (3)

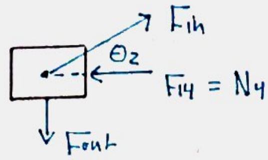


→ here we don't use F_{43}^x & F_{43}^y

Because it is a 2-Force Member.

$F_{43} = F_{in}$

link (4)



$\Sigma F_y = 0$

$F_{out} = F_{in} \sin \theta_2$ #

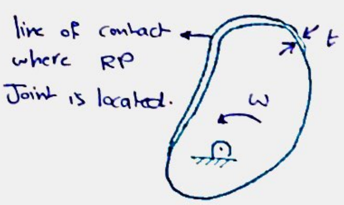
suggested → (11-11)

CAM - FOLLOWER MECHANISM

→ A cam is a general shape Mechanism component that rotates about a fixed axis (usually not the centroid) and forces a driven link (follower) to conform to a definite path of Motion (variable radius Rotation)

→ A cam-Follower Mechanism is considered a Function generator, meaning that we can specify a desired output function of Motion and design the cam's surface to achieve this output.

→ In this course we only deal with one type of cams, which is the Rotating Disk cam, there is other types like cylindrical, ... etc

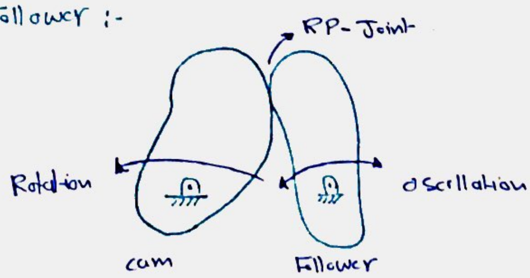


↳ Rotating Disk OR (Plate) cam.

note → We Always assume that the cam rotates with constant c.c.w angular velocity.
(w = +ve constant)

→ A Rotating Disk cam can have the following types of followers:

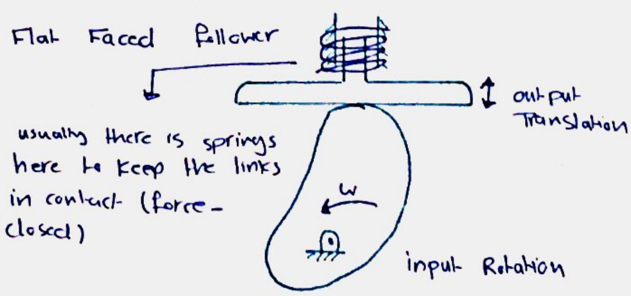
[1] Oscillating Follower :-



[2] Translating Follower :-

common types:

1- Flat Faced follower



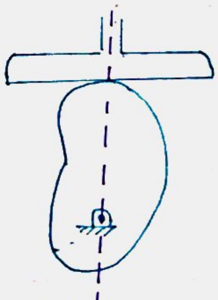
2. Roller follower.

other types of translating followers (not common) :-

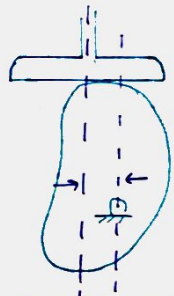
(Sharp - edge (Pen))

(Mushroom Follower)

* Inlined & offset Positions :-



Inlined

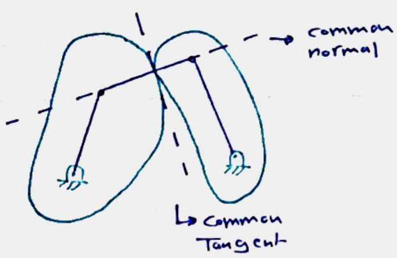


offset

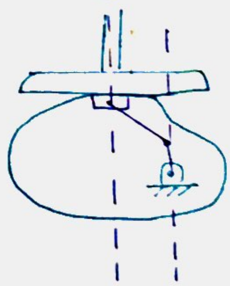
offset (eccentricity)

* Equivalent Mechanisms:

→ oscillating follower ↔ 4-Bar



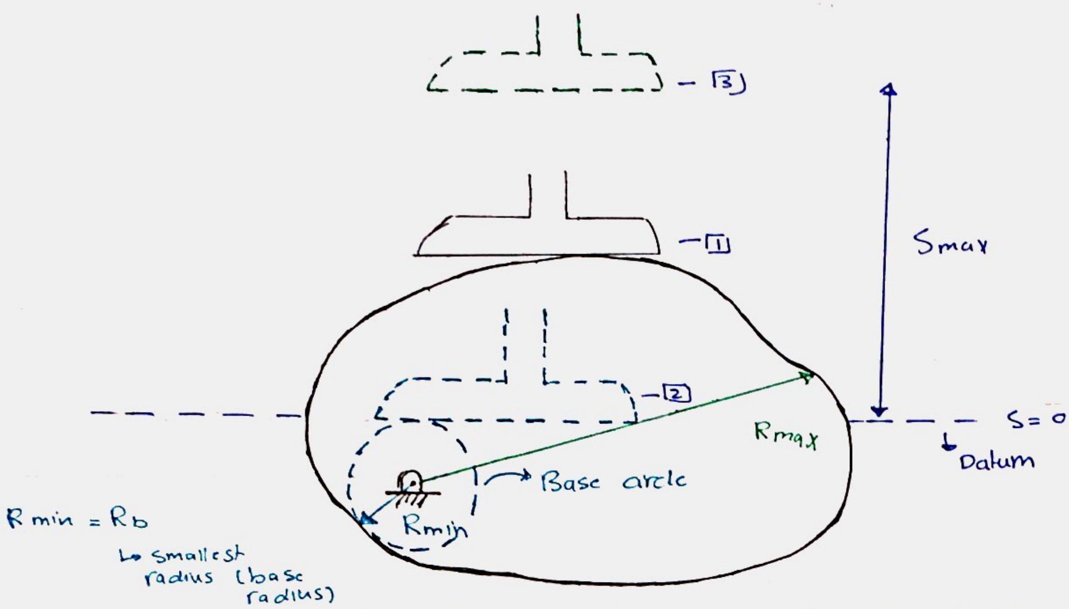
Translating follower ↔ Slider- Crank



Q → can I use a perfect circle as a cam ?

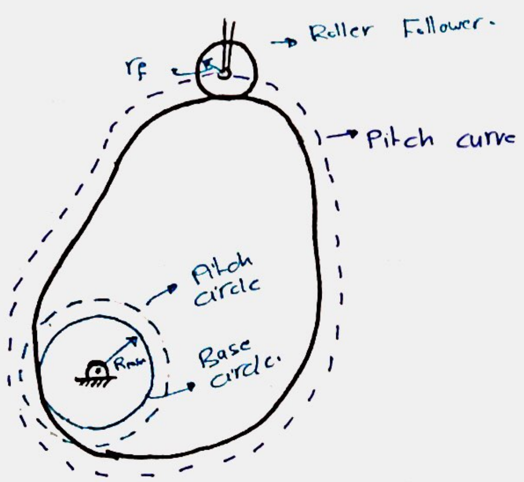
Yes, IF the center of rotation wasn't on the centroid, if it was on the centroid no output motion would occure.

* Positions of the follower:



Assume you have this flat-faced cam follower mechanism, the posit of the follower are as follows:

* position 1 is an arbitrary position
* Position 2 is the lowest position for the follower where (R) is minimum
  - we call the circle with Rmin the "BASE CIRCLE", it's center is the center of rotation of the cam & it is tangent to the follower.
  - we consider this position as the Datum (S = 0)
* Position 3 is the maximum position for the follower (when R is maximum).



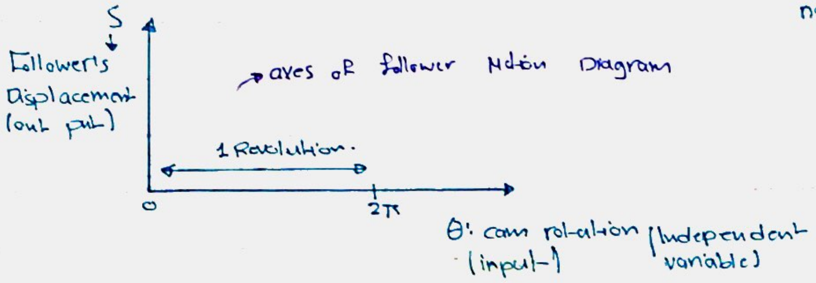
* The Dashed curve represents the curve you would get if you trace the roller's center of rotation as it rolls around the cam, this curve will have the same shape of the cam but with enlarged radius by (r_f) where r_f is the radius of the follower

* The dashed circle is called the Pitch circle it is the same as the Base circle with r enlarged by (r_f)

* Follower Motion Diagram :-

→ we are in interest to analyse the follower's Motion since it is the output Motion, then we use this Analysis to design a cam that would achieve it..

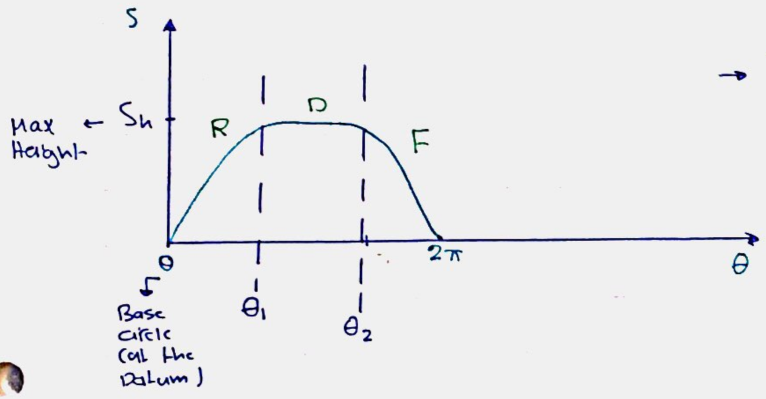
→ The follower Motion diagram represents the follower Motion shape for 1 Rev of the cam. (the Motion will repeat it's self after 1 Rev.



note → S can never be negative since the Datum is the lower position for the follower (R_{min}).

→ We name the diagram according to the regions it consists of :

Example :



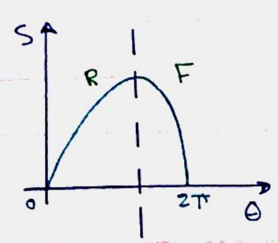
→ This is called a R-D-F Diagram. (For this example).

R → (Rising) - Follower is Rising while cam is rotating. $0 \to \theta_1$

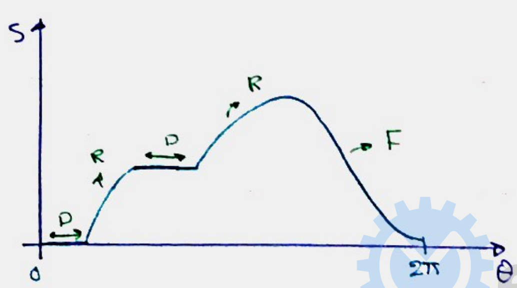
D → (Dwell) - no output Motion while cam is rotating. $\theta_1 \to \theta_2$

F → (Falling) - Follower is falling while cam is rotating $\theta_2 \to 2\pi$

other examples:



R-F Diagram



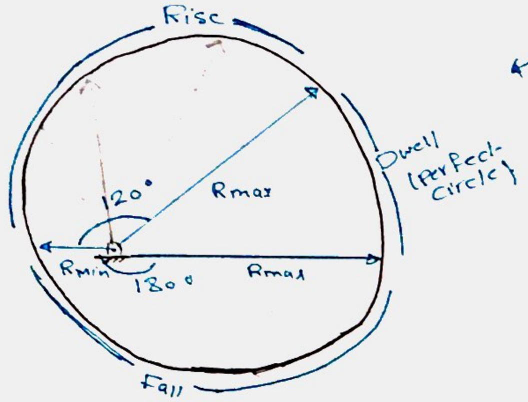
DRDRF - Diagram.

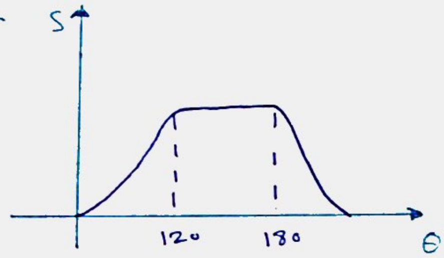
Q → How can (Dwell) happen?

If a part of the cam is a circle, which means no change in Radius & no output Motion.

Q → How is this Diagram used to design a cam?

example:





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* Representing the Motion of the follower:

→ The general form of a follower displacement function:

$$\boxed{S(\theta) = S_i + f(\theta - \theta_i)}$$

S_i, θ_i → S & θ at the end of the previous Interval.

→ The function (f) could be any of the following:

common functions of follower Motion:

1. Uniform Motion.
2. Uniform acceleration.
3. Simple Harmonic Motion.
4. Cycloidal Motion.
5. Polynomial Motion.

↳ We will express them using S V A J diagrams, where:

S → Displacement (S)

V → velocity $d\vec{s}/dt$

A → Acceleration $d\vec{v}/dt$

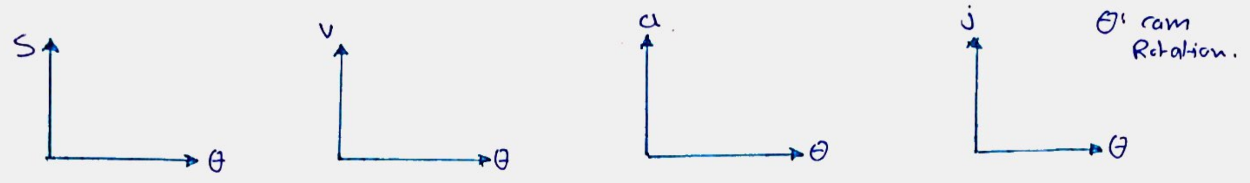
J → Jerk $d\vec{a}/dt$

note → Jerk is an indication of the change in force, How?

→ $\vec{J} = \frac{d\vec{a}}{dt}$

But $\frac{d\vec{F}}{dt} = \underbrace{m}_{\text{constant}} \frac{d\vec{a}}{dt}$ #

So, for each Φ, you'll have 4 Diagrams:



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[1] Uniform Motion:

ex → $V = c$ (constant)

$\int$ Integrate w.r.t θ

$S = c\,\theta$

$f = c(\theta - \theta_i)$

$s(\theta) = S_i + c(\theta - \theta_i)$

→ These two types are not commonly used because they have too much disadvantages.

[2] Uniform Acceleration:

ex → $a = c$ (constant)

$V = c\,\theta$

$a = c\,\theta^2 \rightarrow f = c(\theta)^2$

$S(\theta) = S_i + c(\theta - \theta_i)^2$

* Self - Study → sketch SVAJ Diagram for them.

↳ Do it after other types of Motion.

---

[3] Simple Harmonic Motion: (SHM)

→ Harmonic Motion is a special case of periodic Motion (Repeats it's self)

→ Harmonic means sin, cos or a combination of them with different amplitudes.

→ simple Harmonic is a special case of Harmonic

↳ sin $\boxed{\text{OR}}$ cos with constant amplitude.

$$\boxed{f(\theta) = \frac{h}{2}\left[1 - \cos\frac{\pi\theta}{\beta}\right]}$$

General Form of S.H.M

h → Follower Rise within a specific Interval

β → Cam Rotation within a specific Interval

note: → θ is a variable angle within β

example → if $\beta = \frac{\pi}{2}$, θ is some angle between 0 & $\frac{\pi}{2}$.

Example:

A cam with a flat faced follower will have the following motion for 1 cycle:

	Cam Motion (rotation)	Follower Motion (translation)
1.	$(0 \to 150°)$	Rise of 10 cm (S.H.M)
2.	$(150° \to 240°)$	Dwell
3.	$(240° \to 360°)$	Fall with (S.H.M)

Draw the svaj Diagram.

Solution:

General $\to$ $S(\theta-\theta_i) = S_i + f(\theta-\theta_i)$

for S.H.M $\to$ $S = S_i + \frac{h}{2}\left(1-\cos\left(\frac{\pi}{\beta}(\theta-\theta_i)\right)\right)$

Procedure $\to$ for each interval find $S(\theta)$, differentiate it (3) times to get V, a & j functions, draw the diagrams according to the functions you got.

1. $(0 \to 150°)$ - Rise.

$\theta_i = 0$ $\quad S_i = 0$ $\quad h = 10$ cm $\quad \beta = 150 = \frac{5\pi}{6}$

$S = 5\,[1-\cos(1.2\theta)]$: $0 \leq \theta \leq 150$ $\to$ it's important to specify the interval.

check end points:

$S(0) = 0$ cm $\quad S(\frac{5\pi}{6}) = 10$ cm ✓

$\to$ sketch on the diagram. then check continuity of end points before (V).

$$V = \frac{ds}{dt} = \frac{ds}{dt}\cdot\frac{d\theta}{dt} = w\frac{ds}{d\theta}$$

$V = 6w\sin(1.2\theta)$: $0 \leq \theta \leq 150°$

$V(\theta=0) = 0$ $\quad V(\theta=150) = 0$

$V(\theta=75) = 6w$ $\to$ max. velocity (amplitude of the function) $\sin = 1$

↳ we used $\theta = 75$ because both 0, 150 gave a zero.

$\to$ check continuity at 0 & 150 $\to$ continuous ✓

$$a = \frac{dv}{dt} = \frac{d^2s}{ds^2} = w\frac{dv}{d\theta}$$

$a = 7.2\,w^2\cos(1.2\theta)$: $0 \leq \theta \leq 150$

$a(0) = 7.2\,w^2 = a_{max}$ $\quad a(75) = 0$

$a(150) = -7.2\,w^2 = a_{min}$

* discontinuous at 0 & 150

$j = \frac{da}{dt} = \frac{d^3s}{ds^3} = w\frac{da}{d\theta} = \frac{d^2v}{dt^2}$

↳ note → since the acc. is not continuous at limits, jerk will be undefined on those Points (we say that the jerk is very high when it's undefined).

no equality

$$\# \quad j = \begin{cases} -8.64W^2\sin(1.2\theta) & ; \quad 0 < \theta < 150 \\ \pm\infty & \theta = 0, 150 \end{cases}$$

$j(\theta = 0^+) = 0$

$j(\theta = 150^-) = 0$

$j(\theta = 75°) = -8.64W^2$

2. (150 → 240) - Dwell

$\theta i = 150°$ $\quad$ $Si = 10$ cm $\quad$ $B = 240 - 150 = 90° = \frac{\pi}{2}$

$\therefore S = Si + \cancel{f(\theta - \theta i)}^{0}$

$\# \quad S = 10 \quad ; \quad 150 \leq \theta \leq 240$

$\# \quad V = 0$

$\# \quad a = 0$

$$\# \quad j = \begin{cases} 0 & 150° < \theta < 240° \\ \pm\infty & \theta \to 150°, 240° \end{cases}$$

3. (240 → 360) - Fall

$\theta i = 240$ $\quad$ $Si = 10$ cm $\quad$ $B = 120 = \frac{2\pi}{3}$ $\quad$ $h = -10$ cm

$S = Si + \frac{h}{2}\left(1 - \cos\left(\frac{\pi}{B}(\theta - \theta i)\right)\right)$

$\# \quad S = 5(1 + \cos 1.5(\theta - 240)) \quad : \quad 240 \leq \theta \leq 360$

$S(240) = 10$ cm ← $\quad$ $S(360) = 0$ ←

$V = w\frac{ds}{d\theta}$

$\# \quad V = -7.5w\sin(1.5(\theta - 240)) \quad ; \quad 240 \leq \theta \leq 360$

$V(240) = 0$ $\quad$ $V(360) = 0 = V_{max}$

$V(300) = -7.5W = V_{min}$

continuous at 240 & 360 ←

continued →

$$a = \omega \frac{dv}{d\theta}$$

$a = -11.25\,\omega^2 \cos[1.5(\theta - 240)]$; $240 \le \theta \le 360$

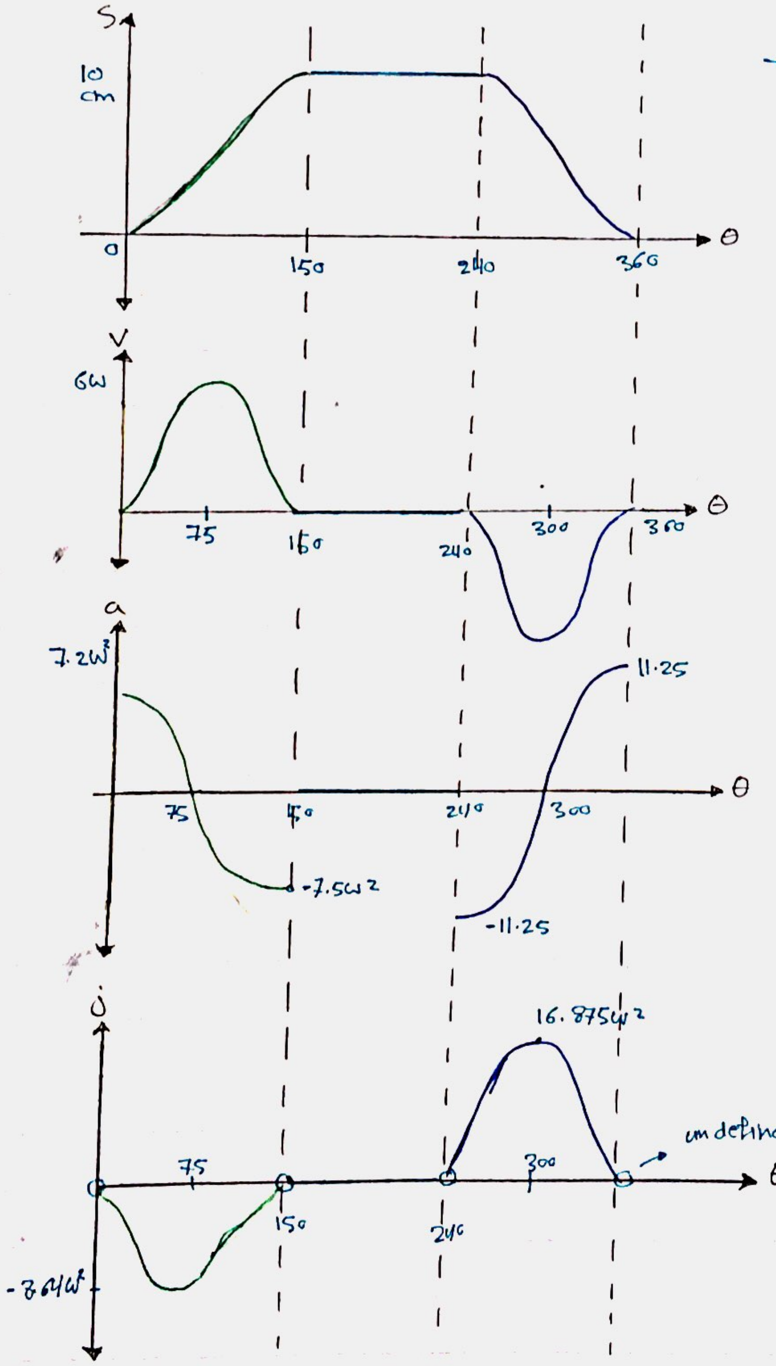
$a(240) = -11.25\,\omega^2 = a_{min}$ $\quad a(300) = 0$

$a(360) = 11.25\,\omega^2 = a_{max}$

discontinuous at $\theta = 240$ & 360 (undefined jerk)

$j = \begin{cases} 16.87\,\omega^2 \sin(1.5(\theta - 240)) & 240^\circ < \theta < 360^\circ \\ \pm\infty & \theta \to 240^\circ, 360^\circ \end{cases}$

$j(\theta \to 240^+) = 0$ $\quad j(\theta \to 360^-) = 0$ $\quad j(300) = 16.875\,\omega^2 = j_{max}$ (cm/s^3)



→ "svaj" Diagram"

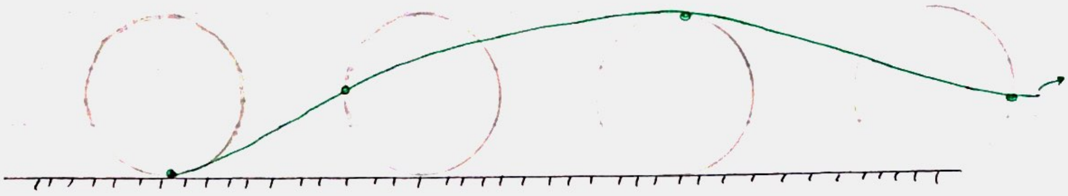
note → Make sure to label all the important points (max, min, end points ...)

→ notice the discontinuities at 150° & 240° (Jumps)

undefined jerk (very high)

4 Cycloidal Function :-

a (cycloid) is the curve traced by a point on a circular disk as the disk rolls along a straight line without slipping .



$$\boxed{f(\theta) = h \left\{ \frac{\theta}{\beta} - \frac{1}{2\pi} \sin\left(\frac{2\pi}{\beta}\theta\right) \right\}}$$

note → you can't make a difference between cycloidal & simple harmonic function by free hand sketching.

Example: - same as the one in S.H.M

0 → 150 Rise by 10 cm (cycloidal)

150 → 240 Dwell

240 → 360 Fall (cycloidal)

Draw s v a j diagram.

S-Diagram → Diagram by Diagram (not intervals)

1 Rise (0 → 150)

$\theta_i = 0$ $S_i = 0$ $\beta = 150$ $h = 10$ cm

$$S = S_i + h \left\{ \frac{\theta - \theta_i}{\beta} - \frac{1}{2\pi} \sin \frac{2\pi}{\beta}(\theta - \theta_i) \right\}$$

$$S = 10 \left\{ \frac{1.2\theta}{\pi} - \frac{1}{2\pi} \sin(2.4\theta) \right\}$$

check end points → $S(0) = 0$ $S\left(\frac{5\pi}{6}\right) = 10$ ✓

2 Dwell (150 → 240) :

$S = 10$ cm : $150 \leq \theta \leq 240$

$V = a = j = 0$

continuous ✓

continued →

[3] Fall : (240 → 360)

$\theta_i = 240°$ $\beta = 120° = \frac{2\pi}{3}$ $S_i = 10$ cm $h = -10$ cm

$S = 10 - 10\left\{ \frac{\theta - \frac{4\pi}{3}}{\frac{2\pi}{3}} - \frac{1}{2\pi}\sin(3(\theta - 240)) \right\}$

check :

$S(240) = 10$ cm ✓

$S(360) = 10 - 10(1 - 0) = 0$ ✓

Velocity :

Rise :

$V = \frac{w\,ds}{d\theta}$

$V = w(10)\left\{ \frac{1.2}{\pi} - \frac{1.2}{\pi}\cos(2.4\theta) \right\}$

$= \frac{12w}{\pi}(1 - \cos(2.4\theta))$: $0 \le \theta \le 150$

Dwell :

$V = 0$: $150 \le \theta \le 240$

Fall :

$V = -10w\left\{ \frac{1.5}{\pi} - \frac{1.5}{\pi}\cos(3(\theta - 240)) \right\}$

$= -\frac{15}{\pi}w\,(1 - \cos(3(\theta - 240))\}$ $240 \le \theta \le 360$

check → $V(240) = 0$ $V(360) = 0$ $V(300) = -\frac{30w}{\pi} = -9.55w = V_{min}$

Acceleration :

Rise :

$a = 9.17w^2 \sin(2.4\theta)$: $0 \le \theta \le 150$

$a(0) = 0$ $a(75) = 0$ $a(150) = 0$ ↱ $2.4\theta = \frac{3\pi}{2} = 112.5$

$a(37.5) = 9.17w^2 = a_{max}$ $a(112.5) = -9.17w^2$ (min)

↳ $2.4\theta = \frac{\pi}{2} \rightarrow \theta = 37.5°$

Dwell :

$a = 0$ $150 \le \theta \le 240$

Fall :

$a = -14.3w^2 \sin(3(\theta - 240))$: $240° \le \theta \le 360°$

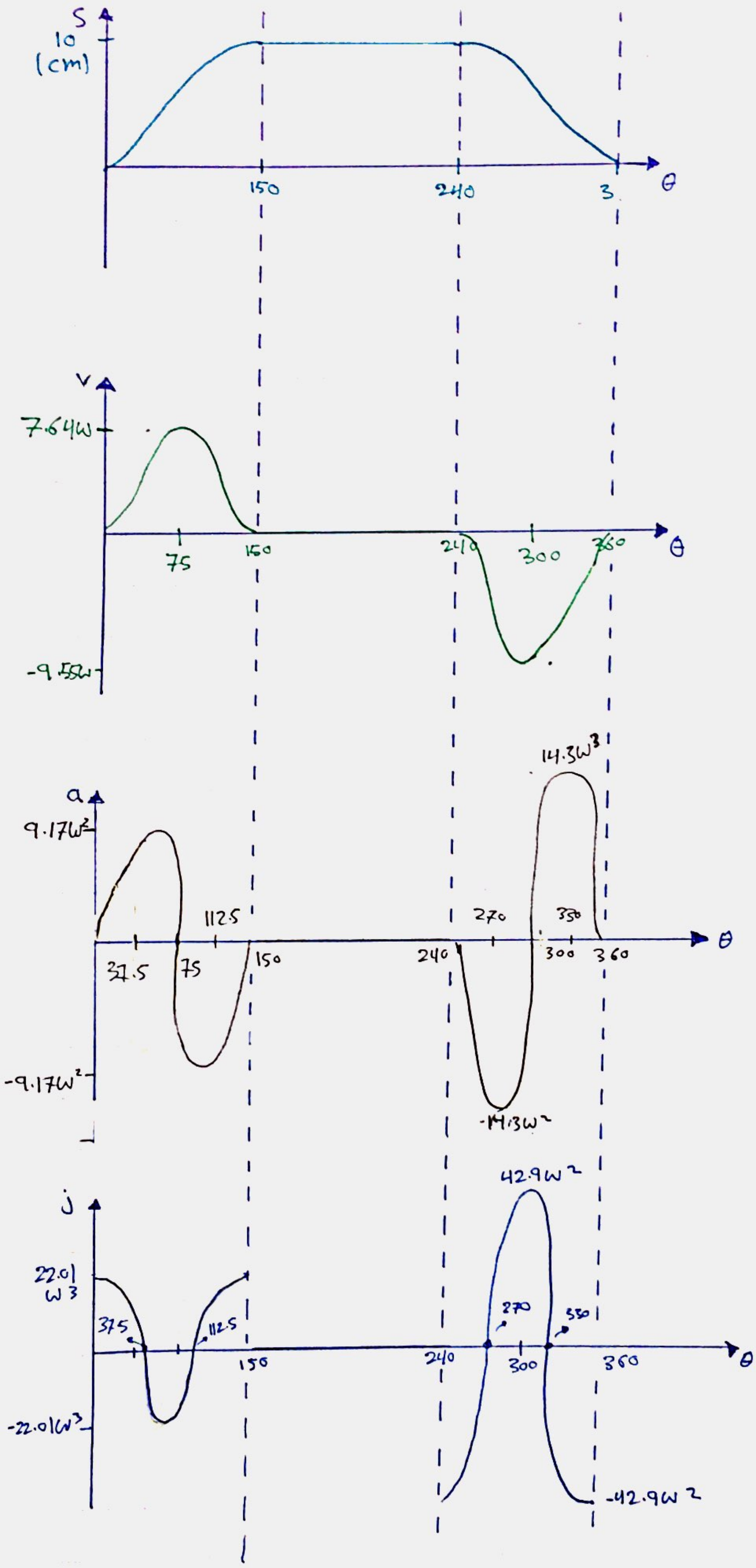
$a(240) = 0$ $a(300) = 0$

$a(360) = 0$

$a(270) = -14.3w^2 = a_{min}$ → $3(\theta - 240) = \frac{\pi}{2} \rightarrow \theta = 270$ ($\sin = 1$)

$a(330) = 14.3w^2 = a_{max}$ → $3(\theta - 240) = \frac{3\pi}{2} \rightarrow \theta = 330$ ($\sin = -1$)

s v a j Diagrams.



→ note that there is no jumps here unlike the S.H.M.

Polynomial Motion :-

→ it has an extra advantage because you can increase the accuracy by controlling the number of terms. (the more terms the more accurate).

$$\boxed{f(\theta) = \sum_{k=0}^{N} C_k \theta^k}$$

C_k → coefficients (unknowns)

no. coeff. = $N+1$

↳ they will be found using Boundary conditions (B.C's)

→ $S = S_i + \sum_{k=0}^{N} C_k (\theta - \theta_i)^k$

→ no. of required conditions = no. of terms

Example:

For a follower having a polynomial function of motion, Find the general displacement function, Assume that the polynomial function has 4 terms.

Solution:

Conditions → $S(\theta_i) = S_i$, $S(\theta_i + \beta) = S_i + h$

$V(\theta_i) = 0$, $V(\theta_i + \beta) = 0$

$S = S_i + \sum C_k (\theta - \theta_i)^k = C_0 + C_1(\theta - \theta_i)^1 + C_2(\theta - \theta_i)^2 + C_3(\theta - \theta_i)^3$

1) $S(\theta_i) = S_i$

$S = S_i + C_0 + 0 + 0 + 0$ → $\boxed{C_0 = 0}$

2) $V(\theta_i) = 0$

$S = S_i + C_0 + C_1(\theta - \theta_i)^1 + C_2(\theta - \theta_i)^2 + C_3(\theta - \theta_i)^3$

$V = \omega (C_1 + 2C_2(\theta - \theta_i) + 3C_3(\theta - \theta_i)^2$

$V(\theta_i) = \omega (C_1 + 0 + 0) = 0$ → $C_1 = 0$

3) $S(\theta_i + \beta) = S_i + h$

$S(\theta_i + \beta) = S_i + \cancel{C_0}^{0} + \cancel{C_1}^{0}(\theta_i + \beta - \theta_i) + C_2(\theta_i + \beta - \theta_i)^2 + C_3(\theta_i + \beta - \theta_i)^3$

$= S_i + C_2\beta^2 + C_3\beta^3 = S_i + h$

$C_2\beta^2 + C_3\beta^3 = h$ — $\boxed{1}$

4) $V(\theta_i + \beta) = 0$

$= \omega (C_1 + 2C_2(\theta_i + \beta - \theta_i) + 3C_3(\theta_i + \beta - \theta_i)^2)$

$2C_2\beta + 3C_3\beta^2 = 0$ — $\boxed{2}$

* solve (1) & (2) → $C_2 = \frac{3h}{\beta^2}$ $C_3 = \frac{-2h}{\beta^3}$

So → $C_0 = 0$ $\quad C_2 = \frac{3h}{\beta^2}$

$C_1 = 0$ $\quad C_3 = \frac{-2h}{\beta^3}$

↳ plug them in the desplacement function:

$$S = S_i + \frac{3h}{\beta^2}(\theta - \theta_i)^2 - \frac{2h}{\beta^3}(\theta - \theta_i)^3$$

→ to check: Find $S(\theta_i)$ → it should give you a zero

$S(\theta_i + \beta)$ → $(S_i + h)$

⋮

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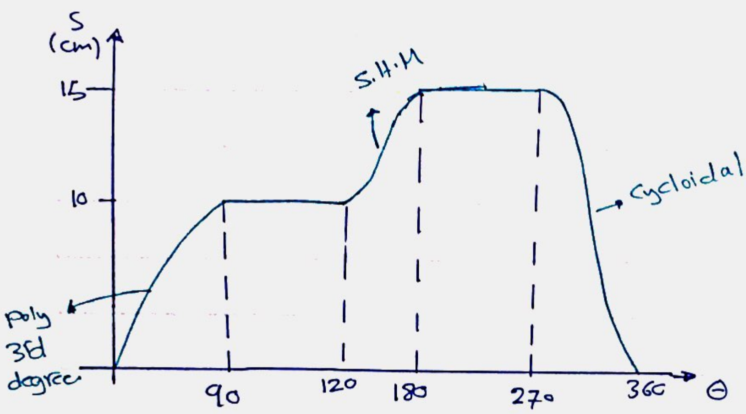
* suggested problem:



1- Draw the S V a J Diagram

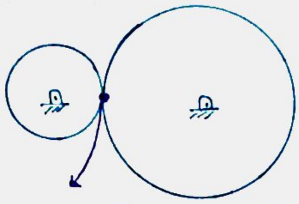
2- Draw the cam profile $\quad R_B = 10\,cm$

GEARS

→ Gears are toothed components, used to transmit power & motion, transmission occurs through meshing of the teeth.

→ We can represent gears by circles:

↳ They are called "pitch circles" and they are tangent at the pitch point.



→ For any two gears, there is: - Driver gear (usually the pinion)
- Driven gear (usually the gear)

* Types of Gears:

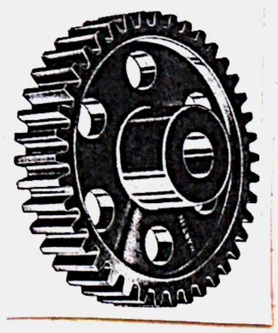
[1] Spur gear:

→ it is the (default - standard - Basic) type of gears.

→ teeth are straight and parallel to the axis of the gear.

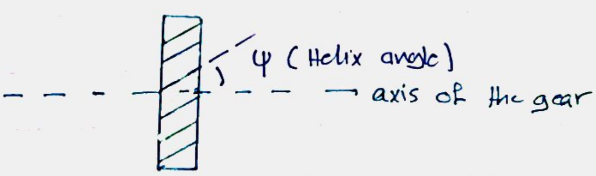
→ During meshings, shafts of the two gears are parallel.

→ advantages: - low cost
- easy to get
- simple to design.



[2] Helical Gears:

→ They are gears by which the teeth are inclined with respect to the axis of the gear by a certain angle that we call (Helix angle)

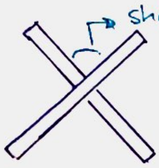


↳ note: for spur gear $\psi = 0$

→ Shafts or axes of the gears could be related in two ways while meshing:

1. Parallel (similar to spur gears)

2. Crossed with an angle called the (shaft angle)



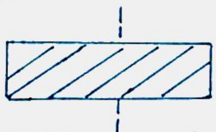
* Default shaft angle is (90°) but it could have any value.

(projection) → notice that they are not really crossed but their projection is.

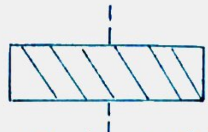
→ Helical shafts are harder to design but they cause less noise & vibration than spur gears.

* Hand of the Helix:

→ it's away to describe how the teeth are inclined, as follows:



Right hand (R.H)



Left hand (L.H)

→ To determine if the helix is a right hand or left hand:

- when the axis of the gear is parallel to the y-axis

Imagine going from left to Right:

→ If Right-hand you'll go up ↗

→ If left-Hand you'll go down ↘

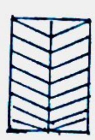
→ determining the hand of the helix is important to know if the meshing is possible or not, as follows:

- For parallel Helical gears meshing can't occure unless the two gears are of opposite hands (R.H & L.H)

- For crossed Helical gears similar or opposite hands can be used.

[3] Herringbone Gears:

→ It consists of two Helical gears of opposite hands joined together on the same shaft (Double Helical Gear)

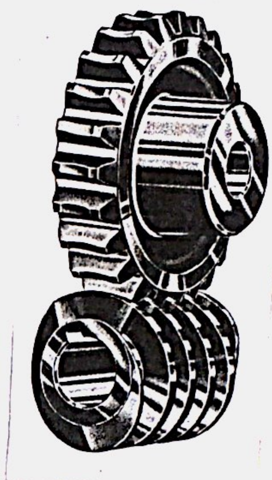


[4] Bevel Gears:

- Teeth of bevel gears are cut on a conical surface (spur's teeth are cut on a cylindrical)
- shafts (axes) of the two gears are intersecting

[5] Worm Gears:

- They can be considered as a special case of crossed Helical gears.
- The Driver is a screw.
- screws have threads not teeth.
- Disadvantage: Friction causes losses due to sliding causing the efficiency to decrease to 70%, while it has higher values for other gears (97% → 99%)

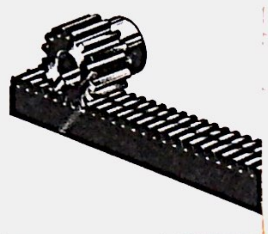


[6] Rack & Pinion:

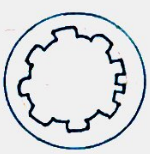
a Rack is a flat gear (a gear with ∞ radius)

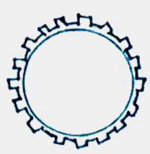
most common application is converting rotational motion to linear motion or vise versa.

example → sterring system in cars.



* Internal & External Gears.

 → "Internal gear"

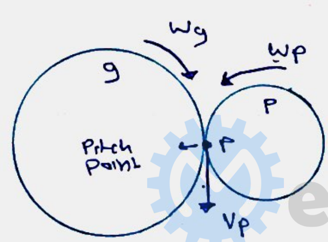
 → "External gear."

* Angular velocity Ratio:

V_p → tangential velocity or Pitch line velocity.

$(V_p)_p = W_p r_p \qquad (V_p)_g = W_g r_g$

$$W_p r_p = W_g r_g \rightarrow \frac{W_g}{W_p} = \frac{r_p}{r_g}$$



$$\boxed{m_v = \frac{W_g}{W_p} = \frac{r_p}{r_g}}$$

→ it could also be $\frac{W_p}{W_g}$, it depends on which one is the output but usually (P) is the input and (g) is the output.

Gear tooth nomenclature (9.3 From Book).

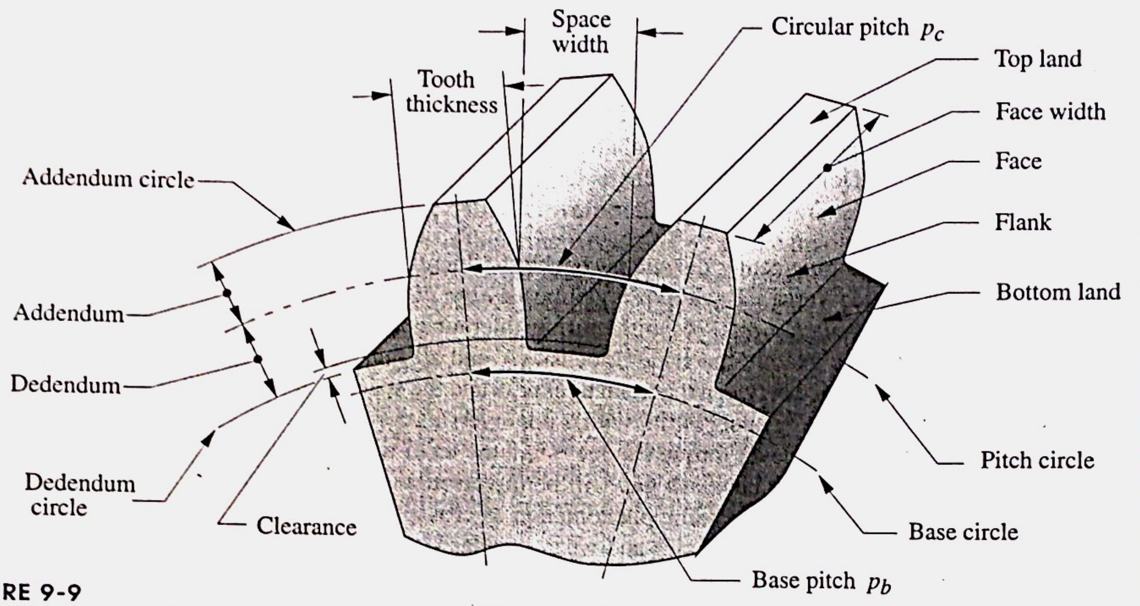


FIGURE 9-9

notes :-

→ Front view is called the Plane of Rotation.

→ on top view you have Top Land which is the upper surface of the tooth.

→ notice the curvature you have on the sides of each tooth, it'll be described later by a certain function.

→ All circles in a gear share the same center which is the center of Rotation.

Pitch Circles → Theoretical circles (not actual or physical) that are tangent to each at the pitch point when two gears are in mesh.

so; you can't locate a pitch circle using one gear by it's self, you have to have two gears in mesh or use certain calculations.

Pitch Point → point of contact between two pitch circles.

* Addendum circle → the outside circle which is in contact with the top land
- it is an actual (physical) circle which you can determine by having only one gear.
- The word Addendum came from (Add) ; we add sth to the pitch circle to reach the addendum circle.

* Addendum ( ) → Radial distance from pitch circle to Addendum circle.

$$a = r_{add} - r_{pitch}$$

* Dedundum Circle → the circle in contact with the bottom land, also called (Root Circle) - it's like the tooth has a root.

From Dedud. : we deduct sth from the Pitch circle to reach the dedundum circle.

* Dedundum(b) → The Radial distance between Pitch Circle & Addendum circle.

$$b = r_{pitch} - r_b$$

→ The tooth is located between the addendum circle & the dedendum circle.

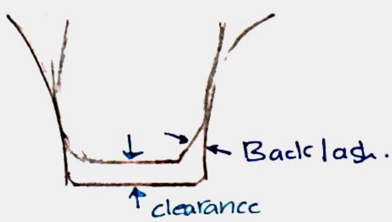
So: Addendum (a) + Dedendum (b) = whole depth of the tooth (h)

Clearance & Backlash:-

clearance → Radial distance by which the top land of the tooth doesn't contact the bottom of the other tooth while meshing (c → small c)

Backlash → tangential distance where no contact occurs measured along the Pitch circle.

→ it is not good in design to have high values of clearance & backlash, it increases noise & vibration.



→ There are standard values for clearance & backlash.

AGMA: American Gear Manufacturers Assosiacion.

↳ (Most popular assocaiation for standards of gears)

Face → The surface from Pitch circle to top land

Flank → The surface from Pitch circle to Bottom land

] → Pitch circle separates the face from the flank

Circular Pitch (P_c) : The Arc distance from one point on a tooth to the identical Point on the other tooth (not necessarily from center to center)

number of P_c's = number of teeth (N)

So: $P_c = \dfrac{\text{circumference of the Pitch circle}}{\text{number of teeth}}$

$$\boxed{P_c = \frac{\pi d_p}{N_p} = \frac{\pi d_g}{N_g}}$$

→ notice that P_c is equal for two gears in mesh.

we can relate the circular Pitch (P_c) to angular velocity Ratio (m_v), as follows:

$(P_c)_g = (P_c)_p$

$$\frac{\pi d_p}{N_p} = \frac{\pi d_g}{N_g} \rightarrow \frac{d_p}{d_g} = \frac{N_p}{N_G} \quad \text{BUT} \quad \frac{d_p}{d_g} = \frac{r_p}{r_g} = \overbrace{\frac{\omega_g}{\omega_p}}^{m_v}$$

So: $$\boxed{\frac{\omega_g}{\omega_p} = \frac{N_p}{N_g} = m_v}$$

→ This is the most common Ratio to use for m_v since it is valid for all types of gears.

* Gear Ratio (m_G) → $$\boxed{m_G = \frac{N_g}{N_p} = \frac{1}{m_v}}$$

* Tooth Thickness → Thickness of the tooth along the Pitch Circle.
* space width → The space between two teeth, that would fit the tooth from the other gear.
* Face width →

circular Pitch (P_c) = tooth thickness + space width.

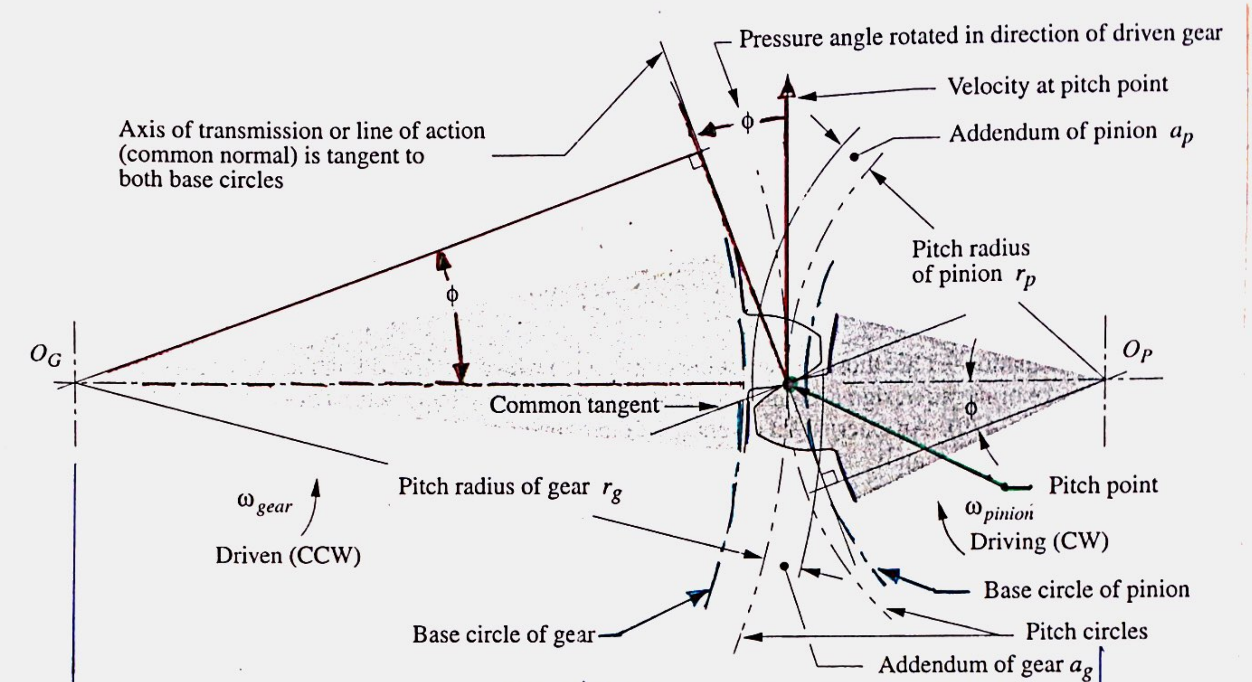


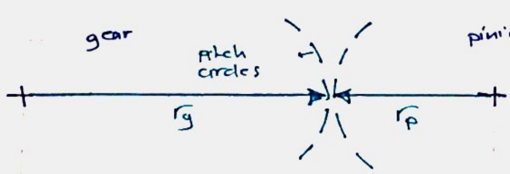
FIGURE 9-6

When you have two gears in mesh:

* center distance → the distance between the centers of the two gears.

center distance (C) = $r_g + r_p$

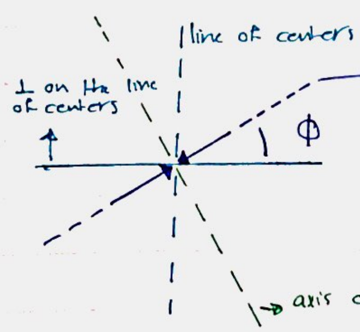
→ note: when we say r_p or r_g by default it means radius of the pitch circle of the gear or the pinion.



line of centers

↳ the line between the centers of the two gears

* Pressure Angle: (ϕ) → The angle between the line of action (common normal) and the line normal to the line of centers (tangent to the pitch circle)



→ line of action or pressure line or common normal or axis of transmission.

→ Forces are transmitted along this line & all points of contact are located on it.

↳ axis of slip

common tangent to the teeth

↳ perpendicular on the common normal.

ϕ is constant for gears unlike cams.

→ now, we can define the pitch point as → the Intersection of line of actions with the line of centers

→ The point of contact between two teeth represents a RP-Joint.

So to draw a pitch circle (graphical approach):
start from the center of rotation to the point of intersection between line of action and the common normal.

* Conjucate Action (Functmatal law of gearing) :- (9.2 From the Book)

→ Conjucate Action is satisfied when tooth profiles of two meshing gears produce a constant angular velocity Ratio.

→ Conjucate Action is an indication of the quality of Meshing.

$$\frac{\omega_{out}}{\omega_{in}} = \text{constant}$$

→ To satisfy this relation, the profile of the tooth must be designed in a certain shape.

→ For example, this kind of profiles doesn't satisfy the conjucate action.

Involute Profile → the profile of the tooth that satisfies the conjucate action conditions.

→ How is an involute profile generated ?
an involute curve must be drawn, and the side of the teeth must be apart of that curve.

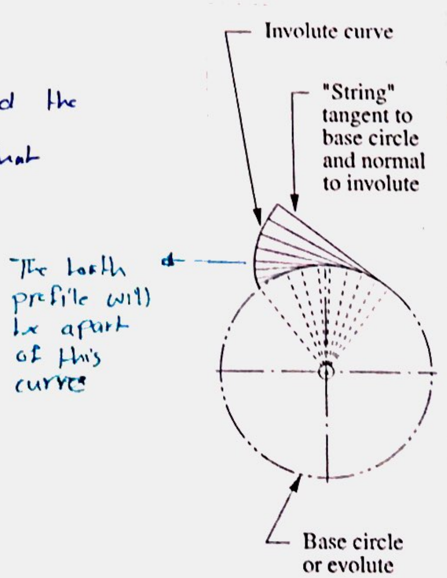


FIGURE 9-5

Base Circle → The location where the Involute curve starts.

The location of the Base circle could be:

1. Before the dedendum circle.
2. on the dedendum circle.

→ In these cases there is no problem in meshing since all the contact will happen on the Involute curve.

3. After the dedendum circle. ↳ look at Fig (9-6)

↳ In this case there is a non-Involute part in the teeth profile which causes a chance of Interference. (Problem !!)

Interference happens when the tooth tries to maintain the constant (mv) by adjusting it's shape which causes permanent failure on the teeth (Premature failure).

* Interference has two conditions:

1. a part of the tooth is non-Involute.
2. there is contact between the two teeth at that Part. (If there is no contact I have no problem).

* To eliminate the chance of Interference you can:

1. locate the base circle before or on the Dedendum circle.
2. Avoid contact
3. Undercutting → Removing a portion of the non-Involute part, which prevents contact, it weakens the tooth but it's better than Interference.

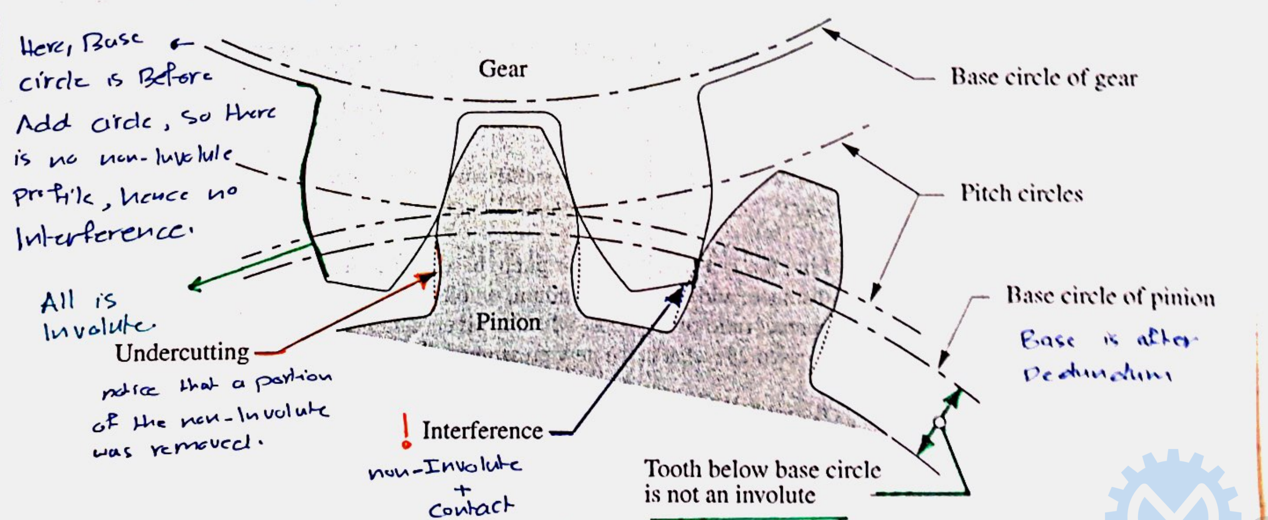
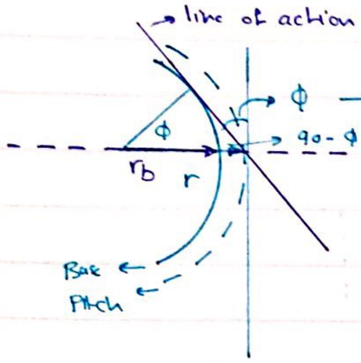


FIGURE 9-12

* Relation Between line of action & Base circles :

→ line of action is always tangent to both base circles.



ϕ → pressure angle (from geometry)

r_b → Base Radius

r → Pitch Radius

$$\boxed{r_{b_g} = r_{p_g} \cos\phi}$$

→ ϕ is the same for both pinion & gear.

↳ Important.

→ we have a term called (Base Pitch) - P_b

Base pitch is the same as circular Pitch but along the base circle instead of Pitch circle.

$d_b = d_p \cos\phi$

$$P_b = \frac{\pi d_b}{N} = \frac{\pi d_p \cos\phi}{N} = P_c \cos\phi \rightarrow \boxed{P_b = P_c \cos\phi}$$

(SIZE INDEX)

(S.I) System	(U.S) system
Module (m) → not meters !!	Diametral Pitch (P_d)
$m = \frac{d}{N}$ (the same for two gears in mesh)	$P_d = \frac{N}{d}$ → teeth / inch
→ According to AGMA standards:	→ standards:
Addendum $a = 1\ m$	$a = \frac{1}{P_d}$
Deddendum $b = 1.25\ m$	$b = \frac{1.25}{P_d}$
Clearance $c = 0.25\ m$	$c = \frac{0.25}{P_d}$
→ $c = b - a$	

Note !!

$P_d \neq \frac{1}{m}$ (since m & P_d each follows a different system of units & different standards you CAN'T ever use $P_d = \frac{1}{m}$) !

↳ m has no meaning in U.S system.

P_d " " " " S.I system.

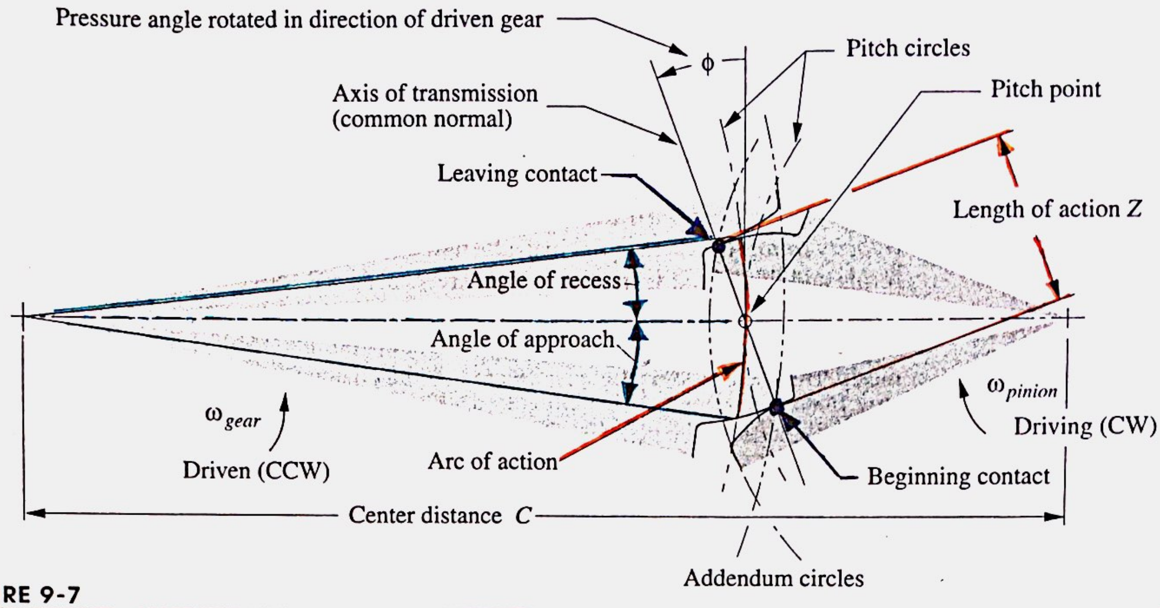


FIGURE 9-7

Beginning Contact → when the Flank (any point on it.) of the driver contacts the tip of the driven.
↳ (Initial contact)

Leaving contact → when the tip of the driver contacts the flank of the driven
↳ (final contact)

(Z)

* length of the line of action is from the initial point of contact to the final point of contact

Angle of approach → The angle between the line of centers & the line from the center of the gear to the point of intersection between the Pitch circle and the profile of the tooth at the Initial contact.

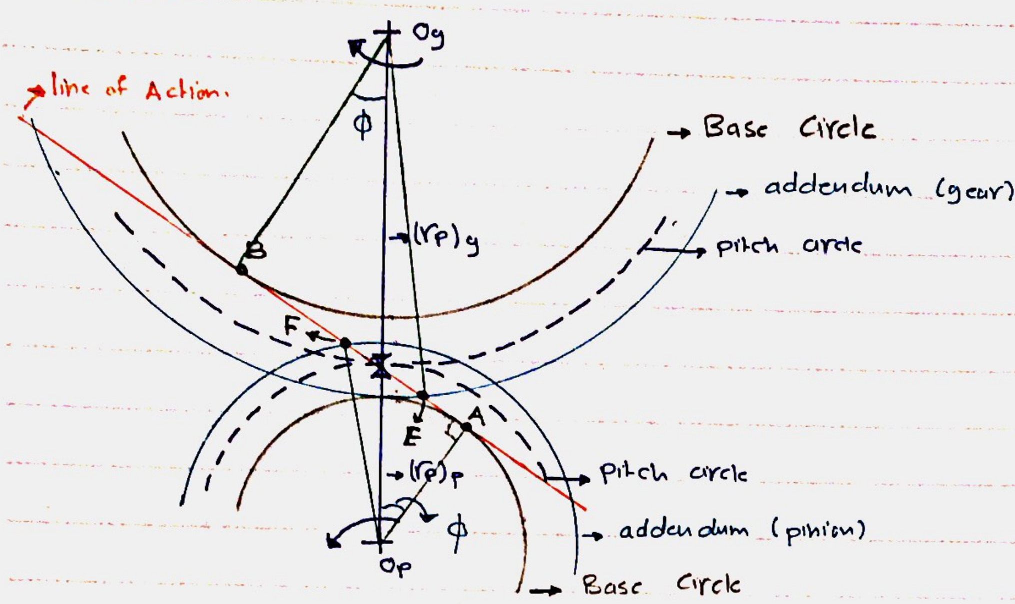
Arc of approach → Arc infront of the angle of approach on the Pitch circle.

Angle of Recess → The angle between the line of centers & the line from the center of the gear to the point of Intersection between the Pitch circle and the profile of the tooth at the final point of contact

Arc of Recess → The Arc infront of the angle of Recess on the pitch circle.

Arc of Recess + Arc of Approach = Arc of Action

↳ it's like a reflection of the line of action but as an arc.



A: First point of tangency (Base of pinion & line of action)

B: Final point of tangency (Base of gear & line of action)

E: First point of contact (Intersection between the a circle of driven & line of action)

F: Final point of contact (Intersection between a circle of the driver & line of action)

$O_pF \rightarrow r_a$ of pinion $\qquad O_gE \rightarrow r_a$ of gear.

$O_pA \rightarrow r_b$ of pinion $\qquad O_gB \rightarrow r_b$ of gear

$O_pP \rightarrow r_p$ of pinion $\qquad O_gP \rightarrow r_p$ of gear.

Recall $\rightarrow (r_b)_p = (r_p)_p \cos\phi \qquad (r_a)_p = r_p + a$

$(r_b)_g = (r_b)_g \cos\phi \qquad (r_a)_g = r_g + a$] → notice that the a is the same for both pinion and gear.

(Z) → length of the line of action

$z = EF = EP + PF$

now, we want to find a formula for (z) (it's important for the contact ratio $m_c = \frac{z}{P_b}$)

continued →

* To find z: (Derivation of the formula) → $z = EP + PF$

→ consider Triangle (O_gEB): (gear side)

$EP = EB - PB$

→ $EB = \sqrt{(O_gE)^2 - (O_gB)^2} = \sqrt{(r_a)_g^2 - (r_b)_g^2}$

→ Consider Triangle (O_gPB):

→ $PB = O_gP \sin\phi = r_g \sin\phi$

So: $EP = \sqrt{(r_a)_g^2 - (r_b)_g^2} - r_g \sin\phi$

now → Do the same procedure for the pinion side, as follows:

$PF = AF - AP$

→ consider Triangle (O_pAF)

$AF = \sqrt{(O_pF)^2 - (O_pA)^2} = \sqrt{(r_a)_p^2 - (r_b)_p^2}$

→ Consider Triangle (O_pAP)

$AP = r_p \sin\phi$

$z = z_1 + z_2 - z_3$, where:

$z_1 = \sqrt{(r_a)_p^2 - (r_b)_p^2} = \sqrt{(r_p + a)^2 - (r_p \cos\phi)^2}$

$z_2 = \sqrt{(r_a)_g^2 - (r_b)_g^2} = \sqrt{(r_g + a)^2 - (r_g \cos\phi)^2}$

$z_3 = r_p \sin\phi + r_g \sin\phi = (r_p + r_g) \sin\phi = C \sin\phi$

→ length of the line of action (z)

- Important -

Question! will Interference happen (in the last sketch)?

Remember → conditions for Interference: 1. Involute Part 2. contact at that part

Interference could happen at initial or final points of contact or both. So you have to check Both:

notice that the contact Point started at (E) while Base circle started at (A) which means that points after (A) are part of the Involute profile, same thing for points F & B.

So → no, no interference.

* In general, To know if there is Interference or not (Rule) :-

→ If the Initial point of tangency (A) was Before the Initial Point of contact (no interference at the start)

→ If the Final Point of tangency (B) was after the final point of contact (no Interference at the end)

Example :

Is there any Interference ?

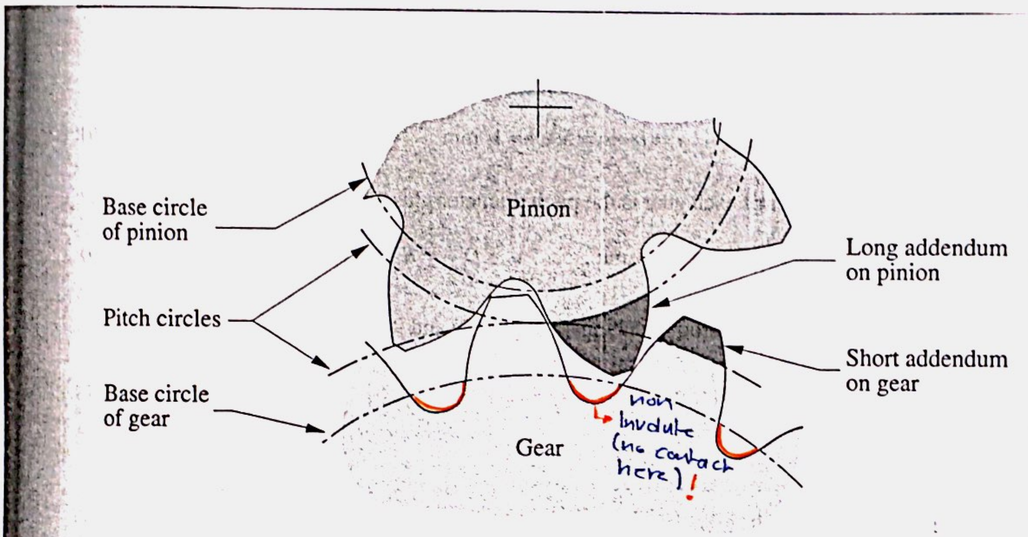


FIGURE 9-13

Answer :- no, even though there is a non-Involute part, no contact will happen at that part so conditions of Interference didn't happen

→ notice that clearance helped in this case.

See → Fig (13-16) Shigley's Book:

→ Interference at Both Initial & Final contact

→ Contact started before we reach the Base circle.

→ Base circle is done before end of Contact now, Recall → arc of action = arc of approach + arc of Recess.

Q → Is arc of action the same as the circular pitch ?

Answer: no, it could be smaller, equal or larger than the circular pitch

For design it's best to have the arc of action larger than the circular pitch

↳ From here, we define Contact Ratio:

* Contact Ratio $(m_c) = \frac{a}{P_c}$ $\begin{matrix} \to \text{Arc of action.} \\ \to \text{Circular Pitch} \end{matrix}$

* cases of (m_c) →

$m_c > 1$	(1.3, 1.4, ...)	Good Design	$a > P_c$
$m_c = 1$	→	Critical	$a = P_c$
$m_c < 1$	→	Bad Design.	$a < P_c$

* we can also find m_c using (the avg no. of teeth in contact)

↳ (weighted avg)

ex:

10 sec → total time.

9.0 sec → 1 Pair in contact

1 sec → 2 Pairs in contact

$$m_c = \frac{1(9.0) + 2(1)}{10} = 1.1$$

* Because it's Better to deal with linear distances, we can express (m_c) as follows:

$$m_c = \frac{Z}{P_b} = \frac{Z}{P_c \cos\phi}$$

→ length of the line of action.

→ Base Pitch.

So → $\boxed{m_c = \frac{a}{P_c} = \frac{Z}{P_b} = \frac{Z}{P_c \cos\phi}}$ → "CONTACT RATIO"

Changing Distance center.

→ Due to some errors while designing the pair of gears we will have:

* Theoritical center Distance → The STANDARD center distance that I want to achieve between the centers of the two gears.

* Actual center Distance → The REAL center distance you end up having between the centers of the gears, which is usually increased by Δc
  Δc (change in center distance)

* Due to the change in center distance, a group of other parameters will change → we are supposed to know how to find each of them after this change occurs
  ↳ look at the following page to know how

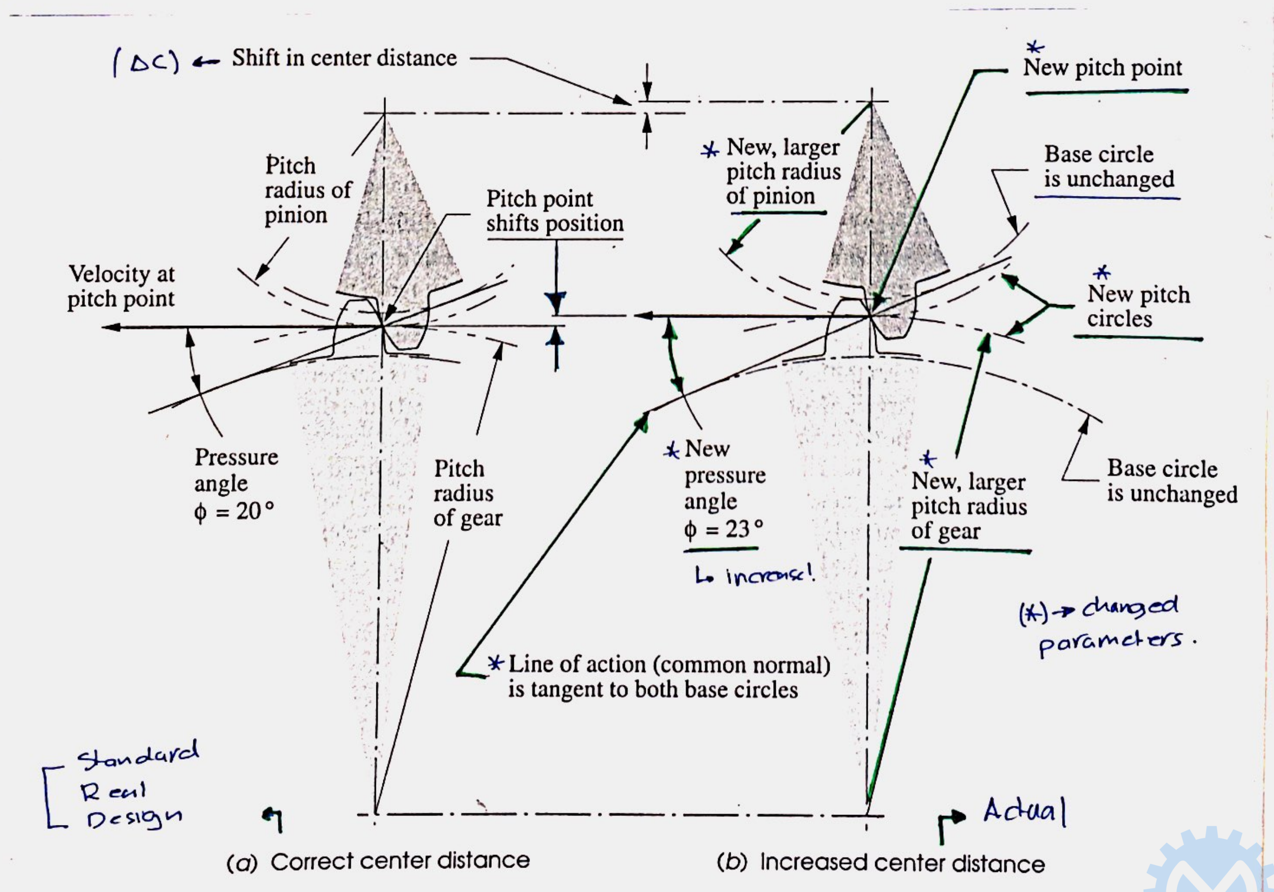


FIGURE 9-8

* The Following Table represents the changed & unchanged parameters in Response to the change in center Distance.

unchanged	changed
N_p & N_g	Pitch Circles
Base Circles	Module ($\frac{d}{N}$ → changed)
Addendum circle (r_a)	center distance (c)
Dedendum circle (r_b)	pressure angle (ϕ)
Velocity Ratio (ω)	length (z) & Direction of line of action (z_3)
Gear Ratio (m_G)	Contact Ratio ($\frac{z}{P_b}$ z → changed, P_b → the same)
z_1 & z_2	angle/arc of approach & Recess.
	Initial & Final Points of contact.

↳ note that all physical parameters (actual) Remains unchanged while theoretical does.

* For calculations:

$$\boxed{c' = c + \Delta c}$$

c' → actual distance Δc → change

c → standard

$$\boxed{\begin{aligned} z &= z_1 + z_2 - z_3 \\ z' &= z_1 + z_2 - z_3' \end{aligned}}$$

note → z_1 & z_2 Remains unchanged because they depend on unchanged parameters (r_a & r_p) while z_3 depends on Pitch Circle.

$$\boxed{\frac{d_p'}{d_g'} = \frac{d_p}{d_g}}$$

→ $m_v = \frac{\omega_{out}}{\omega_{in}} = \frac{d_p}{d_g} = \frac{N_p}{N_g}$ = constant

↳ unchanged

↳ even if d_p, d_g change, the Ratio between them remains constant

$$\boxed{r \cos\phi = r' \cos\phi'}$$

→ r_b : unchanged , $r_b = r\cos\phi = r'\cos\phi'$

↳ base

Solve Example (9-1) ! → very Important

GEAR TRAINS

→ Gear Trains are used to achive velocity difference (change in speed), but most importantly to achive Torque Magnification, as Follows:

(Stages) of a gear train is the number of gears in mesh in that gear train

→ For All gears except worm Type:

Power = Constant

$P_{in} = P_{out}$

$T_{in} W_{in} = T_{out} W_{out}$

$$\underbrace{\frac{T_{out}}{T_{in}}}_{\text{Torque Ratio}} = \underbrace{\frac{W_{in}}{W_{out}}}_{\frac{1}{m_v}} = \frac{d_{out}}{d_{in}}$$

↳ According to these Ratios, If I want to achieve Higher Torque Ratio (Torque Magnification), the out put gear should be the smaller one

→ $\frac{d_{out}}{d_{in}}$ → small value means bigger Ratio.

→ Gear Trains are mainly two types :- 1- Fixed Axis
2- Moving Axis.

[I] Fixed Axis Gear

→ Also called 1 D.O.F or Ordinary Gear Trains. (1 input → 1 output)

→ The Motion of ALL gears is only Rotation.

→ Three Types:

1. simple → no. of gears = no. of shafts Fig (9-28)

2. Compound → at least one shaft Holds more than one gear Fig (9-29)

↳ it needs less space than simple.

3. Revertal compound → the Input shaft is on the same axis as the out put shaft. Fig (9-31)

↳ it needs less space than both simple & compound.

Application of Revertal : Gear Box in cars.

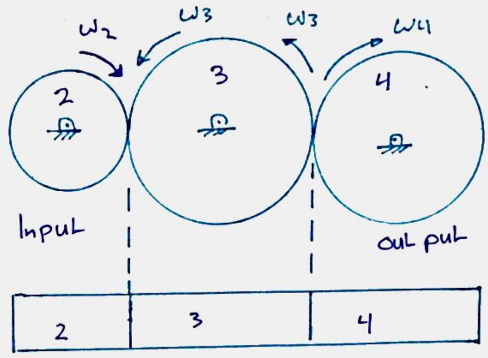
Example (simple gear Train) :-

Find → Angular velocity Ratio.

Solution:

→ note that: no. of gears = no. of shafts = $\boxed{3}$

→ specify the direction of rotation for each gear according to ω_2.



- Top view - Thicknesses.

$m_v = \frac{\omega_{out}}{\omega_{in}} = \frac{\omega_4}{\omega_2}$

step by step:

$(2,3)$: $\frac{\omega_3}{\omega_2} = \underset{\text{opposite direction}}{-}\frac{N_2}{N_3} \longrightarrow \omega_3 = -\frac{N_2}{N_3}\omega_2$ — $\boxed{1}$

$(3,4)$: $\frac{\omega_4}{\omega_3} = -\frac{N_3}{N_4}$ — $\boxed{2}$

Devide $\boxed{2}$ over $\boxed{1}$ → $\omega_4 = \left(-\frac{N_3}{N_4}\right)\left(-\frac{N_2}{N_3}\right)\omega_2$

$\frac{\omega_4}{\omega_2} = \frac{N_2}{N_4} = m_v$ #

Gear (3) is called Idler gear, it has no effect on Torque or velocity Ratio, it's only purpose is to change the direction of Rotation, it works as a drier & driven.

↳ That was just to understand, But we won't be using this procedure every time we want to find mv.

→ we will Define a new Term (Train value) - e

→ same as angular velocity ratio but for gear trains.

$$\text{Train Value} = e = \frac{\omega_L}{\omega_F} = \frac{\omega \text{ of last gear in gear train (output)}}{\omega \text{ of first gear in gear train (input)}}$$

$$= (\pm)\,\frac{\text{Product of driving teeth no.}}{\text{Product of drivin teeth no.}}$$

→ This applies on all ordinary gears.

+ → If First & Last gears Rotates with the same direction.

- → If First & Last gears Rotates with opposite direction.

For the previous example:

$e = \frac{\omega_4}{\omega_2} = \frac{(N_2)(\cancel{N_3})}{(\cancel{N_3})(N_4)} = \frac{N_2}{N_4}$ # again gear (3) cancelled out because it is Idler gear.

Example (simple gear train)

Find Train value (e)

Solution:

input (2)] → notice only 1 input
output (6)] 1 output → simple (1 DOF)

$$e = \frac{\omega_L}{\omega_F} = +\frac{\omega_6}{\omega_2} = \frac{N_2 * N_3 * N_4 * N_5}{N_3 * N_4 * N_5 * N_6}$$

$e = \frac{N_2}{N_6}$ → notice it's the same for mv between 2 & 6 but different sign.

*you can also look at (e) as the product of (mv) for each stage with the other, as follows:

stages = no. of gears in mesh = 4

→ (2 & 3) - (3 & 4) - (4 & 5) - (5 & 6)

$$e = \left(\frac{\omega_3}{\omega_2}\right)\left(\frac{\omega_4}{\omega_3}\right)\left(\frac{\omega_5}{\omega_4}\right)\left(\frac{\omega_6}{\omega_5}\right) = \frac{\omega_6}{\omega_2}$$ # same Result

note → no. of gears = no. of stages −1 (For simple)

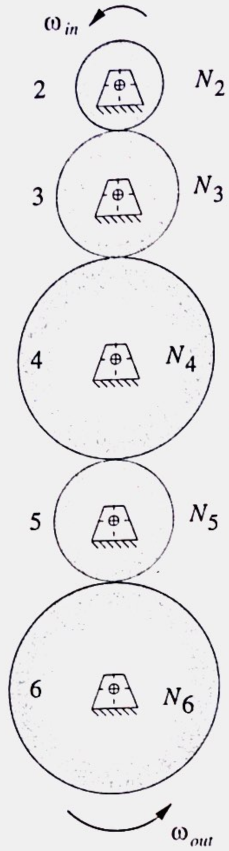


FIGURE 9-28

Example (compound)

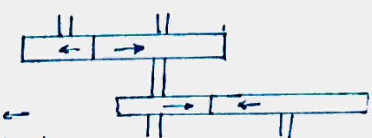
Find Train value

solution:

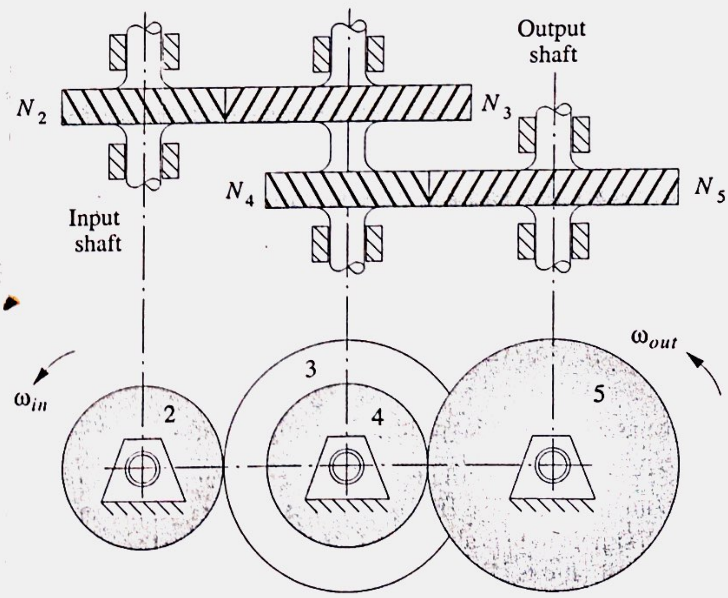
4 gears, 2 stages because 3 & 4 are on the same shaft.

$$e = (+)\frac{\omega_5}{\omega_2} = (+)\frac{N_2}{N_3}\ \frac{N_4}{N_5}$$

1st stage ↩ ↳ 2nd stage

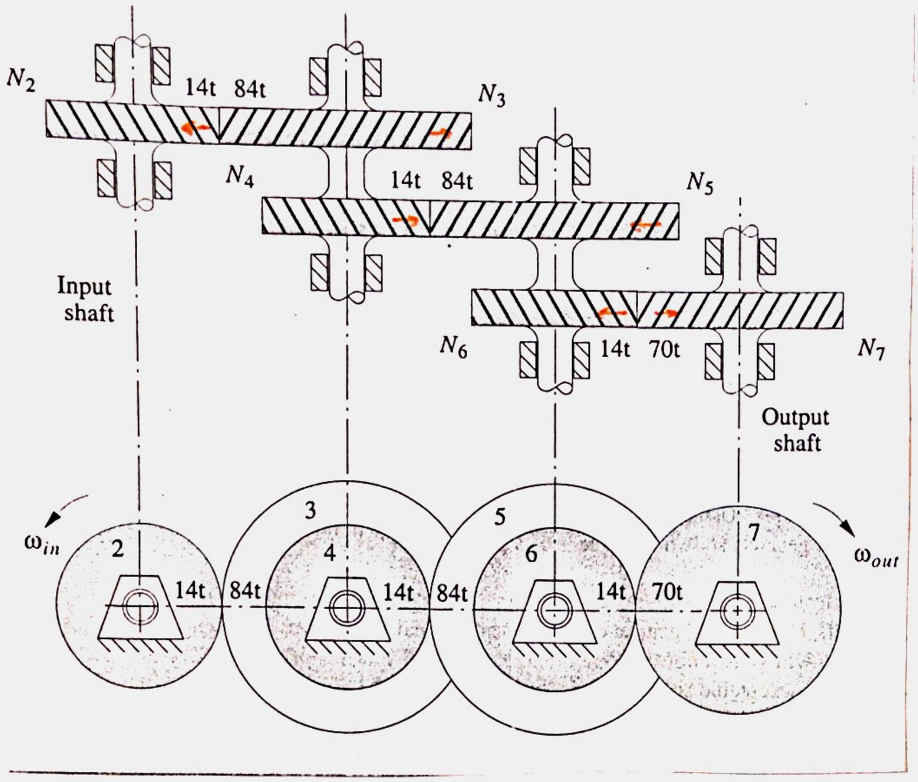


you can use this method to find directions and specify the sign. ←



9-29

note → I would have needed 4 shafts & more space if I used simple gear train to connect the 4 gears.



*Example (compound)

Find Train value.

6- Gears

3- Stages

$$e = -\frac{\omega_7}{\omega_2}$$

$$= \frac{(N_2)(N_4)(N_6)}{(N_3)(N_5)(N_7)}$$

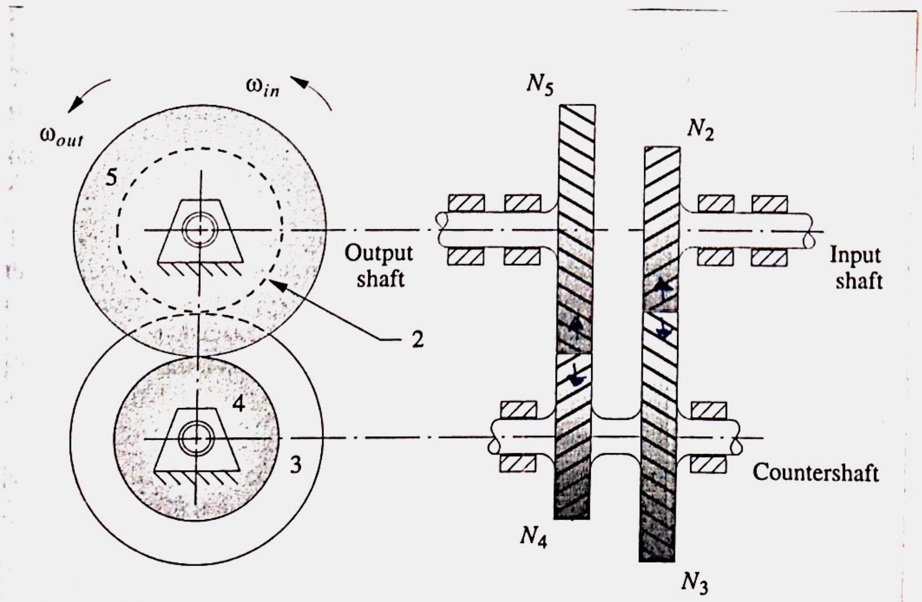


FIGURE 9-31

→ Example

Reverted compound

4- Gears

2- Stages

$$e = +\frac{\omega_5}{\omega_2}$$

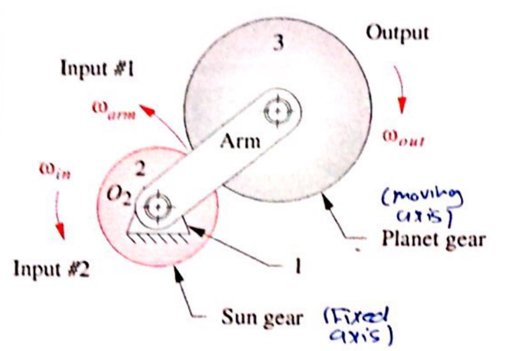
$$= \frac{N_4 N_2}{N_3 N_5}$$

[2] Moving Axis

- Also called (planetary) or (Epeycyclic) or (2 DOF)
- At least one of the gears is in planetary Motion.
- (2 DOF) → this means that the gear train (all) as a Mechanism has 2 DOF . 2 input → 1 output
- Application: Automatic gear.
- Planetary Motion: Rotation about a center and Rotation about the gear's axis (like planets with the sun)

- notice that there is 2 inputs:
  one for the sun & one for planet.

- Arm or (Planet carrier): it is used to force the planet gear to rotate about the sun, BUT at the same time it allows it to rotate about it's self. (Bearing)



(b) Planetary or epicyclic gearset

note → planet gears CAN'T be inputs or outputs because they have Moving axis.

* Train Value for Planetary Gear Trains :

$$\boxed{e = \frac{\omega_L - \omega_A}{\omega_F - \omega_A} = \pm \frac{\text{Product of no. of teeth of Drivers}}{\text{Product of no. of teeth of Drivens}}}$$

notice we look (ω_L, ω_F) w.r.t the Motion of the arm (Relative Motion).

$\pm$ → Imaginary step explained in the example below.

Before you apply the train value equation on any planetary gear train, you HAVE to check 3 conditions:

1. F, L Fixed axes
2. F, L are on the same plane (they have parallel shafts)
3. F, L should mesh with a planet gear (not necessarily the same planet).

Example (Planetary gear train): (ex 9-5)

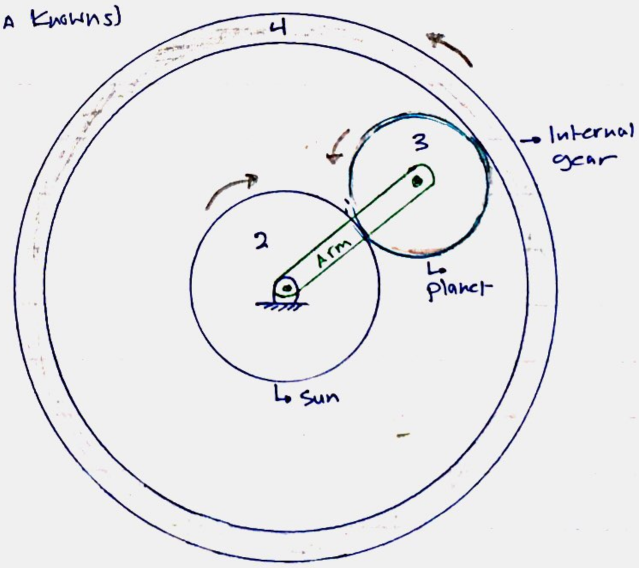
Inputs → to sun and arm (ω_2 & ω_A knowns)

output → 4

the 3 conditions apply ✓

$$e = \frac{\omega_L - \omega_A}{\omega_F - \omega_A} = \frac{\omega_4 - \omega_A}{\omega_2 - \omega_A}$$

$$= - \frac{N_2 N_3}{N_3 N_4}$$



to determine the sign:

Imagine if the Arm was fixed and see whether gear 2 & 4 will have similar or different directions.

similar → (+) different → (−)

In this example different so (−)

note → the shafts are concentric but they have clearance between them (independent Motion)

Problem (25)

5-gears

All conditions Apply

$$e = \frac{W_6 - W_A}{W_2 - W_A} = \frac{N_2 N_3 N_5}{N_4 N_5 N_6} = \frac{N_2 N_3}{N_4 N_6}$$

sign → (+) - same direction

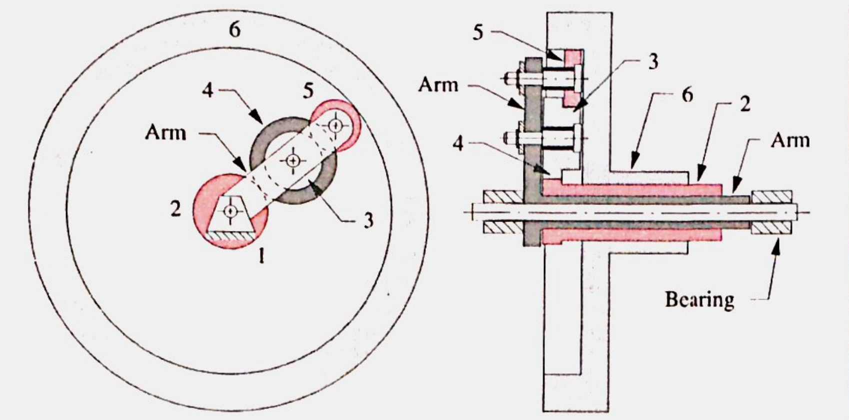


FIGURE P9-1

Problem (26)

5 gears + Arm

5,4 → planets

2,3,6 → Fixed

If you apply the conditions you'll notice that (2) is not in contact with a planet gear.

so: I have 2 trains:

train (1) (2,3) → ordinary

$$W_3 = -W_2 \frac{N_2}{N_3} = \checkmark$$

train (2): 3,4,5,6 & Arm

$$e = \frac{W_6 - W_A}{W_3 - W_A} = + \frac{N_3 N_5}{N_4 N_6} = \checkmark$$

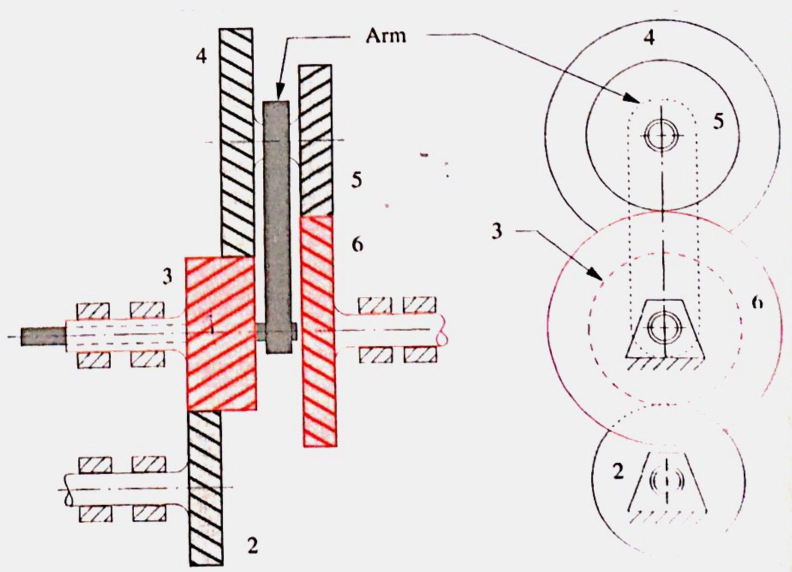


FIGURE P9-2