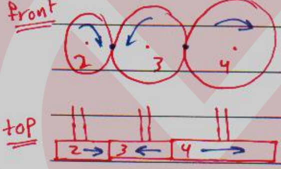
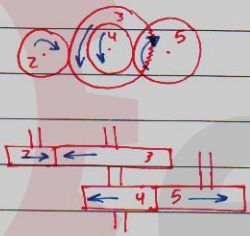
Gear Trains

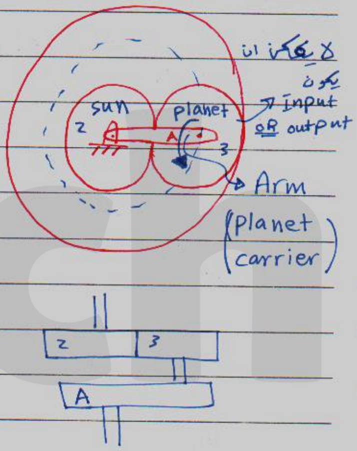
Basic, Fixed Axes or 1.d.o.f
{1in → 1out}

- simple
- 2 stage
- compound





Planetary or Eplicyclic or moving axes or 2.d.o.f
{2 in → 1out}



*** Overal velocity ratio OR Train value :-**

ang. velocity (rad/s)

rotation speed (rpm) (rps)

$$m_V = \frac{\omega_L}{\omega_F} = \frac{\omega_{last}}{\omega_{first}} = \frac{n_L}{n_F}$$

L: last driven.
F: First driver.

$$\frac{\omega_L}{\omega_F} = \pm \frac{\text{Porduct of teeth of driver}}{\text{Porduct of teeth of driven}}$$

Ex:-

$$\frac{\omega_6}{\omega_2} = (+)\frac{\underset{first}{N_2} \cdot \cancel{N_3} \cdot \cancel{N_4} \cdot \cancel{N_5}}{\cancel{N_3} \cdot \cancel{N_4} \cdot \cancel{N_5} \cdot \underset{last}{N_6}}$$

ω_L , ω_F → same direction → (+)

ω_L , ω_F → opossite direction → (−)

$$m_V = m_{V_1} \times m_{V_2} \times m_{V_3} \times m_{V_4}$$

1, 2, 3, 4 → no. of stage.

$$= \frac{\omega_3}{\omega_2} \times \frac{\omega_4}{\omega_3} \times \frac{\omega_5}{\omega_4} \times \frac{\omega_6}{\omega_5} = \frac{\omega_6}{\omega_2}$$

$$= \left(\frac{-N_2}{N_3}\right)\left(\frac{-N_3}{N_4}\right)\left(\frac{-N_4}{N_5}\right)\left(\frac{-N_5}{N_6}\right)$$

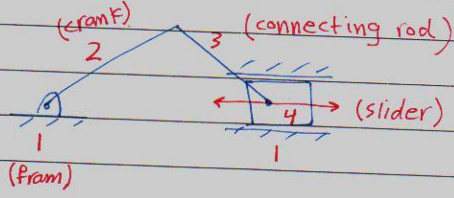
Rolling without sliping → 1. D.O.F

Rolling with sliping → 2. D.O.F

1. D.O.F → Full Joint.

2. D.O.F → half Joint.

Ex:-



* Degree of Freedom (D.O.F) :-

Def :-

Number of independent coordinates (minimum number) required to fully describe the motion of an object.

* curvilinear لها one D.O.F.

* اكبر عدد ممكن ل D.O.F في 3D هو 6.

* اكبر عدد ممكن ل D.O.F لجسم في 2-D هو 3.

* Elastic Body has infinity Degree of freedom.

* Joints [Pairs or Kinamatic pairs or pairing elemants] :-

Elements by which two or more links are joined together in such a way that motion is allowed.

⇒ Classification of Joints :-

[1] Based on type of contact.

a. lower Pair : Joints with surface contact :

1 D.O.F
- Revolute (R) Joint Rot.
- prismatic (P) Joint slide
- Helical (H) Joint

* Joint Order = No. of link connected − 1

* kinematic Chain :-

A system of consisting of a number of links connected by Joints, we have two type:-

1) open chain: at least one link connected with one joint.

2) closed chain :- each link connected at least ~~with~~ two Joint with

* لزيادة D.O.F نقوم بتحويل واحدة من Full Joint الى half Joint ⎫ بمقدار 1
* لتقليل D.O.F نقوم بحذف link ⎭

Mobility (D.O.F) of mechanisim :-

$$\text{Mobility}(M) = 3(\underbrace{L}_{\#\text{of links}}-1) - 2\underbrace{J_1}_{\substack{\#\text{of Full Joint} \\ (1\,D.O.F)}} - \underbrace{J_2}_{\substack{\#\text{of half Joint} \\ (2\,D.O.F)}}$$

$M < 0$ → its not mechansi → structure (undeter.)

$M = 0$ → No motion. (structure)

4-Bar Mechanisim :-

EX :-

$L = 4$

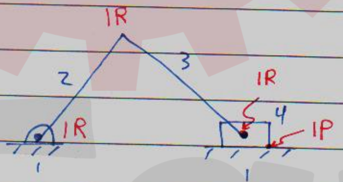
$\left.\begin{matrix} 4R \\ 0P \end{matrix}\right\} \Rightarrow J_1 = 4$

$0\ RP \} \Rightarrow J_2 = 0$

$M = 3(4-1) - 2(4) = 1$

slider - crank Mechanisime :-

EX :-



$L = 4$

$\left.\begin{matrix} 3R \\ 1P \end{matrix}\right\} J_1 = 4$

$0\ RP \} J_2 = 0$

$\therefore M = 3(4-1) - 2(4)$

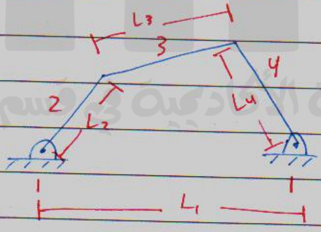
$M = 1$

4 - Bar Mechanisime :-

S : length of shortest link

L : Length of longest Link

P, Q : Intermediat length



* order to form a 4-Bar length $(L < (S + P + Q))$

4-Bar

Grashof :-

at least one link is capable of making Full Rotation.

$(S+L) \leq (P+Q)$

non-Grashof

$(S+L) > (P+Q)$

* Grashof types (condition) :-

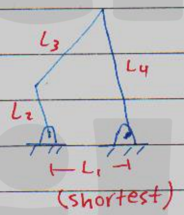
* يعني اذا L_2 : crank بتكون L_4 : Rocker والعكس صحيح

① Crank - Rocker :-

* $(S+L) < (P+Q)$
* the shortest link is either link pivoted to ground ($S = L_2$ or $S = L_4$) which will make Full Rotation.

② Double crank (Drag link) :-

* $(S+L) < (P+Q)$.
* shortest link is the ground.
* links 2 & 4 : full Rotate.



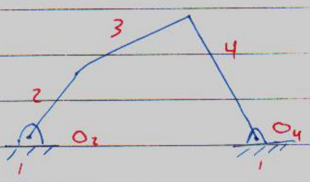
③ Double Rocker :-

* $(S+L) < (P+Q)$
* shortest link is 3 which will make Full Rotation.
* which link shortest (2,4) will oscillation (Rockers)

④ special case Grashof (change point) :-

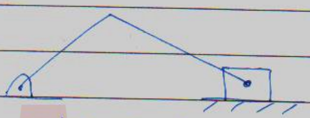
$S + L = P + Q$

اذا تساوت راح يكون واحد من الاحتمالين { Crank - Rocker / double - crank

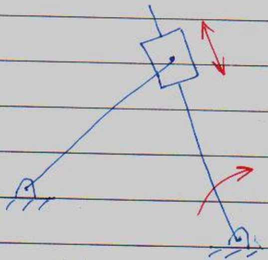


collinear.

* Inversions of Mechanisim :-



slider crank.



sliding - contact

(inversions of slider)

Homework :-

2-8 , 2-15, 2-21

* Kinematic Analysis of Mechanisims:-

1) displacement (position).
2) Velocity .
3) Acceleration.

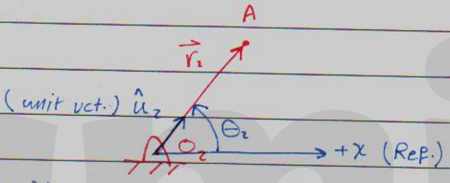
$\{ \hat{u} = \hat{u}_2 = u_{\theta_2} \}$

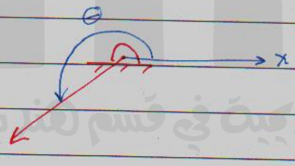
vectors :-

$\overrightarrow{O_2A} = \overrightarrow{r_2}$

$= r_2 \, u_{\theta_2}$

$= r_2 (\cos\theta_2 \, \hat{i} + \sin\theta_2 \, \hat{j})$



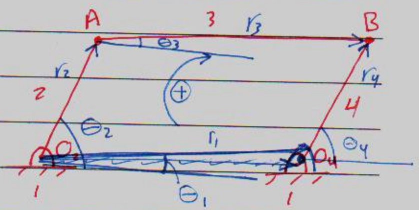


Ex:-

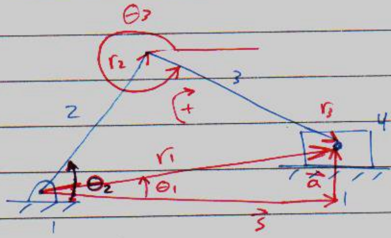
loop closure Equation (vector loop eqn)

$\overrightarrow{O_2A} + \overrightarrow{AB} + \overrightarrow{BO_4} + \overrightarrow{O_4O_2} = \overrightarrow{zero}$

$\vec{r_2} + \vec{r_3} - \vec{r_4} - \vec{r_1} = \vec{0}$



$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = \vec{0}$



Ex:-

$\vec{r_2} + \vec{r_3} - \vec{r_1} = \vec{0}$

$\underset{f}{r_2} \underset{v}{\hat{u}_{\theta_2}} + \underset{f}{r_3} \underset{v}{\hat{u}_{\theta_3}} - \underset{v}{r_1} \underset{v}{u_{\theta_1}} = \vec{0}$

$\vec{r_1} = \vec{s} + \vec{a}$

$r_1 \hat{u}_{\theta_1} = \underset{v}{s} \underset{f}{\hat{u}_0} + \underset{f}{a} \underset{f}{\hat{u}_{90}}$

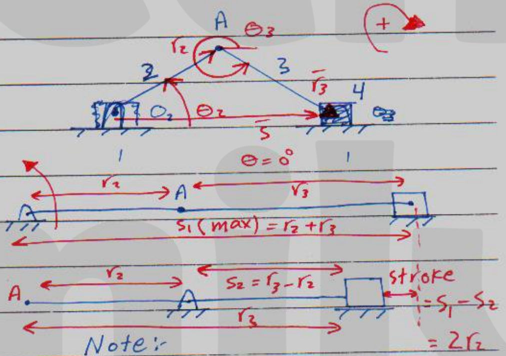
* ايالك ان تعوض قيمة (Input) في بداية الحل
تعوض فقط المعطيات الثابتة (fixed).

$\bar{r_2} + \bar{r_3} - \bar{a} - \bar{s} = \bar{0}$

$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - a \hat{u}_{90} - s \hat{u}_0 = 0$

* Position Analysis: Slider-crank Mech.

$\boxed{1}$ at same level



L.C.E:-

$\vec{r_2} + \vec{r_3} - \vec{s} = \vec{0}$

$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - s \hat{u}_0 = \vec{0}$

Knowns:-

$r_2, r_3, \theta_1 = 0, \theta_2$

Find:- s, θ_3

x:

$r_2 \cos\theta_2 + r_3 \cos\theta_3 + s = 0$ ---- (1)

y:-

$r_2 \sin\theta_2 + r_3 \sin\theta_3 = 0$ ---- (2)

from (2): $\sin\theta_3 = \frac{-r_2}{r_3} \sin\theta_2$

Note:-

Multiple solution:

اذا ظهر اكثر من جواب يمكن الاستعانة بالرسم لأخذ القيمة المطلوبة.

???

$\frac{ds}{d\theta_2} = 0 \rightarrow s_1$ (Max), s_2 (min)

2

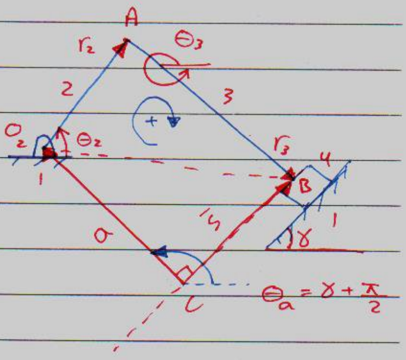
١) لبيت على خط واحد
٢) Roker مائل

- Knowns:-

Fixed: r_2, r_3, γ, a Input: θ_2

- Find:-

s, θ_3



L.C.E:-

$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3}^{?} - \overset{?}{s} u_\gamma + a \hat{u}_{\gamma+90} = 0$

solution:-

Elimination of θ_3:

$r_3 \hat{u}_{\theta_3} = s \hat{u}_\gamma - r_2 \hat{u}_{\theta_2} - a \hat{u}_{\gamma+90}$

vector
Dot each side by itself

$r_3 \hat{u}_{\theta_3} \cdot r_3 \hat{u}_{\theta_3} = (s \hat{u}_\gamma - r_2 \hat{u}_{\theta_2} - a \hat{u}_{\gamma+90}) \cdot (s \hat{u}_\gamma - r_2 \hat{u}_{\theta_2} - a \hat{u}_{\gamma+90})$

$r_3^2 = s^2 + r_2^2 + a^2 \underbrace{- 2 s r_2 u_{\theta_2} \cdot \hat{u}_\gamma}_{-2 s r_2 [\cos\theta_2 \cos\gamma + \sin\theta_2 \sin\gamma]} \cancel{- 2 s a \hat{u}_\gamma \cdot \hat{u}_{\gamma+90}}^{\,0} \underbrace{+ 2 r_2 a \hat{u}_{\theta_2} \cdot u_{\gamma+90}}_{2 r_2 a \sin(\theta_2 - \gamma)}$

$-2 s r_2 [\underbrace{\cos\theta_2 \cos\gamma + \sin\theta_2 \sin\gamma}_{\cos(\theta_2 - \gamma)}]$ $2 r_2 a \sin(\theta_2 - \gamma)$

$s^2 - 2 s r_2 \cos(\theta_2 - \gamma) + r_2^2 - r_3^2 + a^2 + 2 r_2 a \sin(\theta_2 - \gamma) = 0$

$A s^2 + B s + C = 0$

$A = 1$

$B = -2 r_2 \cos(\theta_2 - \gamma)$

$C = a^2 + r_2^2 - r_3^2 + 2 r_2 a \sin(\theta_2 - \gamma)$

$s_{1,2} = \dfrac{-2B \pm \sqrt{B^2 - 4AC}}{2A}$

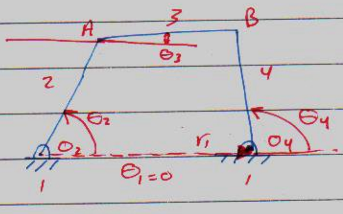
H.W # 1 :-

4-6, 4-7 , Row (a) Table P4-1

Due Thursday 20/6

Suggested problem : 4-10, 12, 18 (f)
a

4-Bar :-



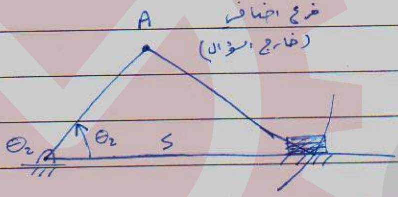
known:

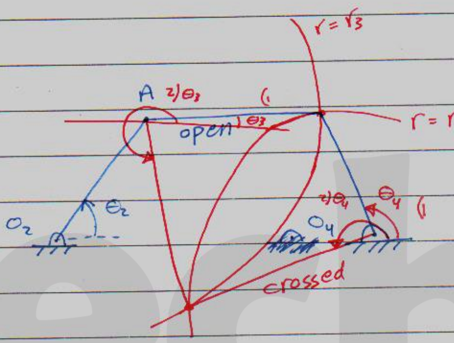
$r_1, r_2, r_3, \theta_1, r_4$ + input θ_2

Find : θ_3, θ_4

solu → Graphical
→ Analytical
→ Numereeal

Graphical :-





Analytical :-

L.C.E :-

$$r_2\,\hat{u}_{\theta_2} + r_3\,\hat{u}_{\theta_3} - r_1\,\hat{u}_0 - r_4\,\hat{u}_{\theta_4} = 0 \quad ---(1)$$

✓ ✓ ✓ ? ✓ ✓ ✓ ?

$$X:\ r_2\cos\theta_2 + r_3\cos\theta_3 - r_4\cos\theta_4 - r_1 = 0$$

$$Y:\ r_2\sin\theta_2 + r_3\sin\theta_3 - r_4\sin\theta_4 = 0$$

To eliminate θ_3 for (1)

$$r_3\,\hat{u}_{\theta_3} = r_1\,\hat{u}_0 - r_2\,\hat{u}_{\theta_2} + r_4\,\hat{u}_{\theta_4}$$

Dot each side by itself

$$r_3^2 = r_1^2 + r_2^2 + r_4^2 - 2r_1r_2\underbrace{\hat{u}_{\theta_2}\cdot\hat{u}_0}_{\cos\theta_2} + 2r_1r_4\underbrace{\hat{u}_{\theta_4}\cdot\hat{u}_0}_{\cos\theta_4} - 2r_2r_4\underbrace{\hat{u}_{\theta_4}\cdot\hat{u}_{\theta_2}}_{\cos\theta_4\cos\theta_2+\sin\theta_4\sin\theta_2}$$

$\cos(\theta_4-\theta_2)$ ✓

⋮

$$A\cos\theta_4 + B\sin\theta_4 + C = 0$$

, $A = 2r_1r_4 - 2r_2r_4\cos\theta_2$

$B = -2r_2r_4\sin\theta_2$

$C = r_1^2 + r_2^2 - r_3^2 + r_4^2 - 2r_1r_2\cos\theta_2$

$\tan \frac{\theta_4}{2} = X$

$\Rightarrow \cos\theta_4 = \frac{1-X^2}{1+X^2}$

$\Rightarrow \sin\theta_4 = \frac{2X}{1+X^2}$

Now,

$$A\left(\frac{1-X^2}{1+X^2}\right) + B\left(\frac{2X}{1+X^2}\right) + C = 0 \quad \Big\} * (1+X^2)$$

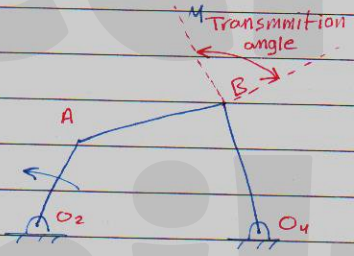
$$\underbrace{(C-A)}_{a}X^2 + \underbrace{2B}_{b}X + \underbrace{(A+C)}_{c} = 0 \xrightarrow{\text{similar}} aX^2 + bX + C = 0$$

$\Rightarrow$ we will find $X_{1,2} = \checkmark$

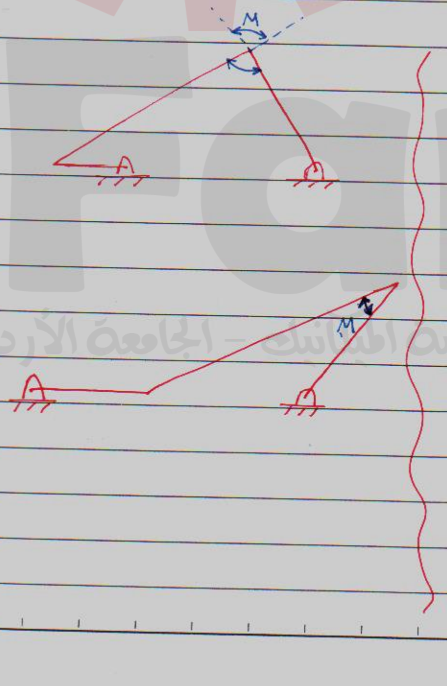
$\Rightarrow$ Now, $\theta_4 = 2\tan^{-1}(X_{1,2})$

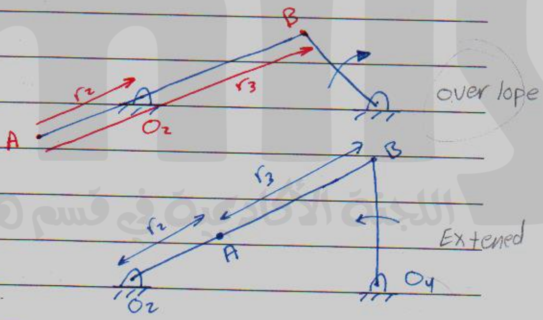
$\Rightarrow$ Now, find $\theta_{3(1,2)}$ from equation (x, y componant).

* Limit Position :-

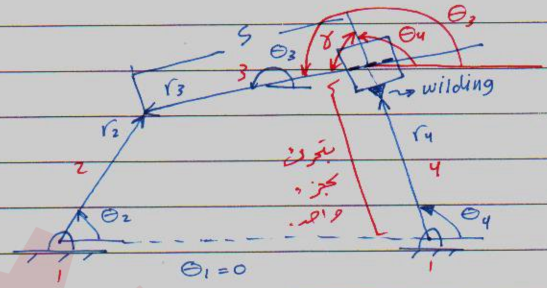


Ex: {crank-Rocker}



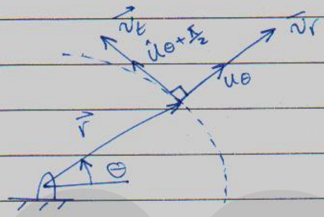


θ_3, S, θ_4



$\theta_3 = \theta_4 + \gamma$

* Velocity Analysis :-



$\vec{v} = \frac{d\bar{r}}{dt}$

$\vec{r} = r\hat{u}_\theta$

$\vec{v} = \frac{d\bar{r}}{dt} = \frac{d}{dt}(r\hat{u}_\theta) = \underbrace{\frac{dr}{dt}}_{\dot{r}}\hat{u}_\theta + r\frac{d\hat{u}_\theta}{dt}$

$\hat{u}_\theta = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

$\frac{d\hat{u}_\theta}{dt} = -\sin\theta\underbrace{\frac{d\theta}{dt}}_{\dot{\theta}=w}\hat{i} + \cos\theta\underbrace{\frac{d\theta}{dt}}_{\dot{\theta}=w}\hat{j}$

$= \dot{\theta}\underbrace{(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})}_{u_{\theta+90}}$

$\hat{u}_{\theta+90} = \cos(\theta+90)\,\hat{i} + \sin(\theta+90)\,\hat{j}$

$= -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

$\therefore \vec{v} = \underbrace{\dot{r}}_{v_r}\hat{u}_\theta + \underbrace{r\dot{\theta}}_{v_t}\hat{u}_{\theta+90}$

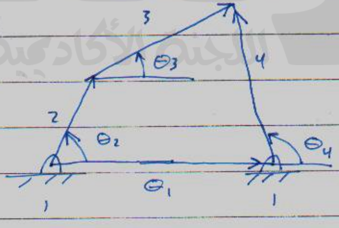
~~Note:-~~

Note:-

$\vec{\omega} = \pm w\hat{k}$, $w \rightarrow$ +ve c.c.w

$= \pm\dot{\theta}\hat{k}$ $\quad\quad w \rightarrow$ -ve c.w

4-Bars :-



Known:-

- fixed quantity
- Input : θ_2 , $\dot{\theta}_2 = w_2$
- complete positions

Find :-

w_3, w_4

L.C.E :-

$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 \hat{u}_{\theta_1} = 0$

V.E :-

$\frac{d}{dt}(L.C.E)$

$$\left(\underbrace{\cancel{\dot{r}_2 \hat{u}_{\theta_2}}}_{0} + r_2 \omega_2 \hat{u}_{\theta_2+\frac{\pi}{2}}\right) + \left(\underbrace{\cancel{\dot{r}_3 \hat{u}_{\theta_3}}}_{0} + r_3 \omega_3 \hat{u}_{\theta_3+\frac{\pi}{2}}\right) - \left(\underbrace{\cancel{\dot{r}_4 \hat{u}_{\theta_4}}}_{0} + r_4 \omega_4 \hat{u}_{\theta_4+\frac{\pi}{2}}\right) - (0) = 0$$

0 لأنه المسافة LinK2 ثابتة

$$\overset{\checkmark}{r_2} \overset{\checkmark}{\omega_2} \overset{\checkmark}{\hat{u}}_{\theta_2+\frac{\pi}{2}} + \overset{\checkmark}{r_3} \overset{?}{\omega_3} \overset{\checkmark}{\hat{u}}_{\theta_3+\frac{\pi}{2}} + \overset{\checkmark}{r_4} \overset{?}{\omega_4} \overset{\checkmark}{\hat{u}}_{\theta_4+\frac{\pi}{2}} = 0 \quad \text{-----} \quad (1)$$

X velocity :-

$-r_2 \omega_2 \sin\theta_2 - r_3 \omega_3 \sin\theta_3 + r_4 \omega_4 \sin\theta_4 = 0$

Y velocity :-

$r_2 \omega_2 \cos\theta_2 + r_3 \omega_3 \cos\theta_3 - r_4 \omega_4 \cos\theta_4 = 0$

sol. #1 :-

$$\begin{bmatrix} -r_3 \sin\theta_3 & r_4 \sin\theta_4 \\ r_3 \cos\theta_3 & -r_4 \cos\theta_4 \end{bmatrix} \begin{bmatrix} \omega_3 \\ \omega_4 \end{bmatrix} = \begin{bmatrix} r_2 \sin\theta_2 \\ -r_2 \cos\theta_2 \end{bmatrix} \omega_2$$

sol. #2 :-

* افترضنا u_{θ_3} لأنه $\theta_3 \cdot \theta_3 + \frac{\pi}{2} = 0$

$[eq.(1)] \cdot \hat{u}_{\theta_3}$

$$r_2 \omega_2 \underbrace{(-\sin\theta_2 \cos\theta_3 + \cos\theta_2 \sin\theta_3)}_{-\sin(\theta_2-\theta_3)} + \underbrace{\cancel{r_3 \omega_3 (0)}}_{0} - r_4 \omega_4 \sin(\theta_3 - \theta_4) = 0$$

$= \sin(\theta_3 - \theta_2)$

$$\omega_4 = \frac{r_2 \sin(\theta_3 - \theta_2)}{r_4 \sin(\theta_3 - \theta_4)} (\pm \omega_2)$$

$\Rightarrow$ we will find θ_3 from eq (Xve or Yvelocity), or by Dot eq. (1) by $(\hat{u}_{\theta_4})$:

$$\therefore \omega_3 = \frac{r_2 \sin(\theta_2 - \theta_4)}{r_3 \sin(\theta_3 - \theta_4)} (\mp \omega_2)$$

* Velocity Ratio (m_v) $= \dfrac{\omega_{out}}{\omega_{in}}$ ← Angular

EX: 4-Bar with 2: in, 4: out

$$m_v = \frac{\omega_4}{\omega_2} = \frac{r_2 \sin(\theta_3 - \theta_2)}{r_4 \sin(\theta_3 - \theta_4)}$$

* Torque Ratio (m_T) $= \dfrac{T_{out}}{T_{in}}$

Power $= \vec{T} \cdot \vec{\omega} = T\omega$

Assuming 100% efficincy

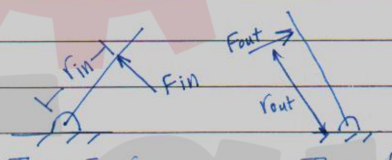
$$T_{in}\,\omega_{in} = T_{out}\,\omega_{out}$$

$$m_T = \frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}} = \frac{1}{m_v}$$

EX: 4-Bar $\quad m_T = \dfrac{\omega_2}{\omega_4} = \dfrac{r_4 \sin(\theta_3 - \theta_4)}{r_2 \sin(\theta_3 - \theta_2)}$

* ~~Ad~~ Mechanical Advantge (m_A) $= \dfrac{F_{out}}{F_{in}}$

EX:-

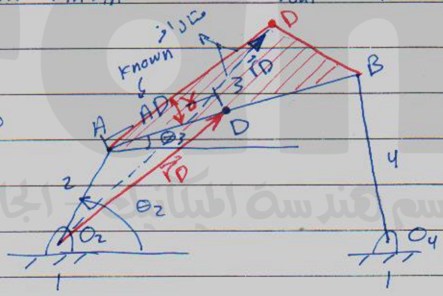


$$m_A = \frac{F_{out}}{F_{in}} = \frac{T_{out}}{T_{in}} \frac{r_{in}}{r_{out}} = m_T \frac{r_{in}}{r_{out}}$$

$$= \frac{1}{m_v} \frac{r_{in}}{r_{out}}$$

EX:-

Find $\vec{V}_D$?



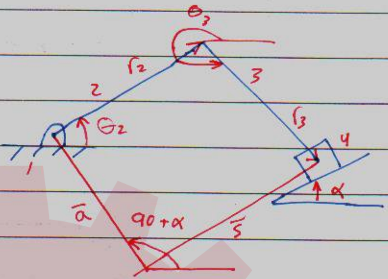
$$\vec{r}_D = \overrightarrow{O_2A} + \overrightarrow{AD}$$

$$\vec{r}_D = r_2 \hat{u}_{\theta_2} + AD\, \hat{u}_{\theta_3 + \gamma}$$

$$\vec{V}_D = \frac{d\vec{r}_D}{dt}$$

$$= r_2 \omega_2 \hat{u}_{\theta_2 + \frac{\pi}{2}} + AD\,\omega_3 \hat{u}_{\theta_3 + \frac{\pi}{2} + \gamma}$$

Example:-



Known :-

- r_2, r_3, a, γ
- Input: θ_2, ω_2
- complete position (s, θ_3)

Find:

$\dot{s}, \omega_3$

L.C.E :-

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - s \hat{u}_{\alpha} + a \hat{u}_{\alpha+90} = \vec{0}$$

velocity . eq :-

$$r_2 \omega_2 \hat{u}_{\theta_2+90} + r_3 \overset{?}{\omega_3} u_{\theta_3+90} + \overset{?}{\dot{s}} \hat{u}_{\gamma} + 0 = 0 \quad ---- (*)$$

Dot eq (*) by u_{θ_3}

$$r_2 \omega_2 \sin(\theta_3 - \theta_2) - \dot{s} \cos(\theta_3 - \gamma) = 0 \Rightarrow \boxed{\dot{s} = \frac{r_2 \sin(\theta_3 - \theta_2)}{\cos(\theta_3 - \alpha)} \omega_2}$$

$$\Rightarrow \omega_3 = \left[\frac{-r_2 \cos(\theta_2 - \alpha)}{r_3 \cos(\theta_3 - \alpha)}\right] \omega_2$$

Example: [sliding - contact Mech.]

L.C.E:-

$$r_2 \hat{u}_{\theta_2} - s \hat{u}_{\theta_3} - r_1 \hat{u}_0 = \vec{0}$$

x. eq :-

$$r_2 \cos\theta_2 - s \cos\theta_3 - r_1 = 0 \quad --- (1)$$

y. eq :-

$$r_2 \sin\theta_2 - s \sin\theta_3 = 0 \quad --- (2)$$

from (1) $\Rightarrow s \cos\theta_3 = r_2 \cos\theta_2 - r_1$

from (2) $\Rightarrow s \sin\theta_3 = r_2 \sin\theta_2$

$\Rightarrow$ Divide (1) / (2) to find θ_3

$\Rightarrow$ then find s.

known :-

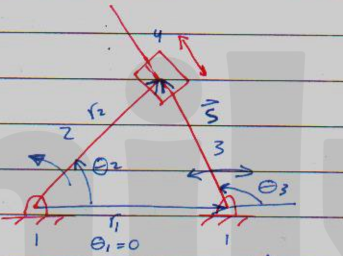
- r_2, r_1, θ_1
- Input :- θ_2, ω_2

Find :

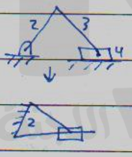
- Position s, θ_3

Find :-

$\dot{s}, \omega_3$



Inversion to



V.E :-

$$\left[r_2 w_2 \hat{u}_{\theta_2+90} - \dot{s}\,\hat{u}_{\theta_3} - \dot{s}\, w_3\, u_{\theta_3+90} = \vec{0} \right] \cdot u_{\theta_3}$$

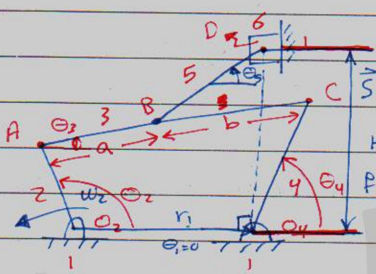
$\cdot u_{\theta_3+90}$

$$r_2 w_2 \sin(\theta_3 - \theta_2) = \dot{s}$$

$$r_2 w_2 \cos(\theta_3 - \theta_2) - s w_3 = 0$$

$$\Rightarrow w_3 = \left[\frac{r_2 \cos(\theta_3 - \theta_2)}{s} \right] w_2$$

Example :-



$a + b = r_3$

Known :-

- Given : $r_1, r_2, \theta_1 = 0, \underbrace{a, b}_{r_3}, r_4, r_5$
- Input : θ_2, w_2

Find :-

$\theta_3, \theta_4, \theta_5, S$

$w_3, w_4, w_5, \dot{S}$

sol :-

Loop I :-

$O_2 \rightarrow A \rightarrow C \rightarrow O_4 \rightarrow O_2$

$$r_2 \hat{u}_{\theta_2} + r_3 \hat{u}_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 u_{\theta_1} = \vec{0} \Rightarrow \theta_3, \theta_4 \quad \text{------ (I)}$$

loop II :-

$O_2 \rightarrow A \rightarrow B \rightarrow D \rightarrow O_4 \rightarrow O_2$

use x,y eq. & عند x eq تصفر

$$r_2 \hat{u}_{\theta_2} + a \hat{u}_{\theta_3} + r_5 \hat{u}_{\theta_5} - S \hat{u}_{90} - r_1 u_{\theta_1} = \vec{0} \Rightarrow \theta_5, S \quad \text{---- (II)}$$

V.E. (I)

$$r_2 w_2 \hat{u}_{\theta_2+90} + r_3 w_3 \hat{u}_{\theta_3+90} + 0 - r_4 w_4 \hat{u}_{\theta_4+90} \rightsquigarrow \text{find } w_3, w_4$$

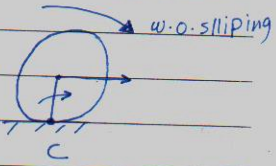
V.E. (II)

$$\left[r_2 w_2 u_{\theta_2+90} + a w_3 u_{\theta_3+90} + r_5 w_5 u_{\theta_5+90} - \dot{S} u_{90} - 0 = 0 \right] \cdot u_{\theta_5}$$

$$\frac{r_2 w_2 \sin(\theta_5 - \theta_2) + a w_3 \sin(\theta_5 - \theta_3)}{\cos(\theta_5)} = \frac{\dot{S} \cos(\theta_5)}{\cos(\theta_5)}$$

Method of instant center :-

* Instant center of rotation in a mechanism is a point common to two links and has the same instant velocity in each link.



* For a planar mechanisim with (L) Links there are ($N_{I.C}$) instant center, given by :-

$$N_{IC} = \frac{L(L-1)}{2}$$

* some instant center's can be found by inspection {Ex: R joints دائمًا, P joints أحيانًا ??}

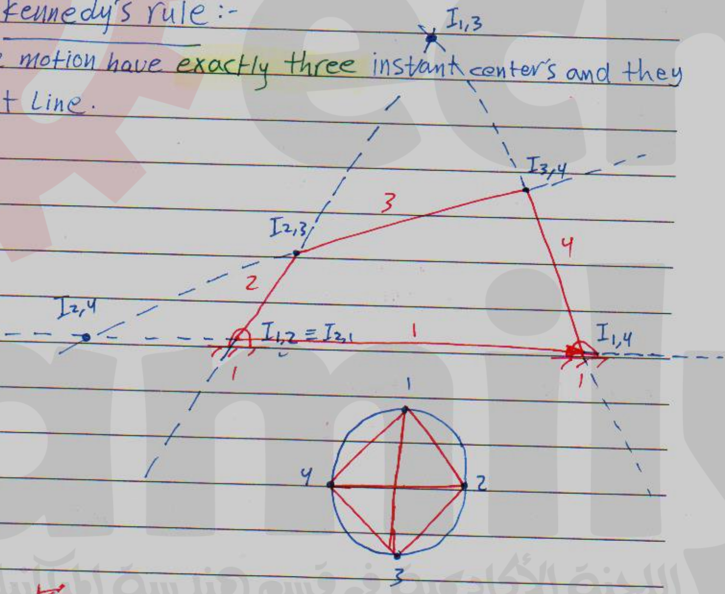
* others are found using kennedy's rule :-

any three bodies in plane motion have exactly three instant center's and they lie on the same striaght Line.

Example :-

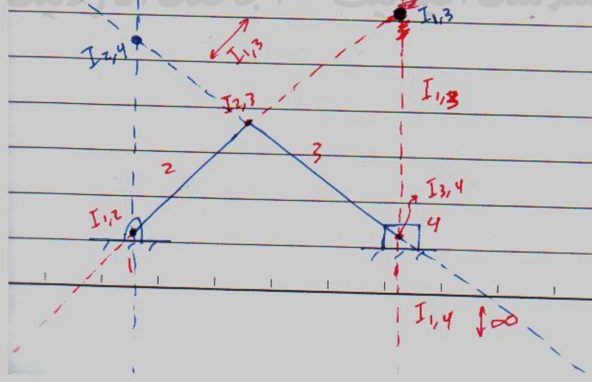
$L = 4$

$N_{I.C} = \frac{4(3)}{2} = 6$

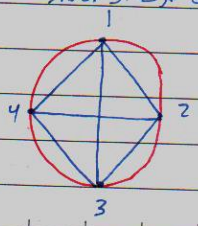


Example #2 :-

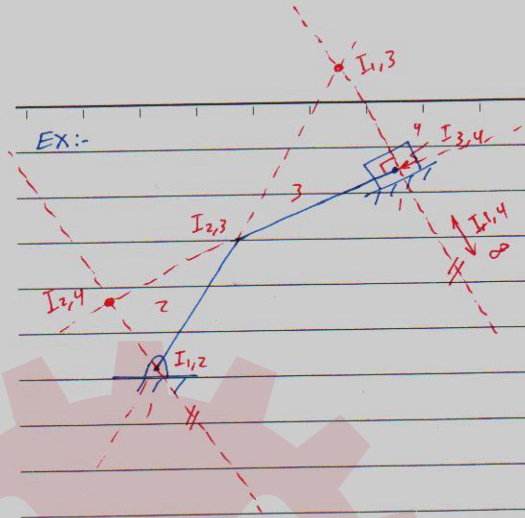
$N_{I.C} = 6$

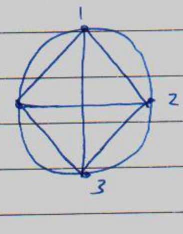


في حالة slider تكون I.C :
عاموديّ على اتجاه حركة ال slider



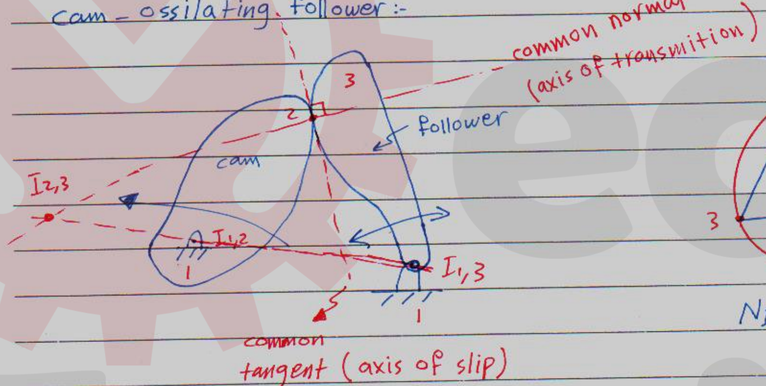
Ex:-

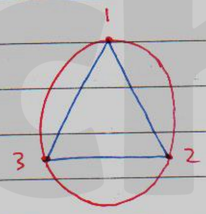




Example:-

cam - ossilating follower :-





$$N_{I.C} = \frac{(3)(3-1)}{2} = 3$$

* suggested problem

6-17, 18(b+c), 21(b), 44

Finding velocities using I.C method :-

at any Joint has (1) ∴ velocity = 0

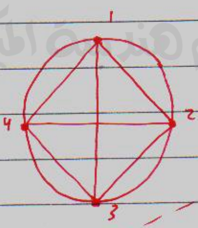
Ex: 4-Bar

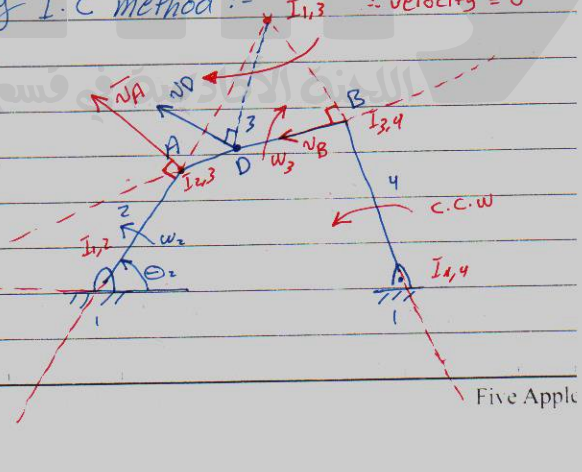
Given:

position

θ_2, ω_2

Find ω_3, ω_4





① $v_A = \omega_2 r_2$

② $\omega_3 = \dfrac{v_A}{AI_{1,3}}$ { $\overline{AI_{1,3}}$: القيمة نحصل عليها من خلال قياس }

← لا تتغير الـ I.C

بتغير الـ Inversion.

لكن تتغير الاتجاه.

③ $v_B = \omega_3 (BI_{1,3})$

④ $\omega_4 = \dfrac{v_B}{r_4}$; ⑤ $v_D = \omega_3 (\overline{I_{1,3}D})$

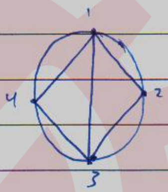
Ex:-

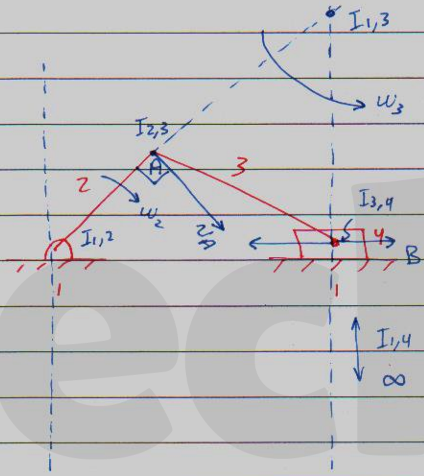
Given:

position, ω_2

find:

ω_3, $\dot{S} \equiv v_B$





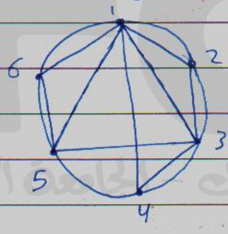
① $v_A = \omega_2 r_2$

② $\omega_3 = \dfrac{v_A}{I_{1,3}A}$, c.c.w

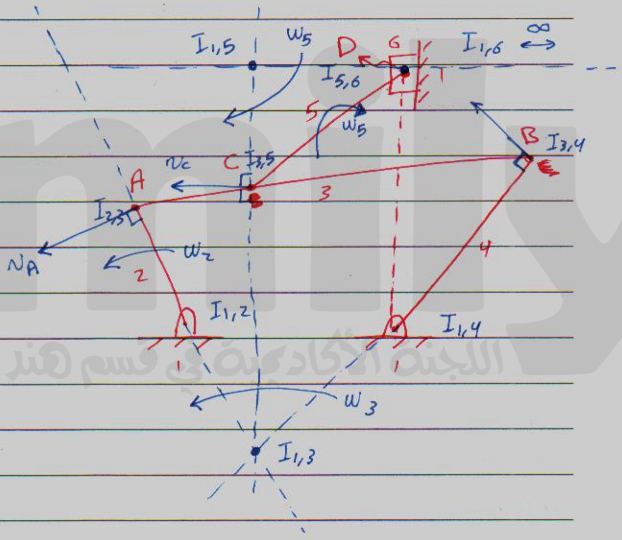
③ $v_B = \omega_3 (\overline{I_{1,3}B})$, dir: $\longrightarrow$

Ex:- find $\omega_3, \omega_4, \omega_5, v_6$

$N_{I.C} = \dfrac{6(5)}{2} = 15$



يجب أن تكون خطوط المثلث مصمتة solid



① $v_A = \omega_2 r_2$

$= \omega_3 (\overline{I_{1,3}A})$; c.c.w

② $v_B = \omega_3 (I_{1,3}B)$

$= \omega_4 r_4 \Rightarrow \omega_4 = \dfrac{v_B}{r_4}$ c.c.w

$v_C = \omega_3\,(\overline{I_{1,3}C}) \leftarrow$

$= \omega_5\,(\overline{I_{1,5}C})$

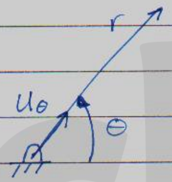
$\therefore \omega_5 = \dfrac{v_C}{(\overline{I_{1,5}C})}$; c.w

$v_6 = \omega_5\,(\overline{I_{1,5}D})$

* Acceleration Analysis :-

$\vec{r} = r\hat{u}_\theta$

$\vec{v} = \dfrac{d\vec{r}}{dt} = \dot{r}\hat{u}_\theta + r\,\overbrace{\omega\,\hat{u}_{\theta+\frac{\pi}{2}}}^{\dot{\hat{u}}_\theta}$



$\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}(\dot{r}\hat{u}_\theta) + \dfrac{d}{dt}\left(r\omega\,\hat{u}_{\theta+\frac{\pi}{2}}\right)$

$= \ddot{r}\hat{u}_\theta + \dot{r}\omega u_{\theta+\frac{\pi}{2}} + \dot{r}\omega u_{\theta+\frac{\pi}{2}}$
$+ r\alpha u_{\theta+\frac{\pi}{2}} + \underbrace{r\omega\,\dot{\hat{u}}_{\theta+\frac{\pi}{2}}}_{-r\omega^2\hat{u}_\theta}$

$u_\theta = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

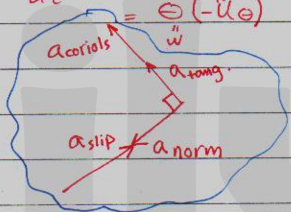
$\dot{u}_\theta = \dot{\theta}(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})$

$u_{\theta+\frac{\pi}{2}} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$

$\dfrac{du_{\theta+\frac{\pi}{2}}}{dt} = -\cos\theta\,\dot{\theta} - \sin\theta\,\dot{\theta}\,\hat{j} = \underbrace{\dot{\theta}}_{\omega}(-\hat{u}_\theta)$

$\vec{a} = (\underbrace{\ddot{r}}_{a_{slip}} - \underbrace{r\omega^2}_{a_{norm}})\hat{u}_\theta + (\underbrace{r\alpha}_{a_{tang.}} + \underbrace{2\dot{r}\omega}_{a_{coriols}})u_{\theta+\frac{\pi}{2}}$

$\underbrace{a_{slip} \quad a_{norm}}_{a_{radial}} \qquad \underbrace{a_{tang.} \quad a_{coriols}}_{a_{transv.}}$



Note :-

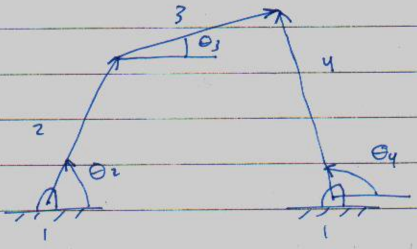
$\vec{\alpha} = \pm\alpha\hat{k}$, $\alpha \rightarrow +$ (c.c.w), $\rightarrow -$ (c.w)

Ex :-

Given :-
pos + vel

Input :-
$\theta_2, \omega_2, \alpha_2$

Find : α_3, α_4



L.C.E :-

$$r_2 u_{\theta_2} + r_3 u_{\theta_3} - r_4 \hat{u}_{\theta_4} - r_1 u_{\theta_1} = 0$$

V.E :-

$$r_2 w_2 u_{\theta_2+\frac{\pi}{2}} + r_3 w_3 u_{\theta_3+\frac{\pi}{2}} - r_4 w_4 u_{\theta_4+\frac{\pi}{2}} - 0 = 0$$

A.E :-

$$-r_2 w_2^2 \hat{u}_{\theta_2} + r_2 \alpha_2 \hat{u}_{\theta_2+\frac{\pi}{2}} - r_3 w_3^2 u_{\theta_3} + r_3 \overset{?}{\alpha_3} \hat{u}_{\theta_3+\frac{\pi}{2}} + r_4 w_4^2 u_{\theta_4}$$
$$- r_4 \overset{?}{\alpha_4} u_{\theta_4+\frac{\pi}{2}} = 0$$

x-Acce :-

$$-r_2 w_2^2 \cos\theta_2 - r_2 \alpha_2 \sin\theta_2 - r_3 w_3^2 \cos\theta_3 - r_3 \alpha_3 \sin\theta_3 + r_4 w_4^2 \cos\theta_4 + r_4 \alpha_4 \sin\theta_4 = 0$$

Y-Acce :-

$$-r_2 w_2^2 \sin\theta_2 - r_2 \alpha_2 \cos\theta_2 - r_3 w_3^2 \sin\theta_3 - r_3 \alpha_3 \cos\theta_3 + r_4 w_4^2 \sin\theta_4 + r_4 \alpha_4 \cos\theta_4 = 0$$

$$\begin{bmatrix} -r_3 \sin\theta_3 & r_4 \sin\theta_4 \\ r_3 \cos\theta_3 & r_4 \cos\theta_4 \end{bmatrix} \begin{bmatrix} \alpha_3 \\ \alpha_4 \end{bmatrix} = \begin{bmatrix} r_2 w_2^2 \cos\theta_2 + r_2 \alpha_2 \sin\theta_2 + r_3 w_3^2 \cos\theta_3 - r_4 w_4^2 \cos\theta_4 \\ r_2 w_2^2 \sin\theta_2 + r_2 \alpha_2 \cos\theta_2 + r_3 w_3^2 \sin\theta_3 - r_4 w_4^2 \sin\theta_4 \end{bmatrix}$$

OR

Dot (A.E) By $\hat{u}_{\theta_3}$

$$-r_2 w_2^2 \cos(\theta_3-\theta_2) + r_2 \alpha_2 \sin(\theta_3-\theta_2) - r_3 w_3^2 + r_4 w_4^2 \cos(\theta_3-\theta_4) - r_4 \sin(\theta_3-\theta_4)\,\alpha_4 = 0$$

$\alpha_4 =$ ______

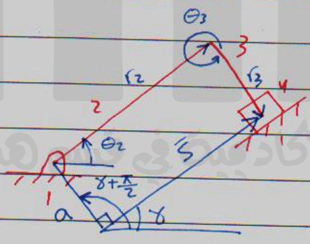
Then find $\alpha_3 =$ ______

Ex :-

Given : pos. + velo.

Input : θ_2, w_2, α_2

Find : $\ddot{s}$, α_3



L.C.E :-

$$r_2 u_{\theta_2} + r_3 u_{\theta_3} - s\, u_\gamma + a\, u_{\gamma+\frac{\pi}{2}}$$

V.E :-

$$r_2 w_2 U_{\theta_2+\frac{\pi}{2}} + r_3 w_3 U_{\theta_3+\frac{\pi}{2}} - \dot{s} U_\gamma + 0 = 0$$

A.E :-

$$\left[-r_2 w_2^2 U_{\theta_2} + r_2 \alpha_2 U_{\theta_2+\frac{\pi}{2}} - r_3 w_3^2 U_{\theta_3} + r_3 \overset{?}{\alpha_3} U_{\theta_3+\frac{\pi}{2}} - \overset{?}{\ddot{s}} U_\gamma = 0\right] \cdot U_{\theta_3}$$

$$\frac{-r_2 w_2^2 \cos(\theta_3-\theta_2) + r_2\alpha_2 \sin(\theta_3-\theta_2) - r_3 w_3^2}{\cos(\theta_3-\gamma)} = \ddot{s}$$

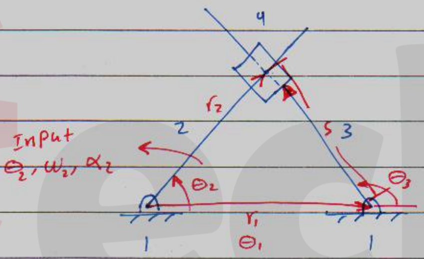
Example :-

Inverted slider crank (sliding contact) :-

Given : pos. + vel

Input : θ_2, w_2, α^2

find : $\ddot{s}$, α_3



L.C.E :-

$$r_2 U_{\theta_2} - S U_{\theta_3} - r_1 U_{\theta_1} = 0$$

V.E :-

$$r_2 w_2 U_{\theta_2+90} - \dot{s} U_{\theta_3} - S w_3 U_{\theta_3+\frac{\pi}{2}} = 0$$

A.E :-

مستحيل يلغوا بعض

$$\underbrace{-r_2 w_2^2 \hat{U}_{\theta_2}}_{\text{normal}} + \underbrace{r_2\alpha_2 U_{\theta_2+90}}_{\text{tangential}} - \underbrace{\overset{?}{\ddot{s}} U_{\theta_3}}_{\text{slip}} - \underbrace{\dot{s} w_3 U_{\theta_3+\frac{\pi}{2}}}_{\text{corioles}} - \underbrace{\dot{s} w_3 \hat{U}_{\theta_3+\frac{\pi}{2}}}_{\text{coriolis}} - S\overset{?}{\alpha_3} U_{\theta_3+\frac{\pi}{2}}$$

$$+ S w_3^2 U_{\theta_3} = 0$$

By Dot (A.E) By U_{θ_3}

$$-r_2 w_2^2 \cos(\theta_3-\theta_2) + r_2\alpha_2 \sin(\theta_3-\theta_2) + S w_3^2 = \ddot{s}$$

then find α_3 By Dot (A.E) By $U_{\theta_3+\frac{\pi}{2}}$

prob. 7-48 :-

Loop I :-

$O_2 \rightarrow A \rightarrow O_4 \rightarrow O_2$

ازا طلب $\boxed{a_A}$ فقط

$$\bar{r}_A = r_2 U_2$$

$$\bar{V}_A = r_2 w_2 U_{\theta_2+\frac{\pi}{2}}$$

$$a_A = -r_2 w_2^2 U_{\theta_2} + r_2\alpha_2 U_{\theta_2+90}$$

loop II :-

$O_4 \rightarrow B \rightarrow C \rightarrow O_4$

$(C-A)x^2+2Bx+(A+C)$
$x_1 = \theta_1, \theta_2$
$x_2 = \theta_1, \theta_2$

L.C.E. I :-

$20.3\, U_{\theta_2} - \overset{?}{S_3}\, U_{\overset{?}{\theta_4}} - 47\, U_{278.5°} = 0$

X :

$S_3 \cos\theta_4 = 20.3 \cos\theta_2 - 47 \cos(278.5)$

Y :

$S_3 \sin\theta_4 = 20.3 \sin\theta_2 - 47 \sin(278.5)$

$\frac{Y}{X}$:-

$$\tan\theta_4 = \frac{20.3 \sin(241) - 47 \sin(278.5)}{20.3 \cos(241) - 47 \cancel{\sin}\ \cos(278.5)}$$

$\theta_4 = -59.7° \times,\ 120.3 ✓$

$S_3 = 33.3$ mm

V.E. I :-

$\left[20.3\, \omega_2\, U_{\theta_2+90} - \overset{?}{\dot{S}_3}\, U_{\theta_4} - S_3\, \overset{?}{\omega_4}\, U_{\theta_4+90} = 0\right] U_{\theta_4}$

$\cancel{20}\ 20.3\, \omega_2 \sin(\theta_4 - \theta_2) = \dot{S}_3$

$\dot{S}_3 = \cancel{\ \ \ }\ -698.2$ (mm/min)

~~A.E.I~~ To find ω_4 Dot V.E By $(\theta_4 + \frac{\pi}{2})$:-

$$\frac{20.3\, \omega_2 \cos(\theta_4 - \theta_2)}{S_3} = \omega_4$$

$\omega_4 = -12.4$ rad/min.

A. E. I :-

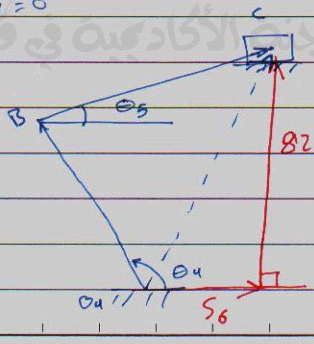
$-20.3\, \omega_2^2\, \hat{U}_{\theta_2} + 20.3\, \alpha_2\, \hat{U}_{\theta_2+90} - \overset{?}{\ddot{S}_3}\, \hat{U}_{\theta_4} - \dot{S}_3\, \omega_4\, \hat{U}_{\theta_4+90} - \dot{S}_3\, \omega_4\, \hat{U}_{\theta_4+90} - S_3\, \overset{?}{\alpha_4}\, \hat{U}_{\theta_4+90}$
$+ S_3\, \omega_4^2\, \hat{U}_{\theta_4} = 0$

Dot A.E I By U_{θ_4}

$-20.3\, \omega_2^2 \cos(\theta_4 - \theta_2) + 20.3\, \alpha_2 \sin(\theta_4 - \theta_2) - \ddot{S}_3 + S_3\, \omega_4^2 = 0$

$\Rightarrow \ddot{S}_3 =$

$\Rightarrow$ Then $\alpha_4 =$



L. C. E II :-

$75.4\, \hat{U}_{\theta_4} + 66.3\, U_{\overset{?}{\theta_5}} - 82.6\, \hat{U}_{90} - \overset{?}{S_6}\, \hat{U}_0 = 0$
$\ \ \ \ $ I

Five Apple

X:-

$75.4 \sin\Theta_4 + 66.3 \sin\Theta_5 - 82.6 = 0$ ($=120.3$)

$\Rightarrow \Theta_5 = 15.3^\circ$

X:-

$S_6 = 75.4 \cos\Theta_4 + 66.3 \cos\Theta_5 = 261.2$ mm

V.E.(II) :-

X. velocity:

$\dot{S}_6 = -75.4 \omega_4 \sin\Theta_4 - 66.3\, \omega_5 \sin\Theta_5 \Rightarrow \dot{S}_6 = 931$ mm/min $= v_c$

y. velocity:-

$0 = 75.4 \cos\Theta_4\, \omega_4 + 66.3\, \omega_5 \cos\Theta_5 \Rightarrow \omega_5 = -7.1$ rad/min

A.E(II) :-

X. Acce. :-

$\ddot{S}_6 = -75.4 \alpha_4 \sin\Theta_4 - 75.4\, \omega_4^2 \cos\Theta_4 - 66.3 \alpha_5 \sin\Theta_5 - 66.3\, \omega_5^2 \cos\Theta_5$

Y. Acce. :-

$0 = 75.4 \alpha_4 \cos\Theta_4 - 75.4\, \omega_4^2 \sin\Theta_4 + 66.3 \alpha_5 \cos\Theta_5 - 66.3\, \omega_5^2 \sin\Theta_5$

$\vec{a}_A = -20.3\, \omega_2^2\, U_{\Theta_2} + 20.3\, \alpha_2\, U_{\Theta_2+90}$

$= -20.3\,(40)^2\, U_{\underset{241}{\Theta_2}} + 20.3(-1500)\, \hat{U}_{\underset{331^\circ}{\cancel{\Theta_2+90}}}$

$= (\quad)\,\hat{i} + (\quad)\,\hat{j}$

$\vec{a}_B = \underbrace{-75.4\, \omega_4^2\, U_{\Theta_4}}_{A} \underbrace{+75.4\, \alpha_4\, U_{\Theta_4+90}}_{B}$

$= (\quad)\,\hat{i} + (\quad)\,\hat{j}$ $\qquad |a_B| = \sqrt{A^2 + B^2}$

$\vec{a}_C = \ddot{S}_6\, \hat{u}_0 = \ddot{S}_6\, \hat{i}$

H.W : 7-20 sun 30/6

Sugg : 7-17

7-52

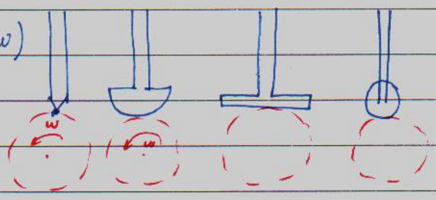
7-54

Chapter 8

Cam Design

Rotating (Disk) cam with Translating follower :-

uniform cam rotation (w is const c.c.w)
input ↗



output ⇒ follower motion displacement
(s)

cam-follower :- function ~~generation~~ or generater.

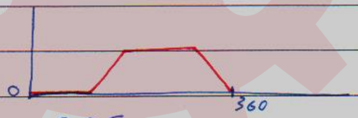
Motion program (of the follower):

- Rise : R
- Dwell : D
- Fall or Return : F

Motion prog. depends on the shape of the ~~cam.~~

Ex:-

1) prog. DRDF



2) prog RDF
3) Prog RF
4) prog DRDFD

* Standard Motion function :- (output oR Follower)

1) uniform motion (const. velocity)
2) uniform acc. (const. acc)
3) simple Harmonic Motion (S.H.M) ⇝ sine &cosine with same Frequency
4) Parabolic motion
5) cycoloidal motion
6) polynomial motion.

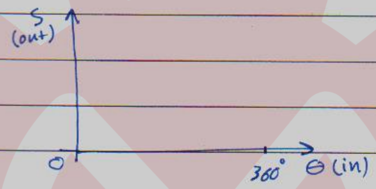
(S V a j) diagrams

dis veloc. acc. jerk

Input : θ (cam rotation)

out put : S (follower disp)

$\Rightarrow S = S(\theta)$

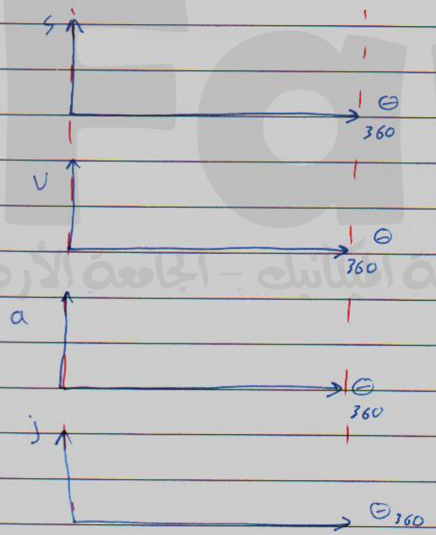


$V = \frac{ds}{dt} = \dot{s}$

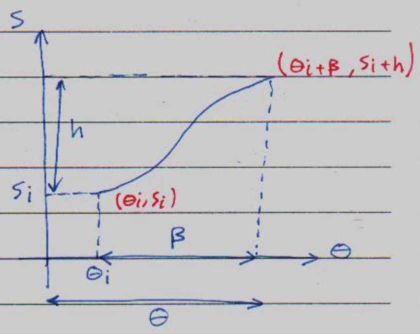
$V = \frac{ds}{d\theta} \frac{d\theta}{dt} = w\frac{ds}{d\theta} = ws'$

$a = \frac{dv}{dt} = \dot{v} = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt} = wv' = w^2 s''$

$j = \frac{da}{dt} = \frac{da}{d\theta} \cdot \frac{d\theta}{dt} = wa' = w^3 s''' = w^3 v''$



Cam general displacement :-



$S(\theta) = S_i + f(\theta - \theta_i)$

Domain: $\left(\theta_i \leq \theta \leq \underbrace{\theta_{i+\beta}}_{\theta}\right)$

Ex:- (S.H.M)

$$f(\theta-\theta_i) = \frac{h}{2}\left\{1 - \cos\left[\frac{\pi}{\beta}(\theta-\theta_i)\right]\right\}$$

$$S = S_i + \frac{h}{2}\{\cdots\}$$

Ex:-

uniform motion :-

$$S = S_i + \frac{h}{\beta}(\theta-\theta_i) = S_i + h\left(\frac{\theta-\theta_i}{\beta}\right)$$

Ex:-

uniform acc. :-

$$S = S_i + \frac{h}{\beta^2}(\theta-\theta_i)^2 = S_i + h\left(\frac{\theta-\theta_i}{\beta}\right)^2$$

Ex:-

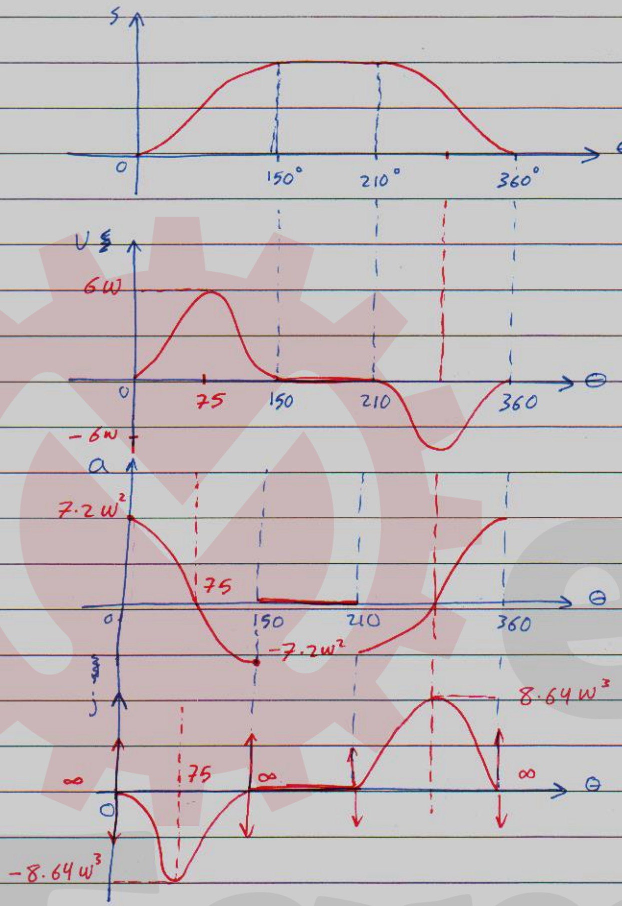
A cam rotating with (w) is to have the following motion for one cycle

* $(0 - 150^\circ)$: Rise of 10 cm with S.H.M
* $(150^\circ - 210^\circ)$: Dwell
* $(210^\circ - 360^\circ)$: fall with simple. H.M

R.D.F

* اذا لم يحدد كم نزل راح يكون نفس المقدار الي ارتفع فيه

$\beta \begin{cases} \to \text{rad.} \\ \to \text{deg.} \end{cases}$



$\boxed{1}$ Rise :-

$h = 10$ cm , $\theta_i = 0$, $\beta = \frac{150}{180}\pi = \frac{5}{6}\pi = 150°$

$s_i = 0$

$s(\theta) = 5\left[1 - \cos(1.2\theta)\right]$

Domain $(0 \leq \theta \leq 150°)$

velocity :-

$V = w \frac{ds}{d\theta}$

$= 6w \sin(1.2\theta)$; $0 \leq \theta \leq 150°$

V_{max} @ $\theta = 75 = 6w$

$v(\theta=0) = 0$, $v(\theta=150) = 0$

accelaration :-

$a = w \frac{dv}{d\theta}$

$= 7.2w^2 \cos(1.2\theta)$

$(0 \leq \theta \leq 150)$

a_{max} @ $\theta = 0 = 7.2w^2$

a_{min} @ $\theta = 150 = -7.2w^2$

$a = 0$ @ $\theta = 75$

jerk :-

$j = w \frac{da}{dt}$

$= -8.64w^3 \sin(1.2\theta)$

$(0 < \theta < 150)$

$j = \begin{cases} -8.64w^3 \sin(1.2\theta) & 0 < \theta < 150 \\ \pm\infty & \theta = 0, \theta = 150 \end{cases}$

$\boxed{2}$ Dwell :-

$S = 10$ cm ; $150° \leq \theta \leq 210°$

$V = 0$; $150° \leq \theta \leq 210°$

$a = 0$; $150° \leq \theta \leq 210°$

$j = \begin{cases} 0 & 150 < \theta < 210 \\ \pm\infty & \theta = 210, \theta = 150° \end{cases}$

$\boxed{3}$ Fall :-

$S_i = 10$, $\theta_i = 210$, $\beta = 360 - 210 = 150^\circ = \frac{5}{6}\pi$, $h = -10$

$$S = 10 + \left(\frac{-10}{2}\right)\left\{ 1 - \cos\left[1.2(\theta - 210)\right] \right\} \quad ; 210^\circ \le \theta \le 360^\circ$$

$$V = w\frac{ds}{d\theta} = -6\sin[1.2(\theta - 210)]$$

$$a = w\frac{dV}{d\theta} = -7.2\,w^2\cos[1.2(\theta - 210)]$$

$$j = \begin{cases} 8.64\,w^3\sin[1.2(\theta - 210)] & ; 210 \le \theta < 360 \\ \pm\infty & ; \theta = 210 , \theta = 360 \end{cases}$$

* Cycloidal :-

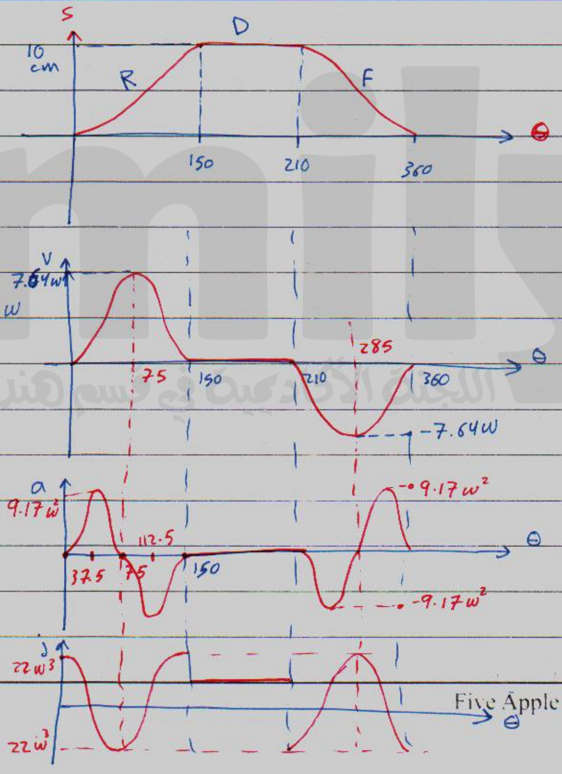
$$S = S_i + h\left[\frac{\theta - \theta_i}{\underbrace{\beta}_{\text{duration}}} - \frac{1}{2\pi}\sin\left[\frac{2\pi}{\beta}(\theta - \theta_i)\right]\right] \quad ; \theta_i \le \theta \le \theta_i + \beta$$

Example :- [same previos example]

$\boxed{1}$ Rise :- $(0 \le \theta \le 150)$

$h = 10$, $\theta_i = 0$, $S_i = 0$, $\beta = 150^\circ = \frac{5}{6}\pi$

$$\Rightarrow S = \frac{12}{\pi}\theta - \frac{5}{\pi}\sin(2.4\theta)$$



$$V = \frac{12}{\pi}w(1 - \cos(2.4\theta))$$

$V(0) = 0$, $V(150) = 0$, $V(75) = 7.64\,w$

$$a = 9.17\,w^2\sin(2.4\theta) = w\,dV/d\theta$$

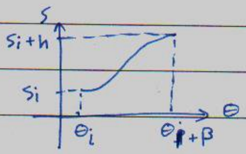
$$j = 22\,w^3\cos(2.4\theta)$$

sygg :- 6 condition
⑤ $a=0$ @ $(\theta=\theta_i)$
⑥ $a=0$ @ $(\theta=\theta_i+\beta)$

Polynomial function :-

$$S(\theta) = S_i + \sum_{k=0}^{n} C_k (\theta-\theta_i)^k$$

Example :- [4-terms] $\Rightarrow$ we need 4-end condition



conditions (end or boundary)

1) $S(\theta_i) = S_i$ 2) $v(\theta_i) = 0$

3) $S(\theta_i+\beta) = S_i+h$ 4) $v(\theta_i+\beta) = 0$

$$S = S_i + C_0 + C_1(\theta-\theta_i) + C_2(\theta-\theta_i)^2 + C_3(\theta-\theta_i)^3 \quad [3^{rd}\text{-order polo.}]$$

① $S=S_i$ @ $\theta=\theta_i \Rightarrow S_i = S_i + C_0 \Rightarrow \boxed{C_0 = 0}$

$$V = C_1 W + 2C_2 W(\theta-\theta_i) + 3C_3 W(\theta-\theta_i)^2$$

② $v=0$ @ $\theta=\theta_i \Rightarrow 0 = C_1 W \Rightarrow \boxed{C_1 = 0}$

③ $S=S_i+h$ @ $\theta=\theta_i+\beta \Rightarrow S_i+h = S_i + 0 + 0 + C_2\beta^2 + C_3\beta^3$

$\Rightarrow C_2\beta^2 + C_3\beta^3 = h$ ---- ①

④ $v=0$ @ $\theta=\theta_i+\beta \Rightarrow 0 = 0 + 2C_2 W\beta + 3C_3 W\beta^2$

$\Rightarrow 2C_2 W\beta + 3W C_3 \beta^2 = 0$ ---- ②

from eq. 2 :-

$$C_3 = -\frac{2\beta C_2}{3\beta}$$

sub (C_3) in eq. 3 :-

$$\boxed{C_2 = \frac{3h}{\beta^2}} \qquad \boxed{C_3 = \frac{-2h}{\beta^3}}$$

$$S = S_i + 3h\left(\frac{\theta-\theta_i}{\beta}\right)^2 - 2h\left(\frac{\theta-\theta_i}{\beta}\right)^3 , \quad \theta_i \le \theta \le \theta_{i+\beta}$$

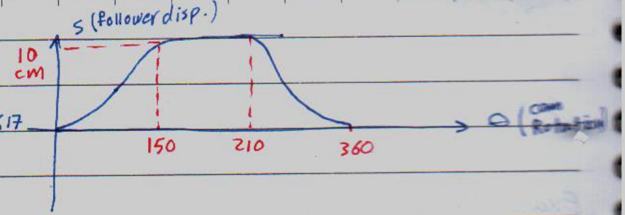
polynomial 3-4-5 → number of coeff. ⇝ $r_0, r_1, r_2 = 0$

Example:-

Rise:-

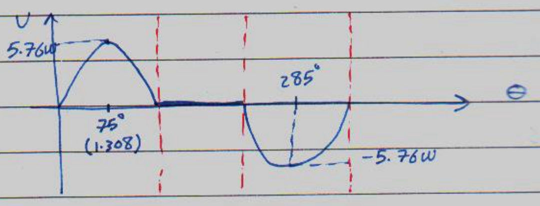
$\theta_i = 0$, $S_i = 0$, $\beta = 150^\circ = \frac{5}{6}\pi = 2.617$

$h = 10\,cm$



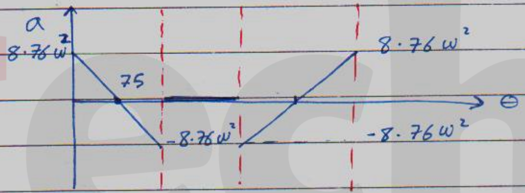
$$S = 30\left(\frac{1.2}{\pi}\theta\right)^2 - 20\left(\frac{1.2}{\pi}\theta\right)^3$$

$$= 4.38\theta^2 - 1.11\theta^3 \quad ; \; 0 \le \theta \le 150^\circ$$



$$V = w\frac{ds}{d\theta} = 8.76\,w\theta - 3.33\,w\theta^2$$

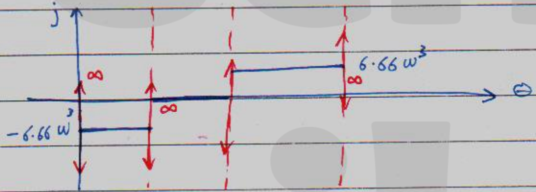
$$a = w\frac{dV}{d\theta} = 8.76\,w^2 - 6.66\,w^2\theta$$



$a = 0$ @ $\theta = 1.308\,rad = 75^\circ$

$a(0) = 8.764\,w^2$, $a(150^\circ) = -8.76\,w$

$$j = \begin{cases} -6.66\,w^3 & , 0 < \theta < 150 \\ \pm\infty & , \theta = 0, \; \theta = 150 \end{cases}$$



Fall:-

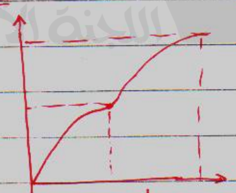
$\theta_i = 210^\circ = 3.665\,rad$, $\beta = 150^\circ = \frac{5}{6}\pi = 2.617\,rad$, $S_i = 10\,cm$, $h = -10\,cm$

$$S = 10 - 30\left(\frac{\theta - 3.665}{2.617}\right)^2 + 20\left(\frac{\theta - 3.665}{2.617}\right)^3$$

Example:-

$$\begin{cases} 0 & ; \; 0 \le \theta \le 60^\circ \\ S.H.M,\; h = 5cm & ; \; 60 \le \theta \le 150 \\ 5\,cm & ; \; 150 \le \theta \le 180 \\ \text{Rise, cyc, 5cm} & 180 \le \theta \le 240 \\ \text{Fall, Pol 5th order} & 240 \le \theta \le 330 \\ \text{Dwell (0)} & 330 \le \theta \le 360 \end{cases}$$

Note



combined Rise.

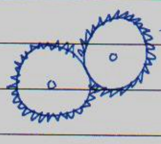
* Gears :-

- joint between two gears → 2-DoF

1) spur Gear :-

① straight teeth and parallel to the axisial direction

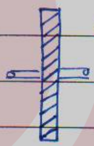
② parallel shaft



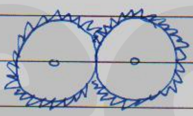
→ pinion (smallest Gear)

2) Helical Gear :-

inclinde to the shaft the teeth

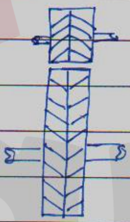


② parallel shaft



3) crossed Helical Gear :-

4) Herring Bone Gear :-

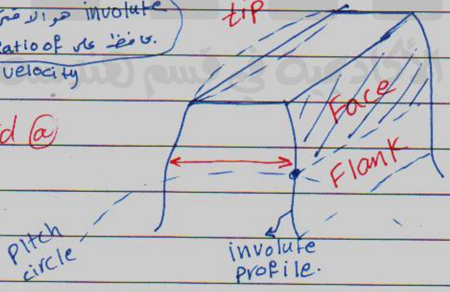


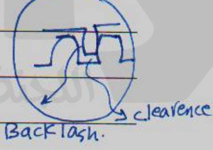
5) Bevel Gear

involute هو الاقتران الوحيد اللي بحافظ على Ratio of velocity ثابتة.

Note :-

thickness variable but standard @ pitch circle





$P.C = \frac{\pi d}{N}$ [mm, in]

P.C → circular pitch

N → # of teeth

P_c = Tooth thickness + space wedth.

$P_b = \frac{\pi d_b}{N}$

P_b → Base pitch

Module (m) :-

$m = \frac{d}{N}$ [mm] only

Diametral pitch (P_d)

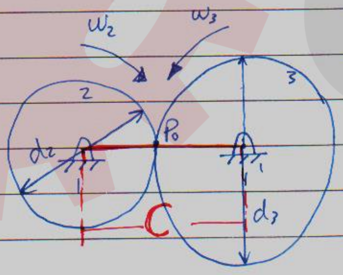
$P_d = \frac{1}{m} = \frac{N}{d}$ [teeth] only

$P_c = \pi m$ [mm]

$P_c = \frac{\pi}{P_d}$ [in]

* Any Gears in mesh should have same m, P_c, P_d

P_o : Pitch point



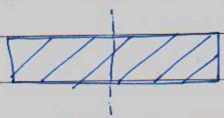
$C = \frac{d_2 + d_3}{2}$

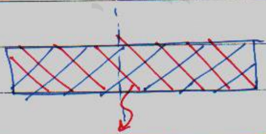
$= m\left(\frac{N_2 + N_3}{2}\right)$

$v_{P_o} = \frac{\omega_3 d_3}{2} = \frac{\omega_2 d_2}{2} \Rightarrow \boxed{\frac{\omega_2}{\omega_3} = \frac{d_3}{d_2}}$

$\frac{\omega_2}{\omega_3} = \frac{\cancel{m} N_3}{\cancel{m} N_2} \Rightarrow \boxed{\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2}}$

Note:-

R.H 

L.H 

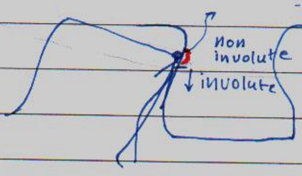
اذا مشينا من باتجاه اليمين ارتفع.

اذا مشينا باتجاه اليسار ارتفع

$$\frac{\omega_2}{\omega_3} = \frac{d_3}{d_2} = \frac{N_3}{N_2} = \underline{\underline{const.}}$$

Fundemantal law of gears.

اذا حدث التصادم في non-involute تحدث شكلة



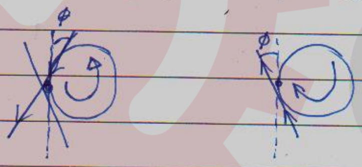
$r_b = r\cos\phi$ ‏ b: basic circle.

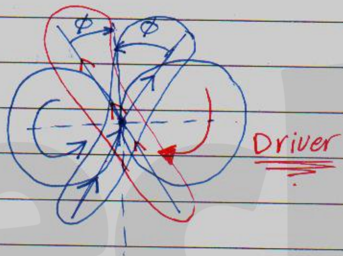
$d_b = d\cos\phi$ ‏ r, d (بدون اي رموز) → pitch circle

كيفية تحديد اتجاه ϕ :

① نحدد اتجاه Driver (c.c.w or c.w)

② يكون اتجاه الزاوية مع اتجاه الدوران





* Notes:-

start motion when tip of Driven contact with the flank of driver

end motion when Flank of Driven contact with the tip of driver.

* begening contact ocurrs when line of action intersect with Addendum circle for the driven

* Angle of approach @ Begining contact.

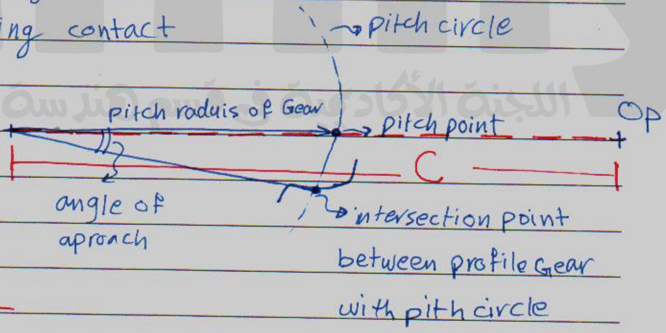
* Angle of recess @ leaving contact
انتهاء / بكون

* Angle of aproach :-

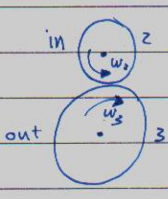
هي الزاوية المحصورة بين :

(C) center distance و نقطة تقاطع

pitch circle مع Gear profile



$m_V = \frac{\omega_{out}}{\omega_{in}} = \frac{\omega_3}{\omega_2} = \frac{d_2}{d_3} = \frac{N_2}{N_3}$



$m_T = \frac{T_{out}}{T_{in}} = \frac{1}{m_V}$

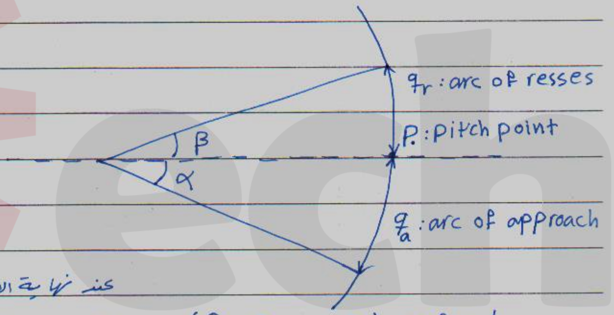
for 100% efficiency
(spur, parallel helical, straight bevel gear)

$(Power)_{in} = (Power)_{out}$

$T_{in}\,\omega_{in} = T_{out}\,\omega_{out}$

$\frac{T_{out}}{T_{in}} = \frac{\omega_{in}}{\omega_{out}} = \frac{1}{m_V}$

$\frac{T_3}{T_2} = \frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{d_3}{d_2}$



if :- $q = P_c$: عند نهاية الاتصال الاول يبدأ الثاني فوراً

$q < P_c$; lead to non-continous motion

$q > P_c$: قبل نهاية اتصال الاول يبدأ الثاني

(two pair of teeth بين contact يصير ٢)

$(q_a, q_r, \alpha, \beta)$ are fixed.

$q = q_a + q_r$

arc of action.

* Contact Ratio (m_p)

Indicates the average no. of pairs of ~~teeth~~ teeth in contact

$[m_p > 1]$; Recomanded: $m_p > 1.5$.

$m_p = \frac{q}{P_c} = \frac{z}{P_b}$

$P_b = \frac{\pi d_b}{N}$; $d_b = d\cos\phi$

$\boxed{P_b = P_c \cos\phi}$

2: pinion
3: Gear.

$$Z = Z_1 + Z_2 - Z_3$$

a: addendum circle of gear
b: dedendum circle of gear

$$Z_1 = \sqrt{(\underbrace{r_2 + a_2}_{r_{a2}})^2 - (\underbrace{r_2 \cos\phi}_{r_{b2}})^2}$$

$$Z_2 = \sqrt{(\underbrace{r_3 + a_3}_{r_{a3}})^2 - (\underbrace{r_3 \cos\phi}_{r_{b3}})^2}$$

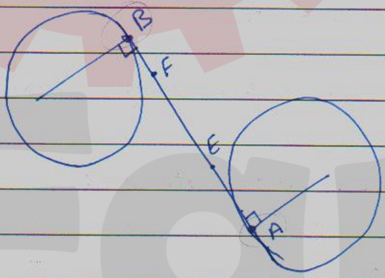
$$Z_3 = (\underbrace{r_2 + r_3}_{C}) \sin\phi$$

Note:-
$P_c, P_b, m, P_d, \phi, a, b$
are the same in mesh of two gear
$\therefore a_2 = a_3$ (في حالة اللي فوق)

AGMA standard:-

$a = 1.0\, m \rightarrow$ (module not meter) $= \frac{1.0}{P_d}$

$b = 1.25\, m = \frac{1.25}{P_d}$



* اذا كانت point of tangent داخل initial & final contact point يوجد Interferance.

* = = تساوي = ✗ يوجد =

$Z = AB$

$= (BC - CP_o) + (AD - DP_o)$

$= BP_o + AP_o$

$= AB$

Whole depth (h_t)

$h_t = a + b$

full depth : $h_t = 2.25\,m = \frac{2.25}{P_d}$ stub : $h_t = 1.8\,m = \frac{1.8}{P_d}$

<u>Note</u>

clearance $(b_3 - a_2)$

$\therefore b - a = 0.25m.$

But:-
$a_2 = a_3 = a$
$b_2 = b_3 = b$

Base raduis, Base pitch, velocity ratio ⇝ will ber remaning const.

Addendum circle, dedendum circle

when center distance change :-

Change	unchange
circular	Base circle
Z_3, Z	Base Bitch
Addendum	Addendum circle, raduis
dedendum	dedendum circle, raduis
contact ratio	Z_1, Z_2

If $Z_1 > Z_3$

$\Rightarrow$ use $Z_1 = Z_3$

If $Z_2 > Z_3$

$\Rightarrow$ use $Z_2 = Z_3$.

Example:-

For $m_G = 4$, $\phi = 20^\circ$, find the smallest no. of teeth of pinion to avoid interferance??

Note:-

$$N_P = \frac{2(1)}{(1+2(4))\sin^2(20)}\left\{4+\sqrt{16+(1+8)\sin^2(\phi)}\right.$$

اذا كان fall depth تكون $k=1$

واذا كان stup تكون $k=1.8$

واذا ما حدد السؤال نوعه تكون fall depth

$= 15.4$

$\Rightarrow N_P = 16$

$\Rightarrow N_G = 64$

Example:-

For $N_P = 13$, $\phi = 20^\circ$, find the largest no. of teeth of gear to avoid interference??

$$N_G = \frac{13^2 \sin^2(20) - 4(1)^2}{4(1) - 2(13)\sin^2(20)} = 16.45$$

$N_G = 16$.

* standard clearance:-

$$(c)_2 = (b)_2 - (a)_3$$

$$c = b - a = (1.25 - 1)m = 0.25m$$

* standard back lash:- $(0.003 - 0.018)$ mm

increase in back lash due to change (ΔC) in C.D:-

$$\Theta_B = 43{,}200(\Delta C)\frac{\tan\phi'}{\pi d'} \quad [\text{minutes}]$$

$d' \rightarrow$ Diameter ~~of bitch~~ circle

Example :- Find :-

$m_G, P_c, P_b, d_2, d_3, r_2, r_3, C, a, b, h_t, C, (d_2)_a, (d_3)_a, m_p$

if, $m = 4mm$, $\phi = 20°$, $N_P = 19$ ("2"), $N_G = 37$ ("3")

$m_G = \frac{N_G}{N_P} = \frac{N_3}{N_2} = 1.947$

Check :-

$\frac{r_3}{r_2} = \frac{d_3}{d_2} = m_G$

⋮

$m_p = \frac{q}{P_c} = \frac{z}{P_b}$

$P_b = 11.809\ mm$

P : pinion : 2

$Z = Z_1 + Z_2 - Z_3$

$Z_1 = \sqrt{(r_P)_a^2 - (r_P)_b^2} = \sqrt{(42)^2 - (38\cos 20)^2} = 22.111\ mm$

$Z_2 = \sqrt{(78)^2 - (74\cos 20)^2} = 35.335\ mm$

OK

$Z_3 = 112 \sin 20° = 38.306\ mm$

$Z = 19.140\ mm$

$\therefore m_p = \frac{19.140}{11.809} = 1.621$

$\% \Delta C = 2\%$

$C = 112\ mm$

$C' = (1.02)(112) = 114.240\ mm$

$\Delta C = 2.24\ mm$

$r_b = r_b'$

$r\cos\phi = r'\cos\phi'$

$\phi' = \cos^{-1}\left(\frac{r_P \cos\phi}{r_P'}\right) = \cos^{-1}\left(\frac{r_g \cos\phi}{r_g'}\right)$

$r_P = 38\ mm$, $\phi = 20°$

r_P' :-

$r_P' + r_G' = 114.24$ --- ①

oR

$m_G = \frac{N_G}{N_P} = \frac{r_G}{r_P} = \frac{r_G'}{r_P'}$ --- ②

from (2): $r_G' = \frac{r_G}{r_P} r_P' = 1.947\, r_P'$

sub. in (1):

$(1 + 1.947)\, r_P' = 114.24$

$\Rightarrow r_P' = \frac{114.24}{2.947} = 38.765$ mm

$\Rightarrow \phi' = \cos^{-1}\left[\frac{38 \cos 20^\circ}{38.765}\right] = 22.89^\circ$

$C' = r_P' + r_G' = 1.02C = 1.02\,(r_P + r_G)$

$\boxed{r_P' = 1.02\, r_P}$ $\boxed{r_G' = 1.02\, r_G}$

Find the new contact ratio m_P'

$m_P' = \frac{Z'}{P_b}$

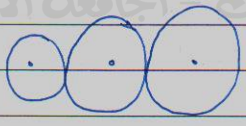
$Z' = Z_1 + Z_2 - Z_3'$

$Z_1' = 114.24 \sin 22.89 = 44.435$

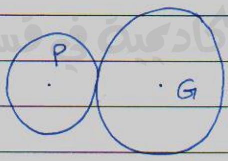
$Z' = 13.011$

$m_P' = \frac{13.011}{11.809} = 1.101$

* Gear trains :-



Multis - stage of reduction



I stage of of reduction

$m_V = \omega_{out} / \omega_{in}$

$m_G = \frac{N_G}{N_P}$

Gear Trains

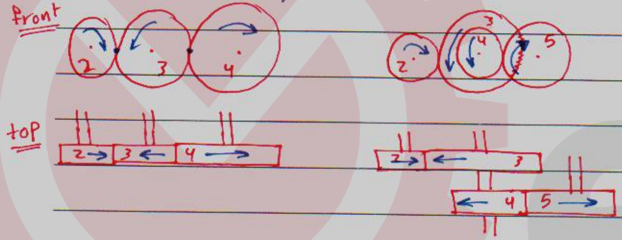
Basic, Fixed Axes or 1.d.o.f {1in → 1out}

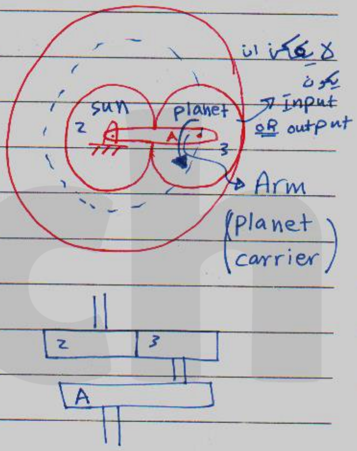
- simple — 2 stage
- compound

Planetary or Epicyclic or moving axes or 2.d.o.f {2 in → 1out}

front

top





* Overal velocity ratio OR Train value :-

$$m_V = \frac{\omega_L}{\omega_F} = \frac{\omega_{last}}{\omega_{first}} = \frac{n_L}{n_F}$$

ang. velocity (rad/s)

rotation speed (rpm) (rps)

L: last driven.

F: First driver.

$$\frac{\omega_L}{\omega_F} = \pm \frac{\text{Porduct of teeth of driver}}{\text{Porduct of teeth of driven}}$$

Ex:-

$$\frac{\omega_6}{\omega_2} = (+)\frac{N_2 \cdot \cancel{N_3} \cdot \cancel{N_4} \cdot \cancel{N_5}}{\cancel{N_3} \cdot \cancel{N_4} \cdot \cancel{N_5} \cdot N_6}$$

first — last

$\omega_L , \omega_F \rightarrow$ same direction $\rightarrow (+)$

$\omega_L , \omega_F \rightarrow$ opossite direction $\rightarrow (-)$

$$m_V = m_{V_1} \times m_{V_2} \times m_{V_3} \times m_{V_4}$$

1, 2, 3, 4 → no. of stage.

$$= \frac{\omega_3}{\omega_2} \times \frac{\omega_4}{\omega_3} \times \frac{\omega_5}{\omega_4} \times \frac{\omega_6}{\omega_5} = \frac{\omega_6}{\omega_2}$$

$$= \left(\frac{-N_2}{N_3}\right)\left(\frac{-N_3}{N_4}\right)\left(\frac{-N_4}{N_5}\right)\left(\frac{-N_5}{N_6}\right)$$