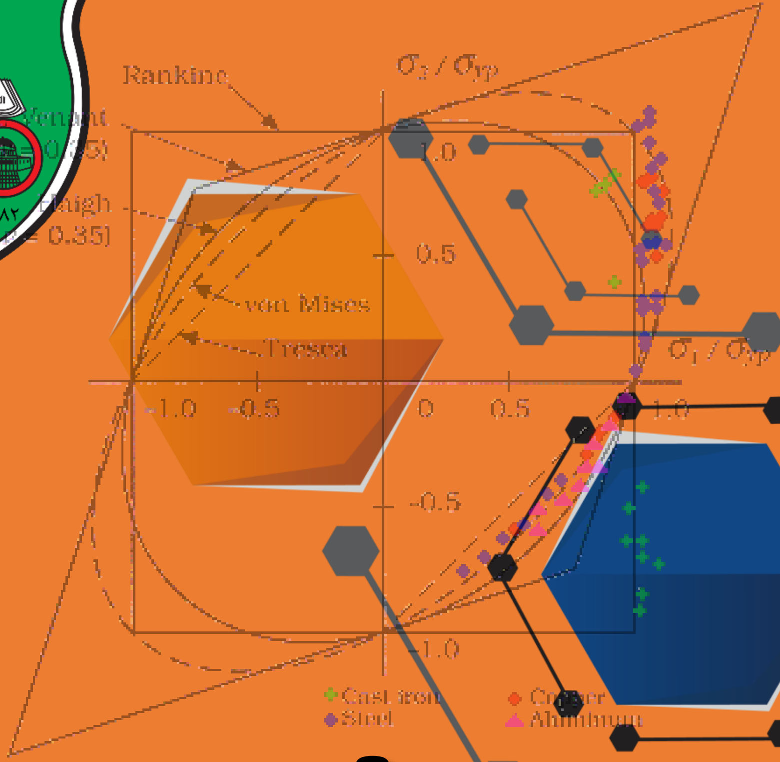

Static Failure Theories

Dr Moudar Zgoul
m.zgoul@ju.edu.jo

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Introduction

What is a static Load?

An ideal static load is one that is applied slowly and is never removed. Some loads that are applied slowly and removed and replaced very infrequently can also be considered to be static.

What examples can you think of for products or their components that are subjected to static loads?

- Consider products that have failed. Did they fail the first time they were used? Or did they fail after some fairly long service? Why do you think they were able to operate for some time before failure?

Introduction

- Can you find components that failed suddenly because the material was brittle, such as cast iron, some ceramics, or some plastics? Can you find others that failed only after some considerable deformation? Such failures are called ductile fractures.

What were the consequences of the failures that you have found? Was anyone hurt? Was there damage to some other valuable component or property? Or was the failure simply an inconvenience? What was the order of magnitude of cost related to the failure?

The answer to some of these questions can help you make rational decisions about design factors to be used in your designs.

It is the designer's responsibility to ensure that a machine part is safe for operation under reasonably foreseeable conditions.

Introduction

This requires that a stress analysis be performed in which the predicted stress levels in the part are compared with the design stress, or that level of stress permitted under the operating conditions.

The stress analysis can be performed either analytically or experimentally, depending on the degree of complexity of the part, the knowledge about the loading conditions, and the material properties.

The designer must be able to verify that the stress to which a part is subjected is safe.

Introduction

The manner of computing the design stress depends on the manner of loading and on the type of material. Loading types include the following:

- Static
- Repeated and reversed
- Fluctuating
- Shock or impact
- Random

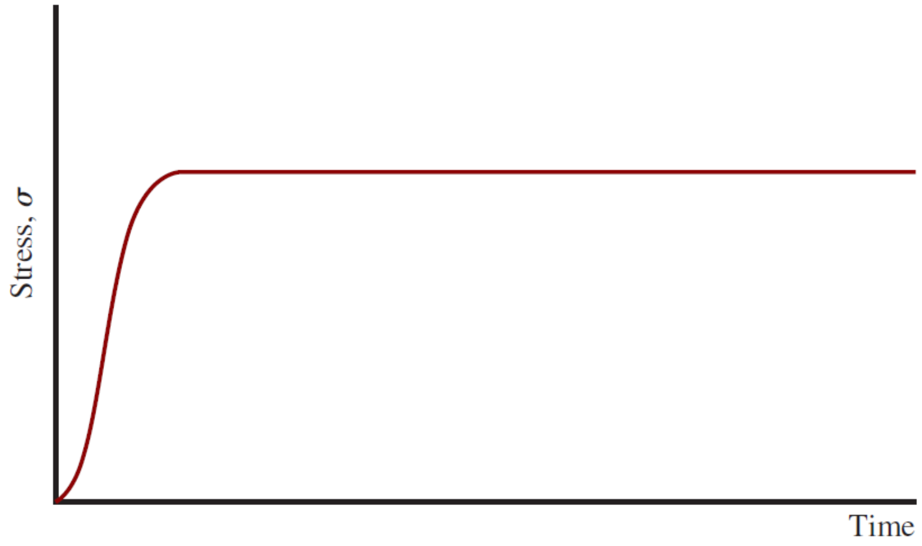
Material types are many and varied. Among the metallic materials, the chief classification is between ductile and brittle materials. Other considerations include the manner of forming the material (casting, forging, rolling, machining, and so on), the type of heat treatment, the surface finish, the physical size, the environment in which it is to operate, and the geometry of the part. Different factors must be considered for plastics, composites, ceramics, wood, and others.

Introduction

Static Stress

When a part is subjected to a load that is applied slowly, without shock, and is held at a constant value, the resulting stress in the part is called static stress.

An example is the load on a structure due to the dead weight of the building materials.



Stress versus time for static loading.

Failure Theories

When a part is designed, it is of primary interest to know if the part will fail during service.

One of the design objectives is to ensure that the material has the strength to sustain the stress. Since materials behave differently under different types of loading conditions, various failure theories have been proposed and tested.

The theories presented are accepted practices for designers to compare some defined stresses to some defined strengths.

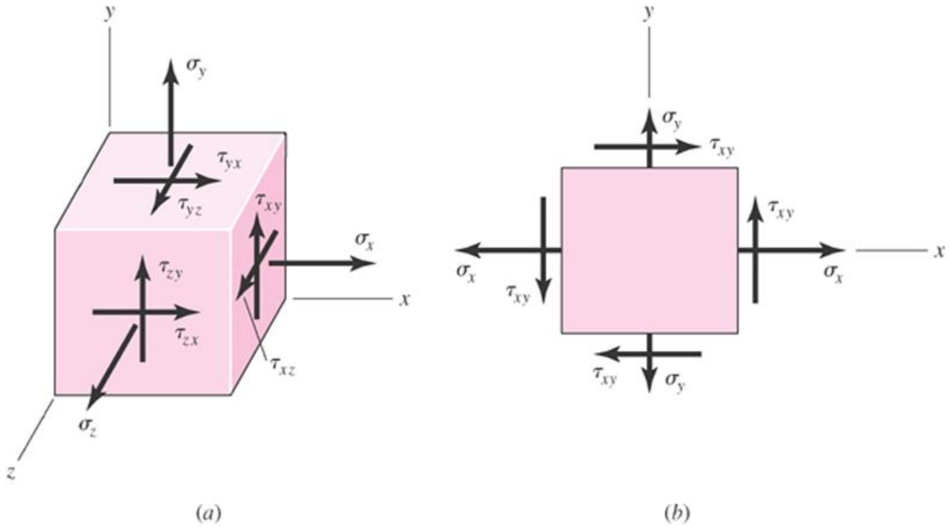
For ductile materials under static loading, a stress element is considered to have failed when yielding occurs. Thus, the failure theories are based on established yield criteria. As brittle materials do not exhibit yielding before fracture, the evaluation of failure is based on postulated fracture criteria.

Need for Static Failure Theories

- Uniaxial stress element (e.g. tension test)

$$n = \frac{Strength}{Stress} = \frac{S}{\sigma}$$

- Multi-axial stress element
  - One strength, multiple stresses
  - How to compare stress state to single strength?



Need for Static Failure Theories

- Failure theories propose appropriate means of comparing multi-axial stress states to single strength

- Usually based on some hypothesis of what aspect of the stress state is critical

- Some failure theories have gained recognition of usefulness for various situations

What is Failure

It is any change in a machine part which makes it unable to perform its intended function.

It is essential to be able to predict the capability of materials to withstand the infinite combination of non-standard loads

What is Static Loading?

No impact, fatigue nor surface wear.

What to avoid?

Unwanted deflection, unwanted elastic instability (such as buckling)

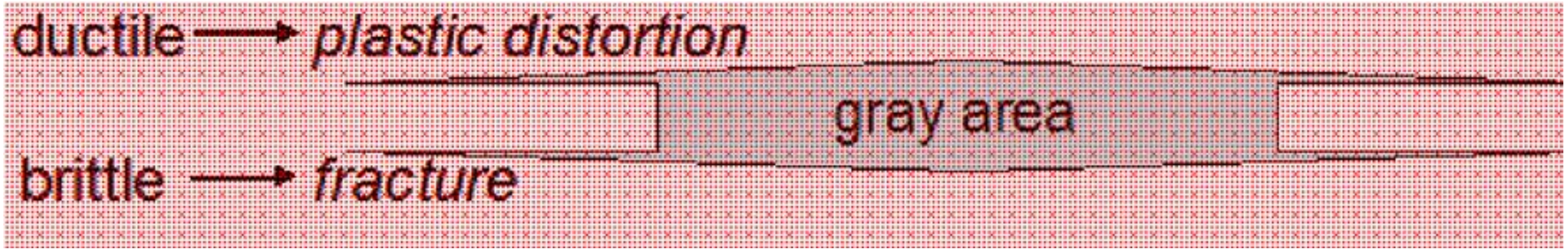
Mechanical Properties of Materials

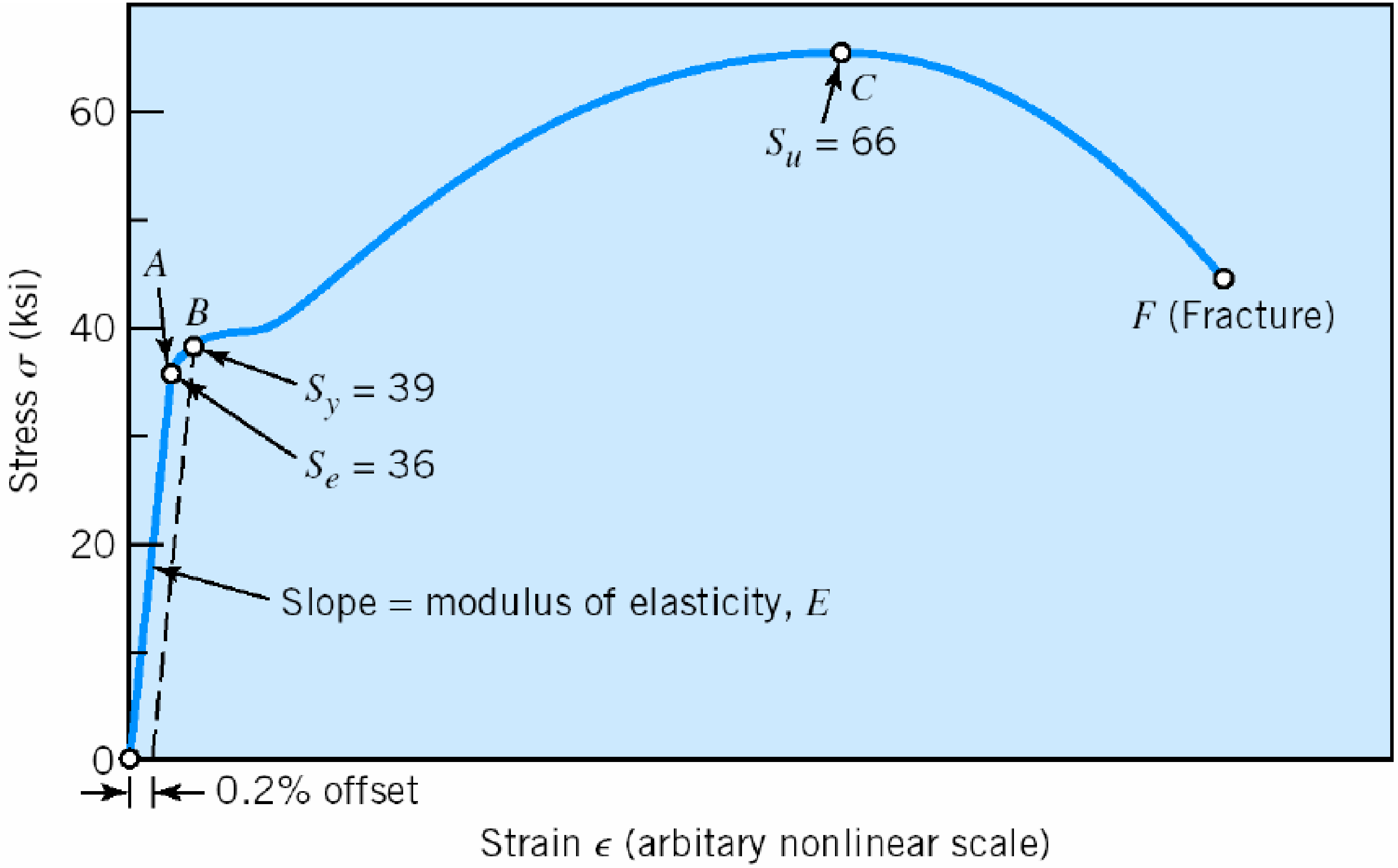
MECHANICAL PROPERTIES	DEFINATION
Strength	Ability to resist stress without failure under static loading.
Fatigue strength	Ability to resist stress without failure under fatigue loading.
Elasticity	Ability to regain original shape after load removal.
Plasticity	Ability to retain permanent deformation after load removal.
Stiffness/Rigidity	Ability to resist deformation under load.

Mechanical Properties of Materials

Ductility	Ability to be drawn into wire or elongated.
Brittleness	Ability to rapture with negligible deformation.
Malleability	Ability to undergo change in size and shape.
Resilience	Capacity to absorb energy when deformed within elastic range.
Toughness	Capacity to absorb energy without fracture.
Hardness	Resistance to penetration, abrasion, or plastic deformation.
Creep	Progressive deformation under load at high temperature.

How much distortion is too much





Factor of Safety

A factor of safety increases the safety of people and reduces the risk of failure of a product.

When it comes to safety equipment and fall protection, the factor of safety is extremely important.

If a structure fails there is a risk of injury and death as well as a company's financial loss.

The safety factor is higher when there is a possibility that a failure will result in these things.

How much distortion is too much

Factor of Safety

While designing a component, it is necessary to provide sufficient reserve strength in case of accident. This is achieved by taking suitable factor of safety (n).

$$n = \frac{Failure\ stress}{allowable\ stress} \quad \text{or } n = \frac{failure\ load}{allowable\ load}$$

The allowable stress is the stress value which is used in design to determine the dimensions of the component. It is considered as a stress, which the designer expect will NOT be exceeded under normal operating conditions.

How much distortion is too much

There is a number of reasons that forces us to implement or introduce a factor of safety. For example:

1. Uncertainty in the magnitude of external forces acting on the component.
2. Variation in properties of material like yield strength or ultimate strength.
3. Variation in the dimensions of the component due to imperfect workmanship.

and many more reasons…

How much distortion is too much

The magnitude of factor of safety depends upon the following factors:

1. **Effect of failure**
2. **Type of load**
3. **Degree of accuracy in force analysis**
4. **Material of component**
5. **Reliability of component**
6. **Cost of component**
7. **Testing of machine elements**
8. **Service condition**
9. **Quality of manufacturer**

Factor of Safety

Typical overall Factors of Safety:

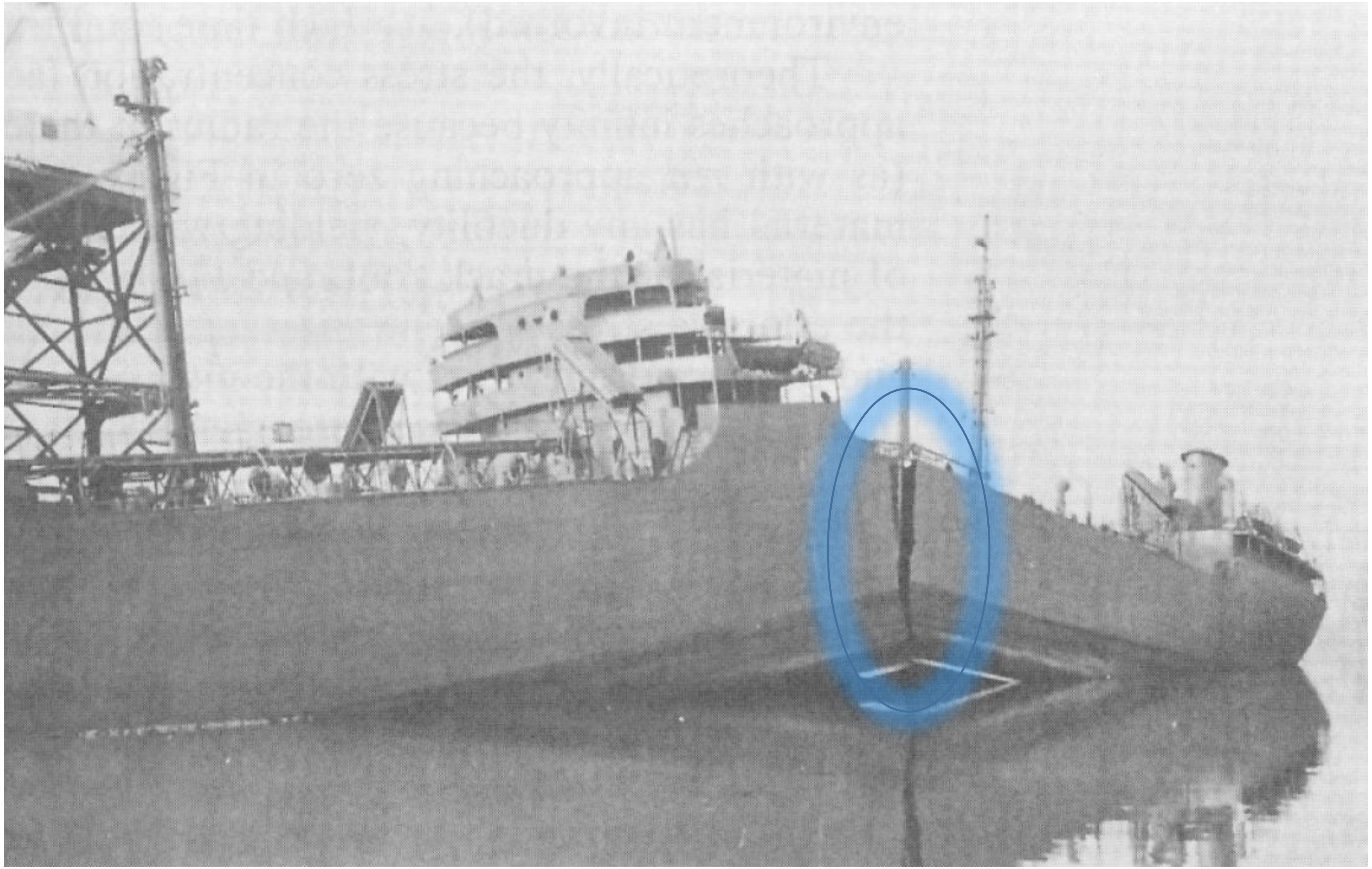
Equipment	Factor of Safety - *FOS* -
Aircraft components	1.5 - 2.5
Boilers	3.5 - 6
Bolts	8.5
Cast-iron wheels	20
Engine components	6 - 8
Heavy duty shafting	10 - 12
Lifting equipment - hooks ..	8 - 9
Pressure vessels	3.5 - 6
Turbine components - static	6 - 8
Turbine components - rotating	2 - 3
Spring, large heavy-duty	4.5
Structural steel work in buildings	4 - 6
Structural steel work in bridges	5 - 7
Wire ropes	8 - 9

Factor of Safety

General recommendations

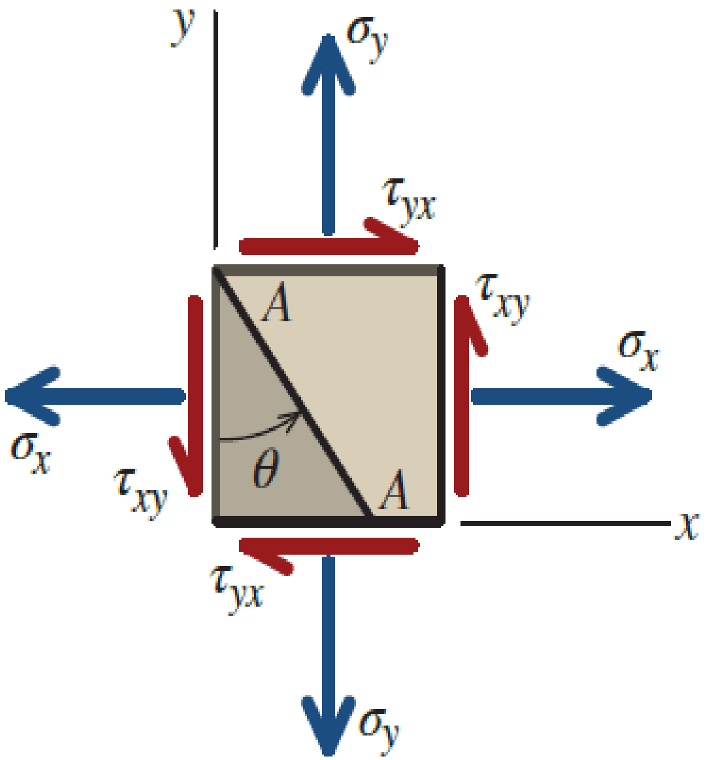
Applications	Factor of Safety - *FOS* -
For use with highly reliable materials where loading and environmental conditions are not severe and where weight is an important consideration	1.3 - 1.5
For use with reliable materials where loading and environmental conditions are not severe	1.5 - 2
For use with ordinary materials where loading and environmental conditions are not severe	2 - 2.5
For use with less tried and for brittle materials where loading and environmental conditions are not severe	2.5 - 3
For use with materials where properties are not reliable and where loading and environmental conditions are not severe, or where reliable materials are used under difficult and environmental conditions	3 - 4

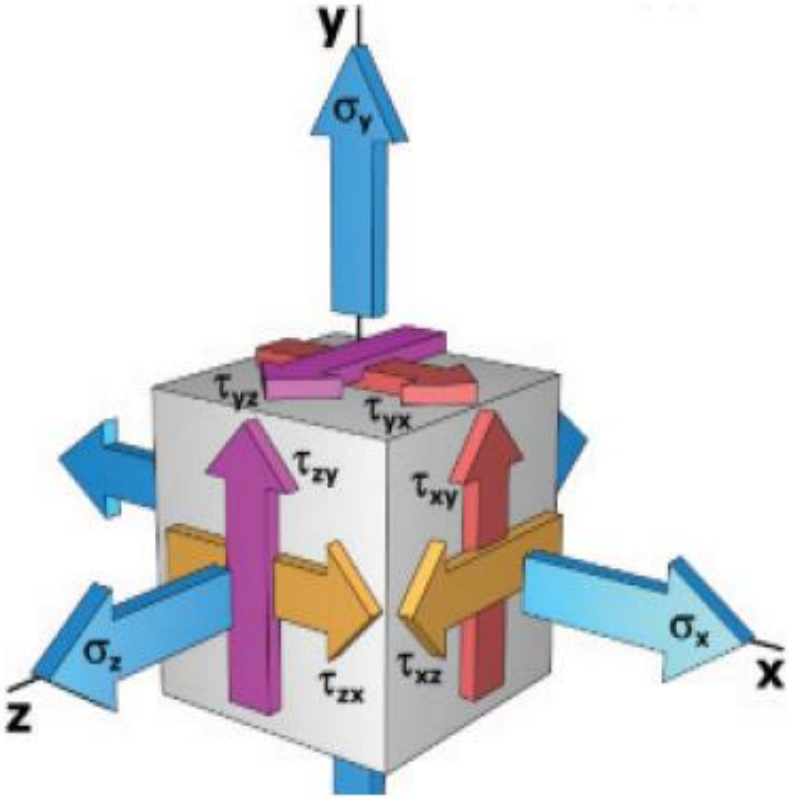
How much distortion is too much



World War II tanker broken in two by a brittle fracture, despite the normal ductility of the used steel.

Mohr's Circle for Plane Stress

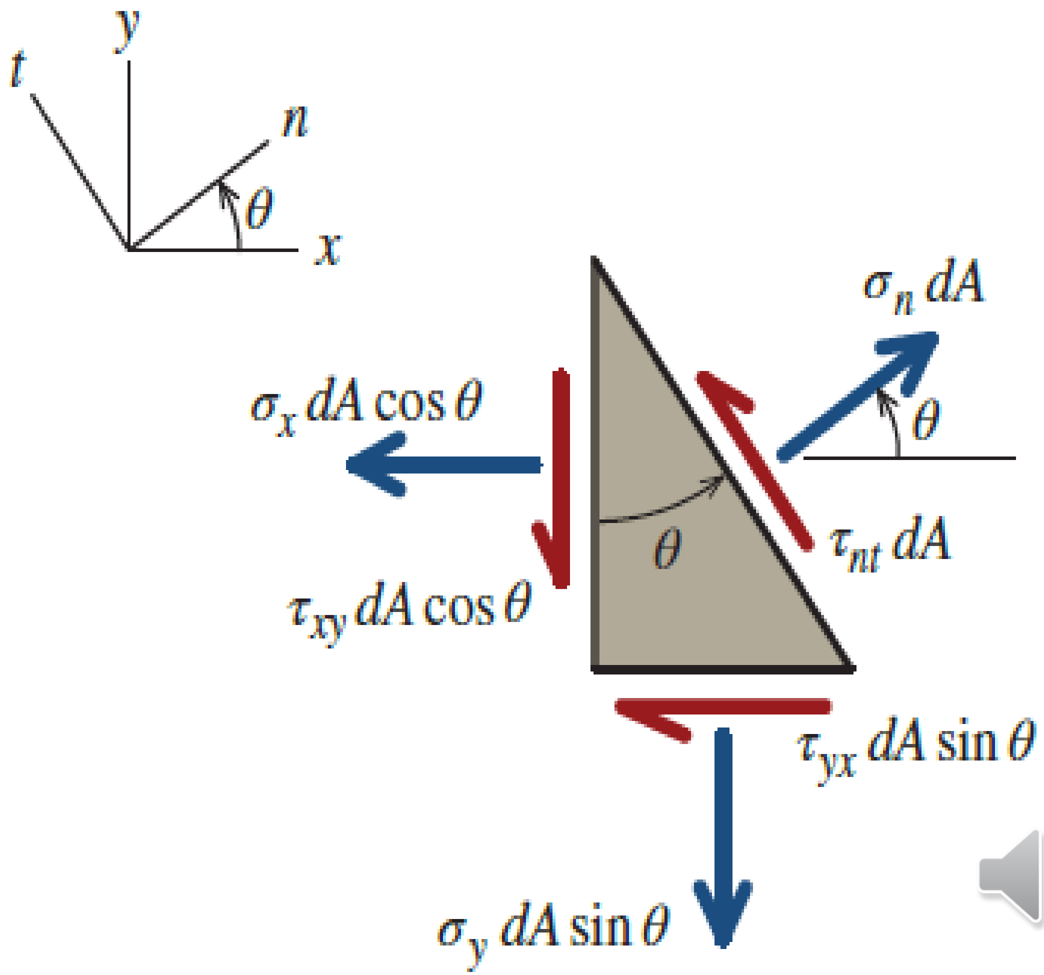




Mohr's Circle for Plane Stress

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

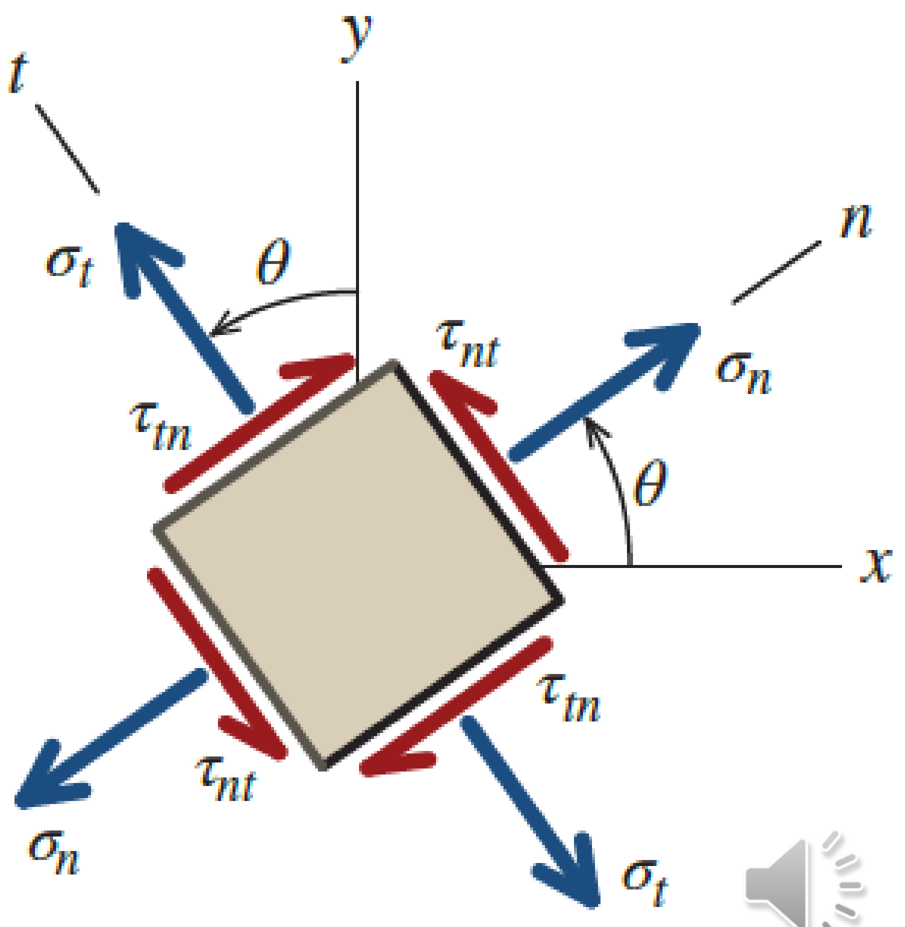


Mohr's Circle for Plane Stress

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_t = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_n + \sigma_t = \sigma_x + \sigma_y$$

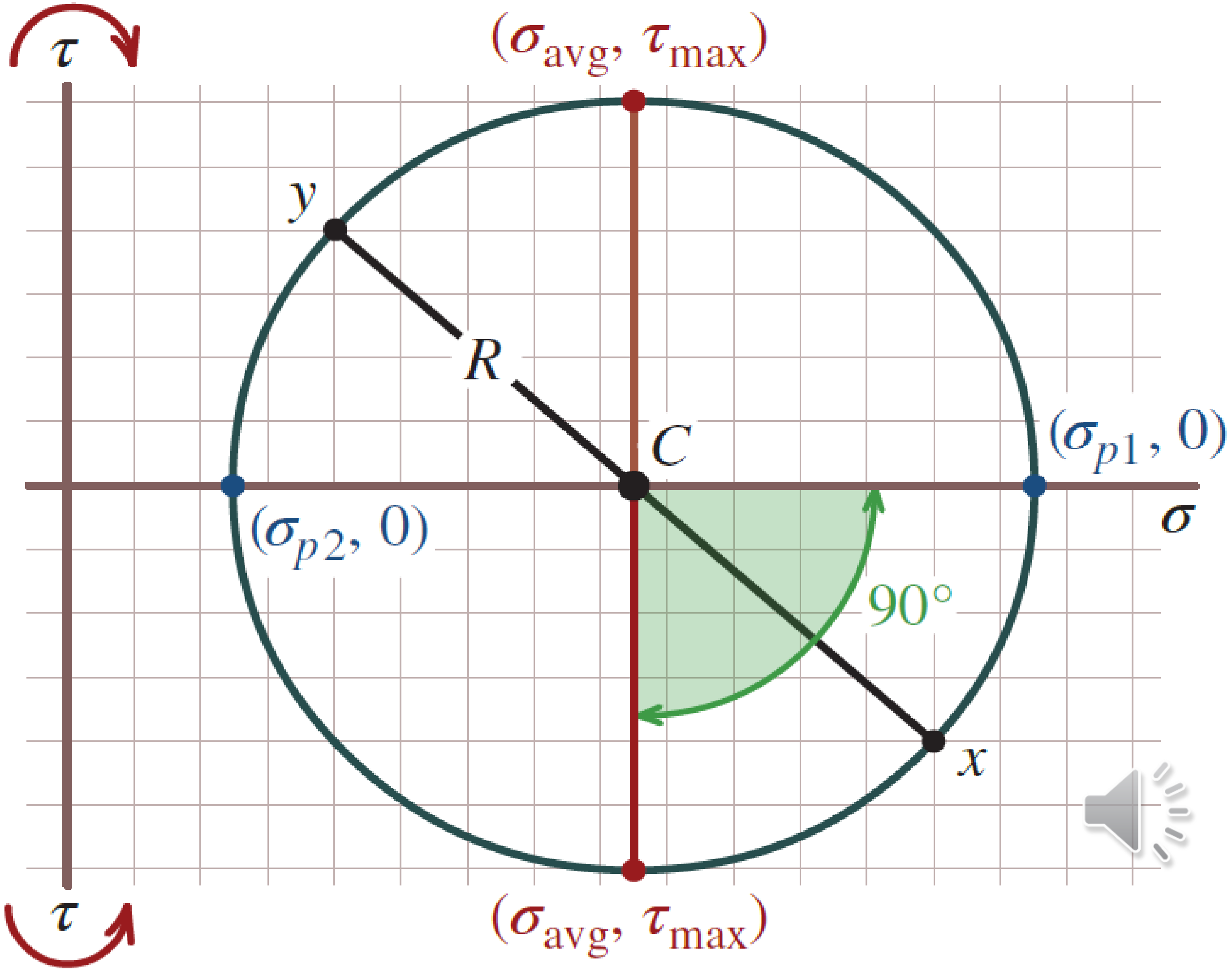


Mohr's Circle for Plane Stress

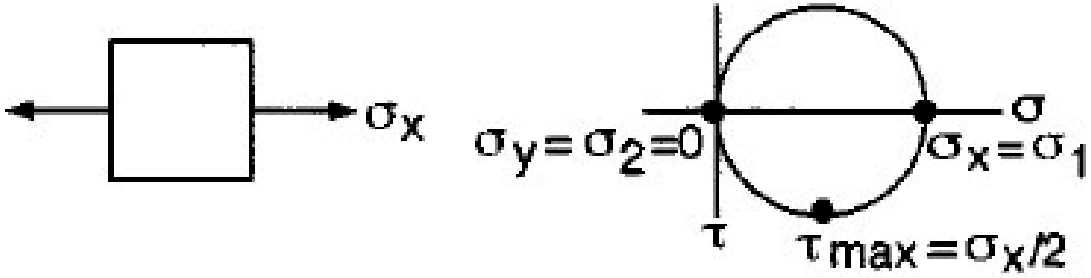
$$(\sigma_n - C)^2 + \tau_{nt}^2 = R^2$$

$$C = \frac{\sigma_x + \sigma_y}{2}$$

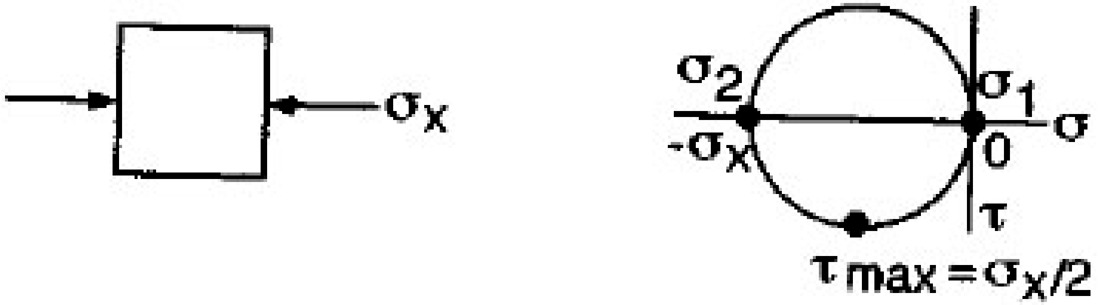
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



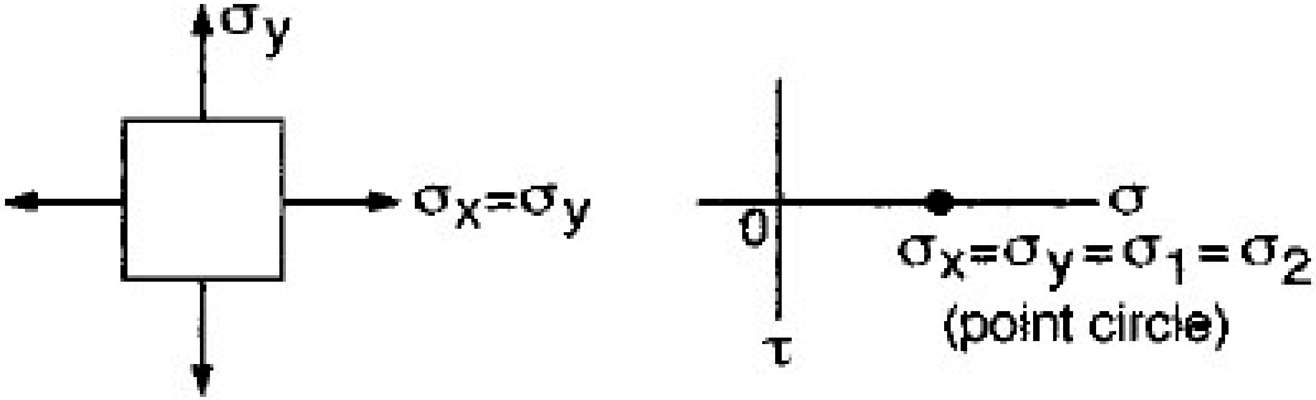
Special Cases of Mohr's Circle for Plane Stress



Uniaxial Tension

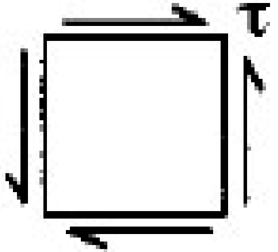


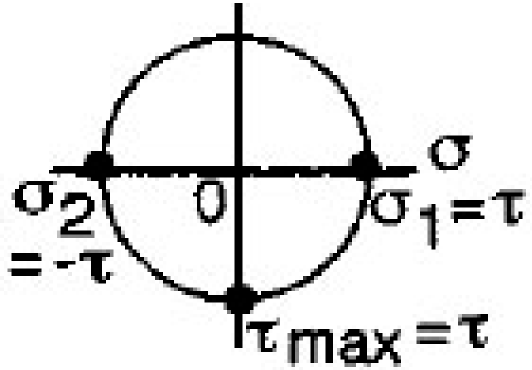
Uniaxial Compression



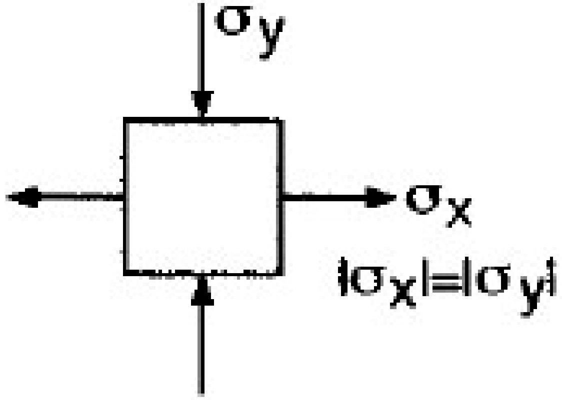
Biaxial Tension

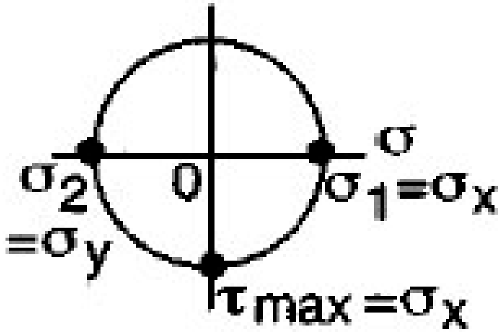
Special Cases of Mohr's Circle for Plane Stress





Pure Shear





Biaxial Tension-Compression

Failure mechanisms

1. Wear and tear
2. Fatigue – cyclic loading
3. Corrosion
4. Creep
5. Buckling
6. Friction, etc..

Failure of materials is usually classified into

1.) Ductile failure (yielding)

2.) Brittle failure (fracture)

A material can fail in brittle or ductile manner or both, depending upon the conditions.

Ductile Vs Brittle Fracture

	Ductile	Brittle
Fracture Stress	Greater than yield strength	Lower than yield strength
Energy Absorption	High	Low
Nature of Fracture	Necking, rough fracture surface, linking up of cavities	No necking, shiny granular surface, cleavage or intergranular
Type of Material	Metals	Ceramics, glasses
Crack Propagation	Slow	Fast
Nature of Failure	Plastic deformation warning, less catastrophic	Little deformation, more catastrophic

Theories of Failure

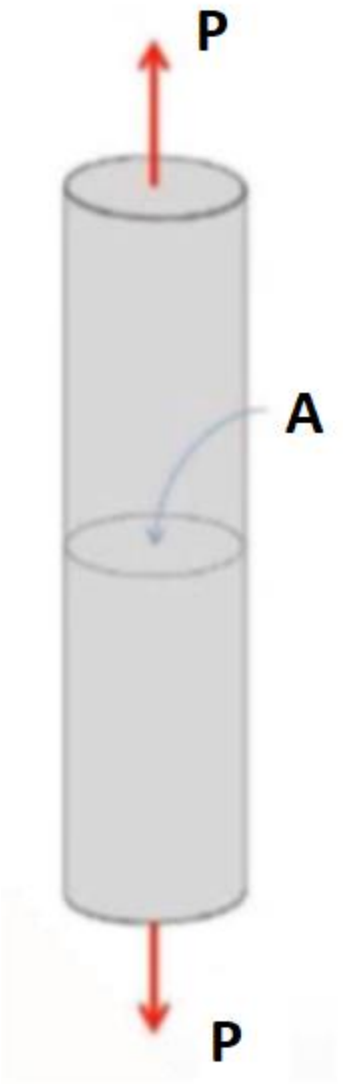
A “theory of failure” is a theory for predicting the conditions under which the (solid) materials fail under the action of external loads.

This, of course, is while ignoring the damage due to any causes internal to the material, for example, phase change.

Developing a “theory of failure” is difficult because determining the type of the predominant stresses – tensile, compressive, shear – is difficult.

Also, whether the predominant type of failure for the material is ductile or brittle is a significant factor ⟹ yield stress or ultimate stress

Theories of Failure in 1D



$$normal\ stress, \qquad \sigma_1 = \frac{P}{A}$$

Criterion:

$$\boxed{\sigma_1 \leq \sigma_{yield}}$$

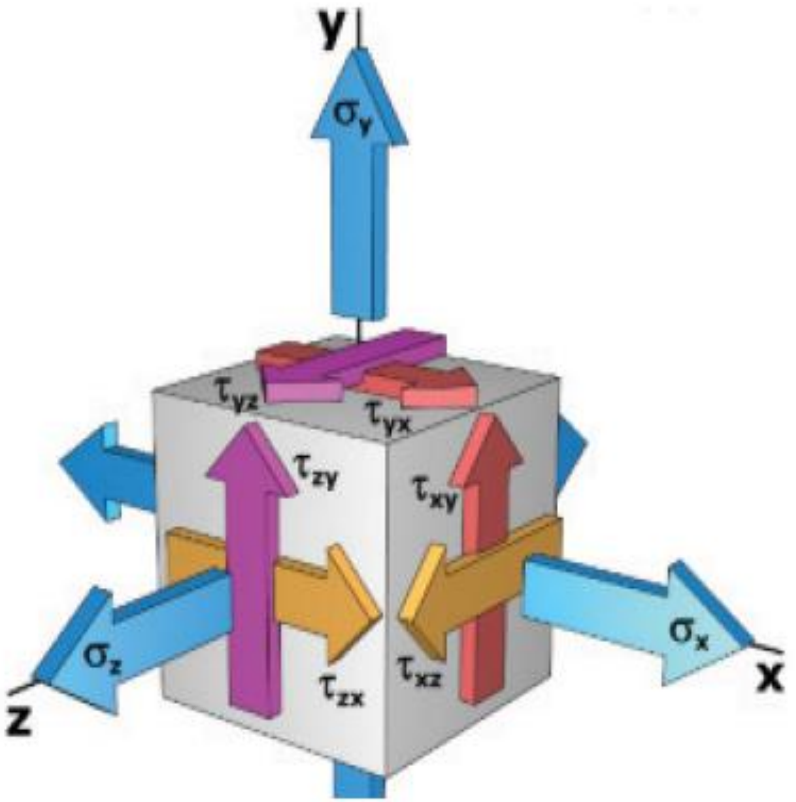
Theories of Failure in 3D

State of stress in 3D is complicated. So, we need better theories.

A good failure theory requires:

- Observation of a large number of test results in different load conditions (empirical knowledge)

- A theory explaining the microscopic mechanisms involved which must be consistent with the observations



Failure Theories

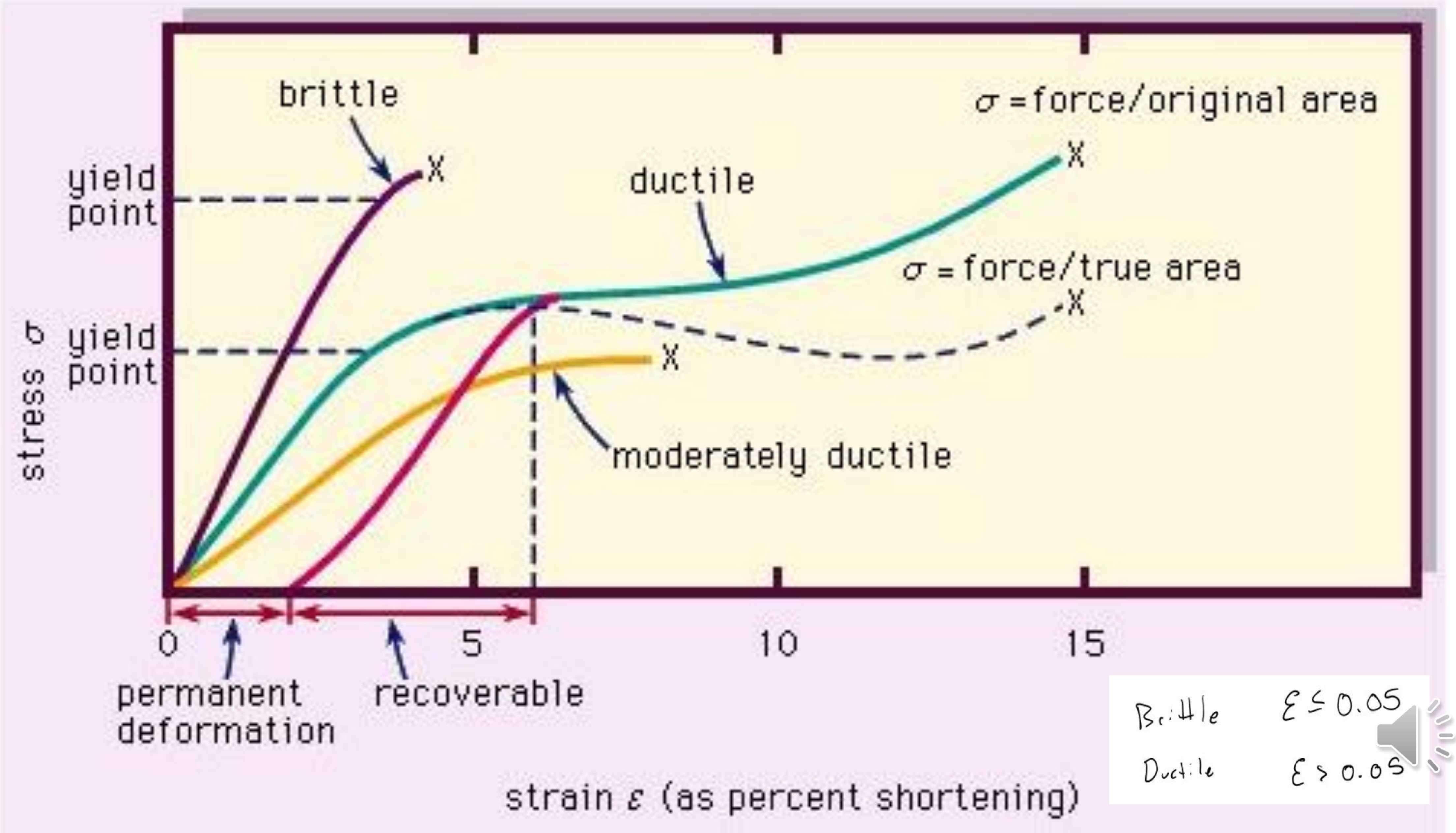
Load type
- Uniaxial
- Biaxial
- Pure Shear

Material Property
- Ductile
- Brittle

Application of Stress
- Static
- Dynamic

Static Loading
- Maximum Normal Stress
- Modified Mohr
- Yield strength
- Maximum shear stress
- Distortion energy

Dynamic Loading
- Goodman
- Gerber
- Soderberg



Brittle $\varepsilon \leq 0.05$

Ductile $\varepsilon > 0.05$

Theories of Failure

- Rankine or Maximum Principal stress theory
- Saint – venant or Maximum Principal strain theory
- Tresca or Maximum shear stress theory
- Von Mises & Hencky or Shear strain energy theory
- *Haigh or Total strain energy per unit volume theory
- Mohr-Coulomb failure theory – cohesive-frictional solids
- *Drucker-Prager failure theory – pressure dependent solids
- *Cam-Clay failure theory - soils

- **Homework: Please find out what these theories are. A brief description on MS Teams**

Theories of Failure

- Many failure theories exist based on critical stress, critical strain, and critical energy.

- Selection of the proper failure theory depends on the stress state, material type, material symmetry, boundary conditions, environment, etc.

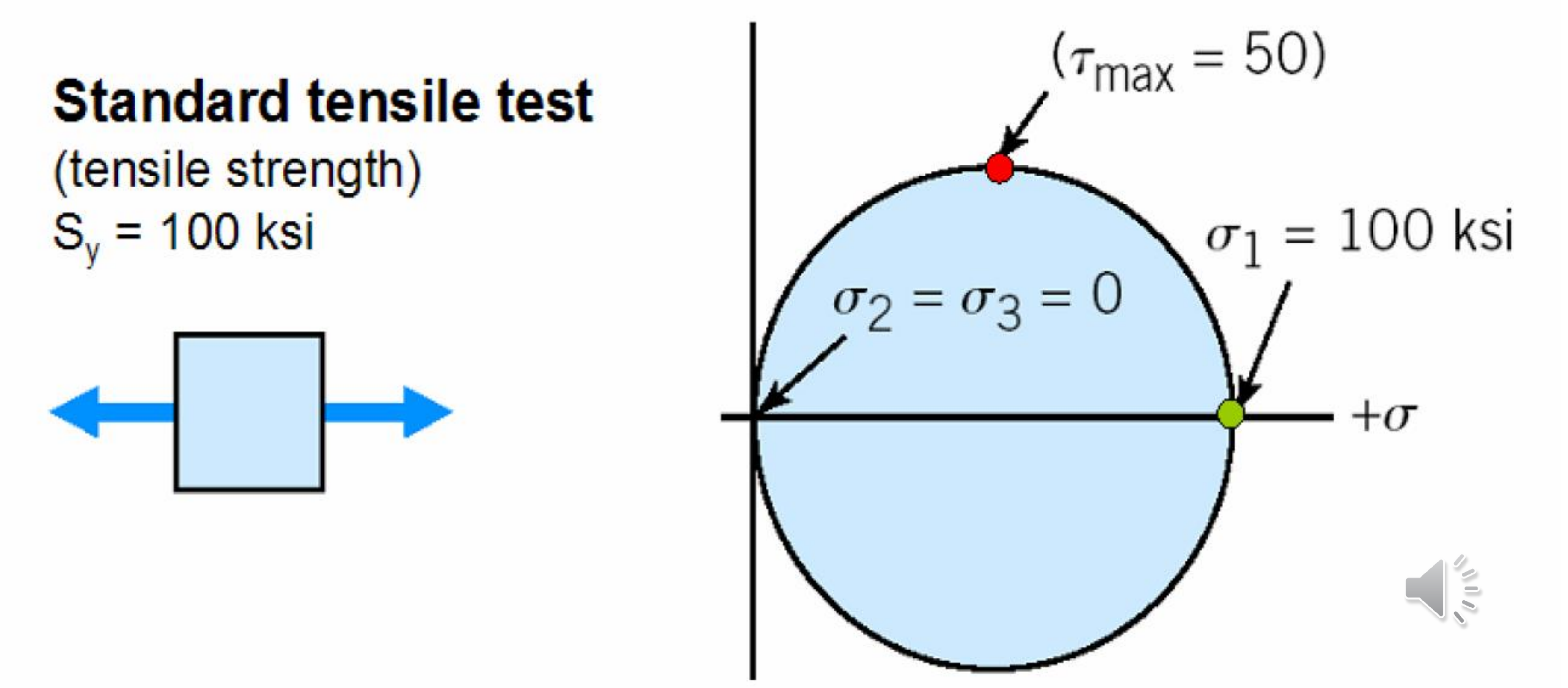
Theories of Failure

	Ductile Material	Brittle Material
Characteristic Failure Stress	Yield Stress	Ultimate Stress
Important Theories	1. Maximum Shear Stress 2. Maximum Octahedral Shear Stress	1. Maximum Normal Stress 2. Modified Mohr.

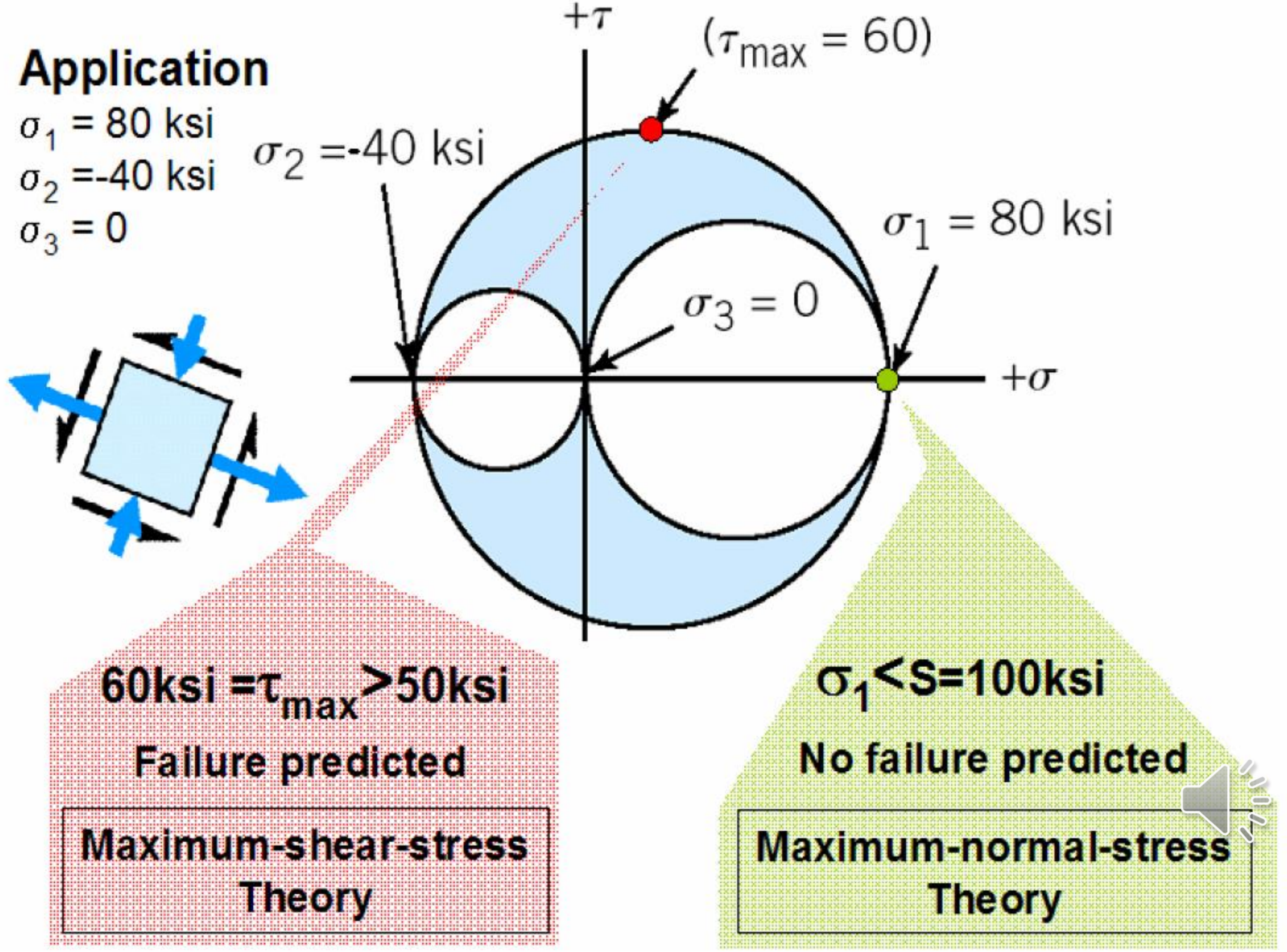
Brittle $\varepsilon \leq 0.05$

Ductile $\varepsilon > 0.05$

Theories of Failure



Theories of Failure

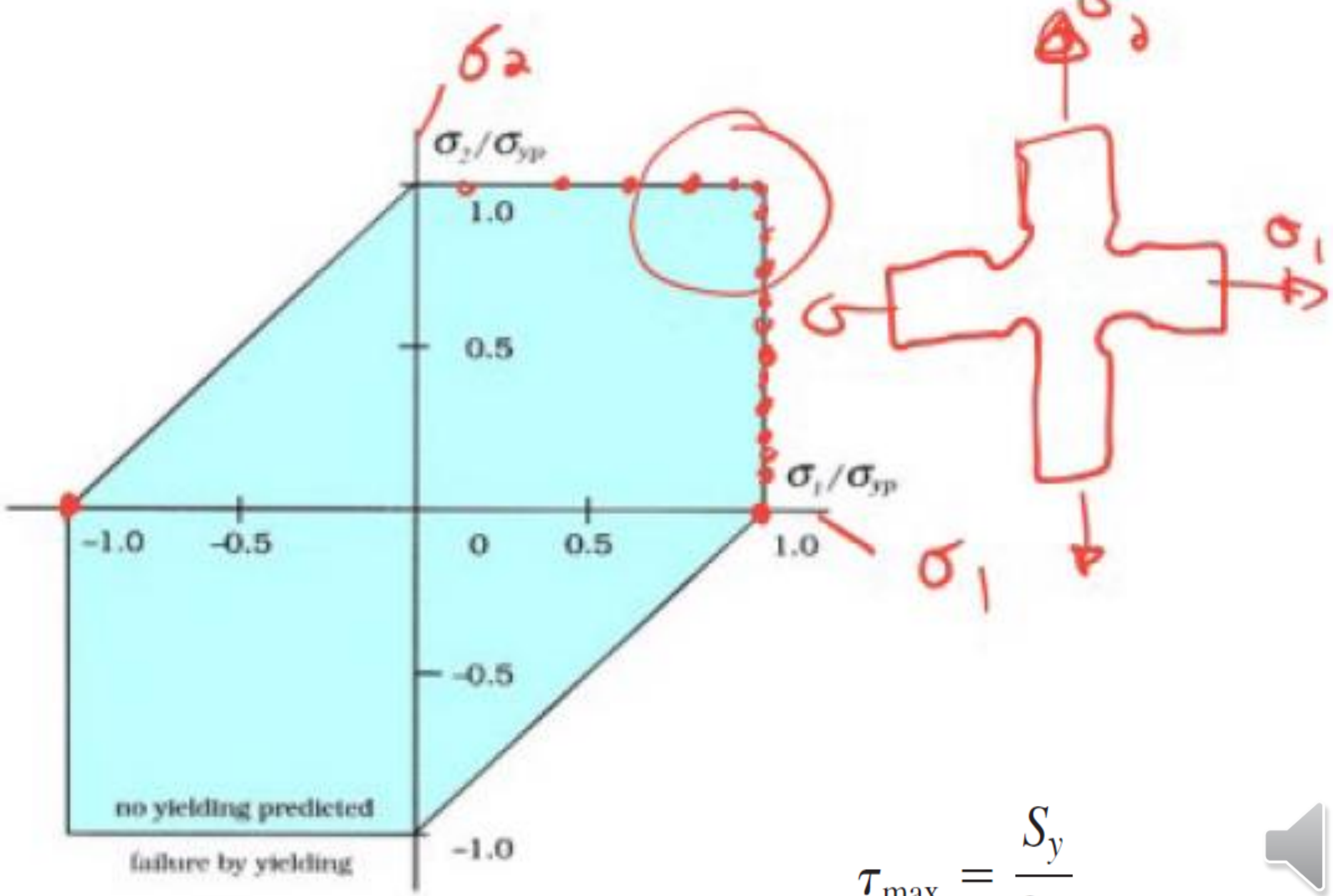


Maximum Shear-Stress-Theory

(C.A. Coulomb 1736-1806, French scientist). Also called: Tresca Theory or Guest's Law

Failure occurs, when maximum shear stress exceeds shear strength in uniaxial tension test.

"Correlates well for ductile yielding"



$$\tau_{max} = \frac{S_y}{2n}$$

Maximum Shear-Stress-Theory

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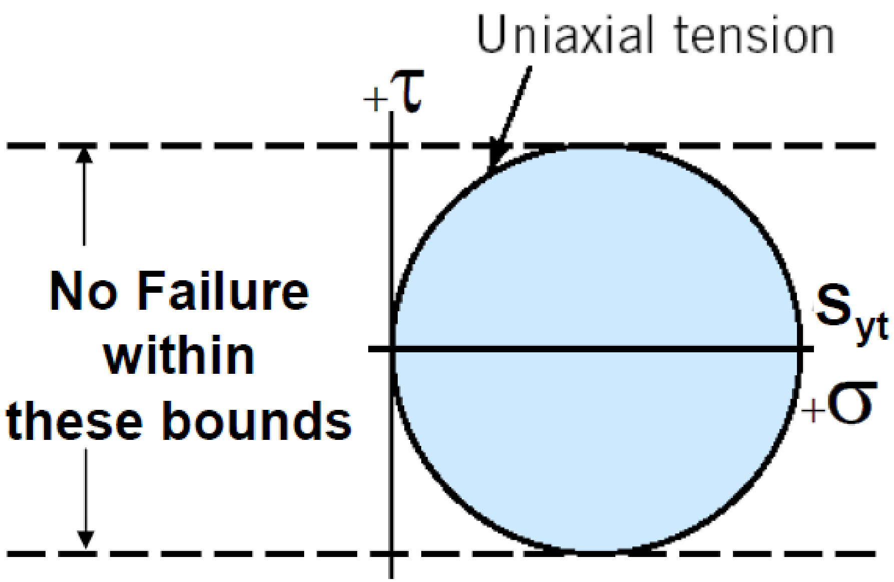
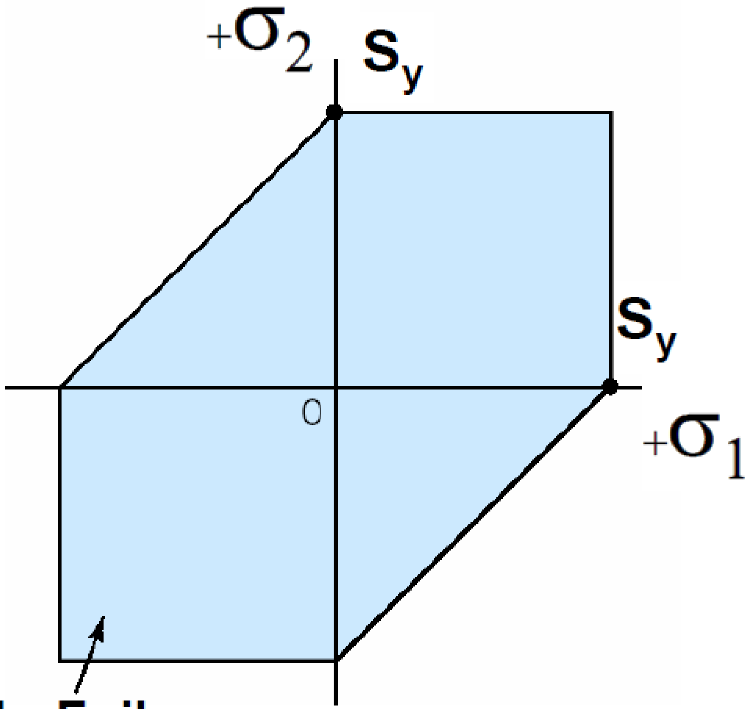


$$\frac{\sigma_1 - \sigma_2}{2} = \tau_{max} = \frac{S_y}{2}$$

Principle Mohr circles



$\sigma_1 - \sigma_2$ plot

$$\tau_{max} = \frac{S_y}{2n}$$

Maximum Shear-Stress-Theory

$$\sigma_1 > \sigma_2 > \sigma_3 \rightarrow \text{3D Mohr's} \rightarrow \tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

$$= \quad \underline{\underline{\sigma}} = \begin{bmatrix} S_y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sigma_1 = S_y$$

$$\sigma_2 = \sigma_3 = 0, \quad \tau_y = \frac{S_y}{2}$$

$$\tau_{max} \geq \tau_y \qquad \frac{\sigma_1 - \sigma_3}{2} \geq \frac{S_y}{2}$$

Maximum Shear-Stress-Theory

For yielding

$$\sigma_1 - \sigma_3 \geq S_y$$

For fracture

$$\sigma_1 - \sigma_3 \geq S_{UT}$$

Criterion for Design

$$\sigma_1 - \sigma_3 = \frac{S_y}{n}$$

n is the design safety factor

Maximum Shear-Stress-Theory

The MSS theory is plotted by plotting three ideal cases on the principal stress axes.

Case 1: $\sigma_A \geq \sigma_B \geq 0$

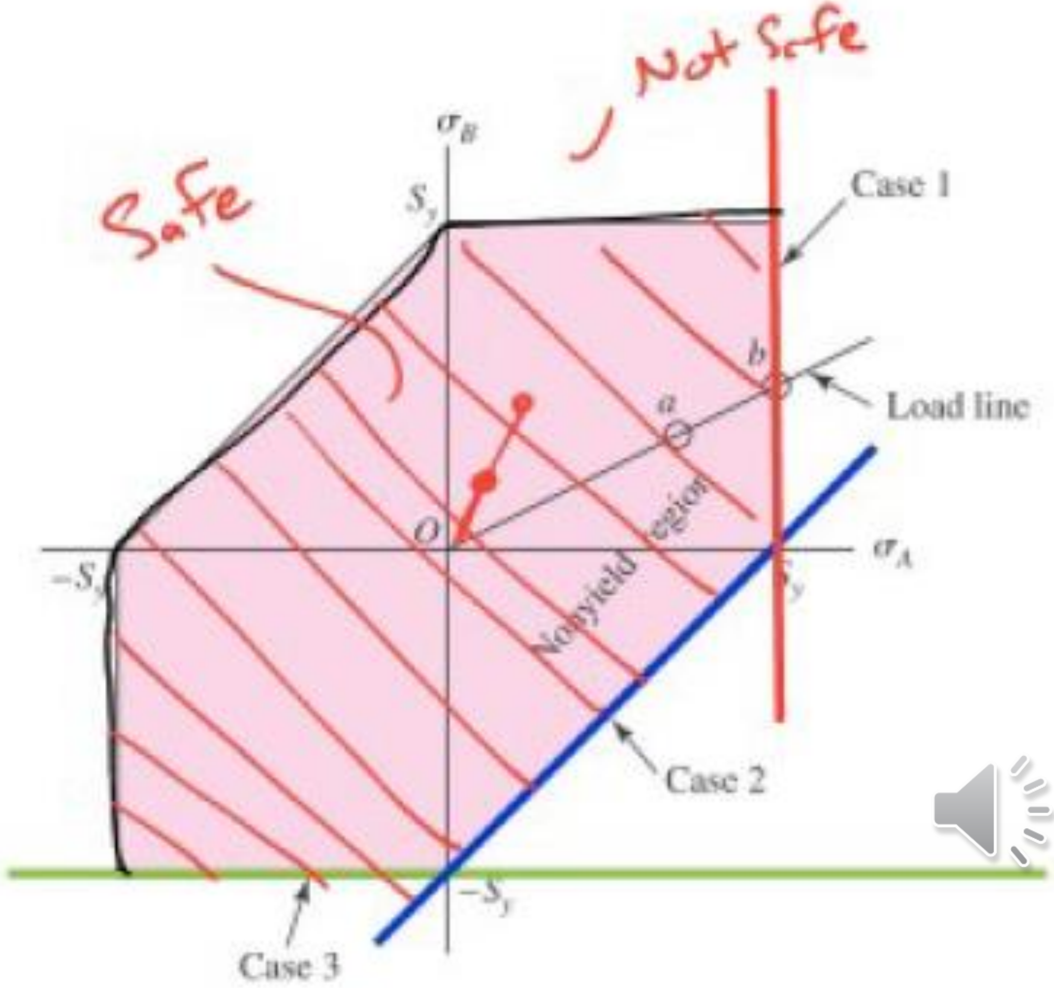
$\sigma_A \geq S_y$

Case 2: $\sigma_A \geq 0 \geq \sigma_B$

$\sigma_A - \sigma_B \geq S_y$

Case 3: $0 \geq \sigma_A \geq \sigma_B$

$\sigma_B \leq -S_y$

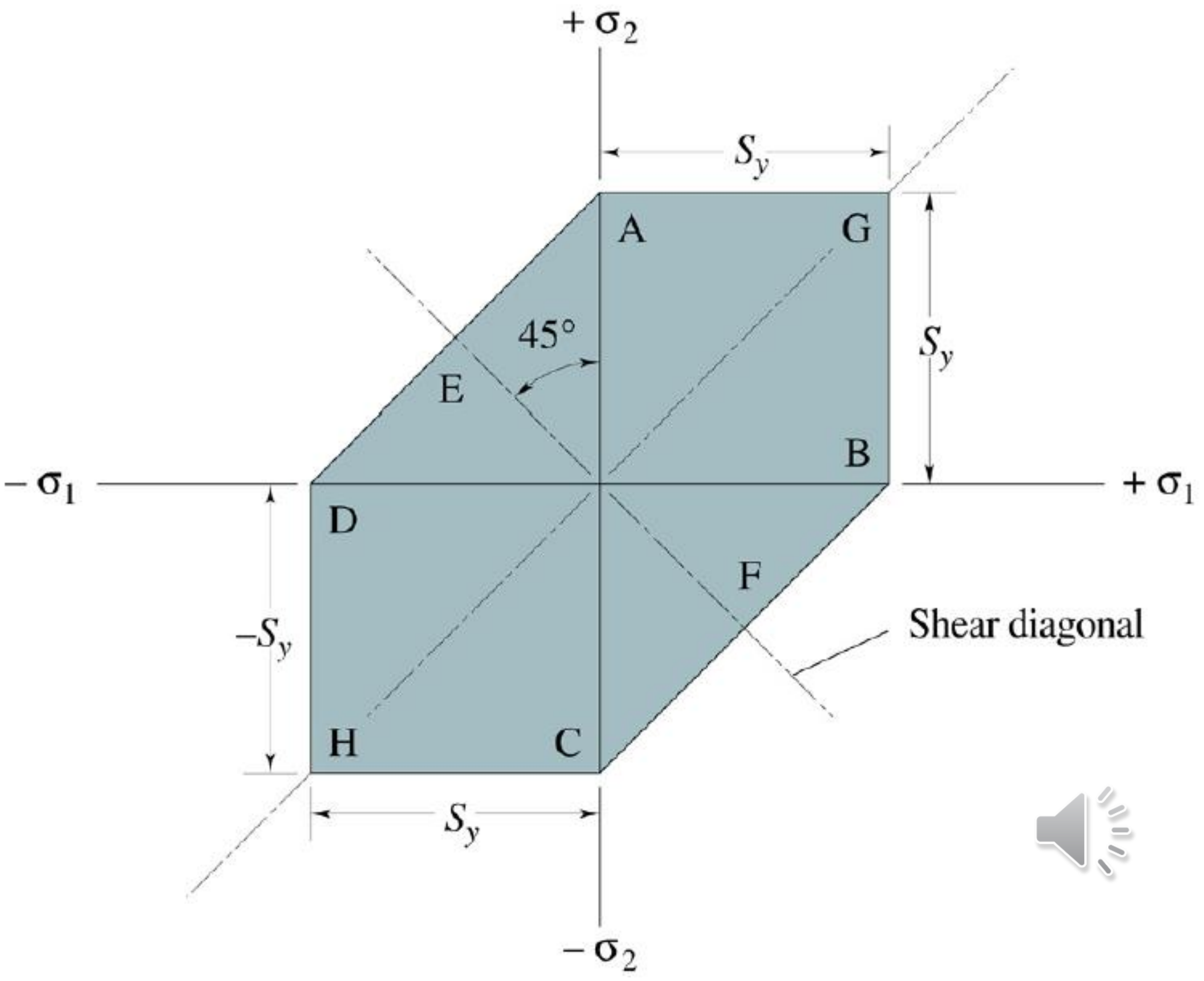


Maximum Shear-Stress-Theory

For a general state of stress:

$$\frac{\sigma_1 - \sigma_2}{2} = \tau_{max} = \frac{S_y}{2}$$

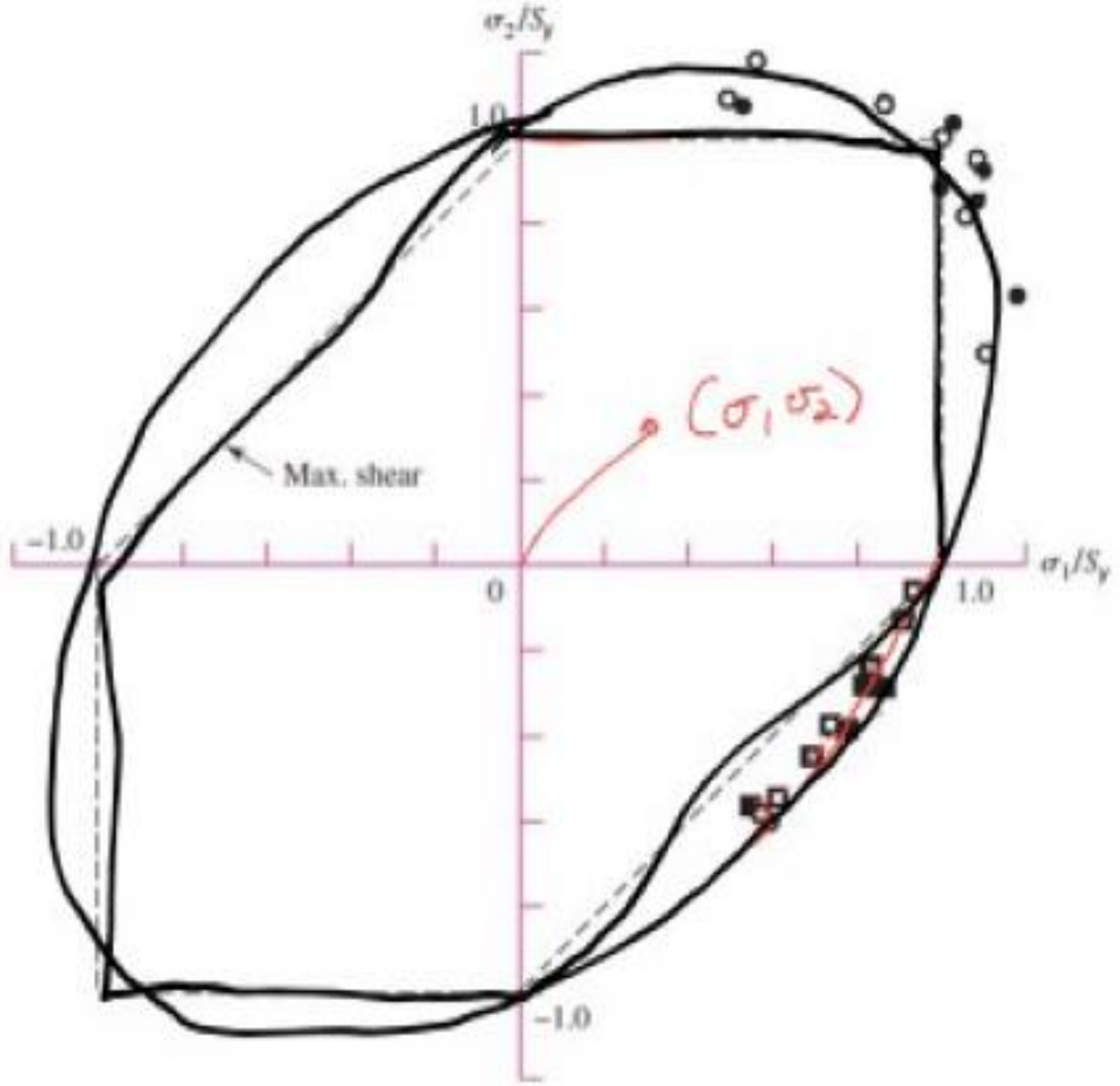
This leads to an hexagonal failure envelop. A stress system in the interior of the envelop is considered SAFE



Maximum Shear-Stress-Theory

MSS plot with real data:

- Comparison to experimental dat
- Conservative in all quadrant.
- Commonly used for design situations.



Maximum Distortion-Energy-Theory

(Maxwell 1856 English, Hueber 1904 Polish, Mises 1913 & Hencky 1925 German/US). Also called: Maximum-Octahedral-Shear Theory.

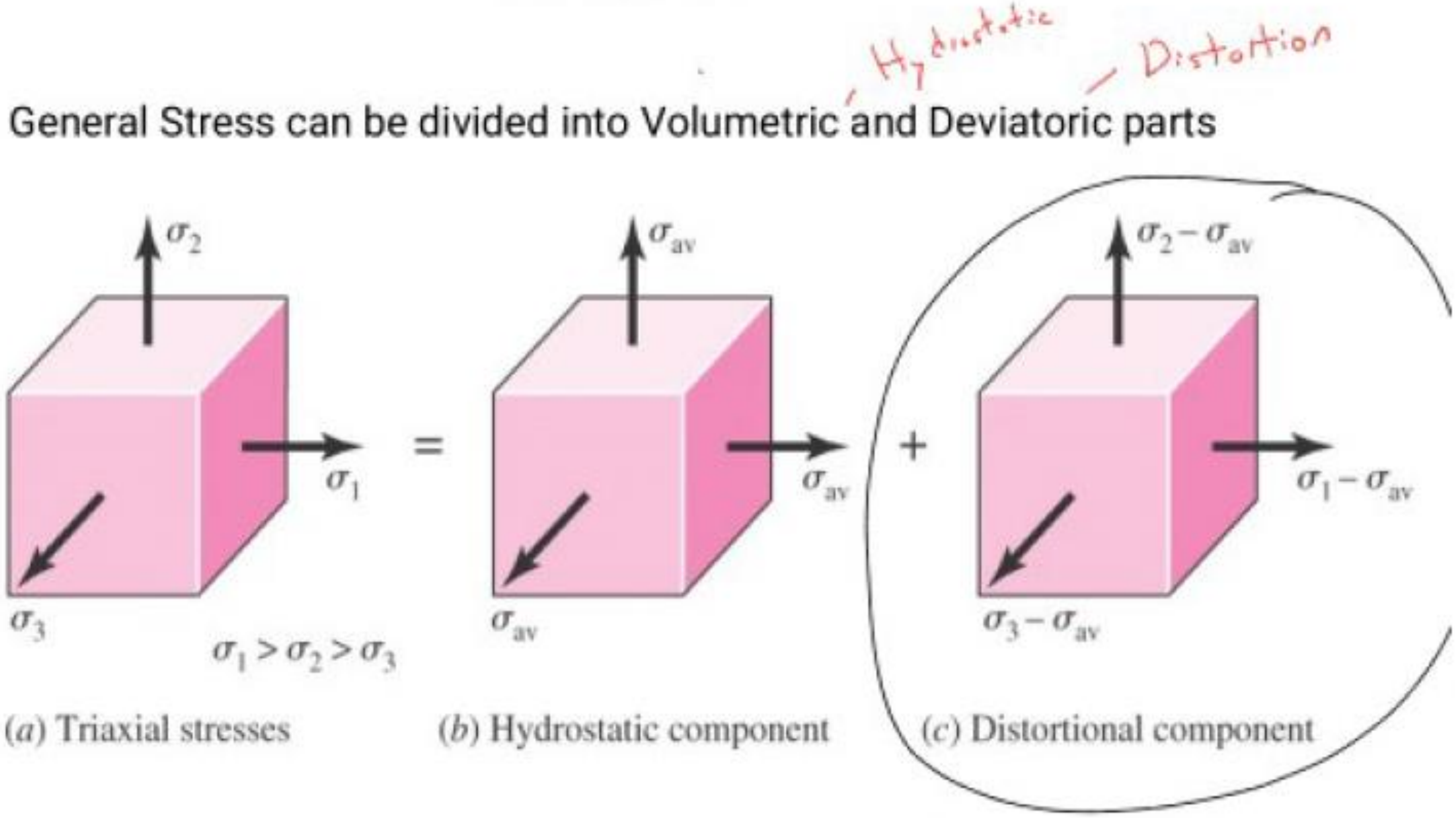
Failure occurs, when distortion-energy in **unit volume** (arbitrary load condition) **equals distortion-energy** in **same volume for uniaxial yielding.**

Resilience is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered.

$$U_r = \int_0^{\varepsilon_y} \sigma d\varepsilon$$

Maximum Distortion-Energy-Theory

General Stress can be divided into Volumetric and Deviatoric parts



(*a*) Triaxial stresses (*b*) Hydrostatic component (*c*) Distortional component

Maximum Distortion-Energy-Theory

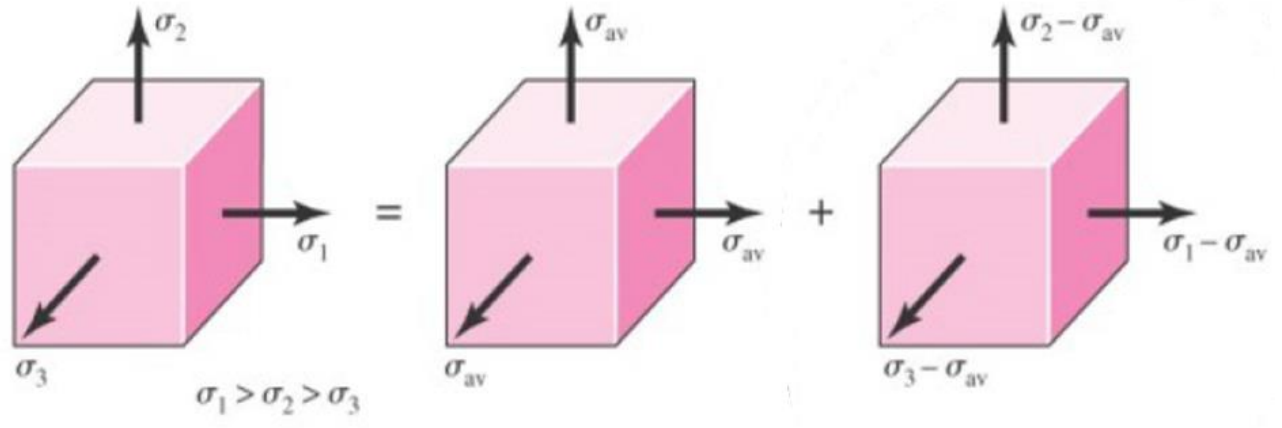
$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ & \sigma_y & \tau_{yz} \\ & & \sigma_z \end{bmatrix}$$

$$\underline{\underline{\sigma}} = \underbrace{\sigma_{avg} \cdot \underline{\underline{I}}}_{\text{Hydrostatic}} + \underbrace{\underline{\underline{\sigma}}_d}_{\text{Distortion}}$$

$$\sigma_d = \underline{\underline{\sigma}} - \sigma_{avg} \cdot \underline{\underline{I}}$$

$$\underline{\underline{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

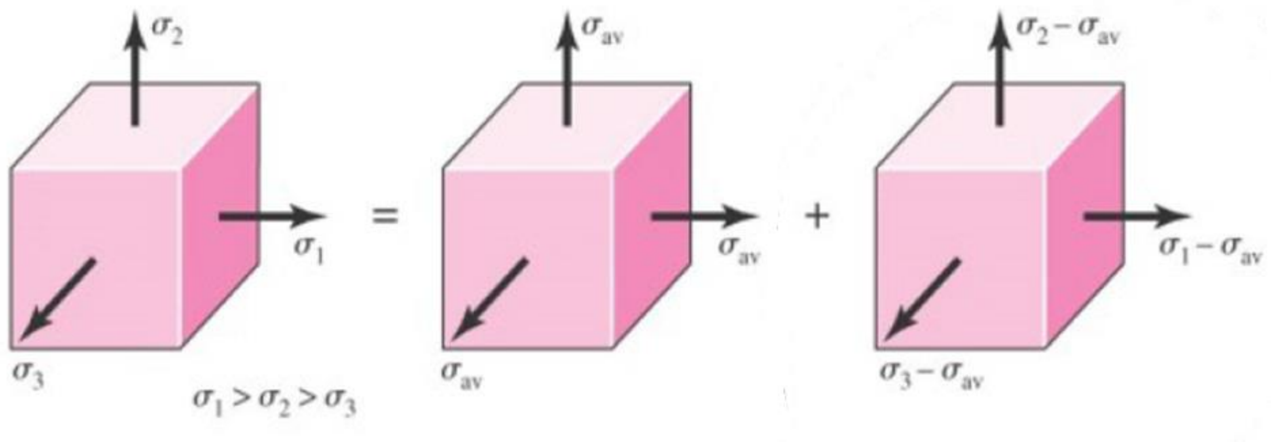
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$



(a) Triaxial stresses (b) Hydrostatic component (c) Distortional component

Maximum Distortion-Energy-Theory

Originated from observation that ductile materials stresses hydrostatically (equal principal stresses) exhibited yield strengths greatly in excess of expected values.



(*a*) Triaxial stresses (*b*) Hydrostatic component (*c*) Distortional component

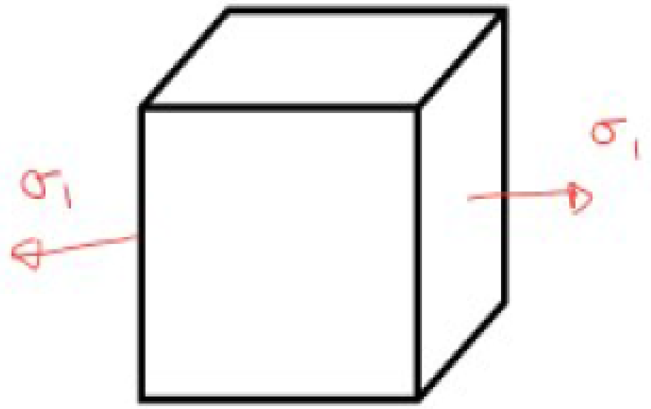
Theorize that if strain energy is divided into hydrostatic volume changing energy and angular distortion energy, the **yielding is primarily affected by the distortion energy**.

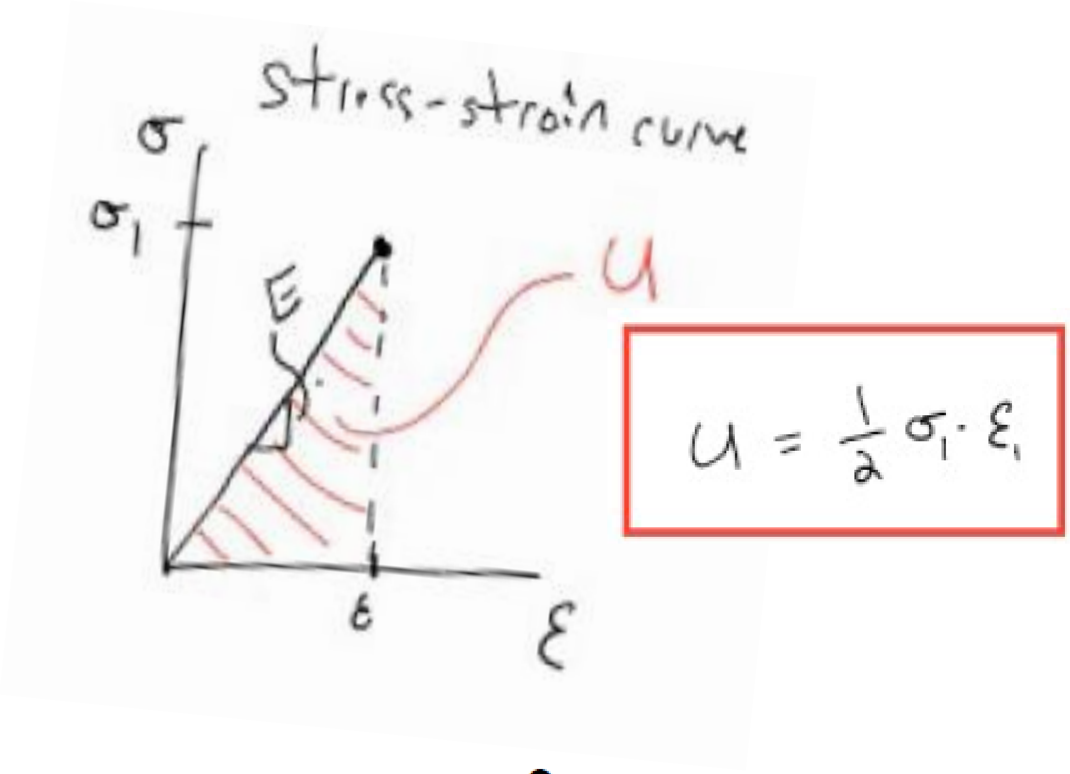
Or:

Failure occurs, when distortion-energy in **unit volume** (arbitrary load condition) **equals distortion-energy** in **same volume for uniaxial yielding.**

Maximum Distortion-Energy-Theory

Consider a Linear Elastic, Isotropic, Homogeneous Material:



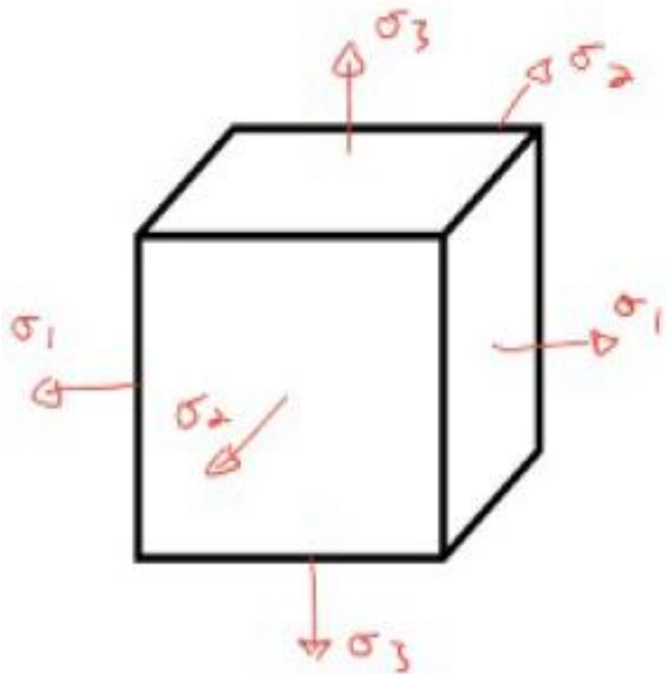


Strain Energy, U

$$U_r = \frac{1}{2}\sigma_y \varepsilon_y = \frac{1}{2}\sigma_y\left(\frac{\sigma_y}{E}\right) = \frac{\sigma_y{}^2}{2E}$$

Maximum Distortion-Energy-Theory

For general 3-D stresses



$$u = \frac{1}{2}\left(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3\right)$$

Applying Hooke's Law

$$u = \frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right)$$

Maximum Distortion-Energy-Theory

$$u = \frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu\left(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1\right)\right)$$

There are two components in this energy a mean component and deviatoric component.

$$u = u_v + u_d$$

Hydrostatic Volumetric Strain Energy can be found by replacing the principal stresses with the average stress.

$$u_v = \frac{3\,\sigma_{av}^2}{2\,E}\cdot(1-2\nu)$$

Maximum Distortion-Energy-Theory

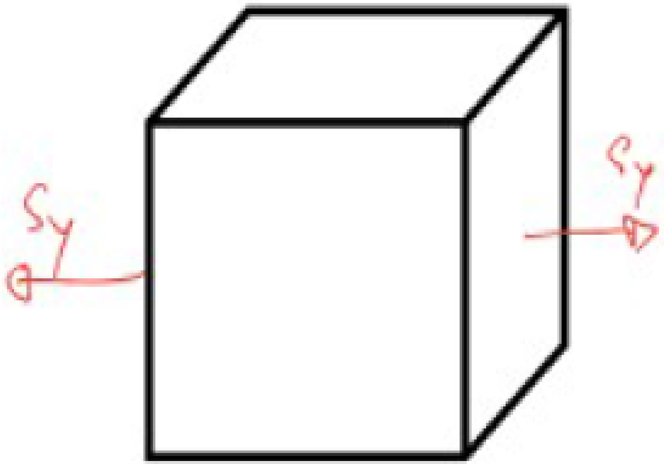
Distortion Energy:

$$U_d = U - U_v = \frac{1+\nu}{3E}\left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]$$

For Uniaxial Yielding:

$$\sigma_1 = S_y \; ; \; \sigma_2 = \sigma_3 = 0$$

$$U_d = \frac{1+\nu}{3E} \cdot S_y^2$$



Maximum Distortion-Energy-Theory

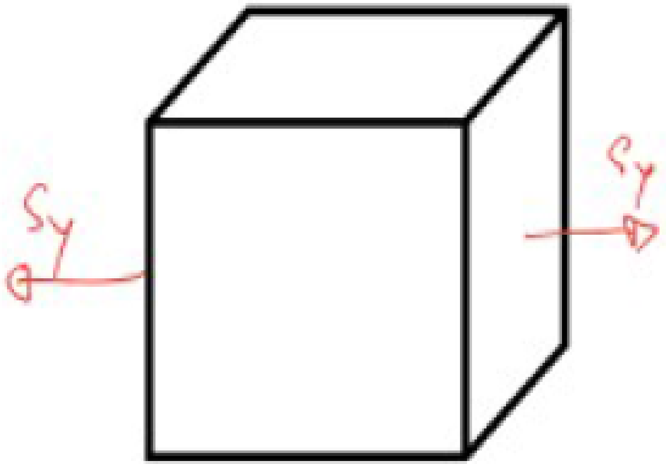
Distortion Energy:

$$u_d = u - u_v = \frac{1+\nu}{3E}\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]$$

For Uniaxial Yielding:

$$\sigma_1 = S_y \,; \quad \sigma_2 = \sigma_3 = 0$$

$$u_d = \frac{1+\nu}{3E} \cdot S_y^2$$



Maximum Distortion-Energy-Theory

$$\sigma_M = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

$$\sigma_{1,D} = \sigma_1 - \sigma_M \qquad \sigma_{2,D} = \sigma_2 - \sigma_M \qquad \sigma_{3,D} = \sigma_3 - \sigma_M$$

The energy due to the mean stress (it gives a volumetric change but not a distortion:

$$u_{Mean} = \frac{1}{2E}\left(\sigma_M^2 + \sigma_M^2 + \sigma_M^2 - 2\nu\left(\sigma_M\sigma_M + \sigma_M\sigma_M + \sigma_M\sigma_M\right)\right)$$

Maximum Distortion-Energy-Theory

$$u_{Mean} = \frac{1}{2E}\left[3\sigma_M^2(1-2\nu)\right] = \frac{1-2\nu}{6E}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2+2\sigma_1\sigma_2+2\sigma_2\sigma_3+2\sigma_3\sigma_1\right)$$

Thus the distortion energy:

$$u_D = u - u_{Mean} = \frac{1+\nu}{3E}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2-\sigma_1\sigma_2-\sigma_2\sigma_3-\sigma_3\sigma_1\right)$$

Compare the distortion energy of a tensile test with the distortion energy of the material:

$$u_{Tensile} = \frac{1+\nu}{3E}S_y^2 = u_D = \frac{1+\nu}{3E}\left(\sigma_1^2+\sigma_2^2+\sigma_3^2-\sigma_1\sigma_2-\sigma_2\sigma_3-\sigma_3\sigma_1\right)$$

Maximum Distortion-Energy-Theory

$$S_y = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1}$$

$$S_y = \sqrt{\sigma_1^2 + \sigma_3^2 - \sigma_3\sigma_1}$$ (Plane stress)

Von Mises effective stress : Defined as the uniaxial tensile stress that creates the same distortion energy as any actual combination of applied stresses.

$$\sigma_{VM} = \sqrt{\frac{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)}{2}}$$

Maximum Distortion-Energy-Theory

for plane stress:

$$\sigma_{VM} = \sqrt{\sigma_x^{\,2} + \sigma_y^{\,2} - \sigma_x \sigma_y + 3\tau_{xy}^2}$$

Case of Pure Shear

$$\sigma_1 = \tau,\ \sigma_2 = 0,\ \sigma_3 = -\tau$$

$$\sigma_{VM} = \sqrt{3}\tau_{xy} \geq S_y$$

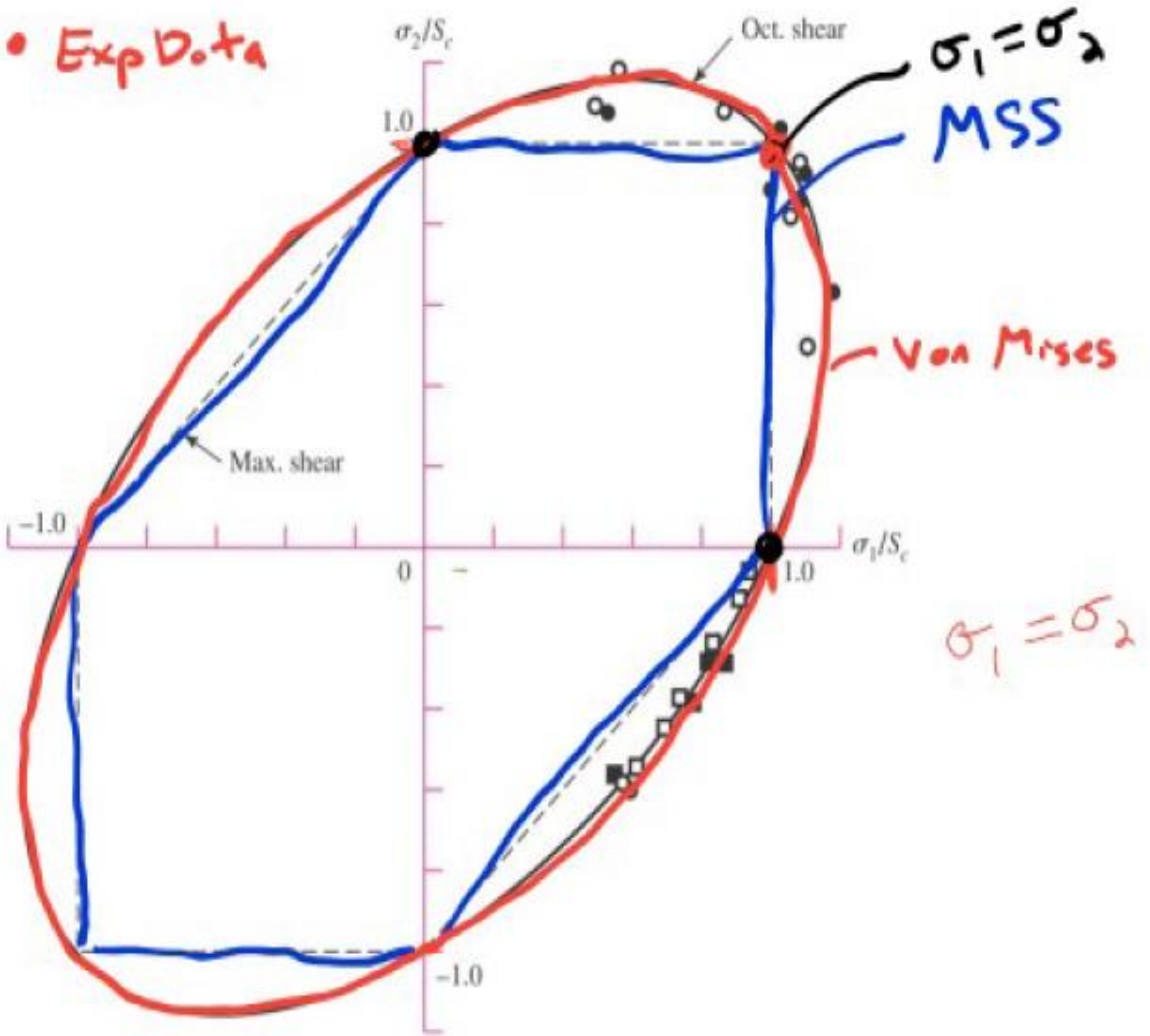
$$\tau_{Max} = \frac{S_y}{\sqrt{3}} = 0.577 S_y$$

Design Equation

$$\sigma_{von} = \frac{S_y}{n}$$

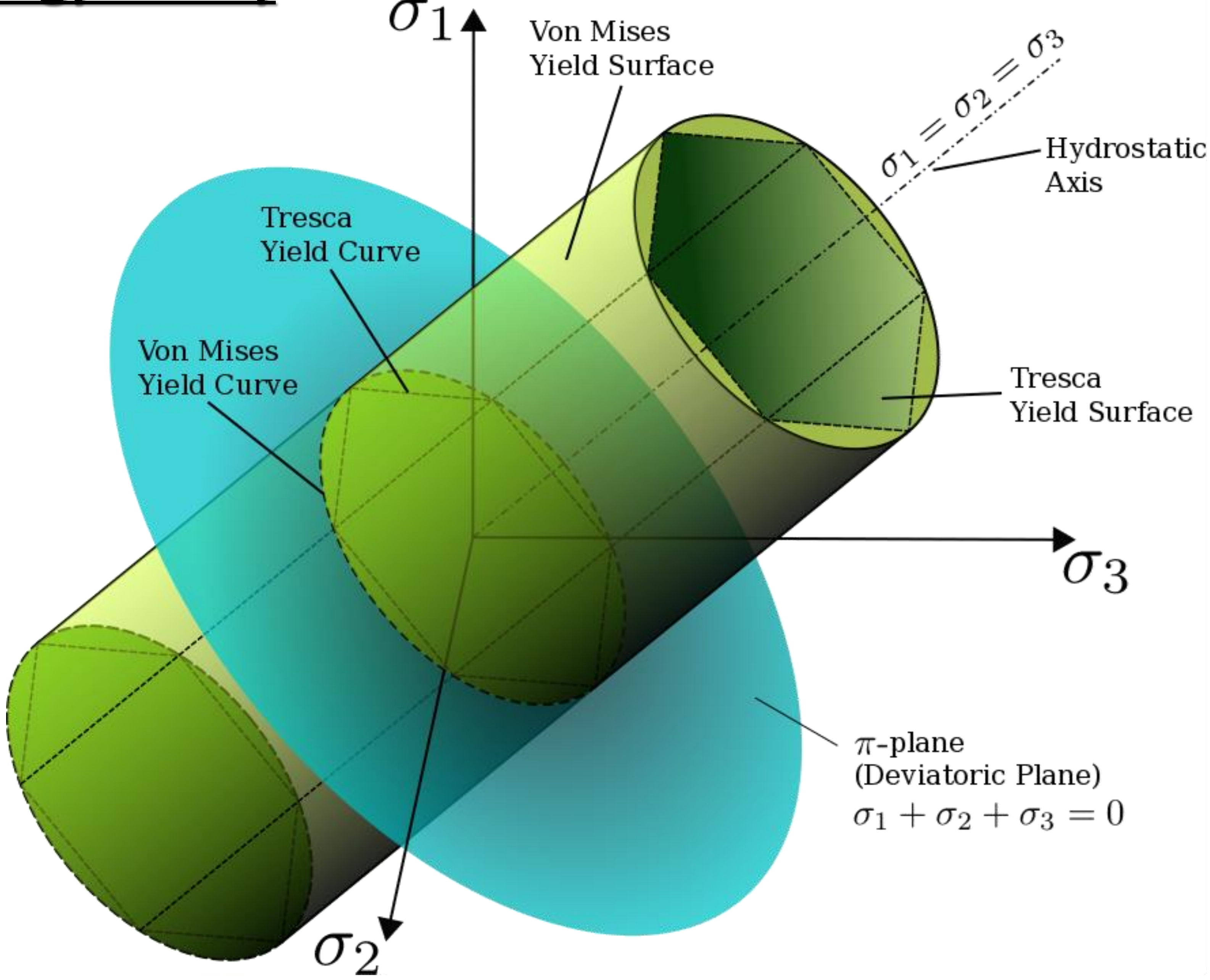
Maximum Distortion-Energy-Theory

What happens when you plot von Mises with real data?



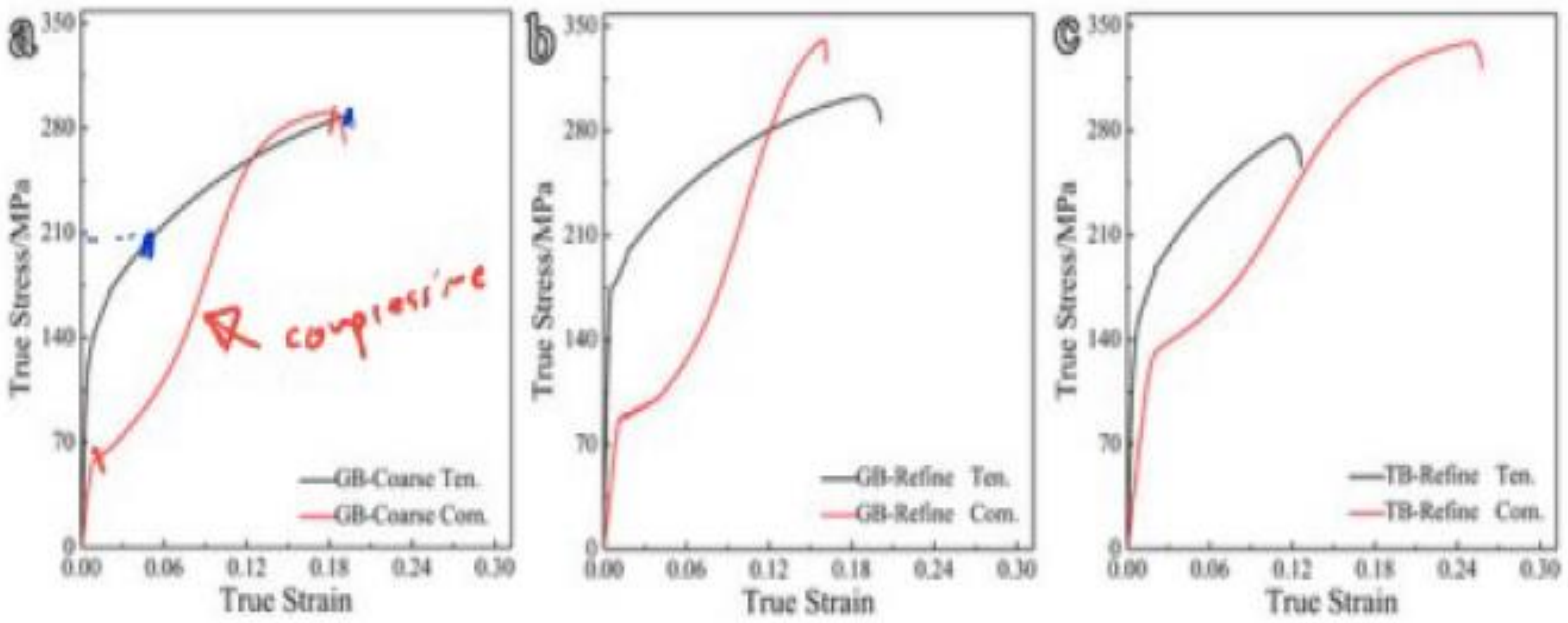
Maximum Distortion-Energy-Theory

For 3D yield surfaces



Coulomb-Mohr Theory

Not all materials have compressive strengths equal to their tensile strengths.



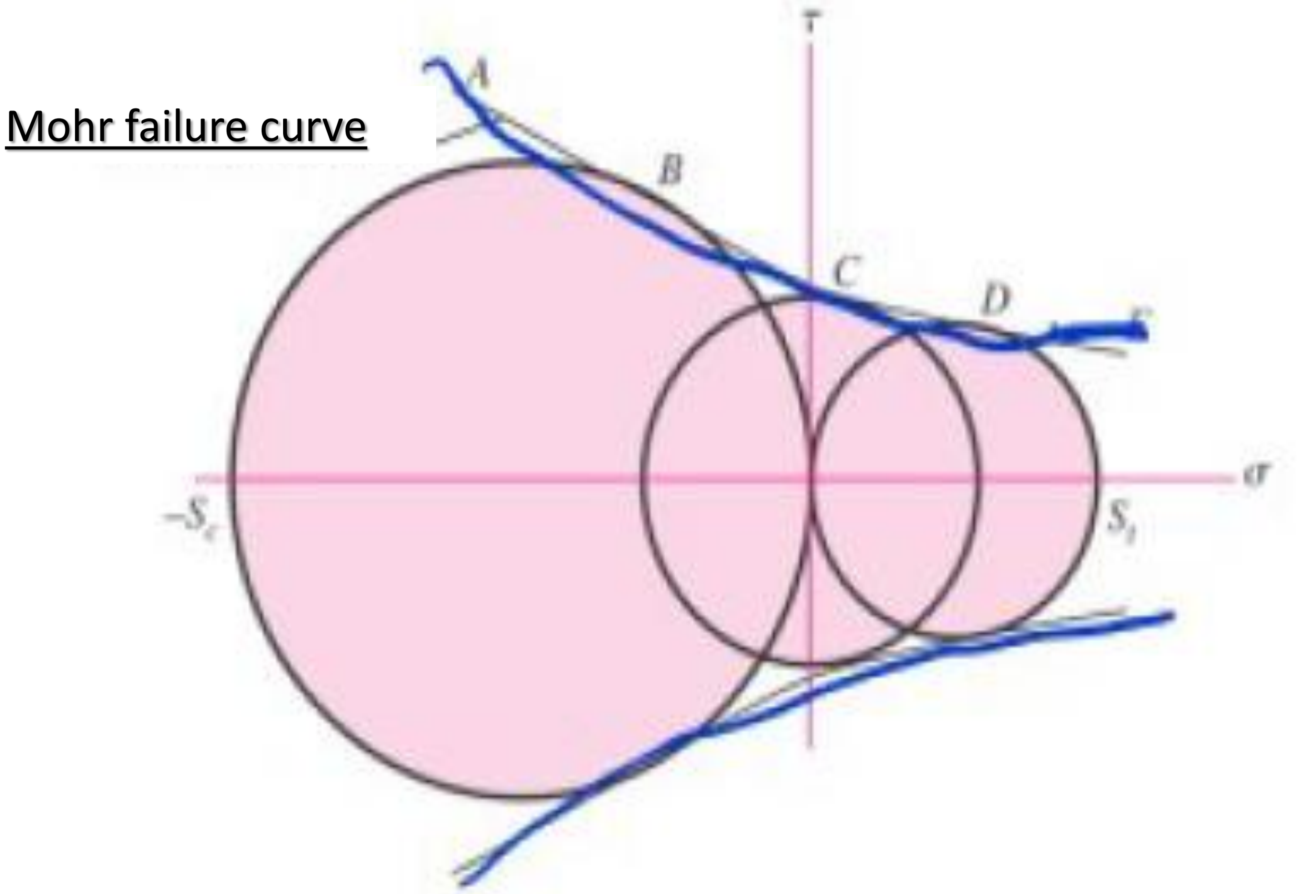
True stress-strain curves under tension and compression along the TD of (a) GB-coarse, (b) GB-refine and (c) TB-refine. Ten. and Com. denote tensile and compressive curves, respectively.

https://www.nature.com/articles/srep29283

Coulomb-Mohr Theory

For such materials, failure is based on three simple tests: Tension, Compression and Shear.

Plotting Mohr's circle for each, the bounding curve defines failure envelope. The curved failure curve is difficult to determine analytically.



Coulomb-Mohr Theory

This theory simplifies to linear failure envelope using only tension and compression tests.

From the geometry:

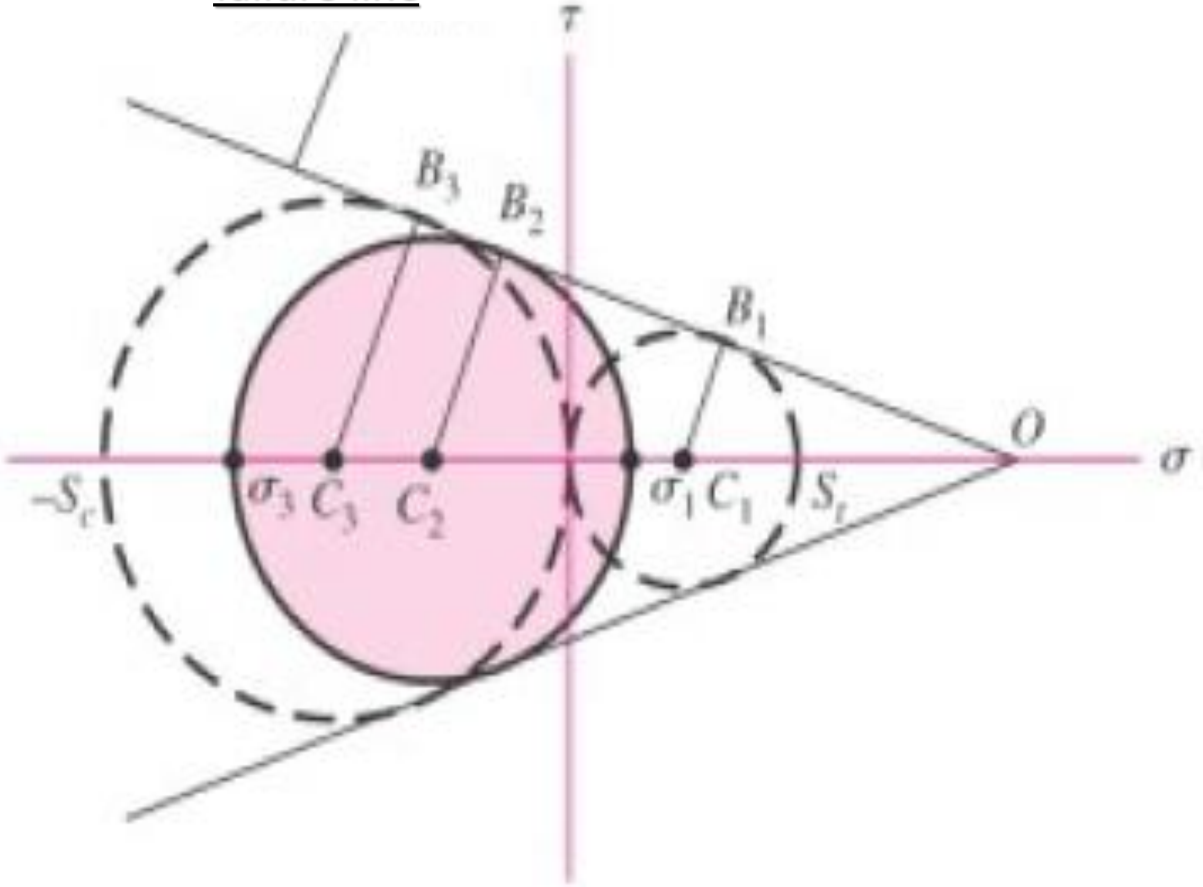
$$\frac{B_2C_2 - B_1C_1}{OC_2 - OC_1} = \frac{B_3C_3 - B_1C_1}{OC_3 - OC_1}$$

$$\frac{B_2C_2 - B_1C_1}{C_1C_2} = \frac{B_3C_3 - B_1C_1}{C_1C_3}$$

$B_1C_1 = S_t/2$, $B_2C_2 = (\sigma_1 - \sigma_3)/2$,

and $B_3C_3 = S_c/2$

Coulomb-Mohr failure line



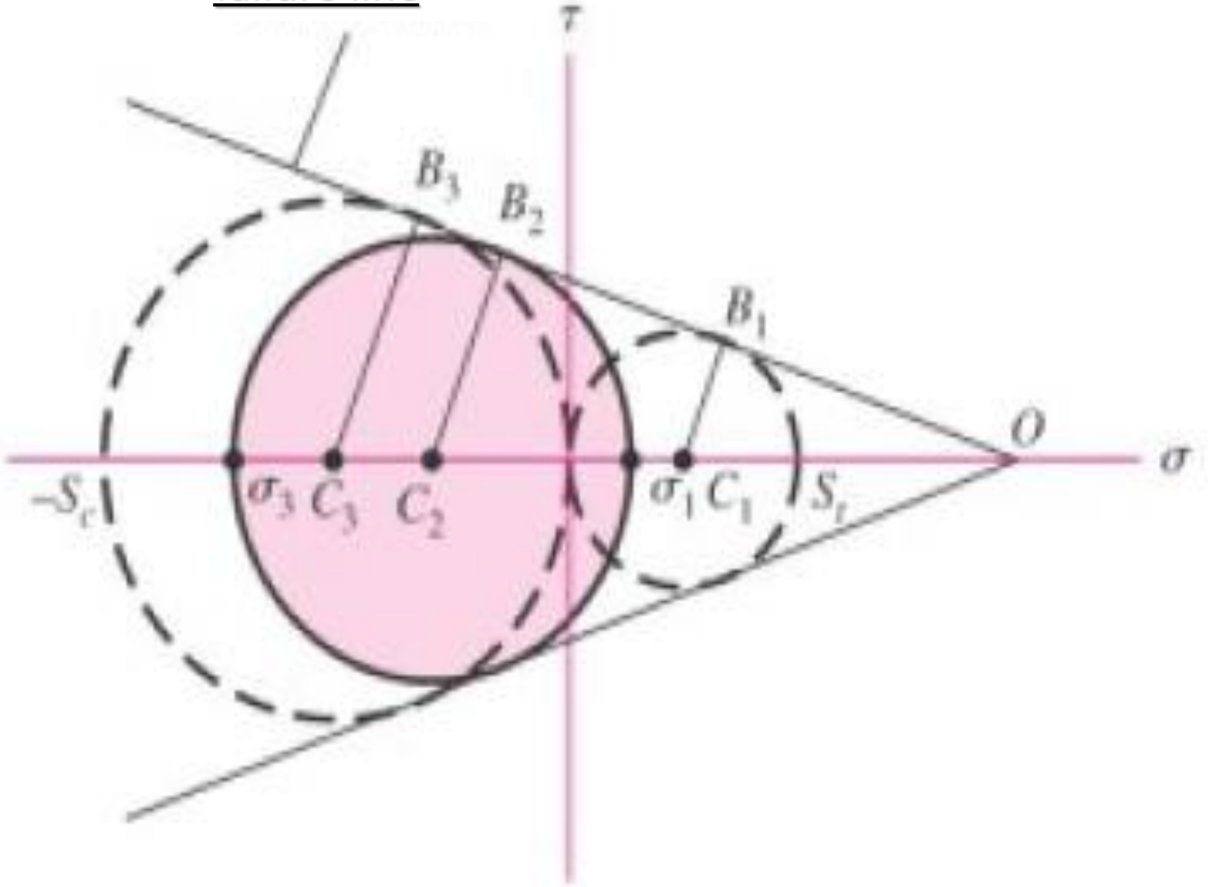
Coulomb-Mohr Theory

$$\frac{\frac{\sigma_1-\sigma_3}{2}-\frac{S_t}{2}}{\frac{S_t}{2}-\frac{\sigma_1+\sigma_3}{2}}=\frac{\frac{S_c}{2}-\frac{S_t}{2}}{\frac{S_t}{2}+\frac{S_c}{2}}$$

$$\frac{\sigma_1}{S_t}-\frac{\sigma_3}{S_c}=1$$

or $$\frac{\sigma_1}{S_t}-\frac{\sigma_3}{S_c}=\frac{1}{n}$$

Coulomb-Mohr failure line



Coulomb-Mohr Theory

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$

or

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n}$$

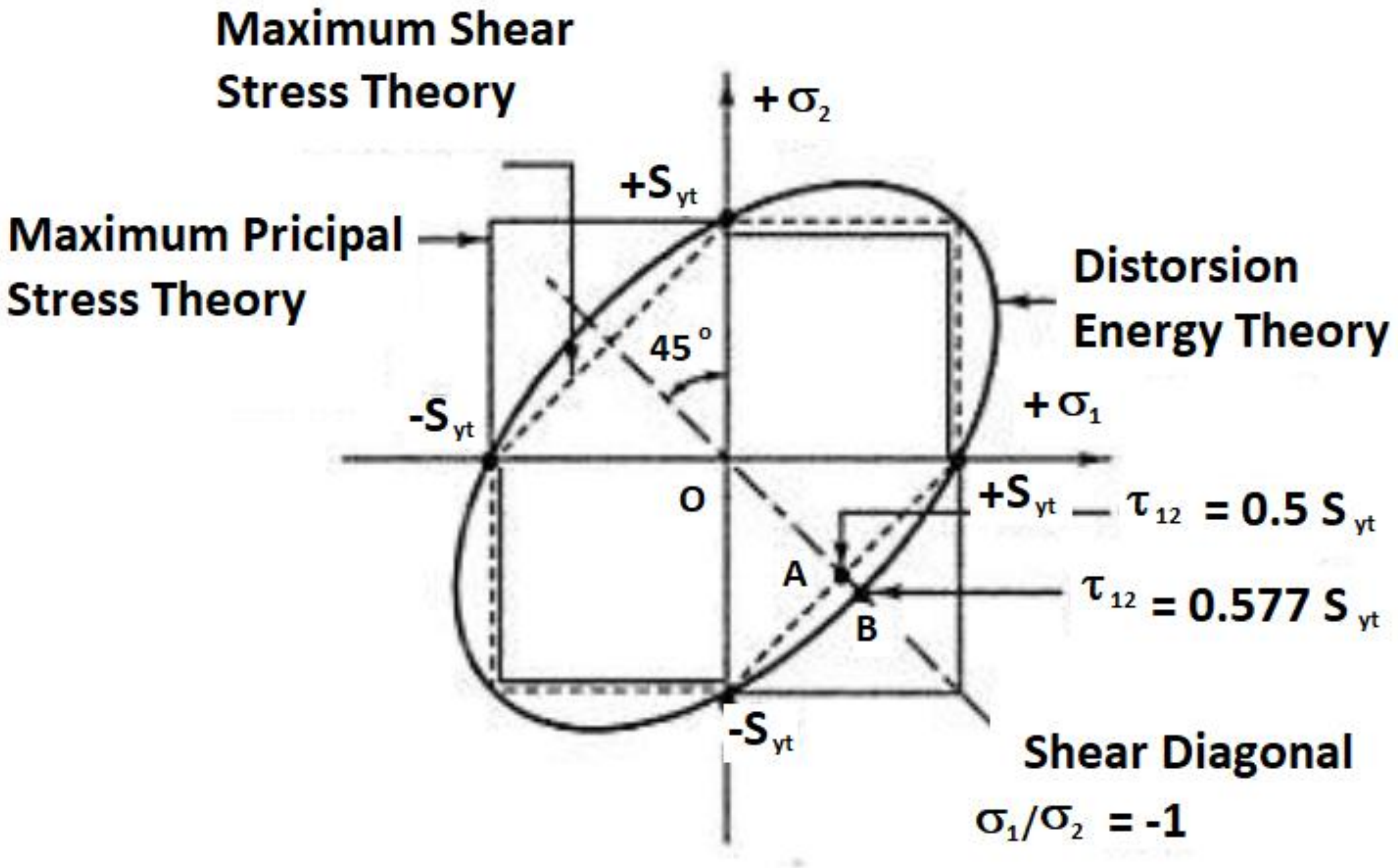
For ductile material, use tensile and compressive yield strengths

For brittle material, use tensile and compressive ultimate strengths

Can be used for both ductile and brittle materials:

- For **ductile** material, use tensile and compressive yield strengths
- For **brittle** material, use tensile and compressive ultimate strengths

Selection and use of Failure Theory



Selection and use of Failure Theory

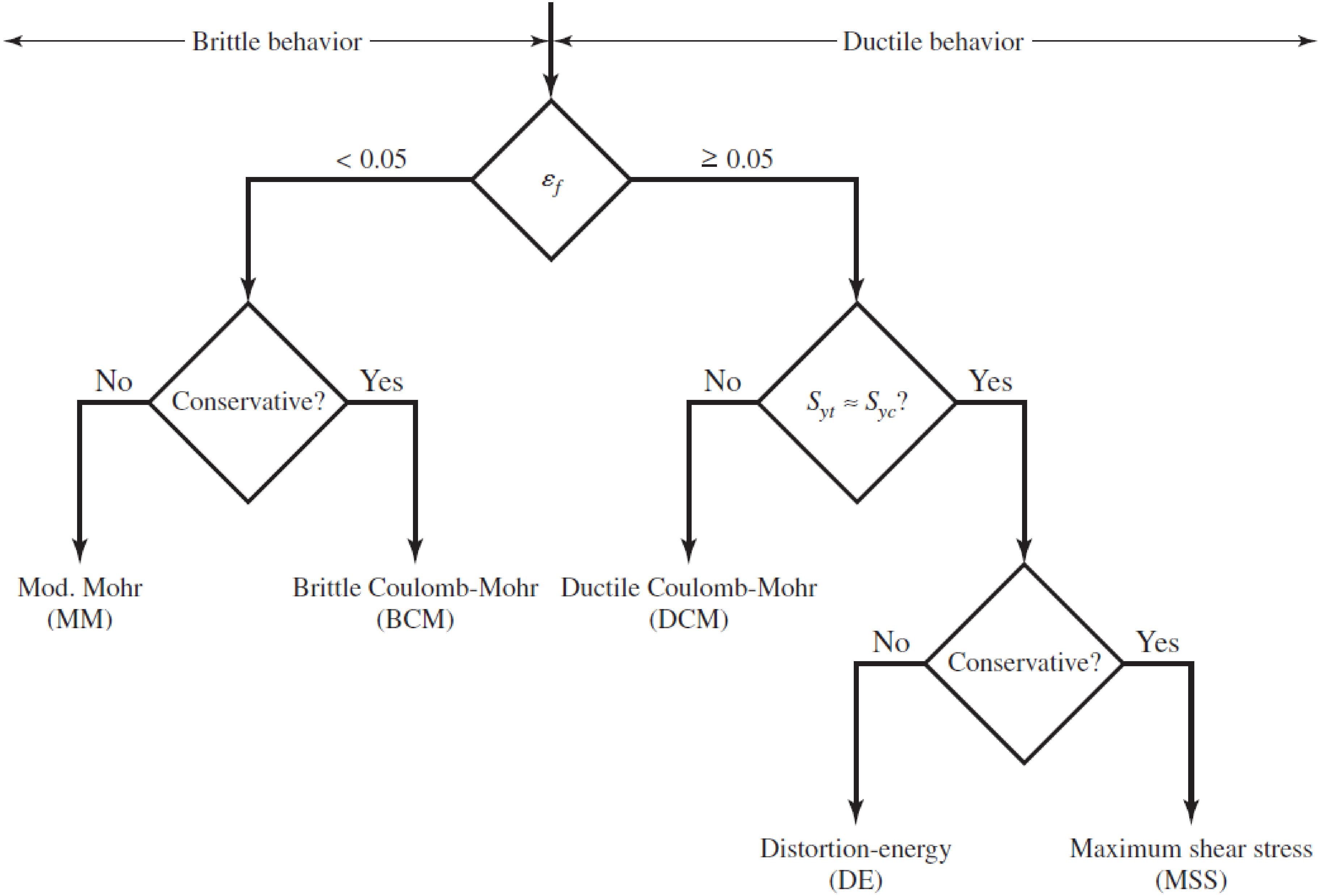
Maximum shear stress theory and distortion energy theory are used for ductile material.

Distortion energy theory predicts yielding with precise accuracy in all four quadrants.

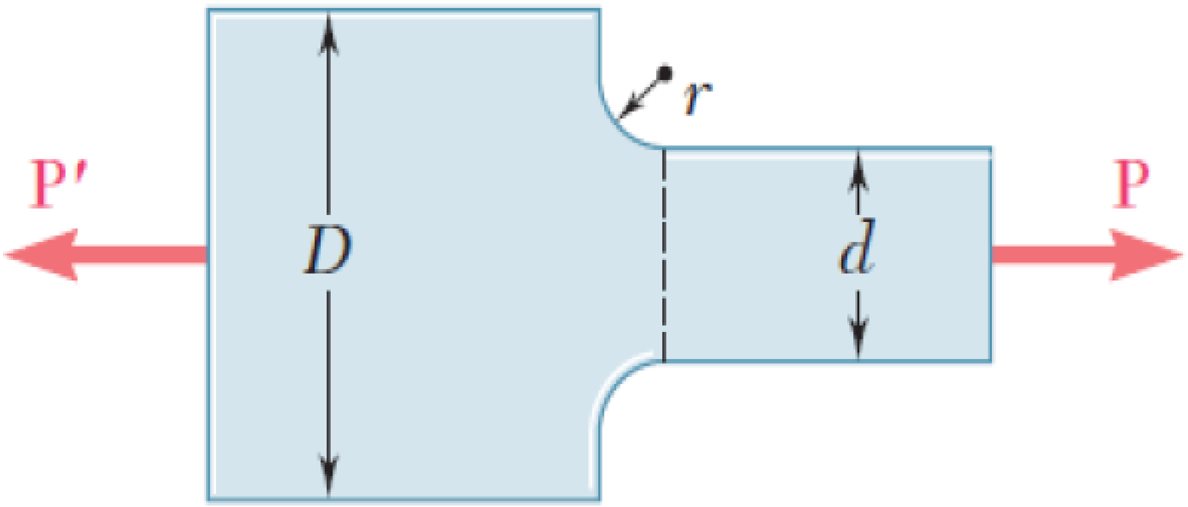
The design calculations involved in this theory are slightly complicated as compared with those of maximum shear stress theory.

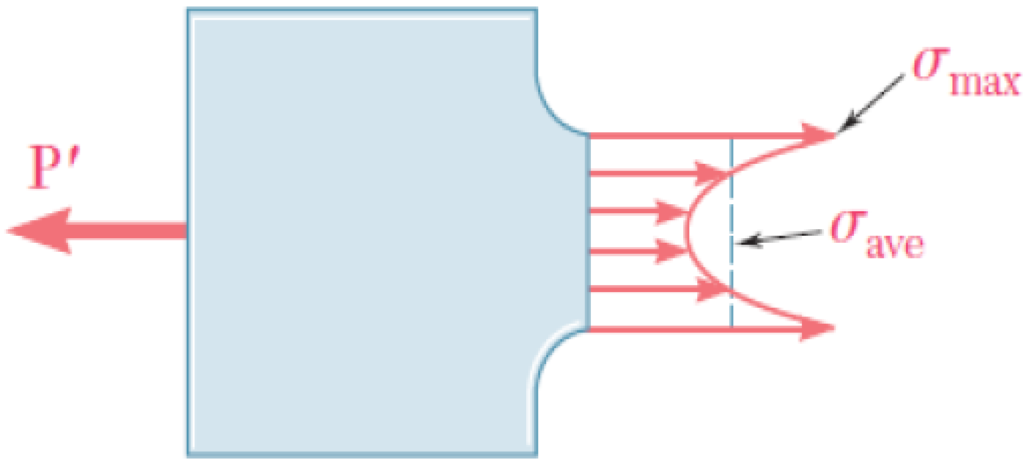
The hexagonal diagram of maximum shear stress theory is inside the ellipse of distortion energy theory. Therefore maximum shear stress theory gives results on the conservative side.

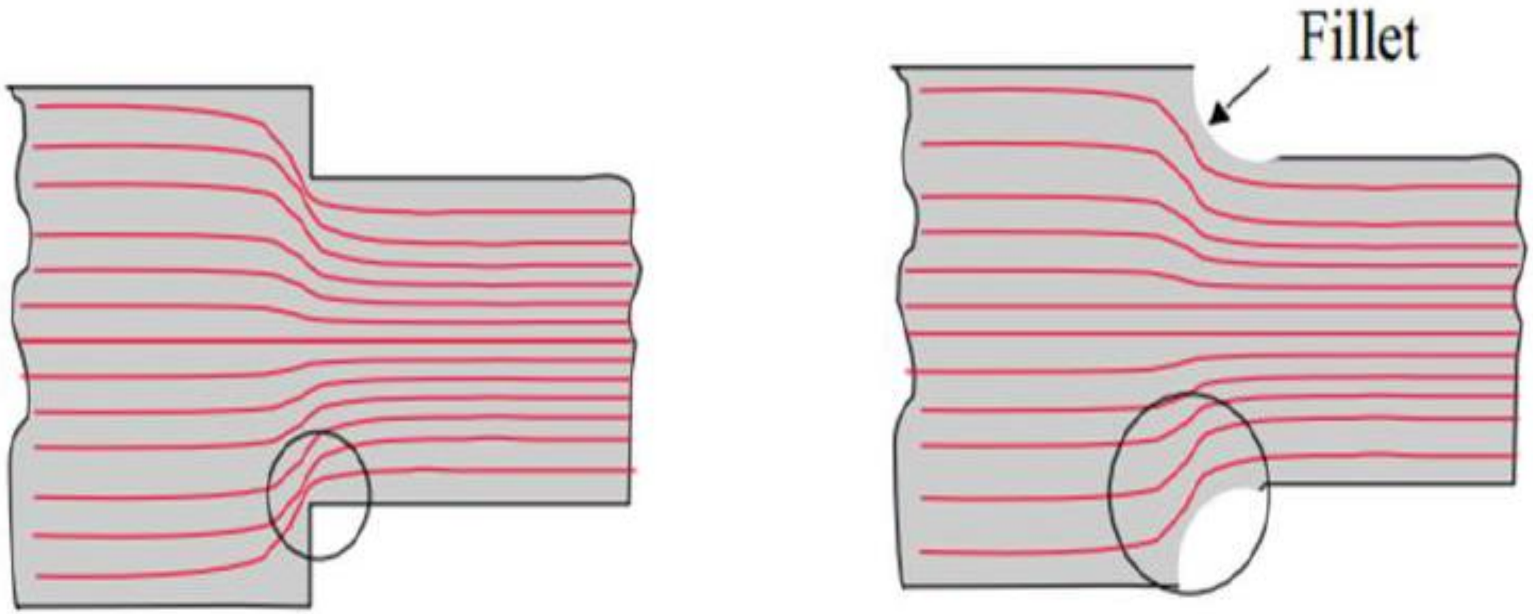
Selection and use of Failure Theory



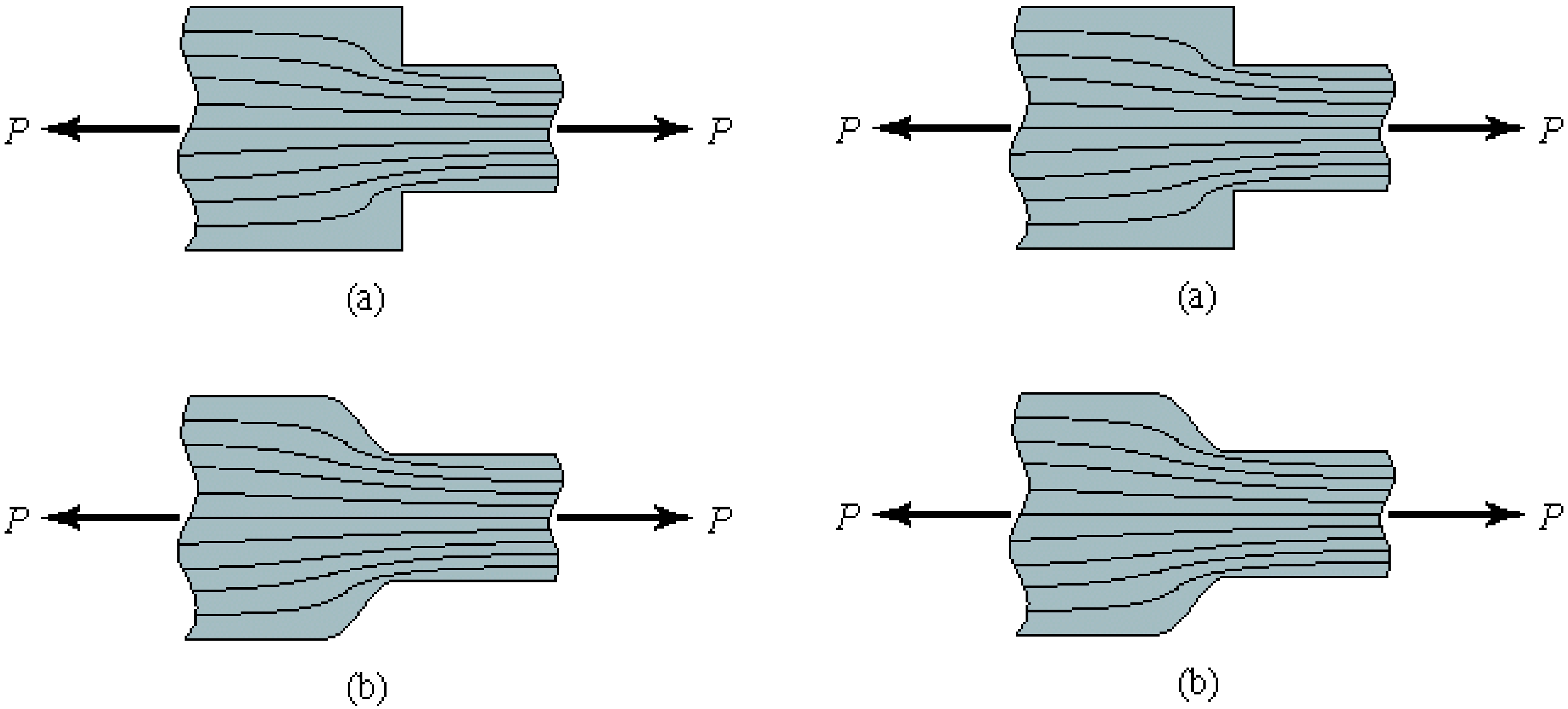
Geometric Discontinuities





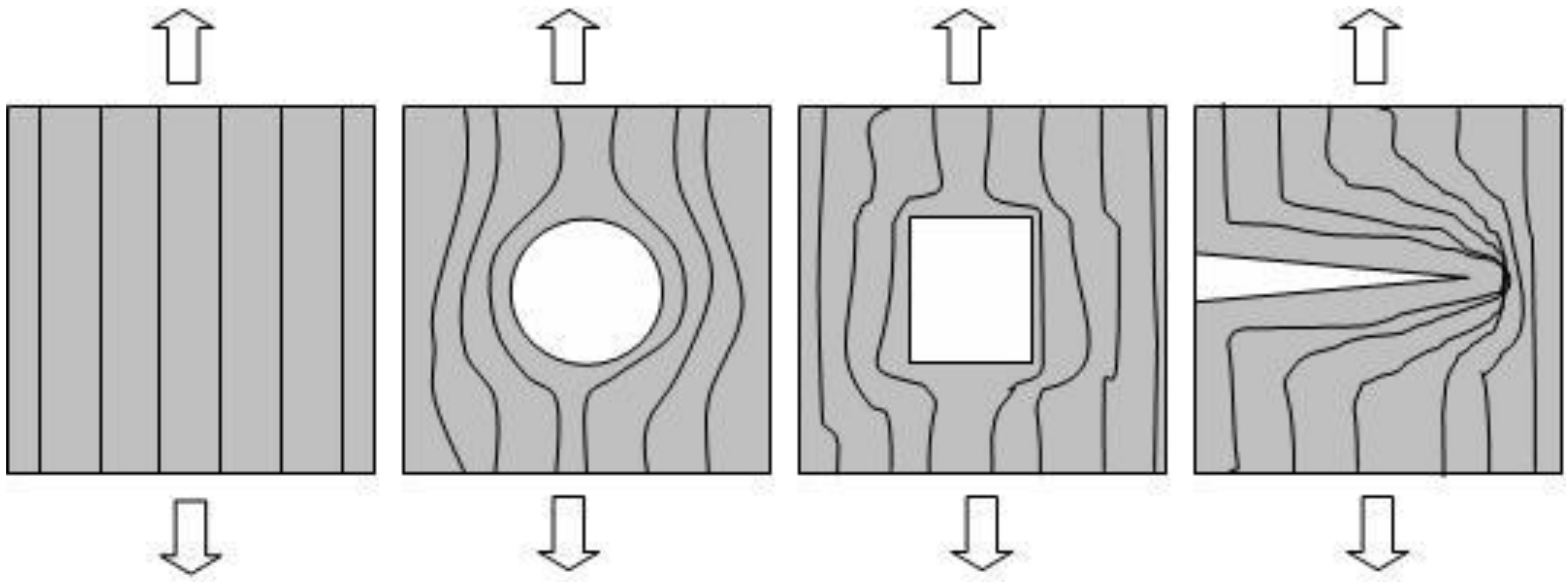


Geometric Discontinuities



Bar with fillet axially loaded showing stress contours through a flat plate for (a) square corners, (b) rounded corners (c) small groove, and (d) small holes.

Geometric Discontinuities

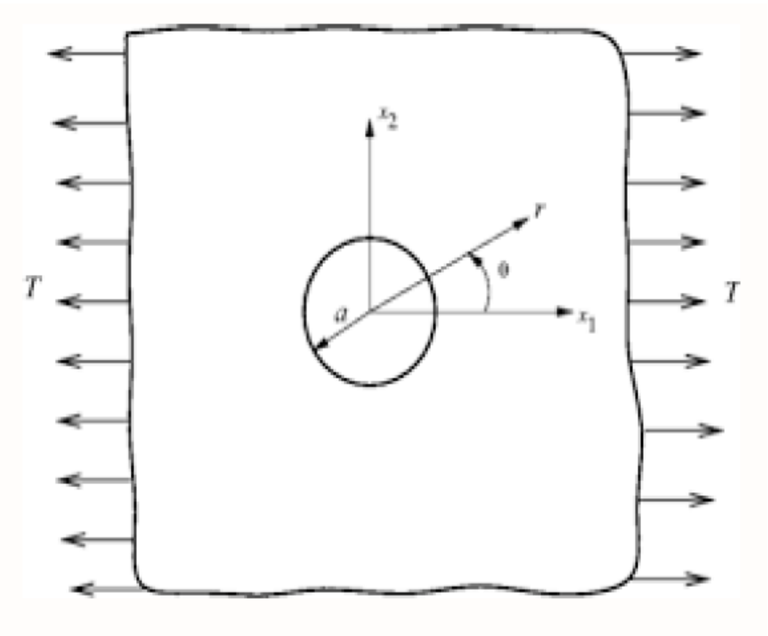


- Localized increase of stress near discontinuities
- K_t is Theoretical (Geometric) Stress Concentration Factor

Geometric Discontinuities

How to calculate concentrated stresses?

- Analytical methods



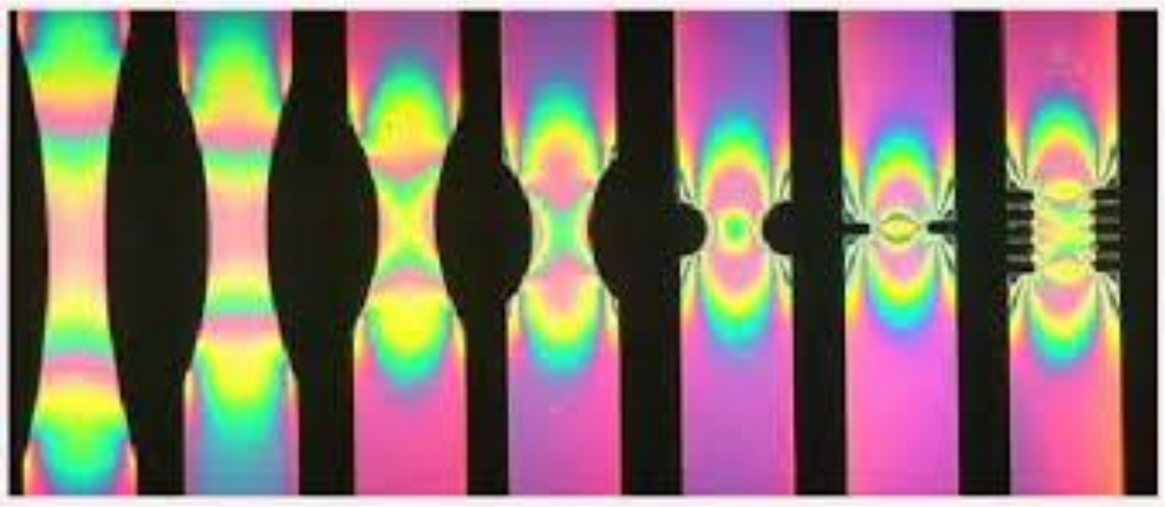
$$\sigma = f(r, \theta, T)$$

Limited to very simple geometries

Geometric Discontinuities

How to calculate concentrated stresses?

- Photoelastic methods - laboratory experiments



Widely used experimental technique before the invention of numerical methods nut it is limited to small components

Geometric Discontinuities

How to calculate concentrated stresses?

A **stress concentration** (often called **stress raisers** or **stress risers**) is a location in an object where stress is concentrated.

An object is strongest when force is evenly distributed over its area, so a reduction in area, e.g., caused by a crack, results in a localized increase in stress.

$$K_t = \frac{\sigma_{max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{max}}{\tau_0}$$

Geometric Discontinuities

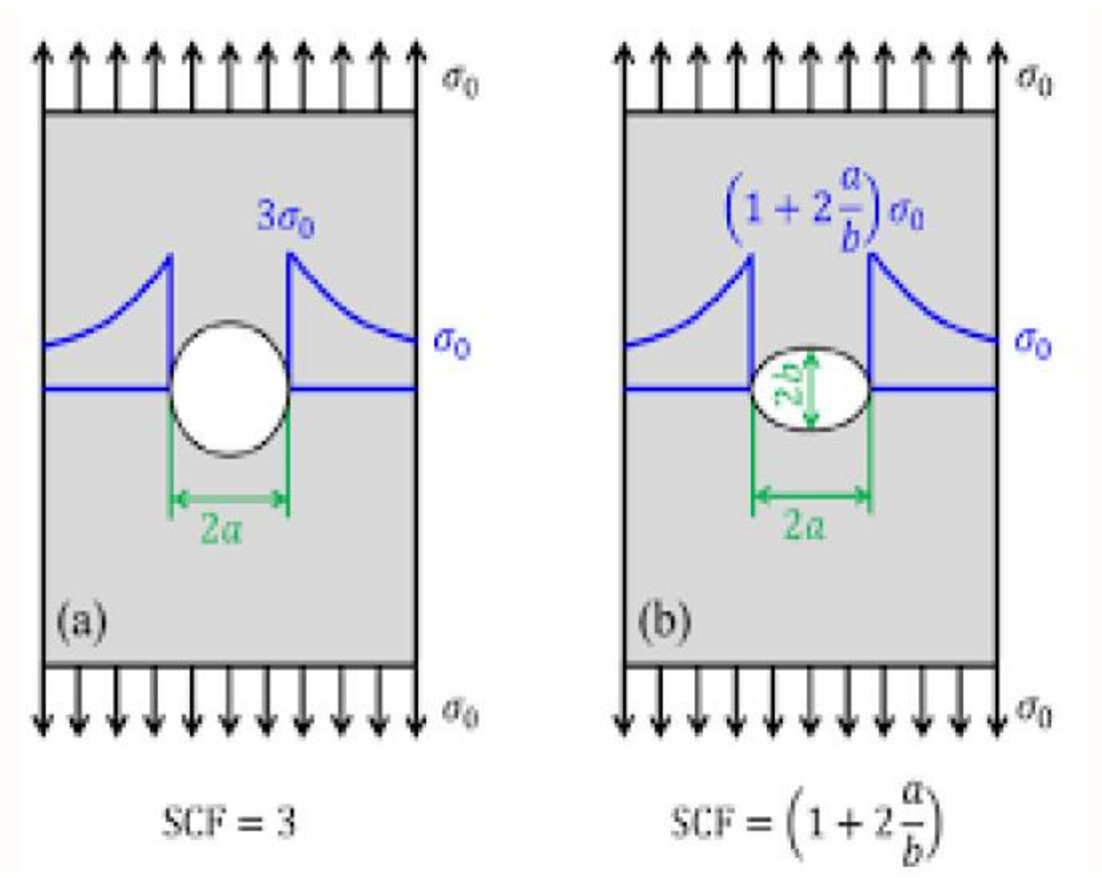
A material can fail, via a propagating crack, when a concentrated stress exceeds the material's theoretical cohesive strength.

The real fracture strength of a material is always lower than the theoretical value because most materials contain small cracks or contaminants (especially foreign particles) that concentrate stress.

Geometric Discontinuities

How to calculate concentrated stresses?

- Empirical data – Stress concentration factors

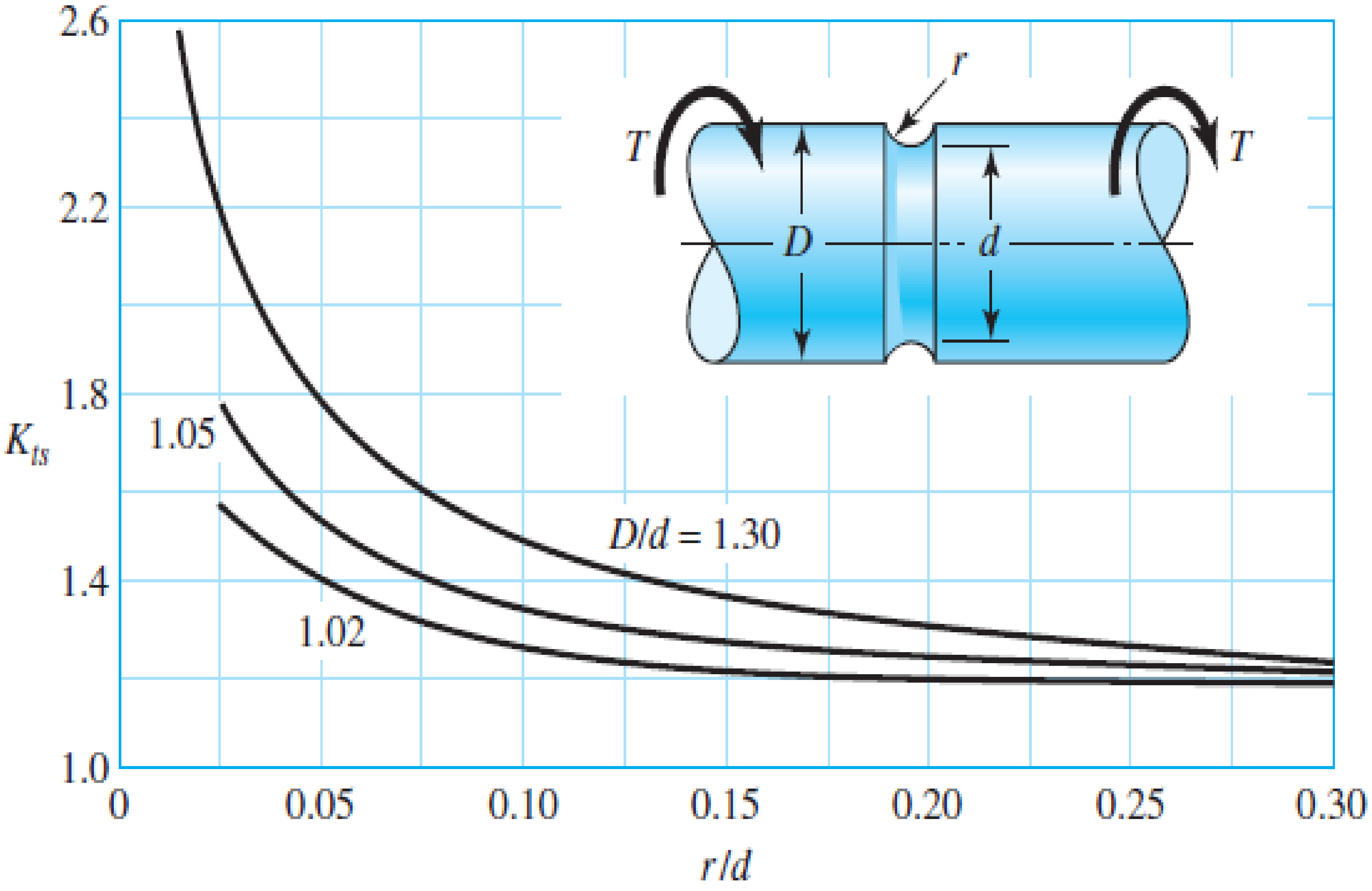


$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$

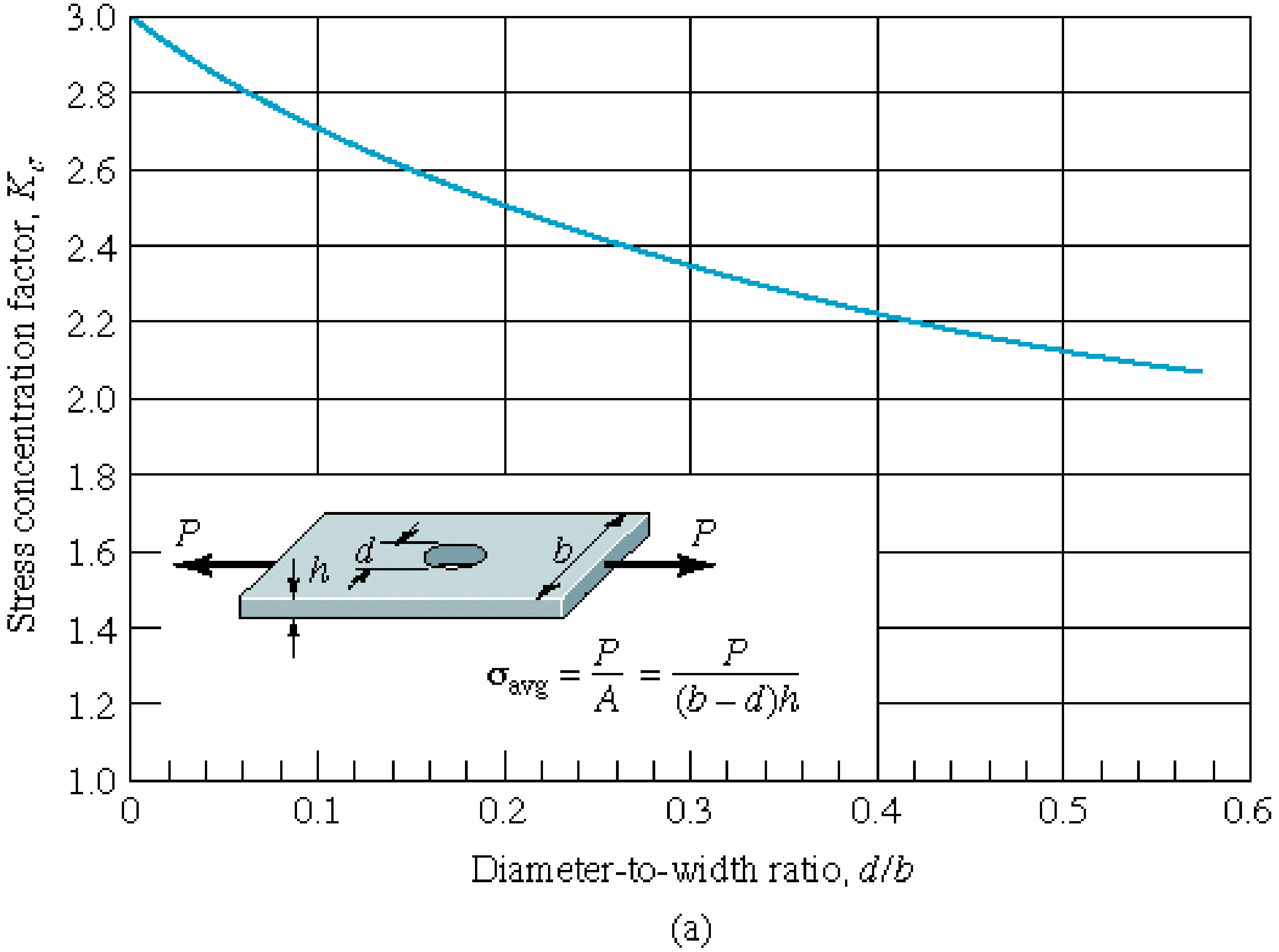
Geometric Discontinuities

Figure A–15–15

Grooved round bar in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.



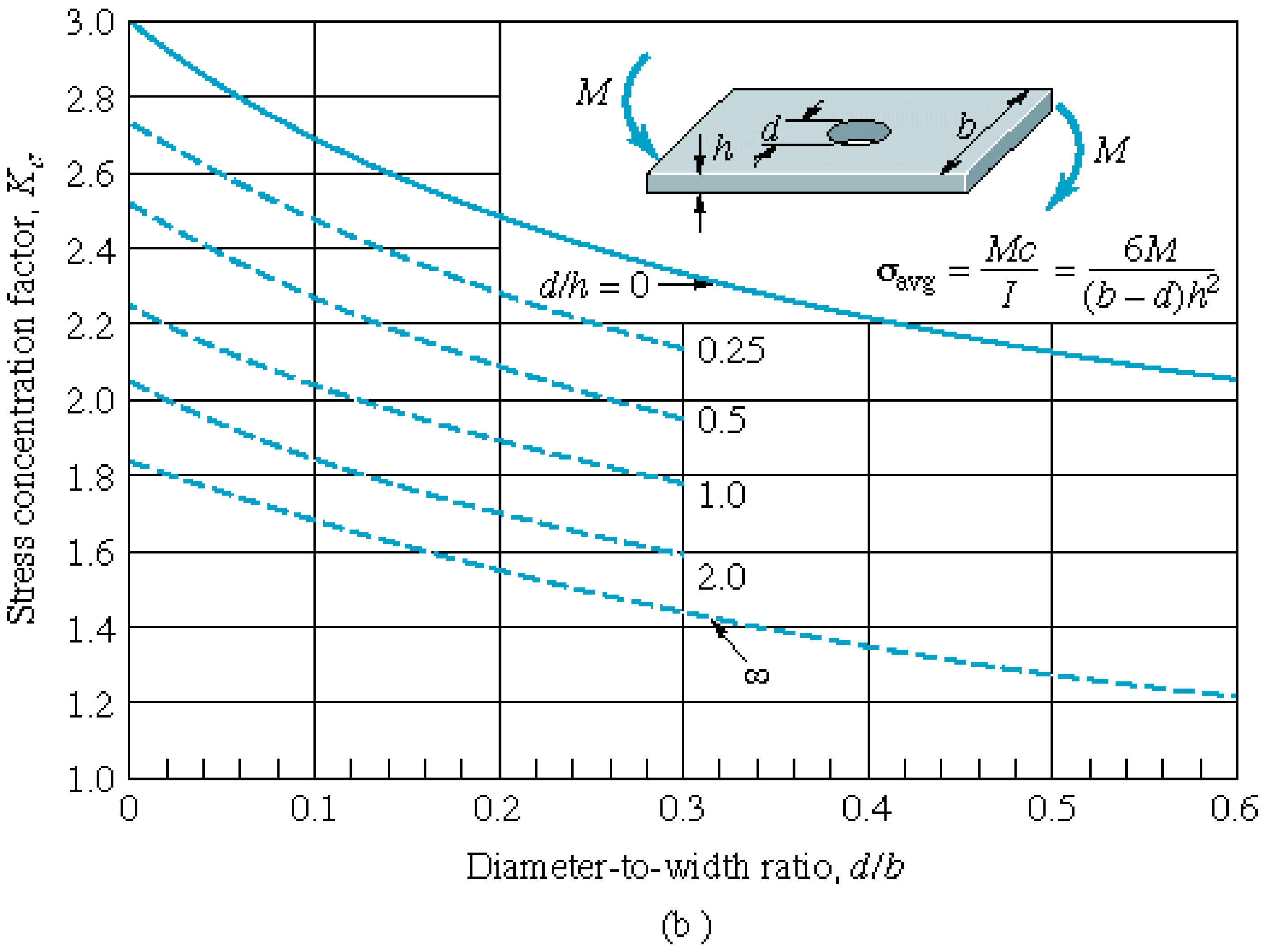
Geometric Discontinuities



(a)

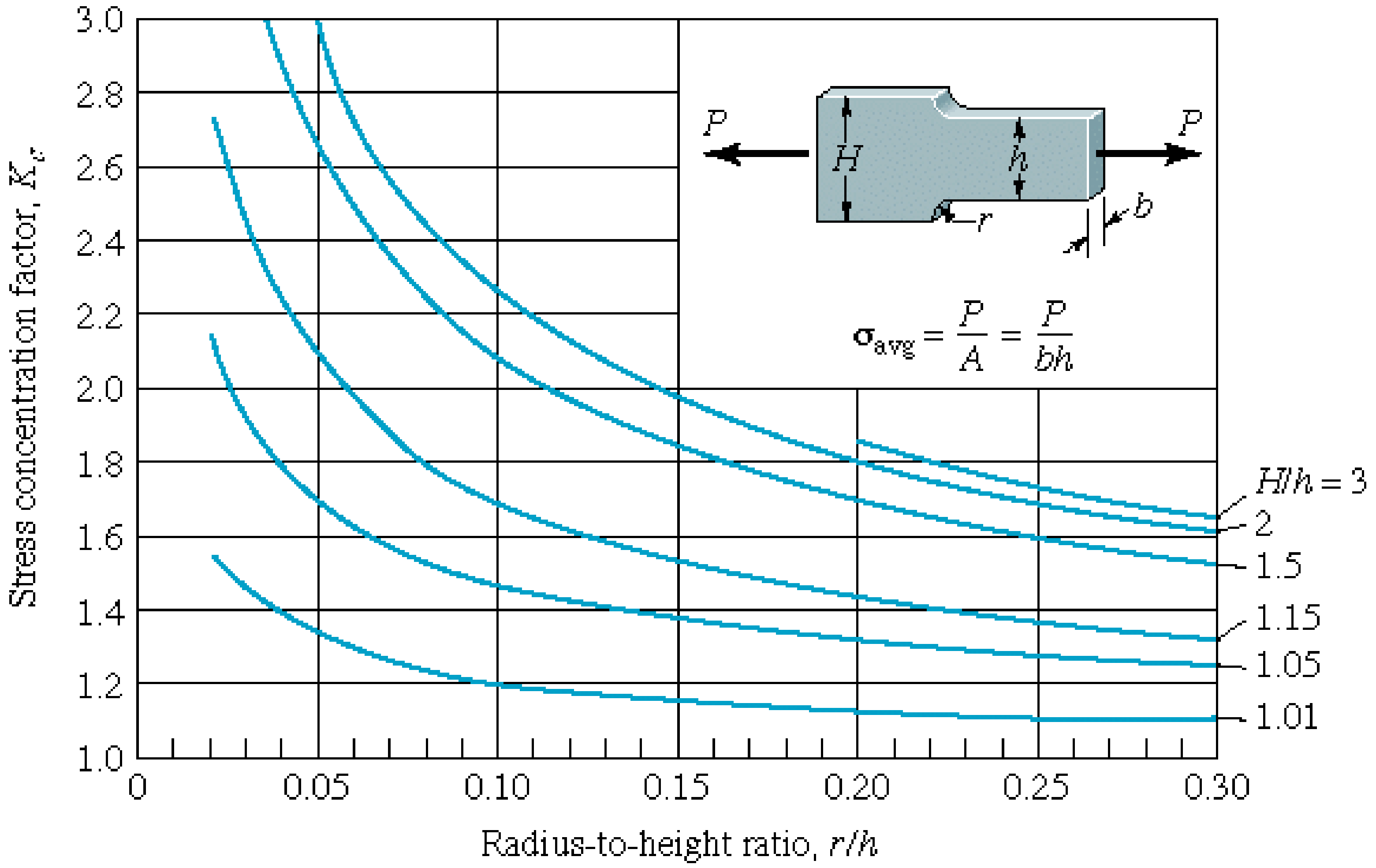
Stress concentration factor for rectangular plate with central hole. (a) Axial Load. [Adapted from Collins (1981).]

Geometric Discontinuities



Stress concentration factor for rectangular plate with central hole. (b) Bending.
[Adapted from Collins (1981).]

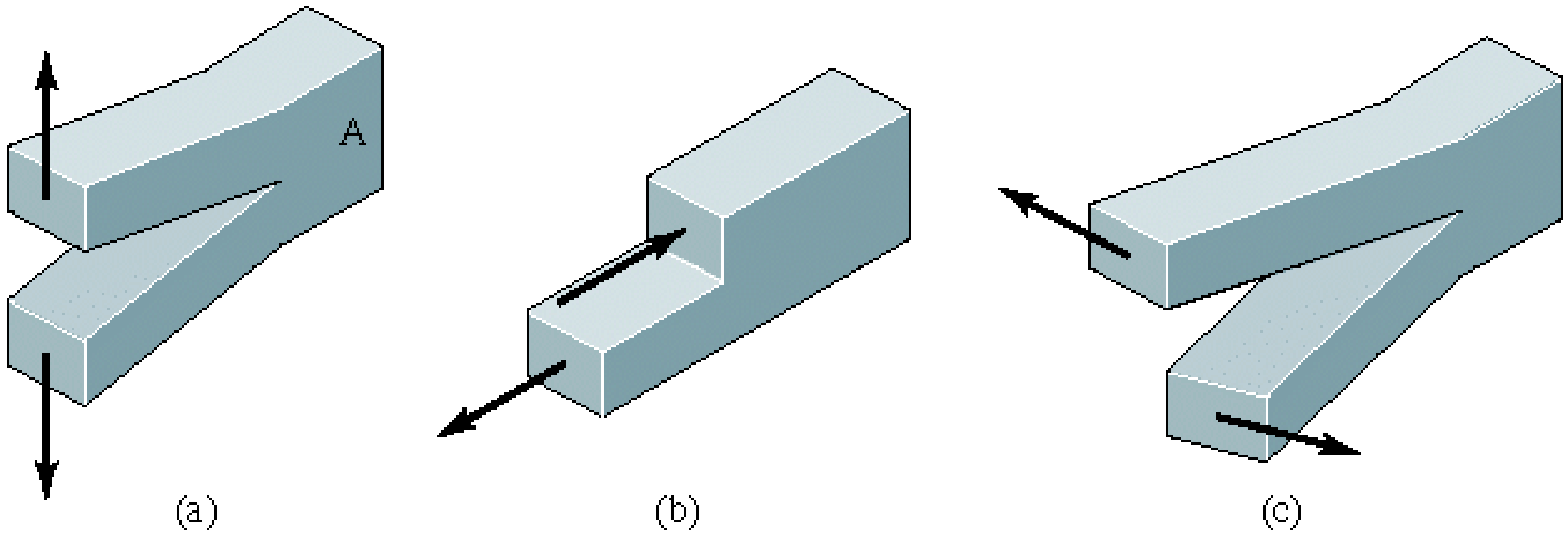
Geometric Discontinuities



(a)

Stress concentration factor for rectangular plate with fillet. (a) Axial Load. [Adapted from Collins (1981).]

Modes of crack displacement



Three modes of crack displacement. (a) Mode I, opening; (b) mode II, sliding; (c) mode III, tearing.

Stress Concentration for Static and Ductile Conditions

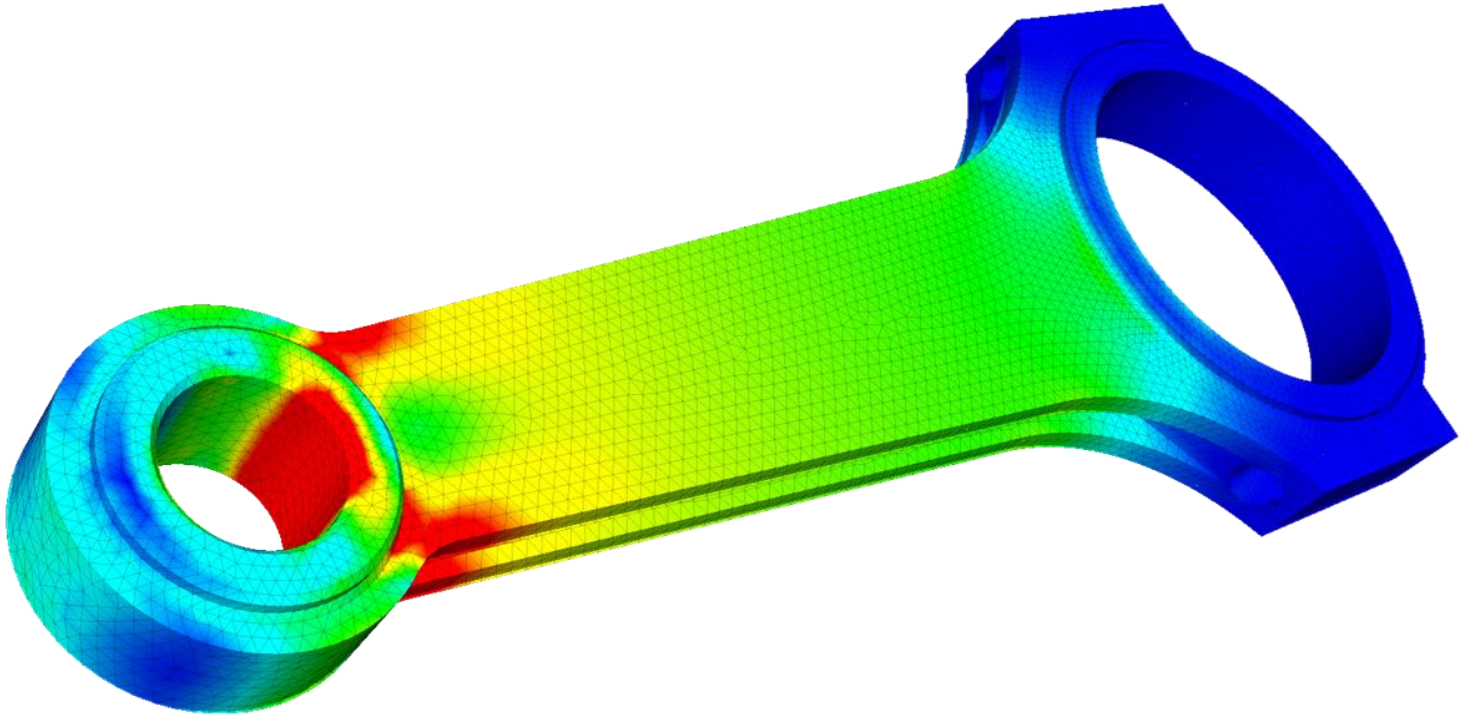
With static loads and ductile materials

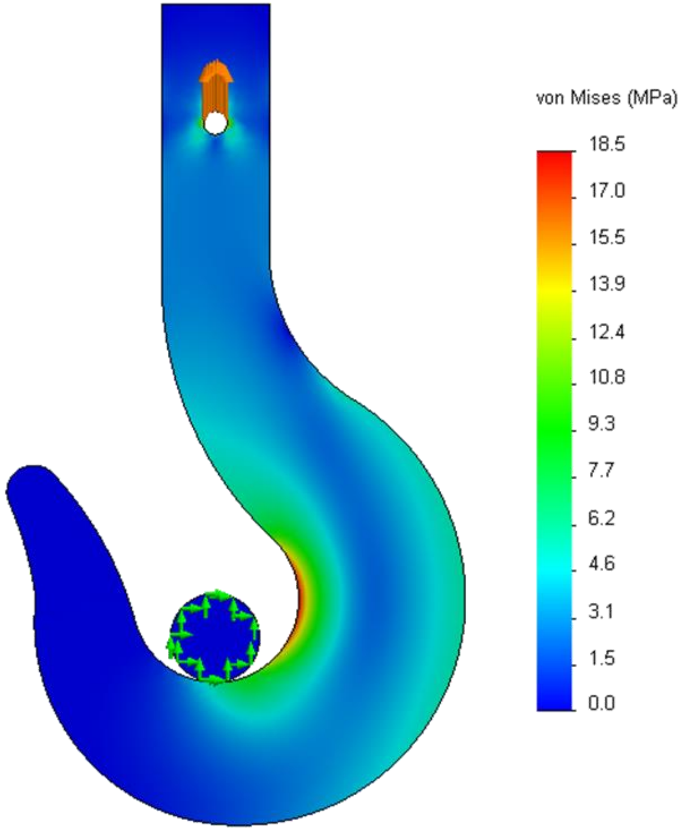
- Highest stressed fibers yield (cold work)
- Load is shared with next fibers
- Cold working is localized
- Overall part does not see damage unless ultimate strength is exceeded
- Stress concentration effect is commonly ignored for static loads on ductile materials
- Stress concentration must be included for dynamic loading.
- Stress concentration must be included for brittle materials, since localized yielding may reach brittle failure rather than cold-working and sharing the load.

Geometric Discontinuities

How to calculate concentrated stresses?

- Virtual methods – numerical techniques (Finite Element Analysis)





Homework

Can stress concentration be avoided?

How to reduce Stress Concentration?

Investigate and report on methods used to reduce stress concentration in mechanical components. (5-10 PAGES maximum supported with equations and figures). Including a list of references.

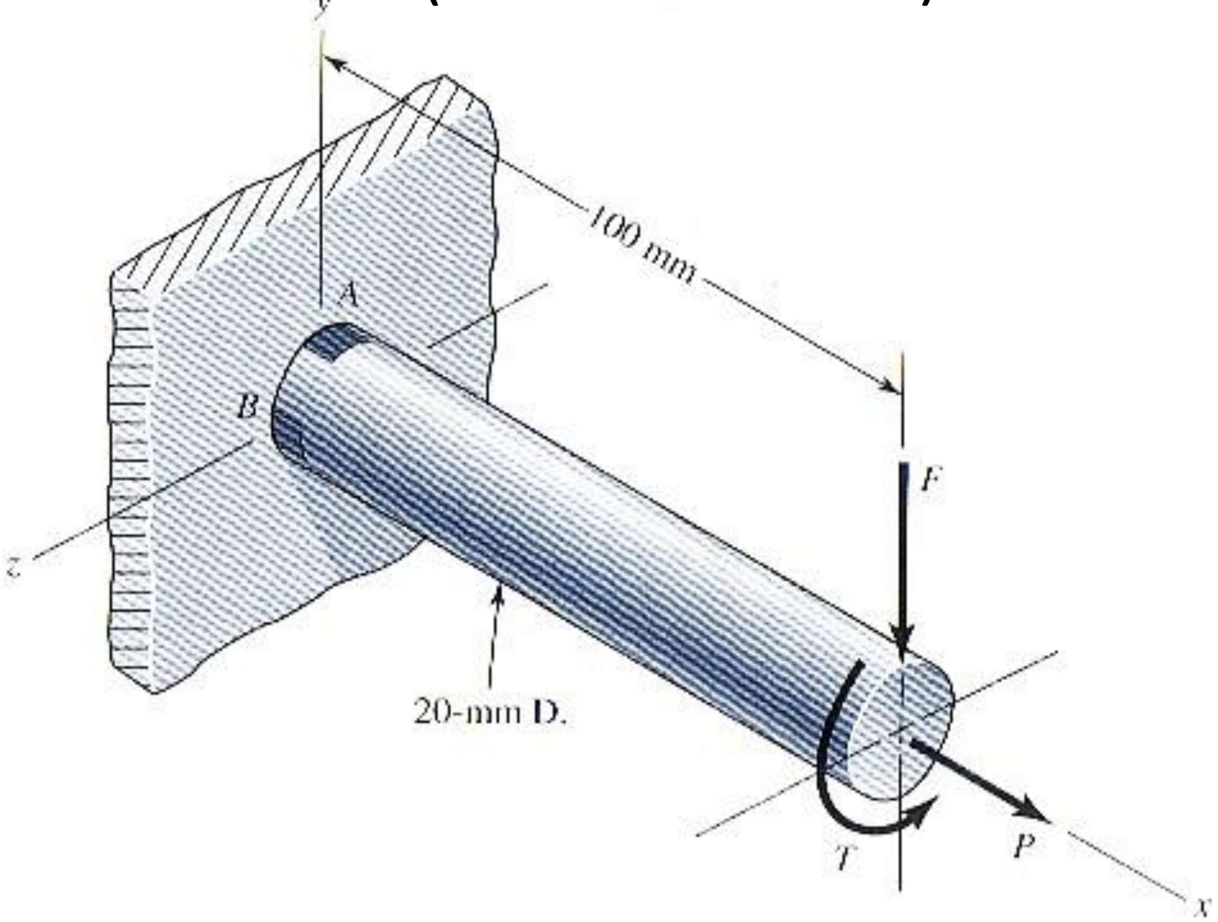
Due (NEXT WEEK).

Submission on MS Teams

Examples

Given:

The shown bar is made of AISI 1020 hot-rolled steel (ductile material)



F = 0.55 kN

P = 8.0 kN

T = 30 Nm

Find:

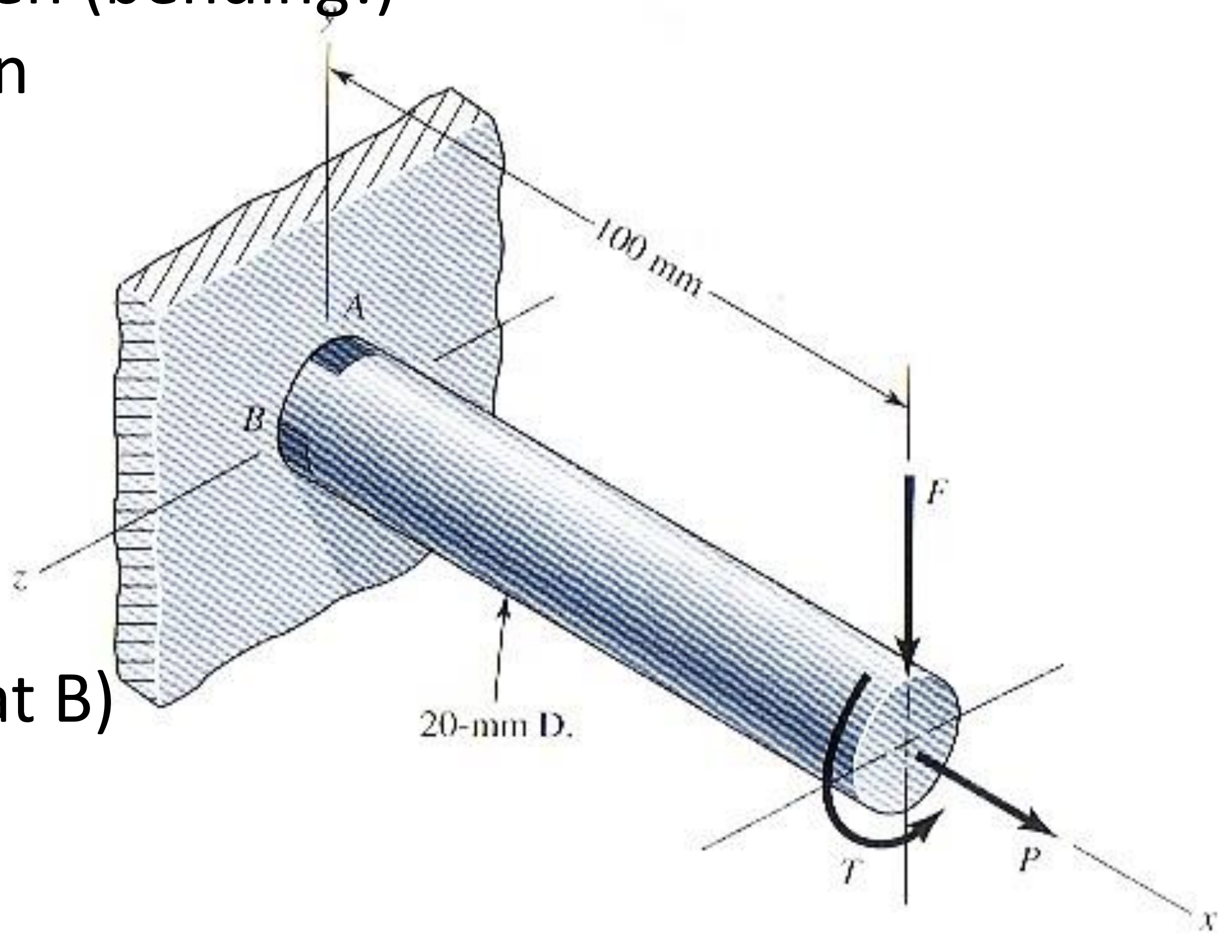
Factor of safety (n)

Examples

Two areas of interest:

A: Top – where max normal stress is seen (bending!)

B: Side – where max shear stress is seen



Consider the types of loading we have:

Axial? Yes – due to P

Bending? Yes – due to M (σ at A, τ at B)

Torsion? Yes – due to T

Examples

Element A

Calculate stresses due to each load

- Axial:

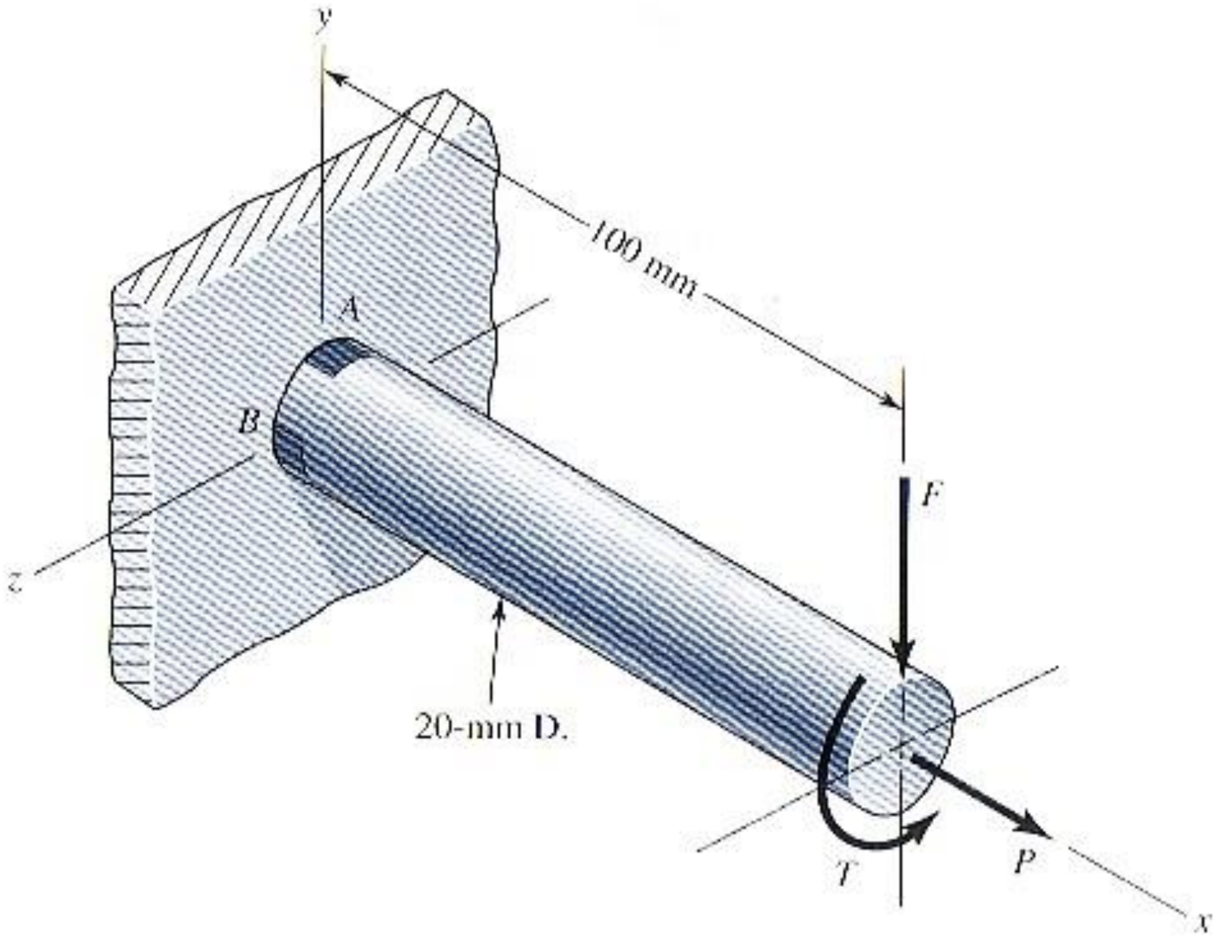
$$\sigma_x = \frac{P}{A} = \frac{P}{\left(\frac{\pi D^2}{4}\right)} = \frac{4P}{\pi D^2}$$

- Bending:

$$\sigma_x = \frac{My}{I} = \frac{(FL)\left(\frac{D}{2}\right)}{\left(\frac{\pi D^4}{64}\right)} = \frac{32FL}{\pi D^3}$$

- Shear:

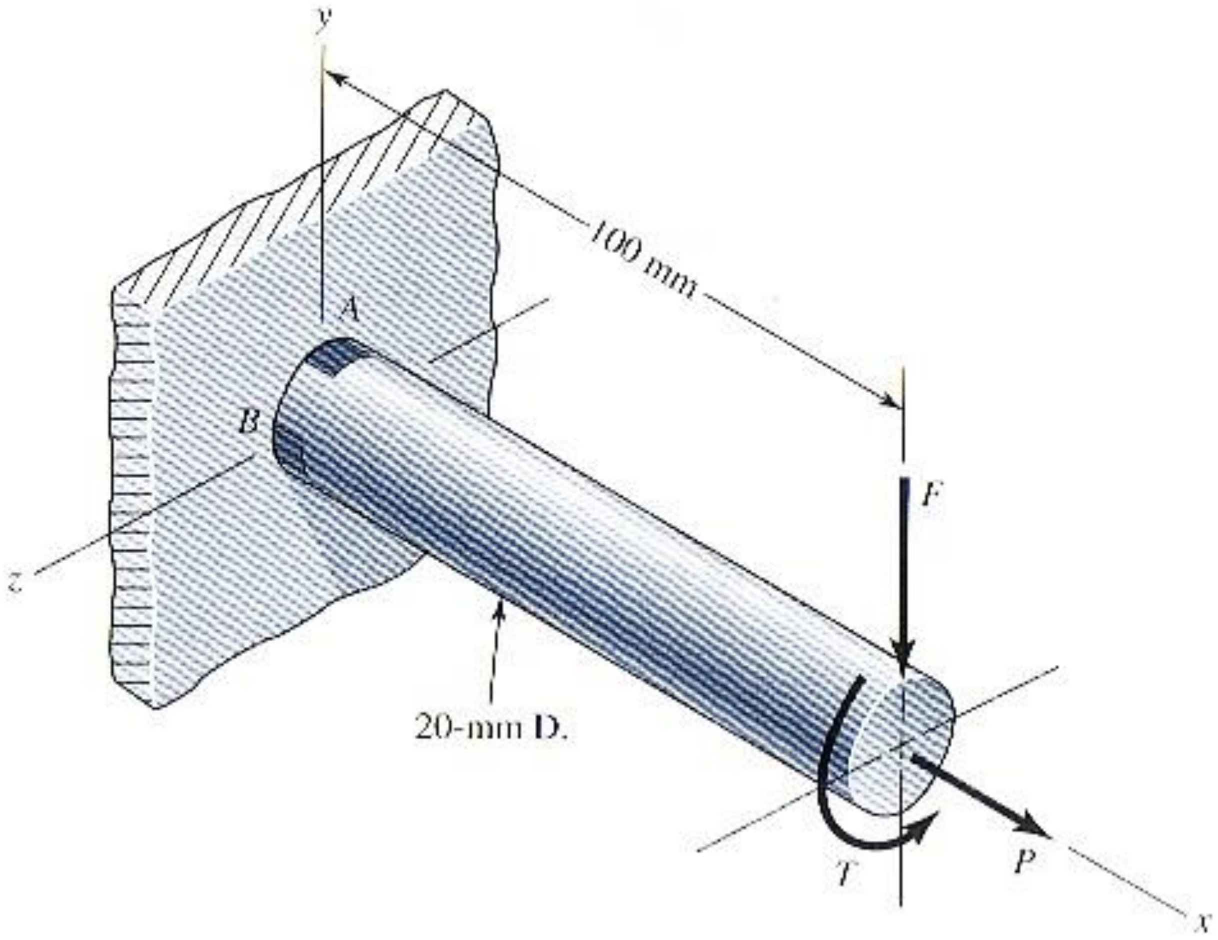
$$\tau_{xy} = 0$$



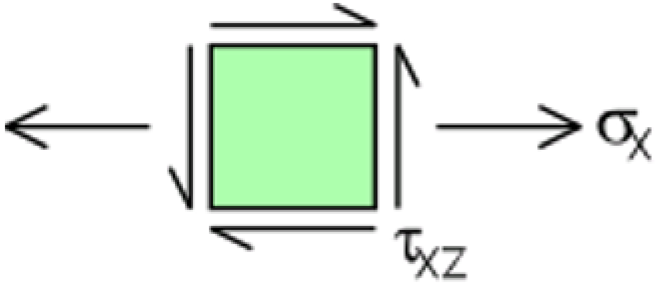
Examples

- Torsion:

$$\tau_{xz} = \frac{Tc}{J} = \frac{(T)\left(\frac{D}{2}\right)}{\left(\frac{\pi D^4}{32}\right)} = \frac{16T}{\pi D^3}$$



Stress element at A

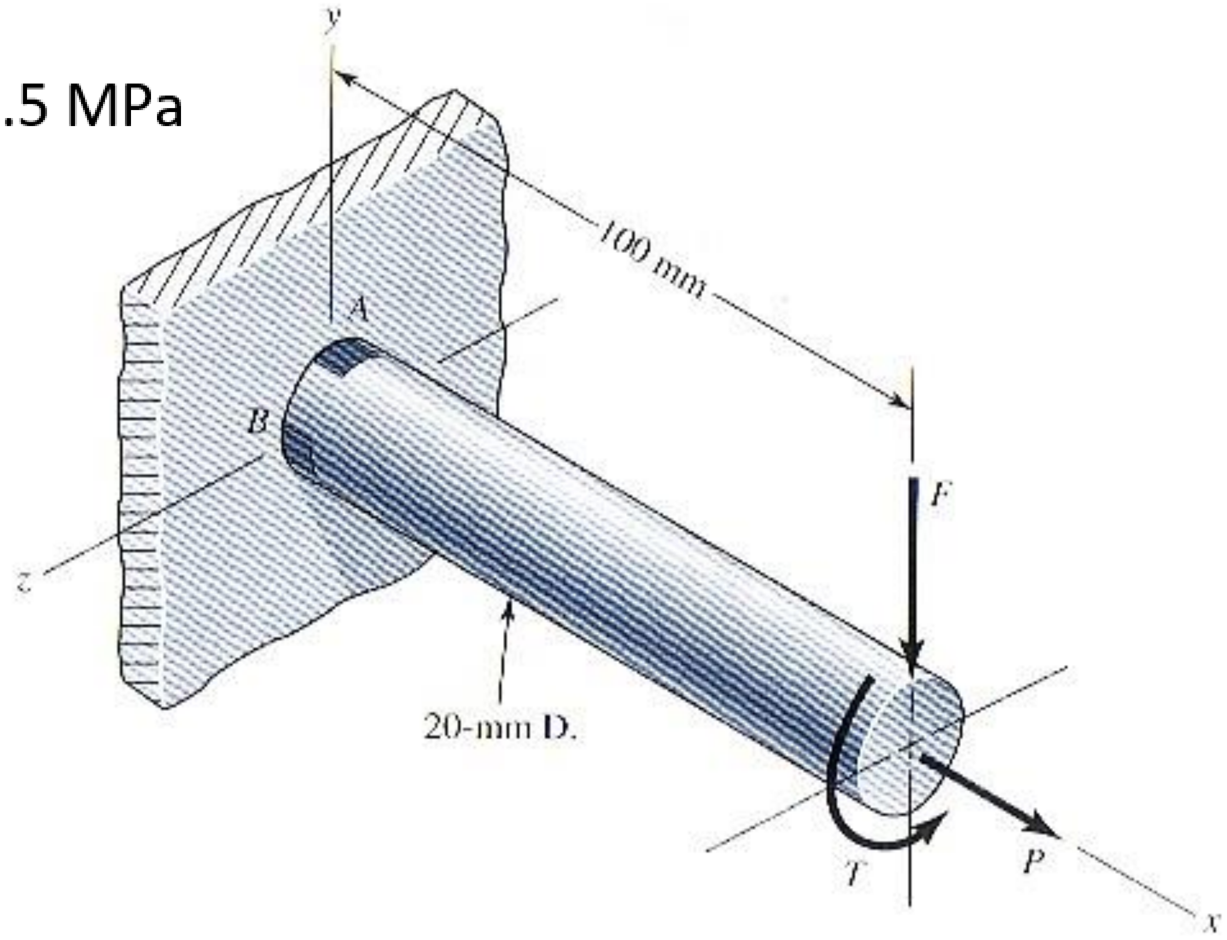


Examples

Sum up stresses due to all the loads

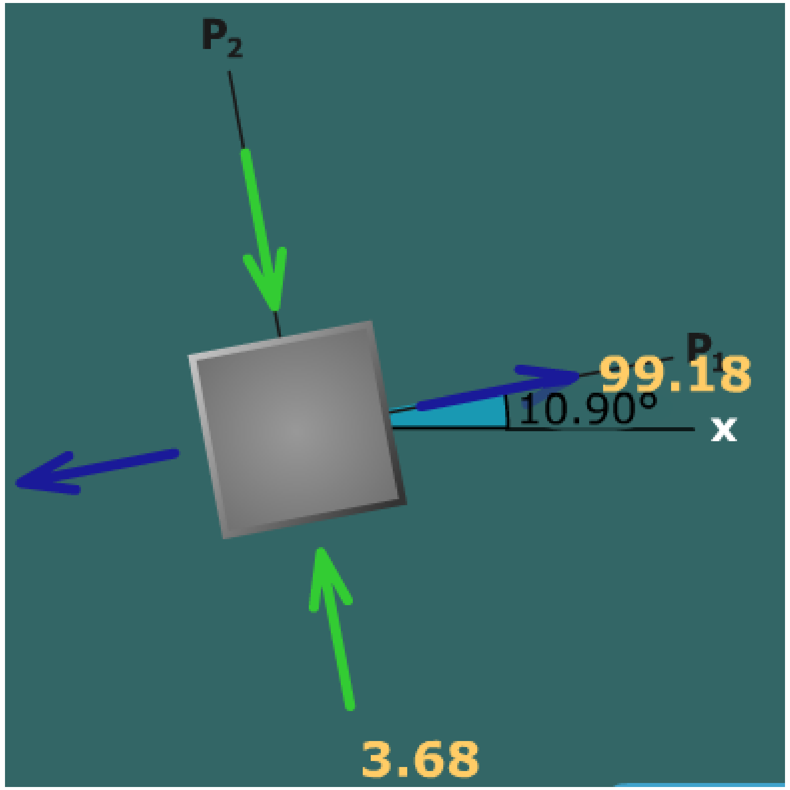
$$\sigma_x = \frac{4P}{\pi D^2} + \frac{32FL}{\pi D^3} = \frac{4PD + 32FL}{\pi D^3}$$ = 95.5 MPa

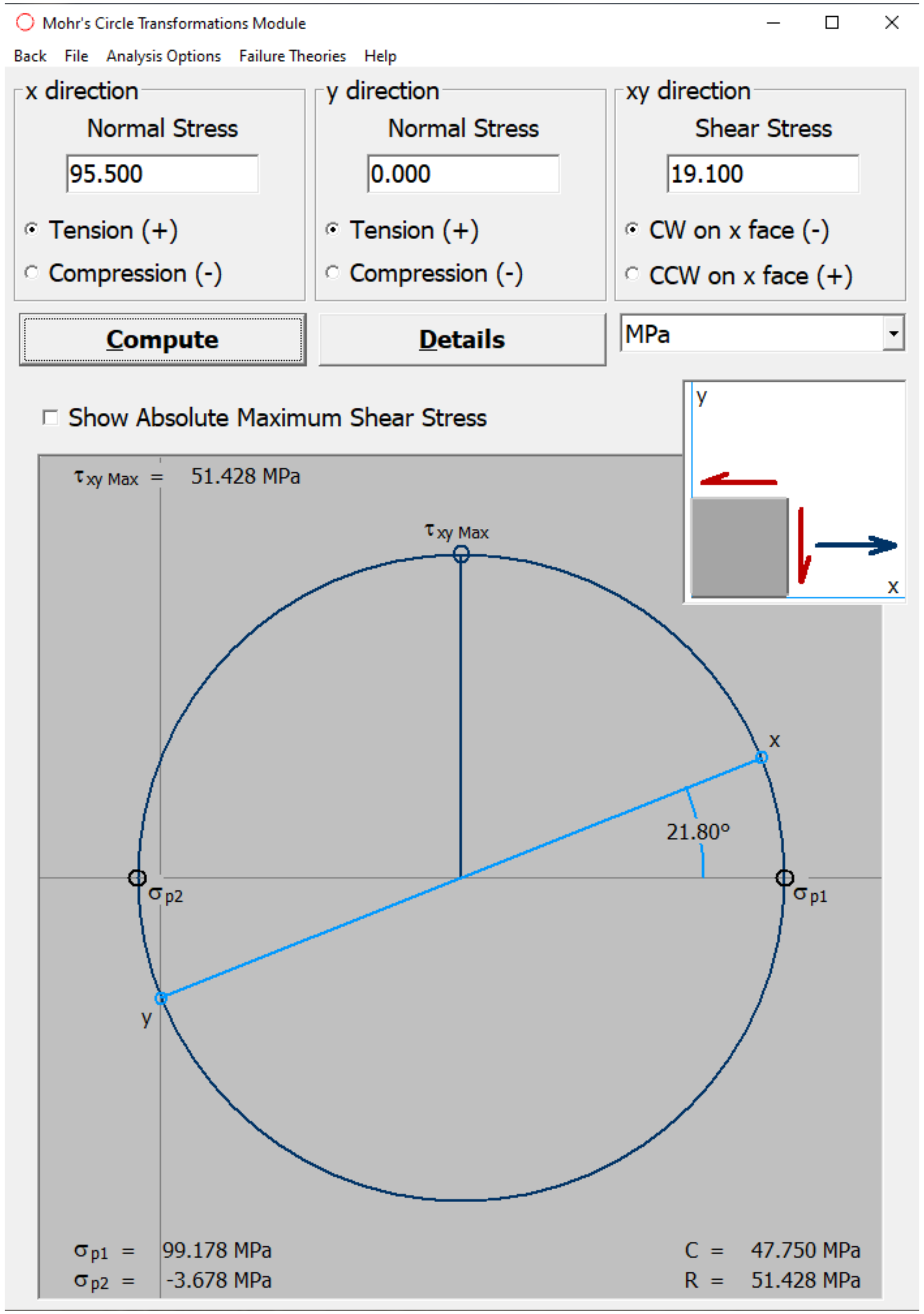
$$\tau_{xz} = \frac{16T}{\pi D^3}$$ = 19.1 MPa



Examples

Mohr's Circle





Examples

Apply a failure theory:

Use DE Theory since no restrictions or other information were given

$$\sigma_e = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2\right]}$$

$$\sigma_e = \sqrt{\frac{1}{2}\left[(99.2 - 0)^2 + (0 + 3.63)^2 + (99.2 + 3.63)^2\right.} \quad = 101 MPa$$

Examples

Apply a failure theory:

Yield strength for the mentioned material is: S_y = 331 MPa:

Factor of safety, n = $\frac{S_y}{\sigma_e} = \frac{331}{101} = 3.28$ (for yield), so SAFE

Examples

Element B

Calculate stresses due to each load

- Axial:
- Bending:
- Shear:
- Torsion: